DEVELOPMENT OF A SURVIVAL BASED FRAMEWORK FOR BRIDGE DETERIORATION MODELING WITH LARGE-SCALE APPLICATION TO THE NORTH CAROLINA BRIDGE MANAGEMENT SYSTEM

by

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ABSTRACT

RAKA GOYAL. Development of a survival based framework for bridge deterioration modeling with large-scale application to the North Carolina bridge management system. (Under the direction of DR. MATTHEW J. WHELAN)

This dissertation presents the development and implementation of a comprehensive automated software framework for probabilistic bridge deterioration modeling that takes into account the time dependent nature of deterioration as well as the impact of various functional, design, and geographic factors on the deterioration rate. Deterioration models are a critical component of the bridge management systems (BMS) used by transportation departments to optimize the allocation of increasingly constrained resources for maintenance, repair, and rehabilitation (MR&R). Since deterioration models are used to predict the MR&R needs at both the bridge and the network levels, the effectiveness of BMS-driven investment decisions related to the repair and preservation of bridge components and, consequently the economy of bridge management actions and safety assurance of the traveling public, is directly affected by the accuracy of the bridge deterioration models. Although probabilistic approaches have been employed for construction of deterioration models, prior studies have largely been constrained by excessive reliance on practitioner opinion surveys and limited application of statistical analytics. Survival analysis-based approaches implemented to date have been parametric in nature and have neither examined the suitability of the pre-existing bridge classifications nor extended the probabilistic methodology to fully realize the predictive potential of such models. In this study, semi-parametric multivariable proportional hazards modeling of survival functions is combined with application of semi-Markovian theory to develop probabilistic deterioration models that reflect the time dependence as well as effects of explanatory variables on deterioration rates of individual bridge components throughout their life cycle. A user-friendly standalone graphical user interface (GUI) is designed for use by transportation personnel to develop and update these models for obtaining future expected condition rating forecasts over specified planning horizons during network-level multi-objective optimization analyses. The developed framework is implemented on North Carolina's statewide bridge database consisting of over 17,000 bridge records spanning 35 years of historical general condition ratings (GCR) assigned during bridge inspections. As a result, significant factors affecting deterioration rates over different bridge components are identified over the life cycle of component and their time-varying influence is quantified in terms of state-dependent hazard ratios. Comparison of the predictive fidelity of the developed probabilistic models to the currently used deterministic deterioration models is used to characterize the improvement in accuracy afforded by the new technique. A strategy for probabilistically incorporating the effects of maintenance action on deterioration rates in the proposed model is discussed as well as potential secondary applications of the developed framework, including quantifying the value of preventative design measures and preservation actions.

DEDICATION

To my husband, Anoop, my daughters, Smiti and Sanjna, and my sisters, Rashmi and Ankur, whose love and faith made it all possible.

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CHAPTER 1: INTRODUCTION

1.1 Background and Motivation

National Bridge Inspection Standards (NBIS) were instituted in early 1970's following the collapse of Silver Bridge in Ohio due to corrosion-induced catastrophic failure. This legislation mandates that all states maintain bridge inventory and inspection records for each and every bridge in their jurisdiction. Each bridge record acts as a historical reference of any changes occurring in the physical condition of the bridge over time. These changes are measured and recorded through periodic inspections that must be performed no less frequently than on a biennial schedule. In this way, the deterioration, if any, of the overall condition of the bridge and its components is monitored so that remedial action can be taken as needed to preserve the bridge in good condition and ensure the safety of the traveling public.

While trying to achieve the objective of maintaining all individual bridges in their inventory, states continuously face the challenge of allocating increasingly limited funds and resources to most efficiently address network-level maintenance and reconstruction as well as anticipate future funding needs. This challenge led to the evolution of Bridge Management Systems (BMS), which are systematic data-driven approaches for using available bridge data, projected costs, and functional needs at the local and network-level to help objectively make such decisions. A BMS helps decision makers to interactively understand the trade-offs associated with allocating constrained funding to rehabilitation or maintenance work versus bridge replacement projects across the entire network of bridges to formulate optimal decisions based on economics, performance, and safety. North Carolina was one of the first states to develop a BMS (Chen and Johnston, 1987). Since then, many states, along with the Federal Government, have developed bridge management systems, although the majority of states now use the AASHTOWare Pontis software for some degree of bridge management (Markow and Hyman, 2009).

The North Carolina Department of Transportation (NCDOT) currently maintains records for 17,046 in-service bridges with each record having over 200 items of operational and functional bridge information, including condition rating data from the most recent visual inspection. The digital recording of National Bridge Inventory (NBI) data for North Carolina bridges began in 1981, so there are now 35 years of bridge records in NCDOT database. NCDOT currently uses a BMS software developed by AgileAssets Inc. However, while this software implements the constrained optimization analysis to provide scenarios for decision-making, the database relies on independent development of both deterioration models for the prediction of bridge maintenance needs and user and agency costs.

The two most important prediction tools of a BMS are bridge deterioration models and bridge-related cost models. Deterioration models are used to project the condition of key bridge components such as the deck, superstructure, and substructure in the future based on the current and historical records of component condition ratings. A condition rating is a number assigned by bridge inspectors to quantify the physical condition of these bridge components after assessing the extent of observable deterioration. For example, within the NBIS, 9 is the condition rating assigned for a new bridge component in excellent condition, whereas 3 denotes extensive deterioration and serious need for rehabilitation or major repairs. The complete scale of condition ratings and associated descriptions of general condition that are applied for bridge deck, superstructure, and substructure condition rating is reproduced in Table 1.1 (FHWA, 1995).

TABLE 1.1: NBI condition ratings for bridge deck, superstructure, and substructure components

Rating	**Condition**	**Description**
9	Excellent	
8	Very Good	No problem noted.
7	Good	Some minor problems.
6	Satisfactory	Structural elements show some minor deterioration
5	Fair	All primary structural elements are sound but may have minor section loss, cracking, spalling, or scour.
4	Poor	Advanced section loss, deterioration, spalling, or scour.
3	Serious	Loss of section, deterioration, spalling, or scour have seriously affected the primary structural components. Local failures are possible. Fatigue cracks in steel or shear cracks in concrete may be present.
2	Critical	Advanced deterioration of primary structural elements. Fatigue cracks in steel or shear cracks in concrete may be present or scour may have removed substructure support. Unless closely monitored, it may be necessary to close the bridge until corrective action is taken.
1	Imminent Failure	Major deterioration or section loss present in critical structural components, or obvious loss present in critical structural components, or obvious vertical or horizontal movement affecting structural stability. Bridge is closed to traffic, but corrective action may put back in light service.
0	Failed	Out of service; beyond corrective action.
N	Not Applicable	

Most states now have over 30 years of inspection data collected to support development of prediction tools for decisions regarding maintenance, repair, and rehabilitation (MR&R) and replacement of bridges. This level of collected data presents extensive data mining opportunities for discovering inter-relationships amongst geographic, functional, and structural characteristics and rates of deterioration to better predict future rehabilitation needs as well as improve understanding of the performance and

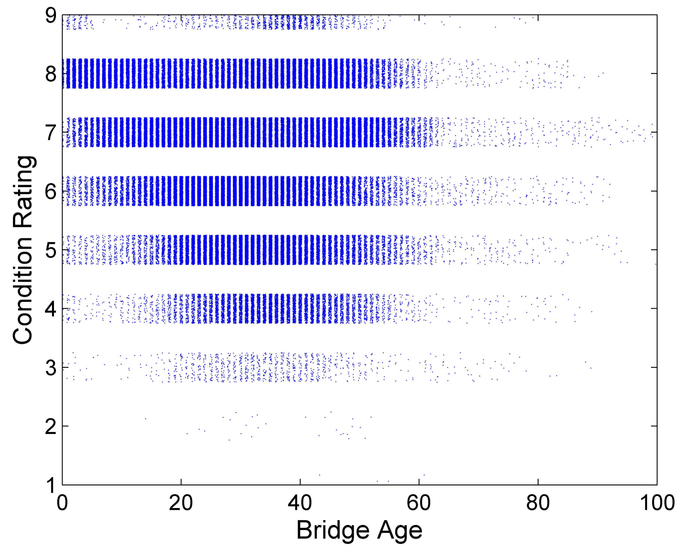

FIGURE 1.1: Deck condition ratings plotted against bridge age for all timber deck bridges in the 1981-2013 NCDOT bridge records

expected service life of different highway bridge designs. However, processing the database of condition ratings and functional descriptions is computationally challenging and requires advanced statistical tools to extract meaningful and reliable information. To illustrate the magnitude and complexity of the deck condition rating data alone, a scatter plot of the entire historical database of deck condition ratings for all of the timber bridges in the NCDOT bridge network is shown in Figure 1.1. Bridge components deteriorate over time and therefore are expected to be correlated with bridge age, however this historical data suggests the need for more advanced analysis than offered by simple regression techniques. Figure 1.2 shows the distribution of the continuous durations at each condition rating observed in the same dataset. These distributions contain significantly more meaningful information that can be leveraged to develop appropriate deterioration models. Over the past several decades, methodologies for developing deterioration models from databases of inspection records have

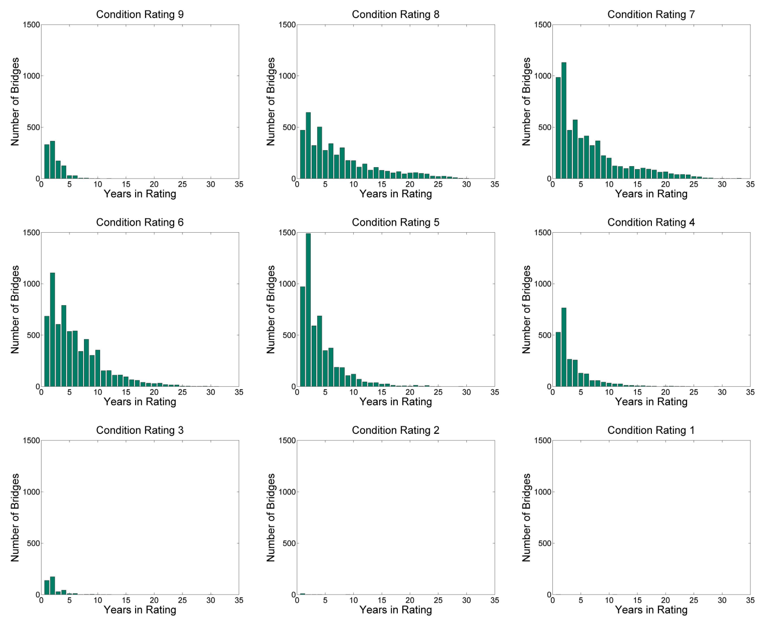

FIGURE 1.2: Distributions of observed continuous condition rating durations for all timber bridge decks in the 1981-2013 NCDOT bridge records

matured from purely deterministic models to probabilistic Markovian methods and, most recently, duration models, which take advantage of the now extensive history of recorded condition ratings.

Ultimately, deterioration models are most useful for the prediction of future MR&R needs when these models are developed after several cycles of inspection data are available to provide a basis of historical performance of bridges in the network. To enhance the fidelity of these predictive models, bridges are often first preclassified into groups specific to their design, functional, and geographic characteristics to independently account for the effects of such external variables on the rate of deterioration. In parallel to the deterioration models, cost models are used for calculating agency and user costs with reference to impacts on detour mileage, accident costs, level of service goals, and associated improvement actions. Both the deterioration and cost models are instrumental inputs for the BMS that then help determine the most beneficial selection of projects to minimize overall network level costs through multi-objective optimization schemes.

1.2 Objectives and Scope

This research seeks to improve the accuracy of deterioration model predictions by developing strategies to overcome challenges identified by previous research. Theoretical contributions are sought through the development of a framework for multivariable probabilistic deterioration modeling and strategies for efficiently implementing these models over large databases. Using this developed framework, applied contributions related to the discovery of external factors affecting deterioration of different bridge components over the service life are sought. The broad components of research associated with each of these objectives are summarized below:

1. Theoretical contributions sought by the research effort:

    - Techniques for multivariable statistical regression of bridge condition rating data using semi-parametric Cox proportional hazards methodology for probabilistic deterioration modeling.
    - Development of a semi-Markov approach for condition rating forecasting based on non-stationary transition probabilities and covariate hazard ratios obtained from multivariable proportional hazards regression.
    - Exploration of the time-dependent nature of transition probabilities in probabilistic deterioration models and the impact of incorporating non-stationary probabilities rather than simplified stationary probabilities in

expected condition rating forecasts over near-term and long-term planning horizons.

2. Applied contributions sought by the research effort:

   - Development of deterioration models for each material-specific general condition rating (GCR) bridge component category in the North Carolina state bridge inventory.
   - Examination of the significant design, functional, and geographic features affecting deterioration rates at each condition state, their impact on expected condition rating forecasts, and the validity of expert elicited *a priori* classification strategies.
   - Quantitative comparison of the predictive fidelity of deterministic deterioration models and probabilistic deterioration models over typical long-term planning horizons.
   - Development of a user-friendly standalone graphical user interface for development and updating of deterioration models by transportation personnel.

1.3 Organization of Dissertation

This dissertation consists of a total of eight chapters including this introductory chapter.

Chapter 2 provides a comprehensive literature review covering the various deterioration modeling approaches developed since the initial conception of bridge management systems. The assumptions, advantages, and limitations of each approach are discussed and a status review of implementations and practices by state and federal transportation agencies is provided.

Chapter 3 explains the theoretical background of the Cox proportional hazards model for survival analysis including model development, model selection, and model assessment. The nature of its application to the modeling of bridge component condition rating durations, including the underlying assumptions and techniques for handling descriptive data types, is discussed with illustrative examples.

Chapter 4 describes the methodology and statistical regression technique developed for construction of multivariable probabilistic deterioration models and for the efficient use of these models in predicting future condition states of various bridge components. The challenges associated with application of proportional hazards modeling to bridge condition rating data are discussed and strategies adopted to overcome them are formulated. The details of the functions executed by various constitutive software routines developed to implement the methodology are presented with the help of flowcharts and algorithms.

Chapter 5 describes the development, layout, and functionalities of the Windows-based standalone graphical user interface designed for implementation of the proposed deterioration framework.

Chapter 6 presents results obtained from successful implementation of the framework on general condition rating databases of deck, superstructure, and substructure components in the North Carolina state bridge inventory. Identified significant factors and their effect on deterioration rates are examined. Additionally, the impact of simplifications to the probabilistic deterioration models are assessed over near-term and long-term planning horizons.

Chapter 7 provides validation of the deterioration models obtained from the developed framework by comparing their predictive fidelity with that of the deterministic models currently used by NCDOT. The impact of incorporating covariates in the probabilistic models is also assessed by comparison with predictions obtained by simplified survival analysis without the proportional hazards model.

Chapter 8 reviews the theoretical and applied contributions of this dissertation and provides recommendations for future work.

CHAPTER 2: LITERATURE REVIEW

Bridge deterioration models represent the estimated deterioration of specific bridge components over time. These predictive models are developed on the basis of historical condition ratings of bridge components characterizing the extent of physically observable signs of deterioration as recorded by bridge inspectors during scheduled biennial inspections. Deterioration models form an important component of bridge management systems by predicting future MR&R needs at the bridge and network level. Consequently, the efficacy of a BMS in optimally allocating MR&R budgets to ensure the preservation of bridge components and the safety of the traveling public is directly affected by the accuracy of the bridge deterioration models. With the increased reliance on optimized, data-driven BMS planning to address infrastructure maintenance needs of large bridge inventories under constrained budgets, the importance of having accurate deterioration models cannot be overemphasized.

Since the introduction of BMS frameworks in the early 1980s, approaches for deterioration modeling have continuously developed in complexity from the earliest purely deterministic methods. Currently, the most widely prevalent in US are the Markov chain based probabilistic approaches, which have also been incorporated in the AASHTOWare Pontis and Bridgit commercial BMS softwares adopted by many states. However, the growth of the historical condition rating database has recently permitted duration-based probabilistic approaches to be investigated as well as the integration of these approaches with the earlier Markovian models. The different strategies for developing deterioration models from condition rating data are discussed in the following sections alongside their assumptions, advantages, and limitations.

2.1 Deterministic Models

Deterioration of bridge components is associated with many factors including age, environment, design characteristics, and traffic conditions. It manifests itself in observable defects like corrosive loss in steel components, delamination in concrete, cracking, and scour of foundation systems. Deterioration models are a way of linking observable symptoms of deterioration to the various explanatory factors affecting deterioration to enable prediction of deterioration behavior and planning of suitable corrective actions. Early studies formulated mathematical relationships between observed deterioration quantified by condition ratings with specific classifiers, such as component and material type, using statistical measures like mean, standard deviation, and linear regression coefficients. These studies ignored the random errors inherent in statistical prediction and therefore all these models are classified as deterministic models. A typical deterministic deterioration model is shown in Figure 2.1, where the ordinate is the condition rating that is plotted against the average age of the bridges at that condition rating, which forms the abscissa.

The earliest deterministic models devised in 1987 for the North Carolina bridge inventory used two parameters: the average age of bridges at a particular condition rating and the average age of bridges when the condition rating dropped by one point (Chen and Johnston, 1987). The researchers did not use data regression as their ef-

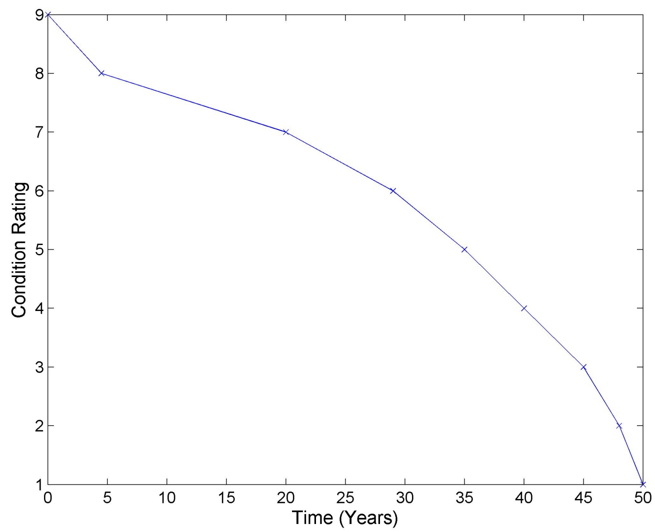

FIGURE 2.1: A deterministic bridge deterioration model.

forts to do so proved ineffective on account of substantial scatter in condition rating data due to alteration in natural deterioration patterns caused by maintenance and repair activities. As an alternative, they used *a priori* classification of bridges and bridge components into categories based on factors believed to significantly affect the deterioration of the particular bridge components. Through this heuristic classification, all of the three primary bridge components analyzed (deck, superstructure, and substructure) were initially grouped by primary material type under the logical expectation that the deterioration rates would be strongly associated with characteristics of the long-term durabilities of the construction materials. As a secondary level of classification, bridge decks were further sub-classified into bins by average daily traffic (ADT), superstructures were sub-classified by both structural design type and highway functional classification, and substructures were sub-classified by geographical region. Statistical analysis of the then-limited historical condition rating data indicated deterioration of bridge condition with age, but ultimately was found unreliable for development of deterioration models due to ill-conditioning of the data caused by characteristics of the bridge age distribution and effects of maintenance activities. The deterioration models ultimately adopted at the time of this study were based on the results of an opinion survey of professional bridge inspectors and supervisors (Chen and Johnston, 1987). These heuristic deterioration models were used in the development of the Optimum Bridge Budget Forecasting and Allocation Module (OPBRIDGE) that produced North Carolina's original BMS (Isa Al-Subhi and Johnston, 1989).

A later study proposed the use of the average change in condition ratings over multiple years to model deterioration and to improve the performance of the North Carolina BMS (Abed-Al-Rahim and Johnston, 1991). The categorization of bridge components on the basis of expected explanatory factors was expanded to include geographic classifications in an attempt to account for the perceived dependence of deterioration rates on the presence of marine environment and de-icing salt applications. The study developed illustrative sets of deterioration models that were consistent in terms of predicting deterioration with respect to various material and environmental factors as well as other considerations (Abed-Al-Rahim and Johnston, 1991). Updating of the bridge deterioration models in OPBRIDGE was, however, implemented much later using the average durations of bridge components at particular condition ratings (Duncan and Johnston, 2002). Both of these models, while still deterministic, had the advantage of using time series data of bridges in addition to the cross-sectional data used exclusively by the earlier models. The NBI data is cross-sectional as it is comprised of inspection records that report only the condition ratings of all the nation's bridges in the current year. Time series data on the other hand represents the historical condition data of a particular bridge as it changes over time. In early studies that were disadvantaged by insufficient time series data for analysis, the cross-sectional condition rating data of all bridges of various ages was aggregated to represent the expected deterioration of a single representative bridge.

2.1.1 Linear and Non-linear Regression Models

During the early 1990's, similar deterministic deterioration modeling studies were carried out using NBI bridge inventories for the whole nation as well as those of individual states, some of which are reviewed here. Linear regression was used in a study by the Transportation Research Center of the U.S. Department of Transportation (DOT) to correlate the relationship of bridge condition ratings with other bridge characteristics recorded in the NBI database in a linear statistical model (Busa et al., 1985). An improved piecewise linear regression was used in other studies performed for the Wisconsin DOT and the New York State DOT (Fitzpatrick et al., 1981, Hyman and Hughes, 1983). Deterioration of bridges in the New York City Metropolitan area was modeled as function of age using two methods: 1) the average rate of change for each condition rating and 2) the average condition rating of bridges of all ages (Yanev and Chen, 1993). Nonlinear regression models were also developed for the first time using time-series data for Pennsylvania bridges (West et al., 1989). Most of these studies developed composite deterioration curves with respect to age with minimal or no classification of bridges into categories based on characteristics like structural design or environment.

In subsequent years, several studies used nonlinear statistical regression along with classification of bridges into relatively homogeneous groups on the basis of potential determinants of deterioration identified through literature review and discussions with the members of the bridge engineering community. To produce a representative sample of diverse environments, one of these studies analyzed superstructure deterioration with respect to age and ADT for all of the steel and prestressed concrete bridges in the national inventory as well as individually for the seven states of Colorado, Illinois, Iowa, North Carolina, Pennsylvania, Tennessee, and Texas (Veshosky et al., 1994). This study found that there was no statistically significant difference in the rates of deterioration of steel superstructures relative to prestressed concrete superstructures. In general, age was found to be the most statistically significant factor followed by ADT, although the impact on the rate of deterioration was found to decrease with time. Another study was performed for bridges within the state of Nevada that correlated condition ratings with age while accounting for all other factors through *a priori* classification of bridges (Sanders and Zhang, 1994). Explanatory factors investigated in these studies included material type, structure type, ADT, maintenance responsibility, rehabilitation status, and geographical region. A particular challenge faced when increasing the number of variables in both of the studies was the reduced number of bridges in each category. The classification of condition rating data into such datasets of limited sample size ultimately compromised the reliability and applicability of the statistical models. To overcome this problem, investigation of some combinations of variables necessarily had to be abandoned whereas others were combined into larger, more generalized groups that would lend themselves to a more statistically significant analysis. This was especially true for Nevada, as it is a sparsely populated state with a relatively small bridge inventory.

2.1.2 Limitations and Contributions of Deterministic Models

While deterministic deterioration models based on simple statistical properties offer relative computational ease, they are associated with some critical inherent limitations. Primarily, they neglect the stochastic nature of the deterioration process as well as the subjectivity and uncertainty present in the condition rating data. For instance, it was found that although the polynomial regression techniques gave reasonable results within the bounds of available data, their projection beyond these bounds could be significantly misleading, thus severely limiting their predictive reliability and usefulness in BMS. Probabilistic models have been shown to provide better extrapolation capabilities and can be easily integrated into dynamic BMS optimization processes resulting in more efficient and effective MR&R strategies (Butt et al., 1987). Furthermore, *a priori* classification of bridges and bridge components may overlook the impact of unobserved or unmodeled factors that influence deterioration rates. Stated another way, the statistical model may ultimately predict the average deterioration for a group of bridges well but inaccurately predict the deterioration of the bridges individually. This phenomenon is evident from a comparative study of deterioration models developed using two different approaches and applied to forty bridges in the Indiana bridge database. It was found that the magnitudes of prediction errors in models based on polynomial regression techniques were much higher compared to those in models based on a probabilistic Markov chain approach (Jiang, 2010).

The comparative lack of accuracy in model predictions has led to the gradual replacement of strict deterministic approaches with probabilistic approaches throughout the majority of state BMS implementations. However, despite their limitations, studies using deterministic approaches succeeded in deriving some common inferences about bridge deterioration behavior. For instance, the statistical analysis of condition rating data revealed that decks deteriorate faster than the superstructure or the substructure components of a bridge (Chen and Johnston, 1987, Sanders and Zhang, 1994). Likewise, similar components may deteriorate at different rates depending upon various factors, including geographical location and ADT (Chen and Johnston, 1987, Veshosky et al., 1994). Decks with higher ADT tend to deteriorate faster than those with lower ADT and, perhaps inter-related, bridges on secondary highway systems comprising local roads and minor collector roads tend to deteriorate at a lower rate than those on primary systems and interstates (Abed-Al-Rahim and Johnston, 1991, Chen and Johnston, 1987). Impact of saltwater in coastal regions, freeze-thaw cycles, and the use of de-icing salts in cold climatic regions were found to measurably exacerbate deterioration (Abed-Al-Rahim and Johnston, 1991, Sanders and Zhang, 1994). Bridges with expansion joints were found to deteriorate faster than continuous span bridges without joints (Yanev and Chen, 1993). With respect to maintenance actions, rehabilitated bridges tend to deteriorate faster than new bridges (Sanders and Zhang, 1994, Yanev and Chen, 1993).

2.2 Markov Chain Models

Probabilistic models aim to capture the stochastic nature of the deterioration process and thereby improve the accuracy of prediction. These models are discrete time and state as the infrastructure condition in these models is represented by discrete condition states at fixed inspection intervals. The earliest probabilistic models considered deterioration as a discrete time Markov process, called a Markov chain, with a finite number of states (Butt et al., 1987, Jiang et al., 1988). The Markov chain models are also called incremental models or state-based models as they model the change in condition or "state" over fixed increments of time. The change in state during a fixed time increment is treated as a random variable that captures the uncertain and random nature of deterioration. Aggregating these random variables over time provides a more realistic representation of deterioration as a stochastic process rather than a purely deterministic one like in the models presented earlier (Madanat and Ibrahim, 1995, Papoulis and Pillai, 2002).

2.2.1 Markov Decision Processes

A key component of the Markovian approach is the definition of the states in the system such that they capture the complete status of the system and all the information necessary for the decision making process. Consideration of all N number of bridges in a particular state inventory, a number of which may run in the tens of thousands, each with n possible states corresponding to the NBI condition ratings, would make the total state space of size n^N, which would be computationally burdensome. This problem has been resolved by pre-classifying the bridges into categories with similar characteristics according to variables like material and design type, traffic loading, and geographical and climatic regions, as described earlier in the deterministic approaches. This allows for a tractable representation of the bridge system. A Markov model is then constructed for each class of bridges with the capability to generate models for individual bridges in each class. As mentioned earlier, this process may result in problems associated with limited data at the lower levels of the classification hierarchy when the number of classes increases to the extent that there are not enough bridges in each class to enable a statistically significant analysis (Scherer and Glagola, 1994).

A Markov process is a stochastic process with the 'Markovian' property or assumption of time-independence in which the conditional probability P of a future condition state depends only on the present state and is independent of the past states. This can be represented for a discrete time, discrete state stochastic process X_t as given below (Morcous et al., 2003).

$$P(X_{t+1} = i_{t+1} | X_t = i_t, X_{t-1} = i_{t-1}, \ldots.., X_1 = i_1, X_0 = i_0)$$
$$= P(X_{t+1} = i_{t+1} | X_t = i_t) \quad (2.1)$$

where i_t is the condition state at time t. In the context of bridge deterioration, the NBI condition ratings ranging from 0 to 9 represent the ten possible states of the bridge component being modeled with state 1 corresponding to a condition rating of 9 and state 10 to a condition rating of 0. The change of state is assumed to occur at discrete time intervals equal to the routine inspection period of 2 years. Consequently, the probabilities $P_{i,j}$ that a bridge component would transition from state i to another state j during a specified period are represented in a transition probability matrix given below.

$$P = \begin{bmatrix} P_{1,1} & P_{1,2} & \dots & P_{1,10} \\ P_{2,1} & P_{2,2} & \dots & P_{2,10} \\ . & . & \dots & . \\ . & . & \dots & . \\ . & . & \dots & . \\ P_{10,1} & P_{10,2} & \dots & P_{10,10} \end{bmatrix} \tag{2.2}$$

where $i = 1, ..., 10$ and $j = 1, ..., 10$. The indices, i and j, can take any value between the lowest and the highest condition state for the particular bridge inventory database. The size of this matrix, however, is specific to the discrete integer range of condition states in the rating system used. For example, the New York State Department of Transportation (NYSDOT) implements its own visual inspection program that assigns condition ratings within the range from 1 to 7, resulting in 7 condition states and therefore a 7x7 transition probability matrix. Similarly, the condition ratings range from 1 to 5 for the Commonly Recognized (CoRe) Structural Elements defined by the American Association of State Highway and Transportation Officials (AASHTO) and the Federal Highway Administration (FHWA), resulting in 5 condition states and a 5x5 transition matrix. Each row of the matrix represents the probability of moving from one state to any other state, including itself. Consequently, the sum of the probabilities in each row should be equal to one. The associated probabilities of each condition rating remaining unchanged between inspections is simply the $P_{i,i}$ probability values, which are found on the diagonal of the transition matrix. The transition matrix has zero values below the diagonal, because it is assumed that the deterioration takes place without rehabilitation and hence the probability of an improvement at any state is zero. Furthermore, for computational simplicity it is routinely assumed that a bridge component would not deteriorate by more than one state in a single inspection cycle. The practical influence of these simplifying assumptions on the transition matrix is shown in the reduced form shown below:

$$P = \begin{bmatrix} P_{kk} & P_{k(k-1)} & 0 & 0 & \dots & 0 & 0 \\ 0 & P_{(k-1)(k-1)} & P_{(k-1)(k-2)} & 0 & \dots & 0 & 0 \\ . & . & . & . & \dots & . & . \\ . & . & . & . & \dots & . & . \\ 0 & 0 & 0 & 0 & \dots & P_{22} & P_{21} \\ 0 & 0 & 0 & 0 & \dots & 0 & P_{11} \end{bmatrix} \tag{2.3}$$

where k is the highest condition state and 1 is the lowest condition state, and $P_{k(k-1)} = 1 - P_{kk}$, $P_{(k-1)(k-2)} = 1 - P_{(k-1)(k-1)}$.......$P_{21} = 1 - P_{22}$ and $P_{11} = 1$. The transition probability of the lowest state P_{11} is one because there is no possibility of transitioning to a lower state that does not exist (Butt et al., 1987, Jiang et al., 1988, Madanat et al., 1995). The transition probability matrix can be used to predict the future condition of a specified bridge component if its present condition is known. The condition at any point in time is represented by a vector; for example, the initial state vector Z_0 for a component in new condition will be [1 0 0 0 0 0 0 0 0 0] for the NBI condition rating scale of 0 to 9 with 9 signifying the good as new condition. Given the transition matrix P as defined above, the future state vector Z_t at time t is obtained using

$$Z_t = Z_0 \cdot (P)^t \tag{2.4}$$

Since the initial state vector is a known quantity, it is necessary to determine the transition matrix to completely define the Markov chain (Jiang et al., 1988). A illustrative Markovian bridge deterioration model is shown in Figure 2.2, where all of the transition probabilities $P_{i,i}$ on the diagonal of a 5x5 transition matrix are 0.8. The Y-axis represents the initial state, Z_0, when the probability of being in condition state, 5, is 1, and that of being in all the other states is zero. A vertical line drawn parallel to the Y-axis at any time, t, on the X-axis represents the state vector, Z_t, comprised of the probabilities of being in each of the 5 states, and the expected condition rating at that time, respectively, using equations 2.4 and 2.8. The accuracy of a Markovian model depends nearly exclusively on the accuracy of the transition matrix. Various methods have been developed to calculate the transition probabilities.

The earliest methods for determining transition probabilities were developed mainly in construction of pavement deterioration models. One of these defined the transition probability, $P_{i,j}$, simply as the percentage or proportion of pavement sections in condition state i that deteriorated to condition state j in one inspection period. Mathematically, this yields:

$$P_{i,j} = \frac{n_{i,j}}{n_i} \tag{2.5}$$

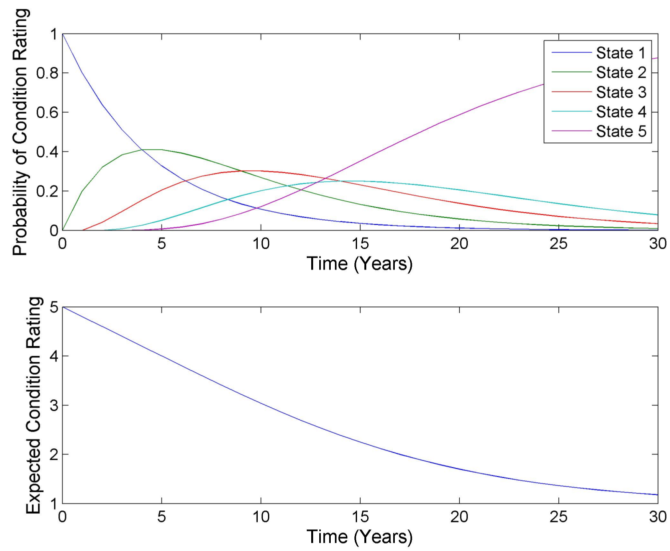

FIGURE 2.2: A Markovian bridge deterioration model.

where n_i is the total number of pavement sections in condition state i and $n_{i,j}$ is the number of pavement sections whose condition state changes from i to j in one inspection period (Scherer and Glagola, 1994, Wang et al., 1994). An inspection cycle is representative of a specified duration of weather and traffic causing deterioration in the pavement condition. In the early models, not only was the duration of the inspection cycles assumed to be the same, but the deterioration contributing factors of weather and traffic were also assumed to be the same in subsequent inspection cycles irrespective of the age of the pavement section. Consequently, the transition probabilities were not expected to change from one inspection cycle to the next. This type of process is deemed a homogeneous or stationary process and known as a Markov Decision Process (MDP) (Frangopol et al., 2004, Jiang et al., 1988).

The assumption of constancy of behavior within inspection cycles relative to factors producing deterioration over the life of a infrastructure component is not realistic as changes occur due to increases in traffic loads or modification of maintenance policies. This inadequacy was recognized after observing the deviation of the actual deterioration curve from the predicted deterioration curve based on MDP for a 30 year life of pavement (Butt et al., 1987). To overcome this limitation, a new model was developed in which the life of the pavement section was zoned into 6-year periods. The deterioration rate was assumed to be constant within each zone and a homogeneous Markov chain with a stationary transition matrix was developed specific to each zone. A non-homogeneous Markov chain was then developed to transition pavement sections from one zone to another. During such transitions, each subsequent zone takes the last state vector of the previous zone as its starting state vector. The deterioration curve developed using this model was found to more closely represent the actual deterioration curve (Butt et al., 1987). This model was also adopted for developing the Markov chain based bridge deterioration models for the Indiana bridge database, which were the earliest models of these kind developed in the U.S. (Jiang et al., 1988, Sinha et al., 1988), and continue to be used in the present-day Indiana Bridge Management System (IBMS) (Sinha et al., 2009).

In the previously mentioned models, a non-linear programming approach was used to calculate the transition probabilities. This approach is known as the expected-value method and is still the most widely used method of calculating Markov chain transition matrix probabilities. In this method, the average condition rating of the bridge components in a particular zone or age group is first determined by applying a polynomial regression to all the bridges in that group in the form,

$$Y_t = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3 \tag{2.6}$$

where Y_t is the bridge component condition rating for a bridge at age t, and β_0, β_1, β_2 and β_3 are regression coefficients to be estimated. The transition probabilities are then calculated by minimizing the distance between the average condition rating $\hat{Y}_t$ obtained through this regression and the theoretical expected value $E(t, P)$ of the condition rating based on the Markov chain at time t for the transition probability matrix P. The objective function to be minimized is thus given by

$$min \sum_{t=1}^{N} |\hat{Y}_t - E(t, P)| \tag{2.7}$$

$$subject\ to\ 0 \leq P_{i,j} \leq 1\ and\ \textstyle\sum_{j=1}^{k} P_{i,j} = 1\ for\ i, j = 1, 2, ..., k$$

where N is the number of years in one age group, and $P_{i,j}$ is the transition probability in the transition probability matrix, P, associated with moving from condition state i to condition state j over the inspection cycle (Butt et al., 1987, Jiang et al., 1988). In terms of equations (2.3) and (2.4), if the condition states are represented in a column vector R, $E(t, P)$ in equation (2.7) is given by (Madanat et al., 1995)

$$E(t, P) = Z_t \cdot R \tag{2.8}$$

The unknown transition probabilities are the decision variables and the maximum number of these that can be estimated using the expected value method is the number of years in each age group (Madanat et al., 1995). The assumption that a bridge component does not deteriorate by more than one state in any one inspection cycle, as mentioned earlier, is helpful in this regard by reducing the probabilities of transition to other states to zero thereby significantly minimizing the number of decision variables that require estimation (Madanat et al., 1995). This assumption was recently applied to element level inspection data to determine transition probability matrices and develop deterioration models for use in Pontis for the Florida Department of Transportation (FDOT). The so-called "One-step method" was found not only to be simpler and require smaller sample sizes, but also more robust while having the same coefficient of determination as the regression model that did not use this assumption (Sobanjo and Thompson, 2011).

A limitation of the expected-value method is that it cannot handle the case where the condition ratings in a particular age group remain the same or tend to increase instead of decreasing. In such a case, the non-linear optimization statement provided in equation (2.7) may result in a unity or close to unity transition matrix P and, consequently, the deterioration curve flattens out at this point. This problem has been resolved by introducing a second level Markov process (Agrawal et al., 2010). In this second level process, the average condition rating, $\hat{Y}_t$, in equation (2.7) is derived from the first level Markov chain predictions instead of the originally used polynomial regression. The objective function is then minimized to determine a single transition matrix for the total number of years in all age groups combined. The deterioration curve generated by this second level transition probability matrix is found to follow the original data but continues realistically to exhibit a decreasing trend even in case where the original curve derived from the first level Markov process stops decreasing beyond a certain age (Agrawal et al., 2010).

2.2.2 Commercial BMS Packages and Element-Level Models

The Markov chain models are widely recognized as better than deterministic models by accounting for the stochastic nature of the deterioration process. Moreover, these models have the advantage of computational simplicity and can be applied to both network level and project level bridge management systems. As a result, MDP-based deterioration models were adopted in the two U.S. national bridge management systems, AASHTOWare Pontis and BRIDGIT, that have been implemented in over forty states since their development in the late 1990s (Golabi and Shepard, 1997, Hawk and Small, 1998). Regarding these two commercial softwares, their difference is based on the optimization strategy employed. Pontis follows a top-down approach by doing network level optimization first before determining needs of individual bridges. BRIDGIT, on the other hand, implements project-level optimization prior to making network level recommendations (AASHTO, 2011b). BRIDGIT is better suited for use by smaller transportation departments with limited staff resources, but it can be run in parallel with Pontis to complement the decision process by providing an independent set of recommendations (Hawk and Small, 1998).

AASHTOWare Pontis requires the use of element level condition rating data and development of deterioration levels for each element. This requires more detailed data than available in the NBI as each bridge component (deck, superstructure, and substructure) is comprised of numerous elements that currently do not get independently recorded condition ratings. Bridge inspections at the element level were formalized by AASHTO in 1997 through its Guide for Commonly Recognized (CoRe) Elements, which has been recently updated (AASHTO, 2011a). Most states did not have sufficient bridge condition rating data for their bridge inventories when they implemented Pontis. To overcome this limitation, Pontis provides for development of the initial transition probability matrix using "expert elicitation" data. Expert elicitation data is comprised of responses from qualified transportation engineers and inspectors to a questionnaire asking for their estimate of transition probabilities for various elements in a bridge inventory. For example, in Florida, this took the form of "do-nothing" probabilities developed by asking bridge engineers to estimate the median number of years, Y, that an element would take to transition out of a given condition state. The estimate was established as the duration at which the probability of staying in the same condition state dropped to 50%. The unknown "stay-the-same" transition probability, $P_{i,i}$ was then calculated using (Sobanjo and Thompson, 2001)

$$P_{i,i}^{Y} = 50\% \tag{2.9}$$

which implies,

$$P_{i,i} = 0.5^{(1/Y)} \tag{2.10}$$

Under the assumptions that an element can transition by only one state at the most in any given inspection cycle and that there is no possibility of transitioning to a better state in absence of any maintenance action, it is possible to ascertain the remaining transition probabilities described in equation (2.3). Transition matrices obtained from this approach were used to develop the first deterioration models in Pontis. However, as new inspection data for element-level condition ratings become available, Pontis uses a Bayesian approach to update the initial transition probabilities. Using this approach, updated posterior transition probabilities are developed by taking a weighted average of the prior transition probabilities and those derived from the observed inspection data (Bulusu and Sinha, 1997, Golabi and Shepard, 1997). This leads to an improvement in the accuracy of the models over time as the process continues (Golabi and Shepard, 1997). The same concept is also used in BRIDGIT (Hawk and Small, 1998).

Recently, with the availability of sufficient element-level inspection data, FDOT estimated its transition probability matrices entirely from historical inspection data using regression and the one-step methods mentioned earlier instead of the expert elicitation process used in the 2001 study (Sobanjo and Thompson, 2011). The median transition times Y were also calculated from the transition probabilities $P_{i,i}$ using the inverse of equation (2.9)

$$Y = \frac{ln\,(0.5)}{ln\,(P_{i,i})} \tag{2.11}$$

It was found that the average ratio of the transition times for the new deterioration models to those of the earlier models was 1.97, indicating that expert opinion tends to overestimate the probabilities associated with deterioration (Sobanjo and Thompson, 2011). The Colorado Department of Transportation also recently estimated its transition probability matrices from its historical data using the percentage prediction method. The median transition times were also calculated using equation (2.11) (Hearn, 2012). The median transition times for prestressed concrete superstructure elements were found to be unreasonably high, often exceeding 100 years, in both of these studies (Hearn, 2012, Sobanjo and Thompson, 2011).

The limitation encountered when applying regression techniques to historical element level inspection data has been the lack of sufficient condition rating data available for each element. Pontis has the ability to handle as many as 160 elements each having up to four deterioration models corresponding to the four specified environments (benign, low, moderate, and severe). To have sufficient sample sizes for meaningful regression analysis, the elements have to be grouped by component (deck, superstructure, substructure) or environment or both. Since grouping results in loss of corresponding sensitivity, for example, collapsing of all environmental categories into one would result in loss of sensitivity to environmental factors, different levels and types of classifications have to be examined to obtain a complete picture (Sobanjo and Thompson, 2011).

It is pertinent to mention the role of the NBI translator at this point. The NBI translator works on the concept of assigning relative weights to the condition ratings of elements constituting a particular bridge component (deck, superstructure, or substructure) and aggregating them to obtain a single condition rating for that bridge component (Sobanjo and Thompson, 2011). An NBI translator program was developed by FHWA to help transportation agencies convert the element level inspection data to the format required for NBI submittals and subsequent consideration for federal funding eligibility (Markow and Hyman, 2009). However, the translator was found to have some shortcomings that resulted in inaccuracies in condition rating prediction, especially for bridges in very good condition. This was because it could not distinguish effectively between the highest (6 to 9) and the lowest (0 to 3) NBI condition ratings. Moreover, it assigned too much weight to the fraction of elements in the poorer condition states thereby resulting in unreasonably rapid deterioration rates associated with the NBI condition ratings (Patidar et al., 2007). These inaccuracies were found to affect all performance measures based on NBI ratings that were developed for use in the optimization programming and budgeting decision support tools in the BMS. This was especially true for newly developed BMS software products like the Multi-Objective Optimization System (MOOS) developed by the National Cooperative Highway Research Program (NCHRP) Project 12-67 (Patidar et al., 2007), and the Project Level Analysis Tool (PLAT) and Network Analysis Tool (NAT), both developed by FDOT (Sobanjo and Thompson, 2011). All of these optimization tools were found to be highly sensitive to any changes in deterioration or unit cost inputs. To overcome these issues, FDOT has further improved upon its version of the NBI translator by applying multiple regression and optimization techniques to two years of bridge inspection data from the Florida bridge inventory to estimate the relative weights of element condition ratings. Reviews of initially translated ratings were performed by studying randomly selected bridges and corrections were applied to the translator algorithms as necessary. Although the final developed version had similar issues with regard to the lowest and the highest condition ratings, it performed significantly better than the FHWA translator and produced more accurate translated ratings when compared to the actual NBI inspected ratings (Sobanjo and Thompson, 2011).

Pontis also has an action effectiveness model to determine the effect of MR&R activities. Any MR&R action is assumed to produce an immediate transition to a better condition state, defined by a set of action effectiveness transition probabilities. These "do something" probabilities are also obtained through the expert elicitation process (Sobanjo and Thompson, 2001). The action effectiveness transition probabilities are used once to arrive at the new condition state vector immediately following the action, after which deterioration is assumed to resume according to the process defined by the deterioration transition probability matrix for the component. Thus, any MR&R action has the effect of resetting the deterioration curve to a prior state in time (Golabi and Shepard, 1997).

Although Pontis has been licensed by 46 states, it is mostly used solely for managing bridge inspection data. Only 17 states, or less than 37%, are using the Pontis BMS capabilities for network level planning, project planning, or both (Markow and Hyman, 2009). Many of these states, including Idaho, Virginia, and South Dakota, have modified and customized the Pontis framework instead of adopting it completely in its original format (FHWA, 2010b,c). The percentage of states using the deterioration modeling capabilities of the BMS is even lower at less than 20%. This has been attributed to various reasons including lack of trained staff for using these models, lack of data analysis and preprocessing tools needed to generate the models, or lack of credibility of the available predictive models (Markow and Hyman, 2009). Some states, including Ohio, Michigan, and New York, develop their own deterioration models outside of Pontis and input them into Pontis for optimization and decision making.

At the national level, a National Bridge Investment Analysis System (NBIAS) is used by FHWA to predict nationwide future bridge conditions and investment requirements based on the complete NBI database. The prediction models use element level data and the Markovian models derived from Pontis. Since the NBI data do not contain element level data, a series of stochastic models, known as the Synthesis, Quantity, and Condition (SQC) models, are applied by NBIAS to "synthesize" element-level condition data from the NBI data (FHWA, 2010a, Markow and Hyman, 2009). These SQC models are based on statistical analysis of over 10,000 bridges nationwide to form a representative sample of various structural and material configurations. These models enable NBIAS to create a statistical model consisting of a typical assortment of elements with estimated quantities and condition state distributions for each structure based on its functional descriptors in the NBI database. NBIAS was first used in 1999 for preparing bridge-related need estimates for the Conditions and Performance report submitted biennially to the U.S. Congress. It has replaced the Bridge Needs and Investment Process (BNIP) model developed earlier by the FHWA in 1991 (FHWA, 2010a, Markow and Hyman, 2009).

2.2.3 Limitations of Markovian Models and Proposed Improvements

Despite the widespread use of Markovian models and the commonly used approaches for estimating transition probability matrices, a number of limitations have been identified in these models. These approaches do not model the effects of various explanatory variables, and therefore, as mentioned earlier, have to rely on pre-defined segmentation of the bridge population into homogeneous categories for meaningful statistical analysis. Moreover, the Markovian assumption of time independence is contrary to the time dependence of the deterioration process. This time dependence can indirectly be taken into account by dividing the bridges within each category further into various age groups. However, this grouping is *ad hoc* and fails to recognize the continuous nature of the underlying deterioration. The use of linear regression to calculate transition probabilities, as described in the expected-value method, is also deemed to be inappropriate by some researchers because the dependent variable, which in this case is the condition rating, is discrete and ordinal, and not continuous as presumed by linear regression (Bulusu and Sinha, 1997, Madanat and Ibrahim, 1995, Madanat et al., 1995, 1997, Mishalani and Madanat, 2002, Morcous et al., 2002).

Different models and approaches for calculating infrastructure transition probabilities have been proposed progressively with a view toward addressing the above-mentioned limitations. The discrete nature of the dependent variable was first addressed through applying Poisson regression instead of linear regression in the estimation of transition probabilities (Madanat and Ibrahim, 1995). In addition to improving the predictive fidelity of the previous model, this model also permitted the development of a relationship between deterioration and the various explanatory variables affecting it. Further, it eliminated the need for segmenting the bridge population into homogeneous groups so that the statistical advantage of having the entire dataset for estimation was obtained. The model was extended into a negative binomial regression model to relax the constraining assumption of equality of variance and mean in Poisson regression. Both models were applied to a subset of bridges in the Indiana State Bridge inventory to estimate the infrastructure transition probabilities. The results were found to be very close to the actual observed frequencies (Madanat and Ibrahim, 1995). Another model developed around the same time also accounted for the ordinality of the dependent variable and the time-dependence of the deterioration process. This model, known as the ordered probit model, was used to derive non-stationary transition probabilities for a subset of bridges also from the Indiana State Bridge Inventory. The results were compared to those obtained from the expected value method by using a chi-square test on a sample of concrete bridge decks in condition state 7. The probabilities calculated using the ordered probit model were found to be more accurate in prediction than prior models (Madanat et al., 1995).

The above mentioned models, however, are still considered deficient in their ability to address the two issues of heterogeneity and state-dependence found in panel data, or longitudinal data (Bulusu and Sinha, 1997, Madanat et al., 1997). Panel data is multidimensional data. It is comprised of data sets combining cross-sectional and time-series data, such as those being used for deterioration modeling where the deterioration behavior of a number of facilities is observed across time (Greene, 1997). Such data may have persistent facility specific unobserved factors, referred to as "heterogeneity", for example, construction quality, that if not accounted for may bias the estimates of model coefficients. State dependence, on the other hand, is when the transition probability of moving from one state to another is dependent on the history of the deterioration. Such dependence is likely to make some facilities more deterioration prone than others in the same condition rating (Madanat et al., 1997). The issue of heterogeneity was addressed by developing the binary probit (Bulusu and Sinha, 1997) and random-effects (Madanat et al., 1997) models. Although no appreciable difference was observed in the coefficient values of explanatory variables, these models were found to improve significantly the goodness of fit and predictive fidelity relative to the previous models (Bulusu and Sinha, 1997, Madanat et al., 1997). The issue of state-dependence is, however, still unresolved. Madanat et al. (1997) found that state dependence was present and correlated heavily with the elapsed time in the condition state. However, once the effect of heterogeneity is accounted for, it is difficult to distinguish between the effects of time nonhomogeneity as captured in non-homogeneous Markov chain models and true state dependence (Madanat et al., 1997).

All of the above-mentioned model improvements were tested only on sample subsets of bridges and have not been applied to complete statewide bridge inventories for actual use in a BMS. However, by investigating and exposing the weaknesses of the state-based models, these models served as precursors to the time-based or duration models discussed in the following section.

2.3 Duration Models

Duration models are those that model the time or duration that a bridge component remains in a particular condition state. In these models, the duration until the occurrence of the event of deterioration to the next lower condition state is treated as a random variable, instead of the event itself as done in the state-based Markovian models. Duration models have been found to better model the stochastic nature of the deterioration process by accounting for duration dependence among other aspects of deterioration that could not be considered in earlier models. The earliest time-based models were the state increment models developed for the pavement management and bridge management systems of the New York State Thruway Authority (NYSTA). In these models, the concept of state transition time was defined as the time between two consecutive changes of state, or in other words, the time taken by a bridge component to transition from an initial condition state to the next lower condition state (Ravirala and Grivas, 1995). A uniform distribution of transition time was assumed between minimum and maximum values of transition time, which were estimated on basis of expert elicitation. This assumed parametric distribution was then used to estimate the cumulative probability of the occurrence of a specified state transition event within any specified time, known as the 'transition probability' (DeStefano and Grivas, 1998). The initial models were verified and enhanced by determining the transition probabilities using a non-parametric Kaplan-Meier approach and adding an elapsed-time parameter, respectively (DeStefano and Grivas, 1998). The revised models were then tested on a subset of 123 bridge decks located on the New York State Thruway and the resulting deterioration models were found to be more accurate than the original models. These models used life data analysis techniques on bridge inspection data for the first time. Previously, these techniques had long been used in engineering for reliability studies of industrial components, in the biomedical field for survival time analysis of patients diagnosed with a disease, and more recently, in the social sciences (Greene, 1997). Life data, or duration data, has typical characteristics like censored observations that were taken into account in this study. Later researchers used survival analysis techniques to further develop the duration models (Mauch and Madanat, 2001, Mishalani and Madanat, 2002). The problem of censored observations in duration data and the basic concepts of survival analysis are described in the following subsections before continuing further with the review of duration models in bridge deterioration.

2.3.1 Censored Data

'Censoring' is the term applied to instances when a particular event is not completely observed, and it is a commonly encountered and unavoidable problem in analysis of any duration data. Bridge condition rating data has a large number of censored observations, the reason being that discrete time measurements are made during the continuously ongoing deterioration process. A commonly occurring type of censored observation is the right censored observation where the observed period is known to be less than a certain value. There are many instances of right censored observations in condition rating data, such as at the beginning and end points of the data. For example, let's consider a bridge component that had a condition rating of 7 at the beginning of the observation period in 1981 and stayed at that rating until 1987 when it changed to 6. In this case, all we know is that the time in condition state 7 was at least 6 years as we cannot say how long it was at that rating before the observation first began in 1981 when the state inventory was initiated. Similar is the case for condition ratings observed at the most recent observation period (currently 2015), when we only know that the observed time in the state is at least as long as the actual time, since the remaining duration in that state has yet to be observed.

Likewise, the condition rating of a bridge component may increase during its lifetime because of maintenance actions. This represents a premature interruption of the natural deterioration processes. For example, if an observed condition rating of 5 increases to 7 due to maintenance, we do not know how long the bridge component would have stayed at rating 5 in the absence of maintenance. Therefore, the actual duration of condition rating 5 for the structure is again not fully observed and only known to be as long as or longer than the observed duration, making it a right-censored observation.

In addition to right censoring of data, bridge condition rating data is also subject to a form of censoring due to the discrete interval of inspection recording. Condition ratings are required to be recorded at least every two years in the USA. Therefore, although deterioration itself is a continuous process, the accuracy of the time measurement is limited to the two year inspection interval. This type of discrete measurement results in a type of incomplete observation of data known as interval censoring. For example, if a bridge component is observed to be at condition rating 6 since 1992, and remains at the same condition rating during inspections in 1994 and 1996, but deteriorates to condition rating 5 in 1998, all we can say is that the time that it stayed in condition rating 6 is between four years and six years.

Presence of censored observations in data does not lend well to deterministic modeling or many conventional statistical regression techniques. However, survival analysis models can account for the effect of censored observations and are therefore suitable for analysis of bridge condition rating data (Greene, 1997, Hosmer and Lemeshow, 1999).

2.3.2 Survival Analysis Concepts

Analysis of duration or life data, known as survival analysis, is a category of statistical analysis that models the time until the occurrence of an event of interest.

In such analysis, the duration observed is referred to as survival time or time until failure. In analyzing bridge condition rating data, this time, T, would be the duration that a bridge component stays at a particular condition rating until it deteriorates to a lower rating. If T has a cumulative distribution function, $F(t)$, at time, t, then the probability that T exceeds t is given by the Survivor or Survival function, $S(t)$, given by (Greene, 1997),

$$S(t) = 1 - F(t) = Pr(T \geq t) \tag{2.12}$$

The survival function, or cumulative survival rate, is a non-increasing function of time that takes a value of one at $t = 0$ and a value of zero at $t = \infty$. Given that the survival time exceeds t or $T \geq t$, the probability that the failure event will occur in the next small interval of time, Δt , or when $t \leq T \leq t + \Delta t$, is given by the hazard function, $l(t, \Delta t)$, where

$$l(t, \Delta t) = Pr(t \leq T \leq t + \Delta t | T \geq t). \tag{2.13}$$

The hazard function is usually characterized by using the hazard rate function, $h(t)$, which is the instantaneous rate of failure at time t and is given by

$$h(t) = \frac{\lim_{\Delta t \to 0}(Pr(t \leq T \leq t + \Delta t)|T \geq t)}{\Delta t} = -\frac{d}{dt} \ln S(t). \tag{2.14}$$

The hazard rate of a bridge deck at a particular condition rating is a measure of the risk of dropping to a lower rating at any given time, t. The hazard rate is also known as the conditional failure rate and depends on when the observation was made. If the hazard rate is constant and does not vary with time, it implies that the process is memoryless, like the Markovian processes discussed earlier. This is also known as duration independence and can be modeled using an exponential distribution,

$$S(t) = e^{-\lambda t} \tag{2.15}$$

where $h(t) = \lambda$ (a constant). In general, the hazard rate function may have an upward or a downward slope depending on whether the risk of failure increases or decreases with time. This is termed as positive or negative duration dependence, respectively (Greene, 1997).

Let $f(t)$ be the probability density function of T associated with $F(t)$. It is the probability of failure in a small interval of time per unit time, also known as the unconditional failure rate. The probability density function, the cumulative density function, survival rate, and hazard rate are related as follows,

$$h(t) = \frac{f(t)}{S(t)} = \frac{f(t)}{1 - F(t)} \tag{2.16}$$

The sum total of risk or hazard up to any time, t, is given by the cumulative or integrated hazard function, H(t), and it is a useful tool in survival analysis. Its relationship to the survival function is given by,

$$H(t) = \int_0^t h(x)\mathrm{d}x = -\ln S(t) \tag{2.17}$$

The cumulative hazard function is zero at time $t = 0$ and infinity at $t = \infty$ (Lee and Wang, 2003).

Duration data can be modeled using non-parametric, fully parametric, or semi-parametric methods. Non-parametric methods are strictly empirical or distribution-free as they are not constrained by any pre-imposed structure. A commonly used non-parametric approach is the Kaplan-Meier estimator, also known as the product limit estimator, which was used for developing duration based bridge deterioration models by DeStefano and Grivas (1998), as mentioned earlier. Although this approach is simple and flexible, it is not possible to relate exogenous explanatory factors to the dependent variable using this approach.

2.3.3 Parametric Duration Models

Parametric models are those that follow a theoretical distribution mathematically defined by one or more parameters. The exponential distribution that applies to the constant hazard rate model is one such parametric distribution. A parametric generalization of the exponential distribution that allows for a duration dependent hazard rate is the Weibull distribution. The Weibull distribution is characterized by a shape parameter, γ, and a scale parameter, λ, that determine the shape and the scale of the distribution, respectively. Estimation of these distribution parameters is done by maximizing the statistical likelihood function. The survivorship function for a Weibull distribution is given by

$$S(t) = e^{-(\lambda t)^{\gamma}} \tag{2.18}$$

A limitation of the non-parametric and parametric distributions relative to the semi-parametric distributions is that they cannot directly model the effect of exogenous variables. This limitation can however be overcome by defining λ of the Weibull distribution as an exponential function of the exogenous variables (Greene, 1997, Mishalani and Madanat, 2002).

It is possible to determine the transition probabilities of Markovian state-based models from those of time-based models. In fact, transition probabilities derived from time based models are found to give more accurate results, particularly when inspection data are available for a sufficiently long and continuous time period (Mauch and Madanat, 2001). Duration models using the parametric Weibull distribution were developed for a subset of reinforced concrete bridge decks in the Indiana State bridge inventory (Mishalani and Madanat, 2002). This study illustrated a methodology for determining the state transition probabilities from transition time distributions. The results highlighted that deterioration rates of bridge components could exhibit different behavior at different condition states. For example, condition state 7 was found to exhibit the Markovian property of duration independence whereas condition state 8 had a hazard rate that was positively duration dependent (Mishalani and Madanat, 2002). All of these studies proposed using estimated duration distributions for computing accurate transition probabilities for the corresponding state-based models in order to construct the deterioration models (DeStefano and Grivas, 1998, Mauch and Madanat, 2001, Mishalani and Madanat, 2002).

Recently duration models using the Weibull distribution were developed for the New York State Department of Transportation (NYSDOT) (Agrawal et al., 2009, 2010). The deterioration models were constructed by calculating the expected duration spent in each condition rating using

$$E(T_i) = \eta_i \Gamma(1 + \frac{1}{\beta_i}) \tag{2.19}$$

These duration based deterioration models were compared to Markovian models developed using the second level Markov process. The Weibull models were found to be more realistic and were therefore adopted for use in the NYSDOT BMS (Agrawal et al., 2009, 2010). A Weibull based enhancement was also used to improve the Markovian deterioration models recently updated for the FDOT database (Sobanjo and Thompson, 2011).

Weibull based models, however, can only model monotonically increasing or decreasing hazard rate functions. They cannot model unimodal distributions frequently found in infrastructure deterioration (Yang et al., 2013). Moreover, they cannot take into account the effect of explanatory variables without preclassification of data.

2.3.4 Semi-Parametric Duration Models

Semi-parametric models, on the other hand, support multivariate analysis while not making any assumptions about the shape of the distribution. A commonly used semi-parametric approach is the Cox Proportional Hazards Model (Cox, 1972), which defines hazard rate, $h(t, \vec{z})$, at time t and for covariates, $\vec{z}$, in terms of two components: 1. A non-parametric baseline hazard function, h_0, which varies only with time, and 2. A time-independent multiplier function using the exponential function to represent the effects of the covariates, $\vec{z}$, through regression coefficients, $\vec{\beta}$, as given by,

$$h(t, \vec{z}) = h_0(t)e^{\vec{z}\vec{\beta}} = h_0(t)e^{(z_1\beta_1 + z_2\beta_2 + \ldots z_n\beta_n)} \tag{2.20}$$

Here, $\vec{z}$ is a row vector of covariates or explanatory factors and $\vec{\beta}$ is a column vector of the corresponding regression coefficients that define the effect of the covariates on the hazard rate. The baseline hazard rate is the underlying model for the default factors or with covariates set to zero. The multiplier function associated with the covariates adjusts the hazard rate proportionally to the values of the covariates. The Hazard Ratio, HR, is defined as the relative risk of instantaneous failure of any two items observed at time t associated with covariate sets $\vec{z^1}$ and $\vec{z^2}$, and is constant, as shown below, thus giving the model its name (Kumar and Klefsjõ, 1994).

$$HR = \frac{h(t, \vec{z^1})}{h(t, \vec{z^2})} = \text{constant} \tag{2.21}$$

Semi-parametric models do not restrict the shape of the distribution but give it better structure than non-parametric models by relating it to various explanatory variables. Model parameters are estimated by maximizing a partial likelihood function derived from the distribution. The use of semi-parametric Cox proportional hazards regression was illustrated for the Indiana state bridge inventory using a subset of reinforced concrete bridge decks in condition states 6, 7, and 8 (Mauch and Madanat, 2001). Different condition ratings were found to have different hazard functions, which served to recognize the change in the nature of deterioration of reinforced concrete from one condition state to the next. For example, for decks in condition state 8 and 7, deterioration may be primarily caused by chemical processes, like chloride ingress and corrosion, whereas for decks at condition state 6, it may be due more to mechanistic processes, like delamination cracking. The regression coefficient estimates were also found to be different and not all parameter estimates were significant for each condition state. Ultimately, the hazard ratios helped quantify the relative effect of explanatory variables on the deck deterioration rate at different condition states

(Mauch and Madanat, 2001) and can be used to improve bridge classification over *a priori* groupings.

To overcome the limitations inherent in fully parametric models, an integrated modeling approach to combine the advantages of semi-parametric and parametric models has also been proposed (Yang et al., 2013). This approach suggests first determining the shape of the distribution using the semi-parametric Cox proportional hazards method, and then fitting a mixed Weibull model to it for ease of determining transition probabilities and application to BMS. The mixed Weibull model was shown to produce significantly better results than the two-parameter Weibull model used in earlier studies (Yang et al., 2013).

2.3.5 Limitations

Duration models are considered appropriate only if more than 20 years of inspection data are available, otherwise state based models are considered more suitable (Mauch and Madanat, 2001). Consequently, it is only recently that sufficient NBI records have been available to facilitate use of these powerful statistical regression models. It is expected that duration modeling will be a very active and productive area of bridge management research over the coming decades as analysts exploit the over three decades worth of condition rating data now available in the NBI. However, for element level data where only 10 years or less of inspection data is available, duration models may not give reliable results. To overcome this limitation, various approaches have been recently suggested. One of these is a backward prediction model that can be used to generate past historical data from available inspection data (Lee et al., 2008).

Likewise, an integrated algorithm that can match a suitable modeling technique to the available data has also been proposed (Bu et al., 2014).

Other bridge deterioration modeling approaches found in the literature review include Artificial Neural Network techniques (Lee et al., 2008), case based reasoning (Morcous et al., 2002), and fault tree modeling (Sianipar and Adams, 1997). A two level approach using probabilistic duration models at the network level and a mechanistic approach at the project level for safety critical bridges has also been proposed to improve the effectiveness of the BMS (Cusson et al., 2011, Lounis and Madanat, 2002, Morcous et al., 2010).

CHAPTER 3: PROPORTIONAL HAZARDS REGRESSION MODELING

The Cox Proportional Hazards Model (PHM), a type of semi-parametric duration model, has been used in this study to model the deterioration rates of bridge components and their dependence on various exogenous explanatory factors. As described in Chapter 2 (Section 2.3.4), this model defines the hazard rate as a multiplicative function of a time dependent non-parametric baseline hazard function and a time-independent exponential function. The time-independent exponential function represents the effects of covariates on the hazard rate, as given by equation (2.20). The following sections describe the model-specific interpretation of survival analysis terminology, model development including model fitting and model selection, and the assessment of overall goodness-of-fit of the model. This description is largely based on comprehensive guidance for survival analysis provided in Hosmer and Lemeshow (1999). In particular, this chapter explains the underlying techniques and assumptions used for development of survival functions to model the probability of time spent by a bridge component in a certain state of deterioration based on historical condition rating data. These functions later form the basis for development of a semi-Markov deterioration model in Chapter 4. This newly proposed deterioration model will reflect the duration dependent nature of transition probabilities as well as the impact of significant exogenous variables on bridge deterioration rates.

3.1 Hazard and Survival Functions

Hazard rate, or failure rate, is the instantaneous rate of failure or transition from one state to another. Hazard rate can be a function of time and include the effect of explanatory factors. If it is assumed, for the sake of simplicity, that the model contains only one covariate, z_1, the hazard rate for the Cox Proportional Hazards Model (Cox, 1972) is given by

$$h(t, z_1) = h_0(t)e^{(z_1\beta)} \tag{3.1}$$

where β is the regression coefficient quantifying the effect of z_1 on the hazard rate. Due to the exponential form of the time independent component of the hazard rate function, the hazard rate is equal to $h_0(t)$ when $e^{(z_1\beta)} = 1$ or $z_1 = 0$. Therefore, $h_0(t)$, known as the baseline hazard function, is the hazard rate of the subject under study when the covariate affecting it takes a value of zero. For example, in investigating fatigue failure of a structural component, consider the effect of a dichotomous variable such as presence or absence of cracking, which takes only two values: $z_1 = 0$ for uncracked components and $z_1 = 1$ for cracked components. In this case, the hazard ratio, originally defined in Section 2.3.4, is given by

$$HR = \frac{h(t,1)}{h(t,0)} = e^{\beta(1-0)} = e^{\beta} \tag{3.2}$$

If β is expressed as logarithm of x,

$$HR = e^{ln(x)} = x \tag{3.3}$$

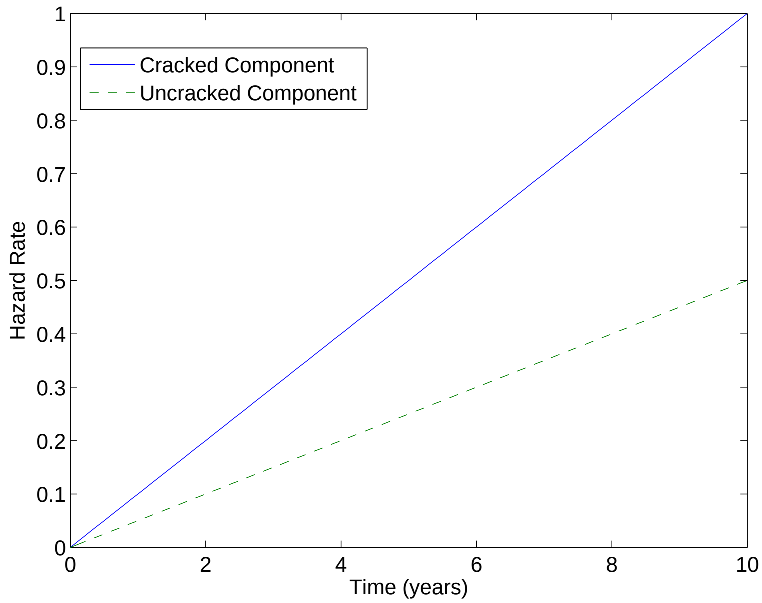

FIGURE 3.1: Hazard functions for fatigue failure of structural components

implying that the risk of failure of cracked components is x times the risk of uncracked components. For instance, $HR = 2$ would indicate that cracked components are likely to fail at twice the rate of failure of the uncracked components. This is illustrated in Figure 3.1 using a hypothetical linear hazard rate function. It can be observed that, at any instance, the hazard rate of a cracked component has a value that is twice that of the hazard rate of an uncracked component. The hazard ratio of the Cox PHM thus lends itself to a quantifiable and easy interpretation of the comparative effect of the covariates under study.

The cumulative hazard function, H, is an integration of the hazard rate based on an assumption of absolutely continuous survival time. For a single covariate PHM,

$$H(t, z_1) = \int_0^t h(t, z_1)\mathrm{d}t = e^{(z_1\beta)} \int_0^t h_0(t)\mathrm{d}t = e^{(z_1\beta)} H_0(t) \qquad (3.4)$$

where H_0 is the baseline cumulative hazard function, which takes the same value

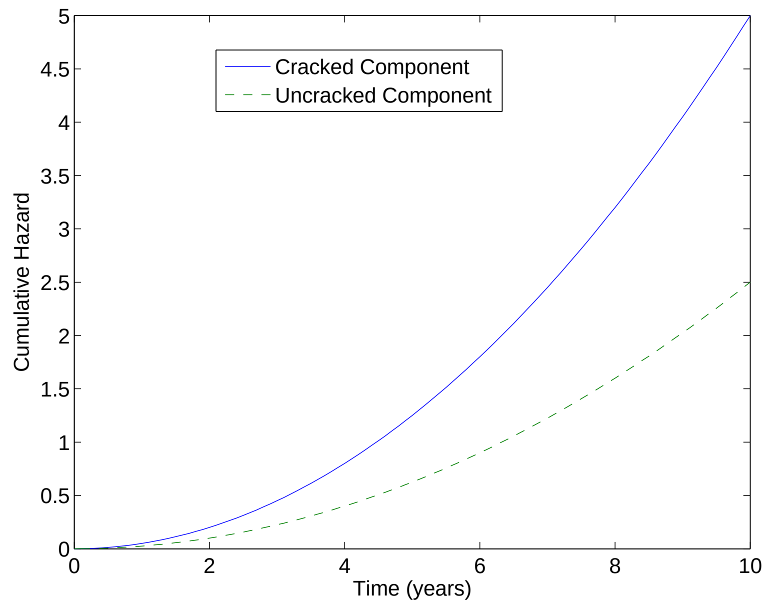

FIGURE 3.2: Cumulative hazard functions for fatigue failure of structural components

at any particular instant of time for different covariates in the same model. The cumulative hazard function incorporating the effect of covariate z_1 can be obtained by multiplying H_0 by the hazard ratio. The cumulative hazard functions for cracked and uncracked components are shown in Figure 3.2. In this example, H for cracked components can be obtained by multiplying H_0 with $HR = 2$ when considering the cumulative hazard function for uncracked components as H_0.

Although the cumulative hazard function is typically not used directly, its importance to survival analysis is that it is the negative logarithm of the survival function, as given in equation (2.17). Alternatively, the survival function can be written in terms of the cumulative hazard function using (Hosmer and Lemeshow, 1999)

$$S(t, z_1) = e^{-H(t,z_1)} = e^{-e^{(z_1\beta)}H_0(t)} = [e^{-H_0(t)}]^{e^{(z_1\beta)}} = [S_0(t)]^{e^{(z_1\beta)}} \tag{3.5}$$

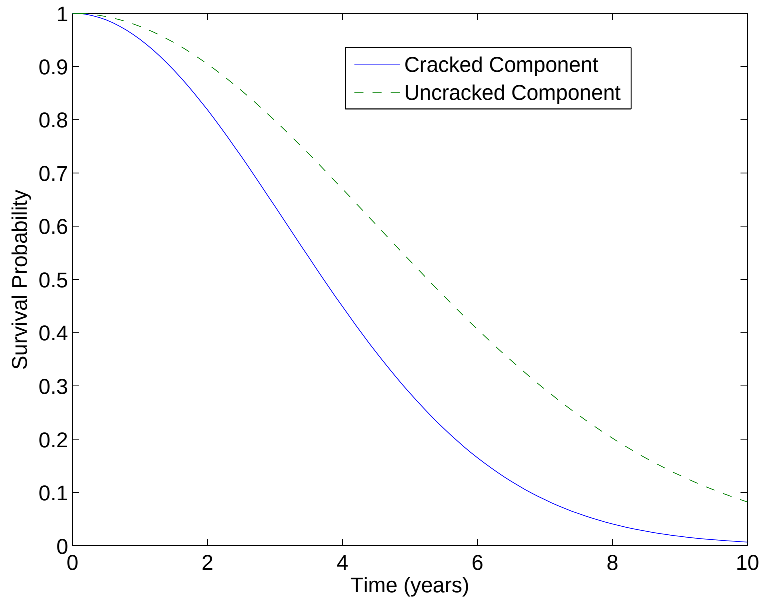

FIGURE 3.3: Survival functions for fatigue failure of structural components

where S_0 is the baseline survival function. Continuing with the structural component failure example, a hazard ratio of $e^{(z_1\beta)} = x = 2 > 1$ implies that $S(t, z_1) < S_0(t)$ since the value of the baseline survival function is always between 0 and 1. This means that the survival probabilities associated with cracked components are lower than the survival probabilities associated with uncracked components. The survival functions for cracked and uncracked components based on the hypothetical hazard rate functions are plotted in Figure 3.3. A comparison amongst Figures 3.1 to 3.3 illustrates the nature of the relationship between the hazard rate function, cumulative hazard function, and survival function.

3.2 Model Fitting

Cox proportional hazards regression is performed using a partial likelihood function based on the concept of maximum likelihood estimation (MLE). MLE employs a likelihood function that represents the probability of occurrence of observed survival time data under the given model. It is employed in survival analysis to estimate the model parameters from limited and potentially censored observations. For censored survival data, $f(t, z)$ is taken as the probability of the survival time being exactly t for uncensored observations and the survival function $S(t, z)$ is taken as the probability of the survival time being at least t for censored observations. The likelihood function, $l(\beta)$, is then obtained by multiplying these probabilities over the entire sample, so that

$$l(\beta) = \prod_{i=1}^{n} \{([f(t_i, z_i)]^{c_i} \times [S(t_i, z_i)]^{1-c_i}\}. \tag{3.6}$$

where $c = 1$ for uncensored observations and $c = 0$ for censored observations. This can be written in terms of the hazard function $h(t, z)$ and the survival function (Hosmer and Lemeshow, 1999), by substitution of equation (2.16), as

$$l(\beta) = \prod_{i=1}^{n} \{[h(t_i, z_i) \times S(t_i, z_i)]^{c_i} \times [S(t_i, z_i)]^{1-c_i}\} = \prod_{i=1}^{n} \{[h(t_i, z_i)]^{c_i} \times [S(t_i, z_i)]\}. \tag{3.7}$$

The estimates of the regression coefficients β, can be obtained by maximizing this likelihood function with respect to β. For computational simplicity, the logarithm of the likelihood function, commonly known as the Log Likelihood function, is used for this purpose, since its maximum occurs at the same value of β. Iterative optimization methods, such as the Newton-Raphson algorithm or its variations, are commonly employed on the gradient of the maximum likelihood equations to determine maximum of the log likelihood function, and consequently the regression coefficients, β (Lawless, 1982).

3.2.1 Maximum Partial Likelihood Estimation

MLE can be used for parametric models that have fully defined and continuous hazard and survival functions. However, the proportional hazards model, being semi-parametric in nature, does not specify a distribution for its error component, which results in an unknown functional form for the baseline hazard and survival functions. Therefore, the MLE defined above cannot be used to estimate β since these regression coefficients are not the only unknown variables within the equation. This problem can be resolved by defining a partial likelihood function, $l_p(\beta)$, developed only with terms of the parameters of interest, β (Cox, 1972, Cox and Oakes, 1984, Hosmer and Lemeshow, 1999). For the proportional hazards model, this partial likelihood function is

$$l_p(\beta) = \prod_{i=1}^{n} \left[\frac{e^{z_i\beta}}{\sum_{j \in R(t_i)} e^{z_j\beta}} \right]^{c_i} \tag{3.8}$$

where each term of the product on the right hand side represents the likelihood of failure estimated at a particular survival time. The survival times are ordered and $R(t_i)$ is the set of subjects at risk at any time t_i, which corresponds to those having a censored or actual survival time equal to or greater than t_i. The log partial likelihood function is given by

$$L_p(\beta) = \sum_{i=1}^{n} c_i \left\{ z_i\beta - ln \left[\sum_{j \in R(t_i)} e^{z_j\beta} \right] \right\} \tag{3.9}$$

The maximum partial likelihood estimation is performed in a similar way to MLE by differentiating $L_p(\beta)$ with respect to β and equating the gradient of the function to zero, or

$$\frac{\partial L_p(\beta)}{\partial \beta} = \sum_{i=1}^{n} c_i \left\{ z_i - \frac{\sum_{j \in R(t_i)} z_j e^{z_j \beta}}{\sum_{j \in R(t_i)} e^{z_j \beta}} \right] = 0 \tag{3.10}$$

It can be observed that the value of β so obtained is such that the sum of the covariate over the uncensored subjects is equal to the sum of the risk-set weighted means of the covariate. The likelihood expressions given above assume that there are no tied survival times in the observed data. Tied survival times are, however, common in applied settings. In such cases, it is assumed that the ties in survival times are due to limited measurement precision and so the r tied values could have been observed in order of any one of the $r!$ possible combinations. The likelihood expression given by equation (3.8) is suitably modified for data with tied times to include each of these arrangements in the denominator (Kalbfleisch and Prentice, 1980). The maximum partial likelihood estimated value of β is generally designated as $\hat{\beta}$. The estimated variance of $\hat{\beta}$ is based on the negative of the second derivative of the log partial likelihood, $-\partial^2 L_p(\beta)/\partial\beta^2$, which is called the observed information, $I(\beta)$. The estimated variance is the inverse of observed information, evaluated at $\hat{\beta}$, or

$$V\hat{a}r(\hat{\beta}) = \left[-\frac{\partial^2 L_p(\hat{\beta})}{\partial \beta^2} \right]^{-1} = I(\hat{\beta})^{-1} \tag{3.11}$$

$I(\beta)$ takes the form of a matrix in models with multiple covariates and is called the observed information matrix for such models. The estimated standard error used for determining the confidence intervals on the estimated coefficients is the positive square root of the variance (Hosmer and Lemeshow, 1999) or

$$S\hat{E}(\hat{\beta}) = \sqrt{V\hat{a}r(\hat{\beta})} \tag{3.12}$$

The estimated variance and standard error, as defined above, are used to assess the significance of individual coefficients within a PHM model as well as the overall significance of the model. Such assessment is required after the preliminary regression of a PHM and subsequently when adjusting the model, to achieve a lean and improved final fit. The two most common statistics used to test the significance of coefficients are the Wald statistic and the partial likelihood ratio, which are described in the following subsections.

3.2.2 Wald Statistic

The Wald statistic, Z, is the ratio of the estimated coefficient to its estimated standard error, or

$$Z = \frac{\hat{\beta}}{\hat{SE}(\hat{\beta})} \tag{3.13}$$

The Wald statistic follows a standard normal distribution under the null hypothesis that the coefficient is not significant and consequently equal to zero. Therefore, its two-tailed p-value can be used to determine if the null hypothesis can be rejected, or in other words, if the coefficient β is significant to the predictive fidelity of the model. For example, if the p-value is less than 0.05, the null hypothesis can be rejected at the 5% level of significance and the coefficient is deemed to be significant with at least 95% confidence. For a significance level of α, the endpoints of a $100(1-\alpha)$ percent confidence interval (CI) for β are obtained using

$$CI = [\hat{\beta} - Z_{1-\alpha/2}\hat{SE}(\hat{\beta}), \hat{\beta} + Z_{1-\alpha/2}\hat{SE}(\hat{\beta})] \tag{3.14}$$

Statistical software used to fit PHM models typically provide the estimated maximum partial likelihood value for each coefficient and the corresponding standard error, Wald statistic, and p-value. For example, considering again the covariate associated with the presence of cracking used earlier in the hypothetical model described in Section 3.1, the estimated coefficient would be $\hat{\beta} = ln(2) = 0.6931$. If the estimated standard error, obtained from a maximum partial likelihood estimate, is 0.1, the Wald statistic is calculated using equation (3.15) as $Z = (0.6931/0.1) = 6.931$. Since the two-tailed p-value for this Wald statistic, as obtained from the standard normal distribution, is less than 0.001, it can be concluded that the coefficient is significant. Consequently, the model would indicate that the presence of cracking in the structural component has a statistically significant effect on the rate of failure under the proportional hazards assumptions. The 95% confidence interval of the coefficient, calculated using equation (3.16), is $(0.6931 \pm 1.96 \times 0.1)$ or $[0.4972, 0.8892]$, and gives an estimate of the expected variability of the parameter under the model assumptions. The confidence interval for the hazard ratio can be obtained by exponentiating the CI bounds for $\hat{\beta}$. The values obtained above are summarized in Table 3.1. In this example, the

TABLE 3.1: Wald statistic and confidence intervals on estimated coefficient

Variable	$\hat{\beta}$	**HR**	**Std. Error**	**Z**	**p-value**	95% **CI**($\hat{\beta}$)	95% **CI(HR)**
Cracking	0.6931	2.000	0.1000	6.931	< 0.0001	$0.4972, 0.8892$	$1.644, 2.433$

confidence interval for the hazard ratio does not contain 1. Therefore, there is greater than 95% confidence that the hazard rate of the components is significantly affected by the covariate. In addition to the null hypothesis test on the Wald statistic, this evaluation of the confidence interval is another way of determining that an individual coefficient is significant to the model fit (Hosmer and Lemeshow, 1999).

3.2.3 Partial Likelihood Ratio Test

In addition to the Wald statistic, the partial likelihood ratio test is routinely used for assessing the significance of individual covariates to the PHM model fit. The log partial likelihood ratio statistic, G, is defined as twice the difference between the log likelihood of a model constructed using the covariate and that of a model not containing the covariate. For a single covariate model,

$$G = 2(L_p(\hat{\beta}) - L_p(0)) \tag{3.15}$$

The statistic G is used to test the null hypothesis on the covariate associated with $\hat{\beta}$. Rejection of the null hypothesis is required to prove the significance of the covariate with coefficient $\hat{\beta}$. In this case, the G statistic follows a chi-square distribution with a single degree of freedom under the null hypothesis that the coefficient is equal to zero. This distribution is used to obtain the p-value for the model's G to determine if the null hypothesis can be rejected or not. For multivariate models, the significance of several covariates can be similarly tested by calculating the likelihood ratio statistic relative to the reduced model not containing these covariates. In this case, the number of degrees of freedom for the chi-square distribution that G will follow under the null hypothesis would be equal to the difference in the number of covariates between the full model and the reduced model. The log partial likelihood ratio test is preferred for PHM regression as it is easier to compute than the Wald test and other available methods, such as the score test (Hosmer and Lemeshow, 1999). This is especially true in the case of multiple covariate models for which computation of the Wald test and the score test requires matrix calculations. Moreover, the log partial likelihood test is considered the best test for assessing the significance of a fitted PHM model (Hosmer and Lemeshow, 1999). The log partial likelihood value is also a statistical measure regularly provided by statistical regression software used for PHM. Its use is illustrated in a subsequent section.

3.2.4 Estimation of the Survival Function

The survival function of a individual subject in a proportional hazards model can be expressed in terms of the covariate vector, z, for the subject, and the baseline survival function, S_0, for $z = 0$, as given in equation (3.5) and repeated as follows

$$S(t, z) = [S_0(t)]^{e^{(z\beta)}} \tag{3.16}$$

The regression coefficients, $\hat{\beta}$, are estimated using the partial likelihood function as explained in the previous subsections. However, to complete the development of the proportional hazards model it is necessary to estimate the baseline survival function. The approach adopted to achieve this is to substitute the maximum partial likelihood estimates, $\hat{\beta}$, for β in the full likelihood function, which is then maximized to obtain estimates of S_0 (Lawless, 1982). This approach is based on the empirical Kaplan-Meier estimator of the survival function given by

$$\hat{S(t)} = \prod_{t_{(i)} \leq t} \left(1 - \frac{d_i}{n_i}\right) \tag{3.17}$$

where $t_{(i)}$ are the rank-ordered survival times in a sample of n independent observations, d_i is the observed number of failures at $t_{(i)}$, and n_i is the number at risk of failing at $t_{(i)}$. The expression $(1 - d_i/n_i)$ represents the estimated conditional probability of survival, $\hat{\alpha}_i$, at the observed ordered survival time $t_{(i)}$. The Kaplan-Meier estimator of the survival function is the product of estimators of the individual conditional survival probabilities. The estimator of the survival function in the proportional hazards model is similarly developed on basis of conditional survival probabilities (Hosmer and Lemeshow, 1999). The conditional survival probability in a PH model is given by

$$\frac{S(t_{(i)}, z)}{S(t_{(i-1)}, z)} = \frac{\{S_0(t_{(i)}\}^{e^{(z\beta)}}}{\{S_0(t_{(i-1)}\}^{e^{(z\beta)}}} = \left\{\frac{S_0(t_{(i)})}{S_0(t_{(i-1)})}\right\}^{e^{(z\beta)}} = \alpha_i^{e^{(z\beta)}} \tag{3.18}$$

where α_i is defined as the conditional baseline survival probability. The differentiation of the PH log likelihood function with respect to α_i yields equation (3.19) that can be solved to obtain the estimator, $\hat{\alpha}_i$, of the baseline conditional survival probability.

$$\sum_{l \in D_i} \frac{e^{(z_l \hat{\beta})}}{1 - \alpha_i^{e^{(z_l \hat{\beta})}}} = \sum_{l \in R_i} e^{(z_l \hat{\beta})} \tag{3.19}$$

where D_i is the set of subjects failing at the ordered survival time $t_{(i)}$ and R_i is the set of subjects at risk of failure at $t_{(i)}$ (Lawless, 1982). In the absence of tied survival times, D_i contains only one subject, and the solution of equation (3.19) is given by

$$\hat{\alpha}_i{}^{e^{(z_l \hat{\beta})}} = 1 - \frac{e^{(z_l \hat{\beta})}}{\sum_{l \in R_i} e^{(z_l \hat{\beta})}} \tag{3.20}$$

In case of tied survival times, however, $\hat{\alpha}_i$ can be obtained by using iterative techniques to solve equation (3.19) (Hosmer and Lemeshow, 1999, Lawless, 1982). As mentioned earlier, $\hat{\alpha}_i$ are determined using the previously estimated $\hat{\beta}$.

The estimator of the baseline survival function is obtained by multiplying the individual estimators of the conditional baseline survival probabilities using

$$\hat{S}_0(t) = \prod_{t_{(i)}} \hat{\alpha}_i \tag{3.21}$$

The estimator of the survival function for any subject associated with a given set of covariates can then be obtained by substituting the corresponding parameter estimates and the estimated baseline survival function in equation (3.16). Software for proportional hazards regression generally provide an estimator of the baseline survival function in addition to the estimated regression coefficients.

3.3 Types of Covariates

Explanatory variables or covariates characterizing descriptive features of a population that may be linked to survival time may be of different data types and unique strategies may be required depending on the nature of the explanatory variables. The statistical regression as well as interpretation of a variable in a proportional hazards model differs depending on whether it is a binary variable, nominal scale variable, or a continuous variable. Commonly encountered variable types and their treatment within proportional hazards regression are discussed in the following subsections with illustrative examples generated using historical condition rating data and bridge records from the North Carolina inventory. The following analysis uses recorded data from 1981-2013 sourced from NC bridge maintenance inventory files, the AgileAssets North Carolina BMS, and FHWA NBI files. Additional information on the source data and features of the bridge records is discussed in Chapter 4.

3.3.1 Binary or Dichotomous Variables

Binary or dichotomous variables are descriptors that take one of only two values, such as the presence or absence of cracking described in the earlier example. An example of a dichotomous variable used within the BMS is the State System classification. In North Carolina, bridges are classified into either State System 1 or 2 depending on the highway functional classification. State System 1 is comprised of bridges on interstate, urban, and primary roads while State System 2 is comprised of those on secondary roads. For PH regression of data with a dichotomous variable, the descriptor field, z, may be simply coded to take binary values, such as 0 for State System 1 and 1 for State System 2 (or not State System 1). The hazard ratio associated with the dichotomous variable is obtained simply by taking the exponential of the coefficient $\hat{\beta}$, or $HR = e^{\hat{\beta}}$, as given in equation (3.2). Further, given the standard error for $\hat{\beta}$, the $100(1-\alpha)$ confidence interval for the hazard ratio at any required level of significance, α, can also be obtained by taking the exponential of the end points of the confidence interval for $\hat{\beta}$, as discussed in Section 3.2.2. For $\alpha = 0.05$ or the 95% confidence interval, this is given by $[e^{(\hat{\beta}-1.96\times SE(\hat{\beta}))}, e^{(\hat{\beta}+1.96\times SE(\hat{\beta}))}]$.

Illustrative survival functions based on PH regression analysis of data from North Carolina bridges with timber decks in condition rating 4, using State System as the only explanatory variable, are shown in Figure 3.4. The value of coefficient, $\hat{\beta}$, obtained from the regression is -0.7550, which provides the difference in log hazard for bridges on State System 2 relative to State System 1. Consequently, the hazard ratio, HR, is 0.4700. This signifies that the risk of a timber bridge deck deteriorating

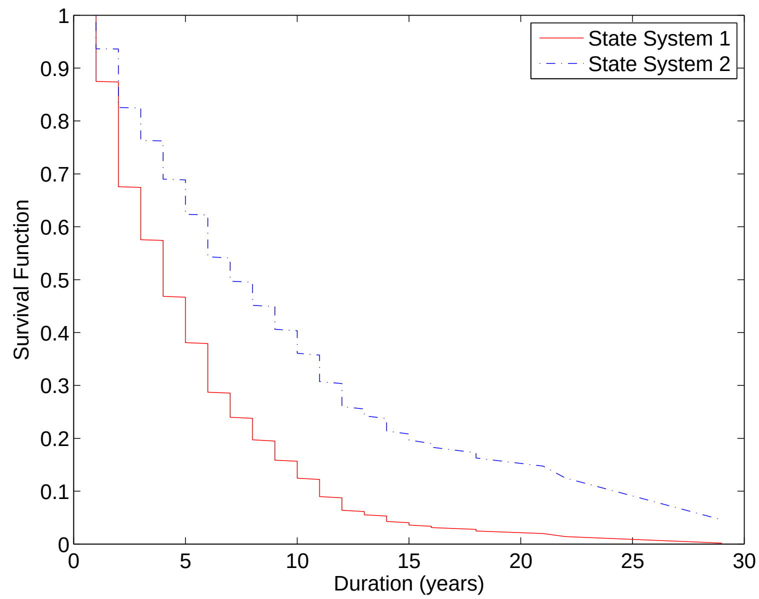

FIGURE 3.4: Survival functions for timber decks in condition rating 4 based on the covariate 'State System'

from condition rating 4 to a lower rating on a secondary road is 0.47 times the risk of a similar bridge deck located on an interstate, urban, or primary road. As previously detailed, the significance of the coefficient $\hat{\beta}$ is commonly assessed using either the Wald statistic and its two-tailed p-value, or the partial likelihood ratio test. In this case, the p-value obtained is less than 0.001 which indicates that the covariate 'State System' is significant at $\alpha = 0.05$ for timber bridge decks in condition rating 4. Furthermore, the standard error of the coefficient estimate is 0.2268 giving the 95% confidence interval for the hazard ratio as (0.3013, 0.7331). The interval for the hazard ratio does not include 1.0, which corroborates the significance of this covariate.

3.3.2 Nominal Scale Explanatory Factors

Nominal scale variables are classified as those that may take one of more than two values, but on a scale that is limited to an integer number of predefined values. In general, if a nominal scale variable is associated with a scale of K predefined values, we can model it using K - 1 ‘design variables’ using a method known as reference cell coding. In this method, one of the levels is established as the reference level against which all other levels are compared (Kleinbaum et al., 2008). An example of such a variable in the NCDOT bridge inventory is geographical region. Depending on county, bridges are classified into one of three regions: Coastal, Piedmont or Mountain. To accommodate this nominal scale variable over the three potential values, reference cell coding can be introduced using two design variables, z_1 and z_2. In this coding, each of these design variables then takes a binary assignment like the dichotomous variable discussed in the earlier section. For example, bridges in the Coastal region could be arbitrarily established as the reference condition, which requires that all coastal bridges are coded with both design variables z_1 and z_2 assigned as zero. The first design variable, z_1, could be associated with bridges in the Piedmont and then the second design variable, z_2, would be associated with bridges in the Mountain region. For bridges in each of these regions, the respective design variable would take an assignment of 1, as illustrated in Table 3.2.

TABLE 3.2: Reference cell coding of a nominal scale variable over a scale of three values using two design variables

Region	$\mathbf{z_1}$	$\mathbf{z_2}$
Coastal	0	0
Piedmont	1	0
Mountain	0	1

The hazard ratio for a proportional hazards model with two design variables, such as the example above, would be given by

$$HR = e^{z_1\hat{\beta}_1 + z_2\hat{\beta}_2} \; where \; z_1 \in [0,1] \; and \; z_2 \in [0,1] \tag{3.22}$$

It follows from equation (3.22) and the coded values in Table 3.2 that the HR is necessarily one for the Coastal region in this model since it is the reference assignment. Likewise, the HR is $e^{\hat{\beta}_1}$ for the Piedmont region and $e^{\hat{\beta}_2}$ for the Mountain region due to the convenience of the binary assignments for the design variables. The results of fitting the Cox PHM to the condition rating 4 data of North Carolina's timber deck bridges using the variable, Region, as coded above in bivariate analysis are shown in Table 3.3.

TABLE 3.3: Illustrative PHM analysis for timber decks at condition rating 4 using reference cell coding

Variable	$\hat{\beta}$	**HR**	**Std. Error**	**Z**	**p-value**	95% **CI**($\hat{\beta}$)	95% **CI(HR)**
Piedmont	-0.394	0.675	0.101	-3.920	< 0.001	$-0.591, -0.197$	$0.554, 0.821$
Mountain	-0.582	0.559	0.107	-5.461	< 0.001	$-0.791, -0.373$	$0.454, 0.689$

As seen from the HR values in Table 3.3, the timber bridge decks in the Piedmont region are 0.675 times as likely to deteriorate from condition rating 4 to 3 in the same time as those in the Coastal region. Similarly, the risk for deterioration of timber decks in the Mountain region is 0.559 times less severe than those in the Coastal region. In this example, the 95% confidence intervals on the hazard ratios of both variables do not include 1.0, indicating that these are significant covariates. Likewise, the p-values of less than 0.001 confirm this. The survival functions for the bivariate analysis associated with geographic region are plotted in Figure 3.5.

3.3.3 Continuous Scale Explanatory Factors

In contrast to nominal scale variables that are associated with a finite number of integer assignments, real valued variables or variables associated with a large number of assignments over either a bounded or unbounded range can be treated as contin-

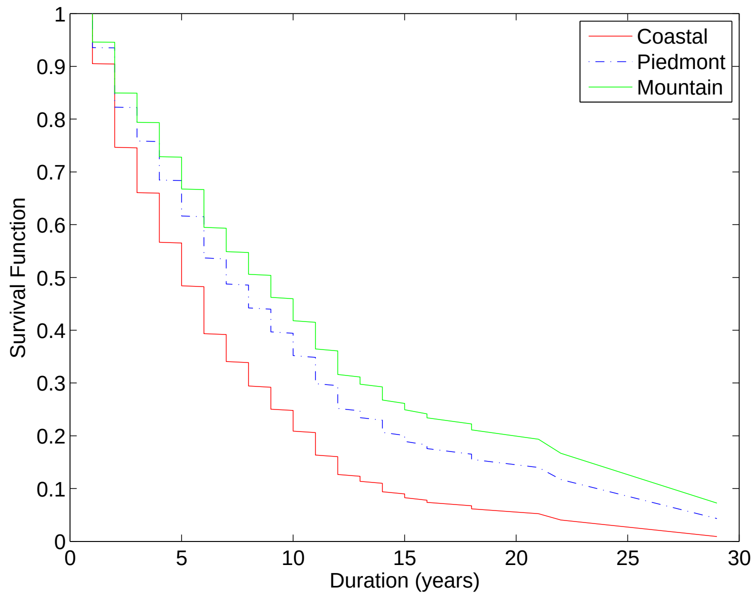

FIGURE 3.5: Timber deck survival functions for condition rating 4 and covariate 'Region'

uous variables (Kleinbaum et al., 2008). For example, Average Daily Traffic (ADT) is an explanatory factor in the bridge database that falls in the latter category and can be considered a continuous variable. With dichotomous variables or nominal scale variables treated as dichotomous variables using reference cell coding, linearity assumptions need not be assessed since each covariate can take only one of two assignments. However, in the analysis of continuous variables, it is important to examine the relationship between the scale of the variable and the effect of the covariate on the log hazard to assess whether the continuous scale sufficiently adheres to linearity assumptions related to β in the proportional hazards models. If the relationship is nonlinear, a linearizing transformation has to be applied to the hazard function to make it linear in the coefficients. This is necessary for correct interpretation of the PHM coefficients. The linearizing transformation, also called the link function, can be determined using the method of fractional polynomials (Royston and Altman, 1997).

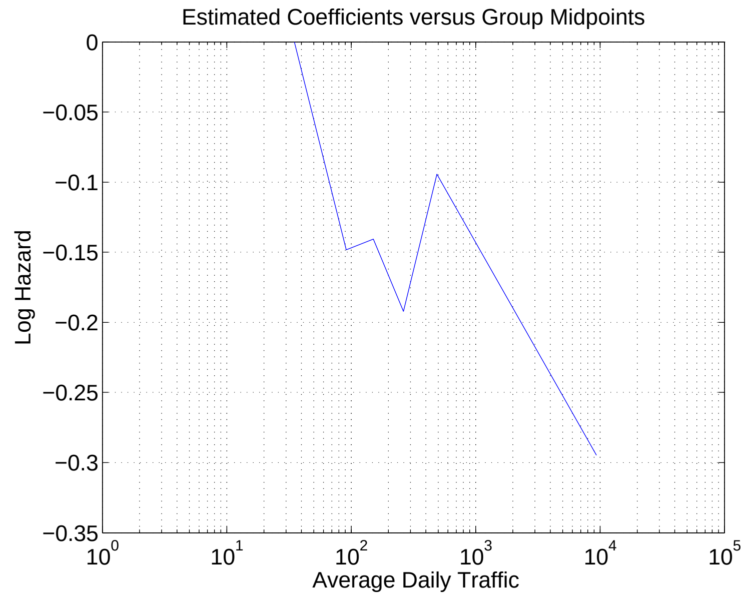

FIGURE 3.6: Estimated coefficients versus group midpoints for average daily traffic categories

This method is illustrated here using a bivariate PHM for the timber bridge deck condition rating data with ADT as the single explanatory variable.

The scale of ADT is first examined using the method of design variables. This was done by grouping ADT values into 6 binned ranges with an equal number of bridges in each category. The continuous variable is thus replaced with 6 design variables representing these categories, as in the nominal scale analysis presented in the prior subsection, and a PH model is fitted to the binned data. The coefficient values are then plotted against the category midpoints to assess the linearity of the log hazard, as shown in Figure 3.6. The figure illustrates that the assumption of linearity is not justified.

The method of fractional polynomials entails defining the covariate z in terms of J functions, each of which is defined in terms of a power p_j of z, where j=1,2, ...,

J. Therefore, if J=1, there will be one function with one value of p. Likewise, for J=2, there will be two functions and therefore two associated powers in the model. For each value of J, the best power coefficients are found by maximizing the log partial likelihood function. As a general rule, a simpler transformation, or lower-order fractional polynomial model, is to be preferred unless a significant improvement is achieved by increasing the level of complexity (Hosmer and Lemeshow, 1999). Table 3.4 presents the powers and log likelihood values for the best fit fractional polynomial models of ADT. The first row represents a model without ADT as a covariate and

TABLE 3.4: Fractional polynomial results for timber deck bridges at condition rating 5 for average daily traffic

	Log-Likelihood	**G for Model vs Linear**	**Approx. p-Value**	**Powers**
Not in Model	−9367.1934			
Linear	−9363.6424	0.0000	0.01393∗	1
$J = 1(2dof)$	−9363.2539	0.7770	0.37807+	0.6
$J = 2(4dof)$	−9363.1755	0.9337	0.81729 @	1.1, −2

∗Compares linear model to model without ADT
+Compares the best J=1 model to one with ADT Linear
@ Compares the best J=2 model to one with ADT Linear

the subsequent rows are for the linear model, first-order, and second-order polynomial models, respectively. The G statistic in column 3 is the partial log likelihood ratio of two models relative to one another. A comparison of p-values of the partial log likelihood ratios of the models indicates that none of the fractional polynomial models affords a significantly better fit than the linear model. It is generally recommended that the fit of models be graphically assessed before making any final decision. The linear, J=1, and J=2 models are plotted in Figures 3.7 , 3.8, and 3.9 respectively. It can be seen that the none of the models captures the impact of ADT as exhibited in Figure 3.6.

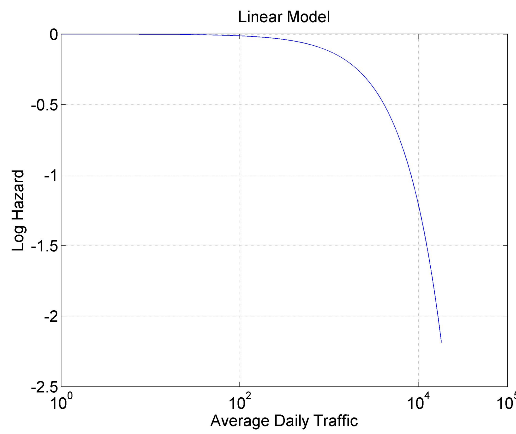

FIGURE 3.7: Linear model for average daily traffic

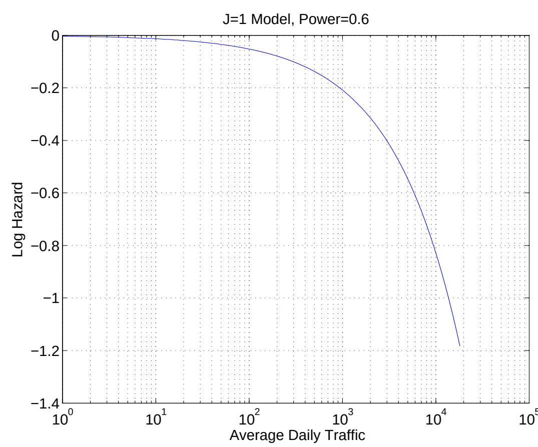

FIGURE 3.8: J=1 fractional polynomial best fit model for average daily traffic

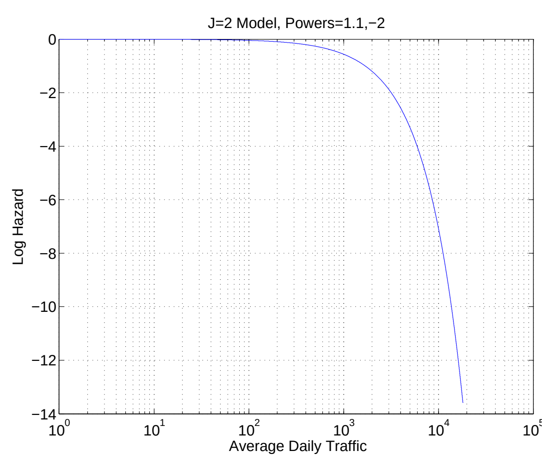

FIGURE 3.9: J=2 fractional polynomial best fit model for average daily traffic

As observed in the above example, an assumption of linearity for ADT treated as a continuous variable was not found to be justified, however even a second order polynomial distribution or J=2 model was not found to be an appropriate fit for the ADT continuous variable. To avoid the complexities involved with modeling of continuous variables as demonstrated above and in the interest of simpler models and ease of interpretation, continuous variables like ADT, Average Daily Truck Traffic (ADTT), Age, and Maximum Span, have been analyzed as categorical or nominal scale variables in this study.

3.4 Multiple Covariate Models

Most practical situations require consideration of more than one covariate affecting the outcome under study. The proportional hazards model allows for construction of such multi-variable models. The advantage of having multiple covariates in a model is that it allows for statistical adjustment of the relative impact of various covariates as well as their possible interactions with one another. Consequently, the resulting statistical inferences are more meaningful in their application (Hosmer and Lemeshow, 1999). This is explained by illustration using the example of the variable 'Geographical Region' affecting the survival time of timber decks in condition rating 4 discussed earlier.

The results of bivariate analyses with Geographical Region as the only factor affecting survival time are presented in Table 3.3. A multivariable PHM regression of the observed survival data for timber decks at condition rating 4 was also performed. In addition to Geographical Region, the variables State System, Maximum

Span, and Age were also included in the analysis. The coefficient values and statistics for the covariate, Geographical Region, obtained through multivariable regression are presented in Table 3.5.

TABLE 3.5: Multivariable PHM results for covariate 'Region' for timber decks at condition rating 4

Variable	$\hat{\beta}$	**HR**	**Std. Error**	**Z**	**p-value**	95% **CI**($\hat{\beta}$)	95% **CI(HR)**
Piedmont	-0.289	0.750	0.104	-2.750	0.006	$-0.492, -0.0827$	$0.611, 0.921$
Mountain	-0.475	0.622	0.110	-4.328	< 0.001	$-0.690, -0.260$	$0.502, 0.771$

By comparison with the values in Table 3.3, it can be observed that the coefficient values obtained from the maximum partial likelihood estimation have undergone a change of 37% for the Piedmont region and 22.6% for the Mountain region, which indicates that the presence of other covariates in the multivariable model has influenced the effect of the covariate Geographical Region on survival time under the proportional hazards assumptions. In practice, it is generally considered that a change greater than 15-20% in the coefficient values indicates a need to include the other covariates in the model (Hosmer and Lemeshow, 1999). Although multivariable PH models are more complex than bivariate PH models, a key advantage of the functional form of the PH model is that the coefficient value and hazard ratio of a covariate in a multivariate model still quantifies the individual effect of the specific covariate on survival time when the values of all other covariates in the model are held constant. In the current example, two bridges of the same age, same maximum span length, and within the same State System can be considered using the condensed results presented in Table 3.5. If one of these bridges is located in the Coastal region and the other in Mountain region, the hazard ratio of 0.622 associated with the design variable for the Mountain region signifies that the bridge in the Mountain region is 62.2% less likely to deteriorate from condition rating 4 to 3 than the bridge in the Coastal region. Although a model with multiple variables is the objective of most statistical analyses, bivariate models discussed earlier are nonetheless important as a starting point for any multivariable modeling to determine the individual impacts on the survival time associated with the various covariates and their significance.

3.4.1 Multicollinearity

Collinearity or multicollinearity amongst independent variables in a multivariable model can be the cause of unstable parameter estimates and high standard errors (Kleinbaum et al., 2008). Therefore investigating multicollinearity is an important step in any multivariable model development process. The intercorrelation between independent variables is generally examined by regressing each independent variable on all other independent variables in the model. If R_j^2 is the squared multiple correlation for an independent variable z_j based on such a regression on the remaining independent variables, a Variance Inflation Factor (VIF) for that variable can be calculated using

$$VIF_j = \frac{1}{1 - R_j^2} \tag{3.23}$$

The value of VIF greater than 10 is generally considered indicative of presence of collinearity (Kleinbaum et al., 2008). The inverse of VIF or $(1 - R_j^2)$, also known as Tolerance, can also be used to assess multicollinearity. Tolerance approaches zero as R_j^2 approaches 1, while VIF goes to infinity. In this study, an algorithm was developed to calculate the values of R_j^2, VIF, and Tolerance for regression of each independent variable included in the model on the remaining variables. This multicollinearity assessment is used to indicate the presence of any pairs of variables for which abnormal values of these statistics were obtained. This analysis ensures that the final multivariable models do not include covariates that are linearly related to each other.

3.4.2 Model Selection

Development of multivariable models can become challenging if there are a large number of explanatory factors to be investigated in the model development. In such cases, it is useful to follow certain model development strategies to select as few significant covariates required for inclusion in a model to adequately predict the underlying behavior captured by the model without sacrificing the accuracy and applicability. These model selection methods depend on comparison of different subsets of variables using selection criteria based on model fit statistics, such as the log partial likelihood ratio discussed in Section 3.2. Model selection is commonly employed in survival analysis and covered in most statistics textbooks (Hosmer and Lemeshow, 1999, Lee and Wang, 2003). Application to the proportional hazards model is similar to other types of multivariate regression modeling and consists of the following basic steps (Kleinbaum et al., 2008):

1. Define a preliminary model with maximum number of covariates

2. Choose a criterion for selecting a model

3. Formulate a strategy for selecting variables

4. Perform regression analyses

5. Assess goodness of fit of the selected model

In this study, the Akaike Information Criterion (AIC), has been adopted for model selection (Lee and Wang, 2003). Its value is calculated using,

$$AIC = L_p(\hat{\beta}) - 2p \tag{3.24}$$

where $L_p(\hat{\beta})$ is the log partial likelihood of the model, $\hat{\beta}$ is the maximum partial likelihood estimator of all parameters in the model, and p is the total number of parameters in the model. The magnitude of $L_p(\hat{\beta})$ is generally directly related to the number of parameters p since the goodness of fit is improved as the number of degrees of freedom in the model is increased. Therefore, by itself, the log partial likelihood measure can only be used as a selection criterion across models that have the same number of variables. In the AIC, it represents the gain as the number of variables included in the model is increased. The second term of the AIC balances this gain by imposing a penalty for increasing the number of variables and is included as a means of directing the selection of an optimally lean model without jeopardizing the predictive capability of the model. The AIC is therefore suitable for models with varying numbers of parameters and is widely used as a selection criterion for multivariable statistical regression.

3.4.3 Model Assessment

Assessment of model fit is required to provide confidence in the validity of the inferences derived from any model under the inherent assumptions introduced by the functional form of the particular model used and regression techniques employed. For inferences to be considered valid, the model must adequately represent the data on which it is based. Proportional hazards modeling, being a semi-parametric method, presents more of a challenge because it does not contain an intercept and, unlike parametric models, therefore does not provide an absolute estimate of mean survival time. Various tests to measure goodness-of-fit of a proportional hazards model have been proposed, however most of these involve complex computations and depend on built-in capabilities of individual software packages used to process the PHM (Hosmer and Lemeshow, 1999). However, in the case of large datasets such as the one used in this study, it is possible to perform a fully stratified analysis and compute Kaplan-Meier estimators of survival functions for all possible combinations of covariates. The similarity of these empirically derived functions to the survival functions obtained from the developed model can serve to assess the adherence of the underlying data to the assumptions in the developed model and therefore verify the correctness of any inferences drawn. Verified models can then be reliably used to describe survival times and calculate median values in absolute terms, as for any parametric model.

3.5 Alternatives to Proportional Hazards Models

In addition to the Cox proportional hazards model adopted in this study and the previously discussed Weibull parametric model that can be modified for multivariable analysis, the accelerated failure time (AFT) model is an alternative survival analysis model suitable for modeling of multivariable survival data. In this model, the effect of the covariates is multiplicative on the survival time, or time to failure, rather than on the hazard function, as in the case of the proportional hazards model. In other words, the covariates, z, directly accelerate, or decelerate, the time to failure, giving the model its name. This model also assumes a baseline hazard function. However, the methods for estimating β in the accelerated failure time model place undue restrictions on the baseline hazard function. The unique advantage of the proportional hazards model is the availability of methods of inference that do not place any restriction whatsoever on the baseline hazard function (Cox and Oakes, 1984, Kalbfleisch and Prentice, 1980, Lawless, 1982). The adherence to relatively weaker model assumptions in comparison to other models, makes the Cox proportional hazards model a more robust choice in the case of infrastructure deterioration modeling where the distribution of survival time is as yet undefined. However, the advantages of parametric approaches in offering increased precision and statistical power make it important to explore the appropriateness of different models. Given the similarities to the Cox proportional hazards model and the more intuitive interpretation of the effect of regression coefficients on survival time, the accelerated failure time model may be appropriate for future research in the area of infrastructure deterioration modeling.

CHAPTER 4: DEVELOPMENT OF A FRAMEWORK FOR PROPORTIONAL HAZARDS DETERIORATION MODELING

The recent applications of survival analysis methodologies for improving the understanding and modeling of bridge deterioration modeling are described in Chapter 2 and can be divided into two general categories. Under one approach, multivariable proportional hazards regression techniques have been employed to study the effect of explanatory factors on deterioration rates at individual condition ratings, although these studies have been limited to small subsets of bridges (Mauch and Madanat, 2001, Mishalani and Madanat, 2002). Under the second approach, univariate Weibull survival functions are developed at each condition rating. These parametric survival functions are then used to obtain the expected durations associated with each rating, from which a final deterministic model was proposed (Agrawal et al., 2009, 2010). The latter approach has produced the only survival-based deterioration models in the country currently in use in a state BMS (NYSDOT). It is notable that, although survival analysis is used in the statistical regression at each condition rating, the second approach does not carry forward the probabilistic nature of the regression model when constructing the deterioration model used for future prediction. Moreover, in this prior work, the univariate survival analysis proposed at each condition rating is not capable of accounting for the individual effects of explanatory factors on the deterioration rate. The framework developed in the current study and described in this chapter overcomes the limitations of all previous models by developing multivariable survival functions for each condition rating and then integrating them into probabilistic deterioration models using the familiar Markov chain approach. By quantifying the effects of the explanatory variables on the deterioration process at each stage in its service life, such models would potentially enhance the predictive fidelity and, consequently, improve MR&R decisions within multi-objective optimization analysis at the network level. Fundamental challenges associated with integrating survival analyses over all condition ratings while incorporating the effect of various explanatory factors throughout the lifecycle of the bridge and developing probabilistic tools into tractable deterioration models suitable for use in a BMS, are summarized in Section 4.1.

A fundamental contribution of this dissertation is the development of a methodology based on proportional hazard modeling that is capable of estimating survival functions associated with the different stages of deterioration throughout the full life cycle of highway bridge components using large databases of historical condition rating data and associated functional, geographic, and design features from the bridge records. Through the analysis routine presented in Section 4.2, regression coefficients that quantify the effect of individual significant exogenous variables on the deterioration rate over each condition rating are obtained to provide unique insight on the factors influencing deterioration rates and characterize how the influence of these factors changes over the life cycle of each bridge. For each subset of bridge components, a set of baseline values is assigned to the significant covariates resulting in baseline survival functions at these values. The proportional hazards model allows scaling of these baseline functions for any individual bridge or category of bridges by a hazard ratio calculated using the vector of descriptive covariates specific to that bridge and their respective regression coefficients. Section 4.3 of this chapter explains how these survival functions can be used to derive non-stationary transition probabilities associated with the change in condition rating of specific bridge components as a function of the duration in each condition rating. These non-stationary transition probabilities are then used to develop Markov chain transition probability matrices leading up to the development of a semi-Markov model of deterioration that not only accounts for duration dependence but can model the effect of various exogenous variables. In this section, a method is derived that permits for easily incorporating the influence of covariates into the transition probability matrices using the hazard ratios formed through the multivariate proportional hazards regression. Owing to its Markovian formulation and the simplicity of applying proportional hazard ratios, the developed model is easily implementable in current bridge management systems, especially those already using Markov chain deterioration models.

4.1 Challenges of PHM Application to Bridge Condition Rating Data

The challenges in applying multivariable PHM regression to bridge condition rating data with a view towards developing an integrated deterioration model across all condition ratings stem from the time dependent influences of explanatory factors on deterioration rates across the different ratings. Furthermore, from a practical implementation perspective, advanced multistate PHM applications are still actively evolving, especially with respect to infrastructure applications, and therefore algo-

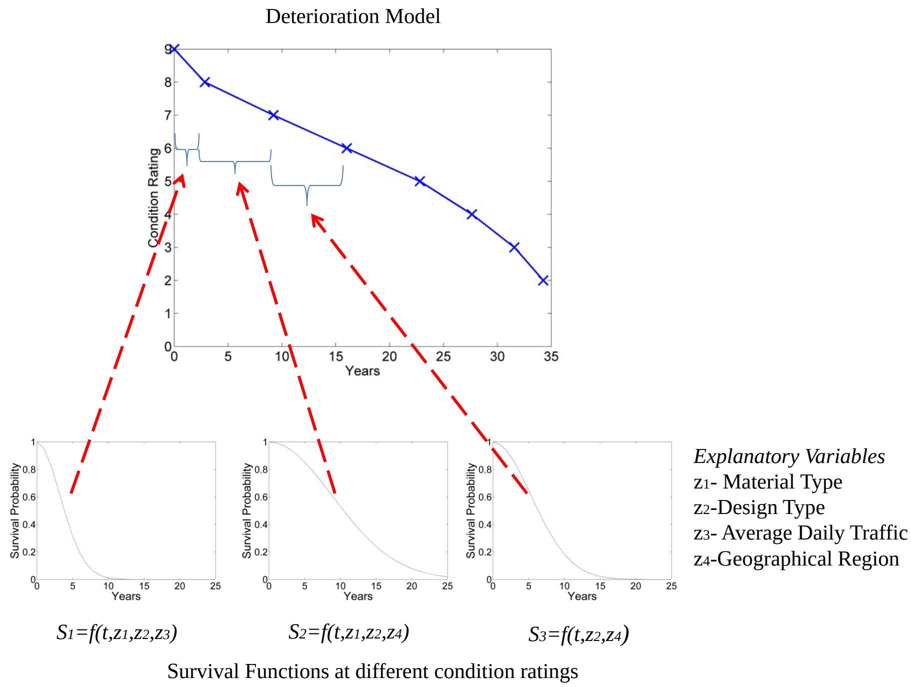

FIGURE 4.1: Deterioration model from survival analysis across individual condition ratings

rithms suitable for bridge deterioration analysis may not be integrated in available software. The phenomenon of time dependency of explanatory factors over the service life a typical bridge component is illustrated in Figure 4.1. As shown, S_1 is the survival function at condition rating 9, which denotes the probability that the specific bridge component will deteriorate to a condition rating lower than 9. It is naturally a function of time, but in this multivariate model it is also dependent on the fictitious explanatory variables z_1, z_2, and z_3. Similarly, S_2 is the survival function associated with condition rating 8, and very likely depends on a different set of fictitious explanatory variables z_1, z_2, and z_4. Likewise, the survival function, S_3, for behavior in condition rating 7 is shown in this example to depend on fictitious explanatory variables z_2, and z_4. Thus the survival functions at individual condition ratings are not only unique but may be affected by a different set of explanatory variables as the underlying physical mechanisms that influence deterioration of specific components may be variable over the service life. Likewise, the same variable may affect the hazard rate of the survival functions of individual condition ratings differently, which would be reflected in changes in the regression coefficients associated with the covariate. This is true across all condition ratings and with any number of relevant explanatory variables. The construction of an overall probabilistic deterioration model for the bridge component under study using multivariate survival analysis requires incorporating these unique survival functions developed from individual condition rating observations. In order to make such an integration possible, it is necessary to devise a common multivariate model structure underlying the survival functions developed for all condition ratings. Some of the challenges encountered in achieving this objective are summarized below:

- Independent variables may have different statistical distributions within observed data at each condition rating. Strategies to ensure uniformity when categorizing continuous scale variables across all condition ratings should be adopted to simplify the structure of the common model.

- There are potentially different subsets of factors expressing statistically significant influence on the deterioration rate at each condition rating.

- Given the large number of potential explanatory factors, efficient strategies for best subset selection of multi-variable PHM models at each condition rating are required.

- Scarcity of records associated with certain material-specific GCRs may preclude identification of any significant factors affecting deterioration at certain condition ratings.

More details about the specific nature of these challenges and the strategies developed to address them are discussed in subsequent sections as measures accommodating them are described.

4.2 Approach for Proportional Hazards Regression of Bridge Condition Rating Data

The general framework developed for generation of multivariable survival functions associated with specific condition ratings is presented in this section. These are used subsequently to develop the multivariable probabilistic deterioration models that form the basis of this dissertation's contributions. The methodology established for proportional hazards regression analysis of bridge condition rating data is presented schematically in the flowchart in Figure 4.2 and described here briefly to provide an overview of the process prior to the detailed descriptions of the individual steps provided in the following subsections. The process begins with the querying and extraction of relevant descriptive and condition-specific data from the bridge database. This data is preprocessed to extract all observations of the response variable, which is the observed continuous duration at the particular condition rating being analyzed, for each bridge in the dataset. Censoring information is compiled in a separate vector of the same size as the response variable but stored as a binary variable of either 0 or 1 depending on whether the observations are classified by the extraction algorithm

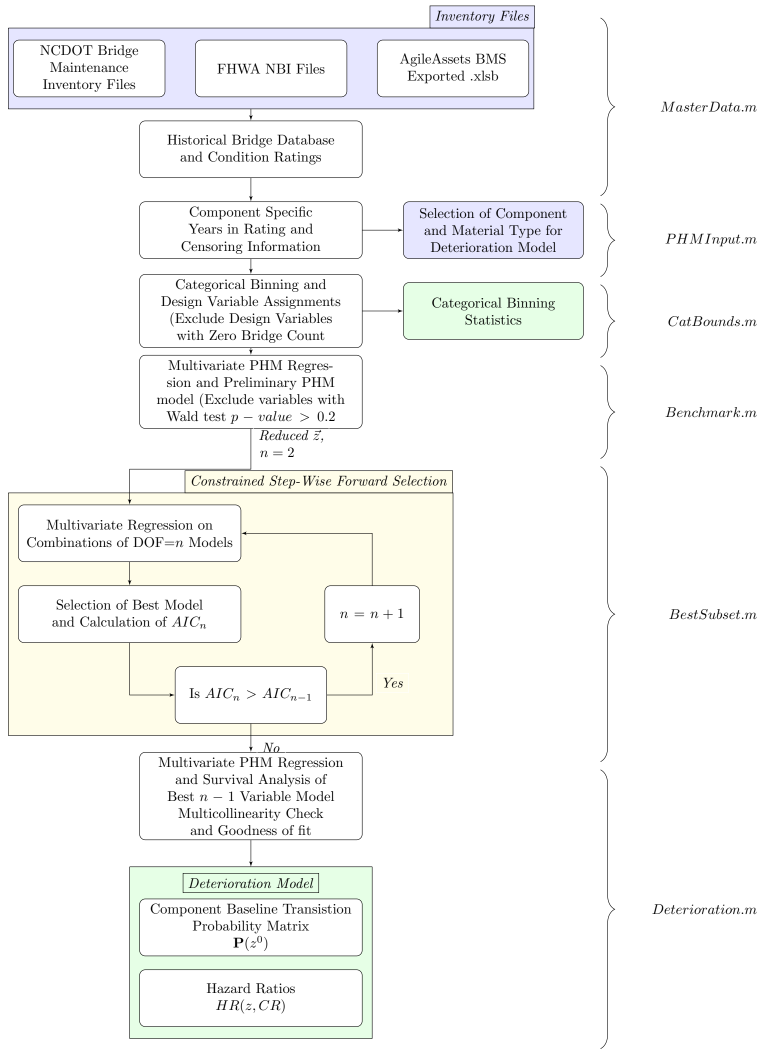

FIGURE 4.2: Proportional hazards model development flowchart

as fully observed or censored. The BMS historical records also contain descriptive information on each structure, such as the design type, functional classification, geographical region, average daily traffic, percent average daily truck traffic, maximum span length, wearing surface and others that could be considered to potentially produce significant influence on deterioration rates of specific bridge components and therefore could be treated as explanatory variables. Each of these variables is organized into categories designated by one or more design variables to which bridges are classified based on either binary or reference cell coding. It is important to note that a distinct set of dependent and independent PH regression input variables are associated with each condition rating for any subset of bridges being analyzed. For example, in analyzing timber deck condition rating durations, the full subset of bridges with timber decks is first isolated from the full bridge and then unique individual sets of variables associated with historical observations within each condition rating are then extracted. The subsequent steps in the processes of multivariable PH regression, best subset selection, and development of survival models are then performed individually on each of these condition-rating specific sets associated with the component subset analyzed.

An initial multivariate PHM regression is carried out using only those design variables that are observed within one or more bridge records for the condition rating under study. From this initial multivariate model, only those variables that are statistically significant with a Wald statistic p-value of ≤ 0.2, are included in the benchmark multivariable model. This benchmark model represents the best possible fit to the observed data under the proportional hazards assumptions with the largest number of degrees of freedom available. A model selection algorithm for determining the best subset of significant variables to achieve an optimal model fit with reduced degrees of freedom is then implemented on this benchmark multivariable model. This algorithm executes a constrained step-wise forward selection strategy based on a combination of maximizing log partial likelihood and minimizing the number of covariates included in the model. The best subset of statistically significant covariates is then included in a multivariable PH regression to estimate the regression coefficients, hazard ratios, and baseline survival function associated with that rating. At this point, this best subset model is also tested for potential multicollinearity issues using the VIF indicator. Additionally, the goodness-of-fit of the final model is assessed by developing Kaplan-Meier estimators on the select categorical data at each rating for qualitative comparison. The survival functions developed using this best subset model incorporate the effects of the most significant explanatory variables on the deterioration rate over an individual condition rating.

The survival function for each condition rating is subsequently used to calculate the transition probabilities associated with staying at the same condition rating or deteriorating to a lower rating at the end of each annual prediction cycle. The transition probabilities associated with all condition ratings at the end of one annual prediction cycle are integrated into a single transition probability matrix applicable to that annual prediction cycle. In this way, a set of non-stationary transition probability matrices is developed, as described in Section 4.3, that are used to develop a complete deterioration model incorporating the input from all condition ratings. This deterioration model fully characterizes the impact of significant explanatory factors over the full life-cycle of the component.

The MATLAB software environment has been employed in this study to develop the new deterioration models as well as package the developed framework into a graphical user interface (GUI) to facilitate its implementation. The development of the graphical user interface is discussed in a subsequent chapter. MATLAB was selected because of its ability to process and organize vast amounts of data, and the flexibility it allows in developing customized models and algorithms through its various toolboxes and design environments including those for statistical analysis, optimization, and GUI development. Over the following sections, the process for developing deterioration models that is implemented in the MATLAB environment is described in the following sections using algorithms. The algorithms represent the substantially more complex MATLAB routines and functions that were developed to execute the various steps in model building and implementation. The MATLAB functions and subroutines are presented in the appendices.

4.2.1 Database Development and Data Preprocessing

The first step in the model building process consists of compiling the database for the statistical regression by extracting relevant records from source data files. In this case, the source data was comprised primarily of the NCDOT Bridge Maintenance Inventory files (1981-2009) and supplemented with exported yearly databases from the NCDOT Agile Assets BMS database (2010, 2012-2015), as well as one year from the FHWA NBI database (2011). Both the NCDOT Bridge Maintenance Inventory files and the FHWA NBI files contain electronically recorded historical inspection data from all of North Carolina's bridge structures in ASCII format. Every bridge structure has a record for each year consisting of over a hundred fields with entries coded in accordance with the NBI Record Format (FHWA, 1995). It is notable that, since the routine developed can manage data from any of these sources, the current code and user interface can be directly applied to any state database using NBI files. The data used in this study spans a period of 35 years from 1981 to 2015, and contains records for over 17,000 individual in-service bridges.

Relevant fields from each of the external databases were imported and organized into a MATLAB Master File as shown in Algorithm 1, procedure [1]. The MATLAB

Algorithm 1 Compiling Material Specific GCR Data Sets for Deterioration Modeling

Input: Source Data: NCDOT Bridge Maintenance Files, NCDOT BMS, and NBI
Output: MATLAB Master File, Material Specific GCR Data Sets ▷ **Material:** Timber, Concrete, Steel; **General Component:** Deck, Superstructure, Substructure

```
procedure [1] ImportData(Source Data)
    MATLAB ← NCDOT Bridge Files ▷ Import Required Fields for all Records
    MATLAB ← NBI: NC
    MATLAB ← NCDOT BMS
    Create separate identifiable records for reconstructed and rebuilt bridges
    return MATLAB Master File
end procedure

procedure [2] DataQuery & Preprocessing(MATLAB Master File)
    for Specified Material and General Component do ▷ Such as Timber Deck
        Extract Records having Material Code
        Extract Records having General Component Condition Ratings
        Eliminate Records not having both of the above
        return Material-Specific GCR Data File
    end for
end procedure
```

structure array so created served as the master database for all subsequent modeling and analyses performed during this study. In addition to the challenges associated with importing and collating large volumes of data selectively from different sources, the identification and rectification of anomalies invariably associated with such large and manually recorded databases was also difficult and time consuming. Examples of some data anomalies discovered during this process and strategies implemented to address them are given below:

- Inconsistencies in the description data recorded for individual bridges over multiple data records are present in the historical database. The inconsistencies consist largely of instances of missing entries as well as some apparent cases of miscoding, such as a recorded change in deck material in one year of the record only to revert back to the original deck material description in the following year. Eliminating all records with apparent recording errors would have reduced the dataset for statistical analysis significantly. Therefore it was decided to use the most frequently recorded value, or mode, across all data years available for a particular structure to determine explanatory variable assignments for fields where consistency was expected.

- For reconstructed bridges, the 'Year Reconstructed' field was erroneously found to be recorded as '0' (indicating no reconstruction) in many instances of yearly records following the reconstruction. This anomaly would impact the calculation of bridge age relative to each observation of condition rating duration and measures to correct these instances needed to be developed.

- In parsing records for the response variable, i.e. the number of continuous years spent at a particular condition rating, it was found that there were some cases where a condition rating was observed for just one year. These observations are treated in the following analysis as anomalies since the typical inspection cycle is biennial, and therefore the minimum expected duration at any condition rating is two years. To address these perceived anomalies, any observations with a duration of less than two years were filtered from the data prior to statistical regression.

- County numbers in the bridge records are based on alphabetical order of counties in the state. A typical anomaly relating to this was discovered in select database years where assignments of county numbers to counties beginning with ‘Mc’, such as McDowell county, were placed ahead of Macon county because of a special alphabetization rule, which considers ‘Mc’ as an abbreviated form of ‘Mac’ and uses the latter for alphabetical sorting. Failure to address this coding inconsistency would have resulted in erroneous interpretation of county codes relating to these counties and importing of mismatched records relating to these counties from the source databases.

- Each bridge in the sourced bridge records is associated with a unique number that is based on the bridge location and is unchanged even when the bridge is reconstructed or rebuilt. For the purpose of condition rating duration analysis, it was decided that rebuilt bridges should be treated as new structures and reconstructed bridges should be identified to explore the potential effects of reconstruction on component deterioration rates. To achieve this, a searching algorithm was written to create separate records for these bridges by developing a new structure number after each indication of rebuilding or reconstruction in the bridge record.

Procedure [2] of Algorithm 1 describes the creation of database subsets from the master file, each containing records pertinent only to a selected bridge component and specified material type. The material type for bridge decks is coded separately in the ‘Deck Structure Type’ field in the NBI, whereas the material type used for superstructure and substructure is coded in the ‘Structure Type, Main’ field. The general condition ratings (GCRs) for each component are coded as ‘Deck’, ‘Superstructure’ and ‘Substructure’. Each material-specific GCR file developed by the algorithm contains complete database records pertaining to the specific component type and material designation.

4.2.2 Structuring Data and Design Variables for PHM Regression

Algorithm 2 presents the master code designed for constructing PHM-based deterioration models for any material-specific GCR from the database subset files, generated above. The master code executes various independent subroutines by calling associated functions from the main code. These subroutines were developed to perform various tasks in the modeling process and correspond to the functions outlined in the algorithm.

Function [1] preprocesses the data contained in the material-specific GCR file to develop the dependent and independent variables for individual Cox proportional hazards regression over each condition rating. For all condition ratings from 4 to 9, the records in the material-specific GCR file are first parsed into a ‘Subset’ containing only the records where the particular condition rating is observed. Each such record in the ‘Subset’ contains the complete set of relevant data fields imported from the source data, including the condition rating fields. The condition rating data in this subset is further preprocessed to determine the values of the response variable, ‘Years In Rating’, which is the observed continuous duration at the particular condition rating. For each bridge, all continuous observations of the condition rating for two or

Algorithm 2 Developing Multivariable Proportional Hazards Deterioration Models: Part I

Input: Material-Specific GCR Data File

Output: Baseline Deterioration Model:Regression Coefficients and Associated Hazard Ratios

```
function [1] PHMInputData(Material-specific GCR Data File)
    for Condition Ratings 4:9 do
        Subset{j} ← All records including the Condition Rating j
        Years In Rating(:, j) ← Maximum continuous duration at the Rating
        Censor(:, j) ← '0' if Rating is completely observed, '1' otherwise
        Age(:, j) ← Years since first built or reconstructed
        return Subset, Censor, Years In Rating, Age
    end for
end function

function [2] Categorical Variable Means(Subset, Age)
    for Condition Ratings 4:9  do
        Categorize ADT, ADTT, Age, Maximum Span into equal frequency bins
        Determine Category Minima for all variables
    end for
    Weight ← Number of Records for each Condition Rating
    Calculate Weighted Mean Category Minima across all Ratings
    return Category Minima: ADT, ADTT, Age, Maximum Span
end function

function [3] Preliminary Multivariate Analysis(Subset, Censor, Years In Rating, Age, Categorical Variable Means)
    for Condition Ratings 4:9  do
        Develop Design Variables and Coding for Covariates:
        (State System, Reconstruction, Number of Spans) ← Binary Coding
        (ADT, ADTT, Age, Region, Wearing Surface,Maximum Span) ← Reference Cell Coding
        Covariate Values for 'Baseline' Hazard ← 0
        Matrix of Coded Design Variables ← 'X'
        Reduced X ← Exclude Variables with no associated bridge records
        [Reduced X, Years In Rating, Censor] ← Baseline Multivariable PH Regression
        return Preliminary PHM Statistics
    end for
end function
Continued in Part II
```

more years are treated as separate records and the 'Subset' is accordingly augmented. Censoring information is compiled in a separate vector, 'Censor', of the same size as the response variable but taking values of only 0 or 1 depending on whether the observations are fully observed or censored, respectively. As explained in Section 2.3.1, all observations that contain records from the end-point years of the database are considered as censored, as it is presumed that the actual duration of the condition rating is longer than observed due to the limited time span of the data recording period. Similarly, all observations where an increase in condition rating is observed after the observation are also considered as censored due to an assumed interruption of the natural deterioration process on account of maintenance intervention. It should be recognized that variability in condition ratings due to subjectivity of the inspection process is known to be a factor in the accuracy of condition rating data (Phares et al., 2004), which is not considered explicitly in this analysis due to a lack of available techniques for accounting for the subjectivity. In our analysis, we assume that the effect of observations prematurely shortened as a result of subjectivity in the rating process is balanced by records either lengthened as a result of this same subjectivity or denoted as censored as a result of a subjective increase in rating rather than actual maintenance. It was found that at condition ratings below 4, not only did the number of records become relatively sparse compared to the other ratings but also 90% to 100% were censored records, which is expected due to the priority given to maintenance of such structures. Due to this reason, it was decided to analyze only condition rating data pertaining to ratings 4 through 9. The preprocessing function also computes 'Age' of the bridge at the beginning of an observed duration, which is not directly available and is calculated based on the data year at the beginning of the observation relative to the year at which the bridge was built or reconstructed.

NBI records contain information about various explanatory variables or covariates, such as design type, functional classification, geographical region, average daily traffic, percent average daily truck traffic, maximum span length, wearing surface, and other information that could be considered to have the potential to significantly influence deterioration rates of specific bridge components. For the purpose of regression, dichotomous variables are assigned binary coding, and nominal scale variables that take more than two values were coded using the reference cell coding technique described in Chapter 3. As mentioned in Section 3.3.3, continuous scale variables like ADT and Age have been treated as categorical variables in this study in the interest of simplicity of implementation and ease of interpretation. These variables take numeric values in their respective database and are reassigned as binary design variables through reference cell coding after being binned into categories before the analysis. The preference is to bin these variables into groups of uniform size. However, the bounds of uniformly distributed bins are not consistent over all of the condition rating data analyzed. To address this challenge, a subroutine, represented by Function [2], was developed to analyze such variables across condition ratings 4 through 9 to determine these bins for each condition rating and then develop an optimal binning strategy for the material-specific GCR based off of a weighted average of the bins for each condition rating. The number of observable bridge records in the material-specific GCR database for each condition rating is used to weight the average. The covariates analyzed in this study along with the associated design variables and reference categories are listed in Table 4.1. The category bounds for continuous variables ADT, ADTT, MaxSpan, and Age shown in this table relate only to timber decks and are updated for every other component model according to the categorical statistics described above.

TABLE 4.1: PHM covariates

Covariate	**Design Variable**		**Reference or Baseline Category**
	Name	**Category**	
State System	StateSystem	State System 2	State System 1
Reconstruction	Reconstruction	Reconstructed	Original or Rebuilt
Region	Piedmont	Piedmont	Coastal
	Mountain	Mountain	
Wearing Surface	MonolithicConcrete	MonolithicConcrete	None
	IntegralConcrete	IntegralConcrete	
	LatexConcrete	LatexConcrete	
	LowSlumpConcrete	LowSlumpConcrete	
	Bituminous	Bituminous	
	Timber	Timber	
	Gravel	Gravel	
	Other	Other	
Average Daily Traffic (ADT)	ADT2	$94 \leq ADT < 204$	$0 \leq ADT < 94$
	ADT3	$204 \leq ADT < 468$	
	ADT4	$ADT \geq 468$	
Average Daily Truck Traffic (ADTT)	ADTT2	$6 \leq ADT < 13$	$0 \leq ADTT < 6$
	ADTT3	$13 \leq ADTT < 29$	
	ADTT4	$ADTT \geq 29$	
Maximum Span (m)	MaxSpan2	$2 \leq MaxSpan < 3$	$0 \leq MaxSpan < 2$
	MaxSpan3	$MaxSpan \geq 3$	
Number of Spans	NumberSpans	Multiple spans	Single span
Age (years)	Age2	$20 \leq Age < 28$	$0 \leq Age < 20$
	Age3	$28 \leq Age < 35$	
	Age4	$Age \geq 35$	

Bivariate analyses of some of these variables were discussed in Chapter 3. Since there are limited functional features available in the database, it was decided to include all of these features in the initial multivariable regression irrespective of the Wald statistic p-values obtained from bivariate analyses. The matrix of design variables coded appropriately as above is represented by 'X' in Function [3]. A reduced X is obtained by excluding variables that do not have a single bridge record with the associated variable expressed. The functional features or covariate values associated with baseline hazard are coded as zero. This reduced subset of covariates is then analysed using proportional hazards multivariable regression to produce the preliminary multivariable model and associated statistics, within Function [3]. This preliminary multivariable model contains the greatest number of degrees of freedom for the material-specific GCR model and serves as a benchmark during the forward selection of a best subsets model by indicating the significance of variables excluded in the subsequent models. This is accomplished using the p-values of the Wald statistic.

4.2.3 Best Subset Selection

A strategy for reduced model selection, summarized in Function [4], is implemented to determine the 'best subset' of parameters to include in the deterioration model. This task requires a balance between the inclusion of a sufficient number of variables to ensure that the predictions are strongly correlated to the underlying data but also minimization of the number of variables to the smallest set needed to accomplish the former objective so that the deterioration model is not unnecessarily large or complicated to implement in practice. In this study, several different approaches for best subset determination were examined, including step-wise forward selection and step-wise backward elimination, before selecting a constrained step-wise forward approach based on the AIC, as discussed in Section 3.4.1.

Under this approach, the variables in the preliminary model that exhibit a p-value of the Wald statistic of more than 20% are deemed insignificant and are removed as potential covariates of the best subset model. For the remainder of the variables that express significance in the benchmark multivariate model, proportional hazards regression is first implemented on all possible combinations of two variables in or-

Algorithm 2 (Continued) Developing Multivariable Proportional Hazards Deterioration Models: Part II

function [4] BEST SUBSET MULTIVARIABLE MODEL(X,Years In Rating, Censor, Preliminary PHM Statistics)
  for Condition Ratings 4:9 **do**
    Eliminate Covariate Design Variables with p-values > 0.2
    N ← Number of Design Variables in Reduced X
    for n = 2:N Variables
      Fit PHM to all possible combinations of n Variables
      Best Combination of n Variables ←Highest $L_p(\hat{\beta})$
      AIC_n ← AIC value of Best Combination of size n
      if $AIC_n > AIC_{n-1}$ **go to** n=n+1 until AIC is maximized
    end
    If a Variable is included in two consecutive Best Combinations, include it permanently ▷ Reduces computation time significantly without affecting accuracy
    Best Subset←Highest AIC value
    return PHM Best Subset
  end for
end function

function [5] PHM COEFFICIENTS & SURVIVAL FUNCTIONS(Best Subset, Years In Rating, Censor)
  for Condition Ratings 4:9 **do**
    [Best Subset, Years In Rating, Censor] ← Baseline Multivariable PH Regression
    return Final PHM Statistics including β and HR
    return Baseline Survival Function
  end for
end function

der to select the combination that provides the best statistical fit to the data. In this routine, the best statistical fit is determined through the log partial likelihood estimate. The number of variables, or the degrees of freedom, in the models is increased incrementally one degree of freedom at a time until the AIC associated with the identified best combination is maximized. The AIC measure is used to assess the adequacy of the best combination models but with inclusion of a penalty term proportional to the number of variables included. In order to reduce the computational complexity and time associated with processing all possible combinations as the model increases in size, a strategy is adopted that constrains the combinations evaluated in each iteration to include any variables that are identified as significant in any two consecutive best combination models. Since the total number of combinations of all sizes possible for N variables is 2^N, reduction of each variable reduces the total number of potential combinations to be evaluated by half. For example, the total number of combinations of all sizes possible for 15 variables is $2^{15} = 32768$. However, as variables are progressively selected to be included in the final best set, the number of combinations to be evaluated reduces to $2^{14} = 16384$ for reduction of one variable, $2^{13} = 8192$ for reduction of two variables and so on. This strategy was found to be especially useful in reducing computation time in processing best subsets for condition ratings 6 to 8 within which it was common to have 15-20 candidate variables in the initial set. To assess potential issues associated with this constrained best subset selection approach, best subsets for ratings with initial sets of up to 10 variables were processed both with and without implementation of this strategy for several material-specific GCR datasets. The resulting best subsets in both the cases including the constrained selection technique and those developed by full evaluation of all possible combinations were found to be exactly the same. Consequently, this strategy has been uniformly adopted in the interest of faster processing times and is assumed to not compromise the selection of the final best subset of covariates.

Function [5] is then used to perform the proportional hazards regression on the identified best subset for each condition rating and obtain the model statistics, including regression coefficients and hazard ratios. For example, Table 4.2 gives the best subset covariates and the corresponding hazard ratios and Wald statistic p-values across condition ratings 9-4 for analysis of NCDOT timber decks. In this table, a '$*$' indicates that the associated covariate is not included in the best subset survival model for that condition rating. It can be observed that some covariates are significant at certain ratings and not at others. Consequently, the final model indicates that these factors influence deterioration of the timber deck only during those periods of the life cycle. This state dependent effect of covariates over the life cycle of the bridge is a unique aspect of the developed framework and a potentially significant improvement to conventional deterioration modeling approaches that rely on a prior bridge classification that is fixed over all condition rating states in the life cycle.

TABLE 4.2: PHM hazard ratios and Wald statistic p-values for timber deck best subset

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
'StateSystem'	*	*	*	*	*	*	*	*	*	*	0.449	0.114
'Reconstruction'	*	*	0.842	0.008	*	*	1.293	< 0.001	0.779	0.048	*	*
'Piedmont'	*	*	*	*	*	*	*	*	1.363	0.001	*	*
'Mountain'	*	*	1.277	< 0.001	0.857	< 0.001	1.143	0.001	1.424	< 0.001	*	*
'ADT4'	*	*	*	*	1.138	0.003	*	*	*	*	*	*
'ADTT3'	*	*	1.140	0.003	*	*	*	*	*	*	*	*
'ADTT4'	*	*	1.296	< 0.001	*	*	*	*	*	*	*	*
'MaxSpan2'	*	*	*	*	1.171	0.003	1.199	< 0.001	*	*	*	*
'MaxSpan3'	*	*	*	*	1.165	0.002	1.194	< 0.001	*	*	*	*
'NumberSpans'	*	*	1.119	0.007	1.284	< 0.001	1.218	< 0.001	*	*	*	*
'Age2'	2.289	< 0.001	2.438	< 0.001	1.715	< 0.001	1.332	< 0.001	1.302	0.028	0.742	0.163
'Age3'	2.503	< 0.001	2.210	< 0.001	1.264	< 0.001	2.076	< 0.001	1.814	< 0.001	*	*
'Age4'	2.445	< 0.001	3.045	< 0.001	0.787	< 0.001	2.265	< 0.001	1.564	< 0.001	*	*

The baseline survival function for each condition rating is also obtained from this proportional hazards regression analysis. The set of baseline survival functions developed for material-specific GCR 'timber deck' over all condition ratings is shown in Figure 4.3 as an example of the baseline survival functions returned. The state dependent effects of covariates are only incorporated into the deterioration model

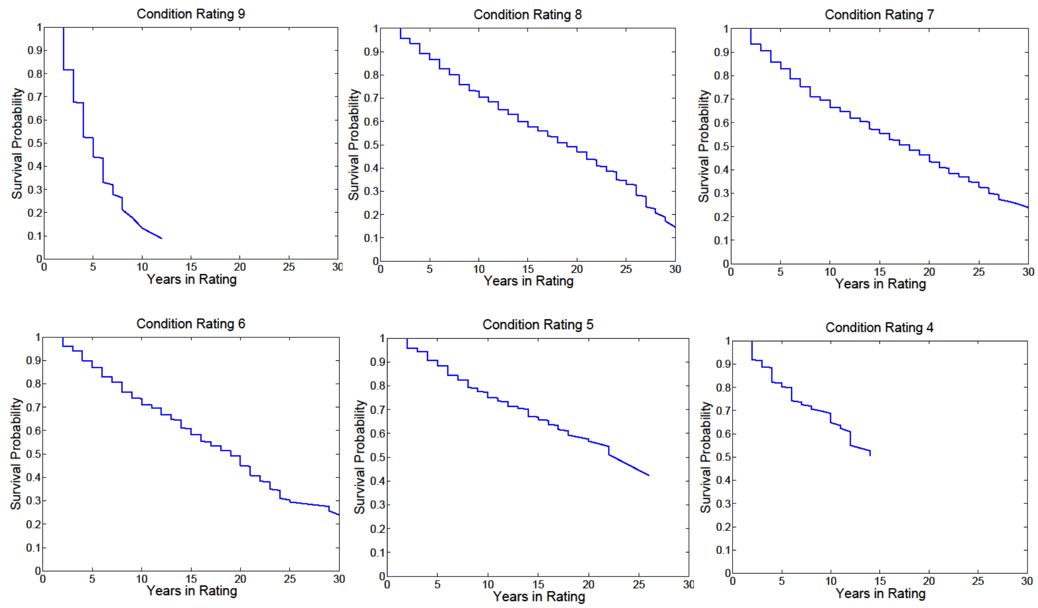

FIGURE 4.3: Baseline survival functions for timber deck across condition ratings 9 through 4

developed from these condition rating survival functions through operations on the transition probabilities estimated from the baseline survival functions. The derivation of this implementation is explained later in Section 4.3.

During model development, the best subset model is tested for multicollinearity between the covariates by calculation of the Variance Inflation Factor (VIF). As mentioned in Section 3.4.1, a VIF value of 10 or greater is generally suggested as indicative of the presence of collinearity. Across the timber deck models developed for all condition ratings and presented for illustration in this chapter, the maximum VIF observed was only 2.2. In subsequent model development performed in later chapters, none of the models in this study were found to exhibit any multicollinearity issues except for the concrete deck model. In the concrete deck model, multicollinearity was observed between only two variables that were removed from the final subset. The goodness-

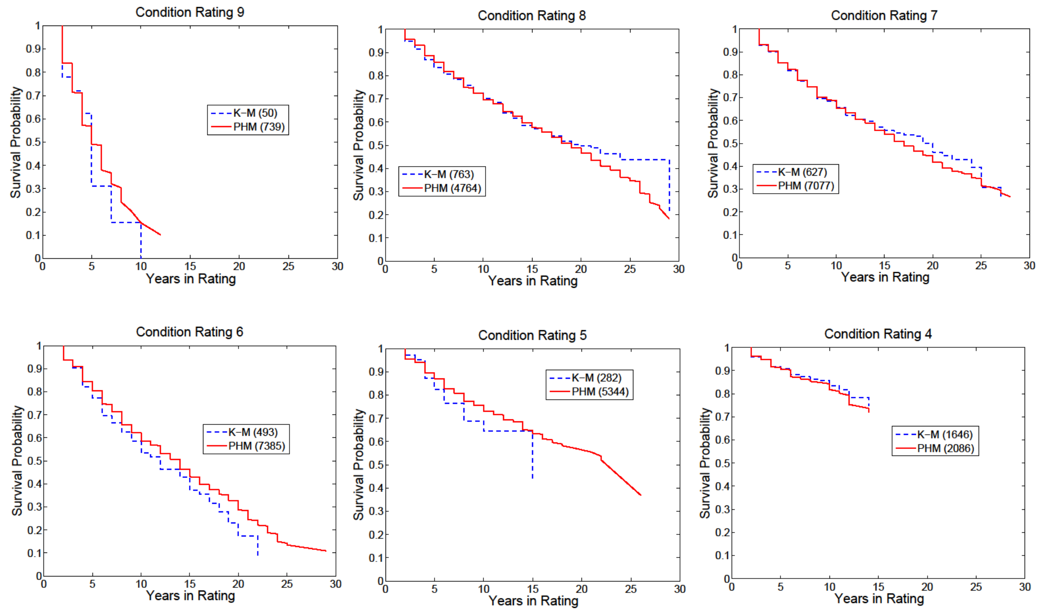

FIGURE 4.4: Timber deck survival functions: model vs empirical

of-fit of these models is also assessed by developing fully stratified analyses. Thus survival functions were constructed for specific subsets of timber deck bridges using the empirical Kaplan Meier product estimator and compared to the corresponding PHM survival functions. Similarity between the two is indicative of the robustness of the model and correctness of the proportional hazards assumption. Figure 4.4 shows such comparisons for timber decks for condition ratings 9 to 4. The number of records used in the analyses are indicated in parentheses against the type of model. Kaplan-Meier (K-M) functions are based on only the records associated with the particular subset of bridges. For example for condition rating 7, the K-M subset is comprised of 627 timber deck bridge records for original/rebuilt bridges of single span, less than 19 years old, and located in Mountain region. The corresponding PHM model on the other hand uses all 7077 records associated with condition rating 7, resulting in a more robust model. Similarly, the K-M model for condition rating 4 uses 1646 records associated with bridges in State System 2 and of age less than 19 years or 27 years and older. The corresponding PHM function for condition rating 4 is based on all 2086 records associated with this rating. The PHM best subset for rating 4 has only two design variables viz. StateSystem and Age2. This minimal stratification accounts for the relatively larger subset of bridges available for K-M modeling at condition rating 4. It can be observed that the PHM survival functions at all condition ratings are similar to the Kaplan-Meier empirical estimates developed for comparable subsets of the database, which demonstrates that the PHM functions can be used correctly for estimation of survival time in a particular condition rating.

4.3 Forward Prediction Using the Developed PHM Deterioration Model

The developed deterioration modeling framework presented in the prior section generated survival functions that characterize the state dependent impact of significant explanatory factors over the life cycle of the component. However, the use of survival functions in forward prediction of condition ratings over a planning horizon has not been adequately addressed in the literature, with the only proposed approaches suggesting a reduction to deterministic models through simple statistics obtained from the survival function (Agrawal et al., 2009, 2010). This section provides another fundamental contribution of the research effort through defining a probabilistic, yet easily implementable, approach for predicting condition ratings over a planning horizon using the Cox Proportional Hazards-based deterioration models detailed in the prior section. This implementation adopts a semi-Markovian approach, associated with calculation of transition probabilities, with proportional hazards effects, and is explained in detail in the following subsections.

4.3.1 Derivation of Transition Probabilities from Proportional Hazards Models

Transition probabilities associated with changes in condition ratings resulting from deterioration processes can be calculated based on survival functions developed across the individual condition states, as mentioned in Section 2.3.3. Transition probabilities calculated by such an approach have been determined to be more accurate than those based on linear regression or expert elicitation methods currently used in most BMS. The calculation of transition probabilities has been illustrated earlier for parametric duration models (Mishalani and Madanat, 2002). This section describes the derivation of non-stationary transition probabilities from PHM survival functions. In this derivation, the conventional assumption that a bridge component does not deteriorate by more than one state in any one inspection cycle has been adopted. This assumption has been routinely employed by past researchers and has been found to result in simpler and more robust models (Madanat et al., 1995, Sobanjo and Thompson, 2011). Moreover, deterioration is routinely assumed to occur without rehabilitation and hence the probability of an improvement in condition state is taken to be zero. An approach for incorporating the probability of condition state improvement due to rehabilitation is outlined in the final chapter of this dissertation as a suggested improved implementation of the developed framework.

For this derivation of the non-stationary transition probabilities, let $S_k(t, \vec{z})$ be the survival function of a material-specific GCR component associated with condition rating k for a bridge described by the vector of covariates $\vec{z}$. At any time t, the value of $S_k(t, \vec{z})$ is the cumulative probability that the structural component will remain in condition rating k up to time t. This probability is naturally 1 at $t = 0$ and decreases with each inspection cycle Δ, as illustrated in Figure 4.3. Therefore, the instantaneous probability that the structural component will remain at the same condition rating over the next annual reporting cycle at any time t (Mishalani and Madanat, 2002) is given by

$$P_{kk}(t, \vec{z}) = \frac{S_k(t + \Delta, \vec{z})}{S_k(t, \vec{z})} = \frac{S_k(t + 1, \vec{z})}{S_k(t, \vec{z})} \; for \; \Delta = 1 \; year \tag{4.1}$$

The step nature of the non-parametric survival function obtained from PHM regression complicates the implementation of the equation (4.1). Although each interval of survival time is associated with a unique survival probability, the instantaneous drop in the survival probability at the end of each interval to the lower step associated with the subsequent survival interval generates a discontinuity in the survival probabilities at that instant when one interval ends and the next one begins. Also, the survival interval represented by one step is not consistently equal to one inspection interval but sometimes spans across multiple or partial inspection intervals. Therefore, a special subroutine had to be designed to extract unique values of $S_k(t, \vec{z})$ for each inspection interval of one year. The maximum survival probability recorded at any year was assigned to that year. The problem of intervals spanning multiple years was resolved by linear interpolation of the survival probabilities associated with the years at the beginning and end of such an interval. The survival probabilities so obtained were then used to calculate the transition probabilities P_{kk} using equation (4.1). Since it is assumed that improvement in condition rating is not possible in the presence

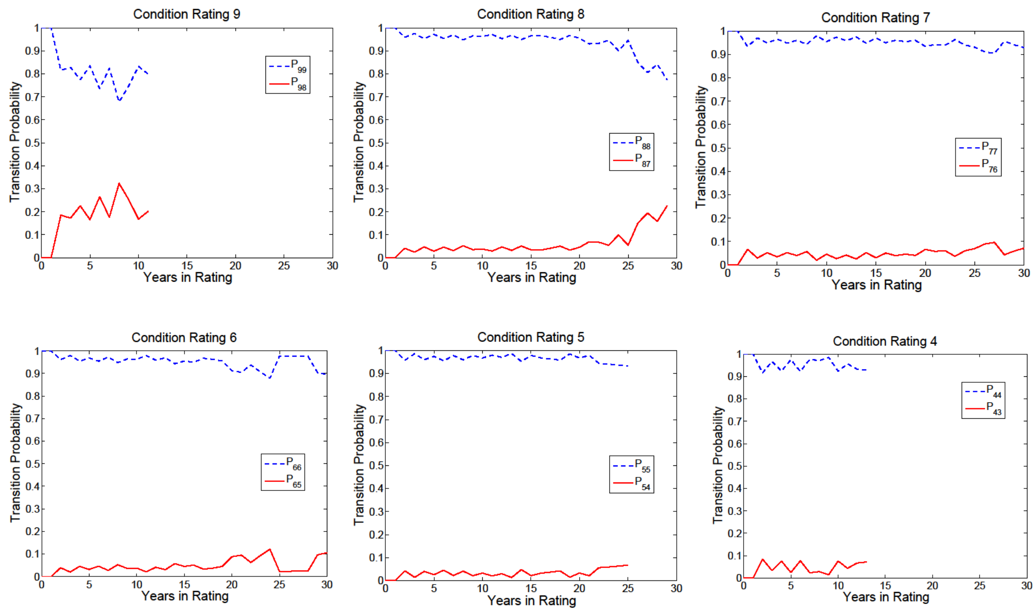

FIGURE 4.5: Baseline transition probabilities for timber deck across condition ratings 9 through 4

of deterioration without rehabilitation, the probability of deteriorating to condition rating $(k-1)$ at any time t is consequently given by

$$P_{k(k-1)}(t, \vec{z}) = 1 - P_{kk}(t, \vec{z}) \tag{4.2}$$

Figure 4.3 shows the previously presented baseline survival functions for the timber deck proportional hazards deterioration model over condition ratings 9 through 4. Transition probabilities calculated from these survival functions using the methodology described above are presented in Figure 4.5. It can be observed that the stay-the-same transition probabilities for each of the condition ratings 8 to 4 are nearly constant with the duration that the deck has been continuously rated in the respective condition rating, with some gradual decrease in the probability after approximately 15-20 years at that condition rating. Only in case of condition rating 9 is a mild decrease in the stay-the-same transition probability observed from the very beginning. It should be noted that the visible increase in stay-the-same transition probabilities in the odd numbered years at each condition rating reflects the biennial frequency of the typical inspection cycle.

The deterioration models developed in this study can account for the duration dependent changes in transition probabilities by developing separate transition probability matrices for each consecutive inspection cycle. However, since the transition probabilities are generally constant with the duration in the condition rating, simplified stationary transition probability matrices may be justified for computational simplicity in most practical applications. It is proposed that the mean value of the transition probabilities over the duration of the survival function be used for this purpose. The implementation of both stationary and non-stationary transition probability approaches for deterioration modeling are described in the next subsection and a comparative analysis is presented.

There is no reference in literature to the use of stationary transition probabilities based on survival analysis. However, use of stationary transition probability matrices in Markovian deterioration modeling have been described in Chapter 2. Stationary transition probabilities associated with staying in the same condition rating can be calculated based on the percentage prediction method (Jiang et al., 1988, Scherer and Glagola, 1994, Wang et al., 1994) or the expected value method (Butt et al., 1987, Jiang et al., 1988, Madanat et al., 1995) explained in section 2.2.1. Stationary transition probabilities for element level models implemented in Pontis are generally based on expert opinion surveys of bridge engineers on account of limited duration of element level data. Only recently have the above-mentioned data based methods been applied to element level data in a couple of state bridge inventories (Hearn, 2012, Sobanjo and Thompson, 2001, 2011). Stationary transition matrices developed in this way were not considered to account for the time dependent nature of deterioration. To provide for time dependence, bridges were divided into several age groups, each of which had its own stationary transition matrix. Non-homogeneous Markov chain methodology was then applied to obtain the deterioration models (Jiang et al., 1988). A major advantage of using survival analysis for the construction of probabilistic deterioration models is its ability to account for the duration dependent nature of deterioration. The calculation of non-stationary transition probabilities and development of non-stationary transition matrices has been illustrated for parametric Weibull survival models (Kallen and van Noortwijk, 2005, Mishalani and Madanat, 2002, Sobanjo, 2011), however, as mentioned earlier, this approach has not yet been implemented in practice.

At this point, it is important to note that the transition probabilities do not yet incorporate the influence of covariates included in the deterioration model on the hazard rate. Considering that the survival functions in Figure 4.3 are the baseline survival functions where all of the covariates, $\vec{z}$, take baseline values, the corresponding transition probabilities reflected in Figure 4.5 can be called the baseline transition probabilities. If z^0 denotes the set of baseline covariates, $S_k(t, z^0)$ can be defined as the baseline survival function and $P_{kk}(t, z^0)$ can be defined as the baseline stay-the-same transition probability for condition rating k. For any other set of covariates, denoted as z^1, associated with assignments in a multivariable model, the PHM allows calculation of a hazard ratio (refer to equation (3.2)),

$$HR_k = e^{\vec{\beta_k}(z^1 - z^0)} = e^{\beta_{k,1}(z_1^1 - z_1^0) + \beta_{k,2}(z_2^1 - z_2^0) + \ldots + \beta_{k,n}(z_n^1 - z_n^0)} \tag{4.3}$$

such that (refer to equation (3.5)),

$$S_k(t, z^1) = S_k(t, z^0)^{HR_k}. \tag{4.4}$$

Therefore, using equation (4.1),

$$P_{kk}(t, z^1) = \frac{S_k(t+\Delta, z^1)}{S_k(t, z^1)} = \left[\frac{S_k(t+\Delta, z^0)}{S_k(t, z^0)}\right]^{HR_k} = \left[P_{kk}(t, z^0)\right]^{HR_k}, and \tag{4.5}$$

$$P_{k(k-1)}(t, z^1) = 1 - \left[P_{kk}(t, z^0)\right]^{HR_k} = 1 - P_{kk}(t, z^1) \tag{4.6}$$

The basic output of a proportional hazards model is comprised of the baseline survival function and the PHM regression coefficients. Equations (4.5) and (4.6) illustrate the ease with which this basic output can be used to calculate the transition probabilities for any individual bridge component, or category of bridge components, associated with a specific set of covariate values, and thus develop the corresponding deterioration models. As reflected in the derivation above, the survival functions are not directly used beyond developing the baseline transition probabilities. Therefore, for practical implementation, the BMS needs only to store baseline transition probabilities for each model and the corresponding hazard ratios associated with the specific bridge covariate assignments over each condition rating. These are small matrices with nominal storage requirements and insignificant computational resources are necessary to carry out the subsequent construction of bridge-specific transition probability matrices.

4.3.2 Transition Probability Matrices

For matrix-based implementation of the probabilistic deterioration models, transition probability matrices introduced in Section 2.2.1, can be developed either as stationary or non-stationary matrices for each year of the planning horizon. Considering a planning horizon of N years, if the duration of each prediction cycle, Δ, is one year, the transition probability matrix for the n^{th} year, where $n = 1, 2, ..., N$, is developed using

$$P_n = \begin{bmatrix} P_{mm}(n,\vec{z}) & P_{m(m-1)}(n,\vec{z}) & 0 & 0 \ldots & 0 & 0 \\ 0 & P_{(m-1)(m-1)}(n,\vec{z}) & P_{(m-1)(m-2)}(n,\vec{z}) & 0 \ldots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \ldots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \ldots & P_{22}(n,\vec{z}) & P_{21}(n,\vec{z}) \\ 0 & 0 & 0 & 0 \ldots & 0 & P_{11}(n,\vec{z}) \end{bmatrix} \tag{4.7}$$

In the above matrix, m is the highest condition state and 1 is the lowest condition state. In developing practical transition probability matrices for implementation in the NCDOT BMS, it is important to recognize that very few bridges are permitted to deteriorate below condition rating 3. There are few records, if any, associated with condition ratings 1 and 2 associated with each material specific GCR and all of these are censored records. Even at condition rating 3, records available for analysis are very limited and nearly all are censored records. Due to this reason, the transition probabilities for condition ratings 1 and 2 were excluded from the transition probability matrix in developing the earliest Markovian bridge deterioration models for the Indiana BMS (Jiang et al., 1988). In development of the current models, it was observed that following this approach caused the deterioration models to unnaturally converge to the lowest condition rating, or condition rating 3, that was included in the transition probability matrix. To avoid this problem, and since insufficient historical condition rating data is available for lower condition ratings to develop meaningful transition probabilities through survival analysis, the stay-the-same transition probabilities for condition ratings 3 to 1 have been prescribed as 0.75 in subsequent models developed in this study. Likewise, NCDOT follows the NBI condition rating scale, where the highest condition rating is 9. Therefore, the baseline transition probability matrix for NCDOT bridge components for the first prediction cycle, which is associated with baseline survival functions and baseline covariate assignments, will be

$$P_1 = \begin{bmatrix} P_{99}(1,z^0) & P_{98}(1,z^0) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_{88}(1,z^0) & P_{87}(1,z^0) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_{77}(1,z^0) & P_{76}(1,z^0) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{66}(1,z^0) & P_{65}(1,z^0) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_{55}(1,z^0) & P_{54}(1,z^0) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & P_{44}(1,z^0) & P_{43}(1,z^0) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.75 & 0.25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.75 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.8}$$

Transition probability matrices can similarly be developed for each year in the planning horizon based on the values of the transition probabilities in that year. A simplifying assumption made in the development of non-stationary transition probability matrices is that the time spent in the initial condition state is zero (Sobanjo, 2011). The application of non-stationary transition probability matrices follows the semi-Markov approach as explained in Section 4.3.3. The current NCDOT database has 35 years of data, which allows for calculation of observed transition probabilities, based on equations (4.1) and (4.2), over the duration of the survival functions, which is less than or equal to the period of observation. Beyond this observed duration, the transition probability is assumed to remain constant for the remaining prediction period. In practical implementation, the planning horizon in long-term analyses does not typically exceed 10 or 20 years. However, longer horizons are used to describe the characteristics of the deterioration model over the full service life of the component. Function [1] of Algorithm 3 presents the calculation of the baseline transition probabilities and construction of transition probability matrices as explained above. These matrices can then be used to predict the future condition of a bridge component if its vector of descriptive covariate assignments and present condition is known.

Algorithm 3 Construction of Transition Probability Matrices and Condition State Prediction over the Planning Horizon

Input: Baseline Survival Functions, z^1, β, HR, N, Initial Condition Rating,R
Output: Baseline Transition Probabilities, Transition Probability Matrices, PHM Deterioration Models, E

function [1] BASELINE TRANSITION PROBABILITIES & MATRICES(Baseline Survival Function)
    for Condition Ratings 4:9 **do**
        Transition Probability of staying at the same rating $\leftarrow P_{kk}$ ▷ Equation 4.1
        Transition Probability of deteriorating to lower rating $\leftarrow P_{k(k-1)} = 1 - P_{kk}$
        return Baseline Transition Probabilities
    end for
    Assemble Transition Probability Matrices $\leftarrow P_n$ ▷ Equation 4.7
    return Baseline Transition Probability Matrices
end function

function [2] FORWARD PREDICTION OF EXPECTED CONDITION STATE OVER PLANNING HORIZON(z^1, β, Transition Probability Matrices, N, Initial Rating, R)
    Baseline Deterioration Model ▷ Equations 4.9 to 4.13
    for other than baseline covariate values: **for** Condition Ratings 4:9 **do**
        Calculate HR ▷ Equations 4.3
    end for
    Adjusted Transition Probability Matrices $\leftarrow P_n$ & HR
    for $n = 1 : N$ **do**
        $Z_n \leftarrow Z_{(n-1)} \cdot P_n$
        $E_n \leftarrow Z_n \cdot R$
        return E_N and Adjusted Deterioration Model
    end for
end function

4.3.3 Probabilistic Prediction of Condition over the Planning Horizon

Using the conventional Markov chain approach, the condition state of a bridge component is represented in the form of a vector for the purpose of developing a probabilistic predictive deterioration model. For example, the initial state vector Z_0 for a newly constructed NCDOT timber deck would be represented by [1 0 0 0 0 0 0 0 0], which indicates the probability of the deck being rated at condition rating 9 being 1 and the probability of the deck being rated at any other condition rating being 0. The predicted state vector at the end of the first year, Z_1, and for consecutive years can be obtained as follows:

$$Z_1 = Z_0 \cdot P_1 \quad \text{after } 1^{st} \text{ year} \tag{4.9}$$

$$Z_2 = Z_1 \cdot P_2 \quad \text{after } 2^{nd} \text{ year} \tag{4.10}$$

$$Z_3 = Z_2 \cdot P_3 \quad \text{after } 3^{rd} \text{ year} \tag{4.11}$$

or recursively for every year until the n^{th} year prediction as

$$Z_n = Z_{n-1} \cdot P_n \quad \text{after } n^{th} \text{ year} \tag{4.12}$$

If stationary transition probability matrices, P, are adopted, the calculation of the predicted state vector does not need to be calculated recursively but can be obtained as

$$Z_n = Z_0 \cdot (P)^n \tag{4.13}$$

The state vector at any year of the planning horizon represents the probability of the bridge component being at each individual condition rating in that year. Using this state vector, it is possible to plot the probability associated with individual condition ratings over the planning horizon. Figure 4.6 shows condition rating probabilities for the timber deck model described throughout this chapter over a prediction period of 120 years, assuming an initial condition rating of 9 and baseline covariate assignments. These condition rating probabilities are based on the non-stationary transition probabilities using the semi-Markov formulation. Figure 4.7 shows the same condition rating probabilities calculated using the stationary transition probability matrix in a Markov Decision Process.

It is observed that the distributions obtained from the stationary transition probability matrix in general have thick tails whereas the distributions obtained from the non-stationary semi-Markov approach are more concentrated around the peak. This is indicative of the relatively smaller uncertainty associated with condition rating duration distributions obtained using the non-stationary approach. This is especially noticeable in case of condition ratings 8 and 7 for which the distributions from the non-stationary approach have almost no tail. The mean durations or expected transition times for the respective condition ratings are accordingly shorter. It is to be noted that this mean transition time is not the same as the time for the expected condition state to be at that rating (Kallen and van Noortwijk, 2005). In either the case of stationary or non-stationary transition probability matrices, if the condition ratings are represented in a column vector $R = [9\ 8\ 7\ 6\ 5\ 4\ 3\ 2\ 1]^T$, the expected condition state at the n^{th} year can be obtained using

$$E = Z_n \cdot R \tag{4.14}$$

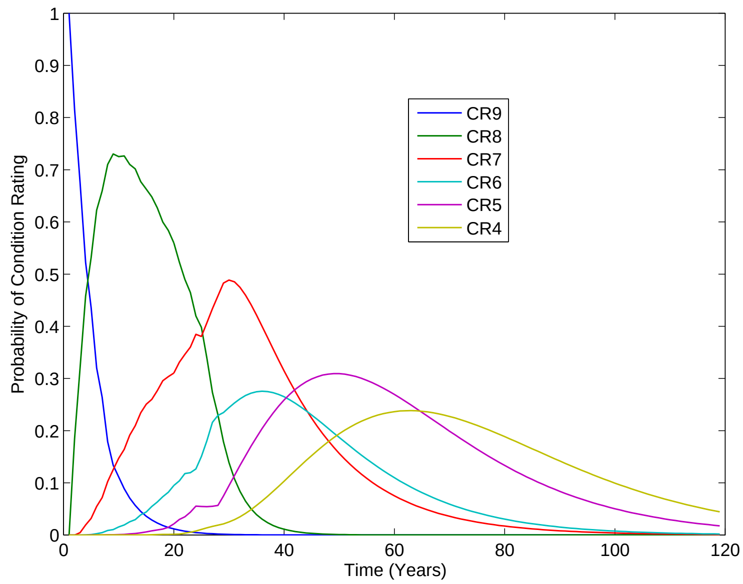

FIGURE 4.6: Condition rating probabilities for timber deck from semi-Markov process

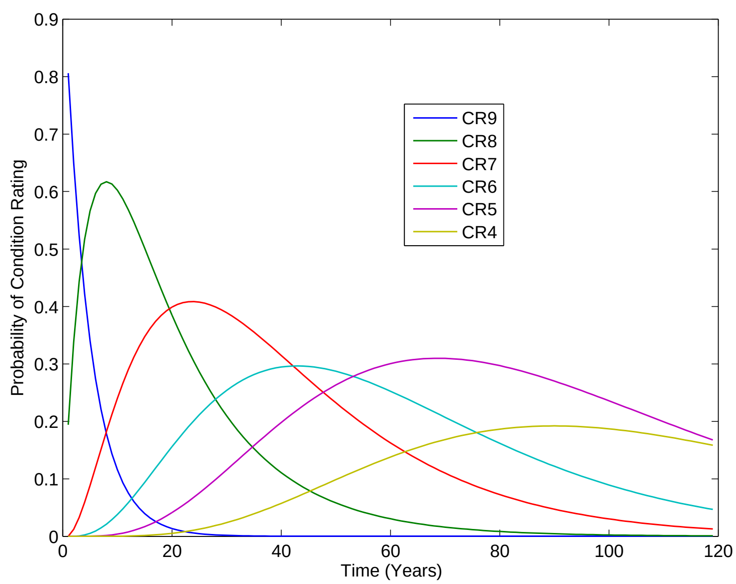

FIGURE 4.7: Condition rating probabilities for timber deck from stationary Markov process

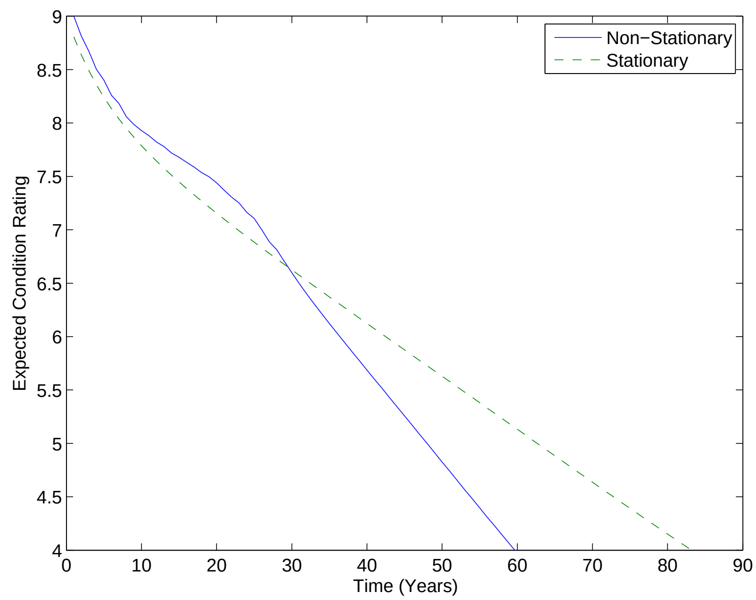

FIGURE 4.8: Expected values of condition rating for timber deck determined using stationary and non-stationary transition probabilities

The predicted expected condition ratings obtained using equation (4.14) can be used in the BMS multi-year optimization iterations, to determine the repair, maintenance, and replacement decisions required at every stage over the planning horizon.

Figure 4.8 shows the expected condition states over the prediction period calculated using the non-stationary and stationary transition matrices. It is observed that the deterioration rates obtained from both approaches are in close agreement over the first 20-30 years of the prediction period. Subsequently, the deterioration rate from the non-stationary matrices is faster and converges to a condition rating of 4 in 60 years. The deterioration rate from the stationary matrices, however, does not converge to the rating 4 even after 80 years, which appears unrealistic, especially for timber decks. This observation is consistent with similar observations of unreasonably high median transition times obtained for prestressed concrete superstructure elements using stationary transition matrices for the bridge inventories in Florida and Colorado,

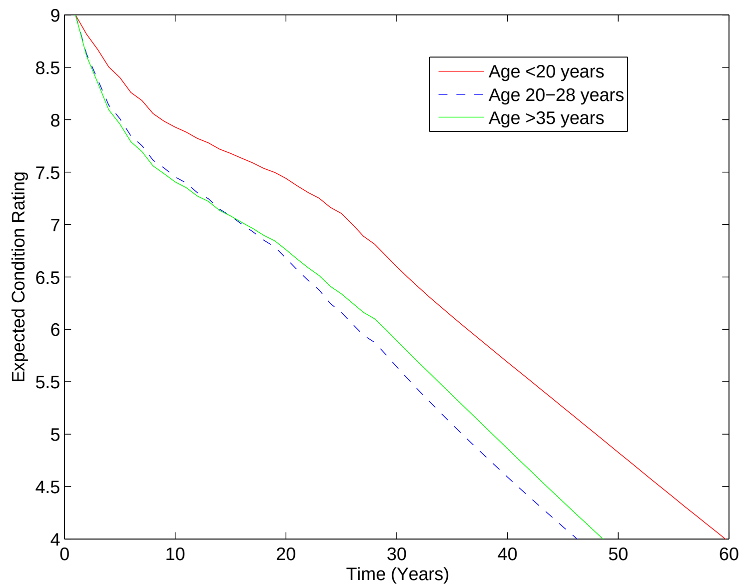

FIGURE 4.9: Expected condition rating prediction of timber deck bridges in different age groups

as mentioned in section 2.2.2 (Hearn, 2012, Sobanjo and Thompson, 2011), and also the observed flattening of the deterioration curve obtained from first level Markov process for the NYSDOT bridge inventory, as mentioned in Section 2.2.1 (Agrawal et al., 2009, 2010). From the above account, it can be justifiably concluded that the use of non-stationary matrices is advisable for longer planning periods to correctly model deterioration behavior.

As discussed earlier, deterioration models can be developed for any category of bridges by scaling the transition probabilities with the respective hazard ratios, as indicated in equations (4.5) and (4.6). Deterioration models for timber decks of different age groups developed in this way are shown in Figure 4.9. This presentation is based on statistically identified significant factors affecting timber deck deterioration, in contrast to the judgment-based *a priori* grouping used in present day BMS. This not only constitutes a breakthrough in modeling infrastructure deterioration,

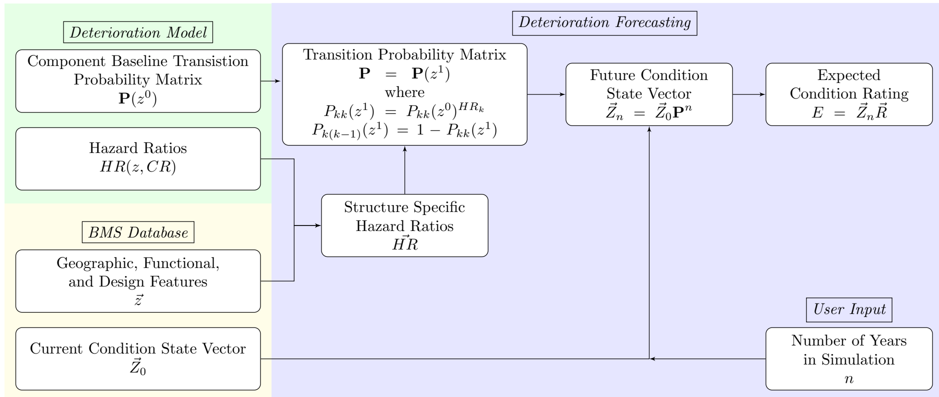

FIGURE 4.10: Proportional hazards model prediction flowchart

but will contribute immensely to understanding the effect of various functional and design features, as well as potentially preservation strategies, on bridge deterioration rates. The final Function [2] of Algorithm 3 represents the subroutines for calculation of hazard ratios for any set of covariates and the development of the final deterioration models for any desired prediction period. The proportional hazards model flowchart shown in Figure 4.10 summarizes the deterioration forecasting process described above. The forecasting period is a user controlled input enabling the program to perform the calculations for bridge-specific transition probability matrices, state vectors, and expected condition ratings over the specified prediction period.

CHAPTER 5: IMPLEMENTATION OF DEVELOPED FRAMEWORK

A Windows-based standalone graphical user interface (GUI) has been developed in the MATLAB software environment for implementation of the deterioration modeling framework described in the previous chapter. The complex software routines developed for deterioration modeling form the basis of the GUI, which translates and abstracts the software code to a series of guided and interactive interfaces. The result is a very accessible and user friendly program suitable for routine use by transportation personnel working with the BMS. The GUI, titled the Bridge Management System - Deterioration Modeling Program (BMS-DMP) is used to produce the results presented in subsequent chapters of this dissertation and can be used in the future to update both deterministic and proportional hazards probabilistic models as additional condition rating data is added to the BMS database each year. The layout and functionalities of the BMS-DMP are presented in the following sections. Using the MATLAB Compiler, the GUI has been developed into a standalone executable that can be installed and operated on workstations without MATLAB installations.

5.1 General Functionality

The GUI design environment (GUIDE) in MATLAB allows construction of a customized user interface with multiple windows that can be opened successively from one main window. Each window can be individually designed to have various types of controls including drop-down menus and buttons as well as to dynamically display output in form of text or images. The Main window of the BMS-DMP is shown in Figure 5.1.

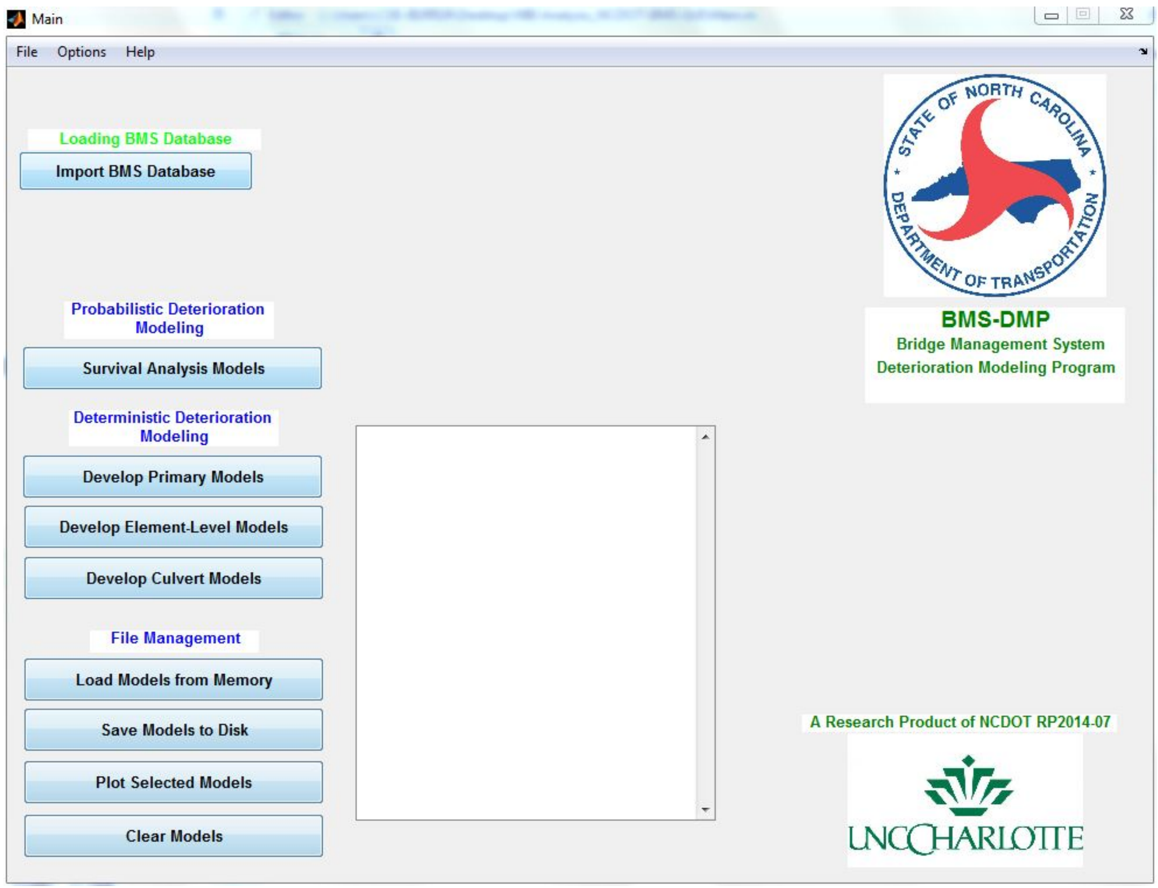

FIGURE 5.1: BMS-DMP Main window

The Main window has three main functionalities: Importing BMS Data, calling Deterioration Modeling subroutines, and File Management. Respective buttons on the front panel activate callback functions associated with each button that execute code in the compiled MATLAB subroutines. For example, the 'Import BMS Database' button executes the function to load the Master Database to be used for developing the deterioration models. In this case, it is also programmed to display the message 'Loading BMS Database', also captured in Figure 5.1, while the function is being

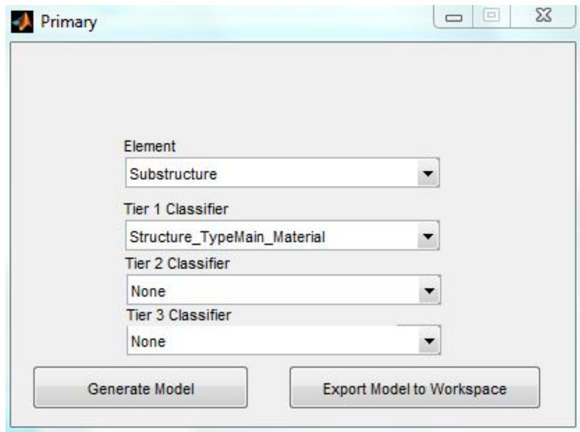

FIGURE 5.2: Primary window: component selection and classification

executed. This is especially helpful when working with large databases and matrices with relatively long file processing times.

5.1.1 Deterioration Modeling Functionality

The deterioration modeling functionality provided by the BMS-DMP allows for implementation of either the survival analysis-based probabilistic modeling techniques developed in this study or the deterministic modeling techniques developed originally by Chen and Johnston (1987) and presently used within the NCDOT BMS. The deterministic deterioration modeling functions can be applied to either primary component, also known as the general condition rating (GCR) data, element-level condition ratings, or culvert condition ratings. The extension of the deterministic deterioration modeling technique to element-level and culvert condition rating data is a new contribution provided by this software. Due to the limited duration of element-level condition rating data available, it is recommended that survival analysis techniques should not be applied yet for the development of element-level deterioration models in North Carolina.

The Primary deterioration models refer to those for those bridge components receiving general condition ratings (GCR): deck, superstructure and substructure. Figure 5.2 shows the window that opens on clicking ‘Primary’ which allows the user to choose the GCR component and classification hierarchy used to build the deterministic models through four drop-down menus. The first menu is for selecting the GCR bridge component and each of the remaining three menus contain the list of available classifiers, including material, design, functional, and geographical features, that are available in the bridge database and are considered to potentially influence the deterioration rate of bridge components. Since the number of bridge classification tiers used in the current North Carolina deterioration models vary by component, the developed interface allows the user to specify between one and three tiers for pre-classifying bridges prior to deterministic regression. Specific to the selected bridge classification tiers, the program generates three types of output: a cascading classification tree plot showing the number of bridge data records available for analysis within each tier, a table for each of the lowest tier categories showing the number of censored and uncensored records available for analysis over each condition rating with the calculated deterministic duration estimate, and a plot of the deterministic deterioration model for each of the lowest tier categories in the specific bridge classification tree. Figure 5.3 shows a typical output for a single tier of the deterministic deterioration modeling routine. When running the program with the specified bridge classification displayed in Figure 5.2, similar output is provided for the remaining categories of the selected Tier 1 Classifier: Structure Type Main - Material (i.e. concrete, steel and prestressed concrete).

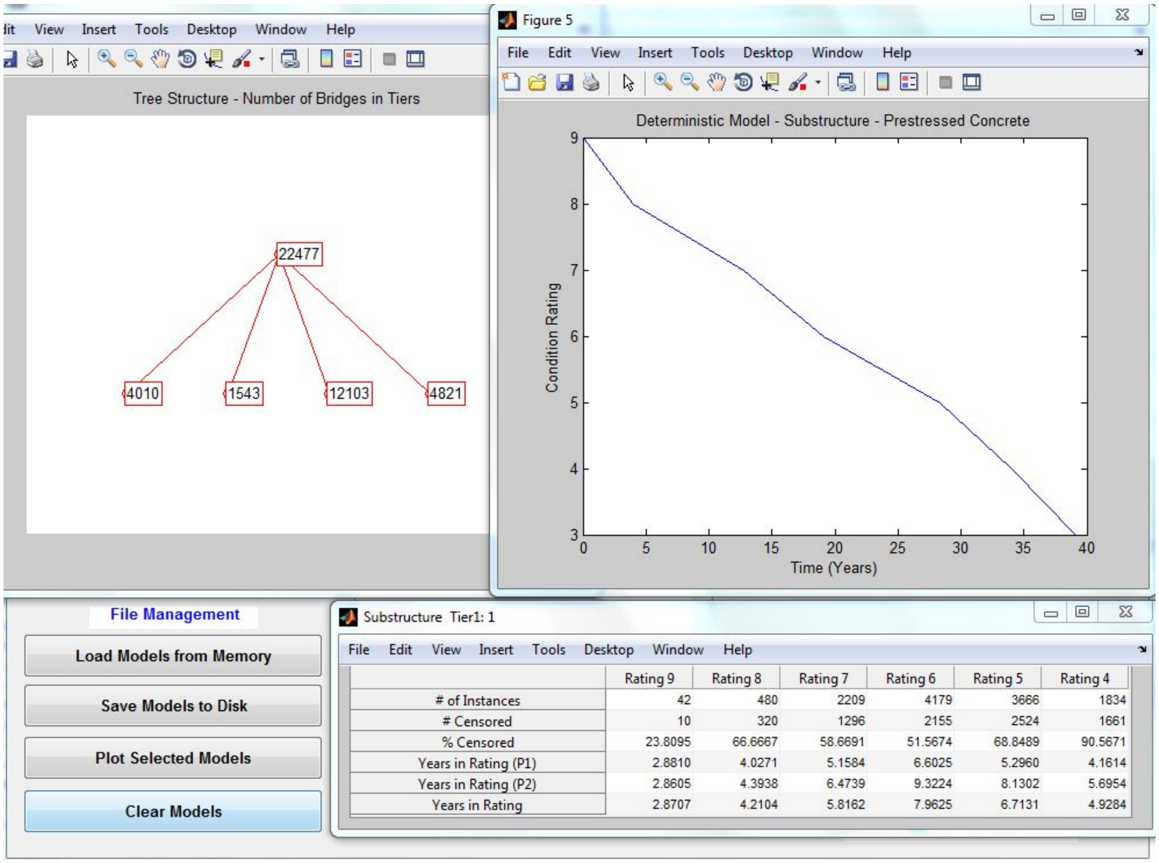

FIGURE 5.3: Representative output from deterministic deterioration modeling functions

Development of element-level and culvert deterministic deterioration models proceeds analogous to the primary components, with the exception of the element and bridge classification tier selection options that are unique to each general category of model. Following generation of any deterministic deterioration model, the regression models and analysis statistics can be exported to the Main window by clicking the 'Export Model to Workspace' button in the respective classification window, as seen in Figure 5.2.

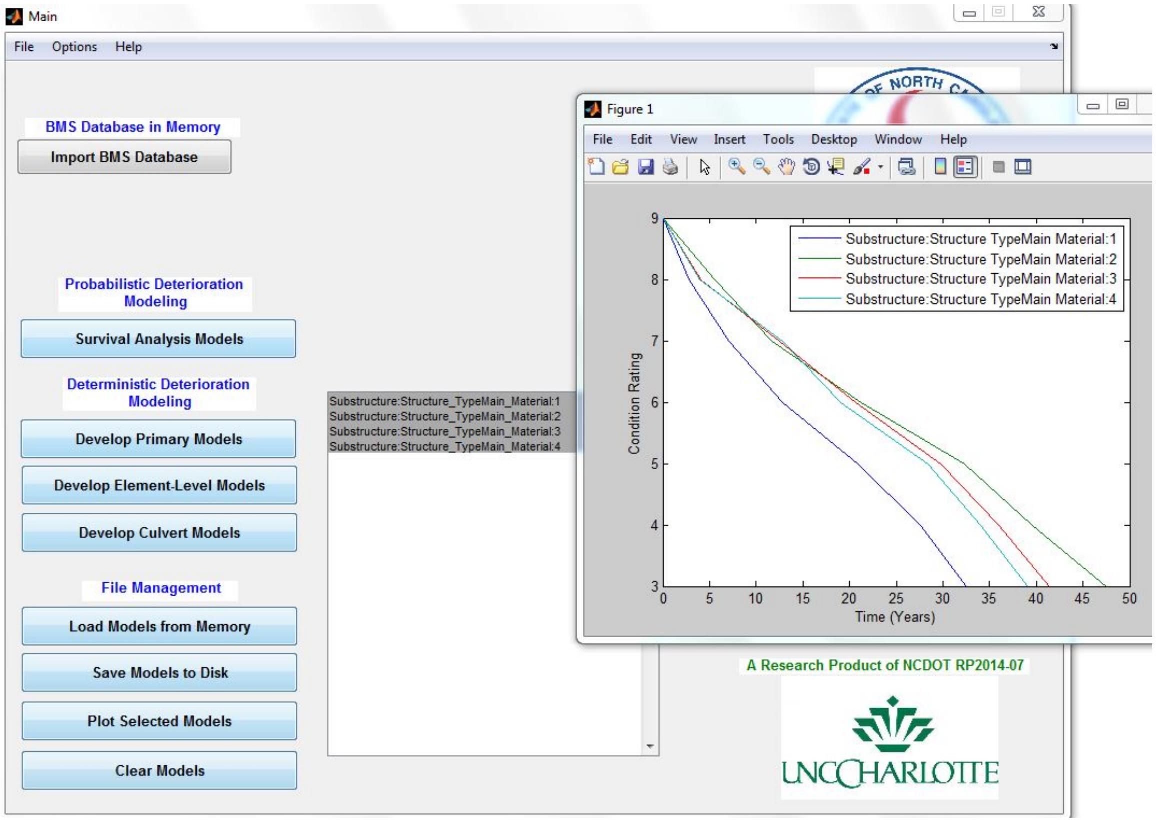

FIGURE 5.4: File management and model comparison

5.1.2 File Management Functionality

The File Management functionality is presented to the user through four push-buttons provided on the Main window. The topmost button permits for the importing of models that may be saved locally to the user's drive during prior usage of the GUI. All models loaded into the workspace memory, either from the local drive or developed in the current session, are listed in the list box to the right of the buttons. The remaining buttons execute functions on the selected models in the workspace. Developed models, exported to the workspace after generation as mentioned earlier, can be saved on the computer using the 'Save Models to Disk' button. The 'Plot Selected Models' button is used to develop comparative plots of selected models from

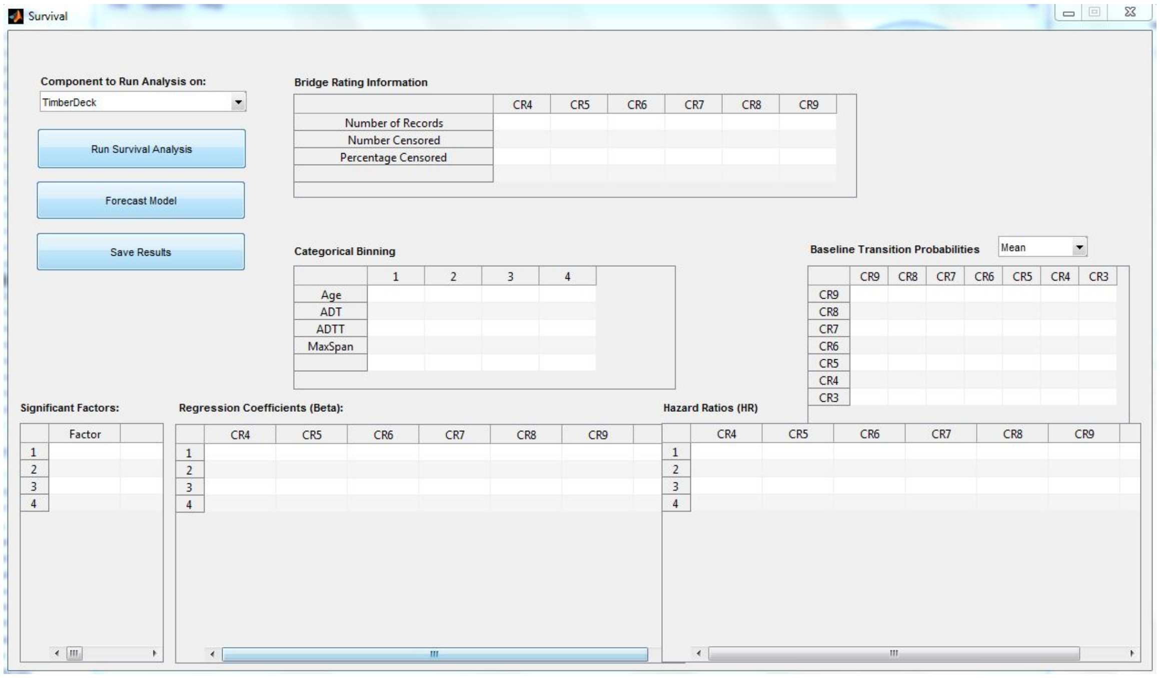

FIGURE 5.5: BMS-DMP Survival window

the workspace. For example, Figure 5.4 shows a subwindow where deterministic substructure deterioration models developed using material type as the sole classifier were exported to the workspace and are comparatively plotted using this function. This functionality is very useful for comparing models and visually assessing the impact of various classifiers on the relative deterioration rate.

5.2 Survival Analysis Functionality

While the developed software permits for development of either classical deterministic deterioration models or probabilistic deterioration models based on the developed proportional hazards approach, the emphasis of this GUI development is placed on the latter approach. Figure 5.5 shows the Survival window that opens on clicking the button ‘Survival Analysis Models’ in the Main window. The Survival window has a component drop down menu at the top left. This menu lists all of the bridge compo-

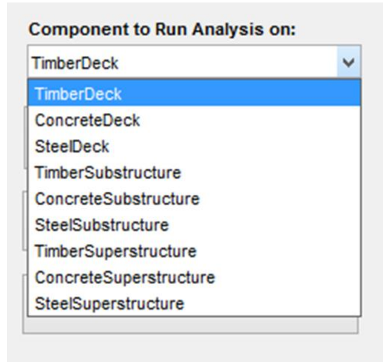

FIGURE 5.6: Survival window: material component dropdown menu

nents of different material types for which survival-based deterioration models can be developed. A screen capture of the component drop down menu is shown in Figure 5.6. In developing this list, the assumption was made that material type would always be used to classify bridges prior to development of the deterioration model. This decision was made in the interest of practical implementation and does not represent an inherent limitation of the developed framework for proportional hazards deterioration modeling. The Survival window has been equipped with three buttons: Run Survival Analysis, Forecast Model and Save Results, each supporting an important capability.

5.2.1 Survival Analysis

The 'Run Survival Analysis' button can be clicked after selecting the material component from the drop down menu. It activates an automated process for developing the baseline deterioration models for that component. This process has been described in detail in Chapter 4. The very first code that is executed after the button-press is for extracting the material component data from the master database previously imported into the program through the Main window (Algorithm 1, procedure [2]).

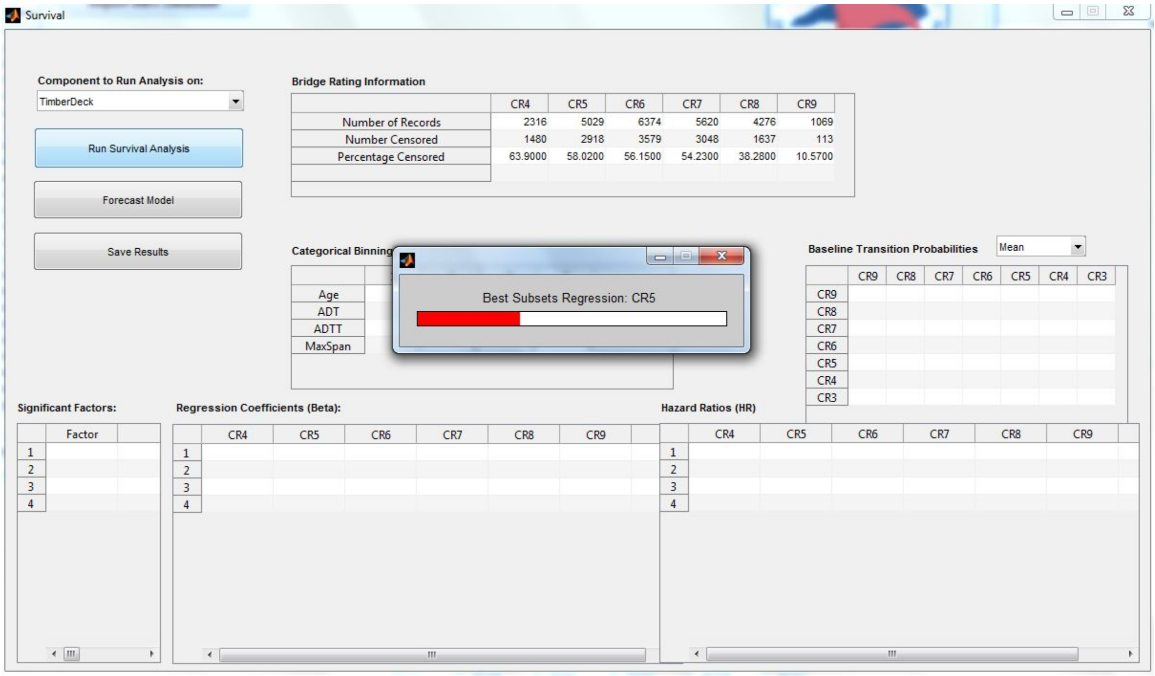

FIGURE 5.7: BMS-DMP performing survival analysis

This is followed by implementation of the PHM master code, represented by Algorithm 2. The various functions of the master code are executed automatically and sequentially through the generation of the final deterioration models. The Survival window is designed with a number of tables that are dynamically programmed to progressively populate the display with critical results from the functions performed to allow for user assessment of data richness, categorical statistics, and regression results. First, the ‘Bridge Rating Information’ table displays the overview of simple statistics related to the richness of the underlying condition rating data used in the survival analysis. This table presents the number and percentage of censored and uncensored bridge records after completion of Function [1], ‘PHM Input Data’, of Algorithm 2. The ‘Categorical Binning’ table subsequently shows the mean categorical bounds calculated over the continuous scale variables: Age, ADT, ADTT, and

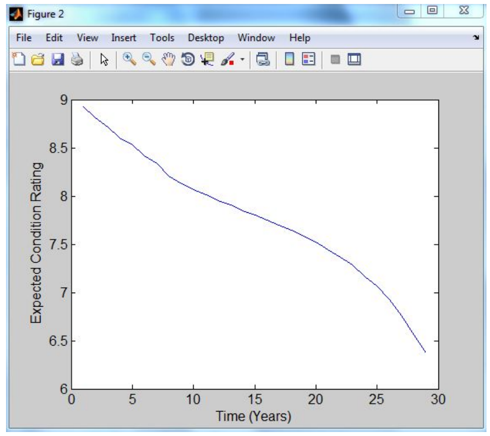

FIGURE 5.8: Baseline expected value deterioration model

MaxSpan. This table reflects the intervals used for reference cell coding developed by Function [2]. The three tables at the bottom of the survival analysis window are programmed to display the significant factors identified by best subset regression, and their respective coefficients and hazard ratios. These tables are populated after the functions for preliminary multivariable model development (Function [3]), best subset selection (Function [4]), and calculation of final PHM coefficients (Function [5]) are executed. These display tables are meant to keep the user informed about essential model attributes during the implementation process. Additionally, to keep track of the progress of the program, individual 'waitbar' displays have been coded in for the more time-consuming functions. Figure 5.7 shows an intermediate screen capture of the Survival window with such a waitbar indicating that the best subset regression (Function [4]) has been completed for condition ratings 4 and 5 and is being carried out currently for rating 6. Following completion of the proportional hazards deterioration modeling, the baseline transition probability matrix is obtained through Function [6] of Algorithm 2 and presented in the remaining table on the survival anal-

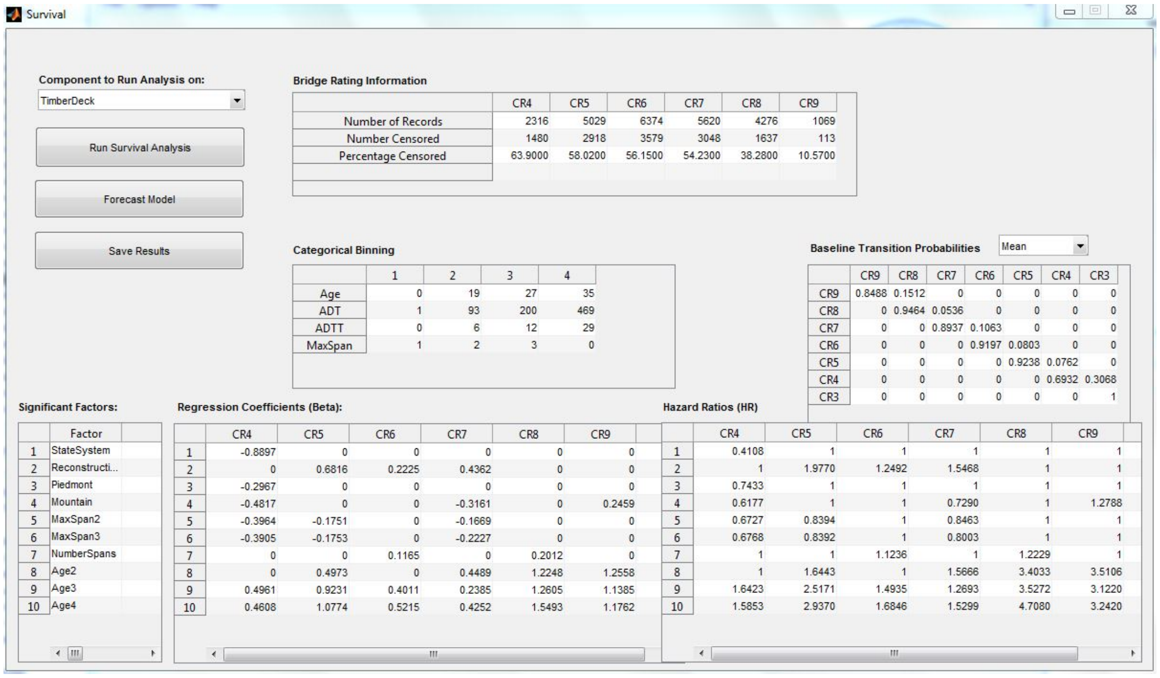

FIGURE 5.9: BMS-DMP: survival analysis completed

ysis window. In addition to these tables, plots of baseline survival functions for each condition rating and transition probability plots are also displayed in individual figures and saved automatically to the local drive. Furthermore, the baseline expected value deterioration model (assuming an initial condition rating of 9), is presented as shown in Figure 5.8. A screen capture of the Survival window after completion of the process initialized by the 'Run Survival Analysis' button is shown in Figure 5.9.

5.2.2 Model Forecasting

The hazard ratios and the baseline transition probabilities can be used to develop deterioration models for any specific bridge or category of bridges as explained in Section 4.3.3 and summarized in Figure 4.10. The Model Forecasting window is envisaged as providing the interface for ease in development of these predictive models in accordance with user specifications. The development of the BMS-DMP GUI is an ongoing process and will be tailored and improved further in accordance with feedback received from its primary users at NCDOT.

CHAPTER 6: APPLICATION TO THE NORTH CAROLINA STATE BRIDGE INVENTORY

The framework for deterioration modeling developed in this study and its implementation was described in Chapters 3, 4, and 5 with the help of the general condition rating (GCR) dataset associated with timber decks in the North Carolina state bridge inventory. This chapter presents selected results obtained from implementing the framework on the remaining GCR components in the NCDOT inventory. The results for material-specific deck, superstructure, and substructure GCR components are organized in separate sections to enable comparative assessments of time-dependent behaviors and effects of significant explanatory factors. Some results for timber decks already presented in previous chapters are reproduced here for uniformity and to facilitate these comparisons. Applied contributions developed through application of the developed methodology are compared to expected responses and observations from prior studies to assess the plausibility of the results and develop conclusions on the factors that most significantly influence deterioration rates of different materials and components.

6.1 Bridge Deck Deterioration Models

Although proportional hazards regression facilitates the data-driven identification of significant bridge features that influence the deterioration rates of specific bridge components, *a priori* preclassification of bridge components by material type was performed prior to all statistical regression. The rationale for this decision is based on an engineering mechanics-based reasoning that deterioration of different materials is naturally driven by different mechanisms and occurs by different processes. Furthermore, this material-based preclassification is consistent with current NCDOT deterioration models and the independent development of material-specific models produces a basis from which the plausibility and consistency of external factor coefficients can be assessed. In the following analysis, bridge decks are classified by primary construction material, which is coded in the Deck Structure Type field in the NBI database. In this coding system, all timber deck structures are categorized under the same code (8). Concrete bridge decks in the state are comprised primarily of decks built with cast-in-place concrete, coded as 1, and a much smaller percentage constructed of precast panels, which are coded as 2. Both of these categories are included within the general concrete deck models developed and analyzed in this study. Similarly, the dataset used for construction of steel deck models includes Deck Structure Types 3 to 6, which represent steel decks with open grating, closed grating, steel plate, and corrugated steel, respectively. This grouping is justified on account of the limited number of bridges in many of the categories associated with this material. The same grouping and an assumption that deterioration rates are more significantly affected by material type itself than deck design type was adopted by previous researchers working on the NCDOT bridge inventory (Chen and Johnston, 1987, Duncan and Johnston, 2002).

Deterministic deterioration models for decks, superstructures, and substructures currently being used in the NCDOT BMS were first developed in 2002 (Duncan and

Johnston) based on the average years in rating spent in each condition rating. The first task implemented in the current study was to develop MATLAB software routines in order to update these models for the twelve years of additional data now available in the NCDOT database. In developing these routines, the procedures originally established by prior NCDOT researchers (Chen and Johnston, 1987, Duncan and Johnston, 2002) were followed as closely as possible to ensure consistent updating of the deterministic deterioration models. Deterministic models are presented here in order to provide a comparative assessment of the survival analysis-based models developed in this study and presented in the subsequent subsections. The updated deterministic models for timber decks, concrete decks, and steel decks, are presented in Figures 6.1, 6.2, and 6.3, respectively. In these figures, the deterioration models were developed after further classification of the bridge records by the ADT ranges currently used in NCDOT deterministic deterioration models, which are presented in Table 6.1. These deterministic models, in particular those for timber deck and concrete decks, illustrate the challenge of *a priori* classification, as the use of ADT as a preclassifier provides insignificant differences between the models for each deck material.

TABLE 6.1: Tier 2 ADT classification of deterministic models under each deck material type

Tier 2	**ADT Range**
1	$0 - 200$
2	$200 - 800$
3	$800 - 2000$
4	$2000 - 4000$
5	> 4000

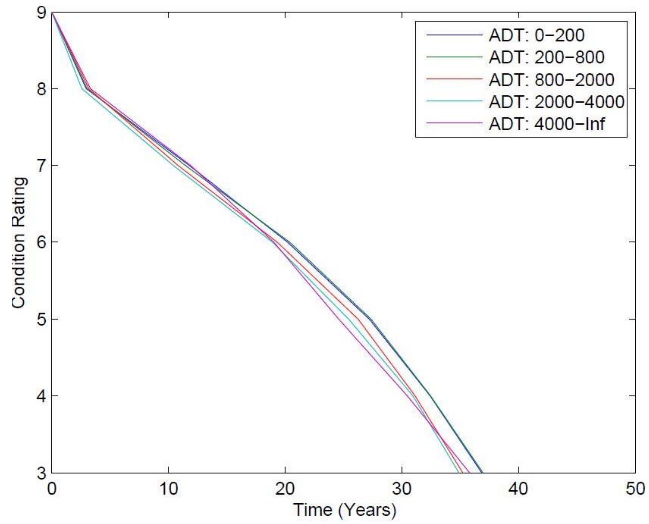

FIGURE 6.1: Timber deck deterministic deterioration models

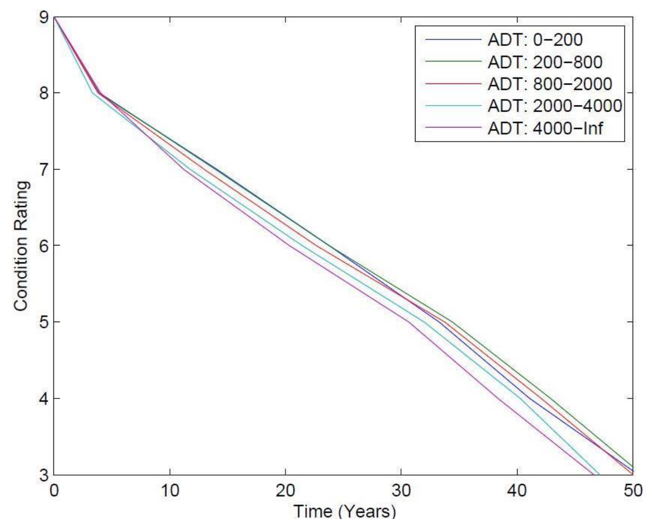

FIGURE 6.2: Concrete deck deterministic deterioration models

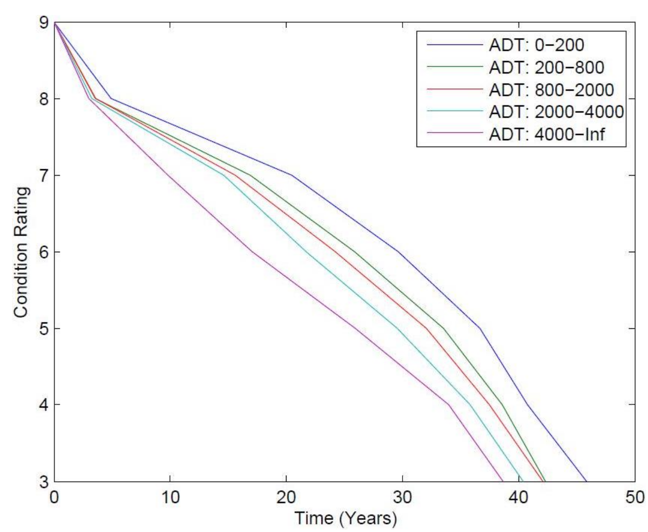

FIGURE 6.3: Steel deck deterministic deterioration models

6.1.1 Data Overview

The sample size of available uncensored data is an important consideration in development of duration-based models. Although the superiority of duration models has been recognized for more than a decade, the availability of records of longer continuous inspection duration have only recently made their implementation feasible because of corresponding increase in the relative quantity of uncensored data. In early illustrative studies performed on concrete deck datasets, a censoring percentage exceeding 70% was observed at all condition ratings (Mauch and Madanat, 2001, Mishalani and Madanat, 2002). Censoring information from recent studies is not available for GCR data, however Weibull-based models that accounted for censoring were found to be more accurate than Markov models in the 2009 NYSDOT study (Agrawal et al., 2010).

To facilitate the development of probabilistic deterioration models using Cox proportional hazards regression, observed historical records for continuous durations spent at individual condition ratings and corresponding censoring information were extracted for each deck category. Tables 6.2, 6.3, and 6.4 present summaries of the distribution and characteristics of this data for timber, concrete, and steel decks, respectively. From these summaries, it can be observed that observations at condition ratings 5 to 8 contribute the greatest percentage of records to each dataset. The percentage of censored records for condition ratings 6, 7, and 8 is generally in the range of 50-65%. Condition rating 5 has a relatively higher percentage of censored records with a maximum censored percentage of 86.6% associated with condition rating data from concrete decks. The reason for this increase in percentage of censoring with lower condition ratings can largely be attributed to the increased priority given to MR&R activities at this state of deterioration, although it is plausible that a greater fluctuation in inspection rating assignments occurs due to an increase in subjectivity at this advanced stage of deterioration, which would contribute to the increase in observed censoring. Records associated with condition rating 9 are relatively limited and are associated with a wide variation in the percentages of censored observations found across the different material types: 10.1% for timber decks, 87.7% for concrete decks, and 67.7% for steel decks. Since censoring due to an increase in condition rating is not possible at condition rating 9, it is most probable that the higher percentage of censored observations present for condition rating 9 is related to a larger percentage of records censored at the beginning or end of the recording period. The lower percentage for timber decks is indicative of the relatively lower construction rates of timber decks in recent years leading to higher ages and, consequently, less likelihood of being in new condition at the end of the recording period. This reasoning is supported by statistical analysis of age distributions for different deck material types, discussed later in this subsection. Observations at condition rating 4 constitute a sufficient number of records for statistical analysis, however it should be noted that over 90% of these records are censored for all deck materials. The impact of censoring is later reflected in higher survival probabilities due to the consequently lower percentages of observed failures or transitions to lower condition ratings associated with uncensored records at the same duration. This can be observed in the survival functions for condition rating 4 in the deck deterioration analysis, presented in a

TABLE 6.2: Timber deck condition rating data overview

Deck Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	745	670	75	10.07
8	4781	3089	1692	35.39
7	7372	2623	4749	64.42
6	7681	2854	4827	62.84
5	5498	1259	4239	77.10
4	2126	169	1957	92.05
3	313	2	311	99.36

TABLE 6.3: Concrete deck condition rating data overview

Deck Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	1419	175	1244	87.67
8	5477	2438	3039	55.49
7	10544	3376	7168	67.98
6	9541	4039	5502	57.67
5	6555	879	5676	86.59
4	1452	94	1358	93.53
3	201	2	199	99.00

TABLE 6.4: Steel deck condition rating data overview

Deck Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	68	22	46	67.65
8	871	437	434	49.83
7	1129	499	630	55.80
6	805	378	427	53.04
5	556	99	457	82.19
4	146	10	136	93.15
3	13	1	12	92.31

later subsection. There are almost no uncensored observations recorded for condition rating 3 in data associated with any of the deck materials, and therefore data from this condition rating does not lend itself to survival analysis. Moreover, the total number of observations available for steel decks at condition rating 3, and for all deck material types for condition ratings lower than 3, is insufficient not only for survival analysis but for any meaningful statistical analysis. Therefore, observations recorded for condition ratings 3 and below are necessarily excluded from the current analyses.

Descriptive variables associated with condition rating data that are recorded on a continuous scale include Average Daily Traffic (ADT), Average Daily Truck Traffic (ADTT), age, and maximum span length. These are divided into categories of approximately equal frequency based on weighted averages computed across bridge records with observed condition ratings 4 to 9, as described in Chapter 4. The individual categories are designated by design variables, which are coded with reference to a baseline category. Table 6.5 shows the lower bounds determined for categorical design variables determined from weighted averaging across all condition ratings. The reference,

TABLE 6.5: Lower bounds of intervals developed for categorical design variables

Deck Type	**Timber**			**Concrete**			**Steel**		
Category	2	3	4	2	3	4	2	3	4
ADT	94	204	468	878	3184	9090	324	669	1677
ADTT	6	13	29	57	226	941	20	42	109
Age(Years)	20	28	35	14	23	33	12	19	27
MaxSpan(m)	2	3	*	4	6	*	3	4	*

or baseline, assignment for each of these descriptive variables includes all values from zero to the lower bound of the second category for that design variable. The complete categorization of design variables for timber deck analysis was previously presented in Table 4.1. The categories for other deck material types can be similarly obtained using the lower bounds provided in Table 6.5 for the reference cell coding of other deck material types. It can be observed that the statistical distributions of the same design variable develop significantly different categorical ranges over each of the deck material types. For example, the reference cell category MaxSpan3 includes bridges with maximum span length of 3 meters or longer when applied to timber decks, but bridges with maximum span length of 6 meters or longer when applied to concrete decks. Consequently, deterioration models for different deck material types adjusted for the same variables are not necessarily quantitatively comparable. Table 6.4 also captures features of the general distribution of bridges in these classifications for each deck material type. For instance, it can be observed from the categorical bounds that the distribution of bridges with timber decks is biased toward older structures with shorter maximum span length and lower ADT and ADTT than concrete deck and steel deck bridges in the state.

6.1.2 Survival Analysis

The most significant, or best subset, variables identified for timber deck deterioration models by proportional hazards survival analysis are presented in Table 6.6 along with the associated hazard ratios and Wald statistic p-values across condition ratings 9 to 4. Corresponding baseline survival functions for the timber deck model over each of these condition ratings are presented in Figure 6.4. Similarly, the best subset statistics obtained for the concrete deck and steel deck models are presented in Tables 6.7 and 6.8 and Figures 6.5 and 6.6, respectively.

The variables with hazard ratios denoted by '*' are not included in the best subset for the particular condition rating. For steel decks, it is observed that no variables are found significant at condition ratings 4 and 9. This is a reflection of the limited number of recorded observations available in these datasets, as noted in Table 6.4. Likewise, for timber decks and concrete decks in condition rating 4, it is observed that there are only two variables in the respective best subset selection. This small set of significant variables is again a reflection of the limited availability of recorded observations and the high percentage of censored records at these ratings, as documented in Tables 6.2 and 6.3.

Comparison of the developed models reveals that the significant explanatory variables identified in the best subsets for individual deck material types all include geographic region, maximum span length, number of spans, and age. However, the influence of these variables on deterioration rate varies across deck material types and also across the individual condition ratings associated with each deck material type. Although the geographic region 'Piedmont' was found to be a significant explanatory variable for deterioration in condition rating 5, the effect of geographic region on timber decks was more notably expressed by whether or not the bridge was located in the mountain region. For concrete decks, the effect of the 'Piedmont' geographic region, although significant is not consistent across condition ratings. It was associated with an increased rate of deterioration in condition ratings 5 and 8 but a decreased rate of deterioration in condition ratings 6, and 9. Similar inconsistency is observed with respect to the 'Mountain' geographic region in the concrete deck models, with an increased rate of deterioration predicted over condition rating 8, but a significantly reduced hazard rate for ratings 7 and 9. Higher rates of deterioration of concrete bridge decks in northern Indiana compared to those in southern Indiana were found in earlier studies (Madanat and Ibrahim, 1995, Madanat et al., 1995, Mauch and Madanat, 2001, Mishalani and Madanat, 2002) and were attributed to the use of deicing salts in cold weather regions that contribute to corrosion of concrete deck reinforcement bars. Similar impact was found in a study on bridge deterioration rates in the state of Nevada with bridges in northern Nevada deteriorating much faster than those in southern Nevada on account of harsher winter environment and, consequently, increased freeze-thaw cycles and salt application (Sanders and Zhang, 1994). In an earlier study done on NCDOT bridges, it is noted that the western divisions of the state's Piedmont region experience more frequent ice and snow compared to the eastern Piedmont divisions, which in turn leads to higher rates of deterioration for these divisions due to the increased use of deicing and anti-icing salts. This study recommended classifying regions into salt/non-salt and marine/non-marine regions instead of Mountain, Piedmont, and Coastal because of striking differences in deterioration rates observed for these classifications (Abed-Al-Rahim and Johnston, 1991). Differences in weather conditions and associated deterioration rates within the same region may be the source of the inconsistent effects of geographic region on deterioration rates in the models developed in the present study. However, the changing mechanisms of the deterioration process itself may be the reason for the observed changes in hazard ratios over the life cycle of individual components. For instance, it is believed that deterioration from condition state 8 to 7 in concrete decks is primarily associated with chemical processes, whereas deterioration from condition state 7 is associated with mechanical processes reflected in spalling of concrete (Mishalani and

Madanat, 2002). For steel decks, ‘Piedmont’ and ‘Mountain’ are found to be significant at condition ratings 7 and 8 and are associated with hazard ratios less than 1. The lower hazard ratio in these geographic regions relative to the coastal region is plausible due to increased susceptibility of steel decks to corrosion in the humid and salt laden environment associated with the coastal region.

From the proportional hazards regression, an increase in maximum span length is observed to increase the rate of deterioration for all deck material types. This increase in deterioration rates with an increased span length has been documented for concrete bridge decks in earlier studies (Freyermuth et al., 1970, Madanat and Ibrahim, 1995, Madanat et al., 1995). The current regression analysis also indicates that multiple span bridges are consistently and often significantly more prone to deterioration than single span bridges. Evidence of increase in rate of deterioration with increase in number of spans has been documented in a number of previous studies (Busa et al., 1985, Madanat and Ibrahim, 1995, Madanat et al., 1995). Multispan bridge decks necessarily include expansion joints with a known propensity for maintenance issues (Chang and Lee, 2002) that are likely to affect the overall general condition rating of the deck. Presence of joints was found to exacerbate deck deterioration in an earlier study (Yanev and Chen, 1993) and serves to support the higher deterioration rate predicted for multispan bridge decks.

Of all of the potential factors included in the best subset selection, bridge age was found to have the greatest impact on the hazard ratio across all deck material types. The positive correlation between the age at inspection and the observed rate of deterioration is well established in deterioration modeling literature (Busa et al.,

TABLE 6.6: Timber deck best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
‘StateSystem’	*	*	*	*	*	*	*	*	*	*	0.449	0.114
‘Reconstruction’	*	*	0.842	0.008	*	*	1.293	< 0.001	0.779	0.048	*	*
‘Piedmont’	*	*	*	*	*	*	*	*	1.363	0.001	*	*
‘Mountain’	*	*	1.277	< 0.001	0.857	< 0.001	1.143	0.001	1.424	< 0.001	*	*
‘ADT4’	*	*	*	*	1.138	0.003	*	*	*	*	*	*
‘ADTT3’	*	*	1.140	0.003	*	*	*	*	*	*	*	*
‘ADTT4’	*	*	1.296	< 0.001	*	*	*	*	*	*	*	*
‘MaxSpan2’	*	*	*	*	1.171	0.003	1.199	< 0.001	*	*	*	*
‘MaxSpan3’	*	*	*	*	1.165	0.002	1.194	< 0.001	*	*	*	*
‘NumberSpans’	*	*	1.119	0.007	1.284	< 0.001	1.218	< 0.001	*	*	*	*
‘Age2’	2.289	< 0.001	2.438	< 0.001	1.715	< 0.001	1.332	< 0.001	1.302	0.028	0.742	0.163
‘Age3’	2.503	< 0.001	2.210	< 0.001	1.264	< 0.001	2.076	< 0.001	1.814	< 0.001	*	*
‘Age4’	2.445	< 0.001	3.045	< 0.001	0.787	< 0.001	2.265	< 0.001	1.564	< 0.001	*	*

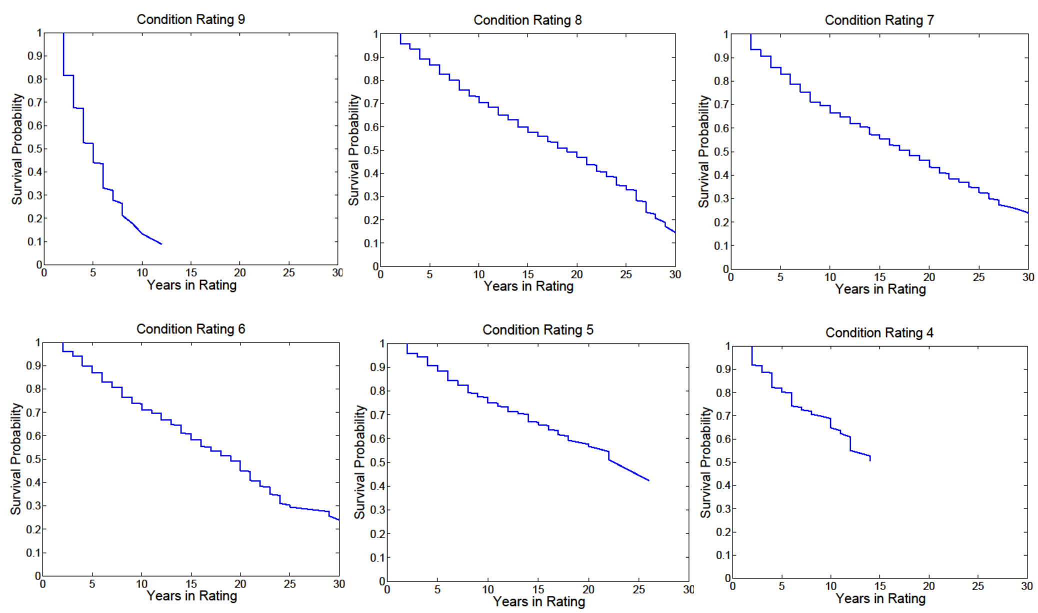

FIGURE 6.4: Timber deck baseline survival functions

TABLE 6.7: Concrete deck best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
'StateSystem'	*	*	*	*	*	*	1.124	0.004	0.772	< 0.001	*	*
'Piedmont'	0.631	0.010	1.224	< 0.001	*	*	0.753	< 0.001	1.434	< 0.001	*	*
'Mountain'	0.460	0.002	1.207	0.003	0.752	< 0.001	0.809	< 0.001	*	*	*	*
'ADT3'	*	*	*	*	*	*	1.131	0.004	*	*	*	*
'ADT4'	*	*	*	*	*	*	1.248	< 0.001	*	*	1.551	0.037
'MaxSpan2'	*	*	1.482	< 0.001	0.804	< 0.001	*	*	*	*	*	*
'MaxSpan3'	0.497	< 0.001	2.179	< 0.001	*	*	1.353	< 0.001	*	*	*	*
'NumberSpans'	*	*	*	*	1.575	< 0.001	1.300	< 0.001	*	*	*	*
'Age2'	4.525	0.003	1.684	< 0.001	1.130	0.011	1.262	< 0.001	*	*	0.257	0.058
'Age3'	*	*	2.285	< 0.001	1.405	< 0.001	1.460	< 0.001	1.692	< 0.001	*	*
'Age4'	*	*	2.280	< 0.001	2.223	< 0.001	2.279	< 0.001	1.363	0.002	*	*

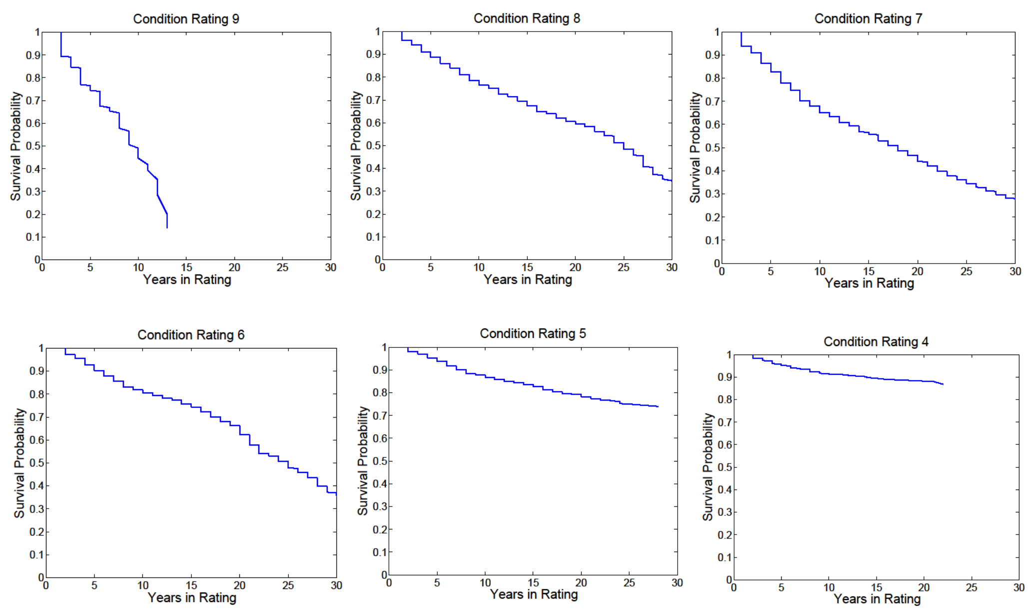

FIGURE 6.5: Concrete deck baseline survival functions

TABLE 6.8: Steel deck best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
'StateSystem'	*	*	*	*	*	*	0.828	0.324	*	*	*	*
'Reconstruction'	*	*	*	*	*	*	1.415	0.006	*	*	*	*
'Piedmont'	*	*	0.705	< 0.001	0.719	0.016	*	*	*	*	*	*
'Mountain'	*	*	*	*	0.624	0.001	*	*	*	*	*	*
'MaxSpan2'	*	*	*	*	*	*	*	*	3.280	0.002	*	*
'MaxSpan3'	*	*	*	*	*	*	*	*	2.837	0.006	*	*
'NumberSpans'	*	*	1.412	< 0.001	1.376	< 0.001	*	*	*	*	*	*
'Age2'	*	*	2.995	< 0.001	1.484	0.002	*	*	*	*	*	*
'Age3'	*	*	3.094	< 0.001	1.613	0.001	1.377	0.019	*	*	*	*
'Age4'	*	*	5.498	< 0.001	1.868	< 0.001	2.593	< 0.001	*	*	*	*

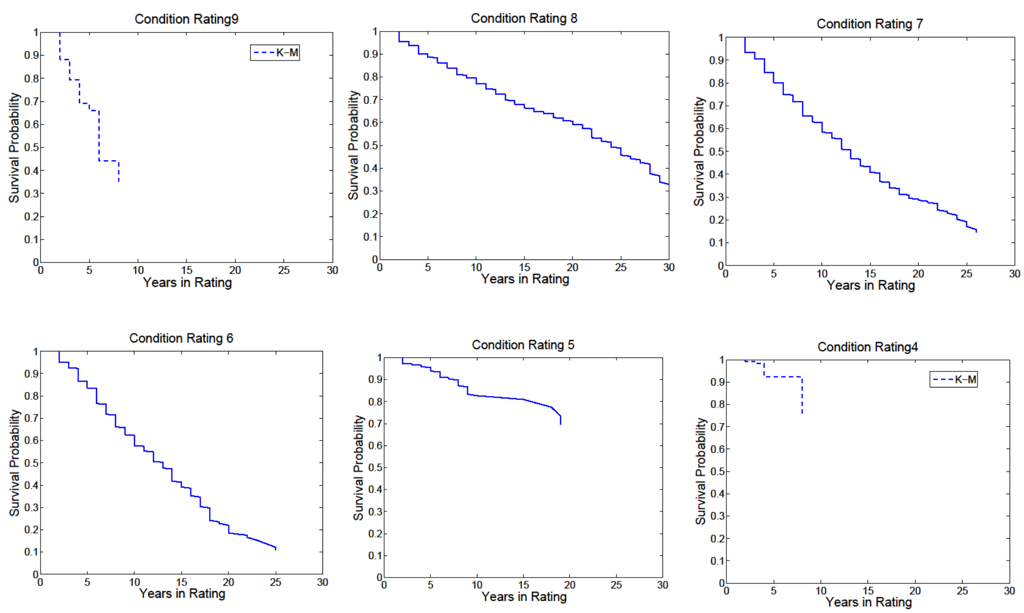

FIGURE 6.6: Steel deck baseline survival functions

1985, Chen and Johnston, 1987, Madanat and Ibrahim, 1995, Madanat et al., 1995). The decreases and increases in rates of deterioration represented by the hazard ratios associated with State System and Reconstruction are also expected and generally corroborated by earlier studies. In this study, the baseline assignment of State System corresponds to State System 1 comprising of interstate, urban and primary roads. Therefore a hazard ratio lower than 1 is indicative of a lower rate of deterioration of secondary or rural roads located on State System 2 in comparison to those located on State System 1. A lower rate of deterioration associated with State System 2 was also observed in an earlier study conducted on North Carolina bridges and is most likely attributable to the lower traffic volumes on secondary roads (Abed-Al-Rahim and Johnston, 1991). It is interesting to note that a reverse phenomenon was observed in studies conducted on Indiana's concrete bridges, with decks of bridges located on interstates and primary roads showing significantly lower rates of deterioration than those on secondary highways (Madanat and Ibrahim, 1995, Madanat et al., 1995, Mauch and Madanat, 2001, Mishalani and Madanat, 2002). This was postulated to be a reflection of the lower design and maintenance standards associated with bridges on secondary roads. In the present study, it can be observed from the p-values that State System is identified as a significant variable only in the case of concrete decks, where it has a hazard ratio slightly higher than 1 at condition rating 6, but a hazard ratio lower than 1 at condition rating 5. Reconstructed bridges have been observed to have higher deterioration rates than original or rebuilt bridges (Sanders and Zhang, 1994, Yanev and Chen, 1993), which is reflected in two out of four of the hazard ratios developed for the effect of reconstruction on the deck deterioration models.

Increase in ADT was found to slightly increase the deterioration rate of timber and concrete decks, but the effect was found to be significant over only one condition rating for each of these components. Similarly, increase in ADTT was also found significant in moderately increasing the deterioration rate, but only in the case of timber decks and only at condition rating 8. Given the *a priori* classification used currently in the protocol for developing deterministic deterioration models for the NDOT BMS, it is important to emphasize the near absence of ADT as an identified significant explanatory factor in the proportional hazards regression. This finding was not unexpected given the nature of the deterministic deck deterioration models, previously presented in Figures 6.1, 6.2, and 6.3, which generally indicate that the use of ADT as a preclassifier for the deterioration models leads to poor development of independent models that clearly distinguish significant factors affecting deterioration. The lack of ADT as a significant factor in the deck deterioration models developed by proportional hazards regression serves to support the validity of the developed framework and benefit offered by the multivariate regression technique. Formal validation of the predictive fidelity of the developed models relative to the deterministic models is documented in Chapter 7.

The survival functions, shown in Figures 6.4, 6.5, and 6.6, are associated with baseline value assignments for the best subset variables for each deck material type. The survival functions are obtained from Cox proportional hazards regression with the exception of the survival functions for steel decks in condition ratings 4 and 9 that were found to have no associated covariates influencing the deterioration rate. The survival functions for these datasets are developed using the Kaplan-Meier empirical estimator and indicated by dashed lines. Observation of the median duration of these survival functions is useful for generally comparing these models with deterministic models currently being used by NCDOT, although it should be noted that the survival functions presented only represent the behavior of bridges in the baseline classification. The longer median duration at each rating in comparison to deterministic models reflects the ability of the survival-based models to account for censoring of the condition rating observations. Condition rating 9 is observed to have a relatively lower duration with respect to all other ratings across all deck material types. This phenomenon is commonly observed in deterioration modeling and is most likely attributable to strict guidelines for assigning a new or excellent rating to bridge decks. In one of the earliest studies on NCDOT bridge deterioration rates, it was observed that if a reinforced concrete component had cracks after construction, its condition rating was automatically recorded as 8 rather than 9 (Chen and Johnston, 1987). In this study, it was also observed that a bridge component would have lower deterioration rates between condition ratings 9 and 6, and the deterioration rate would accelerate if the condition rating was less than 6. The slope of the survival function indicates the rate of deterioration or transition to the lower condition rating. The survival functions obtained from this study corroborate the above-mentioned observation with the exception of the observed behavior in condition ratings 4 and 5 for concrete and steel decks. The flatter slope of survival functions associated with certain condition ratings is indicative of a lower rate of deterioration, which may be partly associated with high censoring percentages. The lower rates of deterioration are also likely to be associated with frequent low-level maintenance work that does not improve the condition rating of the component but might prolong the duration until deterioration to a lower rating (Abed-Al-Rahim and Johnston, 1991). It should be recognized that accounting for the contribution of all maintenance activities is a continuing challenge associated with infrastructure deterioration modeling.

6.1.3 Transition Probabilities and Expected Value Prediction Models

Figures 6.7 to 6.15 show the baseline transition probabilities, mean baseline transition probability matrices, and expected value prediction models developed for baseline assignments of significant covariates associated with individual deck material types. The software framework calculates non-stationary transition probability matrices for each inspection cycle that can be used to produce predictions of the expected condition rating by implementing the deterioration model as a semi-Markov process. The mean baseline transition probability matrix is also used in the following analysis to produce a simplified expected value prediction model using the stationary Markov process. This simplified stationary model is distinguished by the use of a dashed line in Figures 6.9, 6.12 and 6.15. Initial observations on the suitability of the use of stationary transition matrices to simplify the implementation of the deterioration models, as well as potential limitations, were discussed in Section 4.3.3. The comparisons between non-stationary and stationary predictions obtained across the different deterioration models developed in this chapter serve to formulate conclusions and recommendations on the use of stationary transition probabilities instead of non-stationary transition probabilities.

Across all deck material types, it is observed that the expected condition ratings

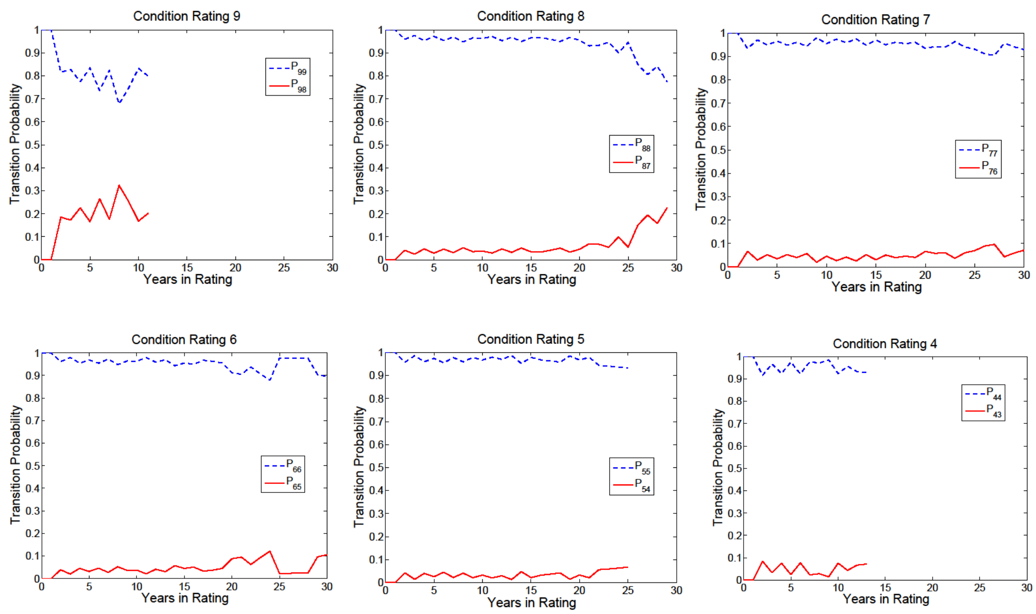

FIGURE 6.7: Timber deck baseline transition probabilities

CR	9	8	7	6	5	4	3
9	0.806	0.194	0	0	0	0	0
8	0	0.937	0.063	0	0	0	0
7	0	0	0.951	0.049	0	0	0
6	0	0	0	0.952	0.048	0	0
5	0	0	0	0	0.966	0.034	0
4	0	0	0	0	0	0.952	0.048
3	0	0	0	0	0	0	0.75

FIGURE 6.8: Timber deck mean baseline transition probability matrix

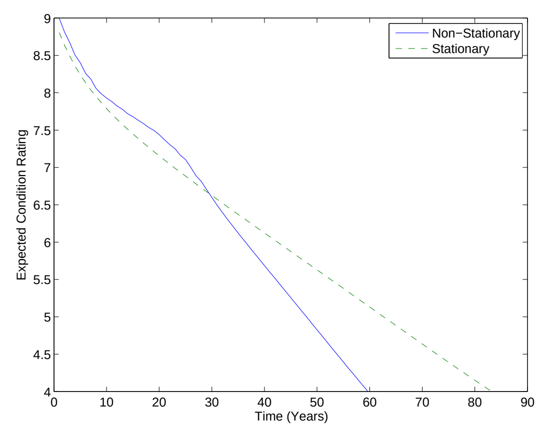

FIGURE 6.9: Timber deck baseline prediction model

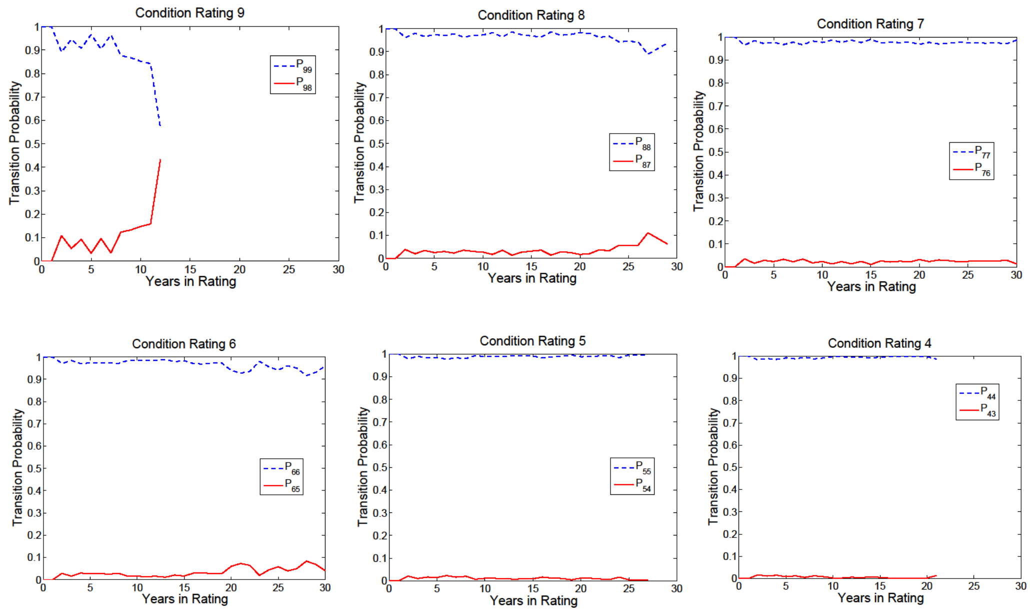

FIGURE 6.10: Concrete deck baseline transition probabilities

CR	9	8	7	6	5	4	3
9	0.882	0.118	0	0	0	0	0
8	0	0.964	0.036	0	0	0	0
7	0	0	0.977	0.023	0	0	0
6	0	0	0	0.967	0.033	0	0
5	0	0	0	0	0.989	0.011	0
4	0	0	0	0	0	0.993	0.007
3	0	0	0	0	0	0	0.75

FIGURE 6.11: Concrete deck mean baseline transition probability matrix

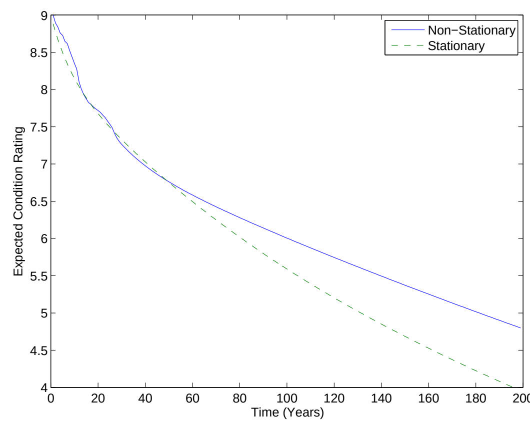

FIGURE 6.12: Concrete deck baseline prediction model

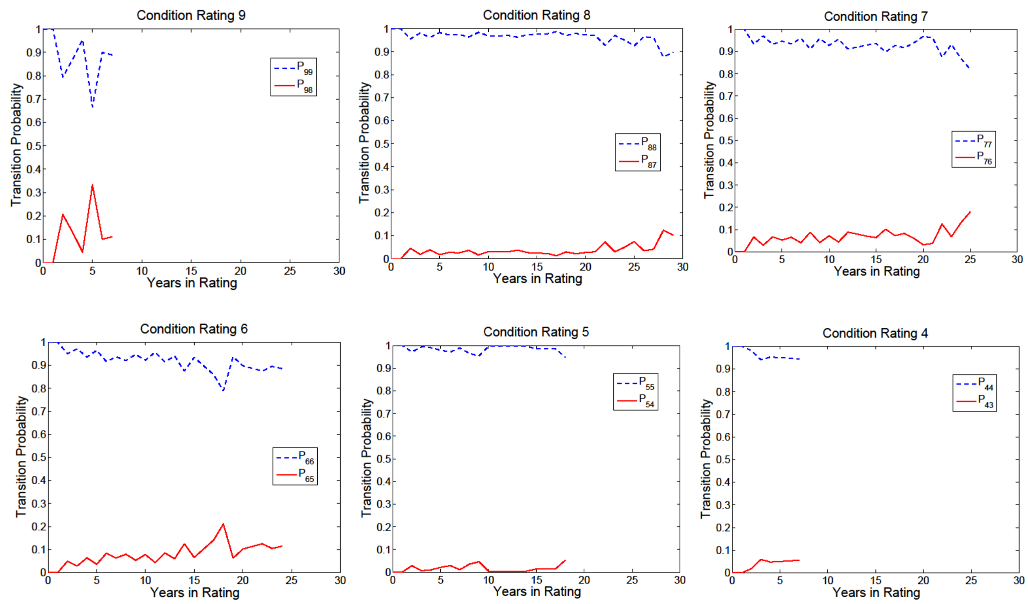

FIGURE 6.13: Steel deck baseline transition probabilities

CR	9	8	7	6	5	4	3
9	0.868	0.132	0	0	0	0	0
8	0	0.963	0.037	0	0	0	0
7	0	0	0.929	0.071	0	0	0
6	0	0	0	0.917	0.083	0	0
5	0	0	0	0	0.983	0.017	0
4	0	0	0	0	0	0.960	0.040
3	0	0	0	0	0	0	0.75

FIGURE 6.14: Steel deck mean baseline transition probability matrix

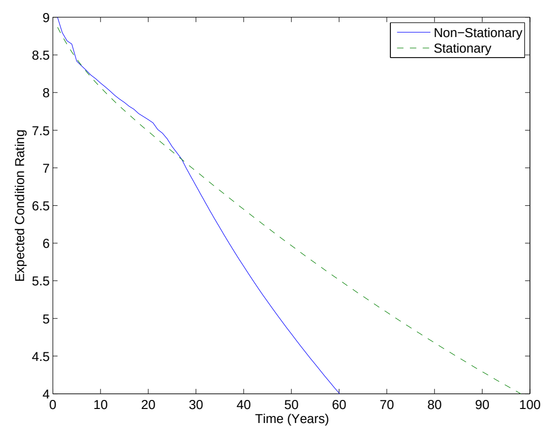

FIGURE 6.15: Steel deck baseline prediction model

predicted by the simplified stationary transition probabilities over an initial period of approximately 20 years agree strongly with those predicted by the non-stationary models. In the longer term, the non-stationary model appears to provide less optimistic, yet more realistic, predictions of expected condition ratings. It is observed from Figure 6.9 that the timber deck with baseline covariate properties deteriorates from condition rating 7 to condition rating 5 within this planning scenario in a period of approximately 20 years for the non-stationary model and in a period of approximately 45 years using the stationary model. Similar difference is observed in steel decks where the relative time associated with deterioration from rating 7 to 5 is approximately 25 years more using the stationary model and planning scenario shown. However, in the case of concrete decks, the non-stationary model provides a more optimistic prediction than the stationary model. The reason for this is the nature of the baseline transition probabilities over the condition rating duration, which are nearly constant for the concrete deck model in ratings 4 through 8, but the stationary model is biased by the lower mean transition probability in condition rating 9. Use of non-stationary transition probability matrices is recommended for greater accuracy in predictions while accounting for non-constancy in transition probabilities and long-term planning periods exceeding 20 years. However for planning periods limited to 20 years or less, stationary transition probability matrices may be adopted without significant loss of accuracy while optimizing the use of computational resources. For BMS that are able to support the computation of non-stationary matrices, decision makers can reserve the option of selecting either model depending on the planning range and purpose of the deterioration forecasting.

Deterioration models can be obtained for bridge components featuring covariate assignments other than the baseline values by using the hazard ratios to scale the baseline transition matrices, as previously explained in Chapter 4. Figure 6.16 shows the effect of different age categories on timber deck deterioration rates. The effect of number of spans on rate of deterioration of concrete decks is shown in Figure 6.17. Figure 6.18 shows the effect of reconstruction on steel deck deterioration rate. The combined effect of two covariates, e.g. age and state system, on concrete deck deterioration rates is illustrated in Figure 6.19. The interested reader can relate the changes in each of these illustrative examples to the hazard ratios associated with each factor presented in Tables 6.6, 6.7, and 6.8. These examples illustrate the depth of information revealed by the models developed in this study using the same databases used for developing the deterministic deck models, which were classified using ADT and expressed far less variable-dependent behavior than expressed in these simple single or two parameter illustrative prediction models.

6.2 Bridge Superstructure Deterioration Models

In the current NCDOT deterministic deterioration model, bridge superstructures are also primarily classified by construction material, which for the superstructure is obtained using the field ‘Structure Type - Main’ in the NBI database record. The four primary main material categories present with any significant number of associated bridge records in the NCDOT BMS are: Timber, Concrete, Steel, and Prestressed Concrete. All structures categorized as timber superstructure carry a code assignment of 7. Concrete and concrete continuous structures, which are coded as 1 and

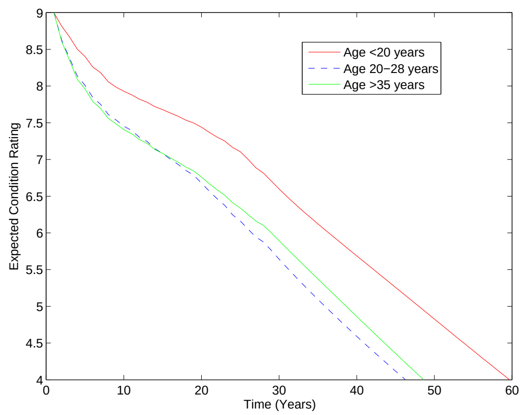

FIGURE 6.16: Effect of age on timber deck deterioration rates

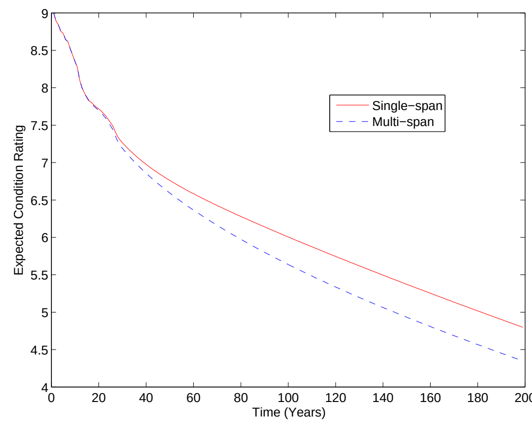

FIGURE 6.17: Effect of number of spans on concrete deck deterioration rates

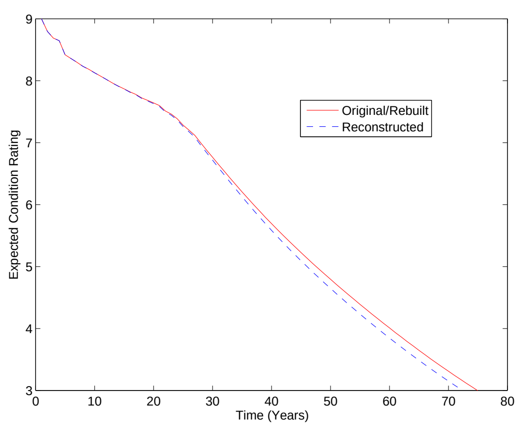

FIGURE 6.18: Effect of reconstruction on steel deck deterioration rates

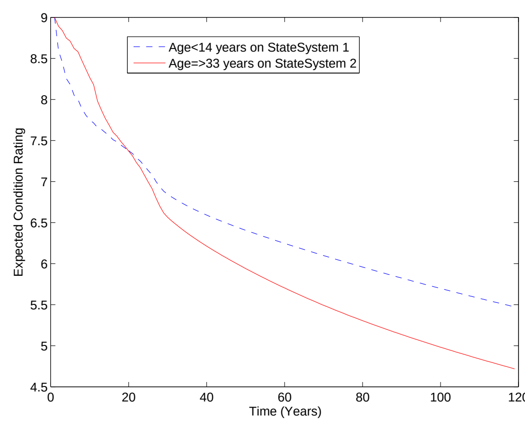

FIGURE 6.19: Effect of age and State System on concrete deck deterioration rates

2, respectively, are both categorized under the general grouping of concrete superstructures for deterioration modeling. Similarly, prestressed concrete and prestressed concrete continuous structures, which are coded as 5 and 6, respectively, are categorized under a single prestressed concrete grouping for deterioration modeling. This category also includes post-tensioned concrete structures, which are not designated separately from prestressed concrete structures in the coding system. The steel superstructure category includes both steel structures and steel continuous structures, which are coded 3 and 4, respectively. Using the same rationale applied for material-specific pre-classification in the deck deterioration analysis, this primary classification was also applied for pre-classification of the superstructure condition rating data prior to application of the proportional hazards regression.

The updated deterministic deterioration models for timber, concrete, steel, and prestressed concrete superstructures are presented in Figures 6.20 to 6.23. These deterministic deterioration models are constructed after pre-classification of the bridge records on basis of State System (Tier 2) and structural design type (Tier 3). The State System categorization is based on the NCDOT practice of dividing highways into two broad categories on the basis of functional classification. State System 1 is comprised of bridges on interstate, urban, and primary roads, while State System 2 is comprised of bridges on secondary and rural roads. This same State System categorization also serves as one of the covariates analyzed for potential influence on deterioration rates using proportional hazards regression. Structural design type used for the third tier of bridge classification is also a broader categorization developed by previous researchers working with the NCDOT bridge inventory (Duncan and

Johnston, 2002) based on the field 'Structure Type - Main' in the NBI database record. The Tier 2 and Tier 3 categories used in the development of superstructure deterministic deterioration models are summarized in Table 6.9. In application of

TABLE 6.9: Tier 2 and Tier 3 classification of deterministic deterioration models under each superstructure material type

Tier2	State System	Tier3	Design Type
1	1	1	Multi-Beam
2	2	2	Slab
		3	Tee-Beam
		4	Truss
		5	Floor-Beam

the deterministic regression procedure to the statewide historical database, it was observed that the number of bridges within several categories at the Tier 3 level was insufficient for statistical analysis. Bridges with timber or steel superstructures were found to be predominantly of the multi-beam design type, while concrete and prestressed concrete superstructures were predominantly designed with slab or tee-beam construction. Deterministic deterioration models for categories with at least 50 bridge records available for deterministic analysis at the Tier 3 level are presented here. The limited diversity of design type within each main material category was the primary reason for not including design type in the variables analyzed for potential effect on deterioration rates of material-specific bridge components within the PHM regression.

6.2.1 Data Overview

The total number of records and censoring characteristics of the datasets associated with individual condition ratings for all four superstructure material categories are presented in Tables 6.10 to 6.12 . As observed for decks, observations at con-

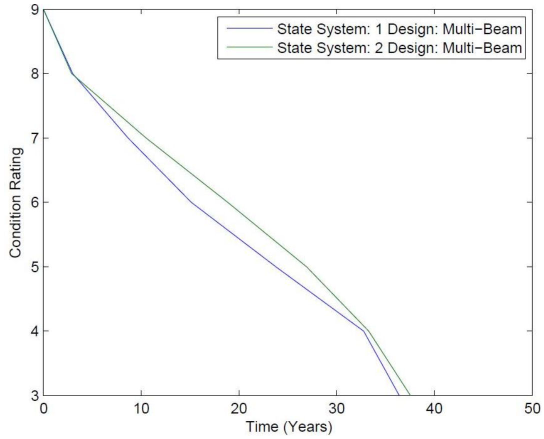

FIGURE 6.20: Timber superstructure deterministic deterioration models

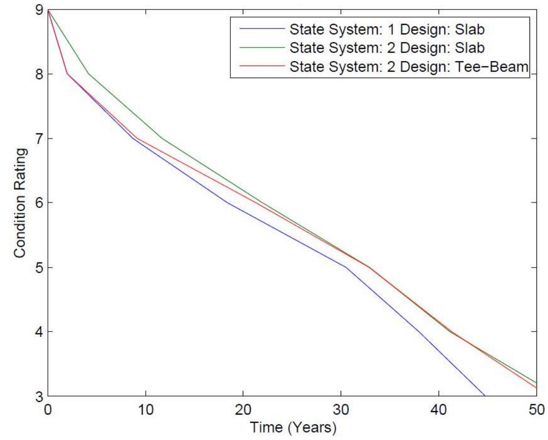

FIGURE 6.21: Concrete superstructure deterministic deterioration models

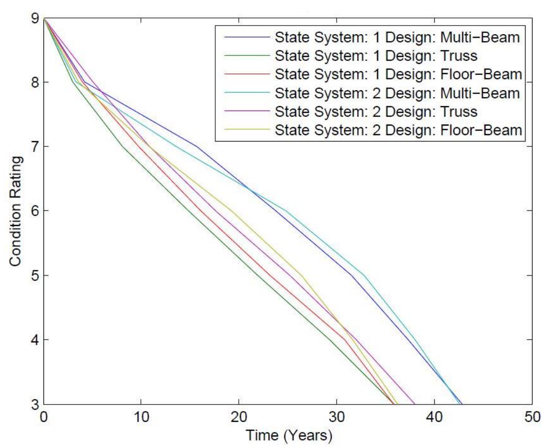

FIGURE 6.22: Steel superstructure deterministic deterioration models

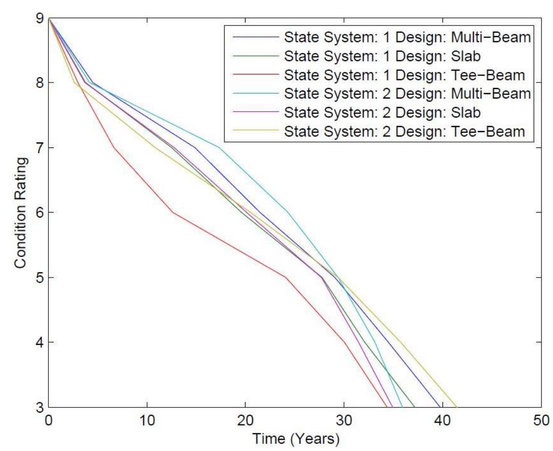

FIGURE 6.23: Prestressed concrete superstructure deterministic deterioration models

TABLE 6.10: Timber superstructure condition rating data overview

Superstructure Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	179	164	15	8.38
8	1123	608	515	45.86
7	3023	1027	1996	66.03
6	3842	1572	2270	59.08
5	2625	591	2034	77.49
4	948	79	869	91.67
3	341	0	341	100.00

TABLE 6.11: Concrete superstructure condition rating data overview

Superstructure Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	11	0	11	100.00
8	315	147	168	53.33
7	1079	412	667	61.82
6	1580	675	905	57.28
5	1301	244	1057	81.25
4	442	33	409	92.53
3	61	1	60	98.36

TABLE 6.12: Steel superstructure condition rating data overview

Superstructure Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	799	117	682	85.36
8	7290	4195	3095	42.46
7	12908	4486	8422	65.25
6	9668	3407	6261	64.76
5	5372	1078	4294	79.93
4	1808	152	1656	91.59
3	376	5	371	98.67

TABLE 6.13: Prestressed concrete superstructure condition rating data overview

Superstructure Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	1335	183	1152	86.29
8	4676	2124	2552	54.58
7	5511	881	4630	84.01
6	1601	626	975	60.90
5	912	123	789	86.51
4	199	15	184	92.46
3	39	1	38	97.44

dition ratings 5 to 9 contribute the greatest percentage of records to the dataset of superstructure condition ratings for each material type. The percentage of censored records is in the range of approximately 45-65% for condition ratings 6 to 8, with the exception of prestressed concrete superstructures at rating 7, which have 84% censored records. The percentage of censored records at condition rating 5 is relatively high ranging from 77.5% for timber superstructures to 86.5% for prestressed concrete superstructures, whereas that for rating 4 is over 90% for all material types. However, these percentages are similar to those previously observed in the deck condition rating data. There are comparatively limited number of records available at condition ratings 3 and 9. The insufficiently small number of uncensored records available at condition rating 3, if any, prohibits the use of survival analysis over this rating. However, there are a sufficient number of total and uncensored records to permit survival analysis over condition rating 9 for all material types, with the exception of concrete superstructures where only eleven observations are present and all are censored.

Table 6.14 shows the lower bounds for the categorical design variables developed for the continuous scale variables: ADT, ADTT, age, and length of maximum span. These bounds were determined by weighted averaging of the observed bridge records extracted across all condition ratings for each superstructure material type. It can

TABLE 6.14: Lower bounds of intervals developed for categorical design variables

Superstructure Type	**Timber**			**Concrete**			**Steel**			**Prestressed Concrete**		
Category	2	3	4	2	3	4	2	3	4	2	3	4
ADT	102	239	555	1419	3815	8263	282	1015	5179	515	1636	5432
ADTT	6	15	34	96	296	788	18	71	454	36	141	738
Age(Years)	22	28	36	32	46	58	17	26	35	7	12	19
MaxSpan(m)	$*$	2	$*$	3	5	$*$	3	5	$*$	4	6	$*$

be observed from the categorical bounds that reinforced concrete superstructures are generally older that the other superstructure materials and tend to carry higher ADT and ADTT. Prestressed concrete superstructures are of more recent construction than the other superstructure material types according to the age categorical distributions. The distribution of timber superstructures is biased toward lower number of spans and lower ADT and ADTT than bridge superstructures of all other material types in the state.

6.2.2 Survival Analysis

The best subset variables identified for timber superstructure deterioration models are presented in Table 6.15 along with their associated hazard ratios and Wald statistic p-values across condition ratings 9 to 4. The corresponding baseline survival functions are presented in Figure 6.24. Similarly, the best subset models, statistics, and baseline survival functions for concrete, steel, and prestressed concrete superstructures are presented in Tables 6.16, 6.17, and 6.18 and Figures 6.25, 6.26, and 6.27, respectively. It is observed that the significant best subset explanatory variables common to all individual superstructure material types include region, length of maximum span, number of spans, and age. However, as quantified by hazard ratios, the influence of these variables varies not only across superstructure material types but also across individual condition ratings associated with each superstructure material type. Both steel and prestressed concrete superstructures in the Mountain and Piedmont geographic regions show overall lower rates of deterioration than those in the Coastal region. Timber superstructures in the Piedmont region also have a lower rate of deterioration than those in Coastal region. Higher rates of deteriora-

TABLE 6.15: Timber superstructure best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
'StateSystem'	*	*	*	*	*	*	1.412	0.004	*	*	*	*
'Reconstruction'	*	*	0.706	0.039	*	*	*	*	*	*	*	*
'Piedmont'	*	*	*	*	*	*	0.746	< 0.001	*	*	0.621	0.055
'ADT3'	*	*	*	*	*	*	*	*	0.453	0.006	*	*
'ADTT3'	*	*	*	*	*	*	*	*	1.937	0.021	*	*
'MaxSpan3'	*	*	*	*	1.305	0.003	*	*	*	*	*	*
'NumberSpans'	*	*	*	*	*	*	1.435	< 0.001	*	*	1.861	0.029
'Age2'	*	*	2.523	< 0.001	1.936	< 0.001	1.207	0.019	*	*	*	*
'Age3'	2.251	0.007	1.716	< 0.001	2.146	< 0.001	1.823	< 0.001	*	*	*	*
'Age4'	1.934	0.012	3.560	< 0.001	*	*	2.408	< 0.001	0.715	< 0.001	*	*

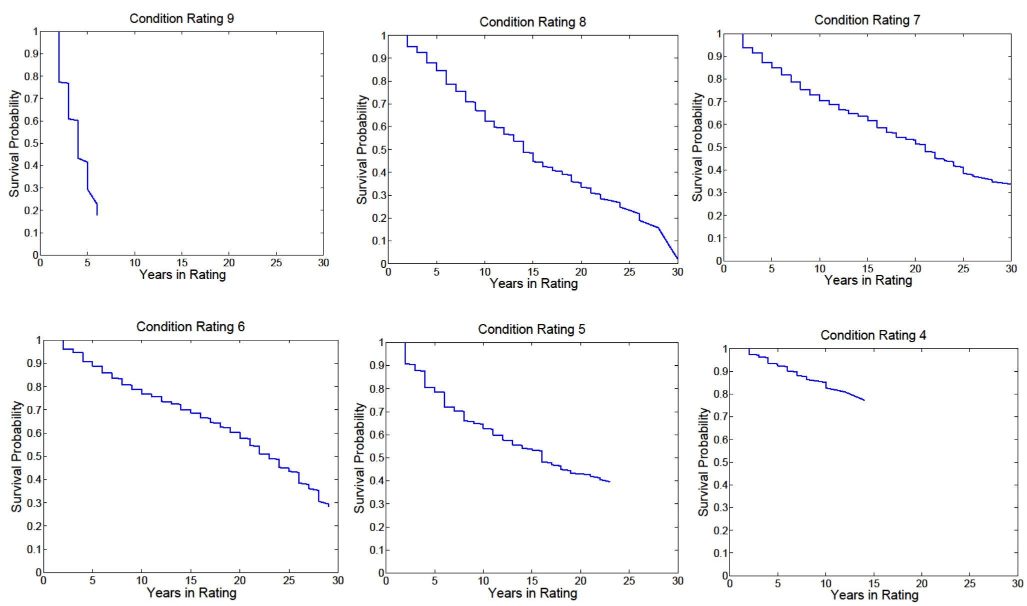

FIGURE 6.24: Timber superstructure baseline survival functions

TABLE 6.16: Concrete superstructure best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
'StateSystem'	*	*	*	*	*	*	*	*	*	*	0.816	0.589
'Piedmont'	*	*	1.534	0.013	*	*	*	*	*	*	*	*
'Mountain'	*	*	*	*	0.791	0.031	*	*	*	*	*	*
'MaxSpan2'	*	*	*	*	*	*	*	*	1.520	0.034	*	*
'MaxSpan3'	*	*	*	*	*	*	*	*	1.996	0.001	*	*
'NumberSpans'	*	*	*	*	1.849	< 0.001	1.621	< 0.001	1.862	0.003	*	*
'Age2'	*	*	*	*	2.059	< 0.001	1.286	0.023	*	*	*	*
'Age3'	*	*	1.512	0.074	2.434	< 0.001	1.920	< 0.001	*	*	0.387	0.078
'Age4'	*	*	*	*	3.001	< 0.001	2.161	< 0.001	*	*	*	*

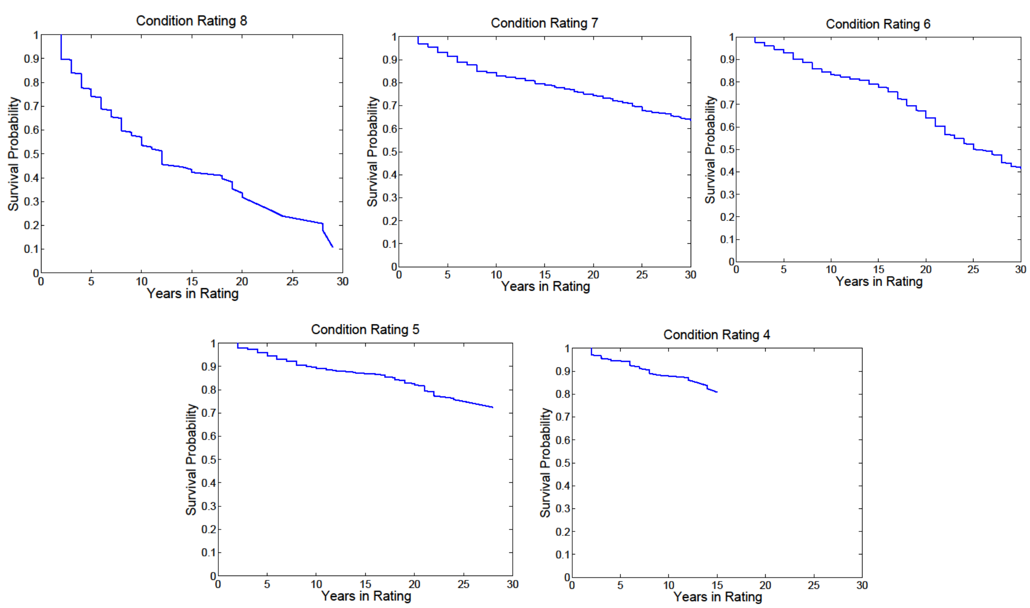

FIGURE 6.25: Concrete superstructure baseline survival functions

TABLE 6.17: Steel superstructure best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
‘StateSystem’	*	*	0.916	0.014	1.162	0.005	0.950	0.265	1.311	< 0.001	*	*
‘Reconstruction’	*	*	*	*	1.507	< 0.001	1.502	< 0.001	*	*	*	*
‘Piedmont’	*	*	1.140	0.005	0.865	0.002	0.889	0.001	*	*	*	*
‘Mountain’	*	*	1.139	0.009	0.803	< 0.001	*	*	*	*	*	*
‘IntegralConcrete’	*	*	*	*	0.082	< 0.001	*	*	*	*	*	*
‘EpoxyOverlay’	*	*	*	*	4.924	< 0.001	*	*	*	*	*	*
‘ADT2’	*	*	*	*	1.140	0.002	1.126	0.006	*	*	*	*
‘ADT3’	*	*	*	*	1.163	0.003	1.121	0.008	*	*	*	*
‘ADT4’	*	*	*	*	1.277	< 0.001	*	*	*	*	*	*
‘ADTT4’	*	*	*	*	*	*	*	*	*	*	0.881	0.551
‘MaxSpan2’	*	*	0.864	< 0.001	0.920	0.026	*	*	*	*	*	*
‘MaxSpan3’	*	*	*	*	0.816	< 0.001	1.287	< 0.001	*	*	*	*
‘NumberSpans’	*	*	*	*	1.245	< 0.001	*	*	*	*	*	*
‘Age2’	*	*	2.451	< 0.001	1.583	< 0.001	*	*	*	*	*	*
‘Age3’	17.228	< 0.001	3.109	< 0.001	2.057	< 0.001	1.396	< 0.001	*	*	*	*
‘Age4’	12.274	< 0.001	3.759	< 0.001	3.008	< 0.001	2.775	< 0.001	1.385	< 0.001	*	*

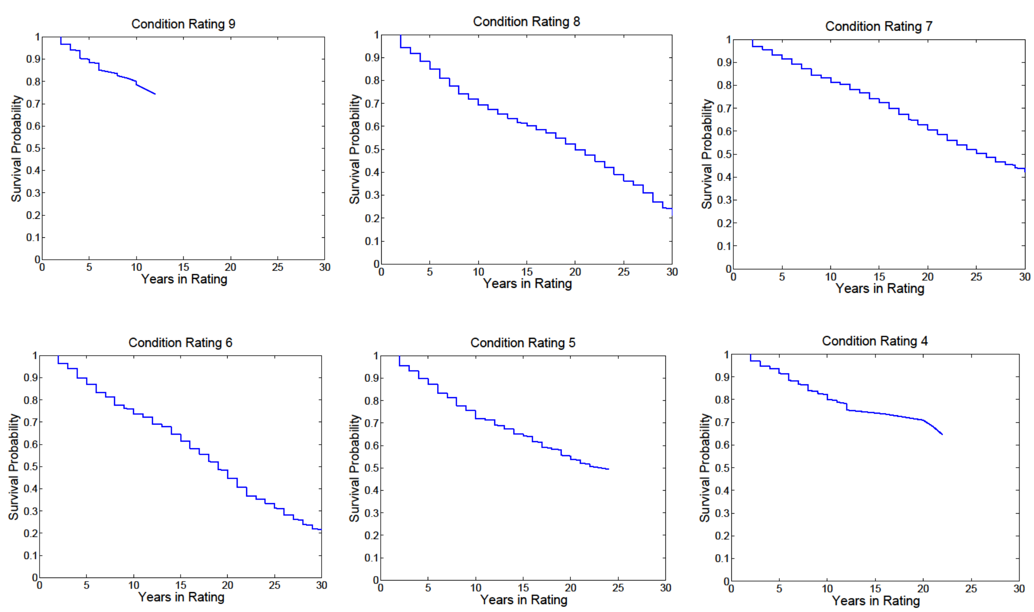

FIGURE 6.26: Steel superstructure baseline survival functions

TABLE 6.18: Prestressed concrete superstructure best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
'StateSystem'	*	*	1.217	< 0.001	0.854	0.055	1.250	0.013	*	*	*	*
'Piedmont'	0.427	< 0.001	*	*	0.689	< 0.001	0.725	< 0.001	*	*	*	*
'Mountain'	*	*	*	*	0.579	< 0.001	*	*	*	*	*	*
'ADT3'	*	*	*	*	*	*	*	*	0.643	0.075	*	*
'ADT4'	0.612	0.006	*	*	*	*	*	*	0.456	0.006	*	*
'ADTT2'	*	*	*	*	1.204	0.022	*	*	*	*	*	*
'ADTT3'	*	*	1.149	0.006	1.334	0.001	*	*	*	*	*	*
'MaxSpan2'	*	*	1.274	< 0.001	0.690	< 0.001	*	*	*	*	*	*
'MaxSpan3'	*	*	1.605	< 0.001	0.489	< 0.001	*	*	*	*	*	*
'NumberSpans'	*	*	*	*	*	*	1.508	0.002	*	*	*	*
'Age2'	9.457	< 0.001	2.440	< 0.001	*	*	*	*	*	*	*	*
'Age3'	10.546	< 0.001	2.010	< 0.001	1.409	0.001	*	*	*	*	*	*
'Age4'	*	*	3.084	< 0.001	2.890	< 0.001	2.462	< 0.001	*	*	*	*

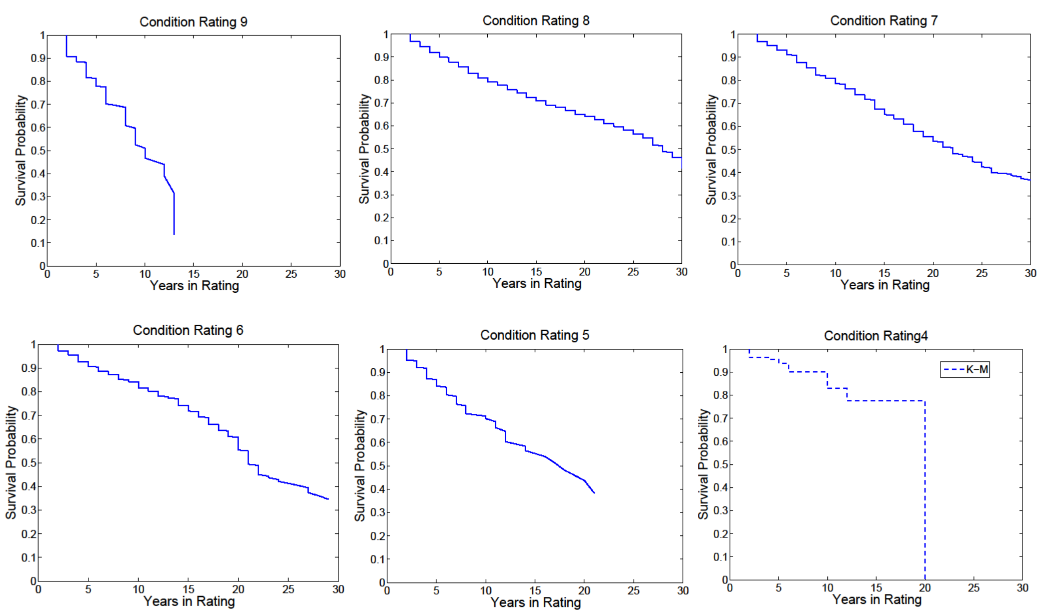

FIGURE 6.27: Prestressed concrete superstructure baseline survival functions

tion in coastal regions have been observed in earlier studies and are attributed to the salt laden atmosphere and humid marine environment in these regions, which exacerbate corrosion-driven deterioration mechanisms (Abed-Al-Rahim and Johnston, 1991, Chen and Johnston, 1987). In general, geographic region was observed to have only nominal effect on deterioration rates of concrete superstructures, as bridges in the Piedmont region were found to deteriorate at a faster rate than the other geographic regions only in condition rating 8. However, due to the total number of observations recorded for this material type in this rating (less than 300), this result should be interpreted cautiously. Likewise, bridges in the Mountain region deteriorate at a slower rate than the other geographic regions only in condition rating 7.

State system and ADT were found to be significant in the case of prestressed concrete, steel, and timber superstructures, although their effect was limited to only one condition rating each in the case of timber superstructures. An increase in ADT was found to slightly increase deterioration rates in steel superstructures at condition ratings 6 and 7. However, the observed effects of ADT were contrary to expectations in prestressed concrete and timber superstructures as an increase in ADT was found to decrease the deterioration rate. Similarly, the effect of State System was also contrary to expectations in these two components and a secondary route highway classification was found to increase the deterioration rate. A similar effect was observed in the case of steel superstructures, in which a secondary highway classification was found to increase the deterioration rate at condition ratings 5 and 7, although the effect of decreasing the rate of deterioration was observed at condition rating 8. A similar behavior was observed for prestressed concrete bridge decks in a previous study on

North Carolina bridges and was attributed to possible variations in the design of prestressed concrete decks (Abed-Al-Rahim and Johnston, 1991). In the current models, the increased rate of deterioration observed in prestressed concrete superstructures with lower ADT secondary route classification may be linked to design differences and related design loads for these low volume rural bridges. ADTT was also identified as a significant explanatory variable in the case of prestressed concrete and timber superstructures, although the effect in timber superstructures was observed only at condition rating 5. However, an increase in ADTT was observed to increase the deterioration rate in both components. The effect of ADTT is plausible and indicates that the volume of heavy truck traffic, as represented by ADTT, may be a better predictor of the deteriorating impact of traffic on roads than ADT, which includes all vehicular traffic.

The presence of reconstruction was found to be influential in the case of steel superstructures as well as timber superstructures, although the effect was again limited to only one condition rating in the case of timber superstructures. In steel superstructures, reconstruction caused an increase in the deterioration rate that was most significant over condition ratings 6 and 7. This increased rate of deterioration in reconstructed bridges is consistent with the observations from the bridge deck models and is supported by the literature (Sanders and Zhang, 1994, Yanev and Chen, 1993). However, timber superstructure components in reconstructed bridges appeared to deteriorate at a much slower rate than those in original or rebuilt bridges at condition rating 8. An increase in rates of superstructure deterioration for multi-span bridges and for bridges in higher age categories were observed across all superstructure materials. Also, increase in length of maximum span was found to consistently increase the deterioration rate in the case of timber and concrete superstructures, but this variable exhibited uneven effects across different condition ratings in the case of steel and prestressed concrete superstructures. Interestingly, the proportional hazards regression of superstructure condition rating data produced the first inclusion of any design variables associated with wear surface in the best subsets. The presence of integral concrete wearing surface and epoxy overlay were identified as significant variables only in the case of steel superstructures and only at condition rating 7. Integral concrete wearing surface was found to significantly decrease the deterioration rate of steel superstructures whereas epoxy overlay was found to increase it substantially. A further investigation revealed that very few records are associated with these wearing surfaces, and therefore, the sparsely observed effects cannot be considered reliable in all instances. A decrease in deck deterioration rates with presence of a protective wearing surface is logical and has been documented in earlier studies (Madanat and Ibrahim, 1995, Madanat et al., 1995, Mauch and Madanat, 2001). However, wearing surface was not found to be included in any of the best subsets for deck deterioration models in this study.

The survival functions shown in Figures 6.24 to 6.27 are associated with baseline value assignments for the best subset variables at each condition rating with sufficient data to permit survival analysis, with the exception of prestressed concrete superstructures at condition rating 4 where no significant variables were identified in the best subsets model. This is due to the limited availability of uncensored observations, as reflected in Table 6.13. The survival function for this model is therefore developed using the Kaplan-Meier empirical estimator. As mentioned earlier, survival analysis could not be performed for condition rating observations of concrete bridge superstructures at rating 9 and, consequently, no survival function can be developed for this data. The insufficient number of observations present for condition rating 9 is not unexpected as the age distribution of concrete superstructures revealed by the categorical bounds presented in Table 6.14, indicates that approximately 75% of concrete superstructures in the statewide inventory are older than the 35 year duration of the inspection rating recording period. A comparison of survival functions reveals similar trends to those observed for the deck models and reflected in the characteristics of the superstructure deterministic deterioration models. The lowest median duration and highest corresponding deterioration rate are observed at condition rating 9 and the highest median durations corresponding to lower rates of deterioration are observed at condition ratings 6 through 8.

6.2.3 Transition Probabilities and Expected Value Prediction Models

Figures 6.28 to 6.39 show the baseline transition probabilities, mean baseline transition probability matrices, and expected value prediction models developed for baseline assignments of significant covariates associated with timber, concrete, steel, and prestressed concrete superstructures in the NCDOT bridge inventory. The expected value predictions obtained using both the non-stationary and stationary approaches are again shown for each material type to continue the assessment of simplified implementation strategies. The stationary model uses the mean transition probability matrix and is distinguished from the non-stationary model by use of a dashed line. As previously observed in the case of deck deterioration models, the expected condition rating predictions obtained from the stationary and non-stationary models for each superstructure material type agree strongly with each other over an initial planning horizon of approximately 20 years. Notably, a less significant difference between the two probabilistic modeling approaches is observed within both the timber and the concrete superstructure prediction models over the long-term planning horizon when compared to those developed for the deck models of the same materials. However, the differences between the stationary and non-stationary model predictions for prestressed concrete superstructures are more than those observed in the case of concrete superstructures over the long-term planning horizon. This variation between long-term predictions obtained from the two modeling approaches depends on the extent of the deviation of the annual transition probabilities from the mean value across different condition ratings and only serves to highlight predictive accuracy limitations associated with the simplified stationary models when applied to any bridge component over long-term planning horizons. One notable feature expressed in the timber and steel superstructure models relative to the deck models is the time required to converge to a condition rating of 4, which is significantly longer than in the predictions obtained from deck deterioration. This confirms the observations concluded by previous studies that decks deteriorate faster than superstructure and substructure components (Chen and Johnston, 1987, Sanders and Zhang, 1994).

The effect of covariates, as quantified by the hazard ratios on the forecasted expected condition ratings over time, is investigated by developing deterioration models for the combinations of best case and worst case scenarios for timber superstructures

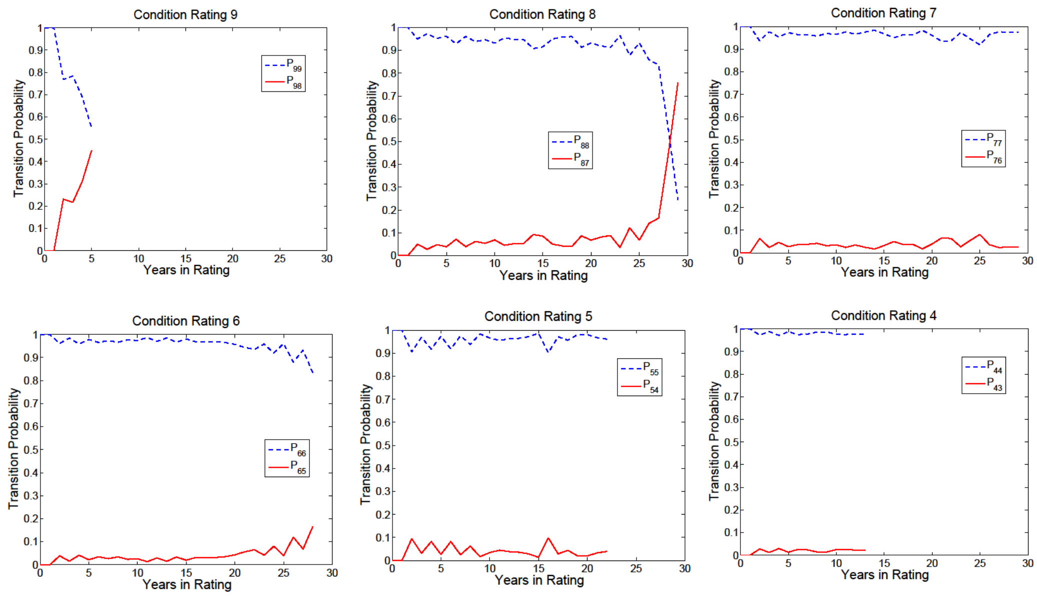

FIGURE 6.28: Timber superstructure baseline transition probabilities

CR	9	8	7	6	5	4	3
9	0.758	0.242	0	0	0	0	0
8	0	0.898	0.102	0	0	0	0
7	0	0	0.963	0.037	0	0	0
6	0	0	0	0.958	0.042	0	0
5	0	0	0	0	0.959	0.041	0
4	0	0	0	0	0	0.980	0.020
3	0	0	0	0	0	0	0.75

FIGURE 6.29: Timber superstructure mean baseline transition probability matrix

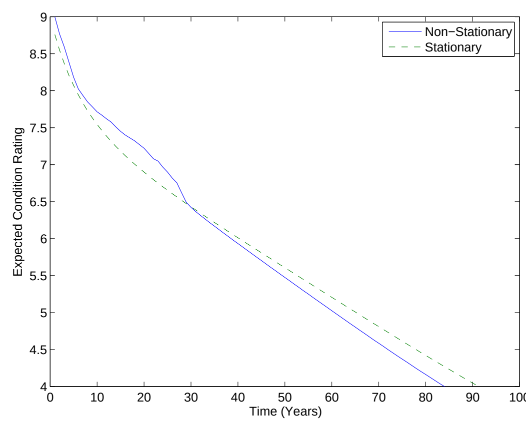

FIGURE 6.30: Timber superstructure baseline prediction model

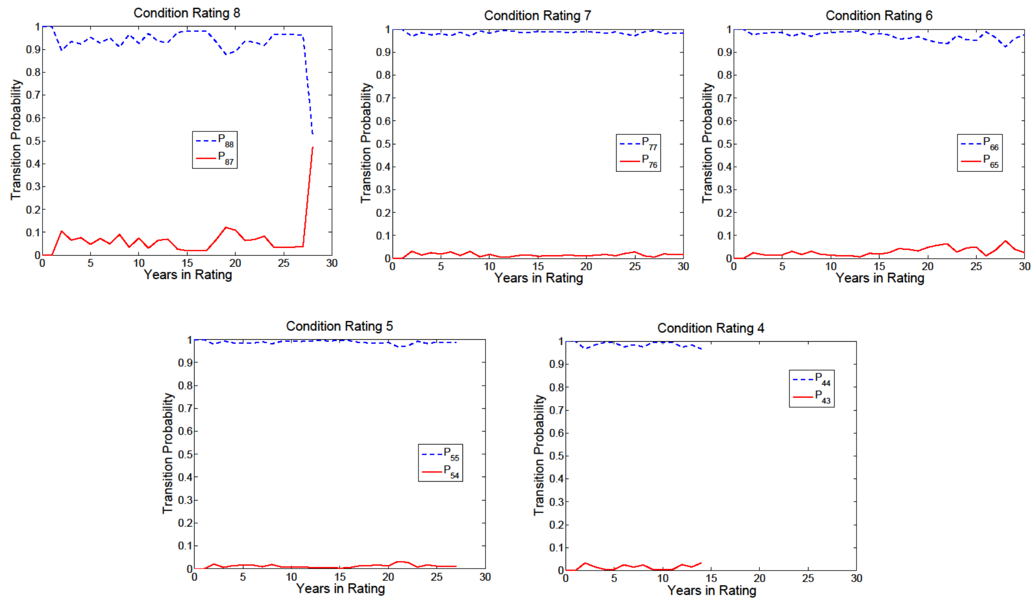

FIGURE 6.31: Concrete superstructure baseline transition probabilities

CR	8	7	6	5	4	3
8	0.929	0.071	0	0	0	0
7	0	0.985	0.015	0	0	0
6	0	0	0.971	0.029	0	0
5	0	0	0	0.988	0.012	0
4	0	0	0	0	0.985	0.015
3	0	0	0	0	0	0.75

FIGURE 6.32: Concrete superstructure mean baseline transition probability matrix

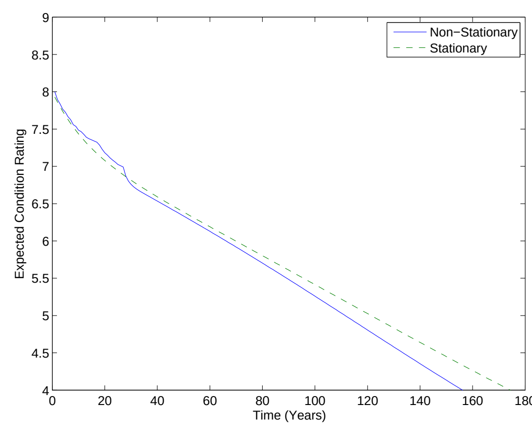

FIGURE 6.33: Concrete superstructure baseline prediction model

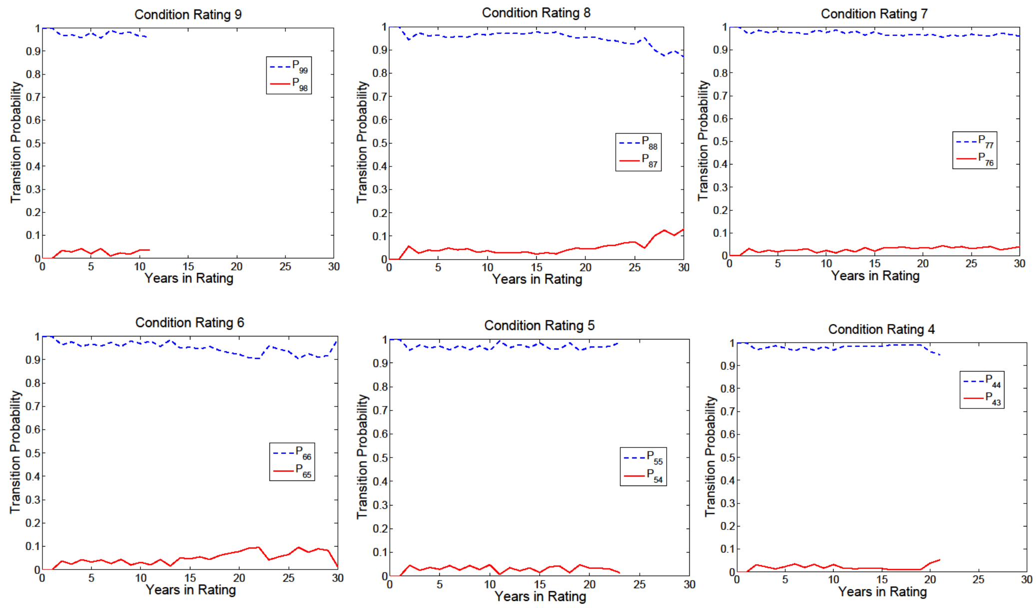

FIGURE 6.34: Steel superstructure baseline transition probabilities

CR	9	8	7	6	5	4	3
9	0.973	0.027	0	0	0	0	0
8	0	0.950	0.050	0	0	0	0
7	0	0	0.972	0.028	0	0	0
6	0	0	0	0.950	0.050	0	0
5	0	0	0	0	0.970	0.030	0
4	0	0	0	0	0	0.979	0.021
3	0	0	0	0	0	0	0.75

FIGURE 6.35: Steel superstructure mean baseline transition probability matrix

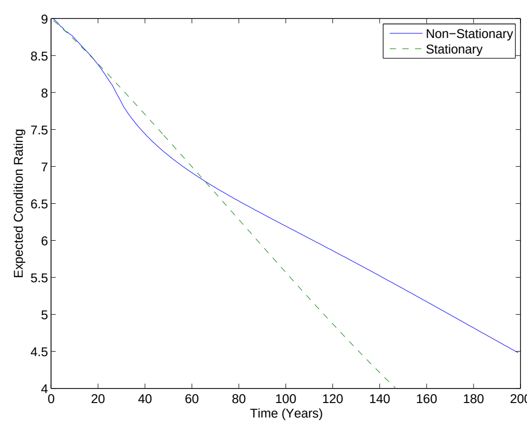

FIGURE 6.36: Steel superstructure baseline prediction model

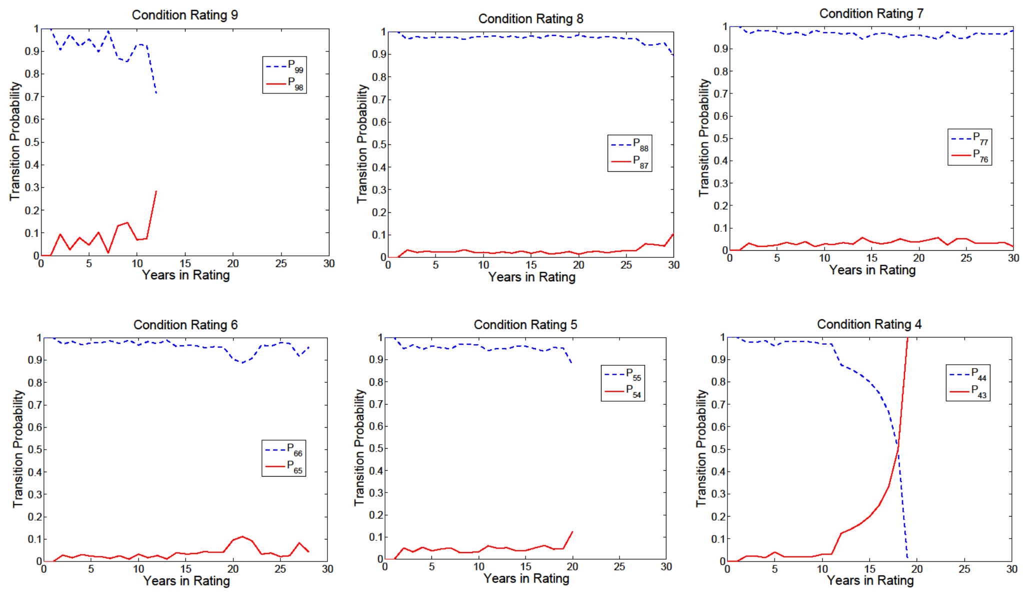

FIGURE 6.37: Prestressed concrete superstructure baseline transition probabilities

CR	9	8	7	6	5	4	3
9	0.911	0.089	0	0	0	0	0
8	0	0.968	0.032	0	0	0	0
7	0	0	0.967	0.033	0	0	0
6	0	0	0	0.963	0.037	0	0
5	0	0	0	0	0.953	0.047	0
4	0	0	0	0	0	0.844	0.156
3	0	0	0	0	0	0	0.75

FIGURE 6.38: Prestressed concrete superstructure mean baseline transition probability matrix

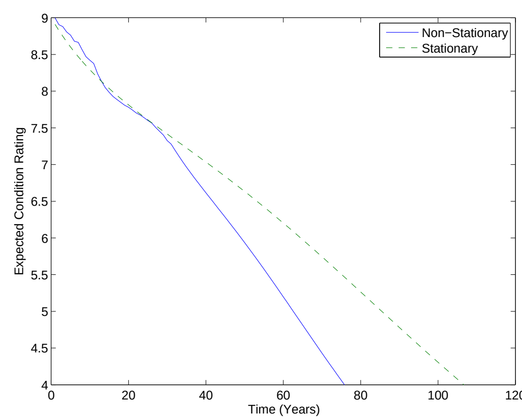

FIGURE 6.39: Prestressed concrete superstructure baseline prediction model

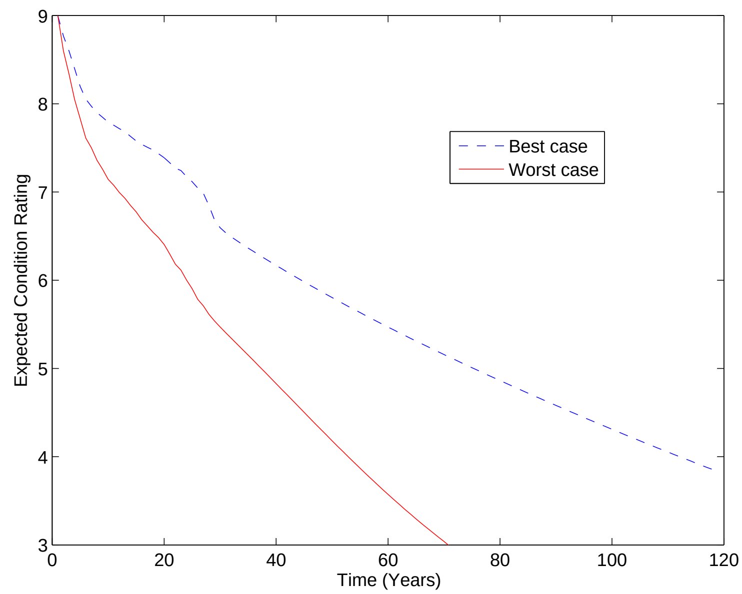

FIGURE 6.40: Timber superstructure forecast models for best case (State System 1, reconstructed, Piedmont region, ADT $239-555$, ADTT < 6, max span < 2m, single span, age < 22 years) and worst case (State System 2, original/rebuilt, Coastal region, ADT < 102, ADTT $15-34$, max span ≥ 2m, multi span, age ≥ 36 years) combination of covariates

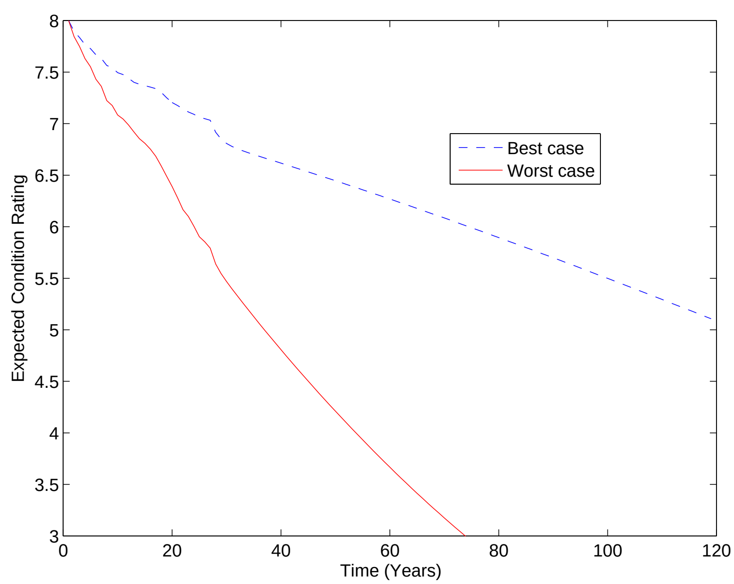

FIGURE 6.41: Concrete superstructure forecast models for best case (State System 1, Mountain region, max span < 3m, single span, age < 32 years) and worst case (State System 2, Piedmont region, max span ≥ 5m, multi span, age≥ 58 years) combination of covariates

and concrete superstructures, shown in Figures 6.40 and 6.41, respectively. It can be observed that the range of variation in superstructure deterioration rates expressed in the developed PHM models is significantly larger in comparison to the variation observed in the respective deterministic models shown earlier. This suggests that the PHM models are more robust at distinguishing the impact of explanatory factors on historically observed deterioration rates of bridge components.

6.3 Bridge Substructure Deterioration Models

Bridge substructures are classified by substructure material type using the fields for abutment material type, pier material type, and 'Structure Type - Main', in the NBI database in this order of priority. Accordingly, the primary classification into four categories, based on the construction material of Timber, Concrete, Steel, and Prestressed Concrete, follows the same approach described for superstructures in Section 6.2. For development of deterministic deterioration models, this primary classification is further classified (Tier 2) on the basis of geographic region, as shown in Table 6.19. Geographic region is also one of the potential variables analyzed in the best subset selection routine of the proportional hazards regression, wherein the Coastal region serves as the baseline or reference category. The updated substructure deterministic deterioration models for individual material types are presented in Figures 6.42 to 6.45.

TABLE 6.19: Tier 2 Region classification applied to each substructure material type prior to construction of deterministic deterioration models

Tier 2	**Region**
1	Coastal
2	Piedmont
3	Mountain

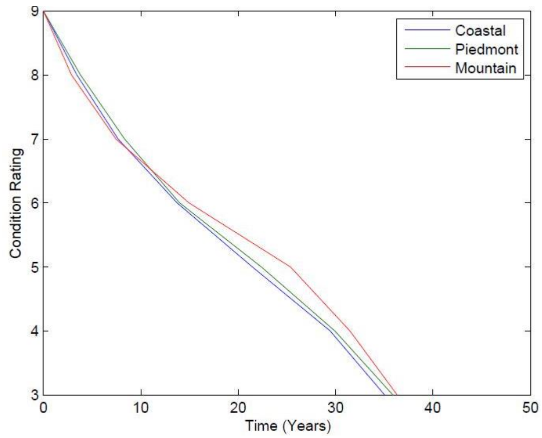

FIGURE 6.42: Timber substructure deterministic deterioration models

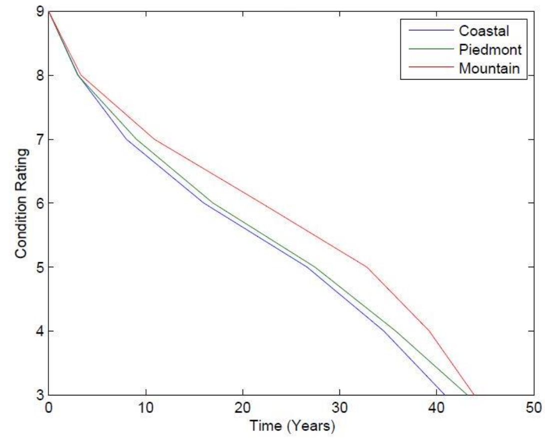

FIGURE 6.43: Concrete substructure deterministic deterioration models

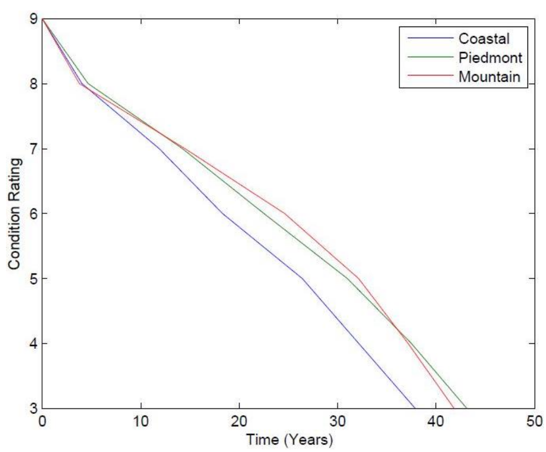

FIGURE 6.44: Steel substructure deterministic deterioration models

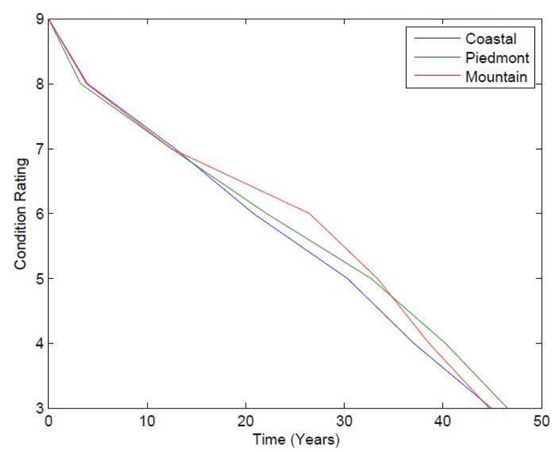

FIGURE 6.45: Prestressed concrete substructure deterministic deterioration models

6.3.1 Data Overview

Tables 6.20 to 6.23 present an overview of the individual continuously observed condition rating datasets available for development of timber, concrete, steel, and prestressed concrete substructure deterioration models. The data is observed to follow similar trends to those described earlier in Section 6.2.1 for superstructure condition rating data. As in the case of superstructures, the majority of the observations are associated with condition ratings 5 to 8, with censoring percentages varying in the range of 50-70% for condition ratings 6 to 8. Censored percentages of observations at condition ratings 4 and 5 are relatively higher and, as previously mentioned, are believed to be reflected in the higher priority for maintenance action as well as in the greater tendency for subjectivity in the inspection process. In the case of condition rating 9, there is a sufficient, yet much lower, number of observations to permit survival analysis across all material types. Table 6.24 presents the lower bounds for the categorical design variables developed for the continuous scale descriptive variables determined by weighted averages over all condition ratings for each substructure material.

6.3.2 Survival Analysis

The best subset variables identified using proportional hazards regression for timber, concrete, steel, and prestressed concrete substructure material types are presented in Tables 6.25 to 6.28 along with the associated hazard ratios and Wald statistic p-values. The corresponding survival functions developed over individual condition ratings are presented in Figures 6.46 to 6.49. The significant explanatory variables

TABLE 6.20: Timber substructure condition rating data overview

Substructure Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	73	55	18	24.66
8	1238	464	774	62.52
7	5545	2706	2839	51.20
6	10705	5639	5066	47.32
5	9093	2940	6153	67.67
4	4695	463	4232	90.14
3	1446	6	1440	99.59

TABLE 6.21: Concrete substructure condition rating data overview

Substructure Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	603	25	578	95.85
8	2185	883	1302	59.59
7	4578	1452	3126	68.28
6	4611	1919	2692	58.38
5	3322	528	2794	84.11
4	945	52	893	94.50
3	168	0	168	100.00

TABLE 6.22: Steel substructure condition rating data overview

Substructure Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	1266	159	1107	87.44
8	5189	2283	2906	56.00
7	6492	1759	4733	72.91
6	3050	1283	1767	57.93
5	1820	228	1592	87.47
4	423	24	399	94.33
3	95	4	91	95.79

TABLE 6.23: Prestressed concrete substructure condition rating data overview

Substructure Condition Rating	No. Total Records	No. Uncensored Records	No. Censored Records	% Censored Records
9	112	33	79	70.54
8	1132	350	782	69.08
7	1833	719	1114	60.77
6	1464	724	740	50.55
5	958	155	803	83.82
4	274	19	255	93.07
3	35	0	35	100.00

TABLE 6.24: Lower bounds of intervals developed for categorical design variables

Substructure Type	**Timber**			**Concrete**			**Steel**			**Prestressed Concrete**		
Category	2	3	4	2	3	4	2	3	4	2	3	4
ADT	124	314	768	289	1100	5102	745	3249	9862	2135	5336	11092
ADTT	8	19	48	19	88	514	54	261	1241	143	456	1387
Age(Years)	21	29	36	15	26	39	10	15	23	15	23	32
MaxSpan(m)	2	3	*	4	5	*	5	8	*	5	7	*

included in best subset selection for deterioration models of all substructure material types include the common variables of reconstruction, geographic region, number of spans and age. An increase in maximum span length is associated with an increase in hazard ratios for all material types, except for prestressed concrete substructures. This trend is similar to that observed in the case of bridge decks. Studies on bridge deterioration behavior have largely been limited to deck deterioration and very few references to the impact of explanatory variables on the deterioration of bridge superstructure and substructure components are available. The general literature related to substructure deterioration is mainly related to the effect of exposure to saltwater, which has been found to significantly exacerbate substructure deteriortion (Abed-Al-Rahim and Johnston, 1991). The effect is reflected in the results of the proportional hazards regression performed in the present study within the often significantly lower hazard ratios predominantly developed for the Piedmont and Mountain regions across all substructure material types. Also notable in the developed hazard ratios is an increase in substructure deterioration rates with an increase in age. This response is consistent with the observed effect of age on deck and superstructure deterioration rates. However, an increase in the number of spans was found to cause an increase in deterioration rates only in the case of timber and concrete substructures, whereas a substantially contrary effect was observed in the case of steel and prestressed con-

TABLE 6.25: Timber substructure best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
'StateSystem'	*	*	2.415	0.002	1.539	< 0.001	1.232	< 0.001	1.094	0.182	*	*
'Reconstruction'	*	*	1.573	0.003	1.230	0.004	1.311	< 0.001	*	*	*	*
'Piedmont'	*	*	*	*	*	*	*	*	1.116	0.011	0.788	0.020
'Mountain'	*	*	*	*	*	*	0.928	0.015	0.777	< 0.001	0.458	< 0.001
'MaxSpan2'	*	*	*	*	1.182	< 0.001	*	*	*	*	0.742	0.026
'NumberSpans'	*	*	*	*	*	*	1.272	< 0.001	1.190	< 0.001	*	*
'Age2'	*	*	1.801	< 0.001	1.581	< 0.001	1.127	0.002	*	*	*	*
'Age3'	1.789	0.209	2.352	< 0.001	1.726	< 0.001	1.427	< 0.001	1.100	0.022	*	*
'Age4'	2.286	0.014	2.834	< 0.001	1.899	< 0.001	2.054	< 0.001	*	*	*	*

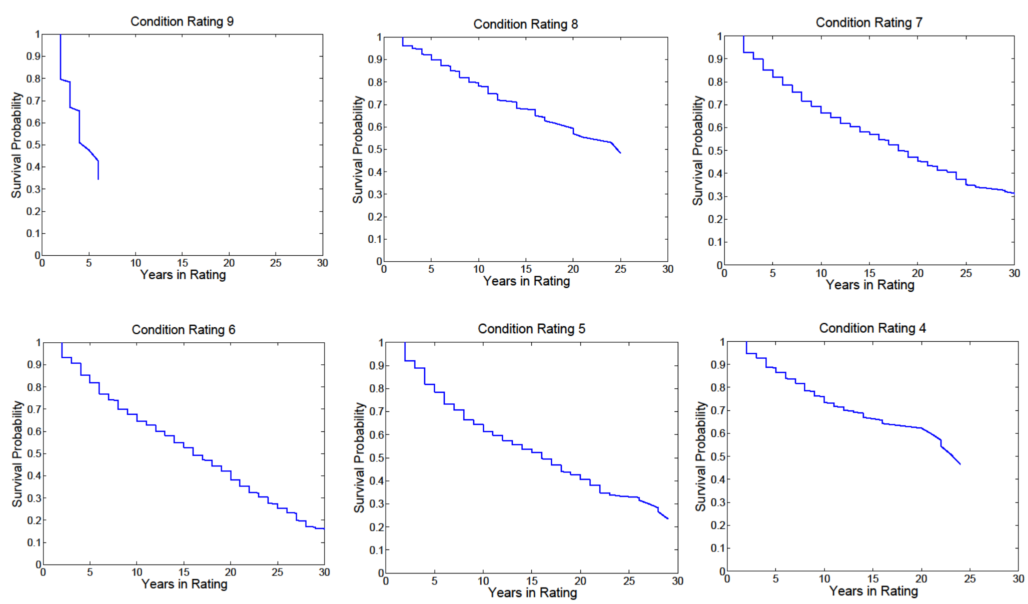

FIGURE 6.46: Timber substructure baseline survival functions

TABLE 6.26: Concrete substructure best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
'StateSystem'	*	*	*	*	*	*	*	*	0.855	0.130	*	*
'Reconstruction'	*	*	*	*	1.422	< 0.001	1.160	0.008	*	*	*	*
'Piedmont'	*	*	*	*	*	*	0.764	< 0.001	*	*	*	*
'Mountain'	*	*	0.730	< 0.001	0.821	< 0.001	0.839	0.015	*	*	*	*
'ADT3'	*	*	*	*	*	*	0.800	< 0.001	*	*	*	*
'ADT4'	*	*	0.728	< 0.001	0.789	0.002	0.747	< 0.001	*	*	*	*
'ADTT2'	*	*	*	*	1.243	< 0.001	*	*	*	*	*	*
'ADTT3'	2.305	0.041	*	*	*	*	*	*	*	*	*	*
'ADTT4'	*	*	*	*	*	*	*	*	0.755	0.022	*	*
'MaxSpan3'	*	*	1.481	< 0.001	*	*	1.171	0.011	*	*	0.490	0.064
'NumberSpans'	*	*	*	*	1.230	< 0.001	1.412	< 0.001	1.674	< 0.001	*	*
'Age2'	7.164	0.001	*	*	1.959	< 0.001	*	*	*	*	*	*
'Age3'	*	*	1.777	< 0.001	2.607	< 0.001	1.218	0.001	1.393	< 0.001	*	*
'Age4'	*	*	*	*	2.580	< 0.001	1.453	< 0.001	*	*	*	*

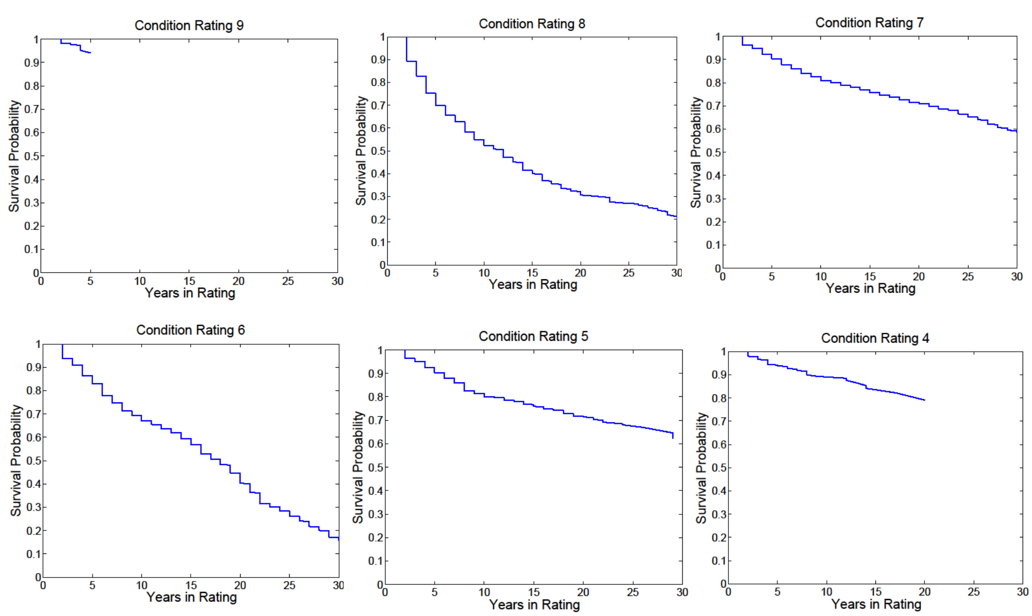

FIGURE 6.47: Concrete substructure baseline survival functions

TABLE 6.27: Steel substructure best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
'StateSystem'	*	*	1.381	< 0.001	*	*	*	*	*	*	2.195	0.067
'Reconstruction'	*	*	*	*	1.747	< 0.001	1.396	0.008	*	*	*	*
'Piedmont'	0.413	< 0.001	1.191	0.007	0.713	< 0.001	0.613	< 0.001	*	*	*	*
'Mountain'	0.489	0.007	0.789	0.001	0.557	< 0.001	0.703	< 0.001	*	*	*	*
'IntegralConcrete'	*	*	0.083	< 0.001	*	*	0.037	< 0.001	*	*	*	*
'LatexConcrete'	*	*	*	*	*	*	1.657	0.008	*	*	*	*
'Timber'	*	*	*	*	*	*	0.543	0.014	*	*	*	*
'ADT2'	1.503	0.024	*	*	*	*	*	*	*	*	*	*
'ADTT2'	*	*	*	*	*	*	*	*	1.418	0.013	*	*
'MaxSpan2'	*	*	1.183	0.002	*	*	*	*	*	*	*	*
'MaxSpan3'	*	*	1.519	< 0.001	*	*	*	*	*	*	*	*
'NumberSpans'	*	*	0.805	0.003	*	*	*	*	0.571	0.003	*	*
'Age2'	14.409	< 0.001	2.268	< 0.001	*	*	*	*	*	*	*	*
'Age3'	*	*	2.563	< 0.001	1.524	< 0.001	1.816	< 0.001	*	*	*	*
'Age4'	*	*	3.590	< 0.001	2.383	< 0.001	2.934	< 0.001	*	*	*	*

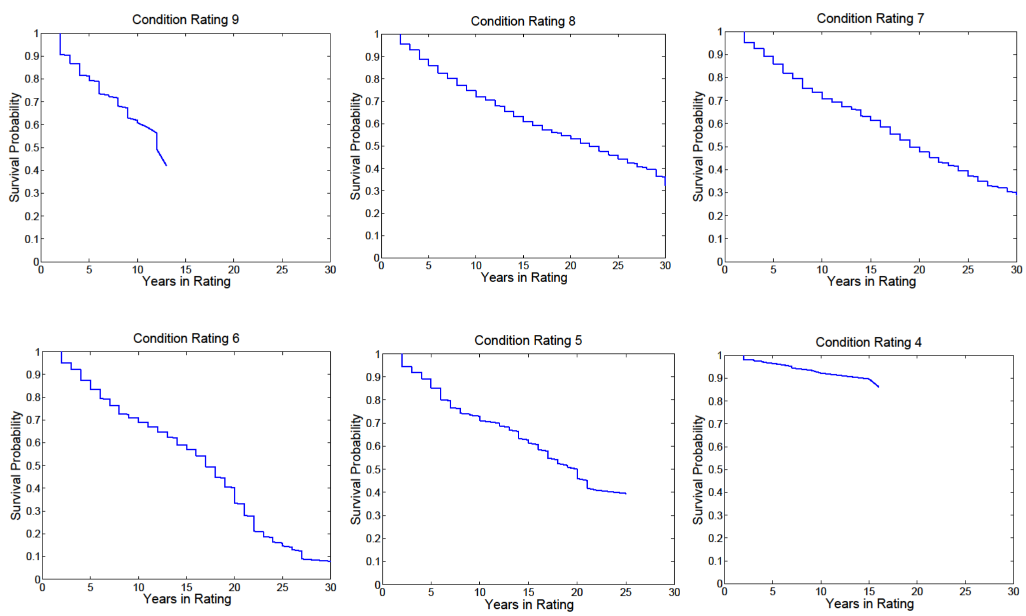

FIGURE 6.48: Steel substructure baseline survival functions

TABLE 6.28: Prestressed concrete substructure best subset covariates, hazard ratios, and p-values

Rating	9		8		7		6		5		4	
Covariates	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**	**HR**	**p**
‘StateSystem’	*	*	*	*	*	*	*	*	1.465	0.069	*	*
‘Reconstruction’	*	*	*	*	*	*	*	*	0.300	0.019	*	*
‘Piedmont’	*	*	*	*	*	*	0.789	0.002	*	*	*	*
‘Mountain’	2.274	0.130	*	*	0.499	< 0.001	*	*	*	*	*	*
‘ADT4’	*	*	*	*	*	*	*	*	1.508	0.022	*	*
‘NumberSpans’	*	*	*	*	*	*	*	*	0.245	< 0.001	*	*
‘Age2’	*	*	1.740	< 0.001	1.295	0.014	*	*	*	*	*	*
‘Age3’	*	*	2.438	< 0.001	1.652	< 0.001	*	*	*	*	*	*
‘Age4’	*	*	5.253	< 0.001	2.750	< 0.001	1.498	< 0.001	*	*	0.626	0.311

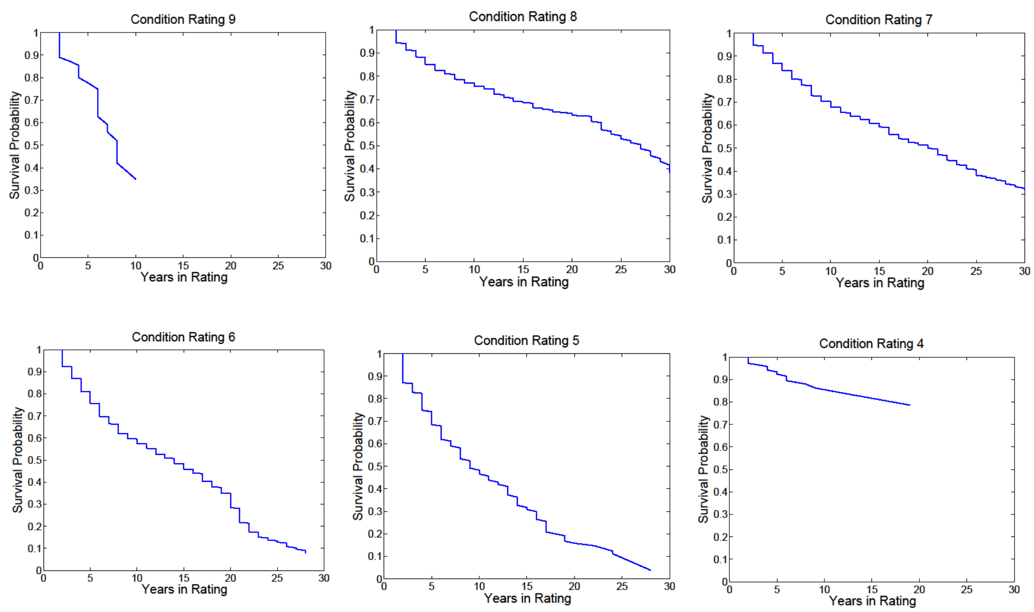

FIGURE 6.49: Prestressed concrete substructure baseline survival functions

crete substructures. Length of maximum span was identified as a significant variable in the case of concrete and steel substructures only and in both components increase in length of maximum span was found to increase the deterioration rate, which is consistent with the effect of these variable across all deck material types. The presence of reconstruction was once again found to most often increase deterioration rates except in the case of prestressed concrete substructures, in which, however, it was found to be significant for only condition rating 5, but with the reverse of the expected effect. Additionally, State System was found to be a significant variable only in the case of timber and steel substructures, for both of which bridges located on secondary routes are found to exhibit a higher deterioration rate compared to those located on interstates and primary routes. A similar observation was previously made for prestressed concrete decks in North Carolina and was attributed to potential variations in the design of prestressed concrete structures for low-volume routes (Abed-Al-Rahim and Johnston, 1991). An increased deterioration rate for concrete bridge decks located on secondary highways relative to interstates was also observed in another state, where the observation was attributed to lower design requirements and maintenance standards on secondary roads (Mauch and Madanat, 2001, Mishalani and Madanat, 2002).

Amongst wearing surface covariates, the presence of an integral concrete wearing surface was found to significantly decrease the deterioration rate of steel substructures in condition ratings 6 and 8. Latex concrete and timber wearing surfaces were also identified as significant, but also only in the case of steel substructures and only at condition rating 6. Timber wearing surface was found to decrease the deterioration rate, however, latex concrete was found to significantly increase the deterioration rate. This is only the second instance where wearing surface is included in the best subset for any of the deterioration models in this study, the first instance being in the case of steel superstructure models as described in Section 6.2.2. As mentioned previously, the wearing surface categories identified as significant for steel superstructure and substructure components are associated with very few records. The majority of records are associated with monolithic concrete and bituminous wearing surfaces, neither of which is included in any of the deterioration model best subsets. This finding, coupled with the fact that wearing surface covariates were not found to be significant in any of the other deterioration models including all the deck deterioration models, is indicative of the lack of reliability of the observed effects in these isolated instances.

6.3.3 Transition Probabilities and Expected Value Prediction Models

Figures 6.50 to 6.61 present the baseline transition probabilities, mean baseline transition probability matrices, and expected value prediction models developed for baseline assignments of best subset covariates associated with individual substructure material types. A comparison of stationary and non-stationary models reveals a similar agreement in predictions obtained from both models for approximately the initial 20 years of the planning horizon. A comparison with deck models reveals that, although decks exhibit a higher deterioration rate than substructures in general, the deterioration rates for timber decks and timber substructures appear to be similar.

Forecasted expected condition ratings over time are developed using hazard ratios to quantify the effect of best case and worst case combinations of covariates on the expected deterioration rates of timber and concrete substructures, shown in Figures 6.62 and 6.63, respectively. Comparison with the respective deterministic models, which are classified only on geographic region, reveals the markedly superior ability of PHM models to distinguish the effect of covariates on substructure deterioration rates.

6.4 Summary of Results and Conclusion

Throughout this chapter, the effects of explanatory factors on deterioration rates across different condition ratings were examined across material-specific GCR component models. General trends in the explanatory factors affecting deterioration rates across all components and their relative impact on deterioration rates are examined and summarized in this section. To reduce the state-dependent hazard ratios to a single index for ease of interpretation, weighted averages of hazard ratios expressed for each covariate across condition ratings 4 to 9 were computed for each material-specific GCR component in this analysis. The weighting is specified in proportion to the number of total records available for individual condition ratings. This weighting scheme reflects the certainty expressed in each hazard ratio and provides weighting factors similar to those that would be developed by weighting based on duration in each condition rating. The weighted mean HR values across all material specific deck, superstructure, and substructure components for the significant variables identified in the proportional hazards regression are presented in Table 6.29. In this table, only factors appearing in at least two material specific component models are presented. In order to rank the explanatory factors based on average significance, the variables

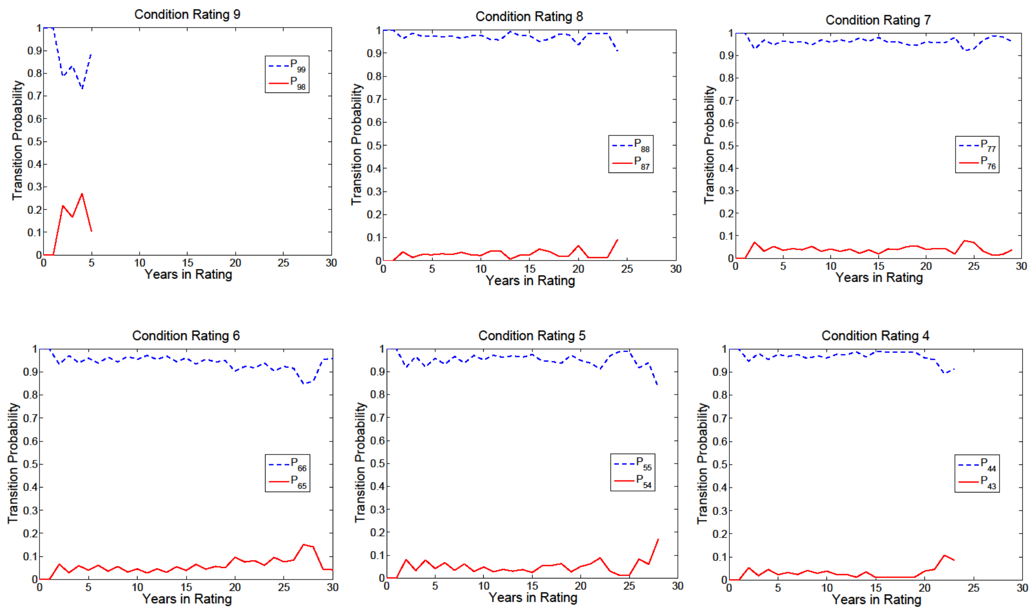

FIGURE 6.50: Timber substructure baseline transition probabilities

CR	9	8	7	6	5	4	3
9	0.849	0.151	0	0	0	0	0
8	0	0.970	0.030	0	0	0	0
7	0	0	0.961	0.039	0	0	0
6	0	0	0	0.940	0.060	0	0
5	0	0	0	0	0.950	0.050	0
4	0	0	0	0	0	0.968	0.032
3	0	0	0	0	0	0	0.75

FIGURE 6.51: Timber substructure mean baseline transition probability matrix

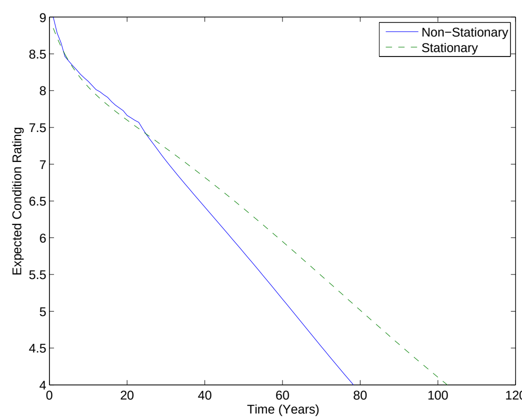

FIGURE 6.52: Timber substructure baseline prediction model

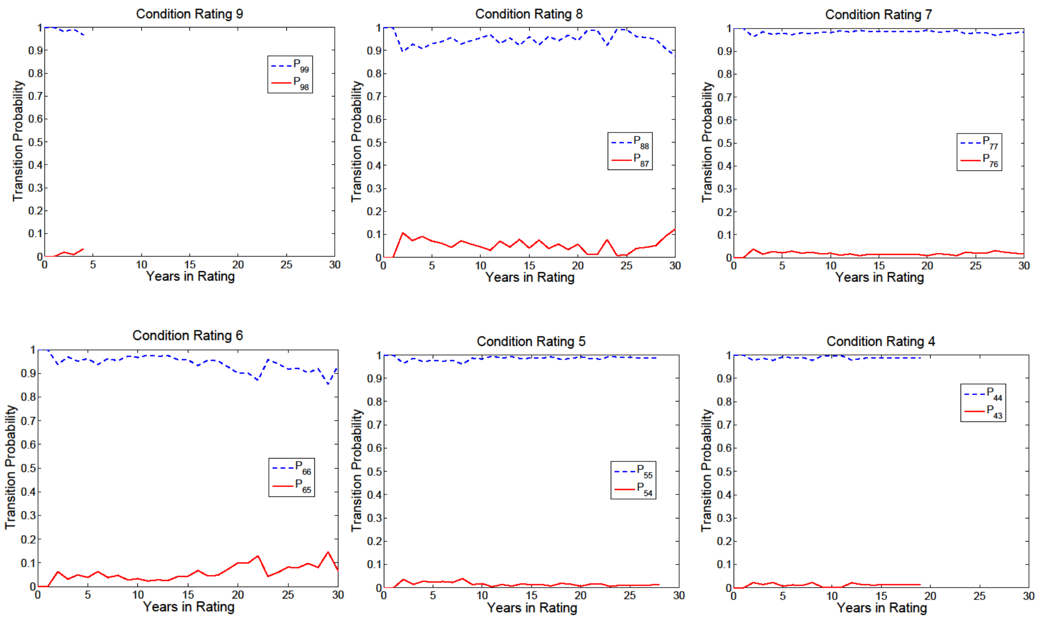

FIGURE 6.53: Concrete substructure baseline transition probabilities

CR	9	8	7	6	5	4	3
9	0.985	0.015	0	0	0	0	0
8	0	0.946	0.054	0	0	0	0
7	0	0	0.983	0.017	0	0	0
6	0	0	0	0.942	0.058	0	0
5	0	0	0	0	0.985	0.015	0
4	0	0	0	0	0	0.988	0.012
3	0	0	0	0	0	0	0.75

FIGURE 6.54: Concrete substructure mean baseline transition probability matrix

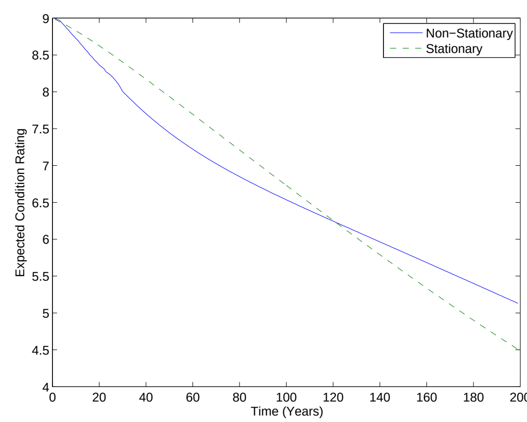

FIGURE 6.55: Concrete substructure baseline prediction model

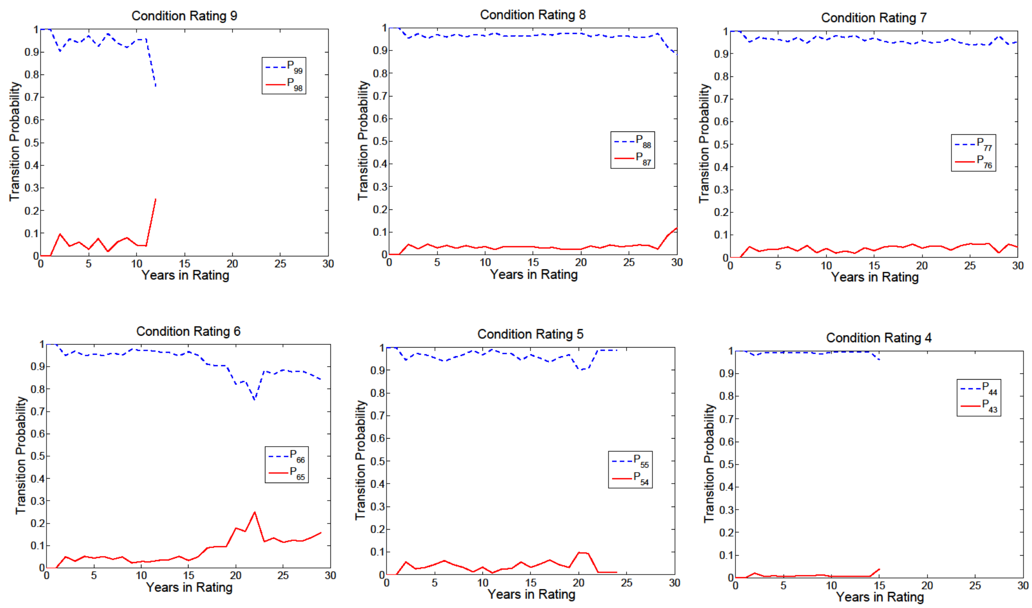

FIGURE 6.56: Steel substructure baseline transition probabilities

CR	9	8	7	6	5	4	3
9	0.933	0.067	0	0	0	0	0
8	0	0.961	0.039	0	0	0	0
7	0	0	0.959	0.041	0	0	0
6	0	0	0	0.918	0.082	0	0
5	0	0	0	0	0.962	0.038	0
4	0	0	0	0	0	0.990	0.010
3	0	0	0	0	0	0	0.75

FIGURE 6.57: Steel substructure mean baseline transition probability matrix

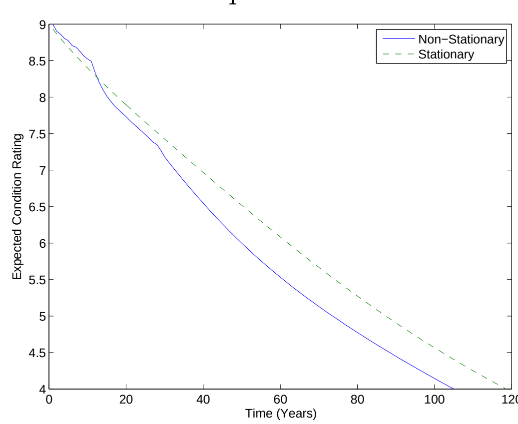

FIGURE 6.58: Steel substructure baseline prediction model

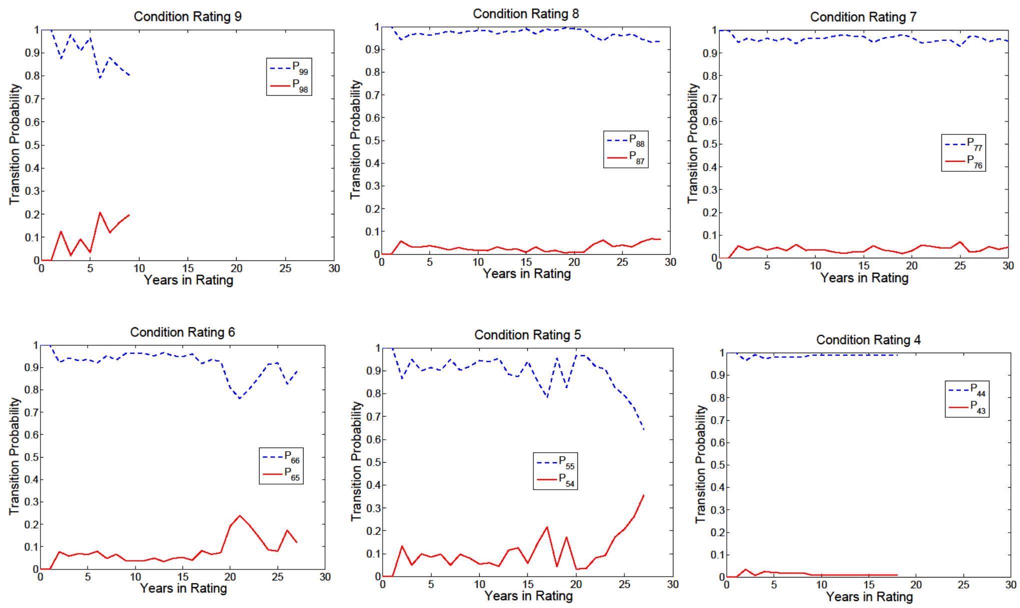

FIGURE 6.59: Prestressed concrete substructure baseline transition probabilities

CR	9	8	7	6	5	4	3
9	0.893	0.107	0	0	0	0	0
8	0	0.970	0.030	0	0	0	0
7	0	0	0.962	0.038	0	0	0
6	0	0	0	0.917	0.083	0	0
5	0	0	0	0	0.890	0.110	0
4	0	0	0	0	0	0.987	0.013
3	0	0	0	0	0	0	0.75

FIGURE 6.60: Prestressed concrete substructure mean basline transition probability matrix

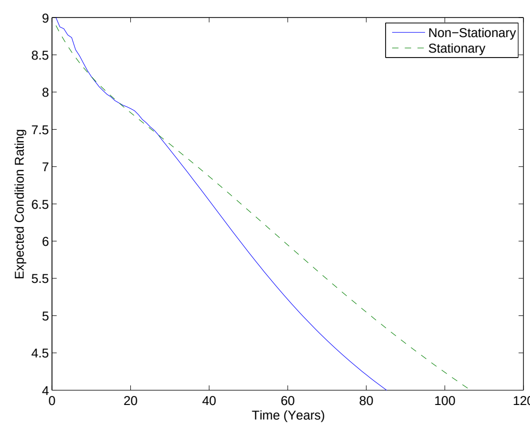

FIGURE 6.61: Prestressed concrete substructure baseline prediction model

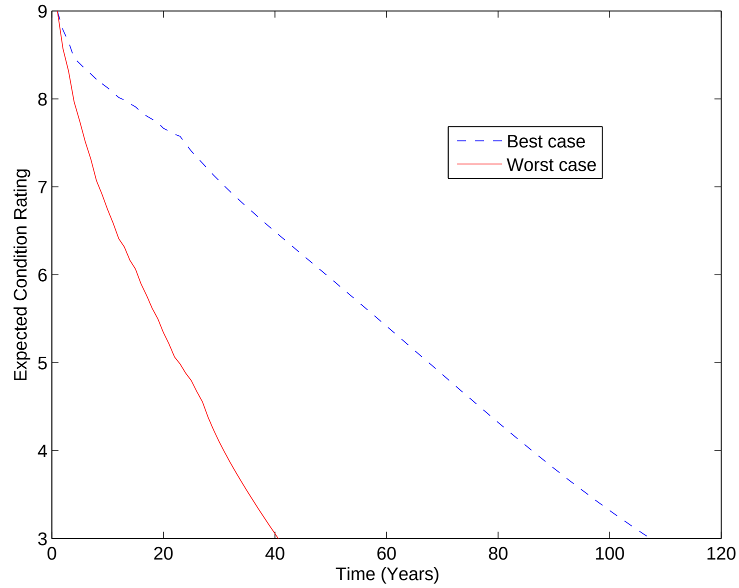

FIGURE 6.62: Timber substructure forecast models for best case (State System 1, original/rebuilt, Mountain region, max span $2-3$m, single span, age <21 years) and worst case (State System 2, reconstructed, Piedmont region, max span <2m, multi span, age≥ 36 years) combination of covariates

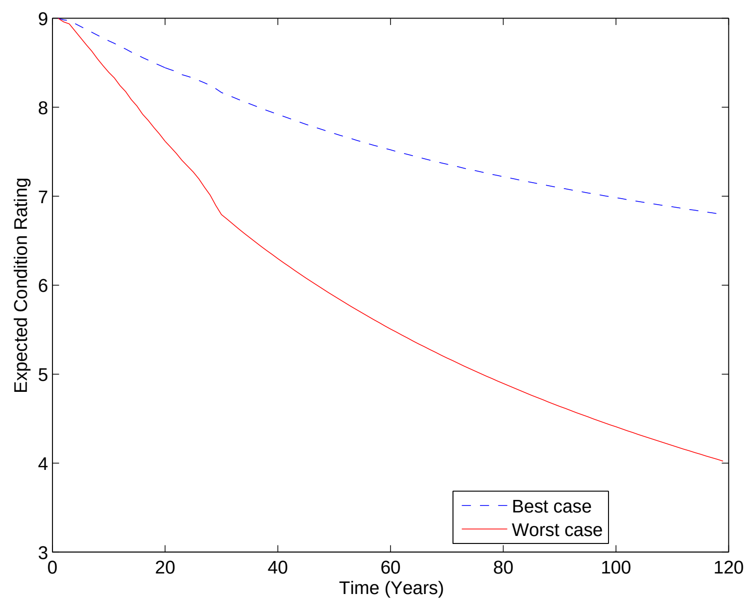

FIGURE 6.63: Concrete substructure forecast models for best case (State System 1,original/rebuilt, Piedmont/Mountain region, ADT≥ 1100, ADTT$19-514$, max span <4m, single span, age <15 years) and worst case (State System 2, reconstructed, Coastal region, ADT<289, ADTT≥ 514, max span ≥ 5m, multi span, age ≥ 39 years) combination of covariates

have been sorted on basis of the mean absolute deviation from unity obtained by averaging across the mean weighted HRs for all material-specific components.

TABLE 6.29: Weighted mean covariate hazard ratios across condition ratings 4 to 9 for all material-specific GCR components

Component	**Deck**			**Superstructure**				**Substructure**			
Material	**Timber**	**Conc**	**Steel**	**Timber**	**Conc**	**Steel**	**Prestr**	**Timber**	**Conc**	**Steel**	**Prestr**
Age4	1.784	1.986	2.729	1.656	1.845	2.963	2.581	1.594	1.574	2.552	2.498
Age3	1.766	1.578	1.789	1.652	1.612	2.210	2.386	1.358	1.700	1.767	1.489
Age2	1.595	1.330	1.639	1.455	1.337	1.478	2.266	1.178	1.499	2.291	1.239
NumberSpans	1.154	1.255	1.219	1.212	1.638	1.083	1.057	1.148	1.320	0.902	0.875
Reconstruction	1.010	1.000	1.093	0.972	1.000	1.301	1.000	1.169	1.165	1.332	0.884
Mountain	1.131	0.884	0.881	1.000	0.952	0.960	0.837	0.830	0.868	0.697	0.866
Piedmont	1.071	1.034	0.839	0.886	1.036	0.952	0.795	1.002	0.933	0.847	0.947
MaxSpan 3	1.096	1.260	1.286	1.079	1.274	1.011	1.001	1.000	1.084	1.148	1.000
Max Span 2	1.099	1.016	1.355	1.000	1.143	0.946	0.970	0.994	1.000	1.052	1.000
State System	0.958	0.991	0.961	1.135	0.983	1.070	1.043	1.258	0.970	1.136	1.077
IntegralConcrete	1.000	1.000	1.000	1.000	1.000	0.687	1.000	1.000	1.000	0.578	1.000
ADT4	1.036	1.091	1.000	1.000	1.000	1.095	0.929	1.000	0.832	1.000	1.084
ADT3	1.000	1.036	1.000	0.878	1.000	1.087	0.977	1.000	0.943	1.000	1.000
ADT2	1.000	1.000	1.000	1.000	1.000	1.080	1.000	1.000	1.000	1.035	1.000
ADTT3	1.024	1.000	1.000	1.209	1.000	1.000	1.178	1.000	1.048	1.000	1.000
ADTT2	1.000	1.000	1.000	1.000	1.000	1.000	1.079	1.000	1.069	1.042	1.000
ADTT4	1.050	1.000	1.000	1.000	1.000	0.994	1.000	1.000	0.950	1.000	1.000

The weighted mean covariate hazard ratios exhibit generally consistent effects across the different components and material types, although the amplitudes of the hazard ratios do vary by both material type and component. In all instances, increased age was associated with the most significant increased rate of deterioration and, on average, the rates of deterioration increase with the age categories. The weighted means for the age covariates vary more significantly by material type than component type. Steel and prestressed concrete components were found to be more significantly affected by age than timber and concrete components. The effect of multispan bridge designs was also found to consistently increase the deterioration rates relative to single span bridge designs with the exception of the steel and prestressed concrete substructure components. Concrete superstructures were found to be affected significantly more by multispan design than the remaining material-specific components, although the effect of multispan design was also notably significant for concrete substructures and concrete decks. The presence of reconstruction was found to exhibit a mild to moderate effect on deterioration rates with the trend toward increasing the deterioration rate for all material-specific components with the exception of timber superstructures and prestressed concrete substructures. Similarly, the impact of geographic region was moderately significant across the majority of models. Predominantly, bridge components in the Coastal region were found to deteriorate at a faster rate than bridge components in the Mountain and Piedmont regions, which were found to generally deteriorate on average at similar rates. This effect was found to be most significantly expressed in steel bridge components, although concrete and prestressed concrete substructures were also found to deteriorate at an accelerated rate in the Coastal Region. Maximum span length was found to be mildly to moderately significant on average across the material-specific components. However, this covariate expresses no clear trends across the different models except that increased span length was associated with increased deterioration rates of all deck materials. Interestingly, bridge decks on structures servicing secondary routes were found to consistently deteriorate at a slightly slower rate than those on interstate, urban, and primary routes. However, the opposite effect was identified for superstructure and substructure components in this study, which were found to exhibit faster rates of deterioration in State System 2, with the exception of concrete superstructures and substructures. The remaining covariates of wearing surface type, ADT, and ADTT were found to exhibit little or no average effect on deterioration rates of the bridge component ratings. The general consistency and presence of clear trends exhibited by the weighted mean covariate hazard ratios across the different components and different material types serves to support the plausibility of the results generated by the developed proportional hazards-based deterioration modeling framework.

CHAPTER 7: MODEL ASSESSMENT

The main objective of deterioration modeling is to predict future condition ratings of bridge components, which is critical to the accurate identification and selection of MR&R projects within the multi-objective constrained optimization techniques used for data-driven transportation planning. Limited MR&R budgets constrain the selection to a limited number of candidate bridges in need of repairs or rehabilitation based on anticipated future needs and performance objectives. The accuracy of condition rating forecasts over the planning horizon directly affects how effectively the benefits of allocating resources for the immediate preservation or replacement of bridges in the inventory is maximized, while anticipating future increases in costs due to postponement of repairs to other structures under the same budget constraints (Patidar et al., 2007). This chapter demonstrates the improvement in accuracy and precision of condition rating predictions obtained through the developed multi-variable proportional hazards based modeling framework described in the previous chapters. An approach for model assessment is presented in the first section followed by comparison of prediction results obtained from the updated deterministic deterioration models and the proposed probabilistic models. The impact of including the multiple covariates identified in the proportional hazards regression on the predictive fidelity is also evaluated by developing simplified probabilistic models based on solely empirical Kaplan-Meier survival analysis. The prediction errors obtained from use of these simplified models are compared with the prediction errors obtained from use of the fully developed PHM models.

7.1 Description of Approach for Model Assessment

Ideally, model assessment should be performed using data independent from the records used to develop the statistical models. However, due to the rate of collection of bridge condition rating data, assessment of the predictive fidelity of the developed deterioration models to future response data is not possible for several years. Consequently, model assessment is performed using a select 15 year time period of condition rating data extracted from the existing NCDOT historical bridge management database. For the assessment results presented in this chapter, the period from 2000 to 2015 was selected. Individual material-specific GCR databases were then parsed for condition rating records available for this period. Only those records that were continuously observed between the starting year and the ending year of the observation period were included. Additionally, to minimize the presence of observations with significant maintenance or reconstruction actions, the data was filtered to include only records where the observed condition ratings either remained the same or decreased from the initial condition rating over the selected observation period. This data preprocessing aims to extract records exhibiting natural deterioration, however it should be recognized that some records without actual maintenance may have been inadvertently removed due to variations in condition rating resulting from inspection subjectivity. Additionally, some records may be present that include significant maintenance that may have prolonged the duration in condition ratings.

The filtering algorithm, however, removed records which exhibited an improvement in condition rating at any point over the observation period. Further, if the condition rating decreased by more than one rating over a single inspection cycle, then the data was likewise treated as anomalous and removed from the dataset used for model assessment.

For each of the continuously observed condition rating records satisfying the filtering criteria, covariate assignments for each record were developed from the descriptive data in the bridge record to facilitate selection of the corresponding deterministic or probabilistic model applicable to each individual bridge component. These included ADT, region, state system, and main structure design type for deterministic models and other variables including reconstruction, ADT, ADTT, wearing surface, length of maximum span, number of spans, and age that are associated with hazard ratios in the probabilistic proportional hazards models. The covariate assignments are consistent with the classification scheme prescribed for the deterministic deterioration models and those associated with reference cell coding for probabilistic models described in Chapter 4.

In addition to the current rating at the start of the observation period, it was necessary to determine the duration already spent by the bridge component at that initial condition rating to incorporate this duration into the predictions generated by the respective deterioration models. Using the sum of this initial duration and the 15 year period of the planning horizon, expected condition ratings at the end of the prediction period (2015) were obtained for each individual bridge record using the deterministic as well as the probabilistic deterioration models. The stationary Markov probabilistic deterioration model was used for all model assessment results presented in this chapter. As demonstrated throughout Chapter 6, the expected condition rating predictions obtained from the stationary models coincide almost exactly with those obtained from the more rigorous non-stationary models over planning horizons less than 20 years. The actual observed condition rating at the end of the 15-year observation period was subtracted from the predicted condition rating obtained from each model to compute the prediction errors associated with each model for all selected records. Statistics of the prediction error distributions are used to reveal the relative accuracy, precision, and confidence afforded by the deterioration models with respect to the actual observed response data.

7.2 Comparison of Predictive Fidelity of Deterministic and Probabilistic Models

Results from implementation of the model assessment routine described above on the timber deck condition rating data in the NCDOT bridge inventory are presented in Figures 7.1 and 7.2. Figure 7.1 shows the histograms of prediction error distributions for deterministic and probabilistic models along with the mean errors and standard deviations. It can be readily observed that the mean error of -0.62 for the probabilistic prediction is much smaller than the mean error of -1.82 for the deterministic prediction, which indicates that the probabilistic deterioration models are significantly more accurate than the deterministic models. The negative error value for both indicates that the predicted condition ratings from both models are lower than the actually observed condition rating. Although both models provide the benefit of generally conservative predictions, the overly conservative deterministic model

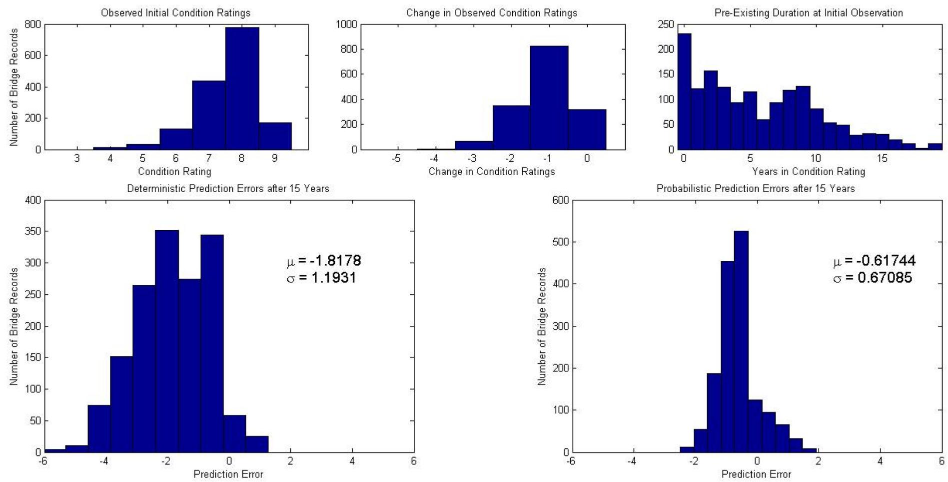

FIGURE 7.1: Histograms of timber deck condition rating data in observation period and prediction errors

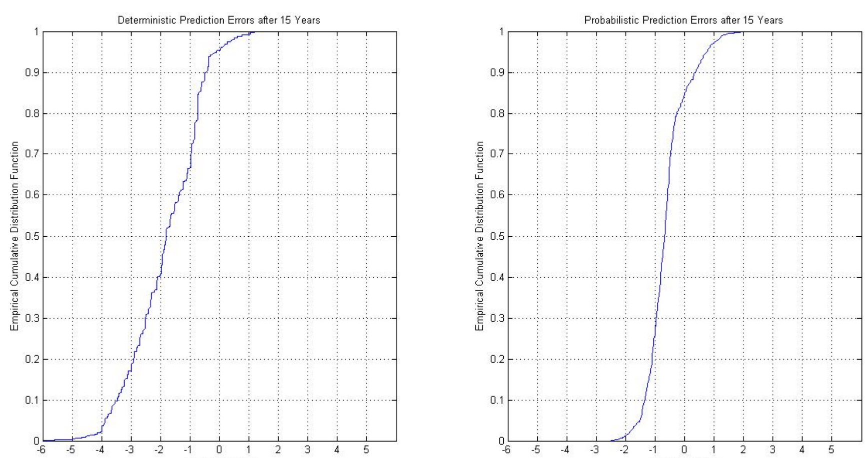

FIGURE 7.2: Empirical cumulative distribution functions associated with prediction errors of deterministic and probabilistic models applied to timber deck condition rating data

predictions can have the undesirable result of overestimating bridge repair needs and, since condition ratings are incorporated into aspects of the user cost models, the associated costs (Patidar et al., 2007). This can become a significant limitation especially when using the latest BMS optimization tools, such as those developed recently by NCHRP and FDOT, which are highly sensitive to any changes in deterioration or unit cost inputs (Patidar et al., 2007, Sobanjo and Thompson, 2011).

The distribution of the prediction errors from the probabilistic model also has a smaller standard deviation than that of the deterministic model, which indicates improved precision and is visible within the spread in the corresponding histograms. Figure 7.1 also provides histograms of features of the observed condition rating data, including the initial condition ratings observed at the starting year, the relative change in condition ratings over the 15 year prediction period, and the continuous durations that condition ratings had already accumulated at the start of the time windowed 15 year observation period. The empirical cumulative distribution function of the prediction errors from each of the models relative to the observed timber deck condition ratings is presented in Figure 7.2. These cumulative distribution functions are particularly useful for estimating the probability that the prediction will be within a prescribed bounded interval over the analyzed planning horizon. Since condition ratings are assigned on an integer scale, the interval of interest might correspond to +/-1 condition rating. It can be observed that this probability is only about 30% for the deterministic timber deck models. On the other hand, the probability that the probabilistic timber deck model will estimate the actual condition rating within +/-1 over a 15 year planning horizon is close to 75%.

Comparative histograms of prediction errors generated over the same planning period for concrete decks and steel decks are presented in Figures 7.3 and 7.4, respectively. It can be observed in both cases that the mean errors and standard deviations are significantly smaller for probabilistic models than for deterministic models thereby confirming the improvements in accuracy and precision. The probability that the respective deterioration models will predict the actual condition rating over the 15 year planning horizon +/-1 condition rating was found to be 22% for deterministic models and 62% for probabilistic models in the case of the concrete deck models. Likewise, it was 29% for deterministic models and 48% for probabilistic models in the case of the steel deck models, as determined from the respective empirical cumulative distribution functions of prediction errors. Similar results were obtained for the cases of superstructure and substructure data as well. These model assessment results therefore present a strong case to support the implementation of probabilistic models over deterministic deterioration models in the interest of significantly improved accuracy and precision in condition rating forecasting. Furthermore, the transition to more accurate and precise deterioration models carries important implications for more efficient management of the complete bridge inventory.

7.3 Evaluating the Effectiveness of Covariate Inclusion in the Probabilistic Models

Multivariable probabilistic deterioration models have been developed for the first time in this study. To evaluate the contribution of covariates in enhancing the accuracy of the models, an approach was adopted that consisted of first developing Markov chain transition probabilities on basis of empirical Kaplan-Meier survival functions

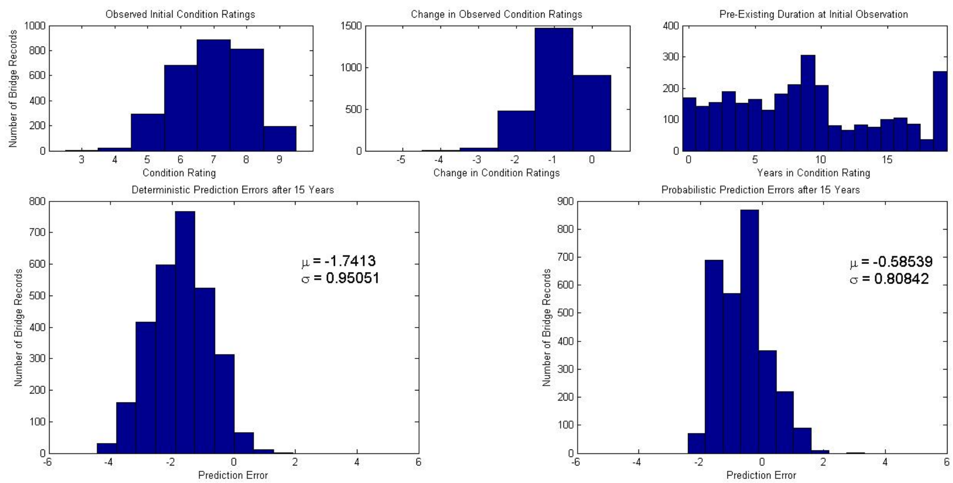

FIGURE 7.3: Histograms of concrete deck condition rating data in observation period and prediction errors

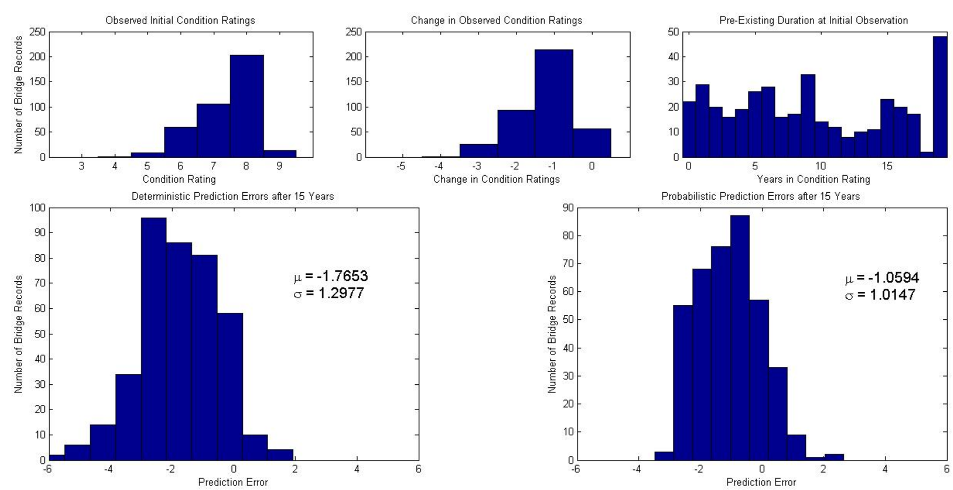

FIGURE 7.4: Histograms of steel deck condition rating data in observation period and prediction errors

across individual condition ratings for the material-specific GCR components. These Kaplan-Meier based transition probabilities utilize the same datasets used for construction of the PHM survival functions but do not explicitly include the effect of covariates. Condition rating predictions were made using the previously described filtered dataset with the Kaplan-Meier deterioration models to permit comparison with the condition rating predictions obtained from the corresponding PHM model that includes covariate hazard ratios. Figures 7.5 and 7.6 show the empirical cumulative distribution functions and the normal probability plots, respectively, associated with the prediction errors from both of these models for the concrete deck dataset. It is observed from Figure 7.5 that the mean errors and mean standard deviation of errors from both the models are comparable, however the PHM model errors appear to be relatively more normally distributed. This is more clearly evident in the normal probability plots presented in Figure 7.6, in which the PHM model residuals exhibit less skew than the Kaplan-Meier model residuals. The closer to normal distribution of PHM model residuals is indicative of its relatively better performance than the Kaplan-Meier model.

It should be noted that, although the Kaplan-Meier model appears to be more accurate by the lower mean error, the mean error is an imprecise measure of accuracy on account of the discrete nature of the recorded condition ratings and continuous nature of the expected value predictions. Furthermore, the accuracy of the prediction ratings is sensitive to the probabilities assigned to the initial state vector. For the generation of all prior predictions, the initial state vector was developed assuming a probability of 1 that the condition rating of the component was directly as observed

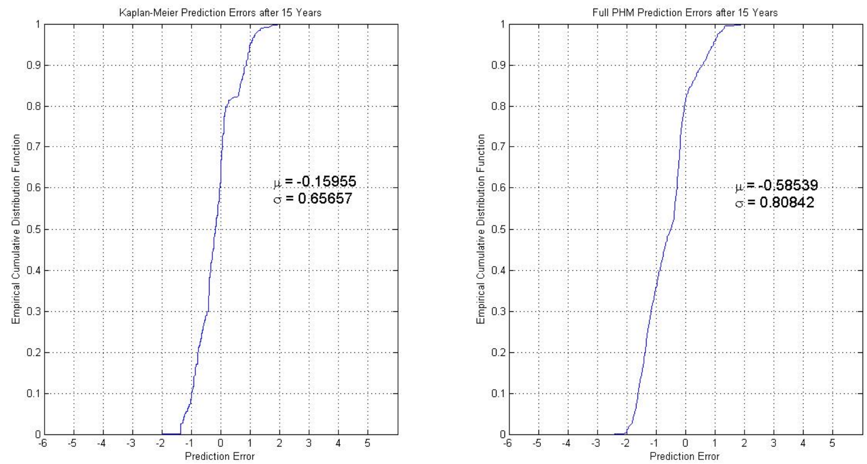

FIGURE 7.5: Empirical cumulative distribution functions associated with prediction errors of Kaplan-Meier and PHM models applied to concrete deck condition rating data

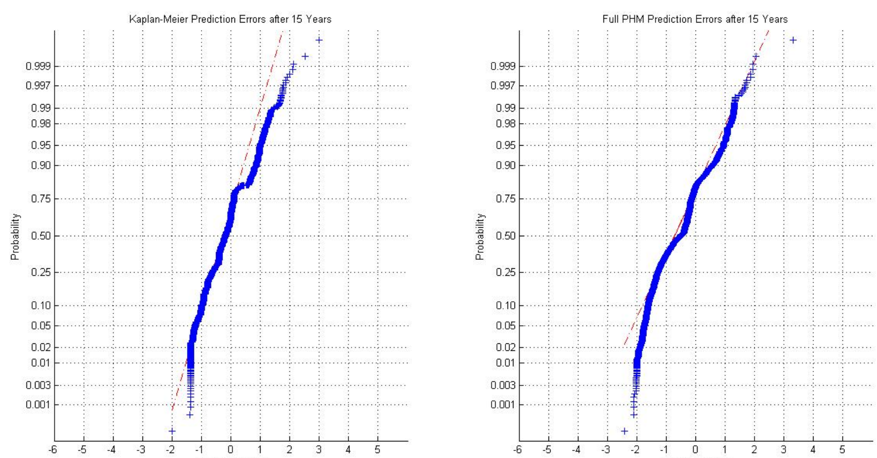

FIGURE 7.6: Normal probability plots associated with prediction errors of Kaplan-Meier and PHM models applied to concrete deck condition rating data

at the beginning of the period of continuous rating. To assess the effect of the initial state vector probabilities on the mean prediction error, an alternative formulation was assessed. Specifically, it was postulated that, since the initial instance of any condition rating occurs at the transition from a higher rating, a valid assumption for the initial state vector would be to divide the probability equally among the observed condition rating and the one step higher rating that was transitioned from. This approach essentially starts the prediction cycle midway between the two condition ratings present at the transition, or with an expected value that is 0.5 condition rating higher than the observed initial rating. The PHM model was executed with this modified initial state vector and a significant improvement was observed in the resulting distribution of errors in predicted condition ratings after a period of 15 years. The results for concrete deck condition rating predictions for the revised initial state vector are shown in Figure 7.7. Comparison with Figure 7.3 shows a favorable reduction in mean error from -0.59 to -0.22, and a lesser but equally noticeable reduction in standard deviation from 0.81 to 0.76 for the revised probabilistic model predictions. The empirical cumulative distribution functions of prediction errors for concrete decks for deterministic models and the revised probabilistic models are shown in Figure 7.8. The probability corresponding to prediction errors with +/-1 condition rating from the observed rating at the end of the 15 year planning horizon is found to have improved to almost 80% from the previously mentioned 62% for the original predictions. Since the deterministic models do not incorporate probabilities, the probability that the deterministic model will achieve accuracy within +/-1 condition rating remains only 22%.

Revised predictions were also obtained using the modified initial state vector for the Kaplan-Meier deterioration models in order continue the assessment of the impact of covariate inclusion on the predictive fidelity of the probabilistic models. The results for concrete deck models are presented in Figure 7.9 in the form of empirical cumulative distribution functions. Comparison with Figure 7.5 shows that the shift in the mean error of the Kaplan-Meier model is of similar magnitude to the shift in the mean error of the full PHM model. However, the shift in the mean of the prediction errors causes the Kaplan-Meier model to develop slightly unconservative predictions with only a modest improvement in absolute accuracy. In contrast, the shift in the mean of the prediction errors in the full PHM model produces a more accurate, while still conservative, estimate of condition rating over the 15 year planning horizon. Similar results were obtained from comparisons developed for other material-specific GCR components.

Lastly, a comparison between the stationary and non-stationary PHM model prediction errors was also performed over the same planning horizon of 15 years. Figures 7.10 and 7.11 present the empirical cumulative distribution functions and the normal probability plots associated with prediction errors from non-stationary and stationary PHM models applied to concrete deck condition rating data. The modified initial state vector with the initial state probabilities split across the transition condition ratings was used for this comparison. It is observed that the mean errors and standard deviations obtained from both of the models are quite similar to each other and suggest that the simplified stationary transition probability implementation performs acceptably well as a substitute for the more computationally complex non-stationary

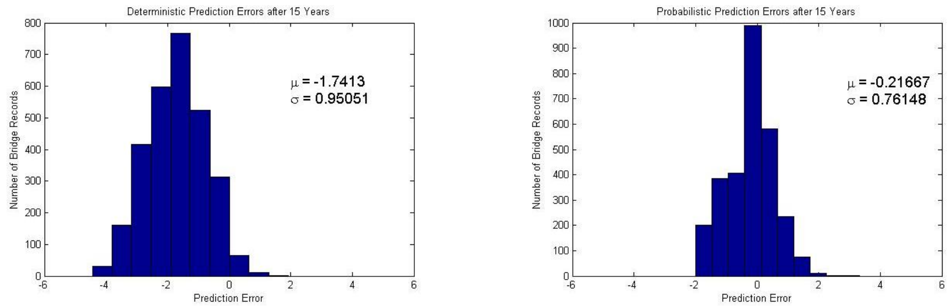

FIGURE 7.7: Histograms of concrete deck condition rating prediction errors obtained with split probability initial state vector

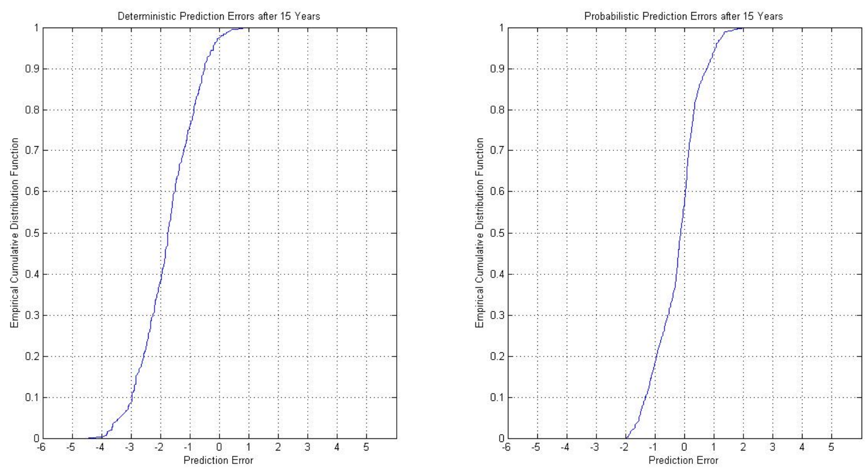

FIGURE 7.8: Empirical cumulative distribution functions associated with prediction errors of concrete deck deterministic and probabilistic models obtained with split probability initial state vector

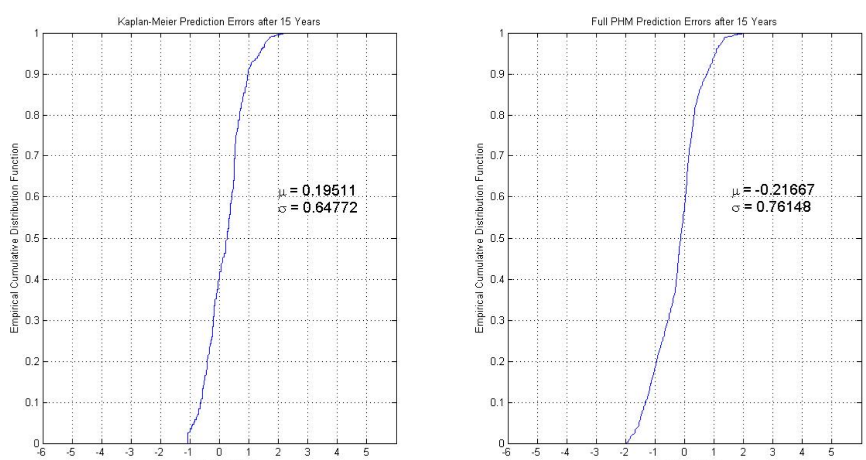

FIGURE 7.9: Empirical cumulative distribution functions associated with prediction errors of concrete deck Kaplan-Meier and PHM models obtained with split probability initial state vector

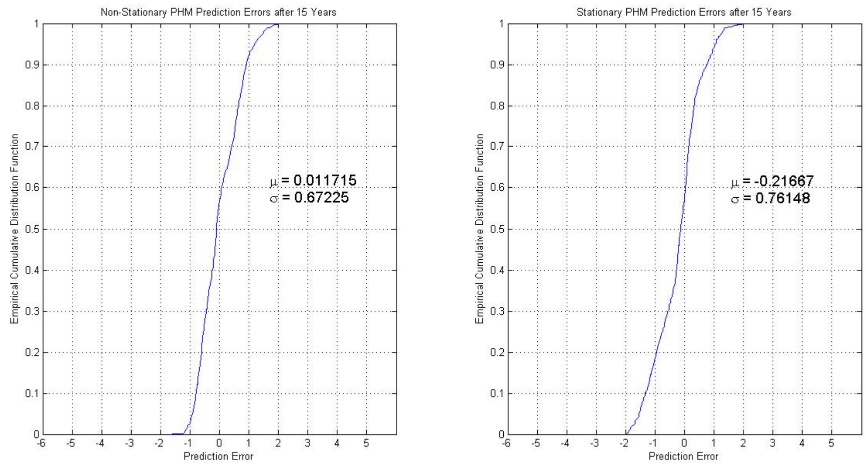

FIGURE 7.10: Empirical cumulative distribution functions associated with prediction errors of non-stationary and stationary PHM models applied to concrete deck condition rating data

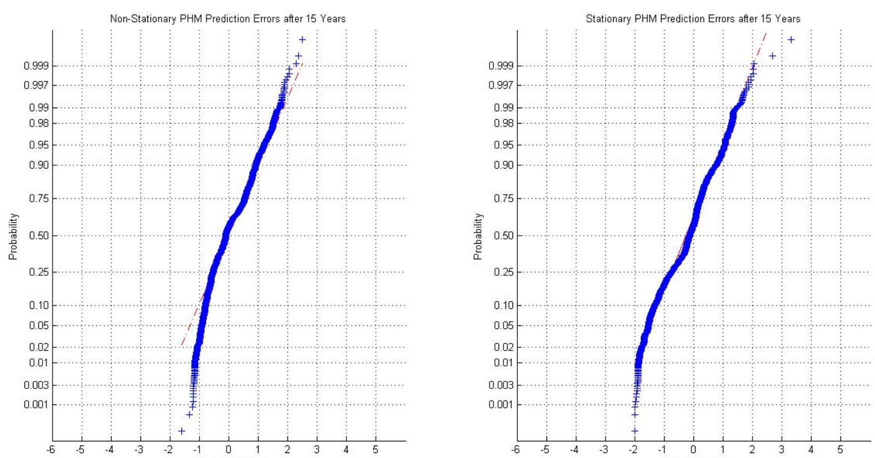

FIGURE 7.11: Normal probability plots associated with prediction errors of non-stationary and stationary PHM models applied to concrete deck condition rating data

model over typical planning horizons. This observation was expected for a planning horizon of 15 years given the correlation between the stationary and non-stationary models demonstrated by the baseline expected condition rating estimates presented in Chapter 6. Both models exhibit strong adherence to the normal distribution, as presented in the normal probability plots shown in Figure 7.11. To further investigate the relative performance of the two models, analysis was also performed using data from a 5-year period (2000-2005), a 10-year period (2000-2010), and the previously presented results from the 15-year period (2000-2015). The modified initial state vector using initial state probabilities split across the transition condition ratings was used for this comparison. The empirical cumulative distribution functions associated with prediction errors from both models for these three prediction periods are shown in Figure 7.12. It is observed that, although the mean error in the stationary PHM model remains essentially constant, there is an improvement in the mean error in the non-stationary models with an increase in length of the prediction period. Furthermore, the normality of the distribution of prediction errors for the non-stationary model improves, while the normality of the distribution of prediction errors for the stationary models degrades. Consequently, while the non-stationary model maintains a high percentage of predictions within +/-1 condition rating of the recorded value as the planning horizon is increased, the stationary model exhibits a moderate reduction in the percentage of predictions accurate within +/-1 rating. This demonstrates that the non-stationary models can achieve strong accuracy and precision over longer duration planning horizons than the simplified stationary models. However, it should be noted that currently the planning horizons used by NCDOT are typically only 5-10 years and, consequently, either approach should offer significantly improved performance over the deterministic models and strong predictive fidelity offering accuracy within +/-1 condition rating greater than 80% of the time.

The lack of substantial difference in model prediction errors developed between the Kaplan-Meier models and the PHM models is most likely related to the nature of the general condition ratings, which reflect the aggregate performance of individual elements comprising a particular bridge component. Since the composition of individual components may vary significantly by the number, design, and potentially even material of elements contributing to the general condition rating, there is likely a large variability in the deterioration rates of components that may share similar covariate assignments. Consequently, deterioration models for GCR components may not warrant such an advanced PHM model to develop reliable condition rating predictions in the current BMS architecture given the resolution of general condition ratings. Nonetheless, the PHM methodology developed in this work provides unique quantitative insight on factors influencing deterioration over the life cycle that may have important applications in decision making related to preservation and project prioritization. Furthermore, the methodology developed may offer significant advantages once sufficient element-level condition rating data becomes available following the federal mandate to collect and record such data (FHWA, 2012). It is very likely that element-level data will offer the granularity to overcome the stochastic characteristics of general condition rating resulting from the aggregation of element ratings and, therefore, the usefulness of covariate inclusion may become more evident. On the other hand, the close agreement between the two models and the difference with

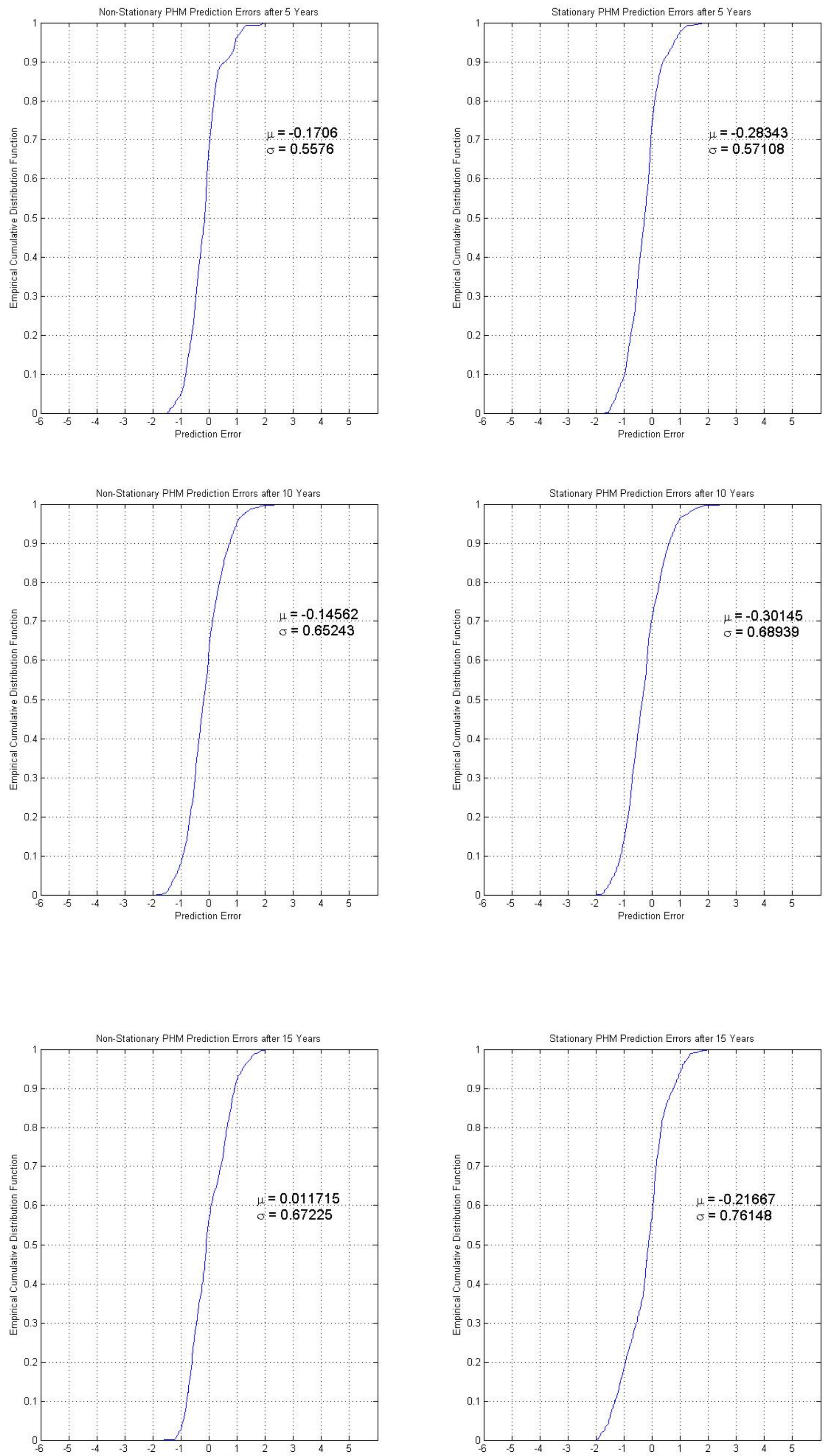

FIGURE 7.12: Empirical cumulative distribution functions associated with prediction errors of non-stationary and stationary PHM models applied to concrete deck condition rating data: Comparison between 5-year (2000-2005), 10-year (2000-2010), and 15-year (2000-2015) prediction periods

deterministic model predictions does testify to the inordinately significant influence of censoring on condition rating model predictions and advantage of duration-based probabilistic models over deterministic models.

CHAPTER 8: CONCLUSIONS AND FUTURE WORK

Deterioration modeling and its implementation in predicting future condition ratings of bridge components presents many challenges owing to the multidimensional nature, subjectivity, aggregation, and variability of bridge inspection data, management and preprocessing demands of such a large database, and limited suitability of available statistical techniques for infrastructure deterioration modeling. This area has seen a significant amount of research since the conception of bridge management systems, however increased accuracy of these models, while facilitating an ease of implementation in current BMS software environments, is necessary to provide the reliable data-driven framework to forecast infrastructure needs and inform project prioritization under increasingly constrained budgets. An exhaustive review of bridge deterioration modeling approaches was performed to identify limitations in current methodologies and formulate potential strategies for addressing these limitations. Through this research, a comprehensive framework for probabilistic deterioration modeling based on the Cox proportional hazards method that combines the advantages of duration-based modeling and semi-Markov theory of prediction was developed and is presented in this dissertation. The developed framework was implemented on and validated using the North Carolina state bridge inventory. The main theoretical and applied contributions of this dissertation are first summarized in this chapter followed by conclusions derived from model implementation and recommendations for future work.

8.1 Summary of Contributions and Significant Research Conclusions

The theoretical contribution of this research is comprised of two main parts. The first is the development of a framework for multivariable statistical regression of bridge condition rating data using the semi-parametric proportional hazards model that yields hazard ratios associated with significant variables affecting deterioration and non-stationary baseline transition probabilities of deterioration over individual condition ratings of a bridge component. This framework is designed to be implemented on large bridge inspection databases and incorporates strategies devised specifically to handle the challenges presented in the statistical analysis of bridge inspection data at every stage from data extraction and preprocessing to the final construction of models. The second part of the theoretical contribution is the formulation of a strategy for predicting expected condition ratings over a specified planning horizon using the non-stationary transition probabilities based on semi-Markov probabilistic theory. A simplified strategy of using the means of transition probabilities over the observed condition rating durations to construct a stationary transition matrix for use in a stationary Markov chain model was also developed. The time-dependent nature of transition probabilities and their impact on probabilistic condition rating predictions for short-term and long-term planning ranges was assessed by implementing both the stationary and non-stationary models on the NCDOT bridge database.

A novel strategy for incorporating the effect of covariates on the condition rating predictions by scaling the transition probabilities with applicable hazard ratios was derived to facilitate the strategy for probabilistically predicting expected condition ratings using the multivariable proportional hazards models. In this way, the complete advantage of the multivariable survival analyses at individual condition ratings was translated probabilistically into an integrated deterioration model over the life cycle of the bridge component. This resulting deterioration model is responsive to the impact of individual covariate assignments and eliminates the dependence on *a priori* classification for improved accuracy of predictions. The entire framework described above was cast into an automated software routine that can run independently from start to finish and produce the desired models based on a minimal one-time input. The Markovian formulation of the prediction model makes it feasible to integrate it into existing bridge management systems, particularly those already using probabilistic models.

Applied contributions of the research were developed through implementation of the above-mentioned theoretical framework. These include the development of proportional hazards based deterioration models for material specific deck, superstructure, and substructure components of the NCDOT bridge inventory. The models are comprised of covariate hazard ratios and non-stationary as well as stationary transition probabilities for baseline assignment of covariates. The significant covariates affecting deterioration at individual condition ratings in the various material-specific component categories were examined to provide an assessment of their overall impact and to identify trends across all categories. A user friendly standalone graphical user interface (GUI) was designed and made fully functional for development and future updating of deterioration models by transportation personnel. Software routines for developing and updating the existing deterministic models were designed and implemented within the GUI. Functionalities for development of new deterministic deterioration models for culverts and element level data were also incorporated in the GUI.

The conclusions derived from implementation of the developed framework and discussed in Chapters 6 and 7 are summarized here. It was found from the survival analysis of individual condition ratings that the rate of deterioration of a bridge component depends on its condition state. This is evident from the difference in survival functions obtained for individual condition ratings associated with the same material-specific component. The transition times at condition rating 9 were observed to be lowest and those at condition ratings 7 and 8 were observed to be the highest across all material-specific components. The influence of covariates on deterioration rate was found to differ not only across material-specific components but also across individual condition ratings and is quantified in the hazard ratios associated with each covariate. The significant variables affecting deterioration of all material-specific components were identified as age, number of spans, reconstruction, region, maximum span length, and state system. Increase in age and number of spans was found to increase the rate of deterioration across all bridge components. Similarly, reconstructed bridges were found to be generally associated with modestly higher rates of deterioration. The impact of region was found to be moderately significant across all component types. In general, and particularly for steel and prestressed concrete bridge components, lower rates of deterioration were found to be associated with the Piedmont and Mountain regions in comparison to the Coastal region. Increase in maximum span length was found to consistently increase deterioration in decks, but exhibited no such clear trends in the case of superstructure and substructure components. Similarly, deck components servicing interstate, urban, and primary routes were prone to slightly higher rates of deterioration in comparison to those on secondary routes, although the opposite effect was identified in the case of most superstructure and substructure components. Decks were found to exhibit higher rates of deterioration than superstructures and substructures of the same material, except in the case of timber substructures which showed a similar deterioration rate to timber decks. Many of the above conclusions are similar to those reached by earlier researchers in the field. These conclusions serve the dual purpose of substantiating the earlier research as well as validating the correctness of the current approach.

A significant constituent of the applied contribution of this research was the quantitative assessment of the predictive fidelity of the developed probabilistic models. This is the first time such an assessment has been performed for duration-based deterioration models with transition probabilities derived from survival analysis. This assessment was done on the basis of observed condition rating data relative to the deterministic model predictions over typical long-range planning horizons using the simplified stationary models. Similar assessment was carried out for stationary versus non-stationary model predictions and multivariable versus univariate probabilistic models.

It was found that there was strong agreement between the expected condition rating predictions obtained from stationary and non-stationary transition probabilities for prediction periods ranging up to approximately 20 years. In the longer term, the non-stationary model appears to generate more realistic, although at times conservative, predictions in comparison to the stationary model. Quantitative assessment of the comparative predictive fidelity of these models over a 15 year planning horizon revealed negligible differences in prediction errors but found that the non-stationary models tended to be more robust than the simplified stationary models. It is therefore recommended that the simplified stationary models may be adopted for planning horizons of 20 years or less in the interest of computational simplicity. However, for longer planning ranges it is advisable to use non-stationary models.

Quantitative assessment of the comparative fidelity of the probabilistic models with respect to the deterministic models currently in use by the NCDOT revealed significant improvement in accuracy associated with the probabilistic models, offering a strong case for NCDOT to adopt the new probabilistic models. The effect of inclusion of covariates on the predictive fidelity of the multivariable models was also assessed. The predictive fidelity of the Kaplan-Meier based univariate models was found to be comparable to the multivariable models although closer examination revealed that the multivariable models were statistically more robust. The probabilistic models were also found to be sensitive to the resolution of the initial condition rating and sensitive to the strategy for assigning condition rating probabilities in the initial state vector. The lack of substantial difference in model prediction errors developed between the Kaplan-Meier models and PHM models has been attributed largely to the nature of the general condition ratings as nominal scale aggregated indexes. Although the developed methodology provides unique quantitative insight into factors influencing deterioration of bridge components over their life cycle and applications to preservation and prioritization decisions, such an advanced PHM model may not be warranted for reliable forecasting of GCR condition ratings. However, it is believed that the recent emphasis on element-level condition rating will eventually offer the granularity of data necessary to fully realize the significant advantages offered by the developed methodology.

A significant advantage of the developed framework is its ability to use both stationary and non-stationary transition matrices for prediction of expected condition ratings in the forecasting algorithm. Development of these transition probability matrices is not strictly limited to the proportional hazards survival-based method for developing transition probabilities utilized in the first part of the framework. Therefore, it is recommended that the Markov chain prediction framework be also used to develop probabilistic models for the element-level and culvert condition rating data, which are of relatively shorter duration than recommended for duration-based analysis. The stationary transition probabilities for these predictions may be developed using the mean durations at each condition rating, as determined through the deterministic modeling approach, in place of the expert opinion-elicited median durations in equations (2.9) and (2.10). In this way, the advantages of accuracy and precision offered by state-based probabilistic models over deterministic models can be obtained for these databases until sufficient duration-based data becomes available.

8.2 Recommendations for Future Work

This study has provided a way of integrating the effect of covariates on condition rating duration probabilities into a semi-Markovian methodology of prediction. The developed methodology has been implemented on the historical NCDOT condition rating database to develop probabilistic models for use in the NCDOT bridge management system. However, investigation of the full extent of the benefits accrued from the developed models and their potential applications has been necessarily limited by time and logistical constraints within the scope of the present study. The recommended directions for future work include the following:

- Investigation of improvement in accuracy and precision afforded by the new models has been limited to comparisons of condition rating predictions with respect to deterministic models. It is important to study the implications of this improved accuracy and precision on the performance of the BMS multi-objective optimization analysis and associated incremental benefits facilitated by the use of the new models.

- The simplified stationary model developed in this study is based on mean transition probabilities obtained from survival analysis. This is different from the state-based stationary Markovian approaches implemented to date that use transition probabilities obtained from linear regression of condition rating data at best and those derived from practitioner opinion surveys at the worst. It is important to assess the difference between the developed duration-based stationary model and the existing approaches.

- As the observed condition rating data over longer planning ranges required for proper model assessment becomes available, comparisons of the predictive fidelity of the developed deterministic and probabilistic PHM models should be reassessed.

- The use of non-stationary probability matrices in the prediction approach permits the possibility of incorporating the effect of maintenance actions through introducing transition probabilities associated with condition rating improvement in the transition probability matrices associated with the corresponding inspection cycle. The pre-requisite for this approach is an estimation of the transition probabilities associated with condition rating improvement for specific maintenance actions. This estimation is possible through pairing of historical condition rating data with maintenance records available in the BMS database. The transition probabilities of improvement can then be incorporated in the transition probability matrix for the annual prediction cycle corresponding to the maintenance action. For example, if the probability of improvement associated with maintenance action, A, performed at condition rating 'k' is I_A^k, the corresponding transition probability matrix, P, can be developed using

$$P = \begin{bmatrix} P_{99} & P_{98} & 0 & \ldots & 0 & 0 & 0 \\ I_A^8 P_{88} & (1-I_A^8)P_{88}+I_A^8 P_{87} & (1-I_A^8)P_{87} & \ldots & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \ldots & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \ldots & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \ldots & I_A^4 P_{44} & (1-I_A^4)P_{44}+I_A^4 P_{43} & (1-I_A^4)P_{43} \\ 0 & 0 & 0 & \ldots & 0 & I_A^3 P_{33} & (1-I_A^3)P_{33} \end{bmatrix} \qquad (8.1)$$

  The matrix formulation can be generalized to include the probability of a particular maintenance action increasing the condition rating by more than 1. This approach needs further investigation to assess its benefits relative to the existing approaches of resetting the deterioration curve to the original condition state in the corresponding year of maintenance action.

- The methodology for developing covariate specific hazard ratios is limited by the availability of variables in the BMS database. This approach has the potential of quantifying the effect of various intervention strategies and preservation treatments provided that strategies are adopted for recording these treatments and actions in the BMS databases.

- The framework developed in this study has been implemented and tested on the North Carolina database of GCR component ratings. It will be beneficial to apply the methodology to NBI databases of other states, since these are readily available from the FHWA. This would be helpful not only in substantiating some of the applied results obtained from the NC database, but also in providing a basis of comparison with other existing models that may be functional in these states.

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APPENDIX A: CONCRETE DECK MASTER DATA CODE

```
function ConcreteDeckMasterData()
%**********************************************************************
% ConcreteDeckMasterData is a representative function for creating master
% database subsets pertaining to any selected bridge component and specified
% material type from the MATLAB Master File (NBI). This code prepares such
% a subset for all bridges with concrete decks including cast-in-place and
% precast panel decks
%**********************************************************************
global NBI
Material=zeros(size(NBI));
%Find all bridges with concrete cast-in-place & precast panel decks
for j=1:length(NBI)
   Temp=double(NBI(1,j).Deck_Structure_Type);
   I=find(Temp==255);
   Temp(I)=NaN;
   Type=mode(Temp);
   I=(Type==1)|(Type==2);
   if sum(I)>0
       Material(j)=1;
   end
end
I= find(Material==1);
Material=NBI(1,I);
 for k=1:length(NBI)
L(k)=length(NBI(k).Data_Year);
 end
nyears=max(L);
%Produce matrix of deck condition ratings
DeckStructureType=zeros(nyears,length(Material));
Rating0=zeros(nyears,length(Material));
for j=1:length(Material)
    Temp=Material(1,j).Deck_Structure_Type;
    DeckStructureType(1:length(Temp),j)=Temp;
    Temp=Material(1,j).Deck;
    Rating0(1:length(Temp),j)=Temp;
end
% Account for condition rating fields left blank
[I,J]=find(Rating0==255);
for k=1:length(I)
    if I(k)>1
    Rating0(I(k),J(k))=Rating0(I(k)-1,J(k));
    else
        Rating0(I(k),J(k))=Rating0(I(k)+1,J(k));
    end
end
save('Databases\ConcreteDeck','Material','DeckStructureType','Rating0');
end
```

APPENDIX B: PHM INPUT VARIABLES CODE

```
function [Censor,YearsInRating,Subset,Age]=FnPHMInput(MTComp)
%*************************************************************************
% PHMInput is a generic function that preprocesses the master data for
% the material specific bridge component (MTComp) to develop the dependent
% and independent variables for individual Cox proportional hazards
% regression over each condition rating (RT).
%*************************************************************************
% MTComp='ConcreteDeck' or 'TimberSuperstructure'....
%**************************************************************************
load(strcat('Databases\',MTComp));
h=waitbar(0,'Processing Condition Rating Data');
for RT=4:9
    waitbar((RT-2)/6,h);
    clear Rating Subset2 Age YearsInRating Censor
    Rating=Rating0;
%Example: Condition Rating Subset that includes RT
%Start by finding all bridges with data for RT
Subset=zeros(size(Material));
for j=1:length(Material)
    if ((sum(Rating(:,j)==RT)>0))
        Subset(j)=1;  %has data for ratings RT
    end
end
I=find(Subset==1);
Subset=Material(I);
Rating=Rating(:,I);
YearBuilt=uint16convert(Subset,'Year_Built',length(Subset));
DataYear=uint16convert(Subset,'Data_Year',length(Subset));
YearRecon=uint16convert(Subset,'Year_Reconstructed',length(Subset));

%Code to ensure that once a bridge has been reconstructed, the year(s) of
%reconstruction are always considered in the age calculation
for j=1:length(YearRecon(1,:))
    ReInd=find(YearRecon(:,j)>0);
    if ~isempty(ReInd)
        ReInstances=unique(YearRecon(ReInd,j));
        clear I2
        for jj=1:length(ReInstances)
            I2(jj)=find(YearRecon(:,j)==ReInstances(jj),1,'first');
        end
        for jj=1:length(ReInstances)
            YearRecon(I2(jj):end,j)=ReInstances(jj);
        end
    end
end

%Same story with YearBuilt
for j=1:length(YearBuilt(1,:))
    ReInd=find(YearBuilt(:,j)>0);
    if ~isempty(ReInd)
        ReInstances=unique(YearBuilt(ReInd,j));
```

```
        clear I2
        for jj=1:length(ReInstances)
            I2(jj)=find(YearBuilt(:,j)==ReInstances(jj),1,'first');
        end
        for jj=1:length(ReInstances)
            YearBuilt(I2(jj):end,j)=ReInstances(jj);
        end
    end
end

%Now develop data for PHM
clear Censor Age YearsInRating
cntr=1;
for j=1:length(Subset)
    clear I I2 I3 I4 I5
    ERating=Rating(:,j);
    IR=find(ERating==RT);
    while length(IR>0)
        DER=diff(IR);
        %*******************************************
        %    CODE FOR ONLY 1 CONTINUOUS RECORD
        %*******************************************
        if isempty(find(DER~=1))
            if length(IR)>1
            YearsInRating(cntr)=length(IR);
            Subset2(cntr)=Subset(j);
            if (YearRecon(IR(1),j))>0
                Age(cntr)=DataYear(IR(1),j)-YearRecon(IR(1),j);
            else
                Age(cntr)=DataYear(IR(1),j)-YearBuilt(IR(1),j);
            end
            if (IR(1)==1)
                Censor(cntr)=1;
            elseif ERating(IR(1)-1)==255
                Censor(cntr)=1;
            elseif IR(end)==length(ERating)
                Censor(cntr)=1;
            elseif ERating(IR(end)+1)>ERating(IR(end))
                Censor(cntr)=1;
            elseif ERating(IR(end)+1)==0
                Censor(cntr)=1;
            else
                Censor(cntr)=0;
            end
            cntr=cntr+1;
            end
            IR=[];
        else
            IE=find(DER~=1);
            if IE(1)>1
            YearsInRating(cntr)=IE(1);
            Subset2(cntr)=Subset(j);
            if (YearRecon(IR(1),j))>0
                Age(cntr)=DataYear(IR(1),j)-YearRecon(IR(1),j);
```

```
            else
                Age(cntr)=DataYear(IR(1),j)-YearBuilt(IR(1),j);
            end
            if (IR(1)==1)
                Censor(cntr)=1;
            elseif ERating(IR(1)-1)==255
                Censor(cntr)=1;
            elseif IR(IE(1))==length(ERating)
                Censor(cntr)=1;
            elseif ERating(IR(IE(1))+1)>ERating(IR(IE(1)))
                Censor(cntr)=1;
            elseif ERating(IR(IE(1))+1)==0
                Censor(cntr)=1;
            else
                Censor(cntr)=0;
            end
            cntr=cntr+1;
            end
            IR(1:IE(1))=[];
        end
    end
end
Subset=Subset2;
cntr

save(strcat('Databases\',MTComp,'\','CR',num2str(RT)),'Censor',...
    'YearsInRating','Subset','Age')
end
close(h)
```

APPENDIX C: CATEGORICAL VARIABLE MEAN BOUNDS CODE

```
function [ADTMin,ADTTMin,AgeMin,MaxSpanMin,CMinTable]=FnCatBounds(MTComp)
%***********************************************************************
% FnCatBounds calculates the lower bounds for the categorical variables ADT,
% ADTT, age, and maximum span, by first binning them uniformly at each
% condition rating (RT) and then taking the weighted average of the bins
% across condition ratings 4 to 9. The number of records in the particular
% material specific bridge component (MTComp) database is used to weight
% the average.
%***********************************************************************
RT=4:9;
ADTBin1Min=zeros(1,length(RT));
ADTBin2Min=zeros(1,length(RT));
ADTBin3Min=zeros(1,length(RT));
ADTBin4Min=zeros(1,length(RT));

ADTTBin1Min=zeros(1,length(RT));
ADTTBin2Min=zeros(1,length(RT));
ADTTBin3Min=zeros(1,length(RT));
ADTTBin4Min=zeros(1,length(RT));

AgeBin1Min=zeros(1,length(RT));
AgeBin2Min=zeros(1,length(RT));
AgeBin3Min=zeros(1,length(RT));
AgeBin4Min=zeros(1,length(RT));

MaxSpanBin1Min=zeros(1,length(RT));
MaxSpanBin2Min=zeros(1,length(RT));
MaxSpanBin3Min=zeros(1,length(RT));

Weight=zeros(1,length(RT));
for k=1:length(RT);
load(strcat('Databases\',MTComp,'CR',num2str(RT(k))),'Subset','Age') ;

%****AVERAGE DAILY TRAFFIC ******
ADT=uint32convert(Subset,'Average_Daily_Traffic',length(Subset));
I=find(ADT>500000);  %Remove Outliers
ADT(I)=NaN;
ADT=nanmean(ADT);
Weight(k)=length(Subset);

NMeanADTCategory=ceil(4*tiedrank(ADT)/length(ADT));
NMeanADTCategory=NMeanADTCategory-1;
ADT1=ADT(find(NMeanADTCategory==0));
ADT2=ADT(find(NMeanADTCategory==1));
ADT3=ADT(find(NMeanADTCategory==2));
ADT4=ADT(find(NMeanADTCategory==3));

if ~isempty(ADT1);ADTBin1Min(k)=min(ADT1);end;
if ~isempty(ADT2);ADTBin2Min(k)=min(ADT2);end;
if ~isempty(ADT3);ADTBin3Min(k)=min(ADT3);end;
if ~isempty(ADT4);ADTBin4Min(k)=min(ADT4);end;
```

```
%*****AVERAGE DAILY TRUCK TRAFFIC*****
PCNTADTT=uint8convert(Subset,'Percent_ADTT',length(Subset));
I=PCNTADTT>100;
PCNTADTT(I)=NaN;
PCNTADTT=nanmean(PCNTADTT);
ADTT=(PCNTADTT/100).*(ADT);

NMeanADTTCategory=ceil(4*tiedrank(ADTT)/length(ADTT));
NMeanADTTCategory=NMeanADTTCategory-1;
ADTT1=ADTT(find(NMeanADTTCategory==0));
ADTT2=ADTT(find(NMeanADTTCategory==1));
ADTT3=ADTT(find(NMeanADTTCategory==2));
ADTT4=ADTT(find(NMeanADTTCategory==3));

if ~isempty(ADTT1);ADTTBin1Min(k)=min(ADTT1);end;
if ~isempty(ADTT2);ADTTBin2Min(k)=min(ADTT2);end;
if ~isempty(ADTT3);ADTTBin3Min(k)=min(ADTT3);end;
if ~isempty(ADTT4);ADTTBin4Min(k)=min(ADTT4);end;

%****AGE******

NAgeCategory=ceil(4*tiedrank(Age)/length(Age));
NAgeCategory=NAgeCategory-1;
Age1=Age(find(NAgeCategory==0));
Age2=Age(find(NAgeCategory==1));
Age3=Age(find(NAgeCategory==2));
Age4=Age(find(NAgeCategory==3));

if ~isempty(Age1); AgeBin1Min(k)=min(Age1);end;
if ~isempty(Age2);AgeBin2Min(k)=min(Age2);end;
if ~isempty(Age3);AgeBin3Min(k)=min(Age3);end;
if ~isempty(Age4);AgeBin4Min(k)=min(Age4);end;

%****Maximum Span Length*****
MaxSpan=uint16convert(Subset,'Maximum_Span_Length',length(Subset));
MaxSpan=nanmedian(MaxSpan)/10;
MaxSpanSet=~isnan(MaxSpan);
NMaxSpanCategory=ceil(3*tiedrank(MaxSpan)/length(MaxSpan));
NMaxSpanCategory=NMaxSpanCategory-1;
MaxSpan1=MaxSpan(find(NMaxSpanCategory==0));
MaxSpan2=MaxSpan(find(NMaxSpanCategory==1));
MaxSpan3=MaxSpan(find(NMaxSpanCategory==2));
if ~isempty(MaxSpan1);MaxSpanBin1Min(k)=min(MaxSpan1);end;
if ~isempty(MaxSpan2);MaxSpanBin2Min(k)=min(MaxSpan2);end;
if ~isempty(MaxSpan3);MaxSpanBin3Min(k)=min(MaxSpan3);end;
end

ADT1Min=round(((ADTBin1Min)*(Weight'))/sum(Weight));
ADT2Min=round(((ADTBin2Min)*(Weight'))/sum(Weight));
ADT3Min=round(((ADTBin3Min)*(Weight'))/sum(Weight));
ADT4Min=round(((ADTBin4Min)*(Weight'))/sum(Weight));
ADTMin=[ADT1Min,ADT2Min,ADT3Min,ADT4Min];
```

```
ADTT1Min=round(((ADTTBin1Min)*(Weight'))/sum(Weight));
ADTT2Min=round(((ADTTBin2Min)*(Weight'))/sum(Weight));
ADTT3Min=round(((ADTTBin3Min)*(Weight'))/sum(Weight));
ADTT4Min=round(((ADTTBin4Min)*(Weight'))/sum(Weight));
ADTTMin=[ADTT1Min,ADTT2Min,ADTT3Min,ADTT4Min];

Age1Min=round(((AgeBin1Min)*(Weight'))/sum(Weight));
Age2Min=round(((AgeBin2Min)*(Weight'))/sum(Weight));
Age3Min=round(((AgeBin3Min)*(Weight'))/sum(Weight));
Age4Min=round(((AgeBin4Min)*(Weight'))/sum(Weight));
AgeMin=[Age1Min,Age2Min,Age3Min,Age4Min];

MaxSpan1Min=round(((MaxSpanBin1Min)*(Weight'))/sum(Weight));
MaxSpan2Min=round(((MaxSpanBin2Min)*(Weight'))/sum(Weight));
MaxSpan3Min=round(((MaxSpanBin3Min)*(Weight'))/sum(Weight));
MaxSpanMin=[MaxSpan1Min,MaxSpan2Min,MaxSpan3Min];

save(strcat('Databases\', MTComp,'CategoryMeans'),'ADTMin','ADTTMin',...
    'AgeMin','MaxSpanMin')
```

APPENDIX D: BENCHMARK MULTIVARIABLE MODEL CODE

```
function [X,XSet,PMulti1,FSetIndex]=FnBenchmark(MTComp)
%**************************************************************************
% FnBenchmark develops design variables and coding for covariates and
% then assembles the coded design variables into a matrix X, for each
% condition rating. A reduced X is obtained by excluding variables that do
% not have even a single associated bridge record. This reduced set of
% covariates is analyzed using Cox proportional hazards multivariable
% regression to produce the preliminary or benchmark model for each of the
% condition ratings (RT) 4 to 9 for any selected material-specific bridge
% component (MTComp).
%**************************************************************************
for RT=4:9;
clearvars -except RT MTComp ;
load(strcat('Databases\',MTComp,'CR',num2str(RT)),'Censor','YearsInRating',...
    'Subset','Age') ;
load(strcat('Databases\',MTComp,'CategoryMeans'));

%*************************************************************************
%  Develop Functional Classifications
%*************************************************************************
DataYear=uint16convert(Subset,'Data_Year',length(Subset));

%****STATE SYSTEM*****
StateSystem=uint8convert(Subset,'State_System',length(Subset));
I=find(StateSystem==3);
StateSystem(I)=NaN;
I=find(StateSystem==0);
StateSystem(I)=NaN;
StateSystem=nanmedian(StateSystem);
I=find(rem(StateSystem,1)>0);
StateSystem(I)=NaN;
StateSystemSet=~isnan(StateSystem);
StateSystem=StateSystem-1;   %Convert 1 or 2 classification to 0 or 1
StateSystemTable=tabulate(StateSystem);

%****AVERAGE DAILY TRAFFIC******
ADT=uint32convert(Subset,'Average_Daily_Traffic',length(Subset));
I=find(ADT>500000);  %Remove Outliers
ADT(I)=NaN;
ADT=nanmean(ADT);
for k=0:1:3
    if k<3
    I=ADT>=ADTMin(k+1);
    I2=ADT<ADTMin(k+2);
NMeanADTCategory(I&I2)=k;
    else I3=ADT>=ADTMin(k+1);
NMeanADTCategory(I3)=k;
    end
end
ADT1=ADT(find(NMeanADTCategory==0));
ADT2=ADT(find(NMeanADTCategory==1));
```

```
ADT3=ADT(find(NMeanADTCategory==2));
ADT4=ADT(find(NMeanADTCategory==3));
ADTTable=tabulate(NMeanADTCategory);

MeanADTCategory=zeros(length(Subset),3);  %Initialize Nominal Scale
for k=1:3
I=NMeanADTCategory==k;
MeanADTCategory(I,k)=1;
end
MeanADTCategorySet=~isnan(MeanADTCategory(:,1));
ADTSet=~isnan(ADT);

%****AGE******
for k=0:1:3
    if k<3
    I=Age>=AgeMin(k+1);
    I2=Age<AgeMin(k+2);
    NAgeCategory(I&I2)=k;
    else I3=Age>=AgeMin(k+1);
    NAgeCategory(I3)=k;
    end
end

Age1=Age(find(NAgeCategory==0));
Age2=Age(find(NAgeCategory==1));
Age3=Age(find(NAgeCategory==2));
Age4=Age(find(NAgeCategory==3));
AgeTable=tabulate(NAgeCategory);

AgeCategory=zeros(length(Subset),3);  %Initialize Nominal Scale
for k=1:3
I=NAgeCategory==k;
AgeCategory(I,k)=1;
end
AgeCategorySet=~isnan(AgeCategory(:,1));
AgeSet=~isnan(Age);

%****ORIGINAL/REBUILT (0), RECONSTRUCTED (1)**********
Recon=uint16convert(Subset,'Year_Reconstructed',length(Subset));
Recon=nanmedian(Recon);
I=Recon>1900;
Recon(I)=1;   %Reconstructed
Recon(~I)=0;  %Original or Rebuilt
ReconSet=~isnan(Recon);
ReconTable=tabulate(Recon);

%****REGION (Nominal Scale: Coastal 0/0, Piedmont 1/0, Mountain 0/1)
NRegion=uint8convert(Subset,'Region',length(Subset));
NRegion=mode(NRegion);
Inan=isnan(NRegion);%gives 1 when NRegion is a nan
NRegion=NRegion-1;
Region=zeros(length(Subset),2);
I=NRegion==1;
Region(I,1)=1;
```

```
I=NRegion==2;
Region(I,2)=1;
Region(Inan,:)=NaN;
RegionSet=~isnan(Region(:,1));
RegionTable=tabulate(NRegion);

%****ADTT ****
PCNTADTT=uint8convert(Subset,'Percent_ADTT',length(Subset));
I=PCNTADTT>100;
PCNTADTT(I)=NaN;
PCNTADTT=nanmean(PCNTADTT);
ADTT=(PCNTADTT/100).*(ADT);
for k=0:1:3
    if k<3
    I=ADTT>=ADTTMin(k+1);
    I2=ADTT<ADTTMin(k+2);
NMeanADTTCategory(I&I2)=k;
    else I3=ADTT>=ADTTMin(k+1);
NMeanADTTCategory(I3)=k;
    end
end
ADTT1=ADTT(find(NMeanADTTCategory==0));
ADTT2=ADTT(find(NMeanADTTCategory==1));
ADTT3=ADTT(find(NMeanADTTCategory==2));
ADTT4=ADTT(find(NMeanADTTCategory==3));
ADTTTable=tabulate(NMeanADTTCategory);

MeanADTTCategory=zeros(length(Subset),3);  %Initialize Nominal Scale
for k=1:3
I=NMeanADTTCategory==k;
MeanADTTCategory(I,k)=1;
end
MeanADTTCategorySet=~isnan(MeanADTTCategory(:,1));
ADTTSet=~isnan(ADTT);

%****Wearing Surface (Nominal Scale)****
WearSurface=uint16convert(Subset,'Wearing_Surface',length(Subset));
WearSurface=nanmedian(WearSurface);
WearingCovars={'None','MonolithicConcrete','IntegralConcrete',...
    'LatexConcrete','LowSlumpConcrete','EpoxyOverlay','Bituminous',...
    'Timber','Gravel','Other'};
Inan=isnan(WearSurface);
Wearing=zeros(length(Subset),9);  %Initialize Nominal Scale
for k=1:9
I=WearSurface>=k*100;
I2=WearSurface<(k+1)*100;
Wearing(I&I2,k)=1;
end
Wearing(Inan,:)=NaN;
WearKey=[0:9];
WearingCovars(1)=[];
WearingSet=~isnan(Wearing(:,1));

NWearing=Wearing;
```

```
for k=1:length(NWearing(1,:))-1
    NWearing(:,k+1)=NWearing(:,k+1)*(k+1);
end
NWearing=sum(NWearing,2);
NWearingTable=tabulate(NWearing);

%****Maximum Span Length*****
MaxSpan=uint16convert(Subset,'Maximum_Span_Length',length(Subset));
MaxSpan=nanmedian(MaxSpan)/10;
MaxSpanSet=~isnan(MaxSpan);
for k=0:1:2
    if k<2
    I=MaxSpan>=MaxSpanMin(k+1);
    I2=MaxSpan<MaxSpanMin(k+2);
NMaxSpanCategory(I&I2)=k;
    else I3=MaxSpan>=MaxSpanMin(k+1);
NMaxSpanCategory(I3)=k;
    end
end

MaxSpan1=MaxSpan(find(NMaxSpanCategory==0));
MaxSpan2=MaxSpan(find(NMaxSpanCategory==1));
MaxSpan3=MaxSpan(find(NMaxSpanCategory==2));
MaxSpanTable=tabulate(NMaxSpanCategory);

MaxSpanCategory=zeros(length(Subset),2);  %Initialize Nominal Scale
for k=1:2
I=NMaxSpanCategory==k;
MaxSpanCategory(I,k)=1;
end
MaxSpanCategorySet=~isnan(MaxSpanCategory(:,1));

%****Number of Spans*****
NumberSpans=uint16convert(Subset,'No_of_Main_Spans',length(Subset));
NumberSpans=nanmedian(NumberSpans);
NumberSpansSet=~isnan(NumberSpans);
I=NumberSpans==1;

NNumberSpansCategory(I)=0;
I2=NumberSpans>1;
NNumberSpansCategory(I2)=1;
NumberSpans1=NumberSpans(find(NNumberSpansCategory==0));% Single Span
NumberSpans2=NumberSpans(find(NNumberSpansCategory==1));% Multi Span

NumberSpansCategory=NNumberSpansCategory;
NumberSpansTable=tabulate(NNumberSpansCategory);
NumberSpansCategorySet=~isnan(NumberSpansCategory);

%****Functional Class*****
FunClass=uint8convert(Subset,'Functional_Class',length(Subset));
FunClass=nanmedian(FunClass);
%
FunClassCovars={'RInterstate','RPrincipalArterial','RMinorArterial',...
    'RMajorCollector','RMinorCollector','RLocal','UInterstate',...
```

```
    'UFreeway','UPrincipalArterial','UMinorArterial','UCollector',...
    'ULocal'};
FunClassKey=[1,2,6,7,8,9,11,12,14,16,17,19];
Functional=zeros(length(Subset),11); % 9 or 11? check - corrected to 11
Inan=~ismember(FunClass,FunClassKey); %Not a member of any classification
for k=1:11
    I=(FunClass==FunClassKey(k+1));
    Functional(I,k)=1;
end
Functional(Inan,:)=NaN;
FunClassCovars(1)=[];
FunctionalSet=~isnan(Functional(:,1));

NFunctional=Functional;
for k=1:length(NFunctional(1,:))-1
    NFunctional(:,k+1)=NFunctional(:,k+1)*(k+1);
end
NFunctional=sum(NFunctional,2);
FunctionalTable=tabulate(NFunctional);

clearvars -except YearsInRating Wearing WearingSet StateSystem ...
    StateSystemSet Region RegionSet Recon ReconSet NumberSpans ...
    NumberSpansSet NumberSpansCategory NNumberSpansCategory ...
    NumberSpansCategorySet MaxSpan MaxSpanSet MaxSpanCategory ...
    MaxSpanCategorySet NMaxSpanCategory NMeanADTTCategory...
    Functional FunctionalSet FunClassKey WearKey Censor ADTT ...
    ADTTSet MeanADTTCategory MeanADTTCategorySet ADT MeanADTCategory ...
    MeanADTCategorySet Subset ADTSet Age AgeCategory AgeTable...
    FunClassCovars WearingCovars NBI StateSytemTable RegionTable...
    NRegion NWearing NWearingTable NFunctional FunctionalTable ...
    NMeanADTCategory NAgeCategory AgeCategorySet RT ...
    MaxSpanTable NumberSpansTable StateSystemTable ADTTTable...
    ADTTable ReconTable MTComp

%**********************************************************************
% INITIAL MULTIVARIATE ANALYSIS
%**********************************************************************
X=[StateSystem;Recon;Region';Wearing';Functional';MeanADTCategory';...
MeanADTTCategory';MaxSpanCategory';NumberSpansCategory;AgeCategory'];
XSet=[StateSystemSet'*ones(1,size(StateSystem,1)),...
    ReconSet'*ones(1,size(Recon,1)),RegionSet*ones(1,size(Region,2)),...
    WearingSet*ones(1,size(Wearing,2)),...
    FunctionalSet*ones(1,size(Functional,2)),...
    MeanADTCategorySet*ones(1,size(MeanADTCategory,2)),...
    MeanADTTCategorySet*ones(1,size(MeanADTTCategory,2)),...
    MaxSpanCategorySet*ones(1,size(MaxSpanCategory,2)),...
    NumberSpansCategorySet'*ones(1,size(NumberSpansCategory,1)),...
    AgeCategorySet*ones(1,size(AgeCategory,2))];
XSet2=sum(XSet==1,2)==36;
FCount1=min( sum((double(X(:,XSet2))'==1)), sum((double(X(:,XSet2))'==0)));
FSet1=FCount1~=0;
FSetIndex=find(FSet1==1);
FSetIndexNull=find(FSet1==0);
[b,logl,H,stats]=coxphfit(double(X(FSetIndex,XSet2))',YearsInRating(1,XSet2)',...
```

```
    'baseline',0,'censoring',Censor(1,XSet2)');
PMulti1(FSetIndex)=stats.p;
PMulti1(FSetIndexNull)=1;
LogLMultiPre=logl;
save(strcat('Databases\',MTComp,'Rating',num2str(RT),'PHData'),'X','XSet',...
    'PMulti1','YearsInRating','Censor','FSetIndex','FSetIndexNull','FSet1',...
    'RegionTable','NWearingTable','FunctionalTable','LogLMultiPre',...
    'MaxSpanTable','NumberSpansTable','StateSystemTable','ADTTTable',...
    'ADTTable','ReconTable','AgeTable')
end
```

APPENDIX E: PHM BEST SUBSET CODE

```
function [FSet,BestCombination]=FnBestSubset(MTComp)
%************************************************************************
% FnBestSubset first removes those variables from the benchmark model that
% exhibit a p-value of more than 20%, and then implements a constrained step-
% wise forward selection approach based on maximizing the AIC. The function
% returns the best subset of variables for each of the condition ratings 4 to
% 9 of any selected material-specific bridge component (MTComp).
%************************************************************************
Ratings=4:9
for md=1:length(Ratings)
load(strcat('Databases\',MTComp,'Rating',num2str(Ratings(md)),'PHData'));

    I=find(PMulti1>0.2);
    FSet=setdiff(1:length(PMulti1),I)
    FSet=setdiff(FSet,14:24)  %Remove functional classification
    clearvars -except FSet PMulti1 X XSet Censor YearsInRating Ratings...
        MTComp md Best BestStats;

    X=X(FSet,:);
    XSet=XSet(:,FSet);
    XSet2=sum(XSet==1,2)==length(FSet);
    [b,logl,H,stats]=coxphfit(double(X(:,XSet2))',YearsInRating(1,XSet2)',...
    'baseline',0,'censoring',Censor(1,XSet2)');
    LogLMulti1=logl;
    PMulti=stats.p;

    N=length(FSet);
    N1=N;
    N1reserve=N1;
    reduction=[];
    Stats=[];
    cntr=1;   %Counter
    statusflag=1;   %1=true (keep running) 0 = false (stop running)
    if N==0
        statusflag=0;  %Don't run loop if Fset is empty
    end
    while (statusflag==1)
    combinations=[];
        if ~isempty(reduction)
        N1=setdiff(1:N1reserve,reduction);
        else
        N1=1:N1reserve;
        end
        for k=N1
            combinations=[combinations;[ones(length(N1),1)*k,[N1]']];
        end
        for j=1:cntr-1-length(reduction)
        temp=combinations;
        cntr2=1;
            for k=N1
                if cntr2==1
```

```
                combinations=[k*ones(size(temp,1),1),temp];
                else
                combinations=[combinations;[k*ones(size(temp,1),1),temp]];
                end
            cntr2=cntr2+1;
            end
        end
        for j=1:cntr-length(reduction)
        I=combinations(:,j)>=combinations(:,j+1);
        combinations(I,:)=[];
        end
        if ~isempty(reduction)
            for k=length(reduction):-1:1
            combinations=[reduction(k)*ones(length(combinations(:,1)),1),...
                combinations];
            end
        end
        if length(FSet)==1
            combinations=1;
        end
        fprintf(strcat('# Sets =',num2str(length(combinations(:,1))),'\r'))
        if length(combinations(:,1))>0
        Stats=[];
        for k=1:length(combinations(:,1))
            if length(FSet)==1
                S=X;
                SSet=XSet2;

            else
                S=X(combinations(k,1:cntr+1),:);
                SSet=XSet(:,combinations(k,1));
                for j=1:cntr
                    SSet=SSet & XSet(:,combinations(k,j+1));
                end
            end

        [b,logl,H,stats]=coxphfit(double(S(:,SSet))',YearsInRating(1,SSet)',...
        'baseline',0,'censoring',Censor(1,SSet)');
        Stats(k,1)=logl;
        Stats(k,2)=2*LogLMulti1-2*logl;
        Stats(k,3)=1-cdf('chi2',-2*logl+2*LogLMulti1,size(X,1)-size(S,1));
        Stats(k,4)=(logl-size(S,1)/2*log(size(S(:,SSet),2)));
        Stats(k,5)=(logl-2*size(S,1));
        end

        [Stats2,I] = sortrows(Stats,-1);
        combinations2=combinations(I,:);
        AIC(cntr)=Stats2(1,5);
        BIC(cntr)=Stats2(1,4);
        pval(cntr)=Stats2(1,3);
        fprintf(strcat('# Parameters =',num2str(cntr+1),'\r'))
        fprintf(strcat('Current AIC =',num2str(AIC(cntr)),'\r'))
        fprintf(strcat('Current BIC =',num2str(BIC(cntr)),'\r'))
        fprintf(strcat('Current p =',num2str(pval(cntr)),'\r'))
```

```
        fprintf(strcat('Best Combination =',num2str(combinations2(1,:)),'\r'))
        Savedcombinations{cntr}=combinations2;
        SavedStats{cntr}=Stats2;
        Best{cntr}=combinations2(1,:)
        BestStats{cntr}=Stats2(1,:)
        if (cntr>1)
            statusflag=(AIC(cntr)>AIC(cntr-1));
            reduction=intersect(Best{cntr},Best{cntr-1});
        else
            statusflag=1;
        end
        if (length(FSet)==1)
            statusflag=0;
        end
        cntr=cntr+1;
        else
            statusflag=0;
        end
        cntr
    if cntr>2
        SavedBest=Best(1,cntr-2);
        SavedBestStats=BestStats(1,cntr-2);
    else
        SavedBest=Best(1,cntr-1);
        SavedBestStats=BestStats(1,cntr-1);
    end

    BestCombination=FSet(1,cell2mat(SavedBest));
    BestCombStats=cell2mat(SavedBestStats);
    end

    if length(FSet)==1
        BestCombination=FSet;
        BestCombStats=[];
    elseif length(FSet)==0
        Best=[];
        BestStats=[];
        BestCombination=[];
        BestCombStats=[];
    end

save(strcat('Databases\',MTComp,'BestSubset',num2str(Ratings(md))),'FSet',...
    'LogLMulti1','PMulti','BestCombination','BestCombStats','Best','BestStats');
end
```

APPENDIX F: PHM BEST SUBSET COEFFICIENTS CODE

```
function [BetaMulti2, HR2]=FnPHMCoefficients(MTComp)
%*************************************************************************
% FnPHMCoefficients performs the proportional hazards regression on the best
% subset for each condition rating of any selected material-specific bridge
% component (MTComp), and obtains the model statistics including
% regression coefficients, hazard ratios, and baseline survival function.
%*************************************************************************
foldername=strcat('Databases_12112014NBI\',MTComp);
Ratings=4:9
for k=1:length(Ratings)
    load(strcat('Databases\',MTComp,'Rating',num2str(Ratings(k)),'PHData'));
    load(strcat('Databases\',MTComp,'BestSubset',num2str(Ratings(k))));
    clearvars -except FSet X XSet Censor YearsInRating Ratings MTComp k...
     foldername BestCombination ;
 if ~isempty(BestCombination)
    XBest=X(BestCombination,:);
    XBestSet=XSet(:,BestCombination);
    XSet2Best=sum(XBestSet==1,2)==length(BestCombination);
    %PHM coefficients
    [b,logl,H,stats]=coxphfit(double(XBest(:,XSet2Best))',...
        YearsInRating(1,XSet2Best)','baseline',0,'censoring',...
        Censor(1,XSet2Best)');
    LogLMulti2=logl;
    PMulti2=stats.p;
    BetaMulti2=b;
    HR2=exp(b);
    SEMulti2=stats.se;
    HMulti2=H;
    HRLB=exp(BetaMulti2-1.96*SEMulti2);
    HRUB=exp(BetaMulti2+1.96*SEMulti2);
    ZMulti2=stats.z; % b divided by se
    CovBMulti2=stats.covb;
    SMulti2=[HMulti2(:,1),exp(-HMulti2(:,2))]; % baseline survival function
    save(strcat('Databases\',MTComp,'BestSubsetCoefficients',...
        num2str(Ratings(k))),'BestCombination','XBest','XBestSet',...
        'XSet2Best','LogLMulti2',...
        'PMulti2','BetaMulti2','HR2','SEMulti2','HMulti2','HRLB','HRUB',...
        'SMulti2','ZMulti2','CovBMulti2','Censor','YearsInRating','FSet');
    figure(k)
    % PHM survival function
    plot(SMulti2(:,1),SMulti2(:,2),'b-','LineWidth',1.5);
    set(gca,'FontSize',16,'color','w','XColor','k','YColor','k');
    set(gca,'XLim',[0,30]);
    set(gca,'YLim',[0,1]);
    title(strcat({'Condition Rating '},num2str(Ratings(k))),'FontSize',20)
    xlabel('Years in Rating','FontSize',20)
    ylabel('Survival Probability','FontSize',20)
    print('-dpdf','-r600',strcat(cd,'\',foldername,'\',...
        'Baseline Survival Function',num2str(Ratings(k))))

    %Kaplan-Meier survival Function when best subset has no variables
```

```
 else [f,x]=ecdf(YearsInRating,'Censoring',Censor,'function','survivor');
     stairs(x,f,'b--','LineWidth',1.5);
     set(gca,'FontSize',16,'color','w','XColor','k','YColor','k');
     set(gca,'XLim',[0,30]);
     set(gca,'YLim',[0,1]);
     title(strcat('Condition Rating  ',num2str(Ratings(k))),'FontSize',20)
     xlabel('Years in Rating','FontSize',20)
     ylabel('Survival Probability','FontSize',20)
     legend('K-M','FontSize',20,'Location','best')
     print('-dpdf','-r600',strcat(cd,'\',foldername,'\',...
         'Kaplan Meier Survival Function',num2str(Ratings(k))))
     BetaMulti2=[];
     LogLMulti2=[];
     PMulti2=[];
     HR2=[];
     SEMulti2=[];
     HMulti2=[];
     HRLB=[];
     HRUB=[];
     ZMulti2=[]; % b divided by se
     CovBMulti2=[];
     SMulti2=[x,f];
     XBest=[];
     XBestSet=[];
     XSet2Best=[];

     save(strcat('Databases_12112014NBI\', MTComp,'\',MTComp,...
         'BestSubsetCoefficients',num2str(Ratings(k))),'BestCombination',...
         'XBest','XBestSet','XSet2Best','LogLMulti2','PMulti2','BetaMulti2',...
         'HR2','SEMulti2','HMulti2','HRLB','HRUB','SMulti2','ZMulti2',...
         'CovBMulti2','Censor','YearsInRating','FSet');
 end
end
for CR=1:6
    load(strcat('Databases\', MTComp,'BestSubsetCoefficients',...
        num2str(CR+3),'.mat'),...
        'BetaMulti2','BestCombination','HR2','PMulti2','HRLB','HRUB');
    TB(BestCombination,CR)=BetaMulti2;
    THR(BestCombination,CR)=HR2;
    TP(BestCombination,CR)=PMulti2;
    THRLB(BestCombination,CR)=HRLB;
    THRUB(BestCombination,CR)=HRUB;
    if CR==1
        Factors=BestCombination;
    else
        Factors=[Factors,BestCombination];
    end
    clear Censor
end
Factors=unique(Factors);
FactorNames={'StateSystem',...
'Reconstruction'     ,...
'Piedmont'  ,...
'Mountain'  ,...
```

```
'MonolithicConcrete',...
'IntegralConcrete'  ,...
'LatexConcrete' ,...
'LowSlumpConcrete'  ,...
'EpoxyOverlay'  ,...
'Bituminous'    ,...
'Timber'    ,...
'Gravel'    ,...
'Other' ,...
'RPrincipalArterial',...
'RMinorArterial'    ,...
'RMajorCollector'   ,...
'RMinorCollector'   ,...
'RLocal'    ,...
'UInterstate'   ,...
'UFreeway'  ,...
'UPrincipalArterial',...
'UMinorArterial'    ,...
'UCollector'    ,...
'ULocal'    ,...
'ADT2'  ,...
'ADT3'  ,...
'ADT4'  ,...
'ADTT2' ,...
'ADTT3' ,...
'ADTT4' ,...
'MaxSpan2'  ,...
'MaxSpan3'  ,...
'NumberSpans'   ,...
'Age2'  ,...
'Age3'  ,...
'Age4'}

TB=TB(Factors,:);
TP=TP(Factors,:);
THR=THR(Factors,:);
THRLB=THRLB(Factors,:);
THRUB=THRUB(Factors,:);
I=find(THR==0);
THR(I)=1;
TFactor=FactorNames(Factors)';
save(strcat('Databases\',MTComp,'BestSubsetCoefficients'),'TB','TP',...
    'THR','THRLB','THRUB','TFactor')
```

APPENDIX G: MULTICOLLINEARITY CHECK VIF CODE

```
function [MVIF]=FnVIF(MTComp)
% FnVIF performs the multicollinearity check by calculating the Variance
% Inflation Factor (VIF) for the final PHM model. It provides a warning if
% VIF exceeds the threshold value of 10 for any two covariates, and also
% provides a summary of the maximum VIF values obtained for the best subsets
% at all condition ratings for a particular material specific bridge
% component (MTComp.

Ratings=4:9
for CR=1:length(Ratings)
    load(strcat('Databases\',MTComp,'BestSubsetCoefficients',...
        num2str(Ratings(CR))));
    X2=XBest;
  if ~isempty(X2)
    rsq=zeros(length(X2(:,1)),length(X2(:,1)));
    for k=1:length(X2(:,1))
        for j=1:length(X2(:,1))
    x=X2(k,:);
    y=X2(j,:);
    I1=isnan(x);
    I2=isnan(y);
    x=x(~(I1|I2));
    y=y(~(I1|I2));
    p = polyfit(x,y,1);
    yfit = polyval(p,x);
    yresid = y - yfit;
    SSresid = sum(yresid.^2);
    SStotal = (length(y)-1) * var(y);
    rsq(k,j) = 1 - SSresid/SStotal;
        end
    end
    Tols=rsq>0.9;
    NI=length(Tols(:,1));
    for k=1:NI
        if sum(Tols(setdiff(1:k-1,k),k)==1)>0
            Ind=find(Tols(setdiff(1:k-1,k),k)==1);
            [k,Ind']
            warndlg(strcat('WARNING: VIF > 10, Collinearity detected between',...
                num2str(BestCombination(k)),' and ',...
                num2str(BestCombination(Ind')),' in Condition Rating ',...
                num2str(Ratings(CR))),'Multicollinearity Check Violation');
        end
    end
    rsq
    rsq=rsq-diag(ones(length(rsq(:,1)),1));
    MVIF(CR)=max(max(rsq));
    MVIF(CR)=1./(1-MVIF(CR));
    clear XBest BestCombination
  else MVIF(CR)=1;

  end
```

```
end
warndlg(['Maximum VIF for Each Rating: ',num2str(MVIF)],...
    'Multicollinearity Check Summary')
save(strcat('Databases\',MTComp,'MaxVIF'),'MVIF');
```

APPENDIX H: TRANSITION PROBABILITIES CODE

```
function [SSTP,TP,SP,RSP]=FnTransitionProbability(MTComp)
%**********************************************************************
% FnTransitionProbability calculates the non-stationary transition
% probabilities associated with changes in condition ratings due to
% deterioration based on the PHM survival function for baseline assignment
% of covariates. The code accounts for the step nature of the survival
% function by assigning the maximum survival probability recorded at any
% year to that year, and by linear interpolation of survival probabilities
% for step intervals spanning multiple years.
%**********************************************************************
foldername=strcat('Databases_12112014NBI\',MTComp);
Ratings=4:9

for k=1:length(Ratings) % condition rating = k+3
    load(strcat('Databases_12112014NBI\',MTComp,'BestSubsetCoefficients',...
        num2str(Ratings(k))));
    clearvars -except SMulti2 Ratings foldername MTComp k
    SP=[]; % Survival probability

     for   i=1:max(SMulti2(:,1)) % years
     Ind1=find(SMulti2(:,1)==i);
     SP=[SP;SMulti2((min(Ind1)),:)];
     end

    for r=1:SP(1,1)
         RSP(r,1)=1
    end

    for j=2:length(SP(:,1))
    mx=SP(j,1)-SP(j-1,1);
    my=(SP(j,2)-SP(j-1,2))/mx;
        if SP(j,1)==j
        r=j;
        else
        r=SP(j-1,1)+1;
        end

     RSP(r,1)=RSP(r-1,1)+my;
        if mx>2
            for n=1:mx-2
            RSP(r+n,1)=RSP(r+n-1,1)+my;
            end
        elseif mx==2
            n=0;
        else
            n=[];
        end
      RSP(r+n+1,1)=SP(j,2);

    end

```

```
    SSTP=[1]; % Stay the the same transition probability at condition
%     rating k+3, at year 0
     for j=1:length(RSP)-1
     SSTP=[SSTP,RSP(j+1,1)/RSP(j,1)];
     TP=1-SSTP; % Transition probability of rating k+3 of deteriorating
%      to rating(k+3)-1
     end
     figure(k+6)
     x=0:length(RSP)-1;

plot (x,SSTP,'b--',x, TP,'r-','LineWidth',1.5)
     set(gca,'FontSize',16,'color','w','XColor','k','YColor','k');
     set(gca,'XLim',[0,30]);
     set(gca,'YLim',[0,1]);
     title(strcat({'Condition Rating '},num2str(Ratings(k))),'FontSize',20)
     xlabel('Years in Rating','FontSize',20)
     ylabel('Transition Probability','FontSize',20)
     legend(['P_',num2str(k+3),'_',num2str(k+3)],['P_',num2str(k+3),'_',...
         num2str(k+2)],'FontSize',20,'Location','best')
     print('-dpdf','-r600',strcat(cd,'\',foldername,'\',...
         'Transition Probabilities at CR',num2str(Ratings(k))))
    save(strcat('Databases_12112014NBI\',MTComp,'\',MTComp,...
        'TransitionProbabilities',num2str(Ratings(k))),'SSTP','TP','SP','RSP');
end
```

APPENDIX I: PHM DETERIORATION PREDICTION CODE

```
function [P,S,Prob,CRM,PCR,PMean]=FnDeterioration(MTComp,Plan,S0)
%**********************************************************************
% FnDeterioration assembles the non-stationary transition probability matrices
% as well as the stationary transition probability matrices based on
% calculated mean transition probabilities over the length of the survival
% function. The code also plots the condition rating probabilities and
% baseline deterioration models for both the stationary and non-stationary
% transition probability approaches.
%**********************************************************************
foldername=strcat('Databases_12112014NBI\',MTComp);
Ratings=4:9
for k=1:length(Ratings)
    load(strcat('Databases\',MTComp,'TransitionProbabilities',...
        num2str(Ratings(k))));
    plan(1,k)=length(SSTP)
    RSSTP{k}=SSTP;
    MeanSSTP{k}=mean(SSTP(2:end))
end

% Diagonal and Off diagonal transition probabilities - non-stationary
for j=1:Plan-1
    PDiag{j}=[];
    POffDiag{j}=[];
    for k=6:-1:1
        if j< plan(1,k)
            PDiag{j}=[PDiag{j},RSSTP{1,k}(1,j+1)];
            POffDiag{j}=[POffDiag{j},1-RSSTP{1,k}(1,j+1)];
        else
            PDiag{j}=[PDiag{j},RSSTP{1,k}(1,plan(1,k))];
            POffDiag{j}=[POffDiag{j},1-RSSTP{1,k}(1,plan(1,k))];
        end
    end
end

% Mean transition probabilities over length of observation - stationary
 PDiagMean=[];
 POffDiagMean=[];
    for k=6:-1:1
        PDiagMean=[PDiagMean,MeanSSTP{1,k}];
        POffDiagMean=[POffDiagMean,1-MeanSSTP{1,k}];
    end

% Transition probability matrices - non-stationary
for j=1:Plan-1
P{j}=diag([PDiag{j} 0.75 0.75 1])+ diag([POffDiag{j} 0.25 0.25],1);
end

% Mean transition probability matrix - stationary
PMean=diag([PDiagMean 0.75 0.75 1])+ diag([POffDiagMean 0.25 0.25],1);

% Baseline deterioration model from non-stationary transition probabilities
```

```
CRM=[9; 8; 7; 6; 5; 4; 3; 2; 1]
for j=1:length(P)
    if j==1
        S{j}=S0*P{j};
    else
        S{j}=S{j-1}*P{j};
    end
    Prob(:,j)=S{j};
    PCR(j)=S{j}*CRM;  %Predicted CR at year j
end
x=1:length(P);

figure(13)
plot(x,Prob(1:6,:)','LineWidth',1)
legend('CR9','CR8','CR7','CR6','CR5','CR4','location','best')
xlabel('Time (Years)','FontSize',12)
ylabel('Probability of Condition Rating','FontSize',12)
set(gca,'FontSize',12)
print('-dpdf','-r600',strcat(cd,'\',foldername,'\',...
    'PHM Non-stationary BL-Probability of CR_PH',num2str(Plan)));

figure(14)
x=1:length(P);
plot(x,PCR)
set(gca,'YLim',[4,9]);
xlabel('Time (Years)','FontSize',12)
ylabel('Expected Condition Rating','FontSize',12)
set(gca,'FontSize',12)
print('-dpdf','-r600',strcat(cd,'\',foldername,'\',...
    'PHM Non-stationary BL-Expected CR_PH',num2str(Plan)))

save(strcat('Databases\',MTComp,'TransitionMatricesPH',num2str(Plan),...
    'Ratings',num2str(Ratings)),'PDiag','POffDiag','P','S','Prob','CRM','PCR');

% Baseline deterioration model from mean stationary transition probabilities
CRM=[9; 8; 7; 6; 5; 4; 3; 2; 1]
for j=1:Plan-1
    if j==1
        S1{j}=S0*PMean;
    else
        S1{j}=S1{j-1}*PMean;
    end
    Prob1(:,j)=S1{j};
    PCR1(j)=S1{j}*CRM;  %Predicted CR at year j
end
x=1:Plan-1;

figure(15)
plot(x,Prob1(1:6,:)','LineWidth',1)
legend('CR9','CR8','CR7','CR6','CR5','CR4','location','best')
xlabel('Time (Years)','FontSize',12)
ylabel('Probability of Condition Rating','FontSize',12)
set(gca,'FontSize',12)
print('-dpdf','-r600',strcat(cd,'\',foldername,'\',...
```

```
    'PHM Stationary BL-Probability of CR_PH',num2str(Plan)));

figure(16)
x=1:Plan-1;
plot(x,PCR1)
set(gca,'YLim',[4,9]);
xlabel('Time (Years)','FontSize',12)
ylabel('Expected Condition Rating','FontSize',12)
set(gca,'FontSize',12)
print('-dpdf','-r600',strcat(cd,'\',foldername,'\',...
    'PHM Stationary BL-Expected CR_PH',num2str(Plan)))

save(strcat('Databases\',MTComp,'TransitionMatricesPH',num2str(Plan),...
    'Ratings',num2str(Ratings)),'PDiag','POffDiag','P','S','Prob','CRM','PCR',...
    'PDiagMean','POffDiagMean','PMean','S1','Prob1','PCR1');

figure(17)
x=1:Plan-1;
plot(x,PCR,'-',x,PCR1,'--')
set(gca,'YLim',[4,9]);
% set(gca,'YLim',[1,9]);
xlabel('Time (Years)','FontSize',12)
ylabel('Expected Condition Rating','FontSize',12)
legend('Non-Stationary','Stationary')
set(gca,'FontSize',12)
print('-dpdf','-r600',strcat(cd,'\',foldername,'\',...
    'PHM Non-Stationary vs. Stationary_BL-Expected CR_PH',num2str(Plan)))
```