

EVALUATING AND EXTENDING THE CONCEPT OF WISDOM OF CROWDS  
IN THE CONTEXT OF PROBLEM SOLVING

by

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## ABSTRACT

MIN SUN. Evaluating and extending the wisdom of crowds in the context of problem solving.  
(Under the direction of DR. MIRSAH HADZIKADIC)

James Surowiecki in his book on the wisdom of crowds [Jame04] wrote about the decisions made based on the aggregation of information in groups. Knowing the many case studies and anecdotes which show the success of wisdom of crowds, he argues that under certain circumstances the wisdom of crowds is often better than that of any single member in the group. This paper provides a new way of problem solving— using the wisdom of crowds (collective wisdom) to handle continuous decision making problems, especially in a complex and rapidly changing world. By extending the concept of Wisdom of Crowds, the method of using collective wisdom is applied to various fields, from Prisoner's Dilemma to simplified stock market. Simulations are built to evaluate this new problem solving method and different aggregation strategies are suggested based on different environments.

## DEDICATION

There are a number of people without whom this thesis might not be completed.

To my parents, Jinfen Sun and Yuehua Gu, for supporting me to fulfill my dream.

To my adviser, Dr. Mirsad Hadzikadic, a knowledgeable, energetic and dedicated mentor, for teaching, guiding, inspiring and encouraging me over the years.

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## CHAPTER 1: INTRODUCTION

Decision making has been the subject of research for a long time. Generally speaking, decision making is the process of selecting one course of action from among several alternatives. It involves using what you know (or can learn) to get what you want [Walk87]. The decision making abilities are related to time preference, risk preference, probability weighting, ambiguity aversion, endowment effects, anchoring, cognitive abilities, and other widely researched topics [Shan05]. Decision making involves expertise, information, experience, emotions, relationships, and goals, thus making it necessary that individuals have systematic tools to deal with any complicated problem. Many computer-based decision support systems [Dani02, Henk85, Efra08, Hols96] have been promoted to help people make decisions in either individual or business enterprise situations. Although computer-based decision support systems have been widely researched and used, managers sometimes feel disappointed with their performance for one or more of the following reasons: 1.) difficulties in collecting useful information in a specific field; 2.) the cost of setting up and updating knowledge databases; 3.) inherent inadequacies in dealing with complex and rapidly changing environments; and 4.) difficulties in determining the proper decision making model/strategy, especially for problems in social sciences or economics that involve numerous human interactions and uncertain personal feelings. With these concerns in mind, a new concept for making decisions is introduced – the (modified) wisdom of crowds.

The idea of using the wisdom of crowds for decision making was originally introduced by J. Surowiecki in a book entitled “The Wisdom of Crowds” [Jame04]. He argues that under certain circumstances the performance of a crowd is often better than that of any single member of the group. This idea appears to be appropriate for explaining the behavior of financial markets, as observed by Nobel Prize-winning economist William Sharpe [Ayse04].

The concept of “wise crowds” might also be useful to decision makers encountering other complex problems. Given that 1.) a properly formed group makes better decisions than individuals do; 2.) aggregation method is related only to the types of groups, not the problem to be solved; 3.) the manner in which one person makes a decision is seldom changed, making decision using “the wisdom of crowds” will save a lot when a decision-making system is built. A decision making system that uses the wisdom of crowds has various benefits, including reducing the cost of collecting information and assembling databases for each field, avoiding frequent data updates and canceling out human bias through information aggregation. The wisdom of crowds has been successfully used in the real world of technology. For example, collective voting has been successfully used by some search engines, including Google [Davi06].

Even though there are many case studies and anecdotes that demonstrate the importance of collective wisdom, there are also authors supporting the opposite conclusion. Some of these opponents are cited in the famous mid-19th century Charles MacKay work, “The Extraordinary Popular Delusions and the Madness of Crowds” [Char41]. For example, the South Sea Company bubble of 1711–1720, the Mississippi Company bubble of 1719–1720, and the Dutch tulip mania of the early seventeenth century. All three cases show that popular delusions began so early,

spread so widely, and had lasted so long. Nonetheless, even in cases where the crowd itself is not smart – as where mobs or crazy "herds" of investors predominate – the collective wisdom can still usually be shown to be superior to the average of all of the crowd members.

In the following sections, Surowiecki's theory – that under certain circumstances, the performance of a crowd is often better than that of any single member of the group – is extended to address a continuous decision making problem, one that deals with a complex and rapidly changing world filled with interactions. The key criteria that separate the wise crowd from the irrational one are investigated using a computer-based simulation. A classical problem in game theory – the Prisoner's Dilemma – is introduced as the context of simulation.

Additionally, the scenario of using the wisdom of crowds for good is extended by adding two other factors into the crowds which make both individuals and crowds "smarter" over time. Those factors are the ability to learn through individual contact and the ability to evolve through generations. Finally, the different types of crowds are investigated and a relationship between the formation of crowds and their performance (aggregation strategies) is suggested for varying environments.

Experiments show that the wisdom of crowds approach is always superior to the average and often to the best performing strategy in the crowd. A step-by-step procedure for making decisions by use of the wisdom of crowds is suggested. Finally, an additional experiment involving a simple stock market shows the possibility of using the wisdom of crowds in different fields, including stock market and trading in general.

## CHAPTER 2: BACKGROUND FOR DECISION-MAKING

### 2.1 Concept of Decision Making

All life is problem-solving [Karl99]. Each day, human beings make thousands of decisions in a variety of situations, in response to issues that range from the very simple to the very complex. Some are conscious and deliberate decisions; others are selections and choices that one makes without much thought or, as one might say, instinctively. When a person falls into sleep out of exhaustion, the body and mind are making a decision driven by the survival instinct. When one refuses to share a scarce amount of a food supply with a friend trapped with that person in a mine or hole, one is giving into a selfish instinct to survive. When one chooses to cooperate with an opponent so as to gain the opponent's reciprocal cooperation, one is deliberately calculating the odds that befriending the adversary will be in one's long-term interest.

Two different persons may make quite different decisions and choices in response to the same situation. You may be able to predict with certainty the response that a well-known friend will make to a given circumstance. On the other hand, a total stranger can either surprise you with his choice or, if you have sufficient clues about his thinking, he may act along the same lines that you can predict.

How do human beings make the numerous decisions that they face each day? Can you predict others' decisions? Scientific researchers may be able to answer part of the question. Decision making has been the subject of research for a long time, and theories have been developed to identify and explain decision-making patterns.

Generally speaking, decision making is the process of selecting one course of

action from among alternatives. It involves using what you know (or can learn) to get what you want [Walk87]. From a cognitive perspective, the decision making process is regarded as a continuous process integrated with the environment. From a normative perspective, analysis of individual decisions is concerned with the logic and rationality of the decision making process and the invariant choice to which it leads [Dani00]. The decision making behavior is also affected by social pressure, time pressure, and other forces.

For a real life decision, one cannot determine whether the correct decision has been made, since the decision maker could not possibly know what would have happened had he or she chosen a different option [Clif02]. But to a certain extent, the concept of decision making today is treated as, or mixed with, problem-solving: individuals believe that each decision is made to achieve some goal, and thus the correctness of the decision can be evaluated by comparing the consequence of the decision to the goal sought. In other words, only the outcome matters. Simple evaluation of the decision is made by understanding the consequences, and evaluating how closely those consequences come to the goal that was intended.

## 2.2 Decision Making Techniques in Everyday Life

When we recognize that individuals cannot always attain what they want, and that a decision can be evaluated by comparing the actual consequences to the intended goal, certain skills and techniques for better decision making emerge. The decision making techniques used in everyday life include:

- 1.) flipping a coin;
- 2.) asking the advice of friends or experts;
- 3.) listing pros and cons, called “Balance Sheet” [Jani77, Whee80];
- 4.) performing a cost-benefit analysis.

The first method is totally random. The result is always fortuitous. And, the total absence of any reasoning or thought in this method guarantees that it will never improve the decision making. While the other three methods are less random than the coin flip method, those other methods are also vulnerable to the influence of bias. Bias is a point of view or personal prejudice that tends to interfere with one's ability to be impartial, unprejudiced, and objective [DICT01]. Sources of one's bias include culture, ethnicity, geography, gender, political philosophy, personal feelings [Wiki01]. Bias may be introduced into the decision in the early stage – when information is collected – and, to that extent, bias may blind one to certain relevant information, or skew one's interpretation and assessment of the value of certain information or data. Bias may also be present throughout the entire process – even after information has been collected – thus subjecting the whole decision process to faulty or unwarranted interpretations.

Various deficiencies are also another major issue that prevents individuals from making better decisions. This can include [Bill06]:

- 1.) insufficient information;
- 2.) insufficient time to review alternatives;
- 3.) insufficient participation of key decision makers;
- 4.) insufficient planning;
- 5.) insufficient communication;
- 6.) insufficient ongoing measurement and management of the decision's implementation.

Since decision making involves various levels and amounts of expertise, information, experience, emotion, relationships, and goals, there are computer-based "decision support systems" that are promoted as useful in helping individuals to make

better decisions without (or with little) human interference.

### 2.3. Current Computer-based Decision Support Systems

Computer-based decision support systems can help individuals collect more-adequate information and make decisions more readily in complicated situations, while at the same time eliminating individual bias. A well-designed decision support system should provide integration and generation of the information, support the exploratory nature of the scientific discovery process, and allow for the development of alternatives and increase the effectiveness of those responsible for decisions. The computer can support and reinforce human judgment in the fulfillment of tasks, which have elements that cannot be specified beforehand [Segr03, Hgso83].

According to Keen[Pete78], the concept of the decision support system first appeared in the late 1960s, and came into its greatest use in the 1980s. Having at its core computer-based information systems, the decision support system evolved as technology advanced. There emerged such tools as data warehousing [Ying02] and OLAP [Bhar01]. Table 1 summarizes the major developments in the evolution of decision support system concept [Dani00b].

Evolution of DSS Concepts			
1960s	1970s	1980s	1990s
MIS and structured reports Interactive systems research Theory development	BrandAid MDS RDBMS	Key books GDSS EIS	Data Warehousing OLAP Data mining

**Table 1:** Evolution of DSS Concepts

As suggested in “*Management Decision System: Computer-Based Support for Decision Making*” written by Michael S. Scott Morton in 1971[Scot71], managers could benefit from using decision support systems. Since then, knowledge-based decision support systems have been widely used. Managers, however, sometimes express disappointment with the performance of such systems for a variety of reasons:



1.) the difficulties in collecting useful information in a specific field; 2.) the cost of setting up and updating knowledge databases; and 3.) the systems' inherent inadequacies to deal with complex and rapidly changing environments.

And, in such areas as the social sciences and economics – which involve numerous human interactions and uncertain personal feelings – a fourth concern has arisen, namely, the difficulty of determining an effective decision making model or strategy that accurately accounts for those subjective factors. With these concerns in mind, a new way of making decisions – termed "the wisdom of crowds" – has emerged, in hopes that it will relax the need to collect information and assemble databases, and instead use "the crowd" to resolve problems that involve numerous human interactions and uncertain personal feelings. The details of the new decision-making algorithm using the wisdom of crowds are introduced in the following chapter

## CHAPTER 3: WISDOM OF CROWDS

### 3.1. Definition of Wisdom of Crowds

As identified by Herbert Blumer, there are four categories of crowds: a casual crowd, a conventional crowd, an expressive crowd, and an acting crowd [Herb69]. A “crowd” in Surowiecki’s book [Jame04], is an acting crowd – any group of persons who can act collectively to make decisions and solve problems. Wisdom of Crowds theory simply suggests that a collective can solve a problem better than most of the individual members of the group acting alone.

As MacKay [Char41] points out, not all crowds (groups) are wise. One need look no further than the stock market and its many examples of fads, market bubbles, and a "herd mentality" in which the majority proves to have been mistaken in its judgment. Consequently, efforts have been made to understand under what circumstances the crowd is wise. Surowiecki suggests the following key criteria to separate wise crowds from irrational ones [Jame04]:

- *Diversity of opinion* - Each person should have private information, even if it is just an eccentric interpretation of the known facts.
- *Independence* – Each person's opinion should not be determined by the opinions of those around them.
- *Decentralization* – The persons comprising the crowd should specialize and draw on local knowledge.
- *Aggregation* - Some mechanism exists for turning private judgments into a collective decision

Three distinct settings have been identified in which crowds may be smarter than the individual members [Jame04]. The first is needle-in-the haystack problem, where some persons in the crowd may know the answer, while many, if not most, do not. The second is a *stated estimation* problem, where some person may "get lucky" and hit the precise answer (while not being aware in advance of the "accuracy"), but on the average, the group performs better than most of the member in the group. Finally, there is a *prediction* problem, where the answer has yet to be revealed [Scot07, Mich07]. For the prediction problem, the unrevealed answer can be either fixed (*e.g.*, the prediction of the next Oscar winner does not change the answer itself) or it can be "fluid" (*e.g.*, the return on your next investment where your action might affect the answer).

A well-known example of the "needle in the haystack" problem is seen in the television game show, "Who Wants to Be a Millionaire?" The contestant is asked a series of multiple-choice questions, ultimately leading to the grand prize of \$1 million. Where a contestant does not know the correct answer to any particular multiple-choice question, she has three options by which to narrow the guess: (1) eliminate two of the four possible answers, (2) call a predetermined "expert" for counsel, and (3) poll the studio audience. Option two – calling the expert – has a respectable record of providing the correct answer two-thirds of the time. Polling the entire studio audience – a group of folks who had nothing better to do that afternoon than to attend the show – has a success rate exceeding 90 percent. Normally the audience was asked for simpler common knowledge questions.

The success of polling lies in the fact that – assuming complete randomness in the answers provided – even a small percentage of the people in the crowd who know the correct answer can add a noticeable advantage to the group's "wisdom". If one

assumes – as with a coin flip – that those in a large audience who have no idea of the correct answer will cancel one another out (half get it right, half get it wrong), then the group's majority decision is determined by those in the audience who do know the answer. If an audience of 100 has 10 members who know the right answer and 90 who have no idea what the right answer is, then if the 90 who are ignorant cancel one another out (45 get it right, 45 get it wrong), then the 10 who know the answer lead the total audience tally to be 55 correct, 45 incorrect.

The “stated estimation” problem normally defines the “guess a quantity or number” scenario. An interesting characteristic of this type of problem is that although one or several of the crowd members may come close to predicting the correct value/quantity of the target variable, none of them knows it for sure when the guess is offered. The well-known example is the “Francis Galton's surprise.” The crowd at a county fair was asked to guess the weight of an ox that was exhibited at the fair. The person with the most accurate answer was promised a prize. Everyone tried his or her best to provide the right answer, while maintaining the secrecy of the guess. The participants included some experts (*e.g.*, butchers) and many non-experts. It was obvious that the experts stood a better chance of winning the prize than the non-experts. However, since the target number was a continuous/real number, the non-experts had a small chance of hitting the most precise number by luck and thereby winning. To his surprise, Galton discovered that the average of all the responses was, in fact, closer to the ox's true butchered weight than the individual estimates of most crowd members, including those made by the cattle experts.

Let's look closer into this stated estimation problem. The collective error can be described as [Scot07]:

$$\text{Collective error} = \text{Average individual error} - \text{Prediction diversity}$$

The average individual error combines the squared errors of all of the participants, while the prediction diversity combines the squared difference between the individuals and the average guess. This equation tells us [Scot07]:

1. The crowd's aggregate prediction is always better than those of most individuals in it, regardless of whether the crowd has a normal or skewed distribution of answers. Sometimes, it can even be better than the best individual, given enough diversity in the right direction.
2. We can reduce the collective error by either increasing the accuracy or increasing the diversity of the crowds.

Other types of problems have been grouped into the third category: the prediction problems. An interesting story is told in Surowiecki's book [Jame04], regarding a submarine lost at sea. The task was to locate the submarine with a very limited knowledge of when and under what weather conditions the submarine went down. A group of specialists with a wide range of expertise was asked to offer their best independent/individual guesses regarding the various scenarios for the submarine's trajectory in its last moments. Although no one knew exactly what had happened, by building a composite picture of the projected movements of the submarine, a remarkably accurate guess was formed and the submarine was found. In this case, even though no single individual in the group knew any of the exact answers, the group as a whole produced them all. This story suggests that even if the crowd is not aware of how much useful information each individual has, the appropriate aggregation of partially available information can provide the best answer.

### 3.2. Using the Wisdom of Crowds vs. Other Decision Making Techniques

#### 3.2.1. Wisdom of Crowds vs. Traditional Group Voting

Voting is a method by which a group – an electorate or a meeting of persons –

can make a decision or express an opinion, often following discussion, debates, speeches, or an election campaign [Wiki02]. Although the wisdom of crowds and group voting may share the same initial knowledge (information from group), they differ in many ways, as shown in Table 2.

Wisdom of Crowd	Traditional Voting
Normally no meetings Little group pressure	Normally face-to-face meetings Under group pressure
Allow the variety	Normally need to reach group consensus
Different aggregate methods to interpret the result	Majority vote under the control of group leader or mediator
Cancel out groupthink through the variety	Groupthink
Equal importance for each member in the group	Normally leader has more importance

**Table 2:** Difference between Wisdom of Crowds and Traditional Voting

### 3.2.2. Wisdom of Crowds vs. Asking the Specialists

Specialists are persons who devote themselves to one subject or to one particular branch of a subject or pursuit. The differences between using the wisdom of crowds and asking specialists are shown in Table 3.

Wisdom of crowds	Specialist
Knowledge in all domains	Knowledge in specific domain
Not time-sensitive	Time-sensitive
Equal importance for each member in the group	High importance for experts
Cancel out bias through variety	Might have bias
Different aggregate methods to interpret the result	Listen to specialist

**Table 3:** Difference between Wisdom of Crowds and Specialist

Compared to the traditional group voting and asking specialists, using the wisdom of crowds to make the decision takes advantage of using collective wisdom

without the group pressure and individual bias.

### 3.3. Using the Wisdom of Crowds – A New Way to Make Decisions

Decision support systems (DSS) are a specific class of computerized information systems that support business and organizational decision-making activities. A properly-designed DSS is an interactive software-based system intended to help decision makers compile useful information from raw data, documents, personal knowledge, and/or business models so that they can identify and solve problems and make decisions [Wiki03].

While widely-used model-driven DSS, data-driven DSS, and knowledge-based DSS suggest making decisions by abstracting information from individual cases, using wisdom of crowds/ collective wisdom provides a different way to make decisions by using aggregated information rather than individual detail. A decision making system using the wisdom of crowds can reduce the cost of collecting information and assembling databases for each field, avoid frequent data updates and cancel out human bias through information aggregation.

In order to demonstrate the wisdom of crowds, we designed and implemented a simulation that can aggregate information from a “crowd” in the context of a problem commonly referred to as the Prisoner’s Dilemma. The Prisoner’s Dilemma is a type of non-zero-sum game developed in game theory. We extended the two-player game into a situation involving hundreds of players (crowd) playing against each other pair-wise. This allows for exploration of various aggregation strategies. This simulation is useful to aid in exploring the effectiveness of the wisdom of crowds when the right answer is not fixed and continuous or serial decision-making is called for. The following sections provide the details of the simulation.

## CHAPTER 4: PRISONER'S DILEMMA

### 4.1. Definition of Prisoner's Dilemma

Since Merrill Flood and Melvin Dresher first articulated the Prisoner's Dilemma (PD) in the 1950s [Floo58], it has been the subject of considerable research, especially after Robert Axelrod introduced the concept of the iterated PD in his book *The Evolution of Cooperation* [Robe84]. The PD is a typical non-zero-sum game explored in game theory. It is based on the well-known expression of PD, the Canonical PD payoff matrix [Robe84], which shows the non-zero net results for the players. In its classical form, the prisoner's dilemma ("PD") is presented as follows:

Player A	Player B		
		Cooperat e	Defect
	Cooperat e	3,3	0,5
	Defect	5,0	1,1

**Table 4:** Definition of Prisoner's Dilemma Problem

In a game, two players are asked to choose to cooperate or defect separately. If both choose to cooperate, each one gains 3 points. If both choose to defect, each one gains only 1 point. If two players choose differently, the one choosing defect gains 5 points while the one choosing cooperate gains none. The goal of the game is to get as many points as possible. Even if it is in both their best interests to cooperate, the two players might still choose to defect.

### 4.2. Strategies for Prisoner's Dilemma

Some of the best-known strategies for solving this game are listed below



[Robe84, Krai95]:

- *Tit-For-Tat* – Repeat opponent's last choice;
- *Tit-For-Two-Tats* – Similar to Tit-For-Tat, except that the opponent must make the same choice twice in a row before it is reciprocated;
- *Grudger* – Cooperate until the opponent defects; then, always defect (unforgiving)
- *Pavlov* - Repeat the last choice if it led to a good outcome;
- *Adaptive* - Start with the set of pre-selected choices (c, c, c, c, c, c, d, d, d, d, d) ; then, after the initial 11 moves, select actions that give the best average score; re-calculated after each move

Finding the strategy to gain the highest number of points is the ultimate problem for the Iterated Prisoner's Dilemma game. Every year, the IPD tournament [Pris01] is held to evaluate strategies from different competitors. Also, the genetic algorithms have been widely used [Axel87, Jenn02] to discover the best strategy. Currently, memory- and outcome-based strategies such as Tit-For-Tat [Pris02] and Pavlov [PRIS02] are regarded as the most effective ones [Foge93, Darw94, Krai93, Krai95].

#### 4.3. Extended Prisoner's Dilemma and Wisdom of Crowds

Extending the “two-player” game to the “many players” context brings about the situation where hundreds of players (a crowd) play together/against each other. With no central control, players begin to “cooperate” or “defect” based on their own strategies. After each round, points are added up for each player. Consequently, a potential “smart” crowd is formed. This decentralization of strategies for playing is interpreted as a set of diverse opinions held in the crowd. Then, a simple polling of playing strategies serves as the aggregation method for understanding the vote/wisdom of the crowd.

As opposed to the needle-in-the-haystack problem and stated estimation problem, PD states a different type of problem -- dynamic prediction problem [Scot07]. The term “dynamic” is used because the outcome is influenced not only by each play, but also by each player’s history of previous predictions. The introduction of this “dynamic” process helps us evaluate performance of various strategies in different crowds over time, which is similar to the decision-making process or cognitive behavior of agents in real life.

Although more complicated, the participating crowd in the context of PD satisfies the key criteria to get a smart crowd:

#### *1.) Diversity and Decentralization*

Page [Scot07] divides diversity into four frameworks:

- Perspective: ways of representing situations and problems;
- Interpretations: ways of categorizing or partitioning perspectives;
- Heuristics: ways of generating solutions to problem;
- Predictive Models: ways of inferring causes and effects.

Decentralization is defined as the dispersion or distribution of functions and powers, specifically, the delegation of power from a central authority to regional and local authorities. Types of decentralization include political decentralization, administrative decentralization, fiscal decentralization and economic decentralization [Akai02, Dubo09, Shar05, Stan05]. As one of the key criteria forming a smart crowd, decentralization emphasizes that individuals are able to specialize and draw on local knowledge [Scot07].

In the PD setting, each agent is given a memory and a strategy. The memory serves to record and accumulate new knowledge, which represents diversity in two ways: the agent’s game history with a certain player, and the accumulation of local

knowledge. The agent's strategy is the ability to choose either to cooperate or to defect based on the information stored in the memory. The strategy also represents diversity in two ways: diversity in the ways of generating solutions to the problem, and diversity in the ability to draw conclusions from the local knowledge, since the agent does so without the preset upper-level/centralized guidance. This diversity and decentralization among the agents is guaranteed through the combination of interpretations and heuristic frameworks described above, as well as through the process of dispersed decision-making.

## *2.) Independence*

Different from the strict "pure" independence required in Surowiecki's theory, the agents playing the PD in our system (the Iterated Prisoner's Dilemma) have limited independence, which allows communication between, and learning from other agents as well as evolution. The agents in our system are regarded as independent players with connections and learning ability.

However, we provide a "controller" to control the level of agent independence in the system, which enables us to experiment with both independence-securing and learning-enabling environments.

## *3.) Aggregation*

Aggregation means combining outputs/solutions from different entities into higher-level entities. Information Aggregation can sometimes obtain more information than the sum of individual cases, by canceling out bias or obtaining hidden information from Privilege database which provides restricted information based on the user's privilege. In the PD game, aggregation assumes deriving a group-level solution by combining the individual members' contributions (or solutions), regardless of whether these contributions are duplicative, contradictory, or

incomplete. The most commonly used methods for this type of aggregation are sampling, polling, and voting. Sampling is an aspect of data collection. A good sampling is the select of an unbiased or random subset of individual observations within a population [Wiki04]. Polling and voting is a method for a group to make a decision or express an opinion.

## CHAPTER 5: METHODOLOGY

### 5.1. Definition of Complex Adaptive Systems (CAS)

In this paper, a simulation using the framework of Complex Adaptive Systems (CAS) is designed and implemented to demonstrate the wisdom of crowds in the context of the Prisoner's Dilemma (PD) problem.

The CAS framework represents a dynamic network of agents (representing cells, species, individuals, firms, nations, etc.) that act in parallel, while constantly reacting to what the other agents are doing [Mmit92, Wiki05].

The term "system" derived from the Greek "systema", is widely used in culture, economics, and biology to describe a set of interacting entities forming an integrated whole. The characteristics of a system include a set of abstract entities, structure, behavior, and interconnectivity. A system may be either simple or complex. A system is considered complex if it is agent-based and exhibits non-linear behavior, feedback loops, self-organization, co-evolution, and emergence [Comp01, Eric99, Fcbi06, Gary98]. Agent in CAS is the smallest unit in the system, which can interact and act based on its rules. Such a system is considered adaptive if it has the capacity to change and learn from experience. In complex systems, the processes occur simultaneously on different scales or levels, and the intricate behavior of the whole system depends on its units in a nontrivial way [Tama02].

Different from traditional multi-agent systems, a CAS is known for its large number of agents, and relatively simple rules, which results in the system's being not complicated but complex and adaptive [Tama02, Mich02].

Complexity is non-deterministic. The emergence of complexity theory shows a domain between deterministic order and randomness that is complex [Paul98]. Randomness and determinism are both relevant to the system's overall behavior. Such systems exist on the “edge of chaos”—they may exhibit almost regular behavior, but also can change dramatically and stochastically in time and/or space as a result of small changes in conditions [Tama02, Perb96]. The research in CAS is to capture the principal laws behind the phenomena.

The theory of and experiments with simple living systems – spanning statistical physics, information theory, self-organized criticality, percolation theory, and fitness landscapes – were introduced in “*Introduction to Artificial Life*” by Christoph Adami [Chri98]. Key concepts and general methods used in studying complexity in statistical physics, evolutionary biology, and economics were revealed for the first time in “*Foundations of Complex-Systems Theories: In Economics, Evolutionary Biology, and Statistical Physics*”, in which the author highlights the features common to each area, and describes how we understand and deal with complexity [Sunn99].

The control in a CAS is distributed. Any coherent behavior of the system has to arise from the competition and cooperation among the agents (constituent parts) themselves. The overall behavior of the system is a result of the decisions made by individual agents in each cycle [Mmit92].

The CAS system often exhibits the property of self-organization and emergency [Haro02, Fcbi06, Stev01]. Self-organization is a process in which the internal organization of the system increases in complexity without being guided or managed by an outside source. Self-organizing systems frequently demonstrate emergent properties [Wiki05]. Every resultant is either a sum or a difference of the co-operant forces: their sum, when their directions are the same; their difference, when their

directions are contrary. The emergent is unlike its components insofar as these are incommensurable; it cannot be reduced to their sum or their difference [Blit92].

## 5.2. Agent-based Modeling in CAS

Bottom-up model is another feature of CAS [Jmep96, Pete02]. A bottom-up approach consists of piecing together systems to give rise to grander systems, thus making the original systems sub-systems of the emergent system. In a bottom-up approach, the individual base elements of the system are first specified in great detail. These elements are then linked together to form larger subsystems, which in turn are linked, sometimes in many levels, until a complete top-level system is formed. This strategy often resembles a "seed" model, where the beginnings are small but eventually grow in complexity and completeness [Wiki06].

An agent-based model (ABM) is a computational model for simulating the actions and interactions of autonomous individuals with a view to assessing their effects on the system as a whole [Wiki07]. The benefits of ABM over other modeling techniques are threefold: 1.) ABM captures emergent phenomena; 2.) ABM provides a natural description of a system, and 3) ABM is flexible [Eric02]. ABM is well suited for modeling a CAS in a bottom-up style.

## 5.3. Implementation of Prisoner's Dilemma in CAS

Examples of CAS include the stock market, social insect and ant colonies, the biosphere and the ecosystem, the brain, the immune system, and any human social group-based endeavor [John01, Kels95, John07].

Hence, it is natural to describe the Prisoner's Dilemma as a complex adaptive system in order to reveal spontaneous reactions among individual players, as well as the wisdom hidden inside the group as a whole.

As mentioned before, the Prisoner's Dilemma is a type of non-zero-sum game

developed in game theory. The basic idea builds upon two suspects charged with having committed a crime. During questioning – which occurs after they have been separated – each must decide whether to “cooperate” with each other or to “defect” by cooperating with the police. Cooperating is the best outcome for both, since they both will go free, given that there is no proof that they actually committed the crime. However, as they do not trust each other, they are enticed to "defect" and confess the crime, in which event each defector will get a lighter sentence than his partner. Of course, the worst-case scenario is if they both defect, thus securing a lengthy prison sentence for both.

In order to establish a crowd, we extended the two-player game into a situation involving hundreds of players (crowd) playing against each other pair-wise. This allows for exploration of various aggregation strategies. The details describing the Prisoner’s Dilemma in this context are introduced in Chapter 6.



## CHAPTER 6: DESIGN OF AGENT-BASED MODEL IN THE CONTEXT OF PRISONER'S DILEMMA

### 6.1. Design of Player-agents

In order to design the crowds as CAS for the Prisoner's Dilemma game, first we need to create individual "player-agents" who can "cooperate" or "defect" when playing the game, based on their own strategy. Since agents play against each other repeatedly without a central control (via random selection), we assign each agent a memory that is used to store information (knowledge) about its previous "matches," such as opponent's last action, points gained overall, etc. The player-agents initially "receive" a randomly allocated strategy that they use to select their actions, based on the information that they have. The strategy may be abandoned or modified later during the learning process, through the interactions with opponents.

The question now becomes: what kind of strategies should be available to the agents? One way to approach this problem is to understand how humans perceive and approach problems. This is obviously related to human personality factors. Raymond Cattelle suggests that there are 16 personality factors [Catt66] that influence human perceptions of and approaches to problems. To keep things manageable in this project, we selected three personality factors to describe the way that people perceive problems: dominance, vigilance, and openness to change.

#### *1.) Dominance*

Agents who are less dominant are deferential, cooperative, averse to conflict, submissive, humble, obedient, easily led, docile, and accommodating. An agent who is perceived as dominant is characterized as forceful, assertive, aggressive,

competitive, stubborn, and bossy.

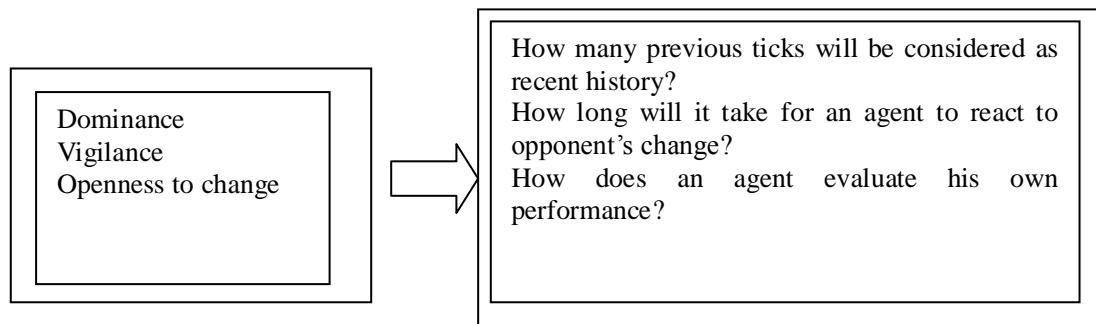
## 2.) *Vigilance*

Agents who are low in vigilance indicate behavior that is trusting, unsuspecting, accepting, unconditional, and easy-going. A highly vigilant agent is characterized as suspicious, skeptical, distrustful, and oppositional.

## 3.) *Openness to change*

Not-so-open-to-change agents are defined as traditional, attached to the familiar, conservative, and respectful of traditional ideas. Highly open agents are defined as analytical, critical, freethinking, and flexible.

In the Prisoner's Dilemma simulation, the action of each agent includes methods for perceiving and solving problems. The methods for perceiving problems can be described by considering the questions described in Figure 1:



**Figure 1:** Personality vs. Action Mode

### a. *How many previous ticks will be considered as a recent history?*

Agents with a “conservative personality” prefer consulting a longer history; otherwise, they are open to change and only care about the most recent history.

### b. *How long will it take for an agent to react to an opponent's change in behavior?*

Agents with a “vigilant personality” are more suspicious of negative behavior. They are also easier to make hostile. Otherwise, they are less sensitive to betrayal.

### c. *How does an agent evaluate its own performance?*

Agents with a “domineering personality” are more aggressive and competitive,

thinking of their opponents relative to their own gain or loss. Otherwise, they care only about their own absolute gain.

The methods for solving problems can be described with the following rules [4]:

- a. Repeat the opponent's last action*
- b. Assume an action opposite to the opponent's last action*
- c. Cooperate*
- d. Defect*
- e. Repeat own last action*
- f. Assume an action opposite to your own last action*

Consequently, in the system, each player-agent is described using a chromosome-like structure: [Mirs07]

Agent Number	Basic Strategy	Limitation	Reaction1	Reaction2
--------------	----------------	------------	-----------	-----------

where –

- *Agent Number* identifies each player.
- *Basic Strategy* indicates the strategy an agent chooses to guide its behavior.
- *Limitation* modifies the Basic Strategy as described below. Taken together, Basic Strategy and Limitation define the situation the agent is facing.
- *Reaction1* defines the behavior of the agent if the situation described by Basic Strategy + Limitation applies in the current case/match.
- *Reaction2* defines the behavior of the agent if the situation described by Basic Strategy + Limitation does not apply in the current case/match.

There are five basic strategies that show how an agent perceives problem:

0. The agent does not care what happened before.
1. The agent takes into consideration the total number of times the opponent

cooperated or defected in the past.

2. The agent takes into consideration whether during the previous X number of matches/time (X defined by Limitation) the opponent cooperated or defected (X times in a row).
3. The agent takes into consideration the average number of points it received previously by cooperating/defecting when playing against the same opponent.
4. The agent takes into consideration whether the number of points it received from the last play is less than three points.

Reaction1 and Reaction2 demonstrate the methods by which an agent approaches problem which can assume one of the following values:

0. Repeat opponent's last action
1. Assume an action opposite to opponent's last action
2. Cooperate
3. Defect
4. Repeat own last action
5. Assume an action opposite to its own last action

For example, Competitor 001 shown below simply repeats the opponent's last action. This is a typical Tit-for-Tat.

001	0	0	0	0
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Competitor 101 repeats the opponent's last action if its opponent cooperated the last two times/matches; otherwise, it chooses an action opposite to its own last action.

101	2	2	0	5
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As can be seen from the above, our agents do not simply "cooperate" or "defect."

They choose to "repeat" or "reverse" an action performed earlier by their opponents or by themselves. This may be more similar to the way people behave in real life.

This process also aggregates redundant strategies often present in evolutionary algorithms.

Another parameter, “forgiveness,” could be added to the chromosome to represent the random or predefined likelihood of cooperation (when defecting for a long time) or defection (to test the opponent after cooperating for a long time). Using “forgiveness,” the chromosome could represent an even greater variety of strategies.

Also, the parameter called “history-weight” is added to the chromosome to represent the different attitudes that agents could have regarding their own history. They may choose to regard every match in their entire history equally, or they may adjust how much emphasis they want to put on either their earlier matches or their more recent ones.

## 6.2. Design of Aggregator-agents

Aggregator-agents are special participants (competitors) in the game. “Aggregator-agents” represents the wisdom of crowds by acting as aggregators of various groups within the crowd of agents.

These aggregators also participate in the game, but they have a different decision-making procedure. The aggregator-agents are given the ability to make their decisions upon consulting with their “advisory group,” formed from the set of player-agents selected by each aggregator-agent.

On each turn, aggregator-agents choose to cooperate or to defect according to the opinions from their chosen player-agent group. Unlike the regular player-agents, aggregator-agents have no strategy that can give them guidance regarding cooperation or defection; their only strategy is to decide (a) which player-agent group they want to listen to and (b) the manner in which they plan to aggregate the group’s advice.

Each Aggregator-agent is described using a chromosome-like structure:

Agent Number	Selection Strategy	Select_Number	Aggregation Strategy
--------------	-----------------------	---------------	-------------------------

where –

- *Agent Number* identifies each aggregator-agent.
- *Selection Strategy* indicates the strategy used to select a player-agent group.
- *Select\_Number* indicates how many player-agents are chosen to form the group; it can be any number between 1 and the total number of player-agents.
- *Aggregation Strategy* indicates the strategy used for aggregation.

There are four selection strategies:

0. The agent chooses the top *Select\_Number* player-agents ranked by points.
1. The agent chooses the bottom *Select\_Number* player-agents ranked by points.
2. The agent chooses the top *N* and bottom (*Select\_Number*–*N*) player-agents ranked by points.
3. The agent chooses *Select\_Number* player-agents randomly.

There are two aggregation strategies:

0. The agent chooses the majority opinion
1. The agent chooses the minority opinion

For example, Aggregator 001 shown below simply has the best ten player-agents as its advisory group, and then chooses the majority opinion.

001	0	10	0
-----	---	----	---

Aggregator 101 shown below simply has the worst ten player-agents as its advisory group, and then chooses the minority opinion (opposite to the majority opinion).

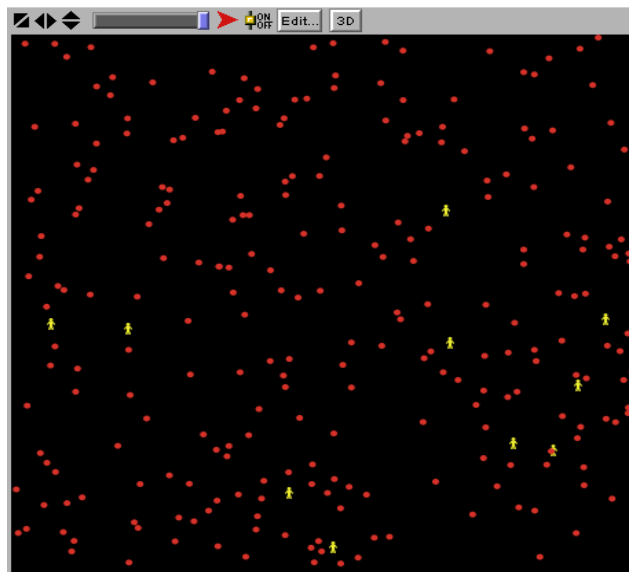
001	1	20	1
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### 6.3. Implementation Using Multi-agents Programming Tools

Netlogo is a multi-agent programming modeling and simulation tool. Using

Netlogo, crowds in the context of the Prisoner's Dilemma are simulated. Agents act and interact autonomously through playing games with each other; at the same time, a view of their effects on the system as a whole (emergency) is shown.

As shown in Figure 2, all agents are scattered randomly in the display area (90\*90 grids in the NetLogo environment) with player-agents represented by red dots and aggregator-agents represented by yellow person-shaped images. A set of basic strategies are assigned randomly to each agent. Agents move randomly in the display area (the speed of agents can be changed via the control panel). If two agents happen to be in the same neighborhood (8-neighbor grid), a meeting is initiated. Agents play a match based on the strategy they follow and the information they have about each other. After each play, the points are added and the agents move on to the next match [Mirs07].



**Figure 2:** Application

## CHAPTER 7: UNDERSTANDING THE CROWDS

### 7.1. Definition of Crowds in the Context of Prisoner's Dilemma

The term "crowd" denotes a group of persons. The individuals in a crowd – *e.g.*, a crowd at a political rally or the audience at a concert – may share a common purpose or a set of emotions. A crowd in the context of the Prisoner's Dilemma simulation is formed by common behavior – playing cooperates or defects intensely with each other. The formation of crowds can be metro-type people or relatively homogenous ones. The diversity of crowds guarantees the fidelity of information input.

As in reality, individuals in the simulated crowds have the ability to learn during the game, and they may be replaced completely under the natural selection rule. The criterion of "independence" is violated and the concept of "Wisdom of Crowds" is extended by introducing communication and learning into the crowds.

### 7.2. Interaction between Individual Player and Crowds

#### 7.2.1. Lucifer Effect or Situational Influence

In order to understand crowds in the context of the Prisoner's Dilemma, we need to examine both the individual players' behavior and their effect on the system as a whole.

Stanford Prison Experiment described in Dr. Philip Zimbardo's book, *The Lucifer Effect: Understanding How Good People Turn Evil*, is a classic simulation study of the Psychology of Imprisonment. In the study, normal college students were randomly assigned to play the role of guard or inmate for two weeks in a simulated prison. The



guards quickly became so brutal that the experiment was terminated after only six days [Phil07].

Unlike the Stanford Prison experiment described above, the crowds' simulation in the context of the Prisoner's Dilemma focuses on situational influences without touching the long discussed but unsolved question about notion of human nature. The crowd simulation experiments show that even a person using a simple strategy/philosophy behaves differently in changing situation. For example, Tit-For-Tat, in which one simply repeats the opponent's last action, actors cooperate when the opponent/situation is friendly, and defect when the opponent/situation is hostile. Without changing the strategy/philosophy, individuals change from the Good to the Evil because of situational influence.

The interactions between individual player and crowd include communication, cooperation, competition, and so on. Learning is required for the goal of evolution, since adaptation is the key point in natural selection as well as in human society [Dawk89, Blum93, Vrie00].

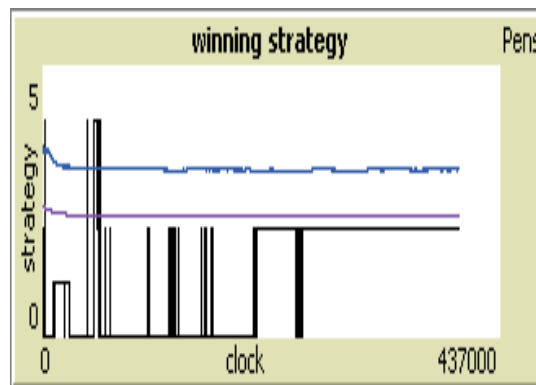
## 7.2.2. People Performance and Crowds Performance

### 7.2.2.1. Player-agents' Performance in Fixed Crowds vs. Evolutionary Crowds

In this experiment, we focus on the player-agents' performance in crowds with different preferences. Three different crowds have been tested: fixed crowds, evolutionary crowds for higher points/gains, and evolutionary crowds for lower points/gains. In a fixed crowd, the crowd composition is unchanged during the whole running time, while in evolutionary crowds, an evaluation will be made at designated stages, and part of the crowd will be replaced with preferred player-agents. For example, in the one for higher points, the chromosome of those player-agents with lower points will be replaced by the one with the highest point. During each round,

250 player-agents have been put into game.

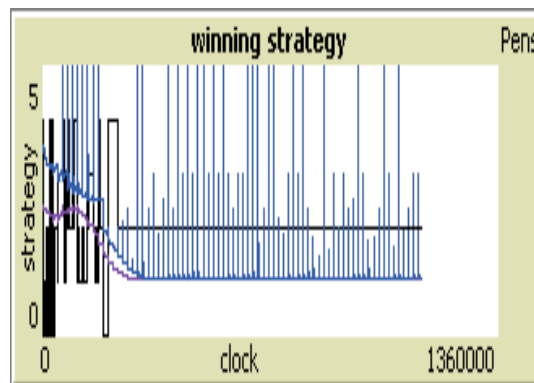
The results for the performance of player-agents are summarized in Figure 3. The jagged *blue line* shows the highest average point (winner's point). For each player-agent, the *average-point* is calculated by dividing the total number of points gained from the previous plays by the total number of plays. The average point is a number between 0 and 5. The *purple line* shows the average of the player-agents' averages. It is computed by dividing the sum of average points by the number of all competitors. The *average average point* is a number between 0 and 5, which outlines the average performance of the whole society. Finally, the *black line* shows the basic strategy chosen by the player-agent winner.



(a) Fixed Crowds



(b) Evolutionary Crowds for Higher Points



(c) Evolutionary Crowds for Lower Points

**Figure 3:** Fixed vs. Evolutionary Crowds

In the three graphs in Figure3, Chart (a) shows smoother lines for the best performance – slightly higher than a score of 3 – and average performance, which is a little lower than 3. In this fixed society, no evolution happens – meaning that the good keep good and the bad keep bad, without any change. The best performer is a greedy player, who takes advantage of the naïve peace player. But, the best performer still ends up with a score that is only a little better than the average people. The best player does not achieve the five points advantage that it expected.

Charts (b) and (c) show more ups and downs as the composition of the crowds changes. In (b), crowds are replaced gradually by those with the highest points. During the run, at the beginning the evolution shows a preference for the greedy player and eliminates the naïve player, which causes the score for the best performance to drop and for the average performance to increase, resulting in smarter overall crowds.

Later, after getting too many greedy players and no naïve ones, the greedy ones die out and are replaced gradually by those who are “smarter” enough to play both cooperation and defection according to the specific situation. At last, the crowds end

up at the score of 3, which suggests a final cooperation in whole society.

In (c), crowds are replaced gradually by those with the lowest points. This is different from what we expected: getting rid of the greedy and replacing with the naïve only helps the average score increase for a short time. Later, the overwhelmed sympathy destroy the more peaceful society by keeping on replacing the best performer with the one with the lowest points. The whole society ends up full of the ones who are afraid of being eliminated by playing cooperate.

Also, something interesting happens in the experiments: the dominant strategies in (b) and (c) are the same, which shows the importance that situational influence has on strategy. The same people can perform quite differently, depending on the particular crowd in which they find themselves.

#### 7.2.2.2. Different Crowds, Different Winners

Crowds can be categorized into different types according to the distribution of strategies, as follows:

- Type I Crowds: *crowds with diverse strategy distribution*
- Type II Crowds: *crowds with homogenous strategy distribution, which include*
  - *no-history crowds: crowds with basic strategy 0, in which player-agents care about no history*
  - *Long-history crowds: crowds with basic strategy 1, in which player-agents care about long-time history*
  - *Latest-history crowds: crowds with basic strategy 2, in which player-agents care about short-time history*
  - *Absolute-gain crowds: crowds with basic strategy 3, in which player-agents care about absolute gains*

- *Relative-gain crowds: crowds with basic strategy 4, in which player-agents care about relative gains*

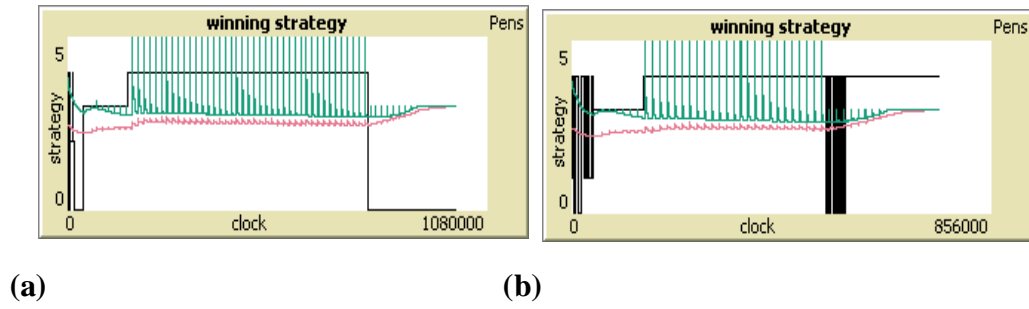
Type I crowds are those with diversity of opinion, which satisfies the criteria for “smart” crowds. Type II crowds are those in which the members share the same perceptions. In reality, Type II crowds can often be found in a crowd where interaction is allowed or group pressure is presented. Type I and II Crowds show different emergency.

Experiments are run in the simulation system several times with randomly established initial settings, including the strategies assigned, play order, and distribution of each strategy in both Type I and II crowds. The results are shown below:

a) Type I Crowds:

Type I crowds are the crowds formed by player-agents whose basic strategy is randomly assigned from 0 to 4. Type I crowds show great diversity in the formation of crowds.

In Figure 4, the jagged *green line* shows the highest average-points (winner’s points). For each player-agent, the *average points* is calculated by dividing the total number of points gained from the previous plays by the total number of plays. The “average points” is a number between 0 and 5. The *red line* shows the average of player-agent averages. It is computed by dividing the sum of average points by the number of all competitors. The *average average points* is a number between 0 and 5. It outlines the average performance of the whole society. Finally, the *black line* shows the basic strategy chosen by the winner.



**Figure 4:** “Cooperate” as the Winning Strategy

The two graphs of Figure 4 (a and b) represent two different runs of the system. Although the final winning strategies are different, the green line and the red line converge into one line at the exact number three, which shows that in the end the highest average points equals the average average points. In other words, in the end, all competitors cooperate and gain three points after each run.

Even the winning strategies are essentially similar to one another. The final winning strategy for the case represented in Figure 4(a) is the agent with the chromosome

	0	0	0	0
--	---	---	---	---

which is a typical Tit-for-Tat strategy: repeat opponent’s last action. A player-agent competitor with this strategy reacts quickly to the actions of others. It is quite defensive when it meets a defector, but friendly when it meets a collaborator. In the beginning of the run, Tit-for-Tat can be neither the best nor the worst strategy. However, when the whole society becomes friendlier, this strategy has a chance to be both the best and the dominating one.

In the graph represented in Figure 4(b), the final winning strategy is the agent with the chromosome

	4	-1	5	0
--	---	----	---	---

The player-agent looks at the points it earned from the last play. If it received

three or more points (it gets three points if they both cooperate, or five points if it defects while the opponent cooperates), it chooses to repeat the opponent's last action (cooperate). However, if it earned fewer than three points from the last play (zero if it cooperates while the opponent defects, or one if they both defect), the agent chooses the action opposite to its own last action. Similarly to the Tit-for-Tat player, the player with this "History Matters" strategy is friendly as soon as the opponent cooperates, and it punishes opponents quickly when it feels betrayed. The only difference is that "History Matters" shows the willingness to cooperate first when it notices that both players are in the situation of defection, and it will try to end the "lose-lose" situation. This approach helps it become a winner earlier than does the Tit-for-Tat.

Our experiment also indicates that player-agents with different strategies not only compete with each other; they are also necessary components for building the whole society. For example, the final winning strategy is seldom that of the initial points leader. The agent with the winning strategy is usually in the middle of the pack at first, but it demonstrates its power when the whole society develops a preference for cooperation. Strategy 3 is important in terms of its ability to change the society from a hostile society into a friendly one. From Figure 4, we can see that the *average average points* began to go up as the highest average points inched down, marking the time when Strategy 3 became the dominant strategy in the society. A player with Strategy 3 cares only about the average points it earned previously by either cooperating or defecting – a relatively stable player who does not change quickly. This makes it a "society changer". Without the participation of players with Strategy 3, Tit-for-Tat or "History Matters" would have little chance to end up being the best strategies.

In the long run, Type I crowds represent themselves as cooperate societies

formed by player-agents who only care about the latest history and react quickly.

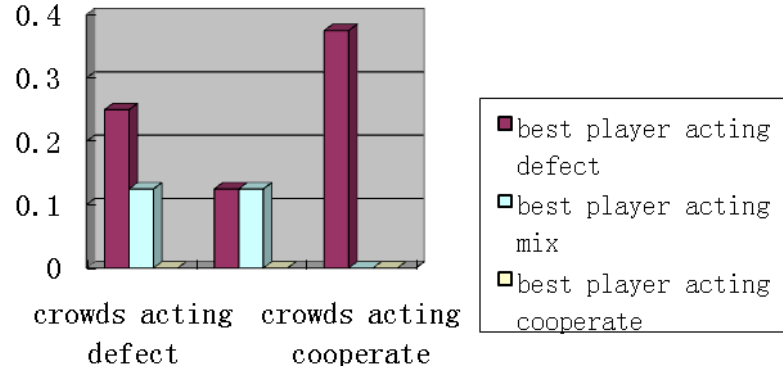
#### b) Type II Crowds

Type II crowds are formed by player-agents who share the same basic strategy selected from 0 to 4. Player-agents in Type II crowds have different ways to approach the problem but use the same way to perceive the world. After a long run in the simulation system, the crowds may show three different tendencies: 1.) most of the player-agents play defect; 2.) most of the player-agents play “mix” (defect and cooperate from time to time); 3.) most of the player-agents play cooperate. The best player (player-agent with the highest points) also has three tendencies: 1.) play defect; 2.) play “mix” (defect and cooperate from time to time; 3.) play cooperate. Thus, the combinations of tendencies for the best player and crowds leave us nine scenarios to expect. Crowds with different basic strategies show different “flavor” as follows:

##### *i. No-history Crowds*

No-history crowds are formed by the player-agents who do not care what happened before. Considering the randomly set initial settings – including the strategies assigned, play order, and distribution of each strategy – the chart in Figure 5 shows the possibility that each of the nine scenarios may occur. As shown in the graph in Figure 5, the possibility that the best player will play defect is 0.75, while the possibility that the crowds will play defect is close to the possibility that the crowds will play “mix” or cooperate. The “oblivious” property of player-agent encourages the best player to play defect and take advantage from others. This action causes chaos in the whole crowds. Whether the crowd is friendly or hostile is unpredictable from the initial setting and is changed by play order, learning speed, and other factors.



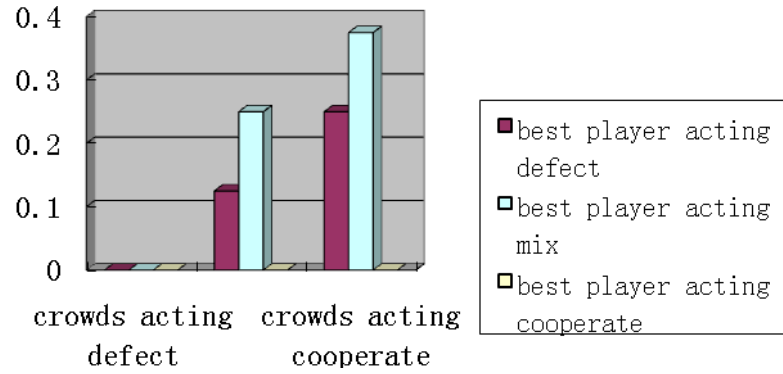


no-history crowds	best player-agent	Crowds	possibility
	mix	Mix	0.125
	mix	Defect	0.125
	mix	Cooperate	0.000
	defect	Mix	0.125
	defect	Defect	0.250
	defect	Cooperate	0.375
	cooperate	Mix	0.000
	cooperate	Defect	0.000
	cooperate	Cooperate	0.0

**Figure 5:** No-history Crowds

## ii. Long-history Crowds

Long-history crowds are formed by the player-agents who take into consideration the total number of times the opponent cooperated or defected in the past. Considering the randomly set initial settings – including the strategies assigned, play order, and distribution of each strategy – the chart in Figure 6 shows the possibility each of the nine scenarios may happen. As shown in the graph in Figure 6, the best player has over 0.60 chances to play “mix” and slightly bigger than the chance to play defect, while the crowds show the tendency to play cooperate over “mix”. The “unforgotten” property of player-agents makes the survivors in the long-history crowds more cautious and friendly, avoiding playing defect. The long-history crowds show the tendency to avoid “pure” defect.



long-history crowds	best player-agent	Crowds	possibility
	Mix	Mix	0.250
	Mix	Defect	0.000
	Mix	Cooperate	0.375
	Defect	Mix	0.125
	Defect	Defect	0.000
	Defect	Cooperate	0.250
	Cooperate	Mix	0.000
	Cooperate	Defect	0.000
	Cooperate	Cooperate	0.000

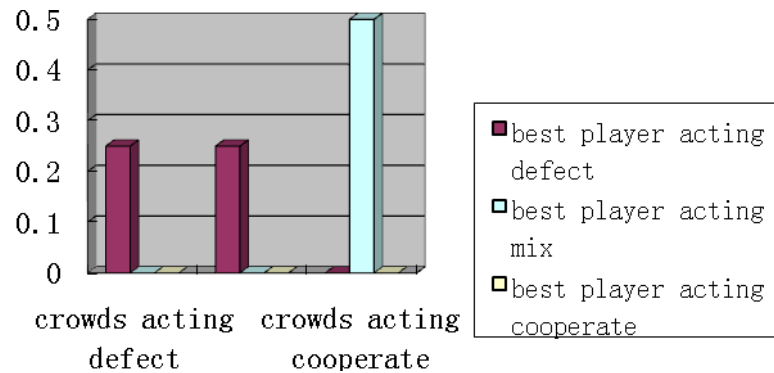
**Figure 6:** Long-history Crowds

### iii. Latest-history Crowds

Latest-history crowds are formed by the player-agents who take into consideration whether during the previous X number of matches/time (X defined by Limitation, normally no more than 3) the opponent cooperated or defected (X times in a row). Considering the randomly set initial settings – including the strategies assigned, play order, and distribution of each strategy – the chart in Figure 7 shows the possibility that each of the nine scenarios may happen. As shown in the graph in Figure 7, best player always plays harsher than the crowds: in cooperate crowds, the best player chooses to play “mix”; otherwise, the best player play defect all the time.

The “Tit-for-Tat” property of player-agents in the “latest-history crowds” makes them react promptly, but at the same time, leaves the whole crowds unstable. Whether the crowd is friendly or hostile is unpredictable from the initial setting and is changed

by play order, learning speed, and other factors.



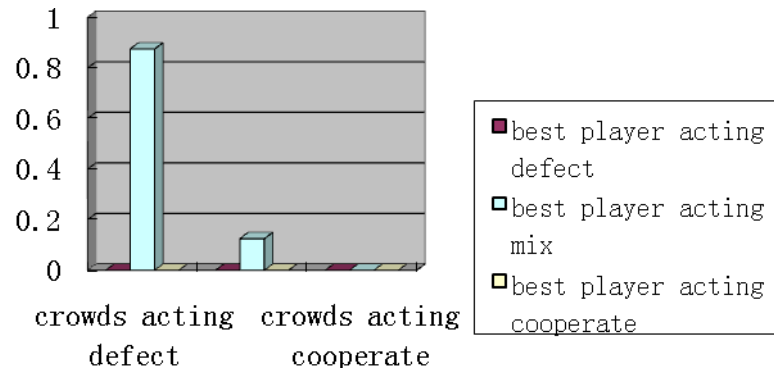
latest-history crowds	best player-agent	Crowds	possibility
	Mix	Mix	0.000
	Mix	Defect	0.000
	Mix	Cooperate	0.500
	Defect	Mix	0.250
	Defect	Defect	0.250
	Defect	Cooperate	0.000
	Cooperate	Mix	0.000
	Cooperate	Defect	0.000
	Cooperate	Cooperate	0.000

**Figure 7: Latest-history Crowds**

#### iv. Absolute-gain Crowds

Absolute-gain crowds are formed by the player-agents who take into consideration the average number of points received previously by cooperating/defecting when playing against the same opponent. Considering the random factor caused by randomly set initial settings – including the strategies assigned, play order, and distribution of each strategy – the chart in Figure 8 shows the possibility that each of the nine scenarios may happen. As shown in the graph in Figure 8, the possibility for the crowds playing defect is over 0.85, while the best player plays “mix” all the time. Actions based on the absolute gain create a hostile environment; and in hostile crowds, in order to obtain higher points, the best strategy

for a player is to find the ally. The absolute-gain crowds show dominance of defect, yet the best player seeks to create an ally, which adds a unique property to the crowds: being friendly to the newcomer.



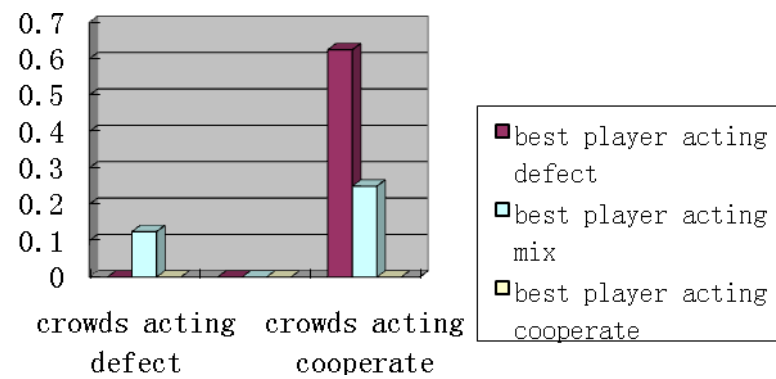
absolute-gain crowds	best player-agent	Crowds	possibility
	Mix	Mix	0.125
	Mix	Defect	0.875
	Mix	Cooperate	0.000
	Defect	Mix	0.000
	Defect	Defect	0.000
	Defect	Cooperate	0.000
	Cooperate	Mix	0.000
	Cooperate	Defect	0.000
	Cooperate	Cooperate	0.0

**Figure 8:** Absolute-gain Crowds

#### v. *Relative-gain Crowds*

Relative-gain crowds are formed by the player-agents who take into consideration whether it received fewer points from the last play, compared to its opponent. Considering the randomly set initial settings – including the strategies assigned, play order, and distribution of each strategy – the chart in Figure 9 shows the possibility that each of the nine scenarios may happen. As shown in the graph in Figure 9, the possibility for the crowds playing cooperate is over 0.85, while the best player swings between playing “mix” and playing defect. The player-agents inside the relative-gain crowds only ask for not being taken advantage by their opponents, and

get a friendly environment in return in long run.



relative-gain crowds	best player-agent	Crowds	possibility
	Mix	Mix	0.000
	Mix	Defect	0.125
	Mix	Cooperate	0.250
	Defect	Mix	0.000
	Defect	Defect	0.000
	Defect	Cooperate	0.625
	Cooperate	Mix	0.000
	Cooperate	Defect	0.000
	Cooperate	Cooperate	0.000

**Figure 9:** Relative-gain Crowds

### 7.2.2.3. New Founding in the Crowds

When observing the crowds, we notice some tipping points in the running period:

i.) Cooperate crowds points threshold; ii.) Cooperate percentage threshold. We will explain the tipping points in the following section.

i.)Cooperate crowds points threshold

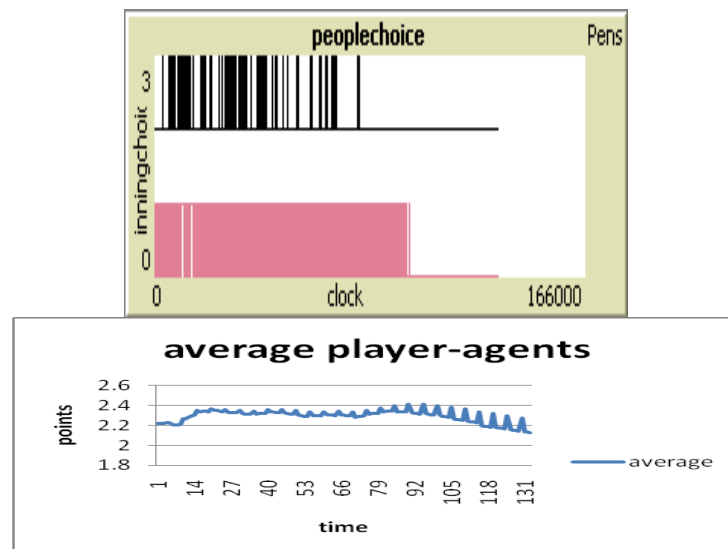
“Cooperate crowds points threshold” is the number used to tell the trend of

crowds. The two graphs in Figure 10 represent two typical scenarios in different runs of the system.

In the “peoplechoice” chart, there are two lines:

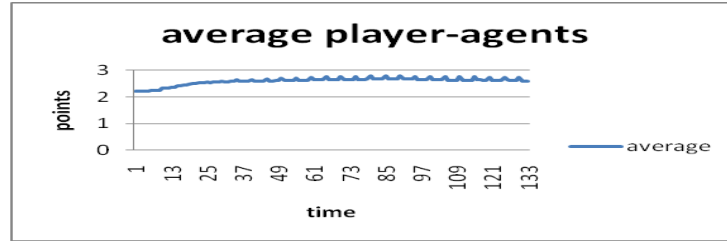
- 1.) pink line between 0 and 1 shows the dominant choice in the crowds during the time, 0 representing defect and 1 representing cooperate
- 2.) black line between 2 and 3 shows the best player-agent choice in the crowds during the time, 2 representing defect and 3 representing cooperate

In the “average player-agents” chart, the performance of the average player-agent during the time is shown.



(a) Defect Crowds





(b) Cooperate Crowds

**Figure 10:** Cooperate Crowds Points Threshold

Experiments show that once the average player-agent points is larger than 2.6, the crowds will eventually show the dominance of cooperate. Let  $x$  represent the possibility, cooperate will happen in the crowds at time  $T$ ;

$$3*x*x + (1-x)*x*5 + (1-x)(1-x)1 = 2.6 \Rightarrow x = 0.69$$

In other words, once the possibility of meeting a cooperative opponent is over 69 percent, the crowds will end up as cooperate. We call the number 2.6 here “Cooperate crowds points threshold”.

#### ii.)Cooperate percentage threshold

“Cooperate percentage threshold” is the number denoting the current status of crowds. It can be used in reality when a person does not want to give out its current choice, but the history data is available. By using history data and “cooperate percentage threshold”, the current status of crowds – i.e., whether most of the crowds cooperate or defect – can be predicted. The two graphs of Figure 11 show in each run the percentage of cooperation previously and actions of best players and crowds in long run.

Average Cooperate Percentage in Different Crowds

No-history	Long-history	Latest-history	Absolute-gain	Relative-gain
0.458	0.819	0.145	0.323	0.897
0.704	0.745	0.349	0.265	0.473
0.399	0.722	0.550	0.292	0.891
0.243	0.537	0.607	0.345	0.908

0.509	0.924	0.605	0.436	0.836
0.695	0.905	0.585	0.540	0.918
0.753	0.511	0.613	0.309	0.859
0.560	0.459	0.606	0.628	0.917

(Best Player Action, Crowds Action) in Different Crowds

No-history	Long-history	Latest-history	Absolute-gain	Relative-gain
(mix,defect)	(mix, cooperate)	(defect, defect)	(mix, defect)	(defect, cooperate)
(defect,cooperate)	(defect, cooperate)	(defect, defect)	(mix, defect)	(mix, defect)
(defect, defect)	(defect, cooperate)	(defect,mix)	(mix, defect)	(defect, cooperate)
(defect, defect)	(defect,mix)	(mix, cooperate)	(mix, defect)	(defect, cooperate)
(defect,mix)	(mix, cooperate)	(mix, cooperate)	(mix, defect)	(mix, cooperate)
(defect, cooperate)	(mix, cooperate)	(defect,mix)	(mix,mix)	(defect, cooperate)
(defect, cooperate)	(mix,mix)	(mix, cooperate)	(mix, defect)	(mix, cooperate)
(mix,mix)	(mix,mix)	(mix, cooperate)	(mix,mix)	(defect, cooperate)

**Figure 11:** Cooperate Percentage Threshold

Experiments show that

- 1.) if average cooperate percentage  $< 0.5$ , most of the crowds play defect;
- 2.) if average cooperate percentage  $< 0.5, 0.6 >$ , most of the crowds play mix;
- 3.) if average cooperate percentage  $> 0.6$ , most of the crowds play cooperate.



## CHAPTER 8: MAKING DECISION USING THE WISDOM OF CROWDS

### 8.1. Aggregator-agent in Type I Crowds

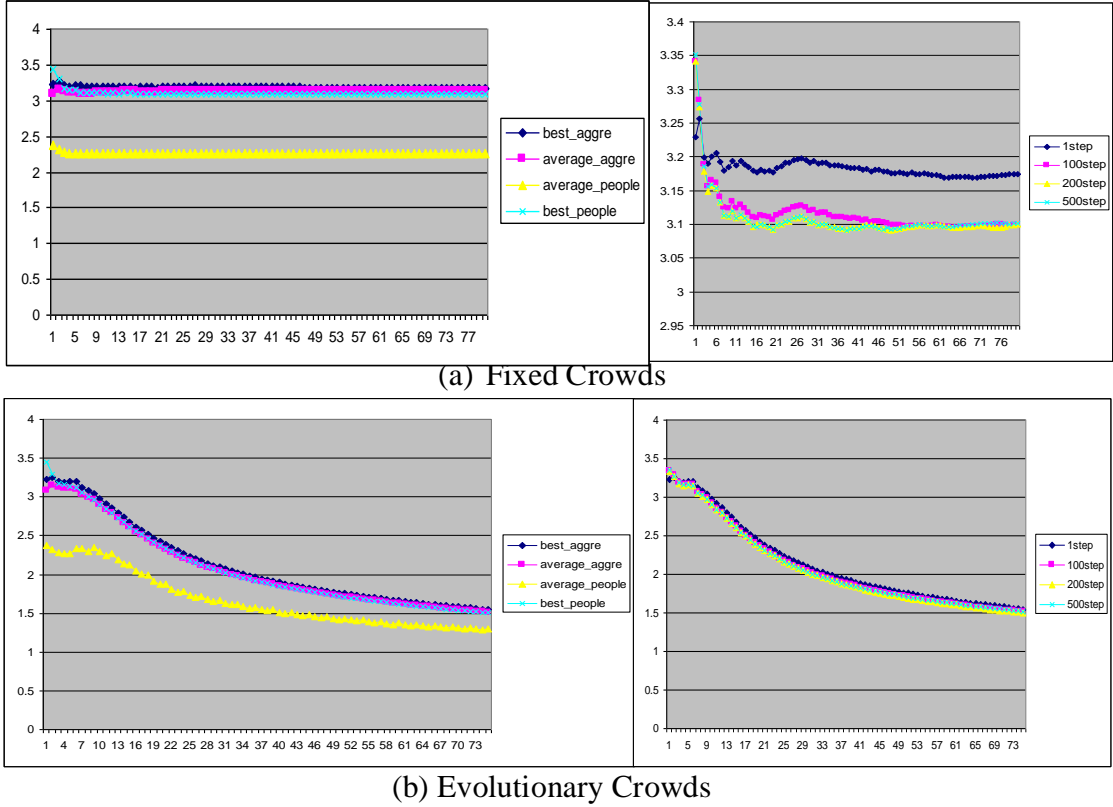
#### 8.1.1. Aggregator-agent Performance V.S. Player-agent Performance

In this experiment, we focus on the effect of controllable factors on the aggregator-agents' performance in Type I Crowds. Given different aggregation methods, the “selected frequency” of replacement, the percentage of replacement, and the fitness function, the performance of the player-agents and the aggregator-agents is recorded and compared.

*a. Given different “select frequency” for aggregator-agent in fixed crowds and evolutionary ones for higher point*

In this experiment, we focus on the effect of different “select frequency” on aggregator-agents in different crowds. During each round, 250 player-agents have been put into game. After player-agents play for a while, aggregator-agents with different “select frequency” are introduced to the game. For example, aggregator-agent whose strategy is to listen to those player-agents with highest point may choose to listen to the group of player-agents with highest point currently or keep the consult group for a while to evaluate average performance.

The “select frequency” shows the ability how an aggregator-agent can choose the currently best people. The results for different select frequency for aggregator-agent in different crowds are shown in Figure 12.



**Figure 12:** Different Select Pace in Crowds

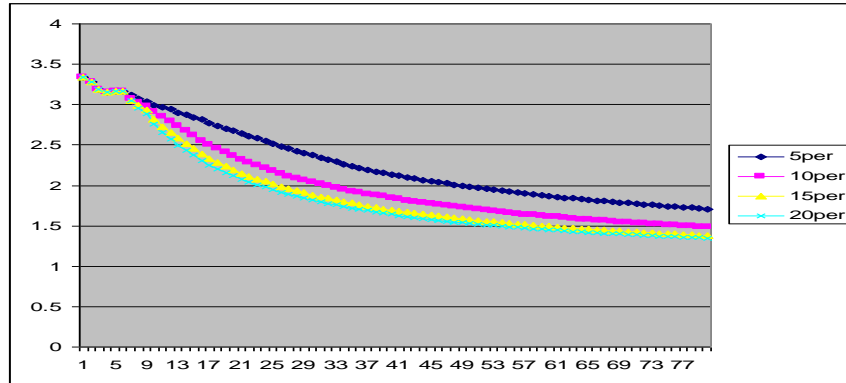
In Figure 12, the charts on the left show the better performance of aggregator-agent than the player-agent, while the charts on the right show the difference having different choosing frequency, 1step means choosing every round, 100 step means choosing every 100 step and keep the same consult player-agent group for 100 runs. Results show that the more often the aggregator-agent check the current best, the more point it can get, while checking will be costly both in the experiment and real life. In (a) fixed crowds, only 3% different between 1 step check and 100 step check, and the 100 step, 200 step, 500 step check has only slightly different, even with 500 step does better job as time goes by. In (b) evolutionary crowds, the different between different frequencies can be ignored. Results show that given expense limitation, frequency for checking won't affect the performance very

much as long as the pace is relatively small to the whole running time.

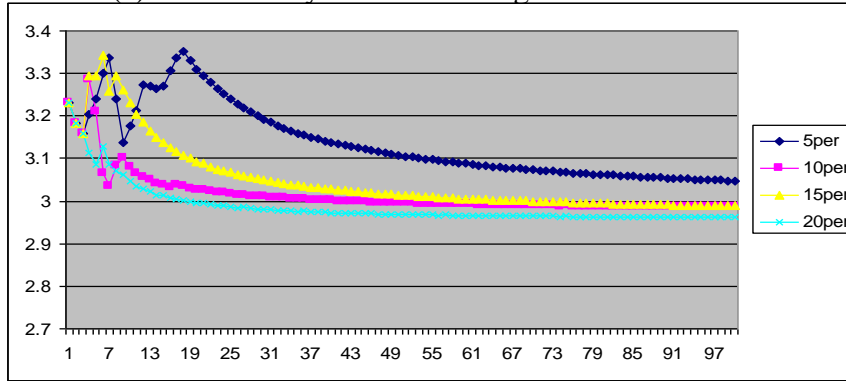
*b. Given different replacing percentage of player-agent in evolutionary crowds for higher point and the ones for lower point*

In this experiment, we focus on the effect of different replacing percentage of player-agents in different crowds and related influence for aggregator-agents. During each round, 250 player-agents have been put into game. In evolutionary crowds, every certain time, evaluation will be executed to replace certain percentage of the crowds with preference player-agents, for example, in the one for higher points, the setting of replacing 10 percentage means 25 player-agents with lowest points will be replaced by the player-agent who has the same chromosome as the one with the highest point.

The results for different select percentage of player-agent in different crowds are shown in Figure 13.



(a) Evolutionary Crowds for Higher Point



(b) Evolutionary Crowds for Lower Point

**Figure 13:** Different Replacing Percentage in Crowds

The replacing percentage shows how fast the components of crowds are changing.

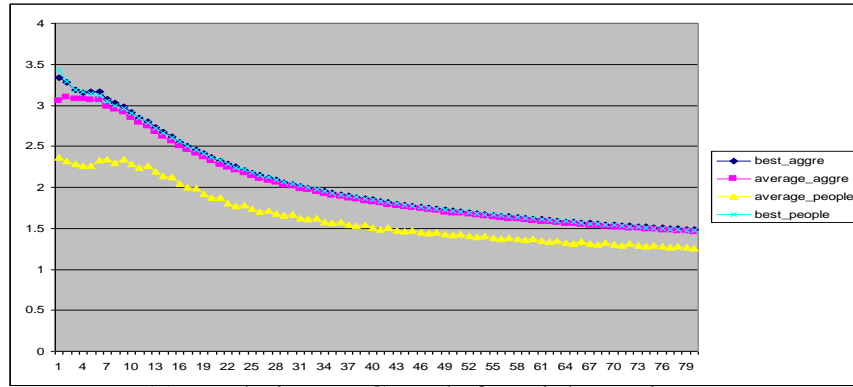
The larger the percentage, the bigger the change in the components of crowds happens. In figure 13, (a) shows the performance of best aggregator in the crowd which player-agents with lower point are replaced by the highest one, while (b) shows the performance of best aggregator in the crowd which player-agents with higher point are replaced by the lowest one. Although (b) shows a period of upward during the run at first, (a) and (b) have the same pattern: The percentage under 20% won't change the trend, the larger the percentage is, the fast the effect of evolution shows.

*c. Given different fitting function for player-agent in evolutionary crowds for higher point and the ones for lower point*

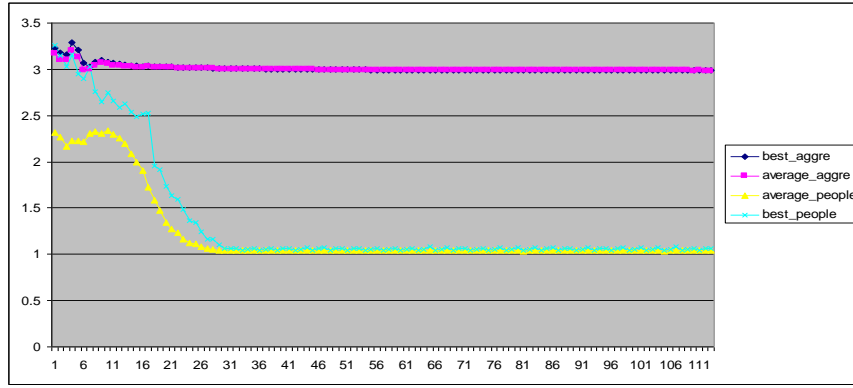
In this experiment, we focus on the effect of different fitting function of player-agents in different crowds and related influence for aggregator-agents.

During each round, 250 player-agents have been put into game. In evolutionary crowds, every certain time, evaluation will be executed to replace certain percentage of the crowds with preference player-agents, for example, in the one for higher points, those player-agents with lowest points will be replaced by the player-agent who has the same chromosome as the one with the highest point, while in the one for lower points, those player-agents with highest points will be replaced by the player-agent who has the same chromosome as the one with the lowest point.

The results for different fitness function for player-agent in evolutionary crowds are shown in Figure 14.



(a) Evolutionary Crowds for Higher Point



(b) Evolutionary Crowds for Lower Point

**Figure 14:** Different Fitness Function for Player-agent

In Figure 14, both charts show the better performance of aggregator-agent than the player-agent in evolutionary crowds. In (a) evolutionary crowds for higher point, which will end up being a friendly crowd suggested by the experiment 1, the interlaced three lines for aggregator-agent with highest point, aggregator-agent with average point and player-agent with highest point and an obvious separated line for player-agent with average point suggest that in the evolution for higher point crowd, the aggregator-agent cannot perform much better than the best individual while it can beat the player-agent with average point easily during most of the time. The aggregator-agents still perform better than most of the crowds. In (b) evolutionary crowds for lower point, which will end up being a more hostile crowd suggested by the experiment 1, the obvious difference between the lines of aggregator-agent and

player-agent suggests that in changing crowds which try to eliminate the better performer, the aggregator-agent can beat the player-agent easily.

d. *Player-agents' and Aggregator-agents' performance with the learning ability in evolutionary crowds*

Adding the learning ability to the player-agents enables them to learn individually to improve their decisions; although this violates part of Surowiecki's criteria –independence – it is common and necessary in real life. Experiments show that by keeping enough diversity of opinion, the aggregate wisdom of the crowd can still perform better than most individual members, even the best individual. In this experiment, we focus on the player-agents' performance with learning ability in evolutionary crowds.

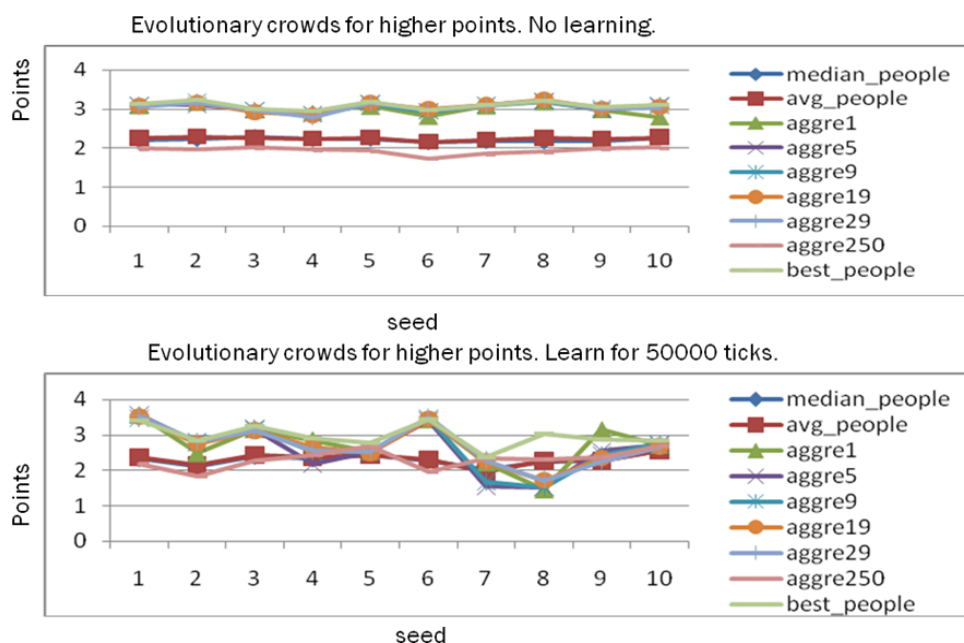
Since the formation of crowds is important to the performance of agents in the Prisoner's Dilemma Problem, we run the experiments 10 times having different random seeds. During each round, 250 player-agents have been placed on the grid. After player-agents have had a chance to play against and learn from each other for certain learning period, aggregator-agents with different strategies are introduced into this game. Aggre1, 5, 9..., 250 represent the aggregator-agent with different aggregation strategies. For example, an aggregator-agent whose strategy is to consult player-agents with the highest scores may choose to follow the advice of the group of player-agents having the current highest score, and we call it aggre1 ... likely a wise strategy for the aggregator-agents. Similarly, best\_people, median\_people, average\_people represent the player-agents. For example, best\_people represents one of the player-agent having the current highest score.

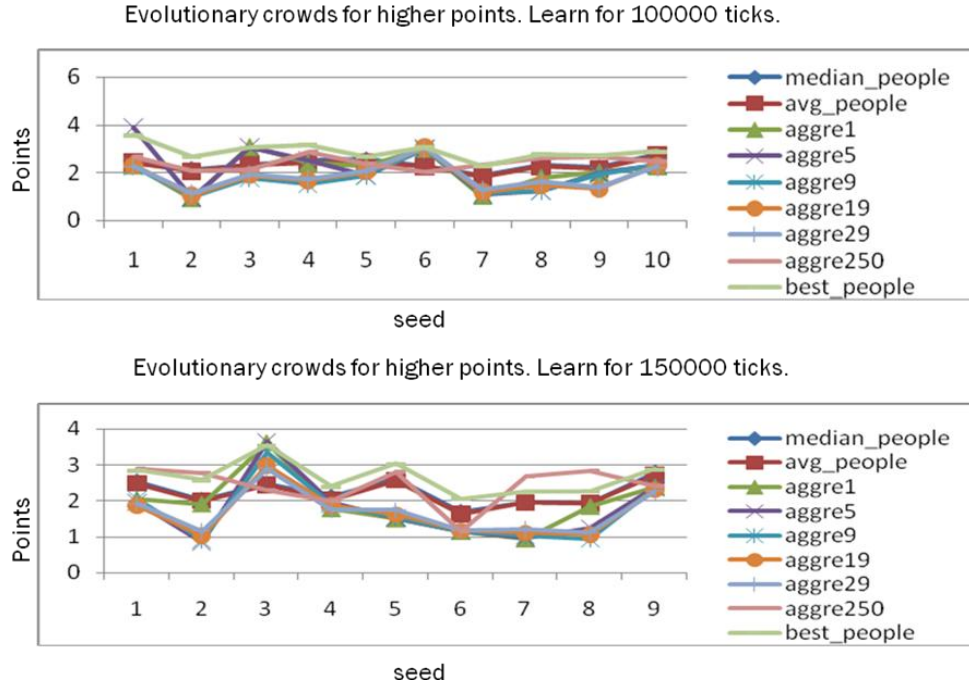
In Figure 15, the charts show the performance of player-agents and aggregator-agents, after certain duration of learning, using 10 different seeds

(formations of crowds).

By introducing the ability to learn, the performance of player-agents and Aggregator-agents show increasing volatility on scoring for different seeds (crowds). When no learning happens, the performance of player-agents and aggregator-agent keeps relatively stable no matter what seed (formation of crowds) is used. Although the line for best-people is always on the top, we observe that the lines for Aggre19 and Aggre29 are close to the one for best\_people, which suggests that the best way to make the decision by using the wisdom of the crowd in this situation is to listen to 10% of the crowd, so that the performance will be similar to the best individual in the crowds but only slightly lower. The Best individual might change for each tick while the performance of aggregators keeps good all the time.

While introducing the learning period, more volatility occurs, and best-people is no longer the all-time winner. In the chart Fig. 5 which shows the situation after learning for 150,000 ticks, the aggregator-player performs better than best\_people six times out of ten. This suggests that more than half the time, making a decision using the wisdom of the crowd is even better than the best individual in the crowds.





**Figure 15:** Performance of Player-agent and Aggregator-agent in Different Seeds, Varying in Duration of Learning

*e. Player-agents' and Aggregator-agents' performance varying in the size of crowds*

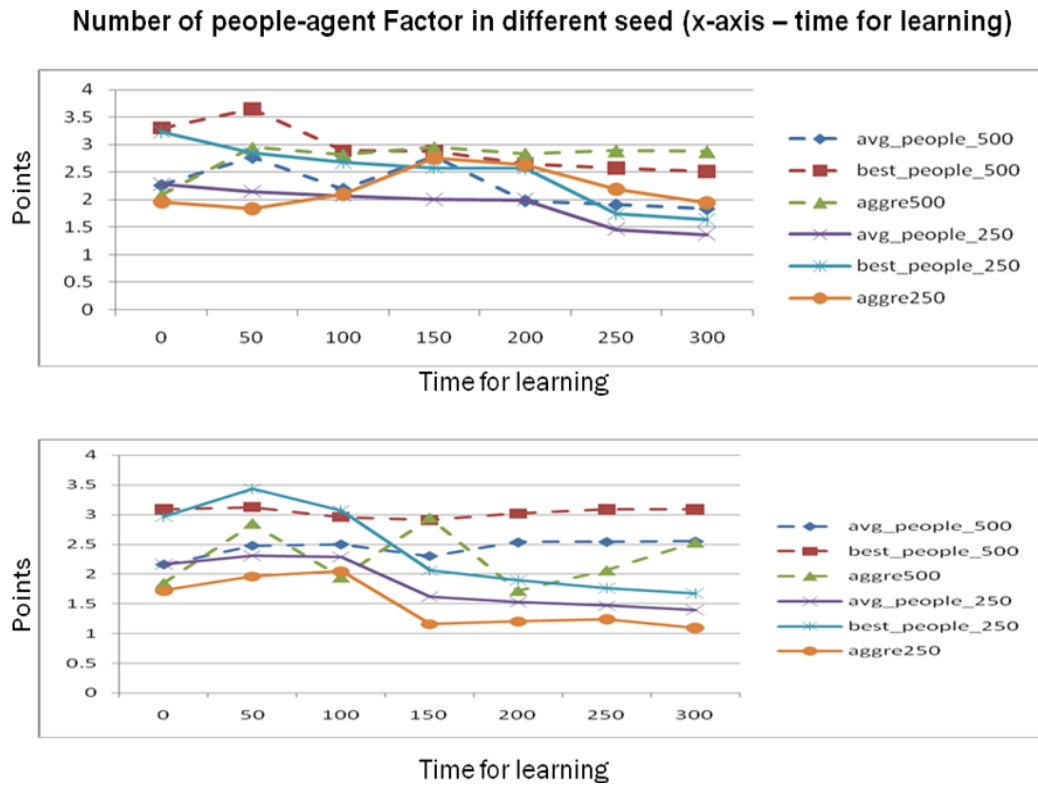
The size of crowds which is related to the diversity of opinions is another factor for agents' performance. In this experiment, we focus on the 'player-agents' and 'aggregator-agents' performance while varying the size of the crowds. Two sets of experiments were run using different random seeds: 250 player-agents and 500 player-agents.

In Figure 16, avg\_people\_250 and best\_people\_250 represent the average player-agent and the player-agent with the current high score in a crowd of 250 player-agents; aggre250 represents the aggregator-agent who chooses the strategy to listen to all 250 player-agents in the crowd; likewise for avg\_people\_500, best\_people\_500 and aggre500.

These charts in Fig. 16 show that despite the different random seeds, the increased size of the crowd (which increases diversity of opinion) results in better performance for both player-agents and aggregator-agents. And for most of time, the



aggregator-agent using the wisdom of the crowd is better than the best player-agents in those crowds. C represents the points gained by Aggregator-agent using the wisdom of crowds; B represents the points gained by Best Player-agent; and A represents the points gained by Average Player-agent.



**Figure 16:** Performance of Player-agent and Aggregator-agent, Varying in Size of Crowds

#### 8.1.2. Findings in Type I Crowds

A simulation using the concept of Complex Adaptive Systems is built to demonstrate the wisdom of crowds, while at the same time Surowiecki's four criteria to form a smart crowd are tested. However, it is hard to imagine a continuous decision-making example where members of the crowd are truly independent from

each other in the real world. Therefore, by partially violating the independence criteria, we added learning ability to the crowd. Experiments show although many aggregation methods we can have, the simplest majority rule still gets its glory in Type I Crowds, and also knowing keep track of the performance of player-agent all the time to form the consulting group may improve the performance of aggregator-agent, given expense limitation, frequency for checking the crowd's performance won't affect the aggregator-agent very much. In all, regardless of the type of crowds and the rate of change in the crowds, aggregator-agent with appropriate aggregation method can always perform better than most of the crowds, even better than the best individual performer nearly all the time in Type I Crowds. And also our experiments show that learning process makes both individual players and the aggregate-players smarter, while still guaranteeing diversity of opinion. Furthermore, these experiments show that in a crowd where the "membership" can be defined dynamically, and where members can communicate with each other and learn from each other, the wisdom-of-crowds approach is superior to the best performing members in the crowd.

## 8.2. Aggregator-agent in Type II Crowds

### 8.2.1. Aggregator-agent Performance vs. Player-agent Performance

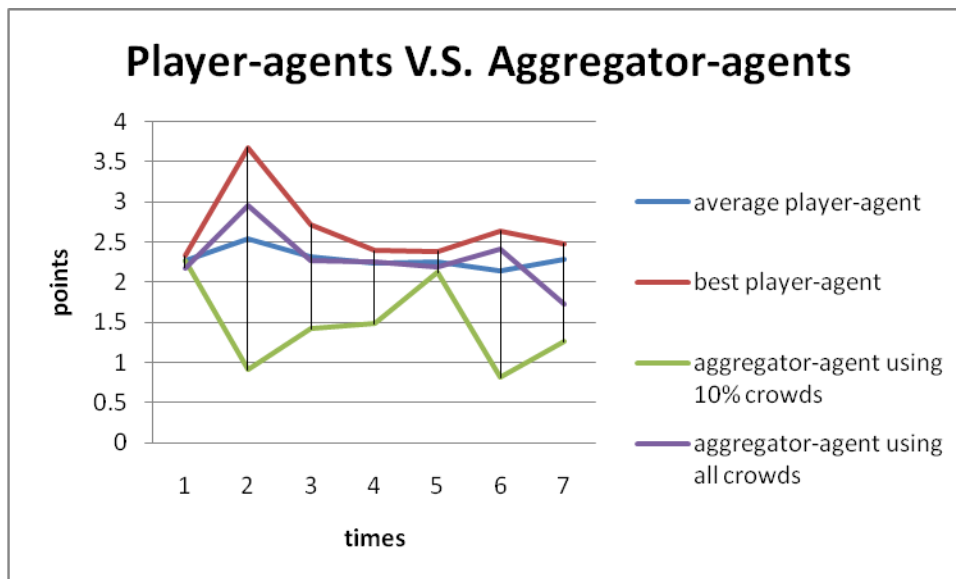
Aggregator-agents using simple polling can perform better than the average in Type I crowds most of the time, as we see from above. But the same aggregation method may not work in all kinds of Type II crowds. Given different aggregation methods, the performance of the player-agents and the aggregator-agents is recorded and compared in Type II crowds.

#### *i. No-history Crowds*

No-history crowds are formed by player-agents who do not care what happened

before. Considering the random factor caused by randomly set initial settings – including the strategies assigned, play order, and distribution of each strategy – the chart in Figure 17 shows the performance of average player-agent, best player-agent, aggregator-agent using all crowds, aggregator-agent using 10 percent crowds. C represents the points gained by aggregator-agent using the wisdom of crowds; B represents the points gained by best player-agent; and A represents the points gained by average player-agent.

As shown in Figure 17, three out of seven times, aggregator-agent using all crowds plays better than the average player-agent, never in seven times does aggregator-agent using 10 percent crowds play better than the average player-agent. In no-history crowds, no good aggregation method is recommended. The wisdom of crowds does not work well, and the best strategy is to play defect.

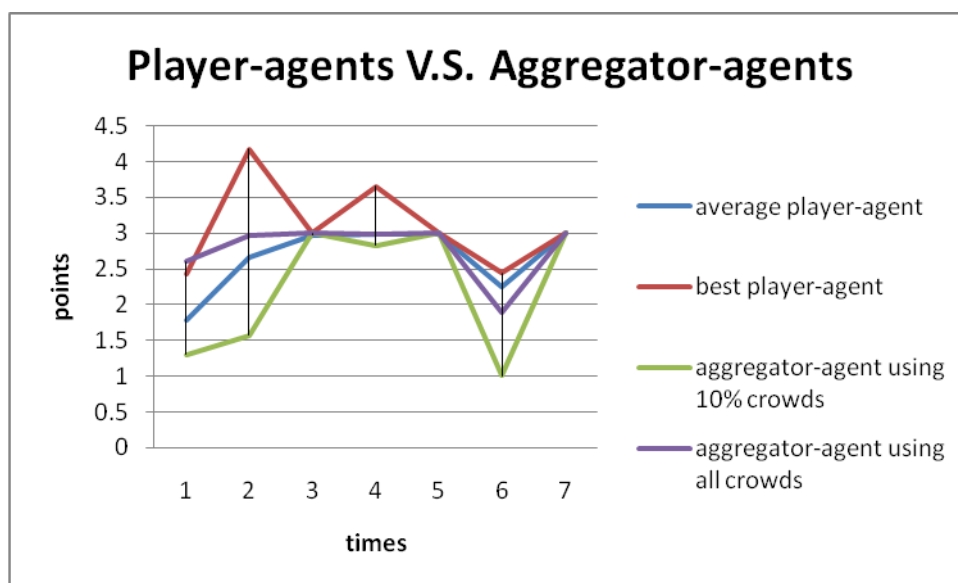


	Aggregator-agent using all crowds			Aggregator-agent using 10% crowds		
	C>B>A	B>C>A	B>A>C	C>B>A	B>C>A	B>A>C
	0	0.43	0.57	0	0	1

**Figure 17.** No-history Crowds

## ii. Long-history Crowds

Long-history crowds are formed by the player-agents who take into consideration the total number of times the opponent cooperated or defected in the past. Considering the random factor caused by randomly set initial settings – including the strategies assigned, play order, and distribution of each strategy – the chart in Figure 18 shows the performance of average player-agent, best player-agent, aggregator-agent using all crowds, and aggregator-agent using 10 percent crowds. C represents the points gained by aggregator-agent using the wisdom of crowds; B represents the points gained by best player-agent; and A represents the points gained by average player-agent. As shown in Figure 18, six out of seven times aggregator-agent using all crowds plays better than the average player-agent, while three out of seven times, aggregator-agent using 10 percent crowds plays better than the average player-agent. In long-history crowds, aggregator-player using all crowds is recommended. The wisdom of crowds works well and the best strategy is to make decisions based on the majority opinion in the whole crowds.

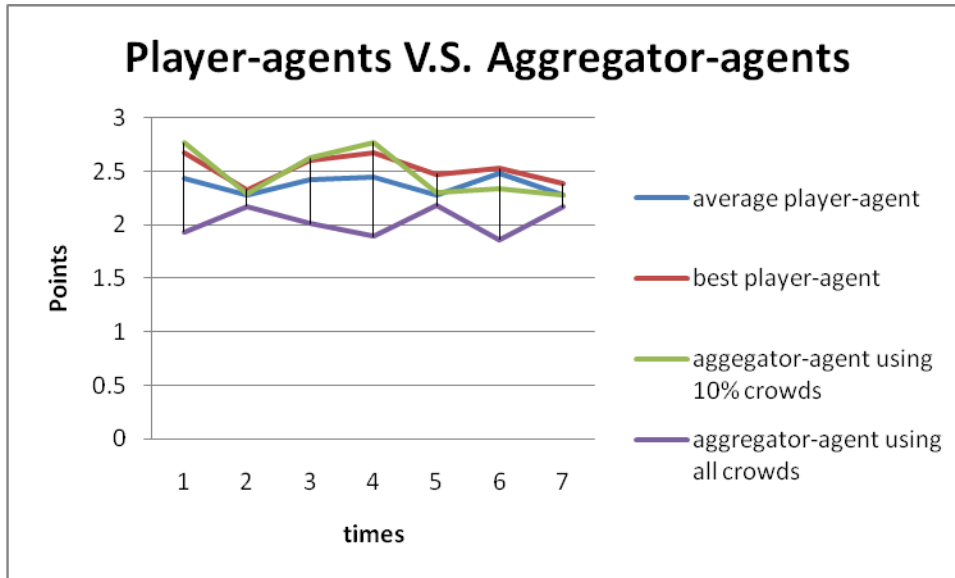


	Aggregator-agent using all crowds			Aggregator-agent using 10% crowds		
	C>B>A	B>C>A	B>A>C	C>B>A	B>C>A	B>A>C
	0.57	0.29	0.14	0.43	0	0.57

**Figure 18:** Long-history Crowds

iii. *Latest-history Crowds*

Latest-history crowds are formed by the player-agents who take into consideration whether during the previous X number of matches/time (X defined by limitation, normally no more than 3) the opponent cooperated or defected (X times in a row). Considering the random factor caused by randomly set initial settings – including the strategies assigned, play order, and distribution of each strategy -- the chart in Figure 19 shows the performance of average player-agent, best player-agent, aggregator-agent using all crowds, and aggregator-agent using 10 percent crowds. C represents the points gained by aggregator-agent using the wisdom of crowds; B represents the points gained by best player-agent; and A represents the points gained by average player-agent. As shown in Figure 19, never in seven times does aggregator-agent using all crowds play better than the average player-agent, while six out of seven times, aggregator-agent using 10 percent crowds plays better than the average player-agent. In latest-history crowds, aggregator-player using 10 percent of crowds is recommended. The wisdom of crowds works well, and the best strategy is to make decisions based on the majority opinion from the top 10 percent crowds.

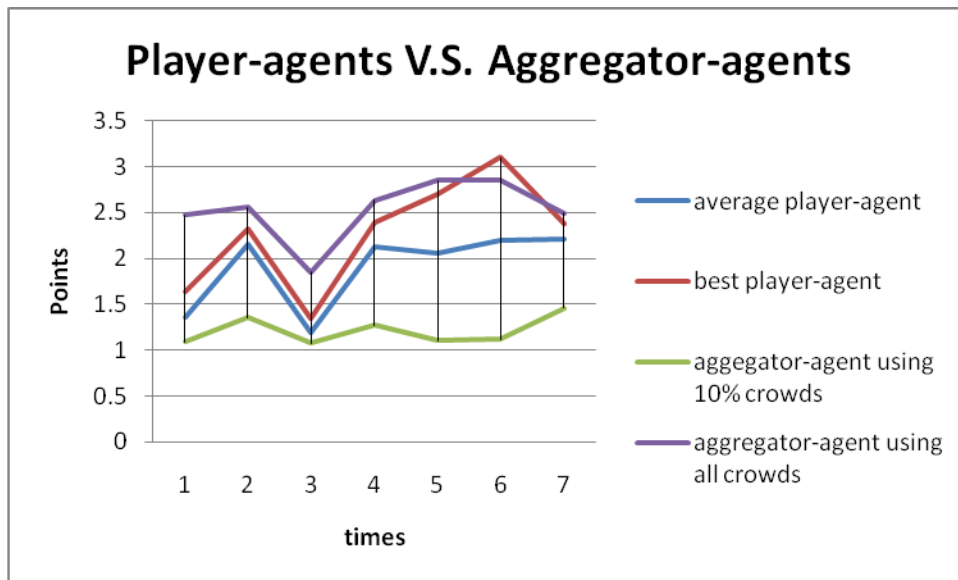


**Figure 19:** Latest-history Crowds

### iii. Absolute-gain Crowds

Absolute-gain crowds are formed by the player-agents who takes into consideration the average number of points received previously by cooperating/defecting when playing against the same opponent. Considering the random factor caused by randomly set initial settings – including the strategies assigned, play order, and distribution of each strategy – the chart in Figure 20 shows the performance of average player-agent, best player-agent, aggregator-agent using all crowds, and aggregator-agent using 10 percent crowds. C represents the points gained by aggregator-agent using the wisdom of crowds; B represents the points gained by best player-agent; and A represents the points gained by average player-agent. As shown in Figure 20, seven out of seven times, aggregator-agent using all crowds plays better than the average player-agent, while never in seven times does aggregator-agent using 10 percent crowds play better than the average player-agent. In absolute-gain

crowds, aggregator-player using all crowds is recommended. The wisdom of crowds works well, and the best strategy is to make decision based on the majority opinion in the whole crowds.



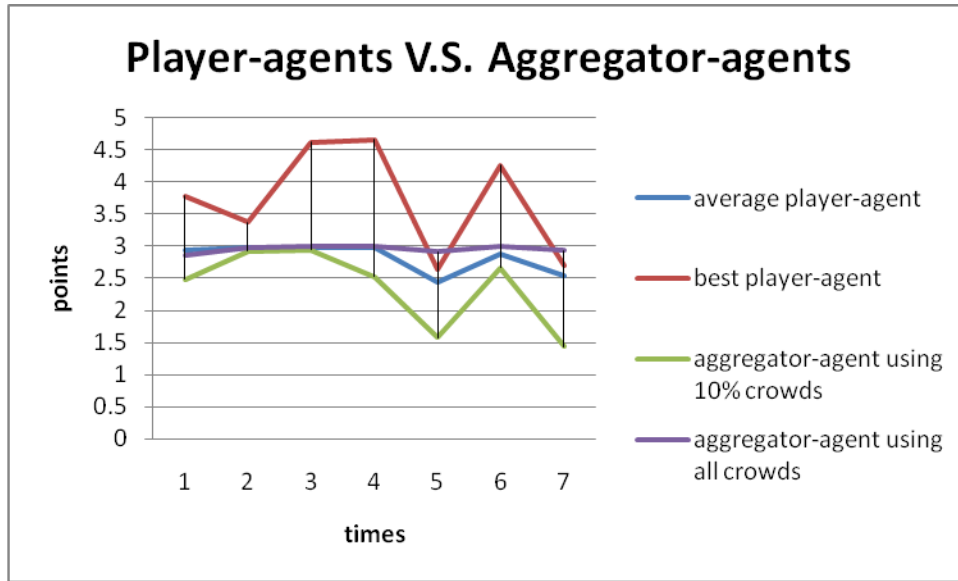
	Aggregator-agent using all crowds			Aggregator-agent using 10% crowds		
	C>B>A	B>C>A	B>A>C	C>B>A	B>C>A	B>A>C
	0.86	0.14	0	0	0	1

**Figure 20:** Absolute-gain Crowds

#### iv. *Relative-gain Crowds*

Absolute-gain crowds are formed by the player-agent who takes into consideration whether it received fewer points from the last play, compared to its opponent. Considering the random factors caused by randomly set initial settings – including the strategies assigned, play order, and distribution of each strategy – the chart in Figure 21 shows the performance of average player-agent, best player-agent, aggregator-agent using all crowds, and aggregator-agent using 10 percent crowds. C represents the points gained by aggregator-agent using the wisdom of crowds; B represents the points gained by best player-agent; and A represents the points gained

by average player-agent. As shown in Figure 21, six out of seven times, aggregator-agent using all crowds plays better than the average player-agent, while never in seven times does aggregator-agent using 10 percent crowds play better than the average player-agent. In long-history crowds, aggregator-player using all crowds is recommended. The wisdom of crowds works well, and the best strategy is to make decision based on the majority opinion from the whole crowds.



	Aggregator-agent using all crowds			Aggregator-agent using 10% crowds		
	C>B>A	B>C>A	B>A>C	C>B>A	B>C>A	B>A>C
	0.29	0.57	0.14	0	0	1

**Figure 21: Relative-gain Crowds**

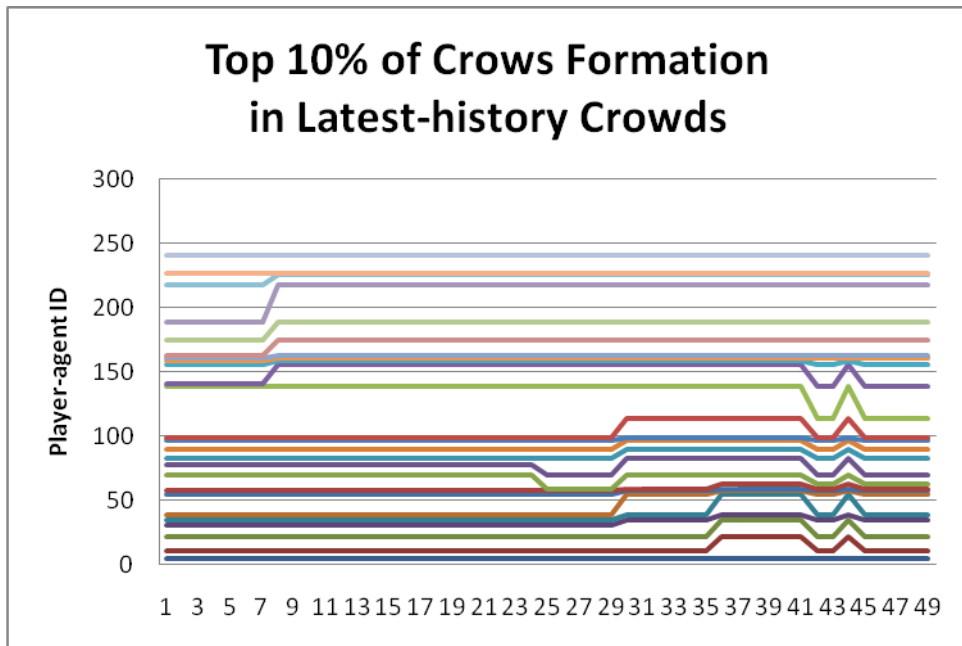
### 8.2.2. Performance of Aggregator-agent and Formation of Top 10 percent Crowds

From the above, we come to the conclusion that using appropriate aggregation method aggregator-agent can always perform better than most of the crowds, even better than the best individual performer in Type II Crowds, except for No-history Crowds. We also notice that in latest-history crowds, aggregator-agent using 10



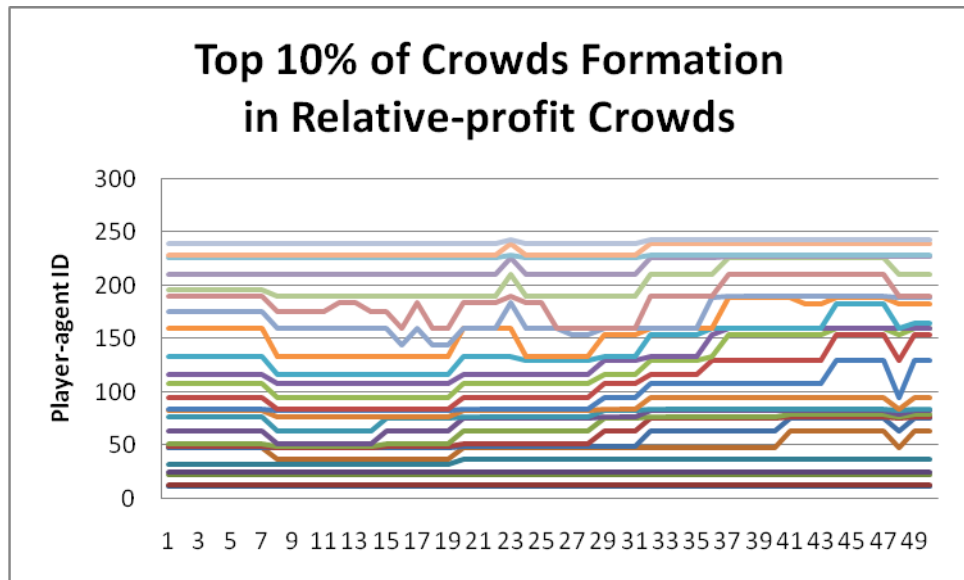
percent crowds as advisory group plays better than the one using the whole crowds, while in other Type II groups, the one using the whole crowds is better. Experiments show that the difference in performance of aggregator-agent is related to the formation of top 10 percent crowds in different Type II crowds.

In latest-history crowds, aggregator-agent using 10 percent crowds as advisory group plays better than the one using the whole crowds. The typical formation of top 10 percent crowds in latest-history crowds is shown in Figure 22. Each line represents one of Top N ( $1 < N < 25$ ) player-agents during the running time. Flatness in the lines suggests the formation of top 10 percent crowds does not change. Up-downs in the lines suggest the formation of top 10 percent crowds change during the time. As shown in Figure 22, the relatively-flat lines suggest the formation of top 10 percent crowds change little during the time. The performances of top 10 percent player-agents are stable, and the formation of top 10 percent crowds remains nearly the same during the running time.



**Figure 22:** Top 10% Crows Formation in Latest-history Crows

The typical formation of top 10 percent crowds in relative-gain crowds is shown in Figure 23. Each line represents one of Top N ( $1 < N < 25$ ) player-agents during the running time. Flatness in the lines suggests the formation of top 10 percent crowds does not change. Up-downs in the lines suggest the formation of top 10 percent crowds change during the time. The relatively-bumpy lines in Figure 23 suggest the formation of top 10 percent crowds in a relative-gain crowd change often during the time. The performances of top 10 percent player-agents are unstable, and the formation of top 10 percent crowds changes during the running time.



**Figure 23: Top 10% Crowds Formation in Relative-gain Crowds**

Based on the experiments' results, we reached the conclusions that –

- 1.) In most of the Type II crowds, the aggregator-agent using all the crowds performs better than the one using top 10 percent crowds as advisory group. Suggestions on the selection of advisory groups for different crowds are shown below.

Type II Crowds	Using all the crowds	Using 10% top of the crowds
No-history Crowds	Yes	
Long-history Crowds	Yes	
Latest-history Crowds		Yes
Absolute-gain Crowds	Yes	
Relative-gain Crowds	Yes	

**Figure 24:** Strategies on Selection of Advisory Group in Type II Crowds

2.) In most of the Type II crowds, aggregator-agent using wisdom of crowds (either all the crowds or top 10 percent crowds) performs better than the average player-agent, even the best player-agent. The only exception is no-history crowds. In a no-history crowds, playing random is the basic strategy for the single Player-agent thus aggregator-player using collective wisdom cannot show its superiority. Aggregator-agents' performance in Type II crowds are shown in Figure 25. C represents the points gained by aggregator-agent using the wisdom of crowds; B represents the points gained by best player-agent; and A represents the points gained by average player-agent.

probability	C > B > A	B > C > A	B > A > C
No-history Crowds	0	0.43	0.57
Long-history crowds	0.57	0.29	0.14
Latest-history Crowds	0.43	0.43	0.14
Absolute-gain Crowds	0.86	0.14	0
Relative-gain Crowds	0.29	0.57	0.14
Average	0.43	0.37	0.20

**Figure 25:** Performance of Aggregator-player

3.) Aggregator-players, using different strategies to select the advisory group, perform differently in each of Type II crowds. Experiments show that the

formation of top 10 percent crowds in different Type II crowds can be the key factor.

In a crowd that has a relatively-stable top 10 percent group, aggregator-agent using the top 10 percent as advisory group does a better job than the one using all the crowds. On the other hand, in a crowd whose top 10 percent formation is changing all the time, using temporary top 10 percent crowds as advisory group would not help the aggregator-agent make a good decision and using all the crowds as advisory group does a better job.

### 8.3. Choosing the Best Strategy to Make Decision in the Crowds

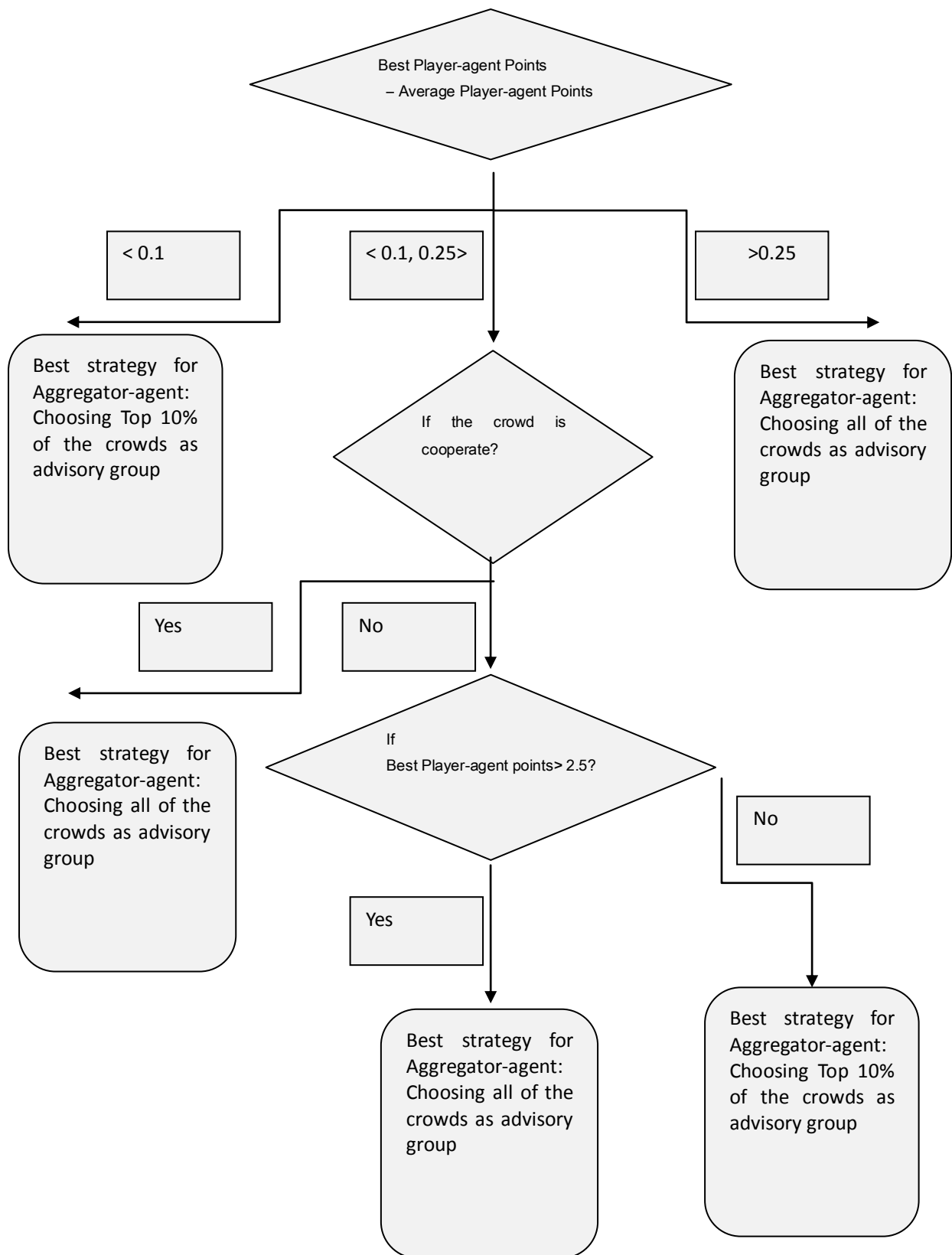
The above experiments show that in both Type I and Type II crowds aggregator-agent using the wisdom of crowds in a proper way can perform better most of the time than the average individual player-agent in the crowds, and can even perform the best some of the time. The formation of the crowds is the key factor to choose the best aggregation method for the aggregator-agent. Experiments in both Type I and Type II crowds show that the formation of crowds can be evaluated by simply observing 1) the performance of top 10 percent crowds or 2) the performance of best player-agent and average player-agent.

As described in 8.2, in a crowd that has a relatively-stable top 10 percent group, aggregator-agent using the top 10 percent as advisory group does a better job than the one using all the crowds. On the other hand, in a crowd whose top 10 percent formation is changing all the time, using temporary top 10 percent crowds as advisory group would not help the aggregator-agent make a good decision and using all the crowds as advisory group does a better job.

Choosing the best aggregation method based on the formation of crowds may not

be easy to apply when the detail of crowds is unknown. There are other ways to choose the best aggregation methods, for example, making decision based on the performance of best player-agent and average player-agent. The strategy is useful when the scores from the best player-agent and average player-agent are available, while the top 10 percent crowds data is too large or too hard to collect and analyze.

Using the points that best player-agents and average player-agents get during the game, we generate rules for the aggregator-agents that enable them to choose the best strategy to make a decision using the wisdom of crowds in all kinds of crowds in the context of Prisoner's Dilemma. Rules are shown below in Figure 26.



**Figure 26:** Procedure Making Decision Using Wisdom of Crowds

The evaluation of the above decision-making procedure is shown in Figure 27. C represents the points gained by aggregator-agent using the wisdom of crowds and A

represents the points gained by average player-agent.

Judging the crowds based on different between Best Player-agent Points and Average Player-agent Points	Probability that $C > A$
$< 0.1$	.83
$< 0.1, 0.25.$	.64
$> 0.25$	.93
Overall	.83

**Figure 27:** Evaluation Decision Making Procedure Using the Wisdom of Crowds

## CHAPTER 9: APPLICATION: USING THE WISDOM OF CROWDS IN A SIMPLIFIED STOCK MARKET

### 9.1. Using the Wisdom of Crowds in Stock Market

The equity markets have long been the subject of research in a number of fields: mathematics finance, computational finance, quantitative behavioral finance, and others. Mathematical finance is a branch of applied mathematics concerned with the financial markets [Wiki08]. Computational finance is a cross-disciplinary field that relies on computational intelligence, mathematical finance, numerical methods, and computer simulations to make trading, hedging, and investment decisions, and to facilitate the risk management of those decisions [Wiki09]. Quantitative Behavioral Finance attempts to quantify basic biases and use them in mathematical models [Gcag99].

By developing theoretical models, mathematical finance, when combined with computational finance, attempts to evaluate real market data and predict stock trends, including crashes. Stochastic analysis and partial differential equations are currently widely used methods, including the Black Scholes Model, the Stochastic Volatility Model, and the Poisson Market Model [Blac73, Slhe93, Neil05, Alan97].

Others see the stock market as a disorganized crowd of individuals, buying and selling, with the sole common purpose of ascertaining the future mood of the market [Jaso04]. Behavioral finance is the study of the influence of psychology on the behavior of financial practitioners and its effect on markets [Sewe01]. It builds models based on analyses of human behavior, especially in financial markets, using



mathematical and statistical methodologies, in conjunction with valuation [Char06, Oliv04].

While most of the research efforts focus on building a more accurate financial model and providing the best strategy to the investors based on their model, the idea of using the Wisdom of Crowds is to help the investor be the better-than-most player for the long run. Using the Wisdom of Crowds in the stock market focuses on the collective human behaviors. And those behaviors may cancel out the individual biases caused by mood, personality, or peer pressure. Understanding the collective wisdom helps the individual investor sense the foreseeable changes in the market before they become a fad.

GSPC -- S&P 500 INDEX is a free-float capitalization-weighted index published since 1957 of the prices of 500 large-cap common stocks actively traded in the United States. The S&P 500 is one of the most widely followed indexes of large-cap American stocks [Wiki10]. It is considered a bellwether for the American economy. Figure 28 shows the profit gained by investing on GSPC or trading GSPC using our strategy in different time periods. The interest rate is 2% per year and no transaction cost is considered. Experiments prove that using the wisdom of crowds is a good strategy in a fast-changing market.

Time period\Strategy	Investing on GSPC	Trading GSPC using WoC
1	118961.1713	385714.087
2	117595.3795	513837.69

**Figure 28:** Comparison between different strategies

## 9.2. Simplified Stock Market

In a simplified stock market simulation, there is only one stock trading. The prices of any particular stock are derived from the actual market prices [Yaho01] and change daily based on the history records; they are not affected by the investors'

actions in the system. 250 individual investors and 42 aggregator investors buy/sell stocks, or just watch in the market. “Individual investors” can buy or sell based on their own strategy. “Aggregator-investors” can buy or sell based on the aggregated information deprived from the crowds.

Since individual investors make their own decisions continuously, we assign each agent a memory that is used to store information (knowledge), not only their previous decision but also their market observations as well, such as the duration of stock price rises since the investor's most recent transaction. The individual investors initially “receive” a randomly allocated strategy that they use to select their actions, based on the information that they have. The strategy may be abandoned or modified during the learning process, based on perceptions of and interactions with other investors.

Aggregator investors are special participants (investors) in the game. The aggregator-investors represent the wisdom of crowds by acting as aggregators of various groups within the crowd of agents. These aggregators also participate in the game, but they have a different decision-making process. The aggregator investors are given the ability to make their decisions after consulting with their “advisory group”, formed from the set of individual investors selected by each aggregator-agent. On each turn, aggregator investors choose to buy, sell, or watch according to the opinions from their chosen advisory group. Unlike the regular individual investors, aggregator investors have no strategy that can give them guidance regarding trading; their only strategy is to decide (a) which individual investor group they want to listen to and (b) the manner in which they plan to aggregate the group’s advice.

In the system, each individual investor is described using a chromosome-like structure:

Agent Number	Buy time	Sell time	Trade percentage
--------------	----------	-----------	------------------

where:

- *Agent Number* identifies each individual investor.
- *Buy time* indicates the standard that an individual investor uses to decide to buy.
- *Sell time* indicates the standard that an individual investor uses to decide to sell.
- *Trade percentage* indicates the percentage of money/stocks that individual investor will trade in one transaction.

*Buy time* suggests when an individual investor decides to buy, based on the previous stock price change. It can be any integer number randomly selected from  $[-N, N]$  ( $N$  is a natural number). In the experiment, *Buy time* is chosen from  $[-4, 4]$ , which can assume one of the following values:

- 4. The individual investor buys when the stock price goes down 4 days in a row.
- 3. The individual investor buys when the stock price goes down 3 days in a row.
- 2. The individual investor buys when the stock price goes down 2 days in a row.
- 1. The individual investor buys when the stock price goes down.
- 0. The individual investor buys or sells randomly.
- 1. The individual investor buys when the stock price goes up.
- 2. The individual investor buys when the stock price goes up 2 days in a row.
- 3. The individual investor buys when the stock price goes up 3 days in a row.
- 4. The individual investor buys when the stock price goes up 4 days in a row.

*Sell time* suggests when an individual investor decides to sell, based on the previous stock price change. It can be any integer number randomly selected from  $[-N, N]$  ( $N$  is a natural number). In the experiment, *Sell time* is chosen from  $[-4, 4]$ , which can assume one of the following values:

- 4. The individual investor sells when the stock price goes down 4 days in a row.
- 3. The individual investor sells when the stock price goes down 3 days in a row.

- 2. The individual investor sells when the stock price goes down 2 days in a row.
- 1. The individual investor sells when the stock price goes down.
- 0. The individual investor buys or sells randomly.
- 1. The individual investor sells when the stock price goes up.
- 2. The individual investor sells when the stock price goes up 2 days in a row.
- 3. The individual investor sells when the stock price goes up 3 days in a row.
- 4. The individual investor sells when the stock price goes up 4 days in a row.

*Trade percentage* suggests the percentage of money/stocks that individual investor will trade in one transaction, which can be one of the integer values between [1, 3]. For example, value 1 means individual investor trades all its money/stocks and value 3 means individual investor trades 1/3 of its money/stocks.

For example, individual investor 001 shown below will buy stocks using all its money when the stock price goes up 2 days in a row, and it sells all its stocks immediately when the stock price goes down.

001	2	-1	1
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In the system, each aggregator investor is described using a chromosome-like structure:

Agent Number	Selection Strategy	Aggregation Strategy	Trade Percentage	Select Number
--------------	--------------------	----------------------	------------------	---------------

Where:

- *Agent Number* identifies each aggregator investor.
- *Selection Strategy* indicates the strategy used to select an individual investor group.
- *Aggregation Strategy* indicates the strategy used for aggregation.
- *Trade percentage* indicates the percentage of money/stocks that aggregator

investor will trade in one transaction.

- *Select\_Number* indicates how many individual investors are chosen to form the group; it can be any number between 1 and the total number of individual investors.

There are three selection strategies:

0. The agent chooses the bottom *Select\_Number* individual investors ranked by their total assets, including money and current value of stocks.
1. The agent chooses the top *Select\_Number* individual investors ranked by their total assets, including money and current value of stocks.
2. The agent chooses all the individual investors in the market

There are two aggregation strategies:

1. The agent chooses the majority opinion.
  0. The agent does not choose the majority opinion. If the majority suggests to buy/sell, the aggregator investor chooses not to trade; otherwise, it chooses to buy/sell randomly.
- *Trade percentage* indicates the percentage of money/stocks that aggregator investor will trade in one transaction.

For example, aggregator investor 001 shown below will choose the top 20 individual investors as its advisory group, and trade all its money/stocks as the group suggests.

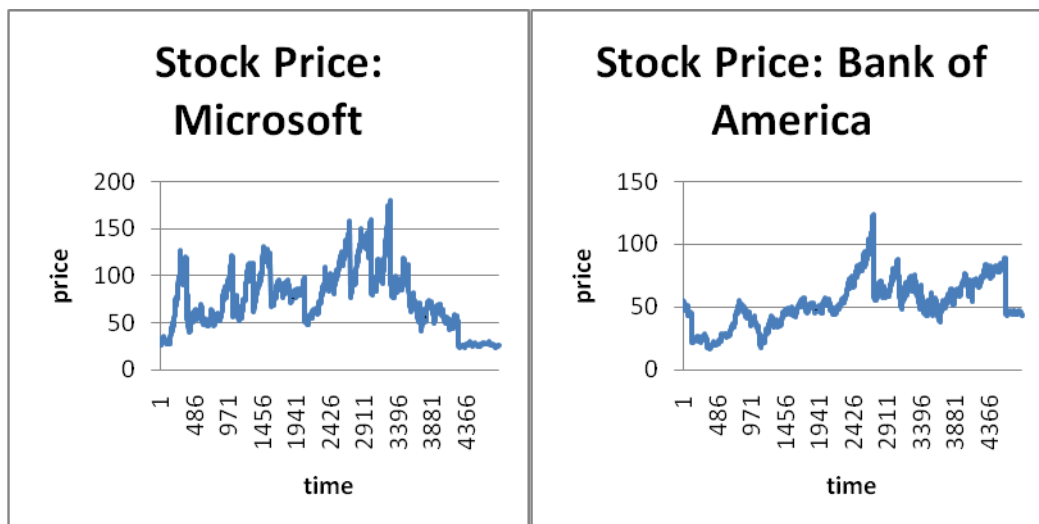
001	1	1	1	20
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### 9.3. Experiments Result

In a simplified stock market, where only one stock is being traded, individual investors and aggregator investors are buying, selling, or not trading, based on their strategies, starting with the same amount of money and stocks. The goal for each

investor is to increase its assets, which includes money and stocks. Individual investors can change their strategies by learning from those around him who perform better, while the performance of aggregator investor will also change along with the whole changing crowds.

In the experiments, there are 250 individual investors and 42 aggregator investors. A set of basic strategies is assigned randomly to each investor. Investors buy/sell stocks or just watch the market, and there is only one stock that is trading. The price of the stock comes from real market data from the past five years. The stock price will change day by day, not as a result of the investor's actions but based on actual historical data. In the experiments we choose two different types of stocks -- Microsoft and Bank of America. As shown in Figure 28, stock price for Microsoft and Bank of America show different pattern. The stock price for Bank of America has relatively smooth up-downs while the one for Microsoft shows dramatic changes in a short time period. Here the stock of Microsoft represents the ones with low velocity, and the stock of Bank of America represents the ones with high velocity.



**Figure 29:** Stock Price: Microsoft and Bank of America

Two different crowds have been tested in the experiments: no-learning crowds

and adaptive/extended crowds. In a no-learning crowd, the crowd composition is unchanged during the whole running time and follows the four key criteria suggested in the book “wisdom of crowds”. Each individual investor use the same invest strategy during the whole running time. While in real life, it’s hard to imagine an investor without consulting with experts or friends. By adding communication within crowds and allowing individual to learn from each other, adaptive/extended crowds violate “independent” criteria -- one of the four key criteria mentioned previously yet is more realistic. For example, after each trading day, an individual investor can choose to learn from others to improve its performance, by adopting other’s strategy. The experiment's results are shown below:

*a. Performance of Individual investors in No-Learning Crowds*

In this experiment, we focus on the performance individual investors in the stock market. During the randomly selected time period, 250 individual investors have been put into game. A set of basic strategies is assigned randomly to each investor and 3 time period is randomly selected. Investors buy/sell stocks or just choose not to trade in the market, and there is only one stock that is trading. The crowd is defined as no-learning crowds. Its composition is unchanged during the whole running time and follows the four key criteria suggested in the book “wisdom of crowds”.

Performances of individual investors are recorded below.

For Microsoft, results are shown below

Time period	Best strategy for Individual investors	Assets for best Individual investors	Assets for average Individual investors
1	(-3,1,1)	19543.55	16427.54
2	(-4,1,1)	16553.48	12280.53
3	(-4,1,1)	15130.83	11229.35

For Bank of America, results are shown below

Time period	Best strategy for Individual investors	Assets for best Individual investors	Assets for average Individual investors
-------------	--	--------------------------------------	---

1	(4,-4,5)	18565.21	14509.64
2	(-3,2,1)	24847.6	21045.47
3	(-4,2,1)	15530.83	13233.79

**Figure 30:** Individual-investors in No-Learning Crowds

As shown in Figure 30, the best strategy for individual investors, assets for best individual investors and assets for average individual investors are recorded. Assets for individual investors are calculated based on the money and stocks the investors owns as well as the current stock price. Experiments show that in a no-learning environment, mostly the best strategy for individual investor is  $(-N1, N2, 1)$  ( $N1$  and  $N2$  are natural numbers), which suggests that

1. Not considering the transaction cost, the best strategy still suggests the investor to trade all its money/stocks every time.

2.  $-N1$  in the individual strategy chromosome indicates the standard that an individual investor uses to decide to buy. For both stocks, the best strategy suggests the investor to buy cautiously after the stock price goes down in a row for days.

3.  $N2$  in the individual strategy chromosome indicates the standard that an individual investor uses to decide to sell. For the stock of low velocity, such as Bank of America, the best strategy would encourage the investors to buy/sell more cautiously – observing until the stock price changes for days. For the stock of high velocity, such as Microsoft stock, it would encourage the investors to sell immediately when the price go up, because the price often change dramatically in a short period as shown in Figure 29.

4. The best strategies for high velocity stocks, such as Microsoft, in different time periods share the same pattern – buy when the price goes down for days and sell immediately when the price goes up.

5. The best strategy is not the same for different stocks and different time periods. There is no all-time individual winner in the stock market.



*b. Performance of Aggregator investors in a No-Learning Setting*

In this experiment, we focus on the performance for aggregator investors in the stock market. During the randomly selected time period, 250 individual investors and 42 aggregator investors have been put into game. A set of basic strategies is assigned randomly to each investor and 3 time period is randomly selected. Investors buy/sell stocks or just choose not to trade in the market, and there is only one stock that is trading. The crowd is defined as no-learning crowds. Its composition is unchanged during the whole running time and follows the four key criteria suggested in the book “wisdom of crowds”

Performances of aggregator investors and individual investors are recorded below.

For Microsoft, results are shown below

Time period	Best strategy for Aggregator investors	Assets for best Aggregator investors	Assets for best Individual investors	Assets for average Individual investors
1	(1,0,1,1)	19567.97	19543.55	16427.54
2	(1,0,1,1)	14246.92	16553.48	12280.53
3	(1,0,1,1)	13919.5	15130.83	11229.35

For Bank of America, results are shown below

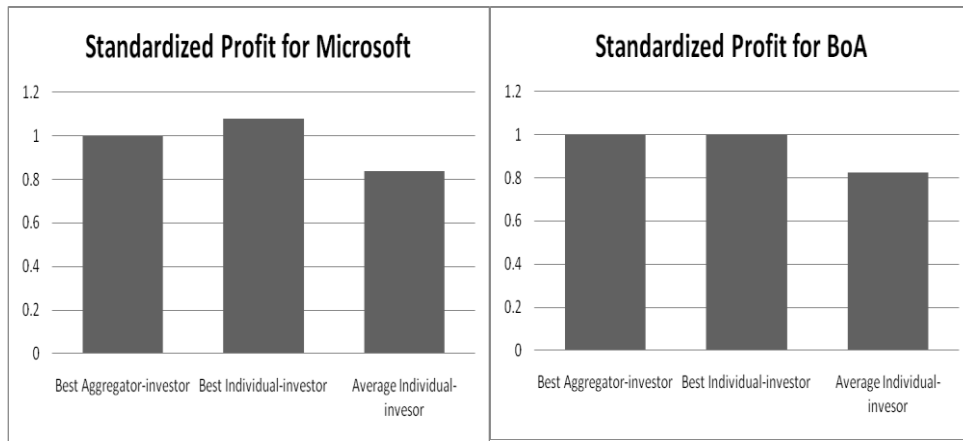
Time period	Best strategy for Aggregator investors	Assets for best Aggregator investors	Assets for best Individual investors	Assets for average Individual investors
1	(2,1,1,0)	18775.74	18565.21	14509.64
2	(1,0,1,21)	28888.79	24847.6	21045.47
3	(1,0,1,11)	13626.98	15530.83	13233.79

**Figure 31:** Aggregator investors in a No-Learning Setting

As shown in Figure 31, the best strategy for aggregator investors, assets for best aggregator investors, assets for best individual investors and assets for average individual investors are recorded. Assets for investors are calculated based on the money and stocks the investors owns as well as the current stock price. Experiments

show that in a no-learning environment, mostly the best strategy for aggregator investor is (1, N1, 1, N2) (N1 and N2 are natural numbers), which suggests that:

1. The aggregator investor should consult with part of the crowds -- the top performers in the crowds.
2. Not considering the transaction fee, the best strategy still suggests the investor to trade all its money/stocks every time.
3. N2 in the strategy chromosome indicates how many individual investors are chosen to form the advisory group. For the stocks with low velocity, such as Bank of America, the best aggregator investor prefers larger advisory group even the whole crowds occasionally, while for the stocks with high velocity, such as Microsoft, it prefer smaller advisory group, for example, having the best individual as adviser.



Standardized Profit for Microsoft			Standardized Profit for BoA		
Best Aggregator investor	Best Individual investor	Average Individual investor	Best Aggregator investor	Best Individual investor	Average Individual investor
1	1.083	0.836	1	0.996	0.824

**Figure 32:** Comparison among investors in a No-Learning Setting

Figure 32 shows that the performance of aggregator investors is better than the performance of the average individual investor, and is close to or even better than the performance of the best individual investor, by following an appropriate aggregation strategy.

*c. Performance of Aggregator investor in Adaptive/Extended Crowds*

In this experiment, we focus on the performance for aggregator investors in the stock market. During the randomly selected time period, 250 individual investors and 42 aggregator investors have been put into game. A set of basic strategies is assigned randomly to each investor and 3 time period is randomly selected. Investors buy/sell stocks or just choose not to trade in the market, and there is only one stock that is trading. The crowd is defined as adaptive/extended crowds. After each trading day, an individual investor can choose to learn from others to improve its performance, by adopting other's strategy

Performances of aggregator investors and individual investors are recorded below.

For Microsoft, results are shown below

Time period	Best strategy for Aggregator investors	Assets for best Aggregator investors	Assets for best Individual investors	Assets for average Individual investors
1	(0,1,1,11)	19014.92	18509.36	16583.95
2	(0,0,1,1)	14081.71	15465.35	12575.09
3	(2,0,1,0)	13233.51	12062.48	11007.71

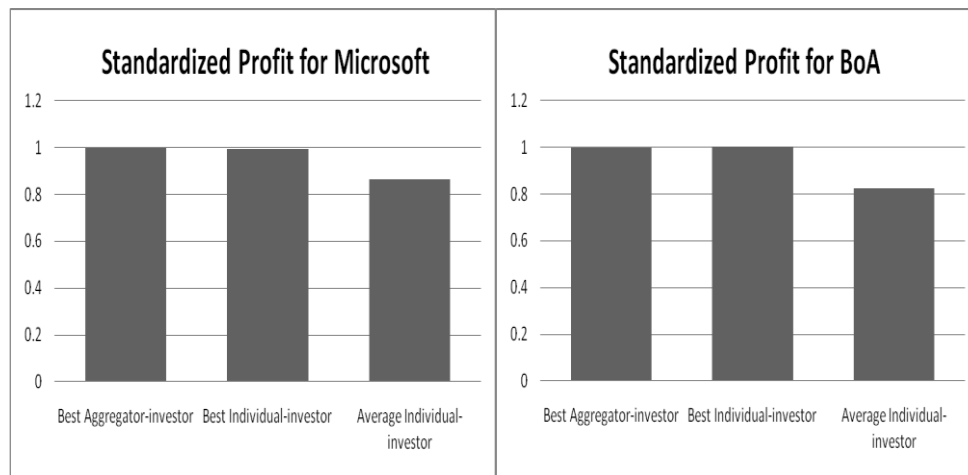
For Bank of America, results are shown below

Time period	Best strategy for Aggregator investors	Assets for best Aggregator investors	Assets for best Individual investors	Assets for average Individual investors
1	(2,1,1,0)	19070.26	18565.21	12873.95
2	(1,1,1,1)	25744.96	26397.37	22023.16
3	(1,1,1,1)	14240.48	14411.08	13467.58

**Figure 33:** Aggregator investors in an Adaptive Setting

As shown in Figure 33, the best strategy for aggregator investors, assets for best aggregator investors, assets for best individual investors and assets for average individual investors are recorded. Assets for investors are calculated based on the money and stocks the investors owns as well as the current stock price. Experiments

show that the best strategies for aggregator investors in different scenarios differ. These can be: 1.) consult with the best individual investor and do what it suggests; 2.) consult with the worst individual investor or group and do what it suggests or not; or 3.) consult with the whole crowds and do what it suggest or not in the market and . The first two strategies above suggest consulting with part of the crowds with better performance. The third strategy suggests consulting with the whole crowds. There is no all-time winning strategy pattern for aggregator investors in a stock market with adaptive crowds.



Standardized Profit for Microsoft			Standardized Profit for BoA		
Best Aggregator investor	Best Individual investor	Average Individual investor	Best Aggregator investor	Best Individual investor	Average Individual investor
1	0.994	0.866	1	1.003	0.825

**Figure 34:** Comparison among investors in an Adaptive Setting

Figure 34 shows that the performance of aggregator investors is better than the performance of the average individual investor, and is close to or even better than the performance of the best individual investor, by following an appropriate aggregation strategy.

*d. Comparison between Individual investors and Aggregator investors*

Figure 32 and Figure 34 show that the performance of aggregator investors is

better than the performance of average individual investor, and close to or even better than the best individual investor, by following an appropriate aggregator strategy.

*e. Effect of Other Factors*

Other factors may also affect the performance of individual investors and aggregator investors, for example, transaction cost, bank interest rate. In economics and related disciplines, a transaction cost is a cost incurred in making an economic exchange. For example, most people must pay a commission to their broker when buying or selling a stock. That commission is a transaction cost of doing the stock deal [Wiki11]. An interest rate is the rate at which interest is paid by a borrower for the use of money that they borrow from a lender. Interest rates are normally expressed as a percentage rate over the period of one year [Wiki12].

In this experiment, we focus on the effect of adding transaction cost and bank interest rate in the stock market. During the randomly selected time period, 250 individual investors and 42 aggregator investors have been put into game. A set of basic strategies is assigned randomly to each investor and 3 time period is randomly selected. Investors buy/sell stocks or just choose not to trade in the market, and there is only one stock that is trading. The crowd is defined as no-learning crowds. The transaction cost is \$10 per-transaction and the interest rate is 0.02/360 per-day.

Performances of aggregator investors and individual investors are recorded below. Experiments show that –

1. The best strategy for individual investors changes as shown in Figure 35,

stock name	Time period	Type	without transaction cost and interest rate	with transaction cost and interest rate
Microsoft	1	No-learning crowds	(-3,1,1)	(-4,4,5)
	2	No-learning crowds	(-4,1,1)	(-4,3,4)
	3	No-learning crowds	(-4,1,1)	(-3,4,4)
	1	Adaptive crowds	(-4,1,1)	(-4,4,5)

	2	Adaptive crowds	(-4,2,1)	(-4,1,5)
	3	Adaptive crowds	(-2,1,1)	(-4,4,2)
Bank of America	1	No-learning crowds	(4,-4,5)	(4,-4,5)
	2	No-learning crowds	(-3,2,1)	(-2,4,1)
	3	No-learning crowds	(-4,2,1)	(4,-4,5)
	1	Adaptive crowds	(4,-4,1)	(4,-4,5)
	2	Adaptive crowds	(1,-1,1)	(-2,4,1)
	3	Adaptive crowds	(-3,2,1)	(4,-4,5)

**Figure 35:** Strategies for Best Individual investors

Best strategy in a more realistic stock market with transaction cost and daily interest suggests that the investors should buy or sell more cautiously, by 1.) trading only part of their money/stocks , for example 1/5 of their money/stocks instead of all each time, despite the transaction cost they have to pay for each trade and 2.) observing the market trend for longer time, for example more than three days before sell, instead of the one or two day preferred by the simplified stock market which has no transaction cost and bank interest.

2. The best strategy for aggregator investors changes as shown in Figure 36.

Stock name	Time period	Type	without transaction cost and interest rate	with transaction cost and interest rate
Microsoft	1	No-learning crowds	(1,0,1,1)	(0,1,1,11)
	2	No-learning crowds	(1,1,1,1)	(2,1,3,0)
	3	No-learning crowds	(1,0,1,1)	(2,0,3,0)
	1	Adaptive crowds	(0,1,1,11)	(0,1,1,11)
	2	Adaptive crowds	(0,0,1,1)	(2,0,3,0)
	3	Adaptive crowds	(2,0,1,0)	(2,0,1,0)
Bank of America	1	No-learning crowds	(2,1,1,0)	(2,1,1,0)
	2	No-learning crowds	(1,0,1,21)	(2,1,1,0)
	3	No-learning crowds	(1,0,1,11)	(1,1,1,21)
	1	Adaptive crowds	(2,1,1,0)	(2,1,1,0)
	2	Adaptive crowds	(1,1,1,1)	(1,1,1,11)
	3	Adaptive crowds	(1,1,1,1)	(2,0,3,0)

**Figure 36:** Strategies for Best Aggregator investors

Adding transaction cost and daily interest change the best strategy for aggregator

investor. As shown in Figure 36, the best aggregation strategy in a more complicated market tends to be in the format of (2,X,X,X) which suggest the investors listening to the whole crowds. On the other hand, the most popular aggregation strategy in a simplified market tends to be in the format of (1,X,X,X) which suggest the investors listening to part of crowds. It may suggest that with the more complexity in the real-world market, the power of using the wisdom of the whole crowd shows more.

### 3. Comparison between Individual investors and Aggregator investors

Figure 37 shows that the performance of aggregator investors is better than the performance of average individual investor, and close to or even better than the best individual investor, by following an appropriate aggregator strategy in a stock market with transaction cost and interest rate.

For Microsoft, results are shown below

Time period	Type	Assets for best Aggregator Investors	Assets for best Individual investors	Assets for average Individual investors
1	No-learning crowds	16647.78	17081.34	15060.98
2	No-learning crowds	12442.96	12800.27	10949.72
3	No-learning crowds	12118.00	11640.13	9406.46
1	Adaptive crowds	16647.81	17081.35	15061.00
2	Adaptive crowds	12417.10	12889.06	10032.26
3	Adaptive crowds	13236.65	12745.83	9845.99

For Bank of America, results are shown below

Time period	Type	Assets for best Aggregator Investors	Assets for best Individual investors	Assets for average Individual investors
1	No-learning crowds	17899.97	16251.03	9630.04
2	No-learning crowds	19706.32	20395.15	17724.76
3	No-learning crowds	12603.61	13437.03	11707.58

1	Adaptive crowds	14151.36	16251.03	7609.00
2	Adaptive crowds	19536.87	20395.15	12274.49
3	Adaptive crowds	12535.66	13437.03	10394.16

**Figure 37:** Comparison among investors in a stock market with transaction cost and interest rate



## CHAPTER 10: LESSONS LEARNED AND FUTURE WORK

In the previous chapters, we use the concept of Wisdom of Crowds to a continuous decision making problem – The Prisoner's Dilemma and a simple stock market model.

Originally introduced by J. Surowiecki, Wisdom of Crowds theory simply suggests that a collective may solve a problem better than most of the individual members of the group acting alone under certain circumstances. In areas such as the social sciences and economics – which involve numerous human interactions and subjective decision making, it is difficult to determine an effective decision making model or strategy that accurately accounts for those subjective factors. Wisdom of Crowds uses "the crowd" to resolve problems that involve numerous human interactions and subjective decision making and relax the need to collect information, assemble and update database. A decision making system that uses the wisdom of crowds has various benefits, including reducing the cost of collecting information and assembling databases for each field, avoiding frequent data updates, and canceling out human bias through information aggregation. The effectiveness of Wisdom of Crowds in decision making depends on the type of crowds and the aggregation method using to obtain collective wisdom. Surowiecki suggests four key criteria to form smart crowds: Diversity of opinion, Independence, Decentralization and Aggregation. We built a simulation, using the concept of Complex Adaptive Systems, to demonstrate the wisdom of crowds, while at the same time testing Surowiecki's four criteria to form a smart crowd.

However, it is hard to imagine a continuous decision-making example where members of the crowd are truly independent from each other in the real world. Therefore, by partially violating the independence criteria, we added learning ability to the crowd. Our experiments show that this addition makes both individual players and the aggregate-players smarter, while still guaranteeing diversity of opinion and the effectiveness using wisdom of crowds. Evolution is also added into the decision-making process of crowds.

In the Prisoner's Dilemma experiments, both the individual players' behavior and their effect on the system as a whole are examined in order to understand the type of crowds and the performance of aggregation methods.

Experiments show that

- 1.) In a fixed crowd, the crowd composition is unchanged during the whole running time. Although no evolution happens (which means the good keep good and bad keep bad without any change), the best performer -- a greedy player who takes advantage of naïve cooperate players still ends up with the score only slightly better than the performance of the crowds, instead of 5 points advantage which it had expected for.
- 2.) In the extended crowds where Player-agents are replaced gradually by those with the highest point, the crowds end up with the score of 3, which suggests that all players accepted cooperation as their mode of operation.
- 3.) In the extended crowds where Player-agents are replaced gradually by those with the lowest score, the whole society ends up consisting of the players who are afraid of being eliminated by playing 'cooperate'.

Also, the fact that the same player can perform differently in different crowds, without changing its action rules, tells us that the Lucifer Effect may not be related to the notion of human nature but only to the interaction of participants.

Experiments in extended crowds including Type I crowds -- crowds with diverse strategy distribution and Type II Crowds -- crowds with homogenous strategy distribution give us a close look at the structure of crowd more precisely by using elements such as size, density, and various kind of behavior settings, including heuristic, behavior pattern, social influence, learning speed. When observing the crowds, we notice some tipping points in the running period: 1.) Cooperate crowds points threshold; 2.) Cooperate percentage threshold. “Cooperate crowds points threshold” is the number used to tell the trend of crowds. “Cooperate percentage threshold” is the number denoting the current status of crowds. By using history data and “cooperate percentage threshold”, the current status of crowds – i.e., whether most of the crowds cooperate or defect – can be predicted. There two variables can help us to predict the crowds trend in advance.

Experiments show that in both Type I and Type II crowds the aggregator-agent, using the wisdom of crowds in a proper way, performs better than the average individual player-agent. It can even perform better than the best performer sometime.

The formation of the crowds is the key factor for choosing the best aggregation method for the aggregator-agent. Experiments in both Type I and Type II crowds show that the formation of crowds can be evaluated by simply observing 1.) the performance of the top 10 percent performers in the crowds or 2.) the performance of the best player-agent and the average player-agent. In a crowd that has a relatively-stable top 10 percent group, the aggregator-agent using the top 10 percent performer as the advisory group does a better job than the one using all the players in the crowds. On the other hand, in the crowd whose top 10 percent formation is changing all the time, using the current top 10 percent performers as the advisory group would not help the aggregator-agent make a good decision. Using all the

players in the crowds as an advisory group is the best possible aggregation method.

Since the detailed information of the crowds is hard to obtain in real life, choosing the best aggregation method based on the formation of crowds may not be realistic sometimes. There are other ways to help to choose the best aggregation methods, for example, by making decision based on the performance of the best player-agent and the average player-agent. The strategy is useful when the scores of the best player-agent and the average player-agent are available, and the top 10 percent performer of the crowd data is too large or too hard to collect and analyze. Using the scores that the best player-agent and the average player-agents obtain during the game, we generate rules for the aggregator-agents that enable them to choose the best strategy for making a decision using the wisdom of crowds in all kinds of crowds in the context of Prisoner's Dilemma.

A simplified stock market system was introduced to demonstrate the utility of wisdom of crowds in real life. Given different stocks, performances of individual and aggregator investors are examined.

In a simplified no-learning stock market -- where the crowd composition follows the four key criteria suggested in the book "wisdom of crowds" and no transaction cost or interest rate is considered, experiments show that

1. The best strategy suggests the investor to trade all its money/stocks every time.
2. For the stock of low velocity, such as Bank of America, the best strategy would encourage the investors to buy/sell more cautiously – observing until the stock price changes for days. For the stock of high velocity, such as Microsoft stock, it would encourage the investors to sell immediately when the price go up, because the price often change dramatically in a short period.
3. For the stocks with low velocity, such as Bank of America, the best aggregator

investor prefers larger advisory group even the whole crowds occasionally, while for the stocks with high velocity, such as Microsoft, it prefer smaller advisory group, for example, having the best individual as adviser.

4. The performance of aggregator investors is better than the performance of the average individual investor, and is close to or even better than the performance of the best individual investor, by following an appropriate aggregation strategy.

In a simplified adaptive/extended stock market -- where the “independent” criterion is violated and communication/learning is introduced, similar conclusion is yield as in simplified no-learning stock market. The performance of aggregator investors is better than the performance of the average individual investor, and is close to or even better than the performance of the best individual investor, by following an appropriate aggregation strategy.

In a more complicated stock market setting, in which bank interest rate and transaction cost are added, experiments show that the best aggregation strategy in such market is to listen to the whole crowds. This may suggest that the greater complexity in the real-world market, the higher the power of using the wisdom of the whole crowd.

In conclusion, in both Prisoner’s Dilemma system and the simplified stock market, experiments show that the wisdom of crowds approach is always superior to the average and often to the best performing strategy in the crowd.

The future work will focus on the application using wisdom of crowds in different fields, including better defined stock market, other types of trading and social problem decision-making.

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