# EFFECTS OF SINGAPORE MODEL METHOD WITH EXPLICIT INSTRUCTION ON MATH PROBLEM SOLVING SKILLS OF STUDENTS AT RISK FOR OR IDENTIFIED WITH LEARNING DISABILITIES

by

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# ABSTRACT

ANGELA IRENE PRESTON. Effects of Singapore model method with explicit instruction on math problem solving skills of students at risk for or identified with learning disabilities. (Under the direction of DR. CHARLES L. WOOD)

Over the last two decades, students in Singapore consistently scored above students from other nations on the Trends in International Mathematics and Science Study (TIMSS; Provasnik et al., 2012). In contrast, students in the United States have not performed as well on international and national mathematics assessments and students with disabilities are not performing as well as their peers without disabilities (NAEP, 2015; Provasnik et al., 2012). In the 1980s, Singapore's Ministry of Education designed Singapore Math (Singapore Math Inc., 2014) as the national curriculum for students in Singapore (Ginsburg, Leinwand, & Anstrom, 2005). As part of Singapore Math, the Singapore Model Method (SMM) is a problem-solving heuristic used to solve math word problems. Currently, very little research supports the use of SMM for problem solving (Mahoney, 2011; Ng & Lee, 2009) and at this time, there are no studies that evaluate the use of SMM for problem solving with students with disabilities. Therefore, the purpose of this study was to evaluate the effects of the SMM with explicit instruction on math problem solving skills of students at risk for or identified with learning disabilities. The researcher designed a nine-stage instructional format that used explicit instruction to teach SMM to seven students to solve single-step math word problems. This study used a multiple probe across participants with an embedded ABCDE design. Students were taught to solve addition and subtraction word problems as well as multiplication and division word problems. Results of the study demonstrated a functional relation between SMM with explicit instruction and all students' mathematics problem solving skills for addition, subtraction, multiplication, and division word problems. Between the pretest and posttest, all students demonstrated major improvements on problem solving skills. Social validity results for students indicated that most students found the steps easy to follow. Discussion of the results as well as specific contributions of the study, limitations of the study, recommendations for future research, and implications for practice are included.

# DEDICATION

I dedicate my dissertation to my husband Nathan and my daughter Eloise. Without their support, I would never have accomplished this goal. My husband stood by my side each day (the good days and bad days) and was always my biggest cheerleader. To my daughter, I hope you find your passion in life and pursue happiness in all you do. In our family lyrics can be more meaningful than words, so I leave it to Michael Hutchence from INXS to express the rest of my gratitude.

Don't ask me
What you know is true
Don't have to tell you
I love your precious heart

I, I was standing
You were there
Two worlds collided
And they could never tear us apart

We could live
For a thousand years
But if I hurt you
I'd make wine from your tears

I told you
That we could fly
'Cause we all have wings
But some of us don't know why

I, I was standing
You were there
Two worlds collided
And they could never ever tear us apart

(Hutchence, 1987)

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# TABLE OF CONTENTS

# CHAPTER 1: INTRODUCTION

## Statement of the Problem

According to the most recent Trends in International Mathematics and Science Study (TIMSS) in 2011, students from Singapore scored the highest of all countries in fourth grade mathematics and second highest on eighth grade mathematics (results from the sixth TIMSS in 2015 are not yet published). Students in Singapore have scored in the top two of all participating nations in fourth grade mathematics each year the country participated in TIMSS since 1995, and in the top three of all participating nations in eighth grade mathematics (Provasnik et al., 2012). In 2011, students in the United States ranked 11th out of 45 countries participating in the fourth grade mathematics assessment and 9th out of 38 countries participating in the eighth grade mathematics assessment (Provasnik et al., 2012). Based on results from the Global Competitiveness Forum (2015), the United States is ranked 3$^{rd}$ on the overall score of the global competitiveness index (GCI); yet, it is ranked 44th for quality of mathematics and science education, and 29$^{th}$ for quality of primary education (WEF, 2015). Considering the United States' ranking as a top global competitive nation, the expectation for mathematics achievement and quality of education should be much higher (similar to the GCI rank). Conversely, Singapore is ranked 2$^{nd}$ on the GCI, with 1$^{st}$ for quality of mathematics and science education, and 3$^{rd}$ in quality of primary education (WEF, 2015). In other words, Singapore's rankings are consistent between the GCI and education system, whereas there is a discrepancy in the United States' rankings between the GCI and education system.

At the national level across the United States, student math achievement in 4th and 8th grade has leveled off over the last 8 years. The National Assessment for Educational Progress (NAEP) provides a math and reading assessment every 2 years to students in all 50 states. Based on the NAEP mathematics assessment from 2015, 4th grade average scores decreased by two points from 242 to 240 from 2013 to 2015, and 8th grade average scores also decreased by three points from 285 to 282 from 2013 to 2015 (NAEP, 2015). Less than half (40%) of students performed at the proficient level or higher in fourth grade math, and only 33% of eighth grade students performed at or above the proficient level in math (NAEP, 2015).

Upon further examination of national student achievement in math, students in the bottom 10th percentile (who may be at risk for disabilities) demonstrated statistically significant decreases in fourth grade scores from 203 to 202 and eighth grade scores from 237 to 235 between 2013 and 2015 (NAEP, 2015). In fact, since 2007, fourth grade students in the lowest 10th percentile have demonstrated a flat trend with scores of 202 in both 2007 and 2015, whereas students in eight grade in the lowest $10^{th}$ percentile have demonstrated slight increases since 2007 and a significant decrease in 2015 with scores of 235 in both 2007 and 2015 (NAEP, 2015). Unfortunately, overall scores for students with disabilities have also shown a flat line and decreasing trend. Fourth grade math scores for students with disabilities remained the same over three assessments between 2011 and 2015 (218), and decreased since 2009 (221). In eighth grade, math scores for students with disabilities decreased by two points from 2013 to 2015 from 249 to 247 respectively (NAEP, 2015).

In an effort to improve the mathematics achievement of students in the United States, the Common Core State Standards (CCSS) were created to improve math skills and raise expectations for all students (Little, 2009). Standards for mathematical content include (a) number and quantity, (b) algebra, (c) functions, (d) modeling, (e) geometry, and (f) statistics and probability. Standards for mathematical practice include eight steps: (a) make sense of problems and persevere in solving them, (b) reason abstractly and quantitatively, (c) construct viable arguments and critique the reasoning of others, (d) model with mathematics, (e) use appropriate tools strategically, (f) attend to precision, (g) look for and make use of structure, and (h) look for and express regularity in repeated reasoning (CCSS, 2010). One notable change in the CCSS was the focus on conceptual understanding, as well as problem-solving applications in real world situations instead of memorization of math facts; however, there is little research on how this change will affect the achievement of students with disabilities (Little, 2009).

Implementation of CCSS has raised expectations for all students, including those who are above average, average, at risk, and who have a disability in math. In order to address students' needs across all levels of achievement, the use of Response to Intervention (RTI), which was previously supported through federal legislation (IDEA, 2004), has been implemented for students across the United States. RTI is a multi-tiered system of supports that uses interventions, assessments, and data-based decision making for the prevention and identification of students with learning disabilities (LD; NCRTI, 2010). RTI provides universal screenings to all students in a school to identify students who are at risk for LD. All students in Tier 1 receive research-based practices through core instruction in general education and are progress monitored using curriculum-based measurements (CBM) to determine if they are responding to core instruction. Using data-based decision making, if students are not making enough progress, they begin Tier 2 services which include small group instruction using research-based and evidence-based practices (NCRTI, 2010). The same process is followed at Tier 2 with progress monitoring and data-based decision making for moving students into Tier 3. Instruction at Tier 3 includes small group, intensive instruction using evidence-based and research-based practices in either general or special education, depending on the model used (NCRTI, 2010). The goal of RTI is multifaceted and was designed to provide better quality instruction to all students, identify and address student academic problems, and contribute to identifying students with LD (NCRTI, 2010).

Early research on RTI mainly focused on beginning reading skills. Researchers saw similarities with early numeracy skills, and over the years began to build a foundation for implementing RTI with mathematics due to RTI's promising effects on student achievement (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008a; Fuchs, Fuchs, & Hollenbeck, 2007a; Fuchs, Fuchs, & Prentice, 2004a; Gersten et al., 2009a). Analysis of RTI and math research has produced recommendations for implementing RTI in math (Gersten et al., 2009a), but more research is needed to determine the specific types of interventions that are necessary within each tier of intervention. This is especially true for determining the types of interventions needed for improving math word problem solving skills due to the recent adoption of CCSS that focuses on application of problem solving skills.

Although research to support specific interventions within the tiers of RTI is somewhat lacking, research has been conducted on effective strategies for teaching math word problem solving skills to students who are at risk for or identified with LD. Based on reviews of problem solving literature, among others, heuristics, representational techniques, cognitive strategies, and explicit instruction, are considered effective strategies for teaching math word-problem solving (Xin & Jitendra, 1999; Zhang & Xin, 2012; Zheng, Flynn, & Swanson, 2012). Heuristics are defined as "a generic problem-solving guide in which the strategy (list of steps) is not problem specific (Gersten et al., 2008, p. 2)." Heuristics assist students in organizing the information and can be used for a variety of problem types. This is different from other strategies that are problem specific. When students are taught multiple heuristic strategies, they have the opportunity to decide which strategy to use to solve the problem (Gersten et al., 2009b). Heuristic strategies are broad and along with lists of steps, and other general plans, may include representational techniques (e.g., schema-based instruction; SBI) and cognitive strategy instruction (CSI). SBI uses schematic diagrams to represent various problem types. Use of diagrams focuses attention to the schema of a word problem by eliminating irrelevant information making it easier for students with LD to understand how to solve the problem (Jitendra & Montague, 2013). Another heuristic-based strategy is CSI. CSI applies one strategy or method to a variety of problems by using cognitive processes including visual representations and metacognitive processes including questioning. CSI includes the use of simple or complex processes, strategies, and thinking skills to solve word problems (Jitendra et al., 2013; Jitendra & Montague, 2013; Montague, Enders, & Dietz, 2011). Finally, explicit instruction is defined as a structured and systematic approach to teaching academic concepts and skills (Archer & Hughes, 2011). Characteristics of explicit instruction include the use of scaffolds to lead students through the steps of learning by providing a purpose for learning, clearly demonstrating the skill, presenting multiple opportunities for practice with ongoing feedback, and meeting mastery criteria (Archer & Hughes, 2011). Explicit instruction is empirically validated for teaching students with disabilities. In a meta-analysis, explicit, systematic instruction demonstrated a strong mean effect size of 1.22, for teaching computation, word problems, and solving math problems in new situations. (Gersten et al., 2009b). SBI and CSI use explicit instruction within their models to enhance the effectiveness of the instruction.

Another method that follows a heuristic model and includes components of SBI and CSI is The Singapore Model Method (SMM; Hong, Mei, & Lim, 2009). SMM is a component of Singapore Math (Singapore Math Inc., 2014) that teaches problem solving. This curriculum was designed by Singapore's Ministry of Education and is used by all schools across Singapore, the leading country in mathematics (Ginsburg, Leinwand, & Anstrom, 2005; Hong et al., 2009). SMM is used to solve problems dealing with the four operations (i.e., addition, subtraction, multiplication, and division), fractions, ratios, percentages, and algebra. SMM uses a part-whole model and comparison model to assist in determining the problem structure and operation necessary to solve the problem (Hong et al., 2009). To further assist students with problem solving, SMM uses heuristics, schematic diagrams, and metacognitive strategies, but does not use explicit instruction to teach the strategy. The most common heuristic derived from SMM is an eight-step procedure applicable to various types of word problems (see Table 1). Similar to SBI, students draw schemas to diagram part-whole and comparison models in order to determine the function needed to solve the problem. Students also use metacognitive skills, similar to CSI, to monitor their ability to solve the problem (Hong et al., 2009). In this way, SMM is a heuristic model that combines features of SBI and CSI.

Table 1: General steps of SMM

| General SMM Steps |
|---|
| 1. Read the entire problem |
| 2. Determine who the problem is about |
| 3. Determine what the problem is about |
| 4. Draw unit bars for each variable |
| 5. Place the numbers |
| 6. Place the question mark |
| 7. Do the computation to the side or underneath the bars |
| 8. Write the answer in a complete sentence |

Despite Singapore's consistently high achievement in math on the TIMSS, there is very little research to support the use of SMM. Ng and Lee (2009) conducted a non-experimental study to evaluate the effects of Singapore Math instruction on 151 students' problem solving abilities. Using a post-test only design, results demonstrated students in the higher level (E1) performed higher than students in the middle level (E2). It was suggested that students in the middle level struggled to understand the word problems. Statistical data were not discussed in the results of this study. Implications for future research included conducting experimental studies that focused on students who are at risk.

In another study on SMM, Mahoney (2011) conducted a delayed multiple baseline across participants. Four students in third and fourth grade, with average math ability, participated in the study. Results demonstrated a functional relation between SMM and students' word problem solving skills in multiplicative comparison problems and fractions. One of the limitations of the study was the researcher did not include students with disabilities.

The What Works Clearinghouse (WWC; 2015) conducted an evaluation of Singapore Math (Singapore Math Inc., 2014), which includes SMM for problem solving instruction, and produced inconclusive results. Three studies conducted on Singapore Math (Singapore Math Inc., 2014) were reviewed but did not meet WWC standards for group design (Blalock, 2011; Goldman, Retakh, Rubin, & Minnigh, 2009; Merchlinsky & Wolanin, 2003) because comparison and treatment groups were not determined equivalent before treatment (WWC, 2015). The remaining studies could not be included in the review for a variety of reasons including use of an ineligible design, evaluation of an outcome not included in the WWC protocol, and nonexperimental reviews of the curriculum. WWC recommended conducting experimental studies that examine the effects of Singapore Math (Singapore Math Inc., 2014) on math achievement skill (WWC, 2015).

Of major concern over the past decade, students who are at risk for or identified with disabilities are making little to no gains in mathematics based on national assessments (NAEP, 2015). Due to the limited amount of research on SMM and the high math achievement of students in Singapore, more research is needed to determine the effects of SMM on students' math word problem solving skills especially for students who are at risk for or identified with LD. Within the Singapore Math curriculum (Singapore Math Inc., 2014), SMM is taught by the teacher guiding students through the process of bar modeling, but a list of specific steps, strategies for error correction or mastery criteria are not provided in the teacher's guide. In the study by Ng and Lee

(2009) it was suggested that students who were streamed into the middle level (E2) had difficulty understanding word problems and struggled to solve word problems using SMM. It can be assumed that students in the lower level (E3, e.g., students at risk for or identified with LD) would perform worse than their peers in the middle level when provided the same instruction on SMM. Because the use of explicit instruction is highly supported for teaching students who are at risk for or identified with disabilities (Gersten et al., 2009b), teaching SMM to this student population may have stronger results if it is taught using explicit instruction. Therefore, research is needed to determine the effectiveness of using explicit instruction to teach SMM to students who are at risk for or identified with LD.

## Purpose of Study and Research Questions

This study addressed the use of SMM with explicit instruction (designed by the researcher) on math word problem solving skills of students with disabilities or who are at risk for a disability in math at the elementary level. The lack of research about Singapore Math (Singapore Math Inc., 2014), SMM (Hong et al., 2009), and the TIMSS data demonstrating Singapore as a leader in math (Provasnik et al., 2012), suggests the need for more research on this program, especially for students with disabilities. Originally, SMM was not taught using explicit instruction. Due to the empirical effectiveness of explicit instruction for students with disabilities in mathematics, the addition of explicit instruction to SMM may provide stronger results. Therefore, the purpose of this study was to determine the effects of SMM with explicit instruction on word problem solving skills of students who are at risk for or identified with LD at the elementary level. The following questions will be addressed in this study:

1. What are the effects of SMM with explicit instruction on addition and subtraction word-problem solving skills of students who are at risk for or identified with learning disabilities?

2. What are the effects of SMM with explicit instruction on multiplication and division word-problem solving skills of students who are at risk for or identified with learning disabilities?

3. What are the effects of SMM with explicit instruction on combined skills of addition, subtraction, multiplication, and division word-problem solving skills of students who are at risk for or identified with learning disabilities?

4. What are student participants' perceptions of SMM with explicit instruction?

5. What are teachers' perceptions of SMM with explicit instruction?

Significance of Study

This study contributed to the body of research on word problem solving interventions for students who are at risk for or identified with a disability by examining the impact of SMM, which is a problem-solving method used by one of the top scoring countries in mathematics, with explicit instruction. Previously, this method was only researched with average to above average students. This study evaluated the effectiveness of SMM, when taught using explicit instruction, on math word problem-solving skills of students who are at risk for or identified with a disability. Another contribution will be provided to teachers within the context of an RTI framework, by demonstrating how to break down a strategy that is typically taught to average and above average students in general education classrooms (Tier 1) and use explicit instruction in small groups (Tier 2) to teach this strategy to students who are at risk for or identified with LD.

Finally, another benefit of this study was that it directly impacted a group of students at risk for or identified with LD. The group of students was taught through explicit instruction to use SMM to solve math word problems. This study also contributed to the research that demonstrates students who are at risk for or identified with LD are capable of learning strategies to solve math word problems. By evaluating SMM with explicit instruction, students who are at risk for or identified with LD improved their ability to solve math word problems. Eventually, this skill may also generalize to other math word-problem solving settings including class work, homework, formative assessments, and state-level standardized summative assessments.

## Delimitations

This study addressed the effects of SMM with explicit instruction on math word-problem solving skills of students who are at risk for or identified with LD. There were delimitations of the study that needed to be addressed in order to assist with interpreting the results. First, this study used a single-case design which means results cannot be generalized to a larger population without multiple replications of this study. Second, another delimitation was the location of the study. This study took place in a suburban school in the southeastern part of the United States and generalization to other parts of the country with different demographics will require additional research. Third, the researcher limited the specific types of word problems being used in this study in order to control the intervention and reduce confounding variables. Finally, this study modifies the commercial SMM curriculum to include explicit instruction. Results of this study may not be generalized to the commercial SMM curriculum that does not include explicit instruction.

## Definitions

Cognitive-strategy instruction (CSI)- CSI applies one strategy or method to a variety of problems by using cognitive processes including visual representations and metacognitive processes including questioning (Jitendra & Montague, 2013).

Explicit instruction- Explicit instruction is defined as a structured and systematic approach to teaching academic concepts and skills (Archer & Hughes, 2011).

Heuristics- Heuristics are a set of steps to problem-solving that are applicable to all types of problems (Gersten et al., 2008).

Learning disability (LD)- "LD is a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, that may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or do mathematical calculations"… "and is not due to a sensory, motor, or intellectual disability, to emotional disturbance, or to environmental or economic disadvantage (PL 108-466, Sec. 602[30]; IDEA, 2004)."

Math word problems- Math word problems refer to math stories that include a missing quantity and require solving a problem using one-step addition, subtraction, multiplication, or division.

Math word-problem solving- The act of solving math word problems.

Math word stories- Math word-stories refer to math stories that include all quantities without any missing information, used in teaching students how to diagram different types of problems.

Mild disabilities- Mild disabilities include learning disabilities, mild intellectual disability, emotional-behavioral disability, and other health impairment (e.g., attention deficit-hyperactive disorder; Gresham & MacMillan, 1997).

Response to Intervention (RTI)- RTI is a multi-tiered system of supports for the prevention and identification of students with a learning disability by providing increasingly intensive interventions through tiers of instruction and progress monitoring students' achievement within each tier of instruction (NCRTI, 2010).

Schema-based instruction (SBI)- SBI uses schematic diagrams to represent various problem types (e.g., change, group, compare, restate, vary) in order to reduce the load on working memory and allow students who struggle with math to become more effective at problem solving (Jitendra & Montague, 2013).

Schemata- Schemata are cognitive structures that are organized hierarchically and stored in long-term memory for use during problem solving (Jitendra & Montague, 2013).

Singapore Model Method (SMM)- SMM is an eight-step heuristic that uses a part whole comparison model, schematic diagrams, and metacognition skills in order to determine the problem structure and operation necessary for solving the problem (Hong et al., 2009).

Students who are at risk for learning disabilities- are students who are scoring below grade level on normative assessments (e.g., Discovery Ed, AIMSweb, Woodcock-Johnson-III), but have not been referred for special education evaluation or did not qualify for special education services.

# CHAPTER 2: REVIEW OF LITERATURE

The purpose of this chapter is to review pertinent literature on LD, response to intervention (RTI), and math word-problem solving. The chapter starts with the history of LD, definition of LD, characteristics of students with LD, students at risk for LD, and the need for early intervention. The second part discusses RTI in math, including components and research to support multi-tiered interventions, assessment in math, and implications for future research. The final part of this chapter includes a review of research on math word-problem solving. This part begins with a review of meta-analyses on math instruction and word-problem solving instruction, then moves into descriptions and research that supports problem-solving heuristics including schema-based instruction, cognitive strategy instruction, as well as research to support explicit instruction in math. The chapter also includes a description of the Singapore Model Method, a review of research, and reasons to support using the Singapore Model Method in the current study. Finally, the chapter concludes with an overall summary of the literature.

## Learning Disabilities

### History of Learning Disabilities

Prior to the beginning of special education as a federal policy in 1975, professionals began to notice a commonality among certain struggling students. These particular students did not have an intellectual disability, but continued to have academic difficulties (Hallahan, Pullen, & Ward, 2013). In the early 1960s, the term learning disabilities (LD) originated in Samuel Kirk's (1962) textbook Educating Exceptional Children, and was quickly adopted by parent and professional organizations. LD became the universally accepted term for students without an intellectual disability who had persistent academic difficulties (Hallahan et al., 2013). With the passing of the Education of All Handicapped Children Act (1975), LD was finally considered an official area of eligibility in special education and fully funded by law (Hallahan et al., 2013).

Even from its inception, LD has been the subject of controversies. The definition of LD and the use of the ability-achievement discrepancy model were continually debated by professional organizations and researchers (Hallahan et al., 2013). Federal regulations deliberately excluded an inclusionary formula in the definition of LD in order to allow states to make those determinations (Fletcher, Lyon, Fuchs, & Barnes, 2006; Hallahan & Mercer, 2002). Subsequently, multiple definitions arose based on educational and medical theories with little consensus among LD organizations (Hallahan et al., 2013).

Definition of LD. LD is difficult to define because it is a hypothetical construct and underachievement, which assists in defining LD, exists on a continuum as opposed to distinct categories (as found in other disabilities; Fletcher et al., 2006). Another difficulty in defining LD was the use of exclusion criteria in lieu of inclusion criteria (Fletcher et al., 2006). Students with LD possessed "unexpected" underachievement that was not due to the personal effects of visual impairments, hearing impairments, physical impairments, intellectual disability, or emotional disturbance, or the external effects of environment, culture, or economic disadvantage (Fletcher et al., 2006). Compounding the difficulties of defining LD, students with LD are a heterogeneous group with commonalities of under achievement and the exclusion criteria (Fletcher, Stuebing, Morris, & Lyon, 2013).

In an attempt to provide consensus, in 1977, the United States Office of Education (USOE) defined LD as follows:

> The term "specific learning disability" means a disorder in one or more of the psychological processes involved in understanding or in using language, spoken or written, which may manifest itself in an imperfect ability to listen, speak, read, write, spell, or do mathematical calculations. The term includes such conditions as perceptual handicaps, brain injury, minimal brain dysfunction, dyslexia and developmental aphasia. The term does not include children who have learning disabilities which are primarily the result of visual, hearing, or motor handicaps, or mental retardation, or emotional disturbance, or of environmental, cultural, or economic disadvantage (USOE, 1977, p. 65083).

The regulations included an ability-achievement discrepancy (without including a formula; Hallahan et al., 2013) for qualifying students with LD by stating, "the child has a severe discrepancy between achievement and intellectual ability…" (USOE, 1977, p. 65083). This definition has remained the central definition of LD since 1977 and is currently found in the Individual with Disabilities Education Improvement Act (IDEA, 2004) with modifications on how students can qualify.

Ability-achievement discrepancy model. Originally, the concept of an ability-achievement discrepancy came from Monroe's (1938) work in the first half of the 20$^{th}$ century. While working on approaches to teaching reading, Monroe established a "reading index" which calculated a discrepancy between a student's actual level of achievement and expected level of achievement (Hallahan et al., 2013). This concept eventually evolved into the practice of identifying an ability-achievement discrepancy for the identification of students with LD.

In order to evaluate the claims of the ability-achievement discrepancy, Rutter and Yule (1975) conducted evaluations to determine if there was a difference between students with “reading retardation” (i.e., reading LD) and students with “reading backwardness” (i.e., low achievement in reading). They hypothesized that IQ scores and reading achievement scores for the entire school population would fall equally along the bell curve. The results of their study found IQ scores matched the bell curve, but reading achievement scores were not equally distributed along the bell curve. They found a “hump” (i.e., more students’ scores) in the below average range for reading achievement than IQ scores. This demonstrated that more students scored below in reading achievement than what would be expected based on their IQ scores falling in the average range. These findings were used to support the inclusion of the ability-achievement discrepancy model for identifying students with LD.

Further evaluation of the ability-achievement discrepancy was conducted. Ysseldyke, Algozzine, and Epps (1983) evaluated the reliability of the ability-achievement discrepancy model by determining the consistency in which students would qualify for LD based on definitions used by states across the United States. In the first part of the study, they found 17 operationalized definitions for qualifying students with LD. They assessed 248 students in general education (with no known disability) using the 17 operationalized definitions. Based on their results, 85% of students in general education, without any known disabilities, qualified for LD on at least one of the 17 operationalized definitions. More staggering is that 68% of students in general education qualified for LD on two or more of the 17 operationalized definitions. Of the 248 students in general education, only 37 students did not qualify for LD. These results demonstrated the ambiguity of the requirements for determining LD identification.

In the second part of their study, Ysseldyke et al. (1983) evaluated the differences between students who were low achieving (LA) and students with LD. They evaluated 99 students, of which 50 had LD and 49 were LA, using the 17 operationalized definitions of LD. Results found 91 of the 99 students qualified with LD on at least one of the 17 operationalized definitions. This meant 92% of students could qualify LD when only 51% of students were actually LD. Results of this study demonstrated the unreliability of the ability-achievement discrepancy model for identifying students with LD.

Since the original research on the ability-achievement discrepancy model was conducted, many researchers have not been able to replicate the findings of Rutter and Yule (1975) with differences between IQ and achievement for students with LD and who are low achievers (Fletcher et al., 2013). This research also led to questioning the validity of IQ testing to predict reading achievement (Fletcher et al., 2013; Scruggs & Mastropieri, 2002; Siegel & Mazabel, 2013). The field of LD began looking for other options for determining LD identification.

To address the overall dissatisfaction of the ability-achievement discrepancy model, in 1997 the National Joint Committee on Learning Disabilities (NJCLD) submitted a letter to the Office of Special Education Programs (OSEP) highlighting the issues of the ability-achievement discrepancy model for identifying students with LD. In response to the letter, OSEP formed the LD initiative (made up of researchers, parents, teachers, advocates, and policy makers) to address the issues by proposing suggestions for the process of identifying students with LD (Bradley & Danielson, 2004; Bradley, Danielson, & Doolittle 2007). One of the more promising suggestions was to use response to intervention (RTI) as a means for identifying students with LD. Research on RTI was funded by OSEP through the National Research Center on Learning Disabilities and by 2004, with the passing of the Reauthorization of IDEA, requirements for LD identification were no longer restricted to an ability-achievement discrepancy, but could now include a student's response to research-based interventions (Bradley & Danielson, 2004; Bradley et al., 2007). A review of research regarding RTI is included later in this chapter. Despite the changes in regulations regarding LD eligibility, the basic characteristics of students with LD have remained the same.

## Characteristics of Students with LD

Students with LD often have difficulties in listening, reasoning, visual perception, auditory processing, memory, attention, and selecting important information (Heward, 2013). These difficulties affect their ability to make adequate academic progress in school in reading, writing, math, and language, and can affect social skills, behavior, attention, and self-esteem (Heward, 2013). The three most common areas of LD are reading, writing, and math. Because the focus of the current study is on LD in math, this discussion will include more in-depth information on math LD.

Characteristics of reading LD. Of those three academic deficits, reading is the most common type of LD as 80% of students with LD have reading difficulties (Heward, 2013). A reading LD may also be referred to dyslexia (Heward, 2013; Siegel & Mazabel, 2013). The International Dyslexia Association and the National Institute of Child Health and Human Development (NICHD) define dyslexia as:

> a specific learning disability that is neurological in origin. It is characterized by difficulties with accurate and/or fluent word recognition and by poor spelling and decoding abilities. These difficulties typically result from a deficit in the phonological component of language that is often unexpected in relation to other cognitive abilities and the provision of effective classroom instruction. Secondary consequences may include problems in reading comprehension and reduced reading experience that can impede the growth of vocabulary and background knowledge. (IDA, 2002; p. 1)

Students with an LD in reading have difficulty with phonological processing, syntactic processing, working memory, semantic processing, morphological awareness, and orthographic processing (Siegel & Mazabel, 2013). Of those processes, phonological processing has the greatest effect on reading ability, as it relates to a student's ability to associate sounds to letters (Siegel & Mazabel, 2013). Academically, students with a reading LD have difficulty with phonological awareness, decoding individual words, word meaning, comprehension, and fluency (Heward, 2013; Siegel & Mazabel, 2013).

Characteristics of written expression LD. Written expression is another type of LD, although not as common as reading. LD in written expression affects approximately 10% of school-aged students (Fletcher et al., 2013). The term "dysgraphia" is often used when discussing LDs in written expression. Specifically, dysgraphia refers to difficulty in the visual and motor processing skills of handwriting and copying text (Fletcher et al., 2013). Students with LD in written expression have difficulty with language, planning, writing ideas, and reviewing text (Graham, Harris, & McKeown, 2013; Heward, 2013). These difficulties can impede students' skills in handwriting, punctuation, vocabulary, spelling, grammar, and expository writing (Heward, 2013).

Characteristics of math LD. Academically, students with math LD typically have difficulties with computation and word-problem solving skills (Fletcher et al., 2006; Vukovic & Sigel, 2010). Students with math LD also have common cognitive characteristics. Research has found these students may have deficits in working memory, processing speed, mathematics background knowledge, phonological processing/language, and possibly visual-spatial abilities (Fletcher et al., 2006; Geary, 2013; Vukovic & Siegel, 2010). Working memory includes the ability to hold on to information while completing a separate task (Geary, 2013). Processing speed is the ability to quickly process new information (Vukovic & Siegel, 2010). Mathematics background knowledge refers to a student's understanding of basic mathematics principles (e.g., counting, number sense; Vukovic & Siegel, 2010). Phonological processing is the ability to understand the structure of language and impacts reading development (Vukovic & Siegel, 2010). Visual spatial ability is processing and analyzing visual-spatial information (Vukovic & Siegel, 2010). Currently, research on visual-spatial ability and its effect on mathematics ability is limited (Geary, 2013; Vukovic & Siegel, 2010). These deficits negatively affect students' abilities to solve computation problems and math word problems.

Characteristics of students with math LD in relation to math word-problem solving. Solving word problems involves the multiple processes of (a) computation, (b) language, (c) reasoning, (d) reading skills, and (e) possibly visual-spatial skills (Geary, 1993). In order to effectively complete math problems, a student must be able to pay attention to the task, alternate between different types of concepts, organize the information, and work efficiently enough to use working memory skills in order to keep from overburdening it with stored information (Fletcher et al., 2006). Completing math word problems also requires students to use language skills to build a model for the problem based on information from the text (Fletcher et al., 2006).

Students with math LD who struggle to follow those processes to solve math word-problems have similar characteristics. Fuchs et al. (2006a) conducted a study to determine which cognitive correlates impact skills in arithmetic, algorithmic computation, and arithmetic word problems. Standardized assessments were administered to 330 third graders on language, nonverbal problem solving, concept formation, processing speed, long-term memory, working memory, attentive behavior, phonological decoding, reading, arithmetic, algorithmic computation, and arithmetic word problems. Results showed performance on arithmetic measures were linked to performance on algorithmic computation and arithmetic word problem measures, but algorithmic computation skills were not linked to performance on arithmetic word problem measures. Specifically related to word problem solving, results indicated attentive behavior, nonverbal problem solving, concept formation, sight word efficiency, and language were correlated to performance on arithmetic word problems. In other words, students who performed poorly on those measures also performed poorly on solving arithmetic word problems.

Problems students with LD encounter. Students with disabilities are not faring as well as their peers without disabilities in high school. According to the NLTS-2 results (2011), the mean GPA (of graded courses) for high school students with disabilities was 2.2 on a 4.0 scale, whereas the mean was 2.7 for non-disabled peers. While in high school, 47.3% of students without disabilities failed at least one graded course and 66.4% of students with disabilities failed at least one graded course. More specifically, 69.1% of students with LD failed at least one graded course (NLTS-2, 2011). These findings indicate not only do students with disabilities have a lower GPA and fail more classes than their peers without disabilities, but also students with LD are more likely to fail courses than all students with disabilities. Subsequently, students with disabilities are twice as likely to drop out of school as students without disabilities (Chapman et al., 2011). When students with disabilities have lower GPAs, fail in school, and higher dropout rates, it is not a surprise that they would continue to encounter difficulties upon entering the workforce.

With the growth of technology, much of our workforce requirements have changed. Specifically, students who struggle with math or have a math LD will be at a disadvantage when entering the workforce (Geary, 2013). These students will also struggle with daily life functions that involve math skills (e.g., money, time, budgets, distance; Geary, 2013). Over the years, students with LD may struggle at school, home, and eventually work, which could lead to a lifetime of frustration. Fortunately, new research is published every year to provide further insights into assisting students with math LD to achieve grade level standards.

## Students at Risk for Learning Disabilities

Originally, the analysis report on the National Education Longitudinal study of 1988 (NCES, 1992) stated that "an at-risk student" was typically identified by his or her likelihood of failing school, and that school failure was generally determined after a student dropped out of school. The authors then expressed that analyzing characteristics of students who dropped out after they dropped out was too limiting, and basing school failure on whether or not someone dropped out was too restricting. Over two decades ago, they began to change the perceptions of at-risk status by expanding their specific definition to include eighth grade students who "failed to achieve basic proficiency in mathematics or reading, or had dropped out of school altogether" (NCES, 1992, p. 2).

Since then, the term "at-risk" has broadened to also include students at risk for identification of LD, students with diverse learning needs (Coyne, Kame'enui, & Carnine, 2011), and students with borderline intelligence (Shaw, 2010). Common factors of students who are at risk for school failure include poverty/low socioeconomic status (SES), low English proficiency, ethnicity (Coyne et al., 2011; Schulte & Stevens, 2015), low levels of parents' education (Coyne et al., 2011), and students with below average intelligence scores (Shaw, 2010). Poverty and low SES have the highest association with risk for failure, but parents' level of education can reduce that effect (Coyne et al., 2011).

Identification methods of students at risk vary based on factors such as age, disability status, and school policies. Often, students who are evaluated for LD using the discrepancy model but do not qualify are considered students at risk, or "slow learners" (Shaw, 2011, p. 12). These students typically have below average IQ scores (between 99-70) with academic achievement scores similar to their IQ scores. Another method for identifying students at risk is to use cutoff scores on normed assessments. Typically, cutoff scores are set at the 10th percentile (similar to prevalence of math LD) or 25th-35th percentile (as commonly used in research on math LD and low achievers; Gersten et al., 2005a; Murphy, Mazzocco, Hanich, & Early, 2007). In order to bring more insight to differentiations within cutoff scores, Murphy et al., (2007) evaluated differences in characteristics of students with math difficulties based on cutoff scores of below the 10th percentile (MD 10) and 11th-25th percentile (MD 11-25) on the Test of Early Math Ability-second edition (TEMA-2) over a 4-year longitudinal study. The authors found that students in the MD-10 and MD-11-25 groups (without a math intervention) were unable to "catch-up" to their peers (above 25th percentile) when receiving typical math instruction over the course of 4 years. Students in MD-10 group's TEMA-2 scores plateaued in second grade and remained at that same level the following year. Specifically, there were no differences in visual-spatial ability between MD-10 group and MD-11-25 group until the final year in third grade, students in MD-11-15 group had poor decoding skills, while students in MD-10 had poor decoding and poor fluency skills (both groups were below non MD students), and both MD-11-25 groups and MD-10 groups had difficulty with working memory skills (MD-11-25 groups took longer to answer, but made fewer errors than MD-10 groups). Overall, differences were identified between the MD-11-25 group and MD-10, but no patterns of poor performance on specific skills were identified as reasons for those differences.

Common characteristics of students who are at risk for school failure include poor vocabulary skills (Coyne et al., 2011; Hart & Risely, 2009), difficulty retaining and generalizing information (Coyne et al., 2011; Shaw, 2010), language-based problems (Coyne et al., 2011), lack of or inefficient use of learning strategies (Coyne et al., 2011), and difficulty organizing new material into previously learned knowledge (Shaw, 2010). Specifically in math, one study indicated students at risk for math disabilities (e.g., low achievers), were found to struggle with fluency in processing number sets, estimation using number lines, and the retrieval speed of addition facts (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). Students who are at risk for math disabilities benefit from explicit instruction (Coyne et al., 2011; Hughes, Witzel, Riccomini, Fries, & Kanyongo, 2014), instruction using concrete-representational-abstract techniques (Hinton, Strozier, & Flores, 2014; Hughes et al., 2014), cognitive/ model-based interventions (instruction on conceptualizing word problems to determine how to solve the problem) with explicit instruction (Hughes et al., 2014), and for solving word problems, computer assisted instruction, representational techniques, and strategy training produced moderate to large effect sizes (Xin & Jitendra, 1999). Over the years, students at risk for and identified with LD may struggle at school, home, and eventually work, which could lead to a lifetime of frustration. Luckily, the field of special education has been moving towards providing interventions in the early years of school in order to prevent failure.

## The Need for Early Intervention

It has become well known that early intervention promotes better outcomes for students with LD. Another pitfall of the ability-achievement discrepancy model was that students often had to “wait to fail” (typically until third grade) before they would qualify with an LD, but researchers have determined early intervention provides better results (Scruggs & Mastropieri, 2002). Therefore, the case was made to move the focus from remediation to prevention and early intervention (Scruggs & Mastropieri, 2002).

In support of this concept, Vellutino et al. (1996) evaluated the effects of an intensive reading intervention on the reading ability of 118 first graders identified as poor readers. Results indicated after the first semester of daily tutoring, 67% of the poor readers scored in the average or above average range on standardized reading assessments. These findings supported the idea that intervening early in reading is an effective method for improving reading achievement.

Similar to students at risk for reading LD, students at risk for math LD will continue to perform below grade level without the use of interventions and preventative practices in math (B. Bryant et al., 2008). Early reading instruction and early numeracy instruction are comparable in that students may not develop necessary skills to be proficient in reading or math without early identification, intervention, and progress monitoring (D. Bryant et al., 2008a). These findings support the case for the implementation of response to intervention (RTI), which includes screenings, early intervention with research-based interventions, in order to promote academic achievement for students with LD or who are at risk of academic failure.

## Summary

The history of LD has been a long journey from conceptualization to application and controversies to necessary changes. The field of LD uses applied research to determine how to meet the needs of students with LD in reading, writing, and math. Students with LD struggle in schools with academics, social skills, attention, and self-esteem. Students at risk for math LD also struggle with academics including generalizing material, fluency, and retrieval of basic facts. The earlier schools intervene on behalf of students, the more growth students make. In the next section, further analysis of interventions for students in math will be reviewed within the context of RTI.

## Response to Intervention in Math

### Definition of RTI

Response to Intervention (RTI) is a multi-tiered system of supports for the prevention and identification of students with LD (NCRTI, 2010). RTI provides universal screenings to all students in a school to identify students who are at risk for LD. At-risk students in Tier 1 receive research-based practices through core instruction in general education and are progress monitored using curriculum-based measurements (CBM) to determine if they are responding to core instruction. Using data-based decision making, if students are not making enough progress, they begin Tier 2 services. Students in Tier 2 receive small group instruction using research-based and evidence-based practices (NCRTI, 2010). The same process is followed at Tier 2 with progress monitoring and data-based decision making for moving students into Tier 3. Instruction at Tier 3 includes intensive intervention using evidence-based and research-based practices in either general or special education depending on the model used (NCRTI, 2010).

RTI is based on the premise that early intervention services can provide necessary support for students who are at risk for LD and need intensive instruction in general education in order to avoid being mislabeled as LD. RTI is also a means for identifying students with LD and providing special education services in Tier 3 (NCRTI, 2010). Due to the various RTI models, the National Center on RTI (NCRTI; 2010) recommended using the terms primary prevention (i.e., Tier 1), secondary prevention (i.e., Tier 2), and tertiary prevention (i.e., Tier 3) when describing the RTI tiers. NCRTI (2010) also expressed despite the number of tiers of instruction, the intensity of the interventions should be aligned with primary, secondary, and tertiary support, but cautioned that the more tiers of instruction used, the more complicated the process becomes.

Currently, more research exists on examining the effects of using RTI on reading achievement than on math achievement (Fuchs, et al., 2004a; Fuchs et al. 2007b; Gersten, Jordan, & Flojo, 2005a). Most of the studies investigating RTI on math achievement focused on improving skills with basic facts and computation using drill and practice. Although the research provided promising results, future research needs to be conducted in other areas of math in order to fully evaluate the effects of RTI on math achievement (Fuchs et al. 2007b). The following sections will address research conducted in the primary, secondary, and tertiary levels, and on assessment procedures. Due to the varying models of RTI (e.g., three-tiered RTI, four-tiered RTI) and its relation to special education, this review will consider Tier 3 synonymous with special education. In order to maintain focus on RTI, only studies that mention RTI, tertiary support, and/or Tier 3 support will be reviewed. Studies that were conducted in a special education setting that do not mention RTI or Tier 3 will be excluded from this review.

## Primary Level: Tier 1

Components of Tier 1 instruction. Effective Tier 1 practices provide the least intensive instruction using evidence-based or research-based core curricula for teaching all students (B. Bryant et al. 2008; NCRTI, 2010). The expectation is that all students will benefit from core curricula that use research-based instructional design principles (e.g., big ideas, explicit instruction, mediated scaffolding; B. Bryant et al. 2008). In order to determine that a curriculum is meeting the needs of students, Tier 1 instruction should be effective for 80% of the student body. For this reason, Tier 1 is known as the primary prevention level (NCRTI, 2010). Tier 1 should also provide culturally and linguistically responsive instruction, differentiated activities as needed to address individual needs in general education, accommodations for students to access the core curriculum, and universal screenings in order to determine present levels of performance for individual students and the student body (NCRTI, 2010). Based on universal screening results, students who are unresponsive to Tier 1 instruction become eligible to receive more intense instruction at Tier 2 (NCRTI, 2010).

Research supporting Tier 1 math instruction. The first three of the following five studies evaluated the effectiveness of Tier 1 math instruction on student math achievement. In contrast, the final two studies evaluated specific curricula for the inclusion of instructional design principles and other effective teaching practices.

First, Fuchs et al. (2004a) evaluated the effects of a problem solving treatment during Tier 1 math instruction on math achievement scores of 201 third grade students identified as not at risk for disabilities (ND), at risk for math disabilities (MD), at risk for reading disabilities (RD), and at risk for math and reading disabilities (MRD). Using a randomized control trial design, teachers were randomly assigned to treatment and control conditions. Treatment included 3 weeks of whole class explicit instruction on basic problem solving skills, then 12 weeks of explicit instruction to teach transfer skills (relating novel problems to skills learned) with self-regulation (self-monitoring strategies). Results demonstrated a statistically significant interaction effect between disability risk group, treatment, and pretest posttest scores on all math assessments.

Further analysis found all groups in treatment made statistically significant improvements in their scores ($p < .05$). Students who were not at risk for disabilities achieved statistically significantly higher problem-solving scores than students at risk for disabilities ($p < .05$). More specifically, students with ND, MD, and RD scored statistically significantly higher than students with MRD ($p < .05$). Students with MD improved the same amount as students with ND in understanding, reducing the achievement gap. This study demonstrated that students who are at risk for MD are capable of similar achievement levels in problem solving as their ND peers when provided explicit instruction and worked examples; however, students at risk for MRD may require even more explicit instruction with problem solving skills.

In order to gain a better understanding of how features of a curriculum affect student achievement, Agodini and Harris (2009) conducted a study to determine the relative effects of four commercial math curricula on math achievement scores from the Early Childhood Longitudinal Study-Kindergarten Class of 1998-99 (ECLS-K) assessment for 1,330 first grade at-risk students in 131 classrooms within 39 schools. The study evaluated Investigations (Russell et al., 2006) a curriculum that uses a student-centered approach, Math Expressions (Fuson, 2006) a curriculum using teacher-directed and student-centered approaches, Saxon Math (Larson, 2004) a curriculum that uses teacher-directed approach with explicit instruction and hands-on activities, and Scott Foresman-Addison Wesley Mathematics (Charles et al. 2005; SFAW), a basal curriculum that uses a teacher-directed approach. Using pre-test posttest group design with random assignment of curricula, findings indicated students who received Math Expressions and Saxon math as their core math program had statistically significant higher scores at 0.30

SDs above students in Investigations and 0.24 SDs above students in SFAW. This implies students who received Math Expressions and Saxon as their math instruction could potentially score 9 to 12 percentile points higher than students receiving Investigations or SFAW math instruction. These results indicated the choice of core curricula in Tier 1 had an effect on first grade students' math improvement and that teacher directed programs with either effective, efficient procedures or explicit instruction had a greater impact on student achievement.

Extending the knowledge base of Tier 1 instruction in math, Clarke et al. (2011) conducted a study to determine the effects of Early Learning Mathematics (ELM), a Tier 1 curriculum with built-in research-based strategies including systematic instruction, explicit instruction, and instructional design principles (e.g., scaffolding, judicious review, strategic integration) for students at risk, on math achievement scores of over 1,300 kindergarten students in general education. Using a randomized control trial, results indicated at-risk students in treatment made statistically significant growth on the Test of Early Math Assessment (TEMA) scores ($t(61) = 3.29$, $p = .0017$) and Early Numeracy Curriculum Based Measurement (EN-CBM) scores ($t(61) = 2.54$, $p = .0138$) when compared to students at risk in the control group receiving typical daily math instruction. Results also determined treatment students who were not at risk for math difficulties did not make statistically significant gains on TEMA scores ($t(61) = 0.05$, $p = .9586$) and EN-CBM scores ($t(61) = -0.05$, $p = .9570$) when compared to students who were not at risk in the control group. During pretest and posttest measures, students were categorized as not at risk or at risk based on scores falling above or below the 40th percentile. A statistically significant difference was found between at-risk students during pre-test receiving ELM who moved into the not at-risk category after posttest, and at-risk students in control classrooms who moved into the not at-risk category ($\chi^2(1) = 5.96$, $p = .0155$), effectively reducing the achievement gap. This study demonstrated ELM, a Tier 1 curriculum that covered critical math content and used research-based strategies, yielded higher math achievement scores for students at risk than conventional math programs and successfully reduced the achievement gap for students at risk.

As demonstrated in the aforementioned studies, Tier 1 instruction is more effective for students struggling in math when core curricula include effective teaching practices (e.g., explicit instruction, teacher-directed instruction, instructional design principles; Agodini & Harris 2009; B. Bryant et al., 2008; Clarke et al., 2011; Fuchs et al., 2004a). In order to evaluate the presence of critical features of instruction within curricula, B. Bryant et al. (2008) reviewed four of the most common kindergarten, first, and second grade basal texts in the state of Texas for the 2004-2005 school year for the inclusion of critical features of instruction. Specifically critical features of instruction included (a) clarity of objective, (b) additional skills/concepts taught, (c) use of manipulatives and representation, (d) instructional approach, (e) provision of teacher examples, (f) adequate practice opportunities, (g) review of prerequisite mathematics skills, (h) error correction and corrective feedback, (i) vocabulary, (j) strategies, and (k) progress monitoring. Basal textbooks were rated on a scale of one to three. A rating of one (i.e., unacceptable) indicated an absence of the critical feature, a rating of two (i.e., approaching acceptable) indicated the critical feature was included but did not meet the acceptable criteria, and a rating of three (i.e., acceptable) indicated the critical feature was consistently included. Results of the evaluation indicated an acceptable rating for one third of critical features in kindergarten lessons, over one fourth of the critical features in first grade lessons, and 16% of the critical features in second grade lessons. Specifically, “strategies” was rated as unacceptable across all grade levels in all basal texts; in kindergarten, “additional skills/concepts taught” and “review of prerequisite skills” were rated as unacceptable in all basal texts; and in second grade, “error correction and corrective feedback” and “progress monitoring” were rated as unacceptable in all basal texts. All other features included varied ratings based on basal text and grade level. Based on these results, the researchers suggested that publishers strive to include more critical features of instruction in order to benefit students who may be struggling in mathematics. They also stressed the importance of having a strong core curriculum in Tier 1 that teachers could use with minimal preparation time.

More recently, Doabler, Fien, Nelson-Walker, and Baker (2012) extended the research on evaluating commercial math programs for the presence of instructional design principles. The researchers evaluated three math programs from the mathematics adoption list in California and Oregon. The first two programs were top sellers on the mathematics market. The last program was a newer curriculum to the United States, but a national curriculum from one of the top performing countries on the Trends in International Math and Science Study (TIMSS). The reviewers chose three topics from second grade textbooks: place value, addition with renaming, and telling time; and three topics from fourth grade textbooks: converting decimals to fractions, multiplication of multidigit numbers, and estimating and measuring area. The textbooks were reviewed for the presence of eight research-based principles of instruction including (a) preteaching of prerequisite skills, (b) teaching of math vocabulary, (c) explicit instruction, (d) selection of instructional examples, (e) math models to build conceptual understanding, (f) multiple and varied practice and review opportunities, (g) teacher-provided academic feedback, and (h) formative feedback loops. Programs were evaluated on a scale of one to four. A rating of one indicated the item was not included in the textbook, a rating of two indicated the item was present less than 50% of the time, a rating of three (i.e., minimal acceptable score) indicated the item was inconsistently applied, and a rating of four (i.e., highest acceptable score) indicated the item met full criteria for the principle. Results found 44% of the summary scores (i.e., average score of all three raters per principle per topic) in the second grade textbook met the minimal accepted score and 33% of the summary scores in the fourth grade textbook met the minimal accepted score. In the second grade textbooks the following principles were found acceptable across all topics: (a) math vocabulary (Program C), (b) explicit instruction (Program C), (c) instructional examples (Programs B and C), and (d) math models (Programs A, B, and C). The remaining four principles did not meet an acceptable score across all topics. In fourth grade textbooks the following principles were found acceptable across all topics: (a) prerequisite skills (Program B), (b) math vocabulary (Program A), (c) math models (Program C), and (d) academic feedback (Program B). The remaining four principles did not meet an acceptable score across all topics. The reviewers stressed the importance of publishers including instructional principles in commercial programs. Due to the onset of the Common Core, they indicated the need for programs to include evidence-based instructional design principles that assist students who struggle with learning math concepts.

Summary of Tier 1 instruction. Research conducted within the guise of Tier 1 has found that including effective teaching methods (e.g., explicit instruction, instructional design principles) within core curricula results in positive student achievement for students struggling in math (Agodini & Harris, 2009; Clark et al., 2011; Fuchs et al., 2004a). Unfortunately, multiple reviews of commercial programs have found that not all programs include these research-based methods (B. Bryant et al., 2008; Doabler et al., 2012). Even though research supports explicit instruction and instructional design principles for teaching math concepts, it appears that publishing companies are not integrating these methods into their curricula. Students in Tier 1 could benefit from the use of these instructional strategies and teachers could easily apply the strategies if they were included in the commercial math programs.

Secondary Level: Tier 2

Components of Tier 2 instruction. Within Tier 2 interventions (i.e., secondary prevention), the intensity of instruction is amplified and students at risk for LD in math typically receive research-based instruction in small groups (NCRTI, 2010). The National Center for RTI (NCRTI, 2010) recommends implementing small group sessions lasting 20 to 40 min, three to four times a week, for 10 to 15 weeks. The small groups should be adult-led and use an evidence-based intervention implemented with fidelity (NCRTI, 2010). Most students who were unresponsive to Tier 1 instruction will make appropriate progress with instruction in Tier 2. Their progress is monitored to determine the effectiveness of the intervention at Tier 2 or their need for even more intensive instruction at Tier 3 (NCRTI, 2010).

Research supporting Tier 2 math instruction. The following articles discuss the effects of different types of intensive small group math instruction in Tier 2 on math achievement of students at risk for math LD.

At the onset of IDEA (2004), the policy supporting RTI, Fuchs et al. (2005) evaluated the effects of Tier 2 math tutoring in number sense and math fact fluency on 561 first grade students' math achievement as measured by Woodcock Johnson-III (WJ-III) Computation, First-Grade Concepts/Application, and basic Story Problems test. Using a randomized control trial design, at-risk students performing in the bottom 25th percentile were randomly assigned to control or treatment in Tier 2 tutoring. The tutoring intervention included instruction of 17 topics on number sense and math fluency (e.g., identifying and writing numbers, sequencing numbers, place value, addition facts) through scripted lessons using the concrete-representation-abstract model. Instruction was provided three times a week to students in groups of two or three for 30 min on the topics and 10 min using a computer software (i.e., Math Flash) to improve math fact fluency. Results indicated at-risk students in treatment performed statistically significantly higher than at-risk students in control (ES = 0.57, p = 0.01) as well as students not at risk (ES = 0.61, p = 0.01) on the WJ-III Calculation assessment. Overall, at-risk students receiving intervention scored higher on all posttest assessments when compared to their at-risk control peers, but below their not at-risk peers. Although at-risk students in treatment did not close the achievement gap with pretest scores 0.60 to 1.48 SDs below peers not at risk and posttest scores 0.41 to 1.35 below students not at risk, the gap did slightly decrease. Unfortunately, the achievement gap widened for at-risk students in control with posttest scores 0.70 to 2.04 SDs below their at-risk peers. These findings support the use of a Tier 2 tutoring program for increasing math achievement for first grade students at risk for math disabilities.

In order to further the research on aligning Tier 1 and Tier 2 instruction, Fuchs et al. (2008) conducted a study to determine the effects of Schema-Broadening (SB) instruction on problem solving measures of 1,141 third graders. Using a randomized control trial design, classrooms were randomly assigned to treatment (Classroom SB) or control (Classroom control). Classroom SB included 3 weeks of general problem solving instruction (designed by researchers) and 13 weeks of SB instruction. SB instruction focused on teaching students to solve word problems by understanding the problem type, identifying the schema based on problem type, solving the problem, and transferring those skills to novel problems with previously learned schemas. Classroom control instruction included 3 weeks of general problem solving instruction (designed by researchers) and 13 weeks of instruction designed by the teacher. After the first 16 weeks, all students were assessed, students with poor math problem solving skills were identified, and identified students were randomly assigned to SB tutoring or no tutoring. Results indicated students at risk receiving SB tutoring aligned with Classroom SB were statistically significantly more effective than students at risk receiving SB tutoring unaligned with traditional classroom instruction (Classroom control). This demonstrated students who were at risk made higher gains in problem solving in math when provided two tiers of instruction. The first tier was classroom instruction incorporating SB instruction (Classroom SB) and the second tier was tutoring with SB instruction (SB tutoring) using the same math concepts being taught during whole class instruction. More importantly, when comparing students at risk and students not at risk who were in

Classroom SB groups without SB tutoring, the achievement gap widened for students at risk by 2.69 standard errors of measurement. Conversely, in the SB tutoring aligned with Classroom SB group, students at risk reduced the achievement gap from their peers not at risk by 1.34 standard deviations. These findings illustrate the importance of including two tiers of aligned problem solving math instruction for third graders.

Providing more insight to Tier 2 instruction, Bryant et al. (2008a) examined the effects of supplemental Tier 2 booster sessions aligned with core math instruction on number, operation, and quantitative reasoning performance of 266 first and second grade students. Core math instruction included 45-60 min lessons on specific weekly skills and supplemental booster sessions were conducted for 15 min three to four times a week. Content for Tier 2 booster sessions focused on number, operation, and quantitative reasoning using explicit, systematic instruction and concrete-semiconcrete-abstract approach. Booster sessions were aligned to core math content and modified if needed (e.g., core math focused on numbers 0-99, booster session focused on numbers 10-20). Student progress was assessed using the Texas Early Mathematics Inventories-Progress Monitoring (TEMI-PM) with the following subscales: (a) magnitude comparisons (MC) identifying the smaller of two numbers, (b) number sequences (NS) identifying the missing numeral in a series of three numbers, (c) place value (PV) stating the amount displayed in a picture of ones and tens blocks, and (d) addition/subtraction combinations (ASCs) solving addition and subtraction facts up to 18. Using regression-discontinuity design, results demonstrated second grade students in treatment had a statistically significant increase in posttest scores on the total standard score of the (TEMI-PM) indicating a main effect ($b = .19$, $p = .018$) for the intervention. First grade students in treatment improved, but not at the statistically significant level. One explanation for the difference in continuity between first and second grade gains was at-risk first graders may need to spend more time in intervention to gain a better understanding of number sense. By second grade, the intervention time was sufficient enough to produce statistically significant gains in number sense for students at risk.

In response to the findings of the previous study (Bryant et al., 2008a), Bryant and colleagues made changes to the intervention and conducted a regression-discontinuity design to evaluate the effects of a Tier 2 math intervention on 161 first graders' acquisition of early numeracy skills the following year (Bryant et al., 2008b). Forty-two participants who scored at or below the $25^{th}$ percentile on total score of the TEMI-PM were eligible for the Tier 2 intervention. The remaining students were not eligible to receive Tier 2 intervention and served as the comparison group using a regression-discontinuity design. In groups of four to five, students received 23 weeks of 20 min sessions, 4 days a week. Lessons were taught using explicit, systematic instruction focused on early numeracy skills, place value, and addition/subtraction combinations. Students were administered a pretest and a posttest using the TEMI-PM. Results indicated a significant main effect ($p = .014$) on the total score was found with a positive effect on the intervention. In subcategories, a program effect was found for Number Sequences and Addition/Subtraction subtests ($p = .048$, $p = .029$ respectively), while Magnitude Comparison demonstrated significant interaction effect ($p = .028$). Results on Place Value did not show any significant effects. Suggestions for future research included evaluating intensive instruction at Tier 3 for students who did not respond to Tier 2 interventions.

In a subsequent study, Bryant et al. (2011) evaluated the effects of an early numeracy intervention program on 224 first grade students' early numeracy skills using a pretest-posttest control group design. Specifically, students were assessed using the TEMI-PM with the following subscales: (a) magnitude comparisons (MC) identifying the smaller of two numbers, (b) number sequences (NS) identifying the missing numeral in a series of three numbers, (c) place value (PV) stating the amount displayed in a picture of ones and tens blocks, and (d) addition/subtraction combinations (ASCs) solving addition and subtraction facts up to 18. Outcome measures were evaluated using the Stanford Achievement Test-Tenth Edition (SAT-10) on math problem solving (MPS) and math procedures (MP) and Texas Early Mathematics Inventories Outcome (TEMI-O) on math problem solving (MPS) and math computation (MC). The intervention was conducted 25 min per day, 4 days per week, for 19 weeks. Each intervention session included a warm-up for 3 min and two 10-min scripted lessons on number and operation concepts and skills using systematic instruction. Results of the study found statistically significant difference on the following skills: NS ($p = <.00001$), PV ($p = <.002$), ASC ($p = <.0001$), and total score ($p = <.01$) with the treatment group outperforming the control group. In regard to MC, no difference was found between the two groups ($p = .16$). Outcome measures found statistically significant differences in favor of the treatment group on the TEMI-O MC ($p = .001$), TEMI-O total score ($p = .05$), and on the SAT-10 MP ($p = .05$). Results did not find statistically significant differences between groups on the TEMI-O PS ($p = .99$), SAT-10 PS ($p = .32$), and SAT total score ($p = .14$). Although problem solving was assessed as an outcome measure, the intervention did not include explicit instruction on problem solving. The researchers hypothesized the effects of the Tier 2 intervention were statistically significant for first graders during this study due to the design of the intervention, systematic instruction, problem construction, visual representations, consistent practice and review, and length of the intervention sessions. They also examined the at-risk status of students by the end of the year and found 45% of students in the treatment group and 22% of students in the control group who were considered at risk at the onset of the study were no longer considered at risk for mathematics difficulties based on results from the TEMI-PM. These students no longer required Tier 2 support. The researchers stressed the importance of further research addressing the long-term effects of Tier 2 support for these same students in subsequent grades.

Summary of Tier 2 instruction. Results from research on the effects of Tier 2 instruction had a positive impact on student achievement (Bryant et al., 2008a; Bryant et al., 2008b; Bryant et al., 2011; Fuchs et al., 2005; Fuchs et al., 2008a). Research conducted by Fuchs (2005) demonstrated the importance of aligning instruction in Tiers 1 and 2 in order to decrease the achievement gap between at-risk students and students on grade level. Research conducted by Bryant (2008a, 2008b, 2011) found second graders saw gains with less time in intervention (e.g., 15 min), than first graders who demonstrated greater gains with more time in intervention (e.g., 25 min). Overall, strong Tier 2 instruction has the capability of improving at-risk students' math skills with the possibility of decreasing the achievement gap.

## Tertiary Level: Tier 3

Components of Tier 3 instruction. Tier 3 is also known as the tertiary level and provides the most intensive instruction in a three-tiered model of RTI (NCRTI, 2010). A smaller portion of students who, based on progress monitoring data, do not respond to Tier 1 or Tier 2 instruction become eligible for intensive instruction at Tier 3. These students continue to receive research-based or evidence-based interventions but more frequently, in smaller groups, and for longer periods of time than what they received in Tier 2. Progress continues to be monitored and the intervention is adjusted based on the student's progress (NCRTI, 2010). In some models of RTI and for the purpose of this dissertation, Tier 3 is synonymous with special education.

Research supporting Tier 3 math instruction. Over the years, researchers in the field of special education have compiled a wealth of research on educational interventions supporting students with mild disabilities. Researchers have found students with mild disabilities benefit from the following instructional methods and interventions: (a) Direct Instruction- an instructional method that efficiently and effectively teaches new content through specific program design, organization of instruction, and student-teacher interactions (Marchand-Martella, Slocum, & Martella, 2004; Przychodzin, Marchand-Martella, Martella, & Azim, 2004); (b) learning strategies- general techniques, strategies, and rules that assist students in learning, problem solving, and working independently (Deshler & Schumaker, 1986); (c) explicit instruction- an approach to teaching that provides instruction in small steps using scaffolds that makes each step/concept clear with consistent assessment of student understanding (a review of literature is provided later in this chapter; Archer & Hughes, 2011; Gersten et al., 2009b); (d) peer assisted learning strategies- a type of class-wide peer tutoring that pairs students together based on skill level to practice academic skills with each other (Calhoon & Fuchs, 2003; Fuchs, Fuchs, & Karns, 2001); (e) mnemonics- versatile strategies used to enhance memorization of various types of information and include using keyword mnemonics, pegword mnemonics, and letter strategies (Scruggs & Mastropieri, 2000; Scruggs, Mastropieri, Berkeley, & Marshak, 2010); (f) instructional design principles- features of instruction that focus on big ideas, conspicuous strategies, mediated scaffolding, strategic integration, primed background knowledge, and judicious review (Coyne, Kame'enui, & Carnine, 2011); (g) cognitive strategy instruction- a strategy that teaches math problem solving using cognitive processes (e.g., visualizations) and metacognitive strategies (e.g., self-questioning) through a consistent routine (a review of literature is provided later in this chapter; Jitendra & Montague, 2013); (h) schema-based instruction- a strategy that teaches math problem solving through determining schema types and representing the problem through a diagram (a review of literature is provided later in this chapter; Jitendra & Montague, 2013); and (i) data-based individualization- a method for individual (i.e., one-on-one) instruction by providing intensive interventions, monitoring progress, and adjusting specific components of the intervention based on the student's progress (Fuchs, Fuchs, & Stecker, 2010; Fuchs, Fuchs, & Vaughn, 2014). Research also supports specialized instruction as opposed to solely inclusive instruction for students with LD or who need Tier 3 supports (Fuchs et al., 2015). The use of these interventions and educational strategies are supported by research and appropriate for students in Tier 3. This study will use explicit instruction, components of instructional design (e.g., conspicuous strategies, strategic integration), components of cognitive strategy instruction, and components of schema-based instruction as part of the intervention designed to teach students to solve math word problems.

To date, there is only one study that investigates a math intervention for students receiving Tier 3 support (and not receiving special education services) in math. Bryant et al. (2016) implemented a multiple baseline across participants design to evaluate the effects of a Tier 3 math intervention on mathematics performance of students with math difficulties. This was a follow-up study to the Bryant et al., (2008b) evaluation of a tier 2 intervention (previously reviewed in this chapter under Research supporting Tier 2 math instruction.). The following year, the team identified 12 second graders from three similar schools who scored below the $10^{th}$ percentile on the TEMI-PM (thorough description in previous section) universal screener and were considered nonresponders from the initial Tier 2 intervention in first grade (Bryant et al., 2008). Students were grouped by school to receive Tier 3 intervention and data were collected twice a week using a researcher-developed probe, Texas Early Mathematics Inventory-Aim Checks (TEMI-AC). The TEMI-AC evaluates magnitude comparisons, number sequences, place value, and addition/subtraction combinations within a 2-min fluency probe. The 10-week intervention was provided 5 days a week for 30 min to students in groups of two or three. Lessons included a warm-up, preview of lesson, modeling, guided practice, and daily check. Content of lessons included counting; comparing number magnitude, quantity, and sequencing; base 10 instruction; and number families. TEMI-AC means were computed for each group by school and visually analyzed for a functional relation between baseline and intervention phases. Results of the study demonstrated a functional relation between baseline and intervention with stable data demonstrating a change in level, trend, immediacy of effect, and nonoverlap. Means at all schools increased between baseline and intervention (School 1’s scores: baseline M= 63, intervention M= 109.8; School 2’s scores: baseline M= 40.9, intervention M= 90; School 3's scores: baseline M= 67.3, intervention M= 118.2). By the end of the intervention 9 out of 12 students were no longer eligible for Tier 3 interventions. Future research included a need for more research on Tier 3 interventions, determining how or if students receiving intensive interventions (e.g., Tier 3) differ from students with LD, generalizing interventions into core instruction, and evaluating longitudinal effects of interventions.

Summary of Tier 3 instruction. Tier 3 provides the most intensive level of instruction within an RTI model. In some models, students receiving instruction in Tier 3 have met eligibility for special education services, while in other models students who are at risk for disabilities receive instruction at Tier 3. Despite the type of model, instruction in Tier 3 should include intensive, research-based practices tailored to students' individual needs (NCRTI, 2010). Interventions can be modified through data-based individualization (Fuchs et al., 2014) and continually progress monitored (NCRTI, 2010).

## Assessment in RTI

Assessment is a critical element of RTI (Bradley et al., 2007; Stecker, Fuchs, & Fuchs, 2008). Initially, students are provided universal screenings to identify students at risk for LD in Tier 1 (Anderson, Lai, Alonzo, & Tindal, 2011; Fuchs et al., 2007b; NCRTI, 2010; Stecker et al., 2008). Screening data are also used to evaluate the effectiveness of the core curriculum at Tier 1 (NCRTI, 2010). Throughout instruction within the tiers, students' progress is monitored to determine if they are responding to the instruction (Anderson et al., 2011; Fuchs et al., 2007b; NCRTI, 2010; Stecker et al., 2008). Progress monitoring data can also be used to modify the intervention based on students' needs (NCRTI, 2010; Stecker et al., 2008). The following three articles evaluated the use of screening measures and progress monitoring tools for use in RTI implementation in math within a general education setting.

Universal screening in math. The first study addressed the ability of universal screenings to predict LD. Fuchs et al. (2007b) conducted a study to evaluate the relation between math screening assessments at the beginning of first grade and the presence of math disability (MD) at the end of second grade on 170 first graders (113 scored in the average range in math, 57 scored below average in math). The researchers used a correlational design to measure the following math screening assessments: (a) Addition and Subtraction Fact Fluency, (b) Curriculum Based Measurement (CBM), (c) Number Identification/Counting, and (d) CBM Concepts/Applications to predict MD for first grade students. Results indicated using all four assessments resulted in accurately identifying 78.2% of students with MD in calculation, with a good AUC coefficient of .847. For identifying students with MD in word problems, the assessments resulted in accurately identifying 74.7% of students with a good AUC coefficient of .806. These results were based on logical regression analysis of scores on the math screening assessments at the beginning of first grade with scores on the WRAT-Arithmetic and Story Problems at the end of second grade. Further analysis of the data indicated CBM Concepts and Applications and CBM Computation to have stronger predictive ability, with CBM Concepts and Applications as the leader of predicting both MD calculation and MD word problems.

Progress monitoring in math. In general, previous research in assessment addressed computation skills. As a result, Leh, Jitendra, Caskie, and Griffin (2007) investigated the ability of the Word Problem-Solving measure (WPS) to reveal growth over time on 77 third graders' math word problem-solving skills. Using a repeated measures design, results indicated that over time students made a statistically significant average rate of growth $F(1, 57) = 30.05$, $p < .001$ when administered biweekly. The WPS overall rate of growth (.24) was less than the CBM computation measure (.36) overall rate of growth. The authors stated one explanation for this difference could be the complexity of skills necessary to complete WPS may affect the growth of progress for word problems. Overall results of the study contributed to WPS credibility as a measure of third-grade students' word problem solving skills.

In the final study, a fifth-grade progress monitoring assessment was evaluated. Anderson et al. (2011) conducted an evaluation to determine the effects of easyCBM® number and operation assessment on measuring accessibility of grade level items based on content standards for low-performing fifth grade students ($n = 380$). Using a pretest only nonexperimental design given to students in two school districts in the fall, descriptive results indicated number and operations items difficulty levels ranged from -1.99 to 2.45 (8 items below 0, 8 above 0), well-fitting outfit statistics of .61 to 1.40, moderate point measure correlations of 0.25 to 0.46, close observed match and expected match, and measurement errors of .05 to .09, which are considered low. Results also found scores on easyCBM® were capable of being sorted by lower end ability percentile ranges of 0 to 5, 5.1 to 10, and 10.1 to 15 for each content standard related test item. This study provided preliminary support for the use of easyCBM® for identifying and progress monitoring low-performing fifth grade students.

Summary of assessment in RTI. Assessment is an integral component of the RTI process (Stecker et al., 2008). Screening identifies those who may be at risk of math difficulties and need more support (NCRTI, 2010; Stecker et al., 2008). Findings from research on screening measures indicate that CBMs for Concepts and Applications have strong predictive ability to identify students who may struggle in math and need targeted math instruction (Fuchs et al., 2007b). Progress monitoring provides consistent examination of a student's response to instruction within all tiers (NCRTI, 2010; Stecker et al., 2008). These data can then be used to determine how to modify instruction in order to intensify the intervention. Research on progress monitoring suggests number and operation CBMs can be used to rank students by percentile at the lower end of achievement (e.g., fifth, tenth, fifteenth percentiles), which assists in identifying the required intensity levels of instruction (Anderson et al., 2011). Other research supports the use of WPS CBMs to measure students' abilities to solve word problems (Leh et al., 2007). These findings contribute to the body of research on using CBMs to track student progress, which is a key component of RTI.

## Future Research

Initially, RTI focused on reading instruction and throughout the years has begun to address skills in math. Although research on RTI in mathematics has come a long way, future research is still needed in many areas. In Tier 1, as publishers continue to update math curricula more research will be needed to evaluate the new curricula for the inclusion of critical features of instruction (B. Bryant et al., 2008). Research is also needed on the effects of Tier 1 instruction on student achievement (Clarke et al., 2011), as well as the effects of Tier 2 instruction aligned with Tier 1 instruction on student achievement (Clarke et al., 2011; Fuchs et al., 2008a). Future research is also needed in Tiers 2 and 3 in order to address enhancing tiered instruction. Researchers need to determine the effects of instructional design on student achievement and manipulate variables within a program to modify the intensity of instruction, thereby increasing student achievement (B. Bryant, 2008; Fuchs et al., 2005). More research is needed in the area of problem solving including real-life problem solving (Fuchs et al., 2008a). Finally, longitudinal studies are needed to evaluate tiered instruction across multiple years. Investigations are needed to determine if students continue to have math difficulties after receiving tiered instruction in previous years (Clarke et al., 2011). Addressing each area of need is out of the scope of one study, therefore this research will primarily focus on enhancing problem solving skills, and aligning a Tier 2 intervention on problem solving with Tier 1 instruction that uses the same problem solving process.

Summary

Despite a slow beginning, the body of research supporting RTI in mathematics continues to expand to include research at Tier 1, Tier 2, Tier 3, and assessment. In Tier 1, research denotes effective core curricula include instructional design principles. Despite this understanding, many core curricula do not include effective instructional design principles. With more research and information disseminating the intent is for publishing companies to address the lack of instructional design principles in newer core curricula. Tier 2 provides strategic interventions for students who are at risk for math difficulty. Research has found providing students with sound Tier 2 interventions aligned with Tier 1 instruction benefits students at risk for math difficulties. Tier 3 is the most intensive tier within RTI and typically denotes placement in special education. A wealth of research has been conducted on effective teaching strategies for students with mild disabilities in special education. Teachers are encouraged to use these strategies in their classrooms in order to promote positive student outcomes. Finally, assessment within RTI addresses screening students and progress monitoring students. Research has found screening measures are capable of identifying students who are at risk for math difficulties. Teachers then provide strategic interventions for those students. Progress monitoring research has found that students' progress can be tracked and monitored over time to determine the effectiveness of interventions. Current research supports the use of RTI in mathematics, but further research is needed to refine procedures for students participating in RTI. Future research should address aligning tiered instruction, intensifying interventions, interventions on word-problem solving, and longitudinal studies that track students' progress based on tiered instruction.

## Word-Problem Solving

Students with disabilities or students who are at risk for disabilities have difficulty solving math word problems (Jitendra & Xin, 1997). Several meta-analyses have reviewed research on effective strategies for teaching math skills to students with disabilities. Initially, Xin and Jitendra (1999) conducted a meta-analysis to determine if instruction in word-problem solving increased students' ability to solve word problems and to determine the types of instruction that are most effective in teaching word-problem solving. The meta-analysis included 25 studies from 1960-1996 based on four types of instruction: (a) representation techniques (i.e., use of pictures/diagrams, concrete manipulatives, verbalizing procedures, and mapping/schema-based instruction); (b) strategy training (i.e., explicit heuristic procedures for problem solving); (c) computer-aided instruction (i.e., computer-based or videodisc math interventions); and (d) other (i.e., no instruction on problem solving, use of calculators, providing only attention, key word instruction or problem sequence). Results indicated that instruction in word-problem solving skills yielded strong outcomes in problem solving for students with learning problems with an overall strong effect size (ES=.89) for group studies, and positive results (PND = 89%) for single case studies when compared to studies in the “other” category. Effective strategies based on group designs included computer assisted instruction (d= 1.80), representational techniques (d= 1.77), and strategy instruction (d= .78); whereas ineffective strategies included using the key word to determine how to solve the problem and providing attention without instruction. Based on single case studies, results indicated that representational techniques (PND = 100%) were more effective than strategy instruction (PND = 87%) and long-term and intermediate-term treatments (PND = 100% and PND = 87%, respectively) were more effective than short-term treatments (PND = 49%).

Jayanthi, Gersten, and Baker (2008) conducted another review of literature on effective math strategies to teach math concepts including word-problem solving. Based on the findings, the authors compiled seven recommendations for teachers to use during math instruction. These recommendations urged teachers to consider (a) using explicit instruction (mean effect size = 1.22), (b) embedding multiple instructional examples in instruction (mean effect size = 0.82), (c) verbalizing decisions and solutions for students (mean effect size = 1.04), (d) providing visual representation (mean effect size = 0.47), (e) using multiple/heuristic strategies (mean effect size = 1.56), (f) providing feedback and ongoing assessment (mean effect size = 0.23), and (g) adopting peer-assisted instruction (mean effect size =1.02) when teaching word-problem solving. More specifically, use of heuristics was reviewed using studies that only measured students’ word-problem solving skills and yielded the highest mean effect size when compared to the other recommendations. Therefore, use of heuristics was found effective for teaching word-problem solving skills.

More recently, Zhang and Xin (2012) conducted a follow-up meta-analysis to the Xin and Jitendra (1999) meta-analysis on word-problem solving interventions. Zhang and Xin reviewed 29 group experimental studies and 10 single case studies from 1996-2009 that evaluated the use of a math instructional strategy on word-problem solving skills of K-12 students with math difficulties. Results indicated that problem structure representation techniques (e.g., schema-based strategies) were considered the most effective strategies for teaching word-problem solving. Based on a review of 16 group design studies, problem structure representation techniques yielded the strongest mean effect size of 2.637. The second most effective strategy was cognitive strategy training with a mean effect size of 1.855 based on a review of 12 studies, whereas assistive technology yielded a mean effect size of 1.218 based on a review of 20 studies. For the four single case studies that used problem structure representation techniques, PND ranged from 95% to 100%. In this follow-up meta-analysis, Zhang and Xin found 20 interventions that used problem structure representation techniques, whereas Xin and Jitendra found only one study that used the problem structure representation technique of schema-based instruction. This study also demonstrated the growth in popularity of structure representational techniques (e.g., schema-based instruction) in the past decade.

In another meta-analysis, Zheng et al. (2012) reviewed 15 studies, between 1986-2009, to determine the effectiveness of instruction in math word-problem solving on students ages 8 to 18 with math and reading disabilities. Studies included group designs, quasi-experimental designs, and single case designs experiments. General findings indicated higher outcomes for students with math disabilities (ES = 1.45) than students with math and reading disabilities (ES = .58). Results also demonstrated positive effect sizes for interventions using explicit instruction components as well as other techniques such as including objectives for the lesson, providing assistance, explaining general concepts, explicit practice, checking student understanding, modeling, cuing use of strategies, and reducing the task into smaller units. Most of the strategies reviewed included strategy instruction paired with explicit instruction and the conclusion was made that explicit instruction is an effective strategy for teaching math word problem solving.

The findings of these meta-analyses suggest that heuristics, representational techniques (i.e., schema-based instruction), cognitive strategies, and explicit instruction yielded large effect sizes when used to teach word-problem solving skills to students at risk or students with math and reading disabilities (Jayanthi et al., 2008; Xin & Jitendra, 1999; Zheng et al., 2012; Zhang & Xin, 2012). The following sections will address research behind heuristics, schema-based instruction, strategy instruction (e.g., cognitive strategies), and explicit instruction and their effectiveness on word-problem solving skills of students at risk or with mild disabilities.

### Problem-Solving Heuristics

The origins of heuristics can be traced back to mathematicians such as Pappus, Descartes, Leibniz, and Bolzano. These famous mathematicians and philosophers worked to build a heuristics system that intended to evaluate the rules and methods of discovery and invention (Polya, 1957). Modern heuristics were based on the work of these classic mathematicians and philosophers led mainly by George Polya in the mid-1940s with his book How to solve it: A new aspect of mathematical method (Polya, 1957). Polya worked to revive and further define heuristics by determining mental operations used to solve problems. He devised a four-step heuristic for all types of mathematics problems: (a) understand the problem, (b) devise a plan, (c) carry out the plan, and (d) look back. This heuristic was also designed to generalize to other subjects (Polya, 1957).

More recently, heuristics have been further defined as "a generic problem-solving guide in which the strategy (list of steps) is not problem specific (Gersten et al., 2008, p. 2)." Heuristics assist students in organizing the information and can be used for a variety of problem types. This is different from other strategies that are problem specific. When students are taught multiple heuristic strategies, they have the opportunity to decide which strategy to use to solve the problem (Gersten et al., 2009b). Heuristics are categorized into four types: (a) representations including diagrams, equations, and lists; (b) estimations including guess and check, and patterns; (c) processes including work backwards, before-after, and act it out; and (d) problem changes including rephrase the problem, simplify the problem, and solve part of the problem (Hong et al., 2010). Solving word problems requires a deeper understanding of concepts than when solving simple algorithms that require computation skills (Giordano, 1992). Students who struggle with mathematics may read a word problem and try to solve it without knowing if the algorithm they chose was appropriate. Heuristics provide students with a procedure to understand the problem and organize the information, thus reducing students' attention to the computational portion of the problem (Giordano, 1992). Heuristics have evolved over the years from Polya's (1957) general four-step plan to four types of heuristics and have found to be more effective when multiple heuristics are combined into one strategy

(Gersten et al., 2009b; Giordano, 1992). The theoretical underpinning of heuristics serves as the foundation for several problem-solving strategies, including schema-based instruction and cognitive strategy instruction.

Schema-based instruction. One highly effective, thoroughly researched strategy adapted from representational heuristics is schema-based instruction (SBI; Jitendara & Montague, 2013). Schemata are cognitive structures that are organized hierarchically and stored in long-term memory for use during problem solving. SBI uses schemas to reduce the load on working memory in order to allow students who struggle with math to become more effective at problem solving. Schemata in SBI are categorized as either additive or multiplicative structures, with change, group, and compare problems used in additive structures, and restate and vary problems used in multiplicative structures (Jitendra & Montague, 2013). Schematic diagrams are used to represent the various problem types. Use of diagrams focuses attention to the schema of a word problem by eliminating irrelevant information making it easier for students with LD to understand how to solve the problem (Jitendra & Montague, 2013). According to Jitendra (2007), SBI is taught using two phases: the problem schema instruction phase and the problem solution phase. The problem schema instruction phase uses complete word problems with only known information to allow students to identify the schema and complete the diagram. This provides students an opportunity to identify the schema and put the information together. The problem solution phase includes an unknown variable for students to solve for the unknown within the schema. SBI also uses a four-step mnemonic to guide students through the steps of SBI called FOPS, with F for finding the problem type, O for organizing the information in the problem using the diagram, P for planning to solve the problem, and S for solving the problem (Jitendra, 2007). SBI is considered a problem structure representational technique because it uses explicit instruction to teach students to use strategies such as schema-based diagrams and mathematical models (Zhang & Xin, 2012).

In order to extend the research base of SBI, Fuchs and colleagues began investigating schema-broadening instruction as another approach to SBI for teaching students to solve word problems. Schema-broadening instruction is similar to SBI but also includes explicit instruction on transfer (Fuchs et al., 2008b). Schema-broadening instruction includes teaching four components: (a) the basic structure of the problem, (b) the schema of the problem type, (c) to solve the problem, and (d) to transfer knowledge of schemas to new problems (Fuchs et al., 2008b). The following studies chronologically review SBI first, then chronologically review schema-broadening instruction.

In one of the preliminary studies on SBI, Jitendra et al. (1998) extended the work from the original single case study on SBI (Jitendra & Hoff, 1996) and used a randomized control trial to evaluate the effects of SBI on the problem solving skills of 58 elementary students in second through fifth grades. Of the 58 participants, the comparison group included 24 typically achieving students and the treatment group consisted of 34 students (i.e., 25 students had disabilities and 9 students were at risk for a math disability). The comparison group received typical classroom instruction in math and participated in the 15 single-step addition and subtraction word problem pretest and posttests only. Students in the treatment group were randomly assigned to two conditions: traditional word problem solving and schema training. All students in both treatment conditions received 40-45 min of instruction for 17 to 20 days by trained investigators.

The traditional word problem condition was based on a basal mathematics program for the first phase of instruction and the second phase included a checklist for solving word problems with five steps, including (a) focusing on understanding the question, (b) locating important data, (c) making a plan by guessing and checking, (d) completing the math problem, and (e) determining if the answer made sense. The schema training condition included two phases. Phase one involved using story situations (no missing information) for students to (a) identify different problem types (i.e., change, group, compare), (b) translate information, and (c) diagram the schema. Phase two included using story word problems for students to solve for the missing information using the same steps as phase one and delivering instruction on how to solve the problem based on the problem type. Results of the study demonstrated that students in traditional word-problem solving group and schema training scored 49% and 51% on the pretest (respectively), 65% and 77% on the posttest, and 64% and 81% on the delayed posttest. Students in the schema training condition showed 26% growth between pretest and posttest, compared to students in traditional word-problem solving group who showed 16% growth. A statistically significant main effect was found with the schema-training group outperforming the traditional word-problem solving group ($p = .02$). The schema training group's scores (77% posttest, 81% delayed posttest) also approached the control group of typically achieving students based on the third grade normative sample of 82% on the posttest. Overall, schema training was found to be an effective strategy for teaching students to solve math word problems.

In a subsequent study, a separate research team, Fuchs et al. (2004b) evaluated the effects of SBI on the problem solving skills of third grade students. Participants included

24 third grade teachers and 366 students. Teachers were randomly assigned to one of three conditions, control, SBI, and SBI plus sorting practice. Student participants received instruction in one of three conditions based on their teacher's condition. Students were categorized as low performing (LO), average performing (AP), or high performing (HI). Instruction for all conditions were provided by the researchers and included three phases of (a) district curriculum, (b) basal text, and (c) 3 weeks of instruction on generic math word-problem solving. Students in SBI and SBI with sorting received instruction on schema; students in SBI with sorting also received guided instruction in schema-based sorting (e.g., sort problems by schema- change, combine, compare). Progress was measured using a word problem pretest and posttest, and a posttest on schema development. Students receiving both types of SBI outscored students in the control group with strong effect sizes for immediate transfer 3.17, near transfer 3.65, and far transfer 1.55. No difference in effect size was found between the SBI condition and SBI with sorting condition. Results of this study contributed to the literature in support of SBI increasing student's word-problem solving skills.

In another study, Jitendra, Star, Dupuis, and Rodriguez (2013a) evaluated the effects of SBI on 1,163 seventh graders' ability to solve ratio, proportion, and percentage word problems using a randomized treatment-control, pretest-posttest group design. The study included students in general education and special education. Students in the treatment group received instruction on SBI in four steps: (a) prime problem structure; (b) map problem using visual representation of schema; (c) explicit instruction using a heuristic (DISC, D- Discover the problem type, I- Identify information in problem to represent in diagram, S- Solve the problem, C- Check the solution); and (d) explicit teaching of multiple methods to solve problems and determine more efficient methods. Students in the control group received typical math instruction based on the district-adopted math curricula. Results found students in treatment outperformed students in control scoring 1.48 points higher on a researcher-designed posttest of problem-solving involving questions on ratio/rates and percent (ES = 1.24). Results of the delayed posttest found students in treatment outperformed students in control by 1.17 points (ES = 1.27). Finally, students in treatment did not outperform students in control on a separate test to measure transfer effects. This study extended previous research and found SBI to be an effective practice for teaching problem-solving skills to students in middle school.

Recently, Jitendra et al. (2013b) conducted a randomized control trial to determine the impact of SBI on math problem solving skills of third grade students. The participants included 109 third grade students who were at risk in math over nine schools. Students were randomly assigned to treatment SBI condition (53 students) or control condition (56 students). Students in the treatment group received 30 min of SBI instruction from trained tutors. Instruction included a pre-unit on whole number combinations (e.g., addition and subtraction combinations, categories, and inverses), but mainly focused on solving one- and two-step addition and subtraction word problems using change, group, and compare problems. Students in the control condition received 30 min of tutoring session from trained tutors. Instruction included topics on place value, addition and subtraction computation, and generic word problem solving based on the strategies in their class textbook. Progress was measured using a 16-problem assessment with one- and two-step addition and subtraction word problems at pretest, posttest, and delayed posttest. Results indicated that students in the SBI condition outperformed students in the control condition. A statistically significant treatment effect was found ($p = .004$). Additionally, students were categorized by risk status (high-risk and low-risk) and results indicated SBI was effective for students in both risk levels.

As an extension of SBI, Fuchs et al. (2006b) investigated the effects of schema-broadening instruction on third-grade students' ability to solve real-life math word problems using a randomized control trial. Participants included 30 third grade teachers in seven schools with 445 students in general and special education. Teachers were randomly assigned into three groups of 10 teachers in control, schema-broadening instruction, and schema-broadening instruction with real-life problems. Students in the control group only received 3 weeks of instruction on a researcher-made general math problem-solving unit. Students in both treatment groups also received 3 weeks of the general math problem-solving unit. Then students in treatment participated in four, 3-week schema-broadening instruction units on (a) shopping list questions, (b) half problems, (c) buying bags, and (d) pictograph problems. Students in schema-broadening instruction with real-life problems also received explicit instruction in solving real-life problems. Instructional sessions lasted 25-40 min. Students were assessed using measures that included immediate transfer, near transfer, and far transfer novel word problems. Immediate transfer novel word problems were closely related to problems used during instruction, near transfer novel word problems varied from problems used during instruction with trivial features (e.g., unfamiliar vocabulary, different questions, addition of irrelevant information), and far transfer novel word problems varied from problems used during instruction in several ways (e.g., presented as a commercial test, multiparagraph narrative, information needed to answer questions were in figures). Each of the four far transfer questions were evaluated and discussed separately in the results. Results indicated students in the treatment groups accelerated at the same rate but beyond their control group peers. A statistically significant difference was found between control and treatment groups on immediate transfer ($p < .001$), near transfer ($p < .001$), far transfer question 2 ($p < .05$) and far transfer question 4 ($p < .05$). No statistically significant differences were found for far transfer question 1, and although the control group and schema-broadening instruction group demonstrated growth on far transfer question 3, the schema-broadening instruction with real-life problems group grew statistically significantly higher than the other groups. These results demonstrated the importance of providing explicit instruction on transferring math word-problem knowledge to novel problems and on real-life problems.

Subsequently, Fuchs et al. (2010) evaluated the effects of schema-broadening instruction on second-grade students' ability to solve three types of math word problems (i.e., total, change, and difference) and use algebraic expressions to represent the three different schemas. The study included typically developing students ($n = 270$) randomly assigned by teacher ($n = 18$) to treatment and control groups. Students in treatment were provided three, 4-week, 45-60 min sessions on each of the types of math word problems and explicit instruction on transfer. Results found a statistically significant difference on overall word-problem skills ($g = .46$) in favor of the treatment group. On the scores assessing difference and change, results found an effect approaching significance ($g = .34$ and $.31$ respectively). Results of the problems with missing information at the beginning of the problem and at the end of the problem were significant ($g =.42$ and $g = .56$ respectively), but problems with missing information in the middle were not significant (g =.26). When solving for “x” students in treatment outperformed their peers in control (g = .87). This study extended the support for schema-broadening instruction to include students in second grade.

Previous studies on SBI resulted in positive increases in math word-problem solving skills for students with disabilities or those who were at risk for disabilities. This research base extends across almost 2 decades of implementing SBI with elementary and middle school students and spans across multiple mathematics subjects.

Cognitive strategy instruction. Another type of highly researched heuristic is cognitive strategy instruction (CSI). CSI applies one strategy or method to a variety of problems by using cognitive processes including visual representations and metacognitive processes including questioning (Jitendra & Montague, 2013: Montague et al., 2011). CSI includes the use of simple or complex processes, strategies, and thinking skills to solve word problems. The seven cognitive processes used in CSI include (a) reading the problem, (b) putting the problem in your own words, (c) visualizing and drawing a schematic representation, (d) hypothesizing and setting up a plan, (e) estimating or predicting an answer, (f) computing the problem, and (g) checking the answer (Jitendra & Montague, 2013). There are also three self-monitoring checks (i.e., Say, Ask, Check) for students to use as they complete the problems (Jitendra & Montague, 2013; Montague et al., 2011). The intention of CSI is to teach students how to problem solve by thinking and behaving like effective problem solvers through the use of explicit instruction (Jitendra & Montague, 2013).

The original instructional model of CSI was Solve It!; it follows the seven cognitive processes in CSI: (a) read the problem, (b) put the problem in your own words,

(c) visualize and draw a schematic representation, (d) hypothesize and set up a plan, (e) estimate or predict an answer, (f) compute the problem, and (g) check the answer (Jitendra & Montague, 2013). Montague et al. (2011) conducted a cluster-randomized design to determine the effects of Solve It! on students' math problem solving skills. The study included 719 students in 40 middle schools (with 20 pairs of matched schools). One school from each pair was randomly assigned to treatment or control and one eighth grade, general education, inclusion teacher (nominated by an administrator) participated in the study. Unavoidable attrition at the beginning of the year resulted in eight schools in treatment and 16 schools in control, with 719 eighth grade students with and without disabilities or who were low achieving participating in the study. Students in the treatment group received Solve It! instruction, and students in the control group received typical math instruction. All students were assessed once a month from October to June using seven curriculum-based measures (CBM) that assessed problem-solving ability. Results showed that students in treatment (n = 319) receiving the cognitive strategy Solve It! made statistically significant growth on math problem solving when compared to typical classroom instruction in math inclusion received by the control group (n = 460).

In order to determine the evidence base of CSI, Montague and Dietz (2009) evaluated the quality of research of CSI on students' mathematical problem solving from 1969 to 2006. Initially 42 studies were eligible for evaluation, but only seven articles met all inclusion criteria (i.e., was a research study; published in a peer-reviewed journal; used CSI for intervention; used group experimental, quasi-experimental, or single case design; included participants with a disability; and dependent variable was mathematical problem solving) with five being single case design studies and two group design studies.

The seven studies included 142 student participants with a mean age of 8-4 to 16-7 years. Most students were identified as LD (n = 110), others with mild intellectual disability (n = 30), and in one study two participants were described as having mild mental retardation. Overall, the group and single case studies indicated that CSI was effective in increasing students' problem solving skills. The evidence-based evaluation used quality indicators based on criteria from Horner et al. (2005) and Gersten et al. (2005b). The results of the evidence-base evaluation indicated all five single case studies met most of the minimally acceptable criteria. Treatment fidelity was not reported in any study and only one study reported scoring interrater agreement on the dependent variable. Of the two group design studies, neither study reported a measure of treatment fidelity, nor did they use multiple measures of the dependent variable. Although CSI was reported as effective for improving students' mathematical problem solving skills, the research does not meet the rigorous standards for an evidence-based practice. The authors recommend future research adhere to the quality indicators proposed by Horner et al. (2005) and Gersten et al. (2005b) with an emphasis on providing treatment fidelity.

After the results of Montague and Dietz's (2009) CSI evidence-based evaluation, Jitendra et al. (2015) evaluated the quality of research on explicit strategy instruction derived from cognitive science (i.e., identifying problem type, use of schematic diagrams, metacognitive strategies, use of transfer features) for determining problem type by examining 18 group design and 10 single-case design studies from 1960 to 2011. The authors used quality indicators based on Gersten et al. (2005b), Horner et al. (2005), and What Works Clearinghouse (2011). The inclusion criteria for the studies included the following: (a) participants were identified with LD or were at risk for LD; (b) the study used an experimental or quasi-experimental group design, or a single case design; (c) treatment group received strategy instruction for determining problem type; (d) the study used a minimum of one experimenter-designed or norm-referenced problem solving measure in math; and (e) the study was published in a peer reviewed journal in English. For the seven group design studies that met standards for high-quality studies the weighted mean effect size was 1.29 and for the seven studies that met standards for acceptable studies, the weighted mean effect size was 1.27. Only one single-case study met expectations on all seven indicators (Horner et al., 2005) and two met acceptable expectations on six of the seven indicators. Results indicated that strategy instruction used to prime the mathematical problem structure (e.g., teach students to determine word problem type) based on group design research is an evidence-based practice.

Overall, these research studies on CSI suggest that CSI is an effective, research-based practice for teaching math word-problem solving skills to students with disabilities or who are at risk for disabilities and an evidence-based practice for priming the mathematical problem structure. In order to address limitations of these studies, future research should follow quality indicators proposed by Horner et al. (2005) for single-case designs and Gersten et al. (2005b) for group designs. Specific focus on treatment fidelity measures of the independent variable and interrater agreement of the dependent variable should be included in future research.

Explicit Instruction

Over the years, research has determined explicit instruction as effective for teaching math to students at risk for or identified with disabilities (Gersten et al., 2009b; Jayanthi et al., 2008; Zheng et al., 2012). SBI and CSI use explicit instruction as a key teaching component. Explicit instruction is defined as a structured and systematic approach to teaching academic concepts and skills (Archer & Hughes, 2011). Characteristics of explicit instruction include the use of scaffolds to lead students through the steps of learning by providing a purpose for learning, clearly demonstrating the skill, presenting multiple opportunities for practice with ongoing feedback, and meeting mastery criteria (Archer & Hughes, 2011). CSI uses explicit instruction as the format for each lesson. Structured lessons are organized and include prompts and cues, distributed practice, modeling, feedback, praise, overlearning, and mastery of content (Jitendra & Montague, 2013).

Previously reviewed meta-analyses indicated the effectiveness of using explicit instruction to teach math concepts including word-problem solving skills. In the aforementioned meta-analysis, Jayanthi et al. (2008) evaluated the effectiveness of math strategies on students' understanding of math concepts including problem solving and determined explicit instruction was an effective strategy with a strong effect size of 1.22. In another previously discussed meta-analysis, Zheng et al. (2012) determined explicit instruction was an effective strategy for teaching word-problem solving skills to students with math disabilities only and math and reading disabilities. These authors concluded, "explicit instruction is an effective approach for teaching students with learning disabilities" (p. 108).

In another meta-analysis, Gersten et al. (2009b) conducted a review of literature on teaching the components of math to students who were at risk and who had LD and found significant effect sizes for using explicit instruction, visual representations, sequence and/or range of examples, student verbalizations, and providing ongoing feedback when teaching the components of math to students with LD. Explicit, systematic instruction yielded the strongest statistically significant mean effect size of 1.22, providing the best results for teaching computation, word problems, and solving math problems in new situations.

The wide research base behind explicit instruction supports the use of explicit instruction to teach math concepts including word-problem solving to students including students with disabilities. Explicit instruction is an effective instructional format for teaching CSI and improving students' skills in math word-problem solving.

Singapore Math

An additional instructional method that includes a heuristic model and is a type of CSI is the Singapore Model Method (SMM; Hong et al., 2009). According to the Trends in International Mathematics and Science Study (TIMSS) in 2011, Singapore was the highest scoring country on fourth grade mathematics and second highest scoring country on eighth grade mathematics. Singapore has scored in the top two of all participating nations in fourth grade math each year the country participated in TIMSS, and in the top three of all participating nations in eighth grade math (Provasnik et al., 2012). Singapore's Ministry of Education designed Singapore Math (Singapore Math Inc., 2014) that includes SMM for teaching word-problem solving and is used in all schools across the country (Ginsburg et al., 2005). The Ministry of Education is the centralized education system that approves all school-based textbook choices, which include the model method as the instructional strategy for teaching word-problem solving (Ginsburg et al., 2005; Ng & Lee, 2009).

The American Institutes for Research (AIR) conducted a longitudinal study on the differences between mathematics education systems in Singapore and the United States, in order to explore reasons behind Singapore's students' mathematics success (Ginsburg et al., 2005). Cultural differences between homogeneity of students, motivation of students, and size of population were discussed, but refuted as the sole reason for Singapore's students' success in mathematics education. In regards to homogeneity of students, 75% of Singaporeans were Chinese, and the remaining 25% were Malaysian and Indian. On the TIMSS, 83% of Malaysian Singaporeans scored above the international average (data were not available for Indian-Singaporean students). Also, English was taught as the primary language of instruction in Singapore and many students spoke a different language at home. In the United States, it is well known that students from minority ethnicities and students whose native language is not English do not score as well as their Caucasian, native English-speaking peers on national assessments of academic achievement (NAEP, 2015). In regards to motivation, families typically value education in Singapore and that encourages students to work hard, but motivation is not the only reason for their success. In the mid-1980s, eighth grade students in Singapore tied students in the United States for 13th place out of 18 countries on the Second International Science Study (SISS; Medrich & Griffith, 1992). The performance of their students on the SISS was an impetus for Singapore's Ministry of Education to restructure their mathematics curriculum. The change in achievement of Singapore students since the 1980s has more to do with the restructuring of the curriculum than motivation, as families in Singapore have consistently valued education. Finally, although the population of Singapore is much smaller than the population of the

United States, at the time of this report, the United States did not have a common set of education standards (CCSS) and suggestions were made to centralize smaller units (e.g., states, large urban districts) to follow common frameworks (Ginsburg et al., 2005). Given these arguments, AIR concluded that the differences in homogeneity and motivation of students and population sizes between Singapore and the United States were not solely responsible for the differences in student achievement in mathematics.

AIR then evaluated differences in mathematics content and textbooks in Singapore and the United States. At that time, three states (i.e., Texas, California, North Carolina) had similar types and numbers of topics and outcomes to Singapore’s framework, and they also organized concepts by grade level. On the contrary, textbooks were markedly different between the two countries. Singapore mathematics textbooks included fewer lessons, but their lessons included five to six times as many pages (12-17 pages) as the reviewed United States textbooks. Concepts were developed through multiple perspectives of problem-based learning to solve problems in more depth. In contrast, the United States textbooks covered three to five times the number of lessons with fewer pages (2-4 pages) in each lesson and exposed students to twice as many topics as were found in the Singapore textbooks. Overall, United States textbooks were found to sacrifice a depth of understanding for breadth of content within the textbooks. One reason for this difference was due to publishing companies attempting to cover wide varieties of topics to be marketable for each state based on their own mathematics standards (Ginsburg et al., 2005). Post CCSS implementation, it will be interesting to see if the market changes enough that content within textbooks shifts to provide depth over breadth.

Along with differences in the content being taught at the elementary levels between the two countries, differences were found in the teacher preparation programs. Before even starting the teacher preparation program, preservice teachers in Singapore were required to demonstrate higher levels of mathematics proficiency than preservice teachers in the United States, and they had a more stringent screening process for admittance into the teacher education program. Despite the fact that some teachers in Singapore only earned a 2-year college certificate, they completed almost twice the number of credit hours in mathematics that were required by a 4-year education degree in the United States. Overall, preservice teachers in Singapore teacher preparation programs receive stronger instruction in mathematics content and pedagogy than similar programs in the U.S (Ginsburg et al., 2005). This brief review explored the differences between Singapore's mathematical framework and United States mathematics content based on the longitudinal study by AIR. This review was necessary to provide an understanding of the cultural similarities and differences between both countries and to provide support for incorporating Singapore's approach to mathematics to student's in the United States. Although this section discussed the mathematic framework of Singapore Math (Singapore Math Inc., 2014), SMM is a component of Singapore Math designed specifically for solving math word problems. Subsequent sections will focus on SMM and word problem solving skills.

Singapore Model Method. SMM was created (within Singapore Math) in the 1980s as a way to increase math word-problem solving skills for younger students in Singapore and was revised in the 1990s and 2000s to include secondary skills (e.g., fractions, ratios, percentages, and algebra) and the Singapore mathematics framework

(Hong et al., 2009). The mathematics framework was designed to improve students' math problem solving abilities and consisted of five principles: (a) metacognition – thinking about and selecting problem-solving strategies; (b) processes – reasoning, heuristics, and modeling; (c) concepts – numeric, algebraic, geometric, statistics, probability, and analytic concepts; (d) skills – developing and applying proficient mathematics skills; and (e) attitudes – beliefs, interests, appreciation, confidence, and perseverance in problem solving. SMM uses the principles within the mathematics framework to effectively teach students how to solve math word problems (Hong et al., 2009).

SMM is used for solving problems dealing with the four operations (i.e., addition, subtraction, multiplication, and division), fractions, ratios, percentages, and algebra. SMM uses a part-whole model and comparison model to assist in determining the problem structure (e.g., schema) and operation necessary for solving the problem (Hong et al., 2009). To further assist students with problem solving, SMM uses heuristics, schematic diagrams, and promotes the use of metacognition skills. The most common heuristic in SMM is a seven-step procedure applicable to all types of word problems. Students use schemas to diagram part-whole and comparison models, as well as metacognition skills to monitor their ability to solve the problem (Hong et al., 2009). In this way, SMM is similar to SBI and is a type of CSI.

Despite the fact that Singapore is consistently one of the top scoring nations in math on the TIMSS, there is very little research to support the use of SMM. In the first study, Ng and Lee (2009) evaluated the effects of SMM on word problem solving skills of 151 Primary 5 students (average age of 10.7 years) in Singapore. Students in Primary 5 are ability-tracked by student performance. This study included students in the top and middle track, but excluded students in the bottom track. Using a qualitative study that involved teacher interviews and analysis of a 10-question word problem assessment completed by students, the study investigated how students used SMM and whether or not they solved the questions correctly. Overall results showed that participating students were successful in using SMM to solve math word problems. SMM was more effective with arithmetic word problems than algebraic problems. Results also demonstrated that SMM was not an all-or-nothing process for determining whether word problems were correct or incorrect. Students could misrepresent the entire problem or one piece of information and finish with the wrong answer. Thirty-three percent of students who were partially correct were not completely correct or completely incorrect. In other words, students made slight errors in representation that altered their final answer, but may have understood the problem and how to solve it otherwise. Other results indicated that students in the top track were able to accurately use SMM, whereas students in the middle track had more difficulty using SMM. The authors did not analyze reasons for those results. Overall, this study demonstrated that SMM was appropriate for average students to use when completing math word problems.

In the second and final study on SMM, Mahoney (2011) conducted a multiple baseline across participants design to evaluate the effects of SMM on solving word problems that included multiplicative comparison problems and fractions. Participants included four general education, third and fourth grade students with average mathematics ability. Baseline sessions were 30 min and consisted of providing students with the word-problem probe. The researcher-designed probes included 10 questions (five multiplicative comparison problems and five fraction problems). The researcher provided the intervention for eight 60-min sessions, with 30 min spent on instruction and 30 min spent on the probe. Instruction followed a model-lead-test format with the researcher demonstrating the first problem, the researcher and student completing the next three problems, and the student completing the last problem independently. Maintenance probes were administered 1 week and 3 weeks after intervention concluded. Results demonstrated a functional relation between SMM and students' word-problem solving skills in multiplicative comparison problems and fractions (PND = 93.75%). Upon further analysis of the study, and although the author did not explicitly state, it seems the study used a delayed multiple baseline for all students rather than a typical multiple baseline. The graphs were not displayed vertically, but separately within the description of each participant. All graphs displayed three baseline points at sessions 1-3, then intervention, and maintenance. A timeline is provided that implies baseline and interventions started and ended at different times.

A review of the only two existing studies on SMM showed that neither study included students with identified disabilities. The first study by Ng and Lee (2009) did not employ SMM as an intervention, but assessed students' use of SMM based on their classroom instruction in SMM. They found students in the middle track had more difficulty than students in the top track. They did not assess students in the low track, but it can be assumed that students in the low track (e.g., students with disabilities) would have had more difficulties than students in the middle track. The second study by Mahoney (2011) only included instruction in multiplicative comparison models and fractions for students without disabilities in general education. Due to the high performance of students in Singapore and moderate performance of students in the

United States on the TIMSS in recent years, word-problem solving instruction using SMM is worthy of investigating with students in the United States. Given that both studies did not include students with disabilities (and in the first study students placed in the middle track struggled to use SMM, Ng & Lee, 2009), and because SMM is part of a general education curriculum (Tier 1 core instruction), it can be assumed that students with disabilities would struggle to solve word problems using SMM without modifications in a general education setting. Thus, modifying SMM to include effective strategies for teaching students who are at risk for or identified with LD (Tier 2 intervention) is worth investigating. Explicit instruction is supported through research as an effective strategy for students who are at risk for or identified with LD (Gersten et al., 2009b; Jayanthi et al., 2008; Zheng et al., 2012). Therefore, in order to include students who are at risk for or identified with LD in the research base of SMM, adding explicit instruction to SMM may provide students with disabilities with an effective instructional strategy for solving math word problems dealing with addition, subtraction, multiplication, and division. Even stronger would be to conduct a study in a school where teachers have been taught to use SMM as core instruction in Tier 1 and identify students who are not proficient in solving word problems based on their current level of instruction. Then provide a Tier 2 intervention using SMM with explicit instruction to teach students who are at risk for or identified with LD to solve math word problems.

## Summary

Researchers have evaluated the effectiveness of multiple instructional strategies for math concepts including problem solving. Review of meta-analyses and research studies demonstrated effective practices in math word-problem solving included use of explicit instruction and heuristics. More specifically, when focusing on problem solving heuristics, SBI and CSI techniques were found effective for at-risk students and students with disabilities. Research is needed using problem-solving interventions within preexisting curricula for increasing students' abilities to solve math word problems. SMM is worthy of further inquiry for the following reasons. First, students in Singapore continue to demonstrate high math achievement compared to students in the United States and around the world. Second, SMM is used to teach students in Singapore math problem-solving skills and is not widely used in the United States. Third, there is a lack of empirical research on the use of SMM. Fourth, research findings promote the use of heuristics, SBI, and CSI for teaching math word-problem solving skills. Finally, SMM includes components of SBI, CSI, and uses a heuristic model for teaching word-problem solving that when taught through explicit instruction, may result in higher problem-solving achievement for students with disabilities. SMM, when combined with explicit instruction to address the word-problem solving skills of students who are at risk for or identified with LD will add to the lack of research behind SMM.

Further research may also justify the use of SMM within a multi-tiered system of support (e.g., RTI). Currently in Singapore, SMM is taught to all students during math instruction. Potentially in the United States, SMM could be taught to all students as Tier 1 instruction in math. If students demonstrate the need for further support in Tiers 2 or 3, SMM could be provided with more explicit instruction across the tiers. Therefore, students with the highest level of need (e.g., students with math LD, students receiving Tier 3 interventions, students at risk for LD) characterized by general difficulties with reasoning, processing, memory, attention, and selecting important information (Heward, 2013), would receive SMM with explicit instruction (which includes research-based strategies for students at risk for and identified with LD such as heuristics and components of SBI and CSI) potentially resulting in higher achievement levels in math problem solving for students who are at risk for or identified with LD.

Summary of the Literature Review

Internationally, Singapore is outperforming all other countries including the United States on mathematics assessments based on the most recent TIMSS (Provasnik, 2012). At the national level, less than half the students in the United States are proficient in mathematics in fourth grade and eighth grade (40% and 33% respectively; NAEP, 2015). Unfortunately, students who are at risk for or identified with disabilities are not fairing as well as their peers without disabilities (NAEP, 2015; NLTS-2, 2011). Students who are at risk for or identified with LD need stronger instruction in math.

Review of literature revealed that students who are at risk for LD in math and students identified with LD in math may have similar characteristics including deficits in working memory (e.g., retaining and generalizing information), processing speed, mathematics background knowledge (e.g., difficulty organizing new material with previously learned information), phonological processing/language (e.g., language-based problems, vocabulary), and possibly visual-spatial abilities (Coyne et al., 2011; Fletcher et al., 2006; Geary, 2013; Shaw, 2010; Vukovic & Siegel, 2010). These issues are compounded for students who are at risk for or identified with LD when they work on word problem solving skills. Research found a correlation between poor performance on arithmetic word problems and issues with attentive behavior, nonverbal problem solving, concept formation, sight word efficiency, and language (Fuchs et al., 2006a).

One method to address these issues is through implementation of an RTI, a multi-tiered system of supports for the prevention and identification of students with LD (NCRTI, 2010). Research validates RTI as a framework for providing students with evidence-based core instruction in Tier 1, research-based interventions in Tier 2, and research-based intensive interventions in Tier 3 (NCRTI, 2010). Research also supports the effectiveness of Tier 2 instruction aligned with Tier 1 instruction on math problem solving skills of students who are at risk for or identified with LD (Bryant et al., 2008a; Fuchs et al., 2008).

Within the essence of RTI lies instruction that has been validated by research. Many studies have been conducted on strategies to assist students who are at risk for or identified with LD in solving math word problems (Jayanthi et al., 2008; Xin & Jitendra, 1999; Zhang & Xin, 2012; Zheng et al., 2012). Four strategies with extensive research support that have been produced positive outcomes for students solving word problems include heuristics, SBI, CSI, and explicit instruction (Fuchs et al., 2010; Gersten et al., 2009b; Jayanthi et al., 2008; Jitendra et al., 2013b; Jitendra et al., 2015; Montague et al., 2011; Zheng et al., 2012). Despite the success of each strategy, a query arises. Are there any strategies that include components of each of these instructional methods for solving word problems?

An investigation into the math curriculum of a country leading the world in education identified SMM as the instruction used by Singapore's education system to teach students to solve math word problems. SMM is a heuristic approach to word problem solving that includes use of schemas and cognitive strategies, but is not taught using explicit instruction (Hong et al., 2009). Currently, there is very little research on the use of SMM designed for students receiving Tier 1 core instruction (Mahoney, 2011; Ng & Lee, 2009). Due to the dearth of research on SMM and research supporting heuristics, SBI, CSI, and explicit instruction, it would be appropriate to evaluate the effectiveness of SMM, which is typically used in core instruction at Tier 1, modified for a Tier 2 intervention as SMM with explicit instruction for students who are at risk for or identified with LD.

# CHAPTER 3: METHOD

The purpose of this study was to determine the effects of Singapore Model Method (SMM) with explicit instruction on word problem solving skills of students at risk for or identified with LD at the elementary level. This chapter will provide a description of the participants, setting, researcher, second rater, third observer, data collection, experimental design, and procedures used to evaluate the research questions.

## Participants

Students. Total student participants for this study included seven, 4th and 5th grade students, ages 9 to 12, who were at risk for or identified with LD. Teachers nominated students based on the following initial criteria: (a) poor performance with math problem solving skills, (b) students identified with LD who received special education services in math, or students identified at risk who received Tier 2 or Tier 3 supports in math (based on a score of 1 or 2 on Discovery Ed), and (c) score of 1 or 2 on the previous year's End of Grade (EOG) math assessment. Exceptions were made for students who recently transferred to the school and did not have Tier status or EOG scores. Eligibility criteria also included a score of 4 (Expanding) or higher (e.g., 5- Bridging, 6- Reaching) on the composite score for the Assessing Comprehension and Communication in English State to State for English Language Learners (ACCESS) test for English Language Learners (ELL; WIDA, 2016). This was to ensure each student had a basic understanding of English because the study focused on comprehending and solving word problems in math. It was important not to fully exclude all ELL participants in the study as ELL participants are often left out of research. Finally, students who scored 50% or less on a researcher-made problem solving pretest would be considered eligible for the study.

Initially, 4th and 5th grade teachers nominated 41 students who met the initial inclusion criteria. Students were provided assent forms and parents of the students were provided informed consent forms prior to participating in any eligibility assessments (see Appendices A and B). Of the 41 students nominated, 13 students received ELL services and 9 of those students scored less than proficient on the ACCESS test (composite scores range 2.2-3.4, Beginning-Developing respectively) leaving 32 students. Parental Consent was returned for 27 students and once the student granted assent, eligibility assessments were administered. Students were given an eight-question word-problem placement test with two addition problems, two subtraction problems, two multiplication problems, and two division problems. Students who scored 50% or below on the placement test were eligible for the study. Of the 41 students originally referred by their teachers, seven students met eligibility criteria and were included in the study. See Table 2 for a breakdown of student demographics.

Table 2: Student demographics

| Student | Gender | Ethnicity | Grade | Disability/ Tier/ ELL Status | Problem-Solving Pretest Score | Previous Year's EOG Score |
|---|---|---|---|---|---|---|
| Ryan | Male | Caucasian | 5th | LD | 50% | No score |
| Aiden | Male | African American | 5th | Tier 2 | 33% | 1 |
| Lisette | Female | Hispanic | 5th | Tier 2/ELL-C | 33% | 1 |
| Raymond | Male | African American | 4th | Tier 3 | 25% | 1 |
| Amber | Female | Hispanic | 5th | Tier 2/ELL-E | 38% | 1 |
| Jasmine | Female | African American | 4th | Tier 2 | 25% | 1 |
| Ricardo | Male | Hispanic | 4th | Tier 2/ELL | 38% | 1 |

Note: ELL-C indicates the student was receiving consultation ELL services, ELL-E indicates student was exited from ELL. No score on Ryan's EOGs because he was a transfer student.

Teachers. Teacher participants included two fourth grade teachers, three fifth grade teachers, and one special education teacher. Teachers were selected to participate in the study based on whether their student(s) met eligibility criteria. Teachers taught specific subjects and students changed classes in fourth and fifth grade. Three teachers taught math and science and two teachers taught language arts and social studies. Prior to the study, all teachers attended a district arranged training by Greg Tang that included Model Drawing (based on Singapore Model Method) over the previous 2 years. Follow-up sessions were offered on professional development days for any teachers needing additional support. That school included Model Drawing as an option for teachers to use to enhance core instruction at Tier 1. Model Drawing included six steps for solving math word problems, but did not include an explicit instruction framework for teaching students who are at risk or identified with LD. See Table 3 for teacher demographics.

Table 3: Teacher demographics

| Teacher | Gender | Grade | Subjects Taught | Attended Model Drawing Training |
|---|---|---|---|---|
| Ms. Neely | Female | 5th | Math/Science | Yes |
| Ms. Forrester | Female | 5th | Lang. Arts/ Soc. St. | No |
| Ms. Coyne | Female | 5th | Math/Science | Yes |
| Ms. Houghton | Female | 4th | Math/Science | Yes |
| Ms. Swanson | Female | 4th | Lang. Arts/ Soc. St. | Yes |
| Ms. Garrison | Female | K-5th Resource Special Education | Math/Lang. Arts Social Skills | Yes |

Setting

The study took place in a public, Title 1, elementary school in a suburban school district in the southeastern United States. The school population was 758 students with the following breakdown of ethnicity: 16.6% African American, 2.7% Asian, 45.6% Caucasian, 31.4% Hispanic, 3.4% two or more ethnicities, 0.1% Native American, and 0.3% Pacific Islander. Of the population, 54% of students received free or reduced-price lunch, 21% of students were English Language Learners, 11% of students were identified with disabilities, and 3% qualified as Academically/Intellectually Gifted. Over the previous 2 school years, teachers at all grade levels received training on using SMM to teach their students to solve word problems.

The study took place in a small classroom designated for remedial instruction. The interventionist and students sat at a U-shaped table in the center of the room facing a large white-board on the wall. Instruction was conducted with students in groups of two using SMM with researcher-designed explicit instruction, researcher-made math word problems, 8" x 11" white boards, and calculators.

Researcher

The researcher served as the interventionist for this study. The interventionist was a doctoral student in her final year of the Ph.D. program with 8 years teaching experience in a special education resource classroom at the elementary level. She held a Master's degree in Special Education and an undergraduate degree in Special Education-Learning Disabilities. The researcher had North Carolina teaching licenses in special education adapted curriculum, and specific learning disabilities.

Second Observer

The second observer earned a Ph.D. in special education and was a master clinician in the special education program at a separate university. Responsibilities of the second observer included completing inter-rater reliability of probes. In order to conduct inter-rater reliability, the second observer scored 30% of probes (n = 104), using the answer key and an unscored photocopy of each probe. The researcher scored the original version of all probes.

Third Observer

The third observer was a special education teacher with a Master's degree in special education at the school in which the study took place. Responsibilities of the third observer included observing lessons for fidelity of implementation of SMM with explicit instruction using a fidelity checklist (see Appendix C). The third observer observed 31.9% of lesson days across phases and students through video-recorded lessons and in-person observations.

Data Collection

Dependent variables. The dependent variable was the score earned when solving math word problems involving addition, subtraction, multiplication, and division. A response was correct when it had a correct number sentence or visual depiction of the number sentence and correct answer for each problem. A correct number sentence contained the equation numbers, operation, equal sign, and an answer. The answer itself did not have to be correct to be counted as a correct number sentence (e.g., 3 + 5 = 8, 3 + 5 = 9). A correct visual depiction of the number sentence included shapes that represented the number sentence (e.g., 2*3=6 would be shown as II II II). Each word problem had a value of three points: two points for a correct number sentence and one point for a correct answer. Students who used a visual depiction of the number sentence and did not include a number sentence received one point for a correct visual depiction of the problem and one point for a correct answer. Students who wrote a number sentence and drew a picture received two points for a correct number sentence because use of a number sentence demonstrated higher order thinking skills verses drawing a visual depiction of the problem. Each probe included four questions for a score of up to 12 points on each probe. Mastery criterion was designated as 10-12 points per probe (83%-100%). There were two possible ways to earn 11 points: (a) making an error calculating the answer, and (b) drawing a correct visual representation with the correct answer (no number sentence). Both options indicated that the student understood the problem and set the problem up correctly, just made a calculation error or did not write the number sentence. In order for a student to receive 10 points, he/she could have followed options (a) and/or (b) above on two problems, or (c) the student got the correct answer, but did not write the number sentence/ wrote an incorrect number sentence (not typical). The most common ways students made errors were options (a) and (b), indicating that in order for a student to meet "mastery" criterion, he/she would have to understand how to solve each problem. Most likely, if a student earned 9 or fewer points (75% or less), that student incorrectly set up and incorrectly solved at least one problem.

Three types of probes were used to measure students' word problem solving skills. Probe-AS included four word problems using two addition and two subtraction problems (see Appendix D). Probe-MD included four word problems using two multiplication and two division problems (see Appendix E). Probe-ASMD served as a type of maintenance with a combined probe that included four word problems using one addition, one subtraction, one multiplication, and one division problem (see Appendix F). The order of problems for each probe was randomly assigned using an online randomization program (i.e., www.random.org). During the Baseline A, Probe-AS and Probe-MD were conducted once a day for 5 days or until the data were stable. Probe-ASMD was conducted once during the first 5 days of Baseline A. During Intervention B (instruction using SMM for addition and subtraction), Probe-AS was administered twice a week (in order to control for threats to internal validity by over testing) and Probe-MD and Probe-ASMD were administered at least once during Intervention B. Throughout Baseline C, Probe-MD was administered across three consecutive sessions. During Intervention D, Probe-MD was administered twice a week (in order to control for a threat to internal validity by over testing) and Probe-ASMD was conducted at least once throughout the phases. Finally, during Maintenance E, Probe-AS, Probe-MD, and Probe-ASMD were conducted one week after completing Intervention D, and at least once a month for the remainder of the study. Data were collected through permanent product recording.

Inter-rater reliability and training. Data were collected twice a week and inter-rater reliability were conducted on 30.2% (n = 104) of the total probes (n = 344) equally distributed across all students and phases. The primary rater scored probes by attaching a sticky note to each probe and recording the score for each question on the sticky note. This procedure allowed the primary rater to avoid writing on any probes. Thirty percent of the probes were then given to the secondary researcher for scoring (without the sticky note attached). Permanent product data were calculated item by item as a frequency count. Percentage of agreement was determined by dividing the number of agreements by the total number of agreements plus disagreements and multiplied by 100. The primary researcher trained the secondary researcher to score math probes. The primary researcher shared a probe scoring protocol with a list of steps and directions for scoring the probes with the secondary observer. Three probes were selected for scoring during training. The secondary observer and primary researcher scored all three probes with 100% agreement.

Procedural fidelity. Procedural fidelity was conducted by the third observer for 31.9% of the session days distributed across phases and participants. The procedural fidelity checklist measured the researcher's fidelity of implementing SMM with explicit instruction. The third observer was trained to teach SMM with explicit instruction and trained to observe lessons using the fidelity checklist. Both the third observer and interventionist observed videos and completed the fidelity checklist. Training continued until they reached 100% agreement on the fidelity checklist for three consecutive videos.

Procedural fidelity for the intervention was calculated by dividing the number of correct steps by the total number of applicable steps and multiplying by 100 (see Appendix C).

Social validity data. Data were collected on social validity with a 10-item questionnaire for students (see Appendix G) and a seven-item questionnaire for teachers (see Appendix H). The student social validity questionnaire was based on a three point Likert scale (1 = no/disagree, 2 = I don't know, 3 = yes/agree). Students were provided the student questionnaire in order to determine their perceptions on whether the SMM procedure was helpful with solving math word problems. The questionnaire statements included: (a) The model method lessons helped me to understand how to do math word problems; (b) The 7 steps in the model method were easy to follow; (c) Drawing the bars helped me understand what the word problem was asking; (d) Labeling the bars with parts and whole, or part, difference, whole helped me solve the problems; (e) The rules about missing the part and missing the whole helped me learn if I should add or subtract and multiply or divide; (f) I am better at solving word problems now; (g) I use the model method to do word problems in my classroom; (h) I would like my classroom teacher to use the model method during math; (i) I would like to learn how to use the model method to solve other kinds of math word problems. See Appendix G for the Student Social Validity form.

At the onset of the study, all teachers of students in the study provided consent to participate in the study (see Appendix I). Upon completion of the study, teachers were provided the social validity questionnaire to determine teachers' perceptions of the progress their students made and perceptions of the use of SMM in the general education or special education classroom. The teacher social validity questionnaire was based on a four point Likert scale (1 = strongly disagree, 2 = disagree, 3 = agree, 4 = strongly agree). The questionnaire statements included: (a) My students learned how to solve word problems from this strategy; (b) I saw a change in my student's math skills in my classroom after receiving the strategy; (c) The 7 steps to the model method are easy to understand; (d) Teaching how to solve word problems will be easier using the model method; (e) This strategy would be feasible to implement in my classroom; (f) I would like to use this strategy in my class; and (g) I would like training on how to use this strategy in my class. See Appendix H for the Teacher Social Validity form.

Experimental Design

The experimental design for this study was a multiple probe across participants with an ABCDE design (Cooper, Heron, & Heward, 2007). A multiple probe across participants is an experimental single-case design and is used when a skill, once learned, cannot or ethically should not be reversed. In this study, all participants began together in the baseline phase. Students were administered a probe (i.e., permanent product) intermittently at a predetermined rate (i.e., twice a week), whereas in a multiple baseline, students would be administered a probe at each session throughout the study. After five baseline data points or when the data were stable, the students with the lowest and/or most stable baseline or demonstrated the most need moved into the intervention. The first pair of students receiving intervention continued to complete the probes and once the students demonstrated an increase in trend or level across a predetermined number of probes, the next pair of students completed a cluster of baseline probes and began the intervention. This process repeated for the remaining students in the study. In order for a single-case design to demonstrate experimental control, the design must include prediction (i.e., the dependent variable data will remain stable without an intervention), verification (i.e., the dependent variable remained stable across participants, settings, or behaviors), and replication (i.e., the independent variable likely produced the change in dependent variable data and is replicated across participants, settings, or behaviors at different times). A multiple probe can be used to determine a functional relation between the independent variable and dependent variable when the data demonstrate either a change in trend, change in level, change in variability, and/or immediacy of effect (Cooper et al., 2007).

The ABCDE design for this study included five phases. See Figure 1 for a visual representation of the design. The first baseline was phase A, and the first intervention was phase B. The second baseline was phase C, and the second intervention was phase D. Maintenance was conducted within phase E. Phase A collected baseline data on Probe-AS (i.e., addition and subtraction) and Probe-MD (i.e., multiplication and division) for all participants. Phase B included the addition and subtraction intervention. Phase C was a repeat of baseline data collection for Probe-MD and phase D included the multiplication and division intervention. Phase E included maintenance data on Probe-AS and Probe-MD, as well as data on the combined skills probe, Probe-ASMD (i.e., addition, subtraction, multiplication, and division). All students began phase A at the same time and baseline data were gathered for each student during the probe sessions. Students remained in baseline for five points, before the first student moved into intervention. Students demonstrating the most need began phase B intervention first. Once students completed stages 1-5 of the intervention and reached mastery criteria (i.e., at least 10/12 points, 83.3%) for at least three data points (not consecutively), all remaining students completed a cluster of probes, and the next two students with the lowest or most stable data began phase B intervention. After meeting mastery criteria for Probe-AS, students from the first group, began phase C and completed Probe-MD for three consecutive sessions. After three probes, those students moved into phase D intervention. Once students completed stages 6-9 (including mixed review word problems based on all types of addition, subtraction, multiplication, and division word problems) and scored at mastery criteria of 83.3% (10/12 points) on Probe-MD for at least three consecutive probes (not consecutive), those students moved into phase E. During phase E, students no longer attended intervention sessions and remained in maintenance for the remainder of the study. In order to measure maintenance, students were administered Probe-AS and Probe-MD monthly (for students who started in the first two groups) and weekly (for students in the final groups). While in phase E, maintenance data were collected using Probe-AS and Probe-MD as well as a combined skills probe, Probe-ASMD. The latter assessed students' ability to discriminate between all four types of word problems. Overall, this research pattern continued until all students completed phases ABCDE and remained in maintenance (see Figure 1).

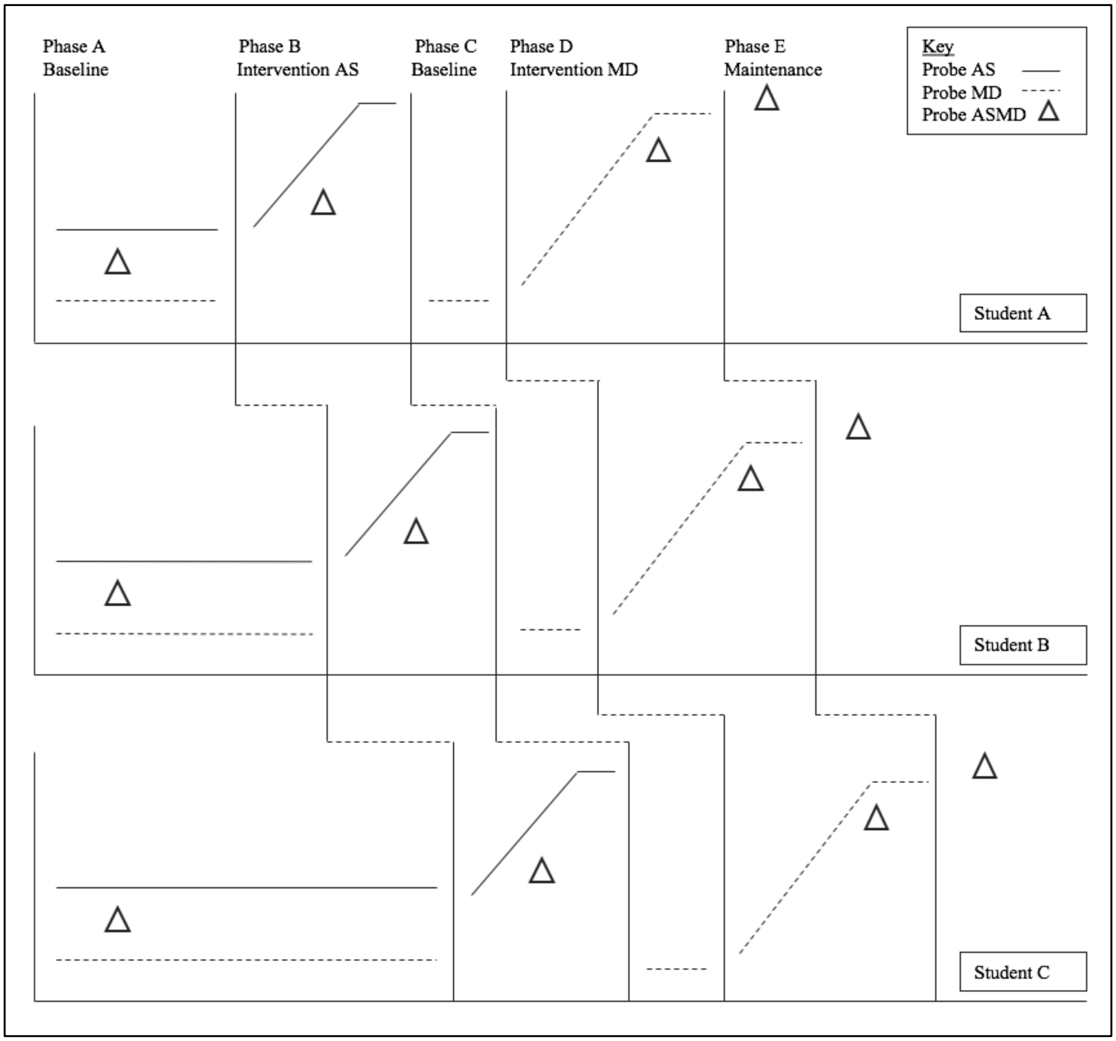



Figure 1: Mock graph of results

Materials

This study included assessment materials and instructional materials. Assessment materials included a researcher-made pretest and posttest, researcher-made probes, and a calculator. Data were collected from the Discovery Ed assessment, but the assessment was administered by school personnel. Instructional materials included a wall mounted white board, 8"x10" white boards, white board markers, graph paper, researcher-made instructional bar modeling worksheets, researcher-made instructional word stories worksheets, researcher-made instructional word problems worksheets, SMM steps displayed on a chart-paper sized poster on the white board, and a calculator.

All word problems for researcher-made assessments were organized by type of question (i.e., addition, subtraction, multiplication, division) and situation (e.g., add to start unknown, take from change unknown) and reviewed by a content expert for content validity. Questions were based on word problems from Singapore Math 70 Must-Know Word Problems (Schaffer, 2009) and included only single-step problems. Addition and subtraction word problems included numbers between 100-999. Situations for addition included: (a) compare, bigger unknown; and (b) take from, start unknown. Students demonstrated mastery of add to result unknown/put together total unknown on preliminary probes before the study began, so those situations were used for instructional purposes and not included in the probes used during data collection. Situations for subtraction included: (a) add to, change unknown; (b) take from, result unknown; (c) take from, change unknown; (d) take apart, addend unknown; (e) compare, difference unknown; and (f) compare, smaller unknown. Multiplication and division word problems included basic math facts with multiples of 2-10. Situations for multiplication included: (a) equal groups, unknown product and (b) arrays, unknown product. Situations for division included: (a) equal groups, size unknown; (b) equal groups, number of groups unknown; (c) arrays, group size unknown; and (d) arrays, number of groups unknown. Random assignment of probe questions was generated using an online list randomizer. The words addition, subtraction, multiplication, and division were entered in the database, depending on the type of probe, and randomized. Situations for subtraction and division were randomized and assigned to probes. The six situations for subtraction were listed and randomized in an online list randomizer, then assigned to probes 1-3 (two problems each probe), then randomized again and assigned to probes 4-6. These steps were repeated until all 25 addition and subtraction probes were assigned subtraction situations. The same procedure was followed for all four types of division situations until all 20 multiplication and division probes were assigned division situations. Addition and multiplication problems had two situations each, so both situations were used for each probe. Instructional worksheets were developed by the primary researcher based on word problems from the content validated assessment questions by changing the names, numbers, and subjects within the problems, but keeping the situations the same.

Procedures

General procedures. Students received typical math instruction with their assigned teachers for the duration of the study. In the general education class, students received instruction during math workshop using a district designed scope and sequence of grade-level math units that aligned with Common Core Standards. Math workshop included a 10-min whole class mini-lesson with three 15-min small group stations (i.e., teacher led station, reasoning, and computation). Some teachers used a similar bar modeling procedure, Model Drawing, (which was not taught explicitly) during core Tier 1 math instruction. Model Drawing instruction at Tier 1 was not effective for students who were nominated by teachers for this study. Students did not receive explicit instruction solving math word problems from their general education teachers.

Pretest. Prior to baseline, students completed a researcher-made math pretest using word problems from the premade lists of questions. This assessment contained eight word problems, with two addition, two subtraction, two multiplication, and two division problems in random order. Students who scored 50% or below were eligible for the study. The same pretest served as the posttest that was conducted at the conclusion of the study to demonstrate student growth across the study.

Probes. During baseline, students followed the general procedures for receiving typical math instruction in the general education class. Students were administered a math probe twice a week. Probe-AS consisted of four questions containing two addition and two subtraction word problems randomized by order (e.g., AASS, ASSA, SASA) and randomized by situation (e.g., compare smaller unknown, take apart addend unknown). Probe-MD consisted of four questions containing two multiplication and two division problems randomized by order (e.g., DMDM, MDDM, DDMM) and randomized by situation (e.g., arrays number of groups unknown, equal groups group size unknown). Probe-ASMD served as one of the maintenance collection measures using a combined probe that consisted of four questions containing one addition, subtraction, multiplication, and division word problems. The order of questions for each probe were randomized by order (e.g., ASMD, MSAD, DASM) using the list randomizer. Students were allowed to use calculators while completing probes throughout the phases.

Baseline. Students completed two baseline phases (i.e., phase A and phase C) during this study. During baseline, students continued to receive typical class instruction in the general education classroom (as described in general procedures). Students did not receive explicit instruction on math word problem solving during baseline.

SMM with explicit instruction. The SMM intervention was conducted 5 days a week for 20 min in groups of two with the interventionist. Steps for SMM were modified for application within nine stages of explicit instruction (see Figures 2 and 3).

1. Read problem

2. Rewrite the question as an answer

3. Circle who/what the problem is about

4. Draw unit bars

5. Adjust unit bars, add question mark

6. Solve

7. Put answer in sentence.

The nine stages of explicit instruction also included five rules (see Figures 2 and 3).

Rule 1: Difference is the space between the part and the whole.

Rule 2: Missing a whole add or multiply.

Rule 3: Missing a part subtract or divide.

Rule 4: Single items add or subtract.

Rule 5: Groups of items multiply or divide.

The researcher taught all the steps and rules using model-lead-test format. The researcher modeled the step or rule, led by assisting the student with the step or rule, and tested by allowing the student to complete the step or rule independently. The researcher included systematic error correction by modeling the steps to determine how to solve the problem and guiding the student through the process. For problems the student completed independently, the researcher used questioning strategies to ask the student to explain how he/she determined how to solve the problem. These strategies was used during all the stages in intervention phases B and D.

Phases of baseline and instruction. Implementation of this study included five phases. Phase A provided baseline for addition/subtraction word problems and multiplication/division word problems. Phase B included instruction on addition and subtraction word-problem solving. Phase C provided a repeat baseline for multiplication/division word problems. Phase D included instruction on multiplication and division word-problem solving. Phase E served as the maintenance phase. A thorough description of each phase will be discussed.

Phase A: Baseline. Phase A began at the same time for all students. Each student was given Probe-AS and Probe-MD for a minimum of five consecutive sessions and Probe-ASMD once. The researcher administered the math probes as a group assessment. All questions were read aloud at least once and repeated upon student request. Students were allowed to use calculators during baseline. Students who remained in phase A while other students started intervention in phase B, completed Probe-AS and Probe MD at least once a month once for the duration of phase A.

Phase B: Instruction addition/subtraction. Each student started with Stage 1 to demonstrate equal bars, unequal bars, and the difference. Given two rectangles on graph paper, the student was taught to point out which bars were equal, which bars had more or less, and which part was the difference between the two unequal bars. Students also learned Rule 1: Difference is the space between part and whole. Stage 1 was necessary to lay the foundation for diagramming the word problems (see Figure 2).

Stage 2 provided instruction on the three types of bar sets (i.e., schemas) for addition and subtraction (see Figure 2). The first type of addition and subtraction problem was part, part, whole for one set of objects diagrammed with one bar. The second type of addition and subtraction problem was part, part, whole (e.g., change problems) for more than one set of objects diagrammed with two bars (e.g., combine problems). The third type of addition and subtraction problem was part, difference, whole diagrammed with two bars (e.g., compare problems). The student was given blank diagrams and learned to label the diagrams with p for part, w for whole, and d for diagram. Teaching three distinct diagrams was a specific modification to SMM based on SBI and use of explicit instruction. Traditional SMM was more general allowing students to draw any type of bars for any problem.

Stage 3 used addition and subtraction math stories without any missing variables and students learned to label the bars with the corresponding numbers (see Figure 2). The first type of addition and subtraction problem was part, part, whole for one set of objects. An example of this type of problem is “Pete has 114 apples (i.e., part). He found 378 more apples (i.e., part). Pete has 492 apples (i.e., whole).” The second type of addition and subtraction problem was part, part, whole for more than one set of objects. An example of this type of problem is “Pete has 235 apples (i.e., part). Rosa has 173 apples (i.e., part). They have 408 apples altogether (i.e., whole).” The third type of addition and subtraction problem is part, difference, whole. An example of this type of problem is “Pete has 235 apples (i.e., part). Rosa has 62 more apples than Pete (i.e., difference). Rosa has 173 apples (i.e., whole).”

Stage 4 provided instruction on steps 1-4 for problem set up (see Figure 2). Students learned the following steps: (1) read problem, (2) rewrite the question as an answer, (3) circle who/what the problem is about, and (4) draw unit bars. The researcher taught each step in sequential order using researcher-made word problems for addition and subtraction.

Stage 5 provided instruction on steps 5-7 for solving addition and subtraction problems using the seven steps of problem solving (see Figure 2). Students learned the following steps in sequential order: (5) adjust unit bars, add question mark, (6) solve, and (7) put answer in sentence. The researcher used researcher-made word problems for addition and subtraction to teach the problem-solving procedure. Students received multiple opportunities for practice and received feedback from the researcher. Students learned two rules: (a) Rule 2: Missing whole, add; and (b) Rule 3: Missing part, subtract. Rules were based on the Number-Family Problem-Solving Strategy (Stein, Kinder, Silbert, and Carnine, 2006).

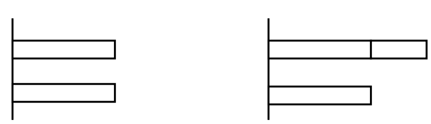
| Stages | Procedures | Criteria |
|---|---|---|
| | Phase B | |
| <u>Stage 1</u><br>Equal bars and Difference | Use pairs of various rectangle sizes on paper. Demonstrate equal, unequal, which is bigger, smaller.<br><br>Model: Use graph paper. Draw two rectangles (e.g., 5 and 3). Show how 5 has 3 (and 2 more). Point out the equal parts (3s), parts of the whole (3, 2) and the whole (5). Explain "difference" (d) is the space between 5 and 3 (how much they are different).<br>Lead and test.<br><br>Rule 1: Difference is the space between part and whole.<br><br>Equal Unequal | 3 independent correct answers: draw and label equal, unequal rectangles, part (p), whole (w), difference (d) |

Figure 2: SMM with explicit instruction procedures phase B

Figure 2: SMM with explicit instruction procedures phase B continued

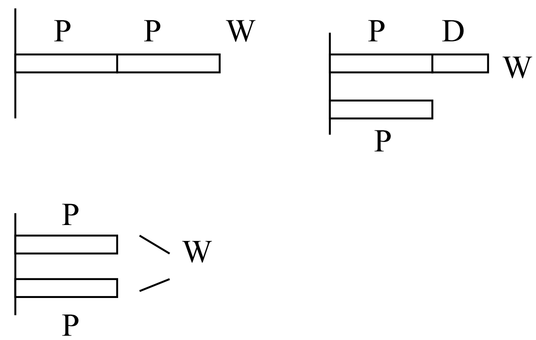
| Stages | Procedures | Criteria |
|---|---|---|
| Stage 2<br>Bar Sets<br>(addition/<br>subtraction) | Model: Draw three types of bar sets. Label part, part, whole; part, part, whole; and part, difference, whole.<br><br>Randomly draw three types of bars. Student labels bars. | Independently and correctly labels all three types of bars twice. |
| Stage 3<br>Math Stories | Use math stories (without a missing variable), place numbers in the corresponding bar sets, model, lead, test each bar set before moving to the next type in the following order.<br><br>One bar (P, P, W); two bars (P, P, W); two bars (P, D, W) | Correctly number three bar sets of each type before moving to the next type.<br><br>Correctly number all types of bar sets with 90% accuracy before moving to Phase 4. |
| Stage 4<br>Problem<br>Setup | Use math word problems.<br><br>1. Read problem<br>2. Rewrite the question as an answer<br>3. Circle who/what the problem is about<br>4. Draw unit bars<br>Demonstrate how to draw one bar for each who/what. | Correctly setup problem 3 consecutive times |

Figure 2: SMM with explicit instruction procedures phase B continued

| Stages | Procedures | Criteria |
| --- | --- | --- |
| Stage 5<br>Addition/<br>Subtraction | 5. Adjust unit bars, add question mark<br><br>Fill the numbers into the corresponding bar set. Adjust the bars to show larger, smaller, or equal bars and numbers. Use the answer sentence (step 2) to determine where to put the question mark. Check if the whole, part, or difference is missing.<br><br>Rule 2: Missing whole, add.<br>Rule 3: Missing part or difference, subtract.<br>Rules 2 and 3 based on Designing Effective Mathematics Instruction (Stein, Kinder, Silbert, and Carnine, 2006)<br><br>6. Solve- Add or subtract<br><br>7. Put answer in sentence.<br><br>Follow steps 5-7 for each type of bar set separately, before combining types of problems. | Complete lessons for each type of bar set. Correctly set up 3 problems for each type before moving to the next type of bar set.<br><br>Solves 85% of the problems correctly on mixed problems to move on to phase 6. |

Phase C: Baseline multiplication/division. Students entered phase C for the second baseline at different times during the study based on the amount of time it took groups of students to progress through phase B. Students were administered Probe-MD for three consecutive sessions prior to receiving instruction on multiplication and division word problems.

Phase D: Instruction multiplication/division. Stage 6 provided instruction on the types of bar sets for multiplication and division and single items versus groups of items (see Figure 3). Students reviewed the concept that bars of equal sizes have the same amount and learned how to label two types of multiplication and division diagrams. The first type included parts and whole for one bar, the second type included parts and whole for more than one bar. Students labeled the two types of multiplication and division diagrams with “p” for part and “w” for whole.

Stage 7 used multiplication and division math stories without any missing variables and students learned to label the bars with numbers (see Figure 3). The first type of multiplication and division diagram was parts and whole for one bar. An example of this type of problem is “There were four plates. Each plate had 3 cookies. There were 12 cookies in all.” The second type of multiplication and division diagram was parts and whole for more than one bar. An example of this type of problem is “Joe, Bianca, and Edwin split 21 pieces of gum equally. They each had 7 pieces of gum.”

Stage 8 included instruction on solving multiplication and division problems using the seven steps of problem solving (see Figure 3). Students completed the first four steps with researcher guidance, before the researcher demonstrated how to complete steps five through seven with multiplication and division problems. The researcher instructed students using researcher-made word problems for multiplication and division. Students received multiple opportunities for practice and feedback from the researcher. During Stage 8, students learned the last four rules: (a) Rule 4: single items add or subtract; (b) Rule 5: groups of items multiply or divide; (c) Rule 6: missing whole, multiply; and (d) Rule 7: missing part, divide. Rules 6 and 7 were based on the Number-Family Problem-Solving Strategy (Stein et al., 2006).

Stage 9 was the final review stage of the intervention (see Figure 3). The researcher provided students with mixed practice of all types of situations for addition, subtraction, multiplication, and division word problems. Students completed this work independently unless they make an error in which the researcher provided corrective feedback and the student reworked the problem. Once the student completed the mixed practice work independently and scored at least 85% correct on three consecutive sessions, that student moved into Phase E- Maintenance. In order to control for a consistent group size during intervention, groups of students moved together through stages into the next phase with the exception of Phase E. Students were able to move into Phase E at their own pace.

The researcher taught all phases of instruction to students in groups of two. All students met a specified criterion within each phase in order to move on to the next phase (see Figures 2 and 3). During instruction, students had a calculator available for computation and steps to SMM were posted at the front of class. The researcher provided immediate error correction by demonstrating the step correctly and required the student to rework the step.

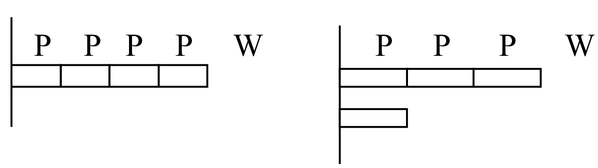
| Stages | Procedures | Criteria |
|---|---|---|
| | Phase D | |
| <u>Stage 6</u><br>Bar Sets (multiplication/ division) | Explain when multiplying and dividing, draw bars a little differently. Each bar is equal to the next bar. Model: Draw two types of bar sets. Label parts and whole. Explain each bar is equal to the next bar.<br><br>Randomly draw two types of bars. Student labels bars.<br><br>Teach difference between single items and groups of items. Single items are by themselves and can have different sizes. Groups have equal amounts, and are in, with, or part of something. | Independently and correctly labels all three types of bars twice. |

Figure 3: SMM with explicit instruction procedures phase D

Figure 3: SMM with explicit instruction procedures phase D continued

| Stages | Procedures | Criteria |
|---|---|---|
| <u>Stage 7</u><br>Math Stories (multiplication/ division) | Use math stories (without a missing variable), place numbers in the corresponding bar sets, model, lead, test each bar set before moving to the next type in the following order.<br><br>One bar (P, P, P, W); two bars (P, P, P, W);<br><br>Review singles and groups. | Correctly number three bar sets of each type before moving to the next type.<br><br>Correctly number all types of bar sets with 90% accuracy before moving to Phase 8. |
| <u>Stage 8</u><br>Multiplication/ Division | Use math word problems.<br>1. Read problem<br>2. Rewrite the question as an answer<br>3. Circle who/what the problem is about<br>4. Draw unit bars<br>5. Adjust unit bars, add question mark<br><br>Read the problem and decide if it is asking about single items or groups of items. Single items add or subtract. Groups of items use multiplication or division.<br><br>Rule 4: Single items add or subtract.<br>Rule 5: Groups of items multiply or divide.<br><br>Use the answer sentence (step 2) to determine where to put the question mark. Check if the whole or part is missing.<br><br>Rule 6: Missing whole, multiply.<br>Rule 7: Missing part, divide.<br>Rules 6 and 7 based on Designing Effective Mathematics Instruction (Stein, Kinder, Silbert, and Carnine, 2006)<br><br>6. Solve-Multiply or divide<br><br>7. Put answer in sentence.<br><br>Follow steps 5-7 for each type of bar set separately, before combining types of problems. | Complete lessons for each type of bar set. Correctly set up 3 problems for each type before moving to the next type of bar set.<br><br>Solves 85% of the mixed multiplication and division problems to move to stage 9. |

Figure 3: SMM with explicit instruction procedures phase D continued

| | | |
|---|---|---|
| Stage 9<br>Mixed-Practice Review | Provide student with addition, subtraction, multiplication, and division word problems to complete independently. | Solves 85% of the problems correctly on mixed problems over three consecutive trials moves into maintenance. |

Phase E: Maintenance. Students entered phase E once they received instruction in phase D and their data for independent practice in stage 9 reached at least 85% accuracy over all types of problems. While in maintenance, students did not attend intervention sessions to receive SMM with explicit instruction. Maintenance data were collected through administration of Probe-AS, Probe-MD, and Probe ASMD to determine if students maintained the skills in word-problem solving with addition, subtraction, multiplication, and division problems. Probe-AS and Probe-MD were given within the first week of maintenance and then once a month for the remainder of the study. Students in the final groups completed maintenance once a week because of time constraints. Probe ASMD was given once during Phase E to determine if students were able to discriminate between problem types upon the completion of all intervention phases.

# CHAPTER 4: RESULTS

This chapter discusses the results of the study, effects of SMM on word-problem solving skills of students identified with and at risk for disabilities. Findings for research questions one, two, and three, which focus on student outcomes, are reported first. Subsequently, research questions four and five, which include social validity questionnaires for students and teachers, are reviewed. Finally, fidelity of implementation data and inter-rater reliability data are included. Results of data collection for all phases are displayed in Figure 4.

## Effects of SMM with Explicit Instruction

Research Question 1. What are the effects of SMM with explicit instruction on addition and subtraction word-problem solving skills of students who are at risk for or identified with learning disabilities?

Students were administered Probe-AS: Addition/Subtraction to assess word problem-solving skills dealing with addition and subtraction problems. Baseline data for Probe-AS were obtained during Baseline A. Probe-AS was recorded as intervention data beginning at Intervention B, and as maintenance data for the remainder of the study. Results of data collection are displayed in Figure 4 (all phases) and Figure 5 (addition and subtraction). Overall, all students' data demonstrated a change in level, trend, and stability of data between baseline and intervention. Findings for groups of students demonstrated consistent data patterns across similar phases between baseline and intervention. Therefore, there was a functional relation between the intervention and dependent variables demonstrated across groups of students. Results of individual student progress are reviewed for each group below.

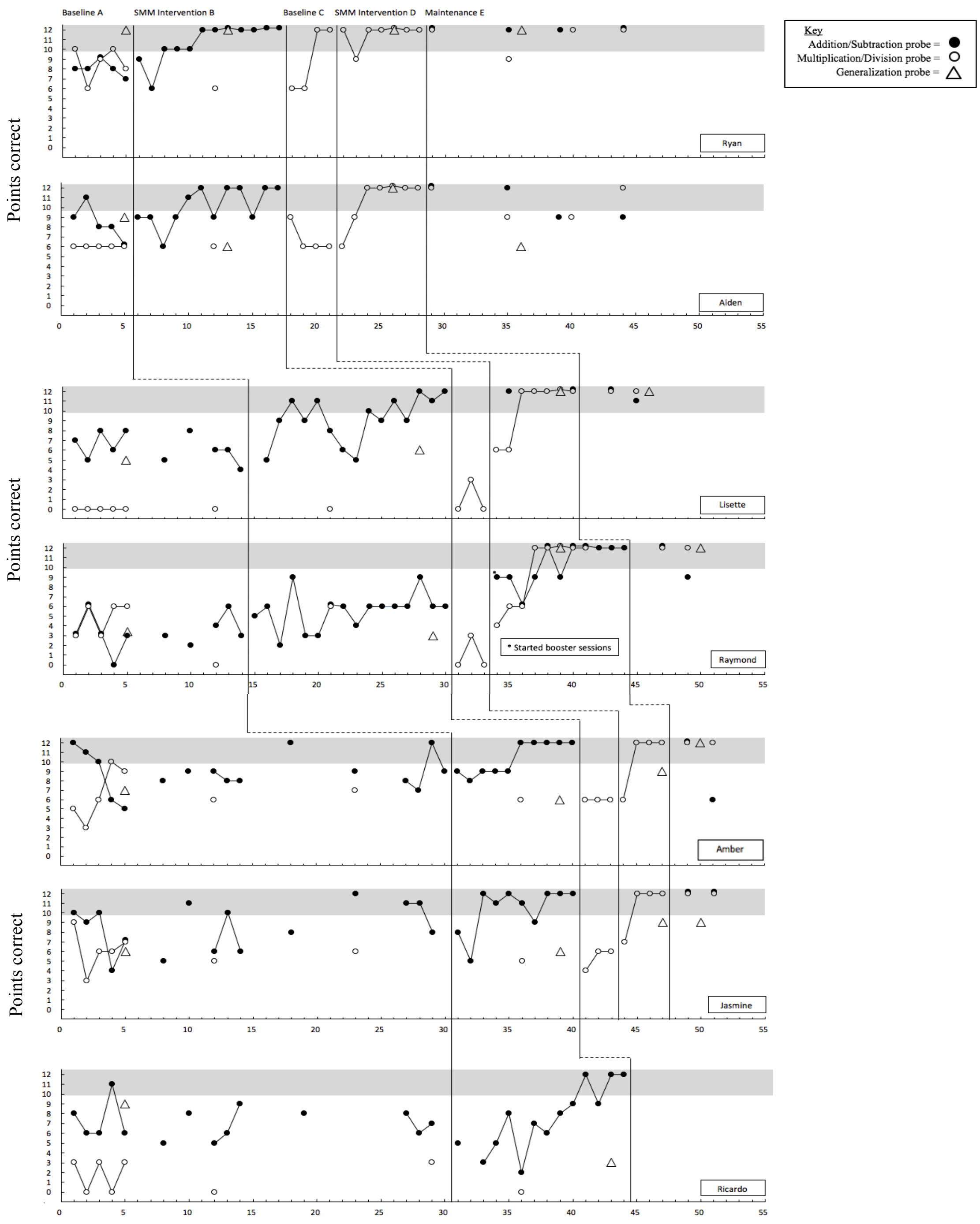



Figure 4: Results across all phases of the study

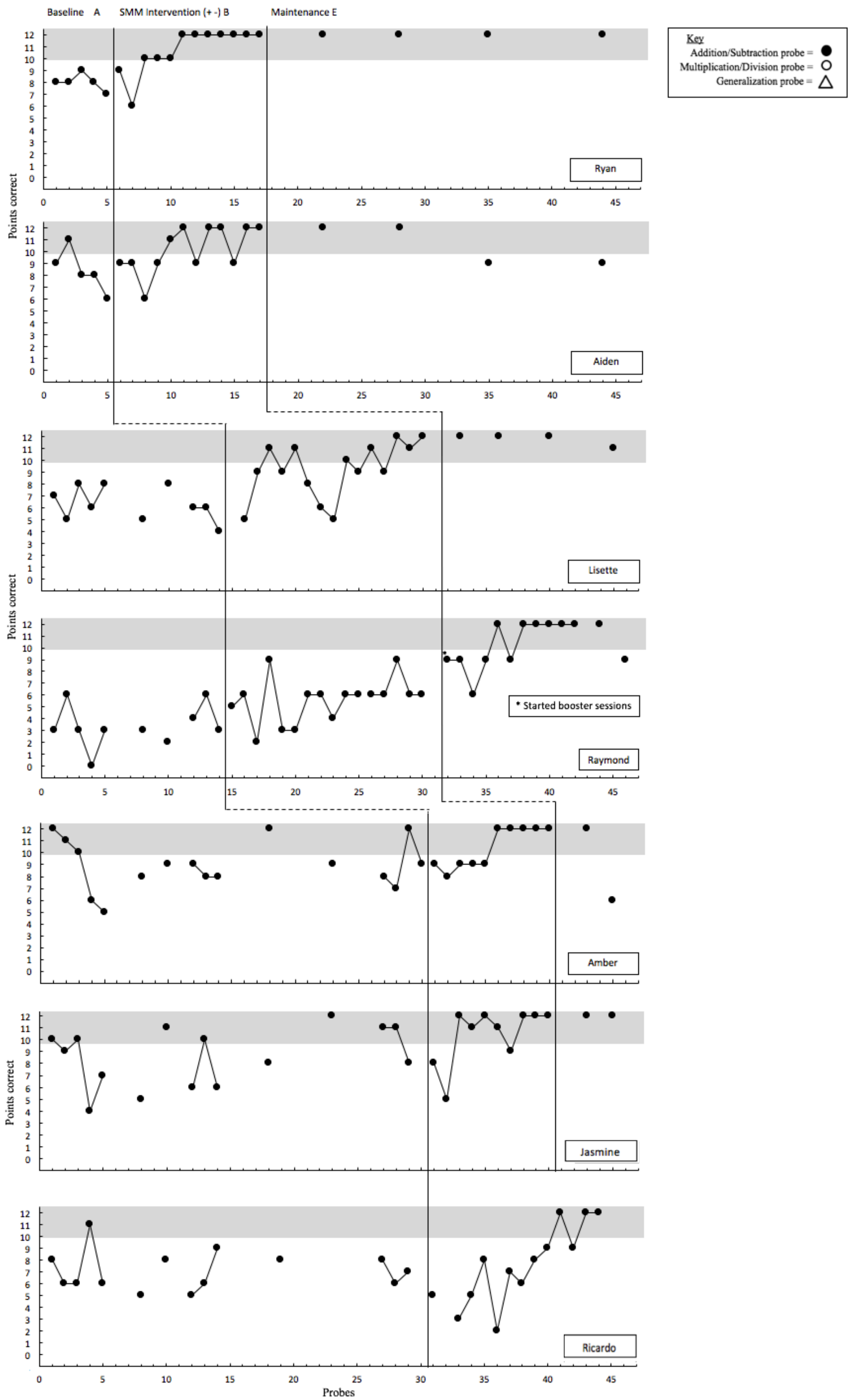



Figure 5: Results of intervention B- addition/subtraction

Ryan and Aiden. The first group to receive Intervention A (addition and subtraction word problems) included two students (Ryan and Aiden) with data showing a decreasing trend in baseline (see Figure 5). Ryan was selected for the first group because his data demonstrated a need for the intervention, and he was the only student already identified with a learning disability. Ethically, it was important to start Ryan in the first group in order to avoid withholding the intervention from the only student with a documented learning disability. During baseline, Ryan's scores ranged from 7-9 points, M = 8.0 points. His scores demonstrated a slightly decreasing trend within baseline. Throughout intervention, Ryan scored between 6-12 points, M = 11.06 points. After an initial decrease, his data demonstrated an increase in trend and level to 10 points and another increase in level that remained stable at 12 points. Ryan gained skills to complete the addition and subtraction probes accurately and consistently prior to completing Phase 5 and remained in intervention until he met the change criteria (three probes scores of 10, 11, or 12 points upon completion of Phase 5). Ryan maintained these skills at 1 week, 1 month, 2 months, and 3 months after completing the intervention scoring at mastery (12 points) on each maintenance probe.

In order to avoid further decreases in demonstrated skill, Aiden was also selected for the first group. During baseline, Aiden's data demonstrated a decreasing trend with scores ranging from 6-11 points, M = 8.4 points. His second baseline data point increased to 11 points and continually decreased over the remaining probes to 6 points. Once in intervention, Aiden's data initially increased, then decreased, and began an increasing trend towards mastery with a range of 6-12 points, M = 10.25 points. Aiden met the change criterion with three scores of 12 (not consecutively) upon completion of phase 5.

His data demonstrated a change in level and trend between baseline and intervention. Aiden maintained these skills demonstrating mastery (12 points) at one week and also one month after completing the intervention. He scored 9 points on the addition/subtraction probe at 2 and 3 months after completing the intervention.

Lisette and Raymond. The second group of students (Lisette and Ryan) began intervention once the first group demonstrated mastery (see Figure 5). In Baseline A, Lisette's data exhibited a range of 4- 8 points, M = 6.3 points. Her data were slightly variable with a decreasing trend. Upon receiving intervention, Lisette's data immediately increased and decreased before gradually increasing and remaining within mastery criteria. Her intervention data demonstrated a range of 5-12 points, M = 9.67 points. Lisette maintained skills at mastery criteria (11-12 points) at 1 week, 1 month, 2 months, and 2 months-1 week after completing the intervention.

Raymond was selected for group two due to his low and variable baseline data. His scores ranged from 0-6 points, M = 3.3 points. After receiving intervention, Raymond's data ranged from 2-9 points, M = 5.56 points. Initially, his data fluctuated and over time remained stable at 6 points. He did not meet mastery criteria for addition and subtraction intervention at the same time as Lisette and remained in intervention to receive booster sessions. Results of Raymond's booster session intervention are included in the results section with Ricardo's data.

Amber and Jasmine. Intervention began for the third group (Amber and Jasmine) of students after Lisette demonstrated mastery of addition and subtraction skills in Group 2 (see Figure 5). Amber and Jasmine were grouped together due to scheduling conflicts with Ricardo. Amber's baseline data were mostly variable and ranged from 5-12 points, M = 8.9 points. Upon entering intervention, Amber's data stabilized at 9 points and increased to 12 points remaining stable at 12 points until she met change criteria after completing Phase 5. The variability of her data decreased with a range of 8-12 points, M = 10.4 points. During maintenance, Amber continued to demonstrate mastery at one month after completion of Phase B (12 points), but at 1 month-1 week, her score dropped below mastery criteria (6 points). Due to time constraints, maintenance was provided weekly during Phase E.

During baseline, Jasmine's data had the highest level of variability compared to other students' data and had a range of 4-12 points, M = 8.5 points. During intervention, Jasmine's data were initially variable with a range of 5-12 points and then stabilized to meet mastery M = 10.4 points. Jasmine's first two scores in intervention were below mastery (8 and 5 points respectively). Her scores on the remaining probes ranged from 9-12 points with seven of the final eight probes meeting mastery. After completing phase 5, Jasmine's scores remained at 12 points consistently. During maintenance, Jasmine demonstrated mastery of addition/subtraction skills at 1 month and 1 month-1 week post completion of Phase B (12 points each). Due to time constraints, maintenance was provided weekly during Phase E.

Raymond and Ricardo. The fourth group (Raymond and Ricardo) began intervention around the same time as the third group (see Figure 5). Raymond was placed with Ricardo for booster sessions and to make a group for Ricardo. They had the same classroom teacher and Ricardo was unable to attend the intervention at the same time as Amber and Jasmine. Booster sessions were an exact repeat of lessons in phases 3-5. After Raymond began attending booster sessions his skill level increased. Initially, his data were variable with a range of 6-12 points, then stabilized at mastery (12 points), M = 10.5 points during booster sessions. Data from Raymond's entire intervention phase ranged from 2-12 points, M = 7.6 points. During maintenance, Raymond continued to demonstrate mastery of addition/subtraction skills 1 week post intervention (12 points), but his score dropped below mastery criteria (6 points) 2 weeks post intervention.

Ricardo's data were variable during baseline with a range of 5-12 points, M = 7.1 points. Upon entering intervention, his data remained variable with a range of 2-12 points, M = 7.5 points. Ricardo met mastery criteria for three probes while working through Stage 5- Solve for Addition and Subtraction Word Problems. He did not receive maintenance probes due to time constraints at the end of the study.

Research Question 2. What are the effects of SMM with explicit instruction on multiplication and division word-problem solving skills of students who are at risk for or identified with learning disabilities?

Students were administered Probe-MD: Multiplication/Division to assess word problem-solving skills on multiplication and division problems. Baseline data for Probe-MD included Baseline A, Intervention B, and Baseline C. Probe-MD was recorded as intervention data beginning at Intervention D and maintenance data at Maintenance E. Results are displayed in Figures 4 and 6. Overall, all students' data demonstrated an increase in level, trend, immediacy of effect, and consistency across similar phases between baseline and intervention for multiplication and division word problems. Therefore, a functional relation was demonstrated across all students between baseline and intervention. Results of individual student progress are reviewed for each group below.

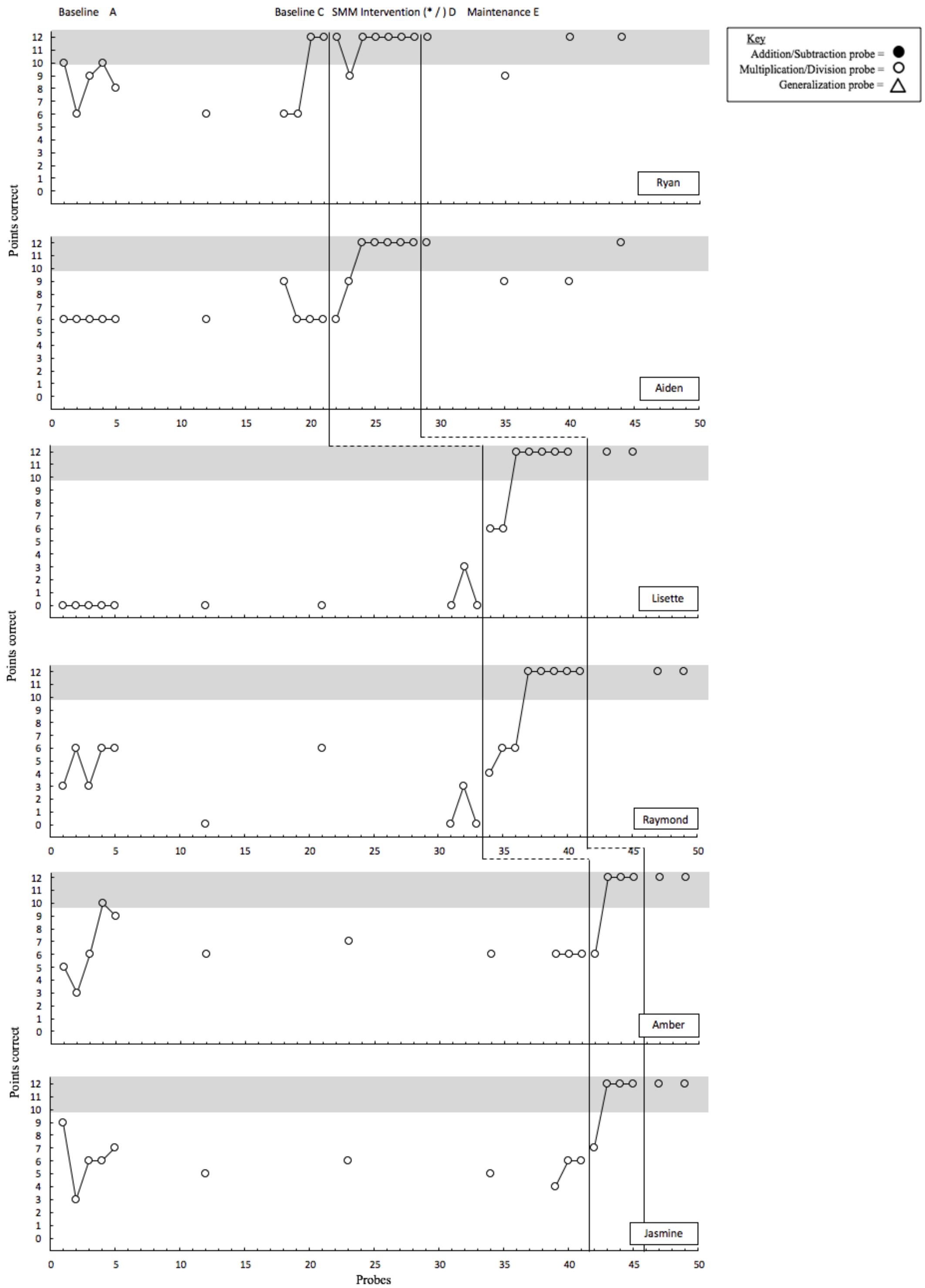



Figure 6: Results of intervention D- multiplication/division

Ryan and Aiden. Upon completion of phases 1-5, Ryan and Aiden began phases 6-9 (see Figure 6 for results). During baseline, Ryan's data were variable with a range of 6-12 points, M = 8.5 points. Just before starting intervention, Ryan completed one probe at mastery therefore a fourth probe was administered to determine if he remained at mastery. His score on the final probe in baseline was also 12 points, although prior probes ranged from 6-10 points. Upon intervention, Ryan's scores remained within mastery level with the exception of the second probe at 9 points. His intervention scores ranged from 9-12 points, M = 11.6 points. With the exception of the probe administered once month after intervention (score of 9 points), Ryan demonstrated mastery of skills at 1 week, 2 months, and 3 months after he completed the intervention (scores of 12 points).

During baseline, Aiden's scores were stable with a range of 6-9 points, M = 6.3 points. Aiden correctly solved two problems during each probe (6 points) in baseline with the exception of one probe in which he correctly solved three problems (9 points). After starting intervention, Aiden's scores demonstrated an increasing trend until he reached mastery, at which point his data remained stable at mastery. During intervention, Aiden's scores ranged from 6-12 points, M= 10.9 points. His data demonstrated an increase in level, trend, and immediacy of effect. Upon entering maintenance, Aiden met mastery criteria twice (score of 12 points) at 1 week and at 3 months after completion of the intervention. However, Aiden did not demonstrate mastery at 1 and 2 months after the intervention (score of 9 points).

Lisette and Raymond. After Lisette demonstrated mastery and Raymond demonstrated improvement in solving word problems using addition and subtraction, Lisette and Raymond began Phases 6-9 (see Figure 6 for results). During baseline Lisette consistently scored 0 points on the multiplication division probes. She completed one question correctly on one probe, because she generalized the SMM steps into a multiplication problem using repeated addition. Lisette's scores in baseline ranged from 0-3 points, M = 0.3 points. After starting intervention, Lisette's scores demonstrated an immediate change in level and trend. Her scores increased to 6 points for two probes and jumped to mastery at 12 points at which point they remained stable for the remaining five probes. Overall, her intervention scores ranged from 6-12 points, M = 10.5 points in intervention. Lisette's intervention data demonstrated an increase in level, trend, and immediacy of effect. During maintenance, Lisette continued to demonstrate mastery of skills with scores of 12 points at 1 and 2 weeks during Phase E (maintenance).

Although Raymond did not meet mastery criteria for addition and subtraction, he remained in a group with Lisette to begin Phases 6-9 (multiplication/division). This decision was made because he was continuing to receive booster sessions for Phases 3-5 (addition/subtraction) and the two skills did not need to be taught sequentially. During baseline, Raymond's scores were variable with a range of 0-6 points, M = 3.3 points. He did not solve more than two questions correct on a single probe during baseline. While in intervention, Raymond's scores ranged from 4-12 points, M = 9.78 points. He demonstrated an increase in level and trend over the first three probes. Raymond met mastery criteria on the fourth probe in intervention (12 points), and remained stable at mastery for the remaining five probes. During maintenance, Raymond continued to demonstrate mastery of the skills with scores of 12 points at 1 and 2 weeks during Phase E (maintenance).

Amber and Jasmine. Once Amber and Jasmine met mastery criteria for Phases 1-5, they began Phases 6-9 (see Figure 6 for results). In baseline, Amber's scores started off variable and stabilized over time with a range of 3-10 points, M = 6.36 points. Her final three consecutive baseline scores were stable at 6 points each. Upon entering intervention, Amber's scores demonstrated an immediate increase in level and trend. She scored 6 points on the first probe in intervention, then increased to mastery at 12 points and remained stable on the subsequent probes. Her data ranged from 6-12 points, M = 11 points, demonstrating a change in level, trend, and immediacy of effect. While in maintenance, Amber continued to demonstrate mastery of the skills with scores of 12 points at 1 and 2 weeks post intervention.

During baseline, Jasmine's data were variable during the first five consecutive probes and became stable over the remaining baseline probes. Jasmine's scores had a range of 0-6 points, M = 5.7 points. After receiving intervention, Jasmine scored 6 points on her first probe and subsequently increased to 12 points stabilizing at mastery for the remainder of the intervention probes. Jasmine's scores ranged from 6-12 points, M = 10.8 points. Her data displayed a change in level, trend, and immediacy of effect. While in maintenance, Jasmine continued to demonstrate skill mastery with scores of 12 points at 1 and 2 weeks post intervention.

Ricardo. Ricardo completed Baseline A and Intervention B only. He was unable to complete Baseline C, Intervention D, and Maintenance E before the end of the study.

Research Question 3. What are the effects of SMM with explicit instruction on combined skills of addition, subtraction, multiplication, and division word-problem solving skills of students who are at risk for or identified with learning disabilities?

Students were administered Probe-ASMD, which assessed combined skills of addition, subtraction, multiplication, and division word problems, during Baseline A, Intervention B, Intervention D, and Maintenance E. Each probe included four problems, one for each basic math function. Given the nature of this probe along with the sequence of instruction, scores were not expected to reach mastery until Intervention D and scores were expected to maintain mastery in Maintenance E (see Figure 4 for results). Students were also provided a pretest and posttest on eight combined word problems (i.e., two addition, two subtraction, two multiplication, two division). The pretest and posttest served as a measure of intervention strength and as a measure of students' ability to discriminate between other combinations of problems (e.g., addition and multiplication problems, subtraction and division problems) as these skills were not assessed on Probe AS or Probe MD (see Table 4).

Table 4: Pretest and posttest scores

| | Pretest Score | Posttest Score | Point Increase |
|---|---|---|---|
| Ryan | 50% (12/24 pts) | 100% (24/24 pts) | +12 pts |
| Aiden | 33% (8/24 pts) | 75% (18/24 pts) | +10 pts |
| Lisette | 33.3% (8/24 pts) | 75% (18/24 pts) | +10 pts |
| Raymond | 25% (6/24 pts) | 75% (18/24 pts) | +12 pts |
| Amber | 37.5% (9/24 pts) | 95.8% (23/24 pts) | +14 pts |
| Jasmine | 25% (6/24 pts) | 87.5% (21/24 pts) | +15 pts |
| Ricardo | 37.5% (9/24 pts) | 58.3% (14/24 pts) | +5 pts |

Ryan and Aiden. On Probe ASMD (see Figure 4), Ryan met the mastery criterion (12 points) for all administrations of the probe (Baseline A, Intervention B, Intervention D, and Maintenance E). His data remained stable across all phases for Probe-ASMD. Ryan scored 50% on the pretest. He correctly solved one addition, one subtraction, and two multiplication problems on the pretest. On the posttest, Ryan scored 100% (an increase of 12 points) and correctly solved all types of problems.

On Probe ASMD, Aiden scored 9 points at Baseline A, 6 points at Intervention B, and increased to 12 points at Intervention D. Aiden did not maintain the skills of differentiating between addition/multiplication problems and subtraction/division problems as he scored 6 points at Maintenance E, one month after completing the intervention. Aiden scored 33.3% on the pretest. He correctly solved two multiplication problems and received partial credit for one subtraction problem on the pretest. On the posttest, Aiden scored 75% (an increase of 10 points) and correctly solved one addition, two subtraction, one multiplication, and two division problems.

Lisette and Raymond. On Probe-ASMD (see Figure 4), Lisette initially scored 5 points at Baseline A, 6 points at Intervention B, and increased to 12 points at Intervention D and Maintenance E. Lisette scored 33.3% on the pretest. She correctly solved two addition problems and received partial credit for one subtraction problem on the pretest. On the posttest, Lisette scored 75% (an increase of 10 points) and correctly solved two addition, one subtraction, two multiplication, and one division problems.

On Probe-ASMD, Raymond initially scored 3 points at Baseline A, 3 points at Intervention B, and increased to 12 points at Intervention D and maintained mastery at Maintenance E (12 points). Raymond scored 25% on the pretest. He correctly solved two addition problems on the pretest. On the posttest, Raymond scored 75% (an increase of 12 points) and correctly solved one addition, one subtraction, two multiplication, and two division problems.

Amber and Jasmine. On Probe ASMD (see Figure 4), Amber initially scored 7 points at Baseline A, 6 points at Intervention B, 9 points at Intervention D, and 12 points at Maintenance E. Amber scored 37.5% on the pretest. She correctly solved two addition problems and one subtraction problem on the pretest. On the posttest, Amber scored 97.8% (an increase of 14 points) and correctly solved one addition, two subtraction, two multiplication, and two division problems. Amber made a calculation error solving the other addition problem and was deducted 1 point.

On Probe ASMD, Jasmine initially scored 6 points at Baseline A, 6 points at Intervention B, and 9 points at Intervention D, and 9 points at Maintenance E. Jasmine scored 25% on the pretest. She correctly solved one addition and one subtraction problem on the pretest. On the posttest, Jasmine scored 87.5% (an increase of 15 points) and correctly solved two addition, two subtraction, two multiplication, and one division problem.

Ricardo. On Probe ASMD (see Figure 4), Ricardo initially scored 9 points at Baseline A, and 3 points at Intervention B. He did not complete phases C, D, or E nor complete the last two ASMD probes. Ricardo scored 37.5% on the pretest. He correctly solved two addition problems and one subtraction problem on the pretest. On the posttest, Ricardo scored 58.3% (an increase of 5 points) and correctly solved two addition, two subtraction, and one multiplication problem. He miscalculated one addition problem and received only 2 points for that problem. As Ricardo only completed the addition and subtraction intervention, it was expected that he would score around 50% on the posttest.

Social Validity

Social validity questionnaires were distributed to students and teachers in order to gain perceptions of the usefulness and effectiveness of SMM with explicit instruction.

Research Question 4. What are student participants' perceptions of SMM with explicit instruction?

In order gain insight on student perceptions of using SMM to solve math word problems, a social validity questionnaire was provided to all seven of the student participants. Results of the student social validity questionnaire are displayed in Table 5. Most students (n = 5) indicated "yes" the lessons were helpful, the steps were easy to follow, the rules helped to understand what math function to use (remaining students [n = 2] selected "I don't know"). Most students (n = 5) indicated they would like to use this model with other types of math problems (one student marked "no," and the other marked "I don't know"). Students were split between "yes" and "I don't know" on whether or not, drawing bars helped with understanding; labeling part, difference, whole helped with understanding; and they were better at solving math word problems. The majority (n = 6) of students indicated they would like their teachers to use this model in their class, and interestingly, even though the teachers have all been trained on this model, all students (n = 7) indicated they had not used this method in their classroom. Although one student wrote down that he/she used this method one time in class.

Table 5: Summary of students' responses on social validity questionnaire

| | No<br>1 | I don't know<br>2 | Yes<br>3 |
|---|---|---|---|
| The model method lessons helped me to understand how to do math word problems. | | 2 | 5 |
| The 7 steps in the model method were easy to follow. | | 2 | 5 |
| Drawing the bars helped me understand what the word problem was asking. | | 3 | 4 |
| Labeling the bars with parts and whole, or part, difference, whole helped me solve the problems. | | 4 | 3 |
| The rules about missing the part and missing the whole helped me learn if I should add or subtract and multiply or divide. | | 2 | 5 |
| I am better at solving word problems now. | | 3 | 4 |
| I use the model method to do word problems in my classroom. | 7 | | |
| I would like my classroom teacher to use the model method during math. | | 1 | 6 |
| I would like to learn how to use the model method to solve other kinds of math word problems. | 1 | 1 | 5 |

Research Question 5. What are teachers' perceptions of SMM with explicit instruction?

In order gain insight on teacher perceptions of the effectiveness and usefulness of SMM to solve math word problems, a social validity questionnaire was provided to the teachers. All teachers were provided a graph/graphs of their students' final data and a link to a video of SMM instruction, including a time stamp for working through one problem. Results of the teacher social validity questionnaire are displayed in Table 6.

Table 6: Summary of teachers' responses on social validity questionnaire

| | Strongly Disagree | Disagree | Agree | Strongly Agree |
|---|---|---|---|---|
| 1. My students learned how to solve word problems from this strategy. | | 1 | 2 | |
| 2. I saw a change in my student's math skills in my classroom after receiving the strategy. | | 1 | 2 | |
| 3. The 7 steps to the model method are easy to understand. | | | 3 | 1 |
| 4. Teaching how to solve word problems will be easier using the model method. | | 2 | | 2 |
| 5. This strategy would be feasible to implement in my classroom. | | 1 | 1 | 2 |
| 6. I would like to use this strategy in my class. | | 2 | | 2 |
| 7. I would like training on how to use this strategy in my class. | 2 | 1 | 1 | |

Overall, teachers' opinions of this strategy varied across most questions. Of the six teacher participants, all teachers answered the background questions on whether or not they received training on Model Drawing (i.e., variation of SMM; yes- n = 5, no- n = 1) and if they were grade level math teachers (yes- n = 3, no- n = 3). Two of the teachers who selected "no" were grade level reading/social studies teachers and did not complete the rest of the survey (even though their homeroom students were participants in the study). Of the three grade level math teachers, all indicated they received training on Model Drawing and used Model Drawing to teach math problem solving (n =3). All seven student participants in the study were taught by the three grade level math teachers. Perceptions were most favorable for item 3. The 7 steps to the model method are easy to understand (n = 4, M = 3.25), and item 5. This strategy would be feasible to implement in my classroom (n = 4, M = 3.25). Teachers opinions were split (disagree- n = 2, strongly agree- n = 2, M = 3) on items, 4. Teaching how to solve word problems will be easier using the model method, and 6. I would like to use this strategy in my class. Items on whether or not a change in student behavior was observed were rated by three teachers and split between disagree and agree (n = 1, n = 2; M = 2.67) respectively for items, 1. My students learned how to solve word problems from this strategy, and 2. I saw a change in my student's math skills in my classroom after receiving the strategy. The least favorable item was 7. I would like training on how to use this strategy in my class (n = 4, M = 1.75).

Fidelity of Implementation

To ensure the SMM with explicit instruction intervention was implemented with fidelity, a fidelity of implementation checklist (see Appendix C) was completed by a trained observer on 23 out of 72 lesson days for a total of 31.9% of session days distributed across groups and phases of intervention. The fidelity checklist included 7 items: (a) Followed SMM steps, (b) Followed SMM procedures, (c) SMM steps posted, (d) Used bar modeling, (e) Used rules, (f) Error correction as needed, and (g) Provided praise. Items were marked Yes, No, NA based on whether or not they were observed during the lessons. Fidelity was scored by adding the number of Yes' and NA's together, then dividing by 7 (the total score). The overall mean fidelity score was 98.1% across lesson days with a range of 85.7%-100%. Of the 23 observations, 20 lessons scored

100% (7/7) fidelity. Three lessons scored at 85.7% (6/7) because the SMM steps were not posted during the lessons.

Inter-rater Reliability

Reliability data were collected on the dependent variables for Probe-AS, Probe-MD, and Probe-ASMD. The interventionist scored all probes without marking on the original probes. A trained rater scored 30.2% (n = 104) of all probes (n = 344) equally distributed across students, phases, and probe types (Probe-AS = 30.1%, Probe-MD 30.4%, Probe-ASMD 30.8%). Answers to word problems were scored based on predetermined criteria (i.e., correct number sentence and answer = 3 points, correct number sentence = 2 points, correct answer = 1 point, correct visual representation of problem with correct answer = 2 points). Inter-rater reliability data were calculated item by item between the trained rater and interventionist. Each problem was worth up to three points. Problems rated the same (agreements) received 3 points, problems not rated the same (disagreements) received 0-2 points. Problems in which there was a difference of one between scores received 2 points, problems in which there was a difference of two between the scores received 1 point, problems in which there was a difference of three between scores received 0 points. This was to account for any agreements within scoring parts along with disagreements. For example, there was a disagreement where the interventionist scored the problem as 3 points because the number sentence was correct (2 points) and the interventionist thought answer was correct (1 point), and the rater scored the same problem as 2 points because the number sentence was correct (2 points) but the rater thought the answer was not correct (0 points). When calculating the reliability score for that problem, the agreement (number sentence correct) received 2 points, while the disagreement (answer correct/answer not correct) did not receive any points. Total agreements were added and divided by the total number of agreements plus disagreements for each probe.

Inter-rater reliability for all probes ranged from 95-100% with a mean of 99%. Inter-rater reliability for Probe-AS ranged from 95-100% with a mean of 98.9%. Inter-rater reliability for Probe-MD ranged from 96-100% with a mean of 99.2%. Disagreements in scoring were attributed to the quality of photocopies of some of the probes. For example, the student erased the answer and rewrote the correct answer. Inter-rater reliability for that probe was not the same because on the photocopy of the probe, the erased portion was visible and it was hard to tell which number the student wanted as the answer. In person, it was clearer that the student erased the incorrect answer and wrote the correct answer.

# CHAPTER 5: DISCUSSION

The purpose of this study was to determine the effects of SMM with explicit instruction on math word problem solving skills of students at risk for or identified with LD. Specifically, this study sought to determine whether providing explicit instruction on the use of SMM would increase students' ability to answer single-step addition and subtraction problems as well as multiplication and division problems. A multiple probe across participants with an ABCDE design was used to demonstrate the effects of the intervention on both dependent variables. The multiple probe across participants with an ABCDE design essentially served as two separate multiple probes implemented within one study. The first multiple probe across participants design (phases ABE) evaluated the effects of the intervention on students' ability to solve addition and subtraction word problems. The second multiple probe across participants design (phases ACDE) evaluated the effects of the intervention on students' ability to solve multiplication and division word problems. Baseline for both dependent variables started at the same time (during phase A) to avoid a delayed multiple probe design for multiplication and division word problems. Results of the study demonstrated a functional relation between the independent variable and the addition and subtraction dependent variable across all seven students. A functional relation was also demonstrated between the independent variable and multiplication and division dependent variable for six of the seven students (the study ended before the final student could receive instruction in phases CDE on multiplication and division). Results also demonstrated an increase between pretest and posttest scores for all students on a combined assessment with addition, subtraction, multiplication, and division word problems. The combined probe shows variable results across all participants during the study. Social validity was collected to gather student opinions of using SMM with generally positive results. Social validity data were also collected on teachers' perceptions of the intervention. Results were slightly favorable, but mixed across respondents. This chapter is organized in sections discussing results of each research question, specific contributions of the study, limitations of the study, recommendations for future research, implications for practice, and a summary.

## Effects of the Independent Variable on the Dependent Variable

Research Question 1: What are the effects of SMM with explicit instruction on addition and subtraction word-problem solving skills of students who are at risk for or identified with learning disabilities?

Results from this study demonstrated a functional relation between SMM with explicit instruction (independent variable) and an increase in students' ability to solve addition and subtraction word problems (dependent variable) across all students, as well as strong maintenance of skills over time. During baseline the prediction was made that students' scores would remain variable for addition and subtraction word problems. Once the intervention was introduced the expectation was that data would begin to increase and stabilize within the mastery criteria range (i.e., 10-12 points per probe). This expectation was verified after the first group of students received intervention and demonstrated an increase in scores into the mastery criteria range. This first group took around 6 weeks to complete Phases A and B. Ryan's scores increased and remained consistently stable.

Aiden's scores increased and met mastery criteria with three of four final scores at mastery, but his maintenance scores remained at mastery at one week and one month after intervention ended. Replication was demonstrated across the remaining students' baseline and intervention phases.

In the second group, Lisette met mastery criteria and maintained those skills over time at a similar rate to the first group (8 weeks). Raymond's data demonstrated an increase between baseline and intervention, but did not meet mastery criteria after completing phase 5 within 2 months. Due to his increasing skills, the decision was made to provide Raymond booster sessions on the addition and subtraction problems while simultaneously moving him into the next phase (Phases C and D) for multiplication and division. As there were seven participants, instead of creating a group of three for the third group, Raymond's booster sessions were provided with Ricardo (a student from the remaining group of three). The other two students (Amber and Jasmine) made the final group. Ricardo received instruction on stages 1-2 while Raymond worked on a mixed-problem review. Stage 1 (equal and unequal bars) and Stage 2 (bar sets) typically took 1-2 days to complete, therefore Ricardo was provided one-on-one instruction for 2 days (although Raymond was sitting next to him working on review problems) and started Stage 3 (math stories) with Raymond on the third day. They remained working together through Stage 4 (problem set-up) and Stage 5 (solve addition/subtraction). Ricardo demonstrated mastery of skills in addition and subtraction problems (within 10 weeks). Once Raymond repeated the same lessons in stages 3-5, his data improved and he demonstrated mastery and maintenance of the skill across time. During the time Raymond was receiving booster sessions, his teacher made a referral for Raymond to be evaluated with a learning disability. Raymond was the only student receiving Tier 3 supports and was not making adequate progress based on the school's RTI standards. At the meeting, Raymond's parents did not provide consent to an evaluation for special education services. Raymond may have taken twice as long to demonstrate mastery of addition/subtraction problems due his need for more repetition and practice (a characteristic of students with LD).

Results for Amber and Jasmine also demonstrated an increase in data between baseline and intervention. Both students demonstrated mastery within 6 weeks of instruction. Data stabilized and demonstrated a change in level between baseline and intervention phases. Skills were maintained over time for both students. Overall, intervention time for addition and subtraction took almost twice as long (6-8 weeks) as the intervention time in multiplication and division (4 weeks). This could be because the addition and subtraction schemas (n = 8) took longer to teach than the multiplication and division schemas (n = 6). The eight schemas used within addition and subtraction were more variable than the six in multiplication and division. For example, schemas in addition and subtraction included add to (combine), take from (change), take apart (combine), and compare with various missing variables. Schemas in multiplication and division included equal groups or arrays with unknown total, unknown groups, or unknown number of groups.

These findings were consistent with other research on SBI (Fuchs et al., 2004; Jitendra et al., 1998; Jitendra et al., 2013b) that demonstrate an increase in skill for solving addition and subtraction word problems by using schemas to diagram the problems, as well as using word stories to teach schemas and number placement before moving to word problems (Jitendra et al., 1998; Jitendra et al., 2013b). Findings from the study are consistent with research on CSI (Jitendra et al., 2013; Montague et al., 2011; Montague & Dietz, 2009) that demonstrate an increase in skill with the use of metacognitive skills to determine the steps to use and to check the answer. This research also supports the findings of other research on the positive outcomes of the use of explicit instruction (Gersten et al., 2009b; Jayanthi et al., 2008; Zheng et al., 2012) to teach math including word problem solving skills.

Research Question 2: What are the effects of SMM with explicit instruction on multiplication and division word-problem solving skills of students who are at risk for or identified with learning disabilities?

Overall, results from this study indicate a functional relation was demonstrated between the intervention and dependent variables of solving multiplication and division math word problems (with the exception of the final student who did not receive any instruction in Phases CD on multiplication and division). This section will discuss results of the six students who completed Phases CDE. Ryan's data were variable during baseline when solving multiplication and division word problems. Just before beginning intervention his data increased to mastery. On the other hand, prediction for Aiden's data to remain stable at baseline was strong, and his data demonstrated a change in level, trend, and immediacy of effect upon entering intervention.

Replication of effect was demonstrated in the following two groups. Lisette's data for multiplication and division were low and stable during baseline. She did not correctly solve any multiplication or division problems with the exception of one problem in Baseline C before starting intervention. Lisette generalized the SMM schema for addition and diagramed a multiplication problem as repeated addition resulting in one correct answer over the entire baseline for multiplication and division. Upon entering intervention, her data demonstrated a change in level, trend, and immediacy of effect and maintained those skills at the mastery level. During baseline, Raymond solved two or fewer problems correct, upon entering intervention his data demonstrated a change in level and trend, with immediacy of effect by the fourth intervention probe. His growth was maintained over time.

The final group replicated the effects seen within the first two groups. Both Amber and Jasmine had variable baseline data that stabilized over time and an immediate change in level and trend with immediacy of effect. The two students also maintained mastery criteria of skill at one and two weeks after intervention.

The strength of the multiplication and division intervention may be due to fewer schemas needed to solve the problems. Most students met mastery criteria by the third probe (Raymond met mastery criteria at fourth probe). Once students started Stage 7 and saw relationships between fact families in word stories, they began applying the multiplication facts to the word problems resulting in immediate increases on probe data. This increase in skill level was supported more when students moved into Stage 8 and began solving multiplication and division word problems.

Findings from this study are supported through research in using SMM to solve multiplicative comparisons (Mahoney, 2011), and using word stories before word problems in SBI (Jitendra et al., 1998; Jitendra et al., 2013b). This research is also consistent with studies that used metacognitive skills to determine the steps and to check answers (Jitendra et al., 2013; Montague et al., 2011; Montague & Dietz, 2009), as well as positive effects of the use of explicit instruction to teach math skills to students at risk for or identified with LD (Gersten et al., 2009b; Jayanthi et al., 2008; Zheng et al., 2012).

Research Question 3: What are the effects of SMM with explicit instruction on combined skills of addition, subtraction, multiplication, and division word-problem solving skills of students who are at risk for or identified with learning disabilities?

Results from this question were demonstrated across two measures. The first measure included a pretest and posttest that included eight questions with two each on addition, subtraction, multiplication, and division word problems. All six students who completed intervention Phases ABCDE made dramatic increases between pretest (M = 34%) and posttest measures (M = 85%). Each student increased the number of correctly solved addition, subtraction, multiplication, and division word problems (with the exception of Raymond who correctly solved the same number of addition problems on the pretest and posttest). On the pretest, none of the students correctly solved division problems. In contrast, all students who completed all phases correctly solved at least one of the two division problems on the posttest. Error analysis of each posttest revealed that all errors were made within the same function family (e.g., addition and subtraction, multiplication and division). This demonstrates that students were able to discriminate across problem types and did not add when the problem required multiplication or divide instead of subtract. The seventh student, Ricardo, increased his performance on the pretest and posttest from 38% to 58%, with an increase of five points between pretest and posttest scores. His score was expected to reach up to 50% on the posttest as he did not complete the multiplication and division intervention (Phases CDE), and did not correctly solve any multiplication or division problems on the pretest.

The other measure used to analyze whether students could solve combined problems of addition, subtraction, multiplication, and division was Probe ASMD. This probe was administered four times (Baseline A, Intervention B, Intervention D, Maintenance E). The prediction was that students' data would be variable on the first two probes and reach mastery criteria by the third and fourth probe. Results were variable across students on Probe ASMD. Three students (i.e., Lisette, Raymond, Amy) followed the predicted path and displayed mastery of skills over time. One student (i.e., Ryan) correctly answered all questions each time. One reason for Ryan's success on Probe ASMD could be because his average baseline scores were over 50% on individual probes (i.e., Probe AS, M = 8.0 pts, Probe MD, M = 8.5 pts). This means he correctly solved at least two to three problems in each probe. Because Probe ASMD used one of each type of problem, and based on the fact that he was correctly solving more than half the problems, it could be by chance the problems chosen for Probe ASMD were problems he understood how to solve. The remaining three students (i.e., Aiden, Jasmine, Ricardo) demonstrated variable results over time. Aiden met mastery on the third Probe ASMD, but missed one problem on the fourth Probe ASMD. The fourth probe was given a month after intervention ended. That could have been too long of a time period for him to maintain the skills and be able to discriminate between problem types. Jasmine made growth across time, but missed one problem on the third and fourth Probe ASMD. Given that Ricardo only completed half the intervention, he made growth beyond his expected level on the two probes administered.

Currently, there is a lack of research that specifically focuses on evaluating if students can solve addition, subtraction, multiplication, and division word problems within one intervention study. This research is the only study that provided preliminary results on a heuristic, schema-based, cognitive-strategy method that uses explicit instruction to teach word problem solving skills for addition, subtraction, multiplication, and division word problems.

## Discussion of Social Validity Findings

Research Question 4: What are student participants' perceptions of SMM with explicit instruction?

In order to determine students' perceptions of using SMM to solve math word problems, a student questionnaire was provided to all student participants. Overall, most students (n = 5) indicated SMM was helpful in solving math word problems, found steps easy to follow, indicated the rules helped know which function to use (i.e., add or subtract, multiply or divide), and would like to use this model to solve other types of word problems. These results are consistent with research that supports teaching rules through explicit instruction benefits student understanding (Stein et al., 2006). Student opinions were split between "I don't know" and "yes" on questions about the components of the intervention (i.e., drawing and labeling bars). Students' opinions were also split between "I don't know" (n = 3) and "yes" (n = 4) on "I am better at solving math word problems." At the time of providing students the questionnaire, three students had not been shown a graph of their results. The students in the first two phases were shown a graph with their results at the end of the interventions. The remaining students saw their graphs a few days after completing the survey. This could have impacted the students' opinions on whether or not they believed they were better at solving math word problems.

Another interesting finding was the majority of students (n =6) indicated they would like their teacher to use this strategy in class. Despite that all teachers had been trained to use Bar Modeling, which is based on SMM, all of the students indicated they did not use this strategy in class. One student wrote that he/she used bars like this one time in class. This finding is consistent with research that indicates professional development alone is not enough for teachers to implement new strategies in class and coaching increases the probability of teachers implementing new strategies (Joyce & Showers, 2002).

Students were also provided the following open-ended questions: I liked, and I did not like. One student commented that he/she liked “when we had to do more people and PDW and the squares” (then drew a picture of the compare schema). Another student stated, “I liked how I draw the bars and multiplying, divideing [sic] subtracking [sic] and adding.” Other comments included liking, “the word problems,” “that it was easy,” “meeting new people in the group,” “doing the math and fegering [sic] (figuring) the answer,” and “I like the part part whole.” Only two students included a statement on what they did not like. One student indicated he/she did not like “puting [sic] answer in a sentens [sic].” The other stated, “I did not like difference.” These findings were important to the study because students with disabilities can be reluctant to use a more efficient strategy over a previously learned (less efficient) strategy regardless of how effective it is (Jitendra & Montague, 2013; Swanson, 1990). This concept is supported in this study because some of these students (who demonstrated major improvements on solving word problems) were unaware of their achievements or why this strategy was more effective without being provided a graph of their progress. Not to mention, all students indicated they did not use this skill in their classrooms.

Research Question 5: What are teachers' perceptions of SMM with explicit instruction?

Overall on a scale of 1-4, teacher perceptions of SMM were split but slightly positive on the effectiveness, ease of use, and desire to use this strategy in the classroom ($M = 2.67$, $M = 3$, $M = 3$) respectively. Of the seven teachers who completed the survey, four teachers taught math (i.e., three general education, one special education) and completed the remaining questions. All teachers ($n = 4$) agreed or strongly agreed that the steps were easy to understand, and most ($n = 3$) agreed and strongly agreed the strategy would be feasible to implement in the classroom. Half the teachers ($n = 2$) strongly agreed that they would like to use the strategy in their classroom and half disagreed ($n = 2$) with that statement.

Despite being provided graphs of each student's individual progress on the word problem solving probes (and that all students demonstrated growth), one grade level math teacher indicated she disagreed that her student made growth and did not see a change in her student's math skills in the classroom. No comments were provided to support her perceptions. Although it is understandable that the student may not have transferred the skills to the classroom and did not demonstrate a change in the classroom, it is unclear why the teacher disagreed that the student made growth based on the graph that demonstrated clear growth between both baseline and intervention phases.

Another interesting finding is that five of the six teachers have been trained in Model Drawing (a heuristic and bar modeling strategy derived from SMM) and all three math teachers indicated they have using Model Drawing to teach math problem solving in class. In contrast, all students indicated “no” they had not used this strategy in class. Explanations include the following: (a) students were out of the room receiving intervention instruction (from this study or other small groups) when the teacher used Model Drawing to teach math problem solving, (b) students did not generalize that SMM was similar to Model Drawing, (c) Model Drawing was not taught explicitly through systematic phases, and (d) Model Drawing was not used consistently enough for students to remember using it. One student indicated on the student survey that he/she used this strategy once in class.

Despite the success of the students receiving SMM, only one teacher selected “agree” for the item, I would like training on how to use this strategy in my class, one teacher selected “disagree,” and two teachers selected “strongly disagree.” Reasons could be because most of the teachers were trained in Model Drawing and although they were informed at the onset of the study, they may not have realized the differences between SMM with explicit instruction and Model Drawing. SMM is taught through phases; uses model, lead, test; includes problem solving rules; mastery criteria to move through stages, while Model Drawing is taught as a heuristic by following steps to solve a word problem. Another reason, supported by research, is more experienced teachers (five or more years experience) do not feel they benefit as much from professional development workshops as observing other classrooms or attending educational conferences (Mahmoudi & Ozkan, 2015).

Two teachers left comments in the additional comments/suggestions box: “Thank you for your work with our students. I can see where the small group instruction benefited my math students.” “I have used this strategy the past two years to help teach math problem solving. I find this strategy to work well when problems are worded a particular way only. I think this is an effective strategy to use at times, but not the only strategy that should be taught. I am grateful to Ms. Preston for the work she has done. I did see growth in student understanding of math problems. I found they could tackle problems with more ease. Thank you."

## Specific Contributions of the Study

This study makes strong contributions to the literature as the only study to develop an instructional format using explicit instruction to teach SMM to students who are at risk for or identified with LD and the only study that evaluates the use of SMM with explicit instruction on math problem solving skills of students who are at risk for or identified with LD. This study also contributes to literature on use of explicit instruction to teach mathematics concepts and on Tier 2 problem solving strategies for students at risk for LD. Finally, this study contributes to the literature by including treatment fidelity measures that were not typically included in previous research (Montague & Dietz, 2009).

First, this investigation is the only study to break down SMM, a problem solving heuristic, into stages of instruction and develop a format that used explicit instruction, bar modeling, rules, schema-based instruction, and strategy instruction to teach math word problem solving skills to students at risk for or identified with LD. This study also used an experimental single-case research design to evaluate the effects of SMM with explicit instruction on word problem solving skills of students who are at risk for or identified with LD. The two previous studies included evaluating SMM with typically achieving students. In a non-experimental posttest only design that evaluated students use of SMM while solving word problems, Ng and Lee, (2009) evaluated only the students streamed in the top achieving and middle achieving levels (i.e., EM1, EM2) and did not include students in the study who were streamed into the lowest achieving level (i.e., EM3). As students in the middle level had more difficulty using SMM than students in the top level, it can be assumed students in the lowest level would have the hardest time using SMM to solve word problems. This assumption supports the idea that breaking down SMM into stages of instruction and incorporating explicit instruction into the design could potentially benefit students with disabilities (or the lowest achieving students). The other study evaluated the use of SMM with typically achieving students in a delayed multiple baseline using SMM to solve problems using multiplicatives and fractions (Mahoney, 2011). Results of the current study demonstrate that through explicit instruction, students who were at risk for or identified with LD were able to learn the steps of SMM to solve word problems involving addition, subtraction, multiplication, and division. An interesting finding was the only student identified with a learning disability had the most stable data during intervention and maintenance. Visual analysis of the results concluded the intervention provided an obvious change in level, trend, and variability across all students completing the intervention phases and most students maintained those skills over time.

Second, this study contributes to the literature that supports teaching mathematics concepts through explicit instruction. There is empirical evidence that indicates using explicit instruction to teach mathematics concepts (including problem solving) is effective for students with disabilities (Gersten et al., 2009b; Jayanthi et al., 2008; Zheng et al., 2012). This study used explicit instruction to teach a heuristic model to solve math word problems. The instructional design behind the explicit instruction included the use of big ideas, model-lead-test, use of rules, multiple exemplars, strategic integration, and conspicuous strategies (Coyne et al., 2011). SMM as a strategy has research that supports its components including heuristics, visual representations, schemas, and cognitive strategies (Gersten et al., 2009b; Giordano, 1992; Jayanthi et al., 2008; Xin & Jitendra, 1999; Zhang & Xin, 2012), but the explicit instruction added to SMM was demonstrated as effective for students at risk for or identified with LD. These results are consistent with other findings in research on the strength of explicit instruction to teach problem solving skills (Jayanthi et al., 2008; Zheng et al., 2012).

Third, this study also adds to the body of research on math word problem solving interventions for students who are at risk for or identified with LD within an RTI framework. The results of this study indicate that students who are at risk for or have LD may increase their math word-problem solving skills when taught a heuristic strategy with schema and cognitive strategies, through explicit instruction. This study was conducted at a school that used bar modeling (derived from SMM) to teach problem solving in general education classes. All three grade-level mathematics teachers indicated they had been trained on bar modeling and used it to teach math problem solving. In other words, bar modeling was used as part of Tier 1 core math instruction in this school. Teachers nominated students who (despite having had instruction on problem solving) were performing below grade level on problem solving skills based on classwork, the End of Grade assessment, Discovery Ed scores, and were receiving or were in the process of receiving Tier 2 interventions. These students also needed a more intensive intervention for mathematics problem solving. SMM with explicit instruction was provided as a Tier 2 support, which was aligned with Tier 1 classroom strategies, for students not making adequate progress on word problem solving skills. All students who received SMM with explicit instruction as a Tier 2 support improved their skills in mathematics problem solving. This contributes to research that supports use of Tier 2 strategies aligned with Tier 1 classroom strategies (Bryant et al., 2008; Fuchs et al., 2008). A caution for this study is that although the teachers indicated they taught students problem solving skills using bar modeling, only one student indicated he or she had used the strategy once before in class.

Finally, this study included treatment fidelity on the independent variable. Montague and Dietz (2009) conducted a meta-analysis and found treatment fidelity was not measured in CSI studies. They recommended future studies include a measure of treatment fidelity in order to improve the quality of research on mathematics word problem solving. This study provided a checklist for fidelity of implementation that could be replicated and/or modified for other research studies using SMM with explicit instruction to teach problem solving skills.

## Limitations of the Study

There are several limitations to this study including the researcher implementing the intervention, choosing specific problem types, researcher-made probes, and a small number of students participating in the study that limit the generalizability of the study. Another limitation discussed will be faulty stimuli that may have contributed to the results of the study.

First, even though having the researcher (who designed the stages of instruction) implement the study increases the likelihood that instruction was provided with fidelity, it also serves as a limitation to this study. The researcher serving as the interventionist does not generalize as well as if teachers implemented the intervention. Based on this study alone, there is no way to determine if the intervention worked well because the researcher was an expert in instructional design or because of the way the instruction was designed (e.g., explicit, systematic instruction, model-lead-test, multiple opportunities to respond). This limitation was adjusted for by a third observer using a fidelity checklist to ensure the researcher was implementing the study with fidelity. The researcher also used detailed plans for each stage of instruction, and used the same word stories and word problems across all groups of students.

Second, the researcher chose specific types of problems for the intervention. Addition and subtraction word problems on various situations used numbers between 100-999. Situations for addition included (a) compare, bigger unknown; and (b) take from, start unknown, while situations for subtraction included (a) add to, change unknown; (b) take from, result unknown; (c) take from, change unknown; (d) take apart, addend unknown; (e) compare, difference unknown; and (f) compare, smaller unknown. Multiplication and division word problems on various situations included basic math facts with multiples of numbers 2-10. Situations for multiplication included (a) equal groups, unknown product and (b) arrays, unknown product, while situations for division included (a) equal groups, size unknown; (b) equal groups, number of groups unknown; (c) arrays, group size unknown; and (d) arrays, number of groups unknown. Controlling for problem types limits the generalizability of this study, but also controls the intervention by limiting confounding variables within the study. As this is a preliminary study on SMM with explicit instruction, it was appropriate to limit the types of problems and situations.

Third, not only did the researcher design the instruction and choose the problem types, the researcher also created the questions used on the probes for assessing the effectiveness of the intervention. The researcher based questions on Singapore Math 70 Must-Know Word Problems (Schaffer, 2009), but varied the names, subjects, numbers, and sentence structure within the problems. Designing the word problems allowed the researcher to control the assessment measures and ensure they matched the instruction. By doing this, the researcher may have unintentionally limited the word problems and generalization should be considered cautiously. This was accounted for in a few ways. First, a content expert validated the content of the probes, and second, within the word problems, the structure of the sentences was varied (e.g., Sam had 235 eggs. Sara had 101 fewer eggs than Sam. How many eggs did Sara have? Sara had 101 fewer eggs than Sam. If Sam had 235 eggs, how many eggs did Sara have?) A more generalized approach would have been to assess students using an applications probe from a standardized curriculum-based measurement (e.g., Math Concepts and Applications; MCAP). Unfortunately, MCAP measures more than specific word problem skills (e.g., fractions, ratios, time, geometry) and would not have demonstrated student growth appropriate for this study.

Fourth, the study was conducted in one school in a suburban area of the southeastern United States, with seven fourth and fifth grade students and six of the seven students finished all phases of the intervention. Despite the strong results based on a visual analysis of data, the limited number of students participating in the study, limit the generalizability of the findings. Single case research provides strong internal validity because visual analysis clearly determines that the changes in the dependent variable are due to manipulation of the independent variable across different students at different times, but external validity is limited (Cooper et. al., 2007). Single case research builds generalizability through replication of studies with students of different ages, students with different disabilities, in various locations, and with different research groups.

Finally, one limitation that may have affected the multiplication division probes was presence of faulty stimuli between multiplication and division problems. Basic multiplication facts (e.g., multiples of numbers 1-9) were used in the multiplication and division problems. One unforeseen limitation was when the problem required multiplication, both numbers were single digits and when the problem required division, one number had a single digit and one number had two digits. If the students realized this, then they would know if they saw a double-digit number they needed to divide, without needing to know why. Most likely this did not happen because students were required to draw out the problems and they used the rules (i.e., missing whole multiply, missing part divide) to determine how to solve the problems. There were also times during maintenance that students wrote the whole number (dividend) and drew it as a part (factor) and incorrectly multiplied instead of divided, which demonstrated a few students were unaware of the presence of faulty stimuli. To alleviate this in the future, multiplication questions should use 0-99 for multiplication/division probes. This leads to the discussion on future research recommendations.

## Recommendations for Future Research

Recommendations for future research include conducting the study in classrooms with teachers implementing the intervention, providing students the opportunity to self-monitor their progress, conducting a group experimental study with more students, and determining if SMM without explicit instruction is appropriate for students in Tier 1, while students in Tiers 2 and 3 require SMM with explicit instruction in order to improve word-problem solving skills.

As this is a preliminary study on using SMM with explicit instruction, the primary investigator also served as interventionist. Future research should allow teachers to implement this intervention in their classrooms. This will assist in determining if the effectiveness of the study was due to the investigator (who designed the stages of explicit instruction for this specific SMM intervention) implementing the intervention, or due to the instructional design and delivery of SMM with explicit instruction. Future research should include single case design (e.g., multiple probe across teachers) using one set of problems (addition/ subtraction, or multiplication/ division). It will also be important to include a stronger fidelity measure with teachers. The current fidelity checklist worked well for this study because the interventionist designed the instruction and understood how to implement it. Future research may want to include observational fidelity that includes specific items for each stage of instruction. Previous research on CSI did not include measures of fidelity (Montague & Dietz, 2009).

If results from teachers implementing one subject with fidelity are successful, then branching out into studying SMM with multiple subjects, multi-step problems, different subjects (e.g., fractions, ratios, percentages), allowing students to self-monitor their progress would be appropriate next steps, or teaching students to generalize problems into novel situations. The more replication of effects using single case design, the stronger external validity becomes for using SMM with explicit instruction. Eventually, group designs (e.g., pre-posttest, repeated measures) may be used to measure the effects of SMM with explicit instruction with larger groups of students. It is important to note, group designs may be harder to execute if the population remains focused on students at risk for or identified with LD. This current study found only seven students eligible across two grade levels.

To address group design issues of small populations, it would be interesting to determine the strength of SMM as a heuristic model used in core math instruction (Tier 1) in a repeated measures group design and then investigate further use of SMM with explicit instruction (Tier 2) for students not making progress using a multiple baseline/probe design. Research supports using strategies in Tier 2 interventions that are aligned with Tier 1 core instruction (Bryant et al., 2008; Fuchs et al., 2008). This study loosely addressed aligning strategies in Tier 1 with 2 interventions, but no data were collected in Tier 1, nor was fidelity of implementation monitored within Tier 1. Future research could provide more insight into the effectiveness of SMM within core instruction at the Tier 1 level (through randomized control trial) and SMM with explicit instruction at the Tier 2 level (through randomized control trial or multiple baseline across teachers).

## Implications for Practice

Results of this study include multiple implications for practice such as addition and subtraction problems may not need to be taught prior to multiplication and division problems, teachers may need to conspicuously point out when students can use SMM with other types of problems, teachers may need to plan ahead for students who may require booster sessions, and SMM and SMM with explicit instruction can be used within the structure of RTI.

First, an interesting implication for practice is that students may not need to master how to solve addition and subtraction problems before learning to solve multiplication and division problems. Raymond began attending two intervention sessions a day and learned how to solve multiplication and division problems while continuing to work on mastering addition and subtraction problems. This is an important finding as it took 6-10 weeks for students to learn and master solving addition subtraction word problems, but only 3-4 weeks to learn multiplication division word problems.

Second, another implication for practice is to provide some type of connection between using SMM to solve word problems during intervention and using SMM to solve word problems during core math instruction. Students need to work on generalizing what they are learning in a small group with how to apply it in the general education setting. This could be accomplished through instruction on transferring learned knowledge into new situations. Research supports instruction on teaching transfer skills assists students in transferring knowledge into new situations (Fuchs et al., 2006b; Fuchs et al., 2010).

Third, teachers and administrators need to plan ahead for students who may need booster sessions before receiving enough instruction to master problem solving using SMM. Not all students will master the content at the same time with the same amount of practice. Within the classroom, if students are group together based on their math skills, those students who need booster sessions could attend multiple groups and receive more repetition using SMM with explicit instruction. Repetition assists students in learning and mastering new content (Nelson, Burns, Kanive, & Ysseldyke, 2012).

Finally, for schools implementing RTI, SMM could be used as core instruction within Tier 1 and SMM with explicit instruction could be used as interventions in Tier 2 or Tier 3. Research supports aligning Tier 1 and Tier 2 strategies (Bryant et al., 2008; Fuchs et al., 2008). Research also indicates that changes in Tier 2 and Tier 3 instruction can include changes to the context of the group and not just the intervention (e.g., smaller group size, longer sessions, more frequent interventions [NCRTI, 2010]). Schools could use SMM in Tier 1 (without explicit instruction), use SMM with explicit instruction in Tier 2 with groups of five to eight students, and for students needing more intensive instruction, SMM with explicit instruction in Tier 3 with groups of two to three students for at least 30 min, five days a week. Providing a structure to use SMM in all tiers of instruction will ensure consistency for students and promote generalization across settings and problem types.

## Summary

The purpose of this study was to evaluate the effects of SMM with explicit instruction on the math problem solving skills of students at risk for or identified with LD. Specifically, this study evaluated SMM with explicit instruction on students' ability to solve addition and subtraction word problems and multiplication and division word problems using a multiple probe across participants with an ABCDE design. Seven students in groups of two (one student received intervention in two groups) were provided explicit instruction on using SMM to solve addition and subtraction word problems before moving into solving multiplication and division word problems. Visual analysis of the results of the study concluded a functional relation was identified between the independent variable and dependent variables for addition/subtraction word problems and multiplication/division word problems across all students completing the intervention. Data displayed a change in level, trend, and variability for almost every student on both probe types. The intervention for multiplication and division seemed to result in quicker changes in level and trend for all students. Moreover, most students indicated SMM helped them to solve word problems and found the steps easy to follow. All but one student indicated they would like their classroom teacher to use SMM in class.

In conclusion, students in Singapore continually outperform their United States peers on the TIMSS mathematics assessments (Provasnik, 2012). Across the United States, students with disabilities are not making as much progress as their peers on national math assessments (NAEP, 2015). Research supports various strategies as effective for teaching mathematics to students with disabilities including explicit instruction, representational techniques, and cognitive strategies (Xin & Jitendra, 1999; Zhang & Xin, 2012; Zheng, Flynn, & Swanson, 2012). Singapore (the country leading all international math assessments) uses SMM to teach students to solve math word problems. Use of SMM with explicit instruction was found to be effective to teach students who were at risk for or identified with LD to solve addition/subtraction and multiplication/division math word problems with demonstration of strong maintenance. This preliminary study on SMM with explicit instruction provides promising results for improving students' problem solving skills, skills that have long-term benefits for all students.

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APPENDIX A: STUDENT ASSENT FORM

UNC CHARLOTTE
College of Education

Department of Special Education and Child Development

9201 University City Blvd, Charlotte, NC 28223-0001
t/ 704.687.8772 f/ 704.687.2916 www.uncc.edu

Student Assent Form for
Participation in Educational Research

August 15, 2015

Dear Student:

My name is Mrs. Preston, and I used to be a teacher at your school. I am also a student at The University of North Carolina at Charlotte. I am working on a study to help students become better at solving math word problems.

If you want to be in my study, we will be learning a new way to solve math word problems. We will do this for the next few months.

Two times a week, I will give you a worksheet with four word problems. I will score your answers and keep the worksheets. Your scores will remain a secret and I will destroy your worksheets once we finish. If you decide during the study that you do not want to be a part of it, then you can stop at any point, and no one will be mad at you.

I really hope this study will help you become better at solving math word problems. If this study works, it may help a lot of students get better at solving math word problems. I will write a report about the study when we are finished, but I will not put your name in the report.

If you want to participate in this study, please sign your name below.

________________________________________ ______________
Signature of Student Date

________________________________________ ______________
Signature of Investigator Date

# APPENDIX B: PARENT INFORMED CONSENT FORM

UNC CHARLOTTE
College of Education

Department of Special Education and Child Development

9201 University City Blvd, Charlotte, NC 28223-0001

Parent Consent Form for
Participation in Educational Research

August 15, 2015

Dear Parents and Guardians,

Your child is invited to participate in a research study entitled "Effects of the Singapore Model Method with Explicit Instruction on Students' Math Problem-Solving Skills." The purpose of this study is to investigate the Singapore Model Method, a problem solving strategy within Singapore Math, and its effects on solving math word problems.

Investigator:
As a previous teacher in your child's school, I, Angela Preston, am conducting this study as part of the requirements for my dissertation in the Special Education Doctoral Program through the University of North Carolina at Charlotte. The responsible faculty member for this study is Dr. Charles Wood in the Special Education Program. Along with a Master's degree from UNC Charlotte, I have eight years teaching experience in Cabarrus County. The first three years I taught in a resource classroom at Cox Mill Elementary and then I spent five years teaching at Carl A. Furr Elementary. I have full support from the principal at Carl A. Furr Elementary to teach math to the students.

Description of Participation:
This study will use the Singapore Model Method to teach problem solving skills to students who need help completing math word problems. Your child will continue to receive his/her daily math instruction with his/her teacher(s) and I will provide your child with supplemental instruction using the Singapore Model Method. Your child will learn how to follow an eight step process for solving math word problems. Each step will walk through how to draw out the problem identifying the part, whole, and what is missing. The drawing will then allow him/her to understand how to solve the problem. This process can be used with all types of math word problems. Two times a week your child will complete a four question math word problem worksheet in order to keep track of his/her progress. These worksheets will be photocopied and scored by two doctoral students. All photocopies will be destroyed immediately following the completion of the study. Some of the lessons will be videotaped in order to ensure that I am following the

correct procedures while teaching the math program. Another doctoral student will watch the videos and record information about my teaching on an observation sheet. Once your child has demonstrated mastery of this skill, he/she will no longer receive daily instruction with the Singapore Model Method, but will be given the four question math word problem worksheet once a week to determine if he/she has maintained that skill.

Eligibility Criteria:
Students will be eligible for the study who score below the 25th percentile on the 4th grade Math Concepts and Application probes and above the 25th percentile on the 3rd grade Math Concepts and Application probes. All eligible students will take an eight question math word problem assessment. Students who score 50% or below will be eligible for the study. If your child does not meet eligibility criteria, he/she will no longer be eligible for the study. At that point, you will be contacted and informed that your child is no longer eligible and will continue to receive regular math instruction in class with his/her teacher. All data collected up to that point will be destroyed.

Length of Participation:
Your child's participation in this project will begin sometime in October, 2014. The study will most likely end in the spring of 2015. Each student will participate in the study for approximately 20 to 35 minutes per day, four days per week.

Risks and Benefits of Participation:
There are no known risks associated with this study. The benefits of this study may include enhanced math problem solving skills and better ability to understand how to set up and solve math word problems dealing with addition, subtraction, multiplication, and division.

Alternatives:
If you choose for your child to not participate in the study, he/she will continue to receive math instruction with his/her teacher.

Volunteer Statement:
Your child is a volunteer. The decision to participate in this study is completely up to you and your child. If you and your child decide for your child to be in the study, your child may stop at any time. Your child will not be treated any differently if you and your child decide not to participate or if your child stops once your child has started.

Confidentiality:
The data collected by the Investigator will not contain any identifying information or any link back to your child or his or her participation in this study. The following steps will be taken to ensure this confidentiality:

- Math tests will be photocopied for data collection. All originals and copies will be destroyed when the study is finished.

- No participant will ever be mentioned by name in the reported results. To ensure this takes place, student numbers and/or pseudonyms will be assigned to each participant during data collection.
- Data collected will be kept in a locked cabinet in my office at UNC Charlotte.

Fair Treatment and Respect:
UNC Charlotte wants to make sure that you are treated in a fair and respectful manner. Contact the University's Research Compliance Office ([redacted]) if you have any questions about how you are treated as a study participant. If you have any questions about the project, please contact Angela Preston at [redacted] or Dr. Charles Wood at [redacted].

Participant Consent:
I have read the information in this consent form. I have had the chance to ask questions about this study, and those questions have been answered to my satisfaction. I am at least 18 years of age, and I agree for my child to participate in this research project. I understand that I will receive a copy of this form after it has been signed by me and the Principal Investigator.

______________________________ ________________
Name of Student Participant (PRINT) DATE

______________________________ ________________
Parent Signature DATE

______________________________ ________________
Investigator Signature DATE

# APPENDIX C: PROCEDURAL FIDELITY CHECKLIST

| Date: | Phase: | | |
|---|---|---|---|
| Item | Observed- Circle one | | |
| Followed SMM steps | Yes | No | NA |
| Followed SMM Procedures | Yes | No | NA |
| SMM steps posted | Yes | No | NA |
| Used bar modeling | Yes | No | NA |
| Used rules | Yes | No | NA |
| Error correction as needed | Yes | No | NA |
| Provided praise | Yes | No | NA |
| Total Yes/ Total Yes + No | | | |

| Date: | Phase: | | |
|---|---|---|---|
| Item | Observed- Circle one | | |
| Followed SMM steps | Yes | No | NA |
| Followed SMM Procedures | Yes | No | NA |
| SMM steps posted | Yes | No | NA |
| Used bar modeling | Yes | No | NA |
| Used rules | Yes | No | NA |
| Error correction as needed | Yes | No | NA |
| Provided praise | Yes | No | NA |
| Total Yes/ Total Yes + No | | | |

| Date: | Phase: | | |
|---|---|---|---|
| Item | Observed- Circle one | | |
| Followed SMM steps | Yes | No | NA |
| Followed SMM Procedures | Yes | No | NA |
| SMM steps posted | Yes | No | NA |
| Used bar modeling | Yes | No | NA |
| Used rules | Yes | No | NA |
| Error correction as needed | Yes | No | NA |
| Provided praise | Yes | No | NA |
| Total Yes/ Total Yes + No | | | |

APPENDIX D: PROBE-AS: ADDITION/SUBTRACTION

Name: ________________________________________

1. There are 18 adults on a train, 6 of them are women. The rest are men. How many men are on the train?

________________________________________

2. Ahmad has 8 marbles. Kate has 5 more marbles than Ahmad. How many marbles does Kate have?

________________________________________

3. Steve has 43 stickers. Oscar has 38 fewer stickers than Steve. How many stickers does Steve have?

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4. Emma bought a video game for $19 and a DVD for $9. How much money did she spend?

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APPENDIX E: PROBE-MD: MULTIPLICATION/DIVISION

Name: ________________________________________

1. There are 6 children on a bus. There are 3 buses. How many children are there?

______________________________________________

2. Mrs. Wright bakes 18 cookies. She puts 3 cookies on each plate. How many plates does she need?

______________________________________________

3. There are 5 cars in a parking lot. Each car has 4 wheels. How many wheels are there altogether?

4. Tom, Max, Yoko, and Julia shared 32 grapes equally. How many grapes did each of them get?

APPENDIX F: PROBE ASMD:

ADDITION/SUBTRACTION/MULTIPLICATION/DIVISION

Name: ________________________________________

1. There are 6 children on a bus. There are 3 buses. How many children are there?

2. There are 18 adults on a train, 6 of them are women. The rest are men. How many men are on the train?

3. Ahmad has 8 marbles. Kate has 5 more marbles than Ahmad. How many marbles does Kate have?

4. Mrs. Wright bakes 18 cookies. She puts 3 cookies on each plate. How many plates does she need?

# APPENDIX G: STUDENT SOCIAL VALIDITY FORM

| | Strongly Disagree | Disagree | Agree | Strongly Agree |
|---|---|---|---|---|
| 1. The model method lessons helped me to understand how to do math word problems. | 1 | 2 | 3 | 4 |
| 2. The 7 steps in the model method were easy to follow. | 1 | 2 | 3 | 4 |
| 3. The word stories with all three numbers helped me learn how the bars and numbers fit together. | 1 | 2 | 3 | 4 |
| 4. Drawing the bars helped me understand what the word problem was asking. | 1 | 2 | 3 | 4 |
| 5. Labeling the bars with parts and whole, or part, difference, whole helped me solve the problems. | 1 | 2 | 3 | 4 |
| 6. The rules about missing the part and missing the whole helped me learn if I should add or subtract and multiply or divide. | 1 | 2 | 3 | 4 |
| 7. I am better at solving word problems now. | 1 | 2 | 3 | 4 |
| 8. I use the model method to do word problems in my classroom. | 1 | 2 | 3 | 4 |
| 9. I would like my classroom teacher to use the model method during math. | 1 | 2 | 3 | 4 |
| 10. I would like to learn how to use the model method to solve other kinds of math word problems. | 1 | 2 | 3 | 4 |

I liked:

I did not like:

# APPENDIX H: TEACHER SOCIAL VALIDITY FORM

| | Strongly Disagree | Disagree | Agree | Strongly Agree |
|---|---|---|---|---|
| 8. My students learned how to solve word problems from this strategy. | 1 | 2 | 3 | 4 |
| 9. I saw a change in my student's math skills in my classroom after receiving the strategy. | 1 | 2 | 3 | 4 |
| 10. The 7 steps to the model method are easy to understand. | 1 | 2 | 3 | 4 |
| 11. Teaching how to solve word problems will be easier using the model method. | 1 | 2 | 3 | 4 |
| 12. This strategy would be feasible to implement in my classroom. | 1 | 2 | 3 | 4 |
| 13. I would like to use this strategy in my class. | 1 | 2 | 3 | 4 |
| 14. I would like training on how to use this strategy in my class. | 1 | 2 | 3 | 4 |

Additional Comments/Suggestions:

APPENDIX I: TEACHER INFORMED CONSENT FORM

UNC CHARLOTTE

College of Education
Department of Special Education and Child Development

9201 University City Blvd, Charlotte, NC 28223-0001
t/ 704.687.8772 f/ 704.687.2916

August 15, 2015

Dear Teacher,

I am currently pursuing a Ph.D. in Special Education at the University of North Carolina at Charlotte. As part of the requirements for my dissertation in the doctoral program I will be conducting Effects of the Singapore Model Method with Explicit Instruction on Students' Math Problem-Solving Skills.

I have chosen to do my research on the Singapore Model Method. I will teach students how to use this math strategy and monitor their progress on solving math word problems. As part of my research I would like to have fourth grade teachers complete a questionnaire about teaching math word problems. The questionnaire should not take more than 20 minutes to complete. All information collected will be kept anonymous and your answers will be discussed in the research article once the study is complete.

You are a volunteer. The decision to participate in this study is completely up to you. If you prefer not to participate in the study, please do not complete the questionnaire. By completing and returning the questionnaire, you are allowing me to use your questionnaire answers in the study.

UNC Charlotte wants to make sure that you are treated in a fair and respectful manner. Contact the University's Research Compliance Office ([redacted]) if you have any questions about how you are treated as a study participant. If you have any questions about the project, please contact Angela Preston at [redacted] or Dr. Charles Wood at [redacted].

I sincerely appreciate you time. If you have any questions regarding this form or the study, please do not hesitate to call me.

Sincerely,

Angela I. Preston

If you want to participate in this study, please sign your name below.

________________________________________ ______________
Signature of Teacher Date