THE EFFECTS OF STUDENTS' PROFILE, COMMUNICATION, AND DISCOURSE IN AN ADVANCED PLACEMENT STATISTICS CLASSROOM

by

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ABSTRACT

NORMA PORTALATIN ROYSTER. The effects of students' profile, communication, and discourse in an advanced placement statistics classroom. (Under the direction of DR. DAVID K. PUGALEE)

This study sought to describe the communicative aspect that should be part of any secondary mathematics classroom and, more specifically, the statistics classroom. Focusing on the observation of students allowed for a detailed account of the learning of students, i. e., what they said and wrote, how they used the materials provided by the teacher, what supports promoted understanding, and what difficulties to learning arose during the lesson. The research questions guiding the study were: (i) In what way is attention to students' statistical thinking evident in lesson planning? (ii) To what extent does discourse in the classroom enable students to communicate effectively in the AP Statistics course? (iii) To what extent does communication, such as expressing organized and precise ideas, and analyzing and evaluating the statistical ideas of others, change over time? (iv) To what extent do students' profiles influence success in the advanced placement statistics course?

Garfield's (2002, section 3, Table 1) general model of statistical reasoning provided a framework on which to base a characterization of students' statistical reasoning. As has been found in earlier studies about teaching statistics (Garfield & Ben-Zvi, in press, Sherin, 2002), this teacher encountered challenges in establishing patterns of discourse to encourage effective student communication in the classroom. It was also found that student profiles were contributing factors in students' success in the statistics course.

DEDICATION

I would especially like to thank my dear husband, David, for providing invaluable encouragement throughout the entire process. To my dear sons, Thomas Wilkins and Robert Benjamin, I dedicate this work. Robert, when it became a burden to continue writing and trying to accomplish this dream I was reminded of your unfulfilled dreams and aspirations because of your untimely death. Here's to you Bubba.

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LIST OF ABBREVIATIONS

ACP	Activity – Computer Printouts
ACT	American College Testing Program
AG	Anger with student engagement
AMC	Activity Modeling Concepts
AP	Advanced Placement
APQD	AP Questions Discussion
APU	AP Questions Useful
ASD	Active Student Discussion
ASQ	American Society for Quality
CAP	Chance and Probability
CBD	Contextual Based Discussions
CDS	Calculator Discussion
CD	Coefficient of Determination
CG	Cooperative Groups
CI	Confidence Interval
CJ	Contextual Justification
CLT	Central Limit Theorem
CS	Confident about success in course
df	Degrees of Freedom
DL	Directed Lesson
DLP	Don’t Like Projects
ELS	Effects of Larger Sample Size

FC	Feelings towards course changed
FR	Frustration with student engagement
FS	Feelings towards course same
FSC	Frustrated with course
EN	Encouraged by student engagement
GPA	Grade Point Average
GWM	Good Writing Mechanics
HQS	Assignment Questions Student
HQT	Assignment Questions Teacher
IS	Insecure about success in course
LP	Enjoy Projects
LS	Like Statistics
LSG	Study Guide Useful
LSRL	Least Squared Regression Line
M	Mean
MMD	Mean and Median Difference
NCJ	Non-contextual Justification
NCTM	National Council of Teachers of Mathematics
ND	Normal Distribution
NPC	Not prepared for class each day
NTD	New Topic Introduction
PC	Prepared each day for class
PL	Pleased with student engagement

PP	Pleased with performance in course
PSGD	Prefer Small Group Discussion
PWGD	Prefer Whole Group Discussions
PWM	Poor Writing Mechanics
RAL	Reads a lot
RL	Recapping of Lesson
RP	Residual Plot
RVL	Read very little
SAT	Scholastic Aptitude Test
SD	Standard Deviation
SDF	Statistics Difficult
SDM	Student Discussion Minimal
SE	Statistics viewed as easy
SG	Small Group Discussion
SK	Skewness
SRS	Simple Random Sample
SPSS	Statistical Package for the Social Sciences
SV	Sampling Variation
TF	Teacher as Facilitator
TRD	Teacher/Researcher Discussion
TU	Textbook Useful
WG	Whole Group Discussion
WDS	Writing different in Statistics

WSS	Writing in Statistics same Statistics

CHAPTER I: THE RESEARCH PROBLEM

1.1 Introduction

Research has focused on many factors related to the effectiveness of the teacher in a secondary classroom and likewise teacher effectiveness in the statistics classroom includes many complex ideas and processes. The complexity of teaching statistics hinges on presenting an increasing choice of “approaches and perspectives” to collecting, to managing, and to “interpreting” and making inferences about the data (Hawkins, 1997). Among factors used to study teacher effectiveness such as teacher personality, behaviors, self-efficacy, subject knowledge and knowledge of pedagogy, researchers have found that teacher behaviors, beliefs, and subject knowledge when linked to their students’ behavior and self-efficacy could be used at various levels of influence as predicators of students’ achievement (Mills, 2007; Muijs & Reynolds, 2003). An important behavior or aspect of teaching for the secondary school teacher is providing an environment that will allow students to develop a deep understanding of the subject matter which will be retained for future assessments (Berensen, Utts, Kinard, Rumsey, & Gaines, 2008). For the Advanced Placement (AP) Statistics teacher it becomes imperative to provide an environment that will allow students to develop a deep conceptual understanding of statistics in order to prepare students to do well enough on the Advanced Placement Statistics examination to earn college credit.

Many factors affect these outcomes including dispositions (Wild & Pfannkuch, 1998, 1999), communication (National Council of Teachers of Mathematics (NCTM) 1989, p. 78; NCTM, 2000, p. 60), and discourse (NCTM 1991, pp. 35, 45). These factors are of special interest in classrooms where the development of conceptual understanding is emphasized. In light of the fact that dispositions are sometimes confused with attitudes, it would be preferable to use students' profiles which will include attitudes among other measures. This broader inclusionary definition of student profile includes multiple factors that will influence students' approaches in regards to classroom involvement and understanding. In an AP Statistics course, a study of the more broadly inclusive profile of the students, will include traditional objective measures (test scores, achievement measures, etc) as well as non-traditional subjective measures (beliefs about statistics, problem-solving in general, self-efficacy beliefs, self-generated reflections of statistics activity, etc), observed communication, and planned discourse. These variables, traditional objective measures, non-traditional objective measures, communication, and planned discourse will provide a snapshot of classroom dynamics and the teaching and learning taking place. Communication in written form "will identify students' strengths and weaknesses" (Samsa & Oddone, 1994). Communication is the process between two or more individuals that will allow reflection on and clarification of their thinking (Cobb, Wood, Yackel, & McNeal 1992) about statistical ideas and concepts. Planned discourse will then be orchestrated by the facilitator and characterized by elicitation, engagement, and challenging the students' thinking (Cobb, Wood, Yackel, & McNeal 1992; Gal, 1997). A methodical and inclusive research of the classroom, the teaching, and the extent of students' learning will provide a deeper insight into the symbiotic relationship between the teaching practice and student learning.

1.2 Focusing on Student's Statistical Thinking

Just how does one go about uncovering the dynamics of the classroom environment? What are the levels of thinking in an advanced placement statistics class? How can the students' levels of statistical thinking grow through instruction? How does a student's profile, communication (both verbal and written form), and discourse initiated in the classroom between teacher and students and among students affect statistical understanding and thinking? Wild and Pfannkuch (1999) explore these questions and point to the instructor as providing contextual knowledge, statistical knowledge, and synthesis of ideas and information as a starting point to developing statistical thinking. Basing their study on literature and in-depth interviews of students and statisticians, Wild and Pfannkuch (1999, p. 224) view enquiry (questioning) as a starting point for linking the parts of statistical thinking, "thinking patterns involved in problem-solving, strategies for problem-solving, and the integration of statistical elements in the problem-solving." The framework that is referenced by Wild and Pfannkuch is organized around the elements of statistical thinking where the thinker works in four dimensions: the investigative cycle, types of thinking, interrogative cycle, and dispositions.

Wild and Pfannkuch (1999) continue detailing these four dimensions with the investigative cycle being that in which the statistician seeks to define the problem, plan the analyses, manage the data, perform the analyses, interpret the results and communicate these effectively. The types of thinking that inherently are part of statistical thinking include: recognition of a need for data, trans-numeration which involves representing the data so as to give new understanding to the data, variation which involves decision making under uncertainty, and contextual and statistical knowledge. The interrogative cycle permeates the entire problem-solving experience. These various components reveal that communication, dispositions, and other external relevant knowledge areas are crucial to this cycle. Statistical thinking at this stage involves either individual or group brainstorming in searching for explanations, planning an approach or checking out an idea. Statistical thinking is followed by seeking internal or external information which at its most basic level entails investigating the data at hand. Interpretation then allows the thinker to process the results of all statistical analyses. Finally, criticizing and judging the results of the analyses allows the thinker to check these against internal and external reference points.

1.3 Effects of Student Profile

The students' profiles will also permeate the entire problem-solving experience. Internal questioning, awareness, observations, and just being curious will initiate a critical aspect of data exploration. As the authors (Wild & Pfannkuch, 1999) discovered, dispositions, or, as this researcher chooses to call, a student's profile, will influence the engagement in the problem at hand and is dependent on the problem. Background knowledge facilitates interest in a problem. As the student becomes "engaged," his or her internal questioning, awareness, observation, perseverance, and curiosity become discerning. However, natural interest, that which is innate becomes an important factor and therefore is significant. If students' perceptions of statistics are related to their experiences, interests, and goals then learning will be predetermined by their social environment (Gordon, 1995). Although AP Statistics is not a required course in high school, many students are aware that many majors at the college level require a statistics course. This does not mean that they will enjoy the experience (Gal & Ginsburg, 1994). Therefore, the instructor's use of strategies to engage the students will be vital. The type of student profile developed during the course or already existent at the onset of the course will affect the student's success in statistics. Equally important will be the experiences that the students have had in the middle school years through which the students form some basic understanding of elementary terminology used in statistics through concrete experiences in handling data (Watson & Kelly, 2008).

1.4 This Study

This study proposes a process that will organize systematically and coherently the communicative aspect that should be part of any secondary mathematics classroom and, more importantly, the statistics classroom.

Focusing on the observation of students allows the researcher to collect a detailed account of the learning of students, i. e., what they said and wrote, how they used the materials provided by the teacher, what supports promoted understanding, and what difficulties to learning arose during the lesson. The more specific and detailed observations allow the discussion to be valuable not only for the teacher but ultimately the students. The discussions allow the teacher to give immediate corrective feedback on the students' statistical thinking and understanding, while the written documents allow the teacher to give feedback on practice in creating scientific papers (Samsa & Oddone, 1994), because these are the likely forms of writing that students will have to complete in their college courses.

Because of the time commitment involved in observing changes both of the instructor and students, this study took place over a time period of the fall semester with a follow-up during the spring semester to determine students' readiness for the advanced placement examination. This type of study was a collaborative process, both the teacher and the participant-observer, the researcher, were instrumental in gathering the observational data. This study made the classroom teacher an essential part of the research in supporting the data collection and giving an additional perspective into understanding the classroom culture and in the analyses (Chance & Garfield, 2001).

Regular meetings were necessary in order to obtain feedback on materials used with the students. The materials were developed as a tool to fostering statistical discourse. A practical and useful study should provide a means to measure and communicate findings about student performance, therefore, a series of artifacts were used to assess student performance and describe students' levels of statistical thinking and reasoning growth in those levels of statistical thinking and reasoning over the semester measured by essay responses, transcripts of student interviews, examples of student problem-solving activities and strategies used, transcript and/or videos of class discussions to evaluate student questions and teacher responses, post-lesson discussions, textbook and resource materials analysis, and teacher-designed and other materials used as authentic assessments. The study allowed time to investigate aspects of pedagogy and content so that the instructor could reflect on the teaching and how the teaching fits in with the basic curriculum of the course and the experiences of the students.

Brown (1992) describes the difficulty of designing a study that will describe completely the classroom setting. In order to describe the classroom dynamics this researcher chose to use the method of triangulation so that data from multiple sources would get rather close to describing the classroom environment. Videotapes were valuable in “documenting conceptual change, in teachers as well as students and they provide a common data base for discussion and reflective action on the part of teachers and researchers” (p. 174). The jigsaw technique, a common cooperative learning instructional strategy by which students learn by creating their own understanding and subsequently teach concepts to their peers, proved to be a rich source and appropriate method (Brown, 1992; Garfield, 1993) to use.

By planning this study process to continue over an extended period, it was expected that each of the lessons would be streamlined to accomplish a particular goal or goals and to benefit the specific needs of the students. The students’ profile consisting of attributes such as interest, curiosity, and perseverance, towards statistical concepts or ideas affects communication in the classroom and ultimately outcomes on assessments and this was observed over the duration of the study.

In general, students demonstrate a variety of levels of statistical thinking and understanding. Contextual knowledge is needed to solve problems in the area of statistics and each statistical investigation adds to that context knowledge, therefore it is important that the statistician have a foundational base from which to draw (Wild & Pfannkuch, 1998). Pfannkuch and Wild (2000, pp. 133-134) explain that “contextual knowledge refers to knowledge about the system, knowledge about the phenomenon being studied and knowledge about the subject matter from which the data are generated.” It is easy to understand how some students would have difficulty understanding a problem with an unfamiliar context and this absence of contextual knowledge manifests itself in many different ways. Some students readily know how to analyze a complex statistical situation and initiate an academic response while others may not even be able to apply a formula or formulae to determine a numerical analysis of a given data set. In much the same way, there are students who are not sure what analysis to use or what is necessary and sufficient to provide evidence for a particular inference. In the midst of this bewilderment, the added pressure of preparing for an AP examination requires competent students that have a deep conceptual understanding of statistics. Some of the answers to preparing students to successfully meet the stated goals and objectives on the AP Statistics examination, which will be the final form of assessment that will effectively evaluate statistical understanding (Garfield & Chance, 2000), might lie in conducting this proposed study.

1.5 Statement of the Problem

Since the inception of the AP Statistics course in a small, rural high school in a mid-Atlantic state, the percentage of students scoring 3 or better has fluctuated from nearly 25 to 56%. Students that enroll in AP Statistics are generally considered proficient mathematics students with a good academic profile representative of those who take challenging courses. Most of these students are considering a career in which a working knowledge of statistics is advantageous. The AP Statistics course, at the high school mentioned above, originally was offered in the spring and did not allow the entire statistics standard course of study to be completed with adequate time left to review for the AP exam. When the AP Statistics course was offered in the fall term it provided the advantage of being able to expose the students to all the concepts and topics that are part of the statistics standard course of study, but it also had the disadvantage that the students completed the course in January and did not take the AP examination until the following May. This situation did not produce an increase in the number of students who were successful in earning a 3 or better. A student that earns a 3 or better on the AP examination is granted college credit at most state-supported universities. The instructor who has taught the AP Statistics course for the last few years is a skillful and experienced teacher and had observed that notwithstanding the students' mathematical ability, the students enrolled in AP Statistics tended to have limited real-world experiences that they could draw from to use in the statistics course. When the students were given problems that necessitated contextual knowledge, such perspectives were clearly absent. The students also failed to verbalize, both orally and in written form, a complete analysis to a given statistical problem, failed to complete assignments because they tended to be lengthy, and failed to attend review sessions provided in the spring to prepare them for the examination in May. These failures were evident in their grades on assessments and the final examination.

1.6 Research Questions

A growing body of literature has identified many factors that affect the outcomes in a statistics course with student dispositions, which this researcher will refer to as student profile (Wild & Pfannkuch, 1998, 1999), communication (NCTM, 1989), and discourse (NCTM, 1991) of special interest in classrooms where development of conceptual understanding is emphasized. In the current study involving an AP Statistics course, the observations of students and teachers provided an uninterrupted sequence of the day-to-day communication and discourse and how these were affected by the students' profiles in the classroom. These observations also sought to reveal the teaching and learning taking place. The study, described in detail in Chapter 3, will focus on the dynamics of the AP course and how students' profiles, communication, and discourse in the classroom contributed to success in the advanced placement statistics course. In particular, the study sought to answer the following research questions:

1. In what way is attention to students' statistical thinking evident in lesson planning?
2. To what extent does discourse in the classroom enable students to communicate effectively in the AP Statistics course?
3. To what extent does communication, such as expressing organized and precise ideas, and analyzing and evaluating the statistical ideas of others, change over time?
4. To what extent do students' profiles influence success in the advanced placement statistics course?

The third and fourth questions sought to document the extent to which the profiles of the students influence success in the classroom and to what extent communication changes over time. Attention was focused on oral as well as written communication. The student profile will be defined broadly and will include students':

1) Dislikes related to statistics topics.

2) Self-perceptions of their reading ability.

3) Written performance on previous standardized tests.

4) Written essays in the statistics course.

5) Reading abilities as measured by prior testing on standardized testing.

6) Reading habits concerning course readings as well as extra-curricular readings as described by students in individual unstructured interviews.

7) Mathematical abilities as measured by prior testing and mathematics courses taken as well as basic mathematical computations needed in a statistics course that are made on statistics assessments.

8) Commitment to the course.

9) Views on the materials used in the class.

10) Homework completion rates.

11) Attitudes towards statistical topics and concepts as described by students in individual interviews.

12) Study habits as evidenced by questioning in the classroom discussions.

13) Views on the importance of this course in relation to their future plans in college.

The first question sought to discover patterns that appear about how the teacher focuses on students' statistical thinking as evidenced in lesson planning by presenting a qualitative description of these patterns. Lastly, the second question sought to discover how the discourse established by the instructor enabled students to communicate effectively the statistical ideas and concepts.

1.6.1 Significance of the Study

This study is unique in its focus upon high school students. In the next chapter, the researcher will discuss studies that focused on statistical thinking, communication, and students' profiles using some objective and subjective measures. However, these studies did not focus on all aspects mentioned above at the high school level. The focus upon high school students enrolled in an AP Statistics course is important for several reasons. First, very little research currently exists that helps to identify the attributes of a successful high school student in statistics. This study will provide a basic description of such students. Second, available research gives few clues as to growth of statistical thinking in high school students through the use of discourse to facilitate oral as well as written communication on the part of the students. The focus on high school students should appeal to both teachers and researchers seeking to encourage better statistically educated students and as such will add to the existing literature.

1.6.2 Limitations of the Study

As researchers learn more about how students learn statistics, each study contributes to a more cohesive and comprehensive body of knowledge, therefore making each study a vital component in the breadth and depth of how such understanding fits into the body of literature. Therefore, although the framework used in this study does not claim to depict all the qualities of a successful statistics student, it does provide insight into how students' statistical thinking develops and how both traditional (test scores, end-of-course examinations, achievement tests, etc.) and non-traditional measures (students' beliefs about statistics and statistical concepts, their self-efficacy beliefs, their self-generated reflections of statistics activity, etc.) contribute to the profile of a student which affects the development of statistical reasoning.

1.7 Historical Notes on Advanced Placement Statistics

The Advanced Placement Statistics course and examination was first offered nationally in 1997. Many years of planning went into designing the examination and creating a syllabus that would cover four areas: data exploration, study design, probability distributions through simulations, and inference (Roberts, Scheaffer, & Watkins, 1999). An advanced placement statistics course fits into the statistics reform already taking place in the K-12 curriculum. This reform was energized by the American Statistical Association's Quantitative Literacy Project in the 1980's (Roberts, Scheaffer, & Watkins, 1999). The literacy project together with the work taking place on the curriculum standards of the NCTM (1989) was a stimulus to the reform in mathematics as well as statistics.

The AP Statistics examination is a closed-book, three-hour paper and pencil test. The first of the examination half consists of 40 multiple-choice questions and the second half consists of four to seven open-ended questions (currently five questions is typical) designed to be answered in about 10 minutes each, and a longer investigative task for which 25 minutes is allotted. It is expected that students will have access to a computer both in and out of the classroom and be able to interpret computer output. An integral part of the AP Statistics course is the graphing calculator with statistical capabilities. Computations per se are not considered sufficient to merit a high score on the free-response section. Interpretations of the results and the use of appropriate statistical language are crucial and necessary for responses to be considered holistically correct. Most colleges and universities will grant college credit if the student completes the course and examination successfully. Over 108, 000 students (an increase of nearly 10,000 students from 2007) took the Advanced Placement Statistics examination in 2008 (College Board, 2008) and over 59% of the students earned 3 or better, which is considered "qualified".

1.8 Organization of the Document

In Chapter 2, three bodies of literature that are relevant to the study are reviewed: the relationship between statistical communication and reasoning, students' profile and its influence on the development of statistical reasoning, and the vital importance of communication in the statistics classroom. In Chapter 3, the design and methodology of the study is described. Inferences derived from data analysis are detailed in Chapter 4. Finally, the results of the study are briefly summarized and situated in the statistics education literature in Chapter 5.

CHAPTER 2: REVIEW OF THE LITERATURE

Do not put your faith in what statistics say until you have carefully considered what they do not say. ~ William W. Watt

2.1 Introduction

Just what is statistical thinking? In 1996 the American Society for Quality (ASQ) published a Glossary of Statistical Terms defining Statistical Thinking as philosophy of learning and action based on the following fundamental principles:

- All work occurs in a system of interconnected processes
- Variation exists in all processes
- Understanding and reducing variation are keys to success"(Mallows, 1998, p. 3; see also Melton, 2004)

Garfield (2002) posits that statistical reasoning involves the ways that individuals reason with statistical ideas and make sense of statistical information. Such reasoning includes the ability to make sound interpretations based on data sets, graphical representations, and statistical summaries. This type of reasoning combines concepts of data and chance, inferences and interpretation of statistical results within an underlying conceptual understanding of fundamental ideas related to distribution, center, spread, association, uncertainty, randomness, and sampling.

Chance (2002) describes statistical thinking as including "what the statistician does." However, the entire process goes beyond solving a problem to also include the generation of new questions. The statistical thinker will be able to move further than what is taught in the class, to instinctively examine and explore the issues and data involved in a specific context. Students may not be able to apply the concept of chance or "statistical reasoning when they should because they have not learned how to turn an everyday situation into a statistical one" (Schwartz and Goldman, 1996, p. S100). It will be necessary for the teacher to strategically incorporate these type of activities into the lessons.

Similarly, statistical literacy is defined by Rumsey (2002) as concerning two distinctive learning consequences: skilled in having "basic understanding of statistical term, ideas, and techniques" as well as being able to "function as an educated member of society in this age of information" (Rumsey, 2002, section 2, ¶ 6).

Finally, Wild and Pfannkuch (1998) propose that statistical thinking is the interplay between the statistical and the contextual. More specifically statistical thinking takes into account variation, constructing and reasoning from models, having both contextual as well as a statistical knowledge base, and synthesis and integration. Recognizing variation as a component of any statistical investigation is what makes the investigation a statistical problem (Pfannkuch and Wild, 2000). The professional statistician initiates the investigative cycle by identifying a problem. The statistician will gain specific knowledge on aspects relevant to the problem and by acting on the knowledge gained will initiate changes that will solve the problem. However, novice statistics high school students do not engage in this type of study at the beginning stage, nor is it a given that they will have had this experience in other coursework. So how does the instructor set the stage for the development of statistical thinking? It is obvious that the discourse that the instructor establishes in the classroom should be the means to facilitate the communication that will be necessary in order to promote the development of statistical thinking.

2.2 Statistical Reasoning and Communication

Parke (2008) suggests a way of integrating statistical communication throughout the course in order to strengthen and enhance statistical reasoning and conceptual understanding as well as increase confidence in performing statistical analyses and discussing it with other students, the teacher, and equally important, on the AP Statistics examination. This integration revolves around weekly assignments, small-group activities (Garfield, 1993), critical analyses of journal articles, and rubric-based scoring activities.

Although from the onset, novice statistics students may not engage in the data collection itself, they will have to be guided by the instructor through the various types of thinking fundamental to statistical reasoning or literacy. This also requires assessing statistical thinking within specific contexts and to evaluate the appropriateness of the students' constructive and interpretive statistical arguments (Gal & Garfield, 1997; NCTM, 2000; Schwartz and Goldman, 1996). This process, although necessary, is difficult. The use of data from a wide field of specialized areas makes understanding of the context of the questions difficult (Hawkins, 1997). Statistical thinking will include recognition of the need for data, trans-numeration, variation, distinctive models, contextual and statistical knowledge, and synthesis (Wild & Pfannkuch, 1998). To recognize the need for data is to learn that anecdotal evidence is insufficient and students must learn why statistical investigations are conducted (Garfield & Gal, 1999).

Trans-numeration involves obtaining data that will capture the essence of the situation at hand. Pfannkuch and Wild (2000) stress that trans-numeration is much more than simply obtaining data that will capture the elements of what is being observed. Graphical representations and statistical summaries are just two examples of trans-numeration.

The representations of the data and the variety of statistical models tried will help communicate a new insight into the situation. That insight may be needed in order to more “accurately” describe a population (Garfield & Gal, 1999). The models are abstractions of what is really occurring and will be a visual representation of the links between the elements of what is being studied (Pfannkuch and Wild, 2000).

Variation is central to learning and making decisions under uncertainty**.** The measure of uncertainty will be understood by understanding probability and the concepts and vocabulary associated with chance, probability, and uncertainty will need to be developed (Garfield & Gal, 1999). Understanding and accepting that variation is what distinguishes statistical thinking from other types of thinking will lead to the next natural step to wonder about the sources of the variation and wonder about the causes of variation (Pfannkuch and Wild, 2000). Even when statisticians deem their contextual knowledge sufficient to understand the problem under study, it is still necessary to use both contextual and statistical knowledge to devise a plan, collect data, fill any disparities in contextual data and mine new information from the data. However, the crucial step is synthesizing the new information with current contextual knowledge and obtaining new and improved context knowledge. The synthesis of information from many sources is the key to statistical thinking (Pfannkuch and Wild, 2000).

In order to understand some of the more difficult statistical concepts some instructors have used computer or calculator simulations as part of the theoretical framework for teaching statistics (delMas, Garfield, & Chance, 1999; Mills, 2003). The use of computer simulations allows the students to develop their own ideas and understanding of statistics topics (Mills, 2003) such as the Central Limit Theorem or confidence intervals and how to interpret them. The learning will be a beneficial process where the task assists in exploration and discovery. Models for study designs and analysis exist to guide the investigation of the statistician and allow the investigator to think in more standard ways. But all statistical thinking comes to fruition with statistical and contextual knowledge and the information gathered from the data. Context-free data does not help the statistics student engage in statistical thinking (Garfield, 1994). Instead it just becomes a number-crunching activity. In fact, students' explanations have used contextual knowledge as part of their justifications for choices when comparing several sets of data when the context is better known (Vermette, Gattuso, & Bourdeau, 2005).

The typical high school student does not possess specific context knowledge for each situation presented in class and for that matter outside of class. A student will not have the benefit of experts in the field on the AP Statistics examination. Therefore, it becomes crucial for the instructor to provide the students with tools that will compensate for this lack of contextual knowledge in order to build their repertoire of tools (Gal, 1997; Garfield, 1994). It will also be beneficial to the student to have the teacher provide experiences that will allow the students to learn how to change everyday situations into statistical events (Schwartz & Goldman, 1996). It then becomes highly important for the student to become involved in strategic thinking: planning what to do and how to do it (Robinson, 1998). In order to increase the success of engagement, projects with student-selected topics can prove a mechanism to practice statistical analyses with a personal benefit (Berensen, Utts, Kinard, Rumsey, Jones, and Gaines, 2008; Davies and Connor, 2005). Without forgetting the ever presence of variation, students should reduce raw data through summaries and displays and begin to note trends and special features of the data (Garfield, 1994; Garfield & Gal, 1999; Konold & Higgins, 2003).

2.3 Student's Profile and the Development of Statistical Thinking

The students' profiles play a role in initiating the thinking processes that students need to possess to help develop statistical thinking and inquiry. The students' profiles then allow for engagement or interest on the part of the student. If the student is not prepared to function within a learning environment that is problem-solving based then his or her experiences will hinder their success in the statistics course (Gal & Ginsburg, 1994). Some of these elements of the student's profile are imagination, skepticism, logical thought, and perseverance (Wild & Pfannkuch, 1998). These elements of the student's profile at first seem to warrant a "sure, of course" retort. Without imagination the statistician cannot "imagine" what is happening, alternative explanations, nor confounding factors from whence the data originated (Pfannkuch and Wild, 2000). Logic, skepticism, and perseverance position the statistician to have no preconceptions, to use common sense, to be curious, and to taking time to do a good job (Pfannkuch and Wild, 2000). The authors suggest engagement is problem specific and more important and much more reliable is to have the student engage in seeking answers to worthy questions which can be taught. Just how can these be taught? Students must learn the role of theory including the statistical tools available and then practice these to give them the experiences they need to apply theory. This practice will need to engage the students (Berensen, Utts, Kinard, Rumsey, Jones, and Gaines, 2008; Davies and Connor, 2005) and then apply this knowledge to real-world situations (Garfield, 1994; Garfield, 1995). Students will need to recognize how, when, and why statistical tools can be used in an investigative process (Chance, 1997; Gardner & Hudson, 1999; Garfield & Gal, 1999; Melton, 2004). Students will also need to learn how to interpret results and any limitations to inferences (Melton, 2004). The instructor will play an important role in directing the statistics learner towards self-explanation (Broers, 2008). The instructor will direct the focus of the students to key ideas by means of a series of questions (Gal, 1997). These key ideas can be captured from the study material. Additionally questions are given to relate the concepts studied followed by mini-projects or some of the many types of assessments that will evaluate the knowledge gained like group projects (Starkings, 1997) case studies, critical analyses of research studies or issues in the newspapers (Garfield, 1994; Garfield & Chance, 2000).

2.4 Communication in the Statistics Classroom

Crucial to any discussion is the ability of the student to communicate effectively the results of their statistical problem-solving. Each time the teacher engages the students in classroom discourse, a complex set of objectives are being articulated via the examples presented to the class and through small-group activities created to enhance communication (Gal, 1997; Garfield, 1993; Garfield, 1994; Garfield, 1995; Parke, 2008). Teachers value communication because good communication skills are essential to teaching and learning in all subject areas (Chance, 1997; Gordon, Reid, & Petocz, 2005; NCTM, 2000). The function of conversation in learning has been described by researchers as enhancing understanding the learning taking place. The view that good statistics students are those that are mathematically strong would be true if statistics was just a number crunching subject (Moore, 1990). Furthermore, statistics offers other "fields of study a coherent set of ideas and tools for dealing with data" (Cobb & Moore, 2000). The focus on variability in data sets statistics apart from mathematics and gives statistics context. A "different kind of thinking is required" because "data are numbers with a context" (Cobb & Moore, 2000). Statistics reform has emphasized that statistics is composed of an investigation, analysis and use of data (Chance, 1997; Garfield, Hogg, Schau & Whittinghill, 2002). The statistics teacher's goal, to promote students who are effective in their environments, requires that the students learn how to be able to take problems from the real world and use the procedures of a statistical investigation followed by the analysis cycle to arrive at conclusions that can successfully be communicated to others in layman's terms (Chance, 1997; Forster et al, 2005; Francis, 2005; Konold & Higgins, 2003; Ritter, Starbuck, & Hogg, 2001; Rumsey, 2002). Rather than focusing solely on the "statistical" the statistics teacher's goal must integrate the "literacy" component of statistical understanding. The communication of the outcomes in words will require the student to express that statistical understanding adequately (Watson & Kelly, 2008).

2.5 Conclusions

Table 2 summarizes research studies on student profile, communication, discourse, and support of statistical reasoning in the classroom that were reviewed.

It becomes obvious that effective communication of statistical findings is an important component of this goal (Gal, 2002). In fact, communication has many facets in statistics courses (Garfield, 1994; Gordon, Reid, & Petocz, 2005) some of which are sufficient command of language to express ideas clearly, the willingness to talk to peers to develop understanding, communication of statistical ideas so that all understand, communication of the statistical ideas or concepts in layman's terms to help understand the mathematics involved, and enthusiasm to learn and understand by staying focused on statistical "talk". Therefore, along with basic statistical learning goals, teachers would like students to come away from the statistics course with stronger attitudinal goals such as considering the need for statistics, the importance of possessing strong work ethics, the need for communicating using statistical language to solve problems, and the need of supporting conclusions (Garfield, 1994; Garfield, 1995). A challenge to this goal would be the students' background in writing and particularly in technical writing. Stromberg and Ramanathan (1996) identify unfamiliarity with technical writing along with lack of understanding the material, lacking the ability to form coherent arguments from the facts, inability to follow instructions, and failure to write several summaries to improve writing as contributing factors to difficulty in the statistics course. The communication of both the statistics and their results forces students to come to a deeper understanding of the problem being studied and the appropriate supporting statistics to any inferences that are stated. The successful communication of statistical findings should involve the reporting of the exploratory analysis techniques used, the check of assumptions associated with any tests performed, and the report of any statistical findings (Robinson, 1998). This framework or scaffold can be applied across different types of data analysis. The researchers (Forster et al, 2005) advocate a second summary in addition to the first structure described above as "technical notes". This is called an "executive summary" and in this report most technical language is avoided. This report includes an introduction, a section to reveal important information about why and how the results are reported, the strength of evidence, quantification of the significant effects, and a summary of findings. These forms of writing should be an integral part of the statistics course (Francis, 2005; Parke, 2008; Watson and Kelly, 2008; Radke-Sharpe, 1991; Samsa and Oddone, 1994). Radke-Sharpe (1991) enumerates major advantages to requiring writing in a statistics course and in particular, “1) It improves writing skills, 2) it focuses on internalization and conceptualization of material, 3) it encourages creativity, and 4) it enhances the ability to communicate methods and conclusions.” These attributes are then conducive to helping students become better prepared to summarize and justify conclusions on the advanced placement statistics examination.

Table 2 Summary of research studies on student profile, communication and discourse, and support of statistical reasoning in the classroom

Study	Student Profile	Communication /Discourse	Support of Statistical Reasoning
Berensen, Utts, Kinard, Rumsey, Jones, and Gaines (2008)			X
Broers (2008)			X
Parke (2008)		X	X
Watson and Kelly (2008)		X	
Davies and Connor (2005)			X
Francis (2005)		X	
Gordon, Reid, & Petocz (2005)		X	
Mills (2003)			X
Gal (2002)		X	
Garfield & Chance (2000)			X
Pfannkuch and Wild (2000)	X		
delMas, Garfield, and Chance (1999)			X
Garfield & Gal (1999)			X
Robinson (1998)		X	
Wild & Pfannkuch (1998)	X		
Chance (1997)		X	
Gal (1997)		X	X
Starkings (1997)			X
Schwartz & Goldman (1996)			X
Stromberg and Ramanathan (1996)		X	
Garfield, 1995		X	
Gal & Ginsburg (1994)	X		
Garfield (1994)		X	X
Samsa and Oddone (1994)		X	
Garfield (1993)		X	X
Radke-Sharpe (1991)		X	

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Goals of the Study

In Chapter 1, this researcher stated that the purpose of this study was to determine how students' success in the AP Statistics course is determined by their profiles, and the communication and discourse that is part of the course. Therefore, the purpose of this study was two-fold. First, the study allowed the researcher to investigate how students' profiles, and discourse and communication in the classroom affected success in the AP Statistics course. Secondly, the researcher used interviews with the teacher to determine how attention to students' statistical thinking in the lesson planning process affected the effectiveness of pedagogical approaches in presenting statistical content. For this study, students' success in the advanced placement statistics course was measured by the end of course grades. Additionally the students' commitment to the course, another aspect of their profile, as well as evidence in growth of communication of statistical ideas was observed during their participation in the review sessions prior to the AP Statistics examination.

Released investigative tasks from prior AP Statistics examinations were used as a context to study whether the students had retained and mastered the concepts studied the previous semester when the students were enrolled in the course. The student achievement assessment data collected during the semester course by the researcher will include unit tests and quizzes, projects, as well as themes observed in the class communication between teacher and students and between students. This served as a basis for the types of investigative tasks and questioning during the review sessions prior to the AP Statistics examination. Limitations in students' statistical thinking observed during the course provided data beneficial to conducting the review sessions tailored specifically to the students.

3.2 Data Collection and Analysis

In order to accomplish those goals and to address the issues of interest, the design of the study was mixed methods with a greater portion of the study qualitative in nature with a smaller quantitative piece. A qualitative design was chosen for the larger piece of the study for several reasons. One of the purposes of the study was to describe how students' profiles affected student performance in a statistics course. Although a survey would have measured attitudinal aspects while enrolled in a course, the researcher desired to broaden the concept of student profile to include much more. A similar broad definition of profile (or disposition) was used by researchers (Johnson & Kuennen, 2006) to determine the most important determinants of student performance in a statistics course. The students' Grade Point Averages (GPA), scores on science portion of the ACT examination, student scores on a mathematics quiz, gender, and the teacher all impacted the students' performance in an introductory statistics course. The students in this study do not normally take the ACT but take the SAT which is comparable. Although students are not tested in science on the SAT, they normally will have taken a science end of grade test in middle grades as well as at least one science course (high school biology) and possibly a second (high school chemistry). Johnson and Kuennen were able to conclude that taking higher level mathematics courses such as calculus, or remedial mathematics courses were only marginally significant determinants to success in statistics. However, scores on a mathematics quiz (basic mathematics skills) were significantly related to students' performance. Therefore, the framework of this study included several components that were intended to describe those student characteristics that were likely to affect performance in statistics. This composite is referred to as student profile. The academic profile of all of the students was studied from documentation available in the student's academic record. The statistical communication was studied via direct observation, by the teacher and the researcher. Videotapes were made of each lesson and subsequently transcribed. The conversations and discussions were rich in detail and also allowed the researcher to observe non-verbal communication such as facial expressions, mannerisms, posture, and interest.

Purposive sampling (Patton, 1990) was used in the selection of the ten students to provide representative cases of the participants. These participants were chosen, with the help of the statistics' course instructor, to represent a cross-section of the students in the class. These students represented those students with above average writing skills, above average mathematics students with lackluster writing skills, students classified as juniors, strong mathematics students who have taken or are taking calculus, lower level academic students, lower level academic students with good work ethics, and average students. These "labels" were based on past performance in prior mathematics courses and performance exhibited since the beginning of the course. The goal in this type of sampling was to select cases that would assist in answering the questions in the study (Patton, 1990). These ten students also participated in one-on-one, non-structured interviews. Only one interview per student was conducted and a follow-up interview was planned if necessary to clarify or expand on a particular topic. The purpose of the interviews was to get additional data pertinent to the student's profile that was not captured by their academic record. This sample was not chosen with the aim of making statistical generalizations to a larger population of all high school students. Instead it was to describe examples of statistical thinking and dissimilar student profiles observed and how these affected communication and discourse in the classroom. Engagement in particular statistical concepts affects outcomes on assessments and communication in the classroom and this was observed over the duration of the study in order to better understand the interplay between the discourse facilitated and orchestrated by the instructor and the responses on the part of the students.

In order to capture accurately what transpired in the classroom, all observations of the class were videotaped. Additionally, all interviews of students were videotaped as well. Transcripts of these videotapes were compiled so that students would have the opportunity to reflect on what was said during the interview and support as well as clarify the intended responses and ideas. The videotapes served as a record of what transpired and served as a source of data to describe the conceptual change in the students.

Background data on the students was also collected as part of the student profile component and included, if available prior performance on a 10th grade writing test, the writing portion of PSAT test, a 7th grade reading test, English I score on the end-of-course test, SAT scores on critical reading, mathematics, and writing, Algebra I score on the end-of-course test, Algebra II score on the end-of-course test, and Geometry score on the end-of-course test. The Geometry assessment score was added because the course requires logical and deductive thinking similar to statistics. Students' background performance data on standardized tests was used to determine if past performance was helpful in understanding current performance in the AP Statistics course. This component of the study was analyzed through a two-sample difference of means t-test using SPSS to compare students that earned a three or better on the AP Statistics examination with those that did not. In addition to the evidence of prior standardized testing, the student profile component included scores on testing in the form of quizzes, projects, and unit tests in the statistics course prior to the beginning of the study. Data items collected during the study included scores on quizzes, projects, and unit tests on the units, Linear Regression, Two-Variable Non-linear Relationships, and Inference for Regression.

Additionally, the unstructured interviews were videotaped and provided data concerning likes and dislikes of statistical topics, how well-read the students considered themselves, the amount of course reading they completed outside of class, the amount of reading they completed outside of class in addition to the course assignments, their commitment to the course, their views on the materials used in the class, and how important they viewed this course in relation to their future plans in college. Well-read was defined to include reading of different genres, beyond what is required in their English course, and to be well-informed beyond the average student.

In order to measure growth in higher order thinking skills, data from the essay responses on each of the unit tests were collected and compared. The comparison was aimed at determining if there was evidence of change over time in terms of their attention to completeness and communication and use of statistical reasoning and concepts. Additionally, evidence of examining data via the process of the investigative cycle, different types of thinking, the interrogative cycle, and dispositions was evident through the essay responses.

The analysis of students' work and their interactions in the classroom with the teacher and between students provided a rich source of data to characterize their statistical reasoning. Chervaney, Benson, and Iyer (1980) and Chervaney, Collier, Fienberg, Johnson, and Neter (1977) identified three steps in the process of problem-solving that students demonstrate in using statistical concepts in specific problem situations: comprehension (relating a problem to another problem or a class of problems), planning and execution (solving problems through the application of appropriate methods), and evaluation and interpretation (interpreting results within the context of the original problem).

Wild and Pfannkuch (1999) view enquiry (questioning) as a starting point for linking the parts of statistical thinking, "thinking patterns involved in problem-solving, strategies for problem-solving, and the integration of statistical elements in the problem-solving" (Wild and Pfannkuch, 1999, p. 224). Students who have completed Algebra II should have experience with problem-solving. However, successful mathematics students do not just receive new information; they process the new information and "restructure the new information to fit into their own cognitive frameworks" (Garfield, 1995, p. 26). The framework that is built to organize the elements of statistical thinking has the thinker working in four dimensions: the investigative cycle, types of thinking, interrogative cycle, and dispositions.

Wild and Pfannkuch's (1999) framework, organizing elements of statistical thinking related to the four dimensions which include the investigative cycle, types of thinking, interrogative cycle, and dispositions, provided one lens through which data will be qualitatively analyzed. Garfield’s (2002, section 3, Table 1) general model of statistical reasoning provided a framework on which to base a characterization of students’ statistical reasoning. The model consists of five levels:

> Level 1. Idiosyncratic reasoning The student knows some statistical words and symbols, uses them without fully understanding them, often incorrectly, and may scramble them with unrelated information. For example, students have learned the terms mean, median, and standard deviation as summary measures, but uses them incorrectly (for example, comparing the mean to the standard deviation, or making judgments about a good mean or standard deviation).
>
> Level 2. Verbal Reasoning The student has a verbal understanding of some concepts, but cannot apply this to actual behavior. For example, the student selects or provides a correct definition but does not fully understand the concepts (for example, why the mean is greater than the median in positively skewed distributions).
>
> Level 3. Transitional reasoning The student is able to correctly identify one or two dimensions of a statistical process without fully integrating these dimensions, such as, that a larger sample size leads to a narrower confidence interval, or that a smaller standard error leads to a narrower confidence interval.
>
> Level 4. Procedural Reasoning The student is able to correctly identify the dimensions of a statistical concept or process but does not fully integrate them or understand the process. For example, the student knows that correlation does not imply causation but cannot fully explain why.

> Level 5. Integrated process reasoning The student has a complete understanding of statistical processes, and coordinates the rules and behavior. The student can explain the process in his or her own words with confidence. For example, a student can explain what a 95% confidence interval means in terms of the process of repeatedly sampling from population.

The classroom data collected by the researcher included work in which the students worked on authentic tasks, wrote short papers on statistical articles, and on enhanced (students must choose the best answer and show discriminating reasoning) multiple-choice questions. This data provided ways to evaluate students’ statistical reasoning.

In order to address the pedagogical issues, lessons across several units, Linear Regression, Two-Variable Non-linear Relationships, and Inference for Regression, were observed and videotaped. The teacher interview sessions with the instructor that followed the delivery of the lesson was an important component of the study. These sessions were used to discuss the positive and negative aspects of each of the lessons within each unit and to analyze how to make the lessons more effective. The discussion that followed the lesson was a critical aspect of the study and the center of attention was the students’ thinking and understanding. The lessons were analyzed in the debriefing session which followed the lessons. The teacher and researcher planned the lessons and the teacher then implemented the lessons. The lessons were videotaped and the focus was two-fold. Through focused observations of the students in the classroom, the researcher collected a detailed account of the learning of students, in other words, what they said and wrote, how the students used the materials provided by the teacher, what supports promoted understanding, and what difficulties to learning arose during the lessons.

The transcripts of the critique of the lesson during the process of the teacher interviews and videotapes of class discussions was used to evaluate student questions and teacher responses, post-lesson discussions, textbook and resource materials, and teacher-designed and used materials. This type of study was a shared process, so both the teacher and researcher were instrumental in gathering the observational data. The study allowed time to investigate aspects of pedagogy and content so that the instructor could reflect on the teaching and see how that fit in with the basic curriculum of the course and the experiences of the students. By allowing the teacher interviews to continue over several units, it was expected that each of the lessons could be streamlined to accomplish a particular goal or goals and to benefit the specific needs of the students. The knowledge gained from these discussions was used in subsequent review prior to the students taking their unit tests. The knowledge gained from these discussions and the reflective practice of the teacher work well within the view of Garfield (1994) “that assessment drives instruction” and “the main purpose of assessment is to improve learning”.

The type of activity, project, or assessment instrument employed by the teacher in order to assess growth in statistical thinking and understanding was included in the final grades on the activity, project, or assessment (Starkings, 1997). The assessments were in the form of unit tests, activities, and/or unit projects. These assessments were used to measure growth in statistical thinking and understanding from those completed earlier in the semester before the intervention was implemented. The observations of student-teacher and student-student interactions provided valuable information about the difficulties encountered by students in understanding the material being presented. Students had to communicate in written form because it provided a better measure of statistical thinking and understanding for successfully communicating the statistical thinking and understanding that were used in designing a plan to answer a statistical question (Radke-Sharpe, 1991). In addition to the statistical thinking frameworks for looking at students' work and interactions, data was analyzed and coded based on homogeneity and heterogeneity in order to include all themes and categories that provide a broader description of the lessons and the students' statistical thinking (Gay, Mills, & Airasian, 2006). This part of the study fits well within the philosophy of the AP Statistics course which stresses that students are to be engaged in constructing their own knowledge. The collaboration between the teacher and the researcher helped the students master the statistical concepts presented in these three units and provided insight into preparing review sessions for the students who were taking the AP Statistics examination in Spring 2008.

The last three units of the course are cumulative and required mastery of previous concepts presented in the course, therefore, the study also provides details about the concepts not yet mastered by the students. Students were expected to work individually or in small groups and plan and perform data collection and analyses. This gave students an opportunity to think through problems, make decisions, and share questions and conclusions with other students as well as with the teacher. An important component of the course was the use of group problem-solving, and writing, as a part of concept-oriented instruction and assessment. This approach to the teaching of an AP Statistics course permitted students to not only draw from their comprehension of context-based knowledge of the statistical problem but also to build connections between different disciplines and with their world outside of school.

In a study where data is to be collected and analyzed to finesse the intended responses and actual experiences of the participants, it is crucial for the researcher to be assured of the reliability of the codings of the data in order to be able to judge the intended responses of the participants. The meaning attributed to each situation by different individuals will be influenced by their backgrounds and experiences. Because the individual responses of the participants are of interest and give an in depth understanding of the experiences of the participants, all participants were given the opportunity to review their tapes and verify the intended meaning of statements or responses. The transcripts of the interviews with the teacher and students, the videotapes, and the audiotapes used pseudonyms or codes so as to not provide any information that would identify the teacher or students. A colleague with no vested interest in the outcomes of this study was asked to review a random selection of approximately 20% of the transcripts of the videotapes, recordings, and interviews to determine reliability measures from the coding of the data. Only the researcher had access to the audiotapes and videotapes of the participants in the study. Qualitative research software was used to analyze the large amount of textual data for emerging themes.

3.3 Study Context

The duration of the entire study was approximately four months. The subjects were students in an AP Statistics class. The researcher was introduced to the participants at the beginning of the semester and the study was explained to them. A letter of introduction was given to each student to take home to parents in September, 2007. This was done subsequent to receiving a letter of approval for the study from the Superintendent. This letter was used to introduce the researcher to the students and parents and to allow for any objections to the students' participation to be addressed in advance of the beginning of the study. A letter of assent was included with each letter of consent for each student enrolled in the AP Statistics course and sent with the students prior to beginning the study. Because the students were between the ages of 16 and 18, both assent and consent forms were necessary. There were no students who are non-English speaking, therefore, a special letter of consent for these students was not necessary. A letter of consent was also obtained from the instructor who teaches the AP Statistics course and with whom the researcher worked. Students who were not willing to participate in the study were excluded from the interview process and were excluded from the videotaping by strategically excluding them from the viewing window or expurged post taping. These students were not penalized in any way and participated fully in the lessons and activities that were part of the course.

Approximately two and one-half months were included in the researcher's observations of classroom lessons, discourse initiated by the instructor, communication between students and between instructor and students, and interviews with the teacher and selected students outside the classroom setting. There were daily visits to the classroom. Each visit to the classroom was approximately one hour and interviews outside the classroom lasted approximately one hour. The last month and one-half of the study was dedicated to interventions. All students who had registered for the AP examination given in May 2008 were invited to participate in study sessions in preparation for the Advanced Placement Statistics examination regardless of whether they particpated in the study. Because this examination was given after the students had completed the course, participation in the review sessions were strictly voluntary.

Students and the instructor were not identifiable because pseudonyms or codes were used to label each student and teacher. Only the researcher had access to the data collected. Interviews and classroom observations were kept on video tapes and/or audiotapes and both video tapes and tape-recordings and transcripts of these were kept secured. These data will be destroyed once the study has been completed and published. Personal data of students were kept anonymous and will also be destroyed once the study is completed and published.

3.4 Implications of the Study

There were several benefits to the subjects, instructor, and the statistics community as the study was completed. The students benefitted by having an opportunity to discuss their statistical reasoning and problem-solving in areas of statistics content identified and appropriate remediation was completed before taking the AP Statistics examination. The instructor gained knowledge of how the lessons needed to be tailored to address students' learning needs. More broadly, the statistics community will gain knowledge of how students' learning is affected by the students’ profile and how students’ communication affects performance on the AP Statistics examination.

CHAPTER 4: RESULTS

4.1 Introduction

The observation of classroom interactions between the teacher and the students served as the primary medium for data collection and was conducted with the expectation of eliciting patterns of statistical reasoning observed in responses to questioning and statistical tasks. As the reader progresses through the lessons described in this chapter, it will be possible to observe the development of thinking in the students and the classroom discourse initiated by the teacher. Although it would have been much easier to collapse these lessons and summarize what transpired, the researcher considered that it would be much more effective to use the transcripts of the lessons to successfully support the patterns of thinking and themes that emerged from the analysis of the transcripts and other data. These tasks involved exploring data, sampling and experimentation, anticipating patterns, and statistical inference as the guiding points of emphasis. The first part of this chapter will report on the patterns of students' responses that were identified in the lessons through quantitative analysis and will be followed by the relative levels of statistical reasoning of each. The discussion of the responses observed will be accompanied by descriptive student responses.

The second part of this chapter will describe interview responses of ten students as well as the profile differences using prior testing and assessments given in the AP Statistics course. There were multiple criteria for selecting these ten students and these students were representative of various sub-groups from the class of thirty students. Two students, female and male, were the only minority students in the class. Two students, male and female, were the only junior class students in the class. The other six students, three male and three female, were identified by the instructor of the course and represented average and better than average writers. This chapter will also report statistical analyses of student academic data and the results of a comparison between students earning a three or better on the AP Statistics examination and therefore earning college credit with those that did not. A quantitative analysis of the interview transcripts provides a mechanism to support and extend the quantitative analysis of the student academic data.

In the third part of this chapter, the findings from the analysis of the interviewed students' two major artifacts will be presented including their growth in statistical reasoning as well as deficiencies that were identified in the analysis of the artifacts.

4.1.1 Coding Schema for Observed Lessons

Data collected from the observed lessons were videotaped and transcribed. The videotaping of the lessons allowed the researcher to separate verbal from non-verbal communication for the teacher and students. The researcher applied a coding system to refer to the videos that helped identify occurrences of the teachers and students actions related to verbal as well as non-verbal communication. The coding system helped separate instances of different teaching approaches such as lecture, cooperative learning, small and large group discussion, and differing strategies of elicitation. Additionally, different activities and strategies to engage the students in the class and to promote discussion relative to a statistical concept were also able to be identified. Although the coding schema sought to separate the occurrences of verbal and non-verbal communication, it was not the intention of making the occurrences mutually exclusive. Table 4.1.1 summarizes the operational codes and examples of how the codes helped to differentiate particular instances in the lessons.

Table 4.1.1 Verbal and Non-Verbal Communication in Lessons

Verbal	Codes
Directed Lesson	DL
Whole Group Discussion	WG
Small Group Discussion	SG
Cooperative Groups	CG
Assignment Questions Student	HQS
Assignment Questions Teacher	HQT
Student Discussion Minimal	SDM
Active Student Discussion	ASD
Recapping of Lesson	RL
Calculator Discussion	CDS
Activity – Computer Printouts	ACP
AP Questions Discussion	APQD
Teacher as Facilitator	TF
Teacher/Researcher Discussion	TRD
New Topic Introduction	NTD
Activity Modeling Concepts	AMC
Contextual Based Discussions	CBD
Non-Verbal	Codes
Encouraged by student engagement	EN
Anger with student engagement	AG
Frustration with student engagement	FR
Pleased with student engagement	PL

4.1.2 Coding Schema for Student Interviews

Data collected from the student interviews were videotaped and transcribed. The videotaping of the interviews permitted the researcher to separate verbal from non-verbal communication in the interviewed students. It also permitted the researcher to gather data about the students that would not appear in their academic record nor be culled from the observed lessons. Table 4.1.2 summarizes the operational codes and examples of how the codes helped to identify explicit as well as non-explicit responses and feelings related to the course. Although the coding schema sought to separate the occurrences of verbal and non-verbal communication in the interviews, it was not the intention of making the occurrences mutually exclusive.

Table 4.1.2 Verbal and Non-Verbal Communication in Interviews

Verbal	Codes
Statistics viewed as easy	SE
Feelings towards course changed	FC
Feelings towards course same	FS
Prepared each day for class	PC
Not prepared for class each day	NPC
Writing different in Statistics	WDS
Writing in Statistics same Statistics	WSS
Like Statistics	LS
Statistics Easy	SE
Statistics Difficult	SDF
Enjoy Projects	LP
Don’t Like Projects	DLP
Study Guide Useful	LSG
Textbook Useful	TU
AP Questions Useful	APU
Prefer Whole Group Discussions	PWGD
Prefer Small Group Discussion	PSGD
Reads a lot	RAL
Read very little	RVL
Non-Verbal	Codes
Frustrated with course	FSC
Pleased with performance in course	PP
Confident about success in course	CS
Insecure about success in course	IS

4.1.3 Coding Schema for Student Artifacts

Data collected from the ten interviewed students included written artifacts that were coded to reflect levels of written expression and levels of statistical reasoning. The written students' work provided an additional point of view from the interviews and lessons. Furthermore, the coding allowed the researcher to determine strengths and weaknesses in students' understanding of statistical concepts related to the Central Limit Theorem and Regression Analysis. The transcripts permitted the researcher a way of identifying and making references to understanding or confusion concerning statistical concepts. Four of the interviewed students were selected to have an additional artifact analyzed, a critical statistical analysis, for growth in written expression in academic language and understanding of statistical concepts related to surveys. The coding schema used in the third artifact was similar to the first two artifacts. Table 4.1.3 summarizes the operational codes and examples of how the codes helped to identify particular instances of understanding or confusion related to statistical concepts as well as ability in written expression using correct English mechanics as well as providing appropriate contextual evidence in supporting their conclusions. Although the coding schema sought to separate the occurrences of conceptual misunderstanding from difficulties in English mechanics it was not always possible to isolate the occurrences so that they were mutually exclusive.

Table 4.1.3 Conceptual Understanding and English Expression

Concept	Codes
Normal Distribution	ND
Central Limit Theorem	CLT
Sampling Variation	SV
Mean and Median Difference	MMD
Standard Deviation	SD
Effects of Larger Sample Size	ELS
Skewness	SK
Chance and Probability	CAP
Coefficient of Determination	CD
Residual Plot	RP
Least Squared Regression Line	LSRL
English Expression	Codes

Table 4.1.3 (continued)

Poor Writing Mechanics	PWM
Good Writing Mechanics	GWM
Contextual Justification	CJ
Non-contextual Justification	NCJ

4.2 In the beginning

The usual pre-lesson procedures in class were to check homework on the overhead and for students to ask questions when their answers are not in agreement with the answers shown. Some of the homework tasks ask the students to work on a study guide and this is completed by students' use of the textbook as they respond to the items and questions.

Lesson 1: Linear Regression

The first lesson observation occurred when the students were working on linear regression. An analysis of the lesson (complete transcript of the lesson can be found in Appendix D, Lesson1) revealed that most questions at the very beginning of class refer to minutiae and not the big picture related to content concepts. For example: Student Z asks, "How do you find SUM on calculator?" In order to focus on the important issues the teacher stressed that the importance should be on what the computations mean. She says, "You can easily compute "r" and "r^2", coefficient of correlation and coefficient of determination, but what do these mean in your own words?"

Although this observation might be influenced by the combination of the researcher being there and videotaping the class, discussion was non-existent at first. Students were either unwilling or unable to answer. The teacher gave the students an appropriate wait time, approximately ten seconds, to answer and gave hints, cajoled, and finally finessed the answer that was partially complete and that was that the coefficient of correlation reveals information about how linear the data and whether the data is very scattered. The teacher paid close attention to the language the students used and recapped the comments made by the students into a cohesive statement which used appropriate statistical language that they would have to learn to use on future assessments and other discussions. After the students had completed a first reading on this material the teacher began to elicit the big ideas from the reading. The teacher explained that she was concerned about their understanding of the concepts and wanted to get a sense of their comprehension. By observing this lesson and other lessons during the following weeks, the researcher noticed that when the students had collectively answered a question, the teacher recapped so that the students would have an opportunity to focus not on the individual answers but on one fairly cohesive set of thoughts. As will become evident in future discussions of the lesson observed, there were a few students who always answered, whether entirely correct or not, a few students that rarely answered, and some students that answered only if called upon. It was difficult to get the majority of students to contribute to the discussion. The teacher had the attention of the majority of students but the students in general did not appear focused when there was the expectation that they should contribute to the discussion. In prior conversations between the researcher and the teacher, the teacher explained that the discourse initiated by her has been fairly successful in this course in the past. As she began to elicit a definition for the coefficient of determination she realized that was not part of the homework and stopped.

One way that the teacher tried to break the monotony of teacher to group discussion was to break the class into groups and have them discuss amongst themselves a particular concept. The students in each group were then responsible for addressing salient points from the reading with the rest of the class. The teacher then became the facilitator and managed the discourse to follow. So, following the discussion of the coefficient of determination, the teacher broke the class into groups and asked two students to lead the groups. These groups were then asked to address the concepts of least square regression line and residuals.

The discussion that ensued was quite revealing. The questions from the homework contained blunders. Before reading the three sentences, the teacher gave fair warning. “Now, I remember seeing something like this besides the textbook. I can’t remember if it is a quiz I give or a test that I give or an AP review. I’ve seen something like this, word of warning.” The teacher then read, “Each of the following statements contains a blunder.”

(a) There is a high correlation between the sex of American workers and their income.

This exercise was designed to have students grow from just idiosyncratic reasoning to verbal reasoning of statistical concepts. The teacher tried to initiate a discussion. Again, discussion seemed to be difficult for these students. At first the students were side-tracked by the word “high” and the teacher explained that they can have a high correlation because a high correlation means that they can have an *r* value close to 1 or -1. Finally, several students saw that gender is categorical and therefore you cannot have a correlation between categorical variables. The teacher wanted to make sure they understood why and several students agreed that they must have numerical data in order to calculate the correlation.

(b) We found a high correlation(r = 1.09) between students' ratings of faculty teaching and ratings made by other faculty members.

The next statement was a recall question intended to have students reach that transitional stage of reasoning where they were able to remember facts or dimensions of a particular concept and then understand the process of interpreting the correlation value. All students responded to this question with "acceptable" answers.

(c) The correlation between planting rate and yield of corn was found to be r=.23 bushels.

The discussion was brief because students were able to recall that r does not have a unit. Students had questions about this exercise. In terms of the AP curriculum, these are considered simple exercises. These questions covered only the definition and possible values of correlation covered in the reading in the previous assignment.

In general, discussions were very difficult to initiate. Either students had not completed their assignments or did not understand some basic ideas that should have been easily clarified by reading the textbook. Student A, who asked to have the exercise clarified, is not a strong mathematical student; the knowledge about whether 1.09 could be a value for the coefficient of correlation or not is considered a basic fact. Equally basic is the fact about whether categorical data could be used to compute the coefficient of correlation. Student J is considered a competent mathematics student but was confused by the wording of the problem and became preoccupied by the adjective "high" instead of looking at the context in which the correlation value was given.

For the next part of the lesson the teacher divided up the class. Because the study guide (Appendix E, Example of Study Guide) asked for definitions, the teacher wanted the students to go beyond the questions on the study guide and read the sections and pick out key points about Least Square Regression Line (LSRL) and residuals.

Once the assignment was completed, the teacher initiated the discussion and recorded the key points. Student L began to explain what she had learned about LSRL

"It says it is a line." Student F added that it was a method for finding a line that summarized the relationship between two variables. Explanatory and response variables were defined and the teacher asked for more important details about the LSRL. Student C added that it made the vertical distances as small as possible. Again, the teacher wanted to know what the LSRL had to do with regression. For this exercise, students did not focus on the reading and tended to be unprepared to answer more than just cursory and basic questions. After many back and forth questions and answers between the teacher and students, several important ideas emerged:

1) The process is intended to try to make the sum of the squares between points and the line as small as possible

2) When finding the regression line, $\hat{y} = a + bx$ and $\begin{array}{l} a = \overline{y} - b\overline{x} \\ b = r\dfrac{s_y}{s_x} \end{array}$.

3) $\hat{y}$ is the predicted response for the *x* variable.

These are basic concepts but they only emerged after much discussion. The next lesson will focus on having the students learn how to use the graphing calculator to do the work for them by facilitating the computations and allow them to concentrate on conceptual understanding.

Following this discussion, it was now the second group's job to lead the discussion on residuals. Student M initiated the discussion on the topic. Student M stated, “Points are not in a straight line but close to a straight line. Points are above and below a straight line.” Student M was confused about the regression line and the concept of residuals. The teacher directed the students to the textbook and explained: “If our data and our regression equation are going in a certain direction then there are some points above our regression equation and there are some points below our regression equation. Your residual is your distance to the line.” What should have been a straightforward discussion becomes troublesome and the teacher expressed some frustration. She recovered and then said that those points above the line have a positive distance and so a positive residual and those below the line have a negative residual. Students were just beginning to verbalize their ideas about these statistical concepts. Their answers did not show a deep understanding of residuals in the context of linear regression. After a much one-sided discussion several ideas were presented:

1) The residual plot should have points that will be normally distributed with some very close to the line and some a little away from the line if the regression line is a good fit.

2) Sometimes the data is a little curved and not noticeable but the residual plot makes it more obvious. When the students see that kind of situation then they should know that a linear equation is not a good fit for that data.

It was not obvious if the students understood the implications of having a random pattern for the residuals and the information that it provides the statistician in deciding whether a linear model is appropriate or not.

After discussing outliers and influential observations, it was then important for the students to learn how to use their calculators to perform linear regression, because the AP examination expects the student to be able to do this quickly and not depend on memorizing formulas to do these calculations. The teacher had set up a graphing calculator which was connected to a TI-Presenter which in turn was connected to a monitor that was visible to the entire class. The teacher always had a student at the presenter desk to perform the calculator calculations while she moved around the classroom helping the students. It becomes even more important to free the teacher to do this because there are 30 students in a course where 18–20 students would be considered ideal. Directions were given and an example, including the residual plot, was completed. The teacher initiated a discussion of what had just transpired.

The teacher emphasized that if they had a perfect graph and equation they could substitute a point to the left and to the right of a point given into the regression equation and those points could be graphed and the line could be drawn between those two points on the graph. Additionally, the residual was going to be the distance from each point to the line. Some of these residuals would have very small absolute values, some would be positive some would be negative. Other residuals would have larger values and these residuals are further away from the line. The students were encouraged to look for a pattern or lack of pattern. Some students observed that it looked similar to the other plot (of the data). The students were hesitant to make a judgment call and therefore were reluctant to talk. One possible reason might be that students were reluctant to answer when they did not know if they will be correct. The teacher summarized the steps and they were given an assignment to work on.

In analyzing what transpired during this lesson, there were several students that always tried to answer and others that did not contribute to the lesson discussion. The students' contributions to the discussion were characteristic of statistical reasoning between Level 1 and Level 3. The teacher was the main contributor to the lesson and initiated the discussions. In conversations with the teacher it became clear that the frustration with the lack of adequate daily preparation for class contributed to the lack of substantive discussions about the statistical concepts being studied. The students answered direct questions concerning definitions, calculations, and procedures to completing the exercises. However, they were reluctant to answer questions that were interpretive in nature. Student W enjoyed contributing to discussions and although he has knowledge of the concepts being studied, he has difficulty using the proper academic language needed to answer questions to the degree needed to obtain full credit on a teacher-made test and more importantly on the AP examination. Student F was unsure of herself even though the teacher can always count on her to have completed the assignment on a regular basis. Her answers tended to be at a level 2 because she could verbalize concepts but was not always sure how to apply the concepts to different statistical situations. It was possible for this student to recite the definition of residual but she was not able to interpret its meaning in the context of a problem. Student G usually had some questions that went go beyond the computational aspects of a problem. Her question concerning the scatter of the residual plot showed some understanding that the residual plot gives information about the appropriateness of the linear model to a set of data. Student G was still operating at a level 3. She may know the different elements of regression but still had some problems integrating them for full understanding and reasoning. Student I will rarely answer in a complete sentence. She is usually seen observing other students answer and trying to be playful. Student K is considered a very bright mathematical student but is often absent and not prepared for class. He is able to do well on tests without attending on a regular basis. He is able to do the computations but may not know how to apply the statistical procedures to a data set. Student K was working at a level where he understood the elements of regression and could process these to complete a regression problem but the conceptual understanding was not yet mastered. Student T is a very well-spoken young lady but does not focus on the details required in statistical procedures. She has contextual knowledge but cannot apply the proper statistical analyses to particular situations. Student V rarely answers but is correct most of the time demonstrating a level 4 of statistical reasoning. He understands the procedures and is well on his way to understanding how to integrate all the dimensions of a regression problem. Student Y will not answer unless asked. He is bright and took AP Calculus as a junior. It was evident from his answers that he did read the assignments and knew the functional and conceptual dimensions of a regression problem. Student Z was also not very active in contributing to classroom discussions. He has also taken AP Calculus. He was able to understand the dimensions of a regression problem but had not integrated them to fully understand how they are interrelated. Although the teacher was near the end of the chapter on linear regression, students were still not clear about basic concepts related to appropriateness of the linear model and the information about the model that is possible to finesse from the residual plot. In spite of the time constraints, the teacher managed to introduce the statistical concepts that she had planned for the lesson and had initiated some discussion even though the students rarely contributed any new ideas or observations.

Lesson 2: Reading Linear Regression Computer Printouts

The following day (Appendix D, Lesson 2) the discussion of the homework made it evident that the students were focused on the discussion but still struggling with the concepts of regression. Student Q asked if the regression line and linear regression line were the same. The problems (except for one problem) given for homework were to be done with the calculator. One exercise, without the use of the calculator built in functions, had been done the previous day in class from start to finish. This experience was important for the students. Even though the graphing calculator is a very important technological tool that makes computations easier, it is also important for the students to understand the statistical formulas they are using in regression. These students have all completed Algebra II and most have completed Pre-Calculus, therefore, a problem involving a formula should not be taxing. One unusual problem that a student asked for the teacher to discuss gave information about the correlation between two variables and the mean and the standard deviation for both variables. A value for the explanatory variable was given and the students were to determine the value for the response variable. The formulas for linear regression are basic formulas similar to what the students have been exposed to in previous mathematics classes and these should be easy to use in answering the question. The third part of the question asked the students to calculate r^2 and to determine if the prediction equation gives a higher or lower value than the actual one. Because the coefficient of determination is only 0.6 that meant that the coefficient of determination was only 0.36 and it was not high. Therefore, the actual value was probably higher.

This went beyond just knowing how to compute the individual values for a regression equation. The level of statistical reasoning that was being asked for was beyond the procedural. If the students are to do well on the AP examination, they will have to think beyond the basic ideas and integrate the elements of this multi-dimensional concept.

One other expectation of the AP Statistics examination is the ability to read computer printouts. The teacher distributed a variety of examples of regression with different types of printouts. In the past the teacher has also had the students work with MINITAB which allows the user to complete statistical analyses. Because this was the first printout that asked the students to extract information, the teacher went through the handout and explained the meaning of slope and showed them how they would have to pick out intercept and slope from the tabular data. These values were labeled constant and variable coefficients. Students will have to interpret these in the context of the problem (data) and will also have to look for r and r^2 (adjusted).

This task seemed to engage the students and future lessons in which this type of activity was used will usually prove to be useful in focusing the students on the concept being studied. Student L discussed the first problem with her partner. She seemed to be able to explain the difference between constants, intercept and slope. She explained that they were not both slopes. Most students appeared engaged, discussing, and working on the problems. The teacher emphasized that patterns were important for determining linearity. She reminded them about the time factor in completing questions. The teacher circulated about the classroom answering questions. The researcher was recruited to go around the room and to also answer questions. Several conversations indicated that the students understood these problems. A discussion ensued as to how to interpret the slope and intercept in the context of the problem. This will be an ongoing problem.

For example, in part b) the value of the slope, b = 233.517, had to be interpreted in the context of the problem. Student bb could not interpret it correctly. Student W tried to answer by saying: As you increase # aircraft means for each year there are 233.517 more aircraft.

A more difficult problem for the students was interpreting the intercept. This is not a statistics skill but one that is developed in Algebra I and II.

For example, student E stated that each year you always had that # aircraft. Student T took a “gamble” and said that in 1990 there were that # of aircraft in service. Year 0 is 1990 and that was the intercept.

The students were starting to answer questions about interpreting the constants in the context of the problem. This type of question was similar to what they will encounter on the AP examination rather than a textbook exercise. Additional questions are asked concerning how to read and interpret computer printouts. They were also beginning to understand that computations alone will not earn them credit on the AP Statistics examination. Student L was beginning to understand linear regression very well. She was able to interpret the values for constant and slope in the context of the problem. She was working with her assigned partner and talking through the process with her and was able to explain the concept satisfactorily. The majority of the students were working through the mechanics of linear regression but were having difficulty with the interpretation and reading the values from the computer printouts. Although these were AP questions from previous examinations, they should have been familiar to the students because they have done these types of problems in their daily assignments. Student T surprised the teacher today and often will surprise the teacher with her insight into problems. Student Q often stresses over how to express the answers to the questions and asks for confirmation from the teacher. Student Q seemed to understand that learning to use the appropriate language will benefit her in the long run.

Lesson 3: Advanced Placement Regression Exercise

The third lesson about regression began with a discussion of AP Question # 4 from 2002. The students received a partial computer printout along with a scatterplot with the least square regression line (LSRL) superimposed on the scatterplot. Additional information about the data was added. The three parts to the question asked the student to:

a) Determine the regression line and define the variables used.

b) Determine the coefficient of correlation and provide an interpretation.

c) Describe if the LSRL would be a good description for a smaller range of values.

The students were able to correctly determine the regression equation but not all students were able to identify the variables. This should not have been a major task because the graph was clearly labeled and students have graphed in all previous mathematics classes.

Because r^2=57.0% the students had to find the square root and then determine that it was positive based on the LSRL. The coefficient of correlation was found to be r=0.75. Student X correctly said that it would be a moderately strong association and you could see it was positive from the graph. The teacher asked if the students could interpret r^2.

Student E gave a partial response when she said that variation was explained (57%) by variables in the graph. The teacher continued negotiating a complete definition of coefficient of determination in the context of the problem with several students until she explained that a more complete answer would be that 57% of operating costs was explained by the change in number of passenger seats.

The third part proved to be more challenging for the students. The following excerpt of what transpired shows the difficulty the students had in trying to explain what they had observed in the scatterplot and interpreting whether parts of the data could be better explained by a better least squares regression model.

> Teacher: Does the LSRL provide a good model? No, data has negative association. Why?
>
> Student aa: No, because points are changing.
>
> Teacher: Explain, since points are not changing. What did change? Did you cover points in range? What is linear regression about? Line should show relationship between points.
>
> Teacher: What do we call this line?
>
> Student bb: Line can be called line of best fit.
>
> Teacher: (Question to Student aa) – Are the middle points still going in the direction of line?
>
> Student aa: I think so.

Student aa hesitated in answering. Student aa did not understand the regression equation and how it related to the data.

Student K explained that the answer should be that it would have a negative relationship or association. The new line would have a negative slope instead of a positive slope. Student W added: It would be a totally new line.

The lesson continued with the discussion of AP Question – 1998#4. The students were given a partial computer printout and a residual plot. Additional information about the problem was added. The two additional parts to the question asked the student to:

a) Determine the regression line and define the variables used.

b) Determine if the prediction for a certain value of the independent variable would be too large, too small, or a prediction could not be made based on the information given.

There was less hesitation in answering the regression exercise this time and the first part was completed quickly. For the second part, students knew to substitute the value but had more difficulty answering the question. The residual plot had been given and had not been used in any part of the question. The students should have surmised that it should be used in this part of the question. This should have follow from what they have done in class. The teacher reminded the students of the equation Residual = Observed – Predicted = some negative value.

Students expressed some confusion with this statement even though there does not seem to be a problem when students are asked to use the calculator to determine the predictor equation. They do not appear to understand that the residuals are the difference between observed and predicted and what this means. This was not the first time the teacher had talked about this concept and they have completed many examples related to this topic. Many students do not understand the idea of using residuals to answer a question, yet they are not asking any questions.

The last question for the day was AP Question 2001#6. The sixth question is typically used to stretch the content beyond the basics. It should take about 25 – 30 minutes to complete. It is intended to use the knowledge of concepts learned in the course in a different scenario. It is worth twice as much as the other problems.

The students were given two sets of data, the regression output for each data set, informed that it was a linear regression model, that the residual plot did not reveal any unusual patterns, and that the assumptions for inference were reasonable. The three parts to the question asked the student to:

a) Use an appropriate display to compare the same variable for the two data sets.

b) Determine if there is a significant relationship between two of the variables in one of the data sets.

c) Determine, if a new observation is given, whether it would belong to one data set or the other.

Only one student had completed this exercise and the teacher had to start by asking the students to put the data into their calculators to begin the analyses. Because a comparison was asked for in the first part, it seemed reasonable that the students should have resorted to using the descriptive graphs studied at the beginning of the course to compare the data sets. A student suggested using a bar graph but he really meant histogram, because the student was using the bar graph to describe numerical data and not categorical data. Students inadvertently interchange the types of graphs, bar graphs with histograms, demonstrating that they do not remember that the use of each is dependent on the type of data. Another student suggested a double bar with the two variables of interest. Still another suggested two different scatterplots. The teacher suggested that because they cannot come to a consensus that perhaps it would be best to put the data into their calculators. This suggestion was presented in order to spark the students' memory concerning boxplots and histograms. The students were quick to remember the 5-number summary as a way to compare the two data sets.

The researcher was encouraged to make a comment about the difficulty in deciding what to do to answer this question. The researcher explained that key words lead you to what you have to do and they should look for those key words. The students should answer the question in the simplest manner after reading the questions carefully. The parameter of interest could be mean, standard deviation, or proportion. In this case they had been asked to compare. So they needed to ask themselves, "How do I compare in the simplest way possible?"

For part c) it seemed reasonable that because the students had not used the information about the regression that this would be the approach to take. Student F substituted in one of the regression equations but did not rule out the other data set. Student T suggested that they should look at both regression lines and determine if the point lay closer to either of the least square regression lines.

After initial successes very few voiced feelings about being confident about the AP Examination. The teacher encouraged them by saying they should feel confident by May.

After the lesson, a discussion with the teacher revealed that the teacher was still perplexed as to why the students are having difficulties using appropriate statistical language when answering questions or when justifying their procedures. The teacher and researcher have noticed that the students are trying to memorize standard statements to make instead of trying to understand what it is they have to show as evidence for or against and supporting it with the statistical values. Students are unclear how to extract information from the prediction equation and residual values. They are able to compute values of interest but not to interpret these in the context of the problem. The majority of the students are between the verbal and transitional levels, levels 2 and 3, respectively. They appear to be trying to memorize instead of applying logic to what they are doing.

Lesson 4: Regression Model Inference

In the following lesson (Appendix D, Lesson 4) the teacher continued with linear regression. The class began discussions concerning inference and its relation to the regression model. They read first in order to answer questions about what they have gleaned from the reading. After a long break while students read, the teacher began by saying, “Since we have been talking about regression and scatterplots, it makes sense to me to talk about regression slope. We are going to be talking about how to run a significance test just like we did to check if we have a significant relationship between our variables–just like we did with regression. It is going to talk about significance just like we looked about having a significant sample in our earlier work.”

Just as in previous days, when a new topic is introduced, the students were hesitant to discuss what they had just finished read. Questions about the meaning and what is being accomplished by doing regression inference did not appear to be evident for the students. The topic of confidence intervals was brought up and a connection to previous examples with confidence intervals was attempted by the teacher. The students struggled with how to explain what they had just read. One student decided to attempt an answer by speculating that it had something to do with the slope of the true regression line. After relating the reading to the example completed for homework, the teacher and students came to an agreement that they were trying to predict the slope of the regression line and that would be β, trying to predict beta and that is done with the sample. The students agreed that when b is obtained from the sample, then it is not β but an estimate of β.

Student W suggested that the confidence interval for β was how well the predicted value matched up to β. The teacher argued that because the true slope is not known then one may not be able to tell that the predicted value matches up with the true slope. The language issue is still a problem with these students. Care is not taken when explaining concepts therefore it can lead to the students making comments or coming up with explanations that are not entirely true. Student T answered that it is an estimate of where beta lies. The teacher continued to query, sensing that the students are having problems describing what the confidence interval actually is telling the statistician, a bit more trying to finesse a good description. Student T proposed a 95% probability. Of course, the teacher retorts that probability is a dirty word (when describing CIs) because it will mislead them.

This is a topic that had been discussed for several weeks earlier in the semester and students had difficulty with the concepts. It is still not clear to the students. The teacher explained that if they are talking about a CI for beta then if an interval is constructed then they hope it will capture beta. She continued explaining that they are trying to make a CI which will capture the true slope β. But of all the CIs constructed, 95% of them will capture the true slope. A statistician will never know if it does or not capture the true value of the slope of the regression line. Finally, the teacher clarified by saying that they should be 95% confident that the CI will contain β. The teacher and students continued refining the concept of CI for the slope of the regression line and the teacher went on to ask what else would occur when conducting inference tests.

The students began to recall and quickly said that the next thing to do was a test of hypothesis on how well the data falls into a linear pattern. The teacher reminded them that all the statements they are making can be answered with what they know already. That is: how well *x* relates to *y*, residuals, strength of the linear relationship, and straight line dependence between *x* and *y*. The teacher made clear that if the students just thought about their statements they would know that they already knew how to answer those questions.

After turning their attention to the reading, the teacher and students finally agreed that what they do not know is whether the slope is equal to zero. The students agreed that would cause serious implications for the linear model because if the slope was equal to zero that would indicate that there was no significant relationship between *x* and *y*. The test of hypothesis would say that the slope is zero against that which says that when *x* changes *y* changes in a particular way. The teacher wrote the following on the overhead projector and made an effort to accompany the hypothesis set-up with the descriptive language.

H_o: β=0 If we cannot reject H_o that means there is no significant relationship between *x* and *y*.

H_a: $\beta \neq 0$ Alternative hypothesis – This is a two-sided test. The context of the problem will dictate the alternative hypothesis.

There seemed to be a better grasp of the idea of test of hypothesis in the context of the slope of the linear regression line as evident through students increased attention and participation in the discussion. The teacher summarized the discussion and said that they were trying to predict the slope with a CI. They knew that the slope could be obtained from the sample so now they were trying to construct a CI that would contain the true slope beta. The second thing they were doing was to try to run a test on the data to see if there was a significant positive or negative linear relationship that drew away from the null hypothesis. There was significant evidence that there was a linear relationship. The teacher emphasized that they will have to say it in order to learn how to say it correctly. After summarizing what they had finally agreed concerning the test of hypothesis and confidence interval for the slope of the regression line, the teacher returned to Student W and emphasized that Student W should not keep from answering even if he is not entirely correct.

The class then began to work on an exercise from start to finish in order to give them the opportunity to remember a lot of the procedures from the previous chapter and project. The example gave the data for the femur and humerus for an extinct bird-reptile looking beast. The students were asked the following:

Part a) Make a scatterplot with femur length as the explanatory variable. Determine the coefficient of correlation and the regression equation. Do you think that femur length will allow a good prediction of humerus length?

The teacher explained that a correct answer was not yes or no. A complete answer will be a yes or no with supporting evidence. A sentence will have to accompany the answer in order to explain the question fully. There was no response at first, but Student R answers that the question is yes because the r^2 value was high. When asked what r^2 meant, the student said that it was how the data fits the line. The teacher was visibly disheartened and asked again, "What does r^2 really does tell us."

Student T mentioned that r^2 told us how close the points were to the linear regression line. The researcher interjected that Student T said something very good when she mentioned how close points were to the linear regression line. This can be said in statistical terms. Student E then asked if it is not the percent of variation in *y* explained by the change in the *x* variable. A bit of a breakthrough, great, so 98% of the variation in *y* can be explained by the change in the *x* variable. Student T had the right idea but Student E said it right by answering with supporting statistical values given in the context of the problem. The second part of the question followed:

Part b) Explain in your own words what beta, the slope of the true regression line, says about the extinct beast.

The students answered that every time the femur length changes by one centimeter the humerus changes because it goes up by 1.197 cm. The students were able to calculate the intercept and slope easily. They have no trouble with the arithmetic involved. They also realized that a negative value for the intercept will cause problems with the model. The rest of the exercise was done quickly because it involved calculator computations.

The students have had a bit of trouble remembering the concepts from the linear regression unit and the tests of hypothesis unit. This unit on inferences about the regression model brought all the individual concepts into play. As has been the problem from the beginning, discussion was predominately one-sided because the teacher hinted, asked for clarification, and recapped the salient points of the discussion. Very few students answered and they usually answered in the form of a word or two. A second example should have helped them recall how to determine the parameters of regression intercept, slope, and standard deviation of the residuals.

Lesson 5: Linear Regression Continues (Short Lesson)

Lesson 5 (Appendix D, lesson 5) commenced with Student U saying that she was confused about how to explain the standard error around the regression line and the standard error around the slope and how to calculate these values. This was not an easy concept to explain, however, Student U does frequently ask these types of questions to clarify her understanding of what the concepts mean and how to perform the necessary calculations. The teacher explained that the value, s, is the standard error about the line. It was the standard deviation of the residuals. The standard error of the slope is SE_b. That value is usually reported in a printout. It is $SE_b = \frac{s}{\sqrt{\sum (x-\bar{x})^2}}$.

After the discussion concerning the clarification of the two concepts surrounding standard error, the teacher turned her attention to checking the assumptions before the linear regression model was calculated. The students responded to her questions that the assumptions included discussing pattern, spread, how well the line fits, checking for normality, and looking for outliers with confidence. The rest of the lesson was spent on exercises to practice the new skills leaned so far.

Lesson 6: Test of Hypothesis on the Slope

The lesson (Appendix D, Lesson 6) started with a discussion prior to a quiz. The students were working through a test of hypothesis of the slope. They were to formulate the null and one-sided alternative hypotheses about the slope, complete the test by reporting the *t*-statistic, the degrees of freedom, and the *P*-value. The final step was to write the conclusion in narrative form. The teacher said that the *t*-value = 7.2557 with the degrees of freedom = 9. The students had a discussion about whether it should be 8 or 9 degrees of freedom. They settled on 8 as the correct degrees of freedom with the teacher's guidance because you lose two degrees of freedom when calculating standard deviation. The teacher asked the students to use their calculator by typing *tcf* (7.2557, 1000, 8) to determine the *t*-value. The interchanges that follow suggest that students are not reading the assigned lessons nor are they retaining major concepts from one unit to another.

Student U: How do we word the results for CI in problem 5?

Teacher: Since slope is 2nd round scores and if increase of 1st round scores increase by 1. The second round scores increase by the slope for every increase of one unit in first round scores.

Student G: How do we get standard error by hand?

Teacher: You need to use the formula.

Teacher: You use residuals found in regression and follow outline on page 759-760 in textbook. Very rarely are you going to have to do it.

This is the same standard deviation formula given earlier in the semester but students did not appear to recognize the similarity.

Student M: What is the standard error about line?

Students raised several questions that indicated their confusion. It is not a difficult assignment and it is expected that students should be able to complete the task with greater ease given that this type of work was done with other distributions.

Student A: How do we shade for the t distribution?

Student A – How do we find p-value?

The teacher repeated the procedure after finding t value etc. though she expressed that they should know this information. She asked for them to hand in homework before taking a quiz. Later on in the class the conversations revolved around a great deal of minutiae. Many of their questions related specifically to information they could find in the assigned readings. Other questions could be answered by recalling what they had done in previous units. None of the questions raised involved new concepts or ideas.

For the next lesson students will be engaged in performing transformations before performing linear regression. This builds on what students have learned about the basics of linear regression; therefore, the lesson should flow more smoothly. All the students in this class have taken Algebra II and most have already finished Pre-Calculus, therefore logarithms should be a familiar topic.

Lesson 7: Exponential Models in Regression

The lesson (Appendix D, Lesson 7) commenced and the students were working on determining the exponential equation after they had found the linearized data and applied logarithms to the data. The process was not going very well as can be seen in this excerpt from this session.

Teacher: To transform from log values back to exponential or in terms of y form we raise the value to power of 10. All terms to a power of 10. And that will cancel the log from each term. In other words it gives $\hat{y}$ equation. Leave it in the form you get.

Student M: I got lost in the process.

Taking values out to 7 or 8 decimal places makes your equation (to Student Z) very different from the one I have to 4 decimal places. If you try to predict you will get different answers.

Teacher: To graph data as a scatterplot you will find regression equation for List 1 and List 3. To graph residual plot use List 1 and RESIDs.

Teacher: Let's look at the residual plots.

Student L: Mine was almost on axis.

The teacher helped Student L see why her plot was not "real good looking". This helped Student L see just the residuals instead of other data as well.

Student F: What is this y equation?

Teacher: To see the equation from y_1 insert 10 ^ (of equation). That is the original curve equation.

The teacher noticed that talking was getting them off task. Students seemed confused and not engaged in the task. Different questions from various students seemed to be contributing to the others getting off task.

Teacher: (Is showing on the overhead calculator.) This is a curve. Now graph scatterplot of original data with curve.

The teacher asked if the equation seemed to fit the data pretty well.

Student T: Is this the same as the exponential regression on the calculator?

Teacher: You should never use the exponential regression button to do what they are trying to do. It will give a different equation. The Standard Course Of Study requires us to use transformation of data (ie. linearize the data) to determine equation for this type of data. That was a model for growth.

The students realized that the regression process is similar for the case of non-linear data. However, they will have to learn to decide when transforming the data will be necessary. They will also have to decide whether to apply logarithms to either both the explanatory and response variables (in the case of the power model) or just the response variable (in the case of the exponential model). A short activity was done in class to show how exponential decay can be modeled.

Activity to Model Decay

Teacher: You should have M&M's and you should have about 40 in the cup. So for trial 0 it gives 40 M&M's. You will continue to keep pouring M&M's onto a plate and those falling with an M up are then put back into the plastic bag. The remaining go in the cup and are tossed on the plate until they are all gone.

This was a very long activity. However, students were very engaged in the activity and produced some interesting scatterplots to mimic exponential decay. This gave the students some characteristics to look for when deciding whether an exponential or power model is more appropriate just by looking at the scatterplot of the data. The activity lasted the rest of lesson with discussion to be completed the following day.

Lesson 8: Linear Regression vs. Non-Linear Regression

After the previous lesson the teacher thought it would be a good follow-up (Appendix D, Lesson 8) to talk about what kind of situations would warrant an exponential model rather than a linear or power model. The teacher explained that most data will be considered linear at first and the residual plot will be looked at to determine the suitability of the linear model. The students were engaged in telling the teacher the process to follow. Some of the students who answer on a regular basis were sure of the steps to follow. The students knew why the steps were used and what to look for. The teacher asked about the coefficient of correlation and tried to persuade the students into thinking that a line fits the data. However, the students noticed that the data had a pattern. Student W suggested looking at the residual plot. Student G committed an error by saying that the residual plot would look linear. The teacher wanted the students to determine if the data was exponential or not. Student cc believed the plot looked distinctively curved and said that implied that a straight line fit was not going to work. Student cc suggested dividing consecutive values of the response variable to see if they obtained the same values. This would be one more clue to using an exponential model.

Student Q read the paragraph that introduced the problem. The problem stated that the data that they would be discussing was about moths it would seem that the exponential model was appropriate. The teacher wanted the students to decide why moth populations grew exponentially. There was much to talk about and the students discussed the life cycles of moths and reproduction. This conversation is essential so that students learn to think about the data before assuming that one model is better than another. Populations usually grow exponentially but other things do also. The students gave some great examples. Student W: bacteria or growth of populations. Student U: technology. The teacher said that it was an excellent example because the number of people owning I-Pods, money or investments should hopefully grow exponentially. The teacher warned them again about using the exponential function built into the calculator. The students seemed to know the process. With the help of the students the teacher went through the whole calculator-based procedure of where the data was and also the logged data. She then performed linear regression on List 1- years and logged data for the acres defoliated by moths.

Student Q was able to verbalize quite easily that there were some residuals above and some below. Student F said that it was a perfect model because points were randomly scattered about the line. The next part of the process was to perform the inverse transformation. The teacher explained that this was the regression equation but that it was log of y-hat not y-hat. The straight line was used to fit the logged data and it needed to be transformed to graph it on the original data. Overall this discussion went well with many students answering and showing confidence with the procedures to use when performing non-linear regression involving an exponential model.

The class now transitioned to work on power regression. Examples were discussed because the teacher wanted them to get an idea of how to determine what type of model it was – whether it was an exponential model versus a power model – because they will have to make that decision on some problems later on. Knowing that students rely on the coefficient of correlation to determine the best model, the teacher suggested looking for information about the data to support their choice of a model. The teacher explained that looking at data in different domains might indicate the data is linear versus curved. However, knowing something about the data might make their decision about the appropriate model easier.

The teacher then presented an example of a power model. This model concerned the weight and length at different ages for Atlantic Ocean rock fish. Weight is related to volume, therefore it made sense that the model should be cubic. The students were again engaged and answered the questions concerning the procedures to follow if it were a linear or an exponential model. Student Z was at the overhead calculator. The students looked at the original data and saw that it was curved. Plotting the explanatory variable against the logarithmic values of the response variable (for exponential model) showed that this model was not working either. After plotting the log of *y* and log of x the students saw that the scatterplot was much better. It will be important for the students to keep track of which lists refer to which data. Student M suggested looking at the residuals. Student Z had graphed them – looked like it was fitting pretty well. Student G asked to look at the residual plot, but asked how that was done. Student Z said to use the logarithmic values for the explanatory variable and RESIDs (residuals). The students are showing progress in characterizing the residual plot. Student K, said, it had no pattern – no distinct pattern. Student M said that it meant that this fit the line of regression. The wording was not the best prompting the teacher to say you are there but think about how you are going to word it. How would you interpret it? You said all the key components.

The teacher repeated what Student M had said: because this residual plot had no distinct pattern it fits the line of regression well. Student M's wording was faulty but he recognized that the pattern of the residuals or lack thereof was a good sign that the model was appropriate. This was a breakthrough for Student M. Some of the other students are having a more difficult time with this and some of the other characteristics of the residual plot.

Teacher: The points are spread more towards the end of the *x* axis. Why do you suppose?

Student I: (a bit frustrated) there are not a lot of short fish.

Teacher: Well, look at the fish, look at the lengths of the fish. Because the *x*-axis is the log(length), look at the lengths of the fish.

Teacher: Are they a little more concentrated towards the end? So, is there a problem with that?

Teacher: Is that going to be a problem in this particular case? Think about it. Is it a problem that most of the points are over to the right?

Student Z: No, it is showing that as they are getting older they don't grow as large, grow as much or as quickly.

Teacher: What do we need to say to interpret this plot? This is a hurdle I can't get you guys over. It doesn't matter what I do, I can't get you over it. It is going to take you to do it. What do you say Student G?

The teacher showed that she was frustrated. She offered hints, leading questions, and exhortations, but they did not make the students think about what was going on. They were not using contextual evidence. The students do not show that their previous mathematics experiences have helped them develop useful discourse skills for communicating mathematically. The following discussion focused on pushing the students to say what is expected on the AP examination.

Student G: What about the scale?

Teacher: Does it have something to do with how large the residuals are? Does it matter? This is to the class, does it matter how large the residuals are? The residual plot, what are we interested in?

Someone in the class says – sum of residuals are zero.

Teacher: We know the sum is zero. That is nothing remarkable. You are never going to find a residual plot where the sum is not zero. That is not really noteworthy. That is something we need to be aware of. What is important? I heard a word.

Student: (Several students blurt out.) scattered above and below the line.

Teacher: They are scattered both above and below the line with (kids add) no distinct pattern. What does that mean?

Student K: I'm not sure if this applies to this chapter there is a curved pattern with a straight line.

Teacher: But this does not have a curved pattern so a line will be an appropriate model for this data. This data is not an appropriate model for this line – that is what you said first of all, I believe. Is that what he said at first?

Student G: The linear model is appropriate for this data with the log *x* and log *y*.

Teacher: You guys are going to have to start saying that. Start saying the whole statement. Don't think that when I read your quiz or your test or your paper that you are writing that I will know what you mean. Say the whole sentence.

Teacher: Student A–Would you like to make an A in this class? Let's suppose she says, I would like to. If you hear I'd like to, but don't hear the question that I asked, and Student G says I'd like to, that could get you into a lot of trouble. I'd

> like to cut class. I'd like to rob a bank. But, instead of I'd like to, she could say, I'd like to make an A in the class. I'd like to make an A in AP Statistics. Get specific. Say the whole "schpeel". Don't assume that just because the question is, "Do you want to get an A in the class?", that you can say, yup. You have to say the whole thing.

The class then tried to determine the equation that would fit the original data. This meant that the linear equation needed to be "untransformed". Once this is done then the equation can be used to make a prediction. The class finished the problem together.

The students are continuing to make breakthroughs. Some students are beginning to see that some of the same questions asked in the study of linear regression need to be answered when doing regression using the transformation of the data. There are moments when the students are involved and focused on the ideas presented by the teacher but they have a long way to go to demonstrate successful levels of thinking for the AP examination. In order to be prepared for the AP examination they will have to ask these pertinent questions internally, just as the teacher has asked them, and they will not be able to rely on the teacher for prompts. The matter of answering in complete sentences with contextual references is still an ongoing problem. Some students are integrating all the processes well enough to understand regression really well and some students can only verbalize what is to be done without understanding how to integrate all the procedures within the context of the problem.

Lesson 9: Regression Continued

The students received their major assignment back on linear regression and the assignment was incomplete for many students (Appendix D, Lesson 9). The teacher knew that it was important to review this one more time because this is a concept that will be tested on the AP examination. The teacher outlined what needed to be shown on any regression problem:

1. Draw the scatter plot and label it.
2. Find the regression equation and write that in context.
3. Find *r* and interpret it in context.
4. Calculate the residuals.
5. Make a residual plot.
6. Write an interpretation of the residual plot.
7. Comment on unusual points and remove them.

The teacher explained that in general, the residuals for each *x*, the explanatory variable, needed to be in a list or graph. Regression needed to be done in context and the equation should have been premised by a sentence of what they were doing. The interpretation of *r* had to be done in context, say the value and if it indicates a positive or negative association and how weak or strong it is. Some variation of this was necessary in order to receive full credit. A comment on r^2 was also appropriate and expected. After choosing a model it was expected that the students would show the residuals in some sort of list or table for each *x* value. There were two reasons for this. First, if they had some sort of unusual point on their plot they could quickly find it on their table. This could be a bad point put in incorrectly or second, it might just be unusual for other reasons.

The teacher explained that a project was going to serve as an assessment of the unit on regression. They were going to look for data and determine the best fit from the models studied so far linear, exponential, and power. The students had been asked to find the data four days previous to today's lesson and only one student had data to show the teacher. A checklist was given to the students (Appendix A) with the required parts. The points needed to be randomly selected and attached to the paper. The paper needed to be written with a word processor and the graphs needed to be inserted in the appropriate section of their report. When it was finished students should have the parts in the order as they appeared on the checklist and logically written. Papers should be done in narrative form. Points will be deducted for each day it is late. A point will be added for each day early.

The review of homework was uneventful at first but students were quicker answering the basic questions of how to conduct regression when working with a power model. The students were answering questions about why they thought one variable was the explanatory or response variable. Student dd chose height as the explanatory variable and weight as the response variable because one weighs more the taller one gets. Students graphed the data and sketched it on their papers. The students were also able to explain when asked how they linearized the data. Students were also able to verbalize which equation was the linearized form and remembered to plot the residual as well as calculate the coefficient of correlation. The students are realizing that the residual plot is the only acceptable way to know if the logarithmic data is linear and that a large coefficient of correlation will not be sufficient by itself to serve as justification. Student S said there was no clear pattern and they (residuals) were randomly scattered above and below the line. That implied that it was a good fit. Many students are doing well with calculating the inverse transformation and using the model to predict. The problem was straight forward and so it was easy for the students.

The second problem discussed presented a situation in which students needed to do more than just follow a series of steps. These are the kinds of problems that go beyond the obvious. The data presented the average salaries for major league baseball players over a series of years 1989 – 1997. The students were asked to determine the ratio of salaries from year to year and decide which year produced the most to disrupt the trend. The ratios were 1.129, 1.539, 1.217, 1.033, 1.061, 0.901, 1.099, and 1.176. One student called out that the years 1994-5 gave a ratio of 0.901 and said that this was the oddest ratio. However, the correct answer was 1990 – 1991 because the ratio was 1.539. This small detail is an example of basic concepts for which the students should show proficiency but many still struggle with the ideas. This lack of attention to details is an impediment that prevents the students from going more in depth in this course. The integration of the basics of the statistical processes with the contextual knowledge that the students need to have along with the attention to the factors in a problem is needed in order to be able to answer questions and appropriately apply statistical reasoning.

The second part of the task asked the students to transform the data as if an exponential model was appropriate and to observe that five of the points lined up nicely but four were out of line. Based on the material previously taught, this type of observation is what students should be able to do without any hints and to think about what is the cause of this change in the data. The third part of the question asked the students to delete the four points that were out of line and to perform least squared regression on the remaining transformed points. The students were to determine how well the least square regression line (LSRL) fit the transformed data, determine the coefficient of correlation, and overlay the exponential equation over the five remaining points. The students did not have a problem with this component as they recalled the basic steps that they had used many times before. The fourth part of the problem asked the students to state how the strike in 1995 affected the overall salaries. The students were able to see that the salaries got back in line. One student responded insightfully about why the average salary was higher than the median salary. Student I said that some make a lot more and that would make salary appear higher because it affects the average.

Overall the students responded better to these types of questions though they had been having difficulty earlier in the unit. The newer exercises are helping them learn what to consider in looking for that contextual evidence.

Lesson 10: Writing a Research Paper

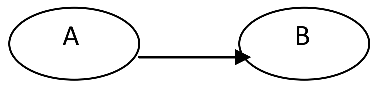
The following lesson (Appendix D, Lesson 10) centered on giving students an example of how to use a narrative to complete the projects that they were working on. The expectation was that the students would see that this was to be an academic paper and not a chat about how this is “cool stuff”. The students thought it sounded like a textbook. That was the whole point of the example. Modeling the expectations for a project on regression is appropriate in a class where students have had little or no experience with reporting the results of statistical analyses associated with regression. The remaining time was spent at the lab helping the students find data for the project. Not many students had found data despite having the assignment for several days.

After the lab session students discussed the difficulties of understanding the difference between causation, common response, and confounding relationships between variables. A graph was used to explain these concepts.

The teacher said that causation can be thought as the following:

Example: A causes B. Higher degree days causes more gas consumption.

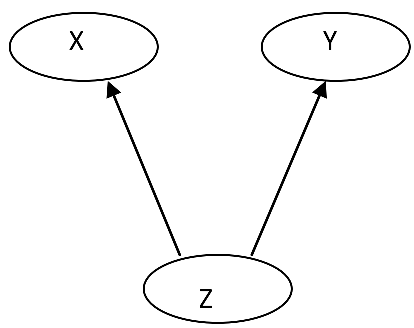
Causation



When trying to determine common response, *x* and *y* appear to be associated and *y* seems to be responding to *x*. However, both are responding to some other variable.

Example: For instance, mathematics and reading scores on the SAT appear to be associated. But increasing the mathematics score because a student got tutoring in mathematics would not necessarily cause the reading score to increase as well.

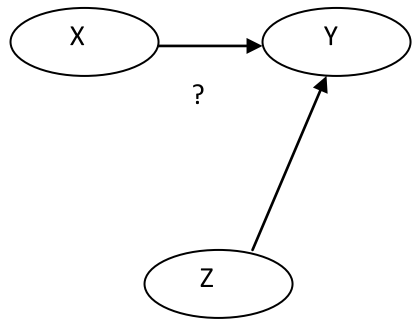
Common Response



The teacher continued by explaining that there was a common response to a lot of other factors. These factors could be home life or educational status of the parents. The higher the parents' educational level the better the vocabulary used at home. Educated parents put more emphasis on homework and they can also help their children with the homework assignments. There are many factors that affect reading and mathematics scores.

On the other hand in explaining confounding there is some type of degree of causation, where *x* is causing *y*. But, there is also something else that might be causing *y*. Confounding means that one cannot really separate what part of the change in *y* is caused by *x* and what part of the change in *y* is caused by *z* the confounding factor. This will be important concepts to understand.

Confounding



There was still a lot of confusion about confounding. The teacher asked the students to recall the examples of confounding that they had read in their textbook. The teacher then asked the students if there were any other situations, they could remember, in which confounding had come into play. There were no responses. They are near the end of the material for the exam that will cover relations in categorical data. The teacher always apologizes to the researcher for the lack of discussion of the students. Nothing short of threats has worked with this class. Only a handful of students answer on a consistent basis though they are not the more accomplished students. This is definitely a phenomenon that needs to be studied.

Lesson 11: Causation, Common Response, and Lurking Variables

In order for students to hear good answers, every assessment and homework exercise from now on is to be read out loud. To encourage more students to answer, an incentive of extra points added to the daily grade will be offered for the rest of the semester. The lesson (Appendix D, Lesson 11) started with a review of a quiz on linear regression and then a discussion concerning the reading and homework from the previous couple of days. The teacher asked about the reading which included examples of lurking variables, causation, common response, extrapolation, and interpolation. Students were asked to define these terms as well as give examples to demonstrate that they really understood them. Student M attempted an answer and stated that a lurking variable could drastically change the data. He did not address the question which was, “What did you learn in this section about interpreting correlation and regression?” However, the teacher decided to continue with the discussion. Student Z responded that variables can change perceived associations. Because it was not clear that this was really understood the teacher asked for clarifications to the answer. Student P showed some understanding of the concepts by answering that the association between x and y is really there or not there but it is hidden by another variable. He continued by saying, “So changes in x may not cause changes in y.” The teacher was delighted by this dialogue. Student L added that association does not imply causation. Student U added more to the discussion by saying that just because two variables are associated it does not mean that one causes the other to change. Sensing that the students do understand this concept but wanting an example for those that may still need one, the teacher asked for a concrete example that the average person would understand. Student L, who usually answers very well, gave an example from the reading, “Just because there is a lot of damage in a fire it doesn’t mean that firefighters cause a lot of the damage when putting out fires.” Several students were in agreement and seemed to understand why the example does not show causation but a common response. Student G surmised that the reason there are more firefighters was probably because the fire was bigger. There was more damage because the fire was bigger not because there were more firefighters.

The next topic of discussion was extrapolation. At first, Student E said that if you use the linear regression line to predict a y value for a particular x value it was not a very good idea to do. Both the teacher and the student knew what she meant but the language was not precise enough to determine what the term meant. The teacher made some comments to get the students thinking: Why do we make a line to predict with if we can't predict with it? You needed to say something … just a little bit more. You said it is not a good idea to make a prediction with the regression line for a value of *x*. Why do we find the regression equation for? What is different about extrapolation? Student E showed she understood the term by clarifying that the prediction is for data outside the range of data. Students were really getting into a real discussion. They were reacting to the teacher's questions and to other student comments.

This was a much more active discussion than during other lessons. It still will be a problem for the students if there is no one there to prompt for complete answers. But hopefully students will learn from these discussions that they need to be more precise with the language. An example was in order and Student W stated, "Say you are trying to predict the future level of US in debt, and you only have a certain range like to go by like from 1990 to now. You want to see what the future debt would be following that pattern it's not necessarily, if you extrapolate too farther out of that date it's not necessarily going to reflect because other changes may happen which might not accurately reflect your prediction." Student W had some good ideas but his lack of precise and concise mathematical language gets in the way of conveying those ideas. A natural question was how far beyond 2007 do you think it would be safe to use that prediction? Student W answered 2008. The teacher added another question, "Before we predict 2009 what might we want to do?" Several students said to use 2008. Some other examples from the homework were discussed that showed how using extrapolation might not make sense.

Students that normally do not answer were involved in the discussion. One possibility of why the students were answering was that this was near the end of the semester and they have finally realized that their grade in the class has worsened by the lack of doing homework each night. This reason emerged in the conversations with the students that were interviewed.

One of the exercises reviewed the influence of variation on correlation. The data gave an average step per second for a group of top female runners. The question asked, "If you had all the data for each runner should the correlation be lower, higher, or the same for the original data." Student E responded that she expected it to be lower because averaging all the data would smooth out the variation. This was a great response. The students seemed to be thinking statistically, making connections between the smaller parts of a statistics problem and understanding the process fully.

After a few more examples from the assignment, it was time to discuss Simpson's Paradox by also using examples from the assignment. Because these students are fairly good mathematics students, they should be able to understand the idea behind combining and disaggregating data. The students seemed to be able to discuss this phenomenon coherently, perhaps of their past experience with tables in mathematics Student Z explained that it was when we look at information and it appeared like in the example that they give about the hospital in the textbook. One hospital had a higher death rate when taken out of the total number of patients. When broken down by poor or good condition of patients the reverse seemed to be true for the hospital. An example from the textbook highlighted this phenomenon. A table of those students who were admitted versus denied admission to the business and law school was categorized by gender and is shown below. From the data it appeared that females had a better chance than males of being admitted into business and law school. In order to see how Simpson's Paradox is demonstrated it was necessary to combine the data.

Business			Law		
	Admit	Deny		Admit	Deny
Male	480(.80)	120	Male	10(.10)	90
Female	180(.90)	20	Female	100(.33)	200

A two-way table made using the combined data is shown below. Individually, a larger percent of the female students who apply are admitted to each of the schools.

	Admitted	Denied	Total
Males	490(.70)	210(.30)	700
Females	280(.56)	220(.44)	500
Total	770	430	1200

Student T understood what the numbers were showing but said it incorrectly: She explained that because there are fewer females applying to the schools there will be fewer females admitted. The teacher corrected her and remarked that there were not fewer females applying to law school. Student T corrected herself and said that as a whole there were more males that applied over all. Student Z added that there were different percentages for male and female applying to the two schools. The teacher remarked that by looking at the percentages even a bit differently one can see those that applied out of the total amount 1200 will give the percentages of those males and females applying and admitted and denied. Student aa calculated that it would be 480/1200 = .40 males in business and 180/1200 = .15 females for business; for law school males = .83, females = .083.

This concept could have been developed even further than it was during this lesson. The important idea was that when taken as whole versus separating the groups the different percentages give a different perspective on the problem. Which is important, to know what percentage of females who apply versus what percentage of the males that apply get accepted into a school, like business or law, or what percentage of the total number admitted are females versus males?

Even though the students think that they have a grasp of the concepts that they should know, they have not had a chance to see AP free-response questions. The next several weeks allowed the students to understand why the language, complete discussions, and being motivated to answer a question regardless of whether they like the problem or not will be important in doing well on the AP examination.

The first problem that they worked on was the 1997 free response question 6. It is a bit different than other regression problems. The teacher asked the students to read the problem completely and decide what the question was asking them to do. The students read the problem but seemed indecisive about what to do to answer the questions.

The students started to answer in much the same way they have approached any topic for the first time, they did not discuss much. Student T's first statement was to tell the class what the problem was about; it is about this guy that wants to sell a car, but does not answer the teacher's question. Student Z repeated the statement and added that it was a 1988 car. The teacher began asking questions and getting short responses from the same students who answer day after day. The teacher called their attention to the three models and the accompanying – scatterplots – and residual plots. The teacher examined the residual plots for them and told the students what they should be observing. Student M said, “You say how does it look, you’re talking about scattered more or..?” This has been an ongoing question that has been explored many times before today. The teacher prompted them to look at the residual plot again and the coefficients of correlation. After returning to the questions the students are asked:

Part a) Use Model 1 to establish an asking price for the car.

Part b) Use Model 2 to establish an asking price for the car.

Part c) Use Model 3 to establish an asking price for the car.

Part d) Describe any shortcomings you see with these three models.

Part e) Use some or all of the given data to find a better method for establishing an asking price for your 1988 automobile. Explain why your method is better.

This problem was presented on the AP examination and gave three different equations with which to predict. One model was not better than the others but the students will have to come up with a better plan. Students looked at the problem, but did not appear to be engaged. Again, they look perplexed though the material is not new. Some students seemed to be working ahead on something related to the problem. The teacher initiated a discussion on depreciation of cars to get the dialogue going. Student T stated that the value of cars depreciates a bit, up to a certain point and then they start appreciating a bit. Student Z pointed out that from the beginning1979 – 1982 there was no increase in price, but from 1991 – 1993 there was. The students then began to warm up to the problem. They all helped analyze the data and came to an agreement that there was a leveling off and there was part of it that looked exponential and fit very well. Then there was a gap because there was a significant change in the slope. The car was a 1988 model and it was very close to the gap. The students decided that the right section was the better portion.

Student G mentioned that if the first 4 points to the left were dropped the rest looked really linear. Those points were for the earliest years, 1979, 1982, 1984, and 1985. Linear regression was done using the remaining points and the teacher observed that the line fit the scatterplot and the residuals were randomly scattered above and below with small residuals. After checking the prediction, the students saw that the value was 4.35 or 4,350 in thousands of dollars. Student G added that after 4 of the oldest years were removed it was more appropriate for the automobile. The rest of the points would not allow the use of extrapolation. This is problem 6 on the AP examination and this problem counts twice as much as the other free response questions on the examination. Student Q said that she was confused about the process. The teacher explained that they selected the section of data that was linear, and they performed linear regression just on that data. The students finished the problem by answering the five parts of this question. Even though the students have expressed a desire to the teacher that they would like to get a good grade on the AP examination, they appear apathetic. The teacher wanted them to be prepared to put into words what they did with the exercise.

This was a very telling part of the lesson regarding where the students are conceptually in their thinking. This is a typical AP question with which students should feel quite comfortable. The students have had much experience with regression yet do not think conceptually about the problem at hand. There are several reasons that students may have for not answering. First, certain students tend to monopolize the discussion. These students are either very able or often somewhat clueless; however, it is the same group of students. Second, prior mathematics experience may not have required the level of thinking and communicating necessary for these students to do well in an AP course. Third, linear regression problems are the only experience with AP questions that has been given to students in this class. Perhaps this experience should have been provided from the onset of the course. As discussed later when describing the results of the interviews, students indicated that such exposure was an experience that they would have wanted from the beginning of the course.

Lesson 12: Simpson's Paradox Continued

The continuation of the lesson (Appendix D, Lesson 12) on Simpson's Rule was carried over due to issues in understanding the concept the previous day. Afterwards the students will have their first chance to look at multiple choice questions and discuss strategies for the AP exam.

In responding to questions about Simpson's Paradox, the teacher said that the best way to understand Simpson's Paradox was to do several problems. Many questions were being asked because the teacher had mentioned a quiz on the material. Questions centered on percentages based on ratios, the difference between a marginal distribution and a traditional distribution, and how the percentages are based on the column sums and row sums. Student M again voiced his concern with the trouble he was having with these types of problems. The teacher explained the answers one more time. Student M did not show that he was actively attending to the explanation. This should be an easy concept because the problem directly asked the student to make a table. Student L said that she also did not understand the percentages. The teacher explained the information again and was quite perplexed at the question. Student S explained how she took each total for each subcategory and found the percentages for each subgroup. This seemed to make more sense to the students.

The teacher was about to continue with the problem on regression that they had started earlier in the week. The majority of the students said that they had completed it. The students were excited because the teacher passed out a set of 35 multiple choice questions that appeared on the 1997 AP Exam. This was the first year that there was an AP examination in Statistics. These will be worked on by the students in pairs over a period of a few days. They will be required to not only find answers but justify their answers. This is an appropriate strategy because they will need to do this on the AP free-response questions. This will also eliminate guessing. This is most important because there is a penalty for every question answered incorrectly on the AP exam. The students seemed anxious to work on their first full multiple choice section.

Lesson 13: Working on AP Questions

The following day (Appendix D, Lesson 13) the students worked on the free response question number 6 started two days earlier. Some students were chosen to write their answers on the board and to show all work. Student G was asked to work on part a), Student R worked on part b), and Student B worked on part c). The questions and answers appear below.

Part a) Use Model 1 to establish an asking price for the car.

Answer: Equation: Price = -58.1+0.719* year, Price = 5.172 or 5,172

Part b) Use Model 2 to establish an asking price for the car. Student R had to get help from the teacher to solve for price.

Answer: ln(price) = -14.9 + .185 year = 1.38, e^1.38 = price = 3.975 or 3,975

Part c) Use Model 3 to establish an asking price for the car.

This was completed by Student B. The equation should have had a square root.

Answer:

$$\hat{y} = -13.313 + .1756(x)$$
$$\hat{y} = sqprice(estimated)$$
$$x = year$$
$$\hat{y} = -13.313 + .1756(88)$$
$$\hat{y} = 2.1384$$

or 2,138.40

The students that had not done the work when it was assigned were confused. The teacher was not sympathetic about their confusion if they had not attempted the assignment. Student B had used 1988 instead of 88 and the teacher had persuaded her to change to 88. Student M had a question and it revolved around the same issue. The students that had substituted 1988 had an answer of 1,371.272. The teacher directed them to the data table. The students answered that the price was given in thousands of dollars. Therefore, the price of the car for 1988 should be $1, 371,272. An answer should be checked in the context of the problem because doing so would have shown the students that answer was not realistic. Looking at the graph, even if they had not recognized that the answer was in thousands of dollars, it would appear that in 1988 cars should be selling in the 1371 dollar range. Neither scenario was realistic in the context of the problem. This has been an ongoing problem and perhaps the students are realizing how important contextual information will be for success on the examination.

The issue with part b) centered on the problem using the natural logarithm instead of logarithm base 10. However, this concept is in the Algebra II curriculum and students should have known how to resolve this surprise. The teacher continued trying to encourage the students to look at their answers in the context of the data.

For part c) the teacher corrected the fact that *y*-hat had to be the square root. She asked for the person who had written the problem on the board to follow with what should be the answer. The error might have been caught if the answer had been checked in the context of the scatterplot.

$$\sqrt{\hat{y}} = -13.313 + .1756(x = year)$$
$$\hat{y} = sqprice(estimated)$$
$$x = year$$
$$\hat{y} = -13.313 + .1756(88)$$
$$\hat{y} = 2.1384$$
$$\mathrm{Pr}\,ice = 4.572$$

or $4,572

For Part d) the students had to describe any shortcomings they saw with the three models. Student G asked if they were talking about the residuals. After being assured that this was the way to go, Student G stated that there was not exactly a random scatter and that the residuals had a pattern. After being asked for clarification Student G believed that the models were not as good as they had thought at first.

After identifying the shortcomings the teacher asked what the students were going to do about it because part e) asked them to use some or all of the given data to find a better method for establishing an asking price for the 1988 automobile and to explain why their method was better. Not many students were sure about their answer but Student Z said that he had done the “whole piece-wise thing”. He said, “I took off the bottom 4 prices, from 1979 to 1985, because they had a different trend than the other side because it starts decreasing." The teacher explained that the trend changed and the prices started decreasing at a more rapid rate. She emphasized that they need to explain why they chose the points. Afterwards Student Z completed least squares regression for the remaining data. Student Z had forgotten to write it on his paper and several students said that they had done the same and had substituted 1988 instead of 88. The next step was to show the new model was better. Several students blurted out that what was needed was to show the residual plot and *r* value. The residual plot should show no pattern and the *r* value should be higher.

The next part of the period was used to discuss the multiple choice handout they had received the day before. The students were able to answer and give justifications. Some examples are:

> Student P, for problem #1, for your multiple-choice, what answer did you choose?
>
> Student P: E.
>
> Teacher: What is a 4% margin of error? How wide a range is a 4% margin of error?
>
> Student Z: Eight.
>
> Teacher: Look at problem #2. This is setting up a hypothesis statement. What is the null hypothesis?
>
> Student T – $\mu = 35$
>
> Teacher: That narrows down our choices for the answer. What would our alternative hypothesis be?
>
> Student W: $\mu < 35$.

Teacher: Problem #3. What does expected number mean Student K?

Student K: What is average based on population proportion.

Teacher: What does expected number remind you of? Expected number, expected value, expected this or that, what does it remind you of? Mean.

Teacher: How do you calculate expected value? This is just #3. We haven't gotten to 5 yet. What did you write for #3, Student M?

Student M –Found the percentages for each and multiplied the values and added up.

Student W – Can calculators be used on multiple choice questions?

Teacher: Yes. Did anyone approach this problem a different way, but the answer choice C is correct?

Student: Student K – He goes through the procedure to calculate expected number again.

Teacher: For problem #4, find a key word in problem #4, expected. How do you find mean of light bulbs if you are given proportions? Mean of binomial is Mu =? Student V answer that.

Student V: 1000 times 0.98

Teacher: That will be 980.

Teacher: Write how to work these out.

Several students continued writing notes on the handout.

Teacher: Problem #5. This is the last problem we're going over. What kind of problem are we talking about here? *Only 3 students raised their hands.* Only 3 people in here can tell me what kind of problem this is? Two more students raised their hands. Tell me something about this problem, Student Q? Two different strains on each leaf, what does that mean? There are two variables. What do you think? What kind of test do we perform on this data? What kind of a t-test?

Student Z: Two-sample t-test.

Teacher: Should this be a two-sample t-test? There are two different strains on the same leaf. That's like I am going to turn that doohickey with my left hand and then turn it with my right hand.

Student Z: It should be a two samples t-test.

Teacher: Student U what do you think? I will read the problem again.

Student C and student bb raised their hands to answer before the teacher got started on her reading. The teacher belabored the sentence until someone could answer. Student G started to ask if the answer is..? The teacher replied that she would not discuss the answer yet. Student bb said it was a matched pairs. The teacher agreed. The question dealt with degrees of freedom. For two samples *df* would be $n – 1$ or the smaller of the two. For matched pairs it would be $n – 1$ so it did not matter because the sample sizes were the same. They were not showing engagement in the discussion. The teacher was expecting a lot of progress to be shown on answering the rest of the questions.

Lesson 14: Hypothesis Testing Review

The teacher had indicated that hypothesis testing was the students' least favorite topic this semester and so the lesson (Appendix D, Lesson 14) started with a review of hypothesis testing.

The class called out different types of inferences that they had studied in the course. Students also gave acceptable examples of the different tests. This took approximately forty-five minutes. They completed a table with all the tests.

t-test for means for quantitative data	z-test for proportions for categorical data	Chi-square observed and expected proportions or numbers more categories Use counts $\sum \frac{(observed - \exp ected)^2}{\exp ected}$	Linear regression t-test bivariate data testing slope to see if linear	Z-test for means Population standard deviation known
2-sample t-test Independent samples	One-proportion z-test	Goodness of fit		2 sample z-test
1-sample t-test	Two-proportion z-test	Homogeneity		1-sample z-test
Matched-pair t-test (difference)		Test of Association		

<u>Recap of Hypothesis Testing</u>

More than half the students contributed to the discussion. The students were exposed to all these tests during the semester and the teacher had explained when each type of test should be used, so the teacher expected them to know examples of each.

This review was followed by an activity that asked the students to determine the appropriate test to use. Students were given 9 different scenarios for which they had to determine which test had to be used. Students were actively engaged in answering the problems. The teacher reminded them to use parameters and to check for assumptions. After the students had been given some time to consider the scenarios the problems were discussed fully. A typical discussion was as follows. The students were really engaged with this type of activity. Everyone had a chance to participate and because they had worked with a partner they were more assured that their answers were correct.

> Problem 1: Discussion: A Z test is appropriate because standard deviation is known. What is the 60 hours per week? Average for California is 60. We will compare mean against 60. We are using z-test. Student E–Null Hypothesis. The teacher asked her exactly what she wrote. HR = 60. We need to write Mean = 60 Mean < 60. Mu must be used to identify parameter. Student A is answering. You must say SRS has been met, n > 40 we don't have to worry about moderate skewness or normality. Only outliers might be a problem. P-value was close to zero, so null hypothesis is rejected and it appears LA doctors are working more than 60 hours.

Lesson 15: Multiple Choice Discussion

For this lesson (Appendix D, Lesson 15) students' work was checked for completion. The teacher asked them to pass in the multiple choice homework. Students were to have worked on the significance tests activity. There was a quick check on the homework. She asked each student to go to her office to discuss homework or lack of it. More than half the students had not completed the assignment. The camera was turned off while the teacher engaged in a rebuking discussion regarding the lack of completing homework. Nearly forty-five minutes was spent on this. She checked all homework answers by giving answers. She went over exercises on the multiple choice exercises about which students had questions. The teacher was furious over lack of work done on homework. Tomorrow they will be asked to explain answers to the multiple choice questions.

A straightforward problem like $1.96\sqrt{\frac{.5*(1-.5)}{n}} \leq .08$ was giving them difficulty. The answer to this problem was that *n* was greater than 150.0625. New material was not presented today because more than half the students had not completed the assignment given several days prior to today's lesson.

Lesson 16: Review of the 2006 Form B Free Response Questions

The lesson today (Appendix D, Lesson 16) was to prove an eye-opening experience for all the students. At the beginning of class a homework check was done and an effort grade proved to be disappointing. Because justifications and work need to accompany the free response questions (2006 Form B) the teacher decided to tell the students what work would give them credit on the exam. She asked if students had accuracy on a minimum of 3 of the 6 problems. Again, the students were not sure of their work. The students were paired in teams of two. The students then exchanged papers so that they could look objectively at their papers. The criteria for grading each of the problems were read out loud. Included in the criteria were details on being awarded partial credit.

#1) Home Sales Problem: A cumulative frequency graph was given on the first problem. Student G was engaged in these review activities which is a change for this student from previous days. Justification was a problem with all the students. They must support answers with one or two sentences. This has been an ongoing problem for students. Students were a bit aghast that they would receive no credit for any work without the identification of the graph as cumulative. It was difficult to understand what students were saying. Students needed to assess if their answer was consistent with the rubric. They seem to begin to understand that they must read the problem carefully. Even though students wanted to copy the answer, the teacher said it was impossible to read every answer verbatim. The first two students, Student bb and Student dd had something written for question #1. Student bb's answer seemed to be fairly complete but not correctly stated. Student cc answered but it was incorrect. The context of the problems is still an issue. Student T answered what was on her partner's paper but the answer was given without context. The students are confused about applying the criteria for grading. The students realized that it will be difficult to get 4 out of 4.

#2) Night and Day Shift problem: This problem dealt with proportions. The instructions did not mention means and specifically mentioned proportions. There were two proportions given. Student S had Student Q's paper which sounded well written. Student Q missed discussing independent samples and in which order she was taking the difference of the proportions. The conditions were mentioned in the question as necessary in order to answer the question completely. These conditions required the students to check that the population was 10 times the sample size. Additionally, the students needed to check that probability p multiplied by the sample size ≥ 5 and $(1 - p)$ multiplied by the sample size ≥ 5. The teacher emphasized that they should say which type of confidence interval was being used. For this example, a two sample z confidence interval should have been constructed. The students were surprised about this and the fact that they must show the formula used or state what is being used in computing confidence intervals with the calculator. Interpretation was expected, that is: based on these samples we can be 96% confident that that the difference of the proportions of the parts specifications for the two shifts is between - .0156 and .0196. In part b) the students were to determine if the difference of the proportions was significantly different from zero. Student Z ran a hypothesis test. A hypothesis test was not allowed according to the rubric, because the question said to use the confidence interval to answer the question. The interval contained zero and the students should have known that it was highly unlikely that the difference would not be zero. The conclusion should have been that it was highly unlikely that the difference of shifts was true.

#3) Distance of Golf Balls problem: normal distribution problem. Part a): What is the probability a ball will travel as far as 280 feet? Students needed to specify that a normal distribution allowed them to use *z*-scores to compute probability. Student S read Student D's work and the answer. She also had drawn a shaded graph to show the probability area for the question. Only about a fourth of the students had this part right. Students had to show correct calculations to get credit. Assumptions are not being stated by the majority of the students.

Part b): Cumulative probability problem : at least 1 of the 5 balls, means one or more. Therefore, it could be set up as $\left(1-\left(1\ -\ \text{prob(none go)}\right)^5\right)$. It was also possible to set this up as a binomial distribution though it would have been more time consuming to work out.

Part c) Student aa read the answer to part c). This was Student X's paper and it was well written. Find $\bar{x}$ given probability. Given a *z*-value for 99th percentile, an inequality was set up for the mean. The teacher explained that it should be finished up by saying that the mean should be set at 284.68 yards to have a 99% probability that a randomly selected ball would reach a maximum distance.

#4) Matched pairs problem: dexterity was measured after participation in a program. It was an obvious matched pairs because each person was tested before and after participation in the program. Student Z had labeled this correctly. Student R read from Student J's paper. The hypothesis was set up, and assumptions had been met. The teacher said that it was not enough. Some of the students argued that because the problem stated that there was a SRS (simple random sample) a normal distribution did not need to be checked. This actually needed to be checked by using stem and leaf plots or another appropriate measure in order to get full credit. That would also address the problem of an outlier if one existed. Student F's paper used a t-test and had included the formula. Only about seven students got this problem correct. The teacher was worried about so few people getting *t*- and *p*-values. The teacher reminded the students that it was important to state conclusions in the context of the problem.

#5) Experimental Design Problem: agricultural problem concerning draft. A plow was to be used on land to determine draft. Draft was explained to the students because they did not know what it was.

Part a) Identify response variable, treatments, and experimental units. Very few students were answering.

Part b) Was randomization used appropriately? Even though the problem stated that the fields had been randomly chosen, the students were not in agreement that this was done properly. Student W wanted to use a plot divided in half. The teacher replied that the randomization was done correctly albeit it was a bad experiment.

Part c) The question asked if replication was used appropriately. The students were able to define replication correctly. The students correctly identified that replication was not done correctly and provided many excellent examples: only two fields were used, the conditions of the field were unknown, and more types of fields should have been used with perhaps different terrains. Students did an outstanding job on this part of the question.

Part d) Why is plot of land a confounding variable? Student W said that each plot could be different so their moisture content could be different. The type of soil could be different as well. The time differential was also mentioned as being problematic.

#6) Binomial probability question – Consumers of juice. The students were tired and it was almost time for class to end. Therefore, this question was not discussed.

Lesson 17: AP Exam Review

Today the students will be discussing (Appendix D, Lesson 17) 2007 Form B AP Exam. These had been graded by the teacher and the best answers were read by the students.

Question 1: This question required the students to a) construct a stemplot from a data set, b) describe the features of the stemplot, and c) discuss how a measure of central tendency could fail to show important characteristics of the plot. Student I displayed the stemplot from question 1 part a) but forgot to accompany the plot with a legend. The majority of the students had forgotten to include this important step. When the students read their discussions of how a measure of central tendency could fail to show important characteristics of the plot there was not one student that had done part b) entirely correct. Student X said that no data fell in the 20s. Student B said that a large portion of data was in the teens. Student U said that there were two curves. Student aa said that there were two clusters with a gap in between. Therefore a combination of answers provided the correct answer. For part c) Student L said that the mean was in the gap and was unrepresentative. Student aa said that there were two groups without the center being in either cluster.

Question 2: A frequency distribution was given showing the total number of cats and dogs owned per household. The students were required to demonstrate that they could evaluate probability statements using the graph. For part a) students were asked to determine the probability that a selected household would be in violation of owning more than three pets. In part b) students were expected to determine, that given that ten households are selected, what the probability would be that exactly two would be in violation of owning more than three pets. In part c) students were given the mean and standard deviation of a relative frequency distribution and were asked to describe a sampling distribution taken from the distribution given. Students were sent to the board to explain their written answers. For part a) Student W was able to answer the probability question. For part b) the students had to calculate the probability that exactly two households were in violation of a local law using the answer to part a). Student T explained that this was a Binomial distribution because it had a set number of trials and trials were independent. The probability would not change for each trial and finally there was a success or failure for each trial. Student G did not understand why she had to multiply by 45 if there were two successes and 6 failures. The students should have remembered that exactly two successes can occur in many ways out of *n* trials. More than half the class had missed answering this part correctly. In part c) the students had to describe the sampling distribution of 150 households' number of pets. Student Y was able to explain that there were samples of 150 households and that they were finding means of those samples. In addition, Student Y explained that the distribution should be nearly normally distributed. This occurred even if the original population was not normal. Furthermore, the standard distribution was $\frac{\sigma}{\sqrt{150}}$, which was the given population standard deviation divided by the square root of the sample size and this value would be smaller than the population standard deviation. Additionally, the mean would stay the same. These three bits of information were crucial to receive points because they were specifically asked for this in the problem. Only about a fourth of the students earned full credit for this part.

Question 3: This question asked students to consider a randomized block experiment with two pro-types of windows. A house was built with 12 windows, six of each type, to test the heat gain. The students were asked to indicate clearly, using the window numbers, how they would block and why. Student V was called to answer how he blocked and he said that he blocked windows by twos. Student K also showed that he blocked by twos. Randomization was a key item to discuss and the pairs of windows needed to be on the same wall. About one-third of the class answered this problem well. Student M explained his answer but it was not correct because he did not block by wall. This part was important in order to get the two types of windows on the same wall. Student T wanted to know if she could get partial credit for the explanation without having the blocking done correctly. The answer was yes as long as the blocking included both types of windows. Student T said that she did not understand blocking at all.

Question 4: This question was based on a given least squares regression line and scatterplot. These students had completed many examples and a project on regression, therefore this should have been an easy problem for the students. The first part a) asked the students to draw the least square regression line (LSRL) on the scatterplot. The majority of the students correctly completed this part. Student S actually calculated the regression line again by reading the points from the scatterplot. However, this involved having to estimate some points, and three points from the data had not been graphed. This information had been divulged in the prompt of the problem. Obviously the student obtained a regression line different from what was given. It was a very time consuming process and unnecessary. Student Z graphed the line by finding two points on the given LSRL by using the table feature of his calculator. The teacher explained that it was better to use the extreme points. The next part asked the students to circle a particular ordered pair on the scatterplot and to show the segment that corresponded to the residual. Related problems had been done numerous times in class discussions. Finally, the students had to give a value for the residual. Student B finished showing the residuals. The residual line should have been done vertically. The concept of residual was explained again by the teacher and Student B. Student E said that the *y* value (observed and expected) had to be calculated for the same *x* value given and expected value came from the calculated regression line. Student bb wrote on the board: Expected – found = residual which gave an incorrect residual sign. It should have read Observed – Expected = residual. The sign had to be correct to earn credit on this part.

Part c) asked students to add a point to the data and to determine if the slope and correlation would change in any way. Student I answered that the point should make the slope of the line be about the same or decrease slightly because the point was in line with the least square regression line (LSRL). The teacher emphasized that the correlation should increase using the formula but because calculations were not required (according to the instructions given) then the students could say that if a point is way out on *x* axis but close to the regression line it should strengthen the correlation.

Question 5: This question was designed to test the student's knowledge about making inferences concerning the difference in two population means after using a cholesterol-reducing medication. Student X and Student B put their work on the board. Student X wrote the correct null and alternate hypothesis and Student B did the same but with a slightly different approach to the null and alternative hypotheses. The teacher emphasized that talking about these might be boring but necessary. The teacher explained that if you have different values for old and new values of drug you wanted to know if it was a significant difference. Student X had written the following: $\begin{array}{l} H_o : \mu_n - \mu_o = 0 \\ H_a : \mu_n - \mu_o > 0 \end{array}$.

Student X forgot to say which μ went with which drug. Student B wrote the following: $\begin{array}{l} H_o : \mu_n = \mu_o \\ H_a : \mu_n > \mu_o \end{array}$. She said she would conduct a 2-sample *t*-test and the medication was more effective than the standard for the alternative hypothesis. She found the *t*–value and the *p*-value. She stated that she obtained this information from her calculator and showed the formula she used. She said she would reject the null hypothesis. The sample sizes were large enough to be able to use the *z*-test. The teacher asked the students to take out their textbooks so they could look at the table of *t*-values. After looking at the *z** line, that is, a *t* –distribution with a large sample size is close to normal distribution. She repeated that she had not taught this particular test yet someone knew how to do this. That is, the normal distribution is acceptable when the sample size is larger than 40. A recap was done. With small samples, a *t*-distribution is more conservative. Several students had questions about using the correct distribution. By being safe the *t*-

distribution only rejects the null hypothesis if there is over-whelming evidence. Because the students are working on free response questions they must include assumptions about random assignment and normal population. Additionally the value of the statistic needs to be given.

Lesson 18: Experimental Design Review

The last question of the test will be the Experimental Design Problem (Appendix D, Lesson 18).

Question 6: This question required that the students be able to make an inference about the difference in two population proportions. A conservation group wanted to determine if small islands or large islands would be better to minimize the possible extinction of certain bird species. In addition, the students must construct a confidence interval for the slope of the regression line for the data of proportion extinct versus ln(area), and use the information from part a) and b) to make recommendations.

For part a) Student D determined that the null and alternative hypotheses should be $\begin{array}{l} H_o : p_1 = p_2 \\ H_a : p_1 < p_2 \end{array}$, large islands and small islands are equally effective at protecting at-risk species versus large islands are more effective than small islands at protecting at-risk species. The teacher explained that p_1 should not be large islands but the proportion of animals that became extinct on either island. Therefore, Student D's work needed more clarification. Student X used p_l and p_s for her proportions and used the 2-proportion *z*-test. The teacher added that Student X should have written out the word proportion instead of prop. Student X also wrote values of *z* and *p* and the conclusion which was to reject the null, meaning extinction proportions are different.

For part b) Student B calculated the 95% CI for the slope of regression line using $b \pm t^{*} s_e$, saying no unusual patterns existed, and df = n – 2 = 11. The teacher said that she should have elaborated on this by checking conditions, linearity, and that standard error should be consistent. The teacher explained her comments by saying, “Look at the residual plot. Our residuals should be normally distributed and could be checked with stem and leaf plot.” There appeared to be some sort of negative slope in the scatterplot, therefore the students should have said that they were 95% confident that the mean proportion of extinct animals was between the computed values of the CI. This would have given the context of problem by alluding to the extinct animals. Contextual references still elude students and they will continue to lose full credit because of this. Three of the four items that the student needed to address had to do with assumptions, mechanics, and contextual reference. Unfortunately, three of the four parts needed to be correct in order to get an answer that was essentially correct.

For part c) the students had to use what they obtained in part b) to answer the question. Student O was able to state this pretty well when she said that every time we add area then the proportion (instead of probability) of species going extinct decreases. The large and small islands were not all the same size and the species are not all becoming extinct at the same rate. As the large islands were getting smaller they were protecting the animals less. Even though the language was not perfect, it seemed clear that the students knew what they were doing.

For part d) the students needed to choose either one large island or several smaller islands. Student U indicated that she would choose the larger island. A good supporting argument was needed. The answer should have included talk about the expectations of preserving the at-risk species would be better on the larger islands.

The next review packet was the 2002 Multiple Choice Test questions. The teacher has explained numerous times that language was important and it was necessary to read the question completely in order to answer the question correctly. It should be clear by now that reading comprehension is crucial to earning full credit on each problem. Questions were still not forthcoming. Approximately one-third of the students were not engaged: some were playing with their hair, others were writing. The types of errors that the students were making are consistent with what has been going on in this course for some time. In general, students are rushing through the questions and not paying attention to the language. If the question is the least bit complicated students have a tendency to not do it or just circle an answer. Each question was analyzed fully by the teacher and answered by a few students.

After a short break the students continued by working on the 2007 AP examination. Question 1 required that the students: part a) explain variability in a control group, part b) use a dot plot (which was given) to compare the effectiveness of an experiment, and part c) based on a given CI comment on the difference of two population means. The teacher started the discussion by asking the connection between standard deviation and variability in the control group. Student T gave a standard answer that the smaller the standard deviation the less variability. The teacher tried again by asking, “If the standard deviation has a given value, how does this value summarize the variability of the control group?” No students were forthcoming with answers. After asking Student bb to answer, which did not advance the discussion, the teacher asked for someone to improve on that answer. Student Z said that he needed to know if the distribution was normal. If that is the case, Student Z said he could use the 68-95-100 rule. That was a good comment because the distribution of the control group looked fairly mound-shaped. The teacher added that the standard deviation measures a typical or average distance between the individual discoloration rating and the mean ratings. This answer should use the contextual information given in the problem but was essentially the definition of standard deviation and how it summarized variability. Student G indicated that she visually calculated the mean of the treatment and control groups and she looked at the proportion of those above the mean. Student T explained that she used the normality of the distribution by looking for the skewness of the distribution. This statement did not take into consideration its relation to the experiment.

For part c) the teacher explained that what was needed was a CI for both groups using a difference. The CI given was completely positive and this could be explained by saying that there is a positive and significant difference at the 0.05 level. No students had attempted to answer this question using a confidence interval even though that was given in the prompt of the question. The teacher explained that zero is not contained in the interval; therefore there is a positive difference in discoloration. This certainly was a match up for the two-sided test. For a one-sided test that would be half the level. The idea of using CI to comment on a hypothesis test had been done many times before. The students did not get it or were not trying.

Lesson 20: Review of 2007 AP, Continued

Continuation of 2007 AP questions was continued (Appendix D, Lesson 20) by discussing Question 2. Question #2 was considered very easy by the students because it was an experimental design question. This question concerned a study that was to be done to compare two dietary supplements and the effectiveness of each in reducing the onset of canine osteoarthritis. Part a) asked the students to discuss the advantage of adding a control group. Part b) asked the students to explain how they would assign 300 dogs to the three groups, supplement 1, supplement 2, and the control group. Part c) asked the students to decide whether to block on clinics or breed of dog.

For all parts students were engaged. Student A answered part a) by saying that a control group would show deterioration in the control group also. Student T added that it would show the negative affects to the dogs' health such as other parts of its well-being and that one would be able to compare dogs without the supplement to the dogs with the supplement. Student T was really engaged in this question. The teacher wanted to go further with this part and asked why it was important to have a control group. Several students answered that a control group helped you identify a lurking variable. The rubric for this question also identified that the normal aging process as detectable by the control group. This researcher believed that students could not finesse this idea simply because they may believe that experiments are short-lived. This researcher's experience working with high school students has been that they expect immediate results. An example: when students have a low grade point average in a class, they become very excited when they get one high grade on a test or quiz because they will expect an immediate improvement to their grade point average. Therefore the improvements in the dogs, if any, would be more noticeable if done over a long period of time because these are not young dogs. The impact of the supplement would be more noticeable by ruling out other factors. Student

W added that the control group would allow the researcher to test normal conditions versus experimental conditions.

For part b) students were asked to be specific. Student cc said that she would assign 0 – 299 to the dogs (should be 000 – 299). Furthermore, she would use a random digit table and she would three digits. The dogs with numbers 000 – 099 would be assigned to the first group. The student needed to address the random assignment therefore the question was asked as to how she would assign them randomly. Student I was asked to comment on assigning the dogs and she said that she just selected the first 100 dogs and assigned it to a group. Of course this would not get credit and the teacher told her. Student V answered that one could just select the first 100 numbers and assign to the first group using a number generator. A number of 374 would get tossed out. Student I insisted that her answer was the same. The teacher commented that her language needed to be clear. The dogs were being given numbers and assigned to a group and both parts needed to be part of the randomization. Student Z commented that by tossing a die a 1 or 2 would be in group 1, 3 or 4 to group 2, and 5 or 6 to group 3. The teacher explained that they needed to be careful with the assignments. She asked them to recall the butterfly problem. Grabbing the first 10 butterflies would choose the weaker and slower ones. The rubric assigned a number from a random number generator until all dogs were assigned using numbers 001-300. A random number generator assigned each dog. Numbers 1-100 got placed in group one etc. This was easier than the random digit table.

Fort part c) students agreed with Student T who wanted to block by dog breed type because of the differences in bone structure and muscles. Blocking by clinics would not work because clinics may have different treatment practices. She continued saying that they (dogs) are prone to diseases according to type of breed. A lively discussion ensued when Student W insisted that blocking by breed would not necessarily have all breeds accounted for. The teacher added that not all breeds needed to be represented because there was the question about what to do with mixed breeds. The researcher added that perhaps the lack of experience with dogs might make you choose clinic for blocking choice. After looking at the rubric the teacher said that a good support to clinic choice was also acceptable if the student was convinced that blocking by clinic allowed for more variability. Student A added that she had never owned a dog and also did not have a close relative that owned a dog so she would have picked blocking by clinics.

Even though students' arguments showed some flaws it was good to see the students get really involved in this discussion.

Question 3 was designed to test students' understanding of the sampling distribution of the sample mean. For part a) students were asked to calculate probabilities concerning sample means. In part b) students were asked to compute a probability given the standard deviation and sample size. In part c) students were indirectly asked to apply the Central Limit Theorem. It was distressing to have Student M not know the formula for z. The teacher asked him to find his formula sheet. Student bb was able to answer part a) quite well. Student bb said that the mean of the length of fish was 8 and so would the sampling distribution and the sampling distribution of sample means was nearly normal with mean 8 then a random sample of fewer fish would have more variability than one with more fish.

For part b) there were many questions. Student K wanted to know if the calculator answer alone was okay. This has been discussed at length as being insufficient. The teacher has mentioned many times that they must accompany calculator answers with work and either a formula or a sketch. Student H answered but did not write probability statement as shown below.

$$\text{Prob}\ (\bar{x} < 7.5) = P\ (z < \frac{7.5-8}{.3}) = P\ (z<-1.667) = 0.0478$$

Student E believed that Student H should have divided by sqrt (50). The prompt informed the students that the standard deviation was for the sampling distribution then 0.3 is the standard deviation of sample with the division by sqrt (50) already done. The students were having difficulty with the concept of variability and the fact that there was more variability in the whole population than there was in small samples taken from the larger population.

In part c) the students were asked to comment on whether they should use the normal distribution in part b) if the original distribution was non-normal. Student J answered no which means that she did not understand the power in the Central Limit Theorem. Student Y disagreed and said that it was okay to use the normal distribution to compute probabilities because the sample size given was 50. The teacher reminded them of the activity/ project they had done earlier in the semester. Student W remembered but agreed he would not have received credit on this question because he could not relate the activity to this question.

Question 4 was a paired-data hypothesis test. Students have had difficulty distinguishing this from a two sample procedure in past examples. Two methods of determining the level of contamination in beef is applied to specimens. Each specimen is tested by both methods. It should be clear to the students that this is a matched pair *t*-test. As has always been the case this semester the students do not check for assumptions to be met. Students should realize that the ten pairs will allow for independence to be met, and that checking the stem plot shows that the population is nearly normal. The teacher explained again, as she has during other times during the semester, that the rubric showed that the students must show correct mechanics in calculating the *t* – statistic. For this example the *p*-value with 9 degrees of freedom needed to be calculated and the students must state answer in context.

Summary of the Classroom Communication and Discourse

The observations of the classroom communications and discourse between the teacher and the students as well as the written artifacts served as a way to document the patterns of statistical reasoning. These tasks involved exploring data, sampling and experimentation, anticipating patterns, and statistical inference as the guiding points of emphasis. Levels of statistical reasoning observed were broad from the most basic idiosyncratic reasoning in which the student knows some statistical words and symbols and uses them without fully understanding them to **i**ntegrated process reasoning in which the student has a complete understanding of statistical processes.

Several levels and forms of elicitation by the instructor were evident in the lessons that are related to the research questions two through three presented in Chapter 1. In order to engage the students in a discussion about each topic studied, the instructor initiated the unit with a study guide (Appendix E, Sample study guide). The study guide reviewed the topics to be studied, asked the students to answer questions related to the concepts to be studied, and reviewed the vocabulary that the students needed to define and be able to use effectively in the discussions to be initiated by the instructor. Furthermore, these discussions served to enhance the general discourse established by the instructor to deepen the students' understanding of the statistical concepts being studied, and hopefully would take the students from level one of statistical understanding, idiosyncratic reasoning in which the student knows some statistical words and symbols and uses them without fully understanding them and perhaps incorrectly to level 5, integrated process reasoning in which the student has a complete understanding of statistical processes and the student can explain the process in his or her own words with confidence. This first form of elicitation was not completely successful in all lessons observed. For example, during Lesson 1, the students were more focused on the minutiae of calculating the coefficients of correlation and determination but were unperturbed by the questions about the meaning of these particular statistics. By Lesson 4 it was still evident that students struggled initially to discuss in a meaningful manner the topics for which they had completed a reading. It was not clear if this form of elicitation was not successful because the students did not read, truly did not understand, did not wish to join in the discussion, or were not used to discussions in a mathematics class.

The next form of elicitation used by the instructor was discussion centered on the textbook exercises. These problems allowed the teacher to gauge the level of understanding of the statistical concepts being studied in each unit. For example, during Lesson 1 it was not clear if the students did not complete the assignment or did not understand some rather simple ideas presented in the reading. During several lessons the researcher observed that students were not discussing the homework exercises as the teacher expected and the instructor would separate the students into pairs or had them work independently to read the previous night’s assignment in order to refocus and engage the students in a discussion post a “second reading”. These secondary attempts were usually more successful. During Lesson 1 this type of activity helped further the discussion on linear regression. The instructor showed patience and determination in finessing the salient ideas. This form of elicitation can be seen in practically all the lessons observed.

Once the instructor determined that the students were more comfortable with the major ideas from the unit it was followed by AP Statistics questions that would allow the students to use the new statistical concepts studied in the unit. These types of questions were presented late in the series of lessons observed by the researcher. The bulk of this type of activity was not initiated until Lesson 11. It became clear, later on, during the students’ interviews, that the students found this activity very valuable. Six of the ten students mentioned the AP questions as being helpful in understanding how to answer questions more completely and wished that they had been presented earlier in the semester.

In particular, the classroom interactions sought to answer questions 2 and 4 and to a lesser degree question 3. Planned discourse was organized by the facilitator (the teacher) and was characterized by elicitation, engagement, and challenging the students’ thinking. The planned discourse was evident in many of the transcribed lessons and was clearly seen in Lesson 7. The teacher challenged the students to think about the data and to not just start computations without regard to the context of the problem. Appropriate tasks and activities were planned with the express purpose of having the students learn how to effectively communicate their solutions to statistical exercises by using statistical reasoning that went beyond idiosyncratic reasoning. Significant attention was paid to the students' statistical thinking. Lesson 20 was one of the most successful for the students and the teacher in terms of richness and outcomes of the discourse. The students thought that the problem concerning dietary supplements and its effectiveness in reducing the onset of canine osteoarthritis was very easy. The students were aware of how to use the control group to make the experiment more effective by allowing the researcher in the study to compare supplements. Although the random assignments were not done perfectly by all the students, the teacher was able to check that the students were using good statistical reasoning to answer questions. Appropriate statistical language was encouraged and modeled by the teacher. As the students became more comfortable with the statistical language, the teacher's expectations increased and more difficult scenarios were used to engage the students to focus on the details such as assumptions, conditions, and best procedures to use in using the data to make statistical inferences with appropriate justifications and contextual references. The more difficult task for the teacher was facilitating discourse that would encourage and enable students to communicate effectively in the statistics course. There were instances in which the students seemed to be progressing towards communicating their solutions to statistical problems effectively and then there would be a setback and the process would start all over again. For example, during Lesson 2 the students were discussing a computer printout about a linear regression question and the students seemed engaged and knew how to choose the intercept and slope. However, they were not able to effectively discuss the slope in the context of the problem. These concepts are studied in Algebra I and II and should be easy for these students. However, on a positive note, this did bring to the forefront the idea that computations alone will not suffice for full credit on the AP examination and that the student will have to accompany their computations with contextual references. Continuing with Lesson 3, the students correctly extracted the equation for the regression line but faltered when trying to answer a question concerning the validity of using the regression for a smaller set of data points. Again, a little later in the lesson the students have no difficulty determining the regression line and defining the variables but had difficulty using the residual plot to compare a prediction to the observed value. They had studied this concept over several lessons and it should have been mastered by this time in the course. In Lesson 20, which the researcher thought was the most successful, instances of faltering and waffling in producing a random assignment in an experimental design was evident. The instructor had to recap the salient ideas in using appropriate assignment procedures in order to randomly assign subjects to specific groups and to be purposefully detailed. This observation was based on the students that were called to answer or chose to answer therefore it was not representative of all the students.

Earlier this researcher mentioned that there was a faction in the class that monopolized the discussions in the class and this group of students usually had very few valuable ideas to add to the discussions. On the other hand there were other students that when called upon provided evidence of understanding and added much to the discussion of a particular concept. This phenomenon was evident in most lessons at the start of a particular unit of study. This does suggest that the teacher did engage the students and hoped that the type of discourse needed in this course would follow however this was not evident throughout all the lessons. Perhaps not all students in this course were at a developmental stage to engage in this type of discourse, that although planned was not the type of discourse, Wild and Pfannkuch's (1999) suggested in their framework for organizing statistical thinking. This idea is related to research question one and will be discussed in the next section.

4.3 The Interviews and Profile Differences

4.3.1 The Interviews

The interviews sought to partly answer the question: To what extent does a student's profile influence success in the AP Statistics course? It was considered essential by the researcher to consider the students' dispositions towards the course and to have the students reflect on whether their dispositions had changed. Data collected for each interviewed student included both video and written artifacts. The videotaped interviews served to separate non-verbal responses from the verbal and because the interviews are social interactions between the researcher and the students, it gives the researcher the opportunity to analyze a dialogue as an observer and for a new interpretation of what transpired during the interview. Written artifacts containing written students' work were also used in the analysis providing additional points of view about the students' conceptual understanding of statistical concepts as well as their ability to write using contextual evidence. The teacher used a variety of strategies to initiate discourse in the classroom and the researcher sought to understand how the students reacted to these various strategies in the context of developing statistical thinking and understanding using the students' perspective. As was evident in the lesson descriptions it was also deemed important by the researcher that the students' actual answers to the interview questions were used rather than summarizing the observed patterns. There were multiple criteria for selecting these ten students. These students were representative of various sub-groups from the class of thirty students. Two students, female and male, were the only minority students in the class. Two students, male and female, were the only junior class students in the class. The other six students, three male and three female, represented average and better writers as classified by the instructor of the class.

The researcher conducted individual student interviews (Appendix C) and these interviews were completed by ten of the students in the AP Statistics course. Each of the interviews lasted about one hour and was conducted approximately two weeks before the end of the semester. The interviews were videotaped and reviewed with the students to verify that the intentions of the responses were accurate. The researcher's questions ranged from questions that asked the participant to explain if their writing in the course had changed over the semester to whether or not they consistently prepared for class by reading the assigned materials and completing the homework. The students were also asked to determine if they thought the course was difficult or not and if that had changed from their initial views of the course. This led to questions about whether their commitment to the course had changed and if the difficulty of the course had changed their habits in preparing each day for class. The researcher was interested in which materials used by the teacher were of most benefit to each of the students and which topics they considered most difficult. Such questions gave the researcher an opportunity to ask additional questions about the students' interest and which strategies used by the teacher to study a statistical concept were of most benefit to them.

Of the ten students interviewed only one, Student T, did not take the AP examination. Student T earned a D$^-$ in the class and earned a C$^-$ on the teacher-made exam. Of the nine students that took the AP examination, five earned college credit. The students that earned credit were, Student B, Student U, Student V, Student Y, and Student Z. These students earned grades in the class of A^-, A^-, B^+, A^+, and C^+, respectively. The four students that did not earn college credit earned grades of either C or D in the class. Based on their classroom performance, it was not a surprise that the students who earned college credit did as well as they did. The four students that did not earn college credit were weaker students and demonstrated difficulty with the material the entire semester. These ten students turned out to be representative of the entire class because half the class that took the AP examination earned college credit as well.

The interviews revealed that an interest in reading did not seem to be a factor in determining how well a student would perform on the exam. However, all the students that performed well on the exam are considered by their teacher to be excellent mathematics students. All the students who earned credit on the exam relied heavily on using the textbook as a resource to understanding the concepts studied in each unit. The interviews revealed details about the students that explain some of the lack of discussion in the classroom as well as the lack of a work ethic demonstrated by them. Six of the students interviewed thought that the AP Statistics course would be easy and the others did not have any expectations as to the difficulty of the course and believed it would just be another mathematics course. They found out too late in the semester that if they had worked a little bit more during the first half of the course they would not have had so much trouble with the AP questions they worked on during the second part of the course.

Student V discussed this during the interviews. This is an excerpt:

a. What was your initial feeling about taking AP Statistics when you first decided to enroll in the course?–I thought it would be easy
b. Have your feelings about AP Statistics changed since then?–A little.
c. How so?–I thought it got a little bit harder. It is not as easy as I thought.
d. What is it that you found harder about it?–Like the tests, that's basically it.
e. Was it that you felt that you were not prepared when you took the test or the test was different from what you thought it would be?–I probably wasn't prepared.
f. Do you feel that writing in statistics is different from writing in your English class?–Definitely.

Student U also addressed her feelings about Statistics:

a. So, you thought AP Statistics would be easier, is that what you thought? – Easier but in a way that I had heard that it was more with words and with ideas that I could understand better.
b. I know that you like that kind of thing. – I'm much better at it.
c. Have your feelings about AP Statistics changed since then? Do you think you made the right decision? – Well, at some point during the semester I started thinking maybe I would have been better off with numbers because I wasn't that good at the words part of it but it's half and half because in a way I like understanding it I like just the content. It is hard. It is harder than I thought it would be, for sure.
d. So, you're feelings might have changed but you came around to saying that this was the right decision? – I think so, yes.

e. Do you find that writing in statistics is different from writing in your English class? – Well, it is very different. In a way it is very similar. I thought too when I took my government class here. Because you have to write in a way that gets straight to the point. You don't do the whole flowery like trying to embellish your sentences. It was like that in history where you just have to state your facts.

f. So, you said that the writing is somewhat different but at the same time because of your government class, you learned not to embellish but try to support your answers? – Yes. I've been working very hard in trying to get my point across in a brief manner. I think that's what it is – concise – yes, concise, that's it.

    g. So, your writing has changed in completing your assignments? - I think so well maybe it hasn't changed but I have just learned to write in a different manner.

Student P added:

a. What was your initial feeling about taking AP Statistics when you first decided to enroll in the course? – I didn't really know what to expect. I thought we would do a lot of probability. Other than that I didn't really know a lot about statistics.

b. Did you feel apprehensive at all about it? – No.

c. Just like any other math course? – Yes.

d. Have your feelings about AP Statistics changed since then? You thought it was a lot about probability and just like any other math course. Have those feelings changed? -Definitely.

e. Can you explain? – It's not really mathish, it's a lot of writing. I didn't think that it would be that involved.

f. Do you find that writing in statistics is different from writing in your English class? –Yes.

g. How so? – Just having to incorporate the numbers and the different formulas and stuff is a lot more complicated. English just kind of flows.

h. So do you normally get good grades in English? – Generally.

i. What kind of a math student do you consider yourself? – Fairly good.

j. Has your writing changed in this class in completing the assignments? – I guess so.

Student A also voiced these words:

a. Tell me, this probably goes with a different answer to the first one, when you thought about taking statistics were there any feelings associated with that besides the fact that you needed to take it or wanted to take? Any feelings about whether it would be easy or hard? - I heard it wasn't difficult. I heard it wasn't easy but it wasn't difficult. It should be okay. But I just heard it had a ton of homework and lots and lots of reading assignments. So I kind of didn't know when I was coming into it. It has got to be like a regular honors class but maybe with more challenges like the homework and tests.

b. Did your feelings change then about AP Statistics? – No, it's not hard but the first semester (sic. Quarter) I didn't apply myself but now once I was sitting through it well fine now I just really need to try in this class. It's not really difficult. In the beginning I thought it was hard because I wasn't really trying as hard as I could but now that I am trying as hard as I could it's not easy but it's not hard.

c. So you can get a handle on the material but now you are going to have to work harder? – Yes.

d. Do you feel that writing in statistics is different from writing in your English class? – No, because writing in statistics is like you are writing a story but you are writing a story about a math problem. In English you are writing a story about a topic a genre.

e. Has your writing changed in this class in completing your assignments? Think about how you first started. – Yeah. When I first started I started with short scripts, just answers, no work, just plain calculations, and that's it. Now it's like I am writing real long paragraphs for each problem or a page paper or something.

Except for one student, all the students would have taken the course even if they had known how difficult it was. About half the students interviewed liked to read other materials besides what is required in English class. All of the students did not realize that academic language was necessary to answer questions in statistics correctly and the context of the problem was crucial. At least one student (Student T) said that she became paranoid about writing too little when answering questions. For the most part these students did not like to answer because they did not like to volunteer answers and only answered when called upon. The lack of adequate preparation each day was also a contributor to not wanting to be part of the discussion concerning the concepts and problems. At least one of the more successful students did not see the purpose of the activities and never did learn its purpose.

The following is an excerpt of the interview with Student V:

> Did the projects help you any?–Not really.
>
> Why?–I don’t know, the one we did about the Central Limit Theorem, I didn’t really understand it from there, I like understood it better like when we read it from the book and talked about it in class and did the problems.
>
> So, you didn’t get it?–No, I didn’t get the point of the projects really.

The following is an excerpt from the Student P interview:

> Who kind of led the project? In other words did you kind of have equal input into the project? – Yes.
>
> Did you know what you were doing? – Not really.
>
> So how did you decide how to learn how to do it? – We just kind of winged it.

An excerpt from the Student A interview:

> How about in completing the projects? Remember she gave a project at the beginning and then she gave a project towards the end. Did that writing change? – Yeah. The project at the beginning I hadn’t got a grasp of the class yet, I mean I did okay on the writing at the beginning but it could have been better and comparing that to the one we just did I could see progression in my work and I’ve seen that the class has advanced my writing skills for the math world and that my writing has improved from the beginning to now.

A comment that was made by several of the interviewees was that the AP questions that were used as a review towards the end of the semester should have been used early in the semester. Relative to that comment most of the students wanted relevant questions that catered to their interests, and second, students thought that the AP questions should have been given earlier in the semester as each topic was covered so that the students would know the expectations of each unit. Additionally, these students felt that a cumulative review for tests would have helped them retain the concepts much better. A few students felt that the whole-group discussions left out most of the students who were intimidated by answering for all to hear. They thought that the smaller groups or pair–discussions were more beneficial to them. Most showed much bravado by stating that they thought they could review all of the material on their own. Even though all of the students promised to come to the review sessions, only two, Student B and Student U, came twice.

4.3.2 Profile Differences

Data of the profile variables were analyzed in order to determine if there were statistically significant differences between the students that earned a grade of three or better and determined to be qualified and those that did not. Twenty-four students from the total of thirty students in the course sat for the AP Statistics examination May 2008. Twelve students earned a 3 or better. There were ten 3s and two 4s. Of the students that did not earn a 3 or better, seven earned 2s and five earned 1s.

A two-sample *t* test was performed to assess whether there were statistically significant differences between the student means on the profile variables: quiz grades, homework grades, test averages, prior testing in Algebra I, Geometry, and Algebra II, prior testing in Biology, seventh grade Reading, grade 10 Writing test, PSAT composite scores, and SAT composite scores. This was done to determine to what extent a student's profile influences success in the AP Statistics course, with earning a three or better defined as a success. The data for each variable was scanned for gross outliers and skewed patterns. Gross outliers were found in the Algebra I and Geometry scores for Group 1 and English I for Group 2. Data with marked skewness was found in Reading 7 for Group 2 and Writing 10^{th} for both groups. The means and standard deviations for the profile variables are presented in Table 2 for comparison by grade earned group.

Table 4.1 Results of a Two-Group Comparison on Fourteen Independent Variables

Means and Standard Deviations for Profile Variables with *t* –test Results for Grade-Earned Groups (N = 24)

	Earned 2 or Less (n = 12)		Earned 3 or Better (n = 12)				Statistically Significant
Variable	M	SD	M	SD	*t* test	df	✓
Homework	81.17	14.38	88.7	9.54	-1.522	22	
Quizzes	81.33	6.71	89.0	5.34	-3.097	22	✓
Test Average	78.71	6.86	87.8	5.77	-3.526	22	✓
*Algebra I	89.92	13.03	93.6	6.54	-.788	19	
*Geometry	87.50	13.71	94.2	7.07	-1.394	20	
*Algebra II	92.42	6.87	96.4	4.06	-1.611	20	
*Biology Raw	68.75	4.41	72.6	2.84	-2.485	21	✓
*Reading 7	87.27	12.01	89.4	9.48	-.441	18	
*Writing 10^{th}	3.25	.62	3.18	.40	.309	21	
*English I	83.64	9.88	93.3	9.03	-2.268	18	✓
*PSAT Sum	2032.8	603.21	2483	320.95	-2.145	17	
*SAT Sum	1679.0	157.06	1856	184.78	-2.424	20	✓
Final Course	82.83	6.59	90.4	5.58	-3.041	22	✓

*Some scores were unavailable

At the alpha level of $\alpha = 0.05$, no significant differences were found between the two groups for the means of the profile variables homework, Algebra Raw Score, Geometry Raw Score, and Algebra II Raw Score, Reading Seventh, PSAT Composite Score, and Writing 10^{th}. However, at $\alpha = 0.05$, statistically significant differences were found for quizzes (for Group 1, M = 81.33, SD = 6.71; for Group 2, M = 89.00, SD = 5.34 with $p = 0.005 < 0.05$). Statistically significant differences were also found for the Biology standardized examination (for Group 1, M = 68.75, SD = 4.41; for Group 2, M = 72.64, SD = 2.84 with $p = 0.021 < 0.05$). Additionally, statistically significant differences were found for the English I standardized examination (for Group 1, M = 83.64, SD = 9.88; for Group 2, M = 93.33, SD = 9.03 with $p = 0.036 < 0.05$). Statistically significant differences were also found for SAT Composite Score (for Group 1, M = 1679.09, SD = 157.06; for Group 2, M = 1856.36, SD = 184.78 with $p = 0.025 < 0.05$), Test Average in the course (for Group 1, M = 78.71, SD = 6.86; for Group 2, M = 87.83, SD = 5.77 with $p = 0.002 < 0.05$), and Final Grade in Course (for Group 1, M = 82.83, SD = 6.59; for Group 2, M = 90.42, SD = 5.58 with $p = 0.006 < 0.05$).

The results for the prior testing in Algebra I, Geometry, and Algebra II were not surprising. Students who enroll in AP Statistics are usually more capable mathematics students and therefore no significant differences should have been expected. The difficulty that the students in the course exhibited in communicating effectively did not seem to be reflected in the scores for both groups of students in Reading 7th and Writing 10th examinations. However, a significant difference between the groups was observed in the scores on the English I examination. A number of factors could have influenced the scores on this examination. These will be explored in Chapter 5. It was interesting that the students in the two comparison groups showed differences in the Biology end-of-course examination as well. The Biology course required the student to complete extensive reading, complete scientific writing, and to pay careful attention to scientific principles as well as using inquiry as the starting point to all investigations. The inquiry process included asking questions that would hopefully stimulate students to think critically and to formulate their own questions. These are also the goals of an AP Statistics course and may indicate that better scores on this assessment shows an attribute that should be part of a student's profile for success in the AP Statistics course. This inference is certainly not part of this study but warrants further investigation in a future study.

4.3.3 Summary of Interviews and Profile Differences

In summarizing the students' interviews and profile differences it was evident that all the students accepted the teacher's approach to instruction. They were compliant in working on projects together, on the study guides, and on the assignments. In relating how the students reacted to the projects, it was clear that although all the students enjoyed working on projects with other students it was not always apparent to the students the purpose of the project. The students that performed well on the AP Statistics examination are excellent mathematics students, however, that did not appear to be a contributing factor to predicting success on the AP Statistics examination. A contributing factor to predicting success on the AP Statistics examination seemed to be a good grade on the Biology end-of-course examination. There were statistically significant differences observed between students that performed well on the Biology end-of-course examination and the AP Statistics examination and those that did not do well on both examinations.

Based on the analysis of the interviews, over half the students procrastinated in completing the assignments. This clearly contributed to the students' lack of participating in the discussions of the assignments and in particular the discussions of statistical concepts initiated by the instructor. Based on the observations of the students' reactions to the teacher's explanations of the justifications required, most of the students did not understand how crucial it was for them to justify answers to questions with appropriate statistical arguments in order to earn full credit on the AP Statistics examination. The interviewed students voiced their concern that AP type questions were not introduced into the Statistics course early enough.

4.4 The Assessments Tell a Great Story

Assessments tell a story of the progression of a student through a course. The grade categorizes the student, but does not tell why they belong in that category or if they belong in that category at all. If the assessment is in the form of a project one can begin to understand if the concepts have been mastered or not. Two projects completed by the interviewed students were analyzed to identify growth in statistical reasoning and writing. Additionally, case studies for four of the interviewed students, Students A, B,W, and Y were used to discuss the levels of statistical reasoning shown across both projects and a critical statistical analysis completed between the two projects mentioned previously.

4.4.1 The First Project

The instructor of this AP course planned projects and activities to encourage growth in writing descriptive narratives that would explain the statistical analyses conducted. The first project was called Exploring the Central Limit Theorem. Two or three students were grouped to complete this project. The students were grouped in order for the students to support and help each other on this first attempt at working collaboratively on conducting statistical analyses. The students were given the steps to follow and were taken to the lab to use MINITAB, statistical software which is user friendly, to help in generating samples of different sizes. The software also allowed the student to construct tables and histograms which could be imported easily into the student's word processing program. This project not only required the student to write a paper but also had the student construct a poster that was also graded. A rubric was given to the student (Appendix B) so that the students knew the requirements beforehand and how they were to be graded.

The purpose of this activity was to lay the ground work for understanding statistical inference. Prior to introducing the Central Limit Theorem, the concepts of sampling distributions and sampling variability had been discussed. The tools used in data analysis to describe any distribution were reviewed. Sample proportions and sample means used to produce sampling distributions had been discussed and how to calculate the mean and standard deviation of these distributions had been introduced as well.

The premise of the project was that when you drew a simple random sample of size n from any population with a mean μ and finite standard deviation σ, then when n is large, the sampling distribution of the sample mean $\bar{x}$ is close to the normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$. The students were given a Binomial distribution with 5 observations and probability 0.1 for a success. With those parameters the students took 20 SRS of 100 observations each from this distribution. The hypothesized probability distribution is:

x	0	1	2	3	4	5
P(x)	0.59049	0.32805	0.7290	0.00810	0.00045	0.00001

The students were then directed to graph the distribution and describe the graph as well as calculate the mean and standard deviation. The investigation asked the students to choose 100 samples of size 2, 5, 10, 15, and 20 from these 20 SRS and compute the averages. The distribution obtained was to be described and graphed.

4.4.1.1 Student A

Student A worked with two other students in completing the lab, writing the paper, and creating the poster. The final grade was 93. The first sentence of the paper demonstrates a lack of care in introducing the reader to the project description: *The Central Limit Theorem states that as a sample size n, increases the sampling distribution of the sample mean* $\overline{x}$ *gets closer to the normal distribution. In our investigation, we were able to observe this fact of probability.*

The student was able to follow the directions well and produce the graphs and calculate the mean, standard deviation, and median of each distribution. Student A chose to calculate the median in order to compare it to the mean and determine how close the distribution that was graphed was to a normal distribution. This was in fact an incorrect hypothesis, because symmetry implies that the median and mean are approximately equal in value but the converse is not necessarily true. The student calculated the mean and standard deviation for each distribution but did not compare it to the hypothesized values according to the conclusion of the Central Limit Theorem. A statement made by the student demonstrates that she is not sure what she is trying to show. *We calculated the means of 100 sets of samples of size N=2 and made a histogram of the means, as shown below. The histogram of this sample has a mean of 0.52, a standard deviation of 0.4868, and a median of 0.5. In addition, the graph is skewed to the right. Using the statistical formulas, we calculated a mean of 0.5 by finding np, and standard deviation 0.4743 by solving* $\sigma/\sqrt{n}$. At times the student was confused whether the samples were taken from the previous data or from the original binomial distribution. For example: *From the data in our histogram, we drew other samples of different sample sizes.* The student calculated the mean and standard deviation for each of the distributions and the hypothesized values but never compared them to reach a conclusion about how the increase in size of the samples taken gradually produced the expected values. Student A can be considered between a level 2 and 3 of statistical reasoning. This student demonstrated a verbal understanding of some concepts, but could not apply this to the actual experiment being conducted. The student provided a correct definition but did not fully understand the concepts. The student stated: *This proves that starting with a non-symmetric distribution, and taking additional samples, will produce a distribution that appears to be approximately normal.* The student was also able to correctly identify one or two dimensions of a statistical process without fully integrating these dimensions, such as, that a larger sample size leads to a smaller standard deviation in a sampling distribution and would therefore produce a more symmetric graph.

4.4.1.2 Student B

Student B worked with one other student in completing the lab, writing the paper, and creating the poster. The final grade was 83. In general, the paper lacked specificity. Statements were made without regard for clear and specific connection to the distribution being discussed. The introduction to the paper was very short and did not tell the reader what the project was all about. An example of a typical sentence is: *For the next graph, we took two adjacent rows in two random samples containing 100 observations. This graph, like the first, is skewed to the right, just not as severely. The spread is between 0 and 2 with a mean of 0.535, a median of 0.5. The standard deviation was 0.4937.*

Because the mean and median are very close to each other, it is safe to say that this graph represents a fairly normal distribution. There was no attempt at looking at the graph itself to see that was not a very accurate description of the graph.

For the third distribution, instead of taking 100 samples of the means of 5 observations, the students took 5 samples of the means of 100 observations. The student stated: *The third graph was derived from the mean of five random samples containing 100 observations.* The student continued to make this mistake throughout the rest of the lab. There was one encouraging note, the student was aware that the conclusions of the Central Limit Theorem were not being met. The student said: *We found that our data did not always support the expected patterns, theoretically, the data will become more symmetric with increased samples. Had we taken many more samples, the final histogram would have been much closer to symmetric.* As can be seen by the above statement, the student alluded to the idea of sampling variability but confused mean and median as to the cause of the changes in the distribution of the graphs. The student made one more error in trying to say that the standard deviation became smaller due to the Law of Large Numbers. The student never made the connection between the conclusions of the Central Limit Theorem and the results that were obtained in the lab. The only observation made was that the standard deviation became smaller and therefore decreased the spread of the data. It also became clear that the student never acknowledged the fact that averages of the sample of size n were being calculated. For example: *The more samples you have, the more compressed the data becomes because the extremes cancel each other out*.

The student demonstrated some verbal reasoning by understanding some statistical terms but confused them on occasion. At this stage in the course, Student B was operating at a level 2 in statistical reasoning. The concept and use of the Central Limit Theorem was not clear to this student.

4.4.1.3 Student P

Student P worked with one other student in completing the lab, writing the paper, and creating the poster. The final grade was 79. The introductory paragraph was fairly lengthy but failed to mention that the samples were sample means taken from a binomial distribution with 5 observations and probability 0.1 for a success. The student said: *We randomly generated data which is pictured below in graph C1*. The student calculated the mean, median, and standard deviation and commented on the size of the standard deviation but with no comparison to the conjectured value. The student commented on the second graph which showed the distribution of 100 samples of the means of samples of size two (this was not clearly stated in the paper). Student P stated: *The second graph, SS=2, used a sample of size two, had a mean of 0.4600 and a median of 0.5000, with a standard deviation of .4643. The shape of the graph is fairly symmetric although skewed to the right with a very wide spread.* The graph was skewed right, yet, the student said that it was “fairly symmetric” which was obviously not correct. As the student explained the procedures the researcher realized that the student was considering that as the median and mean get closer in value that somehow it was correct to say the graph was symmetric. This was clear in her next statement: *The result is due to the difference in the mean and median, which are somewhat closer in this generation, but not enough to produce the desired normal results.* The student seemed to be concentrating on obtaining a symmetric graph and not on comparing the results of the mean and standard deviation to the hypothesized values using the Central Limit Theorem.

When the student produced the third graph with the distribution of the 100 samples of the means of 5 samples there was obvious confusion between spread of the data and the size of the samples. The student may have thought that a larger sample size should produce a greater spread. This idea could have been dispelled if the student had looked at the graph. The student stated: *The mean, median, and standard deviation of this sample was large enough to produce a more even spread, yet, too small to produce normal results. Although the spread is greater the data results are still skewed slightly right*. This was not true because it was actually getting smaller.

The next statement provided stronger evidence about the general confusion of this student with regard to the information that the graphs were showing. The student stated: *Graph SS = 10 has a mean and median which are very similar in value, but the standard deviation is still a bit too great. The sample size will have to be increased a little more in order to decrease the standard deviation enough to produce a normal graph.* The concept of variability was not understood by this student. The student believed that simply adding more data would take care of the value of the standard deviation.

The final two graphs had exactly the same spread and were both smaller horizontally than the original graph shown. The student was still insisting that the spread was wider. However, when constructing her last graph, it all of a sudden became normal with a normal spread. The student contradicted everything she had said previously with the statement: *The standard deviation decreased with each sampling and the spread tightens the normal distribution is obtained.*

The student knew some statistical words but was not able to understand how they were related. This student was still using idiosyncratic reasoning and the concept of how variability plays a role in statistics was absent in this student's understanding of distributions. Student P was confused about the entire process and reveals this in one of her final statements: *This activity proves that the greater the sample size, the more accurate the information that is being depicted. The graphs produced visible evidence that showed how the relationship of mean to median will cause results to drift toward normal.*

4.4.1.4 Student T

Student T worked with one of the better students in the class and therefore should have earned a better grade than an 86. Student T is a good writer therefore most of the points lost was due to the poster part of the project. Student T's introduction was "nearly textbook perfect". She wrote: *In Statistics, the Central Limit Theorem describes a phenomenon that occurs when we draw a simple random sample (SRS) from any population with a given mean, μ, and a standard deviation, σ. When the number of observations taken from the SRS, n, is large, the sampling distribution's mean,* $\overline{x}$ *, draws closer to the normal distribution N(μ,* $\sigma/\sqrt{n}$ *) with mean μ, and standard deviation* $\sigma/\sqrt{n}$ *. In layman's terms, the CLT tells us that if we take a sample from any population and then sample from its distribution, the new sample's distribution will become more and more normal as the sample size increases.* The student continued by describing where her samples would be drawn from and stated the distributions probability distribution.

The student then made a mistake that was very common in these papers; she stated: We asked Minitab to generate 100 random samples taken from the sample means of the distribution graphed with a sample of size two. The work that accompanied the statement was correct. The student also mistook skewed left for skewed right. The student was convinced that as the distribution became more normal the variability was limited. This researcher believes she meant the standard deviation becomes smaller. These examples point to confusion with the concept of variability and the statistic, standard deviation.

Student T believed that the purpose of this project was to prove that the Central Limit Theorem was correct. This idea was prevalent in half the papers of these AP Statistics students. This was one of the first projects, therefore it is easy for students to think that inference involves “proving things”. However, these students should have learned in their mathematics classes that one example does not prove a statement is true. Although this student is beyond the level of understanding verbally, level 2, what she is doing statistically does not make it clear that she has the ability to completely understand the procedures used in applying the Central Limit Theorem.

4.4.1.5 Student U

Student U worked with one student and earned 93 for her project. Almost all the points lost came off the poster presentation. The student’s introductory paragraph was very well written and was one of the few that came closest to saying what they were actually going to do. The student stated: *The Central Limit Theorem states that when a SRS of size n is large in any population with mean* μ *and a finite standard deviation* σ*, then the sampling distribution of the sample means x is close to the normal distribution* $N(\mu,\ \sigma/\sqrt{n}\,)$.The mean and standard deviation of the binomial distribution was calculated correctly. The histogram, mean, median, and standard deviation were calculated using MINITAB.

After taking 100 observations from the first trial the student said: The graph is evidently skewed to the right, with the median of the observations at 0. The mean interestingly is 0.5, but this value could be attributed to chance and should not be construed as a typical occurrence when graphing any one hundred observations. The student also commented on the rather “large” spread from 0 to 2. This researcher has not determined why the students found this a large spread. Student U made one comment which is not too unexpected. She stated: *According to the Central Limit Theorem, this difference (between the theoretical standard deviation of the binomial distribution and the value obtained for the sampling distributions) should diminish with a larger number of observations*. In a subsequent paragraph it was obvious that she meant that the standard deviation would get smaller.

The student continued and generated the other distributions correctly and calculated the mean, median, and standard deviation for each distribution. She also provided a good description of the shape and symmetry of the resulting distribution. As each distribution was graphed and the statistics calculated, the student commented on the mean, median, and standard deviation and its closeness to the conjectured values using the Central Limit Theorem. A typical description was: In our next histogram we graphed 500 observations (5 trials). This histogram is clearly more symmetric that the last, although arguably still slightly skewed to the right side. The mean actually decreased to 0.4680, but still very close to 0.5. The median followed a similar trend. The standard deviation decreased drastically. This is attributed to chance, however, and as long as we see the trend in the overall decrease of the two numbers, the Central Limit Theorem is remains (sic) clearly at work. Although the wording needs a little work it is clear that the student knew the process very well.

Finally, the student recapped the lab experience and what she has learned from the process. It was evident that this student had understood the concepts behind the Central Limit Theorem. Her explanations were easy to follow and great care was taken in understanding the concept of variability and how chance played a role in inferential statistics. The explanation provided in the paper would seem to indicate that this student was operating at a high level using integrated process reasoning to complete the project.

4.4.1.6 Student V

Student V worked together with one student on this project and they earned an 80. Student V is a very capable mathematics student but is reluctant to talk very much in class and it's quite possible that also influenced his desire not to write either. His introductory paragraph was completely missing. The student restated the Central Limit Theorem and stated what he expected the mean and standard deviation to be without prefacing which distribution he was talking about. When discussing the probability distribution for the binomial distribution, he prematurely said that he had noticed that the distribution was skewed right without displaying the graph. Although that is possible to notice, the discussion did not flow well and did not explain the process with reasonable fluidity. A typical paragraph from this student was: We then made a histogram of a sample of 100 with the sample size of 2. This graph also shows a skewness to the right. The mean was 0.475, the standard deviation was 0.4571, and the median was 0.5. The histogram is shown below.

The rest of the paper just listed the steps, very briefly, that the students followed in producing the other graphs and statistics. The student did notice the change in the shape of the distribution from the first to the last graph; however, he failed to compare the values to the expected values for the mean and standard deviation and how this showed the concept behind the Central Limit Theorem.

It was not evident from the written portion of the project whether the student understood the project and its implications in statistics. The analyses were very sparse in detail. Because only quizzes were given for this material and this project served as a summative assessment it was necessary to calculate the average of the quizzes which was 84. From this and his project it is evident that the student did not fully understand this material and was just able to show verbal reasoning, level 2, of concepts in this unit.

4.4.1.7 Student W and Z

Students W and Z worked on this project and earned an 85. Nearly half the points lost was due to the poster presentation. It is interesting to note that Student W participates regularly in discussions in class and Student Z would rather work out all the problems with the graphing calculator and discuss nothing. The introduction was present and fairly straight forward. These students did not say that they would be proving but they would be visually illustrating the Central Limit Theorem. The students state: … *illustrate the Central Limit Theorem by using randomly selected variables that will originally be non-symmetric, and as we begin to take the means of increasing amount rows of more randomly selected variables, we will see how this increase in amount of sample means affects the distribution of our data.* Although this sentence is not clear it shows that the students have an elementary idea of what was to take place. Even though the terminology of the first sentence used was imperfect the students added the correct hypothesis and conclusion of the Central Limit Theorem to complete the introductory paragraph.

Students W and Z described the distribution of the first 100 observations and computed the mean, median, and standard deviation. They did not compare these values to the binomial distribution's expected mean and standard deviation. The students suggested that the distribution would "adjust" as more data was added. It was unclear what adjustments were being discussed. The students stated: *As our original graph clearly illustrates, it is quite unsymmetrical being far skewed to the right. The spread of the data is tightly bound to the three variables. …we should expect that the data has a greater likelihood of being skewed; however, as we continue to observe more data we should see the distribution adjust*. It may be possible that the student was confused about the sample size and the number of observations in the distribution. When the students chose two rows of observations and computed the mean and formed the new distribution the students indicated that they believed that there were 200 observations even though the computer output was letting them know that there were only 100 observations. The students calculated the mean, median, and standard deviation correctly but again did not compare them to the expected mean and standard deviation according to the Central Limit Theorem.

The students now used five rows to compute 100 means and produced the graphs and statistics for the distribution. The students stated: *This third graph encompasses five rows worth of data which allows us to finally see the signs of our data becoming more normal.* The student referred to the data becoming more normal instead of referring to the distribution of the sample means. The students erred, as some of the other students had done, in showing evidence of normality by referring primarily to the statistics and not to the general shape of the distribution. Students W and Z discussed the fact that the graph had increased its spread but it had in fact become smaller. This researcher can only surmise that they meant that there were more bars displayed for the last three distributions. It was not until the last paragraph that the students made some comparisons to the original distribution. The students believed that they drew more samples, when in fact there were still 100 observations. Students W and Z said: …it is safe to say that if we had continued to draw more random samples that the graph would become even more normal.

Students W and Z continued to make the same errors as seven of the other students who assumed that if the mean and median are equal that you can assume normality of the distribution. The concept of the Central Limit Theorem was not clear for these students. Even when they had the correct graphs and statistics they were not able to correctly interpret the information in the context of the Central Limit Theorem. These students were functioning between idiosyncratic and verbal understanding, level 1 and 2, of the concepts they were studying. As a side note, both students completed less than 82% of the homework on this chapter but were able to manage better than an 85% on the quizzes for this material which shows a close relationship to the grade on the project.

4.4.1.8 Student Y and bb

Students Y and bb worked together on this project and they earned a 94. Student Y is a much better student academically, therefore Student bb likely benefitted from working with him. The introduction was a bit weak: *In this investigation, we took random samples from a non-symmetric distribution, B(5, 0.1). Then, we constructed distributions* *of our samples means with gradually increasing samples sizes. The point of the activity was to illustrate the concept of the Central Limit Theorem, meaning that the larger the sample size is, the smaller the standard deviation is, and the closer the mean will approach its true value. The shape of the graph also should be close to the normal distribution as the sample size increases. A smaller sample size gives more variability and a less accurate approximation of the data*. Some students, such as Student Y and bb believed there was only one normal distribution. The researcher did not understand what the student meant by the last statement of his introductory paragraph. The student also did not clarify that sample means were being used to construct the sampling distributions.

The student calculated the standard deviation of the binomial distribution incorrectly by using the formula for the sampling distribution of sample proportions, which is $\frac{p(1-p)}{n}$. The student then had MINITAB generate the 100 sample means of sample size 2 but incorrectly refers to the sampling distribution graph as a graph with sample size 2. The student had the statistical program calculate the mean, median, and standard deviation correctly. The value of the standard deviation was 0.5024 and the student stated that this meant “it has a high variability”. This researcher does not know which value for a standard deviation would be satisfactory and show low variability for this student. The students stated: *The mean of this distribution is 0.51 and the median is 0.5. The standard deviation is 0.5024 meaning it has high variability. It is somewhat skewed right although it should appear normal. It seemed that they were expecting the graph to appear normal with only a sample size of 2 simply because the mean and median were close in value.* However, in the next sentence the students dispelled that idea by stating: *As the sample size increases, you will see how the graph gets closer to the normal distribution*. It is not clear which distribution is “the normal distribution”.

Students Y and bb continued and produced the graphs for sampling distribution of the sample means for sample size of 5, 10, 15, and 20. They were able to have MINITAB calculate the mean, median, and standard deviation (incorrectly using the formula for proportions) for each of the distributions. The students made a telling statement that shows their confusion with the project. They stated: *It (the graph) looks a little skewed left, but overall is about normal. However, its symmetry does not matter as much as the size of its standard deviation.* It was not apparent what value they were expecting.

These students knew some concepts associated with the Central Limit Theorem. There was confusion about variability, standard deviation, normality, and what the mean and median were supposed to tell about the distributions. There was some verbal understanding, level 2, about the process but they were not able to transition to the point of integrating those processes to show correct statistical reasoning when discussing these topics and the processes involved.

4.4.1.9 Summary of the Central Limit Theorem Project

Given that this was the first project given to these statistics students, the expectation on the part of the instructor was that the students would be able to follow the directions given to produce graphs and statistics, use basic yet appropriate statistical language to describe the graphs and statistics produced, and design a poster that would summarize the results. The instructor did not expect perfect papers because this was their first attempt at analyzing data with no guiding questions.

There were eight projects analyzed because two of the ten students whose work was analyzed teamed up to work together. Of these eight projects, three lost credit due to poor poster presentation. Four of the projects had a poor introduction which indicated lack of care in the choice of words used to describe the project on which they were working. All the students were able to produce the graphs and statistics such as median, mean, and standard deviation. Of course, step by step instructions had been given to the students. At least three of the projects mentioned specifically in their paper that when the mean and median are equal the graph of the data will be normal. Four of the projects did not show verbal descriptions connecting the graphical displays to the statistics shown. In several cases (seven projects) there was general confusion about what they were trying to do. They either said they were trying to prove the Central Limit Theorem, incorrectly described what they were doing, or referred to a graph that they had created and described its general shape incorrectly based on what they wanted to have happen rather than what was being shown. The students had a great deal of difficulty handling the concepts of variability, standard deviation and how they could be affected by sample sizes. In three of the projects the students indicated that adding more data or increasing the sample size would decrease the standard deviation or variability. Several students used these statistical terms interchangeably. Four of the projects showed that the students were not looking at the graphs to describe the shape and how the statistics were related to the hypothesized values of the Central Limit Theorem.

4.4.2 The Regression Project

This was one of the last assessments, in the form of a project, given in the course. The project was intended to have students demonstrate mastery in regression analysis.

For this project students were given the choice of choosing their own bivariate data and analyzing it to determine the type of relationship that exists between the two variables. The data could have a linear trend or be non-linear. The rubric (Appendix A) included twenty items that the student needed to address in order to have a chance of earning full credit. The style of the paper needed to be in narrative form and the spelling, grammar, and correct usage of statistical vocabulary was also a component of the grade. Additionally, the teacher gave an overall quality grade that was purely subjective and this was added to the project grade for a total grade out of one hundred points. The expectation was that these students would do an excellent job on this project since the teacher had modeled a regression problem prior to giving the students the project. The students had a week to work on the project and they were given feedback by the teacher if they asked for help.

4.4.2.1 Student A

Student A – earned an 88. The data was of interest to the student because it compared Grade Point Averages (GPA) to ACT. Although it would have been more interesting to have the student compile her own data, that was not a choice the student made. The student used a large font which might have been an indicator of the lack of substantial work was going to be compensated with a large size font and style. The student had difficulty with the construction of tables but was able to capture statistical graphs and incorporate them into the main document quite easily. Appropriate explanatory and response variables were chosen and 23 of the 32 data points were chosen randomly for the final project. The reader was told that the random number generator of the TI-84 graphing calculator was used but the procedure was not explained. The sentence structure was poor and demonstrated that the student was trying to write in an academic language but was unsuccessful. For example: *To correlate with this data so that an appropriate model is selected a scatter plot was created. To interpret the data in more dept (sic) a Residual plot was created and is shown below with a table listing the residuals.* The interpretation that can be made concerning the first statement is that the student wanted to choose an appropriate model and was going to construct a scatterplot in order to make this selection process possible. The second statement referred to the construction of the scatterplot in order to create a visual graph as evidence for choosing a linear model. Even though the data was clearly not linear it was appropriate for the student to continue and after more analyses discard the model for one that was better able to capture the data. The student then indicated that the scatterplot showed a moderately strong positive association between the variables. This was a premature statement because the coefficient of correlation had not been computed yet. The coefficient of correlation was computed by the student as 0.83285 but was called *r* – value. The appropriate name should have been used. The coefficient of determination was named properly however, its interpretation was incorrect. The student said: *The coefficient of determination shows that only 69.3% of the variance represents y.* The reader is left to wonder which variable is *y*. It was evident that the student did not understand what the coefficient of determination measured. Something along the lines of: approximately 69% of the variation in the response can be explained by the explanatory variable was expected. The remaining 31% can be explained by an unknown lurking variable or variables or the inherent variability in the data. The teacher discussed this topic over several weeks. The students had ample time to discuss the project with the teacher and to consult their notes, textbook, and other materials used in the classroom.

The student constructed the residual plot for the first model and indicated that data points were scattered above and below the line with no distinct pattern. The student then discarded the model because it *may not the best choice for this data*. No reason was given for this decision. Immediately, after discarding the linear model the student opted for trying out the power model. No reason was given for this choice either. The exponential model was not chosen because the student said: *if a ratio was taking we are able to see that an exponential model would not work*. Throughout the course, the importance of statements being completed with justifications was emphasized. It does not suffice to say something is or is not appropriate unless it is accompanied by statistical reasoning. It was evident that the scatterplot for the logarithmic values of GPA and ACT looked like the original scatterplot but compacted vertically. The student made another error at this point in the project. The student said that the power model was a better fit because the transformed data (logarithmic values of data points) showed it. However, the transformed data looked linear not curved. The new coefficient of correlation was computed as 0.870889 and the coefficient of determination was calculated as 76%. The residuals were graphed showing that there was not any more scatter above and below the line $y = 0$ than before. However, the values of the residuals were more clustered and quite small compared to the previous model. Some discussion about interpolating and extrapolating a point completed the student’s discussion of her data.

Student A most closely demonstrated statistical reasoning at Level 1: idiosyncratic reasoning and gradually working towards verbal reasoning, Level 2. The student knew how to conduct linear regression but provided no justification for one model over others that were possible. The student should have been aware that she should have discussed the data on the original scatterplot in order to determine an appropriate model to start with and that comments needed to be made about the residual plot as to the merits of using the linear model, but neither commentary was complete or referenced by the correct graph. It was evident that statements that had been used in class and deemed important by the student were used to provide evidence as to the merit or inappropriateness of particular models, but the supporting statistical reasoning and work was not present.

4.4.2.2 Student B

Student B–earned a 97. The data was of interest to the student and it compared mean annual income of an individual living in the United States and the year in which the observation was made. This student was capable of working with data that was more interesting and perhaps less obvious as to the relationship between the variables. Perhaps the student decided to choose an example that was obvious in order to minimize making a serious error in judgment.

The introductory paragraph was well constructed and alluded to the fact that she was expecting a positive correlation due to the increasing cost of living phenomenon. The graph of the data was carefully constructed with appropriate labels and title. After judging the appearance of the data to be almost linear, the student used that fact to proceed and conduct a linear regression using year as the explanatory variable and average annual income as the response variable. The slope was not mentioned by name but the standard LSRL equation was mentioned and the positive value was used to confirm the conjecture stated earlier in her document. The regression line was superimposed on the graph of the data and the coefficients of correlation and determination were calculated. Although these values were not mentioned by name, it was clear from the discussion that the meaning of both were clear to her. Her comments were: *the r – value is equal to .9968. The extreme closeness to 1 in the given data set signifies an exceptionally linear association. The* r^2 *value equals .9937. Approximately 99.3% of the data can be accounted for by the least squares regression line.* The residuals were presented in tabular form and were graphed and explained. The student completed the study by explaining and correctly using the concepts of extrapolation and interpolation to test her model. Finally a confidence interval for the slope and a test of hypotheses provide evidence of an association between year and annual individual income.

In assessing the level of statistical reasoning at which this student is operating this researcher can say that this student has mastered the vocabulary necessary to demonstrate knowledge of regression analysis. The process was clearly and correctly explained and this student is operating at a Level 5: using integrated process reasoning.

4.4.2.3 Student P

Student P–earned a 94. This student's project was unavailable; however the grade indicated that the prerequisite components of the rubric were addressed and present in the project. Prior to turning in the regression project a quiz had been given to the students to measure knowledge of this process. Although the processes required were simpler it should indicate any growth during the period from the quiz to the project. On this quiz Student P earned an 84. The graph of the data was done correctly and labels were appropriate as well as the units along the horizontal and vertical axes. Nonetheless, when asked to transform the data in order to obtain the exponential regression equation, the logarithmic values were not put in table form and the student was unable to give two examples of the transformed data. The student was able to find the LSRL for the transformed data and the coefficient of correlation. However, the equation was not entirely correct in the context of the data. The student was also unable to determine the exponential equation for the original data. No explanation was given of the meaning of the coefficient of correlation nor was it called other than *r*-value. Additional questions on the quiz showed that the student knew the differences between causation, common response, and confounding.

Based on the data obtained about Student P, though limited, one can surmise that she was able to learn from her quiz and used that knowledge to earn a good grade on her project. This would indicate that at a minimum her level of statistical reasoning was beyond procedural reasoning, Level 4, and perhaps is showing some growth beyond that level.

4.4.2.4 Student T

Student T–earned a 77. This student’s project was also unavailable; however the grade indicated that some of the prerequisite components of the rubric were not addressed and were not present in the project. Student T’s quiz on linear regression, as was done with Student P, will be used to show an indication of any growth during the period from the quiz to the project. On this quiz Student T earned a 100. The graph of the data was done correctly and labels were appropriate as well as the units along the horizontal and vertical axes. When asked to transform the data in order to obtain the exponential regression equation, the logarithmic values were placed in table form and the student was able to give all examples of the transformed data. The student was also able to identify an outlier from the transformed data. The student was able to find the LSRL for the transformed data. The equation was given correctly in the context of the data. The student was able to determine the exponential equation for the original data. Additional questions on the quiz showed that the student knew the differences between causation, common response and confounding. Although knowledge of these concepts was not required on the project, it demonstrated that the student knew content beyond the basic steps in regression analysis. Based on the data obtained about Student T one can surmise that she was prepared for her quiz but did not use that knowledge to earn a good grade on her project. Her interview showed that she had come to the point where she dreaded the class and had become obsessed about the wording necessary to receive full credit on her answers. Perhaps she felt that nothing she did would earn her much beyond a passing grade. It was difficult to reconcile that on a quiz she earned a 100 and showed evidence of having a good grasp on linear regression and yet on the other hand earned a 77 on a project that was completed over a week and for which she could have used any resources available to her. It was difficult to determine the level of statistical reasoning with two opposing pieces of data. However based on other data available about this student, she was at the beginning stages, Level 1, of transitional reasoning.

4.4.2.5 Student U

Student U – earned a 94. This student's project was unavailable; however the grade indicated that the student completed the project successfully by addressing the various parts of the rubric. An assessment had been given just prior to starting work on the regression project to measure knowledge of the process used in linear regression. The steps required on the quiz were simpler but can indicate any growth during the period from the quiz to the project. On this quiz Student U earned an 84. The graph of the data was done correctly and labels were appropriate as well as the units along the horizontal and vertical axes. When asked to transform the data in order to obtain the exponential regression equation, the logarithmic values were placed in table form and the student was able to give all examples of the transformed data. The student was also able to identify an outlier from the transformed data. The student was able to find the LSRL for the transformed data. The equation was given correctly in the context of the data. The student was unable to determine the exponential equation for the original data. Additional questions on the quiz showed that the student knew the differences between causation and confounding but not common response. Although knowledge of these concepts was not required on the project, it demonstrated difficulty in going beyond the basic steps in regression analysis.

Based on the data obtained about Student U, one can surmise that she was able to learn from her quiz and use that knowledge to earn a good grade on her project. This would indicate that at a minimum she was operating at a Level 4 of statistical reasoning which is procedural reasoning and perhaps was showing some growth beyond that level.

4.4.2.6 Student V

Student V–earned a 96. This student chose non-linear data comparing number of households from 1890 to 2002. This was not a stretch for this student but it served the purpose of the project. The graph of the data showed a curved trend (perhaps exponential) and the student explained that it was important to know this information and the possible trends because this will eventually affect the natural resources available to the United States.

As mentioned previously, the data was graphed and an exponential trend was noticed. The graph was correctly labeled and both vertical and horizontal axes marked. A LSRL was calculated on the original data with the appropriate context noted. The student noted the r-value of 0.96995 as a strong and positive association between years and number of US households. Additionally, the r^2 value was calculated and correctly interpreted in the context of the problem. Due to the general curved appearance of the data the student continued and calculated the residuals and graphed them against the explanatory variable, years. The student noticed that the residuals had a marked pattern. This information together with the curved trend of the data led the student to try an exponential model for his data. The student was a little premature with the statement: *The best for the data is an exponential model. It goes through about all the points and it has the highest correlation.* He had neither tried the new model nor had he tried other models to rule out which ones were inferior to his choice. The procedure for determining the exponential coefficients was fully explained and accompanied by appropriate graphs and tables. The equation for the logarithmic data was explained in the context of the data as well as the coefficient of correlation and determination. These values were higher than the previous linear model. The exponential form of the equation was explained and superimposed on the original data showing an almost perfect match to the original data. After using the model to test interpolation and extrapolation concepts the student correctly surmised the limitations of the exponential model.

The student had studied the power model as well as the others he tried in his study. It was within the scope of the project to also verify that the power model was a viable alternative. This student had learned the procedures studied in AP Statistics and was definitely using a high level of statistical reasoning in regression analysis. This student demonstrated that he understood that all the processes used to determine whether data can be best represented by one type of regression model over another is an integrated approach. He understood that although a model may not be a perfect model it served the purpose of extrapolating to values close in nature to the data being studied. This student was operating at a Level 5 of statistical reasoning and should be able to demonstrate this understanding of regression in a similar regression analysis question on the free-response question on an AP examination.

4.4.2.7 Student W

Student W – earned a 92. This student is interested in detrimental effects to the environment based on his readings and studies in his Biology course. The data compared the global CO_2 emissions from fossil-fuel burning, cement manufacture, and gas flaring for the years 1751 to 2004. Only the data from 1979 to 2004 was used in his project. The major concern of this student was the global warming due to world-wide dependency on fossil fuels, etc.

The data was graphed and then labeled by hand. The data appeared to be exponential in sections, in particular, 1979 to 1991, 1991 to 1998, and 1998 to 2004. The choice of an exponential model was made with the statement: *From this scatterplot it is safe to assume that this data will be best represented by an exponential model*. An accompanying statement alluded to the exponential growth of the human population and its dependency on fossil-fuels therefore it could be inferred that there would be an exponential growth in the burning of fossil fuels. Although this makes sense, it is not clear that it is a sound statement to make. At first, no statements were made as to the obvious breaks in pattern of the data (there was a statement later on in his paper).

A linear model was examined at first and although the coefficient of correlation was high, .9738, it was discarded after showing the residual plot. Although not mentioned by name, the values of r and r^2 were calculated and correctly analyzed in the context of the data and model. The residuals were calculated and shown in tabular form. The data was separated across pages making the table hard to read. There was an obvious pattern to the residuals and therefore it showed that the linear model was probably not the best choice. The student said: *Based on the residual plot, it appears that there may be an evident pattern from the data, which would prove that the linear model may not be the best model to present this data set*. It was obvious that the student wanted to discard the linear model because of the residual plot but used the word “prove” which is not appropriate in making inferences.

The data was transformed, using the techniques learned in class, and fully explained. The data was graphed along with the new LSRL. Again, this was not appropriate nor revealing because the reader was not shown how the new exponential equation fit the data. The new coefficient of correlation and determination were calculated and were found to be just slightly better than for the linear model. This should have been expected because the residual plot still showed an obvious pattern. The student made a comment about the increase in the coefficient of correlation and how that appeared to make this model the data set best. This student should have tried the power model and should have divided up the data into three sections due to the obvious changes in the original data. At the end of his analyses the student made mention of possible lurking variables that might have influenced the decrease in CO_2 emissions during certain years. For a project that was worth a test grade, it would not be unreasonable to expect this student to consult another source about the reasons for the increases and decreases in CO_2 emissions during certain years.

Although this student presented the items asked for in the rubric it was not clear that the student was able to carry out a regression study and use the tools and techniques studied in the course. There were some obvious steps, as mentioned above, that the student should have commented on or even have done to make this project a great project. The student was able to apply some procedural reasoning (Level 4) but did not demonstrate a complete understanding of the statistical processes when using regression analysis.

4.4.2.8 Student Y

Student Y–earned a 93. The data was of interest to the student because it compared circumference of the Douglas fir to its height. The height was determined by using right triangle trigonometry. This researcher had this student in another AP course and was cognizant of his work ethic and personality. The choice of this study was a bit unusual, for a high school student, but it fit his character. The original study sought to use the information gathered on circumference and height of a tree to determine volume of wood available. The introductory paragraph was well-written and explained why the height was dependent on the diameter and in turn it would be dependent on the circumference making the height the response variable and the circumference the explanatory variable.

The graphs of both the data and the residual plot were imported from Excel and labeled appropriately. The tabular form of the residual data was also incorporated into the document. The data and the methodology used were explained later on in the document but the student failed to concretely express the least squares regression line in the context of the study. The explanation for choosing the power model over the exponential or linear model was fully explained. Only the power model was considered, with a supporting argument that was valid but not stated in a convincing manner. The argument was as follows: *…the data is probably best represented by a power model. A tree will increase in height faster then it increases in circumference when it is young. However, as it grows older, the trunk will expand more than the tree grows in height.* Although the coefficient of correlation and determination were mentioned by name they were not referred in the paper as r and r^2. Although this researcher would not take special issue with this omission, the student lost points on his project for this omission. Both statistics were explained convincingly in the context of the data. The residuals were graphed correctly and a descriptive narrative explained the reason for choosing the power model. The student did not present a thorough argument for choosing this model based on the residual plot. No possible explanation was given as to the tight clustering of negative and positive residuals when the circumference was small and why the residuals were quite negative as the circumference was larger. The original data consisted of only twenty-five observations. The student should have considered whether this data was unusual and was not a random sample after all. The original study, which accompanied the student's report, alluded to this fact. The rest of the student's paper consisted of explaining how to obtain the power model equation. Although the paper was fairly complete according to the rubric this researcher does not find that the student presented a convincing argument as to his choice of a power model. The residual plot was not analyzed for unusual behaviors in the data. Although this student did show an understanding of regression as well as can be expected in an introductory statistics course and would be considered operating at a level 5, it was not clear from analyzing just this paper, that he would "catch" the nuances of the problems inherent in this data set.

4.4.2.9 Student Z

Student Z–earned a 93. The student chose as the data for his project a comparison of year and abortion rates in the United States. The introduction included a historical account of the trends in the abortion rate since Roe vs. Wade. The data was explained relative to the presidency of George H. W. Bush and the cultural shifts that the student attributes to celebrities. This student introduced his personal beliefs and views of Roe vs. Wade in his introduction. The student explained that the data showed two relatively different trends for several time periods. He chose the more current time period of 1983 to 2002 to complete his study.

A linear regression line was calculated that yielded a negative slope. The coefficient of correlation and coefficient of determination were calculated and both of these values were explained correctly in the context of the data. The student decided, based on the residual plot, to attempt to fit an exponential model to the data. The residual plot for the linear model showed a marked pattern that was v-shaped. His statement could have been better stated. It read: *Though the data fits the line of best fit well, the projected* *line is not the best interpretation of the data*. An exponential equation was calculated as an alternative approach and was explained by stating the steps used. This part of his paper did not flow well and was not necessary. It was not clear from the student's narrative if he truly understood the reasoning behind using logarithmic values to transform the data. It was also not evident why the student believed and stated that a value closer to one or negative one for the coefficient of correlation was considered more "accurate". The student checked the residual plot and noticed that the pattern was still noticeable albeit more random. There was still a pattern of positive, then negative, and then finally positive residuals for the data point 1989 to 2002.

It was a positive step for the student to attempt to fit a power equation to the data. He also noticed that the coefficient of correlation and coefficient of determination for the LSRL for the transformed data for this new model were not too different from the values for the exponential model. The residual plot still had a noticeable pattern not too different from the residual plot for the exponential model. It was expected that the student would talk about the limitations of all three models and would have suggested that perhaps the best model had not been found. The student hinted at but was not very obvious as to why he chose just a certain portion of the data. Overall, Student Z understood the procedures to be followed in doing regression analysis and was willing to continue trying to find a better model for the data given. This student also understood that there were other variables that influenced changes in certain variables being analyzed. This student was using a high level of statistical reasoning to complete this project on regression analysis. Student Z was operating at a Level 5 of statistical reasoning and was able to show integrated process reasoning. He was able to go beyond the expected steps in regression analysis project and attempted to explain the meaning behind the characteristics observed in the data.

4.4.2.10 Summary of Regression Project

All ten students were able to earn at least a C^- on this project. All students were able to calculate the coefficients of correlation and determination correctly and interpreted these values correctly with only one student having a bit of trouble correctly interpreting coefficient of determination. From the lesson observations in class it was clear that at least half the class had trouble with interpreting this statistic with confidence and in the context of the problem being discussed. Although there were some cases, two students whose projects were analyzed, in which the students did not present reasons for choosing one model over the other it was clear that it was an act of omission rather than flawed reasoning. At some point these students were able to refer to either a pattern or innate data peculiarities that led to trying a higher order model, exponential or power model, as a better model. With the exception of Student A the students were able to use appropriate statistical language to explain the procedures used to find the linear regression line and discard or choose a model based on the residual plot and coefficient of correlation. Student A knew the statistical jargon that needed to be used but was not successful in using it convincingly in her summaries. Eight of the students were able to demonstrate a level four or better of statistical reasoning which showed that students were able to apply some procedural reasoning, but did not demonstrate a complete understanding of the statistical processes when using regression analysis. Since this was the final project of the semester, the expectations were that they had learned the requirements of the projects throughout the semester. Additionally, the teacher had modeled how the descriptions within the paper should be written and explained. This project should not have been difficult for these students given their mathematical background.

4.4.3 The Case Studies

Several weeks after the first project on the Central Limit Theorem and an assessment on designing and producing samples were returned to the students, a critical statistical analysis assignment was given to the students. The purpose of the assignment was to measure the degree to which the feedback on the first project and first assessment had impacted the students' ability to write statistical statements using contextual evidence and to demonstrate knowledge on basic concepts in designing samples. Additionally, the instructor wanted to determine if the students would be able to draw conclusions from a study that had been published and show high levels of statistical reasoning as evidenced in their analyses. The grade for each student was determined by the degree to which the students were able to address questions concerning: correctly identify the population, sampling frame, and sample using appropriate statistical terminology, correctly discuss survey administration by addressing bias such as undercoverage, nonresponse, and question wording, accurately discuss the statistical techniques employed, accurately discuss the conclusions drawn from the survey, correctly use English mechanics, demonstrate careful organization and appropriate transitions within and between paragraphs

4.4.3.1 Student A

Student A chose to analyze a survey on gender preference of parents carried out by the Gallup Poll during several time periods between 1941 to 2007. It was evident that the sentence structure was weak and lead to ambiguous statements. For example: *I think that Gallop poll did a good job getting age variation but I think that the 18-49 age category should be divided even more so that there is a general conclusion between 18 year olds and 49 year olds.* It seemed to be the intention of this student to say that conclusions specific to each age group would have been more appropriate if divided further into subgroups by differentiating among age groups. The student failed to address the question of population and sampling frame. Student A was confused or did not understand blocking in experiments and stratified samples for surveys. The student said, *They first blocked in genders and then blocked by age categories.* The survey actually did not conduct stratified sampling because the sample was chosen by randomly telephoning and interviewing adults eighteen years and older chosen from a random sample of a Gallup household panel. Information about how the samples were chosen for years prior to 2007 was not explained and this student did not address obvious concerns about possible differences in sampling techniques that might have lead to erroneous conclusions. Student A was not sure about the difference between bias and lurking variables. This was clear from the statement: *The administration of the survey was excellent but there are a lot of lurking variables in the implementation of the survey. Such as bias, nonresponse, undercoverage, and wording of question.* This student knew many statistical terms but could not identify specific examples of each in a study. It was not clear if the student considered bias a separate concept from nonresponse, undercoverage, and wording of questions or if the latter three were examples of bias. It was clear, albeit incorrect, that the student considered lurking variable synonymous to bias, nonresponse, undercoverage, and wording of questions. Student A identified undercoverage as a problem in this poll. The student stressed that because most college students used cell phones and did not live at home they would therefore not be counted in the poll. This student did not understand that the poll was targeting only households and this would of course mean that the survey would not be interested in college students per se. Student A did not show growth in statistical reasoning nor written expression. Her score on her the assessment on this material was a 68. The sentence structure was poor to weak. The student was still functioning at level 1 or 2, between idiosyncratic reasoning to verbal reasoning. This student knew about some statistical concepts but could not select examples of these nor explain them correctly.

4.4.3.2 Student B

Student B chose a study on relationships between siblings to conduct a critical statistical analysis. This study was conducted by the SRBI Public Affairs (Schulman, Ronca, and Bucuvalas, Inc.) to describe relationships between siblings. Student B described the study as follows: *Subjects were given the choices as follows: very close, somewhat close, not very close, and not close at all. The information gathered was then used to compare how close siblings were growing up to how close they were at the time of the survey. After this comparison, they broke down the information by gender to compare responses of men and women*. This student understood the study and was able to convey that information to the reader. Student B was also able to identify the population, sampling frame, and sample and describe it clearly. Student B said: *The sample used in this survey was chosen from the population by conducting a national random sample by telephone. The population of interest in this study was adults of age eighteen or older within the United States. The sampling frame consists of individuals that could have* *contacted through telephone, and the sample was 1, 003 randomly selected subjects.* The sentence structure was to the point and clearly articulated. The student's next paragraph showed that she understood possible sources of bias in this study. A short excerpt demonstrates this understanding: *The actual administration of this survey could have produced inaccurate data through undercoverage and nonresponse. Because this survey was conducted by telephone, not everyone within the population could have been contacted. …Many potential subjects could have refused to answer based on lack of time or just the fact that they did not have interest in the poll in question.* The student described the shortcomings of the graphical displays and was able to describe briefly some conclusions based on the survey results. The following excerpts show the synthesis of the findings: *A table is a good way to display both sets of information, and is what I would have used to represent the responses by gender. If I wanted to use a more graphic form to display the first table, I would use pie charts.* Furthermore: *Based on the information, we can conclude that the majority of people were very close to their siblings, and most maintained that relationship into adulthood.*

Student B was able to demonstrate a complete understanding of this survey and was able to address shortcomings that could have affected the results. The student's writing was grammatically correct and her thoughts flowed logically from one point to the next. She understood the concepts of undercoverage and nonresponse and how they could introduce bias in the study and to explain those concepts in the context of the study. Student B's level of statistical reasoning was at the integrated process level, that is, able to understand fully the processes of a study and to explain them completely in her own words. Student B was able to maintain her level of statistical reasoning from the assessment on these concepts, which was 100, and demonstrate complete understanding of the critical aspects of a study.

4.4.3.3 Student W

Student W chose to discuss a series of surveys that were analyzed by Gallup concerning the opinion of the people of the United States about the Patriot Act and the limitations or possible elimination of civil liberties. The study never specifically discussed how the polls were completed yet the student said: *The surveys attached are based around the people of the United States (sic) opinions on the Patriot Act and the limitation, and in some cases elimination, of American's (sic) civil liberties. Therefore, we know that our population of interest in this situation is the willing, opinionated American citizens. Our sampling frame in this instance is "Civil Liberties" and the sampling that will be used for these surveys will be a voluntary response sample, as most surveys are. In this case a sample is not specifically chosen, as the surveys are free to be filled out by any passerby that may wish to voice their own opinion on that matter*. This student based his statements on his own opinion, not on facts given in the surveys. He did not understand what the population of interest, sampling frame, and sample were for this study of surveys. It later became apparent that the student did not understand the difference between statistical bias and the general use of the term bias. The following excerpt demonstrates this lack of understanding. The student said: *With the advent of political parties in the late 18th century, we've seen an enormous bias towards political parties and their decisions because they have garnered a loyal following from the people. It is quite possible that these surveys are littered with a party bias, and it is therefore not their own opinions the surveys are based on, but it is the opinions of their party*. This student knew some statistical terms related to bias and sampling but was unable to apply them to the study that he analyzed. Therefore, he was still at the verbal reasoning level of statistical reasoning. Based on his score of 64 on the assessment on this material he had not progressed much beyond a verbal reasoning level.

4.4.3.4 Student Y

Student Y chose to analyze a Gallup poll that surveyed a random sample of adults aged 18 years or older to determine how Americans viewed tipping after eating at a restaurant. The student was able to correctly identify the population, sampling frame, and sample. This was evident in his statement: *To begin with, the population of this survey would be the entire American population. However, since the results are based on telephone interviews with American adults over the age of 18, the sampling frame does not include the entire population. It only includes people that have telephones; when people are left out of the sampling frame and have no opportunity to be randomly selected, there is undercoverage…. The information I have about the survey doesn't specify how the phone numbers were obtained, but my guess would be a random phone-dialer.* Although the sentences were a bit awkward it was clear that the student knew how to separate the concepts of potential bias and clearly identified the pertinent aspects of the sampling process using appropriate statistical terminology. Student Y felt confident enough about his contextual knowledge to offer comments about the conclusions which were drawn and the possible reasons for the results. The poll had concluded that, based on the survey, seniors were more tolerant of poor service. Student Y offered these comments: *…I think there are few problems with these statements. For one, people aged 18 and older were surveyed; older adults have had a lot more chances to leave and* *provide accurate information than do teenagers. Also, if the poll comes to the conclusion that senior citizens are more resistant to poor service because their (sic) less likely to leave, is this saying that this generation is more likely to walk out without paying?* Student Y was able to successfully go beyond the conclusions in the survey and to offer suggestions as to other possible reasons for the results. This student was able to maintain the level of statistical reasoning that he demonstrated on the assessment for this material. His critical statistical analysis paper showed that he was confident enough to explain the results of the survey in his own words and was able to apply knowledge of statistical concepts as well as contextual knowledge to explain possible sources of bias and alternative reasons for the results. He demonstrated that he understood the processes and aspects of surveying and was able to show integrated process reasoning or a level 5 of statistical reasoning.

4.4.4 Summary of Findings in Both Projects and Critical Statistical Analysis

The Central Limit Theorem, Linear Regression project, and the critical statistical analysis were the three artifacts chosen to be analyzed for growth in written expression and statistical understanding. Although the topics were very different, there was an expectation that evidence of understanding of the statistical concept of linear regression would show more progression than that of the Central Limit Theorem and the critical analysis paper because it was studied in the latter part of the course.

Student A's Central Limit Theorem was completed with two other students and showed that Student A was between a level 2 and 3 of statistical reasoning for this topic. This student had a verbal understanding of some concepts, but could not apply this to the actual experiment being conducted. The student provided a correct definition but did not fully understand the concepts. The student was also able to correctly identify one or two dimensions of a statistical process without fully integrating these dimensions, such as, that a larger sample size leads to a smaller standard deviation in a sampling distribution and would therefore produce a more symmetric graph. For the second project Student A used a large font which was an indicator of the lack of substantial work. The student was able to capture statistical graphs and incorporate them into the main document quite easily. The student consistently used statements with no accompanying justifications. The student used statements that had been used in class discussions to provide evidence as to the merit or inappropriateness of particular models, but the supporting statistical reasoning and work was not present. Additionally, between projects this student failed to show growth in statistical reasoning after much feedback had been given in the first project. Student A maintained a level of reasoning between idiosyncratic and verbal when working alone. It was not surprising, given the lack of growth from the first to the second project, that although the student made an effort during the entire course to participate in class discussions and complete assignments she earned an 82 for the course and only earned a 1 on the AP Statistics examination.

Student B's Central Limit Theorem project showed that the student made no attempt to look at the graphs in order to accurately describe the graphs. The student had demonstrated some verbal reasoning by understanding some statistical terms but confused them on occasion. The regression line was graphed on the data and the coefficients of correlation and determination were calculated. These values were not mentioned by name but it was clear from the discussion that the meanings of both were clear to the student. The student completed the study by explaining and correctly using the concepts of extrapolation and interpolation to test the model. Finally a confidence interval for the slope and a test of hypotheses provided evidence of an association between year and annual individual income. The lack of correctly expressing what she saw in the graphs was evident in her first project. The student was unable to explain the nuances of the data using correct statistical justifications. In the second project written expression was clearer and coincided with graphical evidence. Additionally, between projects this student showed a measureable amount of growth in statistical reasoning after much feedback had been given in the first project. Student B started the course with a demonstrated level of statistical reasoning considered verbal but was able to learn from her mistakes and later maintained a level of integrated process reasoning, or level 5, through the end of the semester on the last project. The student completed the course with a 94 average and earned a 3 on the AP Statistics examination. From the interview with this student it was apparent to the student that it was necessary to learn from prior mistakes in previous assignments and projects in order to succeed in the course.

Student P completed the Central Limit Theorem project with clear lack of understanding of the concept being simulated. Even though she paraphrased the Central Limit Theorem from the textbook, “the larger the sample size, the closer the sampling distribution of the sample mean $\overline{X}$ is to the normal distribution with the actual mean and standard deviation” it was incorrectly done. Repeatedly she mentioned that samples of different sizes were drawn but never mentioned that the graphs were the means of these samples. When explaining the distribution of sample (means – she did not mention this) of size five she stated that the mean had a value of 0.4940 and mean of 0.4000 and standard deviation 0.3272 and stated: *The mean, median, and standard deviation of this* *sample are large enough to produce a more even spread, yet, too small to produce normal results. Although the spread is greater the data results are still skewed slightly to the right.* This was actually incorrect according to the graph produced. In another part of the paper the student stated: *The sample size will have to be increased a little more in order to decrease the standard deviation enough to produce a normal graph.* This statement was repeated and provided evidence of the student's belief that simply adding more data would take care of the value of the standard deviation which the student believed to be too large. There was no clear statement of how the student would decide how large a standard deviation she considered large enough. The student showed confusion when discussing the graphs of two of different size samples that had exactly the same spread and were both smaller than the original graph shown. The student insisted that the spread was wider. The student realized that her last graph with the largest sample size should be approximately normal but even though that was not what the graph showed it all of a sudden was described as becoming normal with a normal spread. The student knew some statistical words but was not able to understand how they were related. This student was still using idiosyncratic reasoning and the concept of how variability played a role in statistics was absent in this student's understanding of distributions. This student's project was unavailable; however the grade indicated that the prerequisite components of the rubric were addressed and present in the project. Although there was evidence of growth from the first project to the last project, the grade in the class was 82 and her grade on the AP examination was a 2. This grade indicates that the student was possibly qualified in understanding statistical concepts.

Student T admitted to being a good writer and showed that she could write well when she chose to do so. Her first project was well written although she believed that the purpose of this project was to prove that the Central Limit Theorem correct. This idea was prevalent in over half the papers that were analyzed. Her regression project was unavailable but an earlier quiz was analyzed and this student demonstrated that concepts not required on the project were well understood. Therefore, it demonstrated that the student knew content beyond the basic steps in regression analysis. Based on the data obtained about Student T one can surmise that she was prepared for her quiz but did not use that knowledge to earn a good grade on her project. The interview showed that this student had become disenchanted with the course and her progress in the course. The student admitted that she dreaded the class and had become obsessed about the wording necessary to receive full credit on her answers. It was difficult to reconcile that the student earned a 100 on the quiz on linear regression but yet earned a 77 on a project that was completed over a time period of a week. It was difficult to determine the level of statistical reasoning with two opposing pieces of data. However based on other data available about this student she was at the beginning stages, Level 1, of transitional reasoning. This student did not take the AP Statistics examination and earned a 73 in the course.

Student U’s Central Limit Theorem project was very well written and easy to follow. The student recapped the lab experience and what had been learned from the process. It was evident that this student understood the concepts behind the Central Limit Theorem. The explanations were easy to follow and it was clear great care was taken in understanding the concept of variability and how chance plays a role in inferential statistics. This student’s project was unavailable; however the grade indicated that the student completed the project successfully by addressing the various parts of the rubric. A prior assessment had been given just before starting work on the regression project to measure knowledge of the process used in linear regression. The steps required on the quiz were simpler than the project but the student completed a graph of the data correctly and the labels and units were appropriate for the vertical and horizontal axes. When asked to transform the data in order to obtain the exponential regression equation, the student was successful and was able to identify an outlier from the transformed data. The student was able to find the LSRL for the transformed data. The equation was given correctly in the context of the data. The grades on these projects showed consistent growth throughout the course and coincided with a grade of 93 in the course and a 3 on the AP Statistics examination.

Student V’s Central Limit Theorem project demonstrated some of the comments from the interview. This student did not like to read or write. He is a very capable mathematics student but the discussion in his first project did not flow well and did not explain the process with reasonable smoothness. The paper, for the most part just listed the steps that the students followed in producing the other graphs and statistics. The student did notice the change in the shape of the distribution from the first to the last graph; however, he failed to compare the values to the expected values for the mean and standard deviation and how this showed the concept behind the Central Limit Theorem. It was not clear from the written portion of the project whether the student understood the project and its implications in statistics. The analyses were very sparse in detail. In the Linear Regression project the procedure for determining the exponential coefficients was fully explained and accompanied by appropriate graphs and tables. The equation for the logarithmic data was explained in the context of the data as well as the coefficient of correlation and determination. He was able to justify the use of this model as a better choice by alluding to the values being higher than the previous linear model. The exponential form of the equation was explained and superimposed on the original data showing an almost perfect match to the original data. After using the model to test interpolation and extrapolation concepts, the student correctly surmised the limitations of the exponential model. This student learned the procedures studied in AP Statistics and was definitely using a high level of statistical reasoning in regression analysis. This student demonstrated that he understood that all the processes used to determine whether data can be best represented by one type of regression model over another is an integrated approach. He understood that although a model may not be a perfect model it served the purpose of extrapolating to values close in range to the data being studied. This student was able to use what he had learned during the semester and it influenced his writing and how it was necessary in order to earn a better grade. The course grade this student earned was a 92 and earned a 3 on the AP Statistics examination.

The concept of the Central Limit Theorem was not clear for Student W. The student had correct graphs and statistics but was not able to correctly interpret the information in the context of the Central Limit Theorem. In the Linear Regression project the student addressed the items asked for in the rubric but it was not clear that the student was able to carry out a regression study and use the tools and techniques studied in the course. There were some obvious steps beyond the expected steps that the student should have commented on or even have done to make this project a comprehensive one. The student was able to apply some procedural reasoning (Level 4) but he did not demonstrate a complete understanding of the statistical processes when using regression analysis. Additionally, between projects this student showed growth in statistical reasoning after much feedback had been given on the first project. Student W did not maintain a level of procedural reasoning between the critical statistical analysis paper and his final project. There was limited growth in statistical understanding. Even though the student was willing to write extensively the understanding was not clear nor was there evidence of increased growth in the second project. The student earned an 85 in the course and earned a 2 on the AP Statistics examination.

Student Y showed understanding of some concepts associated with the Central Limit Theorem. There was confusion about variability, standard deviation, normality, and what the mean and median were supposed to show about the distributions. There was some verbal understanding, level 2, about the process but he was not able to transition to the point of integrating those processes to correct statistical reasoning when discussing these topics and the processes involved. Part of this student’s problem (he admitted during the interview) can be attributed to not liking to write mathematics. For the second project this researcher found that although the paper was fairly complete according to the rubric, the student did not present a convincing argument as to his choice of a power model. The residual plot was not analyzed for unusual behaviors in the data. Although this student did understand regression as well as can be expected in a high school student he would be considered operating at a level 5, it was not clear from analyzing just this paper, that he would capture the nuances of the problems intrinsic to this data set. Student Y showed growth in addressing a problem and writing up the analyses and showed a grasp of statistical ideas but must show more growth in order to earn a 4 or 5 on the AP Statistics examination. Additionally, between projects this student showed considerable growth in statistical reasoning after much feedback had been given on the first project. Student Y maintained a level of reasoning between procedural and integrated process reasoning when working alone. This student earned the Statistics award and a 97 in the course. He was able to earn a 4 on the AP examination demonstrating that he was well qualified.

Student Z's interpretations were weak in the Central Limit Theorem project. It was not evident that the concepts were understood by Student Z. The student had correct graphs and statistics but was not able to correctly interpret the information in the context of the Central Limit Theorem. The student made considerable progress in the Linear Regression project. The student followed the basic guidelines of the project rubric and decided to go further and attempt to fit a power equation to his data. He was able to notice that the coefficient of correlation and coefficient of determination for the LSRL for the transformed data for this new model were not too different from the values for the exponential model. The residual plot still had a noticeable pattern not too different from the residual plot for the exponential model. This researcher had expected that the student would talk about the limitations of all three models and would have suggested that perhaps the best model had not been found. The student hinted at but was not very clear why he chose just a certain portion of the data. Overall, Student Z understood the procedures to be followed in doing regression analysis and was willing to continue trying to find a better model for the data given. This student also understood that there were other variables that influenced changes in certain variables being analyzed. This student was using a high level of statistical reasoning to complete this project on regression analysis. This student was able to go beyond the expected steps in the regression analysis project and attempted to explain the meaning behind the characteristics observed in the data. This student went beyond the requirements and was willing to add more to communicating and wanting to infer as to the causes of the data changing through time. This student earned an 82 in the course and earned a 3 on the AP examination showing that he is qualified.

Student bb worked with Student Y on the Central Limit Theorem project and showed understanding of some concepts associated with the Central Limit Theorem. There was some verbal understanding, level 2, about the process but he was not able to transition to the point of integrating those processes to correct statistical reasoning when discussing these topics and the processes involved. His first paper was weak but not as poor as his second project. Although it was unavailable for analysis the grade he earned, a 70, indicates poor writing and poor understanding of the regression concepts. His grade on the linear regression quiz was also very low. It was not unexpected that he earned a 71 in the course and earned a 1 on AP examination.

Generally, those students who showed some growth in their writing also showed growth in the understanding of statistical concepts. It was not evident that a good grade in the course translated to a good grade (3 or better) on the AP examination. However, a low grade in the course usually indicated a low grade on the AP examination.

After analyzing both projects for each of the ten students there was evidence of growth in statistical reasoning in several students. Two of the ten students stayed at about level 2 of statistical reasoning demonstrating verbal reasoning or understanding of some concepts, but could not apply this understanding successfully to the actual completion of statistical procedures requested by the exercise or project. The students performed the statistical procedures but did not demonstrate or explain why the procedure was chosen. It was not surprising that one of these students earned a 1 on the AP Statistics examination and the other student opted not to take the exam and earned a D^- in the course. Two unexpected results came from Students P and W. Both students showed considerable growth from about a level 2 to a level 4 in the projects analyzed. Student P was reluctant to join the discussions in class and Student W was always willing to add to the class discussions even though there were instances in which he clearly was unable to grasp the concept being discussed. However, both students did not earn college credit because they earned a 2 and 1 respectively on the AP Statistics examination. Student bb's statistical reasoning growth could not be analyzed because his linear regression project was missing (he earned a 94 on his project). All the other students whose projects were analyzed showed considerable care in completing their second project and showed that they had learned how to address the procedures involved in doing linear regression. The concepts of variability and standard deviation, which appeared in both projects were better understood and explained within the context of the problem in the linear regression project. The statistical language necessary to explain the justifications for their choices of model was also clearer and better written. However, none of these students earned a 5 on the AP Statistics examination and only one earned a 4. This showed some knowledge of statistics but not at the extremely well qualified level.

It was difficult to ascertain if time or the concept contributed to the growth observed in language mechanics or conceptual understanding. The concepts used in the

Central Limit Theorem that require students to use other concepts such as sample, population, distribution, variability, and sampling are abstract ideas that are difficult for students (Chance, B.L., delMas, D., & Garfield, J., 2004). On the other hand, linear regression is more in line with teaching how to obtain the equation of a line given sets of points and is part of the curriculum in Algebra I and Algebra II. It is also correct that in order to determine the linear regression equation it is quite easy with the graphing calculator which is used every day in the course. The appropriate justifications for choosing one model over the other by using the residual plot, coefficient of correlation, and type of data are fairly easy to memorize. The difficulty began to manifest itself when the student had to explain variation in the context of the problem, residuals, and how to cull the data in order to obtain a better model. This difficulty arose in many of the discussions in the class relative to linear regression.

4.4.5 Conclusions

The statistical analysis of the students' profiles suggested characteristics of a student that would be successful in the AP Statistics course as measured by their performance on the AP Statistics examination. The students' profiled suggest to the researcher that except for the end-of-course Biology and English I examinations (p-values, 0.021 and 0.036 respectively) other prior testing (end-of-course assessments) was not statistically significant therefore was not a factor related to success in the AP Statistics course. This statement of course needs to be considered in light of the fact that these students self-selected to enroll in the AP Statistics course due to their abilities in mathematics. Statistical analyses of the differences between students that performed well on the AP Statistics examination and those that did not with respect to their final average in the course, performance on quizzes and chapter tests showed that these (*p*-values of 0.006, 0.005, and 0.002) were indicators of a successful student on the AP Statistics examination.

In summarizing, in the lessons observed in the study, there was evidence of the use of planned discourse that was systematically used by the instructor and was characterized by elicitation, engagement, and challenging the students' thinking. Analysis of each lesson demonstrated that starting with the first lesson observed the teacher elicited the students' ideas about the concepts read in the previous night's assignment concerning coefficient of determination and correlation. They had already read about these topics and the students had written the definitions in their study guide for this unit. She showed that she valued that they can calculate these but also emphasized that the meaning was even more important in the study of linear regression. In order to reach an agreement on the correct meaning of these concepts she continued to ask students to add to the discussion, whether incorrect or just partially correct, in order to come to a consensus as to the final definitions of these concepts. The analysis of these question and answer sessions revealed aspects of discourse such as the types of questions proposed and the forms of student responses. When each student responded the teacher made a conscious decision to attend to what the student said and made inferences as to what the students knew and what they were thinking.

In order to engage the students and take a break from the whole-group discussion, where students may be hesitant to discuss in an open environment, the teacher placed the students in self-selected groups to focus on the concepts of linear regression and how residuals play an important role in determining the appropriateness of the model being studied. After much back-and-forth discussion between the students and the teacher, they were asked to point out the salient points about these two topics and the teacher organized these thoughts and recapped these for the students.

The planned discourse was evident in many other lessons as well. The teacher challenged the students to think about the data before starting any computations. It was crucial to the teacher and ultimately the students to read the data in the context of the problem. Appropriate tasks and activities were planned with the purpose of having the students to learn how to effectively articulate the solutions to statistical tasks by using statistical reasoning that went beyond idiosyncratic reasoning. In the first lesson observed, students had been able to use their graphing calculators to calculate the linear regression line and were able to indicate which value was the intercept and which value was the slope of the line. The teacher used the second lesson to introduce the computer printout with the information about a linear regression equation. The students were then asked to indicate and interpret the intercept and slope of the regression line. The difficult part for the students was interpreting each in the context of the data given. This stretched the students' knowledge from just verbalizing the values of each of these constants to interpreting what they meant in terms of the data itself. The teacher paid attention to the students' statistical thinking throughout the lessons as students verbalized their responses to questioning as well as in any written work completed by the students. For the third lesson the teacher planned still another step in the process of understanding linear regression by introducing previous AP examination questions that asked students questions about linear regression. Each question included a computer printout and a scatterplot of the residuals. Each question had various parts that would broaden the students’ familiarity with the type of thinking they will have to do to go beyond the basic computational questions. The first question asked the students to a) determine the regression line and define the variables used, b) determine the coefficient of correlation and provide an interpretation, and c) describe if the LSRL would be a good description for a smaller range of values. The second question asked the students to a) determine the regression line and define the variables used and b) Determine if the prediction for a certain value of the independent variable would be too large, too small, or a prediction cannot be made based on the information given. This represented yet one more level of difficulty because it asked the students to infer from the information given. In the third question the students are asked to a) use an appropriate display to compare the same variable for the two data sets, b) determine if there is a significant relationship between two of the variables in one of the data sets, and c) determine, if a new observation is given, whether it would belong to one data set or the other. These types of questions were consciously chosen by the teacher to increase the level of difficulty and to force the students to go beyond the transitional level of statistical thinking through procedural level of thinking and towards a deep integrated process reasoning in which the student has a complete understanding of linear regression.

Finally, evidence of change in how the students were able to express organized ideas, precise ideas, and to analyze and evaluate the statistical ideas of others was noted in the two major projects given during the semester. As was discussed earlier, it was difficult to ascertain if time or the concept contributed to the growth observed in the projects. The concepts used in the Central Limit Theorem are abstract ideas that are difficult for students. Linear regression is much easier to carry out with the graphing calculator which is used daily in the course. The appropriate justifications for choosing one model over the other are reasonably easy to memorize. The difficulty begins to be noticeable when the student has to clarify his or her choices in determining a better model - taking into consideration variation in the context of the problem, residuals, and how to select a subset of the data in order to obtain a better model. This difficulty arose in much of the dialogue relative to linear regression and was indicative of the type of difficulty that the students had with communication in general throughout the course. Prior to Lesson 4, the students had experience with computing the linear regression equation and interpreting the variables within the context of the data as well as interpreting the coefficient of correlation and determination. The students had just completed reading about regression inference and how to calculate and interpret a confidence interval about the slope. The students struggled to explain what they had read. They had difficulty making the connection between confidence intervals about the slope with the confidence intervals studied in the unit on hypothesis testing. After much back and forth discussion the students and teacher negotiated an agreement of what was to be the reason for computing a confidence interval about the slope and the reason for this type of confidence interval. The students then were able to correctly assume that the next step would be using a test of hypothesis on the true slope of the regression line. The conversation that followed about how to interpret the slope and how that related to evidence of a linear relationship forced the teacher to again summarize what they had discussed. The teacher summarized the discussion and said that they were trying to predict the slope with a CI. So first, they knew the slope that they obtained from the sample so now they were trying to predict a CI that would contain the true slope beta.

Second, they were trying to run a test on the data to see if there was a significant positive or negative linear relationship that drew away from the null hypothesis. That is, was there significant evidence that there was a linear relationship? The teacher emphasized that the students had to say it to learn how to say it correctly. She specifically called attention to a student and emphasized that the students should not keep from answering even if they were not entirely correct.

Near the end of the semester, in Lesson 16, the students worked on a matched-pairs question. Various student answers were discussed and assumptions were simply stated as being met. The teacher stated that simply saying that assumptions had been met was insufficient. These assumptions not only needed to be checked by using a stem-and-leaf plot or other appropriate measure but all conclusions also needed to be stated in the context of the problem just as she had been trying to "train" the students to employ.

Communication is essential in a Statistics course. Statistics education researchers such as Garfield (1995) believe that "learning statistics means learning to communicate using the statistical language, solving statistical problems, drawing conclusions, and supporting conclusions by explaining the reasoning behind them." The teacher in this study had applied this essential component of learning statistics in the course.

CHAPTER 5: REFLECTION ON THE STUDY

5.1 Introduction

The goal of the present study was to answer four research questions:

To what extent does a student's profile influence success in the AP Statistics course?

To what extent does discourse in the classroom enable students to communicate effectively in the AP Statistics course?

To what extent does good communication, such as expressing organized ideas, precise ideas, analyzing and evaluating the statistical ideas of others, change over time?

In what way is attention to students' statistical thinking evident in lesson planning?

In the last chapter this researcher afforded detailed answers to these research questions. In this chapter, this researcher will offer some observations concerning the students and course, critique the study and discuss its limitations as well as its significance for researchers and teachers.

5.2 Overall Impressions of the AP Statistics Course in this Study

Learning statistical concepts is not easy for students nor is it an easy task for teachers to teach these statistical concepts. Students have difficulties learning the language of statistics that is necessary for them to communicate their findings, and to do it with valid justifications. In this study a detailed picture of high school students' statistical thinking in an AP Statistics course was approached using the current research concerning the fact that many factors affect statistical thinking including dispositions (Wild and Pfannkuch, 1998, 1999), communication (NCTM 1989, p. 78; NCTM, 2000, p. 60), and discourse (NCTM 1991, pp. 35, 45). This served as a basis to conduct classroom observations and these observations served as a primary data collection gathering instrument for the study (Appendix D). This study was designed with the expectation of eliciting patterns of statistical reasoning observed in responses to questioning and statistical tasks. These factors are of special interest in classrooms where the development of conceptual understanding is emphasized.

It was neither feasible nor the focus of this current investigation to do a comparative study to research the effects of classroom teaching in an AP Statistics course in order to decide which factors contributed most to student achievement. There was only one AP Statistics course being taught and separating students in order to conduct this study would not be principled. Controlling the environment, perhaps a laboratory setting, in order to study student learning of specific statistical concepts separates the researcher from the classroom practice and the results may not be immediately applicable to the classroom (Chance & Garfield, 2001). This would also fail to include the instructor who has a vital and critical role in the classroom. Immersing this researcher into the classroom environment allowed for a more reflective study. Although it would not be possible to draw conclusions that would support cause and effect interpretations, it would allow the researcher the opportunity to draw some inferences from the different sources of information that were documented in the study. This researcher's role in the classroom was minimal in order minimize bias. This researcher's own experience as instructor, both at the college level and high school level, affected the view of the classroom that was observed. It is reasonable to expect that this researcher was affected by biases and it is also reasonable that the understanding and interpretation of what was experienced was colored by those biases and previous experiences.

The majority of the students in the statistics course in this study are considered above average when considering their mathematics ability. However, these students are likely to be below average, average, or perhaps above average in other school settings. All the students in this course were accepted to a university (junior level students are seniors this year and will be graduating this year and have been accepted by a university in their home state). As the semester progressed and issues concerning the students' classroom participation emerged, conversations with the teacher became focused on this disquieting situation and the conversations revolved around ways to change this situation. This situation also helped the teacher reflect more on the overall goals and to consider how those goals were to be accomplished. One way to do this was to introduce the AP Statistics questions sooner rather than later. The relationship between the teacher and the researcher influenced the study and the focus on dispositions became less and more focus was given to the dynamics of the classroom and the tools used to encourage the students to learn how to communicate effectively in the classroom and ultimately transfer that to the AP Statistics examination.

5.2.1 The Discourse and Communication in the Classroom

As has been found in earlier studies about teaching statistics (Garfield & Ben-Zvi, in press, Sherin, 2002), this teacher encountered challenges in establishing patterns of discourse to encourage effective student communication in the classroom. Students will be resistant to a non-traditional approach to teaching in which they are to engage in discussions about statistical topics and in which arguments presented are to be negotiated in an open environment. The teacher in this study used questioning to foster conjectures about data and later choose an appropriate form of data analysis. The teacher also required that the students explain their reasoning and justify their answers. Other students were asked to confirm their agreement or disagreement with the conjectures or reasoning being discussed. This was not always an easy role for the teacher because this was not a traditional teacher-lecture instruction. From the onset of the course the teacher had difficulty establishing the kind of discourse that would allow the students to communicate effectively using appropriate statistical language for descriptions of data, reasons for using one test over another in tests of hypothesis, or explaining the difference between concepts such as variability, variance, and standard deviation. It was only near the end of the course, after the students started reviewing for the examination, that the teacher was able to enjoy a very successful lesson. A successful lesson is one in which the students carry the discussion during the lesson and the teacher facilitates the discussion. The students should be able to support their answers with very little prompting by the teacher. Other students should question the statements made and add to the discussion by being actively engaged. The problem discussed during the first part of this lesson centered on experimental design. The students showed genuine interest in making comments that were not trite and contributed to answering the problem at hand. The topic was of interest to the students and the context was familiar to the students. This form of answering had perhaps not been practiced in previous mathematics courses and unfortunately may not be typical of most mathematics courses (Vermette, S., Gattuso, L., & Bourdeau, M., 2005). It was one of the few lessons in which the students were supporting their statements with contextual evidence and not giving one word answers. Because this was one of the last lessons before the end of the course, it is possible that the students' knowledge of the context did contribute to the success of the lesson. This type of open-ended problem is helpful to students in formulating answers and getting immediate feedback (Garfield, 1995) from their classmates. No one answer will be incorrect but the additional comments of the students will help the class come to a consensus as to the best answer to the question or questions.

5.2.2 Class Profile

It was found that student profiles were contributing factors in students' success in the statistics course. The findings in this study indicate that even very able mathematics students have difficulty with learning, using, and verbalizing statistical concepts. One aim of the present study was to find out how students' profiles influence their success in the AP Statistics course but more importantly success on the AP Statistics examination.

The size of this statistics class was above average, 30 students, which is considered large if much verbal feedback as well as written feedback is desired. The usual size recommended is anywhere from 15-20 and to not exceed 25 students. Obviously, if each student asks one question during the class period that would take about 60 minutes for the question to be asked and a response to be made. A smaller than average class is recommended in the AP Teachers Guide. Furthermore, this course was taught in the fall semester. This left approximately four months before the students took the AP Statistics examination. Even though review days, in preparation for the AP

Statistics examination, was offered by the researcher, only four students came the first time, two the next time, and then no other students came for one reason or another. It was very difficult for students to attend these review sessions because of commitments to the courses in which they were now enrolled in for the spring semester.

5.2.3 Classroom Discussions

Generally, the questions in class that the students asked had to do with minutiae, either how the calculator was used to complete a calculation or some small detail about the reading from the night before that could be considered a fact or a definition. It was almost as if no attention was given to the theory or concepts and a lot of time spent on how to do the arithmetic or procedures. From the start of the observations it was a constant struggle for the teacher to get the students to respond appropriately and satisfactorily to a question. Some of the interviewees revealed that they did not do the reading until the very day in which a discussion of the material was to take place. The students put their assignments hurriedly together in order to earn an effort grade and this was reflected in their answers to questioning in class. Once the students got into this pattern or habit it was difficult to overcome what they had missed during the semester and to shed old habits. It was difficult to record the lessons and not feel compelled to intervene to let them know that they were not behaving as if this was an AP course. This environment was much more nurturing than that which they will encounter at the college level if they had to take statistics in college. Patterns of response of diverse degrees were evident within each lesson and among lessons. The analysis of the observed lessons led to the identification of some patterns that permeated the lessons.

5.2.4 Planned Discourse Observed In Lessons

The planned discourse was evident in many of the transcribed lessons and is clearly seen in Lesson 7. The teacher continuously challenged the students to think about the data. It was deemed insufficient by the teacher for the students to commence the computations without regard to the contextual evidence. Appropriate tasks and activities were planned with the express purpose of having the students learn how to communicate effectively their solutions to statistical exercises by using statistical reasoning that went beyond level 1 or idiosyncratic reasoning. Ongoing attention was paid to the students' statistical thinking. Lesson 20 (one of the last lessons observed) was one of the most successful and telling of the growth in statistical thinking of the students. It was also a breakthrough for the teacher as well. The students thought that the problem concerning dietary supplements and its effectiveness in reducing the onset of canine osteoarthritis was very easy. The students were able to verbalize how they were going to use the control group to make the experiment more effective. They surmised that this would allow the researcher, in the study, to compare supplements given to the groups of dogs. It was important to consider good reasons for the random assignments to the various groups. This was not done perfectly by all the students, but it allowed the teacher to check that the students were using good statistical reasoning to coincide with their decisions. Appropriate statistical language was encouraged and modeled by the teacher. In Lesson 1, the teacher has been discussing the reading from the night before and she asks what they have learned from the reading.

Student L begins to explain what she has learned about LSRL

"It says it is a line." Student F adds that it is a method for finding a line that summarizes the relationship between two variables. Explanatory and response variables are defined and the teacher asks for more important details about the LSRL. Student C adds that it makes vertical distances as small as possible. Again, the teacher wants to know what the LSRL has to do with regression. After many back and forth questions and answers between the teacher and students, several important ideas emerge:

1) The idea is to try to make the sum of the squares between points and the line as small as possible

2) When finding the regression line, $\hat{y} = a + bx$ and $\begin{array}{l} a = \overline{y} - b\overline{x} \\ b = r\dfrac{s_y}{s_x} \end{array}$.

3) $\hat{y}$ is the predicted response for the x variable.

Another example of the teacher's care in monitoring and modeling appropriated statistical language can be seen in Lesson 3. After calculating the coefficient of determination, r^2=57.0% the students will have to find the square root and then determine that it is positive based on the LSRL. The coefficient of correlation r=0.75. Student X correctly says that it would be a moderately strong association and you can see it is positive. The teacher asks if the students can interpret r^2. Student E gives a partial response when she says that variation is explained (57%) by variables in the graph. The teacher continues and says that a more complete answer would be that 57% of operating costs are explained by change in number of passenger seats.

The teacher's expectations increased throughout the semester and more difficult statistical problems were used to engage the students and encourage them to focus on the details such as assumptions, conditions, and best procedures to apply in using the data to make statistical inferences with appropriate justifications and contextual references. Much importance was given to justification with an appropriate contextual position as the semester proceeded. The most difficult task throughout the entire semester for the teacher was facilitating discourse that would encourage and enable students to communicate effectively in the statistics course. There were occasions in which the students seemed to be advancing towards articulating their solutions to statistical problems successfully and then the students would regress and the process of reverting back to lower level questions and responses would commence over again. This can be seen in Lesson 3 when the students can complete the mechanics of determining the regression line but cannot interpret the residuals in the context of the problem even though they have done this in previous assignments and exercises. In Lesson 4, the students have difficulty explaining what a confidence interval is showing. This topic has been taught earlier in the semester and tested and the students have difficulty interpreting this concept. This phenomenon can be seen again in Lesson 8. The students worked through an exponential model quickly with little prodding from the teacher and seemed visibly comfortable with the procedures involved. As quickly as the students are presented with an example of a power model the students' reactions are similar to most other times when a new concept is introduced. It almost seems as if they have never discussed regression. The teacher again shows the frustration of asking the students once again to interpret the residuals in the context of the data. She comments, "What do we need to say to interpret this plot? This is a hurdle I can't get you guys over. It doesn't matter what I do, I can't get you over it. It is going to take you to do it." The students do not show that their previous mathematics experiences have helped them develop useful discourse skills for communicating mathematically. Close attention to what the students say reveals that the students are not careful with what they say. The teacher repeats what the student has said, “This data is not an appropriate model for this line – that is what you said first of all, I believe. Is that what he said at first?” Because these observations were based on the students that were called to answer or chose to answer, it was not representative of all the students. This phenomenon was evident in most lessons at the start of a particular unit of study.

5.2.5 Management of the Course

The instructor rearranged the order of the topics presented to the students because this order had been suggested in a workshop that the instructor had attended. The course started quite innocuously with topics dealing with producing data by designing samples, experiments, and simulating experiments. It continued with graphical and numerical descriptions of data but became quite theoretical rather quickly when the topics of probability, random variables, and probability models were introduced. Special distributions such as the binomial and geometric as well as sampling distributions were considered next. The next few topics all related to inference starting with the confidence interval and continuing with tests of hypothesis. The last few topics concentrated on studying linear regression as well as modeling non-linear relationships and inference for regression. The AP Statistics course should cover four broad topics which are (College Board, 2009)

1. Exploring Data: Describing patterns and departures from patterns
2. Sampling and Experimentation: Planning and conducting a study

3. Anticipating Patterns: Exploring random phenomena using probability and simulation

4. Statistical Inference: Estimating population parameters and testing hypotheses

The heart of both descriptive and inferential statistical analysis should be numerical and graphical representations. The data should be organized in such a manner that the investigator can ask questions about the data that will form the basis of answering a hypothesis about the source of the data, a population. This researcher would see that as the very first unit in a statistics course. Cobb and Moore (1997) explain this idea in the following quote.

> "Begin with exploratory data analysis. Experience says otherwise to data production first. It is more concrete. Students like exploratory analysis and the features of randomization and distinguishing between population and sample are not needed. Engaging them early in interpretation of results before the more difficult concepts come along help establish good habits. This prepares them for design and for inference. It also introduces them to the concept of variability and potential for bias. If you introduce them to design before data analysis it is harder for students to understand why design matters. It also gets them ready to tackle the idea of sampling distributions. " (p. 815)

Previously, Moore (1992) had urged instructors to start with data analysis.

> "…data analysis is an essential preliminary step. Students should learn to look at the data. It involves a collection of tools but also strategies for examining data intelligently. Starting with a graph move to numerical

> summaries and finally to perhaps a mathematical model. Gradually develop judgment in choosing tools and forming informal conclusions. Students then describe the data in words backed by their graphs and calculations. This first step in teaching statistics should be followed by the design of data production and formal probability-based inference." (p. 21)

This is supported by Scheaffer as evident in the following excerpt (1992)

> "Students should be taught how to intelligently explore data and to look carefully and critically at all data presented to them. An outline of questions to ask themselves helps the student confront each new data set. Starting with exploratory questions such as source, observed patterns, numerical summaries, trends, and association between variables the students get a feel for the data. Once these sets of questions are answered satisfactorily, a systematic look for pattern and departures from pattern may begin." (p. 70-71)

This should be followed by design experiments and inference. This exploratory approach is the way scientific investigation should begin.

This data should be of interest to the students in order to spark their interest. There is no reason why typical AP Statistics questions could not be used from the onset of this unit. These particular data sets could be used and reused as more advanced topics are introduced in the course.

For example, Question 1 from the 2005 examination showed two sets of data in stem and leaf graph. A series of questions asked the students that year to compare the two data sets, asked if generalizations were possible, and were asked to choose another plan from the description of two different plans to conduct a similar study to compare the caloric intake of adolescents living in rural areas of the United States to the caloric intake of students in an urban area. A random sample of ninth graders in two high schools was chosen.

This question draws on several concepts such as data analysis and sampling design but starts with a very low level computational step. This introduces students to the idea that statistics is not as much computational as it is drawing inferences from the data and that thinking about what has to be accomplished is as important as calculating some statistics for the data itself. The interviewed students thought this type of approach, that is, from the beginning of the course to use typical AP questions, was necessary in order to take data that interested them and use it to infer using appropriate justifications.

The mathematics of chance and variability, probability, should be studied next in a statistics course. The main focus in this unit is to give the students the mathematical words to communicate statistical concepts. This will allow the students a way of measuring variability and identify significant differences between groups.

Inference is the primary focus in a statistics course. By working with observational studies and experiments the students will be able to generalize beyond the sample to the population. At this stage in the course students will have learned how to conduct experiments and will know to properly use experimental design to control for any unforeseen variables such as lurking variables or even confounding. By learning how to produce data the students are now ready to go on to the next stage which is to study statistical inference. The previous work in the course will have laid the groundwork for this unit.

Confidence intervals and hypothesis testing was difficult for the students in the study. The lowest class averages on tests were in Chapters 5, 6, 8, 11, and 12 which cover Samples and Experiments, Foundations of Inference, Binomial and Geometric Distributions, Inference for Distributions, and Inference for Proportions. Neither linear regression and nonlinear relationships, nor inference for regression was tested via an assessment but was graded with the regression project.

The statistical analyses will mean nothing if the reporting is either lacking in substance, unclear, or confusing. The students need to be clear and accurate in writing on the AP examination. Therefore, the students need to explain their procedures, interpret their results, and be able to communicate. Statistics involves explanation, interpretation, and translation. Each student needs to be prepared to justify their choice of procedures used and interpret the results in the context of the data.

If switching the topics in the course did not allow the students to gradually learn the procedures to follow in conducting statistical analyses it could explain the results on the chapter tests as well as the AP examination.

5.3 Critique and Limitations of the Study

Measuring understanding of statistical concepts with a paper and pencil assessment is probably not a best choice. Measuring disposition, or as this researcher chooses to call it, determining the profile of a student, is difficult. Rationalizing whether one form of discourse is more appropriate than another is very personal for each teacher. What may work for one group of students may not work for another. If the teacher and the students do not share the same contextual knowledge of a wide range of topics then it becomes much more difficult to choose data that will be of interest to both parties.

Narrow or divergent views concerning where statistics can be used and whether it should be used at all can hamper the understanding of statistical concepts (Gal & Ginsburg, 1994).

Looking back at the study and the planning for the collection of data in the classroom, this researcher believes that the observations should have started much earlier in the semester – perhaps even the first day. There were variables beyond the control of the researcher and the study in accomplishing this. The development of statistical conceptual understanding could have been followed over a longer period of time and over diverse topics. An appropriate task for each unit of study could have been used to assess understanding of the statistical concepts studied in the unit. These tasks could be given to groups of 3 to 4 randomly chosen students and would have allowed the researcher to videotape their interactions as they discussed the approach they would take in completing the task. This type of task would have allowed the researcher to focus on the differences between the groups of students that would eventually be successful on the AP examination and the course.

A pre- and post-profile survey would have made the interviews unnecessary and would have used a sample of all the students in the course. In the face to face interviews the students may have answered the questions in a manner in which they thought this researcher wanted them to be answered. A structured interview and a survey of dispositional and attitudinal characteristics would have accomplished more with less bias. The topics studied during the semester would have been listed in the survey and there would not have been the necessity of recalling them for the students.

Although there were daily discussions between the teacher and researcher, most of these discussions centered on the students and their lack in volunteering in the discussions and their lack of adequate daily preparation. Additional time to focus on more productive ways to stimulate discussions among the students and to have the teacher's position be a true facilitator could potentially have changed the outcomes on the AP examination. The decision of the teacher to change the order of the units studied proved to be problematic. The decision of the teacher to change the sequence of the units was based on the opinion of another teacher expressed at a workshop. The sequencing of the topics was non-standard and very different from the sequencing suggested by the authors of the textbook and other researchers mentioned earlier (Cobb & Moore, 1997, Moore, 1992, Scheaffer, 1992). Based on this researcher's experience teaching AP Statistics this non-standard sequencing did not serve the students well.

5.4 Summary of Findings and Discussion

The goal of the present study was to answer four research questions.

The first question asked: To what extent does a student's profile influence success in the AP Statistics course?

The statistical analysis of the students' profiles suggests some characteristics of a student that would be successful in the AP Statistics course as measured by their performance on the AP Statistics examination. The students' profiles suggests to the researcher that except for the end-of-course Biology and English I examinations (p-values, 0.021 and 0.036 respectively) other prior testing (end-of-course assessments) was not statistically significant therefore was not a factor related to success in the AP Statistics course. Additionally, statistical analyses of the differences between students that performed well on the AP Statistics examination and those that did not with respect to their final average in the course, performance on quizzes and chapter tests showed that these (p-values of 0.006, 0.005, and 0.002) were indicators of a successful student on the AP Statistics examination.

The second question asked: To what extent does discourse in the classroom enable students to communicate effectively in the AP Statistics course?

Analysis of each lesson demonstrated that the teacher asked the students to discuss the ideas about the concepts read in the previous night's assignment. They had already read about these topics and the students had written the definitions in their study guide for each unit. The teacher valued their ability to calculate but also emphasizes that the meaning is even more important in each unit studied. In order to reach an agreement on the correct meaning of specific concepts, the teacher continued to ask students to add to the discussion, whether correct or not, in order to come to a consensus as to the final definitions of concepts. The analysis of these question and answer sessions reveals aspects of discourse such as the types of questions proposed and the forms of student responses. When each student responds the teacher makes a conscious decision to attend to what the student says and makes inferences as to what the students know and what they are thinking.

In order to engage the students in a less threatening environment, the teacher often places the students in self-selected groups to focus on the concepts being studied. After much back-and-forth discussion between the students, it is then important for the teacher to have the students point out the salient points about the topics and she then organizes these thoughts by recapping these for the students.

> The third question asked: To what extent does good communication, such as expressing organized ideas, precise ideas, analyzing and evaluating the statistical ideas of others, change over time?

Evidence of change in how the students were able to express organized ideas, precise ideas, and to analyze and evaluate the statistical ideas of others was noted in the two major projects given during the semester as well as in many of the discussions in the lessons observed. Although there were observed differences in the ability of the students to communicate effectively, there was improvement in many students. The concepts used in the Central Limit Theorem are abstract ideas that are difficult for students. Linear regression is much easier because appropriate justifications for choosing one model over the other are reasonably easy to memorize. The difficulty was noticeable when the student had to clarify his or her choices in determining a better model - taking into consideration variation in the context of the problem, residuals, and how to select a subset of the data in order to obtain a better model. This difficulty arose in much of the dialogue relative to linear regression and was indicative of the type of difficulty and uneven growth that the students had with communication in general throughout the course.

> The fourth and last question asked: In what way is attention to students' statistical thinking evident in lesson planning?

Attention to students' statistical thinking was evident in lesson planning and more concretely in the lessons observed. The teacher challenged the students to think about the data before starting any statistical analyses. It was crucial to the teacher and ultimately the students to read the data in the context of the problem. Appropriate tasks and activities were planned with the purpose of having the students to learn how to effectively articulate the solutions to statistical tasks by using statistical reasoning that went beyond idiosyncratic reasoning. Each lesson was sequenced to follow logically from the previous lesson with appropriate consideration to the difficulties the students were having with a particular topic or concept. A good example of this strategy can be found in the first lesson observed. The students had been able to use their graphing calculators to calculate the linear regression line and were able to indicate which value was the intercept and which value was the slope of the line. The teacher used the second lesson to introduce the computer printout with the information about a linear regression equation. The students were then asked to indicate and interpret the intercept and slope of the regression line. This broadened the students' knowledge from just verbalizing the values of each of these constants to interpreting what they meant in terms of the data itself. The teacher paid attention to the students' statistical thinking throughout the lessons as students verbalized their responses to questioning as well as in any written work completed by the students. For the third lesson the teacher planned still another step in the process of understanding linear regression by introducing previous AP examination questions that asked students questions about linear regression. Each question includes a computer printout and a scatterplot of the residuals. Each question has various parts that will broaden the students' familiarity with the type of thinking they will have to do to go beyond the basic computational questions. These types of questions are intentionally chosen by the teacher to increase the level of difficulty and to force the students to go beyond the transitional level of statistical thinking through procedural level of thinking and towards a deep integrated process reasoning in which the student has a complete understanding of linear regression.

5.5 General Discussion and Challenges for other Studies

The challenge to understand how students learn statistics is still in its infancy even though much has been published in this area. Each environment whether it be the high school or the university is a special situation that influences the learning processes in the classroom. As more students enroll in AP courses like statistics it becomes imperative to learn why some programs are successful and others are not. Just as calculus was a new frontier in high school more than 50 years ago, statistics is the new challenge for high schools and should be a popular choice for research studies. The research studies on learning mathematics, and in particular calculus concepts in the university setting, have been done and research on the learning of statistics at the university setting are plentiful. The AP Statistics examination is relatively new, administered since 1997, and more students enroll in the high school course in order to earn credit for college and make their transcripts more appealing in college applications. The teachers of these students need to have available to them more research studies that will help them plan a course that will allow their students be more successful. The dynamics in a classroom force certain conditions on teaching these former college level courses in a high school environment. This study explicates some of those issues and provided insight into the dynamics that are related to the development of students' statistical thinking.

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APPENDIX A: REGRESSION PROJECT RUBRIC

SCORING RUBIC (Linear)

Chosen data is appropriate ______

Introduction is complete ______

Scatterplot of original data ______

Conjecture is stated and explained ______

LSRL is stated in context ______

Comments on r and r^2 ______

Residuals given in table ______

Residual plot shown _____

Comment on residual plot ______

Interpolation and Extrapolation ______

T-procedures conditions checked ______

CI calculated and interpreted correctly ___

LinReg T-Test completed all parts:
Hypotheses: ______
Statistic and p-value: ______
Conclusion: ______

Data is attached ______

Paper is written as a narrative ______

Paper flows smoothly between topics ____

Correct statistical vocabulary is used _____

Spelling and Grammar are correct ______

SCORING RUBRIC (Non-linear)

Chosen data is appropriate ______

Introduction is complete ______

Scatterplot of original data ______

Conjecture is stated and explained ______

LSRL is stated in context ______

Comments on r and r^2 ______

Residuals given in table ______

Residual plot shown _____

Comment on residual plot ______

Transformed data is shown ______

Scatterplot is given for transformed data __

New residual plot is shown ______

Comment on the residual plot ______

Inverse transformation correct ______

Interpolation and extrapolation ______

Data is attached ______

Paper is written as a narrative ______

Paper flows smoothly between topics ____

Correct statistical vocabulary is used _____

Spelling and Grammar are correct ______

APPENDIX B: CENTRAL LIMIT PROJECT RUBRIC

Special Problem Scoring Rubric

<u>Poster (40 points)</u>

Title is appropriate (5) ______
Contains all 6 graphs (10) ______
Graphs are labeled appropriately (5) ______
Descriptive statistics are included (5) ______
Graphs are sequential (5) ______
Poster is neat and attractive (10) ______

<u>Paper (60 points)</u>

Introduction (10) ______
Correct analyses of graphs (20) ______
Grammar and spelling (10) ______
Appropriate and complete summary (10) ______
Report clear and unambiguous –
student understands purpose
of the Special Problem (10) ______

Total Points______

APPENDIX C: STUDENT INTERVIEWS

1.–Student V–Junior–Above Average Math Student–Not one to join in discussions voluntarily. He is planning to take AP Calculus next year. He is of Eastern Indian ethnicity.

g. What is your name?–xxxxxxxx

h. What grade are you in?–11th.

i. What was your initial feeling about taking AP Statistics when you first decided to enroll in the course?–I thought it would be easy

j. Have your feelings about AP Statistics changed since then?–A little.

k. How so?–I thought it got a little bit harder. It is not as easy as I thought.

l. What is it that you found harder about it?–Like the tests, that’s basically it.

m. Was it that you felt that you were not prepared when you took the test or the test was different from what you thought it would be?–I probably wasn’t prepared.

n. Do you feel that writing in statistics is different from writing in your English class?–Definitely.

o. How so?–Ah, (he smiles), like I don’t know. You apply math in statistics and in English it’s basically what you think or whatever.

p. So, it’s more specific you think?–Yeah.

q. Has your writing changed in this class in completing your assignments? In other words from when you first started in Statistics to now, has your writing changed.–Yeah.

r. How so?–Like, I never write out my answers in math like in complete sentences or anything and I do now.

s. How about answering questions on a test?–Yeah.

t. How about your homework, more complete than they used to be?–Yeah. More right now.

u. How about on projects?–Yeah.

v. Why did you decide that your writing needed to change?–Because Statistics is different than other math classes which is just like numbers and in statistics you have to explain it.

w. I've observed you, a lot, at least these past few weeks, and I notice that you only answer when called upon even though I think you are a very bright person, can you tell me why?–I'm lazy.

x. So you don't think that you prepared well for class each day?–No.

y. Why?–(He smiles and laughs) I don't know, I don't give it that extra effort, I mean I guess I could but just I just answer when I'm called?

z. What was the most helpful type of oral communication used in the class? For example, when you did the whole class discussion, or you did the homework topics that you went into small groups to discuss it, or which was better the whole group or the small group?–I think the pairs or the small groups were better. In that way everybody could ask what they need to and like the whole class some people could be left out.

aa. Okay, so the pairs method you said was better?–Yeah.

bb. Why did you enroll in the course?–My sister told me to do it; it would be good for college.

cc. Do you still have the same commitment to the course as at the beginning?–Yeah.

dd. What expectations do you have for this course?–I want a 4 or 5 on the exam.

ee. Do you think that you're prepared enough so that your expectations will be met?–Not really I think that I can review the stuff that we did at the beginning.

ff. Do you intend to come to the review sessions that I'm going to offer?–Yes.

gg. What could have been different about this course to make it more worthwhile and make you feel better prepared?–I guess review more.

hh. Like before tests?–Yeah. Like before tests, instead of having review time at the end. Like as we went through more review before every test.

ii. So, I guess I want to just clarify what you're saying. You're thinking that if they had gone over the different types of questions for that particular unit (Student V says yeah as we were doing the unit) in addition to getting ready for the test at the end of each chapter.–Yeah.

jj. I see, okay. So, you felt like you would have benefitted by adding a particular area at the end of each chapter in addition to the review for the test on that chapter.–Yeah. Is that more AP type questions?–Yeah.

    Okay, I see, I understand.

kk. Which materials used in class were of most benefit to you? I know that you do study guides and you got together in groups. Which of the materials that you used were of most benefit–Study guides were.

ll. They were very good?–Yeah, they were.

mm. Did the projects help you any?–Not really.

nn. Why?–I don’t know, the one we did about the Central Limit Theorem, I didn’t really understand it from there, I like understood it better like when we read it from the book and talked about it in class and did the problems.

oo. So, you didn’t get it?–No, I didn’t get the point of the projects really.

pp. Did the textbook help any?–Yeah, it did.

qq. What other resources did you use besides the textbook?–The textbook and that study guide that is basically it.

rr. How much time do you spend on homework, just for this class, on average?–Less than an hour.

ss. How important is this course for your future coursework in college?–It’s pretty important.

tt. What are you going to major in?–I think I want to go into business. I think.

uu. So, someone told you that you needed statistics?–Yes.

vv. Do you think then that you will have to take the course again in college or you expect to get credit?–Oh yeah, I hope I don’t have to take it again in college.

ww. So, you want to not have to take it in college?–Yeah.

xx. So, you are expecting a 3, 4, or 5 on it?–Yeah. But, I think it’s going to be hard, because the exam is all the way in May. I hope that I don’t forget everything by then.

Well that’s what the review sessions are hopefully going to be about.

yy. Did you like working on projects with the other students?–Yeah.

zz. Why?–It's better than working alone.

aaa. It's less work you think?–Yeah.

bbb. What benefits do you see in working with other students on the projects?–Their point of view I guess, you can learn more about what they think.

ccc. Which topics studied so far this semester interested you the most? I guess you have to recall the topics, I guess. There was data collection and studying about graphing. The Normal Distribution, Central Limit Theorem, you did regression, you did inference at the end.–It was the one with sampling that we did. Oh, sampling distributions.–Yeah.

ddd. What interested you the most?–I didn't know that it was that hard to sample people, you have to go through that table, and everything.

eee. Oh, like for the surveys, and all that kind of stuff?–Yes.

You know that that's a very difficult area and people make a lot of money doing that.

fff. Okay, so you like that the best. And so, what did you like the least?–Hmm, the least I guess that would be the z-scores and all those tests. Like the hypothesis testing?–Yeah.

ggg. You didn't like that?–No, I always got confused, because there were so many.

hhh. Which topic studied so far this semester was the most difficult?–The one with all the z-scores and doing all the tests.

iii. Which was the easiest?–Probably the regression stuff we did at the end.

jjj. The transformation one or just the plain old linear regression?–The plain linear regression.

kkk. How much time to you spend completing the reading for this course?–Very little.

lll. How much time to you spend completing the reading during class time?–How much time she gives us, I guess.

mmm. How much extra-curricular reading do you do, in other words, reading that has nothing to do with statistics?–None.

None? Okay.

nnn. What types of materials do you read, when you do read?–Sports magazines. [He was trying to come up with an answer I do not think he likes to read at all.] I don’t read.

ooo. So, you don’t consider yourself a well-read person?–No, definitely not.

ppp. Do you, when you do read you just read sports?–Yeah.

qqq. Any other genre that you read?–No.

rrr. So, if they give you something to read in class, that’s all you read?–Yes.

sss. When you were doing problems in statistics, did you go to any other source to learn how to do the problems or you just stuck to the textbook?–Like other people.

ttt. Other people?–Yes.

uuu. How about other books?–No.

vvv. How about the internet?–No.

www. When you were working on the projects how did you decide how you would even start?–Actually, the Teacher, kind of started us off. So, that helped a lot. That would have been coolest.

xxx. When you were working on tests, on your tests in class, how did you go about even starting on the test?–I like don't know, I guess it depends.

yyy. Did you think about anything as you were trying to decide what to do for each problem?–I thought about what problem it would be like. What you have to do for it.

zzz. I know this is kind of difficult, can you think about the kinds of questions you ask yourself when you are trying to do a problem on the statistics test? Suppose you have to decide between a 2–proportion z-test or a 2-sample t-test versus a difference of 2-samples test how do you do it?–I forgot all that but I am sure you have to see if there is a mean and standard deviation or some things like that.

aaaa. So you are looking for clues?–Yeah.

bbbb. Do you just jump into the questions, or do you read it carefully and try to analyze it first?–I just jump into it.

cccc. If you had to do it all over again, knowing what you know now, would you have taken AP Statistics?–Yeah.

dddd. Why?–I don't think it was that hard, I think if I read everything and done all my homework, it would have been easy.

eeee. So, your way of studying Statistics would have changed?–Yeah.

ffff. Do you believe that you will do well on the AP Statistics test?–Yeah. (Very Confident)

gggg. Why?–The review sessions, hopefully will help (*he did not come to any*) and I am going to study and go over the stuff we did in the past.

hhhh. And, you are a pretty smart fellow, huh?–He laughed.

iiii. So, you plan to come to the review sessions?–Yeah.

jjjj. And, that's because?–The extra help.

kkkk. So, you've got the review book already?–No, I haven't got it. I think it's at home.

llll. So, you do need to get it. Because we are going to be looking at different topics and over AP questions.

2.–Student U–Above Average Student–Fair good writer–Senior

h. What is your name? – xxxxxxxx

i. What grade are you in? – 12th.

j. What was your initial feeling about taking AP Statistics when you first decided to enroll in the course? – Pretty much it came down to Calculus or Statistics I thought I would be better off in Statistics just the content from what I had heard. I thought Calculus would be a bit over my head. I don't have much confidence in myself.

I had confidence in you and I was wondering why you didn't take it.

k. So, you thought AP Statistics would be easier, is that what you thought? – Easier but in a way that I had heard that it was more with words and with ideas that I could understand better.

l. I know that you like that kind of thing. – I’m much better at it.

m. Have your feelings about AP Statistics changed since then? Do you think you made the right decision? – Well, at some point during the semester I started thinking maybe I would have been better off with numbers because I wasn’t that good at the words part of it but it’s half and half because in a way I like understanding it I like just the content. It is hard. It is harder than I thought it would be, for sure.

n. So, you’re feelings might have changed but you came around to saying that this was the right decision? – I think so, yes.

o. Do you find that writing in statistics is different from writing in your English class? – Well, it is very different. In a way it is very similar. I thought too when I took my government class here. Because you have to write in a way that gets straight to the point. You don’t do the whole flowery like trying to embellish your sentences. It was like that in history where you just have to state your facts.

p. So, you said that the writing is somewhat different but at the same time because of your government class, you learned not to embellish but try to support your answers? – Yes. I’ve been working very hard in trying to get my point across in a brief manner. I think that’s what it is – concise – yes, concise, that’s it.

    q. So, your writing has changed in completing your assignments? - I think so well maybe it hasn’t changed but I have just learned to write in a different manner.

r. How about answering on tests? – Pretty much so.

s. How about on projects? – Well, it is the same kind of writing, but putting a lot more thought into it when I am working on projects because I have more time and time helps you a lot.

t. I've have observed that you answer quite a bit. Sometimes they have to ask you to answer but I think for the most part you do answer. Can you describe how you prepare for class each day? – I try, I do my homework every night and I guess I study like when I need to which actually is a lot but that usually determines how much I answer. But, a lot of times from what I've noticed lately that when I do answer it will be wrong so I don't answer after that.

u. What was the best helpful type of oral communication that you used in the class? For example do you like the whole class discussion of the homework topics, or do you like the small group discussion of the homework problems? – I think I found that I do best in the small groups. It depends on the groups though.

v. I noticed that the Teacher did either pairs or she let you get into groups of about two or three, which one do you prefer, the pairs or small groups? – I prefer the pairs. I have one person in the class and that was Student C and we work really well together and that is usually what I go to. What I know she doesn't know and the opposite.

w. Can you tell me why you enrolled in this course, besides the fact that you thought it would be easier? –Well I had to, to graduate. I guess in my mind there was no alternative.

x. Do you think that you still have the same commitment to the course as you did at the beginning? – Well I thought it was a lot easier at the beginning, I don't know about commitment but I've got to work a lot harder. It has gotten more frustrating I think 'cause not that I'm not used to it when you study for things but when you don't understand something it gets hard to stay focused to keep working so hard. I don't think I've lapsed in like commitment but it has kind of fluctuated a little bit.

y. What expectations did you have for this course? – I think, it's probably about what it is now I knew that I would have homework and but I knew it wouldn't be over the top and I knew some things would be difficult. It probably is a little more difficult than I anticipated.

z. What could have been different about the course to make it more worthwhile? – Well, I think what I like a lot of times is very structured courses where you come in and you take notes and you have – when everything is a little, I don't want to say cohesive, but where I see better in my mind what we've been learning if we have something tangible every day. Unless you flip back at it, the way it is now everything, like we've learned everything but it is like scattered all over the place and as far as reviewing it was easier if it were more (she shows a chopping motion).

aa. The previous interviewee said something, that I thought was kind of interesting, he said if that there had been sort of AP type questions at the end of each chapter in addition to the review questions that you normally do to take a test it would be better. Do you feel the same way? – I think that definitely would have helped. That and maybe cumulative as we went along. As much as whenever anyone says cumulative everybody says no but everybody has forgotten. Because now we're bringing stuff back that I do remember.

bb. What materials used in class were of most benefit to you? I know that you used the study guides, I noticed you used some sample questions near the end of the semester, and I also noticed that you used the textbook, projects. Which of the materials that you used were of most benefit – I do like the sample questions that we are doing. That's helping a lot. The textbook obviously. I use that every day. The teacher too, helps. Having a reference who is always there to answer questions.

cc. I notice from your other questions that this question will be easier to answer, how much time did you spend on your homework, just for this class? – It varies a lot, but it is usually between an hour and a half to two hours.

dd. How important is this course for your future coursework in college? – That I don't know so much. Because I don't know much about what I'm going to, well, I think that's another reason I just thought about when I 'm going into college it's not going to be in a math or science related and I know that calculus is very much for that and so I thought statistics would be good because for a little while I thought I would be a lawyer so I thought that would be a good choice too but right now I'm back to I have no idea it will probably help but I don't know.

ee. Do you like working on projects with other students? – Not really.

ff. Can you tell me why? – There are some students that you can work really well with, but not just in our class, but sometimes you get stuck with the people who do nothing. I'm one of those people who is very hands-on and very perfectionist and it usually doesn't work well.

gg. Do you see any benefits for working with other students on projects? – Oh, yeah it helps a lot if it's more like ideas and collaboration but when it comes down to the product that's when it gets a little hard it helps to bounce your ideas off each other when everybody helps each other.

hh. Which topics studied so far this semester interested you the most? – I think the one which I grasped the most that I can think of now was planning, like planning surveys, and collecting data and stuff. Just that I wouldn't say that it was easy but it was straight forward and you understood it and I thought that was interesting.

ii. Which topic studied so far this semester interested you the least? –I don't know. I am sure there were a few but off the top of my head?

jj. You did analyzing data, you did hypothesis testing. – That's what I thought of at first, there is a lot of work but they are kind of interesting.

kk. Which topic studied so far this semester was the most difficult? –It was probably the significance testing.

ll. Which do you think was the easiest topic? – Probably what I said before, the data collection through surveys and also planning experiments.

mm. How much time to you spend completing the readings for this course outside of class? – Whenever they are assigned it usually takes me about an hour.

nn. How much time to you spend completing the readings during class or school time? – As much as I can but I find it a lot harder to focus in class so usually if I can't do it in school I do it at home.

oo. How much extra-curricular reading do you do, has nothing to do with your classes? – If I have a lot of homework then maybe an hour but on any given day a couple of hours.

pp. What types of materials do you read, when you do read? –Novels and literature. On the internet if that counts.

qq. Do consider yourself a well-read person? – Yeah, I think so. Compared to a lot of people I know. It's just sad.

rr. Do you read a variety of genres? –I used to it's a little more down now. I have to do lot work for English so I try to stay within that type of reading.

ss. When you were working on projects, how you would proceed? – I broke it down every step in the planning every day what I was going to do.

tt. When you were working on tests, on your tests in class, how did you go about even starting on the test? –I would read it and tried to remember exactly what we learned about it I have a very photographic memory and I would try to remember the problems we've had about it and the pictures and what it would be like.

uu. Was there a series of questions that you would ask yourself, did you try to analyze how to proceed the problem? – Do I know what I am talking about, probably not.

vv. Did you just jump into questions or did you try to decide how to start the question first? – I think I try to think about it, I think it varies but if I don't know where I am going with something I usually don't do well so I try to focus myself.

ww. If you had to do it all over again, knowing what you know now, would you have taken AP Statistics? –That's really hard. I don't know, thinking maybe calculus would have been different and no other alternative, I don't know which would have been worse. I probably would have stayed in it.

xx. Do you believe that you will do well on the AP Statistics test? – Probably not as of right now, but after studying hopefully yes.

yy. Why not at this point? – I don't think that I have reviewed enough. As I was saying earlier, it is still not all clear in my mind. Where everything kind of fits in. But, I probably will go back and get all my formulas and organize them.

zz. Do you think that you are going to participate in the review sessions? – I think so.

aaa. Is there anything that would keep you from doing it? – Well, I have a psychology class twice a week, Tuesdays and Thursdays. I wouldn't be able to come right away.

bbb. Do you have the review book that the Teacher said to get? – Yes.

That will be one of the resources that we will be using.

3.–Student P –Junior - Not Engaged in the Course – Loves to work on languages and history on her own, especially the Russian language.

a. What is your name? – xxxxxxxx

b. What grade are you in? – 11th.

c. What was your initial feeling about taking AP Statistics when you first decided to enroll in the course? – I didn't really know what to expect. I thought we would do a lot of probability. Other than that I didn't really know a lot about statistics.

d. Did you feel apprehensive at all about it? – No.

e. Just like any other math course? – Yes.

f. Have your feelings about AP Statistics changed since then? You thought it was a lot about probability and just like any other math course. Have those feelings changed? -Definitely.

g. Can you explain? – It's not really mathish, it's a lot of writing. I didn't think that it would be that involved.

h. Do you find that writing in statistics is different from writing in your English class? –Yes.

i. How so? – Just having to incorporate the numbers and the different formulas and stuff is a lot more complicated. English just kind of flows.

j. So do you normally get good grades in English? – Generally.

k. What kind of a math student do you consider yourself? – Fairly good.

l. Has your writing changed in this class in completing the assignments? – I guess so.

m. Do you think you've gotten better as the semester progressed? – Yeah.

n. Or just a little bit better? – Well I guess as you write your writing progresses.

o. How about in answering questions on a test has your writing changed? – Yes.

p. How so? – A lot more detail. Now that I know what to write and what to think about.

q. How about in completing your projects? Was there a lot of writing that went into that? – Yeah. Like the papers that we've written. Those really haven't changed that much.

r. I observed in the class and I notice that you rarely answer of your own free will. In other words, it is usually only when called upon. Can you describe how you prepare for class each day? – I try to do the homework. I do the reading. That's it.

s. How do you feel about answering questions? – I don't like it. I always second guess myself.

t. Is it just in this class or in general? – In general.

u. What was the most helpful type of oral communication used in the class? For example, I saw whole class discussion where everyone is sitting and just discussing things and answering questions, or you look at homework topics in individual groups, either pairs or small groups is what I saw. What was the most helpful of those two types? – I liked the small groups.

v. There were two types, the pairs where the teacher paired you up or there was the one that you chose your own group. Which? – I liked choosing. Like the little groups that you're in.

w. Why did that help more? – I don't know. It was more relaxed, I think.

x. So, maybe the stress of answering a question in front of everybody? – She laughs.

y. Tell me can you remember why you enrolled in the course? – I guess I just needed a class.

z. You had taken Pre-Cal hadn't you? – No. I have it next semester.

aa. Okay, so you just wanted a math course? – Yes.

bb. And you decided to take that one? – Yes.

cc. Were you trying to do Pre-Calculus first or you thought you needed to take Statistics first? – I really didn't think about it. That's just the way it played out.

And statistics is only offered first semester so that's why it worked out that way.

dd. Do you have the same commitment to the course as you did at the beginning? –I guess so. Yeah.

ee. What expectations did you have for this course? I think you have kind of already answered this question. – You didn't have any expectations except maybe it was about probability. – Yes.

ff. Any other expectations that you had? – I really didn't know what to expect.

gg. You really had not talked to anyone about the course? – No.

hh. You said that you expected a lot of probability did the course meet your expectations? – Not really.

ii. It wasn’t as much as what you thought it would be about? – It’s a whole lot more involved.

jj. What could have been different about the course to make it more worthwhile? What could the teacher have done differently and what materials could have been given out that were better for you to make the course better–I don’t know.

Maybe it might come to you later, I might ask you that question again.

kk. Which materials used in class were of most benefit to you? I remember seeing study guides, sample questions from the AP test, of course you have the textbook, you worked on projects, you took quizzes, what materials were of most benefit to you? – I liked the AP questions.

ll. So this is what you are doing now, right? – Yes.

mm. Why? – I guess it gives me something concrete to base what I should know.

nn. It’s telling you kind of what you’re weak in or what you’re kind of stronger in? – Yes.

oo. How much time did you spend on your homework, just for this class and that’s an average? – Not too long.

pp. Would you say half an hour? – Yes, between half an hour and an hour.

qq. So on an average day, that is how much homework you would expect? – Yes.

rr. How important is this course for your future coursework in college? –I never considered being a math major.

ss. Now you don’t necessarily have to be a math major to take statistics. What kind of career are you leaning towards? – I’ve no idea.

tt. What do you like best? – Languages.

What they're usually going to ask you in college is usually to take two semesters of a math courses. Whatever that math course is, it could be this one if this was one of those courses that you get a 3 or better.

uu. How did you like working on the projects with other students? – It was really hard like the actual working together was fine but finding time to get together our schedules created a lot of conflict.

vv. What benefits did you see in working with other students on projects? –You can bounce ideas off each other and it just kind of made the process easier.

ww. In general, do you like working on projects with other people? – Yeah.

xx. Some people don't, they don't like it. – Yeah, I know.

yy. What topic, and this will be a little difficult for you to remember, studied so far this semester interested you the most? Maybe you can kind of go over the AP questions that you are going over now and which are the most interesting to you? –I don't know.

zz. How about let's say regression versus surveying? Or data exploration where you did the graphing at the beginning versus hypothesis testing? – I like stuff dealing with the normal distribution.

aaa. Does that seem easy to you? – I understand it.

bbb. Tell me if you can remember the material that you disliked the most, that you liked least? –Anything with a lot of formulas.

ccc. Well that pretty much covers hypothesis testing? Doesn't it? - Yes.

ddd. Which topic studied so far this semester was the most difficult for you? – Hypothesis Testing.

eee. Which do you think was the easiest topic? – Either normal distribution or linear regression.

fff. How about the part where you did the transformations of the data, was that easy too? – Yes.

ggg. How much time to you spend completing the readings for this course outside of class and school? – I don't know, it depends. I'm a slow reader.

hhh. It takes you a long time to read through these passages? – Yes.

iii. So would you say that on a normal day when she gives you readings to do outside of class it would take you more than an hour? – No, never more than an hour.

jjj. How much time do you spend completing the readings during class or school time? – I usually do the problems in English.

kkk. But the readings would have been done already? – Yes.

lll. How much extra-curricular reading do you do, and that means any kind of reading has nothing to do with your classes? – Not a whole lot.

mmm. Do you normally read? – Like in the summer I read a lot. During the school year I'm sort of busy.

nnn. What materials do you read, outside of class in the summer? –Well this past summer I read a lot of books on Russian. That interested me.

ooo. So that's why you are interested in languages. – Yes. I like languages.

ppp. Do consider yourself a well-read person? Because that's kind of unusual to read about Russian.–Yeah, I think so.

qqq. Do you read a variety of genres? –Anything but I don't like science fiction.

rrr. Do you read trashy novels? – Sometimes.

sss. When you were working on the projects, how did you go about deciding how to get into the investigation? How did you decide how you were going to get started? – I still don't understand.

ttt. You are given a project, if you recall the last one was the one where you had to do a linear regression or one of the others and so when you got the project you decided who you were going to work with oh that was the one that was alone. How did you decide how you were going to get started? - Like with the regression I just looked on the internet for data.

uuu. As far as the procedure itself did you understand the steps that the Teacher gave and told you what to do? – Most of them.

vvv. Now, you did another project and this was with a partner, I know there was project in which you did a poster, was that right, who did you do that with? – Student aa.

www. What was that project about? – It was, dealt with a lot of graphs.

xxx. Was it about taking different samples? – Yes.

yyy. Who kind of led the project? In other words did you kind of have equal input into the project? – Yes.

zzz. Did you know what you were doing? – Not really.

aaaa. So how did you decide how to learn how to do it? – We just kind of winged it.

bbbb. Winged it? – She laughs.

cccc. Was your grade as good as you wanted? – No.

dddd. So maybe winging it didn’t work? – Yes.

eeee. When you were working on tests, how did you decide how to even start to answer a question? What do you normally do? What goes through your mind? – I don’t know.

ffff. Let me give you a for instance. When I am working on a test, I try to recall when the teacher was explaining it or I try to recall from my notes what my notes looked like because I try to memorize things many times. I try to recall what did it look like when I was reading my notes. That brings things to mind. I also may recall maybe an activity that we did that had something to do with that question. – I think about what I wrote down when we were doing study guides and the notes and stuff.

gggg. Do you just jump into the questions or do you make a decision about how to proceed before you answer the question? – I just kind of jump in. Then half way through figure out what I have to do and then I go back and fix it.

hhhh. If you had to do it all over again, knowing what you know now, would you have taken AP Statistics? Nobody is going to know about this. –I probably wouldn’t have taken it as a junior but I was afraid that they wouldn’t have it next year with all the juniors leaving and everything.

iiii. So you did it to get it done on time before maybe the course didn’t exist? – Yes.

jjjj. Do you believe that you will do well on the AP Statistics test? – I think I could do well.

kkkk. What would make you, you think, be prepared to do well on the AP test? – Study.

llll. You do know that I'm going to be doing the review sessions. Do you intend to participate in them? – Yes.

Of course she told you to get the review book. We're going to be using that a lot. We're kind of going to go over questions from each topic. And then kind of put it all together.

mmmm. One thing that someone said before in one of the interviews, is that one thing that they might have changed about the course was that the AP questions should have been given a little bit earlier in the semester. Do you feel the same way? – Yeah.

nnnn. How would that have impacted the course and you? – I felt that the AP Questions were harder than the questions in the book and it would have made me more prepared for tests and stuff.

4 – Student A – Minority (African Ethnicity – not African American) – Hard Worker – Below Average Student

a. What is your name? – Student A

b. What grade are you in? – 12^{th}.

c. What was your initial feeling about taking AP Statistics when you first decided to enroll in the course? – Well, the first thing that I wanted was AP credit. The second thing was that I didn't want it to be a year-long course. Some of the year courses were at the same time as some of the other courses I wanted to take so I took two semester AP courses. Stats and US History.

d. Had you taken AP English last year? - No.

e. How about this year are you taking AP English? – No.

f. So, English is not your thing? – Not really.

g. Tell me, this probably goes with a different answer to the first one, when you thought about taking statistics were there any feelings associated with that besides the fact that you needed to take it or wanted to take? Any feelings about whether it would be easy or hard? - I heard it wasn't difficult. I heard it wasn't easy but it wasn't difficult. It should be okay. But I just heard it had a ton of homework and lots and lots of reading assignments. So I kind of didn't know when I was coming into it. It has got to be like a regular honors class but maybe with more challenges like the homework and tests.

h. Did your feelings change then about AP Statistics? – No, it's not hard but the first semester (sic. Quarter) I didn't apply myself but now once I was sitting through it well fine now I just really need to try in this class. It's not really difficult. In the beginning I thought it was hard because I wasn't really trying as hard as I could but now that I am trying as hard as I could it's not easy but it's not hard.

i. So you can get a handle on the material but now you are going to have to work harder? – Yes.

j. Do you feel that writing in statistics is different from writing in your English class? – No, because writing in statistics is like you are writing a story but you are writing a story about a math problem. In English you are writing a story about a topic a genre.

k. Has your writing changed in this class in completing your assignments? Think about how you first started. – Yeah. When I first started I started with short scripts, just answers, no work, just plain calculations, and that's it. Now it's like I am writing real long paragraphs for each problem or a page paper or something.

l. How about answering questions on a test, has that kind of writing changed? – I would say so, because in the beginning I guess none of us expected to write long paragraphs and her having to say you need to justify everything, write what you've already shown in words, yes I believe it has changed.

m. How about in completing the projects? Remember she gave a project at the beginning and then she gave a project towards the end. Did that writing change? – Yeah. The project at the beginning I hadn't got a grasp of the class yet, I mean I did okay on the writing at the beginning but it could have been better and comparing that to the one we just did I could see progression in my work and I've seen that the class has advanced my writing skills for the math world and that my writing has improved from the beginning to now.

n. I've observed that you answer on occasion. In fact, I see you answering quite a bit. Can you tell me how you prepare for class each day? – I just come in and get out my homework and my book and I'm the one that takes notes on the Post-its. I stick it in my book so if she asks me a question that has to do back with the reading I can just flip through my post-its and then I can say for that particular time or if I have a post-it that has a question mark because I didn't know something then I could just go back and ask. So, I take notes in little Post-its notes all over my book and I have to remove them all tonight.

Well, good for you that sounds like you have a good idea. I hadn't thought about using Post-it notes.

p. I know that there were all different kinds of oral communication that went on in class. For example, you had the whole class discussion where the Teacher would kind of lead the discussion and on the homework especially for a particular topic. Then you also had the small group discussion where you had either the pairs or the little groups. Of those, which was the most helpful? – I like working with the pairs only because when we are in the class discussion there tends to be not a lot of people talking and you hear like the same voices and the same opinions so it's not really like a broad range of different opinions floating around the class if you've got 5 or 6 people doing all the talking, talking back and forth. When you have the small groups, you have that same setting, some of the people talk and others won't talk. When you are working in pairs you're communicating back and forth and you can bounce different ideas off of each other and it's more comfortable for me than the big setting because I don't want to feel lost because when it's just pairs I can say my idea and she can say her ideas and you can just go back and forth between the two of us.

I noticed that you talk and work with Student F a lot.

q. Why did you enroll in the course? I think that we already answered that question right, because you needed a math course? – No, I don't need a math course, I am taking Pre-Calculus as well next semester.

r. Oh, you just needed an AP course? – Right. I wanted to have an AP course.

s. Do you have the same commitment to the course as at the beginning? – No, because at first I wasn’t really committed to the class now my commitment to the class has gotten stronger and now I feel more comfortable in the class because at the beginning I wasn’t comfortable and I couldn’t find by feel for the class but once I started actually studying I could find myself in the class and my commitment was there and could be better.

t. What expectations did you have for this course? I think you talked a little bit about that, you said that there would be a lot of homework. – I didn’t expect the Teacher to be so nice. Because I’ve heard a lot of things. I’ve heard that she was kind of strict, I guess not taking as much crap from us, but she does. I expected it to be challenging, trying to test me, push me, as if I was in college, I expected it to be really, really hard.

u. So, did your course meet your expectations? – Yeah, I’d say so. The Teacher is not what they pictured her to be. To me, I think she’s nice and she’s helpful and friendly but she pushes you to go that extra mile so that you don’t stop half way at the mark, you reach the mark that you need to be.

v. What could have been different about the course to make it more worthwhile, anything you would have changed? – Yeah, when we had a discussion, I’d hope for that if you are taking an AP class that you would have been willing to speak out and talk and have open opinion from everyone and I would also expect to someone else besides your friend or your buddy to get help from. Sometimes I don’t get that opinion, some of the times I do, but if we could have that open communication between me and everyone else in the class I would feel more comfortable that way on like study sessions and chapter questions.

w. So, do you think that you find the, this is the sense that I'm getting, I just want to know that I'm sensing this correctly, you're sensing that the group in general is not open enough for you to feel comfortable to work together with a lot of people? – Right. It's certain people I know that I will be able to work like Student B, or Student dd, or Student F, but some people in the class I just get the feeling that they wouldn't help me like Student Y, and Student bb and those type of people.

x. Able or willing to help? – Yes.

y. What materials used in class were of most benefit to you? I noticed that she gave study guides and you did the sample questions, you used the textbook, you did the projects, quizzes. What materials helped you the most? – I would say the study guides and the book. That's because we made so much reference to the book and we always referred to our study guides for a particular chapter. I know that I can always go back and look at my study guide and refresh my memory.

z. How much time did you normally spend doing homework, just for this class? – About an hour to an hour and a half. Because usually I never do homework, like ever, but this class I found myself having to study, look over notes, so I'd say an hour to an hour and a half.

aa. How important is this course for your future coursework in college? –I don't know. It depends on what I major in college. I'm not sure what I'm going to do right now. But, my neighbor's sister told me that when I go to college that I would need to take statistics and she thought that it would be of benefit to me to take it now that way when I do take my statistics in college I'd know what I was doing and I wouldn't be like a fish in a big pond.

bb. Do you like working on projects with other students? – It depends on the type of student. Because you have some students that'll make you do all the work, actually there are 3 types of categories, there are students that don't allow you to do any work, and those that you can't openly communicate and work on it 50-50.

cc. What benefits did you see in working with other students on the projects? – Instead of you seeing a small picture you saw an even bigger picture. Also, we were able to share ideas and it would kind of help you if you were lost and help you find a way to guide you on getting back on the correct path instead of taking the wrong path.

dd. Which topics studied so far this semester interested you the most?–The tests. That was my favorite.

ee. Hypothesis Tests? – Yes.

ff. That's rare. – I know.

gg. Okay what was the least favorite? –I don't know.

hh. I know that at the beginning you did your data exploration and you did all the kinds of graphs. – I didn't like that. I did not like the Normal Distribution graph. Now, that I understand it a bit better, but at the beginning I didn't like it.

ii. What topic studied so far this semester was the most difficult? – The distributions.

jj. And which topic was the easiest for you? - Probability.

kk. Why do you think it was the easiest for you? – Because it was a concept that we were familiar with in elementary school, middle school, and most of high school.

ll. You were familiar with it? – Yeah.

mm. How much time do you usually spend in completing the readings for this course outside of class and school? – It takes me an hour to 2 hours and that is only because I'll read it and go back reread it but then I'll add my post-it notes and all those questions back into my reading.

nn. How much time to you spend completing the reading during class or during school time? –Like 10 minutes because I cannot concentrate because you have all those conversations going back and forth, back and forth.

oo. How much extra-curricular reading do you do, so that would mean anything outside of what's given to you in classes to do? – A lot. I like to read and I also work at the library so I do a lot of reading outside of class on my own and I usually that's usually what I'm doing in most of my free time when I'm not working or I'm in school or a sporting event or watching my favorite movie.

pp. What types of materials do you read? –Romance, historical fiction, young adult, mysteries.

qq. Do you consider yourself a well-read person? – I would hope so. I think so.

rr. And do you read a variety of genre? – Yes.

ss. What genre do you like the best?–Historical fiction is my favorite.

tt. When you were working on the projects how did you go about deciding how you would proceed with the investigations, I know you had a couple of projects.

How do you even get started? – Well, first I go and find what exactly she, what we're trying like the Central Limit Theorem and I had to go back and reread the chapter to figure out where I was going to start to accomplish investigating that topic, so the first thing I do is go back and try to read or look at our book on that particular topic to help me find a starting place and go ahead just looking at the definition.

uu. When you were working on the tests, how did you decide to start answering the questions? – Usually I'll like read the questions three times and then I'll look at the answer choices and try to go through a process of elimination. Kind of, if it's a formula type question, find the formula, but I usually try to look at the answer choices to see which one it cannot be the answer. But if it's something you've got to plug into a formula I probably do the math and see what answer works and if you can reverse it and put that number into the equation and try that.

vv. Sometimes people ask themselves a series of questions as they are trying to answer a question, for example, you might say, where have I seen this before? Or, in what context did I see this question before? Someone even said that they try to picture when the teacher was talking about the topic. How do you go about, what kinds of questions do you ask yourself? – I probably ask, what did the Teacher say and then I try to think of the responses we gave or I'll just go back to what we were doing in class and try to find a book or the study guide and just tell myself how did I arrive at the answer? What steps do I take to get the answer? How did we go about solving this question?

ww. Let me see if I understand this correctly, so if you're answering a question then what you try to do is picture what you were doing on the homework assignment or in a textbook question to see if you can recall the steps. – Yes.

xx. Do you just jump into questions, or do you kind of decide how to start answering the questions after a while? – I decide how to answer a question after a while. I know that if I jump in I know that I will second guess myself and change the right answer to the wrong answer and vice versa so I take my time.

yy. I know some people that just like to jump in and then change it if they have to. – No, I don't.

zz. If you had to do it over again, knowing what you know now, would you have taken AP Statistics? – Yes.

aaa. Why? –1) Well, I probably still need that AP credit. 2) I like the class, I actually like this class. I thought it was a good class, it taught me a lot and I can actually apply most of the things I learned in this class to the real life setting and it will be beneficial to me in the future, just taking this class. So, I would take it again.

bbb. Do you believe that you will do well on the AP Statistics test in May? – I don't know. I think it will depend on how hard I work, making sure I know what I have to know.

ccc. You know that I will be doing the review sessions this spring? Do you intend to participate? – Yes.

ddd. Why do you want to do it? – Because I don’t know that I could review it all on my own but if I had someone to guide me in the right direction then it would help me improve my score.

5. – Student B –All-around Strong Student

a. What is your name? – xxxxxxx

b. What grade are you in? – 12th.

c. What was your initial feeling about taking statistics? – I thought I was going to hate it. Because I was told that a lot of writing was required. I don’t like writing at all.

d. And you’re not a writer? – No.

e. Have your feelings about AP Statistics changed since then? – Yes.

f. How so? – I don’t know. It’s one of my favorite classes.

g. Is there any particular reason why it’s your favorite? – Because, I understand it pretty well.

h. Do you find that writing in statistics is different from writing in your English class? – Yes.

i. How so? – Because in statistics there is a definite answer.

j. So, you can be more creative in English, is that what you are trying to say? – Yes.

k. Has your writing changed in this class in completing the assignments? – My writing in statistics? Yes.

l. How so? – I have learned to write more technically and be more clear with my answers.

m. How about in answering questions on the test, has your writing changed? – Yes.

n. In much the same way? – Yes.

o. How about in your projects, has that changed? I know you had at least to projects that I know of. – Yes, I think that it has changed.

p. I have observed that you answer quite often. You also answer when called upon, you seem to know what's going on. Can you describe how you prepare for class each day? – I do the homework and the reading.

q. What was the most helpful type of oral communication used in the class? Let me give you an example. You used whole class discussion when you were discussing the homework. You used small groups and she also put you in pairs. Of those three that I can remember what was the one that was most useful?– Whole class discussion.

r. Someone said, in another interview that I did, that they didn't like whole class discussion because it was the same number of people that are answering. Do you feel that that's the case? – Somewhat, but people that are answering have a pretty good idea what their talking about.

s. So, you wouldn't want people to be discussing if they don't know what they are talking about? – No.

t. Why did you enroll in the course? – I wanted to take a math class.

u. Did you think that it would be tough to handle, since you have two, do you have another one? – No, just two.

v. Did you think it was going to be tough? – Yes.

w. Have you been able to manage it? – Yes, I would have been able to manage it better if, I am not sure.

x. Do you still have the same commitment to the course as you did at the beginning? –Probably more so.

y. More so? Why? – Because I feel that I need to do well on the AP exam.

z. What expectations did you have for this course? I think you mentioned that there would be a lot of writing, reading and writing. Did you have any other expectations? – Not really.

aa. Did your course meet your expectations? – Yeah. The reading and writing was not that bad.

bb. What could have been different about the course to make it more worthwhile, you are going to have to think about that a little bit? If you could change anything in the course how would you change it to make it more worthwhile? - I'm not sure.

Maybe it will come to you, if we get back to that one.

cc. What materials used in the classroom were of most benefit to you? I noticed that she, the Teacher used the textbook, she used study guides, she used sample questions from AP exams, she used the projects. What was the most worthwhile material? –The textbook.

dd. How much time do you spend on homework, just for that class? –On average, probably an hour.

ee. How important is this course for your future coursework in college? –I'm not sure.

ff. Do you have any kind of area that you would like to study in college? – Architecture.

gg. I have a feeling that you may find that you're going to need statistics. – Really?

Yes. Especially if there is any engineering component to it.

hh. Did you like working on projects with other students? –Yes.

ii. Why? – Because it gave us a chance to bounce ideas off each other.

jj. What benefits did you see in working with other students on the projects, besides what you just told me? –So it was easier to not have to do all the work by myself.

So you actually worked with someone that was very helpful then.

kk. Which topic so far in this semester interested you the most?–I'm not sure what you are asking.

ll. Well so far this semester you talked about the normal distribution, just distributions in general, you talked about data analysis where you did the graphs and your 5 number summary, you did Hypothesis Tests, you did linear regression, you did inference, you did sampling distributions, there were a lot of topics, which of those interested you the most? – Probably Normal Distributions.

mm. Why? – I don't know why, I just understood it better.

nn. Okay which interested you the least? –The hypothesis tests.

oo. Why? – There was a lot to it.

pp. There were a lot of parts to it.

qq. Which topic studied so far was the most difficult? – Probably that.

rr. And easiest? – Probability.

So you had some exposure to that before.

ss. How much time do you spend completing the readings for this course outside of class and school? – About 30 minutes.

tt. How much time to you spend completing the readings during school? –I don't, I always read the assignment at home.

uu. How much extra-curricular reading do you do, so that means outside of any class any other readings that you do? – Not a lot.

vv. You don't read just for pleasure? – No.

ww. What types of materials do you read when you have the opportunity to read? –Things that people have finished and given me.

xx. Do you consider yourself a well-read person? – Yeah.

yy. If you were to read what kind of genre would you be attracted to? – Fiction.

zz. Do you read a variety of genre, or do you limit yourself? – Probably a variety.

aaa. When you were working on the projects how did you decide how you would proceed with the investigations, if that's what you were doing? Did you do any preparation beforehand, did you just jump right into the investigation or how did you handle it? –I just jump right in.

bbb. When you were working on the tests, you are going to have to recall about tests, how do you usually proceed in answering the questions? –I read it a couple of times to see if I understand it. Then I just start answering it they way that I learned it.

ccc. Do you just jump into questions or do you try to analyze it first? – Analyze it.

ddd. When you look at a problem or question how do you go about deciding how you are going to answer it besides analyzing it? I know that I try to always look for how my notes looked. Some of the people do things differently. How do you go about answering a question that's relatively new? - I think about examples that we've gone over some of the ones like the question.

ddd. If you had to do it over again, knowing what you know now, would you still have taken AP Statistics? – Yes.

eee. Why? –Because I feel like I've learned a lot.

fff. Do you believe that you will do well on the AP Statistics test? –Yeah.

ggg. Why? – Because I will be able to motivate myself to study on my own.

hhh. Are you going to be participating in the review sessions in the spring? – Yes.

iii. Why? –Because I want to do well.

jjj. Have you taken other AP courses before these two? – Chemistry.

kkk. And you did well on that? – 3

6. – Student bb – Below average student – not a hard worker

a. What is your name? – xxxxxxx

b. But they do call you by? – Student bb

c. What grade are you in? – 12^{th}.

d. What was your initial feeling about taking statistics? – Last year I took Pre-Calculus and it was a suggestion that this year I should take AP Statistics. So, besides I wanted the two extra points. For my GPA it looks better.

e. So basically for your GPA? – Yes, and it looks good on you college application.

f. Have your feelings about taking AP Statistics changed? How did you feel in the beginning about taking AP statistics course and then how did it change? – At first when I started taking AP statistics I thought it was hard or difficult or whatever and now at the end It’s not so much harder it just takes a little bit of time.

h. Do you find that writing in statistics is different from writing in your English class? – Yes. Because writing in your English class you can express your thoughts or something but in statistics you have a different vocabulary that you have to use.

i. So by expressing yourself differently you mean? – In statistics you have to say to the point and use the vocabulary like dealing with the questions or whatever you’re just writing whatever you think, your thoughts or whatever.

j. So you have a little more leeway in going off tangents? – Yes.

k. Has your writing changed in this class in completing the assignments? – At the beginning of the year I did not write that much because I didn’t know what to say but now I guess with what we’ve done in class I know more vocabulary and what the Teacher expects so I write a little bit more.

m. How about on your tests? – We haven’t had a test in a while but on our last test I wrote a lot more than I normally do.

n. Did it help? – Yes.

o. How about on your projects? She made a comment about that today. – Yes, at the beginning of the year I was writing like I was in English using vocabulary and that's not statistics and she made a comment today that I wrote better than the paper that I did before.

p. So you think more technical more scientific rather than more flowery. – Yes more instead of more emotions and stuff like that more science, math.

q. Very academic. – Yes.

p. I've observed that you don't answer a lot unless you're called upon. Can you tell me how you prepare for class each day? – How I prepare for class like the night before? If we have homework then I do my homework the night before or when I wake up in the morning because I don't have first period. I'll read the chapter or whatever or study guides or something. It's just when we're in class I don't like to answer questions.

q. Why don't you like to answer questions in class? – Because people that answer questions they talk a lot and I don't like to talk that much lot. If she calls on me I try give the best answer that I could explain I just don't know if other people think the same way I do so I don't know if it will confuse them or whatever and I don't want to confuse myself.

r. What was the most helpful type of oral communication used in the class? Let me explain that. You used whole class discussion when you went over the homework and different topics. You used small group discussion and you also used the pairs. Which one of those did you like the best? - Smaller group discussion because you get different perspectives of a question or an answer.

If all three in group are wrong maybe together you can come up with the right answer. Pairs are good also when you get paired with a smart one. That's what Student Y is.

s. Why did you enroll in this course is that just for the 2 quality points? – Not just for the 2 quality points it looks good on your college application and I know I had to take a math course my senior year and I didn't want to progress in math and then go lower and AP Statistics was higher.

t. Do you still have the same commitment to the course as you did at the beginning of the course? –Probably I commit myself more now because of the AP exam that's coming up compared to the beginning of the year. There wasn't as much riding on my grade as now.

u. What expectations did you have for this course? – I knew that it would be kind of challenging I really did not know what to expect. It would be a good class to take my senior year and prepare for next year.

v. Did your course meet your expectations? – Pretty much. I would say so. I knew it would be challenging and it was at some point and it wasn't in others.

w. What could have been done differently in the course to make it more worthwhile? – If I would have spent more time doing homework and stuff on my own or with a teacher then I would have had a higher grade.

x. Let's turn that around a little bit. Not necessarily what you would have done but what could have been done in the course? Maybe on the part of the teacher or maybe on just the setup of the course. Something like that. – Maybe this is just me but I learn best like work out problems and then while I'm working on the problems like if I have a question it would help me out so when I see another problem like that I would know how to work it out by myself . So instead of doing a lecture and taking notes in class I think we could have a worksheet and then we work out and do the worksheet and the teacher helps us in class individually and on top of that have homework so that the next day we will go over our homework and our new lesson have another worksheet so I could understand how to do the problems.

That is something that is typical of college courses that they don't do that. (He seemed surprised.)

y. What materials used in the classroom were of most benefit to you? I know that you used study guides, sample questions from AP tests, textbooks, projects, all those kinds of things. What were the most useful? –The AP questions because that's what we're going to take at the end of the year so it helps. The study guide helps to get familiar with the vocabulary in the chapter and things like that.

z. How much time do you spend on homework, just for this class? –On average, half an hour to at most an hour if we have project.

aa. How important is this course for your future coursework in college? –Like my major I really don't know what math courses are going to be involved in them but in my freshman year it's going to help a lot.

bb. What is it that you plan to major in? – History.

I'm afraid that they will only ask you for two math courses but if you can get the AP credit that would be great.

cc. How did you like working on projects with other students? –I preferred, I don't want to say I prefer but it is better than working on the projects by yourself but again it different perspectives on what it could be of what to do. Sometimes it would be better than what I would do by myself.

dd. What benefits did you see in working with other students on the projects, I thing you kind of hit on that just the different perspectives? –Uh huh.

dd. Anything else that you can think of? – If you are stuck on an idea then you can bounce ideas off one another. If you're by yourself you might get frustrated and give up. If you don't know it then somebody else can help you.

dd. Which topic studied so far in this semester interested you the most?–What topics did we do this semester.

ee. Well you did distributions, and one of them was the normal distribution, geometric, binomial, you did hypothesis testing, you did inverse, you did where you did the 5 number summary and some graphs? –I would say the last three chapters that were on the normal distributions and hypothesis testing and the surveys and how to collect data and all those things those are interesting because you can see how statisticians and people who can things like that in the real world with what it has to do and how it relates. We were talking the other day about elections and like all the polls and all the things you have to do now, I watch them and I understand them better. How they conducted them and what it actually means.

ff. Okay how about the least? –I really didn't have a least favorite, because everything was about the same. The last three chapters were really interesting.

gg. Which topics studied so far this semester was the most difficult? – I first had trouble forming hypotheses. There are so many different kinds of problems.

hh. Which was the easiest topic? – The topic where we did boxplots and we analyzed the data.

ii. Maybe that was the first chapter. – Yes. (He laughs.)

jj. How much time do you spend reading outside of class for this course? – Whenever we got the study guide at the beginning of the chapter I would to the study guide and I would read the chapter and it would take me 45 minutes to an hour. Then I wouldn't read too much after that.

kk. How much time do you spend reading inside the class? – I read a lot in class, because we have to read the problems and we read the chapter in class when we get the study guides. So we read like 20 minutes every day in class.

ll. How much extra-curricular reading do you do, and that means anything that is not related to any course? – I read a lot. So that would be like an hour or two every night.

mm. What types of materials do you read? – I like historical fiction or non-fiction novels. I like to read a lot about history.

nn. Do you consider yourself a well-read person? – Not compared to some people, but I would say I've read a lot of good books. But, my brother reads like one book every week. Compared to him I don't think I read that much.

oo. Do you read a variety of genres? – If it's not for school I mostly read psychology or about history.

pp. When you were working on the projects how did you decide how you were going to get started on the project? –It depends on how much time we have for the project or what we were supposed to do. If we were supposed to do a paper then would prepare myself do it and finish it in the same time period. I would like to spend more than one day doing it. But if we had to find a survey and write a paper and do something else I would break it into three different days, one thing for each day.

qq. When you were working on the tests, how did you decide how to even get started on the test question? –I read the question and reread the question because at first I'm not quite sure what they're asking me or I don't remember or understand the chapter that well then I read the answers so that I can remember what we learned what the question is asking and the best answer.

rr. Some people like to do note cards when they are getting ready to take a test before the test and then they try to recall almost like a photographic memory and other people like to remember what the teacher did when they were talking about that topic. Other people just try to remember examples that they read about in the book. How do you go about, you said that you were looking for answers when you read the question, what are you looking for when you reread the question? – Key words like for an equation on a note card. I read the question to see if it was similar to a question I read in the book.

ss. So you kind of ask yourself what are the key words, where did I see this before, what formula did this, kind of those questions. – Yes.

tt. Do you just jump into questions? – No, it takes me a while before I answer the question.

uu. Was that the same in Pre-calculus? – No, it wasn't the same because there weren't that many sentences.

vv. If you had to do it all over again, knowing what you know now, would you still have taken AP Statistics? – I still would have taken AP Statistics but I would have spent more time on AP Statistics instead of other things I have done. I would pay attention more in class, did my homework and stuff like that a little bit better than I did before.

ww. Do you believe that you will do well on the AP Statistics test? –I thing I will do well if I prepare myself for the exam. If I don't prepare myself for the exam I don't think I will do that well. If I go to the review sessions and I prepare myself like for past AP tests then I could do well.

xx. That was my next question. Are you going to be participating in the review sessions in the spring? – Oh, yes. (Never showed up for the review sessions)

yy. Okay so you have your review book? – No, ma'am.

Well you need to get one.

7. – Student Y – Strong student. Earned the overall award in AP Statistics. Took AP Calculus and earned a 5.

a. What is your name? – xxxxxx

b. What grade are you in? – 12th.

c. What was your initial feeling about taking AP Statistics? – That it was just another math course and it would be easy.

d. Have your feelings about AP Statistics changed? –It’s a lot harder than I expected, there’s writing and less math.

e. Is that the part that bothers you, the writing? – Yes.

f. Do you find that writing in statistics is different from writing in your English class? – Yes, pretty much. Not extremely different because English is like essays but in Statistics you have to describe in depth.

g. Has your writing changed in this class in completing the assignments? – I don’t know if it has.

h. How about in answering questions on tests? – I guess, I answer a question in depth but she still gets on me.

i. Maybe because she thinks you can do better? – Probably.

j. How about on your projects, has that improved? – No. It actually went down.

k. If you know that this is a requirement, how come it hasn’t improved? – That’s a good question. I don’t know.

l. Do you think it’s your philosophy what a mathematics course should be like? – Yeah.

m. I’ve observed that you answer only when called upon. You don’t usually volunteer. Can you describe how you prepare for class each day? – I do the homework and I understand the topics, I’m not outspoken.

n. How about the readings do you do those? – Yes.

o. What was the most helpful type of oral communication that you did in the classroom? Let me give you an example. They did whole class discussion when you were all going over the homework. You used small groups and you also did pairs. Of those three ways which one did you prefer?–Whole class discussion because when you're in pairs or groups you tend to talk about other topics about your friends rather than about math.

p. Why did you enroll in the course? – To get AP credit for it.

q. Do you still have the same commitment to the course as you did at the beginning? –Yes, it has not changed, not really.

r. What expectations did you have for the course?–I expected it to be easier.

s. Had you talked to anyone who had the course before? – No. Maybe RC, he thought it was easy.

Maybe, you should have questioned that, right.

t. Did your course meet your expectations? You said it was easy, and I think you mentioned it's not as easy as you thought? –It wasn't as easy as I thought because of the writing part.

u. What could have been different about the course, this is the part that you are going to have to think about, to make it more worthwhile? - Like a suggestion for improvement? More book work, I learn better from the book. I don't think I learn as much from the teacher as I do from the book. Many times I read from the book, and we pretty much read every chapter. I guess more sample problems AP style.

v. Okay, earlier in the semester? – Yes.

w. That might have helped? – Yes.

x. Maybe that would have given you an idea that you have to do more writing? – Yes.

y. Even though you don't like it, you said. – Yes.

z. What materials used in the classroom were of most benefit to you? I know that they used the study guides, they used sample questions from AP exams, they used the textbook, and you did projects. Of all of those, which was the most useful?–I think the textbook is the most useful because that's where I learn from reading it.

aa. How much time do you spend on homework, just for this class, on average? – Some nights and hour, no more than an hour.

bb. How important is this course in your future coursework in college? –I know it's required for my degree. But I think that we will go in more depth than here.

cc. What do you plan to major in? – Architecture.

If anything they may ask you to do Calculus but you have that already.

dd. Did you like working on projects with other students? – Yes, when the project was in groups it made it easier to get different perspectives and different person's point of view.

ee. Anything else that made it better? –Not really, but the disadvantage is that it makes it easier to get off topic easily.

ff. Which topic studied so far in this semester interested you the most?–Inference.

gg. Which one interested you the least? –Probability, I don't like probability.

hh. Why don't you like probability? – Because it is so elementary, and a lot of the stuff we have learned before. It just seemed like a review basically.

ii. Which topic studied so far this semester was the most difficult? – Probably inference.

jj. So, in other words you liked the challenge? – Yes.

kk. Which seemed the easiest? – I liked like the normal distribution and stuff like that seemed easy. Probability and stuff with that.

ll. How much time did you spend completing the readings for this course outside of school? – Maybe 5 hours a week.

mm. How much time did you spend completing the readings during class?–Not hardly any time at all because I don't do well in the classroom setting. I can't read in a classroom setting.

nn. How much extra-curricular reading do you do, and by that I mean that it has nothing to do with any of your classes? Just for pleasure. – A couple of hours a week.

oo. What types of materials do you read? –Newspapers, magazines, every now and then I read a book.

pp. Do you consider yourself a well-read person? – Not really.

qq. What do you consider a well-read person? – Someone who loves to read, someone who reads fast, and understands the topics. That's not me.

rr. Do you read a variety of genre? – Yes, I do.

ss. So, what's your favorite? – Action type.

tt. When you were working on the projects, how did you go about deciding how you were going to start? You have a project how do you start it? What is it that you do to get ready for it?–It depends on the kind of project. Go on the internet and just research the topic and find out what you are going to do the project about.

uu. When you were working on tests, how do you proceed? Are you one of those that jumps right into the question or do think a little bit about it? Do you have some plan of action? Do you try to remember what you did in class? –I just skim the question and do the first thing that I think I should do. I don't go back and look at it again because if I did I start to question what I did. If I know the information it's just recalling it. If it's right in my head and I look at the question it just pops up.

vv. If you had to do it over again, knowing what you know now, would you have taken AP Statistics? – Yes.

ww. Why? –Because I think it's a good experience. It's a good challenge even though it's hard. It's the challenge.

xx. Do you believe that you will do well on the AP Statistics test? –If I review I will do good enough to get credit and do well enough to get a 5.

yy. Are you going to be participating in the review sessions in the spring? – Yes.

zz. I'm going to ask this question because you've already done Calculus at this point in time how do you feel relative to how you felt let's say around March or April before you took Calculus. Do you feel more comfortable with this material, less comfortable with this material, or as comfortable with the material as you did in Calculus? - I think about the same. In Calculus we reviewed AP style questions and that's what we're starting to do now. It really helped in Calculus. I hardly reviewed any on my own.

8. – Student Z – Good Math Student – Not a good writer

a. What is your name? – xxxxxx

b. What grade are you in? – 12th.

c. What was your initial feeling about taking AP Statistics? – I thought that it would be a class that would help me a lot for college and I was very excited about it.

d. Have your feelings about AP Statistics changed? –I thought it was going to be a little bit easier but I am still happy that I'm going to get college credit.

e. How about the writing, do you find that your writing in statistics is different from your writing in the English class? –I think it's more uhh, like less open like if you have certain same things like it's more formatted than in English but very much similar.

f. Has your writing changed in this class in completing your assignments? – I think that my writing has become more formatted I think in this class.

g. How about in answering your questions on tests? –I'd say they've become more formatted.

h. How about in completing your projects? – I think that now when I complete projects it's more like technical than it was.

i. I know this seems pretty obvious, but why did you change? – Just because of like in statistics you got to have to follow stuff like by the book so you can get like AP with the correct answer so it has to be formatted and technical.

j. I have observed that you answer quite a bit in class. Can you describe how you prepare for class each day? – Just doing my homework and reading the book and making sure that I know what's going on.

k. What was the most helpful type of oral communication used in class? I'm going to give you some examples. You used whole class discussion where everyone talked about the homework and different topics. There's the small group and there was the paired. Of those, which one did you like the best?–I really liked the whole class because then you get everybody's opinions because when you read the book you might miss something and somebody else might pick it up. It connects better when you do that.

You felt that it was better because you got more opinions and things like that.

l. Why did you enroll in this course? Did you answer it about you initial feeling? Is that what it was?–Pretty much. It's more like looking at your career and college.

m. Do you still have the same commitment to the course as you did at the beginning? –I think I near the end I kind of tapered off the work ethic but when the AP comes up I'm going to make sure I study hard.

n. Why did you enroll in this course? – Mainly to get a feel of statistics and to get myself ready for college.

o. Did the course meet your expectations? – Definitely was perfect.

p. What could have been different about this course to make it more worthwhile? So that's kind of like a thinking question. – I think like maybe going over the homework more in depth, and like writing on the board as you were going over some of the examples. But, for the most part, it was pretty good.

q. What materials used in the classroom were of most benefit to you? For example I noticed the Teacher used the study guides, sample test questions, textbook, projects. Which was the most helpful?–I thought the study guides were very helpful.

r. How much time do you spend on homework, just for this class? –Depending on how many problems between 35 to 45 minutes.

s. How important is this course for your future coursework in college? –I think it will be helpful because, I'm not really exactly sure what I want to do in college, so being that I've always liked math it might count towards a math related career.

t. Do you have an idea of what you want to go into? – I've got a couple like either emergency room surgery, maybe law, maybe economics, I'm not really sure yet.

u. Did you like working on the projects with other students? – I did, it was cool, because you had their input as well as it allowed us to have more contact with each other.

v. So the benefit that you saw was that you had more input from the students, any other things? – It was cool because you could kind of collaborate with your ideas and your results.

w. Which topic studied so far this semester interested you the most?–Probably just the different types of t-tests and z-tests and we had to see how they finally, all the statistics, how they are used on the news and on the internet and all that.

x. Which interested you the least? –Probably the first couple of topics they were kind of bland.

y. Which topic studied so far this semester was the most difficult? – I think when we did the Chi-square since we did like different summaries at the end. That was the hardest one.

z. Which was the easiest? –Probability. We've been doing that since 6^{th} grade.

aa. How much time do you spend completing the readings for this course outside of school? – Chapter readings? Like maybe 15 to 20 minutes.

bb. How much time do you spend completing the readings during school?–About the same.

cc. How much extra-curricular reading do you do, and by that I mean reading that has nothing to do with any of your classes? –I read like a book a month or so.

dd. What types of materials do you read? –Just like different novels fiction, some non-fiction. Just a lot of different stuff.

ee. Do you consider yourself a well-read person? –Yes, I don’t read as much as I used to but I still read.

ff. Do you read a variety of genre, then? – Yes.

gg. When you were working on the projects, how did you go about deciding how you would proceed? You have a project, so what’s your first step?–I would usually look at the paper and what the requirements would be then, I kind of go step by step through the project. Just follow the rubric.

hh. How about delegating jobs, when do you do that? – If you’re going to work on a project with someone, usually we see who comprehended which part the best. Kind of go by that.

ii. Now this is a little bit different. When you were working on tests, how do you go about starting to answer a question? – Like what order do I take the questions?

No. You have a question in front of you, how do you decide how you are going to answer it. What do you look for, what do you think about? – I really just look for key words. I look through the answer choices and see if there are some key words. You have to look for those trigger words that will help you answer the question.

How about thinking back on the lesson, or thinking back on the homework, do you do any of that when you're going to answer a question? – Yes, especially when you have your book or you don't understand the question you go back to the book.

jj. If you had to do it over again, knowing what you know now, would you have taken statistics? –I would have.

kk. Why? –Because I really felt that not only do you get the college credit which is really important with getting all that done but it shows me how college math is going to be. Also, it's a very interesting class.

ll. Do you believe that you will do well on the AP Statistics test? –Yes.

mm. Why? – I'm really going to work hard and study, I'm doing pretty good now that's once I start studying and start memorizing everything.

mm. Will you participate in the review sessions in the spring? – Yes, I will. (*Never showed up*)

nn. Why? – Because up in New York, we do a lot of review and all the review sessions have helped me out a lot.

9. – Student T – Above Average Writer – Not a strong math student – Hates Statistics – Sees it as not relevant to her interests

a. What is your name? – xxxxxxxx

b. What grade are you in? – 12^{th}.

c. Tell me about your initial feeling about taking AP Statistics was like? – Well it was like either this or Pre-Calculus. My counselor like you probably should take AP Statistics. You might get credit for college. You don't like math so it's just as bad as Pre-Calculus, so I said I'll take that.

d. Have your feelings about AP Statistics changed from being something that was almost like forced upon in a way to like you felt like doing? –It's gone from me having no opinion on statistics to me hating it so.

e. Do you find that your writing in statistics is different from your writing in your English class? –In some ways it is and in some ways it's not. I love to write. Like, it's a passion of mine, of sorts. And in some ways it's a lot more you have to think creatively on how am I going to word this and in other ways, it is totally different. Because, I remember when we wrote the first paper we had to write the CSA. All those statistical gibberish I wrote out I hope this makes sense, like it did. I got a 90 on it. But, I'm using words that I really don't understand. I'm not used to that.

f. Has your writing changed in this class in completing the assignments? – I've gone from having to write the minimal to being paranoid in writing every little thing that they might possibly want to know in the problem.

g. How about in answering questions on tests? –Same thing more or less.

h. So therefore it's translated to also when you're completing your projects? – Yeah.

So, you felt compelled to write a lot? – Yeah. And in some ways I don't think I wrote as much as for the CSA, the first one we did.

So, maybe you streamlined it? – A little bit.

i. I have observed that you answer many times. Can you describe how you prepare for class each day? – Well, normally, I'm at the second period, I don't like math and science all that much, and I have Chemistry right before, and at the end of first period, I'm like, huh, I have two periods of math, and I'm kind of like preparing myself for battle, just trying to ride it out, to the sanctuary of the theater, during fourth period. Where I can do stuff that I know exactly what I'm doing.

So, can I say then that you don't prepare for class each day? – Well, you can't prepare your house when it's about to be hit by a hurricane don't you?

j. What was the most helpful type or oral communication used in the classroom? Let me explain what I mean. For example, you used whole class discussion where you discussed the homework and certain topics. You did small groups and you did the paired group. So which one of those, was the most helpful?–I think when I was working in pairs. I would be with a friend and I wouldn't have any problems asking a question. You could sound stupid I have heard them sound stupid before so it's not that bad. If it's something we both don't know then it's like insanity in numbers. Maybe I'm not the only one that's clueless on this question, let's ask the teacher.

k. So, apart from your feelings about statistics, why did you enroll in this course? You did have an option. - Because this had 2 quality points instead of Pre-Calculus. Possible college credit and Pre-Calculus didn't.

l. Do you still have the same commitment to the course as at the beginning? Has that changed any? –Yeah. At the beginning, you know it was kind of like, alright I'm going to do my best in this class, I'm going to try my best. And then as we went along, gosh I hate this class, why did I take it, just going from trying my best to be satisfied with passing.

m. What expectations did you have for this course? – I didn't expect it to be as hard. Normally, math is more solid, with a definite answer, but with this it's more so not.

And, I thought that was the answer, you said that it was that thing, why is it that like that other question.

So, you find that you really can't pinpoint answers that normally would have been easier in a math course. – Yes.

n. Did your course meet your expectations then? – In some ways it did and in some ways it didn't. Like I guess I didn't expect it to be quite as hard but in other ways it did meet my expectations because I expected it to be a math class where I would be bored most of the time.

o. What could have been different about the course to make it more worthwhile? And this you might want to take a little time to reflect. – I don't know, maybe if there's more problems and stuff, activities that related more to my life, things that are more interesting. Like some of the activities we did where we got to pick the stuff and relate it to us. And like some of the other activities, like one that we thought was interesting. Those were more interesting and I was able to pay attention more but some of the others talking about butterflies, talking about companies with samples of juices, I couldn't care less.

Do you understand the reasoning behind having those kinds of examples that may be not interesting to you but nonetheless, they do them in that course? – Yes, I could understand why because not necessarily everything that's interesting to me is relevant to the class. So, there has to be some problems that provide examples of things that we wouldn't find relevant to me.

I guess a way of providing relevance would have been more like that last project that you did, where you picked your data. Maybe that would make it a little bit more relevant.

p. What materials in the class were of most benefit to you? I'm talking particularly about the study guides, sample questions, the textbook, projects, anything that was used in the classroom that you thought were of most benefit to you?–The study guides were useful because they gave me an idea of what I needed to know. That is, the relevant information.

q. How much time do you spend on homework, just for this class? –It depends. Normally, about half an hour to an hour depending on what it is.

r. How important is this course for your future coursework in college? –I don't think it's all that relevant because I want to be a history teacher. Some of it will be relevant but I won't be performing t-tests on things.

You'd be surprised. I'll let you figure that out.

s. Do you like working on a project with other students? –Sometimes I do, like if it's something that I don't understand very well then the other student is useful. But in other ways I don't like trusting my grade to another person.

So, the benefit that you see in working with working with someone else is? – If it's on a subject that they know better than me on it then yeah it's more beneficial. But, if it's something that we're both lost on I'd just rather do it myself.

t. Which topic studied so far this semester interested you the most?–I liked the probability stuff. That was more interesting and I could see some relevance to my life through it. Like how initially it created bicameralism stuff?

x. Which interested you the least? –The hypothesis testing. Half the time like when we were learning it I just would say I want to learn it myself, because it didn't make much sense in class to me, when we were learning it. I was like all Greek to me. I just couldn't see it applying to my life all that much.

y. Which topic studied so far this semester was the most difficult for you? – It's mainly when we started the test stuff because I was doing a lot of the teaching fellows stuff at that time and I was missing a couple of the classes and some of it went over my head.

z. How about the easiest? –Probably a tie between, I guess the Chi-square test actually very easy to me. It made a lot of sense to me because it was nice and neat in grids and had an equation that I knew exactly what it was used for that came easy to me.

aa. How much time do you spend completing the readings for this course outside of class? – It takes me about 15 minutes. It would normally take me about 15 minutes for that sometimes it takes me a little bit longer because sometimes I get distracted just because I'm not that interested after reading it.

bb. How much time do you spend completing the readings inside class?–About 15 minutes.

cc. How much extra-curricular reading do you do, and that means outside of any of your classes? –I read all the time. Over the winter break I could get probably about 5 to 10 books read depending on over the break I could get more books read than I could care to count. I love to read. Like, I want to be a published author some day. If they let me I want to write a book for my senior project, a novella.

dd. What types of materials do you read? –It's a bit of a variety, I prefer things more on the fantasy side of things. I read a little bit of sci-fi, I read, I like books with magic at home. I read a lot of graphic novels. I like historical fiction a lot too. I grew up reading historical fiction so that‘s why I like history so much.

ee. Do you consider yourself a well-read person? –Yes, I like to think that I am.

ff. So, it seems from what you told me that you read a variety of genre. – Yes.

gg. When you were working on the projects, how did you go about deciding how you would just start? Let’s say you were working with someone else. – If I was working with someone else I would like to figure out who’s going to do what first. After that, I decide on the data that we’re going to use, then how we’re going to go about analyzing that.

So you work together with that person on deciding all of that.

hh. Now when you were working on tests, how do you decide how you’re going to get started on answering a question? What do you do? Think about what you normally do? – Try to identify what sort of tests I have to use, what sort of question it is whether it’s experimental design.

So, are you looking for clues, or something? – I’m just deciding what kind of question it is so I can go about solving it.

What I’m trying to think about is sometimes like when you’ve taken Algebra you remember how it was done in the class is that sort of what you do. Or you think, oh this seems to be this type of question that you answer this way. – I normally try to recall examples. But the hardest thing about recalling examples is what all the variables are like yeah, the number minus the number is, but what do they all mean? I have a photographic memory so I can recall the examples but it’s the deciphering part that is the hard part.

ii. Do you just jump into questions or do you try to analyze them or some people like to just go ahead let's start answering? Then if I need to change my mind I'll go back. – I don't just want to just want to jump into a question but the first instinct is just the best. I'll just read the question and think about it for a moment whatever answer seems to be not most right to me I'll just answer it. Then I go on and it might occur to me something I didn't before and I will go back to it.

jj. If you had to do it over again, knowing what you know now, would you have taken AP Statistics? –Probably not.

kk. Probably not, and the reason?–I don't like math all that much and it kind of switched things up on me because I kind of got into a routine of getting at least a C in the math class but it kind of switched things up on me to not have solid answers. Yeah, I didn't like that very much. I kind of had to go a different way of figuring out how to solve all the questions and survive in the class.

ll. You've taken Algebra, Geometry and Algebra II, were those honors courses? – Yes.

mm. Were they here? - Algebra IA I took in the eighth grade, then I had IB when I came here to high school.

nn. Why did you end up in Algebra IA and not just Algebra I? Was that your choice? - I had been misplaced in math class when I was in the sixth grade and my seventh grade teacher the next year realized that I shouldn't have been placed in that class because I had 100.6 in the class so she had me take a placement test and I got placed in Algebra IA.

oo. Then you took Honors Geometry? – Yes. I did pretty well in that.

pp. Did you get an A? – I got an A. I got like a 95 in that.

qq. And, in Honors Algebra II? – Algebra II, I was good in math until Algebra II. Algebra II for some reason just killed me. I somehow managed to pull off a C in the class. I guess it was that I had more work than I was used to. The teacher gave lots of homework.

I can't imagine who she is. But we won't mention her name.

She gave lots of homework, way too much for my taste. Give a kid 10 problems and if they can't get them all right then they're not going to be able to do 50. They will probably do them all the wrong way and if you can do 10 problems you shouldn't have to do more. The teacher gave way too much and I wasn't used to having to do all that much. My teachers before had given homework but not a whole ton. It was only about 20 problems, on average. It was mainly started in class. I wasn't used to doing homework every day out of class. So, it was just a different experience for me doing Algebra II.

So that may have colored your feelings about math.

rr. Now, why are you going to take Pre-Calculus instead of Advanced Functions and Modeling? – Well part of it has to do with the people that normally take those classes. It was refreshing to take classes with those who were my age instead of below me. They are getting more and more immature.

ss. Do you believe that you will do well on the AP test? – I don't know.

tt. So, are you going to be ready for this test on Thursday? – Possibly. Depends on the questions she asks. Some of the free response questions that we went over made sense to me but others did not.

You could work with her on Wednesday.

uu. How about the review sessions in the spring, are you going to participate? – Yes. I probably really need them. (*Did not show up for any review sessions and did not take AP exam*)

vv. So you have your review book already? – Yes.

10. – Student W – Above average student – very willing to participate in class discussions

a. What is your name? – xxxxxx

b. What grade are you in? – 12th.

c. What was your initial feeling about taking AP Statistics? –I needed colleges to see that I was willing to take that advanced kind of math. I didn't feel comfortable with trying to push myself to take Calculus knowing how Pre-Calculus went.

d. Have your feelings about AP Statistics changed since then? –It was definitely a lot harder than I anticipated. But not as bad as I could see going into it.

e. Do you find that writing in statistics is different from your writing in the English class? –Very different.

How so? – My explanations, it's more like you have to follow a set pattern where English you can kind of interpret your own way and be right.

f. Has your writing changed in this class in completing your assignments? –If I were to write answers it would be more of a logical progression instead of just kind of free thought, yes. That has changed a lot.

g. How about on your tests? –With the going over and over every day it keeps getting better and better.

h. How about on your projects? – They could have been better. I'm not really sure but it feels statistics-wise my vocabulary and everything seems to have improved.

i. I have observed that you answer quite a bit. Can you describe how you prepare for class each day? –Generally I just make sure that at least looked at what we're going to talk about so that I make sure that I know some part of it and feel confident to answer or at least attempt to answer. I can't stand to be quiet.

j. What was the most helpful type or oral communication used in the classroom? Let me give you an example. I know you used whole class discussion to go over the homework and certain topics. You used the small group and you also used the paired group or pairing. So which one did you like the best?–At first I went for large group discussion because you get a wide range of how people are looking at things. So you could see how you would interpret that works better for you than what you originally were planning. For the small group or pairing you get a lot less of what you want to learn or get out of it.

k. Why did you enroll in this class? – Not so much for AP credit but to see if I could handle it. From what I saw in AP Chemistry that was hard enough. That's why I am taking three AP classes to see if I can do what I need to do for College.

l. Did the course meet your expectations? – Yes. It was challenging for quite a few chapters and a little laid back for some others. I was coming into it expecting it to be a little more laid back AP class than it was. It was definitely a little bit more challenging.

m. What could have been different about the course to make it more worthwhile?–I'm not entirely sure. Maybe if we are going to prepare AP-wise because I know that the way we did it in Chemistry was always referring back to last year, that was what Mrs. L was doing since science wasn't all year and we didn't have that time. So for those wanting that AP credit have a lot of focus on AP questions always throughout the year.

In other words have AP questions almost from the beginning? – Yes, so you are always focused.

You mentioned another thing, I don't know if that's what you meant. You mentioned something that almost sounded like a review sheet, sort of cumulative right from the beginning. It sounded like it. – Make it so that's done throughout the entire year.

Because it sort of like that at the end? – Yes.

n. Which materials used in the class were of most benefit to you? I'm going to give you an example. I know you used study guides, you used sample questions from the AP test, you used the textbook, and you had projects. Which materials were the most useful? Or did you have any other materials that you used on your own? - Just for overall understanding of the concepts just do a paper. That had you use all of the material, just bring them altogether into a paper and bring into one thing.

o. How much time do you spend on homework, on average? – Depending on what we are doing about an hour at most and thirty minutes at least.

p. How important is this course for your future coursework in college? –I'm not entirely sure. I wish I knew more.

q. What are you planning to major in? – Right now I've got two choices for myself. Next semester I'm going to be taking Med Science II and that will determine if I want to go into a kind of medical field. Another major that I'm also looking at is Psychology, just because it always has been an interest of mine in general.

Well, let me tell you, both of them require statistics. Psychology more so. It's heavy duty statistics.

r. Do you like working on a projects with other students?–Sometimes as long as you're with someone that will do the work. I just don't like having to put a major role into it I'd rather have a shared kind role instead of taken on a deeper role.

s. So, what benefits do you see in working with other students on projects? – As long as it's someone that will also share the work it kind of takes weight off of you evenly but also helps you because you have someone with which to talk about what you're doing.

t. Which topic studied so far this semester seemed the most interesting?–The one that seemed the most interesting just from reading a book was probability in general. Just the probability aspects as the book used it somewhat like a natural force that is always present.

You had thought of it as a theoretical thing and this was like a natural occurrence.

u. What was the least interesting? –I think the first chapter we were doing, that was the one in which I did the worst in, there was no excuse, it was how to determine the different kinds of setting up experimental designs or whatever. It was beneficial to know but it was kind of read through and make sure you know the definitions. It's not all that interesting.

v. Which topic studied so far this semester was the most difficult for you? – I think, which test was it? I don't remember what test it was. Unfortunately it was the one in which I did the worst in. Going into the advanced testing of things it got complicated because you had to narrow it down and you had to know which one to apply where. Was that hypothesis testing? – Just knowing them overall and how to do them.

w. What was the easiest for you? –Once we started doing testing and confidence intervals it was simple and you could tell the difference of which one to use.

x. How much time do you spend completing the readings for this course outside of class? – Not as much as I would have hoped to. But, usually I'd try to get as much done in class so whatever I had left I could finish out of class.

y. That kind of answers my next question, how much time do you spend reading during class?–Whenever she gives us time I try to get all of it done.

z. How much extra-curricular reading do you do, and that means outside of your class requirements? –It used to be way more. Over the years it's kind of lapsed. This year I've been doing a lot more. Since this year I've kind of picked myself back up. I've started reading material that I want to read that's interesting to me. When I get the time I read for a while.

aa. What types of materials do you read? –Right now it's like a philosophy kind of thing. They call it experimental psychology. I read Steven Pinker, a psychologist at Harvard.

bb. Do you consider yourself a well-read person? –For the most part yes.

cc. Do you read a variety of genre? – Yes.

dd. If you were working on a project, or when you were working on a project, how did you decide how you would proceed? – That would generally depend on what I was having to do, the timing. And get it started on a day that I didn't have very much to do. So I would have to know what I would have to put into it.

ee. So you wouldn't consider yourself a jump into the midst person? – Yes, jumping in would end up badly for me.

ff. How about when you were working on your tests, you have a question, how do you go about how you are even going to get started on answering a question? I know this is kind of vague, but some people have a particular way they do things. – I'm not entirely sure. I just have to recall material we did earlier that week.

So you try to recall something you did so it will give you a hint about what the question is? – Yes.

Do you read a question once, twice? – Once if the key words just pop out at me. Or maybe twice or three times if that key word hasn't popped out.

Do you make notes before you take a test? – Not very generally.

Do you see yourself recalling the problems that you did in class, problems that you did for homework, or a particular instance in time when the teacher was reviewing that kind of problem? – Maybe homework

gg. If you had to do it over again, knowing what you know now, would you have taken AP Statistics? –Yeah, I think I would. It's a different kind of math and I prefer it over a lot of other maths because it seems more applicable to what you would see in normal life.

hh. Do you believe that you will do well on the AP Statistics test? – Yes, unless they don't throw any curve balls at me. I've seen it a few times. Based on overall knowledge I think I would do far better than I would have done preparing myself for the Chemistry test. We don't cover as much as for the Chemistry test and we've covered it all.

ii. Are you going to participate in the review sessions in the spring? – Since I have Med Science II as long as the time frame allows me to I will. (*Never showed up*)

jj. You bought the review book yet? – I have my own review book that I will get into now that I am finished with the graduation project.

APPENDIX D: LESSON TRANCRIPTS

Lesson 1 Linear Regression

Student: Student Z asks, "How do you find SUM on calculator?"

Teacher stresses that the importance should be on what the computations mean.

Teacher: You can easily compute "*r*" and "r^2", coefficient of correlation and coefficient of determination, but what do these mean in your own words?

The discussion is non-existent at first. Students are either unwilling or unable to answer. Finally, Student W

Student*:* Student W – *r* gives you an overall idea about what the data looks like. If it is negative or positive association depending on whether *r* is negative or positive. If it is large or small it tells you whether the data is …

Teacher interrupts.

Teacher: What is the key word?

Student W does not get it and blurts out – data.

Teacher: Linear, whether it is linear or not and tight to the line.

The teacher recaps what Student W has tried to answer. The teacher asks for elaboration on this statement and to explain how r tells us how tight to the line (data) is.

Student: Student Z – Closer to one. Student I – or close to -1.

Teacher: It tells us how linear it is. What else? The closer to zero, what does that tell us?

Student: Student W – How much less linear it is. Or what, …the more erratic. Teacher: Scattered is a good word. The more scattered the points are. And, the more randomly they appear on the page. They can still be pretty scattered, if they are pretty scattered but they are going like this up the page. That is kind of linear right. They could be less scattered but going like this (Teacher shows and arc motion) around the page. That's not very linear is it?

It is difficult to get the majority of students to contribute to the discussion. Student W tries often even if his answers are not always entirely correct.

Teacher: "Is everybody getting a feel for that?" We haven't done the problems in the book, but there are some problems that we look at the graph and we try to figure out which equation is the best match for it along with the correlation value.

Teacher has the attention of the majority of students but the students are not focused when they are supposed to be contributing to the discussion.

Teacher: "We are going to be doing that a little bit later. But, those are the kind of things we can get a feel for with r. Okay, now, r^2 explains a little bit more specifically. Does anybody know what r^2 tells us? The Coefficient of determination, did you put it in your notes yet? You've not read that yet? I forgot what we have read and what we haven't read. No, not yet, we will talk about it when we read it. Now, I told you yesterday that I wanted you to be able to tell me today when you came into class what the least square regression line really means and how we can say it is the best fit and what way is it the best fit. Did anybody look that up? Uh, you forgot that because I didn't assign that except for you (pointing to Student F) and you (pointing to Student L). The rest of you haven't read it yet. I am going to assign Student F and Student L to two groups in a minute (showing two major groups) this is going to work out perfectly, to the group that is going to do a little research on that topic. Since you (pointing to the two students already mentioned) have already done some of it. Okay. Then we are going to talk about that equation and then we are also going to talk about something else today called residuals.

That is going to kind of wrap up this chapter. Okay. So, are there any other questions on homework before we go on? Student: Student A asks, “I have a question about #30.”

Teacher acknowledges that she has not put up #30 because she wanted to look in the textbook together with the students in order to discuss the problem.

Teacher: Thank you, Student A. Page 136 is where you will find question 30. Number 30 is kind of a discussion-type question.

She reads the problem: “Each of the following statements contains a blunder.” Now, I remember seeing something like this besides the textbook. I can’t remember if it is a quiz I give or a test that I give or an AP review. I’ve seen something like this, word of warning. Often, we will need to know the answer but also we also need to understand the question because questions may be slightly different the next time, okay.

The teacher is giving fair warning that this type of question will appear on a quiz or test.

She reads the instructions and questions.

3.30 – (a) There is a high correlation between the sex of American workers and their income.

The teacher tries to initiate a discussion.

Teacher: What is wrong with that statement?

She restates the sentence. No one answers.

Teacher: What do you think, why is there a problem with that statement? We have been told there is a problem. Why is there a problem?

Student J attempts to answer.

Student: Student J – I believe that the problem is that there is a high correlation.

Teacher: There is nothing wrong with saying there is a high correlation. No, we can have a high correlation.

Several more hands go up.

Teacher: A high correlation means that we have an *r* value close to 1 or -1. There is something wrong.

Student K, Student I, and Student B raise hands.

Student: Student B – Gender is categorical.

Teacher: What is wrong with that?

Student: Student B – That you cannot have a correlation between categorical variables.

Teacher: Why not? Why can't we have correlation between categorical variables?

Student Z, Student T, and Student A raise hands and Student A

Student: Student A - That we do not have numerical data.

Teacher: Do we use the variables in our formula for correlation? Will *m* for male and *f* for female work in that formula?

Several students shake their heads, no.

Teacher: So, without a number value we cannot calculate the correlation. Okay, very good. Categorical, that is the key.

3.30 – (b) We found a high correlation($r = 1.09$) between students' ratings of faculty teaching and ratings made by other faculty members.

Discussion continues.

Teacher: What is wrong with that one Student bb?

Student: Student bb – *r* can't be greater than 1.

Teacher restates his answer.

Teacher: The maximum is one and the minimum is -1. So we have to stay within that range.

3.30 – (c) The correlation between planting rate and yield of corn was found to be $r = .23$ bushels.

Discussion continues.

Teacher: What's wrong with that?

Student: Student E – *r* doesn't have a unit.

Teacher repeats answer.

Teacher: So it can't be bushels.

Teacher: Thank you, Student A, for drawing my attention to that question. Are there any other questions? I want to assign different rows to two reading assignments.

Teacher divides up the class.

Teacher: I am going to assign different sections of the chapter to the groups and I do not want you to have out your study guides. It is not that I have anything against your study guide. I want you to look beyond the questions on the study guide. Otherwise, you will just focus on the study guide. I want you to read the sections that I assign to you and I want you to pick out key points. And in just a little while we are going to talk about what is important in those two sections that we need to discuss. The first one is LSRL. The rest of you are going to read about residuals. We are going to have a little bit of silent time. You will need paper and pencil and I need for you to have at least 3 important concepts from your reading. Pick out the most important things do not give me a detailed writing of everything that you read in the chapter.

Once assignment is completed the teacher writes the key points.

Teacher initiates discussion. Student L begins to explain about LSRL

Student: Student L – It says it is a line.

Student: Student F – It is a method for finding a line that summarizes the relationship between two variables.

Teacher: Which two variables x, and y or the textbook calls them explanatory and response variables. Which is x?

Student: Student W - Explanatory variable.

Teacher: *Y* is response variable. What else did you find in your reading?

Student: Student C – Makes vertical distances as small as possible.

Teacher: It makes vertical distances from the points to the line as small as possible. Going a little bit further with that, what do the least squares have to do with regression? What is the squares part? Anybody wants to add to that?

Teacher calls on Student Z. He cannot answer. Student Z is a very strong mathematics student, but tends to be unprepared most times because he fails to read the assignment as requested.

Student: Student W – The vertical distance between the point and the line are squared.

Teacher: What are we going to do after adding them all up? What's the point of adding them all up? We are trying to make that number as small as possible. It's the least square. We adjust the line until we get that value as small as possible. Thank goodness, the calculator does that for us. You do not have to keep shifting and measuring and calculating. The calculator does that for us and it finds that line that has the least square value. Are there other comments that you have about the regression line?

Student: Student T – When finding the regression line we get $\hat{y} = a + bx$ and $\begin{matrix} a = \overline{y} - b\overline{x} \\ b = r\dfrac{s_y}{s_x} \end{matrix}$.

Teacher: What is $\hat{y}$? It is the predicted response for the x variable. Once we get the equation we can get the prediction of what y should be for any x value that was not given to us.

Teacher then directs student to a discussion on residuals. Student M initiates discussion on the topic.

Student: Student M – Points are not in a straight line but close to a straight line. Points are above and below a straight line.

It is clear that Student M is confusing the regression line and the idea of residuals.

Teacher directs students to page 156 that has to do with residual plots.

Teacher: Look at the figure at the top of the page. Idealized patterns of least squared residuals. If the residuals look like this then that would mean that the line would be a good model. If our model fits our data well then in our residual plot the points would be randomly scattered both and above and below the line y = 0 with no pattern.

Student: Student G – Why is it all random why can't there be a pattern?

Teacher: If this is our data and our regression equation goes like this then there are some points above our regression equation and there are some points below our regression equation. Your residual is your distance to the line.

Teacher is having trouble with this and says so. She recovers and then says that those points above the line have a positive distance and so a positive residual and those below the line have a negative residual.

Teacher: Ideally if we have enough points they will be normally distributed some very close to the line some a little away from the line. So, that is what our residual plot should look like. Now look at the one at the bottom of the page (156). On the lower values the residuals are all negative and in the middle they are all positive and on the right hand they are all negative again. It tells me that if I graph the original set of data and the regression line. But it is definitely a curved pattern here. What the residual plot does it just emphasizes that curve a little more to make it more obvious. Sometimes our data is a little curved and not noticeable but our residual plot makes it more obvious. When we see that kind of situation then we know that a linear equation is not a good fit for that data at all. We would need some other kind of equation that we will see in Chapter 4. Looking at the top of page 157, these are scattered both above and below, but they are close on one end and large on other end. That does not mean that a linear equation is not a good fit. It might be the best fit we can come up with. But (the residuals) it does tell us some more information. It tells us as we go further out our expected values are further away from our observed values than when we are at the lower values. It is fitting better for lower values than higher values. Any other comments you need to make about residuals? I want everyone to read about influential observations and outliers. They are similar but they are different.

I am not sure that the students understand the implications of having a random pattern for the residuals and the information that it provides the statistician in deciding whether a linear model is appropriate or not. As the discussion continues in other lessons perhaps this becomes clear.

Noise is evident. Students are very noisy after lunch.

Teacher: Let's wrap things up a little bit. I think we have done a pretty good job covering this. We were looking at influential observations. Looking at the top of page 158, child 19 is an outlier and child 18 is in the basic pattern but it is an extreme in the positive direction. It is more influential than child 19. You have to test those. Take out calculators. I want us to do a regression equation and residual plot together.

Student T is to go up to the calculator at the desk.

Teacher: Turn to problem 12 on page 124.

Much fidgeting is going on in the class. Transitions in this class are noisy. Perhaps 30 students in a course where 18 – 20 students is ideal is the culprit.

Teacher: Put data into L_1 and L_2. Those would be Guessed Calories and Correct Calories. Once you are finished do a regression on L_1 and L_2 and put it into y_1. I want you to graph a scatterplot along with your regression equation from y_1. Look up to the TV monitor when you are finished to see if you are in agreement with Student T. On your paper make a sketch of that, it does not have to be perfect and exact but it needs to be reasonable. Make sure that you give me a minimum x and a maximum x on the x –axis and a minimum and a maximum y on the y-axis so I know where those points come from.

Teacher walks around the room to see what students are doing.

Teacher: Make a note that this is problem #12 from page 124 so that you know where this data comes from. I also want you to write your regression equation

$\hat{y}$ = below the graph. You may round to 4 decimal places. Please do not go below 4 decimal places.

Teacher checks that students are doing as asked. Teacher emphasizes to Student M she wants four decimal places. No reason is given for this warning.

Teacher: If you have a perfect graph and equation you can plug in a point to the left and to the right of a point given into your regression equation and those points can be graphed and your line can be drawn between those two points on your graph. So far, is everybody clear on what we have done? Now I want you to look at the residuals. The residual is going to be the distance from each point to the line. Some of these have very small residuals. Or, small absolute values, some are positive some are negative. Others have larger residuals that are further away from the line. Do you see any kind of pattern? Let's look at our residuals plot. Turn off scatterplot and y_1. Now go to y_2 and put $y_2 = 0$. That will give us that horizontal line in the middle of our screen and our residuals will be plotted around that. Go back to Plot 2, turn it on, and we want a scatterplot with L_1 as our x – list and for our y – list, go down to y – list I want you to go to STAT LIST and scroll down to residual – RESID. Now, stop right there for a minute. Every time you calculate a new regression equation the residuals are stored. So, you can't get this list after you do a new problem, this list will be gone. It is there right now because we just did the regression line. So, we did the residual plot while we are here. If we went on to another problem and had to do the residual plot we would have to start over again. Now that you have residual there I want you to ZOOM 9 that and go to the graph. Do you see any kind of pattern?

Student: Student F - It looks like the other plot.

Teacher: It will have some of the same characteristics as the other plot. But, instead of having the line this way it has it along this way. (She shows with her hands.) The distance away from the line is emphasized. What are you noticing about this? Do you have scatter above and below? Would you say that this is a good fit?

There is quiet again when the teacher asks about a judgment call.

Teacher: Without the two points it might be a better fit. It is not all that bad. We can consider that linear is an okay fit. It is not clearly a curve because we have points above and below. Be careful about that idea about a curved pattern. It probably is not the best fit there might be something better or not. You understand how to get a residual plot. To graph that, you need x along x-axis and y along y-axis and you need to have scales. How will you do that? Find the highest and lowest point so you can get some idea of the scale. I would put scale along y-axis and along x-axis. You need to state that these are your residuals. You should label graph with variable names. Homework will be problems #31, 33, 34, 36, 37, 39 similar to what you are doing right now, and 40. The problems 39 and 40 are like this problem.

Student A – asked for problems to be written up on the overhead or white board. Student I – objected to having Student T write problems on board because she could not read them.

Teacher: Start on these now. This pretty much wraps up the chapter. We will have a test next week on this chapter next Tuesday. We will work on some AP items tomorrow.

Student V – had questions on a test that had not been graded. A few started on the homework. Student I can be heard laughing instead of working on assignment. Only about 4 – 6 are working on assignment. Many students are just socializing instead of working on assignment.

Lesson 2 Linear Regression Continued

Check Homework – Majority of students on task. There is playfulness.

Student: Student Q – Does it always give the LSRL – in other words – are regression line and linear regression line the same?

Teacher: Yes, we just call it linear regression line.

Student: Student Z – What about the formula used to find *r* or r^2?

Much time spent on going over homework

Teacher: Just square *r* to find r^2. Calculator gives same answer.

Student C asks question about #36).

The formulas for linear regression are:

$$\begin{gathered} \hat{y} = a + bx \\ a = \overline{y} - b\overline{x} \\ b = r\frac{s_y}{s_x} \end{gathered}$$

. These should be easy to use. What is the problem? These students have finished Algebra II. Student Z has finished Calculus – should know how to handle this type of activity.

Teacher: What is r^2 and what does it mean?

Student: Student Z repeats – What is r^2? What does it mean?

Teacher: You need to know what this means. Our regression line only accounts for 36% = r^2% of variability of *y* values can be associated with changes in *x* values. How does this problem use it to give evidence or disprove grade of 78.2 the predicted value? No, it is not high coefficient of determination. She has good argument. We won't use formula if calculator does the work for us.

Students are worrying about quizzes. Student W is worried and doesn't know this material but won't ask questions.

Teacher – Pass up homework and put name on first page and initials on other pages. We will now do an AP question about regression. You have a computer printout – read question! Residual plot is shown with *x* – years and *y* – residuals. You must be able to pick out a, b, that is, intercept and slope of regression line. Look for these, how do you identify them.

She explains meaning of slope and tells them that they will have to pick out intercept and slope.

Teacher: They are labeled constant and Years - coefficients. You will be asked to interpret slope and intercept. Look at second problem – look for and pick out slope and intercept.

Students ask which is slope – one multiplied by x or variable. The printout gives r and r^2 *(adjusted). She asks if students are out there today. They don't seem to be focused.*

Teacher: Assignment – Work on these actual AP questions that are problems 1, 2, 4, 4, and 6. Problems 1 – 5 on AP test average about 12 minutes apiece on AP tests. Question 6 takes about 25 – 30 minutes. Work quietly in pairs. All are regression questions from different sources.

Student: Student Q. Problem #1: a) Linear Model is appropriate - residuals are scattered above and below the horizontal axis.

Student: Student Q – "I think I know but I don't know how to say it?

She did say it correctly.

Teacher repeats the statement made by student.

Researcher states that it could be said as no discernible pattern. That is, not predictable.

Student: Student bb, in b) Value of b = 233. 517. Interpret in the context of the problem.

Student bb couldn't answer.

Student: Student W – as you increase # aircraft means for each year there are 233.517 more aircraft.

Teacher: That can be better stated. Part c) What is value of intercept? That is the constant value. How do we interpret?

Student: Student E – Each year you always have that # aircraft.

Teacher: Try something else in context of air traffic years.

Student: Student T – In 1990 there are that # of aircraft in service. Year 0 is 1990 and that is intercept. That is where we started off.

Teacher: Why did she say that? Think about that.

Teacher: Part d), what is prediction for 1992, Student I?

Student I had not gotten to that part.

Student: Student A – I just wrote down equation of LSRL and substituted x = 2. Answer is 3,406.

Student Z is called down for not being focused on assignment.

Teacher: What is the actual number? It wasn't predicted. How do you get it?

No one answered. Teacher then said Residual = Observed – Predicted. Understanding this will give you a good chance at answering questions of this type. This is a recurring question and in addition the lack of response to questions has been observed throughout the semester. Teacher asks for them to work on Question #2 until lunch. Students seem to be engaged and quite focused. She briefly explains that data can be read from histogram of frequencies.

Teacher: Why is a scatterplot or histogram valuable?

She explains histogram of frequencies.

Teacher: That is, # of defective items. How many days do you have 1 defective item?

Discussion continues as teacher is working on answer to Question #2. Students Z, -V, and -cc are off task.

Teacher: Symmetry of data is not equivalent to normality. Check conditions for inference. Normality requires continuity and % rules (68-95-100).

Student: Student T – On average there are 3 defective items.

Teacher: What do you mean by average?

Student: Student T – This means that the mean and median are the same.

Teacher: You can get obvious information from histogram – like average and mean are the same but not in scatterplot information. The scatterplot shows negative or positive association. We can answer things such as?

Student: Student bb – Such as number of days increase the number of defective items decrease.

Teacher: What does that mean for a manufacturer?

Class is silent. Students can do the mechanics but cannot go further and explain what the values mean. Student L seems to be able to make that connection.

Student: Student L – Maybe employees are doing better or machinery has been tweaked.

Students are asked which problem they are working on and are told to continue working on the others for homework. Work on #4 (third question) and remainder for homework.

Lesson 3 Linear Regression Continued

Teacher went over homework. They had finished question #2. They are going over AP Question# 4 from 2002.

Teacher: Part a) Define variables for the following: $y = 1136 + 14.63x$

Student: Student V – $\begin{aligned} &y = 1136 + 14.63x \\ &x = \#of\ \ passengers \\ &y = operating\ \cos ts\ \ per\ \ hour \end{aligned}$

Teacher: Or you can say operating cost per hour = 1136 +14.673(#of passengers).

Teacher: You must identify variables to get credit.

Student is not able to say what the variables mean.

Teacher: Coefficient of correlation r = 0.75? Interpret this.

Student: Student W – r is what we want, correlation was given.

Student: Student X – r = 0.75, moderately strong association and you can see it is positive.

Teacher: How do we know that? That must be in the problem.

Student: Student M – Do we need to say positive association or association is positive?

Teacher: Using r^2, did any one interpret that way – positive – how so? It must be mentioned in the description. It is critical to mention this.

Student: Student M -Is positive association better than signed association? Association or relationship?

Teacher: Yes, association is better.

Teacher: What about r^2?

Student: Student E – Variation is explained (57%) by variables.

Teacher: In what?

Student: Student E – graph.

Teacher: You mean variation in y due to x.

Student: Student F – variation 57% of change in # passenger seats

Teacher: That is better said, 57% of Operating Costs are explained by change in number of passenger seats.

Teacher: The relationship is between operating costs and number of passenger seats. For a range, look at the remaining values. Does line in scatterplot still provide a good model? No, data has negative association.

Student: Student aa – No, because points are changing.

Teacher: Explain, since points are not changing. What did change? Did you cover points in range? What is linear regression about? Line should show relationship between points.

Teacher: What do we call this line?

Student: Student bb – line can be called line of best fit.

Teacher: (Question to Student aa) – Are the middle points still going in the direction of line?

Student: Student aa – I think so.

Student aa does not understand the regression equation and how it relates to the data.

Student: Student K – It would have a negative relationship or association. The new line would have a negative slope instead of a positive slope.

Student: Student W – It would be a totally new line.

Teacher: You should write a standalone answer and give in the context of the problem. One should be able to read the sentence and understand what the problem is all about.

Discussion of AP Question – 1998#4.

Teacher: Think about how you will answer part a) like we noted on the board.

a) Student: Student U – $y = -20.583 + 24.392x$

Student: Student L completes the answer and says that x = # teaspoons of weed killer per/gallon of water and y = % of weeds killed

Teacher: Great work Student L. Excellent answer. She reiterates answer.

b) Use equation to predict % of weeds killed with 2.6 teaspoons.

Teacher – will it be larger, smaller, or prediction cannot be made than observed.

Student Z did not answer. Teacher caught him with work not completed.

Student: Student A – Plug in 2.6 into regression equation. She did not have it. I don't have $\hat{y} = don't\ have\ it$.

Student: Student C – Answer is 42.8% of weeds were killed.

Teacher: Given residual values for predicted. To determine whether actual is larger, smaller, or cannot be made, where do we look for residuals?

Student: Student C – sum of squares

Teacher: When looking at the graph do we look at prediction is 42.8 and can we look for 2.6 teaspoons? No, they give us *y* values on *x*-axis it is obvious, what do you think?

Student: Student J – above the point we look for residual.

Teacher: Residual = Observed – Predicted = some negative value. So value of predicted is too large.

Students are simply not getting it. There is no problem when students are asked to use the calculator to determine the predictor equation. However, they do not understand that the residuals are the difference between observed and predicted and what this means. This is not the first time the teacher has talked about this idea and they have completed many examples related to this topic.

Student: Student B – It is too large. Actual observed values are below predicted values.

Student bb asks a question that was just answered. Students have a tendency to not pay attention when others ask a question. Many students don't understand the idea of using residual to answer a question but are not asking any questions.

Teacher: These questions are straight forward. You should know how to answer these.

Teacher: Problem#6 — this is typically used to stretch the content beyond, about 25 – 30 minutes. It is intended to use your application of knowledge to another situation.

Student F is the only one that raises hand that has homework completed. Student I laughs and thinks this is funny.

Teacher – Put information into calculator.

L_1 = GPA

L_2= did successfully complete PHD program (dependent on CEU's).

L_3=did not successfully complete PHD program (dependent on CEU's).

Talking is going on among each other in little groups.

Teacher: First Part – What is an appropriate display of data (graph)?

Student: Student W – A bar graph.

Teacher: If you have the GPA along the bottom then?

Student: Student Z – You can have a double bar of successful and unsuccessful along the side.

Student: Student F – I made 2 different scatterplots.

Teacher: Let's try a boxplot instead of histogram to view comparison between L_2 and L_3. What do you notice about them? Give me the 5-number summary. Histogram needs same scale in order to compare.

Student: Student G – L_2: 2.9, 3.45, 3.6, 3.9, 4.0 – successful and

L_3: 2.9, 3.1, 3.5, 3.6, 3.9 – unsuccessful

Student C has a calculator question. This is a recurring problem. Students interrupt to ask basic questions about calculator usage.

Teacher: What do you see? What is striking? The median of successful is 3rd quartile of unsuccessful. Scatterplot is least favored way of comparison (to Student F). The best way to compare two data sets is with boxplots and bar-graphs. Dotplot is also a simpler way to see trends and skewness. You may see 5-number summary. If you use stemplot you can see if data is skewed. Part b) will be done in Chapter 14- significance tests on linear regression.

Researcher makes a comment – Key words lead you to what you have to do. Look for those key words. Answer the question in the simplest manner after reading the questions carefully. The parameter of interest could be mean, standard deviation, or proportion. In our case we need to compare. So ask yourself, how do I compare in the simplest way possible?

b) Mean # of credit hours versus other variable.

Teacher: Part c) Will a new person be successful if they have GPA 3.5 and CEU's of 14.0?

Student: Student F – Yes, look at $\hat{y}$ value. I substituted in y hat equation.

Teacher: Not exactly where you want to go. You need to look at two different regression lines. You must determine if this person's score lies closer to first or second regression line?

Student: Student T – Look at the two least square regression lines and see if the point lies closer to either line of LSRL.

Very few feel confident about the Advanced Placement Examination. The teacher said they should feel confident by May.

Discussion continues –

The topic is hypothesis testing on proportions. They are going over an assessment. The teacher emphasized that students must be thorough and think about what they were writing.

M&M activity on test

The students were checking whether the bag of M&M contained the appropriate proportions of candy it should.

The teacher explains that they need to work on the hypothesis and make sure it makes sense. It is not one of those that you fill in the blanks. Most people did well on the problem on the test.

Teacher: Some people are so set into, I know what I write, I know what I write that you think we always are rejecting and even when we fail to reject you are so convinced that the alternative hypothesis may be true, if you fail to reject the null hypothesis then you really are saying you don’t have enough evidence to lead you to believe that the alternative hypothesis might be true. You are going to say I fail to reject the null hypothesis that this bag of M&M did not come from the population, it did come from a population that matches the M&M company’s proportion of colors. I fail to reject that. So, it appears that it comes from that population that matches the company’s proportions.

Teacher: If you are not listening to the whole sentence there, it makes no sense at all. If I say the whole sentence and hiccup in the middle of the sentence I would have to say the whole sentence again. You have to really think about it. That is an important part about these problems. This said, you are going to reject the null hypothesis. If you choose not to reject the null hypothesis, sometimes some of you make a mistake along the way, and you get the wrong *p*-value, and based on that *p*-value you should fail to reject the null hypothesis, well if you tell me that you fail to reject the null hypothesis, then, you need to tell me what that means. You are telling me conflicting information sometimes. On this one, you would reject the null hypothesis because it did appear that the bag of M&M's may not have come from a population that matches the company's proportions. Don't get so stuck on writing a set statement but think about what you are saying.

Teacher: Any question about the M&M problem? Moving on to problem #6. This is a problem of association. Are the interventions associated with response or not. So, your null hypothesis is there is no association between the prior contact and survey response and the alternative hypothesis is there is an association. Write out the whole sentence. Are there any questions about that? This time you have a very small *p*-value so you have to reject the null hypothesis. At any alpha level it appears there is an association between prior contact and survey response. Are there any more questions?

Teacher: When I am grading your tests, if you make a mistake, in part b) then based on that part b) -answer, I am going to grade part c) and all the rest of the parts. You can't flip-flop and say something in part b) and change it up in the next part. You have to be consistent with what you found. You have to have convincing evidence all the way through. This is true of the AP exam as well. As long as it is not a major flaw, they will grade that way. Some of you had a major flaw with your Chi-square value, I take that back, if you had a flaw in your hypothesis statement, and I took off for that and if your conclusion couldn't be right because, that was too major a mistake. You need to keep those things straight. If your *p*-value was 0.048 and the real *p*-value was 0.053 then I could take your conclusion based on your *p*-value, if it was a minor error like that.

Students are still having problems with the language issues. They are trying to memorize standard statements to make instead of trying to understand what it is they have to show as evidence for or against and backing it up with the statistical values. Students are still unclear how to extract information from the prediction equation and residual values. They are able to compute values of interest but not to interpret these in the context of the problem. The majority of the students are between the verbal and transitional levels. They appear to be trying to memorize instead of applying logic to what they are doing.

Lesson 4 Linear Regression Continued

Homework is passed up to teacher. The room is quite noisy as usual at the beginning of class. They have been talking about linear regression. They will be talking about Chapter 14 – regression on slope which is the first section in the chapter. They will be reading to answer questions from page 763 – 767. I believe that although logical it is a big jump from the previous chapter studied. Long break while students read.

After reading break

Teacher: Since we have been talking about regression and scatterplots, it makes sense to me to talk about regression for slope. That is the first section in Chapter 14 and we will look at that today. We are going to be talk about how to run a significance test just like we did to check if we have a significant relationship between our variables just like we did with regression. It is going to talk about significance just like we looked about having a significant sample in our earlier work. Was that sample significant, did it tell us if our original assumptions, what we thought to be true were supported. You are going to be reading and answering questions. About 20 – 25 minutes.

Teacher: What does it mean? What are we trying to accomplish with this, by doing regression on slope?

Student: Student K – Not clear.

Teacher: I asked two questions, what are you answering.

Student K was not sure.

Teacher: What kind of Confidence Interval could we be talking about when it has to do with regression?

Student: Student W – How well the line matches the data.

Teacher: What kind of CI could we be talking about, what is it called?

Students don't have the confidence to explain what they read.

Student: Student T – Slope of the true regression line.

Teacher: What about the true regression line, what do they mean by the true regression line? Did we not get the true regression line with our calculator?

Student: Student K – The variables were unknown.

Teacher: What variables were unknown?

One or two students say the variables were α and β.

Teacher: Alpha was the intercept and beta was the slope. When we do a regression equation, with a set of points, we get that equation, but only for our particular scenario. Last night you were working (on the homework) with banks that were failing. The regression equation is only for our data – our random sample could be of 20 failing banks. Last night you were working with banks and their assets. With another 20 random sample points could it give you the same regression equation? No, yes, maybe it would give a different linear regression line. Every time we draw a random sample from the population are our results identical?

Teacher: Are the students or individuals identical? Since we have different individuals then our results would be different. So, what are we back to the CI for the slope of the true regression line, what are we trying to do here? We are trying to predict what? We are trying to predict the parameter of what? For the regression for slope it would be β, trying to predict beta and we did that with our sample. If we get *b* with our sample, we know what *b* is for our points, is that not β? No. It is an estimate of β. Someone said it is an <u>unbiased estimate</u> of beta. But, are they going to be equal? No. It is a good estimate of beta, at least we have somewhere to begin. So what is our confidence interval?

Student: Student W – How well our predicted value matches up to β?

Teacher: What do you mean how well? We don't know the true slope. We don't know. It is an unknown parameter. So what is our CI?

Student: Student K – We are confident that the true slope lies in our interval.

Teacher: What if I get that b = .379, what would confidence interval have to sound like?

Teacher: I am 95% confident, you guys are all about the confidence, you need to be all about the interval. What is the interval really?

Student: Student T – An estimate of where beta lies.

Teacher: Very good. So, our confidence interval is an interval in which we hope beta lies. A 95% CI means what?

Teacher: That 95% of beta is in there and the other 5% is out?

Student: Student T – 95% probability.

Teacher: Probability is a dirty word (when describing CI's). We don't want to say probability with a CI because that misleads us. What does 95% Confidence mean? Surely somebody here has heard me say this once or twice or five times.

Student: Student K – mumbles an answer.

Teacher: This is one of many samples. When we take a sample 95% of the samples that we take and make an interval this wide will actually contain the true slope beta and 5% of the samples that we collect and make an interval with won't capture β. Our interval either contains the true slope, however if we are constructing a 95% CI, then of the 100 we construct 95 will contain the true beta and 5 will not.

This is a topic that has been discussed for several weeks. It is still not clear to the students.

Teacher: If we are talking about a CI for beta then if we make an interval that goes on either side of our prediction, that we hope it will capture beta. Does everyone get an idea what we are talking about here? We are trying to make a CI which we hope will capture true slope β. But of all the CI, 95% of them will capture the true slope. We don't know if it does or not. We are 95% confident the CI will contain β. Does everybody understand that concept? That's not new for this chapter. One thing that we do with inference is that with regression we calculate, a slope with our regression, and then we try to predict the interval that will capture the true parameter beta. What else might we do with it?

Student: Student W – Test.

Teacher: What kind of test?

Teacher: It will be a linear regression t- test. Testing what? What is it testing?

Teacher: (To Student Y) – What do you think Student Y? How well they fall into a linear pattern.

Teacher: That is closer than what you started to say is how well *x* relates to *y*. How linear it is? How do we decide if it is linear or not? You guys just read this. How do you decide if it is linear or not?

Student: Student T – Based on p-value.

Teacher: Look at question 9 and maybe question 8.

Student: Student K – Residuals.

Teacher: Everything in this chapter has something to do with residuals.

Student: Student Z – Mumbles

Teacher: Not really.

Student: Student L – Strength of the linear relationship.

Teacher: Can't you get that from r?

Student: Student F – We are looking for correlation.

Teacher: What else?

Student: Student T – A straight line dependence between *x* and *y*.

Teacher: What does question 8 say?

Teacher: (To Student E) – Read passage.

Teacher: What does a slope=0 mean?

Student M and Z raise hand.

Student: Student M – A horizontal line.

Teacher: What happens to what as *x* changes?

Student: Student M – The variable *y* stays the same.

Teacher: If we had a scatterplot (graphed on board) that had no distinct pattern, what would the expected correlation be?

(Several people) –Zero.

Teacher: If I were fitting a line through that, where would we go?

Student: Student W – A horizontal line.

Teacher: Would a horizontal line be any better than a vertical line?

(Several Students) –Not any better.

Teacher: What is the slope of this line? If there is no pattern at all, that shows a linear relationship, then when we will test against a slope being equal to zero because if the slope is equal to zero that would indicate – No significant relationship between *x* and *y*. You would say that the slope is zero against that which says that when *x* changes *y* changes in a particular way.

H_o: $\beta=0$ If we can't reject H_o that means there is no significant relationship between *x* and *y*.

H_a: $\beta\neq0$ Alternative hypothesis – This is a two-sided test.

The context of the problem dictates the alternative hypothesis.

There is a better feeling for the idea of test of hypothesis in the context of the slope of the linear regression line. More students seem to finally be listening and participating in the discussion.

Teacher: Does everyone get an overview of what we are trying to do now? We are trying to predict the slope with a CI. We know that we have one slope that we got from our sample so we are trying to predict a CI that would contain our true slope beta. The second thing we are doing is that we are going to try to run a test on our data to see if there is a significant positive or negative linear relationship that draws us away from our null hypothesis. What was our next question?

Teacher: What does it mean for a p-value is such that $p<0.05$?

Student: Student W – There is a significant correlation between *x* and *y*. There is significant evidence that there is a correlation between *x* and *y*.

Teacher: (Paraphrasing) – There is significant evidence that there is a linear relationship. You have to say it to learn how to say it correctly. Student W do not keep from answering even if it's not entirely correct.

Teacher: We will do problem #1. It asks you to make a scatter plot with the data on page 760. Let's work though this. For this chapter you are going to feel like you are doing a lot of the same things you did in Chapter 3. First thing is to make a scatterplot. We will do a linear regression equation, and we will do a residual plot. We have to do all the things we did in Chapter 3. I need a calculator person. (Student A has a question). (Student Z becomes calculator person). For this problem you would need to sketch the scatterplot. Go ahead and do that. Put scale on each of your axes. Label them. Anybody having trouble let me know. Students are focused on the graphing and other activities.

Teacher explains the best way to draw graph is to choose two points on the line, draw the line and then fit other points. Teacher has explained all this previously. Teacher: Later you will work on the residual plot.

After a break to complete graph.

Teacher: Part a) Given explanatory variable, coefficient of correlation, regression equation, write yes or no with supporting evidence about prediction ability of equation.

Teacher: The last part of the question is important. Do you think that femur length will allow a good prediction of humerus length? The correct answer to that is not, yes or no. It is a yes or no with supporting evidence. We need a sentence here explaining what is going on. The researcher and I are very concerned that you are not fully answering questions. That seems to be the hurdle that we are going to have to go over. Give me a good full response. I need to hear from someone else in the class besides the usual people. There are many in this class that do not answer. What do you think Student R? Is it a good prediction? Will femur length be a good predictor of humerus length?

No response at first.

Student: Student R – Because our r^2 value is high.

Teacher: What does r^2 value mean.

Student: Student R – It's how our data fits the line.

Teacher: What does r^2 really tell us folks?

Researcher: If you want to answer Mrs. Teacher's question you should talk about r. If you are going to talk about r^2 or the coefficient of determination you should be talking about variation that our regression line captures.

Teacher: Why do you say yes? Anybody want to take a stab at why you say yes?

Student: Student T – Because of the linear data.

Teacher asks for more explanation.

Teacher: That is part of it but there is a lot more.

Researcher: The data does not all fall on the line. You have to tell me how close it is. How do you know that it's close? How do you know? You can't tell me I see it.

Teacher: Remember when we first started looking at scatterplots and we looked at the scatterplot that was enlarged and looked like they were a long way from the line. Then we looked at it after it had been shrunk and they looked closer to the line. So we need a little bit more data on that. We need a little more information. Everybody is scared to death of answering because I keep picking and picking. I will give you break Student T and ask someone else.

Student: Student M – High correlation value.

Teacher: What is a high correlation value?

Student: Student K – Data is less than .01 from being linear.

Teacher disagrees with statement and asks what he means by that. You can say that r is 0.994 and is very close to 1 and indicates a positive slope and the points are relatively linear. Give us facts, always mention the r value is positive or negative. Let's move on to next part of question.

Researcher: Student T said something very good when she mentioned how close points were to the linear regression line. How can you say that another way in statistical terms? What determines whether the y value varies from the predicted value given by the line? What else did you get when you did the linear regression and you got r? You got r^2. What is r^2?

Student: Student E – Isn't it the percent of variation in y explained by the change in the x variable.

Researcher: Great. So 98% of the variation in y can be explained by the change in the x variable. Student T you had the right idea but Student E said it right.

Teacher: Part b), Explain in your own words what beta, the slope of the true regression line, says about Archaeopteryx. Every time the femur length changes by one centimeter what happens to the humerus length?

Student: Student K – It goes up by 1.197 cm.

Teacher: So, our true regression equation says that the femur length changes by one unit the humerus length changes by β units. It may not be exactly 1.197 for the slope of the true regression line. Are there any questions about that? What's your estimate of β from the data? That is what we just said. What is the estimate of α from the data for the true regression line?

Student: Student W – α is -3.6595

Teacher: What does that (α) mean? Shows graph of line and intercept.

Student: Student K – Can't have a negative femur length. You have to start off with the smallest one they found. The value of –3.65 is wrong for a femur length. Femur length is always longer.

Teacher: Can't have negative value for femur length. What is the femur? I've got them backwards. The femur is always larger than the humerus.

Teacher: How do we get residuals quickly?

This was a technology question about getting residuals. The teacher got Student Z to show on the calculator.

Teacher: We use residuals from RESID list. Dump these into a list. You can't do statistics on residuals while still in RESID list. You get a sum of zero almost always – to Student W.

This is also a question that has been answered many times.

Teacher: We can perform statistics on this list and determine the standard deviation σ_x. That will tell us how much points vary around line. We assume normality for population. We get this from residuals.

Lesson 5 Linear Regression Continues

Question centers about problem 3 from activity 14.

Teacher: Your calculator will not do residuals unless you do regression equation.

Students are working on jet-ski problem.

Student: Student U – I have a question about standard error in problem 4. Table shows standard deviation on Ddays (x) and coefficient is 0.1889 standard deviation of Ddays. Standard error is around line. S is standard error around slope. Formula does not give value given in answer.

Actually, s is standard error about the line. It is the standard deviation of the residuals. The standard error of the slope is SE_b. That value is usually reported in a printout. It is

$$SE_b = \frac{s}{\sqrt{\sum (x - \bar{x})^2}}.$$

Teacher: Turn to page 762-3. I thought you would have trouble with this. It does not have to be calculated. The computer printout gives this information. Looking at calculating the standard error we do that with the residuals. On page 763 they have given the standard deviation of Ddays or x values. The coefficient was 0.1899 and standard deviation is given for the slope and there is where you get answer. S is standard error around *y*. What do we look at to know how *y* values differ from what we expect? CI formula for slope has standard error of slope and uses s in the formula given below that on page 762.

Teacher went back to study guide

Teacher: Look at #12 – Predicted values are varying according to a normal distribution. To check if predicted values vary normally we check what?

Student: Student Y – Residuals

Teacher: To check if *y* values vary normally, we make stemplot of residuals. Stemplot of sorted points is easier. Look at residual plot and make stemplot and look for outliers. With a small number of values in our sample we can check residual plot for strange values. Then these procedures could not be done appropriately. The procedure is as follows:

a) Check assumptions by making stemplot of residuals and looking for outliers. It will not be perfectly mound-shaped. These points will not be many so it can't be a perfectly normal.

b) Assumption (to Student cc) mean *y* response has straight line relationship with *x*

Teacher: For example – If x = 5, $\hat{y}(5)$ varies and vice versa. The mean of those values for a particular *x* value will lie on the regression line. Mean of *y* values for a particular *x* values form a straight line. We check this by (she asks Student P) – *no response*.

Student: Student J – Residuals

Student: Student W – No obvious pattern

Student: Student Q – Scattered

Teacher: Check residuals and then you talk about it. Draw residual plot and discuss what it looks like. This discussion is crucial on the advanced placement test.

Student: Student W – Sum of residuals is zero.

Teacher: This is always true.

Discuss pattern, spread, and how well it fits.

c) Standard deviation (she asks Student L) of *y* is the same for all values of *x*.

Teacher: How can we do this? Look at residual plot with even distribution of residuals. We look to see if there is no pattern of getting smaller or larger at ends. Are they balanced? Residuals stay kind of even above and below the horizontal line. If they start spreading out then something is wrong with standard deviation of *y*. We don't want to see for different *x* values having very different *y* values.

They will work on study guide.

Teacher: Work on problems 5 – 9.

Lesson 6 Discussion prior to quiz

They are working on problem 7 from homework.

Teacher: P-value of problem #7 is to be computed. The *t*-value = 7.2557. The degrees of freedom = 9.

The students have a discussion about whether it should be 8 or 9 degrees of freedom. They settle on 8 with the teacher's guidance because you lose two degrees of freedom when calculating standard deviation.

Teacher: Use calculator to do tcf(7.2557, 1000, 8).

Student: Student U – How do we word the results for CI in problem 5?

Teacher: Since slope is 2^{nd} round scores increase of 1^{st} round scores increase by 1. The second round scores increase by slope for every increase of one unit in first round scores.

Student: Student G – How do we get standard error by hand?

Teacher: You need to use formula

Teacher: You use residuals found in regression and follow outline on page 759-760 in textbook. Very rarely are you going to have to do it.

This is same standard deviation formula given earlier in the semester. They should see the similarity.

Student: Student M – What is the standard error about line?

The following questions are things students should know how to do. They have done this with other distributions.

Student: Student A – How do we shade for the t distribution?

Student: Student A – How do we find p-value?

Teacher repeats procedure after finding t value etc. Again this should be known. She asks for them to hand in homework. They are taking a quiz.

Lesson 7 Regression after using logarithms to linearize data

Teacher: To transform from log values back to exponential or in terms of y form we raise the value to power of 10. All terms to a power of 10. Are you guys with me. And that will cancel the log from each term. In other words it gives $\hat{y}$ equation. Leave in form you get.

Student: Student M – I got loss in the process.

Teacher: Taking values out to 7 or 8 decimal places makes your equation (to Student Z) very different from the one I have to 4 decimal places. If you try to predict you get different answers.

Student: Student M – I got lost in the process.

Student I, who is always lost, is talking with girl behind her.

Teacher: To graph data as a scatterplot you will find regression equation for List 1 and List 3. To graph residual plot use List 1 and RESID's.

Teacher: Let’s look at the residual plots.

Student: Student L – Mine was almost on axis.

Teacher helps Student L see why plot was not real good looking. This helps Student L see just the residuals instead of other data as well.

Student: Student F – What is this y equation?

Teacher: To see the equation from y_1 insert 10 ^ (of equation). That is the original curve equation.

Teacher notices that talking is getting them off task.

Teacher: (Is showing on the overhead calculator.) This is a curve. Now graph scatterplot of original data with curve.

Teacher asks if equation fits data pretty well.

Student: Student T – Is this the same as the exponential regression on the calculator?

Teacher: You should never use the exponential regression button to do what they are trying to do. It will give a different equation. The Standard Course Of Study requires us to use transformation of data (ie. linearize the data) to determine equation for this type of data. Questions? Did you get all that. Now, back side of page. That was model for growth.

Activity to Model Decay

Teacher: You should have M&M’s and you should have about 40 in cup. So for trial 0 it gives 40 M&M’s. You will continue to keep pouring M&M’s onto a plate and those falling with an M up are then put back into baggy. The remaining go in cup and are tossed on plate until they are all gone.

This was a very long activity. However, students were very engaged in the activity and it produced some interesting scatterplots to mimic exponential decay.

Teacher: Draw a scatterplot. If you need help just ask. Accustom yourself to getting *r* value when doing regression and writing it down.

The activity lasted the entire rest of lesson and discussion was to be completed on the following day.

Lesson 8 Linear Regression Discussion

They are working on a linear regression problem.

Teacher: What does *r* give you? The appearance seems to be that the line fits the data. There is a pattern to this data. Just by looking at it we can tell.

Student: Student W – We should graph residual plot.

Student: Student G – Residual plot would look linear.

Teacher: What would residuals look like? A line?

Student: Student cc – Looking at graph on Plot 4. The plot looks distinctively curved. So, that also tells us that a straight line fit is not going to work.

Teacher: What else can we do?

Student: Student cc –If you divide.

Teacher: If you take consecutive values and divide you will get the same values. What does that mean? It appears to be exponential. Student I what did you get in part b)? In part b) you got about 3 to 4 for the values. One more clue to exponential. Student Q read aloud. Besides looking at the graph, etc. Why would moths grow exponentially?

Teacher: Student Z – Why does this happen? If they reproduce based on number of crickets available what happens? What if they only have 2 to 3 baby moths? What if they die after reproducing?

Student: (Says out loud.) Then the population would remain constant.

Teacher: They have a short life span. Let's talk about this. Why do they have a short life span? Because they are small? Predators? So, since they reproduce more than themselves it would be exponential. Where else?

Student: Student W – Bacteria or growth of populations.

Teacher: How is China being helped with their tremendous growth in population? Just one child. What will be the effect? Where else is there population growth that is exponential?

Student: Student U – Technology.

Teacher: Excellent. Number of people owning I-Pods, money or investments should hopefully grow exponentially. Those are the kinds of things we are going to look at in the homework. We looked to see what would happen if we took acres (data) and logged (found the logarithmic values) that. We tried to linearize this data. We graphed logged acres against years and it looked pretty linear. Let's go to STAT calculate and look for exponential. One person read about exponential regression.

Teacher: For our class we will use logs to get data linearized. There are some students that use the exponential function and it is wrong. I will not check for answers. Questions?

Student: Student Q – My residual plot of y hat on years does not look like yours.

Teacher: Plot years and RESID's. Someone work on calculator at desk – Student Z.

Student Q still has errors in plot. Teacher goes through whole procedure of where data is and logged data and then does a linear regression on List 1- years and logged data.

Teacher: Once you get regression equation Student Q you now have the RESID's. Do stat plot of list 1 and RESID's.

Student Z is working with calculator at desk.

Teacher: Student Q is that what you got? There are some above and some below. Notice you don't have to have an even number of residuals above and below the line. Points are traced to see residuals. How do we interpret that plot? We did last week and need to do a better job on that.

Student: Student F - Interpretation – That it is a perfect model.

Teacher: Why is it a perfect model?

Student F timidly raises hand. Student M raises hand first but Student F is chosen to answer.

Student: Student F – Points are randomly scattered about the line.

Teacher: Yes, and no obvious pattern, therefore, the regression line – linear regression – is the appropriate model for this set of data. Question about that?

Teacher: Part f. Perform the inverse transformation. Do we know what that is? We must write log of (y hat). We need to raise 10 to that power. Just stop here, it is correct but hard to graph. We plug in year.

Student: Student A – Would you go over that again?

Teacher: This is the regression equation but it is log of y-hat not y hat. Those of you not here on Friday, (she explained how they graphed curve on data). I found the straight line to fit the logged data and transformed it to graph it on the original data.

Student: Student G – How do you get prediction?

Teacher explains and asks for problem number 2. Less than half the class had done #2. Student said they did not understand question – Student I. Student says she had gone through part c.

Teacher: This is similar to problem #1 and maybe we can work on it later.

They are going to work on Power regression. Teacher asks them to look at a page in the textbook that shows examples of this.

Teacher: We measure the length of a fish to calculate its weight rather than knowing the weight of a fish using a power model. The reason I want you to get an idea for this, that is, to determine what type of model it is whether it is an exponential model versus a power model is that you will have to make that decision on some problems a little bit later on: should this be an exponential model or a power model.

Teacher: You can't always trust your *r* value to tell you which is the best model. Because if we have data set that has a small domain, our *x* values are in a small domain, or range. Sometimes our data in a small range, our points will look linear, but when we are talking about the entire spectrum of all the different points we could have it would be an exponential model or power model. So we need to first look at what would be the best candidate for this data. Then, with that candidate we begin to explore to see if we can get a good fit. So for an exponential model we are talking about things like the population, money in a savings account, the number of trees that are exfoliated by the growing population of moths, that kind of thing. A different model, a power model, and I am sure that there are lots more out there, the thing about a power model we are trying to work with cost of an item based on its size, area, or calculating a weight, a three dimensional measurement volume, based on its length, two dimensional something of that nature. In those cases we look at a power model. We want to look at an example together, oh here's that problem, the weight and length at different ages for Atlantic Ocean rock fish. I won't begin to say the name, but I would like you to put these values, clear all of your lists, and put into calculator in L1 and L2.

A student asks about the other parts of the data like age.

Teacher: We want to put length and weight not age.

Teacher: Look at sum of L1, it should be 505.5 right?

Everybody agrees.

Teacher: I said it was 505.5, but I may have made a mistake. Check your data. The sum of L2 is 6759. Check your data, check your sums, if you don't match, you've got a point there in wrong. Now is the time to find it before we start doing all the other stuff with it. Everybody got it. If this were an exponential model what would we do? We would do log of what? Okay, we would take log *y* and then we would fit the length to the log *y*, right? That is what we are going to do first. But, don't put it in L3. I want you to put it in L4. I want you to put the log of L2, which is the log *y*, in L4. Now, let's back up a step. Do scatterplot of L1 and L2.

Student Z is at the calculator.

Teacher: Student Z did the regression with L1 and L2 as well, which you may or may not have done. There it is, L1 and L2. It is clearly curved, right. Student Z clear out your regression equation. Now, plot L1 with L4 and see what happens. A little bit different. That's not working. Sometimes it will look much more linear even when it's a power model. You can imagine that if we only had the first 5 points it wouldn't be so obviously curved. Now, for power model, we are going to go back to our L3 and put in the log of L1. So we have the log of *y* and our log of *x*. So, I want you to go back to L3 and get your log L1. Do a scatterplot with L3 and L4. Is that better? I am going to stop right here and I am waiting for everybody to get caught up. As soon as you get caught up, put your pencil down and look at me.

Some people are still working. Student M has a question about which list she had been talking about.

Teacher: That was L1 and L4. It was *x* and log *y*. Now, it looks linear. Student J and Student C are you there yet, almost? Does everyone have a graph that looks like this? Notice that I've been keeping track on the board of where everything is. I won't do that on your homework. I suggest that you do that on your paper. Just trust me. You will be lost if you don't. Once you get all of this done, the good fit looks like it was the log *x* and log *y*. Now, we will do linear regression on these lists L3, log *x* and L4, log *y*. Now we will put that in y1. Look at your r and r^2 values, pretty good, okay. Does that tell us everything we need to know?

Student: Student M – Yes.

Teacher: What else do you need to know?

Teacher: Student Z has graphed them, looks like it's fitting pretty well there. He is saying residuals. Somebody else is saying residuals.

Teacher: Student G said residuals. Let's look at the residual plot. How are we going to get the residual plot with this set of data?

Student: Student Z – Use L3 and RESID's.

Teacher: Interpret that residual plot. What would you say about that residual plot? Someone, I believe Student K, says, it has no pattern. No distinct pattern. What does it mean to have no distinct pattern? When you say something like it has no pattern, you are kind of implying that you know what I mean by that. Maybe I think you are looking for a pattern and it is good to have a pattern and it is bad to not have a pattern so this one has no pattern so it's bad. You need to tell me what it means that it has no pattern.

Student: Student M– This fits the line of regression?

Teacher: I am asking you. Tell me what you would write on your paper if I said interpret this residual.

Student: Student M – Because this residual plot has no distinct pattern it fits the line of regression well.

Teacher: You are there. Think about how you are going to word it. How would you interpret it? You said all the key components.

Teacher repeats what Student M has said: Because this residual plot has no distinct pattern it fits the line of regression well. Student M is showing some idea of what is going on. His wording is faulty but he recognizes that the pattern of the residuals or lack thereof is a good sign that the model is appropriate. This is a breakthrough for Student M.

Teacher: Let's work on that, I think it still needs a lot of work. But you got the right ideas. Who, can help? You know, if you sit here and never say a word you're not going to ever get better at it. What do you think, Student aa? What would you say? Because that residual plot is scattered, has no pattern, it fits the regression equation well, we need to work on that. Student K got us started. What would you say? Think hard and talk. All of you need to be thinking so if I ask you, then you will have something to say.

Teacher: Student aa add something to that, anything. The points are spread more towards the end of the *x* axis. Why do you suppose?

Student: Student I (a bit frustrated) there are not a lot of short fish.

Teacher: Well, look at the fish, look at the lengths of the fish. Because the *x*-axis is the log(length), look at the lengths of the fish.

Teacher: Are they a little more concentrated towards the end? So, is there a problem with that?

Teacher: Is that going to be a problem in this particular case? Think about it. Don't go off on a tangent, think about what we are talking about. Is it a problem that most of the points are over to the right?

Student: Student Z – No, it is showing that as they are getting older they don't grown as large, grow as much or as quickly.

Teacher: What do we need to say to interpret this plot? This is a hurdle I can't get you guys over. It doesn't matter what I do, I can't get you over it. It is going to take you to do it. What do you say Student G.

The teacher is very frustrated. Hints, leading questions, nor exhortations can make the students think about what is going on. They are not using any contextual evidence that they should know to answer simple questions. Perhaps the lack of participating in a discussion in a mathematics class has not prepared them for this type of course. If this is the case, then more needs to be done at the lower level mathematics courses to prepare them for this type of class and activities.

Student: Student G – What about the scale?

Teacher: Does it have something to do with how large the residuals are. Does it matter? This is to the class, does it matter how large the residuals are? The residual plot, what are we interested in?

Someone in the class says – sum of residuals are zero.

Teacher: We know the sum is zero. That is nothing remarkable. You are never going to find a residual plot where the sum is not zero. That is not really noteworthy. That is something we need to be aware of. What is important? I heard a word.

Student: (Several students blurt out.) – Scattered above and below the line.

Teacher: They are scattered both above and below the line with (kids add) no distinct pattern. What does that mean?

Student: Student K – I'm not sure if this applies to this chapter there is a curved pattern with a straight line,

Teacher: But this does not have a curved pattern so a line will be an appropriate model for this data. This data is not an appropriate model for this line – that is what you said first of all, I believe. Is that what he said at first?

Student: Student G – The linear model is appropriate for this data with the log *x* and log *y*.

Teacher: You guys are going to have to start saying that. Start saying the whole statement. Don't think that when I read your quiz or your test or your paper that you are writing that I will know what you mean. Say the whole sentence.

Teacher: Student A–Would you like to make an A in this class? Let's suppose she says, I would like to. If you hear I'd like to, but don't hear the question that I asked, and Student

G says I'd like to, that could get you into a lot of trouble. I'd like to cut class. I'd like to rob a bank. But, instead of I'd like to, she could say, I'd like to make an A in the class. I'd like to make an A in AP Statistics. Get specific. Say the whole "schpeel". Don't assume that just because the question is, "Do you want to get an A in the class?", that you can say, yup. You have to say the whole thing.

Teacher: Let's move on. You've got that done, Student K, and others established that this is a pretty good model, that a linear equation is a pretty good model for this data based on the residual plot. We also looked at *r* and it looked pretty affirmative, so, now what are we going to do? Now, what is our regression equation? What do we have? Is this what you guys have?

Teacher: That's what's on our calculator. But wait a minute, what is y hat? It's a prediction but it is the log of *y*-hat for predicting and what is *x*? It is the log of *x*. Okay, log *y* hat = number times log *x*. When we write up our equation we need to put that in.

Student: Student M– Is that just for this problem that we do we use log?

Teacher: For all our power models we use logs of both variables. If we use log of both *x* and *y* to get an equation, then we would write this equation. We need to write not only log *y*-hat but also log *x*. Now, we just need to transform this, untransform it, get it curved again, we are going to do that by saying, 10 to the power *y*-hat = 10 to the power. That will give us *y*-hat = 10 raised to -1.89994+ 3.0494. Now, we need to use this to make a prediction we substitute our value here. Like when we are doing does a fish measures 22 inches. We would find the slope. Why don't you try this? Are you going to actually plug it in? Go to your table.

The class finishes the problem together.

The students are continuing to make breakthroughs. Some students are beginning to see that some of the same questions asked in the study of linear regression are asked to be answered when doing regression using transformation of data. There are moments when the students are involved and focused on the ideas presented by the teacher but they have a long way to go. In order to be prepared for the advanced placement examination they will have to ask these pertinent questions and will not be able to rely on the teacher.

Lesson 9 Linear Regression Continued

Teacher: Get out homework. I am passing back to you problem #55 from Chapter 3. Okay, listen there are all kinds of scribbles on your paper. Changes and changes were made to grade this appropriately. I listed the things I asked you to do.

1. Draw the scatter plot and label it.
2. Find the regression equation and write that in context.
3. Find *r* and interpret it in context.
4. Calculate the residuals.
5. Make a residual plot.
6. Write an interpretation of the residual plot.
7. Comment on unusual points and remove them.

Teacher: I wanted to see the residuals for each *x* – explanatory variable. Some had a list and/or graph. You should have a max of 24 points. Max here was 22. You should have score out of 24. The purpose of this assignment is to give you experience and the opportunity to thoroughly explain your problem. We are not there yet and we don't have much more time to do this. Please make it accurate and you need to work on that. You will be drawing your graph on blank paper and they will have to be close to accurate as possible. You can't take forever on that. Regression in context – equation should be premised by a sentence of what you are writing. Interpret *r* in context- say value and if it indicates a positive or negative association and how weak or strong it is. Use some of those terms. Comment on r^2 is also appropriate.

Teacher: The next thing, it really is good to show your residuals in some sort of list or table for each *x* value. Two reasons, that way, if you have some sort of unusual point on your plot you can quickly find it on your table. This could be a bad point put in incorrectly. Other reason, it is unusual for other reasons. Interpret residual plot. Who got 4 points? Very few got 4 points.

Student: Student Q – Residuals are scattered above and below the line y = 0 line with no discernible pattern.

Teacher: Listen to what she has said. This shows that the linear regression line may be a good model for the data. The last thing you were supposed to look at was for points that were outliers or those that might be influential. Lots of you identified those that might be outliers or might be influential but very few went on to explore that. You need to explore that. What makes a point an outlier?

Student: Student F – If removing it changes the regression line.

Teacher: That makes it influential. What makes it an outlier? – Outside the pattern.

Teacher shows a graph on the board. What makes it influential?

Student: Student T – Line significantly changes if you take it out.

Teacher: You must remove one at a time if they are outliers or influential. I wanted to see what you said about that. Discuss it both ways. Any questions? Put papers away. I have some quizzes to give back. Tomorrow is our last make up day before progress reports go out. Let’s go on to talk in just a minute about upcoming assignment.

Teacher: We have not tested on Chapter 3 or Chapter 14 and here we are finishing up Chapter 4. We need another test grade. I need it to be done well. This is your project for regression. Now that we have linear, power, and exponential models you should have more examples of the types of data you can look for. You should be trying to find a set of data.

Not very many had a data selected. Student W had.

Teacher: You have used up 4 days to do this. Read over first paragraph. You need points, randomly selected. This tells you what you need to attach. You will follow one or the other set of instructions. Every part needs to be done completely. You need to go by the checklist. There are twenty items for you to include in your project. I would like you to word process and make graphs and insert in appropriate spot. Follow directions depending on what type of model it is. When you are finished you should have in order as they appear on here and logically written. It should be done completely. You have a checklist to follow. Paper should be done in narrative form. What does that mean?

Teacher: Don’t write #1, #2, etc. Write a sentence and talk and explain what you are doing. Talk in 3rd person, the researcher. Today is Tuesday it is due Wednesday of next week. If you have data now you should be able to knock this out extremely well in a couple of hours. The data finding is the big problem. Go looking for data and start on it. Do not wait for next week. One point is given for every day early. There will be minus 5 points for every day late including weekends.

Student: Student E –What kind of data?

Teacher: Appropriate data for project on linear regression or transforming, at least 20 points in your data and you have to reference where it comes from. This is a suggestion – Look this over tonight because if you don't understand something about it you can ask me tomorrow. Talk to me about this every day if you have to. This does not involve a poster nor is it a big thing. This is not like the senior project. Three hours maximum to complete.

Student: Student Z – How do we find data?

Student: Student V – Use the internet.

Student: Student G – Can we use data out of the book?

Teacher: No, but you can use the book for ideas.

Student: Student W – I have lab data from class.

Teacher: I will need documentation. Let's go over homework.

Student: Student K – log $y = a + \log x$

Teacher: Okay, that should be a power model because you are talking about comparing height and weight.

Student: Student dd – Letter b) Explanatory vs response variable

Teacher: Why did you choose that way? There is a small problem in small sections of class. Eye contact should be on me or work not on each other. Why would you choose height as explanatory variable? And weight as response variable?

Student: Student dd – There are height and weight factors. Explanatory variable is height, weight is response variable because we do weigh more the taller we get.

Teacher: Plot the data, what did it look like? Student M did you put data on paper?

Student: Student M – Yes.

Teacher: Show me the data shape with finger. It looked like an ess going up. Part c) what kind of transformation did you use to linearize this data?

Student: Student aa – I said log *y* = something + something else times log *x*.

Teacher: You gave me the equation of the linearized data. In that equation you told me what you did.

Student: Student aa– I used power regression.

Teacher: Tell me what you did. You took log of y and what else?

Student: Student aa – and log *x*.

Teacher: You took log of both explanatory and response variables. Then you plotted what?

Teacher: (To Student R) – What do you plot to get the linear data? If you plot height against weight it is curved. What are you plotting to get it linearized?

Student: Student H – I plot log ht and log wt which linearizes the data.

Teacher: Log ht and log wt. That is how I knew Student aa did that because of her equation. That is what you were doing to linearize the data. I want you to be able to say that. I linearized the data by taking the log ht and log wt and graphing that as a scatterplot. It became much more linear. Now, do least squared regression and find correlation coefficient. Say your equation again, Student aa.

Student aa has the wrong answer.

Teacher: That is not at all what I have. Something doesn't look right with the equation you gave me Student aa. I've got log *y* hat = -1.391+ 2.0029 log *x*. Is that what you guys have? The next part says to construct residual plot. What was the correlation coefficient?

A student answers r = .9997.

Teacher: Does that mean that we are sure that it is linear? No, it doesn't mean for sure that it is linear. How do we find out if it is truly linear?

Students say, "We need to look at residual plot." They are getting quicker at answering this appropriately.

Teacher: So the next part is the residual plot. You are getting better at looking at residual plot for linearity.

Teacher: Student S – How did your residual plot look?

Student: Student S – There was no clear pattern and they were randomly scattered above and below the line.

Teacher: I agree. (Teacher shows with hands.) Yes, they were sort of scattered. Is that what you have (to the class)? Student S said that there is no distinct pattern. They were randomly scattered above and below the line. Student S does a good job at explaining residuals. That tells her what? What does that mean?

Student: Student S – that it is a good fit.

Teacher: That tells her our new regression line fits our new transformed data pretty well. Is everybody there? Does that make sense? Interpreting the residual plot - you have to say this is what it looks like and this is what it means. Not just this is what it looks like. She did a good job with that. So the equation log y hat = -1.39123 + 2.0029*log x. That is the linear equation that fits the linearized data.

Teacher: We are down to part e). Perform the inverse transformation and write the equation for your model.

Student: Student V – Which two lists are used to find residual plot?

Teacher: I would use my explanatory variable (*x*) and RESID's. For this particular one it's going to be log *x* and RESID's. Is there anything else before we go on to the last part?

Teacher: How do we do the inverse transformation? We have to get rid of what? – the log on the *y*-hat. You do that by raising both sides of the equation to power of 10. Some students are doing well with this. It gets down to *y*-hat = 10 raised to (-1.39124+2.0029log *x*). Use your model to predict how many pounds a 5 foot 10" adult would have to weigh in order to be classified as seriously overweight. Do the same for a 7' individual. The five foot 10" would have to weigh 201.5 pounds. Plug in 5 feet 10" into your inverse transformed equation. A seven foot weighs 290.3 pounds. Do you have any questions about that problem?

Teacher: For Problem #6. Raise your hand if you completely finished #6. (A few raised hand). Good. How did the data look when you plotted it in part a? Show me with your finger in the air – Student X. You were not here. Student C – You didn't do that, you didn't graph the data?

Student I is called, and makes a weak attempt at showing a curved data, and laughs.

Teacher: Is this your original graph? The original scatterplot might be looked at on calculator.

Student O shows how hers looks. Teacher shows how hers looks.

Student: Student I – I am a bad drawer.

Teacher: Look at your scatterplot, on your calculator, to get a general idea of what it should look like. It is definitely curved. It says that the points display neither a linear nor and exponential trend. Nevertheless extend the table by recording the ratios of salaries from year to year. Which year most disrupts the trend? So if you do the ratios, what year did things get out of whack?

A student blurts out between 1994-5.

Teacher: I don't agree. For 1990-1 it was a much more disruptive ratio. My ratio was 1.539. My ratio for 1994-5 was .901. Look carefully at this, guys. It can be confusing at times.

She explains that the one disappearing in the data is not enough to say that was a disrupting change.

Teacher: Part b) says to transform the data as if exponential.

Student: Student H– We will perform log of just *y*.

Teacher: Observe that 5 of the points line up nicely but the remaining 4 appear to be out of line. Which four points are not in a linear pattern? If you take a look at that what do we find? Even though this was not exponential (salaries could be exponential) it is appropriate to look at it as if it were and see what happens. This makes it easier to see which years were out of line. What was happening around that time? I am thinking there was a strike.

Teacher: Take a look at that and moving on to part c) it says delete the four points that are out of line. Perform least squared regression on the remaining transformed points. How well does it fit this? Here is the regression on just the remaining 5 points. Everything else was pretty linear.

Teacher: Part d) says there was a strike, how did it affect the overall salaries? Just momentarily not for the future.

Teacher: Part e) average salaries have been higher than median salaries, how do you explain this? How many baseball players are on a typical baseball team? About 30- ish players, how many make a lot of money? How many make a smaller amount, but still a lot of money. So why does that make the average salary higher than the median?

Student: Student I– That some make a lot more that would make salary appear higher. It affects the average.

Student M has a question about ratios. This has been covered in Algebra II, so this is not a new topic.

Teacher: You take consecutive ratios of data. The similar ratios point to an exponential model. They could be done all in either order lower/higher or higher/lower. It also is used to check for points that are out of line, especially for this problem.

Student: Student A – What about question 4.2?

Teacher: Many people had not finished that yesterday. I asked you to try to finish that yesterday for today.

Student: Student F – I have a question about part h).

Teacher: Compare prediction for 1997 to actual value for year 1997. Use your equation and plug in 1997 and compare to what it should be. Oh, you used the coded year of 17. I got 6.1144, and comparing to actual, which was 5.413, we are off by a lot. Not a good estimate.

Lesson 10 Regression using transformation on Power Models

Teacher: I collected that yesterday, there was a few of you with all kinds of issues. You were out on that day or out on another day. If you had part of it because you were absent then when you resubmit it, give me all of it again so that I can look at all of it again.

Today there are a couple of you that need to stay afterwards. I have this quiz that I need to give back.

Teacher: Let's look at your homework for a minute. Problem 12, as I started looking at problem 12, I bet everybody in here did this problem backwards from the way that I would do it. Problem #12 dealt with heart weight and length of the cavity of the left ventricle on mammals. I bet since heart weight was on the left and length of cavity was on the right you took them as explanatory and response in that order. If that is what you did then your final equation should be log (ltcav) =5+.316*log (ht wt). I have a good reason for doing it the other way. Generally we use the height to calculate the weight. Even though those were reversed in the columns, it made sense to me to reverse them for my explanatory and response variables. If you did it the first way, don't fear. But, I do want you to start thinking about what makes sense.

Teacher: For problem #12, I sat down last night and I pretended that this was your project. I thought – What do I want my students to show me on the project? I first looked for a short narrative. So, I wrote on paper and I will read it to you which is slightly boring, I want you to hear what it means to not write this paper in first person. I started out with an explanation. My title was:

An analysis of the data of heart weight and length of the cavity of the left ventricle.

Then I copied the data that I was using. So, anyone reading this would know what I was talking about.

Then I looked at this and I said that the heart weight which is a three dimensional property should be proportional to the cavity length of the left ventricle a one dimensional property. So, a power model is usually appropriate in this kind of setting.

Then I did a scatterplot.

A scatterplot of the original is shown below. Although the data appears to be exponential a power function model makes more sense in this setting. The regression equation for this data is should be in the form weight = Constant value * length$^{\text{some power}}$. That is the type of equation that should give us a good representation of that model.

The data needs to be transformed, a regression must be performed an appropriate least squared regression line. To accomplish this, the log of the cavity length and the log of the heart weight must be calculated. The log values are shown below. Plotting the log weight vs. the log length a linear pattern further indicates that a power model is indicating an appropriate choice for this data.

What would confirm that the power model is a good model?

Someone answers – the residuals.

The least squared regression line log y-hat = -.1364+ 3.1387(log x). This equation can be calculated by performing linear regression on the transformed points. The new correlation is 0.997. The residual plot shown on the next page confirms that the power model is appropriate.

I've calculated my residuals and I have shown them to you.

I show the residual plot and then I say:

Because the residual plot shows the residuals randomly scattered above and below the line with no discernible pattern it is confirmed that a linear model log y-hat = = -.1364+ 3.1387(log x) is a good fit for the transformed data.

It is still fitting the line.

Performing the inverse transformation as follows provides a power function that is a reasonably good fit for the mammal heart data.

I show you how it fits the transformed data and how it fits the original data.

Finally, the predicted heart weight in grams *y*-hat= 10^ (-.1384+log cavity length of the left ventricle)

So then I have to put it back in the context of the problem.

Teacher: Did I expect you to do all of that on your papers last night? No. But, do I expect you to do that and more on your little assessment project? Does it sound like I am chatting with the researcher and talking about how this is cool stuff?

Someone says – It sounds like a textbook.

Teacher: That's exactly right. It sounds like a textbook, because it is an educational document. You are writing a mathematical paper, something that could be published in a statistical paper. Not, like something that you could pick off the floor, somebody's friendly letter. Does this give you a little bit of an idea of what I am looking for in your project? I wanted to give you a good model of what I was looking for and more, because I asked you for a few more things for the data that you are using. How many of you said that you were awesome with EXCEL? Students L, Z, and T. Well I've never done graphs with EXCEL and I kept having a few problems. I couldn't get the label to stretch all the way along the graph. I spent a good amount trying to fix that and I finally gave up. I just typed it up. I glued it on. But that worked too, and then I could make a photocopy and you would never know it has been glued. Hint!! Including the time I spent doing that and keeping in mind that I am the teacher and I can work pretty fast through a lot of this, I knew what to do. It took less than an hour. You can do this kind of paper without spending the entire rest of your life on it. I do expect a high quality paper.

Teacher: With that in mind I have done something nice, I hope, for you. I scheduled the computer lab for today. How many of you have found some data?

Not many raise their hands. I can't see any raised hands. One child had exponential data. He knows this because he looked at it.

Teacher: You might want to look at the nature of the data as well. I want you guys, first of all one last thing about these problems; I can talk about these in the computer lab. I suggest that you take your book, your study guide, your calculator, and some paper.

IN THE LAB

All are trying to find data. Teacher is helping some students. They were looking at many sites. They were on task but I don't know how successful they were.

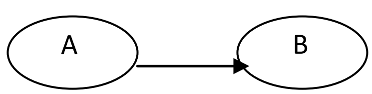
Back in the Classroom

Study guide for Chapter 4 (Section 2): Causation, Common Response, and Confounding Relationships Between Variables.

Student: Student F – I have a question about causation, common response, and confounding.

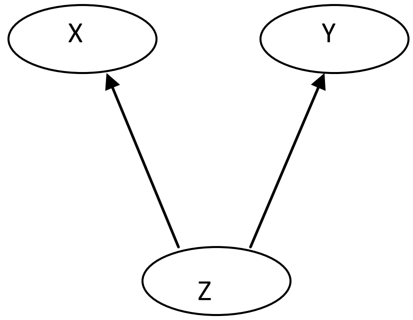
Teacher: The book does it this way, some rendition of this.

Causation



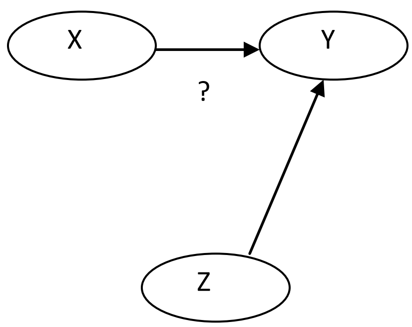
A causes B. The Sanchez family degree days causes gas consumption.

Common Response



It looks like *x* and *y* are associated and *y* is responding to *x*. But what is happening is that they are both responding to something else. For instance, mathematics and reading scores on the SAT look associated. But increasing your math score because you got tutoring in math would not cause your reading score to increase. They are a common response to a lot of other factors like your home life or educational status of your parents. The higher you parent's education the better the vocabulary used at home. They put more emphasis on homework and they can help you with the homework. There are a lot of factors that affect reading and math scores.

Confounding



There is some type of degree of causation, where *x* is causing *y*. But, there is also something else that is causing y. Confounding means that we can't really separate what part of the change in *y* is caused by *x* and what part of the change in *y* is caused by *z* the confounding factor. This is important to get under your belt.

There is always a lot of confusion about confounding.

Are there other questions from your study guide? Can you think of anything that we have talked about in the past or previously where confounding has come into play? This class doesn't talk a lot, that is scary because you need to do that on the advanced placement exam.

Teacher: The last section of Chapter 4 really does not have anything to do with the first two sections. It is the final section of the book. Do I hear a cheer? It is talking about relations in categorical data. We have already done a little bit of work on this when we were working on probability. It would have been better to do this section before we did our sections on conditional probability. We will be talking about conditional distributions in this section. It will be very similar to conditional probability. You will have as your reading assignment pages 215 – 226. Work on problems 19 – 24. Those are not long drawn out calculations, instead they are fairly simple, answer these questions problems. I want you to make good use of your time and tonight to seriously find some data. You have homework tonight, find your data. I am going to limit your out of class work so you can work on your project. That does not mean you go home tonight and say I don't have any homework and I can put it off until Tuesday night. If you have any questions on the problems assigned in this chapter let's get those differences ironed out today. Go ahead and get started and work quietly.

The teacher always apologizes for the lack of discussion of the students. Only a handful of students answer on a consistent basis and they are not your more able students.

Lesson 11 Discussion of Prior Assessment and Homework

Teacher: We want to get better at it. Therefore, every time I give you a quiz or a test I like to go over it. I am trying to tweak the way you say things. Did I ask anyone in here to read your null and alterative hypothesis on your paper? Student L, even if I gave you full credit on this problem, I would like for you to listen to what Student L has said because Student L probably had the best hypotheses in the classroom. You need to write two things with your hypotheses, you need to write it with symbols, and then you need to explain what that means in the context of the problem.

Student L reads her work.

Student: Student G – I have the same exact thing.

Teacher: Others could have been stronger.

Teacher: Beta should not be, not equal to zero in the alternative hypothesis. The reason being, that they are interested in the child's growth and her height will not be diminishing at this early age. Now, when you get to be very elderly, your height sometimes goes down because of arteriosclerosis. At a young age she should be increasing in height. You are expecting as her age goes up her height should go up, so it would be a positive linear relationship.

Teacher: I also asked for you to write in #2 the equation of least squared regression line and when you do that you always need to identify what the variables mean. You can do that in the statement itself or write your statement and then say *y*-hat = blah blah. Someone that made this up yesterday went way overboard with that. I believe it was Student T. Just went overboard. You need to say what Student G? Calculator gives equation.

Student: Student G – I got it wrong.

Teacher: Okay Student H.

Student: Student H – (reads answer)

Teacher: That was a good answer, I need a better answer. Who can give me a stronger answer for that? I am going to start a little extra credit program that is based on voluntary strong answers. I will not tell you how much credit. I am not getting much response from this class.

Nobody wants to answer. Finally, Student L answers. Teacher says it's a bit stronger.

Teacher: We can find predicted height. What is the equation? What is a residual = observed – predicted. We can get predicted height.

Teacher: The t-statistic is calculated in what way? Most everybody got this right. I am asking for a volunteer. How did you calculate this? Yes, Student Q.

Student: Student Q – reads answer.

Teacher: How many of you included a graph with your answer? If you had a t-distribution where would 18.78 be located with 4 degrees of freedom?

Student: Student E–It would be way down in the right part of the graph. It would be way down in the tail.

Teacher: You can sketch and say where it is on the graph.

Teacher: Number 4 there are 4 degrees of freedom.

Teacher: Number 5 the probability of $t > 18.785$ is very close to zero. Don't write 2.363 E -5 ever. You can write 0.00002 not zero. Don't write calculator speak. Don't say it's zero, but when you talk about it you can say that it is approximately zero. So, what are we going to do and why are we going to do it?

Student: Student F –reads answer.

Teacher: We know there is a linear relationship. We don't know if she is on target with her height. Any other questions about 6?

Teacher: Did I ask someone to read number 7? Student Y, did I ask you to read 7? No, what was your t* value? 2.766. What were your two values for your confidence interval? She reads them -0.3664 and 0.43996, what does that mean? What was the context? Since, none of you will answer, I will answer. This means that for every year of child's age, every year that she gets older, her height increases somewhat between .3833 and .43996 units. Is that what you did? How many had context written on their paper? How many points were associated with the word context? Are there any questions or comments about 14A?

Teacher: Student L read your answer about Confidence Interval.

Teacher: On the back side, you guys did pretty well on the scatterplots. How would you write the regression equation for #2 in the context of the problem?

Student: Student E– reads answer.

Teacher: That was a little iffy. If you drink enough alcohol or wine, will you decrease the death rate more and more? No. So, this really needed a little bit more work. *She rereads question*. You would say average amount of alcohol per person. It sounds like you are extrapolating. The null and alternative hypotheses, what would you say? Student U.

Student: Student U– reads answer

Teacher: That was good, any questions about the null and alternative hypotheses? Student U has reversed the sign on alternative hypotheses. Some of you are reversing those. For slope the null is always that there is no association, and the alternative is that there is some type of association. Most people did well on 4, and 5. Student Y read answer to 6.

Student: Student Y – reads answer – it is very low.

Teacher: Please go from small to large on answer. What about the context notation?

Teacher: Get homework out. We will read our homework questions and answers out loud. First problem was 19. Before you read problem 19 what kind of things did you read in this section? This was a reading from the night before last. What did you learn in this section about interpreting correlation and regression?

Student: Student M – A lurking variable can drastically change your data.

He did not address question.

Teacher: A lurking variable can change your data. What does that cause, what does it make us think? Give us some examples. A little more concrete. Think about it a little more and I'll come back to you.

Student: Student Z – Variables can change perceived associations.

Teacher: You need to speak out loud.

Student: Student Z – Variables can change perceived associations.

Teacher: You are throwing around some words like confounding and common response. I need a clear answer.

Student: Student P –Association between x and y is really there or not there but is hidden by another variable. So changes in *x* may not cause changes in *y*.

Teacher: I need to clap. What is another way of saying that?

Student: Student L – Association does not imply causation

Teacher: What does that really mean? Let's break it down for those that cannot get a grip on that.

Student: Student U: Just because two variables are associated it does not mean that one causes the other to change.

Teacher: I still want to hear more. I am not abstract. I want a real example. Give me something real to think about. I am pretending to be your average person. Give me something concrete to think about.

Student: Student L – Just because there is a lot of damage in a fire it doesn't mean that firefighters cause a lot of damage when putting out fires.

Teacher: One of the homework problems in the book asked you to look at the association between the number of firefighters at a fire and the damage caused by the fire. If the number of firefighters implies the damage, is there a relationship there? Does the number of firefighters cause the damage? What is the underlying factor there?

Student: Student G – The reason there are more firefighters is probably because the fire is bigger. There is more damage because the fire is bigger not because there are more firefighters.

Teacher: What do we call that? *Student G didn't know*. The answer is common response. That was much better.

Teacher: I really understand that now. What else did we learn in this section that might be important?

Student: Student E – We learned about extrapolation. If you use the linear regression line to predict a y value for a particular x value and it is not very good.

Teacher: Why do we make a line to predict with if we can't predict with it? You needed to say something just a little bit more. You said it is not a good idea to make a prediction with the regression line for a value of x. Why do we find the regression equation for? What is different about extrapolation?

Student: Student E –It is outside your range of data.

Teacher: Yes, it goes outside your range of data. If we go beyond our x values, either way, you would be extrapolating. I don't know if I understand when not to do that? Give me an example of when it would not be a good idea to do that.

Students are really getting into a real discussion. They are reacting to the teacher's questions. This is a better discussion than during other lessons. It still will be a problem for the students if there is no one there to prompt for complete answers.

Student: Student W –Say you are trying to predict the future level of US in debt, and you only have a certain range like to go by like from 1990 to now. You want to see what the future debt would be following that pattern it's not necessarily, if you extrapolate too farther out of that date it's not necessarily going to reflect because other changes may happen which might not accurately reflect your prediction.

Student W has some good ideas but his language gets in the way of his good ideas.

Teacher: How far beyond 2007 do you think it would be safe to use that prediction?

Student: Student W – 2008.

Teacher: Before we predict 2009 what might we want to do (*several students said use 2008*). Excellent.

Teacher: Give me another example of when we could get some really strange answers maybe from the homework.

Student: Student Z – When the slope is negative and you would be going towards the negative numbers and it could not happen.

The teacher got excited because that meant he looked at the homework.

Student: Student A – Using the data about farms in the US we would end up with negative number of people farming if we predicted for 2000. The data was from 1935 to 1980. This is a poor choice to extrapolate.

Teacher: Even with a positive slope we could not predict far back. We do not know what was happening way back then to affect data.

Teacher: Let's go through the homework, omitting the ones we have talked about. Relationship was negatively related in number 19 – r^2 was 97.7% variation in people who farmed could be explained by change in time. We could go to zero farms but not negative farms. Number 20 – Read it Student dd. I love this problem. What is the explanatory variable?

This student normally does not volunteer to answer.

Student: Student Z – The tea

Teacher: What would be the response variable?

Student: Student F – The cheerfulness of the patients

Teacher: What is the lurking variable?

Student: Student L– The visits by the college students

Teacher: The fact that people are visiting weekly. How many of you do some kind of service at nursing homes? *Several raised hands*. Those people love for anybody to come to visit them. For number 21 – this relates to something we studied a while back. The data gives an average (emphasized) steps per second for a group of top female runners. If you had all the data for each runner should the correlation be lower, higher, or the same for the original data.

Student: Student E – I expect it to be lower because averaging all the data will smooth out the variation.

Teacher: By using all the data, there is a lot more variation than using just the averages. The next few problems are numbers 22 – 24. These are about causation, common response, or confounding variable. Draw a diagram.

Students F, Z, and K volunteer to go to the board –

> Student Z – Confounding– Number – 24 – big hospitals treat serious cases, more specialists, more diagnoses
>
> Student F – Common Response – Number 23 – both shoe and age increase along with reading comprehension
>
> Student K – Common Response – Number 22 – positive correlation

Teacher: You can sometimes have more than one of these problems with variables. Today we will look at something new and different before lunch. Maybe we are not. *She had lost some papers*. Work on section 4.3 problems.

After a short break.

Teacher: What is Simpson's Paradox?

Student: Student Z – It's when we look at information and it appears like in the example that they give about the hospital. One hospital has a higher death rate when taken out of the total number of patients. When broken down by poor or good condition of patients the reverse seems to be true for the hospital.

Teacher: It is a relationship that when you have information put together it looks like you have one type of relationship but when you break it apart you have a different type of relationship. Using your categorical data you kind of have to be careful about what the relationship seems to be. You need to look at the whole picture and then you need to look at it separately. We will take a look at the example on page 224. Look at problem 40 and 41.

Teacher: Problem 40 (to Student H) says what? Read the paragraph.

Student: Student H –*Reads the problem about school admittance into law and business schools by female and male students.*

Teacher: We have two schools and males and females that are admitted and denied. Do part a) which is to make a two-way table. This is part of homework. How is your table looking?

	Admitted	**Denied**	
Males	490	210	700
Females	280	220	500
	770	430	1200

280/500 =.56   490/700=.70   220/500=.44   210/700=.30

With the original table 480/600 = .8   180/200 = .9   10/100 = .1   100/200 = .33

Teacher: When you calculate it this way, is there a higher percentage of males or females that are admitted to both schools? Yes, females. Now, look at your data with the combined admitted and denied for both schools and try to determine why that happens.

Student: Student T – Since there are fewer females applying to the schools there will be fewer females admitted.

Teacher: There are not fewer females applying to law school.

Student: Student T – As a whole. There are more males that apply over all.

Student: Student Z – There are different percentages for male and female applying to the two schools.

Teacher: Looking at those that applied out of the total amount 1200 what are the percentages of those male and female applying and admitted and denied?

Student: Student aa – 480/1200 = .40.

Teacher: For business, 180/1200 = .15; for law school males = .83, females = .083.

Teacher: There are different results depending on how you look at it. For homework, do problems 41, 65. Work the problem and check answers in the back.

Change of pace now –This next problem was the 1997 free response number 6. It is a bit different than other regression problems.

Teacher – let's look at AP problems that deal with linear regression or maybe some other type of regression where we might linearize the data. It is a little different from what we've looked at in class and different from the textbook problems. So this was actually on the AP exam in one of the years, I think 2002 or 1997. I'd like you to take a look at this. I did reduce it so that you need to see all of it together on the front sheet. The most important thing is I put the questions on the back so that means you have to flip it back and forth. Now, taking a look at that question, at the data, what's the problem? Read over the question. What are they even talking about? That's of course what you should do on every problem. What's going on there?

Students are reading it over but seem indecisive.

Teacher: So, what's the situation?

Student: Student T – It is about this guy that wants to sell a car.

Teacher: What's the situation that's being explored?

Student: Student Z –This guy wants to sell a 1988 car.

Teacher: Keep in mind that this is an old item so a 1988 was an old car but it was not as old as it is now. What else?

Student: Student Z – He has to figure out how much he wants to sell it for.

Teacher: He wants to figure out how much he wants to sell it for. What did he do?

Student: Student Z – He checked to see what cars were selling for.

Teacher: He checked to see what cars were selling for. What are other people selling their cars for? With that in mind he's gathered up some data and he's tried to get some type of a model. Now, if you'll look at your paper, there is more than one model there. What do you see? Model #1 is what kind of model?

Student: Student W - Scatterplot.

Teacher: We have a scatterplot. Look at your regression equation, what does it say? Price = -58.1+0.719* year. So, what does that tell you that it is? It is a linear equation. You have the scatterplot, and what is the plot to the right of the scatterplot? Yes, residual plot.

Same people are calling out answer. Look at the residual plot.

Teacher: Does that say anything to you?

Student: Student Z – It is above and below and back above.

Teacher: Notice that this is not the line y = 0. So, it helps to put that line in. Now, look at the next one at the top of the second half of the page. What kind of equation is that?

Student: Student Z – Exponential.

Teacher: Instead of using logs they are using ln (x). This is the same kind of thing, as exponential. Look at your residual plot. Does it look any better? Get a straight edge, the edge of a sheet of paper, and put your line y = 0.

Teacher: Does it look any better? Not, much, maybe a little bit. Move on to the next one. What do you think that means?

Everyone seems perplexed.

Teacher: What is model 3?

Student: Student Z – Square model.

Teacher: What does that sqr mean?

Student: Student Z – Square of price

Teacher: Square of the price? So, look at your residual plot with that. Is it any better? Not a whole bunch. A little bit maybe.

Student: Student M–You say how does it look, you're talking about scattered more or?

Teacher: What should you be looking for on that residual plot?

Student: Student M – if it is scattered,

Teacher: It should be randomly scattered above and below. Is this getting better?

Student: Student W–Yes.

Teacher: Not a tremendous amount. Maybe a little bit.

Student: Student W– It's better than the other ones.

Teacher: Alright, what else do we look at besides the residual plot? It is what you usually want to look at instead of the residual plot.

Student: Student W – Correlation.

Teacher: Correlation, let's see what the correlation looks like. 88.5 %, 94.6%, and 91.9%. Based on those it looks like model 2 would be better. Now, I want you to turn your page and look at the questions on the backside.

Part a) Use Model 1 to establish an asking price for the car.

Part b) Use Model 2 to establish an asking price for the car.

Part c) Use Model 3 to establish an asking price for the car.

Part d) Describe any shortcomings you see with these three models.

Part e) Use some or all of the given data to find a better method for establishing an asking price for your 1988 automobile. Explain why your method is better.

Student T is a bright young lady. She is not 100% engaged in what the class is doing.

Teacher: So this problem was presented on the AP exam and it gave you three different equations to predict with. While all of them are okay, none of them are great and it is asking you to come up with a better plan. Look at your data again. Look at the original scatterplot; that would be your first model. What do you see going on there?

Students are looking at the problem, but are not engaged. Again, they look perplexed.

Teacher: It looks exponential. So, we tried that with model 2. It did get better. But, they're saying there's a better plan. What do you think?

This material is not new they should whip this out quickly. Some students seem to be working ahead on something related to the problem.

A student blurts out power equation.

Teacher: What do you think? Think outside the box. Let's think about cars. I drive a 2001. When my 2001 was new, it was worth the amount of money I paid for it. The next year, what happened?

Several students say it depreciated.

Teacher: The next year it depreciated some more. Now, this story is not true anymore, but my husband still maintained our 1988 Astro van. We kept it for many, many, years it worked quite well to move my son back and forth to college. It finally got to a certain price and probably from the year 2003, 2004, 2005, or 2006 it wouldn't have really mattered the year I sold it I could have gotten the same amount of money for it. We did sell it eventually for $500. We were glad to get it. What happens, older cars, what happens with cars?

Student: Student T – Their value depreciates a bit, up to a certain point they start appreciating a bit.

Teacher: Yes, if you get into antiques they will start appreciating again. But, how about my 1978 Oldsmobile, do you think it is worth a tremendous amount?

Student: Student T – It will

Teacher: I don't have one, but if I did, it would not be worth a tremendous amount. We are looking at cars from 1990 back to 1979. So, how old was the oldest? What happens eventually? The cars kind of level off and will you ever have to pay a negative amount for a car? I guess if you pay someone to haul it off. There has to be some limit to how low you are going. An old car is an old car. There might be several years span and they are all about the same price. Look at this data and see if you see anything like that. What model are you looking at?

Student: Student Z –From the beginning.

Teacher: From 1979 – 1982 was there a tremendous increase in price?

Students answered no.

Teacher: From 1991 – 1993 was there much increase in price.

Students answered yes.

Teacher: So there is a leveling off there. Are you guys seeing that? So, what can we do about that? It's sometimes an exponential and it fits very well. Does it look like it goes up like an exponential should? It is kind of broken. Are you saying that because there is a gap or because there is kind of a significant change in how the slope changes? The slope changes abruptly. Kind of a shallow slope and then it kind of goes up quickly. I want to sell a 1988 automobile. That's the gap in there, or close to it. What part of the line would reflect that best?

Student: Student Z – The right section.

Student: Student G – It looks like if you took out the first 4 points to the left it would be really linear.

Teacher: Yes, if you took out those four points it looks linear. Is that 1990, 1991, 1992, and 1987? No, that would be 1979, 1985, these are not in numerical order. Pick out the ones that need to be picked out to accomplish that, Student B.

Student: Student G – 1979, 1982, 1984, and 1985.

Teacher: Which need to be dropped, the oldest ones right? Mark those on your graph. Put that data into your calculator. Look to see what happens with that data. Once you graph those points what would you want to do next Student G.?

Student: Student G – Do linear regression

Teacher circulates to see students' work.

Teacher: Find your regression equation and the *r* value. See how well it fits your scatterplot. Look at your residuals. Does your line seem to fit well? What do you notice about the residuals? Who's got a residual plot? *A few students raise their hands.* What do you notice about the residual plot? It's randomly scattered finally. It's really randomly scattered. Not above the line, then below the line, and then above the line. How close, how big are those residuals? Small – looking at Student G's paper. Fairly, they are pretty small. *Several students agree*. So, that's a better fit. This will be a good equation to use to predict the cost of the 1988 automobile. Check to see how much it costs. What did you find Student Z?

Student: Student Z – 4.35

Teacher: That is 4.35. That would be 4,350. Cars back then didn't cost as much as they do now. Does anybody have a question about what we are talking about? This makes sense to you? How would you explain on the AP exam what you did and why you did it? Student G, it was your idea. Do you want to explain it? I have to give you some points, girl.

Student: Student G – Taking out those four points would you say they were influential or outliers? They were outside the rest of the pattern.

Teacher: They wouldn't be influential or outliers. They seem to be on a pattern of their own. They are different from this pattern.

Student: Student G – I removed 4 of them, the oldest years, to make it more appropriate for the automobile. The rest of them would not allow us to extrapolate.

Teacher: *Teacher is coaxing the correct response above*. Help her out she has done a great job. This is problem 6 on the AP test, this problem counts more than any other problem on the whole exam. If you nail it you've got some good points covered. You guys could nail this. You just need to think about what you need to say now. Student G has already done the hard work she figured out what she needed to do to take out those points and get that equation that is a much better equation, it fits well, and has a nice residual pattern, so what would you say about that.

Student: Student Q– I am confused. Did we take data out that were non-linear and made it linear?

Teacher: Not really, we don't want to say that we linearized the data. But, we selected the section of data that was linear, and we did our regression just on that data (*she means on the remaining*). Because that data was on either side of where you want to predict, then it was a reasonable thing to do. What do you think Student M?

Student: Student M – We selected and discarded the undesirable ones

Teacher: I am not comfortable with saying the undesirable ones. We removed data that really was not in the same trend with the part of the data that we needed to predict from. We removed it because it wasn't linear with it and didn't seem to fit with our other data. I am not comfortable with undesirable ones. It was changing what our prediction should be. It didn't fit our other models well. We could get a much better linear model than that. I guess I am picky about my words. I want you to complete that problem, calculating the price for the other models.

Teacher: Then think about how you are going to put that in words. If I ask you to read your words tomorrow I don't want you to stammer I just want you to have something you can read. I may call on someone that never answers. Tomorrow, the first thing that we will do is go to the computer lab. So, meet me in the computer lab.

This is a typical question with which students should feel quite comfortable The students have had much experience with regression yet will not think about the problem at hand.

Lesson 12 Marginal percentages are discussed. Multiple Choice Questions will be discussed as well.

Class has come back from Lab.

Teacher: These two problems are dealing with the display tables, manipulating that data and finding marginal percentages and initial percentages. Problem 4.41 should look something like this.

4.41 Table

Death

Penalty

Defendant	Yes	No	
White	19	141	160
Black	17	149	166
	36	290	326

Overall the percentages are:

19/160=11.9% of White defendants receive death penalty

17/166=10.2% of Black defendants receive death penalty

Victim	White Defendant Death Penalty	Black Defendant Death Penalty	
White	12.6%	17.5%	Higher
Black	0%	5.8%	Higher

In cases involving White victims 30/214=14% of defendants got the death penalty. While in cases involving Black victims only say 6/112=5.4% getting the death penalty. White defendants killed Whites 94.3% of the time but were less likely to get the death penalty than Blacks who killed Whites.

Teacher: Responding to idea of Simpson's Paradox we see that generally it shows up when the groups are disproportionate. When data is together versus apart the results are confusing.

Teacher: The best way to understand Simpson's Paradox is to do several problems.

Student: Student cc – How did you get the top table and how did you come up with percentages.

Teacher explains the ratios she computed based on what was asked.

Teacher: In problem 4.65 I have overhead with answers for you to look at. Since the percentages are used then you will have round off error. What's the difference between a marginal distribution and a traditional distribution? Does anybody have an answer to that?

Student: Student cc – You have rows and columns of data and variables are displayed separately.

Teacher: What if I asked you for a marginal percent? Marginal percent of singles is all the singles out of the total in the table. Whereas in a traditional distribution percent of singles would be the percentage of the 25 year olds that are single we would take that out of singles column. In other words it would be the percentages by column. Do you have questions? Quiz is next.

Student: Student M – I have trouble with these. *This question came after teacher mentioned quiz. He had question on 4.41.*

Teacher explains the answers. I don't think he still understands. Student M does not listen even when he asked for an explanation.

This is an easy concept because the problem asks the student to directly make a table.

Student: Student L – I also do not understand the percentages.

Teacher explains again. Teacher seems perplexed at the question.

Student: Student S – Explains how she took each total for each victim and found the percentages getting death penalty or not.

This seems to make more sense to the students. This is a breakthrough for Student S who rarely answers unless asked.

Teacher: Other questions?

Teacher: We also had looked on Friday, on an AP item. Did you complete it? *Some students seemed to have worked on it for homework. It was the models of cars. She says to put it away for now until later. Quiz now.*

Teacher: You may not have time to finish it before lunch.

AFTER LUNCH

Teacher has passed a set of 35 multiple choice questions. They were on the 1997 AP Exam. This was the first year they had an AP exam in Statistics.

Teacher: Only two sets have been released and you will have to sign a paper saying that you will not share any information about the content of the multiple choice questions when you take your test. They use the same mc questions for several years in a row. This coming year there will be all new questions that no one has seen because they are due to release the test from this year. You have lots of multiple choice questions in your review book. These we know were actually on an AP exam. What I want to do over a period of a couple of days is to work in pairs on these. Not just to figure out which is the correct answer but also write in the margin or where ever you have room your justification for your answer.

Teacher: If you tell me that your answer to #1 is b) I will say how do you know that? Explain to me how you got that answer. Don't just go through and circle answers. I want for you to be sure why you chose that answer and why you didn't choose the others. For those of you that take my teacher – made exam I will be selecting multiple choice questions from this set and the other set that we will work on a little bit later on. So if you have these 75 questions and I select 25 you really have the potential of getting them all right. So what we are going to do right now is to divide up into pairs and I want you to be two people not a clump of 4 people. I want you to spread out and if you don't mind sitting on the floor you can sit up here. Since Student bb is not here we will have one person that will be the odd person out. So, one person will have to work alone today. That is what we will do for the remainder of today and I will circulate to see if you have any questions. I don't care who you pair off with, but haste.

The students seemed anxious to work on their first full multiple choice section.

Lesson 13 Continuation of Advanced Placement Exam 1997.

Teacher: Let's start with the problem about selling the car. I would like for you to pull that out and let's discuss it. I will put problem numbers on board and Student G do part a), Student R do part b), and Student B do part c). Not just write the answer but how you did it.

Part a) Use Model 1 to establish an asking price for the car.

Answer: Equation: Price = -58.1+0.719* year, Price = 5.172 or 5,172

Part b) Use Model 2 to establish an asking price for the car. – *Student R had to get help from the teacher to solve for price.*

Answer: ln(price) = -14.9 + .185 year = 1.38, e^1.38 = price = 3.975 or 3,975

Part c) Use Model 3 to establish an asking price for the car.

Answer: This was overdone by Student B. Also, the equation should have a square root.

$$\begin{aligned}
&\hat{y} = -13.313 + .1756(x) \\
&\hat{y} = sqprice(estimated) \\
&x = year \\
&\hat{y} = -13.313 + .1756(88) \\
&\hat{y} = 2.1384
\end{aligned}$$

or 2,138.40

Teacher: There seems to be a lot of confusion about what was going on here. First of all, as I look around, I see a lot of a lot didn’t go on. That was a problem. If you didn’t complete the problem then you don’t know what you know and what you don’t know. I don’t know what to do with you. You’ve gotten really lazy on me. You need to complete the problem. Let’s look at what’s going on here. With the first problem, Student B plugged in 88 and I asked her if she would like to fix it. She says that it is not wrong. There is an issue of rounding. Student M has a question.

Student: Student M – How could you get that answer?

Teacher: That’s what I wanted to talk about. First of all, some of you were getting this answer: 5.172. Either answer is okay as long as you show where you are getting your answer. Now, how many of you used 1988 instead of 88? What did you get for your answer Student M? (*Student M: 1,371.272*). Let’s think about our problem. Looking at your graphs what does your information tell you in this problem? What do we know about the problem itself? Look at your table, what's the asking price that's given? Are they selling cars for $6.70 and $8.70? What is that? Students answer thousands of dollars. So if we are putting in as thousands of dollars, we should be getting out thousands of dollars. If Student M's answer is thousands of dollars, then we are going to ask $1, 371,272 for our car. We would love to sell it but I don't think anyone is going to buy it. So, first of all look at your answer in the context of the problem. That answer was not realistic. Here's the second thing, looking at the graph, even if you had not recognized that the answer was in thousands of dollars, if you thought that this was already in thousands, would it appear that 1988 cars should be selling in the 1371 dollar range? Does that look realistic? No. It was over 2500 in the scatterplot, you see that? So this answer wouldn't be realistic in the context of the problem. Our answer that we get should be similar to the answers in the table, 5.172 thousands. You should change answer to thousands, $5,172. Looking at the second part we did all of our logarithms with, log base 10. This is ln. *Teacher goes over procedure for part b.* It is advisable to look at scatterplot to see if this price is well within the values given and fall on the curve. Is it a reasonable answer? It would not look like an outlier.

Student: Student A – Are we looking at the ln graph?

Teacher: We are looking at the original scatterplot. Looking at the last model, should *y*-hat be square root. She asks who did this one. It should be:

$$\sqrt{\hat{y}} = -13.313 + .1756(x = year)$$
$$\hat{y} = sqprice(estimated)$$
$$x = year$$
$$\hat{y} = -13.313 + .1756(88)$$
$$\hat{y} = 2.1384$$
$$\Pr ice = 4.572$$

or $4,572

The student still misses the squared part of the equation.

Teacher: Does that once again appear to be reasonable? Notice, that on all of these problems you did not have to linearize the data. Different models had already been done.

Teacher: Part d) Describe any shortcomings you see with these three models.

Teacher: What kind of shortcomings are there to see with any of these models? The first thing to mention is that model 2 has the lowest price.

Student: Student G – Are they talking about the residuals?

Teacher: Yes, they are. So what would you say is the shortcomings about the residuals?

Student: Student G – They are not exactly a random scatter and they have a pattern.

Teacher: Every one of the residual plots has a pattern of some sort. What does that tell us?

Student: Student G –They are not as good models as we might think.

Teacher: They are not necessarily modeling the data as we well as we would like them to be.

Student: Student V – I don't have a question but says I don't see a pattern.

Teacher: First of all Student V, go to your residual plot, and draw a horizontal line at zero. Do you see that to the left you start above the horizontal line with positive residuals, then you go below the line and then you come back up above the line? That tells us that for our small values we are underestimating and then we are under the curve, and then go back above. So, there has got to be some curved pattern that fits it better than these. Or maybe, a different pattern altogether. If you look at every one of these you start out above, you go below, then you go back above, every one of them. So, you identified the shortcomings, what do you do about it? Using part e) use some or all of the given data to find a better method for establishing an asking price for your 1988 automobile. Explain why your method is better.

Teacher: What can you do about it? Can you think about a model that will work better? What did you do? Did anybody do anything? Who actually has something that you think is a reasonable answer? Student Z and G, we talked briefly about it, I don't think we did it. But, we talked about it. Student Z, Student W, and four or five of you, starting with Student Z, tell me what you did?

Student: Student Z – I did the whole piece-wise thing. I took off the bottom 4 prices, from 1979 to 1985.

Teacher: Why did you take those off?

Student: Student Z –Because they had a different trend than the other side of the data.

Teacher: What happens after 1985?

Student: Student Z – It starts decreasing.

Teacher: The trend changes and they start decreasing at a more rapid rate. You need to explain why you chose the points that you chose. What did you decide?

Student: Student Z – What did I do after that? I did a LSRL for that data.

Teacher: Then what? Did you write the regression line on your paper?

Student: Student Z – No, on the calculator.

Teacher: You need to write the new regression line on your paper because you need it to make a prediction on the car price. Who has the regression equation?

Student: Student B – I used 1988.

Teacher: It will give a totally different answer. Your slope is pretty close. Your intercept is different because you used 1988.

Student: Student A – My answer is different and I plugged in 1988.

Teacher: You should also show why that model is better. How would you show that? Several students blurt out that you need to show the residual plot and *r* value. Teacher: Your residual plot should show a nice pattern. Your *r* value should be much higher. In fact, this says that your *r* value should be at least .97 or .98 before you consider it a good fit. That might tell you something Student cc. We might not get a really good fit from our live data.

Teacher: In the coming weeks we are going to be doing a lot of advanced placement items. I am going to randomly check these. You can't not do them.

Teacher: You are telling me you're going to take the AP exam but you are not going to practice for it? That's like expecting to star on the varsity basketball and skip the practice all the time. They are not going to let you play if you don't go to practice. You are going to have to practice the problems if you are going to get any better at it. I can tell you the more you do the more comfortable you will get with them. The better you will be able to answer the problems.

Teacher: So you can put that away, don't get rid of it, you may see that problem again on your exam, when you take your exam for this class. You may see something similar on your AP exam. Put that in a nice safe place, not stuffed in your book-bag. Get out your multiple choice items. It looks like we got a few multiple-choice items done yesterday. I am not surprised, some of you didn't get a lot done because we were busy talking about the projects. That's understandable. Others, you were busy talking. Student P, for problem #1, for your multiple-choice, what answer did you choose.

Student: Student P – E.

Teacher: I want everyone to familiarize yourself with problem #1. What is a 4% margin of error? How wide a range is a 4% margin of error?

Student: Student Z – Eight.

Teacher: Are there any questions about #1. Look at problem #2. This is setting up a hypothesis statement. What is the null hypothesis?

Student: Student T – $\mu = 35$

Teacher: That narrows down our choices for the answer. What would our alternative hypothesis be?

Student: Student W – $\mu < 35$.

Teacher: Let's look back at the other answer choices. It would not be totally wrong to use $\leq$ or $\geq$ in our null hypotheses. We always used equal to. It is okay to have the null hypothesis be the opposite of your alternative hypothesis. So if you see that don't flip out. Some people do it that way. It's perfectly fine. Think about what your alternative hypothesis should be and your null hypothesis should be equal to that value and it could also have the opposite of the alt hypothesis. Sometimes problems will have $\overline{x}$ instead of μ. We never use $\overline{x}$, we are always testing the population parameter. We know exactly what our sample mean is so we are never going to have to test what it is. We are using our sample to test our population mean. That would also be true for p and σ. Are there any questions about that?

Teacher: Problem #3. What does expected number mean Student K?

Student: Student K – What is average based on population proportion.

Teacher: What does expected number remind you of? Expected number, expected value, expected this or that, what does it remind you of? Mean.

Teacher: How do you calculate expected value? This is just #3. We haven't got to 5 yet. What did you write for #3, Student M?

Student: Student M –Found the percentages for each and multiplied the values and added up.

Student: Student W – Can calculators be used on multiple choice questions.

Teacher: Yes. Did anyone approach this problem a different way, but the answer choice C is correct?

Student: Student K – *He goes through the procedure to calculate expected number again.*

Teacher goes through the whole procedure using the idea of expected values and asks if there are any questions about either way to do problem.

Teacher: For problem #4, find a key word in problem #4, expected. How do you find mean of light bulbs if you are given proportions? Mean of binomial is Mu =? Student V answer that.

Student: Student V – 1000 times .98

Teacher: That will be 980

Teacher: Write how to work these out.

Several students are writing notes to themselves on the handout.

Teacher: Problem #5. This is the last problem we're going over. What kind of problem are we talking about here? *Only 3 students raised their hands.* Only 3 people in here can tell me what kind of problem this is? *Two more students raised hands.* Tell me something about this problem, Student Q. Two different strains on each leaf, what does that mean? There are two variables. What do you think? What kind of test do we perform on this data? What kind of a t-test?

Student: Student Z– Two-sample t-test.

Teacher: Should this be a two-sample t-test? There are two different strains on the same leaf. That's like I am going to turn that doohickey with my left hand and then turn it with my right hand.

Student: Student Z – It should be a two samples t-test.

Teacher: Student U what do you think? I will read the problem again.

I believe Student C has raised her hand to answer before the teacher got started on her reading. Student bb raises his hand also. She belabors the sentence until someone can answer. Student G starts to ask if the answer is..? The teacher says she will not discuss the answer yet. Student bb says it is a matched pairs. Teacher agrees. The question deals with degrees of freedom. For two samples df would be n – 1 or the smaller of the two. For matched pairs it would be n – 1 so it does not matter because the sample sizes are the same. Answer is A. Researcher adds comment about wording of problem.

Teacher: There are a new set of rules. Work on multiple choice questions in class – all alone. You cannot just circle an answer. You must have reasoning behind it. You can use your book, or notes. If you are really stuck I can direct you to something that you can look up. Remember that your project is due tomorrow. If you don't get finished with these problems it will not be for homework. I need for you to make a lot of progress on this today. There is no group work because you were not making progress during your group work yesterday. It should take you 90 minutes for the whole section. You would have taken 3.5 hours at your rate from yesterday.

Lesson 14 Review of hypothesis testing

Teacher: I want to check to see more or less where you are on the multiple choice questions. Since most are in the 20 range, homework is to finish all 35.

Teacher: Today in class we will look at something different. I have a question: this particular review activity deals with different types of inference that we have done in this course. So let's list the different kinds of tests, significance tests. We are not going to talk, in this portion, about Confidence Intervals. Name the different kinds of tests that you can think of that we have done in this course.

Students are giving examples of the different tests. Students are answering out loud. Students gave good examples of each type. This took a good while. They completed a table with all the tests.

Many students contributed to the discussion.

One-sample t-test – Used with matched pairs when running t-test on difference of two sets

You can use z-test when population standard deviation is known

Most of the time standard deviation is unknown. If not known and population is large then degrees of freedom would be large and it would give you a t-value very close to z-value

We test for means when collecting quantitative data.

<u>Recap</u>

t-test for means for quantitative data	z-test for proportions for categorical data	Chi-square observed and expected proportions or numbers more categories Use counts $\sum \frac{(observed - expected)^2}{expected}$	Linear regression t-test bivariate data testing slope to see if linear	Z-test for means Population standard deviation known
2-sample t-test Independent samples	One-proportion z-test	Goodness of fit		2 sample z-test
1-sample t-test	Two-proportion z-test	Homogeneity		1-sample z-test
Matched-pair t-test (difference)		Test of Association		

We are using categorical data when we use test of proportions

Students were exposed to when you use each type of test so she expects them to know examples of each.

Teacher asks for examples: How many blue marbles? *This was insufficient*. How many blue marbles in the bag? *It was still not good*. What proportion of the marbles is blue? That is better. Would you like get rid of taco bar at school? The question would then lead to ask what proportion of the population would like to get rid of the taco bar.

Teacher: Chi-square test: Data was proportional but you had to observe to see if it was the expected. It is also true that you had more than two categories. We look at counts to see how many fall in each category. Reviewing will not be easy after these next few weeks. Homogeneity – whether two variables are related to each other or not – expected value is difficult to get sometimes. Goodness of fit test. Graph looks different for these last tests. Test of slope is for bivariate data. We are testing whether slope is zero or not.

Activity: Determine the test.

Students are given 9 different scenarios and they will be determining which test will be used.

Teacher: Do tests by calculator and then by hand to practice. Instructions given are that you must tell what type of test, the hypothesis set up, the p-value, and decision.

Students are actively engaged in answering problems.

Teacher: Use parameters because you may not get credit. You should be checking for assumptions. For test it must be an SRS, size of sample indicates we don't have to worry about normality. You are to work in class and try to finish so that it can be reviewed before you leave.

After students have worked a while the following problems are discussed fully.

Problem 1Discussion: Z test is appropriate because standard deviation is known.

What is the 60 hours per week? Average for California is 60. We will compare mean against 60. We are using z-test. Student E – Null Hypothesis. Teacher asks her exactly what she wrote. HR = 60. We need to write Mean = 60 Mean < 60. Mu must be used to identify parameter. Student A is answering. You must say SRS has been met, n > 40 we don't have to worry about moderate skewness or normality. Only outliers might be a problem. P-value was close to zero, so null hypothesis is rejected and it appears LA doctors are working more than 60 hours.

Problem 2 Discussion: It has the standard deviation of population so z-test is appropriate.

Teacher then tells them to work on other problems in class.

Lesson 14

Teacher asks them to pass in the multiple choice homework. Students are to work on significance tests activity.

There is a quick check on homework. Get textbook out and paper. Use notebook. They will be reviewing probability. Silently, look through Chapters 6 – 8 and write down key formulas.

Teacher was checking homework while they worked on the assignment above. They should also check through study guide notes as well.

She asked students to go to her office to discuss homework or lack of it.

Camera was turned off in order to chastise students. Much time was spent on this.

She checked all homework answers by giving answers.

She went over exercises on the multiple choice about which students had questions. The teacher was furious over lack of work done on homework.

Tomorrow they will be asked to explain answers to these.

A simple problem like $1.96\sqrt{\frac{.5*(1-.5)}{n}} \leq .08$ *was giving them problems. With this lack of skills it's no wonder they are doing so poorly. Answer was greater than 150.0625.*

That was problem #33. Someone then asks about problem #32.

That would take a long time so it will be done tomorrow. Tomorrow problems will be put on the board.

Lesson Discussion

Homework check was done at the beginning of the period. She asked about effort grade. She said she was disappointed with those that had not given it 100% effort. The AP has to show work for credit. She asked if students had accuracy on a minimum of 3 of the 6 problems. Again the students were not sure of their work. They had to have worked on 6 questions for homework. About 70% needs to be done to get a 5. They will grade their papers by rows in teams of two. They then exchanged papers so that they could look objectively at their papers. The criteria for grading each of the problems were read out loud. Included in the criteria was a way of getting partial credit.

#1) Sales problem: Cumulative graph is given on first problem. Student G is really engaged in these review activities. Justification is a problem with all the students. They must support answers with one or two sentences. This has been an ongoing problem for students. Students were a bit aghast that they would get no credit for any work without the identification of the graph as cumulative. It is difficult to understand what students are saying. Students need to assess if their answer is consistent with the rubric read. They are just beginning to understand that they must read the problem carefully. Even though students want to copy the answer the teacher said it was impossible to read every answer verbatim. The first two students, Student bb and Student dd had something written for question #1. *This girl's answer seemed to be fairly complete but not correctly stated. Student cc answered but was incorrect. The context of the problems is still an issue. Student T answered what was on her partner's paper but was given without context. I believe that explaining the way of grading the answers is beyond the students'*

comprehension and is making them more confused. I would have said give your answer in context or it is wrong. The students realize that it will be difficult to get 4 out of 4.

#2) Shift problem: The problem is about proportions. The instructions do not mention about means so it is hinting at proportions. There are two proportions given. Student S has Student Q's paper and sounds well written. Student Q missed talking about independent samples and in which order she is taking the difference of the proportions. The population needs to be 10 times sample size and needs to check that p sample size ≥ 5 and (1-p) * sample size ≥ 5. Teacher emphasizes that one should say what you are using. Confidence Interval is size 96%. Must show formula or state what it is 96% difference of proportions using the calculator. Interpretation is expected – Based on these samples we can be 96% confident that that the difference of the proportions of the parts specifications for the two shifts is between - .0156 and .0196. Must comment on right sample size, state interval, and interpret. Part b) Will the difference significantly different from zero? Student Z ran a hypothesis test. That was not allowed. Zero is in the interval. Because the interval contains zero we know that it is highly unlikely that the difference would not be zero. Therefore, it is highly unlikely that the difference of shifts is true.*

#3) Distance problem: Normal distribution problem. Student bb read the problem.

Part a): What is the probability a ball will travel as far as 280 feet? Probability is asked for. Students must specify normal distribution. Student S reads her answer. Person had a normal distribution drawn and shading beyond a point. The teacher asked about standardizing using z. Her z value was = 1.1428. This was Student D's work. Key words are standard deviation, mean, and normally distributed. Student N had correct setup. A *couple of students had this part right. Probability was 0.1265 for ball going across. Students must show correct calculation. Assumptions were not stated by all.*

Part b): Cumulative probability problem – At least 1 of the 5 balls, means one or more.

$\left(1-\left(1\ -\ \text{prob(none go)}\right)^{5}\right)$ *=0.4932. You can also set it up as a binomial distribution. However, it will be more time consuming to work out.*

Part c) Student aa reads answer to part c). This is Student X's paper and it was well written. Find $\overline{x}$ *given probability. Given a z value for 99 percentile, an inequality is set up for mean. Teacher explains that it should be finished up by saying: Mean should be set at 284.68 yards to be 99% that a randomly selected ball will reach maximum distance. A student asks how to set it up.*

#4) Matched pairs problem: Dexterity after a program. Student Z has labeled this correctly. Student R reads from Student J's paper. Hypothesis was set up, and assumptions had been met. Teacher said that it was not enough. Students say SRS (this was given in the problem) and normal distribution. These need to be checked by checking stem and leaf plots. That would also address the problem of outlier. If students addressed the idea of outlier, after thinking there was one, but continued anyway you will be fine. If fact, you should continue just in case you made a mistake. There was a t-test that needed to be done. Formula was shown as required. This was Student F's paper. About 7 – Not many people got this correct. A few people thought there was an outlier (Student A and aa). T-value was done by Student F 3.54 and showed it (this is the mechanics) and p – value = 0.002. Only 7 people got this correctly done. Teacher is worried about so few people getting t and p value. It is important to state conclusion in the context of the problem. Null was rejected.

#5) Experimental Design Problem: Agricultural problem concerning draft. Student cc reads problem. Plow used on land to determine draft. Draft is explained to students. Part a) response variable – the draft. Identify treatments – two hitches, old and new. Experimental units are plots of land. Very few are answering.

Part b) Was the randomization used appropriately? – They randomly chose fields. It was not according to the students. Student W wants to use plot in half. It might be better to randomly choose the plots. The plots were chosen randomly. Teacher says that it was done correctly albeit a bad experiment.

Part c) Teacher asks what replication means. Student says – repetition of experiment or to copy the experiment. Was replication used appropriately? It was not. Only two fields were used and we don't know the conditions of the field. Maybe more fields should have been used – perhaps of different terrains. The experiment was not replicated properly. Teacher talked about planting in their soil at home and the problems encountered.

Part d) Why is plot of land a confounding variable? Student W says that each plot could be different so their moisture content could be different. The type of soil could be different as well. The time differential is also a problem. Very little time left for the last problem.

#6) Binomial probability - Consumers of juice. Part a) Probability is 0.5. H_0*: p=.5. Ha: p≠0. Part b) a 1- proportion z-test is not possible because of sample size and np < 10. Here assumptions are not met. Teacher reads out binomial probabilities. Some students are not listening. Can a significance level of .05 exist? No person answers. Teacher says no. This was a confusing problem. Part e) Test part a) hypothesis.*

Lesson 17 Discussion

New Day – Starting with 2007 Part B AP Exam

Teacher gave out some papers that were make-ups. She gave percentage grades based on what they had done. She calls students with best answers to answer orally. Orange circles were the best answers. Some were not circled and it did not mean it was a bad answer. She explained how the grading is done.

Question 1: Student I is to put up the stemplot from question 1 part a). No one has done part b) correctly. Use stem plot to describe distribution. Student X's paper was well written. She said that, nothing fell in the 20's. Student B said - a large portion of data was in the teens. Student U said that there were two curves. Student aa said that there were two clusters with gap in between. Part c) Why would it be misleading to report only a measure of center. Student L says that the mean is in the gap and is unrepresentative. Student aa says that there were two groups without the center being in either cluster. Key needs to accompany stemplot. Each number should be stated as was taught. Shape rather than order of values is what is important in a stemplot. Split stem could also be drawn by separating into groups of the ones, twos, etc. Teacher commented on grading of question. A few people got full credit for some of the parts.

Question 2: These are probability questions. a) Student W, b) Student T, and c) Student Y go to board and explain answers. Student W – Probability that they have certain number of hits. Student T – This was a Binomial distribution because it had set number of trials and trials were independent. The probability will not change for each trial and finally there is a success or failure for each trial. Student G does not know why she had to multiply by 45 if there were two successes and 6 failures. Nearly no one had this problem *correctly done. Sampling distribution was in last part. Student Y – samples of 150 households. They were finding means of samples. Student Y explains that the distribution should be nearly normally distributed. This occurs even if original population is not normal. Standard distribution is standard deviation / square root (150), which of course is smaller. Mean stays the same. These three bits of information were crucial to receive points. Not many got full number of points.*

Question 3: Teacher asks them to look at their papers. This is a blocking problem. Randomized block experiment involved with window types and heat. Student V is called to answer how he blocked. He blocked windows by twos. Student K is also showing the blocking in part a. Teacher says that it is important to block alike. Criteria mentioned how to choose them by <u>wall</u>. Randomization is also a key item to discuss. The pairs need to be on the same wall. Student R's answer was also done correctly. Blocking procedure needs to be explained very carefully and specifically. The parts of the problem had to include blocking by window and randomly assigning by window type. A few students did well. Student M explained his answer and it was not correct. He apparently did not block by wall. It was not the intention to find the best window of all. Student T wanted to know if she could get partial credit for the explanation without having the blocking done correctly. Student T says she does not understand blocking.

Question 4: Teacher says most people got part a) done correctly. The regression line had to be put on the graph. Student S shows problem, but done the difficult way. Student S estimated points on the graph and did a regression so that she could find how to graph. Her regression line was a bit different from the given regression line given. The reason the line was not exactly the same was because three points had not been given. It was *very time consuming and unnecessary. Student Z goes up next. Student Z graphed the line by finding two points on the line.* $y = 35.1 + .4278x$*. Student Z went to table and used 50. Teacher explains that extreme points are better. At 50, the y value is 56.4. Teacher explains to use x = 80 for next point. The y value is 69-ish very close to 70. Then the student could use a straight edge to draw line. Guidelines say that you just draw line. This is an easy part and I don't know why they would not do this an easy way. The next part asked to circle two points 67, 61 on the scatterplot. Student Z shows point on scatterplot and then you were to draw residual for that point. Student B finished showing the residuals. The residual line should be done vertically. Residual is continued to be explained by teacher and shown by Student B. Student U asked about residuals. Student E said that the y value (observed and expected) had to be for the same x value. Expected value comes from calculated regression line.*

Student bb writes on the board: Expected – found = residual. Which gives incorrect residual sign.

Observed – Expected =residual. 61-63.79=-2.79. Sign had to be correct.

Part c) Adding a new point to the set, students need to determine if the slope changed in any way. Student I answers that the point should make line have about the same or decrease slightly. The teacher emphasizes that the correlation should increase using the formula or say that if point is way out on x axis but close to line it should strengthen the correlation. There weren't any calculations necessary.

Question 5: They have done this before. This should not be difficult. Student X and Student B were going to the board. Student X would put null and alternate hypothesis and Student B would do the whole problem with a slightly different approach to the null and *alternative hypotheses. Apparently Student X wrote a lot of good work. Teacher emphasizes that talking about these might be boring but necessary. If you have different values for old and new values of drug you want to know if it is significant. Student X has the following:* $\begin{array}{l} H_o : \mu_n - \mu_o = 0 \\ H_a : \mu_n - \mu_o > 0 \end{array}$ *. Student X forgot to say which mu goes with which drug .*

Student B wrote the following: $\begin{array}{l} H_o : \mu_n = \mu_o \\ H_a : \mu_n > \mu_o \end{array}$ *. She said she would conduct a 2-sample t-test. The medication is more effective than the standard for the alternative hypothesis. She found t–value and found p-value. She got all of this off calculator and showed what formula she used. She said she would reject the null hypothesis. The sample sizes are large enough to be able to use z-test. Teacher asks students to take out books so they could look at table C. Students are looking at Table C, look at z* line , t –distribution with large sample size is close to normal distribution. She repeats that she has not taught this test. Normal distribution is acceptable when sample size is larger than 40.*

A recap is done.

With small samples t-distribution is more conservative. Several students have questions about using the correct distribution. By being safe t-distribution only rejects the null if you have over-whelming evidence. Because they are working on free response questions they must include assumptions about random assignment and normal population. Value of statistic needs to be given. Tomorrow they work on problem 6 which is the design problem.

Lesson 18 Discussion of the Experimental Design Problem

Question 6: Island Question

Question is read by teacher.

Is the risk of extinction the same for at-risk species on both large and small islands.

Support – significance test needs to be done.

a) Student dd – *performs the test and gets z value and computes probability*

Student D – Null and alternative hypotheses – $\begin{matrix} H_o : p_1 = p_2 \\ H_a : p_1 < p_2 \end{matrix}$, large islands and small islands are equally effective at protecting at risk species versus large islands are more effective than small islands at protecting at risk species. *P_1 should not be large islands but the proportion of animals that went extinct on either island. She should be talking about proportions and she could then add what she said above. She needs more clarification.*

Student X – *She used P_l and P_s for her proportions. She used the 2-proportion z-test. She should write out proportion instead of prop. She also wrote value of z and p and the conclusion which is to reject the null means extinction proportions are different. The teacher says this would happen at any alpha level.*

b) *Student B worked on part b) on the back board. – 95% CI for slope of regression line* $b \pm t^* s_e$ *, no unusual patterns, df = n-2=11. Teacher says that she should elaborate on this. Check conditions, linearity, standard error should be consistent and look at residual plot, our residuals should be normally distributed checked with stem and leaf plot. There appears to be some sort of negative slope – I am 95% confident that the mean proportion of extinct animals is between some values given. This gives the context of problem by alluding to extinct animals.*

c) Student O – *we see that every time we add area the proportion (instead of probability) of species going extinct decreases. The large and small islands are not all the same size* *and the species are not all going extinct at the same rate. As the large islands are getting smaller they are protecting the animals less.*

d) Student U – *Which would you choose? – I would choose larger islands because – a good supporting argument is needed here. The answer should talk about expectations of preserving the at-risk species is better on the larger islands.*

The entire group of questions was reviewed in detail. Students are just not asking questions

Continuation of Question 6 on 2006 Advanced Placement Examination

Drink problem concerning juices. This should be a Binomial Distribution.

They are going to finish part d. Prob of 0.5 is to be questioned. Students are not engaged. This problem is difficult for most people to follow. They couldn't care less about this problem.

Teacher: Part e) Use the Binomial Distribution to do this part. –Probability of being outside of 6 and below 2. You add probabilities of 0, 1, 2, 6, 7, and 8. Small samples are usually leading to not rejecting the null hypotheses. To make a better study, we would like to enlarge the sample size, if n = 80 then 20 would prefer one juice, show the results and z –value = -4.47 with p-value close to zero. This sample size is very large and you could do it with a smaller sample size.

The 2002 Multiple Choice Test–Special Notes on Errors

She is going over the percentages of students that had the questions correct versus when it was actually given.

She is showing that the language is important to read to answer correctly. So the reading comprehension is crucial. Questions are still not forthcoming

Null hypothesis should always have an equal sign.

Expected value is evaluated by multiplying probabilities by number in each group. It should be an easy question. They have seen this before many times. Student W explains how to do this.

Scale only is changed when units are changed, the correlation does not change and so relative clustering is the same. They should have seen this with other examples given in class.

N- 1 is the degrees of freedom. Many students had this wrong when choosing degrees of freedom.

As the sample size gets larger the standard deviation gets smaller. Standard deviation is divided by square root of n. They have seen this multiple times.

Student X is going to show work for a problem because she is one of the few that got it right – Problem is about area under the curve between two probabilities. Even though she asks for questions the students are not asking for explanations concerning answers. Teacher asks for clarification.

Continuation of discussion on the multiple choice questions

The t-distribution always has a larger standard deviation than the normal distribution. They have discussed problems through 20.

As the sample size gets larger and the degrees of freedom gets larger the distribution gets closer to normal.

Student U asks about distribution of colors of eyes given– A Chi-square test is asked for. Sum of differences of observed – expected squared and divided by expected. When Chi square value is small, the less likely you are to reject the null hypothesis – our p-value is *large. Student T answers because she understands this topic well. She is the one playing with her hair. Teacher reminds them of what the distribution looks like. Student U should know better, she has seen this before.*

Quite a few people are not engaged. Some are playing with their hair, others are writing, or just looking bored.

Question #20 was missed quite a lot. Teacher asks students to read next question quietly. She shows how to read a question that Student Z just read.

Teacher: By emphasizing some of the words you will think correctly. There was no cause and effect so scatterplot is not likely. When comparing salaries a scatterplot is least useful.

Question #21 was also missed quite a lot of students. *This is a bad question – CI for slope of a linear regression line = coefficient of size (slope) +/- std deve * t- value (20 df). Silly error was made because they used the y-intercept instead of coefficient of slope.*

Question #23 was missed by a few students. Given two events with positive probabilities tell which statement is correct. If events are mutually exclusive – if event A happens then event B can't happen then they are automatically dependent. Independence means one event does not depend on another. Mutually exclusive means disjoint and therefore are dependent.

If events are independent they are mutually exclusive – not true

If not independent they are mutually exclusive – not true

If events are mutually exclusive they cannot be independent – true

If events are not mutually exclusive then they could be independent they might not be.

Question #24 asks about explanation of significance test. This time this class missed this at a higher rate with their notes than the test takers that year. Student V reads results from the test. There was a P-value of 0.24. The students are asked to choose correct conclusion. One important idea is brought by Student F and that is that the p-value is large and not sufficient to reject null hypothesis and so there are not significant results. So remaining answers to choose from are either c, d, or e.

In c), we are given some information about a two-sided test which is immaterial. What does a value of 0.24 p - value mean? Teacher screams that it is the area in the tails and it is 0.24 – area under the curve that is shaded. If null hypothesis is true you would get this result only 24 % of the time. The students need to know this. This is very basic material.

Question #25 was done well by quite a few.

Teacher: Your homework is to dig and figure out why the answers are right.

They must know answers for #21 to end for tomorrow. They must know why. They should take off covers and also open textbooks and clean out any papers homework tomorrow.

Lesson 19 Discussion of Multiple Choice Questions

Teacher is still working on multiple choice questions. Teacher is asking for questions on these multiple choice questions. She is chastising them for not putting forth effort.

Question #36 – Student T

You’ve got 1, 2, and 3. Random samples are taken. Probability – Drawing with replacement.

Question #35 – Student V

Hypothesis test question is asking about the power of the test. You would want to increase sample size or alpha value.

Question #34 – Student J

Teacher says wording is tricky. She asks her to read it. This question is about correlation which is .428 and you need to square it. That gives the r^2 *value of .183. About 18% of the variation in blood pressure can be explained by the linear relation between blood pressure and caffeine. This language is crucial language to know. This is done by a process of elimination.*

Explanation – for every unit that x increases we know what y changes by one unit for a perfect correlation. For 18% variation a change in x only explains 18% of change in y. The other 82% is not explained, other factors are involved. The teacher has explained this concept a multitude of times. That is, a variation of 18% in blood pressure can be explained by caffeine.

Question #38 – Student M

Student A had the same concern. She explains that at first she chose a binomial distribution. She then saw that it was a proportion test. Several answer choices fell in this category. There were some proportions given. Student I is not paying attention. Student cc has nothing on her desk. Student G is clueless writing on the board. Z = (.4 - .35)/(sqrt(.35.65/500)=2.344. How do we know this is a z-test? We know this because there was no mean in problem.*

Question #26 – Student J

This is a sample size question. There was a mistake in z-value in order to get sample size. Student J writes on the board. She writes $z^*\sqrt{\frac{p^*(1-p^*)}{n}} \le m$. *Then she wrote 2.056 instead of 2.88. She needed to use .3 instead of .5 for probability.*

Question #27 – Student L

This is a cumulative graph. How many scored between 35 and 48? Teacher shows how to disaggregate the data in the graph to answer the question. The lower 50% had more variability than the other part.

Question #39 – Sally and Betty Problem

Sally and Betty are lab partners. At first this problem seems impossible! How could a two-tailed test reject what a one-tailed test failed to reject? Answer: if the one-tailed test is shaded the wrong way! Only z = -1.980 a sufficient value to reject a two-tailed test. If z = -1.980 is shaded greater than, then the one-tailed test fails to reject! Pretty tricky!

Most contested question on the 2002 test.

The students need to know 39 from this test and 35 questions from the other Multiple Choice Test that has been discussed fully.

Continuation

Teacher: What is the connection between standard deviation and variability in the control group?

Student: Student T – The smaller the standard deviation the less variability.

Teacher: The standard deviation has a value of 2.141. How does this value summarize the variability of the control group? Please underline standard deviation and variability.

No students are forthcoming with answers. She asks Student bb to answer. He makes a comment. The answer is not good.

Teacher: Can someone improve on that answer?

Student: Student Z – I need to know if the distribution is normal.

Again she comments on voice range of students.

Student: Student Z – You need to use the 68-95-100 rule.

Teacher: That was a good comment. The standard deviation measures a typical or average distance between the individual discoloration rating and the mean ratings.

Student: Student O – I used a dot plot analysis. Low scores indicate little discoloration.

Teacher: I want you to reconsider that answer.

Student: Student G – I visually calculated the mean of the treatment and control groups. I looked at proportion of those above the mean. So that may be a way to answer the question.

Student: Student T – I used the normality of the distribution. I looked for the skewness of the distribution.

Teacher: You have to be careful about determining skewness in this problem. This problem relates to strawberry discoloration.

Teacher: You need a 95% CI–for both groups using difference. Since CI is positive between (.16, 2.72), there is a positive and significant difference at the 0.05 level. *No students had attempted to answer using a confidence interval. The teacher explains that zero is not contained in interval. So there is a positive difference in discoloration. This certainly is a match up for the two sided test. For a one-sided test that would be half the level. The idea of using CI to comment on hypothesis test has been done many times before. The students don't get it.*

They will be working on problem #2 – to turn in at the end of class.

Lesson 20 continuation of advanced placement questions

Question #2: Students said it was easy because it was an experimental design question. They will work on part a) first. Student A reads question – Dog health question.

Student: Student A – Part a) What would be the advantage of adding a control group to the design of this study? *This is a dog supplement study concerning glucosamine. A control group would show deterioration in control group also.*

Student: Student T – It would show the negative affects to the dogs' health such as other parts of its well-being. You can compare dogs without the supplement to the dogs with the supplement.

Teacher: What is important about having a control group? A control group helps you identify lurking variable (students blurt out the answer). The rubric says that the normal aging process can be detected with the control group. The improvement of using a supplement will then will be more noticeable. These are not young dogs. So the impact of the supplement would be more noticeable by ruling out other factors.

Student: Student W – The control group will allow the researcher to test normal conditions versus experimental conditions.

Teacher: Part b) A control is added to two groups and you are asked to explain how you would assign to the groups. You need to be specific.

Student: Student cc – I would assign 0 – 299, *should be 000 – 299. Teacher adds the fact that if she is using the random digit table she must use three digits.* The dogs with numbers 000 – 099 would be assigned to the first group.

Teacher: How would you assign them randomly? Student cc, your plan has holes in it. You could use random number generator from calculator. You could consult a random digit table to assign the dogs.

Teacher: Student I how did you assign the dogs to the groups?

Student: Student I– I just selected the first 100 dogs and assigned it to a group.

Teacher: That is incomplete and would not get credit.

Student: Student V – Just select the first 100 numbers and assign to the first group using a number generator. A number of 374 would get tossed out.

Student: Student I – That was what I said.

Teacher: Your language needs to be clear. We are giving numbers to dogs and assigning them to groups. Not only do dogs get assigned to groups but treatment is assigned to groups would be a better randomization.

Student: Student Z – I would toss a die, 1 or 2 is group 1, 3 or 4 group 2, 5 or 6 group 3.

Teacher: Be careful with the assignments. Remember the butterfly problem. Grabbing the first 10 butterflies would choose the weaker and slower ones. The rubric assigns a number from random number generator until all dogs are assigned using numbers 001-300. A random number generator assigns each dog. Numbers 1-100 get assigned to group one etc. This is easier than the random digit table.

Teacher: Part c) Investigators want to block on clinics and the other investigators on breed. Which is the better blocking variable?

Student: Student T – I would block by dog breed type because of differences in bone structure and muscles. Blocking by clinics will not work because clinics have different treatment practices. They are prone to diseases according to type of breed.

Student: Student W – Blocking by breed will not necessarily have all breeds accounted for.

Teacher: Not all breeds need to be represented. Also, what do you do with mixed breeds?

Researcher: Lack of experience might make you choose clinic for choice. Student W thought blocking on breed would not cover each type of breed.

Teacher: The answer key says a good support to clinic choice is also acceptable if you are convinced that allows for more variability. Rubric says that blocking on breed gives more variability.

Student: Student A – I have never owned a dog. I also do not have a close relative that owns a dog so I would have picked blocking by clinics.

Even though students' arguments show some flaws it was good to see students get really involved in this discussion. The factor to get these students interested is most certainly interest in the problem. Question #3: Students have 15 minutes to finish. Fish problem – Concerning length.

Teacher: Student M what is the formula for z?

Student: Student M – I don't know.

Teacher: Find your formula sheet. You will have this on your exam.

Part a) Student bb had a good answer. His answer is inaudible.

Teacher: A drawing can accompany explanation to clarify what is being said. Sampling distribution of sample mean is nearly normal with mean 8 with the other having more variability. When n = 15 the area under the curve is larger to the right of a particular value.

Part b) There are a lot of questions. Student H answers the question.

Student: Student K – Is the calculator answer alone okay.

Teacher: A sketch should accompany a calculator answer.

Teacher: Standard deviation .3 for the sampling distribution is given, for mean 8 for sample size 50.

Student: Student H answers but did not write probability statement as shown below.

Prob ($\overline{x}$ < 7.5) = $P(z < \frac{7.5-8}{.3}) = P(z <-1.667) = 0.0478$

Student: Student E – Shouldn't she have divided by sqrt (50)?

Teacher: The information given says that the standard deviation is of sampling distribution so .3 is standard deviation of sample with the division by sqrt (50).

Student: Student K – How do you figure out standard deviation of population?

Teacher: This is not possible. To find sample standard deviation you would multiply answer by sqrt(50).

There is much more variability in the whole population than there is in small samples taken from the larger population. – This seems to be a difficult concept for the students.

Teacher: Part c) If distribution is instead non-normal; could you still use normal distribution?

Student: Student J – I believe no.

Student: Student Y – It is okay because of sample size of 50.

Teacher: Remember the activity you did about the Central Limit Theorem.

Student: Student W– I remember but I don't think I would have gotten credit on this question.

Question #4 – A slight problem is evident concerning this.

Matched Pair t-test. – E. Coli test. Must identify test by name or by formula.

Teacher: Assumptions need to be checked. The ten pairs will allow for independence to be met, and population is nearly normal by checking stem plot. It will not be terribly normal but skewness and outliers should be ruled out.

Teacher: The rubric says that you must show correct mechanics in calculating t statistic = 1.46. You end up with .1793 = p-value with 9 degrees of freedom. Students must state answer in context. Null hypothesis is not rejected. The two specimens are not terribly different.

Teacher is disappointed in how little they are working.

APPENDIX E: STUDY GUIDE EXAMPLE

The Study Guide is a document that introduces the students to the vocabulary and concepts in the chapter. A study guide is given for each chapter in the textbook.

What follows is a facsimile of the study guide used for Chapter 3. The original document was downloaded from the internet by the teacher of the course for use in the classroom.

Chapter 3: Examining Relationships

Key Vocabulary:

- response variable — explanatory variable
- independent variable — dependent variable
- scatterplot — positive association
- negative association — linear
- correlation — r-value
- regression line — mathematical model
- least-squares regression line — $\hat{y}$ "y-hat"
- SSM — SSE
- r^2 — coefficient of determination
- residuals — residual plot
- influential observation

Calculator Skills:

- seq(X,X,min,max,scl) — 2-Var Stats
- sum — Diagnostic On
- $\overline{x}, s_x, \overline{y}, s_y$ — Clear All Lists
- residual plot

3.1 Scatterplots (pp.107-127)

1. What is the difference between a *response variable* and an *explanatory variable*?
2. How are response and explanatory variables related to *dependent* and *independent* variables?
3. When is it appropriate to use a *scatterplot* to display data?
4. Which variable always appears on the horizontal axis of a scatterplot?
5. Explain the difference between a *positive association* and a *negative association*.

3.2 Correlation (pp.128-136)

1. What does *correlation* measure?
2. Explain why two variables must both be *quantitative* in order to find the *correlation* between them.
3. What is true about the relationship between two variables if the *r-value* is:

   a. Near 0? b. Near 1? c. Near -1? d. Exactly 1? e. Exactly -1?

4. Is *correlation* resistant to extreme observations? Explain.
5. What does it mean if two variables have *high correlation*?
6. What does it mean if two variables have *weak correlation*?
7. What does it mean if two variables have *no correlation*?

3.3 Least-Squares Regression (pp.137-163)

1. In what way is a *regression line* a *mathematical model*?
2. What is a *least-squares regression line*?
3. What is the formula for the equation of the *least-squares regression line*?
4. How is *correlation* related to *least-squares regression*?
5. What is the formula for calculating the *coefficient of determination*?
6. The r^2 value shows how much of the variation in one variable can be accounted for by the linear relationship with the other variable. If $r^2 = 0.95$, what can be concluded about the relationship between x and y?
7. Define *residual*.
8. If a *least-squares regression line* fits the data well, what characteristics should the *residual plot* exhibit?
9. What is meant by an *influential observation*?