ROUTING LINE PERSONNEL FOR RESTORATION OF DISRUPTED POWER DISTRIBUTION NETWORK

by

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ABSTRACT

BHAVISH GOKUL GOLLA. Routing line personnel for restoration of disrupted power distribution network (Under the direction of Dr. CHURLZU LIM)

Natural disasters such as storms, floods, earthquakes cause a great deal of damage to human lives and properties, and this damage is aggravated by disruptions to some critical infrastructures, such as transport system, electricity and water supplies. The primary impact of natural disasters on power systems would be severe power outages. When the period of power outage is prolonged the suffering of the customers becomes far more worsened. Consequently, it is desired to restore power distribution systems promptly. In this thesis, we are concerned with a so-called repair crew scheduling problem that minimizes the total restoration time by adequately scheduling repair personnel.

Given information about fault locations and estimated repair times along with travel time data between locations, we investigate mathematical optimization models, specifically two mixed-integer programming (MIP) models. The first MIP model is formulated in analogy to the identical multiple machine scheduling problem, while the second model is proposed in an effort to reduce solution times. Noting that the repair crew scheduling problem is NP-hard, a heuristic method is also proposed. A numerical study was conducted to compare computational performances of the aforementioned exact models and the heuristic method. The computational results revealed that the second MIP model is promising when time allows. It is also observed that the heuristic method can be practically implemented under time pressure. Based on this observation, we propose an implementation plan that combines the second MIP and the heuristic method in a dynamic circumstance.

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CHAPTER 1: INTRODUCTION

Natural disasters such as storms, floods and earthquakes can cause a great deal of damage to human lives and properties, and such damages can be far worsened when essential infrastructures such as transport system, electricity, and water supply are disrupted for a prolonged period. According to the report prepared by the Executive Office of the President in 2013, weather-related power outages costed an annual average of $18 billon-$33 billion to the US economy, during the period 2003-2012. In 2008, the year in which Hurricane Ike occurred, the cost estimates ranged from $40 billion to $75 billion. In the year of Hurricane Sandy, 2012, the cost was estimated to be between $27 billion-$52 billion. These cost estimates were initially collected by major electric companies using customer surveys and later annually compiled by Sullivan [1] based on value-of-service (VOS) data. These annual estimates are then used to calculate a range of the inflation-adjusted average annual costs [2]. As the damage is significantly exacerbated overtime, shortening restoration time can play a critical role to effectively reduce the damage costs. Depending on the severity of the storm and resulting impairment, power outages can last a few hours or extend to periods of several days. This in turn can have real economic effects, as power outages can impact businesses (primarily through lost orders and damage to perishable goods and inventories), and manufacturers (mainly through downtime and lost production, or equipment damage) [3]. Table 1 shows the costs per outage per customer used in the Tobit regression equation to estimate the total cost of power interruptions to U.S. electricity customers [4].

Table 1: Estimated Cost-per-Outage-per-Customer for the U.S. (costs shown in U.S, 2002)

Duration	**Residential**	**Commercial**	**Industrial**
0 sec	$2.18	$605	$1,893
1 hour	$2.70	$886	$3,253
Sustained Interruption	$2.99	$1,067	$4,227

Table 2 displays the costs per customer per outage presented in 2014 [5].

Table 2: Cost-per-Outage-per-Customer (2014)

	Residential	Commercial	Industrial
Cost per customer	$23	$4,257	$35,757

This thesis is concerned with minimizing the time to restore infrastructure after a major storm event via mathematical optimization, called mixed integer programming (MIP). In particular, this thesis considers power distribution system that needs to be repaired after disruption caused by natural disasters or malicious attacks by terrorists. However, it should be noted that the approach investigated in this thesis can be also applied to restore other types of damaged infrastructure systems.

1.1. Problem Overview

When a power distribution system is severely damaged, a number of faulty components of the system need to be repaired. For example, Duke Energy, a utility company that supplies electricity in most part of North and South Carolina States, had to restore power for more than a million customers who lost power due to Hurricane Matthew in September 2016 [6]. Resources to repair damages include repair agents and various types of repair vehicles. Given a limited amount of resources, the optimization problem in this thesis is to minimize the time to repair all faults, which are scattered in a wide geographical region. Hence, the solution is in a form of scheduling available repair crews while estimates of repair times and travel times are under consideration.

1.2. Approach

In order to find a scheduling solution, two exact MIP models and one heuristic method have been investigated in this thesis. The first MIP model was adapted from [7], while the second alternative MIP model is proposed in this thesis. The third method was proposed in consideration of NP-hardness of the problem and the need for a real-time decision making environment.

To compare the efficacy of the three considered methods, a numerical study was conducted on two sets of test problems. The first set of test problems were generated as relatively small scale in order to ascertain the performance of two exact methods and the difference in solution quality between the exact method and the heuristic approach. The second set of test problems were generated based on the scenario similar to the past damage data resulted from severe storm incidences, and used to test three methods under realistic scenarios.

1.3. Organization of thesis

This thesis is organized as follows. Chapter 2 provides reviews on studies relevant to the repair crew scheduling. In Chapter 3, three aforementioned solution methods will be described in detail. A numerical study and its results are presented in Chapter 4, dynamic implementation of the problem offered in Chapter 5, which is followed by conclusion in Chapter 6, where the thesis is summarized, and potential future research directions are outlined.

CHAPTER 2: LITERATURE REVIEW

Although there are some studies related to the system restoration after major outage events, most literature is focused on the estimation of restoration time. For example, Liu et al [8] introduced so-called accelerated failure time (AFT) models to estimate the duration of each probable storm-caused electric power outage. These outage durations were then used to estimate restoration times. AFT falls into the category of survival analysis model (i.e., statistical analysis specifically for time-to-event data, in this case, time until an outage is restored, or outage duration). This model was applied to hurricane and ice storm events for three major electric power companies on the East Coast of United States. By means of a large dataset that includes the companies' experiences in six hurricanes and eight ice storms, AFT models were fitted and used to forecast the duration of each probable outage in a storm. Restoration curves were then estimated for each county in the companies' service areas by aggregating those estimated outage durations and accounting for variable outage start times. This technique can be applied as a storm approaches, before damage assessments are available from the field, thus helping to better inform customers and the public of expected post-storm power restoration times. Results of model applications suggest they have promising predictive ability.

When a high impact low frequency (HILF) event occurs, it is a complicated process to deal with the power restoration tasks once the event has moved out of the region. In order to optimize the post-earthquake restoration of the electric power system, Xu et al [9] developed a stochastic integer program to determine how to schedule inspection, damage assessment, and repair tasks. The objective of the optimization is to minimize the average time duration of customers being without power. While this restoration model is adapted from the way that the Los Angeles Department of Water and Power (LADWP) restoration process currently works, this optimization model can be used to determine if different inspection, damage assessment, and repair schedules would improve the restoration process. The models assume that all generation stations will be on immediately after an earthquake, that load balance is never a constraint during the restoration, and that the household-based estimate of number of customers is accurate. The study focuses only on the performance of restoration processes with different inspection, damage assessment, and repair schedules. It does not consider the possibility of optimizing the amount of repair material stored for use in a post-earthquake restoration situation. Despite these limitations, the results suggest that the optimization modelling approach presented could potentially be used to achieve some improvements in post-earthquake restoration.

Observing that the scheduling problem considered in this thesis resembles the sequence-dependent vehicle routing problem, this chapter provides reviews on the literature relevant to the vehicle routing problem, including the traveling salesman problem, vehicle routing problem, multi-depot multiple traveling salesman problem and multiple machine scheduling problem.

2.1. Traveling Salesman Problem

The traveling salesman problem (TSP) is one of challenging optimization problems, and can be considered as the foundation to many other techniques driven on solving complex optimization problems, such as transportation, network flow, vehicle routing, crew scheduling, etc. TSP refers to a salesman, traveling to a given number of cities, visiting each one of the cities once starting from his city and returning to the same home city, with an objective to minimize the total cost of the trip. TSP is an NP-hard optimization problem and has its applications in various fields, (e.g., drilling of printed circuit boards [11], overhauling gas turbine engines [12], the order-picking problem in warehouses [13], and vehicle routing problem [14]).

One particular extension of TSP is called multiple traveling salesman problem (*m*TSP), where multiple salesmen travel from the same home city. Applications of *m*TSP are mostly in routing and scheduling problems, which include, printing press scheduling problem [15, 16], school bus routing problem [17], crew scheduling problem [18, 19, 20], interview scheduling problem [21, 22], and mission planning problem [23, 24, 25].

Dantzig, Fulkerson and Johnson [26] are credited to the first formulation of the TSP as an integer program in 1954. Since then, there have been numerous mathematical formulations of TSP and *m*TSP. Some notable studies include MIP formulations for the *m*TSP, for both symmetrical and asymmetrical cost structures [27], tree based formulation and three-index based formulations [28], comparison of eight distinct formulations of the TSP as an integer program [29] and survey of different integer programming formulations of the TSP [30].

2.2. Vehicle Routing Problem (VRP)

The vehicle routing problem can be described as follows. Consider a depot with one or more vehicles, and set of n customers. The objective is to find minimum cost routes for this fleet of vehicles to serve n customers in a timely manner. Demand of each customer must be supplied exactly once, by only one vehicle. It is assumed that the demand of any customer does not exceed the vehicle capacity. The total distance of any given route cannot exceed a pre-specified bound. The solution will be answering the question of “which vehicle travels to which customer(s) and in what order?”. When the number of vehicles in the above-mentioned problem is one, the capacity of the vehicle is infinity, and the distance is unbounded, this problem reduces to TSP. One of the earliest studies considered the routing of a fleet of gasoline delivery trucks between a bulk terminal and several service stations supplied by the terminal and proposed a VRP model [32].

There are quite a few variants in the VRP. When only the capacity of vehicles is constrained, the problem becomes Capacitated VRP (CVRP). When customers must be served within predefined time windows, without violating the capacities of vehicles and total trip time, the problem is called VRP with Time Window (VRPTW).

Dynamic vehicle routing problem (DVRP), also referred to as on-line vehicle routing problem, has taken its prominence recently due to the advancement in information and communication technologies that allow information to be obtained and processed in real-time. In DVRP, some of the predetermined visits are known in advance before the driver starts his/her working day, but as the day progresses, new orders arrive and the system has to incorporate them into an evolving schedule. There is an underlying assumption that the crew dispatcher and the driver have an on-going interaction throughout the day. The dispatcher can periodically communicate to the drivers about the new visits assigned to them. In this way, each driver always has knowledge about the next customers assigned to him/her [33].

2.3. Multi-Depot Multiple Traveling Salesman Problem

The multi-depot multiple traveling salesman problem (MDMTSP) is another variant of the TSP that is closely related to the problem considered in this thesis and deserves more attention. In MDMTSP, more than one salesman travels to a given set of cities, starting from a set of depots, with multiple salesman at each depot. Each city must be visited exactly once by only one salesman. MDMTSP can be categorized into a fixed destination version and a non-fixed destination version. In the fixed destination version, all salesmen return to the same depot from which they started. In the non-fixed destination version, all salesmen need not return to the same depot from which they started, but the number of salesmen starting and ending at that particular depot should remain the same [34]. The objective of the MDMTSP is to minimize the total cost of the routes, where the cost can be measured in terms of cost, distance, or time.

The problem has some real applications and is closely related to other important multi-depot routing problems, e.g., the multi-depot vehicle routing problem [36] and the location routing problem [37]. There are other applications as in the motion planning of a set of unmanned aerial vehicles [38, 39, 40, 41] and the routing of service technicians where the technicians are leaving from multiple depots [42].

2.4. Multiple Machine Scheduling

Repair crew scheduling problem also resembles multiple machine scheduling by considering crews as machines and repairs as jobs. Multi-machine scheduling (MMS) can be defined as the action of assigning a number of jobs to a number of performing machines such that certain performance demands like time or cost effectiveness are fulfilled. Consider a set of jobs that must be run and a set of machines available to execute the jobs. Jobs may be classified into groups of similar job types. Each machine can run only one job type at a time, and each machine may have some unique characteristics, such as production rate. Each job has a due date, on which production should have been completed, and certain jobs may be restricted to a subset of the available machines. When a machine changes from one job type to another, there may be some setup time which depends on the similarity of the job types. Machine characteristics may also command batch size preferences if, for example, very small batches result in higher defect rates. When the machines are not located in the same facility, the cost of transporting the product to its destination is machine-dependent, which implies a job-machine assignment cost.

Several related problems have been broadly studied in the literature [45]. Scheduling jobs on a single machine to minimize (weighted) tardiness is studied in [46, 47, 48, 49, 50]. The extension to multiple machines is considered in [51, 52, 53]. Scheduling subject to sequence dependent setup times is considered in [54] for single machine; in [55, 56] for parallel identical machines. Du and Leung [57] proved that minimizing the total tardiness on one machine is NP-hard. Further complexity results on machine scheduling problems can be found in [58].

CHAPTER 3: MODEL DESCRIPTION

This chapter presents two MIP models to represent the repair crew scheduling problem. The first MIP model is a straight-forward adaptation of Guinet [7], while the second one is a new MIP model proposed in this thesis. In addition to these exact models, a heuristic solution method is proposed in consideration of NP-hardness of the problem and the need for a dynamic decision making environment.

3.1. Adaptation of Job Scheduling with Sequence-Dependent Setup Time

The repair crew scheduling problem can be considered as a job scheduling problem, where each job corresponds to repairing a fault and machines are repair crews. It is assumed that crew teams have identical resources. Travel times of repair crew depend on the locations of faults that are repaired consecutively. Regarding travel times as setup times, therefore, the problem becomes a job scheduling problem with sequence-dependent setup times on identical machines. With these similarities between repair crew scheduling and job scheduling with sequence-dependent setup time, the MIP model from machine scheduling [4] has been adapted in repair crew scheduling and altered slightly to incorporate multiple starting depots.

To present the first MIP model, the following notation will be used

3.1.1. Problem parameters

$N = number\ of\ repairs\ (faults)$

$K = number\ of\ crews$

$L = number\ of\ operation\ centers$

$K(l) = set\ of\ crew\ indice\ in\ operation\ center\ l$

$p_i = estimated\ repair\ time\ at\ fault\ location\ i$

$T_{ij} = estimated\ travel\ time\ from\ i\ to\ j$

$T_{li} = estimated\ travel\ time\ from\ operation\ center\ l\ to\ the\ fault\ location\ i$

$M = a\ large\ number\ (\gg 1)$

3.1.2. Variables

$x_{ijk} = 1\ if\ the\ fault\ location\ j\ is\ processed\ directly\ after\ fault\ location\ i\ by$

$crew\ k;\ 0\ otherwise$

$x_{0jk} = 1\ if\ the\ fault\ location\ j\ is\ the\ first\ repair\ job\ to\ be\ processed\ by\ crew\ k;$

$0\ otherwise$

$x_{i0k} = 1\ if\ the\ fault\ location\ i\ is\ the\ last\ repair\ job\ of\ crew\ k;$

$0\ otherwise$

$a_i = completion\ time\ of\ repair\ at\ fault\ location\ i$

$R = total\ restoration\ time$

With above notation, the problem can be written as follows:

$$\text{Minimize } R \tag{1}$$

subject to

$$\sum_{\substack{i=0 \\ i\neq j}}^{N}\sum_{k=1}^{K} x_{ijk} = 1 \qquad \forall j = 1,\ldots,N \qquad (2)$$

$$\sum_{\substack{i=0 \\ i\neq h}}^{N} x_{ihk} - \sum_{\substack{j=0 \\ j\neq h}}^{N} x_{hjk} = 0 \qquad \forall h = 1,\ldots,N, \quad \forall k = 1,\ldots,K \qquad (3)$$

$$\sum_{j=0}^{N} x_{0jk} \leq 1 \qquad \forall k = 1,\ldots,K \qquad (4)$$

$$a_j \geq a_i + T_{ij} + p_j + \left(\sum_{k=1}^{K} x_{ijk} - 1)\right) \times M$$

$$\forall i = 1,\ldots,N, \ \forall j = 1,\ldots,N, \ j \neq i \qquad (5)$$

$$a_j \geq T_{lj} + p_j + \left(\sum_{k\in K(l)} x_{0jk} - 1)\right) \times M$$

$$\forall j = 1,\ldots,N, \ \forall l = 1,\ldots,L \qquad (6)$$

$$R \geq a_i \qquad \forall i = 1,\ldots,N \qquad (7)$$

$$x_{ijk} \in \{0,1\} \qquad \forall i = 1,\ldots,N, \ \forall j = 1,\ldots,N, \ \forall k = 1,\ldots,K$$

$$a_i \geq 0 \qquad \forall i = 1,\ldots,N \qquad (8)$$

$$a_0 = 0 \qquad \forall l = 1,\ldots,L \qquad (9)$$

The objective function (1) is a single variable $R_{,}$ which will represent the makespan of job scheduling problem (or total restoration time in our application). A set of constraints are listed in (2) to (9) which must be satisfied while minimizing the objective function. Each fault is repaired by one crew and only once, which is ensured by constraint (2). Constraint (3) ensures that once crew k repairs fault h, it leaves for another fault location on return to an operation center. Constraint (4) is enforced to avoid assigning a crew to multiple faults initially. Constraints (5) and (6) impose the lower bound on completion time at each fault location, which depends on the processing time, travel time, completion time of the previous repair and the order of repairs assigned to the crew. Constraint (7) sets the lower bound on the maximum completion time, which must be greater than or equal to completion time at each fault location i. Completion time at each fault location is greater than or equal to zero as in constraint (8) and constraint (9) initializes completion time at operations center as 0.

3.2. Proposed MIP Model

This model was proposed in this thesis to help improve the solution effort of the repair crew scheduling problem when compared to the first MIP model.

In addition to the previously introduced notation, the following notation will be used to present the proposed model.

3.2.1. Variables

$a_{ijk} = 1\ if\ the\ fault\ location\ j\ is\ repaired\ after\ fault\ location\ i\ (not$

$necessarily\ immediate)\ by\ crew\ k;\ 0\ otherwise$

$x_{ik} = 1\ if\ the\ fault\ location\ i\ is\ assigned\ to\ crew\ k;$

$\quad\quad 0\ otherwise$

$y_{ik} = repair\ starting\ time\ at\ fault\ location\ i\ by\ crew\ k$

$C_k = upper\ bound\ on\ cycle\ time\ of\ crew\ k$

Other parameters are defined under the first MIP formulation.

The proposed formulation can be written as follows:

$$\text{Minimize } R \qquad (10)$$

subject to

$$\sum_{k=1}^{K} x_{ik} = 1 \qquad \forall i = 1, \dots, N \qquad (11)$$

$$y_{ik} \geq T_{li} - (1 - x_{ik}) \times M \qquad \forall i = 1, \dots, N,\ \ \forall l = 1, \dots, L,\ \ \forall k \in k(l) \quad (12)$$

$$y_{ik} + p_i + T_{ij} \leq y_{jk} + \left(1 - a_{ijk}\right) \times M \qquad \forall i, j = 1, \dots, N,\ \ i \neq j,\ \ \forall k \in k(l) \qquad (13)$$

$$a_{ijk} \leq x_{ik} \qquad \forall i, j = 1, \dots, N,\ \ i \neq j,\ \ \forall k = 1, \dots, K \qquad (14)$$

$$a_{ijk} \leq x_{jk} \qquad \forall i, j = 1, \dots, N,\ \ i \neq j,\ \ \forall k = 1, \dots, K \qquad (15)$$

$$a_{ijk} + a_{jik} \geq x_{ik} + x_{jk} - 1 \qquad \forall i, j = 1, \dots, N,\ \ i \neq j,\ \ \forall k = 1, \dots, K \qquad (16)$$

$$C_k \geq y_{ik} + p_i \qquad \forall i = 1, \dots, N,\ \ \forall k = 1, \dots, K \qquad (17)$$

$$R \geq C_k \qquad \forall k = 1, \dots, K \qquad (18)$$

$$R \geq 0 \tag{19}$$

$$C_k \geq 0 \qquad \forall k = 1, \ldots, K \tag{20}$$

$$y_{ik} \geq 0 \qquad \forall i = 1, \ldots, N, \;\; \forall k = 1, \ldots, K \tag{21}$$

The objective function (10) minimizes R, which results in the total restoration time. An objective value will be returned with the minimum time by when all the repairs can be completed by the crews. No fault location can be visited more than once. Each repair is to be processed by one crew and only once, which is ensured by constraint (11). Constraints (12) and (13) ensure that repair can be started after considering travel time to that location and, if not traveled from operation center, completion time of previous fault. Constraints (14) (15) (16) specify that faults i and j can be repaired by crew k only when they are assigned to same crew k. Constraint (17) enforces a lower bound on the cycle time of crew k. Constraint (18) terms the restoration time constraint.

3.3. Heuristic Method

The repair crew scheduling is NP-hard, and it will be difficult to find an optimal solution within a reasonable amount of time when the problem size is increased. Hence, a heuristic method is proposed to cope with NP-hardness of the problem. This heuristic method is adapted from [59], which is concerned with job scheduling on identical machines without setup times. The proposed heuristic is described in what follows.

Heuristic method:

Initialization step – For each fault location i, compute adjusted processing times;

$$\overline{p_i} = p_i + \left(\sum_{j=1, j \neq i}^{N} T_{ji} + \sum_{l=1}^{L} T_{li} \right) / (N - 1 + L)$$

Step 1 – Sort p_i in the descending order.

Step 2 – Assign the fault with longest p_i to crew 1, second longest to crew 2 and so on until all crews are assigned.

Step 3 – The fault with the next longest p_i is assigned to the crew that finishes its repairs first. Repeat this until all faults are assigned

For illustration, consider the following example. There are three crews, T1, T2 and T3 and 10 faults with sorted $p_i\{100, 90, 45, 20, 18, 15, 13, 12, 11, 10\}$

Step 2 gives:

T1: 100, T2: 90, T3: 45

Then, for next 7 faults, search for the crew that completed assigned repairs so far. In this example, T3 is next.

T1: 100, T2: 90, T3: 45+20

Next, to assign the fault with the adjusted processing time 18, search for the crew with minimum cycle time. That is, T3 (with 45+20=65) is next again.

T1: 100, T2: 90, T3: 45+20+18

In the same manner, subsequent assignments are:

T1: 100, T2: 90, T3: 45+20+18+15

Next,

T1: 100, T2: 90+13, T3: 45+20+18+15

Next,

T1: 100, T2: 90+13, T3: 45+20+18+15+12

Next,

T1: 100+11, T2: 90+13, T3: 45+20+18+15+12

Finally,

T1: 100+11 = 111, T2: 90+13+10 = 113, T3: 45+20+18+15+12 = 110

The resulting adjusted makespan for this problem using this heuristic will be 113.

CHAPTER 4: NUMERICAL STUDY

This chapter presents a numerical study conducted on two sets of scenarios to investigate the numerical performance of optimization methods presented in the previous chapter. For the comparative study of the above listed three solution methods, we have generated 20 small-sized scenarios and 20 relatively large-sized scenarios. Each small-sized scenario consists of 12 fault locations and four crew teams originated from three operation centers, while a large-sized scenario involves 56 fault locations and 17 crew teams from the same three operation centers. Large-sized scenarios were first generated to reflect a HILF event that has actually occurred in the past. Then, the small-sized scenarios were generated by scaling down these realistic scenarios. The way these scenarios were developed will be discussed in the following two sections.

4.1. Generation of Large-Sized Test Scenarios

In order to develop realistic fault scenarios, severe storm days in the past have been examined first. From a utility company, a set of fault data for 1,368 days during the period from April 2010 – July 2014 was obtained. It consists of a total of 65,635 faults that have occurred in the West Carolinas. The number of faults per day is displayed in Figure 1.

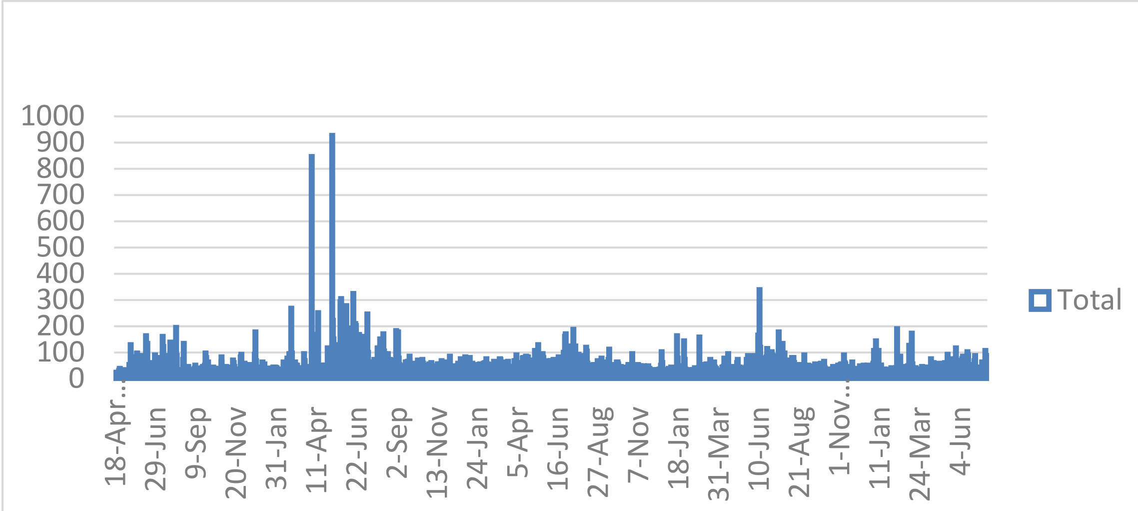

Figure 1: Number of faults per day from April 2010 to July 2014

Table 3: Number of days for ranges of daily faults

Bin	Range of Number of Faults			
	0-99	**100-199**	**200-799**	**800 and above**
2010(Apr-Dec)	215	11	0	0
2011	282	27	10	2
2012	307	13	0	0
2013	311	9	1	0
2014(Jan-Jul)	171	9	0	0
Frequency	1286	69	11	2

Among 1,368 days, there have been 1,286 days with 0-99 faults per day, 69 days with 100-199 faults per day, 11 days with 200-799 faults per day, and two days with more than 800 faults per day (see Table 3).

Note that there are two particular HILF events in 2011, which are specifically April 5th with a total of 847 faults and May 11th with 928 faults. In order to develop a realistic scenario for the Repair Crew Scheduling problem, the storm day of April 5th in 2011 was considered. On this particular day, a total of 270 circuits were involved with faults. Weather information between April 3rd - 5th and the impact of this HILF event was traced back [60] and are summarized below.

- April 3rd – 255 large hail reports, 75 damaging wind reports with wind gusts up to 80 mph hit parts of Kansas and Missouri.
- April 4th – 46 tornado reports, 90 reports of large hail, 1,318 damaging wind reports and 1,476 storm reports with wind gusts up to 90 mph.
- Nearly one million residents suffered from electricity loss (260,000 in the Carolinas).
- April 5th – Wind gusts of nearly 70 mph hit parts of North Carolina and a few other states which led to additional downed trees, power outages and damaged homes.

When faults occur, operation centers of the utility are primarily responsible for repairs. Figure 2 displays the distribution of operation centers in the Carolinas.

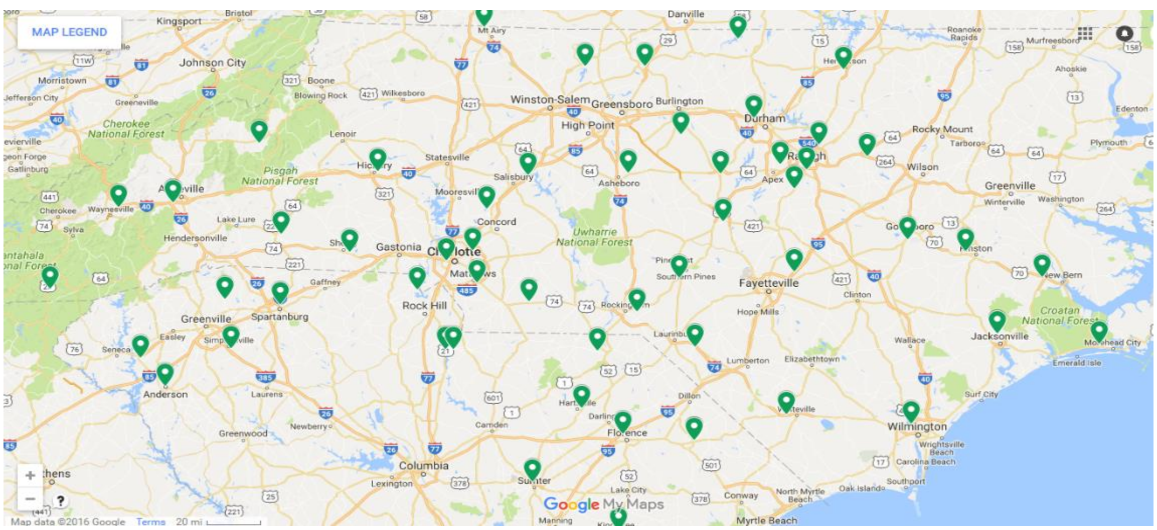

Figure 2: Operations centers of the utility company in the Carolinas

With additional information about substations provided by the utility company, it was attempted to re-create fault scenarios that have similar scales as the actual HILF event that occurred on April 5th, 2011 in and around the Greater Charlotte area.

In consequence, a total of 56 faults were generated using uniformly distributed random variates around eight substations in the Greater Charlotte area. In particular, based on the substation information (coordinates) provided by the utility company, fault location coordinates have been randomly generated [61] within a few miles radius of each substation. Considering the substation location as a center point, if there are less than five faults at that substation, faults have been generated within a 6-mile radius of the substation. And if five or more faults are involved with a substation, faults have been generated within 10-mile radius of the substation. In addition, there are three operation centers that are responsible for the restoration of faults in the Greater Charlotte area, namely, Little Rock, Newell and Matthews. Figure 3 displays locations of 56 randomly generated faults (colored in red and categorized based on their processing times) for one instance of the large-sized scenarios. Locations of three operation centers are colored in green.

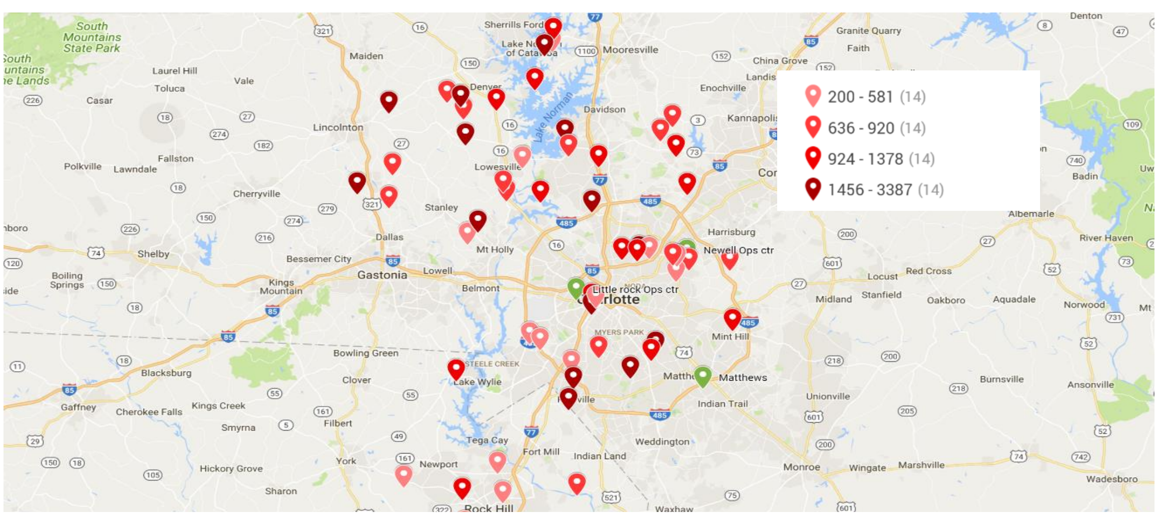

Figure 3: Locations of 56 faults and three operation centers (case of large-sized scenario 1)

Table 4: Estimated repair times at each fault location in minutes (case of large-sized scenario 1)

Fault location	**Estimated Repair Time**	**Fault location**	**Estimated Repair Time**	**Fault location**	**Estimated Repair Time**
1	340	**20**	1102	**39**	1212
2	461	**21**	838	**40**	1047
3	2272	**22**	807	**41**	856
4	2523	**23**	250	**42**	1218
5	679	**24**	249	**43**	1176
6	636	**25**	694	**44**	1070
7	994	**26**	812	**45**	863
8	924	**27**	712	**46**	930
9	1361	**28**	771	**47**	3365
10	1378	**29**	890	**48**	3387
11	1756	**30**	920	**49**	458
12	1530	**31**	1483	**50**	411
13	1579	**32**	1492	**51**	394
14	1456	**33**	2635	**52**	356
15	260	**34**	2364	**53**	581
16	200	**35**	2343	**54**	661
17	418	**36**	2531	**55**	1244
18	215	**37**	678	**56**	1351
19	967	**38**	579		

Tables 4 and 5 display the estimated repair times and travel times between locations, respectively, for scenario 1 of the large-sized scenarios. The estimated repair times were randomly generated based on the actual outage duration data of April 5th 2011. Due to a space limit, an example of only 10 fault locations are displayed in Table 5, where L, N and M represent the three operation centers, Little Rock, Newell and Matthews, respectively (complete estimated travel times can be found from Appendix 1). To generate travel times, the distance (in miles) between fault locations is calculated using Manhattan distance, i.e., l_1 - norm, between two coordinates, and then the travel times are computed using the distance and the speed of repair trucks, where the speed is set as 31.9 mph [62]. In [62], a

Python package, CitySpeed, is used to gather statistics on the efficiency and complexity of road networks using online mapping services. This can be used to create maps showing how fast you can drive around a particular city.

Table 5: Estimated travel times from operation centers to fault locations and between fault locations in minutes (for a sample of 10 fault locations in the case of large-sized scenario 1)

Locations	L	N	M	1	2	3	4	5	6	7	8	9	10
L		29	42	17	15	26	25	15	28	26	37	17	4
N	29		27	47	44	23	33	35	2	26	22	13	27
M	42	27		43	29	16	16	26	25	15	17	40	37
1	17	47	43		14	27	26	16	45	27	43	34	19
2	15	44	29	14		20	13	8	42	18	40	31	17
3	26	23	16	27	20		10	12	22	2	19	24	22
4	25	33	16	26	13	10		10	32	7	29	24	21
5	15	35	26	16	8	12	10		34	11	32	23	11
6	28	2	25	45	42	22	32	34		24	20	15	26
7	26	26	15	27	18	2	7	11	24		22	25	22
8	37	22	17	43	40	19	29	32	20	22		35	33
9	17	13	40	34	31	24	24	23	15	25	35		15
10	4	27	37	19	17	22	21	11	26	22	33	15	

The number of repair trucks at each operation center have been allotted as follows:

7 bucket trucks at Little Rock Operation Center

6 bucket trucks at Newell Operation Center

4 bucket trucks at Matthews Operation Center

It should be noted that this problem assumes that only bucket trucks of the utility company are utilized. This can be called as Aggregate Scheduling (or Aggregate Planning), where we aggregate the different types of trucks to a single representative unit, which can perform all the actions. In production, design and other fields, this concept helps in developing techniques for aggregating units of production, and determining suitable production levels based on predicted demand for aggregate units [63]. The purpose of this is to be able to develop a top-down plan for the entire problem, where bucket truck denotes a single representative repair unit which performs repairs of representative fault types.

4.2. Generation of Small-Sized Test Scenarios

Due to the NP-hardness of the problem, the exact methods may not be practical when the size of the problem becomes large. Small-sized scenarios were also generated in order to examine whether the exact MIP formulations provide optimal solutions within a reasonable amount of time, and whether the heuristic method provides quality solutions. For this set of scenarios, a total of 12 faults were randomly generated in the Greater Charlotte area covering eight substations in each scenario. Figure 4 displays the 12 randomly generated faults (colored in red and categorized based on their processing times) for one of small-sized scenarios and the operation centers colored in green.

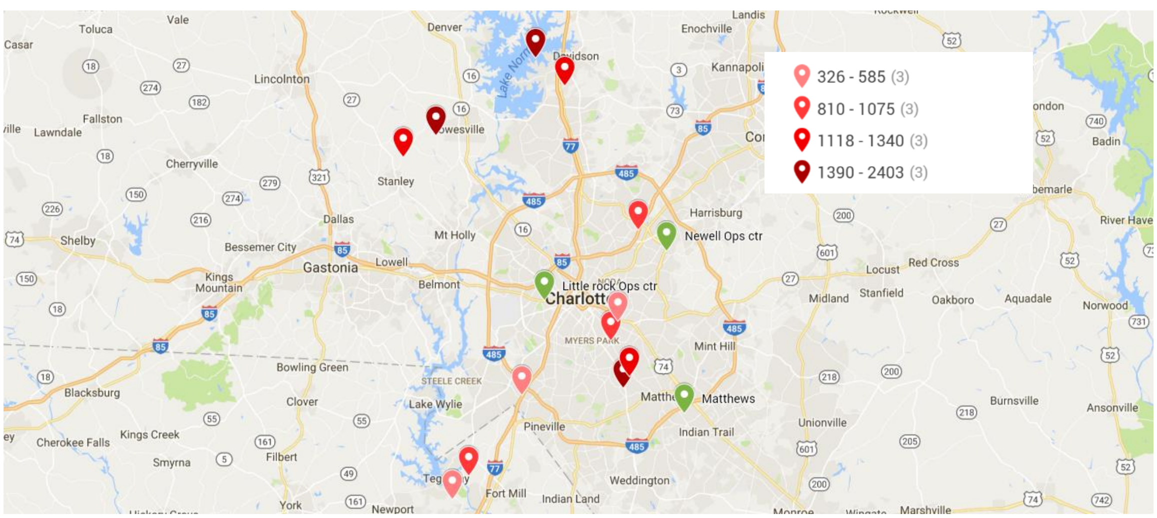

Figure 4: Locations of 12 faults and three operation centers (case of small-sized scenario 1)

Table 6: Estimated repair times at each fault location in minutes (case of small-sized scenario 1)

Fault location	**Estimated Repair Time**
1	375
2	2403
3	1075
4	1340
5	326
6	934
7	1258
8	1563
9	1118
10	1390
11	585
12	810

Table 7: Estimated travel times from operation centers to fault locations and between fault locations in minutes (case of small-sized scenario 1)

Locations	L	N	M	1	2	3	4	5	6	7	8	9	10	11	12
L		29	42	18	27	18	26	16	27	46	44	35	37	46	39
N	29		27	47	28	23	25	19	8	61	58	42	52	75	68
M	42	27		32	15	24	15	25	35	88	85	69	79	54	48
1	18	47	32		19	24	22	28	45	56	53	53	52	28	21
2	27	28	15	19		9	3	11	26	73	70	55	64	47	41
3	18	23	24	24	9		9	4	21	64	61	45	55	52	45
4	26	25	15	22	3	9		10	23	73	70	54	63	50	43
5	16	19	25	28	11	4	10		17	62	60	44	53	56	49
6	27	8	35	45	26	21	23	17		53	50	34	44	73	66
7	46	61	88	56	73	64	73	62	53		9	39	38	59	58
8	44	58	85	53	70	61	70	60	50	9		30	29	56	56
9	35	42	69	53	55	45	54	44	34	39	30		9	81	74
10	37	52	79	52	64	55	63	53	44	38	29	9		79	73
11	46	75	54	28	47	52	50	56	73	59	56	81	79		6
12	39	68	48	21	41	45	43	49	66	58	56	74	73	6	

Tables 6 and 7 display an example of the estimated repair times for faults and travel times between locations, respectively. The data set corresponds to the scenario 1 of small-sized scenarios. As before, L, N and M represent the three operation centers, Little Rock, Newell and Matthews, respectively in Table 7. Similar to the generation of large-sized scenarios, estimated travel times have been calculated based on the distances between locations and vehicle speed of 31.9 mph.

The number of repair trucks for the small-sized scenarios were set as follows:

2 bucket trucks at Little Rock Operation Center

1 bucket truck at Newell Operation Center

1 bucket truck at Matthews Operation Center

4.3. Results of Numerical Study

This section provides and compares the results of all the three solution methods implemented for solving both small-sized and large-sized scenarios. The MIP models have been coded in GAMS [64] and solved by the IBM ILOG CPLEX solver [65] using the NEOS server [66, 67]. In the reported results, EXACT1, EXACT2, and HEURISTIC refer to the first MIP model, proposed MIP model, and heuristic method, respectively.

4.3.1. Results – small-sized scenarios

Table 8: Restoration times for small-sized scenarios

	Restoration times (minutes)			Optimality percentage gap
Scenarios	EXACT1	EXACT2	HEURISTIC	$\left(\frac{HEURISTIC - EXACT2}{EXACT2}\right) \times 100$
Scenario 1	3345	3329	3496	5.02
Scenario 2	3337	3337	3626	8.66
Scenario 3	3361	3361	3587	6.72
Scenario 4	3072	3072	3269	6.41
Scenario 5	3352	3340	3641	9.01
Scenario 6	3401	3390	3632	7.14
Scenario 7	3939	3939	4072	3.38
Scenario 8	3772	3768	4005	6.29
Scenario 9	3356	3356	3582	6.73
Scenario 10	3390	3390	3685	8.7
Scenario 11	3611	3599	3777	4.95
Scenario 12	3102	3102	3455	11.38
Scenario 13	4037	4037	4165	3.17
Scenario 14	3533	3529	3832	8.59
Scenario 15	3336	3336	3502	4.98
Scenario 16	3800	3789	3982	5.09
Scenario 17	3112	3112	3360	7.97
Scenario 18	2939	2931	3196	9.04
Scenario 19	2978	2978	3218	8.06
Scenario 20	3367	3367	3575	6.18

Table 8 provides the restoration times obtained from all three methods for small-sized scenarios.

GAMS has a default 1000 seconds resource limit, which was employed in this study. Consequently, the models taking more than 1000 seconds have not necessarily returned the optimal solution. In that case, it provides the best solution until that point of execution.

Table 9: Execution times for small-sized scenarios

	Execution time (seconds)		
Scenarios	EXACT1	EXACT2	HEURISTIC
Scenario 1		4.536	
Scenario 2		3.05	
Scenario 3		5.459	
Scenario 4		7.712	
Scenario 5		10.304	
Scenario 6		10.167	
Scenario 7		15.027	
Scenario 8		7.699	
Scenario 9		11.793	
Scenario 10	>1000	5.08	<1
Scenario 11		5.649	
Scenario 12		42.346	
Scenario 13		15.797	
Scenario 14		3.346	
Scenario 15		5.669	
Scenario 16		30.486	
Scenario 17		7.42	
Scenario 18		13.033	
Scenario 19		25.679	
Scenario 20		2.745	

Observe that EXACT2, i.e., the proposed MIP model, provided optimal solutions within the time limit for all 20 small-sized scenarios. However, EXACT1 was not terminated within the time limit for all scenarios. Table 8 displays restoration times obtained from three methods. From the table, one can observe that best solutions reported from EXACT1 were indeed optimal for 12 scenarios, while it provided non-optimal solutions for eight scenarios (scenarios 1, 5, 6, 8, 11, 14, 16, 18).

The average solution time of EXACT2 was 11.649 seconds and ranged from 2.745 to 42.346 seconds as reported in Table 9. The heuristic method took less than a second, because of the simplicity of its implementation. Optimality percentage gap that is defined as $\left(\frac{HEURISTIC-EXACT2}{EXACT2}\right) \times 100$ is reported in the last column of Table 8. The average optimality gap was 6.87% and ranged from 3.17% to 11.38%.

4.3.2. Results – large-sized scenarios

Table 10: Restoration times for large-sized scenarios

	Restoration times (minutes)		
Scenarios	EXACT1	EXACT2	HEURISTIC
Scenario 1	9122	4010	4115
Scenario 2	9062	4534	4132
Scenario 3	8795	3983	4070
Scenario 4	8199	4052	3986
Scenario 5	8151	3989	4020
Scenario 6	8719	4023	4043
Scenario 7	6842	4699	4074
Scenario 8	7660	4234	4106
Scenario 9	7960	4056	4100
Scenario 10	8518	3870	4017
Scenario 11	8718	4046	4073
Scenario 12	8314	4206	4049
Scenario 13	8252	4101	4007
Scenario 14	8460	4595	4091
Scenario 15	7872	4262	4054
Scenario 16	8970	3970	4022
Scenario 17	8248	3898	4028
Scenario 18	8501	4289	4019
Scenario 19	6951	4008	4032
Scenario 20	6266	3999	4093

Table 10 provides the restoration times obtained from all three methods for large-sized scenarios, with the best of solutions being highlighted for each scenario.

Both EXACT1 and EXACT2 have taken more than 1000 seconds for all the 20 large-sized scenarios. Due to their termination rule after 1000 seconds, the best solutions until that point of execution are reported. EXACT2, i.e., the proposed MIP model, consistently outperformed EXACT1 for the large-sized scenarios. Of all the three methods, the HEURISTIC method attained shortest restoration times for nine of the 20 scenarios, while EXACT2 provided best solutions for the other 11 scenarios.

Table 11: Optimality gap for large-sized scenarios

Optimality percentage gap
$\left(\frac{EXACT1 - best\ of\ EXACT2\ and\ HEURISTIC}{best\ of\ EXACT2\ and\ HEURISTIC}\right) \times 100$
127.48
119.31
120.81
105.69
104.34
116.73
67.94
86.56
96.25
120.1
115.47
105.33
105.94
106.8
94.18
125.94
111.59
111.52
73.43
56.69

In order to measure the relative performance of EXACT1 compared to the other methods, optimality percentage gap, that is defined as $(\frac{EXACT1 - best\ of\ EXACT2\ and\ HEURISTIC}{best\ of\ EXACT2\ and\ HEURISTIC}) \times 100$ is calculated for the large-sized scenarios and presented in Table 11. Here, best of EXACT2 and HEURISTIC indicates the best solution between both methods for that particular scenario. On average, EXACT1 resulted in 103.6% more restoration time than the best of the other two methods.

As far as the comparison of EXACT2 and HEURISTIC is concerned, the average optimality gap for the 11 scenarios for which EXACT2 performed better, was 1.75% and the average optimality gap for the nine scenarios where HEURISTIC performed better was 6.69%.

CHAPTER 5: DYNAMIC IMPLEMENTATION OF REPAIR CREW SCHEDULING

In the previous chapters, we have compared three optimization methods for scheduling repair crews based on estimated repair times and travel times. During a restoration process such estimated repair times will be provided by damage assessors after inspecting faults in affected areas. Once a repair crew team visits a fault location, these estimates may be changed or updated to reflect the current status of the fault repair progress. As we have seen from the numerical results in Chapter 4, EXACT2 can provide optimal scheduling solutions when time allows, while the heuristic method can quickly furnish an approximated solution under time pressure. With this observation, we propose a dynamic scheduling procedure in this chapter for practical implementation of optimization methods discussed. The main idea is to use the EXACT2 model for an initial scheduling and HEURISTIC is subsequently employed to reflect changes made in repair time estimates in real time. In this dynamic implementation, we assume that the schedule is periodically updated based upon new estimates at that time. Furthermore, it is reasonable to assume that no preemption is allowed. That is, when the schedule is updated, repair jobs that are being performed must be completed without interruption.

5.1. Dynamic Repair Crew Scheduling

This section presents the step by step procedure that is proposed for dynamic implementation. Suppose that the schedule update is performed every T time units.

Step 1: Input initial estimates of repair times for each fault and travel times for each pair of fault locations. Solve EXACT2 to obtain the initial crew schedule.

Step 2: Deploy repair crews according to the current schedule.

Step 3: Collect updated estimates for next time period T.

Step 4: Apply Modified HEURISTIC to revise the repair crew schedule.

Step 5: Return to Step 2.

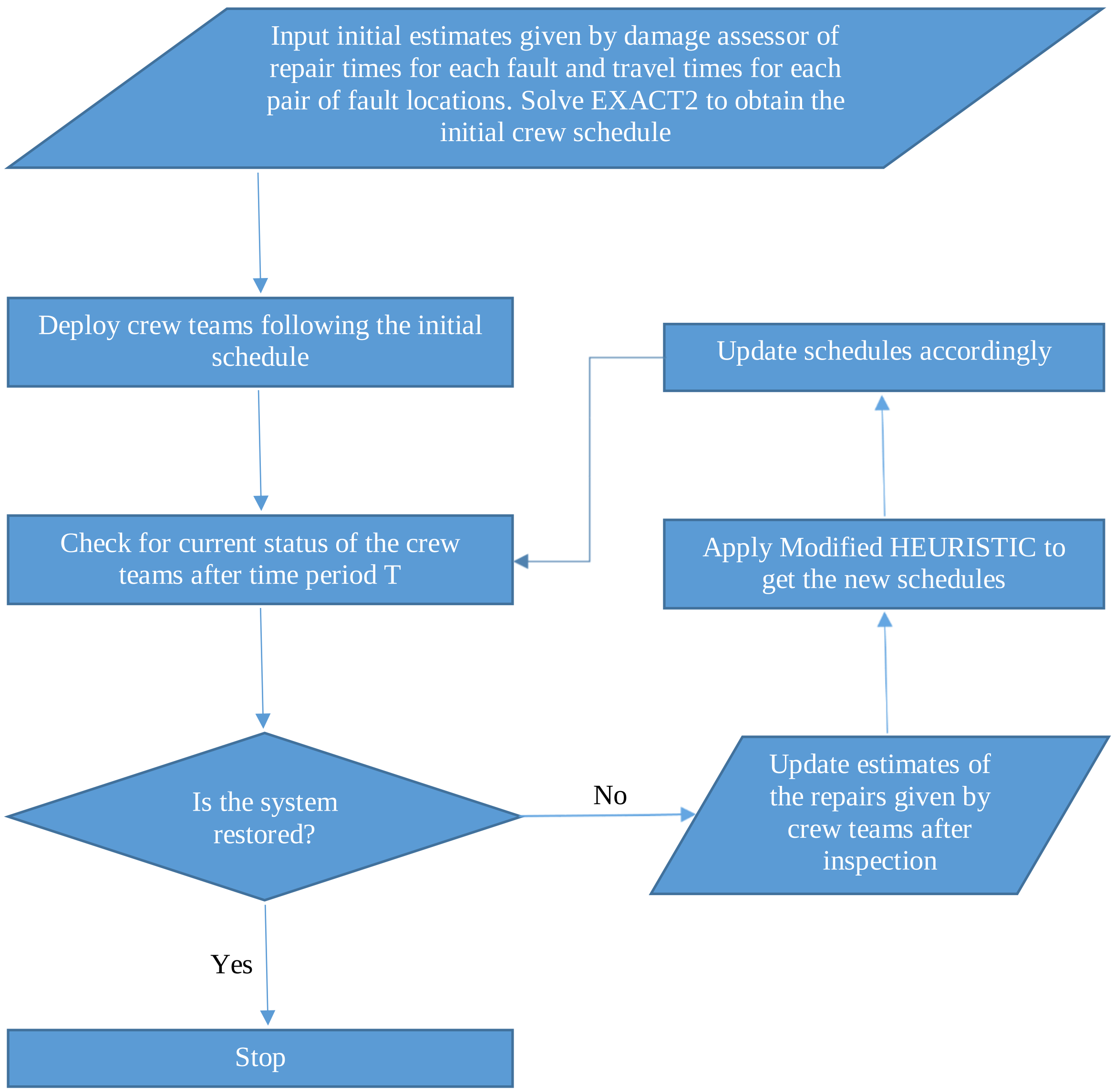

Figure 5: Proposed procedure for dynamic implementation of repair crew scheduling

Note that the proposed dynamic repair crew scheduling employs Modified HEURISTIC in Step 4. The following notation will be used to present this algorithm.

N = Total number of faults to be restored

K = Number of crew teams

C^t = Set of faults restored by time t

N^t = Set of faults to be restored at time t i.e., $(N - C^t)$

N_0^t = Set of faults being restored at time t

U^t = $N^t - N_0^t$

$\bar{P}_i^t$ = Adjusted processing time of fault i for $i \in U^t$

R_k^t = Estimated residual repair time of the fault that crew k currently performs at time t.

C_k^t = Estimated completion time of crew k

Using these notations, Modified HEURSTIC can be formally described as follows.

Step 0: Initialize $C_k^t = t + R_k^t$

Step 1: If U^t is empty, stop. Otherwise, find k^* = $arg\ min\,\{C_k^t : k \in K\}$ and i^* = $arg\ max\,\{\bar{P}_i^t : i \in U^t\}$

Step 2: Assign fault i^* to crew k^*. Update $C_{k^*}^t$ = $C_{k^*}^t + \bar{P}_{i^*}^t$ and U^t = $U^t - \{i^*\}$

Step 3: Return to Step 1.

5.2. Illustration of Dynamic Repair Crew Scheduling

To illustrate the aforementioned dynamic repair crew scheduling, consider the input data of scenario 1 of the large-sized scenarios. Initial estimated repair times for 56 fault locations are

Table 12: Initial estimates for 56 fault locations

1	340	**20**	1102	**39**	1212
2	461	**21**	838	**40**	1047
3	2272	**22**	807	**41**	856
4	2523	**23**	250	**42**	1218
5	679	**24**	249	**43**	1176
6	636	**25**	694	**44**	1070
7	994	**26**	812	**45**	863
8	924	**27**	712	**46**	930
9	1361	**28**	771	**47**	3365
10	1378	**29**	890	**48**	3387
11	1756	**30**	920	**49**	458
12	1530	**31**	1483	**50**	411
13	1579	**32**	1492	**51**	394
14	1456	**33**	2635	**52**	356
15	260	**34**	2364	**53**	581
16	200	**35**	2343	**54**	661
17	418	**36**	2531	**55**	1244
18	215	**37**	678	**56**	1351
19	967	**38**	579		

After solving EXACT2, the initial schedule is obtained as in Figure 6.

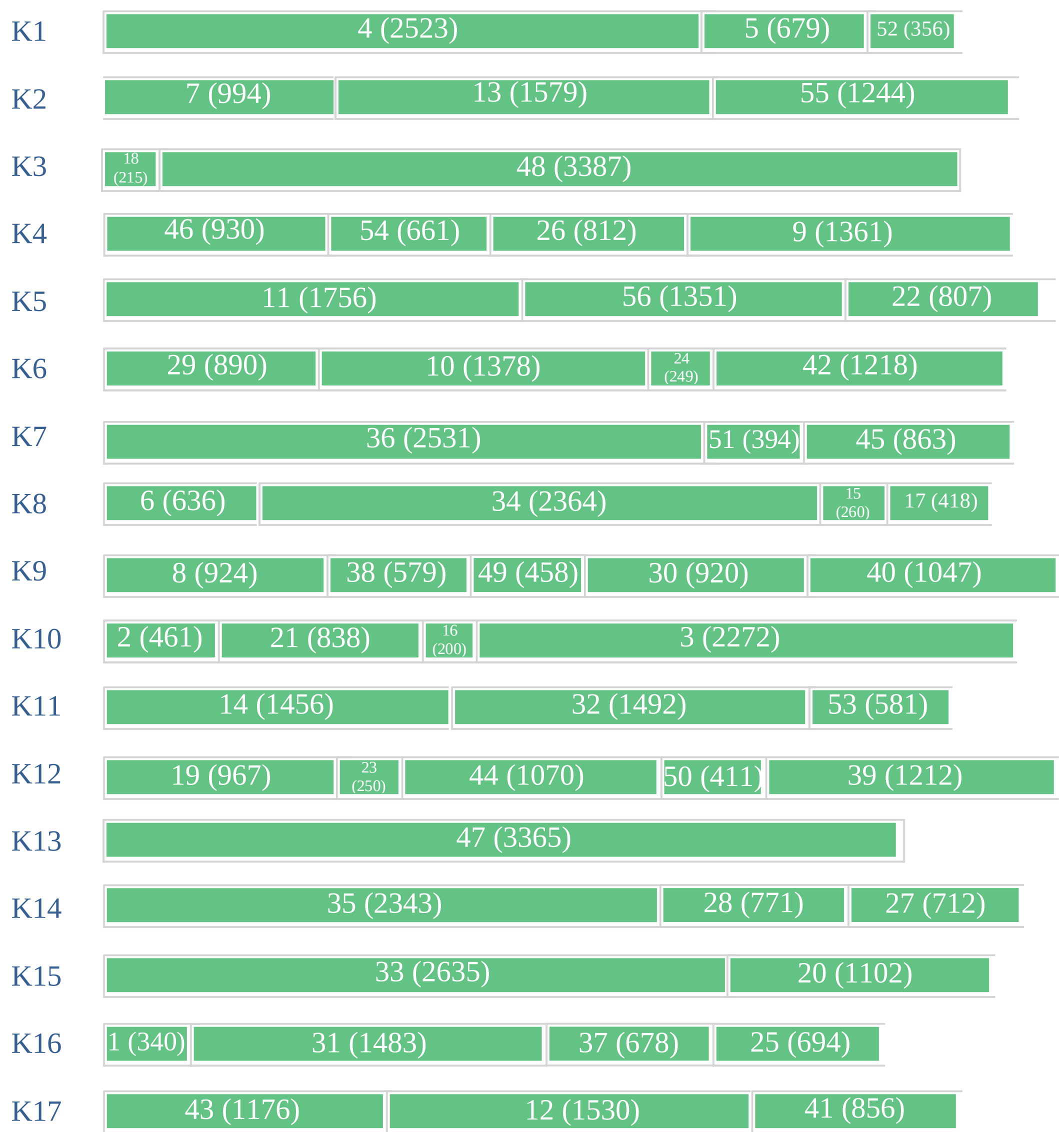

Figure 6: Initial crew schedules provided by EXACT2 (with associated fault numbers, and estimated repair times in parenthesis)

Consider T = 120 minutes and suppose that 120 minutes elapsed, i.e., t=120 at which updated estimates are available. For instance, crew team K3 had an estimated time of 215 minutes for its initial repair (18) has now completed it in 55 minutes and is currently working at the next fault location according to the initial schedule.

Figure 7 displays the updated repair times at t=120 along with old estimates.

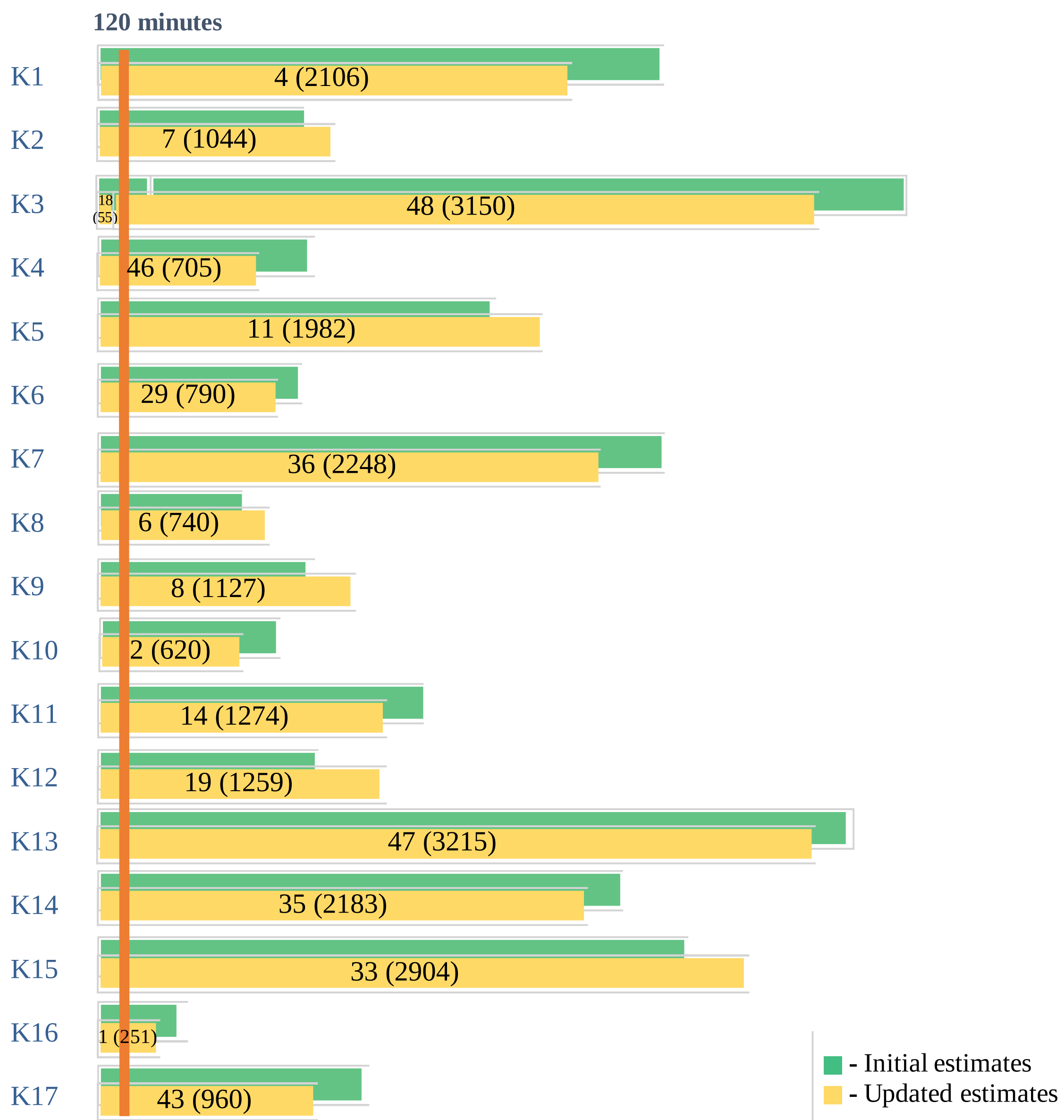

Figure 7: Status of crew teams after initial 120 minutes (with associated fault numbers, and estimated repair times in parenthesis)

Now, to obtain the schedule via Modified HEURISTIC for the remaining faults, adjusted repair times are calculated as in HEURISTIC described In Chapter 3 by adding the average of travel times for each fault location to the updated estimated repair times. The next set of schedules for crew teams, along with the order they were assigned, is presented below

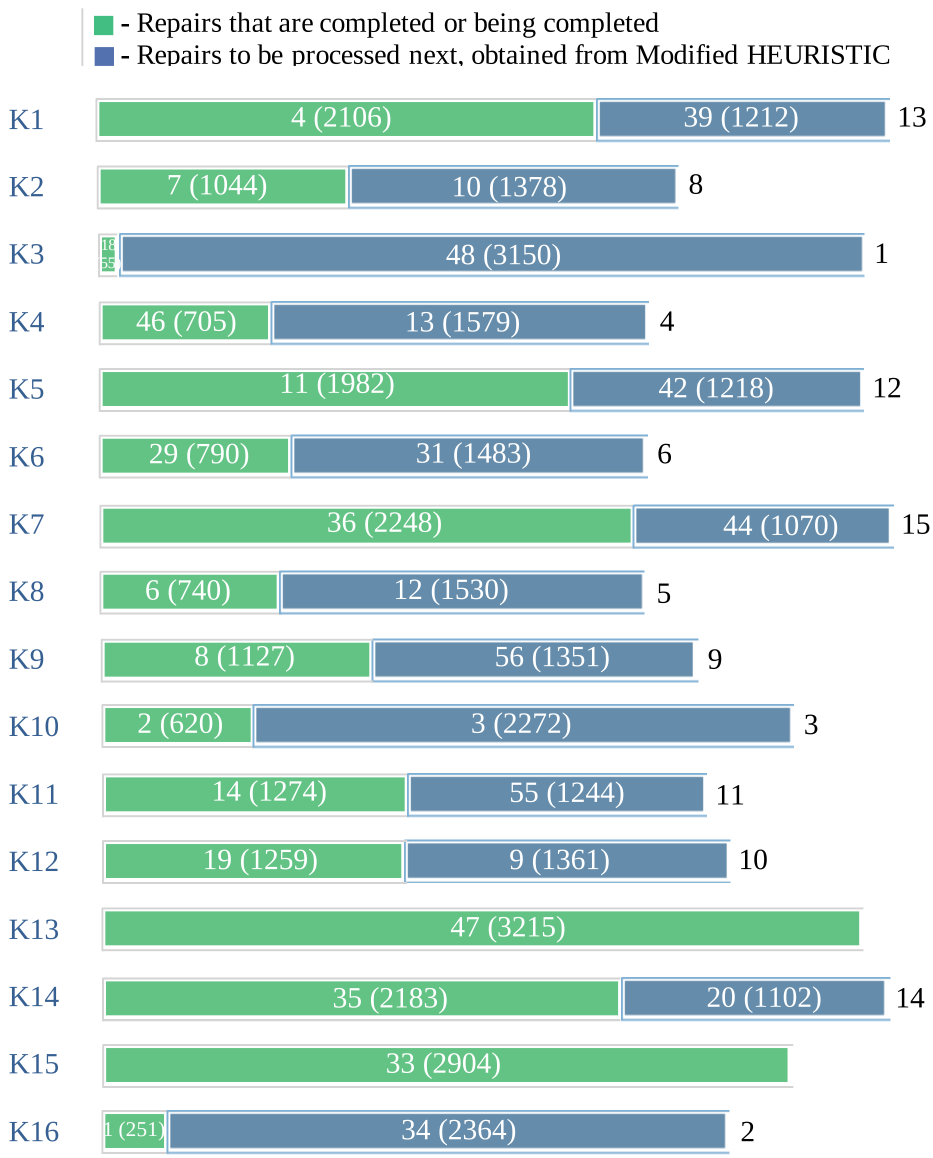

Figure 8: Next set of schedules obtained from Modified HEURISTIC at t=120 (with associated fault numbers, and estimated repair times in parenthesis)

The resulting schedule obtained from Modified HEURISTIC is displayed in Figure 9.

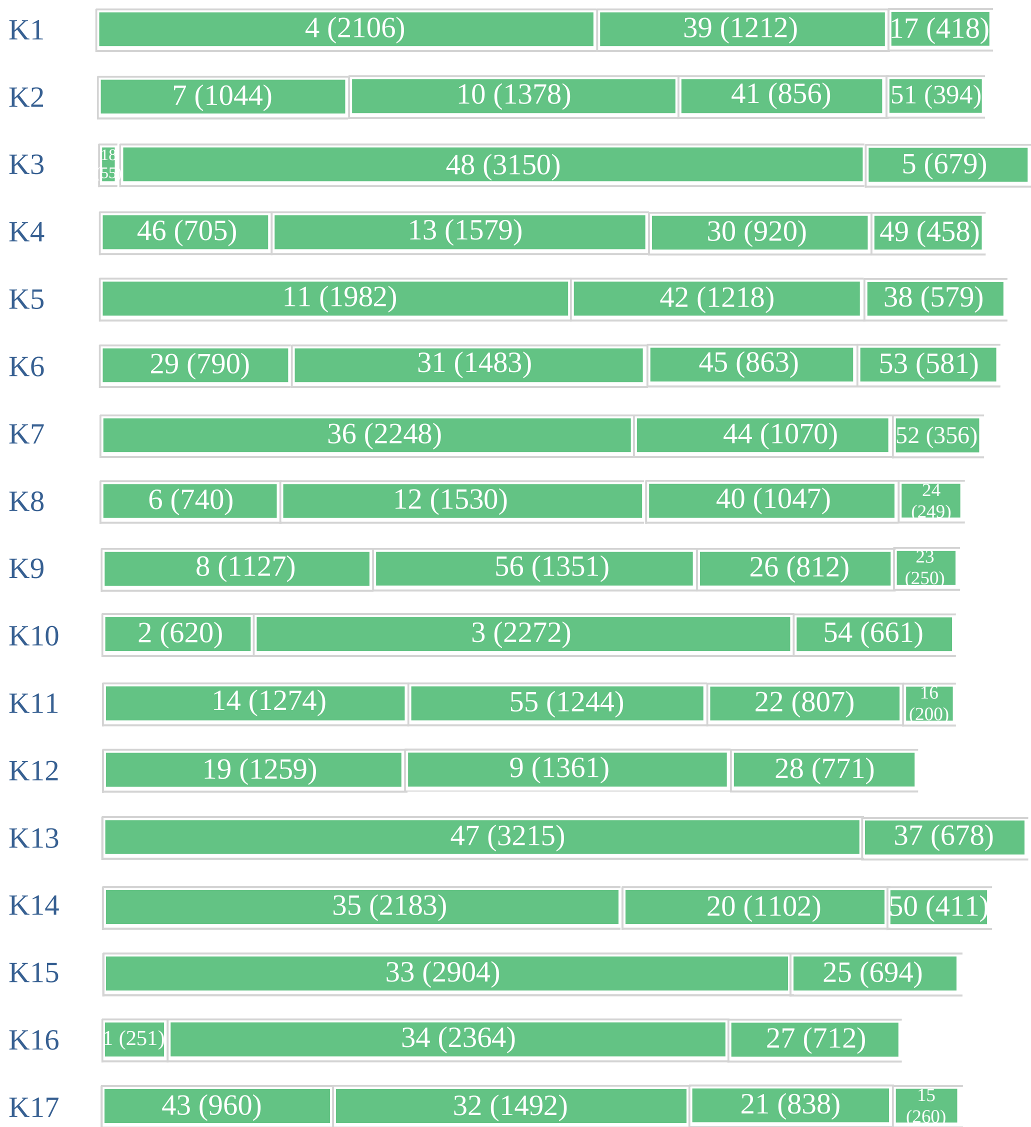

Figure 9: Updated schedule obtained from Modified HEURISTIC at t=120 (with associated fault numbers, and estimated repair times in parenthesis)

In the same manner, the estimates are updated after every time period T until all faults are restored.

CHAPTER 6: CONCLUSION AND FUTURE STUDY

This chapter presents the conclusion of this thesis and potential future research possibilities that can be implemented in this field of study.

6.1. Conclusion

During and after any high impact low frequency (HILF) event, human lives and properties can be devastated to a considerable extent, and they can be far worsened when essential infrastructures such as transport system, electricity, and water supply are disrupted for a prolonged period. Repair crew scheduling aims to minimize the time to restore power after any HILF event via mathematical optimization techniques and heuristic method. To solve this problem, we have implemented three solution methods, which consist of two mixed-integer programming (MIP) models and a heuristic method. The first MIP model was adapted from a solution method for a job-scheduling problem that resembles the crew scheduling problem considered in this thesis. The second model was developed in an effort to improve the computational performance of the first model. In addition, a heuristic method has also been considered to compare the efficiency of results between different models. A real-life storm day of April 5th 2011 has been considered in this thesis along with the data provided by the utility company. All the three methods have been solved for the small-sized and large-sized scenarios and the results have been compared to analyze the performance.

For the small-sized scenarios, EXACT2, i.e., the proposed MIP model, provided optimal solutions within the time limit for all 20 scenarios. EXACT1 reported optimal solutions for 12 scenarios, however, it has exceeded the time limit of 1000 seconds while executing those scenarios. And for the large-sized scenarios, HEURISTIC method yielded optimal solutions for nine scenarios while EXACT2 delivered optimal solutions for 11 scenarios. Looking at the results extracted, one can observe that EXACT2 has performed better for both small-sized and large-sized scenarios. And, in case of the large-sized scenarios, even though the EXACT2 has provided best solutions for 11 of them, it has taken more than 1000 seconds for all of their execution. And for those 11 scenarios, the heuristic method has an average optimality gap of 1.75%, but for the nine scenarios where the heuristic method executed better, EXACT2 has an average optimality gap of 6.69%.

Considering the facts that heuristic method has taken minimal execution time and that this thesis is primarily focused on the extreme events, it is concluded that the HEURISTIC method can be practically utilized under the large-scale HILF events. However, it should be noted that EXACT2 will provide an optimal solution when time permitted and has a great potential to outperform HEURISTIC as advanced computational power emerges as well as state-of-the-art MIP formulations become available. In addition to this, we have proposed the dynamic implementation in Chapter 5, which provides a solution strategy in case of storm events and is a combination of EXACT2 and HEURISTIC methods. As the restoration process begins once the storm has passed away, EXACT2 can be solved beforehand to provide the initial crew schedules. After updating the repair times for each time period T, HEURISTIC can be quickly solved to obtain the next set of schedules.

6.2. Future Study

Further studies in Repair Crew Scheduling problem include the implementation of various kinds of trucks available at the utility company. This thesis assumed only bucket trucks and that all the repair equipment will be present in this truck. Repair Crew Scheduling can be made more realistic by analyzing the trucks and utilizing them based on the type of repair that is to be restored. Another topic that can be worked upon in the future would be prioritizing the faults and restoring them first, based on their significance. This prioritization helps in restoring power to critical facilities such as hospitals, police and fire stations, etc. Prioritization and including the different types of repair trucks can make this problem complete, to help the utility companies in restoring the power outages during HILF events.

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APPENDIX A: TRAVEL TIME

Travel times for scenario 1 of the large-sized scenarios:

	L	N	M	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56
L		29	42	17	15	26	25	15	28	26	37	17	4	17	6	22	20	23	26	22	5	19	42	36	26	35	32	56	60	62	32	54	35	63	51	32	59	72	32	28	51	46	29	51	51	25	47	46	53	19	52	69	52	48	17	66	37	60	39
N	29		27	47	44	23	33	35	2	26	22	13	27	46	29	51	10	6	3	8	27	10	12	10	4	50	47	71	74	77	47	69	49	78	65	47	74	87	47	43	66	22	35	28	66	40	62	28	68	28	66	98	82	77	46	95	65	89	68
M	42	27		43	29	16	16	26	25	15	17	40	37	26	36	31	37	25	28	35	36	37	39	27	29	77	73	98	101	104	74	96	76	105	92	74	100	113	73	70	93	49	62	55	93	67	89	55	95	55	93	78	61	57	39	74	45	68	50
1	17	47	43		14	27	26	16	45	27	43	34	19	17	18	20	38	41	44	40	20	37	59	53	43	34	31	55	59	61	31	53	34	62	50	31	58	71	45	43	59	64	47	69	50	29	48	64	61	37	57	52	35	31	3	48	38	42	21
2	15	44	29	14		20	13	8	42	18	40	31	17	4	15	8	35	38	41	37	17	34	56	50	40	48	44	69	72	75	45	67	47	76	63	45	71	84	44	41	64	61	44	66	64	38	60	61	66	34	64	55	38	34	10	51	24	45	24
3	26	23	16	27	20		10	12	22	2	19	24	22	23	20	28	21	18	20	19	20	21	36	30	20	61	58	82	85	88	58	80	60	89	76	58	84	98	57	54	77	41	46	46	77	51	73	41	79	39	77	75	58	54	23	71	42	66	45
4	25	33	16	26	13	10		10	32	7	29	24	21	13	20	18	24	27	30	26	20	23	45	40	30	61	57	81	85	88	58	80	60	88	76	57	84	97	57	54	76	50	46	55	76	51	73	50	78	39	77	65	49	44	23	62	32	56	35
5	15	35	26	16	8	12	10		34	11	32	23	11	11	9	16	27	30	33	29	10	26	48	42	32	50	47	71	75	78	48	70	50	78	66	47	74	87	47	44	66	53	36	58	66	41	63	53	68	29	67	63	46	42	13	59	30	54	32
6	28	2	25	45	42	22	32	34		24	20	15	26	45	28	50	12	5	3	10	25	12	14	8	4	52	49	73	76	79	49	71	51	80	67	49	75	89	48	45	68	24	37	30	68	42	64	30	70	30	68	97	80	76	44	94	64	88	67
7	26	26	15	27	18	2	7	11	24		22	25	22	21	20	26	22	20	23	19	21	21	38	32	22	61	58	82	86	89	58	80	61	89	77	58	85	98	58	55	77	43	47	48	77	51	74	43	79	39	78	73	56	52	24	69	40	63	42
8	37	22	17	43	40	19	29	32	20	22		35	33	42	31	47	32	20	23	30	31	32	34	12	24	72	69	93	96	99	69	91	71	100	88	69	96	109	69	65	88	44	57	50	88	62	84	50	90	50	88	94	78	73	42	91	62	85	64
9	17	13	40	34	31	24	24	23	15	25	35		15	34	16	39	4	15	12	6	14	3	25	23	11	37	33	58	61	64	34	56	36	65	52	34	60	73	33	30	53	30	22	35	53	27	49	30	55	15	53	86	69	65	33	82	53	77	55
10	4	27	37	19	17	22	21	11	26	22	33	15		19	2	24	18	21	24	20	1	17	40	34	24	39	36	60	64	66	36	58	39	67	55	36	63	76	36	32	55	44	27	49	55	29	51	44	57	17	56	71	54	50	19	68	38	62	41
11	17	46	26	17	4	23	13	11	45	21	42	34	19		17	5	37	40	43	39	20	36	59	53	43	51	48	72	76	78	48	70	51	79	67	48	75	88	48	44	67	64	46	69	67	41	63	63	69	37	68	52	35	31	14	48	21	43	25
12	6	29	36	18	15	20	20	9	28	20	31	16	2	17		22	20	23	26	22	2	19	41	36	25	41	38	62	65	68	38	60	40	69	57	38	65	78	38	34	57	46	29	51	57	31	53	46	59	19	57	69	53	48	17	66	37	60	39
13	22	51	31	20	8	28	18	16	50	26	47	39	24	5	22		42	45	48	44	25	41	64	58	48	54	51	75	79	81	51	73	54	82	70	51	78	91	51	47	70	69	51	73	70	44	66	68	72	42	71	47	30	26	17	44	18	38	28
14	20	10	37	38	35	21	24	27	12	22	32	4	18	37	20	42		12	9	2	18	1	21	20	8	40	36	61	64	67	37	59	39	68	55	37	63	76	36	33	56	26	25	31	56	30	52	26	58	18	56	89	73	68	37	86	57	80	59
15	23	6	25	41	38	18	27	30	5	20	20	15	21	40	23	45	12		3	10	21	12	18	13	4	52	48	72	76	79	49	71	51	79	67	48	75	88	48	45	67	23	37	30	67	42	64	29	69	30	68	92	76	71	40	89	60	83	62
16	26	3	28	44	41	20	30	33	3	23	23	12	24	43	26	48	9	3		7	24	9	15	11	1	49	45	70	73	76	46	68	48	77	64	46	72	85	45	42	65	21	34	27	65	39	61	27	67	27	65	95	79	74	43	92	63	86	65
17	22	8	35	40	37	19	26	29	10	19	30	6	20	39	22	44	2	10	7		20	3	19	18	6	42	39	63	66	69	39	61	41	70	58	39	66	79	39	35	58	24	27	29	58	32	54	24	60	20	58	91	75	70	39	88	59	82	61
18	5	27	36	20	17	20	20	10	25	21	31	14	1	20	2	25	18	21	24	20		17	39	33	23	41	37	61	65	68	38	60	40	68	56	37	64	77	37	34	56	44	27	49	57	31	53	44	58	19	57	72	55	50	19	68	39	62	41
19	19	10	37	37	34	21	23	26	12	21	32	3	17	36	19	41	1	12	9	3	17		22	20	8	40	37	61	64	67	37	59	39	68	56	37	64	77	37	33	56	27	25	32	56	30	52	27	58	18	56	88	72	67	36	85	56	79	58
20	42	12	39	59	56	36	45	48	14	38	34	25	40	59	41	64	21	18	15	19	39	22		23	16	38	53	58	62	65	37	61	37	65	53	48	61	74	34	31	53	9	23	16	53	30	50	15	55	22	54	111	94	90	58	107	78	101	80
21	36	10	27	53	50	30	40	42	8	32	12	23	34	53	36	58	20	13	11	18	33	20	23		12	60	57	81	85	87	57	79	59	88	76	57	84	97	57	53	76	32	45	38	76	50	72	38	78	38	77	105	88	84	52	102	72	96	75
22	26	4	29	43	40	20	30	32	4	22	24	11	24	43	25	48	8	4	1	6	23	8	16	12		48	45	69	72	75	45	67	47	76	63	45	71	85	44	41	64	21	33	26	64	38	60	26	66	26	64	95	78	74	42	91	62	86	64
23	35	50	77	34	48	61	61	50	52	61	72	37	39	51	41	54	40	52	49	42	41	40	38	60	48		25	21	27	27	9	34	8	38	16	21	24	37	13	11	27	33	15	37	16	10	17	32	30	22	25	83	66	62	37	80	72	74	53
24	32	47	73	31	44	58	57	47	49	58	69	33	36	48	38	51	36	48	45	39	37	37	53	57	45	25		24	28	31	16	22	17	31	19	4	27	40	38	36	53	58	40	62	31	22	42	57	55	31	50	58	55	49	34	55	69	49	28
25	56	71	98	55	69	82	81	71	73	82	93	58	60	72	62	75	61	72	70	63	61	61	58	81	69	21	24		25	6	24	31	21	35	5	24	3	16	24	28	30	49	36	43	8	31	20	43	32	43	28	81	79	73	58	78	93	71	50
26	60	74	101	59	72	85	85	75	76	86	96	61	64	76	65	79	64	76	73	66	65	64	62	85	72	27	28	25		24	27	7	25	11	20	28	26	12	40	38	55	60	42	64	33	34	44	59	57	46	52	61	83	76	62	81	96	74	51
27	62	77	104	61	75	88	88	78	79	89	99	64	66	78	68	81	67	79	76	69	68	67	65	87	75	27	31	6	24		30	31	28	35	12	30	4	13	31	34	30	55	42	49	11	37	20	49	33	49	28	80	86	79	65	84	99	77	54
28	32	47	74	31	45	58	58	48	49	58	69	34	36	48	38	51	37	49	46	39	38	37	37	57	45	9	16	24	27	30		25	2	31	18	11	27	40	23	21	37	42	25	47	19	7	26	42	39	19	35	74	57	53	34	70	69	65	44
29	54	69	96	53	67	80	80	70	71	80	91	56	58	70	60	73	59	71	68	61	60	59	61	79	67	34	22	31	7	31	25		25	9	27	22	33	18	47	45	61	66	49	71	39	31	51	66	64	41	59	55	78	71	56	76	91	69	46
30	35	49	76	34	47	60	60	50	51	61	71	36	39	51	40	54	39	51	48	41	40	39	37	59	47	8	17	21	25	28	2	25		29	16	12	24	37	22	20	36	41	24	46	17	9	25	41	38	21	34	75	58	54	37	71	71	66	44
31	63	78	105	62	76	89	88	78	80	89	100	65	67	79	69	82	68	79	77	70	68	68	65	88	76	38	31	35	11	35	31	9	29		31	31	37	21	51	49	65	70	53	75	43	38	55	70	68	50	63	64	87	80	65	85	100	78	54
32	51	65	92	50	63	76	76	66	67	77	88	52	55	67	57	70	55	67	64	58	56	56	53	76	63	16	19	5	20	12	18	27	16	31		19	8	21	20	22	34	44	30	44	13	25	24	39	37	37	32	76	74	68	53	73	87	67	46
33	32	47	74	31	45	58	57	47	49	58	69	34	36	48	38	51	37	48	46	39	37	37	48	57	45	21	4	24	28	30	11	22	12	31	19		27	40	34	32	48	53	36	58	26	18	38	53	51	26	46	62	56	49	34	59	69	53	32
34	59	74	100	58	71	84	84	74	75	85	96	60	63	75	65	78	63	75	72	66	64	64	61	84	71	24	27	3	26	4	27	33	24	37	8	27		15	27	30	28	52	38	46	8	34	18	46	31	46	26	82	82	76	61	80	96	74	52
35	72	87	113	71	84	98	97	87	89	98	109	73	76	88	78	91	76	88	85	79	77	77	74	97	85	37	40	16	12	13	40	18	37	21	21	40	15		40	44	44	65	51	59	22	47	33	59	46	59	41	73	95	89	74	93	109	87	63
36	32	47	73	45	44	57	57	47	48	58	69	33	36	48	38	51	36	48	45	39	37	37	34	57	44	13	38	24	40	31	23	47	22	51	20	34	27	40		3	19	25	11	24	19	16	16	19	21	19	20	96	80	75	44	93	69	87	66
37	28	43	70	43	41	54	54	44	45	55	65	30	32	44	34	47	33	45	42	35	34	33	31	53	41	11	36	28	38	34	21	45	20	49	22	32	30	44	3		23	21	8	26	23	14	19	21	25	15	23	94	78	73	42	91	65	85	64
38	51	66	93	59	64	77	76	66	68	77	88	53	55	67	57	70	56	67	65	58	56	56	53	76	64	27	53	30	55	30	37	61	36	65	34	48	28	44	19	23		44	31	38	22	30	11	38	2	38	2	111	94	90	58	107	88	101	80
39	46	22	49	64	61	41	50	53	24	43	44	30	44	64	46	69	26	23	21	24	44	27	9	32	21	33	58	49	60	55	42	66	41	70	44	53	52	65	25	21	44		17	6	44	35	40	6	46	27	45	116	99	95	63	112	83	106	85
40	29	35	62	47	44	46	46	36	37	47	57	22	27	46	29	51	25	37	34	27	27	25	23	45	33	15	40	36	42	42	25	49	24	53	30	36	38	51	11	8	31	17		22	31	18	27	17	33	10	31	98	82	77	46	95	66	89	68
41	51	28	55	69	66	46	55	58	30	48	50	35	49	69	51	73	31	30	27	29	49	32	16	38	26	37	62	43	64	49	47	71	46	75	44	58	46	59	24	26	38	6	22		38	40	34	5	40	32	38	120	104	99	68	117	88	111	90
42	51	66	93	50	64	77	76	66	68	77	88	53	55	67	57	70	56	67	65	58	57	56	53	76	64	16	31	8	33	11	19	39	17	43	13	26	8	22	19	23	22	44	31	38		26	11	38	24	38	20	89	75	68	53	85	88	79	58
43	25	40	67	29	38	51	51	41	42	51	62	27	29	41	31	44	30	42	39	32	31	30	30	50	38	10	22	31	34	37	7	31	9	38	25	18	34	47	16	14	30	35	18	40	26		22	35	33	12	28	80	64	59	28	77	62	71	50
44	47	62	89	48	60	73	73	63	64	74	84	49	51	63	53	66	52	64	61	54	53	52	50	72	60	17	42	20	44	20	26	51	25	55	24	38	18	33	16	19	11	40	27	34	11	22		34	13	34	8	100	83	79	49	97	84	91	70
45	46	28	55	64	61	41	50	53	30	43	50	30	44	63	46	68	26	29	27	24	44	27	15	38	26	32	57	43	59	49	42	66	41	70	39	53	46	59	19	21	38	6	17	5	38	35	34		40	27	39	115	99	94	63	112	83	106	85
46	53	68	95	61	66	79	78	68	70	79	90	55	57	69	59	72	58	69	67	60	58	58	55	78	66	30	55	32	57	33	39	64	38	68	37	51	31	46	21	25	2	46	33	40	24	33	13	40		40	5	113	96	92	60	110	90	104	83
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