THREE ESSAYS IN ASSET PRICING UNDER MODEL UNCERTAINTY

by

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ABSTRACT

JULIA JIANG. Three essays in Asset Pricing under Model Uncertainty. (Under the direction of DR. WEIDONG TIAN)

This dissertation consists of three essays on asset pricing under model uncertainty. The first essay examines how aversion to uncertainty about the information transfer across firms affects asset prices in an equilibrium. I show that a firm's stock price reacts more strongly to the bad news than the good news from its *economically linked* firms, and there is *price inertia* if the news is not strong enough. Moreover, I show that equilibrium prices do not always fully incorporate relevant firm-specific news. The stock price movement displays overreaction and underreaction, depending on the magnitude of the news, the information quality, the strength of the economic link, the firm size, and the firm risk. The model further explains the asymmetric pattern of financial time series, including the expected stock return and volatility, and the correlation and covariance. The model offers several testable predictions, which are consistent with recent empirical studies on how asset prices and returns are affected by the firm-specific news.

In the second essay, I construct an equilibrium model in the presence of correlation uncertainty and heterogeneous ambiguity-averse investors, in which correlation uncertainty and asset characteristics jointly affect asset prices and trading activities. The price of low-weighted volatility assets declines, the retail investor holds a smaller position when the correlation uncertainty goes up; meanwhile, the price of high-weighted volatility assets increases, and the retail investor holds a larger position. The institu-

tional investor provides liquidity and holds a well-diversified portfolio. Moreover, all risky assets comove more under higher correlation uncertainty. This model explains several empirical puzzles including under-diversification and limited participation, flight-to-quality, and asset comovement.

In the third essay, I present a dynamic equilibrium model of financial innovation when the investor is more prone to time inconsistency than the innovator. I analyze the effects of heterogeneous beliefs among agents on the security's viability and equilibrium pricing. I demonstrate that the market of forward-type securities is more resilient to the underlying market movement compared to the market of option-type contracts, where securities are vulnerable to the underlying market condition and may disappear with a drastic market movement. The model is extended to examine some complex financial instruments with multiple tranches, such as collateralized debt obligations (CDOs) and the pattern of financial innovation under model uncertainty. The analysis explains the recent boom and bust of securitization market and the high-yield puzzle of senior tranche of CDOs during its heyday.

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1.1 Introduction

Firms do not exist as independent entities in financial market, but are linked to each other through many types of relationships.[1] The information transfers literature finds that the news about one firm affects the valuation of its economically linked firms in a nontrivial way.[2] The aim of this paper is to examine how an investor's aversion to *uncertainty* about the information transfer affects the asset prices in an equilibrium.

In this paper, the investor views the signal about one firm's future cash flows as *imprecise* or *uncertain* signal to the other firm: good or bad news for a firm does not always indicate good or bad news for the economic-related firms. I develop an equilibrium model to investigate how the uncertainty about the effect of information transfer contributes to each firm's stock price as well as the asset pricing implications.[3] Previous empirical studies find that the non-announcing firm's stock price movements can either over- or underreact (Ramnath, 2002; Thomas and Zhang, 2008; Rama-

[1] The economic links considered in this paper include the customers-suppliers relation in a supply chain (Pandit, Wasley, and Zach, 2011; Cheng and Elsman, 2014), peer firms in the same industry (Ramnath, 2002; Thomas and Zhang, 2008), or a firm and its blockholder (Ramalingegowda et al., 2012).

[2] The information transfer phenomenon has been studied extensively in accounting and finance literature. Firth (1976), Foster (1981), Clinch and Sinclair (1987), and Freeman and Tse (1992) study the effect of earning announcement of one firm to the other firms in the same industry. Han and Wild (1997), Kim et al (2008), and Glesason et al (2008) study the information transfer effect of management earning forecast. Even though I focus on firm-specific news in this paper, the information transfer around specific firm-specific events are also studied in literature.

[3] To the best of my knowledge, this paper is one of the first to investigate the information transfer effect in an equilibrium model.

lingegowda et al., 2012; and Chen and Eshleman, 2014). I expand this literature and show that the stock price's over-or underreaction is characterized by several ingredients, including the sign and the magnitude of the news, the information quality, the strength of the economic link, the level of uncertainty, the firm size, and the firm risk. In addition, I present several testable predictions on stock price reaction to news. Compared to most of the previous theoretical models (Daniel, et al., 1998, Barberis, et al., 1998, and Hong and Stein, 1999) that consider only one risky asset in the economy, my model is able to explain the stock price underreaction and overreaction to news *across* firms. Moreover, it provides new insights to understand the pervasive asymmetric patterns of financial time-series, including the correlation, the covariance, the expected returns and volatilities.

Specifically, a representative investor observes a piece of news about the future payoff of the "*announcing*" firm, and this news conveys information about the "*non-announcing*" firm indirectly through the correlation channel between the two firms.[4] However, the investor is unable to precisely estimate the impact of news transferred from a related firm and averse to Knightian uncertainty[5]. Due to the uncertainty originated from the information transfer, the investor cautiously processes the news effect across firms, and considers a set of plausible correlation structures in the prior

[4]For illustration purpose, I denote the announcing firm as the one that receives the news concerning the future payoff about itself. It is not required that the news has to be actually announced by the announcing firm, but can come from the financial analysts' reports, or a specific event that directly affects the announcing firm's future payoff. The non-announcing firm is just a firm that is economically related to the announcing firm, of which the future payoff is indirectly affected by the piece of news.

[5]Knightian uncertainty, or ambiguity, is defined as uncertainty about the probabilities over payoffs. Ambiguity is distinguished from risk, which is uncertainty over payoffs (Savage 1954). Another way to think about the difference between risk and ambiguity is risk is when one does not know the outcome but understands the odds of each outcome. Ambiguity on the other hand is a situation where one does not have enough information to understand the odds of each outcome.

distribution of firms' payoffs. The investor evaluates the outcome regarding to each correlation structure and makes the investment decisions based on the correlation structure that yields the lowest expected utility. This max-min approach of decision-making under Knightian uncertainty is axiomatized by Gilboa and Schmeidler (1989) and its dynamic extension is developed in Epstein and Schneider (2008). The validity of this investor preference facing Knightian uncertainty is consistent with experimental evidence by Ellsberg (1961) and more recent portfolio choice experiments such as Ahn, Choi, Gale, and Kariv (2011) and Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010).[6]

The equilibrium characterization of the information transfer under uncertainty is intuitive and straightforward. Facing the correlation uncertainty, the investor tends to consider the worst-case scenario to determine the news effect across the firms as well as the firm valuations. Suppose the two firms are positively correlated, when there is bad news about the announcing firm, the investor would think this is also bad news to the non-announcing firm, and believe the news would affect the non-announcing firm in a similar way. In other words, the worst case scenario is when the two firms are highly correlated to each other and the news about one firm is highly relevant to the other. Therefore, the equilibrium prices are determined by the highest plausible correlation coefficient. On the other hand, when there is good news about the announcing firm, the investor would think this good news is not that good to the non-announcing firm. In such cases, the equilibrium prices are determined by the

[6]This behavior is also consistent with recent research in neuroeconomics that finds that when subjects are faced with decisions under ambiguity, the areas of the brain associated with fear and survival instincts are activated (Hsu, Bhatt, Adolphs, Tranel, and Camerer 2005; Smith, Dickhaut, McCabe, and Pardo 2002).

lowest plausible correlation coefficient. Overall, the endogenous correlation structure in the equilibrium corresponds to the *highest* correlation coefficient under bad news, and the *lowest* correlation coefficient under good news. When the news is not strong enough, the endogenous correlation is negatively determined by the magnitude of the news.

Based on the decreasing endogenous correlation structure conditional on the news, I present several important asset pricing implications of the information transfer. *First of all*, I show that the stock price reacts more strongly to the bad news than the good news from a related firm, that is, there is an asymmetric effect of information transfer. When the news from the related firm is not strong enough (to convey whether this is good or bad news), the stock price shows no reaction, and a "*price inertia*" feature is obtained. Intuitively, if the news decreases, an investor requires a lower price as compensation for the lower posterior mean in order to hold the risky assets. However, the aversion to uncertainty dictates the investor to revise his belief about the correlation upwards if the signal drops. The news effect from two directions counterbalances each other. The lower posterior mean that would require a drop in the equilibrium price is exactly offset by the lower risk premium that would require an increase in the price. As a result, the price does not change. Condie, Gauguli and Illeditsch (2015) demonstrate that the stock price shows a lack of reaction, when the investor has concern about the predictability of news regarding the firm *itself*, in a single period model with only one risky asset. Instead, I provide a dynamic model to show how the stock price can display lack of reaction towards news from a related firm. Furthermore, I show that both the asymmetric effect and the price inertia effect are more significant when the ambiguity increases.

Secondly, I show that the firm's stock price could under- or overreact to the news about the related firm, and this over- and under-reaction is determined by several important factors, including the strength of the economic link, the firm capitalization, the information quality, and the level of correlation uncertainty. I show that the price change displays underreaction when the economic link is strong, and overreaction otherwise. This model offers alternative explanations about individual firm's stock price reaction in information transfer literature. Cheng and Eshleman (2014) proposes a moderated confidence hypothesis that, psychologically, investors have difficulty judging the precision of signals, therefore systematically bias their estimates of signal precision toward the unconditional mean. As a result, the investors overweight imprecise signals, resulting in non-announcing firm's stock prices overreaction to the news (as in Thomas and Zhang, 2008). On the other hand, the investors underweight precise signals so the non-announcing firm's stock prices underreact to the news, as documented in Ramnath (2002). My model explains the empirical evidences in Cheng and Eshleman (2014) from an uncertainty perspective. When the signal is precise, it is shown that the autocorrelation of the non-announcing firm is positive, thus the stock price displays underreaction; and the autocorrelation of the non-announcing firm is negative if the signal is very imprecise. Specifically, I characterize the condition under which the autocorrelation of the non-announcing is positive (negative), based on the strength of the economic link, the firm capitalization, the information quality and the level of correlation uncertainty.

Thirdly, the model also demonstrates the information transfer effect on the announcing firm's stock price and the price movement. Intuitively, the better the news the higher the firm's stock price, but the information transfer effect on the announcing firm is quite different from that on the non-announcing firm. The marginal effect of the good news and the bad news on the stock price is symmetric, because the news conveys direct information about the announcing firm. However, when the news is not strong enough, the announcing firm's stock price is more sensitive to the marginal change of news than that when the news is strong. This is because in the equilibrium, when the news about the announcing firm is not strong, the non-announcing firm is lack of reaction, resulting in a larger investor demand for the announcing firm's stock. Moreover, I show that the announcing firm's stock price generates predictability of the non-announcing firm's stock price by examining the cross-correlation under certain conditions.

In addition to the individual firm effect, I also investigate the information transfer effect on the portfolio with all firms. The entire portfolio can also overreact or under-react to the firm-specific news. If the signal is precise, the entire portfolio under-react to the news. If the non-announcing firm is viewed as a representative of all other firms in the market, my model explains the well-documented stock market underreaction (see for instance Jadedeesh and Titman, 1993, 2001). In general, if either the signal is precise or the economic link is strong, the model implies an under-reaction of the entire portfolio. I also derive precise conditions under which the portfolio overreacts to news (DeBondt and Thaler, 1985).

My model is helpful to understand the price momentum and reversal in the financial market from the information transfer perspective. Jadadeesh and Titman

(1993, 2001), Lo and Mackinlay (1998), among many others, document positive serial correlation. Previous literatures explain that the momentum of short-term stock continuation because of investor's underreaction to new information (Chen et al, 1996; Barberis, Sheifer and Vishny, 1998; Daniel et al 1998, 2001), investor inattention (Hong and Stein 1999), and investor's information uncertainty (Zhang, 2006). My model documents that the information transfer effect also contributes to the short-term stock market continuation under certain circumstances. On the other hand, the auto-correlation of individual firm's stock price can be positive or negative, and the cross-correlation is helpful to explain the largely undereaction of the stock market (Lo and Mackinlay, 1990). My model offers several new testable predictions in this regard. I present concrete conditions on some fundamental elements - the strength of economic link, firm capitalization, information quality and the level of correlation uncertainty - about the positiveness or negativeness of the auto-correlation of each firm and the cross-correlation between firms. Moreover, the underreaction or overreaction increases with the risk of the asset and ambiguity aversion in the model, which is consistent with the empirical findings in Williams (2015).

Fourth, I examine the risk premium and the expected stock returns. The excess risk premium is generated due to the correlation uncertainty. Similar to the stock price reaction to the news, I also show that the conditional expected stock return of each firm displays different patterns with respect to the news, depending on how the news predicts the assets' future payoffs. The conditional expected return of the announcing firm's stock price always decreases with respect to the news. But the conditional expected return of the non-announcing firm's stock price is not monotonic in general except for a relatively weak economic link.

Lastly, the model also provides new insights to understand the asymmetric pattern of the financial time series, including correlation, covariance, and volatility. Since a high correlation is always associated with the arrival of bad news and a low correlation corresponds to a piece of good news instead, the model explains the asymmetric volatility patterns of the stock price return. The asymmetric volatility is robust and persistent for the announcing firm's stock. The asymmetric property of the non-announcing firm's stock return volatility holds largely, however, due to the price inertia, it may display the opposite asymmetric feature when the news is not strong enough. The asymmetric pattern of the covariance pattern is also consistent with the asymmetric property of the correlation and volatility. I further quantify the measures for the asymmetries conditional on the news and show that the asymmetric pattern of financial time series is more pronounced when the news is strong.

This paper contributes to the literature which explores the asset pricing implications of the firm-specific news. Bernard and Thomas (1990), and Abarnamell and Bernard (1992) report that investors do not seem to completely adjust their earnings expectations based on the error in their earnings expectation, and this underreaction to earnings information leads to predictable stock returns. Sloan (1996) shows that the stock price fails to reflect fully information contained in the accrual and cash flow components of current earnings. Zhang (2006) explains the short-term stock underreaction by the information uncertainty factor. Caskey (2009) develops an equilibrium model with heterogeneous ambiguity-averse investors, and shows that prices underreact to overall aggregate signal but overreact to some signal components. Therefore, Caskey (2009) can explain the stock price overreaction to the non-cash portion of profits and underreaction to the cash portion. By contrast, I consider the firm-specific news instead of aggregative signals, and develop an equilibrium model of information transfer across firms when the investor is ambiguity-averse to the relevance of the information. More importantly, the news impact on the valuation of the announcing firm is jointly determined by the news impact on the valuation of the other relevant firms in equilibrium.

The paper is closely related to a strand of literature on economic links. Cohen and Frazzini (2008) find evidence of return predictability across economically linked firms and stock prices do not promptly incorporate relevant firm-specific news. Patton and Verardo (2012) investigate the announcing firm's stock beta with the release of firm-specific news. They found that when the earning announcements have larger positive or negative surprise, investor can extract more information from the other firms and the aggregate economy, and the stock beta is larger. Cohen and Lou (2002) document substantial return predictability from the set of easy-to-analyze firms to other set of complicated firms, which requires more sophisticated analysis to incorporate the information into prices. To some extent my model is similar to Cohen and Lou (2012), in which the same piece of information affects two sets of firms. My model provides explanations for their findings of return predictability across firms if we view the easy-to-analyze firm as the announcing firm and the other complicated firm as the non-announcing firm. My model also contributes to the economic link literature by investigating the information transfer effect on stock comovement (correlation and covariance) in addition to the expected stock return and volatility. [7]

Since this paper focuses on the information transfer under uncertainty, my model is starkly different from the theoretical models proposed in the behavioral finance literature. Daniel et al. (1998) develop a model based on overconfidence and self attribution bias, in which investors hold too strong beliefs about their own information, thus overreact to the private signals and underreact to public signals. Barberis, Shleifer, and Vishny (1998) suggest that stocks react more strongly to bad news than to good news mainly because investors change their sentiments based on the past streams of realizations, and discount recent information. Hong and Stein (1999) consider a model of information diffusion, in which some investors underreact to the news and other trend followers overreact to the news. By contrast, the firm-specific news in my model is public and the public news can be virtually testable. I show that the stock price overreaction or underreaction can be generated by the level of the correlation uncertainty and other firm-specific elements, instead of purely relying on the psychological bias. The behavioral finance literature also document the asymmetric phenomenon of financial time series. For instance, Hong and Stein (1999) argues that investor heterogeneity is central to the asymmetric phenomenon. Ang, Bakaert, and Liu, (2005), and Ang, Chen, and Xing (2006) study loss aversion and disappointment aversion preference, in which investors care differently about downside losses than the upside gains. My model provides an alternative explanation for the asymmetric pattern of the financial time series from the uncertainty perspective. I further

[7] Kelsey, Kozhan and Pang (2011), Peng and Johnstone (2016) also find the asymmetric pattern in price continuation and implied volatility.

demonstrate that the asymmetry effect is persistent under all market conditions and become more significant as the ambiguity increases. Conrad, Cornell, and Landsman (2002) find that individual stocks do indeed react more strongly to bad earnings announcements versus good earnings announcements in good times, as measured by the equity market valuation, but not in bad times.

To empirically test the model predictions in this paper requires a proxy for the information transfer uncertainty, alternatively, correlation uncertainty. Inspired by Zhang (2006) and Bloom (2009) that study the information uncertainty and the macroeconomic uncertainty, the correlation uncertainty in this paper can be tested empirically using the dispersion among analyst forecasts, or the volatility of correlation between the dividends to measure. Zhang (2006) suggests several measures of information uncertainty for the announcing firm, and a similar methodology can be applied to measure the correlation uncertainty. For instance, the ratio of the firm size of the announcing firm to the non-announcing firm, and the ratio of the firm ages can be used as a proxy to test my model prediction. Since my model predictions document the effect of the uncertainty about information transfer on the stock prices and asset returns, I can also empirically examine the changes of those proxies.[8]

This paper draws from many important contributions of asset pricing under ambiguity literature and adds some new contributions in this area. Epstein and Schneider (2002), Caskey (2009), and Illeditsch (2011) address the conditional distribution of signals in an information ambiguity setting.[9] My model departs from the information

[8] A complete test of my model predictions is beyond the scope of this paper and is left for future study. Some relevant empirical evidences are presented in Section 4.

[9] Caskey (2009) considers an ambiguous-averse investor who follows Klibanoff, Marinacci, and Mukerji's (2005) smooth ambiguity aversion preference and a Savage investor who has expected

ambiguity literature in the sense that the information quality is known, instead, the correlation structure among risky assets is ambiguious. To examine the joint distribution for multiple assets' random payoffs, many previous research have investigated the ambiguity on the marginal distribution.[10] In this regard, Jiang and Tian (2016) might be the most relevant study in which the authors study the correlation uncertainty and its asset pricing implications by fixing the marginal distribution. But my model is remarkably different from Jiang and Tian (2016) in that the current paper focuses on the effect of economic shocks and its implications for conditional asymmetric properties, whereas Jiang and Tian (2016) characterize an equilibrium model with heterogeneous correlation ambiguity among investors to explain under-diversification and limited participation puzzle, and flight-to-quality and flight-to-safety.

The rest of this paper is organized as follows. In Section 2, I introduce the model in a dynamic framework of correlation uncertainty. Section 3 characterizes the equilibrium. In Section 4 I present model predictions and supporting empirical evidences. Section 5 concludes and the proof details are provided in Appendixes.

1.2 Model with Correlation Uncertainty

There are two time periods. Investors trade at time $t = 0$ and $t = 1$ and consumption occurs at the terminal time $t = 2$. There are two risky assets and one risk

utility with respect to a unique prior belief. Each investor observes informative signals on one risky firm (asset) and the uncertainty on the information quality allows the ambiguity-averse investor prefer to trade based on aggregated signals that reduce ambiguity at the cost of a loss in information. Similar to Caskey (2009), Illeditsch (2011) considers a setting of an ambiguity-averse investor with a random payoff on one risky asset subject to an uncertain shock. Illeditsch (2011) shows that the desire to hedge the information uncertainty leads to excess volatility. In my model, there are two risky assets and the representative investor is ambiguity-averse to the correlation estimation.

[10]For example, Boyle, Garlappi, Uppal, and Wang (2012), Cao, Wang, and Zhang (2005), Easley and O'Hara (2009), and Garlappi, Uppal, and Wang (2007) investigate expected return parameter uncertainty. Easley and O'Hara (2010) and Epstein and Ji (2013) discuss volatility parameter uncertainty.

free asset. The risk-free rate is set to be zero. Each risky asset denotes a stock of a full-equity firm which pays dividend $\tilde{d}_i$ at the terminal time. The total supply of asset $i = 1, 2$ is denoted by $\overline{\theta}_i$.

The dividends are revealed at the terminal time. The marginal distribution of $\left(\tilde{d}_1, \tilde{d}_2\right)$ is known and $\tilde{d}_i \sim \mathcal{N}(\overline{d}_i, \sigma_i^2), i = 1, 2$. A piece of public news about the first firm (the announcing firm) arrives at time $t = 1$, and this news is interpreted as [11]

$$\tilde{s} = \tilde{d}_1 + \epsilon, \tag{1}$$

where ϵ has a normal distribution with zero mean and variance σ_ϵ^2. ϵ is independent of $\tilde{d}_1$ and $\tilde{d}_2$.

A representative investor makes use of the news $\tilde{s}$ for the valuation of the non-announcing firm's stock price. For instance, the investor performs a regression as follows

$$\tilde{d}_2 = \alpha + \beta \times \tilde{s} + \epsilon_2. \tag{2}$$

But the investor is *uncertain* about the impact of news on the announcing firm. In other words, β is a plausible set, rather than a precise number. Since $\beta = \rho \frac{\sigma_1 \sigma_2}{\sigma_s^2}$, where ρ is the unconditional correlation coefficient between $\tilde{d}_1$ and $\tilde{d}_2$, a range $\beta_a \leq \beta \leq \beta_b$ corresponds to a set of unconditional correlation coefficients $\rho_a \leq \rho \leq \rho_b$, where $\beta_a = \rho_a \frac{\sigma_1 \sigma_2}{\sigma_s^2}$ and $\beta_b = \rho_b \frac{\sigma_1 \sigma_2}{\sigma_s^2}$. Therefore, the uncertainty about information transfer is the same as the *correlation uncertainty* betweens firms.

[11]Equivalently, this news can be used to forecast the future payoffs of the announcing firm such as $\tilde{d}_1 = a \times \tilde{s} + \epsilon_1$, where ϵ_1 is independent of the news, and $a = \sigma_1^2 / \sigma_s^2$.

Specifically,

$$\begin{bmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ \tilde{s} \end{bmatrix} \sim \left(\begin{bmatrix} \overline{d}_1 \\ \overline{d}_2 \\ \overline{d}_1 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \sigma_1^2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & \rho\sigma_1\sigma_2 \\ \sigma_1^2 & \rho\sigma_1\sigma_2 & \sigma_s^2 \end{bmatrix} \right), \rho_a \leq \rho \leq \rho_b. \tag{3}$$

$\mathcal{M}$ is a set of distribution of $(\tilde{d}_1, \tilde{d}_2, \tilde{s})$ given in (3) for all $\rho_a \leq \rho \leq \rho_b$. Given the uncertainty about the information transfer effect, or equivalently, the correlation uncertainty, the investor is ambiguity-averse in the sense of having multiple-prior utility in Epstein and Schneider (2007) and Wang (2003) as follows,

$$U_t = \min_{m_t \in \mathcal{M}_t} \mathbb{E}_{m_t}[u(C_t) + \alpha U_{t+1}], \tag{4}$$

where $u(\cdot), C_t$, and α are the standard utility function, consumption at t and the subjective discount factor respectively. For simplicity I assume that $u(W) = -e^{-\gamma W}$, $\alpha = 1$ and there is no consumption prior to the terminal time. Let $\mathcal{M}_t$ and m_t denote the set of models considered by the investor at time t and a specific model within that set, respectively. $E_{m_t}[\cdot]$ is the expectation given the beliefs generated by model m_t.

Precisely, the investor at time $t = 0$ is aware of the news coming and the set of models is

$$\mathcal{M}_0 = \left\{ m_\rho : (\tilde{d}_1, \tilde{d}_2) \text{ has a Gaussian distribution via (3), written as } m_\rho, \rho \in [\rho_a, \rho_b] \right\}. \tag{5}$$

The set of models $\mathcal{M}_1$ at time $t = 1$ is

$$\mathcal{M}_1 = \left\{ m(s) : m(s) \text{ is the conditional distribution of } (\tilde{d}_1, \tilde{d}_2) \text{ under } m \in \mathcal{M} \text{ given } s \right\}. \tag{6}$$

The model significantly differs from the previous studies about information ambiguity. Epstein and Schneider (2008), and subsequent studies such as Caskey (2009), Illeditsch (2011), Kelsey, Kozhan and Pang (2011), and Zhou (2015), all investigate the ambiguity about the news quality in the sense that variance of the signal, σ_ϵ, moves within a plausible range, while the correlation structure of asset payoffs is given as exogenous.[12] In contrast, the investor in my setting has no ambiguity about the news quality. In fact, the ambiguity is about the relevance of news across firms; alternatively, the ambiguity about the asset payoffs' correlated structure. By its very construction, $\mathcal{M}_0$ and $\mathcal{M}_1$ together satisfy the dynamic consistency condition in Epstein and Schneider (2007) and Wang (2008).

1.3 Characterization of Equilibrium

In this section I first characterize the equilibrium at $t = 1$. Before doing so, I first solve the optimal portfolio choice problem for the representative investor, by characterizing the optimal demand and the worst-case correlation coefficient between the asset payoffs when the asset prices are given exogenously. The characterization of the equilibrium at $t = 0$ is presented afterwards.

[12] See also Mele and Sangiorgi (2015), Condie and Ganguli (2011), and Condie, Ganguli and Illeditsch (2015) for the ambiguity about information quality in a rational equilibrium model.

1.3.1 Optimal Portfolio Choice

By abuse of notation I use p_i to represent the price at time $t = 1$ in this section. Under the CARA utility assumption, the optimal portfolio choice problem under consideration is

$$\max_{\theta} \min_{\rho\in[\rho_a,\rho_b]} \mathbb{E}_\rho \left[u(W_2)|\tilde{s} = s\right] = u\left(\max_{\theta} CE(\theta)\right) \tag{7}$$

where $W_2 = W_1 + \theta_1(\tilde{d}_1 - p_1) + \theta_2(\tilde{d}_2 - p_2)$ and $\theta = (\theta_1, \theta_2)$ is the demand vector on the risky assets, and $CE(\theta) = \min_{\rho\in[\rho_a,\rho_b]} CE(\rho, \theta)$ is the certainty equivalent of the multi-prior expected utility (MEU) investor for a given demand vector θ. $CE(\rho, \theta) = \mathbb{E}_\rho \left[W_2|\tilde{s} = s\right] - \frac{\gamma}{2} Var_\rho \left[W_2|\tilde{s} = s\right]$ denotes the certainty equivalent of a standard expected utility (SEU) investor with the belief that the correlation structure of the asset payoff is ρ. For a SEU investor, there is no uncertainty about the effect of information transfer.

Let us start with the computation of the certain equivalent of a MEU investor. If there is no holdings on the second risky asset ($\theta_2 = 0$), then any correlation coefficient $\rho \in [\rho_a, \rho_b]$ solves $CE(\theta)$. On the other hand, if $\theta_2 \neq 0$, let $\phi = \frac{\sigma_1^2}{\sigma_1^2+\sigma_\epsilon^2}$, and

$$\hat{\rho}(s;\theta) = \frac{\sigma_1\theta_1}{\sigma_2\theta_2}\frac{1-\phi}{\phi} - \frac{1}{\gamma\theta_2}\frac{s-\bar{d}_1}{\sigma_1\sigma_2}, \tag{8}$$

then [13]

$$CE(\theta) = \begin{cases} CE(\rho_a, \theta), \text{if } \hat{\rho}(s;\theta) < \rho_a \\ CE(\rho_b, \theta), \text{if } \hat{\rho}(s;\theta) > \rho_b \\ CE\left(\hat{\rho}(s;\theta), \theta\right), \text{if } \rho_a \leq \hat{\rho}(s;\theta) \leq \rho_b. \end{cases} \tag{9}$$

The intuition of (9) is as follows. Without loss of generality I assume a positive holding on the second risky asset ($\theta_2 > 0$), the worst-case correlation coefficient depends on the trade-off between the effect of news on the portfolio mean and the portfolio variance. For the portfolio mean, the correlation coefficient has a positive effect if and only if the signal is greater than its expected value, which indicates good news for the first firm.

$$argmin_{\rho \in [\rho_a, \rho_b]} \mathbb{E}_\rho[W] = \begin{cases} \rho_a, \text{if } s > \overline{d}_1 \\ \rho_b, \text{if } s < \overline{d}_1, \end{cases}$$

When $s = \overline{d}_1, \mathbb{E}_\rho[W]$ is independent of the correlation coefficient. For the portfolio variance, it depends on the correlation structure. It is easy to see that,[14]

$$argmax_{\rho \in [\rho_a, \rho_b]} Var_\rho[W] = \mathcal{L}\left(\rho_a, \rho_b; \frac{\sigma_1 \theta_1}{\sigma_2 \theta_2} \frac{1-\phi}{\phi}\right).$$

Put it together, the overall effect of the correlation on $CE(\rho, \theta)$ depends on both the news and the correlation structure. Specifically,

$$argmin_{\rho \in [\rho_a, \rho_b]} CE(\rho, \theta) = \mathcal{L}\left(\rho_a, \rho_b; \hat{\rho}(s;\theta)\right).$$

[13] See Appendix A for its proof.

[14] $\mathcal{L}(\rho_a, \rho_b; x)$ is x truncated by ρ_a and ρ_b on both sides.

For the MEU investor, the correlation structure used to compute the certain equivalent is *negatively* determined by the news s. I will show the same insight in equilibrium in the next subsection.

Let $S_i = (\overline{d}_i - p_i)/\sigma_i$ be the unconditional Sharpe ratio of asset $i = 1, 2$. In solving the optimal portfolio choice problem for the MEU investor, I use

$$\tau(x,y) = \begin{cases} \min\left\{\frac{x}{y}, \frac{y}{x}\right\}, & \text{if } xy > 0, \\ \max\left\{\frac{x}{y}, \frac{x}{y}\right\}, & \text{if } xy < 0, \\ 0, & \text{if } xy = 0. \end{cases} \tag{10}$$

to describe the dispersion between x and y.

Proposition 1 *Let $\theta(\rho)$ denote the optimal demand when the correlation coefficient between asset payoff is ρ for a SEU investor, i.e.,*

$$\theta(\rho) = \frac{1}{\gamma}\Sigma_\rho^{-1} \times \begin{bmatrix} \overline{d}_1 + \phi(s - \overline{d}_1) - p_1 \\ \overline{d}_2 + z_\rho \phi(s - \overline{d}_1) - p_2 \end{bmatrix}, \tag{11}$$

where

$$\Sigma_\rho = \begin{bmatrix} \sigma_1^2(1-\phi) & \rho\sigma_1\sigma_2(1-\phi) \\ \rho\sigma_1\sigma_2(1-\phi) & \sigma_2^2(1-\rho^2\phi) \end{bmatrix}, \tag{12}$$

$z_\rho = \rho\frac{\sigma_2}{\sigma_1}$. *Assume that at least one unconditional Sharpe ratio is not zero, $\rho^* = \mathcal{L}\left(\rho_a, \rho_b; \tau(S_1, S_2)\right)$, then $\theta(\rho^*)$ is the optimal demand of the representative investor under correlation uncertainty and ρ^* is its corresponding worst-case correlation coefficient.*

To explain its intuition, I assume that the investor has no knowledge at all about the correlation coefficient. When two Sharpe ratios are close to each other, it indicates that both assets offer very similar investment opportunities, thus the higher correlation the smaller the diversification benefits. The worst-case scenario is associated with the highest possible correlation coefficient. On the other hand, when two risky assets generate fairly opposite investment opportunities, the diversification benefit is increasing with respect to the asset correlation, therefore, the worst-case scenario of a mean-variance utility is obtained at the lowest correlation coefficient. Therefore, the worst-case correlation coefficient must be $\tau(S_1, S_2)$, a *similarity measure* of Sharpe ratios, as documented in Proposition 1.

In the optimal portfolio choice problem of a MEU investor, the worst-case correlation structure depends on the dispersion of the unconditional Sharpe ratios, since the unconditional Sharpe ratios are given exogenously. In equilibrium, the stock prices depend on the news so as to the unconditional Sharpe ratio, as a consequence, the worst-case correlation coefficient relies on the news. This is the objective of the next subsection.

1.3.2 Equilibrium at $t = 1$

Let $n \equiv \frac{\sigma_1 \overline{\theta}_1}{\sigma_2 \overline{\theta}_2} \frac{1-\phi}{\phi}$. The number n can be written as $\frac{\sigma_\epsilon^2}{\sigma_1 \sigma_2 \overline{\theta}}$ or alternatively $\left(\frac{\sigma_\epsilon}{\sigma_1}\right)^2 \frac{\sigma_1 \overline{\theta}_1}{\sigma_2 \overline{\theta}_2}$, where $\overline{\theta} = \frac{\overline{\theta}_2}{\overline{\theta}_1}$ denotes the ratio of firm 2's share to the firm 1's share. Notice that σ_i is the asset price volatility, a product of the return volatility and the stock price. Therefore, $\sigma_i \overline{\theta}_i$ equals a product of the firm capitalization and its return volatility. Consequently, $\frac{\sigma_2 \overline{\theta}_2}{\sigma_1 \overline{\theta}_1}$ is the ratio of the firm capitalization times the ratio of return volatility.

Proposition 2 *1. (*The Endogenous Correlation*) The endogenous correlation coefficient between the asset payoffs conditional on $\tilde{s} = s$ is $\rho(s)\sqrt{\frac{1-\phi}{1-\rho(s)^2\phi}}$, where $\rho(s)$ is the worst-case correlation coefficient that is determined explicitly as follows.*

- *For all* bad news $s < s^L \equiv \overline{d}_1 + \gamma\sigma_1\sigma_2\overline{\theta}_2\,(n - \rho_b)$, $\rho(s) = \rho_b$;
- *for all* good news $s > s^H \equiv \overline{d}_1 + \gamma\sigma_1\sigma_2\overline{\theta}_2\,(n - \rho_a)$, $\rho(s) = \rho_a$;
- *for all moderate news* $s \in [s^L, s^H]$,

$$\rho(s) = \frac{1}{\sigma_1\sigma_2\overline{\theta}_2}\left\{\overline{\theta}_1\sigma_\epsilon^2 - \frac{s - \overline{d}_1}{\gamma}\right\}. \tag{13}$$

2. (The Endogenous Asset Prices) *The endogenous stock price is given by*

$$p_i(s) = \mathbb{E}_{\rho(s)}\left[\tilde{d}_i|\tilde{s} = s\right] - \gamma Cov_{\rho(s)}(\tilde{d}_i, \tilde{d}), i = 1, 2, \tag{14}$$

where $\tilde{d} = \overline{\theta}_1\tilde{d}_1 + \overline{\theta}_2\tilde{d}_2$.

The intuition of Proposition 14 follows from the above calculation of certainty equivalent for a MEU investor. Since the market demand must be the market supply $\overline{\theta}$, the worst-case correlation coefficient in the equilibrium has the same expression as the solution to the certainty equivalent, by replacing θ with $\overline{\theta}$ in Equation (9).

As explained above, the endogenous correlation structure in the equilibrium is influenced by the nature of the news. If the signal conveys bad news about the announcing firm, the MEU investor will interpret this news as highly relevant to the non-announcing firm, and the worst case scenario is when the correlation is the highest. On the other hand, if the signal conveys good news about the announcing firm, the investor will interpret that this good news has nothing to do with the non-announcing firm, so the endogenous correlation structure corresponds to the lowest plausible one. When the news is not strong enough, which falls in $[s^L, s^H]$, the endogenous correlation coefficient is negatively determined by the magnitude of the news s due to the worst-case consideration of the investor. Overall, the worst-case correlation structure between the asset payoffs has a negative relationship with the news in the equilibrium.

Remarkably, the range of the moderate news, $s^H - s^L$, is a proportion of $\rho_b - \rho_a$, which measures the degree of ambiguity about the news. A higher degree of the correlation uncertainty indicates a wider range of the moderate news, and a more significant decreasing shape of the endogenous correlation coefficient.

The equilibrium is obtained by examining the role of the signal and how ambiguity aversion revises the investor's belief in interpreting the relevance of news. To illustrate, first consider a situation when the news is extremely useless; then $\sigma_\epsilon = \infty$, $\phi = 0$ and $s^L = \infty$. The worst-case correlation coefficient should always correspond to the highest plausible estimation ρ_b that minimizes the equilibrium utility of the representative investor.[15] As a result, each stock price is given by $\overline{d}_i - \gamma Cov_{\rho_b}\left(\tilde{d}_i, \tilde{d}\right)$ as in a standard CAPM model (Cochrane, 1992).

After a piece of news $\tilde{s} = s$ about the first firm is revealed on the market, the investor evaluates the trade-off between the diversification benefit and the correlation

[15] A similar result is reached by Jiang and Tian (2016) in their equilibrium analysis. However, they derive the endogenous correlation structure for heterogeneous investors under the setting of Knightian uncertainty on correlation without signaling.

uncertainty. When the news is good, the impact of news indicating a low correlation dominates the impact of the ambiguity concern indicating a high correlation, therefore the correlation structure in equilibrium corresponds to the lowest estimation. On the other hand, a piece of bad news intensifies the investor's concern on the correlation estimation, thus compounds her worst case belief to the highest correlation structure. Therefore, $\rho(s)$ is decreasing with respect to the news s.

By similar intuition, the endogenous correlation coefficient $\rho(s)$ decreases, as presented in Equation (13),

1. if the signal has a better quality, in the sense that σ_ϵ is smaller;

2. if the firm 2's capitalization is larger relative to the firm 1; or

3. if the firm 1's volatility is larger.

The stock price in Proposition 14 is written as $p_i(s) = \mathbb{E}_{\rho(s)}[m_{1,2}\tilde{d}_i|\tilde{s} = s]$, where $m_{1,2}$ is the stochastic discount time factor from time $t = 1$ to $t = 2$,

$$m_{1,2} = \frac{e^{-\gamma\tilde{d}}}{\mathbb{E}_{\rho(s)}[e^{-\gamma\tilde{d}}|\tilde{s} = s]} \tag{15}$$

is the marginal utility of the representative (MEU) investor on the portfolio $\tilde{d}$. Compared with the model of the SEU investor, the correlation structure between asset payoffs depend on the news.

Finally, it is important to compare Proposition 14 with Proposition 1. Assuming ρ^* is given in Proposition 1, by equation (C-11), the unconditional Sharpe ratios are

$$S_1 = \gamma\left\{\sigma_1(1-\phi)\overline{\theta}_1 + \rho^*\sigma_2(1-\phi)\overline{\theta}_2\right\} - \frac{\phi}{\sigma_1}(s - \overline{d}_1), \tag{16}$$

and

$$S_2 = \gamma \left\{ \rho^* \sigma_1 (1-\phi)\overline{\theta}_1 + \sigma_2 (1-\rho^{*2}\phi)\overline{\theta}_2 \right\} - \frac{\rho^*\phi}{\sigma_2}(s - \overline{d}_1). \tag{17}$$

By Proposition 1, the worst-case correlation coefficient ρ^* must satisfy

$$\rho^* = \mathcal{L}\left(\rho_a, \rho_b; \tau(S_1, S_2)\right), \tag{18}$$

which is a highly nonlinear equation since S_1, S_2 depend on ρ^* in Equation (16) and Equation (17). If the representative investor chooses any number either smaller or larger than the fixed point in Equation (18), the investor scarifies her expected (multi-prior) preferences by Proposition 1. Therefore, in equilibrium, the endogenous correlation coefficient must be the fixed point of Equation (18). By solving the fixed point problem in Equation (18), $\rho(s) = \rho^*$ is obtained in Proposition 14.

Proposition 3 (The Decreasing Correlation Principle) *The endogenous correlation coefficient between the asset payoffs conditional on $\tilde{s} = s$,*

$$corr(\tilde{d}_1, \tilde{d}_2 | \tilde{s} = s) = \rho(s)\sqrt{\frac{1-\phi}{1-\rho(s)^2\phi}}, \tag{19}$$

is decreasing *with respect to the magnitude of the news $\tilde{s} = s$.*

As will be shown later, the decreasing correlation principle is the central result that generates several important model predictions. It states the correlation structure between firms' payoff is asymmetric conditional on the news, and this asymmetric correlation structure further yields asymmetric effects on the stock prices and the returns.

To illustrate the decreasing correlation principle numerically, Figure 1.1 depicts the worst-case correlation coefficient $\rho(s)$ (top panel) and the endogenous conditional correlation $corr(\tilde{d}_1, \tilde{d}_2|\tilde{s} = s)$ (bottom panel) with respect to the news s for $\rho_a = 0.4 - \epsilon, \rho_b = 0.4 + \epsilon$, for $\epsilon = 0.05$, and $\epsilon = 0.1$. Other parameters are $\sigma_1 = 3$, $\sigma_2 = 2$, $\sigma_\epsilon = 1\%$; $\overline{d}_1 = 0$, $\overline{d}_2 = 0$, $\overline{\theta}_1 = 1$, $\overline{\theta}_2 = 1$, and $\gamma = 1$. Since ϵ measures the level of uncertainty, the higher the investor's uncertainty about the impact of news, the more significant the decreasing pattern of the correlation.

To summarize, the endogenous correlation coefficient between asset payoffs decreases,

- if the signal has better quality, in the sense that σ_ϵ decreases;
- if the firm 2's capitalization is larger relative to the firm 1; or
- if the firm 1's volatility is larger.

1.3.3 Equilibrium Prices at time $t = 1$

In this section I investigate how the news and the correlation uncertainty jointly affect stock prices. For illustration purpose, I consider a positively correlated structure (that is, $\rho_a \geq 0$).[16]

Proposition 4 *1. The better of the news, the higher the price of each risky asset.*

2. The price of the non-announcing firm reacts more strongly to the bad news than the good news. Moreover, when the news is moderate, the price stays constant.

[16] In a negatively correlated structure, the results of the second stock price can be modified easily. I discuss the negatively economic-linked firms in Section 4.4.

3. With other parameters being fixed, for the good news, the better the quality of news the higher the stock prices. However, for the bad news, the better the quality of the news the lower the stock prices.

Propositions 4 (1) is intuitive. The better the news about the future payoff of the announcing firm, the higher the stock price of both firms. However, the price reaction of the announcing firm and the non-announcing firm is significantly different. Precisely,

$$\frac{\partial p_1(s)}{\partial s} = \begin{cases} \phi, \text{ if } s < s^L, \\ 1, \text{ if } s^L \leq s \leq s^H, \\ \phi, \text{ if } s > s^H. \end{cases} \tag{20}$$

$$\frac{\partial p_2(s)}{\partial s} = \begin{cases} \rho_b \frac{\sigma_2}{\sigma_1} \phi, \text{ if } s < s^L, \\ 0, \qquad textif s^L \leq s \leq s^H, \\ \rho_a \frac{\sigma_2}{\sigma_1} \phi, \text{ if } s > s^H. \end{cases} \tag{21}$$

Intuitively, since the investor is not sure how to interpret the news from one firm to the other firms, the ambiguity aversion leads the investor to react more strongly to a signal which conveys bad news than a signal that conveys good news. Thus the impact of the news is *asymmetric* given a piece of good news versus bad news. As a consequence, the price effect on the non-announcing firm is stronger for bad news than good news. To illustrate from a hedging perspective, let us assume the "true" correlation coefficient is ρ_0, but the investor only knows that $\rho_a \leq \rho_0 \leq$

ρ_b, without knowing the distribution of the correlation coefficient. The right delta hedging ratio for the second risky asset using the first risky asset is $\rho_0 \frac{\sigma_2}{\sigma_1}$ (See Anderson and Danthine, 1981). Clearly, $\rho_a \frac{\sigma_2}{\sigma_1} \leq \rho_0 \frac{\sigma_2}{\sigma_1} \leq \rho_b \frac{\sigma_2}{\sigma_1}$. Hence, the investor's stronger (weaker) reaction to the bad (good) news is consistent with the under-hedge (over-hedge) of the risk in the non-announcing firm against the announcing firm.

A striking result of the information transfer under uncertainty is that the non-announcing firm's stock price stays constant when the news is not strong enough. The intuition is as follows. When the signal is not strong enough, conveying neither good nor bad news, the investor does not know how to interpret the news to the non-announcing firm; hence, the price shows no response to the news. Precisely, within the moderate range, $s^L \leq s \leq s^H$, the stock price stays unchanged, resulting from a counterbalance between the impact of news and the impact of correlation uncertainty. In fact, by straightforward calculation,

$$p_2(s) = \overline{d}_2 - \gamma\sigma_2^2\phi, \forall s \in [s^L, s^H]. \tag{22}$$

Equation (22) demonstrates an important "inertia" property on the risky asset under the ambiguity environment with a piece of news. Using the incomplete preference of Bewley (2002), Easley and O'Hara (2009) identify the portfolio inertia. Cao, Wang and Zhang (2005), Epstein and Schneider (2007) demonstrate that portfolio inertia occurs in risk-free portfolio. Epstein and Wang (1995), Illeditsch (2011), and Jiang and Tian (2016) prove the portfolio inertia for risky portfolios under different frameworks of ambiguity. Condie, Ganguli and Illeditsch (2015) identify *inertia to information* in an economy with one risky asset. The authors show that the stock price stay constant when there is uncertainty for this firm's *own* information. In my setting, I show that the stock price could stay constant facing the news about its *related firm*, which I call "price inertia".

To illustrate the intuition behind the price inertia, first consider a SEU investor whose correlation belief about the asset payoffs is exactly ρ. The equilibrium asset prices are given as $p_i^{SEU} = \mathbb{E}_\rho[\tilde{d}_i|\tilde{s} = s] - \gamma Cov_\rho(\tilde{d}_i, \tilde{d}), i = 1, 2$. Clearly, the SEU investor under the bad news requires a lower price as compensation for the lower posterior mean in order to hold the risky assets. However, this is no longer true for the MEU investor since ρ becomes a plausible range of numbers instead of a fixed number. The MEU investor revises her belief (estimation) about the correlation upwards if the signal drops. The effect of correlation on volatility counterbalances the effect of news on the mean. As a result, the price does not change because the lower posterior mean that would require a drop in the equilibrium price is exactly offset by the lower risk premium that would require an increase in the price.

The price effect to the announcing firm is also remarkable in equilibrium. For the announcing firm, since the signal conveys direct information about its future payoff, the impact of the news on the asset price is *symmetric* give a piece of good news versus bad news. However, the investor demand is stronger on the announcing firm, resulting from the non-announcing firm's lack of reaction facing moderate news, so the supply-demand equation enforces a stronger marginal price effect on the announcing firm.

Figure 1.2 presents the above results about endogenous stock prices graphically with regard to the news impact. The announcing firm's stock price is increasing with the news all the time. For the second firm, when $s < s^L = 6.55$, and $s > s^H = 9.05$, the stock price is always increasing; however, the stock price keeps constant as the magnitude of news s is within $[6.55, 9.05]$.

Proposition 4 (3) highlights the effect of the news quality joint with the magnitude of the news. The good news that is precise leads to a larger price increase, while the bad news that is precise leads to a larger price decrease.

I summarize my model predictions as follow.

Model Prediction I.

1. *When the news conveys direct information about the future payoff, the stock price is more sensitive to a piece of moderate news than the profound news (good or bad). The stock price reaction to the good news and the bad news is symmetric.*

2. *When the news conveys indirect information about the future payoff, the stock price reacts more*<u>*strongly*</u> *to the bad news than the good news. The stock price shows* **lack of reaction** *when the news is moderate.*

1.3.4 Equilibrium at $t = 0$

To finish the characterization of the equilibrium, I derive the equilibrium price at $t = 0$. By the dynamic consistency property of the multi-prior expected utility, the dynamic optimal portfolio choice problem is

$$\max_{D} \min_{\rho \in [\rho_a, \rho_b]} \mathbb{E}[J(W_1, s)]$$

where D is the number of stocks at time $t = 0$ and $J(W_1, s)$ is the derived expected utility conditional on $\tilde{s} = s$ at time $t = 1$,

$$J(W_1, s) = \max_{\theta} \min_{\rho \in [\rho_a, \rho_b]} \mathbb{E}_{\rho}[u(W_2)|\tilde{s} = s].$$

The equilibrium asset prices at time $t = 0$ are given by the next result.

Proposition 5 *The stock price of firm i at time $t = 0$ is $p_i = \mathbb{E}[m_{0,1} p_i(s)]$, where*

$$m_{0,1} = \frac{e^{-\gamma(p_1(s)\overline{\theta}_1 + p_2(s)\overline{\theta}_2 + \frac{\gamma}{2}\overline{\theta}' \Sigma_{\rho(s)} \overline{\theta})}}{\mathbb{E}\left[e^{-\gamma(p_1(s)\overline{\theta}_1 + p_2(s)\overline{\theta}_2 + \frac{\gamma}{2}\overline{\theta}' \Sigma_{\rho(s)} \overline{\theta})}\right]}. \tag{23}$$

is the stochastic discount factor in the first time period, $\rho(s)$ is given in Proposition 14, and $p_i(s)$ is the asset price at time $t = 1$ given in Proposition 4. Moreover, $m_{0,1}$ is strictly decreasing *with respect to the news s.*

By Proposition 5, the log of the price kernel, *Log*$(m_{0,1})$, is in essence (up to a constant) the mean-variance utility of the portfolio, $\mathbb{E}_{\rho}(s)[\tilde{d}|\tilde{s} = s] - \frac{\gamma}{2} Var_{\rho(s)}(\tilde{d}|\tilde{s} = s)$. Moreover, the pricing kernel is *log-convex* with respect to $\tilde{s} = s$. By contrast, the log of the price kernel in the first time period in a standard dynamic equilibrium model is a linear function of the news. Gollier (2011) also demonstrates the non-linear feature of the log of the pricing kernel in a discrete version of the smooth ambiguity model.

Model Prediction II.

The price increases on average in each time period. Precisely, $p_i < \mathbb{E}[p_i(s)|\tilde{s} = s]$ and $p_i(s) < \mathbb{E}_{\rho(s)}[\tilde{d}_i]$ for each $i = 1, 2$.

1.4 Model Implications

This section presents further model implications. I first present the model prediction for the stock prices. Next I discuss the implications for the risk premium and conditional risk premium. In the end, I examine the conditional correlation and covariance between two stock returns as well as the conditional return volatility.

1.4.1 The stock price reaction

I first study the stock price reaction by examining the autocorrelation of stock price changes.

Proposition 6 *1. For the announcing firm, the price change in two consecutive time periods is* <u>*negatively*</u> *correlated.*

2. For the non-announcing firm, the autocorrelation of the price changes is <u>*positive*</u> *if* $\rho_a \geq \frac{n}{2}$*;* <u>*negative*</u> *when* $\rho_b \leq \frac{n}{2}$*.*

To understand Proposition 6, we first consider the situation of a SEU investor who has the correlation coefficient belief about asset payoffs as ρ, in which each stock price is $P_i^{SEU} = \mathbb{E}[\tilde{d}_i|\tilde{s} = s] - \gamma Cov_\rho(\tilde{d}_i, \tilde{d})$. And $corr(\Delta P_1^{SEU}, \Delta P_2^{SEU}) = 0$, the stock price changes of each firm between the first two periods are *independent*, a weak form of market efficiency. In other words, the firm-specific news has been fully incorporated in the stock prices.

By contrast, the stock prices do not fully reflect the relevant news, given the information transfer effect under uncertainty, implying stock predictability in a rational equilibrium model.[17] There are several remarkable aspects in Proposition 6. First of all, the price changes of the announcing firm in consecutive time periods are *NOT* independent anymore due to the correlation uncertainty in equilibrium. Precisely, the announcing firm has a short-term overreaction but reversal in the nest time period, as the autocorrelation of the price changes is negative and the correlation between the short-term price changes with the long-term price change is positive.[18] This short-term reversal property of the announcing firm is remarkable because there is no concern about the information quality for this firm itself, rather, it follows from its information transfer concern to other firms through an equilibrium mechanism. Indeed, the short-term overreaction of the announcing firm is associated with the lack of reaction of the non-announcing firm and the overreaction of the announcing firm when the news is not strong enough.

Second, the autocorrelation of the price changes for the non-announcing firm has different sign as the announcing firm and different predictability implications due to the correlation uncertainty. When ρ_a is large enough, it means that the economic link between two firms is relatively strong, the price changes in the first two time periods are *positive* correlated, thus, there is a underreaction for the non-announcing firm. Moreover, there are positive correlation between the price changes in the short-term period and in the long-term period. Accordingly, the model explains price momentum for both risky assets in this situation.[19] That is, a good (bad) investment in the first

[17]Other rational equilibrium models explain the stock predicability includes Johnson (2005), Vaynos and Wooley (2012).

[18]It means that $corr\left(p_1(s) - P_1, \tilde{d}_1 - P_1\right) > 0$. Its proof is given in the proof of Proposition 6 in Appendix.

[19]See Daniel, Hirshleifer and Subrahmanyam (1998), Barberis, Shleifer and Vishny (1988), and Hong and Stein (1999) for an explanation of the momentum from a behaviorial finance perspective

time period continues to be good (bad) in the second time period.

However, it is not always the case that the non-announcing firm displays a momentum. For instance, when the economic link is virtually weak (for a small ρ_b), the model implies a negative autocorrelation of the price changes for the non-announcing firm. This negative autocorrelation (overreaction) does not guarantee a reversal at time $t = 2$ since the prices at future time periods $t = 1$ and $t = 2$ not necessarily move in the same direction, which is different from the short-term reversal to the announcing firm. Moreover, when $\rho_a < \frac{n}{2} < \rho_b$, the autocorrelation for the non-announcing firm can be either positive or negative, depending on other model parameters. Overall, there is rich predictability structure for the non-announcing firm due to the correlation uncertainty.

Proposition 6 is helpful to study whether the stock price overreacts or underreacts. Thomas and Zhang (2008), Ramnath (2002) report stock price can be either overreaction or underreaction for peer firms in the same industry, and Cheng and Eskhmena (2014) document similar findings for firms in the supply-chain. Based on the moderate confidence hypothesis, Cheng and Eskhemna (2014) suggest price underreaction and the post-earnings announcement drift when the signal is precise; and price overreaction when the signal is imprecise. In fact, their findings are two special cases in Proposition 6. When the signal is extremely precise, ϕ is close to one and $n \sim 0$; thus, $\rho_a \geq \frac{n}{2}$ holds, and the positive autocorrelation indicates a stock price underreaction. On the other hand, if the signal is sufficiently imprecise, ϕ is close to zero, $n \sim \infty$ and $\rho_b \leq n$ holds naturally. Hence, by Proposition 6, the non-announcing firm's

such as overconfidence, investor sentiment and gradual response to the information.

stock price displays a negative autocorrelation, and stock price overreacts. Figure 3 explains the price momentum and reversal under the presented conditions. If the economic link is relatively strong, in the sense that the lowest correlation is a relatively large number, it indicates a larger region of the very good news, where the stock price reacts less strongly to the news. Therefore, on average, the non-announcing firm's stock price underreacts (momentum). The bottom panel explains the reversal when the economic link is weak. Moreover, as shown in Figure 4, the higher the correlation uncertainty, the stronger underreaction or overreaction of the stock price.

The next proposition is about the predictability across firms (or the portfolio).

Proposition 7
1. *The correlation between the price changes of the announcing firm with the price changes of the non-announcing firm in the subsequent time period is* positive *when* $\rho_a \geq \frac{n}{2}$ *and* *negative* *when* $\rho_b \leq \frac{n}{2}$.

2. *The autocorrelation of the price changes of the portfolio* $\tilde{d}$ *is* *positive* *if* $\rho_a \geq n$; *negative* *when* $\rho_b \leq n$.

3. *The correlation between the price changes of the announcing firm with the price changes of the portfolio in the subsequent time period is* *positive* *when* $\rho_a \geq n$ *and* *negative* *when* $\rho_b \leq n$.

Proposition 7 (1) reports the cross-correlation between the announcing firm's stock price changes with the non-announcing firms's stock price changes. It shows that the announcing's stock price has predictability about the non-announcing stock price if the economic link satisfies certain conditions. Proposition 7 (1) is related to recent empirical evidences in Cohen and Frazzini (2008), and Chen and Lou (2012). They document strong predictability from one firm to another firm when the firm-specific news is revealed.

I further investigate how the price changes of the portfolio $\tilde{d}$ is affected by the correlation uncertainty. Because of the predictability component on each firm, naturally, the autocorrelation between the price changes of the portfolio is non-zero. In fact, for $\rho_a \geq n$, there is an under-reaction on the whole portfolio since the underreaction on the non-announcing firm dominates the overreaction on the announcing firm; similarly, for $\rho_b \leq n$, there is an over-reaction on the portfolio.

Since the price changes of the portfolio is equivalent to the price changes in each firm, the autocorrelation between the price changes of the portfolio also depends on the cross-autocorrelation between two firms, in addition to the first order autocorrelation in each firm. Similar to Lo and MacKinlay (1990), the cross-autocorrelation between two firms is given in Proposition 7 (1). In fact, if we consider the price changes of the announcing firm in the latter time period, this cross-autocorrelation is negative because of the overreaction of the announcing firm. On the other hand, the another cross-autocorrelation becomes positive for $\rho_a \geq \frac{n}{2}$ by using the same insight on the underreaction of the non-announcing firm. When the firm-specific news is precise, Proposition 7 (2) implies an underreaction of the market portfolio (Jadedeesh and Titman, 1993, 2001; Lo and Mackinlay, 1988); but the imprecise firm-specific news could lead an overreaction of the market (DeBondt and Thaler, 1985).

Proposition 7 (3) demonstrates the predictability of the portfolio under specific news. When $\rho_a \geq n$ ($\rho_b \leq n$), we see that the cross-correlation between the announcing firm's stock price and the portfolio is positive (negative). Therefore, the announcing firm's stock price is useful to predict the portfolio price. In this regard, Proposition 7 (3) is related to Patton and Verardo (2012), which shows that the specific firm news yields market predictability.

In addition to the information quality, Proposition 6 - 7 state concrete conditions about other elements that explain stock price over- or underreaction. For instance, when the announcing-firm is significantly smaller than the non-announcing firm, the non-announcing firm's stock price displays underreaction (n is close to zero in this situation). By contrast, if the non-announcing firm is significantly smaller than the announcing firm, the non-announcing firm's stock price displays overreaction.

I present the model predictions for the stock price reactions to news as follow.

Model Prediction III.

1. *There is short-term underreaction (momentum) for the non-announcing firm's stock price, if one of the following conditions holds:*

    - *the news is very precise;*
    - *the announcing firm is very risky;*
    - *the non-announcing firm's size is much larger than the announcing firm.*

    In particular, there exists short-term momentum of the market (while the non-announcing firm denotes all other firms and its size is much larger than the announcing firm).

2. *There exists short-term overreaction for the non-announcing firm's stock price,*

if one of the following conditions holds:

- *the news is very imprecise;*
- *the announcing firm's size is much larger than the non-announcing firm.*

1.4.2 Risk premium and Conditional risk premium

This subsection discusses about the risk premium and the conditional risk premium of each asset. Let $\tilde{R}_i = \frac{p_i(s)-p_i}{p_i}$ be the return in the first time period, and $\tilde{R}_i(s) = \frac{\tilde{d}_i-p_i(s)}{p_i(s)}$ be the returns of asset i, conditional on the news $\tilde{s} = s$.

Proposition 8 *1. Each risky asset has a* <u>*positive*</u> *excess risk premium due to correlation uncertainty. Moreover, the higher the correlation uncertainty the higher the excess risk premium.*

2. The conditional risk premium of the announcing firm is always decreasing with respect to the news. The conditional risk premium of the non-announcing firm is also decreasing with respect to the news when $\rho_b \leq \frac{n}{2}$, but the news effect to non-announcing firm's conditional risk premium in not monotonic in general.

The first part of Proposition 8 is consistent with vast uncertainty literature to demonstrate excess risk premium. I provide another source of excess risk premium for each firm under from the uncertain information transfer perspective, even though the quality of the news is certain. The model also implies that the higher uncertain on the information transfer the higher the excess risk premium for each firm.

Proposition 8 (2) reports the effect of the news on the conditional risk premium in each firm. Intuitively, the better the news the higher the price, and thus the smaller the conditional risk premium. The second part of Proposition 8 justifies this intuition for the announcing firm always, and for the non-announcing firm largely.

1.4.3 Asymmetric effects to asset returns

In this subsection I explain the asymmetric properties of asset return. For simplicity I assume that both the expected value of the asset payoffs are reasonable large such that

$$\frac{d_1}{\gamma} > \sigma_1^2(1-\phi)\overline{\theta}_1 + \rho_b\sigma_1\sigma_2(1-\phi)\overline{\theta}_2; \frac{d_1}{\gamma} > \rho_b\sigma_1^2\overline{\theta}_2, \tag{24}$$

$$\frac{d_2}{\gamma} > \rho\sigma_1\sigma_2(1-\phi)\overline{\theta}_1 + \sigma_2^2(1-\rho^2\phi)\overline{\theta}_2, \rho \in \{\rho_a, \rho_b\}; \frac{d_2}{\gamma} > \rho_b\sigma_2^2\overline{\theta}_2. \tag{25}$$

Assumptions (24) - (25) are minor conditions which ensure positive asset prices in equilibrium at the absence of firm-specific news.

Proposition 9 *1. The conditional correlation, corr* $\left(\tilde{R}_1, \tilde{R}_2|\tilde{s}=s\right)$ *is* <u>*decreasing*</u> *with respect to the news* s *except for the region* $min(s_1, s_2) < s < max(s_1, s_2)$*, where* s_1 *and* s_2 *be the unique solution of* $p_1(s_1) = 0$ *and* $p_2(s_2) = 0$.

2. *The conditional volatility* $Var(\tilde{R}_1|\tilde{s}=s)$ *is decreasing for* $s \geq s_1$.

3. *The conditional volatility* $Var(\tilde{R}_2|\tilde{s}=s)$ *is decreasing for* $s \geq s_2$ *except for the region* $s^L \leq s \leq s^H$.

4. *The conditional covariance* $Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s}=s)$ *is decreasing with respect to* s *for all* $s \geq max(s_1, s_2)$.

Proposition 9 demonstrates a robust asymmetric pattern of asset correlation and it follows from the deceasing correlation principle of the endogenous correlation coefficient. The correlation of asset returns is larger under a bad news than a good news. Therefore, assets are more likely to comove under very bad firm-specific news. Specifically, the conditional correlation between asset returns is

$$corr\left(\tilde{R}_1, \tilde{R}_2 | \tilde{s} = s\right) = corr\left(\tilde{d}_1, \tilde{d}_2 | \tilde{s} = s\right) sign(p_1(s)p_2(s)). \tag{26}$$

By assumption (24) - (25), we have $p_1(s^L) > 0, p_2(s^L) > 0$, so $s_1, s_2 < s^L$. The product of these two asset prices is always positive except for the region $min(s_1, s_2) \leq s \leq max(s_1, s_2)$.

Similar to asymmetric correlation discussed above, Proposition 9 (2)-(3) present a robust asymmetric stock volatility pattern conditional on the firm-specific news. For the announcing firm, its conditional stock volatility conditional on the bad news is always higher than the good news. Note that the region $s \geq s_1$ includes all signals which lead to positive stock prices, thus the conditional volatility is always decreasing as long as the stock price is positive.

For the non-announcing firm, the model also implies the asymmetric property of its stock volatility. The information transfer channel also affects the stock volatility in addition to the stock price and its return. Proposition 9 (3) states that the conditional volatility is decreasing with respect to the news, except for a small hump due to the price inertia feature under the moderate news. Therefore, the better the news the smaller the non-announcing form's stock volatility, vice versa

Likewise, the model predicts an asymmetric pattern for the covariance. Proposition 9 (4) shows that the covariance under bad signals is always higher than under good ones. The asymmetric property of the covariance is largely consistent with the asymmetric property of the correlation and volatility.

The model predictions about the conditional correlation, conditional covariance and volatility are summarized below.

Model Prediction VI.

1. *The better the news, the* <u>*smaller*</u> *the conditional correlation of stock returns for almost all types of news.*

2. *The better the news, the* <u>*smaller*</u> *the conditional covariance of the stock returns.*

3. *For the announcing firm, the better the news, the* <u>*smaller*</u> *the conditional volatility of the stock return.*

4. *For the non-announcing firm, the better the news, the* <u>*smaller*</u> *the conditional volatility of the stock return, except for a particular range of the news.*

Kroner and Ng (1993) investigate the conditional covariance between a large-firm and a small-firm time series. By calibrating a M-GARCH model, the authors find the asymmetric pattern of the conditional covariance conditional on information including firm-specific news. Brooks and Del Negro (2006) document the asymmetric pattern of the conditional correlation between international stocks. See also Conrad et al (1991), Campbell and Hanschel (1992).

Since the model does not specific the characteristics of the firms, to some extent these two risky assets can be also used to represent equity portfolios or industry-sectors, and the industry-specific news in one industry can be transferred into another industry. In this way, my model predictions include the asymmetric pattern of the conditional condition/covariance between portfolios. Indeed, Ang and Chen (2002) document the asymmetric property of conditional correlation between US portfolios, Hong, Tu and Zhou (2007) find the asymmetric property of conditional covariance and betas for US portfolios, Bakaert and Wu (2000) also find the asymmetric conditional variance between Japanese portfolios. Even though these authors do not compute the conditional statistics based on the specific news as I proposed in the model, I argue that these conditional events used in calculation are related to some industry news, and good (bad) industry news are associated with high (low) asset return. Therefore, Proposition 9 are also supported by these empirical findings at the portfolio level. Other relevant empirical studies are presented in Table 1.

1.4.4 Measurements of Asymmetric Patterns

In this subsection I investigate further about the asymmetric pattern of financial time series. Inspired by the previous studies, such as Longin and Solnik (2001), Ang and Chen (2002), and Ang and Bekaert (2002), I also use the exceedance level, for all $c \geq c_0$,[20] to measure the asymmetric pattern of conditional covariance and conditional volatility, where

$$c_0 = max\left\{\gamma\sigma_1\left(\sigma_1\overline{\theta}_1\frac{1-\phi}{\phi} - \rho_a\sigma_2\overline{\theta}_2\right), \gamma\sigma_1\left(\rho_b\sigma_2\overline{\theta}_2 - \sigma_1\overline{\theta}_1\frac{1-\phi}{\phi}\right), 0\right\}.$$

Proposition 10 *Assume that* $\rho_a + \rho_b \geq \frac{1-\phi}{\phi}\frac{\sigma_2\overline{\theta}_2}{\sigma_1\overline{\theta}_1}$. c^* *denotes a specific number that is greater than* c_0 *given in Appendix B. Then,*

[20] The results hold for any positive exceedance level c with relatively involved technical arguments. The proof for any $c \geq c_0$ is simpler but the main insights of the model are preserved.

1. $Var(\tilde{R}_1|\tilde{s} \geq \overline{d}_1 + c) < Var(\tilde{R}_1|\tilde{s} \leq \overline{d}_1 - c), \forall c \geq c^*$

2. $Var(\tilde{R}_2|\tilde{s} \geq \overline{d}_1 + c) > Var(\tilde{R}_2|\tilde{s} \leq \overline{d}_1 - c), \forall c \geq c^*$.

3. $Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = \overline{d}_1 + y) < Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = \overline{d}_1 - y), \forall y \geq c^*$.

So far we consider the information transfer effect for positive economic link. I want to point out that in some particular situations the economic link can be negative in the sense that the plausible correlation coefficients between asset payoffs are negative. For instance, if two firms are competitors in the same industry, good news for one firm may indicate bad news for another. It is also well-documented that gold as well as bond market is often negatively correlated with the equity market. Then, it is also interesting to consider the negative economic link in my model.

To finish my discussion in this section, I briefly explain the main results of information transfer under uncertainty for negative economic link. Both the optimal portfolio and the characterization of the equilibrium are given the same as in Proposition 1- 8. My model implications largely hold and can be easily modified to reflect the negative correlated environment.

As an illustration, I present one result on the asymmetric patterns in a negatively correlated economic link situation. For simplicity, I assume that both firms contribute comparable risks to the market in the sense that

$$\frac{1 - \rho_a^2 \phi}{|\rho_a|} < \frac{\sigma_2 \overline{\theta}_2}{\sigma_1 \overline{\theta}_1} \leq \frac{2}{|\rho_a + \rho_b|}. \tag{27}$$

Proposition 11 *Consider the information transfer in a negative correlated situation, and (27). c^* is a specific positive constant such that $c^* \geq c_0$.*

1. For the first asset, $\mathbb{E}[\tilde{R}_1|\tilde{s} > \overline{d}_1 + c] > \mathbb{E}[\tilde{R}_1|\tilde{s} < \overline{d}_1 - c]$*, but for the second asset,* $\mathbb{E}[\tilde{R}_2|\tilde{s} > \overline{d}_1 + c] < \mathbb{E}[\tilde{R}_2|\tilde{s} < \overline{d}_1 - c], \forall c \geq c^*$*.*

2. $Var(\tilde{R}_i|\tilde{s} > \overline{d}_1 + c) < Var(\tilde{R}_i|\tilde{s} < \overline{d}_1 - c), i = 1, 2, \forall c \geq c^*$*.*

3. $Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} > \overline{d}_1 + c) > Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} > \overline{d}_1 - c), \forall c \geq c^*$ *if and only if the following condition holds:*

$$\rho_b(\overline{d}_2 - \alpha_b) + \rho_a(\overline{d}_2 - \beta_b) + \frac{\sigma_2}{\sigma_1}\rho_a\rho_b(2\overline{d}_1 - \alpha_a - \alpha_b) > 0. \tag{28}$$

where $\{\alpha_a, \alpha_b, \beta_a, \beta_b\}$ *are defined in Appendix B.*

Proposition 11 presents the asymmetric pattern of the expected return, the stock volatility and the return covariance. As regard to the expected return, the right tail is heavier than the left tail. This asymmetric feature is intuitive because $\tilde{s}$ coveys direct information about the first risky asset, thus good news always leads to a higher expected return. Since the second asset is negatively correlated with the first asset, it will display the opposite pattern.

For the volatility of returns, the left tail is always heavier than the right tail on the first risky asset, which is consistent with what have been documented empirically in literature. It is interesting to examine the volatility of the second risky asset in the negatively correlated economy. In contrast to Proposition 10, the model shows that the volatility on the right tail for the second risky asset is heavier than the left tail under certain condition. Together by Proposition 10 and Proposition 11, the volatility pattern of the second risky asset is complicated, resulting from the price inertia. Due to the same reason, the asymmetric pattern of the covariance is also complicated. I find that the covariance on the right tail is not necessarily smaller than the covariance on the left tail, under certain economic condition such as Equation (28). The intuition is as follows. Under the correlation uncertainty, the second risky asset price overreacts to the bad news; and the overreaction on the second risky asset is so high that it yields a higher right tail covariance in a negatively correlated environment.

After comparing all the conditional asymmetric measures, I present the last prediction below.

Model Prediction V.

The asymmetric pattern of conditional variance, conditional covariance and conditional correlation is pronounced facing a higher degree of uncertainty.

My model predictions provide theoretical grounding for Williams (2015), which empirically examines the role of news to macro-uncertainty in shaping the responses of stock market participants to firm-specific earnings news. The investors' uncertainty increases under stronger macro-uncertainty environment, thus deeply affects their behaviors facing good and bad firm-specific news. Williams (2015) documents that the asymmetric effects are more pronounced for firms whose prior returns are more correlated with macro-uncertainty.

1.5 Conclusion

In this paper, I develop a theory of information transfer under uncertainty framework, to study how the stock prices respond to relevant firm news in an equilibrium. Assuming the investor are averse to the uncertainty about news impact across firms, my model suggests that the level of uncertainty contributes to the stock price comovement, and the information transfer effect is significant. The non-announcing firm's stock price reacts more strongly to bad news than good news, and shows a lack of reaction when the news is not strong enough. Moreover, the non-announcing firm's stock price movement underreacts when (1) the quality of the news about the announcing firm is good, or (2) the announcing firm's size is significantly smaller than the non-announcing firm's size, or (3) the non-announcing firm is very risky, or (4) the economic link is relatively strong. The non-announcing firm's stock price movement overreacts otherwise. The model offers several testable predictions about stock price momentum and reversal for individual firm's price as well as the stock market. The model provides alternative explanation on the stock market anomalies from the correlation uncertainty perspective.

My model also explains the persistent asymmetric pattern of conditional correlation and covariance between firms or equity portfolios through the transfer of firm-specific news or industry news. Specifically, the conditional correlation and conditional variance are larger under bad news than good news. This paper also presents similar asymmetric pattern of the conditional volatility or conditional stock return. Furthermore, a larger uncertainty about the information transfer leads to a more pronounced asymmetric pattern of the financial time series.

The analysis in this paper demonstrates that the information transfer under uncertainty has a significant impact on the stock prices and stock price movements, which enable us to understand stock momentum and reversal at both the firm-level and the market-level. The information transfer under uncertainty also has a substantial effect on the correlation and covariance structure of stock returns, thus further helps in understanding the excess comovement. Both the stock return and stock volatility are influenced by the information transfer channel. Further empirical tests for my model predictions are left for future work.

Appendix A: Equilibrium

To prove Proposition 1, we first state a simple lemma on the function $\tau(x,y)$ as below.

Lemma 1 *For any $t \in [-1,1]$, $t \leq \tau(x,y)$ if and only if $(xt-y)(yt-x) \geq 0$; $t < \tau(x,y)$ if and only if $(xt-y)(yt-x) > 0$.*

Proof of Equation (9).

Conditional on the news $\tilde{s} = s$, the posterior joint distribution for the random payoffs $\tilde{d}_1, \tilde{d}_2$ is normal. Moreover, the conditional expected payoffs is

$$\mathbb{E}_\rho[\tilde{d}|\tilde{s} = s] = \begin{bmatrix} \overline{d}_1 + \phi(s - \overline{d}_1) \\ \overline{d}_2 + z_\rho\phi(s - \overline{d}_1) \end{bmatrix}, \tag{A-1}$$

and the conditional covariance matrix is given by (12). It is easy to obtain

$$\begin{aligned} CE(\rho,\theta) &= W_0 + \left(\overline{d}_1 + \phi(s - \overline{d}_1) - p_1\right)\theta_1 + \left[\overline{d}_2 + \phi z(s - \overline{d}_1) - p_2\right]\theta_2 \\ &\quad -\frac{\gamma}{2}\sigma_1^2\theta_1^2(1-\phi) - \frac{\gamma}{2}\sigma_2^2\theta_2^2(1-\rho^2\phi) - \gamma\rho\sigma_1\sigma_2\theta_1\theta_2(1-\phi). \end{aligned}$$

Without loss of generality we assume that $W_0 = 0$ in the proofs below. When $\theta_2 = 0$, $CE(\rho,\theta)$ is clearly independent of ρ. The maximum certainty equivalent $CE(\theta)$ among $\theta_2 = 0$ is

$$\max_{\theta_2=0} CE(\theta) = \frac{1}{2\gamma}\frac{\left(\overline{d}_1 + \phi(s - \overline{d}_1) - p_1\right)^2}{\sigma_1^2(1-\phi)}. \tag{A-2}$$

If $\theta_2 \neq 0$, as a function of ρ, $CE(\rho,\theta)$ is a quadratic and convex function with a global minimal value at $\hat{\rho}(s;\theta)$, where

$$\hat{\rho}(s;\theta) = \frac{\sigma_1}{\sigma_2}\frac{1-\phi}{\phi}\frac{\theta_1}{\theta_2} - \frac{1}{\gamma\theta_2}\frac{s-\overline{d}_1}{\sigma_1\sigma_2}. \tag{A-3}$$

Hence $CE(\theta)$ is given by Equation (9). Moreover, the maximin value of B is reduced to be the maximum of the following four values

$$B = \max\{B_0, B_1, B_2, B_3\} \tag{A-4}$$

where $B_0 \equiv \frac{1}{2\gamma}\frac{\left(\overline{d}_1+\phi(s-\overline{d}_1)-p_1\right)^2}{\sigma_1^2(1-\phi)}$, $B_1 \equiv \max_{\hat{\rho}(s;\theta)<\rho_a,\theta_2\neq 0} CE(\rho_a,\theta)$, $B_2 \equiv \max_{\hat{\rho}(s;\theta)>\rho_b,\theta_2\neq 0} CE(\rho_b,\theta)$, and $B_3 \equiv \max_{\rho_a\leq\hat{\rho}(s;\theta)\leq\rho_b,\theta_2\neq 0} CE(\hat{\rho}(s;\theta),\theta)$.

$\square$

Proof of Proposition 1.

The proof is divided into several steps.

Step 1. We apply a dual approach to the optimal portfolio choice problem.

By direct computation,

$$\frac{\partial CE(\rho,\theta)}{\partial\rho} = \phi\frac{\sigma_2}{\sigma_1}(s-\overline{d}_1)\theta_2 + \gamma\sigma_2^2\theta_2^2\rho\phi - \gamma\sigma_1\sigma_2\theta_1\theta_2(1-\phi),$$

and

$$\frac{\partial^2 CE(\rho,\theta)}{\partial\rho^2} = \gamma\sigma_2^2\theta_2^2\phi > 0.$$

Then, $CE(\rho,\theta)$ is quasi-convex with respect to ρ for each demand vector θ.

On the other hand, given a ρ, the Hessian matrix of $CE(\rho,\theta)$ with respect to θ is

$$H \equiv \begin{bmatrix} -\gamma\sigma_1^2(1-\phi) & -\gamma\rho\sigma_1\sigma_2(1-\phi) \\ -\gamma\sigma_1\sigma_2(1-\phi) & -\gamma\sigma_2^2(1-\rho^2\phi) \end{bmatrix}$$

For any $x = (x_1, x_2) \in \mathbb{R}^2$,

$$xHx' = -\gamma \left\{ \sigma_1^2(1-\phi)x_1^2 + 2\rho\sigma_1\sigma_2(1-\phi)x_1x_2 + \sigma_2^2(1-\rho^2\phi)x_2^2 \right\} < 0$$

because the determinant is

$$4\rho^2\sigma_1^2\sigma_2^2(1-\phi)^2 - 4\sigma_1^2\sigma_2^2(1-\phi)(1-\rho^2\phi) = 4\sigma_1^2\sigma_2^2(1-\phi)(\rho^2-1) < 0.$$

Therefore, H is negative definite; thus, $CE(\theta, \rho)$ is quasi-concave with respect to θ for each ρ. Hence, we are readily to apply the Sion's minimax theorem, yielding

$$B = C \equiv \min_{\rho} \max_{\theta} \left[\mathbb{E}_\rho(W_1|\tilde{s} = s) - \frac{\gamma}{2} Var_\rho(W_1|\tilde{s} = s) \right] \tag{A-5}$$

Moreover,

$$\max_{\theta} \left[\mathbb{E}_\rho(W_1|\tilde{s} = s) - \frac{\gamma}{2} Var_\rho(W_1|\tilde{s} = s) \right] = \frac{1}{2\gamma} b'\Sigma_\rho^{-1} b$$

where b is $\mathbb{E}_\rho[\tilde{d}|\tilde{s} = s] - p$ in Equation (A-1) and Σ_ρ is the conditional covariance matrix stated in Equation (12).

Step 2. Derive the value B.

Let $G(\rho) \equiv b'\Sigma_\rho^{-1}b$ and $\rho^* \equiv argmix_{\rho\in[\rho_a,\rho_b]} G(\rho)$. Then $B = \frac{1}{2\gamma} \min_\rho G(\rho) = \frac{1}{2\gamma} G(\rho^*)$. We derive B explicitly and show that $\mathcal{L}(\rho_a, \rho_b; \tau(S_1, S_2)) = argmix_{\rho\in[\rho_a,\rho_b]} G(\rho)$.

By direct calculation, we obtain

$$G(\rho) = \frac{A_0 + A_1\rho + A_2\rho^2}{1-\rho^2} \tag{A-6}$$

where

$$A_0 = \sigma_2^2 \left(\overline{d}_1 - p_1 + \phi(s - \overline{d}_1)\right)^2 + \sigma_1^2(1-\phi)\left(\overline{d}_2 - p_2\right)^2,$$

$$A_1 \quad = \quad -2\sigma_1\sigma_2(1-\phi)\left(\overline{d}_2 - p_2\right)\left(\overline{d}_1 - p_1\right) = -2(1-\phi)\sigma_1^2\sigma_2^2 S_1 S_2,$$

and

$$A_2 = -\left\{\sigma_2^2\phi^2(s-\overline{d}_1)^2 + \phi\sigma_2^2\left(\overline{d}_1 - p_1\right)^2 + 2\phi\sigma_2^2\left(s-\overline{d}_1\right)\left(\overline{d}_1 - p_1\right)\right\}.$$

It follows that

$$A_0 + A_2 = (1-\phi)\{\sigma_1^2(\overline{d}_2 - p_2)^2 + \sigma_2^2(\overline{d}_1 - p_1)^2\} = (1-\phi)\sigma_1^2\sigma_2^2(S_1^2 + S_2^2) \geq 0. \quad \text{(A-7)}$$

Moreover,

$$A_0 + A_2 \geq |A_1|. \quad \text{(A-8)}$$

By simple calculation, we obtain

$$G'(\rho) = \frac{A_1 + 2\rho(A_0 + A_2) + \rho^2 A_1}{(1-\rho)^2} \quad \text{(A-9)}$$

Let $\Delta \equiv 4(A_0 + A_2)^2 - 4A_1^2$ and $\Delta \geq 0$ by virtue of Equation (A-8). If $S_1 = S_2 = 0$, we see that $G(\rho) = A_0 = \sigma_2^2\phi^2(s - \overline{d}_1)^2$ for all $\rho \in [\rho_a, \rho_b]$ and

$$B = \frac{1}{2\gamma}\sigma_2^2\phi^2(s-\overline{d}_1)^2, \text{ if } S_1 = S_2 = 0. \quad \text{(A-10)}$$

We consider four different cases.

Case 1. $\Delta = 0$ *and* $|A_1| > 0$.

When $\Delta = 0$, then $|S_1| = |S_2|$. If $A_1 > 0$, that is, $S_1S_2 < 0$, then $G'(\rho) > 0$ always, thus $\rho^* = \rho_a$. Since $\tau(S_1, S_2) = -1$ in this case, $\mathcal{L}(\rho_a, \rho_b; \tau(S_1, S_2)) = \rho_a$. If $A_1 < 0$, or equivalently, $S_1S_2 > 0$, then $G'(\rho) < 0$ always, then $\rho^* = \rho_b$. Furthermore, in this case $\tau(S_1, S_2) = 1$. Hence $\mathcal{L}(\rho_a, \rho_b; \tau(S_1, S_2)) = \rho_b$.

Case 2. $A_1 = 0$.

$A_1 = 0$ if and only if $S_1S_2 = 0$. Hence $\tau(S_1, S_2) = 0$. Since $A_1 = 0$, then

$$G'(\rho) = \frac{2(A_0 + A_2)\rho}{(1-\rho)^2}$$

Recall that $A_0 + A_2 \geq 0$, and $A_0 + A_2 = 0$ if and only if $S_1 = S_2 = 0$. By assumption, either $S_1 \neq 0$ or $S_2 \neq 0$, thus $A_0 + A_2 > 0$. Then $G(\rho)$ increases when $\rho \geq 0$; decreases when $\rho \leq 0$. Hence $\rho^* = \mathcal{L}(\rho_a, \rho_b; 0)$. Then we have proved that when $A_1 = 0, \Delta > 0$, $\rho^* = \mathcal{L}(\rho_a, \rho_b; \tau(S_1, S_2))$.

From now on we assume that $\Delta > 0$ and $A_1 \neq 0$. Let

$$\kappa = \frac{A_0 + A_2}{A_1} = -\frac{S_1^2 + S_2^2}{2S_1S_2},. \tag{A-11}$$

and by (A-8,)$|\kappa| > 1$. Let $\alpha = -\kappa - \sqrt{\kappa^2 - 1}, \beta = -\kappa + \sqrt{\kappa^2 - 1}$. Then

$$G'(\rho) = \frac{A_1}{(1-\rho)^2}(\rho - \alpha)(\rho - \beta). \tag{A-12}$$

Case 3. $A_1 > 0$.

In this case, $S_1S_2 < 0$. Since $\kappa > 0$ we have $\kappa > 1$. Then $\alpha = -\kappa - \sqrt{\kappa^2 - 1} < -1$ and $-1 < \beta < 1$. By Equation (A-12), $\rho^* = \mathcal{L}(\rho_a, \rho_b; \beta)$. Moreover, we verify that

$$\beta \;=\; -\kappa + \sqrt{\kappa^2 - 1} = \frac{-(S_1^2 + S_2^2)}{-2S_1S_2} + \frac{|S_1^2 - S_2^2|}{-2S_1S_2} = \tau(S_1, S_2).$$

Case 4. $A_1 < 0$.

In this case, $S_1S_2 > 0$. Moreover, $\kappa < 0$ so $\kappa < -1$. We can easily check that $\beta > 1$ and $-1 < \alpha < 1$. Hence, by Equation (A-12), $\mathcal{L}(\rho_a, \rho_b; \alpha) = argmin_\rho G(\rho)$, and

$$\alpha = -\kappa - \sqrt{\kappa^2 - 1} = \frac{S_1^2 + S_2^2}{2S_1S_2} - \frac{|S_1^2 - S_2^2|}{2S_1S_2} = \tau(S_1, S_2).$$

To summarize, we have proved that $\rho^* \equiv argmin_{\rho\in[\rho_a,\rho_b]} G(\rho) = \mathcal{L}\left(\rho_a, \rho_b; \tau(S_1, S_2)\right)$, and the maximum value is

$$B = \frac{1}{2\gamma} G\left(\rho^*\right) = CE\left(\rho^*, \theta(\rho^*)\right). \tag{A-13}$$

Step 3. Given the demand vector $\theta(\rho)$, we investigate $\hat{\rho}(s, \theta(\rho))$.

By using the formula (11), the demand on the second risky asset is

$$\theta(\rho)_2 = \frac{S_2 - \rho S_1}{\gamma(1-\rho^2)\sigma_2}$$

and the demand on the first risky asset is

$$\theta(\rho)_1 = \frac{(S_1 - \rho S_2) + \rho\phi(S_2 - \rho S_1)}{\gamma(1-\rho^2)(1-\phi)\sigma_1} + \frac{1}{\gamma}\frac{\phi}{1-\phi}\frac{s - \overline{d}_1}{\sigma_1^2}.$$

Clearly, $\theta(\rho)_2 = 0$ if and only if $S_2 = \rho s_1$. Moreover, for $S_2 \neq \rho S_1$, we obtain

$$\begin{aligned} \frac{\theta(\rho)_1}{\theta(\rho)_2} &= \frac{\sigma_1\sigma_2^2\{S_1(1-\rho^2\phi) - S_2\rho(1-\phi)\}}{\sigma_1^2\sigma_2(1-\phi)(S_2-\rho S_1)} + \frac{\sigma_2^2\phi(1-\rho^2)}{\sigma_1^2\sigma_2(1-\phi)(S_2-\rho S_1)}(s-\overline{d}_1) \\ &= \frac{\sigma_2}{\sigma_1(1-\phi)}\frac{S_1 - \rho S_2 + \rho\phi(S_2-\rho S_1)}{S_2 - \rho S_1} + \frac{\sigma_2}{\sigma_1^2}\frac{\phi}{1-\phi}\frac{1-\rho^2}{S_2-\rho S_1}(s-\overline{d}_1). \end{aligned}$$

Then

$$\frac{\sigma_1}{\sigma_2}\frac{1-\phi}{\phi}\frac{\theta(\rho)_1}{\theta(\rho)_2} = \rho + \frac{1}{\phi}\frac{S_1-\rho S_2}{S_2-\rho S_1} + \frac{1-\rho^2}{S_2-\rho S_1}\frac{s-\overline{d}_1}{\sigma_1}.$$

Furthermore,

$$\frac{1}{\gamma\sigma_1\sigma_2}\frac{s-\overline{d}_1}{\theta(\rho)_2} = \frac{1-\rho^2}{S_2-\rho S_1}\frac{s-\overline{d}_1}{\sigma_1}. \tag{A-14}$$

Therefore, by the definition of $\hat{\rho}(s,\theta)$, we obtain

$$\hat{\rho}(s;\theta(\rho)) = \rho + \frac{1}{\phi}\frac{S_1-\rho S_2}{S_2-\rho S_1}. \tag{A-15}$$

Step 4. The characterization of the optimal demand.

Assuming first that $\theta(\rho^*)_2 = 0$, then $S_2\rho^* - S_1 = 0$. By assumption, either $S_1 \neq 0$ or $S_2 \neq 0$, we see that $S_2 \neq 0$. Therefore $\rho^* = \frac{S_1}{S_2} \in [\rho_a, \rho_b]$ if $\theta(\rho^*)_2 = 0$ holds. By the proof in Step 3, $B = CE(\rho^*, \theta^*)$. Since $\theta_2^* = 0$, $CE(\rho, \theta^*)$ is independent of ρ. Hence $B = \min_\rho CE(\rho^*, \theta^*)$ and thus $\max_\theta \min_\rho CE(\rho, \theta^*) = CE(\rho, \theta^*)$. Therefore, θ^* is the optimal demanding vector.

We next assume that $S_2\rho^* - S_1 \neq 0$, that is, $\theta_2^* \neq 0$. There are three cases about ρ^* because of the characterization of ρ^* in Step 2.

Case 1. $\rho^* = \tau(S_1, S_2) \in [\rho_a, \rho_b]$.

In this case, $\rho^* = \frac{S_1}{S_2}$ since $\rho^* \neq \frac{S_2}{S_1}$. By Equation (A-15), $\hat{\rho}(s; \theta(\rho^*)) = \rho^*$. Then, we have $B = CE(\rho^*, \theta(\rho^*)) = CE\left(\hat{\rho}(s; \theta(\rho^*)), \theta(\rho^*)\right) = CE\left(\theta(\rho^*)\right)$ by Equation (9). Since $CE\left(\theta(\rho^*)\right) = \max_\theta CE(\theta)$, then $\theta(\rho^*)$ is the optimal demanding vector.

Case 2. $\rho^* = \rho_a > \tau(S_1, S_2)$.

By Lemma 1, $\rho_a > \tau(S_1, S_2)$ is equivalent to $(S_1 - \rho_a S_2)(S_2 - \rho_a S_1) < 0$, thus $\frac{S_1 - \rho_a S_2}{S_2 - \rho_a S_1} < 0$. By Equation (A-15), we have

$$\hat{\rho}(s, \theta(\rho_a)) = \rho_a + \frac{1}{\phi} \frac{S_1 - \rho_a S_2}{S_2 - \rho_a S_1} < \rho_a. \tag{A-16}$$

Then, by Step 1,

$$CE\left(\theta(\rho_a)\right) = CE\left(\rho_a, \theta(\rho_a)\right) = B = \max_\theta CE(\theta). \tag{A-17}$$

Thus, $\theta(\rho_a)$ is the optimal demanding vector.

Case 3. $\rho^* = \rho_b < \tau(S_1, S_2)$.

By Lemma 1, since $\rho_b < \tau(S_1, S_2)$, then $(S_1 - \rho_b S_2)(S_2 - \rho_b S_1) > 0$; thus

$$\frac{S_1 - \rho_b S_2}{S_2 - \rho_b S_1} > 0.$$

By using Equation (A-15) again,

$$\hat{\rho}(s, \theta(\rho_b)) = \rho_b + \frac{1}{\phi}\frac{S_1 - \rho_b S_2}{S_2 - \rho_b S_1} > \rho_b. \tag{A-18}$$

Hence

$$CE\left(\theta(\rho_b)\right) = CE\left(\rho_b, \theta(\rho_b)\right) = B = \max_{\theta} CE(\theta), \tag{A-19}$$

so $\theta(\rho_b)$ is the optimal demanding vector.

The proof of Proposition 1 is finished. □

Proof of Proposition 14.

In equilibrium, the optimal demand $\theta(\rho^*)_2 = \overline{\theta}_2 > 0$. Hence by the characterization of the optimal demand, and assuming the endogenous Sharpe ratios are not equal to zero simultaneously and the endogenous correlation is ρ^* (which is determined in equilibrium below), the asset prices are determined by Equation (11) for $\theta(\rho^*) = \overline{\theta}$, yielding Equation (B-18) and (**??**). Then, the endogenous Sharpe ratio, given the endogenous correlation coefficient ρ^*, is

$$S_1 = T_1(s, \rho^*) \quad \equiv \quad -\frac{\phi}{\sigma_1}(s - \overline{d}_1) + \gamma\left\{\sigma_1(1-\phi)\overline{\theta}_1 + \rho^*\sigma_2(1-\phi)\overline{\theta}_2\right\} \tag{A-20}$$

and

$$S_2 = T_2(s, \rho^*) \quad \equiv \quad -\rho^*\frac{\phi}{\sigma_1}(s - \overline{d}_1) + \gamma\left\{\rho^*\sigma_1(1-\phi)\overline{\theta}_1 + \sigma_2(1-\rho^{*2}\phi)\overline{\theta}_2\right\} \tag{A-21}$$

With this characterization, we first show that one of the (endogenous) Sharpe ratios must be non-zero. Otherwise, both $S_1 = S_2 = 0$ in Equations (A-20) and (A-21) imply that

$$0 \quad = \quad S_2 - \rho^* S_1 = \gamma\sigma_2\overline{\theta}_2[1 - \rho^{*2}],$$

which contradicts to the assumption that $\overline{\theta}_2 > 0$ and $|\rho^*| < 1$. Therefore, we are able to apply Proposition 1 for the following characterization of the endogenous correlation coefficient in the subsequent proof.

To proceed, we define

$$J(s, \rho^*) = \tau\left(T_1(s, \rho^*), T_2(s, \rho^*)\right) \tag{A-22}$$

to represent the dispersion of (endogenous) Sharpe ratios. By the characterization of the worst-cast correlation coefficient in Proposition 1, there are three different situations we investigate in details below. Notice that $J(s, \rho^*)$ depends on the endogenous correlation coefficient.

- If $\rho_b < J(s, \rho^*)$, then $\rho^* = \rho_b$.
- If $\rho_a > J(s, \rho^*)$, then $\rho^* = \rho_a$.
- If $\rho_a \leq J(s, \rho^*) \leq \rho_b$, then $\rho^* = J(s, \rho^*)$.

Case 1. $\rho_b < J(s, \rho^*)$

By Lemma 1, $\rho_b < J(s, \rho^*)$ if and only if

$$(T_1(s, \rho_b)\rho_b - T_2(s, \rho_b)) \times (T_2(s, \rho_b)\rho_b - T_1(s, \rho_b)) > 0. \tag{A-23}$$

By straightforward calculation,

$$T_1(s,\rho_b)\rho_b - T_2(s,\rho_b) = \gamma\sigma_2\overline{\theta}_2(\rho_b^2 - 1) < 0, \tag{A-24}$$

and

$$\frac{T_2(s,\rho_b)\rho_b - T_1(s,\rho_b)}{1-\rho_b^2} = \frac{\phi}{\sigma_1}(s-\overline{d}_1) + \gamma\{\rho_b\sigma_2\overline{\theta}_2\phi - \sigma_1\overline{\theta}_1(1-\phi)\}$$

Then $\rho_b < J(s,\rho^*)$ holds if and only if $T_2(s,\rho_b)\rho_b - T_1(s,\rho_b) < 0$, alternatively, $s < s^L$.

Therefore, for any news $s < s^L$, the endogenous correlation coefficient for the representative investor with correlation uncertainty is the highest plausible correlation ρ_b.

Case 2. $\rho_a > J(s,\rho_a)$

By Lemma 1 again, $\rho_a > J(s,\rho_a)$ holds if and only if

$$(T_1(s,\rho_a)\rho_a - T_2(s,\rho_a)) \times (T_2(s,\rho_a)\rho_a - T_1(s,\rho_a)) < 0. \tag{A-25}$$

Similarly, we have

$$T_2(s,\rho_a) - T_1(s,\rho_a)\rho_a = \gamma\sigma_2\overline{\theta}_2(1-\overline{\rho}_a^2) > 0$$

and

$$\frac{T_1(s,\rho_a) - T_2(s,\rho_a)\rho_a}{1-\rho_a^2} = -\frac{\phi}{\sigma_1}(s-\overline{d}_1) + \gamma\{\sigma_1\overline{\theta}_1(1-\phi) - \rho_a\sigma_2\overline{\theta}_2\phi\}.$$

Therefore, $\rho_a > J(s,\rho_a)$ holds if and only if $s > s^H$. Hence, for any news $s > s^H$, the endogenous correlation coefficient for the representative investor with correlation uncertainty is the smallest plausible correlation ρ_b.

Case 3. $\rho^* = J(s, \rho^*)$.

We examine the fixed point problem of the Equation $y = J(s, y)$ for $y \in (-1, 1)$, in which $J(s, y)$ is defined similarly as in Equation (A-36) by simply replacing ρ^* by the variable y. By Lemma 1, $y = J(s, y)$ holds if and only if

$$(yT_1(s, y) - T_2(s, y)) \times (yT_2(s, y) - T_1(s, y)) = 0. \tag{A-26}$$

By calculation,

$$T_2(s, y) - yT_1(s, y) = \gamma\sigma_2\overline{\theta}_2(1 - y^2) \neq 0, \forall |y| < 1. \tag{A-27}$$

and for $|y| < 1$, we have

$$\frac{T_1(s, y) - yT_2(s, y)}{y^2 - 1} = \frac{\phi}{\sigma_1}(s_1 - \overline{d}_1) - \gamma\sigma_1\theta_1(1 - \phi) + \gamma y\sigma_2\overline{\theta}_2\phi.$$

Therefore, the solution of the Equation $y = J(s, y)$ in the range $(-1, 1)$ is

$$y(s) \equiv \frac{\sigma_1}{\sigma_2}\frac{\overline{\theta}_1}{\overline{\theta}_2}\frac{1 - \phi}{\phi} - \frac{1}{\gamma}\frac{1}{\sigma_1\sigma_2\overline{\theta}_2}\left(s - \overline{d}_1\right). \tag{A-28}$$

Moreover, this solution $y(s) \in [\rho_a, \rho_b]$ if and only

$$\gamma\sigma_1\left(\sigma_1\overline{\theta}_1\frac{1 - \phi}{\phi} - \rho_b\sigma_2\overline{\theta}_2\right) \leq s - \overline{d}_1 \leq \gamma\sigma_1\left(\sigma_1\overline{\theta}_1\frac{1 - \phi}{\phi} - \rho_a\sigma_2\overline{\theta}_2\right). \tag{A-29}$$

That is, $s^L \leq s \leq s^H$.

Therefore, we have proved that for any news $s \notin \mathcal{U} \bigcup \mathcal{V}$, the investor chooses the endogenous correlation coefficient $\rho^* = y(s)$ in Equation (A-28), which is the same as $\hat{\rho}(s; \overline{\theta})$.

To summarize, we have determined $\rho(s)$ as claimed. Since $\rho(s)$ is clearly decreasing with respect to $\tilde{s} = s$, so is the conditional correlation between asset payoffs, $corr(\tilde{R}_1, \tilde{R}_2|\tilde{s} = s)$. Finally, the asset price are derived in Proposition 1 given $\rho(s)$ in equilibrium.

$\square$

Proof of Proposition 4.

(1) For $s \in [s^L, s^H]$, and $\frac{\partial \rho^*}{\partial s} = -\frac{1}{\gamma\sigma_1\sigma_2\overline{\theta}_2}$, it follows from Theorem 14 that

$$\frac{\partial p_1(s)}{\partial s} = \phi - \gamma\sigma_1\sigma_2\overline{\theta}_2(1-\phi)\frac{\partial \rho^*}{\partial s} = 1.$$

It suffices to consider the intermediate region. By Theorem 14, we obtain

$$\begin{aligned} p_2(s) &= \overline{d}_2 - \gamma\sigma_2^2\overline{\theta}_2 \\ &\quad + \rho\frac{\sigma_1}{\sigma_2}\phi(s - \overline{d}_2) + \gamma\sigma_2^2\phi\overline{\rho}^2 - \gamma\rho\sigma_1\sigma_2(1-\phi)\overline{\theta}_1 \\ &= \overline{d}_2 - \gamma\sigma_2^2\overline{\theta}_2 \end{aligned}$$

in which the Equation (13) is used.

(2) By Proposition 14, a direct computation yields

$$\frac{\partial p_1(s)}{\partial \phi} = \begin{cases} s - \overline{d}_1 + \gamma\sigma_1(\sigma_1\overline{\theta}_1 + \rho_b\sigma_2\overline{\theta}_2), \text{ if } s < s^L, \\ \frac{\gamma\sigma_1^2\overline{\theta}_1}{\phi^2}, \text{ if } s^L \leq s \leq s^H, \\ s - \overline{d}_1 + \gamma\sigma_1(\sigma_1\overline{\theta}_1 + \rho_a\sigma_2\overline{\theta}_2), \text{ if } s > s^H. \end{cases} \tag{A-30}$$

and

$$\frac{\partial p_2(s)}{\partial \phi} = \begin{cases} \rho_b \frac{\sigma_2}{\sigma_1}(s - \overline{d}_1) + \gamma \sigma_2 \rho_b (\sigma_1 \overline{\theta}_1 + \rho_b \sigma_2 \overline{\theta}_2), \text{ if } s < s^L, \\ 0, \quad \text{if } s^L \le s \le s^H, \\ \rho_a \frac{\sigma_2}{\sigma_1}(s - \overline{d}_1) + \gamma \sigma_2 \rho_a (\sigma_1 \overline{\theta}_1 + \rho_a \sigma_2 \overline{\theta}_2), \text{ if } s > s^H. \end{cases} \tag{A-31}$$

□

Proof of Proposition 5.

By Proposition 1, and straightforward calculation, we see that

$$J(W_1, s) = u\left(W_1 + \frac{1}{2\gamma}(\mathbb{E}_{\rho^*}[\tilde{d}|\tilde{s} = s] - p(s))' \times \Sigma_{\rho^*}^{-1} \times (\mathbb{E}_{\rho^*}[\tilde{d}|\tilde{s} = s] - p(s))\right) \tag{A-32}$$

where ρ^* is given in Proposition 1. Moreover, the second term in $J(W_1, s)$ is independent of the initial wealth W_1 at time $t = 1$.

The optimal portfolio choice problem at time $t = 0$ can be written as

$$U_0 = \max_D \mathbb{E}\left[u(W_0 + (p(s) - p) \cdot D + \cdots)\right] \tag{A-33}$$

where $\cdots$ represents the second term in $J(W_1, s)$ which depends on s, but independent of D. Therefore, the first-order condition in solving U_0 with respect to D yields

$$\mathbb{E}\left[e^{-\gamma(W_0 + (p(s) - p) \cdot D + \ldots)}(p_i(s) - p_i)\right] = 0, i = 1, 2. \tag{A-34}$$

We make use of Equation (A-34) to derive the equilibrium price at $t = 0$. Since in the representative investor setting, the market demand at time $t = 1$ is $\overline{\theta}$, and by using Proposition 14, we have the conditional asset payoff in equilibrium is

$$\mathbb{E}_{\rho(s)}[\tilde{d}|\tilde{s}=s]-p(s)=\gamma\times\begin{bmatrix}Cov(\tilde{d}_1,\tilde{d})\\ \\ Cov(\tilde{d}_2,\tilde{d})\end{bmatrix}, \tag{A-35}$$

then by direct calculation, we obtain

$$J(W_1,s)=W_1+\frac{\gamma}{2}\overline{\theta}'\Sigma_{\rho(s)}\overline{\theta} \tag{A-36}$$

where $\rho(s)$ is given in Proposition 14.

Since in equilibrium at time $t=0$, the optimal demand $D=\overline{\theta}$, and the investor's initial endowment is θ_i units of asset i for $i=1,2$, then Equation (A-34) implies that

$$p_i=\frac{\mathbb{E}\left[e^{-\gamma(p(s)\cdot\overline{\theta}+\frac{\gamma}{2}\overline{\theta}'\Sigma_{\rho(s)}\overline{\theta})}p_i(s)\right]}{\mathbb{E}\left[e^{-\gamma(p(s)\cdot\overline{\theta}+\frac{\gamma}{2}\overline{\theta}'\Sigma_{\rho(s)}\overline{\theta})}\right]}. \tag{A-37}$$

By straightforward calculation, we have

$$-\frac{1}{\gamma}\frac{\partial Log(m_{0,1})}{\partial s}=\begin{cases}\phi\overline{\theta}_1+\phi\rho_b\frac{\sigma_2}{\sigma_1}\overline{\theta}_2, \text{if } s<s^L\\ \\ \overline{\theta}_1-\frac{1}{\gamma}\frac{\phi}{\sigma_1^2}(s-\overline{d}_1), \text{if } s^L\le s\le s^H\\ \\ \phi\overline{\theta}_1+\phi\rho_a\frac{\sigma_2}{\sigma_1}\overline{\theta}_2, \text{if } s>s^H.\end{cases} \tag{A-38}$$

Therefore, $m_{0,1}$ is decreasing with respect to s. Moreover, the pricing kernel is log-convex with respect to $\tilde{s}=s$.

□

Lemma 2 *Assume Y is an arbitrary random variable, $M,N:\mathbb{R}\to\mathbb{R}$ are two in-*

creasing functions, then $Cov(M(Y), N(Y)) \geq 0$. Moreover, $Cov(M(Y), N(Y)) > 0$ if both $M(\cdot)$ and $N(\cdot)$ are strictly increasing on a subset $B \subseteq \mathbb{R}$ such that $Y^{-1}(B)$ has a positive measure.

Proof. Without loss of generality we assume that $N(x) = x$. Choosing an independent copy Y' of the random variable Y, then because of the increasing property of $M(\cdot)$, we obtain $\mathbb{E}\left[(Y - Y')(M(Y) - M(Y'))\right] \geq 0$. Therefore, $Cov(Y, M(Y)) \geq 0$. Moreover, if $M(\cdot)$ is strictly increasing on $B \subseteq \mathbb{R}$ such that $Y^{-1}(B)$ has a positive measure, we obtain $\mathbb{E}\left[(Y - Y')(M(Y) - M(Y'))\right] > 0$, so $Cov(Y, M(Y)) > 0$.

Proof of Proposition 6.

First notice that $Cov(X, Y) = Cov(X, \mathbb{E}[Y|\mathcal{F}])$ for any random variable $X \in \mathcal{F}, Y \in \mathcal{G}$ and $\mathcal{F} \subseteq \mathcal{G}$. To simplify $a \sim b$ denotes $a = bc$ for a positive number c.

(1). By Proposition 14

$$\begin{aligned} corr\left(\Delta p_{01}, \Delta p_{11}\right) &\sim Cov(\Delta p_{01}, \Delta p_{11}) \sim Cov\left(p_1(s), \tilde{d}_1 - p_1(s)\right) \\ &\sim Cov\left(p_1(s), \mathbb{E}[\tilde{d}_1|\tilde{s} = s] - p_1(s)\right) \\ &\sim Cov\left(p_1(s), Cov_{\rho(s)}(\tilde{d}_1, \tilde{d})\right). \end{aligned}$$

Because of the decreasing correlation principle, $Cov_{\rho(s)}(\tilde{d}_1, \tilde{d})$ is decreasing with respect to s. On the other hand, the asset price $p_1(s)$ is increasing with respect to s (Proposition 4), therefore, Lemma 2 implies that $corr\left(\Delta p_{01}, \Delta p_{11}\right) < 0$. Moreover,

$$corr\left(p_1(s), \tilde{d}_1\right) \sim Cov\left(p_1(s), \tilde{d}_1\right) \sim Cov\left(p_1(s), \mathbb{E}[\tilde{d}_1|\tilde{s} = s]\right).$$

Since both $p_1(s)$ and $\mathbb{E}[\tilde{d}_1|\tilde{s}=s]$ are increasing with respect to s, a positive property of $corr\left(p_1(s), \tilde{d}_1\right)$ follows from Lemma 2.

(2) By straightforward calculation, we have

$$\frac{\partial}{\partial \rho(s)} Cov_{\rho(s)}\left(\tilde{d}_2, \tilde{d}\right) \equiv \begin{cases} 0, & \text{if } s < s^L, \\ \frac{2(s-\overline{d}_1)}{\gamma\sigma_1\sigma_2\overline{\theta}_2} - n, & \text{if } s^L \leq s \leq s^H, \\ 0, & \text{if } s > s^H. \end{cases} \tag{A-39}$$

where $n = \frac{\sigma_1}{\sigma_2}\frac{\overline{\theta}_1}{\overline{\theta}_2}\frac{1-\phi}{\phi}$. By using the decreasing correlation principle again, we have

- If $\rho_a \geq \frac{n}{2}$, then $Cov_{\rho(s)}\left(\tilde{d}_2, \tilde{d}\right)$ is increasing with respect to s.
- If $\rho_b \leq \frac{n}{2}$, then $Cov_{\rho(s)}\left(\tilde{d}_2, \tilde{d}\right)$ is decreasing with respect to s.

I first assume that $\rho_a \geq \frac{1}{2}n$. By a direct calculation, $\rho(s)(s-\overline{d}_1)$ is increasing with respect to s. Then by the same idea in (1), we obtain

$$corr\left(p_2(s) - P_2, \tilde{d}_2 - p_2(s)\right) \;\sim\; Cov\left(p_2(s), \rho(s)(s-\overline{d}_1)\right) > 0,$$

as the asset price is also increasing when s moves in a positive economy. Moreover,

$$corr\left(p_2(s), \tilde{d}_2\right) \sim Cov\left(p_2(s), Cov_{\rho(s)}(\tilde{d}_2, \tilde{d})\right)$$

which is positive by Lemma 2.

Finally, assuming $\rho_b \leq \frac{1}{2}n$, then Lemma 2 yields

$$corr(p_2(s) - P_2, \tilde{d}_2 - p_2(s)) \sim Cov\left(p_2(s), Cov_{\rho(s)}(\tilde{d}_2, \tilde{d})\right) < 0. \tag{A-40}$$

However, the function $\rho(s)(s-\overline{d}_1)$ is decreasing over the region $s^L \leq s \leq s^H$ but increasing otherwise. Therefore, the sign of $Cov\left(p_2(s), \rho(s)(s - \overline{d}_1)\right)$ could be positive or negative depending on model parameters (market situations). $\square$

Proof of Proposition 7.

(3) Notice that $\overline{\theta}_1 p_1(s) + \overline{\theta}_2 p_2(s)$ is the time 1 value of the portfolio. By using the same idea as in (1), the autocorrelation of the price changes of the portfolio has the same sign as the covariance between $\overline{\theta}_1 p_1(s) + \overline{\theta}_2 p_2(s)$ and $Var_{\rho(s)}(\tilde{d})$. By straightforward calculation, the conditional variance $Var_{\rho(s)}(\tilde{d})$ is a positive linear transformation of $2n\rho(s) - \rho(s)^2$, which is decreasing in a range of good news, $s > \overline{d}_1$, and increasing in a range of bad news, $s < \overline{d}_1$.

Assuming $\rho_a \geq n$, then $Var_{\rho(s)}(\tilde{d})$ is increasing with respect to s. Since $\overline{\theta}_1 p_1(s) + \overline{\theta}_2 p_2(s)$ is also an increasing function of the news, Lemma 2 yields the positive autocorrelation as desired. On the other hand, if $\rho_b \leq n$, then $Var_{\rho(s)}(\tilde{d})$ is decreasing with respect to s. We apply Lemma 2 again to obtain the negative autocorrelation.

(4) We consider the cross-autocorrelation when the non-announcing firm price changes first. This cross-autocorrelation has the same sign as the covariance between $p_2(s)$ and the conditional covariance $corr_{\rho(s)}\left(\tilde{d}_1, \tilde{d}\right)$. Since both are increasing with respect to s, this cross-autocorrelation must be positive. The another cross-autocorrelation when the announcing firm price changes first has the same sign as the covariance between $p_1(s)$ and the conditional covariance $Cov_{\rho(s)}\left(\tilde{d}_2, \tilde{d}\right)$, and this conditional covariance is increasing for $\rho_a \geq \frac{n}{2}$ and decreasing for $\rho_b \leq \frac{n}{2}$ as shown above. Then we apply Lemma 2 to finish the proof.

$\square$

Proof of Proposition 8.

The first part is the same as Model Prediction II, which follows from Proposition 14 and Proposition 5.

For the second part, the conditional expected return is

$$\mathbb{E}[\tilde{R}_i|\tilde{s} = s] = \frac{Cov_{\rho(s)}(\tilde{d}_i, \tilde{d})}{p_i(s)}. \tag{A-41}$$

Because each stock price is increasing with respect to s, thus the decreasing principle yields decreasing property of $Cov_{\rho(s)}(\tilde{d}_1, \tilde{d})$. The remaining proof follows from the pattern of $Cov_{\rho(s)}(\tilde{d}_2, \tilde{d})$ as shown in Proposition 7. $\square$

Proof of Proposition 9.

We first prove the property for a positively correlated economy. In this case, it is east to see that $s_1, s_2 < s^L$ under the property of the endogenous asset price. By the definition of the asset return and Equation (12), the conditional correlation coefficient between asset return is

$$corr\left(\tilde{R}_1, \tilde{R}_2|\tilde{s} = s\right) = sgn(\rho_1(s)\rho_2(s))corr\left(\tilde{R}_1, \tilde{R}_2|\tilde{s} = s\right) = sgn(\rho_1(s)\rho_2(s))\frac{\rho(s)\sqrt{1-\phi}}{\sqrt{1-\phi\rho(s)^2}},$$

where $sgn(y)$ represents the sign function. By Proposition 4, $p_1(s) > 0$ if and only $s < s_1$, $p_2(s) < 0$ if and only if $s > s_2$. Therefore, $p_1(s)p_2(s) > 0$ as long as s does not belong to a small region $[\min(s_1, s_2), \max(s_1, s_2)]$.

Next, we consider the negatively correlated economy. By Proposition 4, $p_1(s)$ is increasing, thus $s_1 < s^L$. By Proposition 4 again, $p_2(s)$ is decreasing and $p_2(s^H) = p_2(s^L) > 0$, thus $s^H < s_2$. Then $p_1(s) > 0, p_2(s) > 0$ for $s \in [s_1, s_2]$, thus the conditional correlation of asset returns is decreasing in the range $s_1 < s < s_2$. In the region $s \leq s_1$, the conditional correlation is a constant since $\rho(s) = \rho_a$; in the region

$s \geq s_2$, $\rho(s) = \rho_b$. The proof is completed. The proof follows from the properties of the asset prices in Proposition 4 and the decreasing correlation principle. $\square$

Appendix B: Measuring the Asymmetric Dependence

Before proving Proposition 10, we need a couple of lemmas first.

The first lemma is well known.

Lemma 3 *For any pair of random variables X, Y and a real number c, and $f(y)$ be the density (marginal) distribution of the random Variable Y, we have*

$$\mathbb{E}\left[X|_{Y\leq c}\right] = \frac{\int_{-\infty}^{c} \mathbb{E}\left[X|Y=y\right] f(y)dy}{\int_{-\infty}^{c} f(y)dy}, \tag{B-1}$$

and

$$\mathbb{E}\left[X|_{Y\geq c}\right] = \frac{\int_{c}^{\infty} \mathbb{E}\left[X|Y=y\right] f(y)dy}{\int_{c}^{\infty} f(y)dy}. \tag{B-2}$$

The next lemma, which is interesting in its own right, is about the conditional covariance and conditional variance between two random variables, conditional on one event defined by another random variable.

Lemma 4 *Given a pair of two random variables X_1, X_2, a random variable Y with values in real numbers and a positive number c, let $f(y)$ be the density (marginal) density function of Y, then*

$$Cov(X_1, X_2|_{Y\geq c}) \quad = \quad \frac{1}{2}\frac{\int_{c}^{\infty}\int_{c}^{\infty} h(y,z)f(y)f(z)dydz}{\left(\int_{c}^{\infty} f(y)dy\right)^2},$$

where

$$\begin{aligned} h(y,z) \quad \equiv \quad & Cov(X_1, X_2|Y=y) + Cov(X_1, X_2|Y=z) \\ & + \left(\mathbb{E}[X_1|Y=y] - \mathbb{E}[X_1|Y=z]\right)\left(\mathbb{E}[X_2|Y=y] - \mathbb{E}[X_2|Y=z]\right). \end{aligned}$$

In particular, the conditional variance of X_i conditional on $Y \geq c$

$$\begin{aligned} Var(X_i|_{Y\geq c}) &= \frac{\int_c^\infty Var\left(X_i|Y=y\right)f(y)dy}{\int_c^\infty f(y)dy} \\ &\quad + \frac{1}{2}\frac{\int_c^\infty\int_c^\infty \left(\mathbb{E}[X_i|Y=y]-\mathbb{E}[X_i|Y=z]\right)^2 f(y)f(z)dydz}{\left(\int_c^\infty f(y)dy\right)^2}. \end{aligned}$$

If Y is a symmetric random variable in the sense that $f(y)=f(-y)$, then

$$Cov(X_1,X_2|_{Y\leq -c}) = \frac{1}{2}\frac{\int_c^\infty\int_c^\infty h(-y,-z)f(y)f(z)dydz}{\left(\int_c^\infty f(y)dy\right)^2}$$

The conditional variance of X_i conditional on $Y \leq -c$

$$\begin{aligned} Var(X_i|_{Y\leq -c}) &= \frac{\int_c^\infty Var\left(X_i|Y=-y\right)f(y)dy}{\int_c^\infty f(y)dy} \\ &\quad + \frac{1}{2}\frac{\int_c^\infty\int_c^\infty \left(\mathbb{E}\left[X_i|Y=-y\right]-\mathbb{E}\left[X_i|Y=-z\right]\right)^2 f(y)f(z)dydz}{\left(\int_c^\infty f(y)dy\right)^2}. \end{aligned}$$

Proof:

By using Lemma 3 for $\mathbb{E}\left[X_i|_{Y\geq c}\right]$ and $\mathbb{E}\left[X_1X_2|_{Y\geq c}\right]$, we have

$$\begin{aligned} \mathbb{E}\left[X_1X_2|_{Y\geq c}\right] &= \frac{\int_c^\infty \mathbb{E}\left[X_1X_2|Y=y\right]f(y)dy}{\int_c^\infty f(y)dy} \\ &= \frac{\int_c^\infty \left\{\mathbb{E}\left[X_1|Y=y\right]\mathbb{E}\left[X_2|Y=y\right]f(y)+Cov(X_1,X_2|Y=y)f(y)\right\}dy}{\int_c^\infty f(y)dy} \end{aligned}$$

and

$$\mathbb{E}\left[X_1|_{Y\geq c}\right]\mathbb{E}\left[X_2|_{Y\geq c}\right] = \frac{\int_c^\infty \mathbb{E}\left[X_1|Y=y\right]f(y)dy\int_c^\infty \mathbb{E}\left[X_2|Y=z\right]f(z)dz}{\left(\int_c^\infty f(y)dy\right)^2}.$$

Then

$$Cov\left(X_1,X_2|_{Y\geq c}\right) = \frac{\int_c^\infty Cov(X_1,X_2|Y=y)f(y)dy}{\int_c^\infty f(y)dy} + I$$

where

$$I \equiv \frac{\int_c^\infty \int_c^\infty \mathbb{E}\left[X_1|Y=y\right]\mathbb{E}\left[X_2|Y=y\right]f(y)f(z) - \mathbb{E}\left[X_1|Y=y\right]\mathbb{E}\left[X_2|Y=z\right]f(y)f(z)dydz}{(\int_c^\infty f(y)dy)^2}.$$

In the expression of I above, we interchange the variable between y and z, thus the numerator of I is

$$\begin{aligned}
&\int_c^\infty \int_c^\infty \mathbb{E}\left[X_1|Y=y\right]\left\{\mathbb{E}\left[X_2|Y=y\right] - \mathbb{E}\left[X_2|Y=z\right]\right\} f(y)f(z) \\
&= \int_c^\infty \int_c^\infty \mathbb{E}\left[X_1|Y=z\right]\left\{\mathbb{E}\left[X_2|Y=z\right] - \mathbb{E}\left[X_2|Y=y\right]\right\} f(y)f(z) \\
&= \int_c^\infty \int_c^\infty -\mathbb{E}\left[X_1|Y=z\right]\left\{\mathbb{E}\left[X_2|Y=y\right] - \mathbb{E}\left[X_2|Y=z\right]\right\} f(y)f(z) \\
&= \frac{1}{2}\int_c^\infty \int_c^\infty \{\mathbb{E}\left[X_1|Y=y\right] - \mathbb{E}\left[X_1|Y=z\right]\}\{\mathbb{E}\left[X_2|Y=y\right] - \mathbb{E}\left[X_2|Y=z\right]\}f(y)f(z)dydz
\end{aligned}$$

where the last Equation follows from the average of the integrands on the above two Equations. By the same idea, we have

$$\frac{\int_c^\infty Cov(X_1, X_2|Y=y)f(y)dy}{\int_c^\infty f(y)dy} = \frac{\int_c^\infty \int_c^\infty Cov(X_1, X_2|Y=y)f(y)f(z)dydz}{(\int_c^\infty f(y)dy)^2}$$

and

$$\begin{aligned}
&\int_c^\infty \int_c^\infty Cov(X_1, X_2|Y=y)f(y)f(z)dydz \\
&= \int_c^\infty \int_c^\infty Cov(X_1, X_2|Y=z)f(y)f(z)dydz \\
&= \frac{1}{2}\int_c^\infty \int_c^\infty \left(Cov(X_1, X_2|Y=y) + Cov(X_1, X_2|Y=z)\right) f(y)f(z)dydz.
\end{aligned}$$

We have thus proved the formula for the conditional covariance conditional on $Y \geq c$.

By changing the variable y by $-y$, z by $-z$ in the last Equation, and $f(y) = f(-y), f(z) = f(-z)$, then we obtain the formula of $Cov(X_1, X_2|_{Y\leq -c})$ as required.

□

Lemma 5 *Given three variables X_1, X_2, and Y, if $h(-y,-z) > h(y,z), \forall y, z \geq c$, then $Cov(X_1, X_2|_{Y\geq c}) < Cov(X_1, X_2|_{Y\leq -c})$. Moreover, if for each $y \geq c$, $Cov(X_1, X_2|Y = -y) > Cov(X_1, X_2|Y = y)$, and*

$$(\mathbb{E}[X_i|Y=-y] - \mathbb{E}[X_i|Y=-z])^2 > (\mathbb{E}[X_i|Y=y] - \mathbb{E}[X_i|Y=z])^2,$$

then $Var(X_i|_{Y\geq c}) < Var(X_i|_{Y\leq -c})$ for $i = 1, 2$.

The result holds if all inequality "$<$" is replaced by "$>$".

Proof: This lemma follows from Lemma 4. □

Proof of Proposition 10.

By Lemma 5, it suffices to check several conditions for $X_1 = \tilde{R}_1, X_2 = \tilde{R}_2$ and $Y = \tilde{s} - \overline{d}_1$.

Step 1. We show that $Var(\tilde{R}_1|\tilde{s} = \overline{d}_1 + y) < Var(\tilde{R}_1|\tilde{s} = \overline{d}_1 - y), \forall y \geq c_1$, where c_1 is a positive constant that is greater than c_0.

By Equation (12), it equivalents to show that $p_1(\overline{d}_1 + y)^2 > p_1(\overline{d}_1 - y)^2, \forall y \geq c_1$. By Proposition 14, $y \geq c_0$ implies that $p_1(\overline{d}_1 + y) = \overline{d}_1 + \phi y - \alpha_a, p_1(\overline{d}_1 - y) = \overline{d}_1 - \phi y - \alpha_b$, where

$$\alpha_a \equiv \gamma\sigma_1(1-\phi)\left\{\sigma_1\overline{\theta}_1 + \rho_a\sigma_2\overline{\theta}_2\right\}, \alpha_b \equiv \gamma\sigma_1(1-\phi)\left\{\sigma_1\overline{\theta}_1 + \rho_b\sigma_2\overline{\theta}_2\right\}.$$

By Assumption (24), we have $\overline{d}_1 > \alpha_a, \overline{d}_1 > \alpha_b$. Then it is easy to show that

$p_1(\overline{d}_1 + y)^2 > p_1(\overline{d}_1 - y)^2, \forall y \geq c_1$ where

$$c_1 \equiv \max\left\{c_0, \frac{(\overline{d}_1 - \alpha_b)^2 - (\overline{d}_1 + \alpha_a)^2}{2\phi(2\overline{d}_1 - \alpha_a - \alpha_b)}\right\}.$$

Step 2. We show that $\rho_a^2 p_2(\overline{d}_1 - y)^2 \leq \rho_b^2 p_2(\overline{d}_1 + y)^2$, for all $y \geq c_2$ where c_2 is a positive constant that is greater than c_0.

By Proposition 14, for any $y \geq c_0$, $p_2(\overline{d}_1 + y) = \overline{d}_2 + \rho_a \phi y \frac{\sigma_2}{\sigma_1} - \beta_a, p_2(\overline{d}_1 - y) = \overline{d}_2 - \rho_b \phi y \frac{\sigma_2}{\sigma_1} - \beta_b$, where

$$\beta_a \equiv \gamma\sigma_2\left\{\rho_a\sigma_1(1-\phi)\overline{\theta}_1 + \sigma_2(1-\rho_a^2\phi)\overline{\theta}_2\right\},$$

and

$$\beta_b \equiv \gamma\sigma_2\left\{\rho_b\sigma_1(1-\phi)\overline{\theta}_1 + \sigma_2(1-\rho_b^2\phi)\overline{\theta}_2\right\}.$$

By assumption (25)), $\overline{d}_2 > \beta_a, \overline{d}_2 > \beta_b$, then for all $y \geq c_2$, it is easy to see that $\rho_a^2 p_2(\overline{d}_1 - y)^2 \leq \rho_b^2 p_2(\overline{d}_1 + y)^2$, where

$$c_2 = \max\left\{c_0, \frac{1}{2\phi\rho_a z_b}\frac{\rho_a^2(\overline{d}_2 - \beta_b)^2 - \rho_b^2(\overline{d}_2 - \beta_a)^2}{\rho_a(\overline{d}_2 - \beta_a) + \rho_b(\overline{d}_2 - \beta_b)}\right\}.$$

Step 3. We show that for all $y \geq c_3$, $Cov(\tilde{R}_1, \tilde{R}_2 | \tilde{s} = \overline{d}_1 + y) < Cov(\tilde{R}_1, \tilde{R}_2 | \tilde{s} = \overline{d}_1 - y)$, where c_3 is a specific positive number that is greater than c_0.

By Step 1 and Step 2, $p_1(\overline{d}_1 - y)^2 \leq p_1(\overline{d}_1 + y)^2$ and $\rho_a^2 p_2(\overline{d}_1 - y)^2 \leq \rho_b^2 p_2(\overline{d}_1 + y)^2, \forall y \geq \max(c_1, c_2)$. Then $\rho_a^2 p_1(\overline{d}_1 - y)^2 p_2(\overline{d}_1 - y)^2 \leq \rho_b^2 p_1(\overline{d}_1 + y)^2 p_2(\overline{d}_1 + y)^2$. Let

$$c_3 = \max\left\{c_1, c_2, \frac{\overline{d}_1 - \alpha_b}{\phi}, \frac{\overline{d}_2 - \beta_b}{z_b\phi}\right\}.$$

Then for $y \geq c_3$, $p_1(\overline{d}_1 - y) < 0$ and $p_2(\overline{d}_2 - y) < 0$, thus $p_1(\overline{d}_1 - y)p_2(\overline{d}_2 - y) > 0$.

Therefore,

$$\rho_a p_1(\overline{d}_1 - y)p_2(\overline{d}_1 - y) < \rho_b p_1(\overline{d}_1 + y)p_2(\overline{d}_1 + y),$$

yielding $Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = \overline{d}_1 + y) < Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = \overline{d}_1 - y)$.

Step 4. We investigate the function $\mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + z]$ and show that $Var(\tilde{R}_1|\tilde{s} > d_1 + c) < Var(\tilde{R}_1|\tilde{s} < d_1 - c), \forall c \geq c_3$.

For any $y, z \geq c_3$, a direct computation yields

$$\begin{aligned} \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + z] &= \frac{\overline{d}_1 + \phi y}{p_1(\overline{d}_1 + y)} - \frac{\overline{d}_1 + \phi z}{p_1(\overline{d}_1 + z)} \\ &= \frac{\alpha_a \phi(z - y)}{p_1(\overline{d}_1 + y)p_1(\overline{d}_1 + z)}. \end{aligned}$$

Similarly,

$$\mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 - y] - \mathbb{E}[\tilde{R}_1|\tilde{s} = \tilde{d}_1 - z] = \frac{\alpha_b \phi(y - z)}{p_1(\overline{d}_1 - y)p_1(\overline{d}_1 - z)}$$

By Step 1, we have $p_1(\overline{d}_1 - y)^2 < p_1(\overline{d}_1 + y)^2, p_1(\overline{d}_1 - z)^2 < p_1(\overline{d}_1 + z)^2$, then, by using $\alpha_a < \alpha_b$, for all $y, z \geq c_3$, we have

$$\Big(\mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + z]\Big)^2 < \Big(\mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 - y] - \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 - z]\Big)^2. \tag{B-3}$$

By using the last formula, Step 1, and Lemma 5, we prove

$$Var(\tilde{R}_1|\tilde{s} > d_1 + c) < Var(\tilde{R}_1|\tilde{s} < d_1 - c), \forall c \geq c_3. \tag{B-4}$$

Step 5. We show that $\rho_a p_2(\overline{d}_1 - y)^2 > \rho_b p_2(\overline{d}_1 + y)^2$, and $(1 - \rho_a^2\phi)p_2(\overline{d}_1 - y)^2 > (1 - \rho_b^2\phi)p_2(\overline{d}_1 + y)^2$, for any $y \geq c_4$ where c_4 is a number that is greater than c_0.

In fact, for any $y \geq c_0$, we the express $\rho_a p_2(\overline{d}_1 - y)^2 - \rho_b p_2(\overline{d}_1 + y)^2$ as a quadratic function of y with the leading term being $\rho_a z_b^2 \phi^2 - \rho_b z_a^2 \phi^2 = \rho_a \rho_b \left(\frac{\sigma_2}{\sigma_1}\right)^2 (\rho_b - \rho_a) > 0$. Similarly, the leading term of the quadratic function of $(1 - \rho_a^2 \phi) p_2(\overline{d}_1 - y)^2 - (1 - \rho_b^2 \phi) p_2(\overline{d}_1 + y)^2$, as a function of y, is $(z_b^2 - z_a^2)\phi^2 > 0$. Then there exists a $c_4 \geq c_0$ such that $\rho_a p_2(\overline{d}_1 - y)^2 - \rho_b p_2(\overline{d}_1 + y)^2 > 0$, and $(1 - \rho_a^2 \phi) p_2(\overline{d}_1 - y)^2 - (1 - \rho_b^2 \phi) p_2(\overline{d}_1 + y)^2 > 0$.

Step 6. We investigate the function of $\mathbb{E}[\tilde{R}_2 | \tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_2 | \tilde{s} = \tilde{d}_1 + z]$ and show that $Var(\tilde{R}_2 | \tilde{s}_1 > \overline{d}_1 + c) > Var(\tilde{R}_2 | \tilde{s}_1 > \overline{d}_1 - c), \forall c \geq c_4$.

First, by Step 5, and Equation (12), $Var(\tilde{R}_2 | \tilde{s} = \overline{d}_1 + y) > Var(\tilde{R}_2 | \tilde{s} = \overline{d}_1 - y), \forall y \geq c_4$.

Second, for any $y, z \geq c_3$, by Proposition 14, we obtain

$$\mathbb{E}[\tilde{R}_2 | \tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_2 | \tilde{s} = \tilde{d}_1 + z] = \frac{\beta_a z_a \phi (z - y)}{p_2(\overline{d}_1 + y) p_2(\overline{d}_1 + z)},$$

and

$$\mathbb{E}[\tilde{R}_2 | \tilde{s} = \overline{d}_1 - y] - \mathbb{E}[\tilde{R}_2 | \tilde{s} = \tilde{d}_1 - z] = \frac{\beta_b z_b \phi (y - z)}{p_2(\overline{d}_1 - y) p_2(\overline{d}_2 - z)}.$$

Therefore,

$$\left(\mathbb{E}[\tilde{R}_2 | \tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_2 | \tilde{s} = \tilde{d}_1 + z]\right)^2 = \frac{\beta_a^2 z_a^2 \phi^2 (z - y)^2}{p_2(\overline{d}_1 + y)^2 p_2(\overline{d}_1 + z)^2}$$

and

$$\left(\mathbb{E}[\tilde{R}_2 | \tilde{s} = \overline{d}_1 - y] - \mathbb{E}[\tilde{R}_2 | \tilde{s} = \tilde{d}_1 - z]\right)^2 = \frac{\beta_b^2 z_b^2 \phi^2 (z - y)^2}{p_2(\overline{d}_1 - y)^2 p_2(\overline{d}_1 - z)^2}.$$

By Step 5, $\rho_a p_2(\overline{d}_1 - y)^2 > \rho_b p_2(\overline{d}_1 + y)^2$, and $\rho_a p_2(\overline{d}_1 - z)^2 > \rho_b p_2(\overline{d}_1 + z)^2$, $\forall y, z \geq c_4$, then

$$\rho_a^2 p_2(\overline{d}_1 - y)^2 p_2(\overline{d}_1 - z)^2 > \rho_b^2 p_2(\overline{d}_1 + y)^2 p_2(\overline{d}_1 + z)^2.$$

Since $\rho_a + \rho_b \geq \frac{1-\phi}{\phi}\kappa$, $\beta_a \geq \beta_b$, thus

$$\Big(\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 + y] - \mathbb{E}[\tilde{R}_2|\tilde{s} = \tilde{d}_1 + z]\Big)^2 > \Big(\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 - y] - \mathbb{E}[\tilde{R}_2|\tilde{s} = \tilde{d}_1 - z]\Big)^2.$$

Then by Lemma 5, we have proved that $Var(\tilde{R}_2|\tilde{s}_1 > \overline{d}_1 + c) > Var(\tilde{R}_2|\tilde{s}_1 > \overline{d}_1 - c)$, for any $c \geq c_4$. $\square$

Proof of Proposition 11.

(1) We apply Lemma 3 for the asymmetric expectation of the first risky asset. Under assumption (27), $\alpha_a + \alpha_b > 0$, thus we can prove that $\mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 + y] > \mathbb{E}[\tilde{R}_1|\tilde{s} = \overline{d}_1 - y]$ for any $c \geq c_0$.

For the asymmetric variance of the first risky asset, by the same argument of Step 1 and Step 4 in the proof of Proposition 10, and using $\alpha_a^2 < \alpha_b^2$, we can prove that $Var[\tilde{R}_1|\tilde{s} > \overline{d}_1 + c] < Var[\tilde{R}_1|\tilde{s} > \overline{d}_1 - c]$.

(2). By the condition in this part, we know that $\beta_a > 0, \beta_b > 0$. Since $\rho_a < \rho_b \leq 0$, we can show that $\mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 + y] < \mathbb{E}[\tilde{R}_2|\tilde{s} = \overline{d}_1 - y]$ by a similar method as in Proposition 11 (1). We also note that $\beta_a < \beta_b$ because of $\rho_b \leq 0$. Then $\beta_a^2 < \beta_b^2$.

For the asymmetric variance, we first show that $Var(\tilde{R}_2|\tilde{s} = \overline{d}_1 + y) < Var(\tilde{R}_2|\tilde{s} = \overline{d}_1 - y)$, that is, $(1 - p_a^2\phi)(p_2(\overline{d}_1 - y)^2 < (1 - p_b^2\phi)(p_2(\overline{d}_1 + y)^2$. Since $\rho_b^2 < \rho_a^2$ in the negatively correlated environment, we see that the above inequality holds for any $y \geq c_1$ which is a specific number that is greater than c_0.

By using the same proof in Proposition 10, and $\beta_a^2 < \beta_b^2$, it suffices to show that

$$\frac{\rho_a^2}{p_2(\overline{d}_1+y)^2 p_2(\overline{d}_1+z)^2} < \frac{\rho_b^2}{p_2(\overline{d}_1-y)^2 p_2(\overline{d}_1-z)^2}.$$

By a similar argument as in the positive environment, we can show that $\rho_a p_2(\overline{d}_1 - y)^2 > \rho_b p_2(\overline{d}_1+y)^2$, and $\rho_a p_2(\overline{d}_1-z)^2 > \rho_b p_2(\overline{d}_1+z)^2$. Since $\rho_a < \rho_b \leq 0$, we obtain

$$\rho_a p_2(\overline{d}_1-y)^2 \rho_a p_2(\overline{d}_1-z)^2 < \rho_b p_2(\overline{d}_1+y)^2 \rho_b p_2(\overline{d}_1+z)^2.$$

Therefore, we have completed the proof of $Var(\tilde{R}_2|\tilde{s} > \overline{d}_1 + c) < Var(\tilde{R}_2|\tilde{s} < \tilde{d}_1 - c)$ for all $c \geq c_2$, c_2 is a specific number that is greater than c_0.

(3). Notice that for large enough y, $p_2(\overline{d}_1+y) < 0, p_2(\overline{d}_1-y) > 0$. Under the condition (28), we can show that

$$\frac{\rho_a}{p_1(\overline{d}_1+y)p_2(\overline{d}_1+y)} < \frac{\rho_b}{p_1(\overline{d}_1-y)p_2(\overline{d}_1-y)}, \forall y \geq c^*.$$

Therefore, $Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = \overline{d}_1+y) < Cov(\tilde{R}_1, \tilde{R}_2|\tilde{s} = \overline{d}_1-y), \forall y \geq c^*$. Moreover, the inequality on the conditional covariance is just opposite if $\frac{\sigma_2}{\sigma_1}\rho_a\rho_b(2\overline{d}_1 - \alpha_a - \alpha_b) + \rho_b(\overline{d}_2 - \alpha_b) + \rho_a(\overline{d}_2 - \beta_b) > 0$.

To proceed, we show that $\alpha_a\beta_a\rho_b < \alpha_b\beta_b\rho_a$ under condition (27). By direct calculation, $\alpha_a\beta_a\rho_b - \alpha_b\beta_b\rho_a$ equals to

$$\kappa(\rho_b - \rho_a)\left\{1 + \rho_a\rho_b(2\phi-1) + \rho_a\rho_b\kappa\phi(\rho_a+\rho_b)\right\}.$$

Since $2 + (\rho_a+\rho_b)\kappa > 0$, $1+\rho_a\rho_b(2\phi-1)+\rho_a\rho_b\kappa\phi(\rho_a+\rho_b)$ is increasing with respect to ϕ, and it equals to $1-\rho_a\rho_b > 0$ for $\phi = 0$, then $1+\rho_a\rho_b(2\phi-1)+\rho_a\rho_b\kappa\phi(\rho_a+\rho_b) > 0, \forall\phi \in (0,1)$. Therefore, $\alpha_a\beta_a\rho_b > \alpha_b\beta_b\rho_a$.

By using the fact that $\alpha_a\beta_a\rho_b < \alpha_b\beta_b\rho_a$, we can show that

$$\sqrt{\alpha_a\beta_a(-\rho_a)}p_1(\bar{d}_1 - y)p_2(\bar{d}_1 - y) > \sqrt{\alpha_b\beta_b(-\beta_b)}p_1(\bar{d}_1 + y)p_2(\bar{d}_1 + y), \forall y \geq c^*.$$

Then for all $y, z \geq c^*$, $\alpha_a\beta_a(-\rho_a)p_1(\bar{d}_1-y)p_2(\bar{d}_1-y)p_1(\bar{d}_1-z)p_2(\bar{d}_1-z)$ is smaller than $\alpha_b\beta_b(-\rho_b)p_1(\bar{d}_1+y)p_2(\bar{d}_1+y)p_1(\bar{d}_1+z)p_2(\bar{d}_1+z)$. It implies that the cross dispersion of the expected returns of risky assets on two good news, y and z, is greater than the cross dispersion of the expected returns of risky assets on two mirror bad news, $-y$ and $-z$. Therefore, Lemma 5 derives the asymmetric covariance as required. $\square$

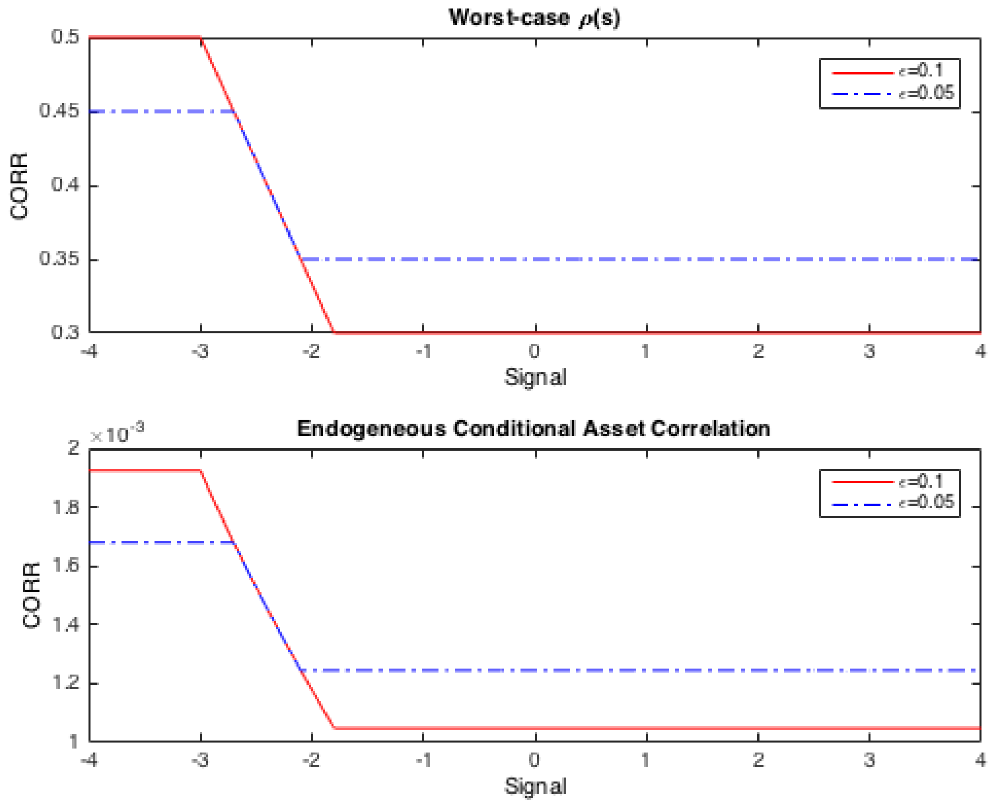

Figure 1.1: Asymmetric correlation in equilibrium

This figure demonstrates the asymmetric correlation between the asset returns in a positively correlated economy. A higher correlation is associated with the bad news. The top panel displays the worst-case correlation coefficient $\rho(s)$ in the equilibrium, and the bottom panel displays the endogenous correlation $corr(\tilde{d}_1, \tilde{d}_2|\tilde{s} = s) = \rho(s)\sqrt{\frac{1-\phi}{1-\rho(s)^2\phi}}$, with respect to the news s. The parameters in this figure are $\rho_a = 0.4 - \epsilon, \rho_b = 0.4 + \epsilon, \epsilon \in \{0.05, 0.1\}$. Other parameters are $\sigma_1 = 3\%$, $\sigma_2 = 2\%$, $\sigma_\epsilon = 1\%$; $\overline{d}_1 = 0$, $\overline{d}_2 = 0$, $\overline{\theta}_1 = 1$, $\overline{\theta}_2 = 1$.

Table 1.1: An Overview of empirical studies

This table lists a sample of studies on the asymmetries of conditional correlation, conditional variance/covariance and beta. These studies typically use regime-switching GARCH and copula models to measure correlation and volatility. Most of the explanations in the literature are statistical. Typically they test pairwise correlation or covariance between U.S. and international market indices or focus on testing the asymmetric reaction to shock at firm level.

Study	Asymmetry Measure	Presence of Asymmetry	Models/Explanations
Ang and Chen (2002)	Conditional correlation	US Portfolios	GARCH-M
Ang and Bekaert (2002)	Conditional correlation	US-UK-Germany	Regime-switching GARCH
Bae, Karolyi, and Stulz (2003)	Counts of extreme return coincidence	International Finance Corporation indices	Multinomial logistic regression approach
Baele, Bekaert and Inghelbrecht (2010)	Correlation	Stock-bond return	Dynamic factor model
Longin and Solnik (1995, 2001)	Conditional correlation	US-UK, France, Germany, Japan Equity Index	Extreme Value Theory
Patton (2006)	Conditional correlation	Mark-Dollar, Yen-Dollar	Copula
Cappiello, Engle and Sheppard (2006)	Conditional correlation	World-wide Equity and Bond Index	DCC-GARCH
Kroner and Ng (1998)	Conditional covariance	A large-firm and a small-firm return series	Multivariant GARCH
Bekaert and Wu (2000)	Conditional covariance	Japanese Portfolios (Nikkei 225)	Volatility feedback effect
Moskowitz (2003)	Conditional covariance	US portfolio	Unspecified
Brooks and Del Negro (2006)	Correlation& Beta	Firm-level international stocks	Unspecified
Christoffersen, Errunza, Jacobs and Langlois (2012)	Correlation & Covariance	Developed and Emerging Market	Dynamic Copula
Hong, Tu and Zhou (2007)	Conditional correlation, Variance and Beta	US Portfolios	Mixed Copula

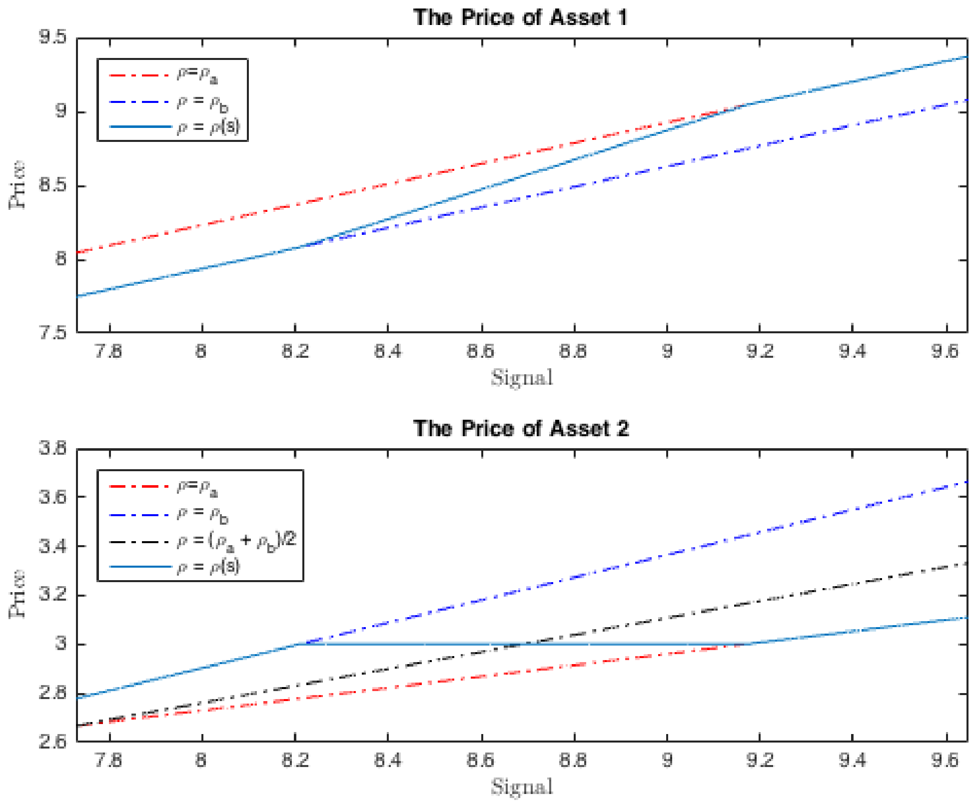

Figure 1.2: The endogenous asset prices in equilibrium
This figure demonstrates how news affects the endogenous asset price in equilibrium. The parameters in this figure are $\rho_a = 0.4$, $\rho_b = 0.8$, $\sigma_1 = 12\%$, $\sigma_2 = 10\%$, $\sigma_\epsilon = 8\%$; $\overline{d}_1 = 10$, $\overline{d}_2 = 2$, $\overline{\theta}_1 = 100$, $\overline{\theta}_2 = 10$, $\gamma = 2$. $s^L = 8.20$, $s^H = 9.17$.

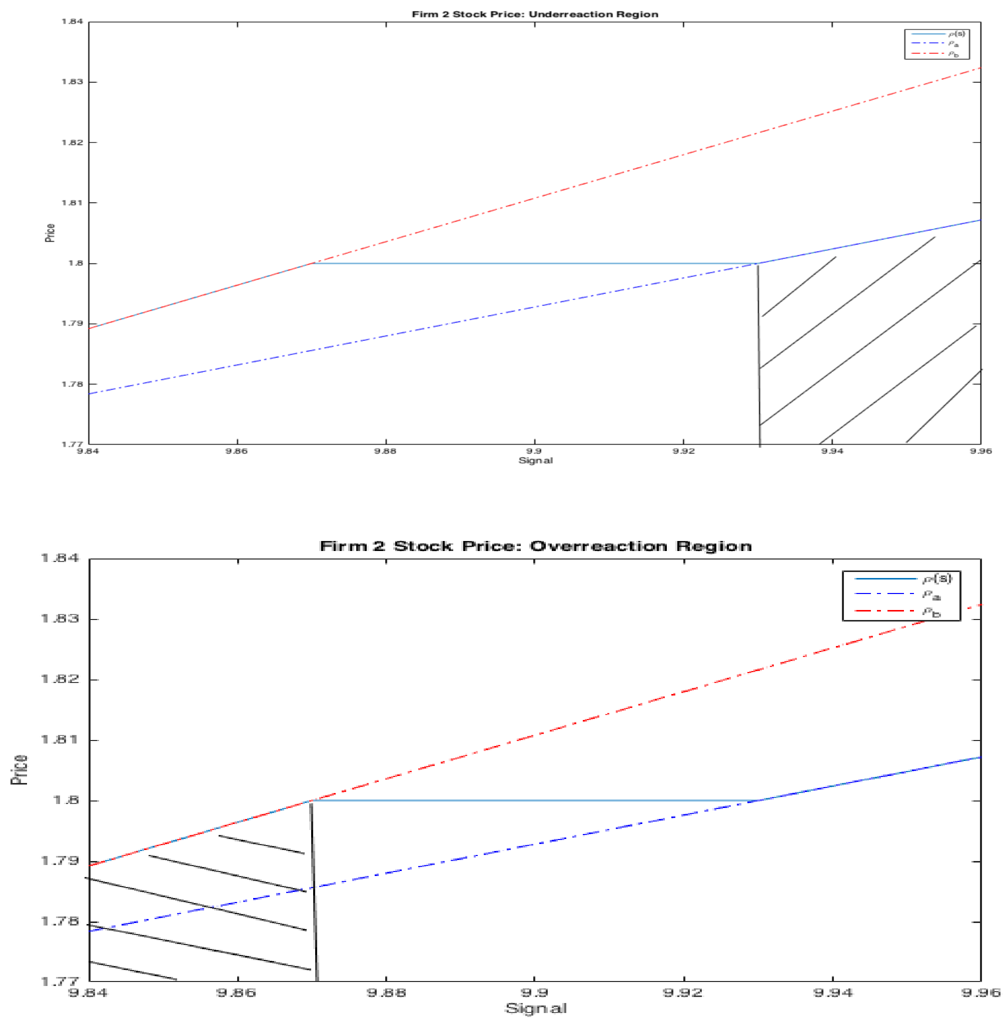

Figure 1.3: Under and Over-reaction of Stock Prices
This figure demonstrates how stock price momentum (underreaction) and reversal (overreaction) is generated under certain conditions in the model. In the top panel, when ρ_a is large enough, that is when s^L is small, it is more likely to enter the shaded area (a lower price sensitivity with respect to ρ_a); therefore, the stock prices underreact on average. The idea is the same for the bottom panel when ρ_b is small enough.

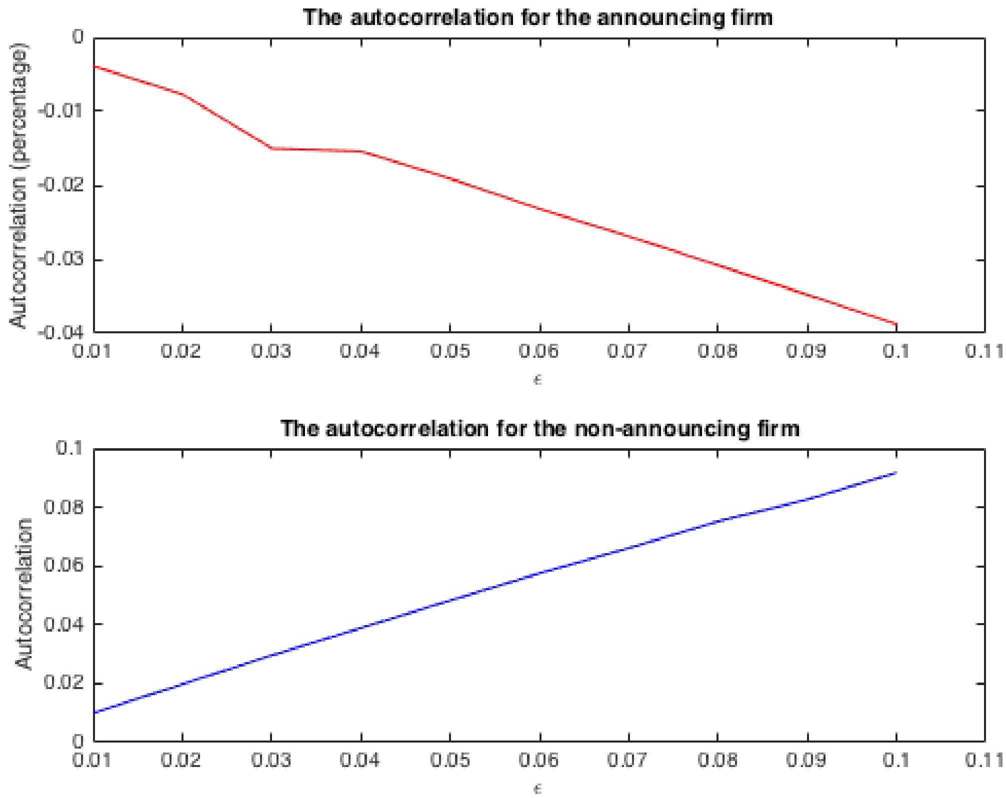

Figure 1.4: Autocorrelation

This figure demonstrates the effect of correlation uncertainty on the price autocorrelation of the two firms. As shown, the higher the correlation uncertainty the more significant the autocorrelation for each firm. The parameters in this figure are $\rho_a = 0.4-\epsilon, \rho_b = 0.4+\epsilon, \epsilon \in \{0.05, 0.1\}$. Other parameters are $\sigma_1 = 3$, $\sigma_2 = 2$, $\sigma_\epsilon = 1\%$; $\bar{d}_1 = 0$, $\bar{d}_2 = 0$, $\bar{\theta}_1 = 1$, $\bar{\theta}_2 = 1$. Notice that in this situation $n = 0.0016\%$, which is extremely small compared with the plausible unconditional correlation coefficient.

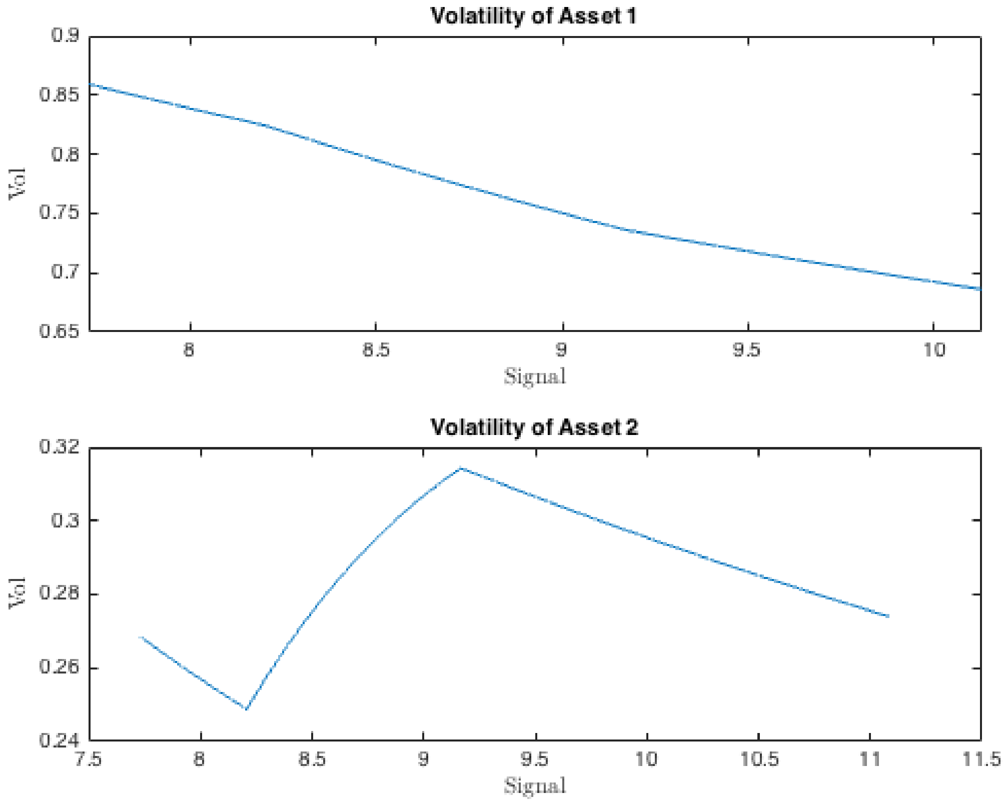

Figure 1.5: The asymmetric volatility in equilibrium
This figure demonstrates the asymmetric volatility of risky assets in equilibrium. The top panel displays the asymmetric volatility of the first risky asset, that is, a higher volatility under bad news than in good news. The bottom panel displays a complicated asymmetric volatility pattern of the second risky asset. The parameters in this figure are $\rho_a = 0.2$, $\rho_b = 0.7$, $\sigma_1 = 25\%$, $\sigma_2 = 10\%$, $\sigma_\epsilon = 5\%$; $\overline{d}_1 = 10$, $\overline{d}_2 = 5$, $\overline{\theta}_1 = 10$, $\overline{\theta}_2 = 100$, $\gamma = 2$. $s^L = 8.20$, $s^H = 9.17$.

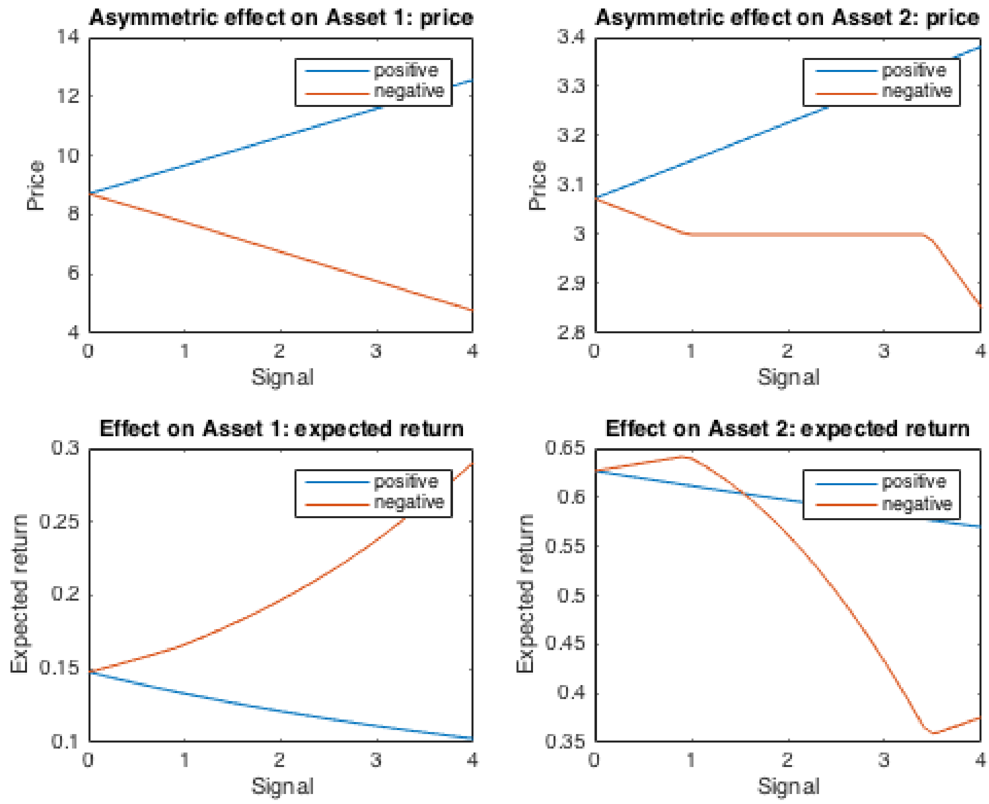

Figure 1.6: Measures of asymmetric patterns of asset prices
This figure displays the comparison between the asset prices and the expected returns conditional on a piece of good news, $y = \tilde{s} - \overline{d}_1 > 0$, and a piece of bad news $-y$ correspondingly. The parameters in this figure are $\rho_a = 0.2$, $\rho_b = 0.7$, $\sigma_1 = 25\%$, $\sigma_2 = 10\%$, $\sigma_\epsilon = 5\%$; $\overline{d}_1 = 10$, $\overline{d}_2 = 5$, $\overline{\theta}_1 = 10$, $\overline{\theta}_2 = 100$, $\gamma = 2$. $s^L = 6.55$, $s^H = 9.05$. In this case $s^L - \overline{d}_1 = -3.45$, $s^H - \overline{d}_1 = -0.95$. I plot graphs along positive signals of $\tilde{s} - \overline{d}_1$, which are greater than -0.95, thus $\rho(s) = \rho_a = 0.2$.

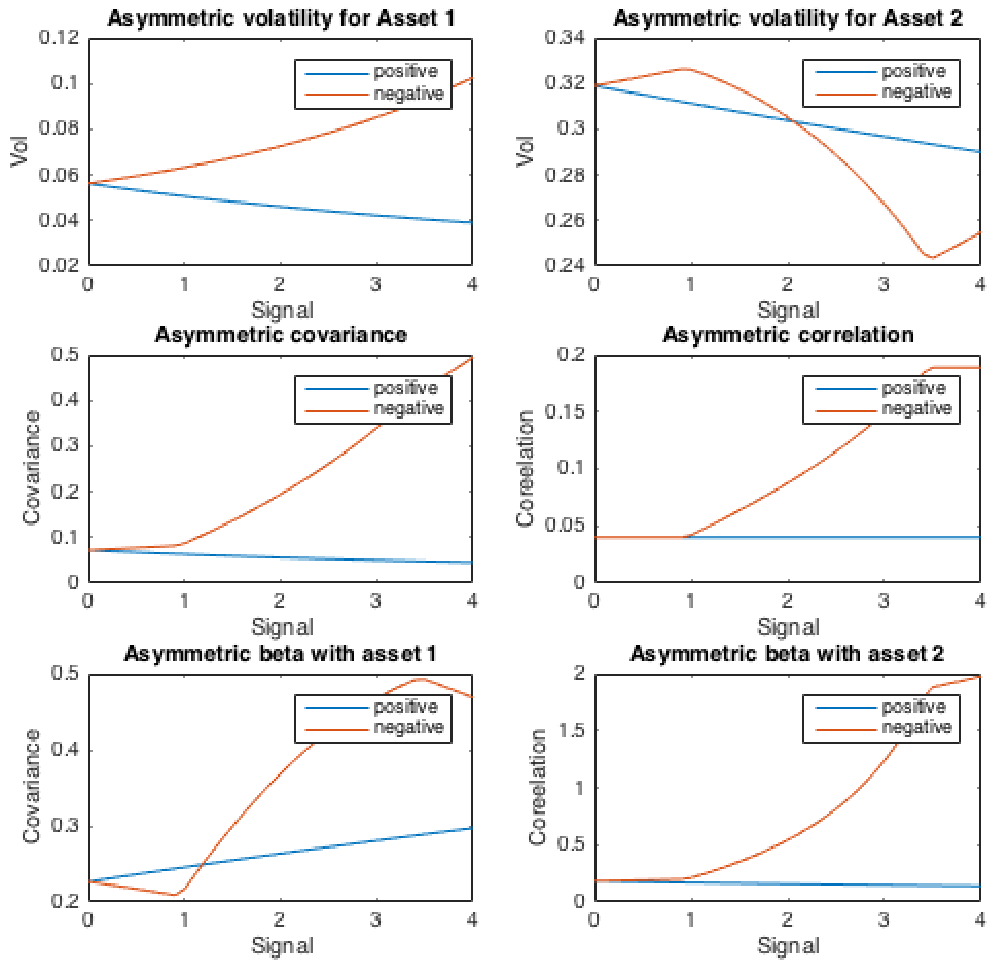

Figure 1.7: Measures of asymmetric patterns of asset returns
This figure displays the comparison of the conditional statistics - volatility, covariance, correlation, and beta, conditional on a piece of good news, $y = \tilde{s} - \overline{d}_1 > 0$, and a piece of bad news $-y$ correspondingly. The parameters in this figure are $\rho_a = 0.2$, $\rho_b = 0.7$, $\sigma_1 = 25\%$, $\sigma_2 = 10\%$, $\sigma_\epsilon = 5\%$; $\overline{d}_1 = 10$, $\overline{d}_2 = 5$, $\overline{\theta}_1 = 10$, $\overline{\theta}_2 = 100$, $\gamma = 2$. $s^L = 6.55$, $s^H = 9.05$. In this case $s^L - \overline{d}_1 = -3.45, s^H - \overline{d}_1 = -0.95$. I plot graphs along positive signals of $\tilde{s} - \overline{d}_1$, which are greater than -0.95, thus $\rho(s) = \rho_a = 0.2$.

CHAPTER 2: CORRELATION UNCERTAINTY, HETEROGENEOUS INVESTORS, AND ASSET PRICES

2.1 Introduction

A principal purpose of research in finance is to study the correlated structure among financial assets since Markowitz (1952)'s seminal work on portfolio choice and Ross (1976)'s arbitrage pricing theory. Our objective in this paper is to develop an asset pricing model on the correlated structure to explain several stylized facts in financial markets, including *under-diversification and limited participation* (Calvet, Campbell, and Sodini, 2009; Dimmock, Kouwenberg, Mitchell, and Peijnenburg, 2016; Vissing-Jorgensen, 2002), *flight-to-quality and flight-to-safety* (Baele, Bekaert, Inghelbrecht, and Wei, 2013; Baur and Lucey, 2009), and *comovement and contagion* (Barberis, Shleifer, and Wurgler, 2005; Chen, Singal, and Whitelaw, 2016; Pindyck and Rotemberg, 1993). Our model offers novel predictions for optimal portfolio choice and asset pricing.

The correlated structure among assets is not often unique in an asset pricing model as well as in practice for several reasons. First of all, estimating the correlated structure is challenging from both the statistical and econometric perspective (Chan, Karceski, and Lakonishok, 1999; Ledoit, Santa-Clara, and Wolf, 2003). The correlated structure is more difficult to estimate than a marginal distribution, due to lacking enough market data sources, limitations in the estimation methodology, instability or complications in the correlation structure (Bursaschi, Porchia, and Trojani, 2010; Engle, 2002).[21] Moreover, the increasing interdependence of financial markets brings in additional estimation concerns (Forbes and Rigobon, 2002). From an economic perspective, it is also very likely that an investor faces uncertainty on the estimation of a correlated structure, and this uncertainty is interpreted by the investor's non-standard preference (Ellsberg, 1961; Bewley, 2002). In this paper we demonstrate profound asset pricing implications based on the non-uniqueness assumption of the correlated structure.

We construct an equilibrium model with *heterogeneous correlation uncertainty* among investors. In a multiple-priors setting of Gilboa and Schmeidler (1989), each investor's concern over the correlated structure is interpreted by ambiguity-aversion about the correlated structure. Investors are heterogeneous in terms of ambiguity aversion, reflecting their various levels of sophistication in dealing with statistical data and estimation methodology.[22] To concentrate on the role of correlation ambiguity, investors in our model have perfect knowledge of the marginal distributions for all assets,[23] that is, they merely have concerns about the correlation structure.

[21] Alternatively, the correlated structure or the covariance matrix can be estimated from the option market. See Buss and Vilkov (2012); Kitiwiwattanachai and Pearson (2015). However, the complexity of the correlation process remains in spite of this implied estimation methodology.

[22] A growing body of research in asset pricing applies the multiple-priors framework of Gilboa and Schmeidler (1989) in which investors are heterogeneous in terms of ambiguity aversion. See, Cao, Wang, and Zhang (2005); Easley and O'Hara (2009, 2010); Epstein and Miao (2003); Garlappi, Uppal, and Wang (2007); Uppal and Wang (2003). Moreover, there is both laboratory evidence and non-laboratory empirical evidence of ambiguity aversion heterogeneity. See Bossaerts, Ghiraradato, Guarnaschelli, and Zame (2010); Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2016).

[23] By a copula theory (see McNeil, Frey, and Embrechts, 2005), the joint distribution for several random variables is characterized by the marginal distribution of each random variable, and a copula function that purely determines the correlation structure. In other words, the correlation structure can be independent of the marginal distributions. It has been well studied in classical statistical literature to characterize the "best" joint distribution with fixed marginal distributions (Strassen, 1965; White, 1976).

In this paper, the equilibrium under correlation uncertainty is characterized by determining both the optimal correlated structure and the optimal portfolio for each investor simultaneously. There are two kinds of equilibriums in which the heterogeneity of investors' correlation uncertainty plays a critical role. Specifically, when the dispersion of correlation uncertainty among investors is small, each investor chooses the highest available correlation in a *full participation* equilibrium. On the other hand, when the uncertainty dispersion is large, while the institutional investor still chooses the highest possible correlation coefficient, the retail investor's choice of correlation coefficient is no longer relevant to the equilibrium, and a *portfolio inertia* occurs. This portfolio inertia feature is derived in the context of both portfolio choice and equilibrium under correlation uncertainty, resulting in the emergence of a *limited participation* equilibrium.

In addition to the characterization of the equilibrium, we further demonstrate how the correlation uncertainty affects asset prices, risk premiums, Sharpe ratios, and betas in different manners, depending on characteristics of individual assets. In our model, we identify each asset using the *weighted volatility*, a product of size and return volatility, which is equivalent to a *risk-adjusted size factor*.[24] We also use *eta*, the ratio of the weighted volatility over the total weighted volatility of all assets, to measure its *relative risk contribution to the entire market*. We show that for a low-eta asset in the heterogeneous equilibrium, the higher the correlation uncertainty, the lower the

[24] The risk-adjusted size factor or the weighted volatility captures a trade-off between asset quality versus firm size. The negative relation between size and quality has been empirically documented. In particular, small firms tend to be "junk". See Ang, Hodrick, Xing, and Zhang (2006, 2009). Asness, et al.(2016) use a wide variety of quality measures over different countries and industries and find robust effect of the size factor after controlling for quality.

price and the higher Sharpe ratio. On the other hand, the price of a high-eta asset increases with correlation uncertainty. Each asset's beta and the correlation with the market also display different cross-sectional pattern according to the change of the correlation uncertainty.

As the model implication, our findings offer new insights into the understanding of the following stylized facts: under-diversification and limited participation puzzle, flight to quality and flight to safety, and asset comovements.

First of all, our characterization of the equilibrium provides novel explanation of the *under-diversification* and *limited participation puzzle*. We show that the institutional investor chooses a smaller correlation coefficient than the retail investor under all circumstances. As a result, the institutional investor always holds a well-diversified portfolio, whereas the retail investor holds an under-diversified portfolio. In some cases, the retail investor even has limited participation due to the portfolio inertia. Our model predicts that the retail investor's portfolio will be less risky because his higher correlation uncertainty will yield higher implicit risk aversion,[25] but the institutional investor's portfolio will achieve better performance than the retail investor.

Second, the model generates *flight-to-quality* and *flight-to-safety* endogenously from a correlation uncertainty perspective. When the dispersion among investors' correlation estimation is large, the institutional investor's position on the low-eta asset increases whereas the retail investor's corresponding position decreases. In contrast, the retail investor demands more positions for high-eta assets. Since a large ambiguity

[25] For the discussion that ambiguity leads to risk aversion implicitly, see Garlappi, Uppal, and Wang (2007); Gollier (2011); Wang and Uppal (2003).

dispersion among investors often comes in a stressed economy, the high-eta assets can be used to hedge against the "catastrophic economic shock" or "macroeconomic uncertainty". Therefore, the model offers an alternative explanation of flight-to-safety or flight-to-quality episodes resulting from a high demand of high-eta assets from the retail investor, and its price moves up sharply in a very weak economic situation.[26] Our model is similar to Caballero and Krishnamurthy (2008) in the Knightian uncertainty setting, but our focus is particularly on the correlation uncertainty.

Third, our model helps in understanding *asset comovement* from an investment perspective. In our model, with increasing correlation uncertainty, the relative Sharpe ratio of high-eta asset decreases whereas the relative Sharpe ratio of low-eta increases; thus, the dispersion of Sharpe ratios *decreases* endogenously. Previous studies document that assets move closely together in a downside market and move apart in an upside market (Barberis, Shleifer, and Wurgler, 2005; Basak and Pavolva, 2013; and Kyle and Xiong, 2001).[27] As a complementary analysis, our model suggests that risky assets are forced to comove more with larger correlation uncertainty, because of similar investment trading opportunities in terms of Sharpe ratios (Xiong, 2001).

Last but not least, our model implies that high *trading volume* is always associated with a high degree of correlation uncertainty or with large heterogeneity in correlation

[26]Caballero and Krishnamurthy (2008) develop an information amplification mechanism with uncertainty averse investors, and in the worst-case scenario of uncertainty, the investors choose to invest only on safe assets. Guerrieri and Shimer (2014) argue that adverse selection is another source of illiquidity. Vayanos (2004) constructs a balance sheet model in which the investor prefers to more liquid and less risky assets when the balance sheet is tight during periods of market stress.

[27]Barberis, Shleifer, and Wurgler (2005) propose three sources of frictions and suggest investor sentiment explanation for stock comovement. Basak and Pavolva (2013) develop models of asset class effect or community effect, in which comovement is implied by the correlated demand that is unrelated to fundamentals. Kyle and Xiong (2001) describe a wealth effect of convergence traders that creates contagion, thus assets become more volatile and more correlated.

estimation among investors, for low-eta as well as high-eta assets. When economic situations are very weak, the retail investor easily panics and overreacts to the market thereafter purchasing a significant amount of high-eta assets and selling low-eta assets. If we interpret the heterogeneity as disagreement, our results explain recent empirical findings by Carlin, Longstaff, and Matoba (2014).

Our model draws from many important works of asset pricing under ambiguity literature. Boyle, Garlappi, Uppal, and Wang (2012), Cao, Wang, and Zhang (2005), Easley and O'Hara (2009), and Garlappi, Uppal, and Wang (2007) investigate expected return parameter uncertainty. Easley and O'Hara (2010) and Epstein and Ji (2013) discuss volatility parameter uncertainty. In an information ambiguity setting, Epstein and Schneider (2008) and Illeditsch (2011) address the conditional distribution ambiguity of the signals. In all these previous studies, the correlation structure is always given as exogenous. Instead, we allow ambiguity to exist in the correlation structure while the marginal distribution is known. In this regard, our model creates a situation in which an ambiguity-averse investor views the overall market as highly ambiguous rather than made up of individual stocks, such as in Boyle, et al. (2012), and Uppal and Wang (2003). Therefore, correlation uncertainty can be viewed to some extent as systemic risk uncertainty because the ambiguity in the overall market is attributed to the macroeconomic uncertainty or the aggregate liquidity risk (see Dicks and Fulghieri, 2015). The asset pricing models under ambiguity or model uncertainty also include the works of Bossarts et al (2010), Maenhout (2004), Routledge and Zin (2014), and Drechsler (2013).

Our work is also closely related to the literature in the household portfolio choice problem and in the limited participation literature. Previous studies such as Cao, Wang, and Zhang (2005), Easley and O'Hara (2009), and Wang and Uppal (2003) focus on negligible positions on assets for which the marginal distribution is ambiguous. In contrast, we examine the optimal portfolio and compare it with a well-diversified market portfolio. To quantify the under-diversification in a precise manner, we hinge upon a dispersion measure inspired by the portfolio selection literature (Hennessy and Lapan, 2003; Ibragimov, Jaffee, and Walden, 2011). Our portfolio analysis is consistent with Calvet et al.(2009), Dimmock et al.(2016), Hirsheleifer, Huang, and Teoh (2016), and Polkovnichenko (2005). As shown in the model, the optimal portfolio is less diversified with higher perceived correlation uncertainty across investors.

The rest of the paper is organized as follows. Section 2 presents our correlation uncertainty model. In Section 3, we study the portfolio choice problem under correlation uncertainty and the equilibrium under heterogeneous correlation uncertainty is characterized. In Section 4 we discuss the joint effect of correlation uncertainty and asset characteristics on asset prices, risk premiums, correlations with the market portfolio and betas. Section 5 presents further implications of our model, including the under-diversification and limited participation, flight to quality and flight to safety, and asset comovement. We also use the 2007-2009 financial crisis as an example to illustrate these empirical implications. Section 6 concludes. Proofs and technical arguments are in the Appendices.

2.2 A Model of Correlation Uncertainty

There are N risky assets and one risk-free asset in a two-period economy (date $t = 0$ and $t = 1$). The payoffs or dividends of these N risky assets are $\tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_N$, respectively, at time $t = 1$. The risk-free rate is zero which serves as a numeraire. The per capita endowment of risky asset i is $\overline{x}_i, i = 1, \cdots, N$. Each risky asset can be viewed as an investment asset, an equity portfolio, an investment fund, or a market portfolio in an international market.

To focus entirely on the correlated risk and its effect on asset pricing, we investigate the correlation structure instead of the joint distribution of $(\tilde{a}_1, \cdots, \tilde{a}_N)$. In our specification of the correlation matrix, we employ Engle and Kelly (2012)'s dynamic equicorrelation (DECO) model, in which any two distinct risky assets have a same correlation coefficient ρ, i.e., $corr(\tilde{a}_i, \tilde{a}_j) = \rho$ for each $i \neq j$.[28] Engle and Kelly (2012) show that the (block) DECO estimation of U.S. stock return data can display a better fit for the data than a general dynamic conditional correlation (DCC in Engle, 2002) model. We assume that $(\tilde{a}_1, \cdots, \tilde{a}_N)$ has a multivariate Gaussian distribution, to be consistent with Cao, Wang, and Zhang (2005), and Easley and O'Hara (2009, 2010). Therefore, each investor is confident in the estimation of expected mean $\overline{a}_i$ and variance σ_i^2 for each risky asset $i = 1, \cdots, N$; but they are seriously concerned about the estimation of ρ, which represents the ambiguity aversion on the correlated structure. Since the correlation coefficient between the payoffs is identical to the correlation

[28]This assumption on the correlation structure can be relaxed in a block equicorrelation model, in which all risky assets are grouped into several sectors and the assets within each sector have close pairwise correlation coefficients. Although the details are not presented here, we extend the presented setting using two block equicorrelation examples in Engle and Kelly (2012), and the main insights of correlation uncertainty on equilibrium are largely the same. The details are available upon request.

coefficient between asset returns, our assumption on the correlation uncertainty is equivalent to the ambiguity being purely on asset returns' correlation structure, and the investor having no ambiguity on the marginal distribution of the asset return. In this paper we confine ourselves to a *nonnegative correlated* financial market.[29]

There is a group of investors in this economy. Each investor has the CARA-type risk preference to maximize the worst-case diversification benefit

$$\min_{\rho} \mathbb{E}^{\rho}\left[u(W)\right], u(W) = -e^{-\gamma W}, \tag{B-5}$$

where ρ runs through a plausible correlation coefficient region, $\mathbb{E}^{\rho}[\cdot]$ represents the expectation operator under corresponding correlation coefficient ρ, and γ is the investor's absolute risk aversion parameter. We assume that each investor has the same absolute risk aversion. In this regard, our setting is significantly different from Ehling and Heyerdahl-Larsen (2016), which studies the correlation structure through the channel of heterogeneity in risk aversion.

There are two types of investors, *institutional investors* and *retail investors*. The percentage of institutional investors in the market is ν and the percentage of retail investors is $1-\nu$. For the institutional investor, the plausible correlation coefficient region is $[\rho_1^{min}, \rho_1^{max}]$; the correlation coefficient region for the retail investor is $[\rho_2^{min}, \rho_2^{max}]$.[30] In our model, $[\rho_1^{min}, \rho_1^{max}] \subseteq [\rho_2^{min}, \rho_2^{max}]$, which reflects the fact that the estimation on correlation coefficients for the institutional investor is more

[29] A positively correlated structure is driven by common shocks or factors in a financial market. Both the diversification benefits and synchronization are more critical in a positively correlated economy than in a negatively correlated environment. In fact, our results in this paper hold when all correlation coefficients are strictly larger than $-\frac{1}{N-1}$ from a technical point of view.

[30] It is well known that the plausible linear correlation coefficient between any two variables X and Y is an interval, $[\rho^{min}, \rho^{max}]$. See NcNeil, Frey, and Embrechts (2015).

accurate than the retail investor. For each investor, his correlation coefficient region consists of two components: the *benchmark* correlation and the degree of correlation *uncertainty*. The benchmark correlation efficient is implied by $\frac{\rho^{min}+\rho^{max}}{2}$. The degree of correlation uncertainty is measured by $\frac{\rho^{max}-\rho^{min}}{2}$, which indicates how far plausible correlation coefficients move above and below the benchmark. In most situations, econometricians are able to find the benchmark correlation coefficient through the calibration to a stochastic matrix process, and treat it as a market reference with some estimation errors (Buraschi, Porchia, and Trojani, 2010; Chan, Karceski, and Lakonishok, 1999; Engle, 2002). We use ρ^{avg} to denote the benchmark and ϵ indicates the degree of uncertainty.[31]

In our model, two types of investors can have either different or same benchmark correlation coefficient. When investors agree on the benchmark, the plausible correlation coefficient for the institutional investor is $[\rho^{avg} - \epsilon_1, \rho^{avg} + \epsilon_1]$, and the retail investor's plausible correlation coefficient is $[\rho^{avg} - \epsilon_2, \rho^{avg} + \epsilon_2]$ and $\epsilon_1 < \epsilon_2$.[32] An extreme situation is $\epsilon_1 = 0$, in which the institutional investor becomes a Savage investor with perfect knowledge about the correlation structure.

[31] Previous literature measuring macroeconomic or aggregate uncertainty use proxies such as the VIX index, volatility of the CRSP value-weighted index, the Chicago Fed National Activity index (Bloom, 2009; Jurado, Ludvigson, and Ng, 2005). We assume the degree of correlation uncertainty is positively associated with these uncertainty indexes.

[32] Alternatively, by adopting Cao et al. (2005), we can assume that there is a continuum of investors, say, $[\rho^{avg} - \epsilon, \rho^{avg} + \epsilon]$, each type of investor's correlation uncertainty is captured by the parameter ϵ while ϵ is uniformly distributed among investors on $[\bar{\epsilon} - \delta, \bar{\epsilon} + \delta]$ with a density of $1/(2\delta)$. The main insights of this setting are fairly similar to ours whereas the impact of institutional investors in our current setting has a clearer expression. Our setting is reminiscent of Easley and O'Hara (2009, 2010) on the heterogeneity of investors' ambiguity aversion.

2.3 Equilibrium

This section presents the characterization of equilibrium under correlation uncertainty. We start with the portfolio choice problem.

2.3.1 Optimal Portfolio Choices

Let x_i be the number of shares on the risky asset i, $i = 1, \cdots, N$, and W_0 is the initial wealth of the investor. Then the final wealth at time 1 is

$$W = W_0 + \sum_{i=1}^{N} x_i(\tilde{a}_i - P_i),$$

where P_i is the price of the risky asset i at time $t = 0$. Assuming the plausible range of the asset correlation coefficient is $[\rho^{min}, \rho^{max}]$, and there is no trading constraint, the optimal portfolio choice problem for the investor is

$$\max_{x \in \mathbb{R}^N} \min_{\rho \in [\rho^{min}, \rho^{max}]} \mathbb{E}^{\rho}\left[-e^{-\gamma W}\right]. \tag{B-6}$$

Under the CARA preference and the multivariate Gaussian distribution assumption of the asset returns, it is standard to reduce this optimal portfolio choice problem to be

$$A \equiv \max_{x} \min_{\rho \in [\rho^{min}, \rho^{max}]} CE(x, \rho) \tag{B-7}$$

where $CE(x, \rho) = (\overline{a} - p) \cdot x - \frac{\gamma}{2} x^T \cdot D^T \cdot R(\rho) \cdot D \cdot x$ is the mean-variance utility of the investor when the demand vector on the risk assets is $x = (x_1, \cdots, x_N)^T$, $D = (d_{ij})$ is a diagonal $N \times N$ matrix with entries $d_{ii} = \sigma_i$ for each $i = 1, \cdots, N$, and $R(\rho)$ is a correlation matrix with a common correlation coefficient ρ. We use $\cdot^T$ to denote the transpose operator of a matrix. Thus the certainty-equivalent of the ambiguity-averse investor is

$$CE(x) = \min_{\rho \in [\rho^{min}, \rho^{max}]} CE(x, \rho). \tag{B-8}$$

It is clear to obtain

$$CE(x) = \begin{cases} CE(\rho^{max}, x), & \text{if } \sum_{i \neq j} \sigma_i x_i \sigma_j x_j > 0, \\ CE(\rho^{min}, x), & \text{if } \sum_{i \neq j} \sigma_i x_i \sigma_j x_j < 0, \\ \sum_{i=1}^{N} \left((\overline{a}_i - p_i) x_i - \frac{\gamma}{2} \sigma_i^2 x_i^2 \right), & \text{if } \sum_{i \neq j} \sigma_i x_i \sigma_j x_j = 0. \end{cases} \tag{B-9}$$

The insights of Equation (B-9) are appealing and we illustrate it by first take an example of $N = 2$. If $x_1 x_2 = 0$, then either $x_1 = 0$ or $x_2 = 0$ and the choice of correlation coefficient in computing $CE(x)$ is irrelevant. When $x_1 x_2 < 0$, it is a pair trading or a market-neutral strategy, thus the investor chooses the smallest possible correlation coefficient for the diversification benefits. If $x_1 x_2 > 0$, the portfolio yields a synchronization strategy; the highest correlation coefficient serves as the worst-case optimal for the certainty-equivalent under correlation uncertainty.

For a financial market with risky assets $N \geq 3$, the intuition of Equation (B-9) is similar. If the holding positions are largely in the same direction, the investor chooses the highest correlation coefficient for the worst-case scenario because of the ambiguity aversion. If the holding positions on the risky assets are opposite, the optimal correlation coefficient to compute the certainty-equivalent is the one that maximizes the diversification benefits, therefore it must be the smallest possible correlation coefficient. Finally, if *limited participation* occurs in the sense that $\sum_{i \neq j} (\sigma_i x_i)(\sigma_j x_j) = 0$,[33] the

[33] This equation worth further comments. For $N \geq 3$, and if each x_i is non-negative, then $\sum_{i \neq j} (\sigma_i x_i)(\sigma_j x_j) = 0$ ensures that there is *at most* one non-zero position x_i; however, if short-sell is allowed, it is possible that each x_i is non-zero in the equation $\sum_{i \neq j} (\sigma_i x_i)(\sigma_j x_j) = 0$. For instance,

choice of the correlation coefficient is irrelevant to compute the certainty-equivalent as $CE(\rho, x) = \sum_{i=1}^{N} \left((\overline{a}_i - p_i)x_i - \frac{\gamma}{2}\sigma_i^2 x_i^2\right)$ for all possible $\rho \in [\rho^{min}, \rho^{max}]$.

For later purpose, we elaborate the certainty-equivalent by introducing a *dispersion measure*, $\Omega(w)$, of a vector $w = (w_1, \cdots, w_N)$ with $\sum_{i=1}^{N} w_i \neq 0$. Let

$$\Omega(w) \equiv \sqrt{\frac{1}{N-1}\left(N\frac{\sum_{i=1}^{N} w_i^2}{(\sum_{i=1}^{N} w_i)^2} - 1\right)}. \tag{B-10}$$

Clearly, $\Omega(w)^2$ is up to a linear transformation of the Herfindahl index $\sum_{i=1}^{N} w_i^2$ for $\sum_{i=1}^{N} w_i = 1$. A formal justification of $\Omega(\cdot)$ being a dispersion measure is presented in Appendix C.[34]

By using the dispersion measure $\Omega(\cdot)$, we now reformulate the certainty-equivalent of the ambiguity-averse investor as[35]

$$CE(x) = \begin{cases} CE(\rho^{max}, x), & \text{if } \Omega(\sigma x) < 1, \\ CE(\rho^{min}, x), & \text{if } \Omega(\sigma x) > 1, \\ \sum_{i=1}^{N} \left((\overline{a}_i - p_i)x_i - \frac{\gamma}{2}\sigma_i^2 x_i^2\right), & \text{if } \Omega(\sigma x) = 1. \end{cases} \tag{B-11}$$

Therefore, the optimal portfolio choice problem for the ambiguity-averse investor is to solve

$$A = \max_x \left\{ \max_{\Omega(\sigma x)<1} CE(\rho^{max}, x), \max_{\Omega(\sigma x)>1} CE(\rho^{min}, x), \max_{\Omega(\sigma x)=1} CE(\rho, x) \right\}. \tag{B-12}$$

Proposition 12 (Optimal Portfolio Choice) *Let $s_i = (\overline{a}_i - p_i)/\sigma_i$ be the Sharpe ratio*

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for $N = 3$ and choose $\sigma_1 = \sigma_2 = \sigma_3$, for $x_1 = 2, x_2 = 2$, and $x_3 = -1$, $\sum_{i \neq j} (\sigma_i x_i)(\sigma_j x_j) = 0$.

[34] The dispersion measure has been applied in the portfolio selection context. See Hennessy and Lapan (2003); Ibragimov, Jaffee, and Walden (2011).

[35] For any two $1 \times N$ vectors s and t, st denotes $(s_1 t_1, \cdots, s_N t_N)$, and its dispersion is written as $\Omega(st)$. $\sigma = (\sigma_1, \cdots, \sigma_N)$.

of asset i and $\Omega(s)$ be the dispersion of the Sharpe ratios vector $s = (s_1, \cdots, s_N)$ of all risky assets. $\hat{\Omega}(s) \equiv \frac{1-\Omega(s)}{1+(N-1)\Omega(s)}$. For each $\rho \in [\rho^{min}, \rho^{max}]$, $x_\rho \equiv \frac{1}{\gamma} D^{-1} R(\rho)^{-1} s^T$ is the optimal portfolio in the absence of uncertainty when the correlation coefficient is ρ. We assume that $\sum_{i=1}^{N} s_i \neq 0$.

1. *If all plausible correlation coefficients are strictly larger than $\hat{\Omega}(s)$, then the investor chooses the optimal correlation coefficient $\rho^* = \rho^{min}$, and the optimal demand is $x^* = x_{\rho^{min}}$ in Problem (B-6).*

2. *If all plausible correlation coefficients are strictly smaller than $\hat{\Omega}(s)$, then the investor chooses the optimal correlation coefficient $\rho^* = \rho^{max}$, and the optimal demand is $x^* = x_{\rho^{max}}$ in Problem (B-6).*

3. *If $\rho^{min} \leq \hat{\Omega}(s) \leq \rho^{max}$, then the investor is irrelevant to choose any correlation coefficient in $[\rho^{min}, \rho^{max}]$ as the optimal one, and the optimal demand is $x_{\hat{\Omega}(s)}$ in Problem (B-6).*

Proposition 12 determines the optimal correlation coefficient and the optimal demand in the certainty-equivalent simultaneously. When all available correlation coefficients are large enough, $\rho^{min} > \hat{\Omega}(s)$, $\Omega(\sigma x_{\rho^{min}})$ is strictly larger than 1. According to the above discussion of the certainty-equivalent, the optimal correlation coefficient is the lowest plausible one to maximize the diversification benefits.[36] On the other hand, if all available correlation coefficients are small, $\rho^{max} < \hat{\Omega}(s)$, then $\Omega(\sigma x_{\rho^{max}})$ is

[36] By Equation (B-11), $(\rho^{min}, x_{\rho^{min}})$ solves $CE(\rho^{min}, x)$ under the demand constraint $\Omega(\sigma x) > 1$. By subtly analyzing the dual-problem of Equation (B-6), ρ^{min} is the best possible correlation coefficient for the ambiguity-averse investor in this situation; thus, $(\rho^{min}, x_{\rho^{min}})$ is the solution of Problem (B-6). The details are given in Appendix B.

strictly smaller than 1. Since the worst-case scenario in this case is a large correlation coefficient, the investor chooses the highest possible correlation coefficient.

Proposition 12 is particularly interesting, when the investor's correlation uncertainty is large in the sense that $\rho^{min} \leq \hat{\Omega}(s) \leq \rho^{max}$, for several reasons. First, the investor holds a limited participation portfolio, because $\sum (\sigma_i x_i^*) \left(\sigma_j x_j^*\right) = 0$ and the dispersion of $x_{\hat{\Omega}(s)}$ is one. Especially for $N = 2$, $x_1^* x_2^* = 0$ ensures either $x_1^* = 0$ or $x_2^* = 0$, which is a typical example of limited participation. Second, the optimal demand x^* is unique. Any other demand vector leads to a smaller maxmin expected utility in Problem (B-6). Lastly, while the investor's optimal demand is uniquely determined, the choice of optimal correlation coefficient is irrelevant, a *portfolio inertia* occurs.[37] This portfolio inertia feature yields important asset pricing implications in equilibrium as shown in the next section.

2.3.2 Characterization of Equilibrium

We first characterize the equilibrium in a baseline model with one representative investor as follows.

Proposition 13 *(*Homogeneous Equilibrium*) Assume the plausible correlation coefficient is* $[\rho^{min}, \rho^{max}]$ *in a homogeneous environment. There exists a unique uncertainty equilibrium in which the representative investor's endogenous correlation coefficient is the* highest *plausible correlation coefficient* ρ^{max}*. The price of the risky*

[37] It has been well documented that high ambiguity might result in portfolio inertia since Dow and Werlang (1992), Epstein and Schneider (2008), and Illeditsch (2011). However, this feature does not emerge naturally in the Gilboa-Schmeidler's maxmin expected utility setting where either the mean or the volatility is unknown and the worst-case scenario corresponds to the extreme parameters, for example, Garlappi, Uppal, and Wang (2007), Easley and O'Hara (2009), and Epstein and Ji (2013).

asset i is given by

$$p_i = \overline{a}_i - \gamma\sigma_i(1-\rho^{max})\sigma_i\overline{x}_i - \gamma\sigma_i\rho^{max}\left(\sum_{n=1}^{N}\sigma_n\overline{x}_n\right). \tag{B-13}$$

Proposition 13 follows easily from Proposition 12. Since the optimal portfolio must be the market portfolio $\sum_{i=1}^{N}\overline{x}_i\tilde{a}_i$ in equilibrium, there is no short position in the optimal portfolio of the representative investor, thus $\Omega(\sigma x^*) = \Omega(\sigma\overline{x}) < 1$. According to Proposition 12, the representative investor must choose the highest possible correlation coefficient to hedge the worst-case scenario of uncertainty.

To highlight the effect of correlation uncertainty, we write $\rho^{min} = \rho^{avg} - \epsilon, \rho^{max} = \rho^{avg} + \epsilon$. Therefore, the risk premium $\overline{a}_i - p_i$ can be written as a sum of two components:

$$\begin{aligned} \overline{a}_i - p_i &= \overbrace{\gamma(1-\rho^{avg})\sigma_i^2\overline{x}_i + \gamma\sigma_i\rho^{avg}\sum_{n=1}^{N}\sigma_n\overline{x}_n} \\ &\quad + \epsilon\gamma\left(\sigma_i\sum_{j\neq i}\sigma_j\overline{x}_j\right) \end{aligned} \tag{B-14}$$

where the first component represents the risk premium in the absence of correlation uncertainty, and the second one is the correlation-uncertainty premium.

Equation (B-14) is useful for explaining the equity premium puzzle arising from a positive uncertainty premium. Jeong, Kim, and Park (2015) find that the ambiguity aversion is both economically and statistically significant by calibrating a multiple-priors recursive utility model and that the estimated ambiguity aversion explains up to 45 percent of average equity premium. For illustrative, we report in Table 2.1 the percentage of the correlation-uncertainty premium to the uncertainty-free component, $\gamma(1-\rho^{avg})\sigma_i\overline{x}_i + \gamma\rho^{avg}\sum_{n=1}^{N}\sigma_n\overline{x}_n$, under parameters $\sigma_1 = 9\%, \overline{x}_1 = 1, \sigma_2 = 10\%, \overline{x}_2 =$

$5, \sigma_3 = 12\%, \overline{x}_3 = 10.5$ and $\gamma = 1$, and then $\Omega(\sigma\overline{x}) = 0.5558$. Assume that $\rho^{avg} = 0.4$, without correlation uncertainty, $(s_1, s_2, s_3) = (0.79, 1.49, 2.70)$. Letting the degree of uncertainty ϵ move between 0 to 0.2, we observe in Table 2.1 that the correlation-uncertainty premium increases by a reasonable amount. For instance, when $\epsilon = 0.08$, the correlation-uncertainty premium adds about 17 %, 16 %, and 12 % to each asset respectively. With a high uncertainty $\epsilon = 0.2$, the correlation-uncertainty premium is quite significant for each risky asset, adding 40 % to the reference correlation coefficient. As a consequence, the average equity premium is increased by about 40 percent when the degree of correlation uncertainty is 20 %.

We next characterize the general equilibrium under heterogeneous correlation uncertainty. Similar to the baseline model, it is vital to determine the optimal correlation coefficient for each investor in characterizing the equilibrium. In contrast to the homogeneous equilibrium, there are "two" kinds of equilibrium in a heterogeneous environment.

Define two auxiliary functions which capture the parameters in the heterogeneous setting. Let

$$m(x,y) \equiv \frac{\nu}{1-x} + \frac{1-\nu}{1-y}, \tag{B-15}$$

and

$$n(x,y) \equiv \frac{\nu x}{(1-x)(1+(N-1)x)} + \frac{(1-\nu)y}{(1-y)(1+(N-1)y)}. \tag{B-16}$$

Proposition 14 *(*Heterogenous Equilibrium*)*

1. Full Participation: *If*

$$\rho_1^{max} \geq \frac{1}{N-1}\left\{\frac{\nu}{1-\nu}\frac{\Omega(\sigma\overline{x})}{1-\Omega(\sigma\overline{x})}N-1\right\}, \tag{B-17}$$

or if ρ_2^{max} is strictly smaller than $K(\rho_1^{max})$, there exists a unique equilibrium in which each investor chooses the corresponding highest *correlation coefficient, respectively. The function $K(\cdot)$ is given by Equation (B-16) in Appendix B.*

2. Limited Participation: *If ρ_1^{max} does not satisfy Equation (B-17) and ρ_2^{max} is larger than $K(\rho_1^{max})$, there exists a unique equilibrium in which the institutional investor chooses the highest possible correlation coefficient ρ_1^{max} and the choice of the retail investor is* irrelevant. *The retail investor's optimal demand is uniquely determined by the endogenous Sharpe ratios in the equilibrium (see Equation (B-19) below).*

3. *In either equilibrium, the market price for asset i is, for each $i = 1, \cdots, N$,*

$$p_i = \overline{a}_i - \frac{\gamma\sigma_i}{m(\rho_1^{max}, \rho_2)}\left(\sigma_i\overline{x}_i + \frac{n(\rho_1^{max}, \rho_2)}{m(\rho_1^{max}, \rho_2) - Nn(\rho_1^{max}, \rho_2)}\sum_{j=1}^{N}\sigma_j\overline{x}_j\right). \tag{B-18}$$

where $\rho_2 = \rho_2^{max}$ in the full participation equilibrium and $\rho_2 = K(\rho_1^{max})$ in the limited participation equilibrium. Furthermore, each risky asset is priced at discount in equilibrium. That is, $p_i < \overline{a}_i$ and $s_i > 0$ for each $i = 1, \cdots, N$.

Following the terminology in Cao, Wang and Zhang (2005), and Easley and O'Hara (2009), we name a full participation equilibrium if all investors participate in the market. If investors are relatively homogeneous, either both ρ_1^{max} and ρ_2^{max} are large or small, all investors participate in the market by choosing the corresponding highest possible correlation coefficient under the uncertainty concern. There are two particular cases in which Equation (B-17) holds, thus a full participation equilibrium prevails. First, $\rho^{avg} \geq \frac{1}{N-1}\left\{\frac{\nu}{1-\nu}\frac{\Omega(\sigma\overline{x})}{1-\Omega(\sigma\overline{x})}N - 1\right\}$, which often applies to a certain asset class in which each pair of assets displays a high correlation by nature, eg. stocks in one sector, or bonds with different maturities. Second, the benchmark correlation coefficient is small, but each investor has high correlation uncertainty. This condition is often true in a stressed economy. As long as Equation (B-17) is satisfied, each investor chooses the highest possible correlation coefficient in a full participation equilibrium regardless of the uncertainty dispersion between investors.

In contrast to the full participation equilibrium, a limited participation equilibrium is generated in Proposition 14 (2), when there is a large amount of heterogeneity of correlation estimation among investors,. For instance, when there are many institutional investors and a high risk dispersion such that a total sum of ν and $\Omega(\sigma\overline{x})$ is greater than 1, Equation (B-17) fails. Furthermore, if the retail investor is very uncertain about the correlated structure, any choice of the correlation coefficient in his plausible range is feasible but irrelevant in the limited participation equilibrium.

By a limited participation equilibrium, we mean the retail investor's choice of the correlation coefficient is irrelevant to the equilibrium; and a limited participation is equivalent to anti-diversification (Goldman, 1979) only when $N = 2$. Hence, Proposition 14 (2) is consistent with an endogenous limited participation equilibrium as shown in Cao, Wang, and Zhang (2005) when investors are heterogeneous in terms of expected mean uncertainty with two risky assets. But the retail investor *does* participate for $N \geq 3$ in a limited participation equilibrium, and his optimal demand, $x^{(r)}$, is uniquely determined by the endogenous Sharpe ratios in the equilibrium such that

$$x_i^{(r)} = \frac{1}{\gamma\sigma_i}\frac{1+(N-1)\Omega}{N\Omega}\left(s_i - \frac{1-\Omega}{N}S\right) \tag{B-19}$$

where $\Omega = \frac{1-K(\rho_1^{max})}{1+(N-1)K(\rho_1^{max})}$, each s_i is determined by Proposition 14 (3), and S is the total sum of all s_i. Proposition 14 (2) offers remarkable new insights regarding the limited participation and under-diversification issue, which are discussed in Section 4.

2.4 Equilibrium Analysis

In this section we conduct a detailed analysis examining how the correlation uncertainty affects the equilibrium with respect to different asset characteristics, which are presented as follow.

Let $\hat{\sigma}_i$ be the return volatility of asset i. Then the payoff volatility $\sigma_i = \hat{\sigma}_i p_i$ and thus $\sigma_i\overline{x}_i = \hat{\sigma}_i\left(p_i\overline{x}_i\right)$. Since $p_i\overline{x}_i$ is the market capitalization of asset i, $w_i \equiv \frac{p_i\overline{x}_i}{\sum_{i=1}^N p_i\overline{x}_i}$ represents the "size factor" of asset i.[38] Therefore, $\sigma_i\overline{x}_i$ is proportional to $\hat{\sigma}_i w_i$, a *product of the volatility and the size factor* and we call it a *risk-adjusted size factor* or *weighted volatility*. In contrast to a simple risk factor, $\hat{\sigma}_i w_i$ is large only when both the size and the volatility are large or at least one factor is extremely large; and $\hat{\sigma}_i w_i$ is small if both factors are small or at least one is very small. Hence, the risk-adjusted size factor is able to capture the trade-off between risk and size. Furthermore, we introduce $\eta_i \equiv \frac{\hat{\sigma}_i w_i}{\sum_{n=1}^N \hat{\sigma}_n w_n}$, a proxy to represent the individual asset's

[38] As documented in Moskowitz (2003), the firm size is a significant factor for predicting future covariation. Ang, Hodrick, Xing, and Zhang (2006, 2009) argue that a small risk asset tends to have high quality.

risk contribution to the market.[39] However, it is worth noting that the eta concept is always relative. By a low eta we mean its weighted volatility is small when comparing with the weighted volatilities of all other risky assets in the market.[40]

2.4.1 Risk Premium and Asset Price

The effects of correlation uncertainty and the asset characteristics on the risk premium and the asset price are given by the next proposition.

Proposition 15 *1. The Sharpe ratio $s_i \geq s_j$, if and only if $\eta_i \geq \eta_j$. Moreover, s_i is larger than the average Sharpe ratio, $\frac{S}{N}$, if and only if $\eta_i \geq \frac{1}{N}$.*

2. For asset i with $\eta_i < \frac{1}{N}$, the higher the correlation uncertainty, the larger its Sharpe ratio and the smaller its price; the effect of correlation uncertainty on the Sharpe ratio and the price is opposite *if η_i is large.*

Proposition 15 (1) displays the symmetric property between the Sharpe ratio and the eta of an individual asset. It states that the a larger Sharpe ratio always corresponds to a higher eta among all risky assets. Thus, one asset with a higher eta is more attractive than the other asset with a small eta from an investment perspective. By the same reason, a risky asset's Sharpe ratio is above the average Sharpe ratio only when its eta is above the average level $\frac{1}{N}$. We divide assets into high-eta assets with relatively high η, and low-eta assets with smaller η.

Proposition 15 (2) demonstrates the effect of the correlation uncertainty on the

[39] We demonstrate that in Appendix A the set of η_i, the correlation coefficient $corr(\tilde{R}_i, \tilde{R}_m)$, and the weighted beta $\beta_i w_i$, are mutually determined by each other in an equicorrelation model.

[40] More precisely, $\eta_i \leq \alpha$, if and only if its weighted volatility, $\hat{\sigma}_i w_i \leq \frac{\alpha}{1-\alpha} \sum_{j \neq i} \hat{\sigma}_j w_j$. In particular, $\eta_i < \frac{1}{N}$ if and only if $\hat{\sigma}_i w_i \leq \frac{1}{N-1} \sum_{j \neq i} \hat{\sigma}_j w_j$.

risk premium. We decompose the Sharpe ratio into two components:

$$s_i = \frac{S}{N} + \overbrace{\frac{\gamma L}{m(\rho_1, \rho_2)} \left(\eta_i - \frac{1}{N} \right)} \tag{B-20}$$

where $L = \sum_{i=1}^{N} \sigma_i \overline{x}_i$ is the aggregate market volatility if all assets are perfectly correlated, ρ_1 and ρ_2 are the endogenous pairwise correlation coefficient of the institutional investor and the retail investor in equilibrium. In the decomposition of Equation(B-20), the first component is the *average Sharpe ratio*, and the second represents how much it differs from the average Sharpe ratio. We call the second component a *specific Sharpe ratio*[41]. The specific Sharpe ratio of asset i is proportional to the difference between the eta and the average level, $\eta_i - \frac{1}{N}$.

As observed in Equation (B-20) and shown in Proposition 15 (2), the correlation uncertainty affects the Sharpe ratios of high and low-eta assets very differently. For a low-eta asset (say, its eta is smaller than average), the correlation uncertainty always *increases* the Sharpe ratio through two distinct channels: the increase on the average Sharpe ratio and the increase on the specific Sharpe ratio. As correlation uncertainty increases, the asset prices drop.

However, for assets whose eta is larger than average, the effect of correlation uncertainty is not so straightforward because of the opposing effects of the average Sharpe ratio and the specific Sharpe ratio. As the uncertainty increases, the average Sharpe ratio always increases, but the specific Sharpe ratio decreases. When the eta is large enough in certain circumstance, the negative effect of the specific Sharpe ratio dominates the positive effect of the average Sharpe ratio, thus reaching an overall negative

[41] See Simsek (2013) for a similar concept

effect on the risk premium and the Sharpe ratio, and the asset price increases.

2.4.2 Correlation and Beta

In the next result, we explain how the asset characteristics and the correlation uncertainty jointly influence an asset's correlation with the market and the beta.

Proposition 16 *Let $\tilde{R}_i = (\tilde{a}_i - p_i)/p_i$ be the asset i's return, and $\tilde{R}_m$ be the market portfolio return, $\tilde{R}_m = \sum_{i=1}^N w_i \tilde{R}_i$.*

1. *The correlation between asset return and the market portfolio is* positively *associated with the asset eta. Specifically,*

$$corr(\tilde{R}_i, \tilde{R}_m) \geq corr(\tilde{R}_j, \tilde{R}_m) \text{ if and only if } \eta_i \geq \eta_j, \forall i, j = 1, \cdots, N. \quad \text{(B-21)}$$

Moreover, when the etas of risky assets display in a reasonable range where $\eta_1 \geq \cdots \geq \eta_N$ and $\frac{2\eta_N}{1-\eta_N} \geq \frac{\eta_1}{1-\eta_1}$,

$$\frac{\partial \left(corr(\tilde{R}_i, \tilde{R}_m) \right)}{\partial \rho} > 0, \forall i = 1, \cdots, N. \quad \text{(B-22)}$$

2. *The weighted beta is* positively *associated with the asset eta. For an asset with a very large or small eta, its beta is* positively *associated with the endogenous correlation coefficient. But for an asset with an intermediate level of eta, its beta is* negatively *related to the endogenous correlation coefficient.*

The first part of Proposition 16 concerns the asset characteristics and the correlation with the market portfolio. Similar to the properties of the Sharpe ratio in Proposition 15, the higher the eta the higher the correlation with the market portfolio. When the dispersion of etas, $\Omega(\eta)$, is small such that all etas are within a reasonable range, the correlation of each asset with the market portfolio is positively related to the correlation uncertainty. In this case, these assets' correlations with the market are higher in a recession period than in a boom period.[42] But the effect of uncertainty on the correlation $corr(\tilde{R}_i, \tilde{R}_m)$ is complicated when there are significant differences among asset characteristics, i.e., when $\Omega(\eta)$ is high. To illustrate, we consider three assets with $\eta_1 = 0.1, \eta_2 = 0.3$ and $\eta_3 = 0.6$, and the benchmark correlation is $\rho^{avg} = 0.5$ in Figure 2.1. As shown in Figure 2.1 (a), the correlation with the market return for asset 1 and asset 2 increase with respect to the correlation uncertainty, ϵ. However, the higher ϵ the less the high-eta asset (asset 3) is correlated with the market portfolio.

By a similar method, we also study the joint effect of correlation uncertainty and asset characteristics on the weighted beta. Proposition 16 (2) follows from the formula of asset's beta:[43]

$$\beta_i = \frac{\eta_i}{w_i} \cdot \frac{\rho + \eta_i(1-\rho)}{\rho + (1-\rho)\sum_{i=1}\eta_i^2}, \tag{B-23}$$

We illustrate this result using a simple market with $N = 2$. By straightforward calculation, we have

$$\frac{\partial \beta_i}{\partial \rho} = \frac{\eta_i}{w_i} \cdot \frac{(1-\eta_i)(1-2\eta_i)}{(\rho + (1-\rho)\sum_i \eta_i^2)^2}. \tag{B-24}$$

For a low-eta asset with $\eta_i < \frac{1}{2}$, its beta is higher in a weak market than in a strong market because the correlation coefficient is higher in the weak market. For the high-

[42] The counter-cyclical correlation pattern is well documented in Ang and Bekaert (2000), Ang and Chen (2002), Cappiello, Engle, and Sheppard (2006), Das and Uppal (2001), Longin and Solnik (2001), Patton (2006) among others.

[43] See Appendix A, Proposition 20.

eta asset with $\eta_i > \frac{1}{2}$, it is vice versa for the same reason. We can observe this feature in Figure 2.1 (b), in which the weighted beta for asset 3 is decreasing with respect to the correlation uncertainty. Therefore, we demonstrate that the weighted beta and the correlation with the market return are jointly affected by the correlation uncertainty and asset characteristics.

2.4.3 Effects of Institutional Investor and Risk Distribution

In addition to the degree of correlation uncertainty, there are other parameters that play important roles in the equilibrium analysis, such as the proportion of the institutional investors ν and the risk distribution $\Omega(\eta)$. For this purpose, we use $K(\rho, \nu, \Omega(\eta))$ to highlight the impact of ν and $\Omega(\eta)$ in this subsection. For simplicity we assume that the institutional investor has a perfect estimate about the correlation structure ρ and the retail investor's correlation uncertainty is ϵ.

First of all, the number of institutional investors in the market is a critical factor determining which kind of equilibrium it will be. Note that $K(\rho, \nu, \Omega(\eta))$ is a decreasing function of ν. When ϵ is small in the sense that $\epsilon + \rho \leq \lim_{\nu \to 1} K(\rho, \nu, \Omega(\eta))$, a full participation equilibrium occurs. Let us assume the retail investor has a reasonable large uncertainty about the market such that $\epsilon + \rho > \lim_{\nu \to 1} K(\rho, \nu, \Omega(\eta))$. For a small number of institutional investors, $\epsilon + \rho \leq K(\rho, \nu, \Omega(\eta))$, a full participation equilibrium is generated. However, if more and more institutional investors participate in the market, such that $\epsilon + \rho > K(\rho, \nu, \Omega(\eta))$, a limited participation equilibrium prevails. In an extreme case, $\nu \to 1$, the limited participation equilibrium becomes a homogeneous equilibrium, as in Proposition 13.

In addition, the proportion of institutional investors has significant effects on individual assets. Consider a low-eta asset with $\eta_i < \frac{1}{N}$, it is easy to see

$$\frac{\partial}{\partial \nu}\left(s_i\right) = \frac{\partial}{\partial \nu}\left(\frac{S}{N}\right) + \frac{\partial}{\partial \nu}\left(\frac{\gamma L}{m}\right)\left(\eta_i - \frac{1}{N}\right) < 0. \tag{B-25}$$

Therefore, the risk premium of low-eta assets drops with the increasing number of institutional investors. Similarly, the risk premium of high-eta assets increases with ν. In this regard, Equation (B-25) is closely related to the findings in Gompers and Metrick (2001), which empirically document that the small-company stock premium drops due to increasing demand from institutional investors. By the same reasoning, the institutional investors' demand for a high-eta firm would increase the premium. Equation (B-25) asserts that the low-eta firm premium decreases and the high-eta firm premium increases with the number of the institutional investors present.

Lastly, the eta distribution $\Omega(\eta)$ also has great impact on the heterogeneous equilibrium. In one case, when each asset has similar weighted volatility risk (eta) in the market, a full participation equilibrium is generated. If each asset contributes the same risk, $\Omega(\eta) = 0$, then $\Omega(s) = 0$. In the other case, when assets offer a skewed eta distribution such that $\Omega(\eta)$ is close to one, a limited participation equilibrium is obtained according to Proposition 14. The intuition is simple. We observe that $\lim_{\Omega(\eta)\to 1} K\left(\rho, \nu, \Omega(\eta)\right) = \rho$. Therefore, any retail investor must hold a limited portfolio for a skewed enough eta distribution. This analysis demonstrates another channel for limited participation phenomena, that is, limited participation can be caused by a large risk dispersion among asset characteristics.

To summarize, we use Table 2.3 to report the conditions in which a full equilibrium and a limited equilibrium is generated. There are three panels in Table 2.3. In Panel A, we discuss the equilibrium cases with varying levels of correlation uncertainties, while ν and $\Omega(\eta)$ are fixed. We conclude that only when ρ is small and $\rho+\epsilon \geq K(\rho)$, a limited participation will occur. Otherwise, a full participation equilibrium prevails. Panel B presents the impact of the percentage of the institutional investor, ν, when other parameters, ρ, ϵ and $\Omega(\eta)$ are fixed. Clearly, if ν is small such that $\frac{\nu}{1-\nu} \leq \frac{1+(N-1)\rho}{N}\frac{1-\Omega(\eta)}{\Omega(\eta)}$, in particular for $\nu = 0$ with only retail investor, there is a full participation equilibrium. For other ν, if the correlation uncertainty is small, the market is close to a homogeneous environment, yielding a full participation equilibrium. It is interesting to examine the situation when ϵ is relatively large such that $\epsilon + \rho > \lim_{\nu\to 1} K(\rho, \nu, \Omega(\eta))$. In this case we define ν^* by the equation $K(\rho, \nu^*, \Omega(\eta)) = \rho + \epsilon$. Hence, a limited participation equilibrium appears if $\nu \geq \nu^*$, or equivalently, $\rho + \epsilon > K(\rho, \nu, \Omega(\eta))$. Similarly, we characterize the equilibrium under conditions of the risk distribution $\Omega(\eta)$ in Panel C, when other parameters ρ, ϵ and ν are fixed. Ω^* satisfies the equation $K(\rho, \nu, \Omega^*) = \rho+\epsilon$. If $\Omega(\eta) \geq \Omega^*$, we obtain a limited participation; otherwise, a full participation equilibrium occurs.

2.5 Implications

We present several asset pricing implications and testable properties on the optimal portfolios in this section. The heterogeneity of correlation uncertainty is shown to be one fundamental channel to explain the following stylize facts in the financial market.

2.5.1 Limited Participation and Under-diversification

We start with the limited participation and under-diversification puzzle by comparing each investor's optimal portfolio with the market portfolio.

Proposition 17 1. *(*Under-diversification and well-diversification*) Compared with the market portfolio* $\sum_{i=1}^{N} \overline{x}_i \tilde{a}_i$*, the retail investor has an* under-diversified *portfolio while the institutional investor has a* well-diversified *portfolio. As a consequence, the institutional investor always holds a better diversified portfolio than the retail investor.*

2. *(*Portfolio Risk*) The institutional investor holds a* riskier *portfolio than the retail investor.*

3. *(*Portfolio Performance*) The institutional investor has a* better *portfolio performance in the sense that the Sharpe ratio of the optimal portfolio is strictly larger than that of the retail investor. Moreover, the institutional investor has a higher maxmin expected utility than the retail investor.*

In Proposition 17, the dispersion measure $\Omega(\cdot)$ is essentially used to measure the diversification extent of each portfolio. The market portfolio $\sum_{i=1}^{N} \overline{x}_i \tilde{a}_i$ serves as the benchmark to compare with each investor's optimal portfolio. $x^{(s)}$ denotes the optimal demand vector for institutional investor. Our model demonstrates in a precise manner that the institutional investor has a better diversified portfolio than the market portfolio, because $\Omega\left(\sigma x^{(s)}\right)$ is smaller than $\Omega(\sigma\overline{x})$. On the other hand, $\Omega\left(\sigma x^{(r)}\right)$ is larger than $\Omega(\sigma\overline{x})$, the retail investor's optimal portfolio is less diversified. As a consequence, the institutional investor holds a well-diversified optimal portfolio and the retail investor's optimal portfolio is under-diversified. Moreover, the retail investor can hold a limited participation portfolio when his correlation uncertainty is large enough so that $\Omega\left(\sigma x^{(r)}\right) = 1$. In short, Proposition 17 (1) helps in understanding the under-diversification and limited participation puzzle by quantifying the difference between the investor's optimal portfolio and the market portfolio.

Our approach is novel compared with the extant theoretical studies that posit under-diversification from perspectives such as model misspecification (Easley and O'Hara, 2009; Uppal and Wang, 2003), heterogeneous beliefs (Hirsheleifer, Huang, and Teoh, 2016; Mitton and Vorkink, 2008), and costly information (Van Nieuwerburgh and Veldkamp, 2010). For instance, Easley and O'Hara (2009) demonstrate that limited participation occurs in the presence of the marginal distribution ambiguity while assets are assumed to be independent. Uppal and Wang (2003) consider the ambiguity of both the joint distribution and the marginal distributions from a portfolio choice perspective. They show numerically that, when the overall ambiguity of the joint distribution is high, a small ambiguity difference on the marginal return distribution will result in an under-diversified portfolio. However, we are the first to demonstrate that under-diversification can be generated endogenously from the dispersion of correlation uncertainty, even without ambiguity on any marginal distribution. Furthermore, a well-diversified portfolio is associated with a better estimation of the correlated structure or smaller degree of correlation uncertainty. Otherwise, under-diversified or even a limited participation portfolio is obtained in equilibrium.

Our result is also related to several empirical studies of household portfolio choice.

Calvet, Campbell, and Sodini (2009) present evidence suggesting that the position of equity held in individual stocks on top of well-diversified portfolio (mutual fund or a market portfolio) is a reasonable proxy for portfolio under-diversification. By using the under-diversification measure proposed in Calvet et al. (2009), Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2016) further examine ambiguity-averse investors who view the overall market as a highly ambiguous under-diversified portfolio. They find that a one standard deviation increase in ambiguity aversion leads to a 38.9 percentage point increase in the proportion of equity allocated to individual stocks for those who view the overall market as highly uncertain. However, institutional investors with good knowledge about the market allocate little to individual stocks. These empirical findings are consistent with Proposition 17 because we also use the market portfolio as a benchmark and the overall market uncertainty is interpreted as the correlation uncertainty.

To explain our result for the under-diversification and limited participation numerically, we draw in Figure 2.2 the dispersion of the optimal portfolios for both investors in a full participation equilibrium. As drawn in the upper plot, the dispersion of the institutional investor's optimal portfolio, is always smaller than the corresponding dispersion of the retail investor in the lower plot, given the same degree of correlation uncertainty. To demonstrate the result completely, we also consider a limited participation equilibrium and compute the optimal portfolio's dispersion for each investor, in Table 2.5. As shown, the retail investor's dispersion is fairly close to one, which reflects his extremely under-diversified portfolio. By contrast, the dispersion of the institutional investor's optimal portfolio is between 0.625 and 0.629, indicating more diversified holdings.

Proposition 17 (2) states that the institutional investor is willing to choose a riskier portfolio than the retail investor due to the dispersion of correlation uncertainty among investors. The intuition is simple. Since ambiguity aversion leads to risk aversion, the retail investor behaves in a more risk-averse way; hence, he has smaller risk in his optimal portfolio. Proposition 17 (2) demonstrates that ambiguity aversion leads to risk aversion in the correlated structure,[44] and a higher correlation uncertainty yields a higher risk aversion; thus the corresponding optimal portfolio is less risky. Specifically, the variance of $\sum_i \tilde{a}_i x_i^{(s)}$ is strictly larger than the variance of $\sum_i \tilde{a}_i x_i^{(r)}$. Finally, as shown in Proposition 17 (3), the institutional investor holds a better performed optimal portfolio in terms of Sharpe ratios.

To conclude, our comparative portfolio analysis demonstrates that a robust limited participation and under-diversification phenomenon resulting from a large variation among investors' correlation estimations.

2.5.2 Flight to Quality and Flight to Safety

After examining the optimal portfolio as a whole, we study the trading positions as well as the trading volume of each individual asset in the optimal portfolio. By investigating how the correlation uncertainty affects the trading position and trading volume, our analysis helps explain the flight to quality and flight to safety phenomenon.

For simplicity we assume that the institutional investor is a Savage investor who knows the true correlation coefficient ρ, and the retail investor's plausible range of

[44] It is well known that uncertainty aversion yields risk aversion in the ambiguity literature. See Cao, Wang and Zhang (2005); Easley and O'Hara (2009, 2010); Garlappi, Uppal, and Wang (2007); Gollier (2011).

correlation coefficient is $[\rho-\epsilon, \rho+\epsilon]$. To study precisely how the degree of correlation uncertainty affects risk-sharing among investors, we assume that each investor initially holds a market portfolio (without the correlation uncertainty).

Proposition 18 1. *(*Portfolio Position*) For a low-eta asset i with $\eta_i < \frac{1}{N}$, the holding of the institutional investor* increases *and the holding of the retail investor* decreases *as the correlation uncertainty* increases. *For the holding of a high-eta asset, the effect of correlation uncertainty on both types of investors is* opposite.

2. *(*Trading Pattern*) Put*

$$J(\epsilon, \nu) = \frac{1}{1 + (N-1)\rho + (N-1)\nu\epsilon}.$$

The institutional investor always sells *high-eta assets satisfying $\eta > J(\epsilon, \nu)$ and* purchases *low-eta assets with $\eta < J(\epsilon, \nu)$; The retail investor always purchases high-eta assets with $\eta > J(\epsilon, \nu)$ and sells low-eta assets with $\eta < J(\epsilon, \nu)$.*

3. *(*Trading Volume*) The higher the correlation uncertainty, the larger the trading volume for the institutional investor, $\left| x_i^{(r)} - \overline{x}_i \right|$, and the retail investor, $\left| x_i^{(s)} - \overline{x}_i \right|$.*

As demonstrated in Proposition 18 (1), the correlation uncertainty affects each investor's position differently. When the retail investor's perceived degree of uncertainty increases, the institutional investor holds larger positions on low-eta assets and smaller positions on high-eta assets. On the contrary, the retail investor holds smaller positions on low-eta assets, and purchases more shares of high-eta assets. The property of the portfolio position under correlation uncertainty is displayed in Figure 2.3.

We investigate next the trading pattern and trading volume assuming that each investor's initial position is a well-diversified market portfolio. As shown in Proposition 18 (2), the retail investor sells low-eta assets and purchases high-eta assets. Correspondingly, the institutional investor buys low-eta assets and sells high-eta assets. Moreover, Propositions 18 (3) states that the trading volume for each investor increases on almost all assets regardless low-eta or high-eta. Put differently, the more uncertain the retail investor is on the correlated structure, the more trading or overreaction occurs in the market.

Propositions 18 (1)-(3) together describe a flight-to-safety or flight-to-quality episode under correlation uncertainty. When investors have different beliefs and the correlation uncertainty is high, investors' trading activities influence the asset prices significantly - a price decline with a fire sale of one asset class is associated with an increase in price and trading volume of another asset class during the same time period. Caballero and Krishnamurthy (2008), Guerrieri and Shimer (2014), and Vayanos (2004) characterize the flight-to-quality in contexts of model uncertainty, adverse selection, and liquidity risk, respectively. Our model complements these previous studies to demonstrate that correlation uncertainty generates flight-to-quality endogenously.

Proposition 18 (3) has another interpretation when we view ϵ as one form of disagreement between the investors. Under this interpretation, Proposition 18 (3) shows that a larger trading volume is associated with a larger disagreement between the institutional investor and the retail investor. Higher volatility is also associated with larger disagreement. Our finding is consistent with the empirical evidence in Carlin, Longstaff, and Matoba (2014).[45] In this paper, they argue that high volatility itself does not lead to higher trading volume, rather it is only when disagreement arises in the market that higher uncertainty is associated with more trading. Proposition 18 (3) presents a theoretical explanation of their findings through the correlation uncertainty mechanism. Moreover, our model reinforces the positive relation between the trading volume and the correlation uncertainty. Because the correlation uncertainty is positively associated with the aggregate market volatility, the model is also consistent with the market microstructure literature on the positive relation between volatility and trading volumes.

2.5.3 Asset Comovement

Finally, we examine how the correlation uncertainty impacts the asset comovement through the Sharpe ratios, from an investment perspective.

Proposition 19 *1. The relative Sharpe ratio* $\frac{s_i}{S}$ *always* decreases *with respect to the correlation uncertainty and* increases *with more institutional investors when* $\eta_i > \frac{1}{N}$*; and it displays* opposite *monotonic feature when* $\eta_i < \frac{1}{N}$*;*

2. $\Omega(s)$ *depends* negatively *on the correlation uncertainty, but it* increases *with respect to the number of institutional investors.*

Proposition 19 provides a comparative analysis of the relative Sharpe ratio $\frac{s_i}{S}$ and the dispersion of Sharpe ratios $\Omega(s)$, assuming the correlation uncertainty changes or

[45] According to the construction of disagreement index in Carlin, Longstaff, and Matoba (2014), the disagreement largely depends on the dispersion of investors' forecast, which is also often used to measure the ambiguous level.

the percentage of institutional investors varies. Proposition 19 (1) follows from the decomposition of the relative Sharpe ratio $\frac{s_i}{S}$,[46]

$$\frac{s_i}{S} - \frac{1}{N} = \frac{\Omega(s)}{\Omega(\eta)} \left(\eta_i - \frac{1}{N} \right). \tag{B-26}$$

Equation (B-26) reveals how the relative Sharpe ratio departs from $\frac{1}{N}$ is proportional to the distance between its eta and $\frac{1}{N}$. The sensitivity of the relative Sharpe ratio with respect to the degree of correlation uncertainty relies on how large its eta is to be, that is, the sign of $\eta_i - \frac{1}{N}$. This sensitivity is negative for a high-eta asset and positive for assets with low-eta. With increasing correlation uncertainty, $\frac{s_i}{S}$ decreases if $\eta_i > \frac{1}{N}$ and increases if $\eta_i < \frac{1}{N}$.

Proposition 19 (1) is important for examining the effect of correlation uncertainty on different assets. For a low-eta asset, its relative Sharpe ratio increases as the perceived correlation uncertainty grows; however, the relative Sharpe ratio decreases for a high-eta asset. In other words, high correlation uncertainty makes a high-eta asset less attractive, and at the same time, the low-eta asset becomes relatively more attractive. In the end, all assets comove under high correlation uncertainty, as presented in Proposition 19 (2). Figure 2.4 displays the dispersion of all Sharpe ratios with respect to the degrees of uncertainty in a heterogeneous equilibrium model.

[46]By a multiplication version of Equation (B-20), we obtain $\frac{s_i}{S} - \frac{1}{N} = \kappa \left(\eta_i - \frac{1}{N} \right)$, where κ is one number that is independent of the asset characteristics. Let $y_i = \kappa \left(\eta_i - \frac{1}{N} \right)$. Then $\sum_{i=1}^{N} y_i = 0$. By the definition of the dispersion, we obtain

$$\begin{aligned} \Omega(s)^2 &= \Omega \left(\frac{s}{S} \right)^2 = \frac{1}{N-1} \left(N \frac{\sum_{i=1}^{N} (y_i + \frac{1}{N})^2}{(\sum_i^N (y_i + \frac{1}{N}))^2} - 1 \right) \\ &= \frac{1}{N-1} N \sum_{i=1}^{N} y_i^2 = \kappa^2 \frac{N}{N-1} \left(\sum_{i=1}^{N} \eta_i^2 - \frac{1}{N} \right) = \kappa^2 \Omega(\eta)^2. \end{aligned}$$

Then $\kappa = \frac{\Omega(s)}{\Omega(\eta)}$.

Equation (B-26) is also useful in understanding the effect of correlation uncertainty on the comovement (contagion) pattern in the market. On one hand, fixing the risk of each asset, the higher $\frac{\Omega(\eta)}{\Omega(s)}$, the smaller the dispersion of the Sharpe ratio; thus, the higher the likelihood the assets move together from an investment perspective. On the other hand, fixing the Sharpe ratio of each asset, the higher $\frac{\Omega(\eta)}{\Omega(s)}$, the larger dispersion of the individual risks is. Therefore, we argue that $\frac{\Omega(\eta)}{\Omega(s)}$ can measure the contagion of the market.[47] Furthermore, $\frac{\Omega(\eta)}{\Omega(s)}$ depends only on each investor's endogenous correlation, and is independent of each asset's marginal distribution.

2.5.4 Empirical Implications

Our model offers several important empirical implications to the financial markets. As we have explained above, the model provides a new approach to explain under-diversification or limited participation, flight to quality, and asset covomement. It also generates some testable cross-sectional predictions on assets and portfolios with different characteristics.

Table 2.4 summarizes our model implications when the correlation uncertainty increases in three different categories. Panel A presents the effect of the correlation uncertainty at the market level. We consider three popular measures of asset comovement: the aggregative market volatility, the pairwise market correlation and the dispersion of Sharpe ratios. In Panel B, we present the cross-sectional effect on the in-

[47] It is easy to derive

$$\frac{\Omega(\eta)}{\Omega(s)} = \frac{\frac{\nu}{1-\rho_1} + \frac{1-\nu}{1-\rho_2}}{\frac{\nu}{1+(N-1)\rho_1} + \frac{1-\nu}{1+(N-1)\rho_2}}.$$

In particular, $\frac{\Omega(\eta)}{\Omega(s)} = \frac{1+(N-1)\rho}{1-\rho}$ in a homogeneous equilibrium increases with the correlation coefficient ρ. It displays similar property of the contagion measure in Forbes and Rigobon (2002).

dividual asset given different asset characteristics, including the asset prices, the risk premiums, Sharpe ratios, the relative Sharpe ratios, the correlations with the market portfolio and the weighted betas. The high-eta and low-eta asset is largely opposite on each element discussed here. Finally in Panel C, we compare the institutional investor and retail investor, in terms of their optimal portfolio, holding position, and trading volume on individual assets. The under-diversification and limited participation puzzle as well as the flight to quality phenomenon are shown.

While a complete empirical test of our model is beyond the scope of this paper, we take the 2007-2009 financial crisis as one example to further illustrate our model implications.[48]

In the period of the 2007-2009 financial crisis, the investor was very uncertain about the entire financial market. Following Bloom (2009), and Baele et al. (2013), we use the Chicago Board Options Exchange Volatility Index (VIX) to measure ambiguity in the overall financial market. Figure 2.5 displays the VIX as well as S&P 500 index from 2006 to 2016. As shown clearly, the VIX is extremely high for all of 2008, representing a very high degree of uncertainty in the market. We also observe that the VIX index and S&P index move in opposite directions consistently over the entire period from 2006 to 2016.

To conduct a general analysis in the heterogeneous environment, we consider two types of asset classes. We treat the entire stock market as one asset class and the fixed income market (in particular, the Treasury market) as another asset class.[49] Although

[48]Given the results in this paper, our model can also be used to discuss other recent flight-to-quality phenomenon such as Black Monday 1987, the Russian debt default and Long-term capital management (LTCM) in 1998, the September 11 in 2001, among many others.

[49]During financial crisis periods, the stock market typically displays a significant decline and the

the volatility of the stock market is larger than the volatility of the fixed income market, the volume of the fixed income market is much larger. The stock market can be seen as a low-eta asset and the fixed income market as a high-eta asset. To illustrate, we follow McKinsey Global Institute research (www.mckinsey.com/mgi) and report, in Figure 2.6, the global stock market and the fixed-income market (including public debt, financial bonds, corporate bonds, securitized loan, and unsecuritized loans outstanding) between 2005 to 2014. The total volume of the fixed-income market is about four times larger than that of the stock market. Given that the volatility of the stock market is around three times that of the fixed-income market according to historical data (Reilly, Wright, and Chan, 2000), the weighted volatility of the stock market is about 75 percent of the weighted volatility of the fixed income market. Therefore, the stock market can be viewed as a low-eta asset class while the fixed income market, as another asset class, is a high-eta asset class.

The asset price movement during the period of the 2007-2009 financial crisis is consistent with Proposition 15. The price of the low-eta asset, the stock market, drops significantly during the financial crisis. At the same time, for high-eta assets, we see that the Treasury rates drop and the government bond prices increase, in Figure 2.7.

We now consider, our model's implications to flight-to-quality and flight-to-safety. Proposition 18 states that the institutional investor holds more on the stock market since she has perfect knowledge of the overall market, and the price decline of the stock

Treasury yield rallies in a short-term period. Baele et al.(2013) empirically characterize flight-to-quality episode using equity index and the Treasury bond. We follow Baele et al.(2013) to compare the equity market and the Treasury market in our explanations market virtually follows from the retail investor's over-selling on the stock market. Moreover, the more uncertain the retail investor is about the entire market, the fewer positions he holds on the stock market; in turn, the institutional investor holds more equity positions. Similarly, for high-eta assets (for instance, government bonds), which are used to hedge against "economic catastrophe risk" (a safe haven), the retail investor holds more and more positions.

During the 2007-2009 financial crisis time period, when the retail investor had a very high perceived degree of ambiguity aversion for the entire financial market, dramatic trading activities and extreme price declines took place on the stock market and a substantial price increase pattern emerged for government bonds, especially the Treasury bonds, in a short time period. Moreover, Proposition 18 (3) explains the huge volume of trading during this time period due to a high correlation uncertainty.

2.6 Conclusion

In investigating the complicated correlation structure among asset classes and the nature of the well-documented stylized facts on correlated structure, this paper develops an equilibrium model in the presence of correlation uncertainty in which two types of investors have heterogeneous beliefs in their correlation estimation. We find that those correlation-related phenomena can be inherently connected through the disagreement among investors on the correlation structure and the asset characteristics, when the marginal distribution of each risky asset is perfectly known. Our model demonstrates that correlation uncertainty is an essential factor in studying asset prices, volatilities, and correlations, which can not be fully explained by fundamentals.

Specifically, the choice of correlation coefficient for the retail investor is shown to be irrelevant even though his optimal portfolio is uniquely determined in equilibrium where: 1) the disagreement on correlation estimation is large, 2) more institutional investors emerge in the market, or 3) the dispersion of asset risks is high. Otherwise, each investor chooses the corresponding highest plausible correlation coefficient. Our portfolio analysis demonstrates that the institutional investor always holds a diversified portfolio versus the retail investor, who is under-diversified. The optimal portfolio becomes less diversified when the perceived level of the correlation uncertainty increases. This equilibrium model is helpful for explaining several empirical puzzles heretofore presented concerning correlation, including under-diversification, flight-to-quality, and asset comovement.

Appendix A: Asset Characteristics

In this appendix we explain the economic insights of the asset-characteristic parameter η_i in an equicorrelation model. We show that the set of $\{\eta_i\}$, $\{corr\left(\tilde{R}_i, \tilde{R}_m\right)\}$, and the weighted beta $\{\beta_i w_i\}$, are determined each other, thus η_i characterizes the sensitivity of $\tilde{R}_i$ with respect to the market portfolio return $\tilde{R}_m$ and the weighted beta. Our result does not depend on any distribution assumption of the asset return $\tilde{R}_i$.

Proposition 20 *In an equicorrelation model with $corr(\tilde{R}_i, \tilde{R}_j) = \rho, \forall i \neq j$, the market portfolio return $\tilde{R}_m = \sum_{i=1}^N w_i \tilde{R}_i$.*

1. *Given η_i, $i = 1, \cdots, N$, the correlation coefficient between individual asset return with the market portfolio return, $corr(\tilde{R}_i, \tilde{R}_m)$, is given by the following equation*

$$corr(\tilde{R}_i, \tilde{R}_m) = \frac{\rho + \eta_i(1-\rho)}{\sqrt{\rho + \sum_{i=1}^N \eta_i^2 (1-\rho)}}. \tag{A-1}$$

Conversely, let $\alpha_i = corr(\tilde{R}_i, \tilde{R}_m), i = 1, \cdots, N$ with $\sum_{i=1}^N \alpha_i \neq 0$, then

$$\eta_i = \frac{\alpha_i}{\sum_{i=1}^N \alpha_i} \cdot \frac{1 + (N-1)\rho}{1-\rho} - \frac{\rho}{1-\rho}. \tag{A-2}$$

2. *The weighted beta for asset i is*

$$\beta_i w_i = \frac{\eta_i(\rho + \eta_i(1-\rho))}{\rho + \sum_{i=1}^N \eta_i^2 (1-\rho)}. \tag{A-3}$$

Conversely, given a set of weighted beta, $\{\beta_i w_i, i = 1, \cdots, N\}$, we obtain

$$\eta_i = \frac{-\rho + \sqrt{\rho^2 + 4(1-\rho) V \beta_i w_i}}{2(1-\rho)}, i = 1, \cdots, N \tag{A-4}$$

where V is solved by the following equation

$$\sum_{i=1}^{N} \sqrt{\rho^2 + 4V(1-\rho)\beta_i w_i} = 2(1-\rho) + N\rho. \tag{A-5}$$

Proposition 20 (1) determines explicitly the correlation coefficient $corr(\tilde{R}_i, \tilde{R}_m)$ in terms of its eta and the dispersion of etas. In particular, if each individual asset has the same weighted volatility, it has the same eta and each $\eta_i = \frac{1}{N}$ due to the fact that $\sum_{i=1}^{N} \eta_i = 1$. Moreover, $\Omega(\eta) = 0$ if each eta is $\frac{1}{N}$. Then, for each asset $i = 1, \cdots, N$, it has the same correlation coefficient with the market portfolio by We also see that η_i is up to a linear transformation of the relative correlation with the market portfolio $\frac{corr(\tilde{R}_i, \tilde{R}_m)}{\sum_{i=1}^{N} corr(\tilde{R}_i, \tilde{R}_m)}$. Similarly, Proposition 20 (2), demonstrates that the weighted beta $\beta_i w_i$ is determined by the eta, and vice versa.

Proof: (1) Since $corr(\tilde{R}_i, \tilde{R}_j) = \rho, \forall i \neq j$, we have

$$Cov\left(\tilde{R}_i, \tilde{R}_m\right) = Cov\left(\tilde{R}_i, \sum w_j \tilde{R}_j\right) = \left(\sum_j w_j \hat{\sigma}_j\right) \hat{\sigma}_i \rho + w_i \hat{\sigma}_i^2 (1-\rho). \tag{A-6}$$

It follows that

$$corr(\tilde{R}_i, \tilde{R}_m) = \frac{\left(\sum_j w_j \hat{\sigma}_j\right) \rho + w_i \hat{\sigma}_i (1-\rho)}{\hat{\sigma}_m}, \tag{A-7}$$

where $\hat{\sigma}_m$ is the volatility of the market portfolio. Equation (A-6) yields

$$\hat{\sigma}_m^2 = (\sum_j w_j \hat{\sigma}_j)^2 \rho + \sum_j w_j^2 \hat{\sigma}_j^2 (1-\rho). \tag{A-8}$$

By using the dispersion $\Omega(\eta) = \Omega(w\hat{\sigma})$, we obtain

$$\hat{\sigma}_m = \left(\sum_{j=1}^{N} w_j \hat{\sigma}_j\right) \sqrt{\rho + \frac{(N-1)\Omega(\eta)^2 + 1}{N}(1-\rho)}. \tag{A-9}$$

By plugging Equation (A-9) into Equation (A-7), we derive Equation (A-1) as desired.

Conversely, let x denote the denominator in Equation (A-1), then by Equation (A-1) again with $\alpha_i = corr(\tilde{R}_i, \tilde{R}_m)$, we obtain

$$\rho + \eta_i(1-\rho) = \alpha_i x. \tag{A-10}$$

By using $\sum_{i=1}^{N} \eta_i = 1$, it follows that

$$\sum_{i=1}^{N} \alpha_i x = N\rho + 1 - \rho, \tag{A-11}$$

yielding

$$x = \frac{1+(N-1)\rho}{\sum_{i=1}^{N} \alpha_i}. \tag{A-12}$$

Equation (A-2) follows from Equation (A-10).

(2) By definition, the asset beta is given by

$$\beta_i = \frac{Cov(\tilde{R}_i, \tilde{R}_m)}{Var(\tilde{R}_m)} = \frac{corr(\tilde{R}_i, \tilde{R}_m)\hat{\sigma}_i}{\hat{\sigma}_m},$$

then

$$\beta_i w_i = corr(\tilde{R}_i, \tilde{R}_m)\frac{\hat{\sigma}_i w_i}{\hat{\sigma}_m}.$$

By plugging equation (A-8) into the last equation and using equation (A-1), we obtain equation (A-3). Conversely, given a set of weighted beta, and employing equation (A-1), we have

$$\eta_i = \frac{-\rho + \sqrt{\rho^2 + 4V(1-\rho)\beta_i w_i}}{2(1-\rho)}, i = 1, \cdots, N \tag{A-13}$$

where $V = \rho + \sum_{i=1}^{N} \eta_i^2(1-\rho)$. It suffices to derive the constant V by using the weighted betas. In fact, by squaring both sides of equation (A-4), we have

$$\eta_i^2 = \frac{2\rho^2 + 4(1-\rho)V\beta_i w_i - 2\rho\sqrt{\rho^2 + 4(1-\rho)V\beta_i w_i}}{4(1-\rho)^2},$$

then

$$\sum_{i=1}^{N} \eta_i^2 = \frac{2N\rho^2 + 4(1-\rho)V - 2\rho\sum_{i=1}^{N}\sqrt{\rho^2 + 4(1-\rho)A\beta_i w_i}}{4(1-\rho)^2}.$$

Since $V = \rho + \sum_{i=1}^{N} \eta_i^2(1-\rho)$, replacing $\sum_{i=1}^{N} \eta_i^2$ by $\frac{V-\rho}{1-\rho}$ in the last equation, we derive the equation of V as desired. $\square$

Appendix B: Proofs of Propositions 12 - 19

We start with several simple lemmas without proofs.

Lemma 6 *Suppose A is an invertible $N \times N$ matrix, and u, v are $N \times 1$ vectors. Suppose further that $1 + v^T A^{-1} u \neq 0$. Then the matrix $A + uv^T$ is invertible and*

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1} u v^T A^{-1}}{1 + v^T A^{-1} u}. \tag{B-1}$$

Lemma 7 *Let $\rho \neq 1, \rho \neq -\frac{1}{N-1}$, x_ρ is defined in Proposition 12. Then*

$$\Omega(\sigma x_\rho) = \Omega(s) \frac{1 + (N-1)\rho}{1 - \rho}. \tag{B-2}$$

Lemma 8 *Let*

$$G(\rho) = \frac{S^2}{N} \left(\frac{(N-1)\Omega(s)^2}{1-\rho} + \frac{1}{1 + (N-1)\rho} \right).$$

Then $argmin_{\rho \in [\rho^{min}, \rho^{max}]} G(\rho)$ is given by, when $\Omega(s) \neq \frac{1}{N-1}$,

$$\rho^* = \begin{cases} \rho^{min}, & \text{if } \rho^{min} > \tau(\Omega(s)), \\ \rho^{max}, & \text{if } \rho^{max} < \tau(\Omega(s)), \\ \tau(\Omega(s)), & \text{if } \tau(\Omega(s)) \in [\rho^{min}, \rho^{max}]. \end{cases} \tag{B-3}$$

If $\Omega(s) = \frac{1}{N-1}$, then ρ^* is given similarly in which $\tau(\Omega(s))$ is replaced by $\frac{N-2}{2(N-1)}$.

Lemma 9 *Assume that $\kappa = \frac{\nu a + (1-\nu) b}{\nu c + (1-\nu) d}$ with $a, b, c, d > 0$ and $\nu \in (0, 1)$. Then*

$$\min \left\{ \frac{a}{c}, \frac{b}{d} \right\} \leq \kappa \leq \max \left\{ \frac{a}{c}, \frac{b}{d} \right\}. \tag{B-4}$$

The both inequalities are strictly if $\frac{a}{c} \neq \frac{b}{d}$.

For simplicity, let τ be a linear fractional transformation: $\tau(t) \equiv \frac{1-t}{1+(N-1)t}$ for any real number $t \neq -\frac{1}{N-1}$.

Proof of Proposition 12.

By Sion's theorem (1958),

$$\begin{aligned} A &= \max_{x\in\mathbb{R}^N} \min_{\rho\in[\rho^{min},\rho^{max}]} \left\{ (\overline{a}_i - p_i)x_i - \frac{\gamma}{2}\sum_{i,j=1}^{N} x_i x_j \sigma_i \sigma_j R(\rho)_{ij} \right\} \\ &= \min_{\rho\in[\rho^{min},\rho^{max}]} \max_{x\in\mathbb{R}^N} \left\{ (\overline{a}_i - p_i)x_i - \frac{\gamma}{2}\sum_{i,j=1}^{N} x_i x_j \sigma_i \sigma_j R(\rho)_{ij} \right\}. \end{aligned}$$

It is well known that $\max_{x\in\mathbb{R}^N} \left\{ (\overline{a}_i - p_i)x_i - \frac{\gamma}{2}\sum_{i,j=1}^{N} x_i x_j \sigma_i \sigma_j R(\rho)_{ij} \right\}$ is given by $\frac{1}{2\gamma}G(\rho)$, where

$$\begin{aligned} G(\rho) &= s^T R(\rho)^{-1} s = \frac{N\sum_{n=1}^{N} s_n^2 - (\sum_{n=1}^{N} s_n)^2}{N(1-\rho)} + \frac{(\sum_{n=1}^{N} s_n)^2}{N(1+(N-1)\rho)} \\ &= \frac{S^2}{N}\left(\frac{(N-1)\Omega(s)^2}{1-\rho} + \frac{1}{1+(N-1)\rho} \right). \end{aligned}$$

Therefore, $A = \min_{\rho\in[\rho^{min},\rho^{max}]} \frac{1}{2\gamma}G(\rho)$.

By Lemma 3, we obtain

$$A = \frac{1}{2\gamma}G(\rho^*) = CE(\rho^*, x_{\rho^*}). \tag{B-5}$$

(1). If $\rho^{min} > \tau(\Omega(s))$, then by definition $\rho^* = \rho^{min}$. Moreover, by Lemma 2, $\Omega(\sigma x_{\rho^{min}}) > 1$ and thus $\max_{\Omega(\sigma x)>1} CE(\rho^{min}, x) = CE(\rho^{min}, x_{\rho^{min}})$. Therefore, $A = CE(\rho^{min}, x_{\rho^{min}})$, and by using Equation (B-12), the solution of the problem (B-6) is given by $\rho^* = \rho^{min}$, and $x^* = x_{\rho^{min}}$.

(2). If $\rho^{max} < \tau(\Omega(s))$, then by definition $\rho^* = \rho^{max}$. By Lemma 2, $\Omega(\sigma x_{\rho^{max}}) < 1$

and $\max_{\Omega(\sigma x)<1} CE(\rho^{max}, x) = CE(\rho^{max}, x_{\rho^{max}})$. Hence $A = \max_{\Omega(\sigma x)<1} CE(\rho^{max}, x) = CE(\rho^{max}, x_{\rho^{max}})$. By using Equation (B-12), $\rho^* = \rho^{max}, x^* = x_{\rho^{max}}$ is the solution of the portfolio choice problem (B-6).

(3). Assume that $\rho^{min} \leq \tau(\Omega(s)) \leq \rho^{max}$. Then $\rho^* = \tau(\Omega(s))$, and by Equation (B-5), $A = CE\left(\tau(\Omega(s)), x_{\tau(\Omega(s))}\right)$. Moreover, by Lemma 2, $\Omega\left(x_{\tau(\Omega(s))}\right) = 1$. By straightforward calculation, we have

$$A = \frac{(\sum_i s_i)^2}{2\gamma}\left(\frac{1+(N-1)\Omega(s)}{N}\right)^2.$$

On the other hand, for each x^* with $\Omega(\sigma x^*) = 1$ and $CE(\tau(\Omega(s)), x^*) = \max_{\Omega(\sigma x)=1} CE(\tau(\Omega(s)), x^*)$, we have $CE(\tau(\Omega(s)), x^*) = CE(\tau(\Omega(s)), x_{\tau(\Omega(s))})$, and because of the uniqueness x_ρ for maximizing $CE(\rho, x)$, $x^* = x_{\tau(\Omega(s))}$. Therefore, we have shown that the unique demand for the ambiguity-averse investor is $x_{\tau(\Omega(s))}$, but the investor is irrelevant to choosing any correlation coefficient $\rho \in [\rho^{min}, \rho^{max}]$ since $CE(\rho, x_{\tau(\Omega(s))}) = A$ for each $\rho \in [\rho^{min}, \rho^{max}]$.

The proof of Proposition 12 is completed. $\square$

Proof of Proposition 13.

The optimal demand x^* is presented by Proposition 12. By the market-clearing condition, $x^* = \overline{x}$ in equilibrium. Then $\Omega(\sigma x^*) = \Omega(\sigma\overline{x})$. Since $\Omega(\sigma\overline{x}) < 1$, the optimal demand in equilibrium satisfies $\Omega(\sigma x^*) < 1$. Then, by Proposition 12 again, the optimal correlation coefficient is $\rho^* = \rho^{max}$, the highest possible correlation coefficient.

$\square$

In what follows, for simplicity reason we do not distinguish investor $j = 1, 2$ or $j = s, r$ for institutional investor and retail investor, respectively.

Proof of Proposition 14.

The optimal demand of type j investor is $x^{(j)} = \frac{1}{\gamma}\sigma^{-1}R_j^{-1}s$ and R_j corresponds to an endogenous correlation coefficient $\rho_j \in [\rho_j^{min}, \rho_j^{max}]$. Notice that $x^{(j)}$ is unique regardless of the optimal correlation coefficient in Proposition 12 (3) and in this case let $\rho_j = \tau(\Omega(s))$. In equilibrium, we have $\nu x^{(1)} + (1-\nu)x^{(2)} = \overline{x}$. Then

$$\frac{1}{\gamma}(\nu R_1^{-1} + (1-\nu)R_2^{-1}) \cdot s = \sigma\overline{x}. \tag{B-6}$$

Let $X = \nu R_1^{-1} + (1-\nu)R_2^{-1}$, $m \equiv m(\rho_1,\rho_2) \equiv \frac{\nu}{1-\rho_1} + \frac{1-\nu}{1-\rho_2}, n \equiv n(\rho_1,\rho_2) = \frac{\nu\rho_1}{(1-\rho_1)(1+(N-1)\rho_1)} + \frac{(1-\nu)\rho_2}{(1-\rho_2)(1+(N-1)\rho_2)}$ and notice that $m - Nn = \frac{\nu}{1+(N-1)\rho_1} + \frac{1-\nu}{1+(N-1)\rho_2} > 0$. Then by Lemma 1, X is invertible and its inverse matrix is

$$X^{-1} = \frac{1}{m}\left(I_N + \frac{n}{\kappa m}ee^T\right) \tag{B-7}$$

where $\kappa \equiv \kappa(\rho_1,\rho_2) = \frac{m(\rho_1,\rho_2) - Nn(\rho_1,\rho_2)}{m(\rho_1,\rho_2)}$. Therefore, $s = \gamma X^{-1} \cdot (\sigma x)$. By straightforward calculation, we obtain the following fundamental relation between the dispersion of Sharpe ratios and the dispersion of risks:

$$\Omega(s) = \kappa\Omega(\sigma\overline{x}). \tag{B-8}$$

Assume first that $\Omega(\sigma\overline{x}) = 0$, then each $\sigma_i\overline{x}_i = c$ and Equation (B-8) ensures that $\Omega(s) = 0$ and $\tau(\Omega(s)) = 1$. Therefore, each investor chooses her highest correlation coefficient in equilibrium by Proposition 12. Moreover, all Sharpe ratios are the same and are equal to

$$s_i = \frac{c}{m}\left(1 + \frac{nN}{m - nN}\right) = \frac{c}{m - nN}. \tag{B-9}$$

We next assume that $\Omega(\sigma\overline{x}) \in (0,1)$ and characterize the equilibrium in general.

By using Proposition 12, there are five different cases regarding the equilibrium. By direct computation,

$$\kappa = \kappa(\rho_1, \rho_2) = \frac{\frac{\nu}{1+(N-1)\rho_1} + \frac{1-\nu}{1+(N-1)\rho_2}}{\frac{\nu}{1-\rho_1} + \frac{1-\nu}{1-\rho_2}}. \tag{B-10}$$

Case 1. We first assume $\tau(\Omega(s)) \leq \rho_2^{min} < \rho_1^{min}$, which ensures that $\Omega(s) \geq \tau(\rho_2^{min}) > \tau(\rho_1^{min})$. By Proposition 12, $\rho_i = \rho_i^{min}, i = 1, 2$. By Equation (B-8),

$$\kappa = \kappa(\rho_1^{min}, \rho_2^{min}) = \frac{\Omega(s)}{\Omega(\sigma\overline{x})} > \Omega(s) > \tau(\rho_1^{min}), \tau(\rho_2^{min}), \tag{B-11}$$

which is impossible by Lemma 4.

Case 2. In this case, $\tau(\rho_2^{min}) > \Omega(s) \geq \tau(\rho_1^{min})$. Then by Proposition 12, $\rho_1 = \rho_1^{min}$ and we can choose $\rho_2 = \tau(\Omega(s))$. We obtain

$$\kappa = \kappa(\rho_1^{min}, \tau(\Omega(s))) > \kappa\Omega(\sigma\overline{x}) = \Omega(s) \geq \frac{1-\rho_1^{min}}{1+\rho_1^{min}(N-1)}, \kappa > \Omega(s) = \frac{1-\tau(\Omega(s))}{1+(N-1)\tau(\Omega(s))}. \tag{B-12}$$

Hence, it is impossible by using Lemma 4 again.

Case 3. We prove that it is impossible that $\tau(\Omega(s)) \in [\rho_1^{min}, \rho_1^{max}]$ in equilibrium. Otherwise, the optimal holding of each investor is $x_{\tau(\Omega(s))}$ by Proposition 12. Then the market-clearing condition yields that $x_{\tau(\Omega(s))} = \overline{x}$. However, by Lemma 2, $\Omega(\sigma x_{\tau(\Omega(s))}) = 1$ but $\Omega(\sigma\overline{x}) < 1$. Therefore, Case 3 is not possible in equilibrium.

Case 4. We characterize the equilibrium in which $\rho_1^{max} < \tau(\Omega(s)) \leq \rho_2^{max}$. By Proposition 12, $\rho_1 = \rho_1^{max}$, we can choose $\rho_2 = \tau(\Omega(s))$, and the optimal holding for the retail investor is $x_{\tau(\Omega(s))}$. Moreover,

$$\tau(\rho_2^{max}) \leq \Omega(s) < \tau(\rho_1^{max}). \tag{B-13}$$

By Equation (B-8) and direct computation, we have

$$\kappa = \kappa(\rho_1^{max}, \tau(\Omega(s))) = \frac{\Omega(s)}{\Omega(\sigma\overline{x})} = \frac{\frac{\nu}{1+(N-1)\rho_1^{max}} + \frac{1-\nu}{N}(1+(N-1)\Omega(s))}{\frac{\nu}{1-\rho_1^{max}} + \frac{1-\nu}{\Omega(s)N}(1+(N-1)\Omega(s))}. \tag{B-14}$$

By solving the last equation in $\Omega(s)$, we obtain

$$\Omega(s) = \frac{\frac{\nu}{1+(N-1)\rho_1^{max}}\Omega(\sigma\overline{x}) - \frac{1-\nu}{N}(1-\Omega(\sigma\overline{x}))}{\frac{\nu}{1-\rho_1^{max}} + \frac{(1-\nu)(N-1)}{N}(1-\Omega(\sigma\overline{x}))}. \tag{B-15}$$

Define

$$K(\rho) = \frac{\frac{1}{1-\rho} - \frac{\Omega(\sigma\overline{x})}{1+(N-1)\rho} + \frac{(1-\nu)(1-\Omega(\sigma\overline{x}))}{\nu}}{\frac{1}{1-\rho} + \frac{(N-1)\Omega(\sigma\overline{x})}{1+(N-1)\rho}}, \tag{B-16}$$

then $K(\rho_1^{max}) = \frac{1-\Omega(s)}{1+(N-1)\Omega(s)}$. $\Omega(s) \geq 0$ ensures that

$$\rho_1^{max} \leq \frac{1}{N-1}\left\{\frac{\nu}{1-\nu}\frac{\Omega(\sigma\overline{x})}{1-\Omega(\sigma\overline{x})}N - 1\right\}. \tag{B-17}$$

Moreover, the left side of Equation (B-13) is translated as $\rho_2^{max} \geq K(\rho_1^{max})$, and the right side of Equation (B-13) is $K(\rho_1^{max}) \geq \rho_1^{max}$ which holds always. Then there exists a unique equilibrium in Case 4, a limited participation equilibrium, under conditions presented in Proposition 14.

Case 5. We characterize the equilibrium in which $\tau(\Omega(s)) > \rho_2^{max}$.

By Proposition 12, $\rho_1 = \rho_1^{max}, \rho_2 = \rho_2^{max}$, and $\Omega(s) < \tau(\rho_2^{max}) < \tau(\rho_1^{max})$, which is equivalent to $\kappa(\rho_1^{max}, \rho_2^{max})\Omega(\sigma\overline{x}) < \frac{1-\rho_2^{max}}{1+(N-1)\rho_2^{max}}$. By straightforward computation, this condition equals to $\rho_2^{max} < K(\rho_1^{max})$.

To the end, we note that when ρ_1^{max} is large enough such that

$$\frac{\Omega(\sigma\overline{x})\nu N}{(1-\nu)(1-\Omega(\sigma\overline{x}))} \leq 1 + (N-1)\rho_1^{max}, \tag{B-18}$$

or equivalently,

$$\rho_1^{max} \geq \frac{1}{N-1}\left\{\frac{\nu}{1-\nu}\frac{\Omega(\sigma\overline{x})}{1-\Omega(\sigma\overline{x})}N-1\right\},$$

then $\rho_2^{max} < K(\rho_1^{max})$ holds naturally since $\rho_2^{max} < 1$. Then we have characterized the equilibrium in Proposition 14.

Finally, by using this equilibrium characterization, we see that each $s_i > 0$. Therefore, each risky asset is priced at discount in equilibrium. $\square$

Proof of Proposition 17.

(1). We compare the investors' optimal portfolios with the market portfolio in terms of the dispersion measure $\Omega(\cdot)$. First, Equation (B-8) states that $\Omega(s) = \kappa(\rho_1, \rho_2)\Omega(\sigma\overline{x})$. In the full participation equilibrium, Lemma 2 implies that $\Omega(s) = \Omega(\sigma x^{(j)})\frac{1-\rho_j}{1+(N-1)\rho_j}$. Then, by Lemma 4 we have

$$\Omega(\sigma x^{(s)}) < \Omega(\sigma\overline{x}) < \Omega(\sigma x^{(r)}). \tag{B-19}$$

The proof in the limited participation equilibrium is the same. Since $\rho_1^{max} \leq \tau(\Omega(s))$, Lemmas 2 and 4 together imply that $\Omega(\sigma x^{(s)}) < \Omega(\sigma\overline{x})$. Moreover $\Omega(\sigma\overline{x}) < 1 = \Omega(\sigma x^{(r)})$.

(2). Let $X^{(s)} = \sum_i \tilde{a}_i x_i^{(s)}$ be the optimal portfolio of the institutional investor and $X^{(r)}$ is the optimal portfolio of the retail investor. We have $Var(X^{(s)}) = \frac{1}{\gamma^2}s^T R(\rho_1^{max})^{-1}s$ where the correlation coefficient is ρ_1, and $Var(X^{(r)}) = \frac{1}{\gamma^2}s^T R(\rho_2)^{-1}s$ with the correlation coefficient ρ_2. By using the same notation in Lemma 3, we have

$$Var(X^{(s)}) - Var(X^{(r)}) = \frac{1}{\gamma^2}\left\{G(\rho_1^{max}) - G(\rho_2)\right\}. \tag{B-20}$$

Applying Proposition 14, since the portfolio choice is under the worst-case correlation uncertainty, $G(\rho_1^{max}) > G(\rho_2)$ holds. Therefore, $Var(X^{(s)}) > Var(X^{(r)})$.

(3). $\mathbb{E}[X^{(s)}] = \sum_{i=1}^{N} x_i^{(s)}(\overline{a}_i - p_i) = (\sigma x^{(s)})^T s = \frac{1}{\gamma} s^T R(\rho_1^{max})^{-1} s$. By (2), the variance of $X^{(s)}$ is $\frac{1}{\gamma^2} s^T R(\rho_1^{max})^{-1} s$. Then the Sharpe ratio of the portfolio $X^{(s)}$ is

$$SR(X^{(s)}) = \sqrt{s^T R(\rho_1^{max})^{-1} s}.$$

Therefore, $SR(X^{(s)}) > SR(X^{(r)})$ follows from $G(\rho_1^{max}) > G(\rho_2)$. Moreover, by the proof of Proposition 12, the institutional investor's maxmin expected utility is $CH(\rho_1^{max}, x_{\rho_1^{max}}) = \frac{1}{2\gamma} G(\rho_1^{max})$. Again, the fact that $G(\rho_1^{max}) > G(\rho_2)$ ensures that the institutional investor has a higher maxmin expected utility than the retail investor.

□

Proof of Proposition 15.

By the characterization of the Sharpe ratios in Proposition 14, we obtain

$$s_i = \frac{S}{N} + \frac{\gamma}{m(\rho_1, \rho_2)} \left(\sigma_i \overline{x}_i - \frac{L}{N} \right). \tag{B-21}$$

Both (1) and (2) follow from Equation (B-21) easily. □

Proof of Proposition 18.

(1). By using the characterization of Sharpe ratios in Proposition 14, we can prove that

$$\frac{s_i}{S} - \frac{1}{N} = \frac{m(\rho, \rho+\epsilon) - Nn(\rho, \rho+\epsilon)}{m(\rho, \rho+\epsilon)} \left(\eta_i - \frac{1}{N} \right). \tag{B-22}$$

We can easily show that $\frac{s_i}{S}$ is increasing with respect to ϵ when $\eta_i < \frac{1}{N}$. Since the average Sharpe ratio S is always increasing with respect to the level of correlation uncertainty, and

$$\gamma\sigma_i x_i^{(s)} = \frac{1}{1-\rho} S \left(\frac{s_i}{S} - \frac{\rho}{1+(N-1)\rho} \right), \tag{B-23}$$

thus $\frac{\partial}{\partial\epsilon}\left(x_i^{(s)}\right) > 0$. By using the market- clearing equation, $\nu x_i^{(s)} + (1-\nu)x_i^{(r)} = \overline{x}_i$, we have $\frac{\partial}{\partial\epsilon}\left(x_i^{(r)}\right) < 0$.

If η_i is large, the level of correlation uncertainty to s_i is negative, by Proposition 15 (4), but S is positively relates to ϵ. Therefore, Equation (B-23) implies that $\frac{\partial}{\partial\epsilon}\left(x_i^{(s)}\right) < 0$ and $\frac{\partial}{\partial\epsilon}\left(x_i^{(r)}\right) > 0$.

(2). By using the expression of $x_i^{(s)}$ and the expression of Equation (B-23) and the expression of s_i, S, it is easy to check that $\gamma\sigma_i x_i^{(s)} < \gamma\sigma_i\overline{x}_i$ is equivalent to $\eta_i > J(\epsilon,\nu)$. Because of the market-clearing condition, $x_i^{(r)} < \overline{x}_i$ if and only if $\eta_i < J(\epsilon,\nu)$.

(3). We next examine the trading volume. First, the retail investor buys the high-eta asset with $\eta > J(\epsilon,\nu)$, so the trading volume is $|x_i^{(r)} - \overline{x}_i| = x_i^{(r)} - \overline{x}_i$. Then, by Proposition 18 (1),

$$\frac{\partial}{\partial\epsilon}\left(x_i^{(r)} - \overline{x}_i\right) = \frac{\partial}{\partial\epsilon}\left(x_i^{(r)}\right) > 0.$$

However, for the low-eta asset, the naive trading volume is $\overline{x}_i - x_i^{(r)}$ since he needs to sell the initial position, thus, by Proposition 18 (1), we obtain

$$\frac{\partial}{\partial\epsilon}\left(\overline{x}_i - x_i^{(r)}\right) = -\frac{\partial}{\partial\epsilon}\left(x_i^{(r)}\right) > 0.$$

By the similar argument, we can show that, for the institutional investor,

$$\frac{\partial}{\partial\epsilon}\left|\overline{x}_i - x_i^{(s)}\right| > 0.$$

$\square$

Proof of Proposition 16.

(1) The first part follows from the expression of $corr(\tilde{R}_i, \tilde{R}_m)$ in terms of η_i in Appendix A. Let $a \equiv \sum_{i=1}^N \eta_i^2 = \frac{(N-1)\Omega(\eta)^2+1}{N}$. By Equation (A-1), it suffices to show that

$$\frac{\partial}{\partial \rho}\left(\frac{(\rho + \eta_i(1-\rho))^2}{\rho + a(1-\rho)}\right) > 0. \tag{B-24}$$

By straightforward calculation, we see that this partial derivative is a positive number times $2a(1-\eta_i) - (1-a)\eta_i + (1-a)(1-\eta_i)\rho$. Under conditions on $\eta_i, i = 1, \cdots, N$, we obtain

$$\frac{2a}{1-a} = \frac{2\sum_{i=1}^N \eta_i^2}{\sum_{i=1}^N(\eta_i - \eta_i^2)} \geq \frac{2\eta_N}{1-\eta_N} \geq \frac{\eta_i}{1-\eta_i}, i = 1, \cdots, N. \tag{B-25}$$

Equation (B-22) follows easily.

(2) By using the expression of the weighted beta in Proposition 20 (2), we obtain the positive effect of the correlation coefficient on the weighted beta follows from the fact that

$$\begin{aligned} \frac{w_i}{\eta_i}\frac{\partial \beta_i}{\partial \rho} &= \frac{\partial}{\partial \rho}\left(\frac{\rho + \eta_i(1-\rho)}{\rho + \sum_{i=1}^N \eta_i^2(1-\rho)}\right) \\ &= \frac{\sum_{j\neq i}\eta_j^2 + \eta_i^2 - \eta_i}{\left(\rho + \sum_{i=1}^N \eta_i^2(1-\rho)\right)^2}. \end{aligned}$$

We see that the numerator $\sum_{j\neq i}\eta_j^2 + \eta_i^2 - \eta_i$ being positive if and only of when η_i is small or large, and in this case, its beta is positively associated with the endogenous pairwise correlation coefficient. For a moderate level of eta, the numerator is negative, this, its beta is negatively associated with the correlation coefficient. □

Proof of Proposition 19.

(1). It follows from the fact that $\frac{\partial \kappa(\rho_1,\rho_2)}{\partial \rho_1} < 0$, $\frac{\partial \kappa(\rho_1,\rho_2)}{\partial \rho_2} < 0$, and $\frac{\partial \kappa(\rho_1,\rho_2)}{\partial \nu} > 0$.

(2). In the limited participation equilibrium, we have

$$\Omega(s) = \frac{\frac{\Omega(\sigma\overline{x})\nu N}{1+(N-1)\rho_1^{max}} - (1-\nu)(1-\Omega(\sigma\overline{x}))}{\frac{\nu N}{1-\rho_1^{max}} + (1-\nu)(1-\Omega(\sigma\overline{x}))(N-1)}. \tag{B-26}$$

Clearly, $\frac{\partial \Omega(s)}{\partial \rho_1^{max}} < 0$ and $\frac{\partial \Omega(s)}{\partial \nu} > 0$. The proof of the full participation equilibrium follows from the fact that $\frac{\partial \kappa(\rho_1,\rho_2)}{\partial \rho_1} < 0$, $\frac{\partial \kappa(\rho_1,\rho_2)}{\partial \rho_2} < 0$, and $\frac{\partial \kappa(\rho_1,\rho_2)}{\partial \nu} > 0$.

Appendix C: A Dispersion Measure

A function $f : (X_1, \cdots, X_N) \in \mathbb{R}^N \to [0, 1]$ is a *dispersion measure* if it satisfies the following three properties:

1. (Positively homogeneous property) Given any $\lambda > 0$, $f(\lambda X_1, \cdots, \lambda X_N) = f(X_1, \cdots, X_N)$;

2. (Symmetric property) Given any translation $\sigma : \{1, \cdots, N\} \to \{1, \cdots, N\}$, $f(X_1, \cdots, X_N) = f(X_{\sigma(1)}, \cdots, X_{\sigma(N)})$;

3. (Majorization property) Assuming $(X_1, \cdots, X_N)$ weakly dominates $(Y_1, \cdots, Y_N)$, then $f(X_1, \cdots, X_N) \geq g(X_1, \cdots, X_N)$.

By a vector $a = (a_1, \cdots, a_N)$ weakly dominates $b = (b_1, \cdots, b_N)$ we mean that

$$\sum_{i=1}^{k} a_i^* \geq \sum_{i=1}^{k} b_i^*, k = 1, \cdots, N \tag{C-1}$$

where a_i^* is the element of a stored in decreasing order. When $f(Y) \leq f(X)$ for a dispersion measure we call Y is more dispersed than X under the measure f. One example is the portfolio weight in a financial market, so the dispersion measure captures how one portfolio is more dispersed than another. Samuelson's famous theorem (Samuelson, 1967) states that an equally-weighted portfolio is always the optimal one for a risk-averse investor when the risky assets have IID return. Boyle et al. (2012) also find that an equally-weighted portfolio beats many optimal asset allocations under parameter uncertainty.

This paper concerns one example of a dispersion measure.

Lemma 10 *Given a non-zero vector* $X = (X_1, \cdots, X_N)$*,*

$$f(X_1, \cdots, X_N) = \sqrt{\frac{1}{N-1}\left(N\frac{\sum X_i^2}{(\sum_i X_i)^2} - 1\right)}$$

is a dispersion measure.

Proof: Both the positive homogeneous property and the symmetric property are obviously satisfied. To prove the majorization property, we first assume that, because of the positively homogeneous property, $\sum X_i = \sum Y_i = 1$. Let $g(x_1, \cdots, x_N) = N(\sum_i x_i^2) - (\sum_i x_i)^2$. Notice that $g(\cdot)$ is a Schur convex function in the sense that

$$(x_i - x_j)\left(\frac{\partial g}{\partial x_i} - \frac{\partial g}{\partial x_j}\right) \geq 0, \forall i, j = 1, \cdots, N.$$

Then by the majorization theorem, (Marshall and Olkin, 1979), $g(X) \geq g(Y)$. Then $f(X) \geq f(Y)$. $\square$

(a) The correlations with the market (b) The weighted betas with the market

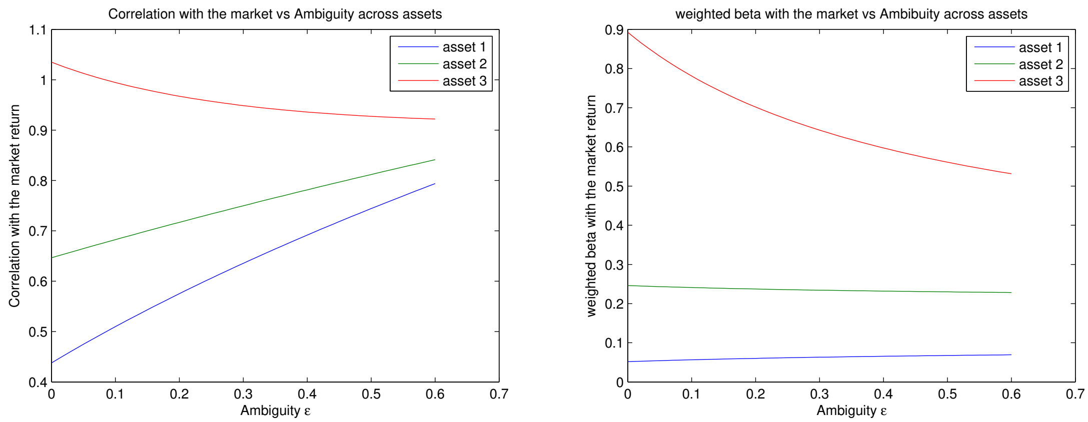


Figure 2.1: Correlation with the market return and the weighted beta
This figure shows how correlation of an asset with the market return as well as the weighted beta change with respect to the correlation uncertainty depending on asset characteristics. Given the input parameters that asset 1 has $\eta = 0.1$, asset 2 has $\eta = 0.3$, asset 3 has $\eta = 0.6$. In this case, asset 1 is the low eta asset, and asset 3 the high eta asset.

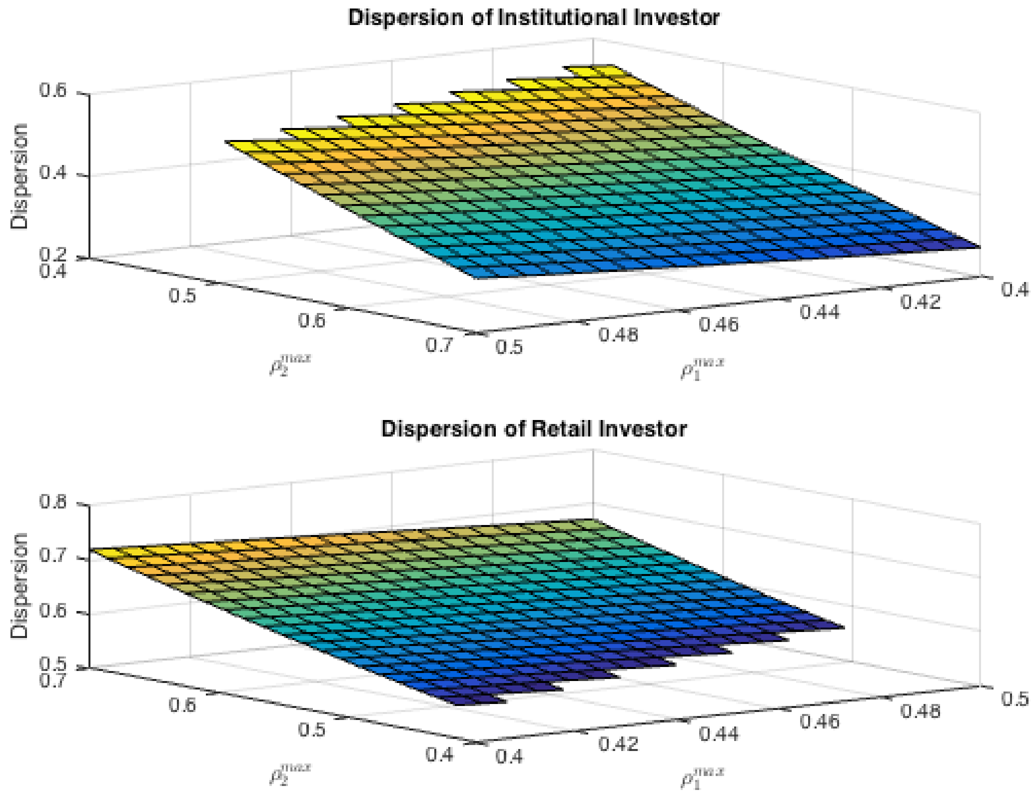


Figure 2.2: Dispersion of Optimal Portfolios of investors

This figure explains precisely that the institutional investor holds a well-diversified portfolio while the retail investor's portfolio is under-diversified. It demonstrates the dispersion of the optimal portfolio for the institutional and retail investor respectively in a full participation equilibrium with respect to the level of correlation uncertainty. Therefore, the figure also explains that an increasing perceived level of correlation uncertainty induces a less diversified optimal portfolio for each investor. The dispersion of the institutional investor's optimal portfolio is smaller than the corresponding dispersion of the retail investor's optimal portfolio, for all possible values of ρ_1^{max} and ρ_2^{max}. Moreover, the institutional investor's optimal portfolio has a smaller dispersion than that of the market portfolio, whereas the retail investor's optimal portfolio has a larger dispersion than that of the market portfolio. The parameters in this figure are $N = 3$, $\nu = 0.3$, $\sigma_1 = 9\%$, $\bar{x}_1 = 1$; $\sigma_2 = 10\%$, $\bar{x}_2 = 5$; $\sigma_3 = 12\%$, $\bar{x}_3 = 10.5$. Note that the dispersion of the market portfolio is $\Omega(\sigma\overline{x}) = 0.56$, and $\frac{1}{N-1}\left\{\frac{\nu}{1-\nu}\frac{\Omega(\sigma\overline{x})}{1-\Omega(\sigma\overline{x})}N - 1\right\} = 0.3$. A similar analysis in a limited participation equilibrium is reported in Table 2.5.

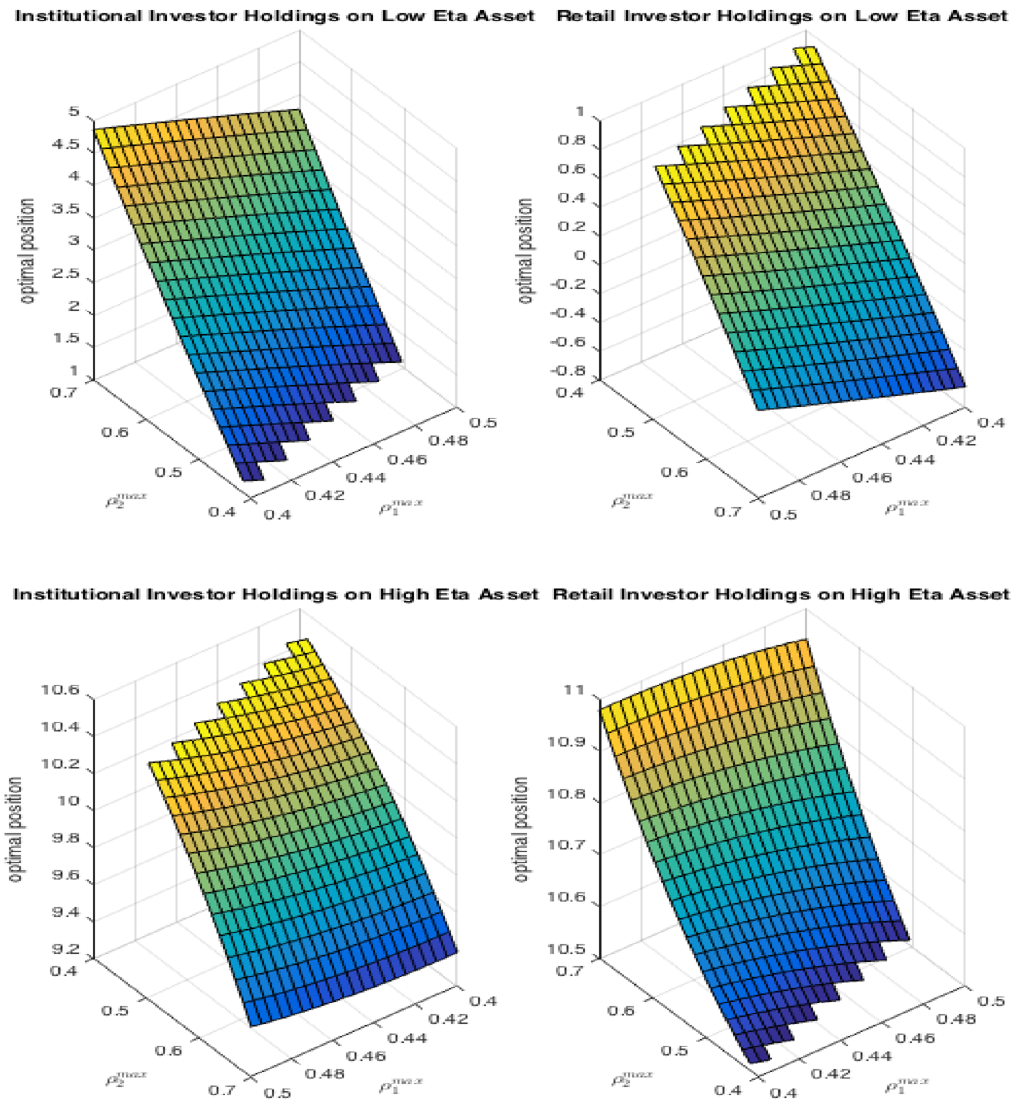


Figure 2.3: Optimal Holdings of investors

This figure explains the flight-to-quality episode from the correlation uncertainty perspective. It reports the optimal holdings of the institutional and retail investor on low-eta asset 1 and high-eta asset 3 respectively when the level of uncertainty changes. This figure demonstrates how the trading positions and trading volumes are affected by the level of uncertainty for different asset. The parameters in this figure are $N = 3$, $\nu = 0.3$, $\sigma_1 = 9\%$, $\bar{x}_1 = 1$; $\sigma_2 = 10\%$, $\bar{x}_2 = 5$; $\sigma_3 = 12\%$, $\bar{x}_3 = 10.5$. Notice that $\Omega(\sigma\overline{x}) = 0.56$ and $\frac{1}{N-1}\left\{\frac{\nu}{1-\nu}\frac{\Omega(\sigma\overline{x})}{1-\Omega(\sigma\overline{x})}N - 1\right\} = 0.3$. $\eta_1 = 0.048 < \frac{1}{N}, \eta_3 = 0.68 > \frac{1}{N}$. A similar analysis in a limited equilibrium model is reported in Table 2.5.

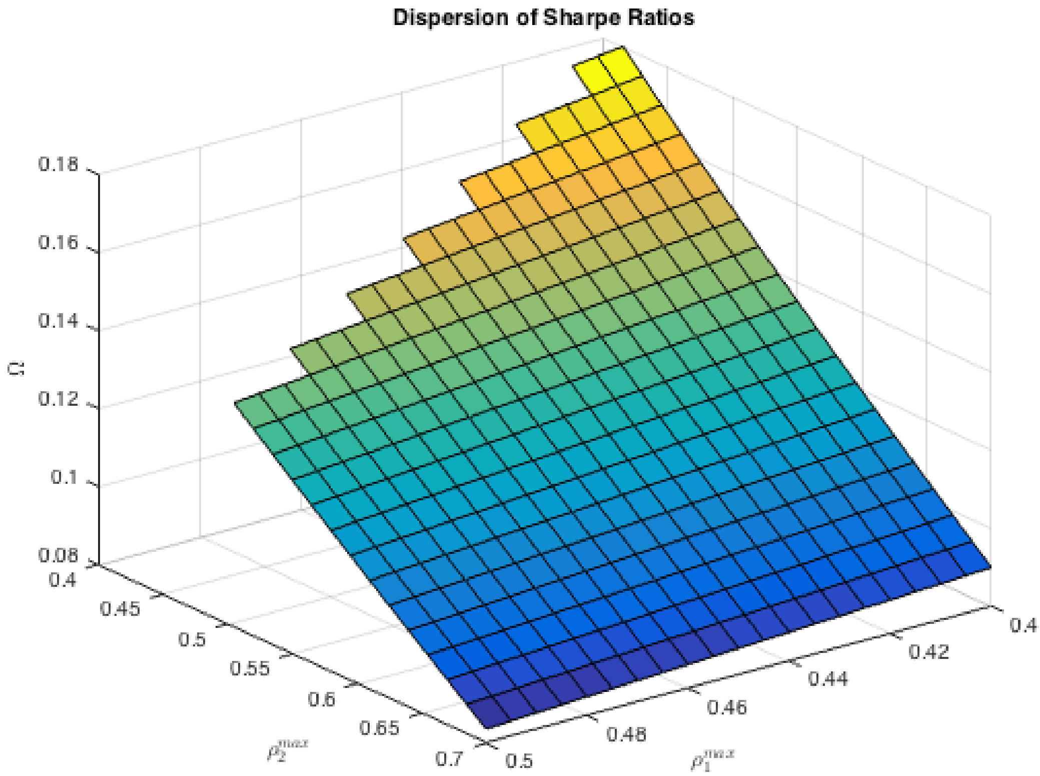


Figure 2.4: Dispersion of Sharpe Ratios
This three-dimensional figure explains the comovement phenomenon from an investment perspective. It plots the dispersion, $\Omega(s)$, of Sharpe ratios in a full participation equilibrium. As shown in this graph, the dispersion of all Sharpe ratios decreases with the increasing of ρ_1^{max} and ρ_2^{max}, of sophisticated and retail investors respectively; this implies that all risky assets intend to comove together when the level of uncertainty is high. The parameters in this figure are $N = 3$, $\nu = 0.3$, $\sigma_1 = 9\%$, $\bar{x}_1 = 1$; $\sigma_2 = 10\%$, $\bar{x}_2 = 5$; $\sigma_3 = 12\%$, $\bar{x}_3 = 10.5$. Notice that $\Omega(\sigma\overline{x}) = 0.56$ and $\frac{1}{N-1}\left\{\frac{\nu}{1-\nu}\frac{\Omega(\sigma\overline{x})}{1-\Omega(\sigma\overline{x})}N - 1\right\} = 0.3$. The same decreasing property of the dispersion of Sharpe ratios in a limited participation equilibrium is reported in Table 2.2.

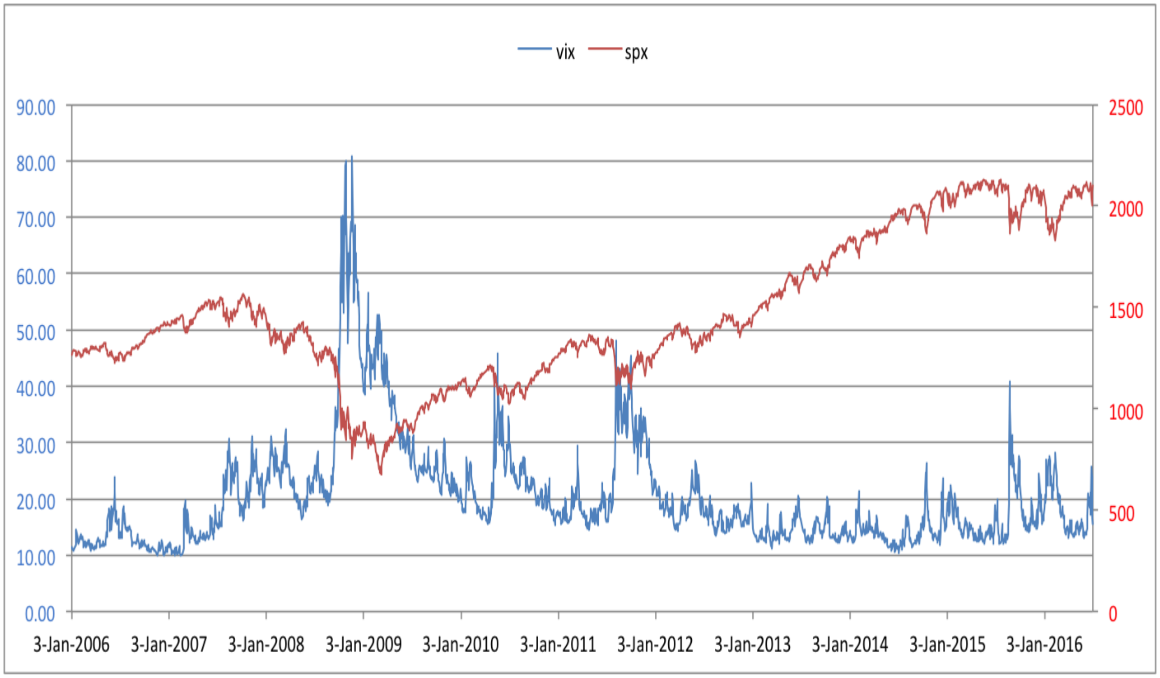


Figure 2.5: VIX vs S&P 500 index

This figure displays the VIX and S&P 500 index from Jan. 2006 to Jan. 2016. As documented in Bloom (2009), VIX measure the macroeconomic uncertainty as well as the ambiguity on the entire financial market. The high level of VIX in 2007-2009 reflects to higher level of Knightian uncertainty. It is clear that the stock market largely moves in opposite direction with the VIX, in particular, in 2007-2009. Source: *Chicago Board Option Exchange.*

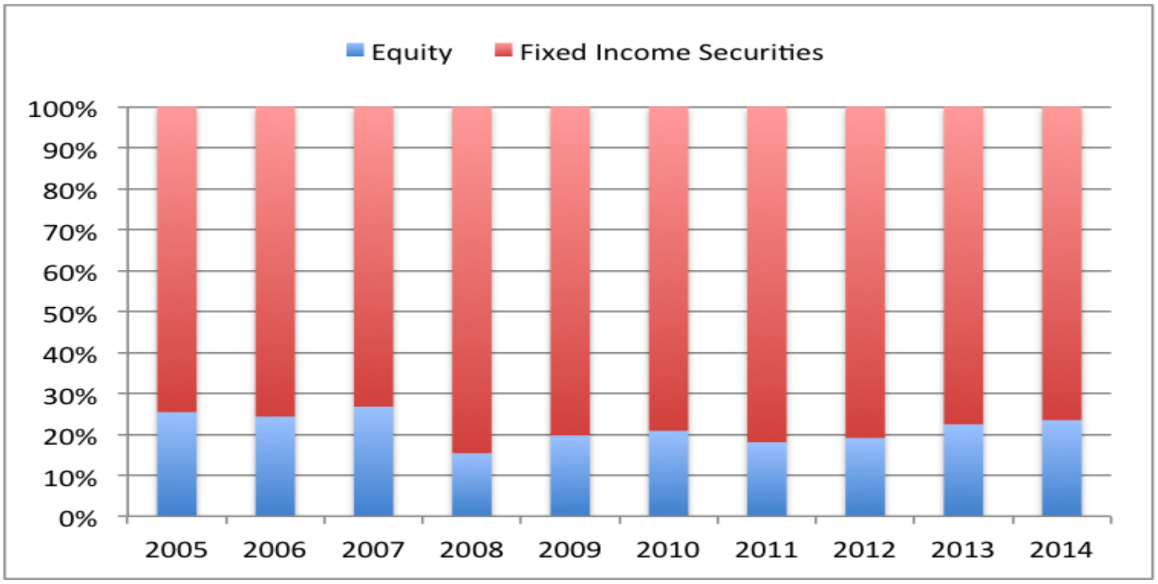
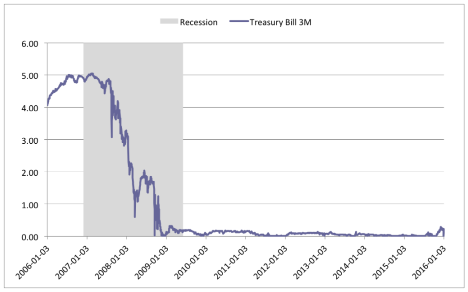
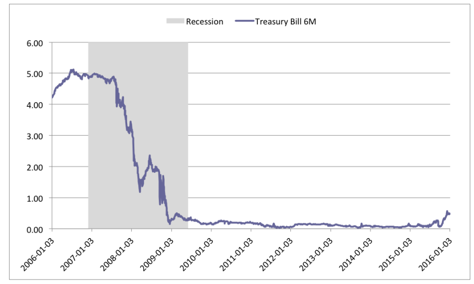


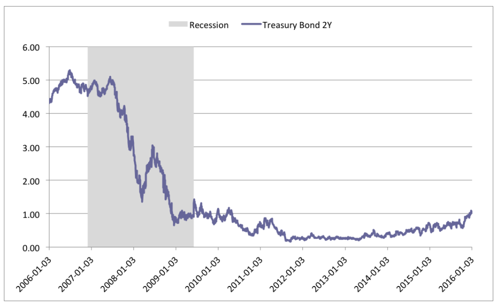
Figure 2.6: Comparison between equity market and debt/loan markets.
The objective of this figure is to show that the stock market is a low-eta asset class compared to the fixed income market which is a high-eta asset class. In the graph, we report the weight of the equity market compared to the fixed income market between 2005 to 2014. "Fixed Income Securities" includes "Public Debt market", "Financial Bonds", "Corporate Bonds", "Securitized Loan Market", and "Unsecuritized Loans Outstanding Market". As shown, the fixed income market is about four time size large of the stock market in term of market capitalization. Since the stock volatility is around three time of the fixed income volatility (Reilly, Wright and Chan, *Journal of Portfolio Management*, 2000), the eta of the stock marker is about 75% of the eta of the fixed income market. Source: *McKinsey Global Institute research.*



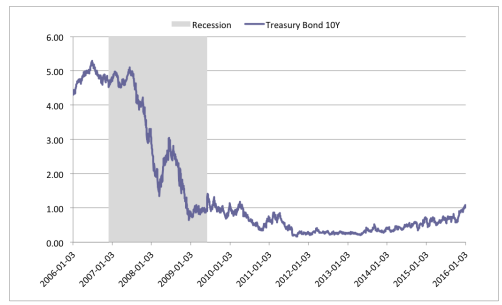
(a) 3-month Treasury Bill Daily Rate



(b) 6-month Treasury Bill Daily Rate



(c) 2-year Treasury Bond Daily Rate



(d) 10-year Treasury Bond Daily Rate

Figure 2.7: Treasury Rates
This figure presents daily Treasury rates with different maturities (3 months, 6 months, 2 years and 10 years respectively) from Jan. 2006 to Jan. 2016. Treasury market is well documented as "uncertainty hedging" or "safe heaven" assets. As depicted in this graph, the Treasury rates decrease significantly and the prices increase during the recession of 2007-2009. Source: *Federal Reserve Bank of St. Louis.*

Table 2.1: Sharpe Ratios and Correlation Uncertainty Premium in a Homogeneous Model

This table reports the Sharpe ratios, the dispersion of all Sharpe ratios, and the correlation uncertainty premium in a homogeneous environment. The parameters are: $N = 3, \sigma_1 = 9\%, \overline{x}_1 = 1, \sigma_2 = 10\%, \overline{x}_2 = 5, \sigma_3 = 12\%, \overline{x}_3 = 10.5$. Note that the dispersion of the risks is $\Omega(\sigma\overline{x}) = 0.56$. The reference correlation coefficient is $\rho^{avg} = 40\%$. By "premium column", we mean the percentage of the correlation uncertainty premium over the Sharpe ratio without the correlation uncertainty, that is, for the benchmark correlation coefficient.

ϵ	s_1	s_2	s_3	$\Omega(s)$	Premium Asset 1	Premium Asset 2	Premium Asset 3
0.00	0.79	1.49	2.70	0.54	NA	NA	NA
0.02	0.83	1.55	2.80	0.51	4.43%	4.14%	3.95%
0.04	0.86	1.61	2.91	0.48	8.87%	8.27%	7.90%
0.06	0.90	1.67	3.02	0.46	13.30%	12.41%	11.85%
0.08	0.93	1.74	3.12	0.43	17.73%	16.54%	11.85%
0.10	0.97	1.80	3.23	0.40	22.17%	20.68%	19.76%
0.12	1.01	1.86	3.34	0.38	26.60%	24.81%	23.71 %
0.14	1.04	1.92	3.44	0.38	31.03%	28.95%	27.66%
0.16	1.08	1.98	3.55	0.34	35.47%	33.08%	31.61%
0.18	1.11	2.04	3.66	0.31	39.90%	37.22%	35.56%
0.20	1.15	2.11	3.76	0.29	44.33%	41.36%	39.52%

Table 2.2: Sharpe ratios in a limited participation equilibrium

This table reports all Sharpe ratios in a limited participation equilibrium, including the Sharpe ratios, the relative Sharpe ratios and the dispersion of Sharpe ratios when the level of correlation uncertainty of institutional investor changes. The parameters are: $N = 3, \nu = 0.6, \sigma_1 = 2\%, \overline{x}_1 = 0.5, \sigma = 8\%, \overline{x}_2 = 3, \sigma_3 = 12\%, \overline{x}_3 = 10.5$. Note that the dispersion of the risks is $\Omega(\sigma\overline{x}) = 0.7631$. The benchmark correlation coefficient is $\rho^{avg} = 40\%$.

ρ_1^{max}	$K(\rho_1^{max})$	s_1	s_2	s_3	S	s_1/S	s_2/S	s_3/S	$\Omega(s)$
0.400	0.557	0.70	0.82	1.36	2.89	0.244	0.285	0.471	0.210
0.405	0.561	0.71	0.83	1.36	2.90	0.245	0.286	0.469	0.207
0.410	0.566	0.72	0.84	1.36	2.92	0.246	0.287	0.467	0.204
0.415	0.570	0.73	0.84	1.36	2.93	0.247	0.287	0.465	0.201
0.420	0.575	0.73	0.85	1.37	2.95	0.249	0.288	0.463	0.198
0.425	0.579	0.74	0.86	1.37	2.96	0.250	0.289	0.461	0.195
0.430	0.583	0.75	0.86	1.37	2.98	0.251	0.289	0.460	0.192
0.435	0.588	0.75	0.87	1.37	2.99	0.252	0.290	0.458	0.190
0.440	0.592	0.76	0.87	1.37	3.01	0.253	0.291	0.456	0.187
0.445	0.596	0.77	0.88	1.37	3.02	0.255	0.291	0.454	0.184
0.450	0.601	0.78	0.89	1.37	3.04	0.256	0.292	0.452	0.181
0.455	0.605	0.78	0.89	1.37	3.05	0.257	0.292	0.452	0.179
0.460	0.609	0.79	0.90	1.38	3.06	0.258	0.293	0.449	0.176
0.465	0.613	0.80	0.90	1.38	3.08	0.259	0.294	0.447	0.174
0.470	0.618	0.80	0.91	1.38	3.09	0.260	0.294	0.446	0.171
0.475	0.622	0.81	0.92	1.38	3.11	0.261	0.295	0.444	0.169
0.480	0.626	0.82	0.92	1.38	3.12	0.262	0.295	0.442	0.166
0.485	0.630	0.83	0.93	1.38	3.14	0.263	0.296	0.441	0.164
0.490	0.635	0.83	0.93	1.38	3.15	0.264	0.297	0.439	0.161
0.495	0.639	0.84	0.94	1.38	3.16	0.265	0.297	0.438	0.159
0.500	0.643	0.85	0.95	1.39	3.18	0.266	0.298	0.436	0.156

Table 2.3: Limited and Full Equilibrium

This table reports the conditions under which the full equilibrium and the limited equilibrium prevails. For simplicity we assume that the institutional investor knows the perfect correlation ρ and the retail investor's degree of correlation uncertainty is ϵ. In Panel A, we characterize the equilibrium under conditions of the correlation uncertainty while ν and $\Omega(\eta)$ are fixed. Panel B discusses the equilibrium cases under conditions of ν, given other parameters, ρ, ϵ and $\Omega(\eta)$, are fixed. ν^* is defined by the equation $K(\rho, \nu^*, \Omega(\eta)) = \rho + \epsilon$. In Panel C, we characterize the equilibrium under the conditions of the risk distribution $\Omega(\eta)$ when parameters ρ, ϵ and ν are fixed. Ω^* satisfies the equation $K(\rho, \nu, \Omega^*) = \rho + \epsilon$.

Panel A: Conditions on ϵ

	$\rho_1^{max} \geq \frac{1}{N-1}\left\{\frac{\nu}{1-\nu}\frac{\Omega(\sigma\overline{x})}{1-\Omega(\sigma\overline{x})}N - 1\right\}$	$\rho_1^{max} < \frac{1}{N-1}\left\{\frac{\nu}{1-\nu}\frac{\Omega(\sigma\overline{x})}{1-\Omega(\sigma\overline{x})}N - 1\right\}$
$\rho_2^{max} \geq K(\rho_1^{max})$	full participation	limited participation
$\rho_2^{max} < K(\rho_1^{max})$	full participation	full participation

Panel B: Conditions on ν, where $\rho + \epsilon = K\left(\rho, \nu^*, \Omega\right)$

	$\frac{\nu}{1-\nu} \leq \frac{1-\Omega(\eta)}{\Omega(\eta)}\frac{1+(N-1)\rho}{N}$	$\frac{\nu}{1-\nu} > \frac{1-\Omega(\eta)}{\Omega(\eta)}\frac{1+(N-1)\rho}{N}$
$\rho + \epsilon \leq \lim_{\nu\to 1} K(\rho, \nu, \Omega(\eta))$	full participation	full participation
$\rho + \epsilon > \lim_{\nu\to 1} K(\rho, \nu, \Omega(\eta))$:		
$\nu < \nu^*$	full participation	full participation
$\nu \geq \nu^*$	full participation	limited participation

Panel C: Conditions on $\Omega(\eta)$, where $\rho + \epsilon = K\left(\rho, \nu, \Omega^*\right)$

	$\frac{\Omega(\eta)}{1-\Omega(\eta)} \leq \frac{1-\nu}{\nu}\frac{1+(N-1)\rho}{N}$	$\frac{\Omega(\eta)}{1-\Omega(\eta)} > \frac{1-\nu}{\nu}\frac{1+(N-1)\rho}{N}$
$\Omega(\eta) < \Omega^*$	full participation	full participation
$\Omega(\eta) \geq \Omega^*$	full participation	limited participation

Table 2.4: Model Implication

This table summarizes our model implications. We report the model implications in three different categories when the correlation uncertainty increases. Panel A presents the effect of the correlation uncertainty at an overall market level. We consider three popular measures of *asset comovement*: the aggregative market volatility, the pairwise market correlation and the dispersion of Sharpe ratios. In Panel B, we present the cross-sectional effect on the individual asset given different asset characteristics, including the asset prices, the risk premiums, Sharpe ratios, the relative Sharpe ratios, the correlations with the market portfolio and the weighted betas. In Panel C, we compare the institutional investor and retail investor, in terms of their optimal portfolio, holding position, and trading volume on individual assets, which explains the *under-diversification or limited participation*, and *flight to quality* in our model. For simplicity, we assume that institutional investor knows the exact correlation coefficient ρ while the degree of correlation uncertainty for the retail investor is ϵ.

Panel A:Effect on the overall market level when ϵ increases

Aggregate market volatility: $\sum_{i=1}^{N} \sigma_i \overline{x}_i$	increase
Pairwise market correlation: ρ	increase
Dispersion of Sharpe ratios: $\Omega(s)$	increase

Panel B: Effect on the individual asset when ϵ increases

	High-eta asset	Low-eta asset
Asset price: p_i	increase	decrease
Risk premium: $\overline{a_i} - p_i$	decrease	increase
Sharpe ratio: s_i	decrease	increase
Relative Sharpe ratio: $\frac{s_i}{S_i}$	decrease	increase
Correlation with market: $corr(\tilde{R}_i, \tilde{R}_m)$	increases or mixed	decrease or mixed
(Weithted) Beta: $\beta_i \omega_i$	increases or mixed	increases or mixed

Panel C: Effect on portfolio holdings and volume when ϵ increases

	Institutional investor	Retail investor
Optimal portfolio: $\Omega(\sigma x)$	well-diversified	under-diversified
High-eta (holding)	decrease	increase
Low-eta (holding)	increase	decrease
High-eta (trading volume)	increase	increase
Low-eta (trading volume)	increases	increase

Table 2.5: Optimal portfolios in a limited participation equilibrium

This table reports a limited participation equilibrium, including the investor's optimal position and the corresponding dispersion of the optimal portfolio. It further demonstrates that the institutional investor holds a well-diversified portfolio and the retail investor always holds a less-diversified portfolio. The parameters are: $N = 3, \nu = 0.6, \sigma_1 = 2\%, \overline{x}_1 = 0.5, \sigma_2 = 8\%, \overline{x}_2 = 3, \sigma_3 = 12\%, \overline{x}_3 = 10.5$. Notice that the dispersion of the risks is $\Omega(\sigma\overline{x}) = 0.7631$. The benchmark correlation coefficient is $\rho^{avg} = 40\%$. For each $i = 1, 2, 3$, $x_i^{(r)}$ and $x_i^{(s)}$ denote the optimal holding of the retail investor and the institutional investor on asset i. $\Omega^{(r)}$ represents the dispersion of the retail investor's optimal portfolio while $\Omega^{(s)}$ is the dispersion of the institutional investor's optimal portfolio.

ρ_1^{max}	$K(\rho_1^{max})$	$(x_1^{(r)}, x_1^{(s)})$	$(x_2^{(r)}, x_2^{(s)})$	$(x_3^{(r)}, x_3^{(s)})$	$\Omega^{(r)}$	$\Omega^{(s)}$
0.400	0.557	(-6.478, 5.152)	(11.277, 9.982)	(1.791, 3.806)	1.000	0.629
0.405	0.561	(-6.484, 5.156)	(11.287, 9.976)	(1.792, 3.805)	1.000	0.629
0.410	0.566	(-6.489, 5.159)	(11.297, 9.969)	(1.794, 3.804)	1.000	0.628
0.415	0.570	(-6.495, 5.163)	(11.306, 9.962)	(1.796, 3.803)	1.000	0.628
0.420	0.575	(-6.500, 5.167)	(11.316, 9.956)	(1.797, 3.802)	1.000	0.628
0.425	0.579	(-6.506, 5.171)	(11.326, 9.949)	(1.799, 3.801)	1.000	0.628
0.430	0.583	(-6.512, 5.175)	(11.336, 9.943)	(1.800, 3.800)	1.000	0.628
0.435	0.588	(-6.517, 5.178)	(11.346, 9.936)	(1.802, 3.799)	1.000	0.627
0.440	0.592	(-6.523, 5.182)	(11.356, 9.930)	(1.803, 3.798)	1.000	0.627
0.445	0.596	(-6.529, 5.186)	(11.365, 9.923)	(1.805, 3.797)	1.000	0.627
0.450	0.601	(-6.534, 5.190)	(11.375, 9.917)	(1.807, 3.796)	1.000	0.627
0.455	0.605	(-6.540, 5.193)	(11.385, 9.910)	(1.808, 3.795)	1.000	0.627
0.460	0.609	(-6.546, 5.197)	(11.395, 9.903)	(1.810, 3.794)	1.000	0.626
0.465	0.613	(-6.551, 5.201)	(11.405, 9.897)	(1.811, 3.793)	1.000	0.626
0.470	0.618	(-6.557,5.205)	(11.415, 9.890)	(1.813, 3.791)	1.000	0.626
0.475	0.622	(-6.563, 5.208)	(11.424,9.884)	(1.814, 3.790)	1.000	0.626
0.480	0.626	(-6.568. 5.212)	(11.434, 9.877)	(1.816, 3.789)	1.000	0.626
0.485	0.630	(-6.574, 5.216)	(11.444,9.871)	(1.817, 3.788)	1.000	0.626
0.490	0.635	(-6.580, 5.220)	(11.454, 9.864)	(1.819, 3.787)	1.000	0.626
0.495	0.639	(-6.585, 5.223)	(11.464, 9.858)	(1.821, 3.786)	1.000	0.625
0.500	0.643	(-6.591, 5.227)	(11.474,9.851)	(1.822, 3.785)	1.000	0.625

CHAPTER 3: VIABILITY OF FINANCIAL INNOVATION UNDER MODEL UNCERTAINTY

3.1 Introduction

Financial innovation has played an important role in the development of financial markets (see, e.g., (Allen & Gale, 1994) and (Tufano, 2003)). Stemming from mortgage-backed securities, the 2007-2008 turmoils in the credit market, especially collateralized debt obligations (CDOs), have raised serious questions about the efficacy and frailty of new financial products. In particular, some of the problems may be attributable to the uncertainty about the underlying asset process faced by issuers, investors and rating agencies alike (see, e.g.,Coval et al. (2009a)). This uncertainty can lead to dispersed beliefs about the valuation of related derivative securities and, in a sharp market downturn, cause a dramatic decline in trading activities in these securities.[50]

Regardless of the discussion on its success or failure, new financial products are created and issued repeatedly and massively in the financial market; thus, it is important to study the financial innovation process to understand its efficacy and viability. However, many previous studies in security design are developed in a static framework, other than a few on the financial innovation process. [51] The objective of this paper

[50]See, for instance, (Bhattacharya & Spiegel, 1991), (Bhattacharya, Reny, & Spiegel, 1995), (Morris, 1994) and (Massa, 2002).

[51]See (Bettzüge & Hens, 2001), (Calvet, Gonzales-Eiras, & Sodini, 2004), (Person & Warther,

is to develop a dynamic equilibrium model of financial innovation and to study how the viability of financial innovation interacts with the heterogeneous beliefs and the underlying market movement in a dynamic process.

In our dynamic equilibrium model, an innovator issues a new financial product that pays off in each period, and the demand for the financial product is determined by a representative investor who is risk averse and optimally allocates his wealth into the new security in a dynamic setting. The model uncertainty about the underlying fundamental process is represented by an unobservable mean of the unconditional distribution, and all agents have their own prior beliefs for the underlying process and revise their expectations based on realized payoffs through Bayesian updating. [52]

A novel component in our dynamic equilibrium model of the financial innovation is that the investor is more prone to time inconsistency than the innovator. Due to the increasing complexity of the financial innovation ((Carlin, 2009)) as well as the reason that the innovator might intentionally ignore risks in the financial innovation ((Gennaioli, Shleifer, & Vishny, 2012)), the investor could distort his optimal investment decision after learning more about the financial innovation product in the market. Therefore, the investor may find his precommitment on the investment of the financial product is not optimal in the future. To capture this time-inconsistent feature of the investor, we assume that the long-term investor has a dynamic mean-

1997) and (Plosser, 2009).

[52] Our equilibrium approach is similar to (Allen & Gale, 1994). The portfolio choice setting has been studied in (Barberis, 2000) and (Kandel & Stambaugh, 1996). Our setting can be also viewed as a discrete-time version of the model in (Buraschi & Jiltsov, 2006) for the option market under model uncertainty.

variance preference.

We *first* characterize the unique equilibrium of financial innovation in a dynamic setting. We derive the intertemporal hedging term for the investor and show that this hedging term is proportional to the covariance between the payoff of the financial innovation and the cumulative gains of the investment over the investment horizon in equilibrium.[53] As a result, our characterization of equilibrium allows us to investigate the viability of financial innovation dynamically. Intuitively, a financial innovation is *viable* if it can maintain a high level of market activities, while a *vulnerable* innovation may fail to survive under adverse market movements with low or no trading volume. For simplicity we call the financial innovation viable as long as there exists equilibrium trading volume between the innovator and the investor, and we derive the sufficient and necessary condition for the market viability. In this way, our characterization of the dynamic equilibrium provides an endogenous pattern of the financial innovation evolution. [54]

Second, this paper examines how different payoff structures of financial securities along with heterogeneous beliefs affect the conditions of market breakdown for these securities. A number of authors have discussed conditions for market failure in the presence of asymmetric information or ambiguity (see, e.g., (Bhattacharya & Spiegel, 1991), (Bhattacharya et al., 1995) and (Dow, 1998)), however, the focus of our analysis is to examine how the financial innovation itself interacts with the underlying

[53]The intertemporal hedging demand for a dynamic mean-variance investor is first shown to be related to the gain on the risky asset over the investment horizon in a continuous-time framework by (Basak & Chabakauri, 2010). We derive the precise relation between the intertemporal hedging demand and the cumulative gain in the remaining investment time periods in equilibrium.

[54](Bettzüge & Hens, 2001) study the success and failure of financial innovation but the market participation role is defined exogenously.

market changes. We consider several basic financial innovations, which are the building blocks of complex financial innovations and can be used to generate the entire space of contingent claims. We show that the market of forward-type securities is rather resilient. The market of option-type contracts, *however*, appears to be more vulnerable in that the supply of the security may dry up quickly when the underlying asset value experiences a dramatic negative shock. The nonlinearity feature of the payoff structure makes the security issuance highly sensitive to the market condition. The key insight of this analysis is the *stability* of financial innovation, which emerges from the above dynamic equilibrium. By a *stable* financial innovation we mean there remains a high level of market activities when the market condition changes within a small range. In other words, a stable financial innovation must be still viable under a slight market movement. We demonstrate that the forward contract market is stable whereas the option market is not always the case.

Third, we extend our dynamic model to encounter more complicated financial innovation and examine how tranching structure and the underlying market jointly affect the market equilibrium. A securitization product like CDOs often consists of multiple tranches and can be viewed as a package of option contracts written on the underlying collateral pool. We elaborate the tranche structure of CDO innovation and develop a dynamic equilibrium of the entire securitization. We find that many CDOs structures emerge during the strong market for the underlying assets, while drying in a weak underlying asset market. When the market experiences a substantial adverse movement, expectations of security payoffs from different agents can shift quite dramatically, injecting some degree of market instability and even causing a market breakdown with no offering of new securities. Our analysis is reminiscent of the situation in the CDO market during the credit crisis in 2008-2009. Originators of CDOs would expect to earn profits from selling the call option (the equity tranche), but they may suffer a loss from selling other option contracts (senior and mezzanine tranches) at prices investors would be willing to pay. If the loss can be offset by the profit from selling the equity tranche, there are incentives for financial innovators to issue and market CDOs. Hence, our result provides one potential explanation for the high yields puzzle observed from the highly rated senior and, sometimes, mezzanine tranches during the heyday of the CDO market. While Coval et al.(2009a) offer an explanation based on the mis-representation of ratings and the unpriced risk of economic catastrophe, our analysis indicates a complementary interpretation from the supply side.[55]

Our work contributes to the dynamic equilibrium with a time-inconsistent investor. (Basak & Chabakauri, 2010) demonstrates that the intertemporal hedging demand for a dynamic mean-variance investor is related to the gain of the investment on the risky assets over the investment horizon. (Gernadier & Wang, 2007) and (Luttmer & Mariotti, 2003) develop the equilibrium for the subjective discounting investor. We first derive the intertemporal hedging demand in equilibrium recursively, and demonstrate that this intertemporal hedging demand plays a crucial role in the analysis of the financial innovation process.

Our research is also related to the model uncertainty literature. (Barberis, 2000) considers the portfolio choice problem for a long-term investor under model uncer-

[55]See also Coval et al.(2009b). (Franke, Herrmann, & Weber, 2007) document the empirical evidences in the European CDO market.

tainty and compares several portfolio choice strategies on the investment of the risky assets. (Buraschi & Jiltsov, 2006) develops an equilibrium analysis for the option market under model uncertainty. We develop a similar Bayesian learning framework, but our setting is more general on the model uncertainty issue and our focus is also different from the previous literature. Precisely, we consider the financial innovation as a derivative on a non-tradeable state variable, and we allow the agents to have uncertainty on both the mean and the variance of the state variable instead of the uncertainty on the risky assets. We also compare the impact level from different sources of model uncertainty, and we find that the uncertainty from the mean (or the drift term) of the state variable is more profound on the equilibrium than the variance uncertainty.

The rest of paper is organized as follows. In Section 3.2, we describe the model structure and we present the dynamic equilibrium in Section 3. In Section 4, We provide the equilibrium analysis for several fundamental types of financial innovations in greater length. We extend our analysis to more complicated financial innovation like collateralized debt obligations and examine the viability of tranching structure under different market scenarios in Section 3.5. We conclude in Section 3.6. All proofs are collected in the Appendix.

3.2 The Model

There are two kinds of agents in the model: the innovator (or seller) and a homogeneous group of investors (or buyers). We use a representative investor to represent a homogeneous group of investors for each financial innovation in this section and extend the setting to securitization with different groups of investors for each tranche in Section 5.[56] We use $i \in \{n, v\}$ to indicate the type of agents; "n" and "v" denotes the innovator and the investor, respectively.

Two marketable assets are traded in the model. One is a risk-free security with a zero risk-free rate of return. The other security is risky and represents an example of financial innovation, in light of the growing volume of financial innovations. The financial innovation in our analysis is written on some state variables. Typical examples include the pass-through or traded securities written on the mortgage pools while the mortgage pools of a commercial bank can not be traded in the market (see (Riddiough, 1997) and (Friewald, Hennessy, & Jankowitsch, 2016)). As another example, a financial option written on a rare event such as weather risk, catastrophic risk or systemic risk is a marketable security on a non-traded state variable (see, for instance, (Banerjee & Graveline, 2014)). While we do not exclude the tradable state variable such as equity index, exchange-traded fund (ETF), equity index option or option written on ETF as financial innovation, we present the theory for the non-tradable state variable specially for two reasons. First, as we will explain below about the construction of the state variable, the transaction cost might prevent the trading directly from buying or selling individual assets in an equity portfolio or a pool of mortgages/collaterals. Second, we allow for a linear payoff structure (forward contract) of the financial innovation in our setting. For instance, since the variance swap is a sequence of forward contracts written on the realized variance, the realized

[56]Under this setting we highlight the heterogenous beliefs between the innovator and the investor, instead of heterogeneous beliefs among investors as in (Axelson, 2008), (Garmaise, 2001) and (DeMarzo, 2004).

variance is treated as a state variable that reveals the same tradable components as the variance swap.[57]

There are T-periods in which $t = 0, 1, \cdots, T$ in the model. At the beginning of each time period, t, the innovator issues a financial innovation product which matures at the end of the time period, $t+1$, and sells it to the investor in the market. The innovator keeps updating the market information to issue the same financial innovation in this dynamic market. This issuance process reflects the simple feature of financial innovation in the real market with the same payoffs continuously. Our dynamic model explicitly accounts for the effect of market movement on the financial innovation or vice versa.

3.2.1 The innovator and the investor

The innovator is risk neutral. Since the innovator issues the innovation product in each time period, she makes the issuance decision at the beginning of each time period in order to maximize her expected profits. We assume that the cost for the issuance of N units of the security is $C(N) \equiv \alpha + \beta N$ to reflect both of its fixed cost and floating cost components. There is also an initial fixed cost of developing the product incurred at time $t = 0$, which we denote as D.

The investor in the market is a risk averse long-term buyer. The investor analyzes the optimal decision by considering variance and expectation, separately; in other words, he follows the mean-variance analysis of Markowitz (1952) in a dynamic

[57] In other words, if we consider a marketable state variable x and an option on x, there are essentially two types of derivatives in the market simultaneously, namely, forward and option. It is plausible to extend the theory into a situation with multiple innovations.

framework.[58] More specifically, the investor maximizes

$$\mathbb{E}[W_T] - \frac{\gamma}{2} Var[W_T] \tag{C-2}$$

where W_T is the final wealth of the investor at time T. [59]

A novel ingredient in this model is that the investor is more prone to the time inconsistency than the innovator due to the following aspects of the financial innovation. (1) As shown in (Gennaioli et al., 2012), sellers in most financial innovation issue financial securities while deceiving certain risk in order to attract investors' demand from safer cash flow, and when investors eventually learn the risk afterwards, they will fly back to the safety security, and (2) the increasing complexity ((Carlin, 2009)) of the financial innovation largely contributes to a longer process for the investor to understand both the design and the mechanism of financial products than the innovator, for example, securitization and structured products. Given the potential neglected risk (or uncertainty) of financial innovation and its complexity nature, the optimal decision for the investor is likely to be distorted after market information is revealed, and therefore, the time inconsistent feature on the financial innovation investment emerges. Recent experiments also document that time inconsistent preference is not rare. Actually it is a fundamental challenge in current literature to disentangling time preference from risk preference ((Halevy, 2015) and (Miao & Zhong, 2015)).

[58]See (Bossaerts, Preuschoff, & Quartz, 2008) for recent evidence of neuroscience regarding the human brain on the risky gambles and investment. (Cochrane, 2014) also formulates and studies the general intertemporal portfolio choice problem in a mean-variance setting recently.

[59]From a technical perspective, this preference assumption is equivalent to a CARA-preference when the final wealth has a normal distribution. While this assumption is standard in literature such as (Calvet et al., 2004), the wealth distribution in our setting can be heavily skewed due to the structure of the financial innovation, regardless of Gaussian distribution of the state variables or not.

For this reason, we employ the dynamic mean-variance to cast the time inconsistent feature of the investor.[60] From the investor's perspective, the expected variance of the portfolio with financial innovation products is reduced when the investor becomes more familiar with the financial innovation product itself.[61] To illustrate, let U_t denote $\mathbb{E}_t[W_T] - \frac{\gamma}{2}Var_t[W_T]$, the mean-variance objective function computed at time t. At the next time period, $t+\Delta t$, its mean-variance value is $U_{t+\Delta t} = \mathbb{E}_{t+\Delta t}[W_T] - \frac{\gamma}{2}Var_{t+\Delta t}[W_T]$. We note that

$$U_t = \mathbb{E}_t\left[U_{t+\Delta t}\right] - \frac{\gamma}{2}Var_t\left[\mathbb{E}_{t+\Delta t}[W_T]\right] \neq \mathbb{E}_t\left[U_{t+\Delta t}\right] \tag{C-3}$$

in which the variance of the expected wealth, $Var_t\left[\mathbb{E}_{t+\Delta t}[W_T]\right]$, quantifies the investor's incentive to adjust the optimal strategy. The smaller the variance of the expected wealth in the next time period, the smaller incentive to disobey the strategy that is optimal in the previous time. On the other hand, when the variance of the expected wealth in the next time period is large, the investor is more motivated to reevaluate and modify the strategy to take account of this time inconsistent feature of the mean-variance objective. Hence, instead of sticking to his pre-commitment on the financial innovation, the investor might distort his optimal decision with more market information revealed.

3.2.2 Belief structure on the state variable

The setting is a standard Bayesian learning model as in (Barberis, 2000) and (Kandel & Stambaugh, 1996). It is also a simpler yet discrete-time version of the

[60] Alternatively, some authors make use of the hyperbolic discount factors to address the time inconsistent preference. See (Gernadier & Wang, 2007), (Harris & Laibson, 2001) and (Luttmer & Mariotti, 2003).

[61] Formally, $Var_t[W_T] = \mathbb{E}_t[Var_{t+1}(W_T)] + Var_t[\mathbb{E}_{t+1}(W_T)] > \mathbb{E}_t[Var_{t+1}(W_T)]$.

framework in (Buraschi & Jiltsov, 2006) on the option markets with heterogeneous beliefs under model uncertainty.

Without loss of generality (see below) we assume that x is a one-dimensional state variable. We further assume that the fundamental distribution for x is normal, i.e., $x = \mu + \eta$, where μ is unobservable and η represents shocks or signals that are independently and identically normally distributed with a zero mean. The variance of η is known to be σ_η^2. Each agent correctly models the distribution of η, except that each has a different estimation of the expected value μ at time $t = 0$. The agent has model uncertainty concern on the distribution of the state variable. Specifically, different agents have different priors of μ, which is believed to be normally distributed as

$$\mu_i \sim \mathcal{N}(\alpha_{i0}, \sigma_{i0}^2), \tag{C-4}$$

where $i \in \{n, v\}$. In the ensuing analysis, $\mathbb{E}_i[\cdot]$ is the expectation taken over the posterior distribution of x for agent i. We assume $\sigma_{n0} \leq \sigma_{v0}$ in order to capture sophistication differential of agents regarding the model uncertainty. In general, the model uncertainty feature between these two type of agents is displayed as follow.

Uncertainty	Innovator	Investor
Mean	α_{n0}	α_{v0}
Volatility	σ_{n0}	σ_{v0}

For each t, let $\mathcal{F}_t = \mathcal{F}\{x_1, \cdots, x_t\}$ be the information set after observing data, $x_1, \cdots, x_t$. The posterior beliefs for the agent $i \in \{n, v\}$ about the state variable right after time t are normally distributed with mean $\alpha_{i,t}$ and variance $\sigma_{i,t}^2$, where

$$\alpha_{i,t} = \alpha_0 \frac{\frac{1}{\sigma_0^2}}{\frac{1}{\sigma_0^2} + \frac{t}{\sigma_\eta^2}} + \overline{x}_t \frac{\frac{t}{\sigma_\eta^2}}{\frac{1}{\sigma_0^2} + \frac{t}{\sigma_\eta^2}} \tag{C-5}$$

and

$$\sigma_{i,t}^2 = \frac{1}{\frac{1}{\sigma_0^2} + \frac{t}{\sigma_\eta^2}}, \tag{C-6}$$

where $\overline{x}_t = \frac{x_1 + \cdots + x_t}{t}$. We literally use the conditional expectation $\mathbb{E}_{i,t}[\cdot]$ and the conditional variance $Var_t[\cdot]$ to reflect the Bayesian learning in this setting after independently observations $x_1, \cdots, x_t$ for agent i. When t approaches to infinity, $\alpha_{i,t}$ will converge to the same posterior beliefs about the distribution of the state variable.[62]

3.2.3 The financial innovation

At last, we explain the financial innovation to complete the description of the model setting. Since the financial innovation is motivated largely to increase the efficiency of financial market, we argue that the financial innovation can be written as $f(x)$ on a one-dimensional state variable x. To demonstrate it, we make use of the spanning theorems stated in (Ross, 1976) and (Tian, 2014). Assuming the market information set, $\mathcal{F}\{y_1, \cdots, y_N\}$, is generated by some variables $y_1, \cdots, y_N$, but these state variables $y_1, \cdots, y_N$ are not necessarily traded assets. It is shown that in (Ross, 1976) and (Tian, 2014) the information set $\mathcal{F}\{y_1, \cdots, y_N\}$ can be generated by simple index options with a payoff $f(\sum_{i=1}^N c_i y_i)$ and the payoff function $f(\cdot)$ has one argument only. More specifically, (Ross, 1976) shows that $f(\cdot)$ can be a

[62](McKelvey & Page, 1986) shows that if there are sufficiently many public information of aggregate statistics, rational individuals start out with different priors beliefs will converge to a common posterior belief. Therefore, it is reasonable to assume no sufficient learning opportunities and hence the issuers would maintain different beliefs in studying the survivor of financial innovation as in this paper.

simple call or put option in a finite sample space setting; and (Tian, 2014) extends Ross's spanning theorem for an arbitrarily sample space with a general class of the payout function $f(\cdot)$. Therefore, we can interpret the financial innovation as an index option $f(x)$ on a one-dimensional state variable x.

3.3 Equilibrium of Financial Innovation

We characterize the equilibrium of financial innovation in this section.

3.3.1 Characterization of equilibrium

The dynamic portfolio choice problem under time inconsistent preference has been studied in (Strotz, 1955), (Harris & Laibson, 2001), (Gernadier & Wang, 2007) and (Basak & Chabakauri, 2010). The idea is to formula the problem as an inter-personal game in which all future "selves" of the investor are considered simultaneously. By assuming that future selves choose strategies that are optimal for future selves, despite being suboptimal from the standpoint of the current self, the optimal investment strategy is the subgame perfect Nash equilibrium strategy among all selves.

We start with a definition of the dynamic equilibrium of financial innovation.

Definition 1 *The set of* $\{(\Phi_t^*, p_t), t = 0, 1, \cdots, T-1\}$ *is an* equilibrium, *where* Φ_t^* *is the equilibrium volume of the financial innovation issued at time* t *and* p_t *is its time-t price if (1) each* Φ_t^**, given the price vector* $(p_t, t = 0, 1, \cdots, T-1)$*, is a pure-strategy Nash equilibrium in the intra-personal game with the mean-variance preference, that is,* Φ_t^* *is an optimal response of the self* t *to the strategy* Φ_s^* *of all future selves* $s > t$*,* *(2) for each* $t = 0, 1, \cdots, T-1$, Φ_t^* *maximizes the gains of the innovator*

$$\max_{\theta} \mathbb{E}_{n,t}\left[\theta(p_t - f(x_{t+1})\right] - C(\theta)$$

given the price vector p_t*, and (3)* $\Phi_t^* \geq 0$*. If the financial innovation has limited liability, that is,* $f(x) \geq 0$*, then we also require non-negative price* $p_t \geq 0$ *in the equilibrium.*

In the condition (1) of the equilibrium, assuming the time t-self's initial wealth is W_t and θ_t is the demand on the financial innovation, the final wealth W_T is written as $W_T = W_t + \theta_t(f(x_{t+1}) - p_t) + \sum_{s>t} \Phi_s^*(f(x_{s+1}) - p_s)$ where the second term is the gain of the time t-shelf in time period $[t, t+1]$ and the last term represents the cumulative gain of future selves $s > t$. Then, Φ_t^* solves the investment problem (C-2). The condition (2) is evident since the financial innovation is issued in each time period, thus, the innovator determines the optimal supply volume of the financial innovation into the market to maximize his own expected utility. As we specify the innovator as the seller, the supply Φ_t^* is always non-negative. By the same reason, if the financial innovation has limited liability, its price p_t in each time period must be non-negative in equilibrium.

The equilibrium of the financial innovation is characterized in detail by the next proposition.

Proposition 21 *A unique equilibrium of the financial innovation exists. Precisely,*

1. *In the last time period,*

$$\Phi_{T-1}^* = \frac{1}{2\gamma} \frac{\left(\mathbb{E}_{v,T-1}\left[f(x_T)\right] - \mathbb{E}_{n,T-1}\left[f(x_T)\right] - \beta\right)^+}{Var_{v,T-1}\left[f(x_T)\right]} \tag{C-7}$$

and if $\Phi^*_{T-1} > 0$, *then*

$$p_{T-1} = \frac{\mathbb{E}_{v,T-1}\left[f(x_T)\right] + \mathbb{E}_{n,T-1}\left[f(x_T)\right] + \beta}{2}. \tag{C-8}$$

2. *For each* $t = 0, 1, \cdots, T-2$, *let*

$$\Theta_t = \sum_{s=t+1}^{T} Cov_{t,v}\left(f(x_{t+1}), \mathbb{E}_{t+1,v}\left[\Phi^*_s\left(\mathbb{E}_{s,v}[f(x_{s+1})] - p_s\right)\right]\right), \tag{C-9}$$

then

$$\Phi^*_t = \frac{1}{2\gamma}\frac{\left(\mathbb{E}_{t,v}\left[f(x_{t+1})\right] - \mathbb{E}_{t,n}\left[f(x_{t+1})\right] - \beta - \gamma\Theta_t\right)^+}{Var_{t,v}\left[f(x_{t+1})\right]}; \tag{C-10}$$

and if $\Phi^*_t > 0$, *then*

$$p_t = \frac{\mathbb{E}_{t,v}\left[f(x_{t+1})\right] + \mathbb{E}_{t,n}\left[f(x_{t+1})\right] + \beta}{2} - \frac{\gamma}{2}\Theta_t. \tag{C-11}$$

The intuition of the equilibrium is as follows. In the last time period, the investor is a myopic investor, so both the price and the volume of the financial innovation are determined in a standard single-period rational equilibrium framework. Namely, the price is an average of the reservation prices of the agents, $\mathbb{E}_{n,T-1}[f(x_T)] + \beta$ for the innovator and $\mathbb{E}_{v,T-1}[f(x_T)]$ for the investor to participate in the market. The optimal volume of the financial innovation is positive as long as the investor's reservation price is greater than the innovator's reservation price.

However in a dynamic setting, the investor is no longer myopic and the intertemporal hedging demand is a substantial component in both the volume and the price of the financial innovation. Specifically, the intertemporal hedging term in equilibrium is given by the last term on the right side of equation (C-10) if we decompose the expression. Note that this hedging term is proportional to the covariance of the risky asset to the cumulative gain or loss over the remaining investment horizon. (Basak & Chabakauri, 2010) demonstrate that in a continuous-time framework the hedging demand of the dynamic mean-variance is derived by the expected total gain or losses from the investment on the risky asset over the investment horizon. Therefore, equation (C-10) is consistent with the hedging demand term in (Basak & Chabakauri, 2010), Proposition 1, in a discrete-time framework. Moreover, we demonstrate precisely that the hedging term is actually determined by the covariance between the financial innovation with the total gain or loss over the investment horizon in equilibrium.

According to Proposition 21, when the financial innovation product is negatively correlated to the anticipated gains over the remaining investment horizon, the hedging demand is high, therefore the equilibrium volume is high; and since the product becomes attractive in the market, its price is going high as well. On the other hand, if the financial innovation is positively related to the anticipated gains of the investment on the financial innovation in the future, both the hedging demand and the equilibrium demand is low, so as to the price. More intuitions about this intertemporal hedging demand will be explained in the following sections.

3.3.2 Viability and Stability in Equilibrium

Given the equilibrium characterization, it is natural to examine under what circumstances the financial innovation is viable as well as stable.

Definition 2 *The financial innovation at time t is* viable *if its equilibrium volume is* *positive. A viable financial innovation market at time t is* stable *if the viable property is preserved under a small movement of the market situation.*

For a viable financial innovation market, the equilibrium volume must be positive; in other words, there exists trading volume and liquidity on this financial innovation product. Proposition 21 describes the viability condition in equilibrium, that is, the financial innovation at time t is *viable* if and only if

$$\mathbb{E}_{t,v}\left[f(x_{t+1})\right] - \mathbb{E}_{t,n}\left[f(x_{t+1})\right] - \beta > \gamma\Theta_t. \tag{C-12}$$

Equation (C-12) states that to ensure the market viability at any time t, the reservation price for the investor must cover the hedging demand in addition to the reservation price of the innovator. If the the difference of the reservation prices for both agents is too small or the hedging demand of the financial innovation is too high, the financial innovation would be not viable or there is no trading activity. It is appealing that the financial innovation's viability depends endogenously on the observed market data as well as the posterior beliefs of both agents in the market. By contrast, (Calvet et al., 2004) develops a dynamic evolutionary approach in which the market participant role is given exogenously.

Given the heterogeneous beliefs about the state variable, the agents might have different probability on the market viability at time t, which affects the investment and issuance policy prior to time t. For instance, the innovator's (unconditional) probability about the market viability is computed by her prior distribution about the state variable, and her conditional probability is computed by her posterior distribution about the state variable accordingly. The market is viable only when both agents participate in the market. It is helpful to derive the probability of market viability for each agent endogenously since the prior distribution can be estimated in the model calibration.

Our definition on a stable financial innovation is also intuitive. Similar to the stability analysis in (Calvet et al., 2004), we expect that a small movement of the market situation will not yield a dramatic impact on the financial innovation's viability. The technical formulation is as follows. Assume $\mathcal{F}_t$ is generated by x_t, or alternatively, the state variable follows a Markov process, the equilibrium volume is written as $\Phi_t^*(x_t)$. The financial innovation market at time t under market observation x_t is *stable* if

$$\frac{\partial \Phi_t^*(x_t)}{\partial x_t} > 0 \tag{C-13}$$

in a small region of x_t (locally).[63]

The above characterization of the equilibrium also enables us to examine the pattern as well as the process of the financial innovation, $\Phi_t^*(x_t), t = 0, 1, \cdots, T-1$. In particular, a richer class of pattern including boom-bust of the financial innovation can be generated in this equilibrium model. As we will see in the next section, the equilibrium volume process depends on both the market condition and the structure of the financial innovation itself. The pattern can be quite different for different financial innovations under the same market condition. Alternatively, the equilibrium volume displays different shapes with changing market situation for the same financial innovation.

[63]Assuming the market is viable at x_t, that is, $\Phi_t^*(x_t) > 0$, then the locally stable condition ensures that $\Phi_t^*(x) > 0$ when x is close to x_t enough.

3.3.3 An example of two-period equilibrium

To explain the intuitions we consider in particular the equilibrium in a two-period market, $t = 0, 1, 2$. At time $t = 1$, each agent observes the realization of x_1 and updates expectation accordingly. The posterior mean and posterior variance are

$$\alpha_{i1} = \alpha_{i0} + \frac{\sigma_{i0}^2}{\sigma_{i0}^2 + \sigma_\eta^2}\left(x_1 - \alpha_{i0}\right), \qquad \sigma_{i1}^2 = \frac{\sigma_{i0}^2 \sigma_\eta^2}{\sigma_{i0}^2 + \sigma_\eta^2}. \tag{C-14}$$

Therefore, the expectation of agent i of x conditional on observing x_1 is based on the posterior distribution of μ. The expected return of the state variable under market information x_1 at time $t = 1$ is larger than α_{i0} when the market moves positively, i.e. $x_1 > \alpha_{i0}$; and $x_1 < \alpha_{i0}$ vice versa. The conditional variance of the state variance is reduced because of updated market information. Both the posterior mean and posterior variance affect the investor demand at time $t = 1$, as well as at time $t = 0$ in equilibrium.

Let

$$\mathcal{A} \equiv \{\mathbb{E}_v[f(x_2)|x_1] > \mathbb{E}_n[f(x_2)|x_1] + \beta\} \tag{C-15}$$

and

$$\Theta \equiv \frac{1}{4\gamma} Cov_v \left(f(x_1), \frac{\left(\mathbb{E}_v[f(x_2)|x_1] - (\mathbb{E}_n[f(x_2)|x_1] + \beta)\right)^2}{Var_v[f(x_2)|x_1]} I_{\mathcal{A}} \right). \tag{C-16}$$

Proposition 22 *1. The financial innovation market is* viable *at time $t = 1$ if, and only if the event $\mathcal{A}$ occurs; and if the market is viable, then the volume Φ_1^**

of the financial innovation is

$$\frac{\mathbb{E}_v[f(x_2)|x_1] - (\mathbb{E}_n[f(x_2)|x_1] + \beta)}{2\gamma Var_v[f(x_2)|x_1]},$$

and its price at $t = 1$ is

$$\frac{\mathbb{E}_v[f(x_2)|x_1] + (\mathbb{E}_n[f(x_2)|x_1] + \beta)}{2}.$$

2. *The market is viable at time $t = 0$ if and only if $\mathbb{E}_v[f(x_1)] - \mathbb{E}_n[f(x_1)] - \beta > \gamma\Theta$. The volume Φ_0^* of the financial innovation at time $t = 0$, when the market is viable, is*

$$\frac{\mathbb{E}_v[f(x_1)] - (\mathbb{E}_n[f(x_1)] + \beta) - \gamma\Theta}{2\gamma Var_v[f(x_1)]}$$

   and its price is

$$\frac{\mathbb{E}_v[f(x_1)] + (\mathbb{E}_n[f(x_1)] + \beta)}{2} - \frac{\gamma\Theta}{2}.$$

3. *(*Boom*) The financial innovation is viable at time 0 and continuously viable at future time when $\mathbb{E}_v[f(x_1)] - \mathbb{E}_n[f(x_1)] - \beta > \gamma\Theta$ and the event $\mathcal{A}$ occurs.*

4. (Bust*) The financial innovation is viable at time 0 but the market goes under at future time when $\mathbb{E}_v[f(x_1)] - \mathbb{E}_n[f(x_1)] - \beta > \gamma\Theta$ and the event $\mathcal{A}$ does not occurs.*

5. *The financial innovation is only issued at time $t = 1$ under event $\mathcal{A}$ when $\mathbb{E}_v[f(x_1)] - \mathbb{E}_n[f(x_1)] - \beta \leq \gamma\Theta$; There is no financial innovation market at all if $\mathbb{E}_v[f(x_1)] - \mathbb{E}_n[f(x_1)] - \beta \leq \gamma\Theta$ and the event $\mathcal{A}$ is not realized.*

According to Proposition 22, the dispersed beliefs among the innovator and the investor is crucial to understand the market viability. If the beliefs are fairly close, or alternatively, the level of uncertainty among the agents are similar, the financial innovation is unlikely to be successful from the market perspective. To ensure a reasonable financial innovation market, the level of model uncertainty across the agents must be quite different.

To understand the dynamic feature of the financial innovation, it is useful to examine the equilibrium at time $t = 0$ in this example. The difference in the reservation prices between the innovator and the investor must be greater than $\gamma\Theta$ to ensure the viability of the financial innovation. Notice that Θ equals to the covariance between the gain of the current self of the investor and the (expected) P& L of the future self of the investor.

$$\begin{aligned} \Theta &= Cov_v\left(f(x_1), \Phi_1^*(f(x_2) - p_1)\right) = Cov_v\left(f(x_1), \Phi_1^*(\mathbb{E}_{v,1} f(x_2) - p_1)\right) \\ &= Cov_v\left(f(x_1) - p_0, \Phi_1^*(\mathbb{E}_{v,1} f(x_2) - p_1)\right). \end{aligned}$$

Therefore, $-\frac{\Theta}{2Var_v(f(x_1))}$ is the intertemporal hedging demand for the representative time-inconsistent investor. The intuition is straightforward. When the expected gain on the investment of the financial innovation in two time periods are positively correlated, it means that a high gain (loss) in the first period likely leads to the high gain (loss) in the second time period, then the demand on the risky asset should be low due to the hedging consideration. By the same token, when a high gain (loss) in the first period likely ensures a small gain (loss) in the next time period, that is, $\Theta < 0$, the hedging demand should be high, so as to the equilibrium volume.

Our results also explain that the liquidity in equilibrium not only depends on the heterogeneous beliefs among the agents but also on the hedging demand, regardless of the time preference of the investor.

Proposition 22 presents the condition under which the equilibrium volume changes, and a particular pattern of the financial innovation occurred. For instance, the financial innovation can be issued continuously ("boom") or not ("bust") under different market conditions. It is also possible that the financial innovation product is viable again when the market condition changes. In this two-period model, it could be the case that initially there is no financial innovation market at time $t = 0$, while this financial innovation becomes viable at a future time $t = 1$.

There are some remarkable implications of the equilibrium analysis. First, the market viability is independent of the risk aversion of the investor at both time periods. No matter how large the investor's risk aversion is, the viability of market depends entirely on agents' beliefs and the payoff structure of the financial innovation. This independent property indeed demonstrates that the heterogeneous beliefs among the agents is pivotal to analyze the financial innovation's viability. In a similar way, the equilibrium price process of the financial innovations is irreverent to the risk aversion of the investors, and again, the heterogeneous beliefs among the agents determine the price process in equilibrium to a large degree.

Second, the risk aversion parameter affects the volume of the financial innovation negatively. The more risk averse of the investor,the smaller of the volume in each time period. This is intuitive since risk averse investor always intends to have safer cash flow, therefore, the demand on the financial innovation is reduced. Both the option volumes and the option prices are jointly affected by the model uncertainty for a CRRA investor in (Buraschi & Jiltsov, 2006). Proposition 22 further demonstrates that the volume and the price can be separately influenced by the concern on the model uncertainty.

Last, it is worth mentioning that the investor is not “myopic” due to the Bayesian learning on the expected return of the state variable. This point is substantial even when the investor's wealth is normally distributed, for instance, when the state variable has a normal distribution and $f(x)$ is a forward contract, since the time inconsistent feature hampers the standard myopic property of the long-term investor with time-consistent preference. We next move to discussions of several examples of financial innovation.

3.4 Examples of Financial Innovation

In this section, we examine the viability of financial innovation under different market conditions with several classical examples, which are the building blocks of most recent financial innovations and spans the contingent claim spaces in a general security market. We first consider a forward-type financial innovation with a linear payoff structure $f(x) = ax$, where a is a positive percentage and x is the underlying variable. Next we consider some examples of non-linear payoff structures; namely, a standard call option with a payoff $\max\{x - L, 0\}$, with a constant strike L; a capped forward with a payoff $\min\{x, K\}$ with a constant cap K, which is equivalent to a bond position and a short position in a put option; and a spread with a payoff $\max\{x - K, 0\} - \max\{x - L, 0\}$, which is a combination of longing a call option and writing another call option.[64] For simplicity we consider the two-period model in Section 3.3.

3.4.1 Forward-type Contract

The first example of financial innovation has a linear payoff structure $f(x) = ax$ where a is a positive parameter, possibly capturing the effects of haircuts and/or counter-party risk. It includes forward contracts or swap contracts written on non-tradable state variable, for instance, variance swap and correlation-related trading ETFs.

The following proposition establishes the conditions for the existence of the forward market.

Proposition 23 *Let*

$$A = \frac{\alpha_{n0} - \alpha_{v0} + g(\sigma_{v0})\alpha_{v0} - g(\sigma_{n0})\alpha_{n0} + \beta/a}{g(\sigma_{v0}) - g(\sigma_{n0})}$$

and

$$g(x) = \frac{x^2}{x^2 + \sigma_\eta^2}.$$

1. *The probability of market viability at time $t = 1$ for the innovator is $N\left(\frac{\alpha_{n0}-A}{\sqrt{\sigma_{n0}^2+\sigma_\eta^2}}\right)$.*

   The probability of market viability for the investor, however, is $N\left(\frac{\alpha_{v0}-A}{\sqrt{\sigma_{v0}^2+\sigma_\eta^2}}\right)$.

2. *The forward-type market at $t = 0$ is viable if and only if*

$$a(\alpha_{v0} - \alpha_{n0}) - \beta > \gamma\Theta$$

[64] These are typical examples to generate the contingent claim spaces. See (Tian, 2014) for many other examples.

where

$$\begin{aligned}\Theta &= \frac{a(\sigma_{v0}^2+\sigma_\eta^2)\left(g(\sigma_{v0})-g(\sigma_{n0})\right)^2}{2\gamma\sigma_\eta^2\left(1+g(\alpha_{v0})\right)} \\ &\quad\times\left\{(\alpha_{v0}-A)N\left(\frac{\alpha_{v0}-A}{\sqrt{\sigma_{v0}^2+\sigma_\eta^2}}\right)+\sqrt{\sigma_{v0}^2+\sigma_\eta^2}n\left(\frac{\alpha_{v0}-A}{\sqrt{\sigma_{v0}^2+\sigma_\eta^2}}\right)\right\}.\end{aligned}$$

In the equilibrium, the volume is

$$\Phi_0^* = \frac{a(\alpha_{v0}-\alpha_{n0})-\beta-\gamma\Theta}{2\gamma\left(\sigma_{v0}^2+\sigma_\eta^2\right)}$$

and the price is

$$p_0 = \frac{a(\alpha_{n0}+\alpha_{v0})+\beta-\gamma\Theta}{2}.$$

3. *There exists the forward-type market at $t=1$ if and only if $x_1 > A$, and if so, the volume is*

$$\Phi_1^* = \frac{g(\sigma_{v0})-g(\sigma_{n0})}{2\gamma a\sigma_\eta^2(1+g(\sigma_{v0}))}\times(x_1-A)$$

and the price is

$$p_1 = \frac{a(\alpha_{n1}+\alpha_{v1})+\beta}{2}.$$

4. *The forward contract market is stable at time $t=1$ as long as it is viable.*

Proposition 23 describes the equilibrium of forward contract, as a financial innovation, on a state variable x. The viable condition is simple. The forward contract market is viable in the second time period if and only if the state variable moves beyond a threshold, A, $x_1 > A$, and the volume is $x_1 - A$ up to a constant multiplier. By natural, its price is a linear function of x_1 at time $t = 1$. We call A or $A(\alpha_{n0}, \alpha_{v0}, \sigma_{n0}, \sigma_{v0})$ a *participation boundary,* which is similar to the notion used in (Person & Warther, 1997) in their analysis of the boom and bust pattern in the adoption of financial innovation. Moreover, the forward contract market is viable in the first time period as long as the condition $a(\alpha_{v0} - \alpha_{n0})) - \beta > \Theta$ holds, which depends on the prior estimation on the state variable by both the innovator and the investor, that is, $\{\alpha_{v0}, \alpha_{n0}, \sigma_{v0}, \sigma_{n0}\}$.

We study the effects of these model uncertainty parameters on the equilibrium of the forward contract. First, the probability of market viability for each agent, as stated in Proposition 23(1), depends on all agents' beliefs on the market through the participation boundary A. Figure 1-2 display the probability of market viability for each agent respectively. In Figure 1, $a = 1, \beta = 0.05, \sigma_\eta = 10\%$, and the investor's prior estimated parameters are $\alpha_{v0} = 1.5, \sigma_{v0} = 17.5\%$. The probability of market viability for the innovator is drawn when α_{n0} and σ_{n0} varies. Fixing σ_{n0}, the higher α_{n0} the smaller the probability. This fact is easy to understand. The market is viable if and only if $\alpha_{v1} - \alpha_{n1} > \beta$ (assuming $a = 1$); thus the higher α_{n0} the smaller the difference, $\alpha_{v0} - \alpha_{n0}$, in the agents' beliefs. Thus the likelihood of market viability becomes smaller. The effect of the volatility uncertainty is similar. The higher σ_{v0}, the smaller the difference of the posterior variance, $g(\sigma_{v0}) - g(\sigma_{n0})$, the smaller difference between the reservation prices. Hence, the probability of market viability is smaller. Figure 2 illustrates the effect of the investor's beliefs on the market viability. The contract parameter and cost parameter in Figure 2 are the same as in Figure 1, and $\alpha_{n0} = 1.5, \sigma_{n0} = 13\%$. Let $\sigma_{v0} \in (13\%, 18\%), \alpha_{v0} \in (1.55, 1.75)$, by the same reason as in Figure 1, the probability for the investor is increased with respect to each parameter, α_{v0}, σ_{v0}. We also note that the expected mean uncertainty has a stronger impact on the agent's views about the market viability than the volatility uncertainty in most situations.

Second, whether the market is viable or not depends jointly on the market situation (the state variable) and the heterogeneous beliefs. The market situation must be strong, that is, $x_1 > A$, besides, $\sigma_{v0} > \sigma_{n0}$. The second condition is important as it points out that the volatility uncertainty plays a crucial role in the market viability. If there is no volatility uncertainty, it is easy to see that the forward contract is either always viable or never viable. In other words, the variance of the market viability follows from the volatility uncertainty. When σ_{v0} is smaller than σ_{n0}, we can verify that the market is viable if and only of $x_1 < A$, given the market situation is weak.

Third, the forward contract market is always stable due to its linear payoff structure. The stability of the forward contract market follows from Proposition 23 easily:

$$\frac{\partial \Phi_1^*}{\partial x_1} = \frac{g(\sigma_{v0}) - g(\sigma_{n0})}{2\gamma a \sigma_\eta^2 (1 + g(\sigma_{v0}))} > 0.$$

A stable financial innovation market should be viewed as a desirable feature since a small movement of the market does not influence the market viability, thus reduces the possibility of liquidity as well as financial crisis. A large movement of the market though, from $x_1 > A$ to $x_1 < A$, would fundamentally change the market viability. We will show that the option market is not stable in general due to the non-linear payoff structure shortly below. Our results could explain why the forward contract and the swap market overall (in financial markets including fixed income, equity, credit, commodity, etc.) is successful and dominates the option market in terms of volume, because of its robust property of the viability and the stability.

We further draw the volume and the price of the forward contract at time t with respect to the underlying market situation x_1 and the innovator's prior mean α_{n0}.[65] Figure 3 displays the equilibrium volume and Figure 4 displays the equilibrium price under the same set of parameters: $\alpha_{v0} = 1.5, \sigma_{v0} = 17.5\%, \sigma_{n0} = 17\%, \sigma_{\eta} = 10\%, \beta = 0.05, a = 1$ and $\gamma = 2$. As expected, the impact of the market situation on both the volume and the price is positive and dominates in the forward contract market.

Finally, we study the effect of the intertemporal hedging demand of the time-inconsistent investor on the market equilibrium at time $t = 0$. If the investor is a myopic investor, the market equilibrium is irrelevant to the market situation. However, giving the Bayesian learning when the agent has model uncertainty concern, the investor is not myopic anymore. (Barberis, 2000) compares several portfolio strategies for a CARA investor in this setting and find out numerically that model uncertainty does affect the long-horizon investment. Similar to (Basak & Chabakauri, 2010), we find out that the intertemporal hedging demand is closely related to the expected gain of the investment over the investment horizon for the mean-variance investor. Moreover, in contrast to the purely portfolio choice setting in (Barberis, 2000) and (Basak & Chabakauri, 2010), we show that in equilibrium the intertemporal hedging demand is exactly proportional to the covariance of the risky asset's payoff with the expected cumulative gain on the financial innovation over the investment horizon. Proposition 23 allows us to study the market equilibrium at time $t = 0$ and the effect

[65] The impacts of other prior estimation parameters, $\alpha_{v0}, \sigma_{v0}, \sigma_{n0}$, can be studied in a similar manner.

of model uncertainty analytically. To illustrate, Figure 5 displays the equilibrium volume and Figure 6 shows the equilibrium price of the financial innovation at $t = 0$ when the innovator's prior estimation parameters change. From both figures we see the mean uncertainty, $\alpha_{v0} - \alpha_{n0}$, has a more substantial effect on the equilibrium than the volatility uncertainty, $\sigma_{v0} - \sigma_{n0}$, in general.

In Table 4, we list the prior estimation of the innovator to show when the market viable condition of the forward contract market is satisfied. The other input parameters in Table 3 are the same as in Figure 1 - 4: $\alpha_{v0} = 1.5, \sigma_{v0} = 17.5\%, \sigma_\eta = 10\%, \beta = 0.05$ and $a = 1$. As shown in Table 3, the heterogeneous beliefs on the mean and the volatility affect the market viability differently. Furthermore, the market viability is much less sensitive to volatility uncertainty than mean uncertainty. For example, when α_{n0} does not deviate too much from α_{v0}, say $\alpha_{n0} = 1.45$, approximately three percent drop to α_v, the market is not viable no matter how $\sigma_{v0} - \sigma_{n0}$ changes. However, when α_{n0} is small enough, the market is always viable despite the variation in σ_{n0}. Therefore, the heterogeneous beliefs in prior mean estimation has a more profound effect than the volatility.

3.4.2 Option-type Contracts

We next examine several examples of option-type contracts. We consider three kinds of option-type contract: call option (or equity-like), cap forward (or debt-like) and spread option with payoff structure $f_1(x) = \max\{x - L, 0\}$, $f_2(x) = \min\{x, K\}$, $f_3(x) = \max\{x - K, 0\} - \max\{x - L, 0\}$, respectively.[66] Hence, we use security "j",

[66] It is well known that call option and a cap forward resemble equity or a debt security in security design and these securities are shown to be optimal security under certain conditions in the asymmetric information setting or rational belief/equilibrium setting. See (Axelson, 2008), (Garmaise,

$j = 1, 2, 3$, to represent the call option, cap forward and spread option below.

Proposition 24 *Regardless of the viability for the option market at time $t = 0$,*

1. *The probability of market viability of security j at time $t = 1$, for agent $i \in \{n, v\}$, is*

$$Prob_i \left\{ \mathbb{E}_v[f_j(x)|x_1] > \mathbb{E}_n[f_j(x)|x_1] + \beta \right\}$$

where $Prob_i\{\cdot\}$ represents the prior probability for the agent $i \in \{n, v\}$.

2. *When the option market of security j is viable at time $t = 1$, the volume and the price of the security is, respectively*

$$\frac{\mathbb{E}_v[f_j(x)|x_1] - \mathbb{E}_n[f_j(x)|x_1] - \beta}{2\gamma Var_v[f(x)|x_1]}; \frac{\mathbb{E}_v[f_j(x)|x_1] + \mathbb{E}_n[f_j(x)|x_1] + \beta}{2}.$$

3. *The market of security 1 (call option) is viable if $x_1 >> 0$ and breaks down if $x_1 << 0$.*[67]

4. *The market of security $j = 2, 3$ (capped forward and spread) breaks down with extreme market movement of the state variable, that is, either $x_1 >> 0$ or $x_1 << 0$; and the market also breaks down if the innovator has very different prior belief from the investor about the market.*

5. *Each security market, in particular for the capped forward and the spread option, is not necessarily stable regardless of its viability at time $t = 1$.*

2001) and (DeMarzo, 2004).

[67] We use $x >> 0$ to represent sufficiently large x while $x << 0$ sufficient small x.

Proposition 24 demonstrates the *asymptotic* property of the market viability for each security j.[68] First, it illustrates the similarity between the call option and the forward contract in the strong situation. Since a deep in-the-money call option is close to a forward contract, the market continuously exists in the second time period with a large upward trend in the underlying variable, x_1.

Second, Proposition 24 states that under extreme downside movement of the underlying state variable, none of security j is viable anymore. In other words, the security issuance for option-type market vanishes in a stressed time period. Despite the complexity of the option payoff structure, the underlying market situation plays a key role in a falling issuance pattern of security. We have seen the same property for the forward contract in Proposition 23. Overall, it is a robust argument that the market breaks down anyway when the underlying state variable is in an extremely bad situation.

Third, the payoff structure of the security does not yield implication about the market viability when the underlying state variable is going in an extremely good direction, that is, $x_1 >> 0$; both the capped forward or the spread option market are not necessarily viable in contrast with the call option market. Nevertheless, within a moderate range of the market realization of x_1, the capped forward and the spread option displays different viable pattern.

In contrast with the forward-type market, Proposition 24 demonstrates that the option market is not stable. We use the capped-forward (security 2) to illustrate this

[68] The analytical expressions about the probability of market viability and the volume and the price of security $j = 1, 2, 3$, are given in Appendix A, Lemma 12.

remarkable point. As explained in Proposition 22, the market is viable at time $t = 1$ if and only if

$$K(x_1) \equiv \mathbb{E}_{1,v}[x_2 - (x_2 - K)^+] - \mathbb{E}_{1,n}[x_2 - (x_2 - K)^+] - \beta$$

is positive. The difference of the reservation price is drawn in Figure 7, given its analytical expression in Appendix A, Lemma 2. It is shown that $K(x_1)$ is negative when x_1 is large enough and positive when x_1 is fairly small. The volume at time $t = 1$ is

$$\Phi_1^*(x_1) = \frac{max\{K(x_1), 0\}}{2\gamma Var_{1,v}(f(x_2))}$$

and its analytical expression is also available in Appendix A, Lemma 2. We can check whether this volume function is monotonically increasing with respect to the market state variable x_1. As shown in Figure 8, the volume is not stable under certain parameters: $\alpha_{n0} = 5, \alpha_{v0} = 7, \sigma_{v0} = 17.5\%, \sigma_{n0} = 17\%, \beta = 0.05, \sigma_\eta = 10\%$, and the strike price of this capped forward is $K = 1$. Therefore the shape of the equilibrium volume for the spread option is more complicated; the stability property of the option market is not persistent as what we have demonstrated in the forward contract market.

We have examined the market dynamics for individual security with simple payoff structures. We then move to the market securitization in which the innovator issues several different tranche simultaneously.

3.5 Implication for Asset-Backed Securities

Our analysis in the previous section has implications for a complex financial innovation: asset-backed securities. In essence, asset-backed securitization is to pool together underlying assets and issue a prioritized structure of claims, known as *tranches*, against these collateral pools. A prototype of asset-backed securities in structured finance is collateralized debt obligations (CDOs). There are three prioritized tranches in a typical CDO structure. The tranche with the least priority and bearing the first brunt of losses is called the *equity tranche*, the tranche with the highest priority is the *senior tranche*, and the tranche with an intermediate priority is the *mezzanine tranche*. We abstract the following discussion of CDO structure from practical institutional intricacies in issuing, monitoring and managing CDO securities.

In its barest form, the payoff to the equity tranche of a CDO structure is represented by a call option, $max\{x-(F-K),0\}$, where x is the value of the underlying collateral pool, F is its face value, and K is the detachment point designating the amount of losses born by the equity tranche. The payoff to the senior tranche is represented by a capped forward contract, $min\{x,F-L\}$, where L is the second detachment point designating the maximum level of losses before the senior tranche will be hit. The payoff to the mezzanine tranche will have a cash flow of $max\{x-(F-L),0\}-max\{x-(F-K),0\}$, similar to a spread contract. The table below summarizes the seniority and the payoff structure of a standard CDO. y represents the loss of the collateral pool.

Seniority	Loss	Payoff
Senior	$max\{y-L,0\}$	$min\{x,F-L\}$
Mezzanine	$max\{y-K,0\}$ $-max\{y-L,0\}$	$max\{x-(F-L),0\}$ $-max\{x-(F-K),0\}$
Equity	$\min\{y,K\}$	$max\{x-(F-K),0\}$

We consider an innovator who decides to issue the tranches together through a special purpose entity, and the tranches have payoff structure $f_1(x), f_2(x)$ and $f_3(x)$ respectively. The trigger points K and L are fixed and known even though it could be solved optimally from the innovator's perspective. For simplicity, we assume the issuance takes place simultaneously. The investors, however, have different risk preferences. There are three different groups of investors, and each group represents a particular tranche security. For this purpose, we assume each representative investor has a mean-variance utility and their risk aversion parameters are written as γ_i, corresponding to the tranche with the payoff $f_i(x)$. We use $\mathbb{E}_{v,i}[\cdot], Var_{v,i}[\cdot]$ to represent the expectation and variance of the payoff for the corresponding investor i-type investor, $i \in \{1,2,3\}$. In terms of the cost structure, we assume $C(N) = \alpha + 3\beta N$.[69]

The dynamic equilibrium for the CDO market under our setting is given by the next proposition.

Proposition 25 *1. The CDO market is* viable *at the second time period if*

$$\sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x)|x_1] - \alpha_{n1} > 3\beta;$$

[69] α represents a fixed cost for setting up the structure, while βN dentes the variable cost for each tranche, hence $\alpha + 3\beta N$ is the total cost.

and if so, the equilibrium volume of the CDOs is given by

$$\Phi_1(p) = \frac{\sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x)|x_1] - \alpha_{n1} - 3\beta}{2\sum_{i=1}^{3} \gamma_i Var_{v,i}[f_i(x)|x_1]}, \tag{C-17}$$

and the equilibrium prices of each tranche at time $t = 1$ is given by

$$p_{1,i} = \mathbb{E}_{v,i}[f_i(x)|x_1] - \gamma_i \Phi_1(p) Var_{v,i}[f_i(x_2)|x_1], i = 1, 2, 3. \tag{C-18}$$

We use $\mathbb{E}_{v,i}[\cdot], Var_{v,i}[\cdot]$ to represent the conditional expectation and conditional variance for the investor $i \in \{1, 2, 3\}$ after Bayesian learning based on his own prior belief on the market state variable.

2. *The CDO market is* viable *at time $t = 0$ if*

$$\sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x_1)] - \alpha_{n1} - 3\beta > \sum_{i=1}^{3} \gamma_i Cov_{v,i}\left(f_i(x_1), \Phi_1(p)(\mathbb{E}_v[f_i(x_2)|x_1] - p_{1,i})\right).$$

If there exists the equilibrium market, then the equilibrium volume and the corresponding tranch price are given by

$$\Phi_0(p) = \frac{\sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x_1)] - \alpha_{n0} - 3\beta - \sum_{i=1}^{3} \gamma_i Cov_v\left(f_i(x_1), \Phi_1(p)(\mathbb{E}_v[f_i(x_2)|x_1] - p_{1,i})\right)}{2\sum_{i=1}^{3} \gamma_i Var_{v,i}[f_i(x_1)]} \tag{C-19}$$

and

$$p_{0,i} = \mathbb{E}_{v,i}[f_i(x_1)] - \gamma_i \Phi_0(p) Var_{v,i}[f_i(x_1)] - \gamma_i Cov_{v,i}\left(f_i(x_1), \Phi_1(p)(\mathbb{E}_v[f_i(x_2)|x_1] - p_{1,i})\right). \tag{C-20}$$

3. The probability of the CDO market viability is

$$Prob_i \left(\sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x)|x_1] - \alpha_{n1} > 3\beta \right), i \in \{1, 2, 3, n\}$$

under his own prior belief for individual agent (innovator n and investor $i = 1, 2$ and 3).

The CDO structure is to some extent similar to individual financial innovation market of Section 3, but the viability condition in each time period depends on all investors because the volume of each tranche must be the same. In a special case where each investor has the same prior belief on the state variable but risk-aversion [70], the CDOs market's viability resembles to the forward contract as in Proposition 23. In fact, notice that $\sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x)|x_1] = \mathbb{E}_v[x|x_1] = \alpha_{v1}$ in this situation, and CDOs market is viable at the second time period (when each investor has the same prior belief) if and only if

$$\alpha_{v1} - \alpha_{n1} > 3\beta$$

or equivalently

$$x_1 > \frac{\alpha_{n0} - \alpha_{v0} + g(\sigma_{v0})\alpha_{v0} - g(\sigma_{n0})\alpha_{n0} + 3\beta}{g(\sigma_{v0}) - g(\sigma_{n0})}.$$

It is worth further noting that the above viability condition is stronger than the viability condition in a typical forward contract because of the issuance cost. More importantly, the equilibrium price of one tranche depends on all investors' risk preference.

[70] An important question is to study how the prior belief and risk aversion affect the investor's selection in individual tranche market, which is beyond the scope of this paper.

Similarly, the CDO market is viable during the first time period if

$$\alpha_{v0} - \alpha_{n0} - 3\beta > \Theta^s$$

where

$$\Theta^s \equiv \sum_{i=1}^{3} \frac{\gamma_i^2}{2} Cov_v \left(f_i(x_1), \frac{(\alpha_{v1} - \alpha_{n1} - 3\beta)^2 Var_v[f_i(x_2)|x_1] I_{\{\alpha_{n1} - \alpha_{v1} - 3\beta > 0\}}}{(\sum_i \gamma_i Var_i[f_i(x)|x_1])^2} \right)$$

is the corresponding covariance risk for all investors in the market.

Furthermore, the market viability is significantly affected by the investors' prior beliefs even though their risk preferences are the same. First, the viability condition of CDOs is more sensitive to the market realization of x_1 given the complex pattern of $\sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x)|x_1]$[71]. Second, the presence of the heterogeneous beliefs makes the CDO market fragile with the new realization of the market variable x_1 and this heterogeneity has a commanding influence. By the same reason, the probability of market viability for each agent varies considerably. Last but not the least, the equilibrium price of each tranche depends on agents' prior beliefs rather than agents' risk preference.

Proposition 26 *Regardless the viability at time $t = 0$, there always exists securitization when the underlying market situation is strong. Securitization market collapses in a very week underlying market situation.*

Proposition 26 has important implication about the securitization market. It states that when the underlying market is very favorable to all agents, the CDOs market is always viable. The market is favorable when x_1 is very large, or even when x_1

[71]The expression is given in Appendix A, Lemma 12.

is moderate but the prior beliefs of all agents satisfy that (according to Proposition 25) $\sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x_1)] - \alpha_{n1} - 3\beta > 0$. It thus explains the continuous and boom pattern of CDOs market in a favorable scenario. On the other hand, when the underlying market is not favorable in the sense that a very small x_1, or the agents' priors (after learning more about the market and potential ignoring risk) satisfy that $\sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x_1)] - \alpha_{n1} - 3\beta < 0$, the CDOs market is not viable anymore. Overall, Proposition 26 explains the boom-burst pattern of the CDOs market under different market situation.

Surprisingly, Proposition 26 indicates that the securitization structure is similar to the forward market (when all agents have the same prior beliefs), or a market of call option with small strike price. In other words, the equity tranche issuance dominates the issuance in the securitization structure.

We perform a comparison analysis to which each tranche is issued separably in Section 3.4. Our next observation is that the securitization volume $\Phi(p)^*$ is a weighted average of volume $\Phi_i(p)^*$ for $i = 1, 2, 3$, volume of tranche $f_i(x)$ when each market is viable. Specifically, the securitization volume

$$\Phi(p)^* = \alpha_1\Phi_1(p)^* + \alpha_2\Phi_2(p)^* + \alpha_3\Phi_3(p)^*, \alpha_1 + \alpha_2 + \alpha_3 = 1 \tag{C-21}$$

is a weighted average of the volume of each tranche, where

$$\alpha_i = \frac{\gamma_i Var_{v,i}[f_i(x)|x_1]}{\sum_{i=1}^{3} \gamma_i Var_{v,i}[f_i(x)|x_1]}, i = 1, 2, 3.$$

In general, we see that [72]

$$\Phi(p)^* \le \alpha_1\Phi_1(p)^* + \alpha_2\Phi_2(p)^* + \alpha_3\Phi_3(p)^*.$$

It means that the securitization volume is actually bounded by the weighted average of equilibrium volume of each tranche, assuming this tranche can be traded separably.

Intuitively, both the mezzanine tranche and the senior tranche are less volatile than the equity tranche. Hence, the weights α_2 and α_3 should approach to zero when the underlying market is strong. Consequently, the securitization structure resembles the equity tranche in terms of volume. The next proposition formally justifies the intuition.

Proposition 27 *In the very strong underlying market, the equilibrium volume of the securitization structure can be as large as possible, in the sense that*

$$\lim_{x_1\to\infty} \Phi(p)^* = \infty. \tag{C-22}$$

Moreover, the securitization volume is close to the equity tranche volume in the sense that

$$\lim_{x_1\to\infty} \{\Phi(p)^* - \Phi_1(p)^*\} = 0, \tag{C-23}$$

[72]

$$\begin{aligned}
\Phi(p)^* &= \frac{\left(\left(\sum_{i=1}^{3}\mathbb{E}_{v,i}[f_i(x)|x_1] - \mathbb{E}_n[f_i(x)]\right) - 3\beta\right)^+}{2\sum_{i=1}^{3}\gamma_i Var_{v,i}[f_i(x)|x_1]} \\
&\le \frac{\sum_{i=1}^{3}\left(\mathbb{E}_{v,i}[f_i(x)|x_1] - \mathbb{E}_n[f_i(x)] - \beta\right)^+}{2\sum_{i=1}^{3}\gamma_i Var_{v,i}[f_i(x)|x_1]} \\
&\le \alpha_1\Phi_1(p)^* + \alpha_2\Phi_2(p)^* + \alpha_3\Phi_3(p)^*
\end{aligned}$$

where in the first equation we apply for fact $\sum_i f_i(x) = x$ again, we make use of the fact that $(x+y+z)^+ \le x^+ + y^+ + z^+$ in the second inequality and use Proposition 24 in the last part.

and

$$\lim_{x_1 \to \infty} \{\Phi(p)^* - \Phi_i(p)^*\} = \infty, i = 2, 3. \tag{C-24}$$

This proposition is helpful to distinguish the role of each tranche. When the underlying market is strong, even though the issuance on a senior tranche, or a mezzanine tranche, is not desirable, security issuance through securitization as a whole is still optimal. To market the senior and mezzanine tranches, the innovator has to sell them at a discount and thus increases the yields. This analysis offers a plausible explanation of the high-yield puzzle of the senior tranche. Although it might be better to retain, the senior and the mezzanine tranches are largely sold, as long as the potential losses can be offset by the gains from selling the equity tranche and from the benefits of not holding any of these tranches in their inventories. This is reminiscent of the situation during the boom of the CDO market. (Coval, Jurek, & Stafford, 2009a) offer an explanation of this high-yield puzzle based on the mis-representation of ratings and the unpriced risk of economic catastrophe. Our analysis presents an alternative explanation from the issuer's perspective.

3.6 Conclusion

We have presented a dynamic equilibrium model of financial innovation (in the form of new security) to analyze the effects of heterogeneous beliefs among agents on the security's equilibrium pricing and viability. We characterize the equilibrium of financial innovation when the investor is more prone to time inconsistent than the innovator given the salient features of innovation. We show that both volume and price of the new security are sensitive to the divergent beliefs of participating agents.

We further confirm that the intertemporal hedging demands of the investor plays a crucial role in the equilibrium analysis.

As complex financial innovation is usually a combination of basic financial contracts, we study in detail forward contract and option-like securities. We demonstrate the viability of forward- and option-like securities under varying market scenarios. We show that the market for option-type contracts can be vulnerable under the adverse market movement. In contrast, the market for forward-type contracts is resilient and stable. We also examine the viability of the securitization since the aforementioned derivatives can resemble individual tranches of a securitization structure. We find out that given the model uncertainty concern, the securitization is to a large extent more robust than individual tranche. Overall, this study sheds some light on the impact of the model uncertainty factor on pricing and trading of complex financial securities, such as CDOs, and helps us further evaluate the efficacy of these securities for sharing and managing risk.

Appendix A. Proofs

We start with the proof of a special case before proving a general result.

Proof of Proposition 22.

(1) We start with the characterization of Nash equilibrium at time $t = 1$. At time $t = 1$, the innovator determines the supply of the financial innovation with payoff $f(x_2)$ at maturity time $t = 2$ while the investor's future self at time $t = 1$ sends his optimal demand schedule at $t = 1$.

The investor's wealth at time $t = 2$ is $W_{v2} = W_{v1} + \Phi_1(p_1)(f(x_2) - p_1)$, where W_{v1} is the wealth of the investor at time $t = 1$. The optimal allocation problem for the investor's future self at time $t = 1$ is

$$\max_{\Phi_1(p_1)\geq 0} \left\{ \mathbb{E}_v \left[W_{v2} | x_1 \right] - \frac{\gamma}{2} Var_v \left[W_{v2} | x_1 \right] \right\}.$$

Hence, the optimal demand schedule for the investor's future self at time $t = 1$ is

$$\Phi_1(p_1) = \frac{(\mathbb{E}_v[f(x_2)|x_1] - p_1)^+}{\gamma Var_v[f(x_2)|x_1]}, \tag{A-1}$$

where the positivity restriction $(\cdot)^+$ reflects the no-short-sale constraint on the financial innovation. For the security to be viable, $\Phi_1(p_1)$ must be positive, it thus follows from the equation (A-1) that

$$p_1 = \mathbb{E}_v[f(x_2)|x_1] - \gamma Var_v[f(x_2)|x_1]\Phi_1(p_1). \tag{A-2}$$

On the other hand, the innovator at time $t = 1$ solves the problem

$$\max_{\Phi_1(p_1)\geq 0} \left\{ \mathbb{E}_n \left[\Phi_1(p_1)(p_1 - f(x_2)) | x_1 \right] - C(\Phi_1(p_1)) \right\}. \tag{A-3}$$

By plugging equation (A-2) into the problem (A-4) of the innovator, the first order condition with respect to $\Phi_1(p_1)$ ensures that the equilibrium at $t = 1$ exists if, and only if the market clears at $\{p_1, \Phi_1(p_1)\}$, where the equilibrium price p_1 and the equilibrium volume $\Phi_1(p_1)$ are given by

$$\Phi_1(p_1) = \frac{\mathbb{E}_v[f(x_2)|x_1] - (\mathbb{E}_n[f(x_2)|x_1] + \beta)}{2\gamma Var_v[f(x_2)|x_1]}, p_1 = \frac{\mathbb{E}_v[f(x_2)|x_1] + \mathbb{E}_n[f(x_2)|x_1] + \beta}{2} \tag{A-4}$$

as long as $\mathbb{E}_v[f(x_2)|x_1] > \mathbb{E}_n[f(x_2)|x_1] + \beta$.

(2) The investor's current self at time $t = 0$ determines his optimal demand by taking account of the optimal demand strategy for his future self at time $t = 1$. Therefore, the optimal problem for the current self is to maximize

$$\max_{\Phi_0(p_0)\geq 0} \mathbb{E}[W_2] - \frac{\gamma}{2} Var[W_2] \tag{A-5}$$

where $W_2 = W_0 + \Phi_0(p_0)(f(x_1) - p_0) + \Phi_1(p_1)(f(x_2) - p_1)$ and $\{\Phi_1(p_1), p_1\}$ are determined in equation (A-4). The second term of the terminal wealth, $\hat{\epsilon} \equiv \Phi_1(p_1)(f(x_2) - p_1)$, is known for the current self and $\hat{\epsilon}$ denotes his future self's P&L in the second time period.

It is easy to derive that, by using the first order condition in the above optimal problem for the current self,

$$\Phi_0(p_0) = \frac{(\mathbb{E}_v[f(x_1)] - p_0 - \gamma Cov_v\left(f(x_1), \hat{\epsilon}\right))^+}{\gamma Var_v[f(x_1)]}, \tag{A-6}$$

and the financial innovation is viable at time $t = 0$ if $\mathbb{E}_v[f(x_1)] - p_0 > \gamma Cov_v\left(f(x_1), \hat{\epsilon}\right)$. To determine the equilibrium price p_0, the innovator's optimal supply at time $t = 0$

is

$$N^* = argmax_{N\geq 0}\mathbb{E}_n\left[N(p_0 - f(x_1)) - C(N) - D\right]. \tag{A-7}$$

In equilibrium, $N^* = \Phi_0(p_0)$, and therefore by equation (A-6), we have

$$p_0 = \mathbb{E}_v[f(x_1)] - \gamma Cov_v\left(f(x_1), \hat{\epsilon}\right) - \gamma N^* Var_v[f(x_1)]. \tag{A-8}$$

By plugging the above relationship between p_0 and $\mathcal{N}^*$ into (A-7) and investigating the first order condition with respect to the supply N again, the time consistent optimal strategy at time $t = 0$ is

$$\Phi_0(p_0) = \frac{\mathbb{E}_v[f(x_1)] - \mathbb{E}_n[f(x_1)] - \beta}{2\gamma Var_v[f(x_1)]} - \frac{Cov_v\left(f(x_1), \hat{\epsilon}\right)}{2Var_v[f(x_1)]} \tag{A-9}$$

as long as $\mathbb{E}_v[f(x_1)] - \mathbb{E}_n[f(x_1)] - \beta > \gamma Cov_v(f(x_1), \hat{\epsilon})$; and if so, the price of the financial innovation at time $t = 0$ is

$$p_0 = \frac{\mathbb{E}_v[f(x_1)] + \mathbb{E}_n[f(x_1)] + \beta}{2} - \frac{\gamma Cov_v\left(f(x_1), \hat{\epsilon}\right)}{2}. \tag{A-10}$$

To finish the proof, notice that $Cov(x, yz) = Cov(x, y\mathbb{E}\left[z|\mathcal{F}\right])$, for variables $x, y \in \mathcal{F}$, $z \in \mathcal{G}$ and $\mathcal{F} \subseteq \mathcal{G}$. Then

$$\Theta \equiv Cov_v\left(f(x_1), \hat{\epsilon}\right) = Cov_v\left(f(x_1), \Phi_1(p_1)(\mathbb{E}_v[f(x_2)|x_1] - p_1)\right) \tag{A-11}$$

and by straightforward calculation, we have

$$\Theta = \frac{1}{4\gamma} Cov_v\left(f(x_1), \frac{\left(\mathbb{E}_v[f(x_2)|x_1] - (\mathbb{E}_n[f(x_2)|x_1] + \beta)\right)^2}{Var_v[f(x_2)|x_1]} 1_{\mathcal{A}}\right). \tag{A-12}$$

The proof is finished. □

Proof of Proposition 21.

The uniqueness follows from the existence of the equilibrium recursively. The first part of the proof is similar to Proposition 22, (1), and logical of the second proof is the same as to Proposition 22, (2) by incorporating all time consistent equilibrium for the investor's future selves at $s = t+1, \cdots, T$ and applications of the law of iterated expectation. Moreover, if the financial innovation has limited liability, and $\Phi_t^* > 0$, then $\mathbb{E}_{t,v}[f(x_{t+1})] - \mathbb{E}_{t,n}[f(x_{t+1})] - \beta - \gamma\Theta_t > 0$ by equilibrium, so

$$\mathbb{E}_{t,v}[f(x_{t+1})] + \mathbb{E}_{t,n}[f(x_{t+1})] + \beta - \gamma\Theta_t > 2\left(\mathbb{E}_{t,n}[f(x_{t+1})] + \beta\right) \geq 0.$$

Therefore, $p_t \geq 0$. $\square$

Proof of Proposition 23.

By Proposition 21, the market is viable if and only if $\mathbb{E}_v[ax_2|x_1] > \mathbb{E}_n[ax_2|x_1] + \beta$. Notice that $\mathbb{E}_i[x_2|x_1] = \alpha_{i1}$ and $Var_i[x_2|x_1] = \sigma_\eta^2(1 + g(\sigma_{i0}))$ for $i \in \{n, v\}$. Therefore, the market is viable is only of $x_1 > A$. By Proposition 21, it remains to compute the covariance

$$\begin{aligned} \Theta &= \frac{1}{4} Cov_v\left(ax_1, \frac{(a(\alpha_{v1} - \alpha_{n1}) - \beta)^2}{a^2 Var_v[x_2|x_1]} 1_{\{x_1 > A\}}\right) \\ &= \frac{1}{4a\sigma_\eta^2(1 + g(\sigma_{v0}))} Cov\left(x_1, (a(\alpha_{v1} - \alpha_{n1}) - \beta)^2 1_{\{x_1 > A\}}\right). \end{aligned}$$

By the Stein's lemma that $Cov(x, G(x)) = Var(x)\mathbb{E}[G'(x)]$ for a normal distributed variable x and the dominated convergence theorem, we have

$$Cov\left(x_1, (a(\alpha_{v1} - \alpha_{n1}) - \beta)^2 1_{\{x_1 > A\}}\right) = 2(\sigma_{v0}^2 + \sigma_\eta^2)a^2(g(\sigma_{v0}) - g(\sigma_{n0}))^2 \mathbb{E}[(x_1 - A)1_{\{x_1 > A\}}]$$

and the expression of Θ follows from the formula $\mathbb{E}[\zeta^+] = \mu N\left(\frac{\mu}{\sigma}\right) + \sigma n\left(\frac{\mu}{\sigma}\right)$ for a normal distributed variable ζ with mean μ and variance σ^2. $\square$

To prove Proposition 24, we present several lemmas with independent interests, whose proofs are straightforward and omitted.

Lemmas

The first lemma is a standard fact on the normal distribution.

Lemma 11 *Given a normally distributed random variable ζ with mean μ and variance σ^2, we have*

a. $\mathbb{E}[n(\zeta)] = \frac{1}{\sqrt{1+\sigma^2}} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$,

b. $\mathbb{E}[\zeta n(\zeta)] = \frac{\mu}{(1+\sigma^2)^{3/2}} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$,

c. $\mathbb{E}[N(\zeta)] = N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$

d. $\mathbb{E}[\zeta N(\zeta)] = \mu N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) + \frac{\sigma^2}{\sqrt{1+\sigma^2}} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$,

e. $\mathbb{E}[\zeta^2 N(\zeta)] = (\mu^2+\sigma^2)N(\frac{\mu}{\sqrt{1+\sigma^2}}) + \frac{\mu\sigma^2}{\sqrt{1+\sigma^2}}\frac{2+\sigma^2}{1+\sigma^2} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$.

f. $\mathbb{E}[\zeta^2 n(\zeta)] = \frac{\mu^2+\sigma^2(1+\sigma^2)}{(1+\sigma^2)^{3/2}} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)$.

The second lemma concerns about both the (conditional) expected return and the (conditional) variance of each option contract $f_j(x), j = 1, 2, 3$ for agent $i \in \{n, v\}$.

Lemma 12 *1. For the forward contract,* $\mathbb{E}_i[x|x_1] = \alpha_{i1}, \mathbb{E}_i[x] = \alpha_{i0}$, *and* $Var_i[x|x_1] = \sigma_\eta^2(1+g(\sigma_{i0})), Var_i[x_1] = \sigma_\eta^2 + \sigma_{i0}^2$.

2. For the call option,

$$\mathbb{E}_i[f_1(x)|x_1] = \sigma_\eta \left\{\mu N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) + \sqrt{1+\sigma^2} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right)\right\}, \quad \text{(A-13)}$$

and

$$\mathbb{E}_i[f_1(x)^2|x_1] = \sigma_\eta^2 \left\{ (1+\mu^2+\sigma^2) N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) + \mu\sqrt{1+\sigma^2}\, n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) \right\} \quad \text{(A-14)}$$

where $\mu := \frac{\alpha_{i1}-L}{\sigma_\eta}$ *and* $\sigma^2 := \frac{\sigma_{i0}^2}{\sigma_{i0}^2+\sigma_\eta^2}$. *Moreover, both* $\mathbb{E}_i[f_1(x)]$ *and* $\mathbb{E}_i[f_1(x)^2]$ *are calculated similarly in which* μ *and* σ *are replaced by* $\frac{\alpha_{i0}-L}{\sigma_\eta}$ *and* σ_{i0}^2*, respectively.*

3. *For the capped forward contract,* $\mathbb{E}_i[f_2(x)|x_1] = \mathbb{E}_i[x] - \mathbb{E}_i[(x-K)^+|x_1]$*, and its conditional variance is*

$$\begin{aligned} Var_v[f_2(x)|x_1] &= Var_v[x|x_1] + Var_v[(x-K)^+|x_1] \\ &\quad + 2\{\mathbb{E}[x|x_1]\mathbb{E}[(x-K)^+|x_1] - \mathbb{E}[x(x-K)^+|x_1] \end{aligned}$$

in which

$$\mathbb{E}[x(x-K)^+|x_1] = \mathbb{E}[(\sigma_\eta^2 + \sigma_\eta K\zeta + \sigma_\eta^2\zeta^2)N(\zeta)] + \mathbb{E}[(\sigma_\eta^2\zeta + K\sigma_\eta)n(\zeta)] \quad \text{(A-15)}$$

where $\zeta \sim \mathcal{N}\left(\frac{\alpha_{i1}-K}{\sigma_\eta}, \sigma^2\right)$. *The unconditional mean and unconditional variance of* $f_2(x)$ *is computed similarly.*

4. *For the spread option, its (conditional) mean follows from the computation of the call options; its conditional variance is*

$$\begin{aligned} Var_v[f_3(x)|x_1] &= Var_v[(x-K)^+|x_1] + Var_v[(x-L)^+|x_1] \\ &\quad - 2\{\mathbb{E}[(x-K)^+(x-L)^+|x_1] - \mathbb{E}[(x-K)^+|x_1]\mathbb{E}[(x-L)^+|x_1]\}. \end{aligned}$$

Conditional Variances and Expectation

This table displays the market effects on the conditional expectation and conditional variance for non-linear contracts (call option, capped forward and the spread option) as well as the forward contract. It demonstrates clearly the difference of those conditional expectation and conditional variance for different securities under different market situation, either $x_1 \gg 0$ or $x_1 \ll 0$.

Security	Payoff	Variance		Expectation	
		$x_1 \uparrow \infty$	$x_1 \downarrow -\infty$	$x_1 \uparrow \infty$	$x_1 \downarrow -\infty$
Call option	$(x-K)^+$	$\sigma_\eta^2(1+\sigma^2)$	0	∞	0
Capped Forward	$\min\{x, K\}$	0	$\sigma_\eta^2(1+\sigma^2)$	K	$-\infty$
Spread	$(x-K)^+ - (x-L)^+$	0	0	$L-K$	0
Forward	x	$\sigma_\eta^2(1+\sigma^2)$	$\sigma_\eta^2(1+\sigma^2)$	∞	$-\infty$

Moreover, for $K < L$,

$$
\begin{aligned}
\mathbb{E}[(x-K)^+(x-L)^+|x_1] &= \mathbb{E}[(\sigma_\eta^2\zeta + \sigma_\eta(L-K))n(\zeta)] \\
&\quad + \mathbb{E}\left[(\sigma_\eta^2 + \sigma_\eta^2\zeta^2 + \sigma_\eta(L-K)\zeta)N(\zeta)\right]
\end{aligned}
$$

where $\zeta \sim \mathcal{N}\left(\frac{\alpha_{i1}-L}{\sigma_\eta}, \sigma^2\right)$. The expression of $Var_i[f_3(x)]$ is similar.

The *asymptotic* properties of the conditional means and conditional variances are given in the subsequent lemma.

Lemma 13 *The asymptotic properties, when $x_1 \uparrow \infty$ and $x_1 \downarrow -\infty$, of the conditional means and conditional variances for the securities with payoff $f_1(x) = (x-L)^+$, $f_2(x) = \min\{x, K\}$, $f_3(x) = (x-K)^+ - (x-L)^+$ are given by Table 1. In this table the conditional mean and conditional variance are calculated for agent $i \in \{m, n, v\}$, and $\sigma^2 := \frac{\sigma_{i0}^2}{\sigma_{i0}^2 + \sigma_\eta^2}$.*

The next lemma concerns the sensitivity regard to the mean and the variance.

Lemma 14 *Given a normally distributed random variable ζ with mean μ and variance σ^2. Then*

$$\frac{\partial\{\mathbb{E}[\zeta N(\zeta)] + \mathbb{E}[n(\zeta)]\}}{\partial \mu} = N\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) \tag{A-16}$$

and

$$\frac{\partial\{\mathbb{E}[\zeta N(\zeta)] + \mathbb{E}[n(\zeta)]\}}{\partial \sigma} = \frac{\sigma}{\sqrt{1+\sigma^2}} n\left(\frac{\mu}{\sqrt{1+\sigma^2}}\right) \tag{A-17}$$

where $n(\cdot)$ and $N(\cdot)$ is the normal density function and the cumulative normal distribution of a standard normal variable.

At last, lemma 5 is used to analyze the market equilibrium under extreme market condition.

Lemma 15 *Let $F(x) = x\sum_{i=1}^{m} a_i N\left(b_i + c_i x\right), c_i > 0$. Then*

$$\lim_{x\to-\infty} F(x) = 0, \tag{A-18}$$

and

$$\lim_{x\to+\infty} F(x) = \begin{cases} \infty, & \sum_{i=1}^{m} a_i > 0 \\ -\infty, & \sum_{i=1}^{m} a_i < 0 \\ 0, & \sum_{i=1}^{m} a_i = 0 \end{cases} \tag{A-19}$$

Prop of Proposition 24.

The first part of Proposition 24 follows from Proposition 21 while the analytical expressions of both the volume and the price follow from Lemma 11 and Lemma 12.

The probability of the market viability can be estimated via the explicitly expression of the conditional expectation of security $j = 1, 2, 3$.

To investigate the call option, write

$$H(x_1, g) \equiv \mathbb{E}_v[(x - L)^+|x_1] - \mathbb{E}_n[(x - L)^+|x_1] - \beta$$

where $g = g(\sigma_{n0})$. By Lemma 14, we have

$$\frac{\partial H(x_1, g)}{\partial x_1} = \sigma_\eta \left\{ N\left(\frac{\mu_v(L)}{\sqrt{1+g_v}}\right) - N\left(\frac{\mu_n(L)}{\sqrt{1+g}}\right) \right\} > 0 \qquad \text{(A-20)}$$

when x_1 is large enough. Moreover, Lemma 15 yields

$$\lim_{x_1 \to \infty} H(x_1) = \infty, \lim_{x_1 \to -\infty} H(x_1) = -\frac{\beta}{\sigma_\eta}. \qquad \text{(A-21)}$$

Let

$$v(g) := \sup \{y < 0 : H(x_1, y) < 0\}.$$

$v(g)$ is finite because of (A-21). By Lemma 14 and the implicit function theorem, we can derive an ordinary differential equation of $v(g)$. The boundary value $v(0)$, when $g \downarrow 0$, is solved by the equation $H(x_1, 0) = 0$. Using Lemma 14 again we obtain $\frac{\partial H(x_1,0)}{\partial x_1} > 0$. Hence $v(0)$ is unique.

We consider both the capped forward and the spread option security. Let $K(x_1, g)$ and $M(x_1, g)$ be $\mathbb{E}_v[f_j(x)|x_1] - \mathbb{E}_n[f_j(x)|x_1] - \beta, j = 2, 3$ respectively. By Lemma 15, we obtain

$$\lim_{x_1 \to \infty} K(x_1, g) = -\frac{\beta}{\sigma_\eta}, \lim_{x_1 \to -\infty} K(x_1, q) = -\infty, \lim_{x_1 \to \infty} M(x_1, g) = \lim_{x_1 \to -\infty} M(x_1, g) = -\frac{\beta}{\sigma_\eta}.$$

Hence, for a fixed $g = g(\sigma_{n0})$, $K(x_1, g) < 0$ for either $x_1 \gg 0$ or $x_1 \ll 0$. Therefore, the capped call market breaks down in a strong or a weak underlying market. Moreover, it can be shown that $K(x_1, g) < 0$ when g is very small. Therefore, if the market participants have far different priors on the underlying market, the capped call market would break down in the second time period.

Similarly, the spread option market breaks down for $x_1 \gg 0$ or $x_1 \ll 0$. Moreover, Lemma 14 ensures that $M(x_1, 0)$ is increasing with respect to x_1. Therefore, $M(x_1, g) < 0$ for a small g. When the innovator and the investor have far different priors on the underlying, the spread option market breaks down in the second time period. The proof is completed. □

Proof of Proposition 25.

We start with the equilibrium for investor $i \in \{1, 2, 3\}$ at time $t = 1$. Similar to the equilibrium in Section 3.2, the demand is characterized by

$$\phi = \frac{(\mathbb{E}_{v,i}[f_i(x_2)|x_1] - p_{1,i})^+}{\gamma_i Var_{v,i}[f_i(x_2)|x_1]}. \tag{A-22}$$

Therefore, the demand-price relationship, in equilibrium, is

$$p_{1,i} = \mathbb{E}_{v,i}[f_i(x_2)|x_1] - \gamma_i \phi Var_{v,i}[f_i(x_2)|x_1]. \tag{A-23}$$

We next plug the above demand-price relationship into the optimization problem for the innovator at time $t = 1$,

$$\max_{\phi \geq 0} \mathbb{E}_n \left[\sum_{i=1}^{3} \phi(p_{1,i} - f_i(x_2)) \right] - \alpha - 3\beta\phi$$

and the first-order condition with respect to ϕ ensures that

$$\phi^* = \frac{\sum_{i=1}^{3}(\mathbb{E}_{v,i}[f_i(x_2)|x_1] - \mathbb{E}_n[f_i(x_2)|x_1]) - 3\beta}{2\sum_{i=1}^{3}\gamma_i Var_v[f_i(x_2)|x_1]}. \tag{A-24}$$

Notice that $\sum_{i=1}^{3} f_i(x) = x$, the equilibrium at time $t = 1$ is obtained.

We next derive the time consistent optimal strategy for investor i at time $t = 0$. In the Nash equilibrium, her wealth at time $t = 2$ is

$$W_{0,i} + N(f_i(x_1) - p_{0,i}) + \hat{\epsilon}_i(\equiv \phi_1(p)(f_i(x_2) - p_{1,i})).$$

Therefore, the demand-price relationship should be

$$p_{0,i} = \mathbb{E}_{v,i}[f_i(x_1)] - \gamma_i N Var_{v,i}[f_i(x_1)] - \gamma_i Cov_{v,i}(f_i(x_1), \hat{\epsilon}_i), \tag{A-25}$$

and we plug this relationship into the innovator's optimal problem

$$\sum_{i=1}^{3} N \cdot (p_{0,i} - \mathbb{E}_n[f_i(x_1)]) - \alpha - 3\beta N.$$

The first order condition with respect to N ensures the time consistent equilibrium at time $t = 0$. The proof is finished. $\square$

Proof of Proposition 26.

By Proposition 25, there is issuance of a securitization structure at time $t = 1$ if and only if

$$L(x_1, g) \equiv \sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x)|x_1] - \alpha_{n1} - 3\beta > 0.$$

By virtue of Lemma 12 and Lemma 14, for any $g \in (0, g_v)$,

$$\lim_{x_1 \to -\infty} L(x_1, g) = -\infty, \lim_{x_1 \to \infty} L(x_1, g) > 0. \tag{A-26}$$

Hence, the CDO market is not viable in a very stressed time period, but it is viable in a strong market. The proof is finished. $\square$

Proof of Proposition 27.

By Lemma 13, $\sum_{i=1}^{3} \gamma_i Var_{v,i}[f_i(x)|x_1] \to \gamma_1 \sigma_\eta^2 (1 + \sigma_{v_1}^2)$, when $x_1 \to \infty$. Then, by Proposition 25 and the same derivation of Proposition 26, we have

$$\lim_{x_1 \to \infty} \Phi(p)^* = \infty. \tag{A-27}$$

Let $\Phi_1(p)^*$ denotes the optimal volume of the equity tranche separably, which is determined by (A-1) for the payoff structure $f(x) = (x - L)^+$. By Lemma 13 again, we have

$$\begin{aligned}
\lim_{x_1 \to \infty} \{\Phi(p)^* - \Phi_1(p)^*\} &= \lim_{x_1 \to \infty} \left\{ \frac{\sum_{i=1}^{3} \mathbb{E}_{v,i}[f_i(x)|x_1] - \alpha_{n1} - 3\beta}{2\gamma_1 Var_{v,1}[f_1(x)|x_1]} - \Phi_1(p)^* \right\} \\
&= \lim_{x_1 \to \infty} \left\{ \frac{L + \mathbb{E}_{v,1}[f_1(x)|x_1] - \alpha_{n1} - 3\beta}{2\gamma_1 Var_{v,1}[f_1(x)|x_1]} - \Phi_1(p)^* \right\} = 0.
\end{aligned}$$

By using Proposition 24, there exists no issuance of the senior tranche and the mezzanine tranche in a strong underlying market. The remaining proofs follows from $\lim_{x_1 \to \infty} \Phi(p)^* = \infty$. $\square$

Table 3.1: The forward-type market Viable condition at $t = 1$

This table reports the sensitivity of the market viability for the forward-type market with respect to the expected mean α_n and and volatility σ_n of the innovator. The parameters are $\alpha_v = 1.5$, $\sigma_v = 0.175$, $\sigma_\eta = 0.1$, $\beta = 0.05$, $a = 1$. Notice that the market is viable if and only if $a * (\alpha_{v0} - \alpha_{n0}) - \beta > \Theta$.

α_n	σ_n	A	$\gamma\Theta$	Boundary	Viability
1.45	0.17	4.903	0.0000	0.000	0
1.45	0.15	2.063	0.0000	0.000	0
1.45	0.13	1.750	0.0002	0.000	0
1.4	0.17	3.726	0.0000	0.050	1
1.4	0.15	1.813	0.0000	0.050	1
1.4	0.13	1.602	0.0007	0.049	1
1.35	0.17	2.548	0.0000	0.100	1
1.35	0.15	1.563	0.0002	0.100	1
1.35	0.13	1.454	0.0019	0.098	1
1.3	0.17	1.370	0.0000	0.150	1
1.3	0.15	1.313	0.0009	0.149	1
1.3	0.13	1.306	0.0039	0.146	1
1.2	0.17	-0.985	0.0003	0.250	1
1.2	0.15	0.813	0.0030	0.247	1
1.2	0.13	1.010	0.0090	0.241	1
1.1	0.17	-3.340	0.0007	0.349	1
1.1	0.15	0.313	0.0052	0.345	1
1.1	0.13	0.714	0.0144	0.336	1

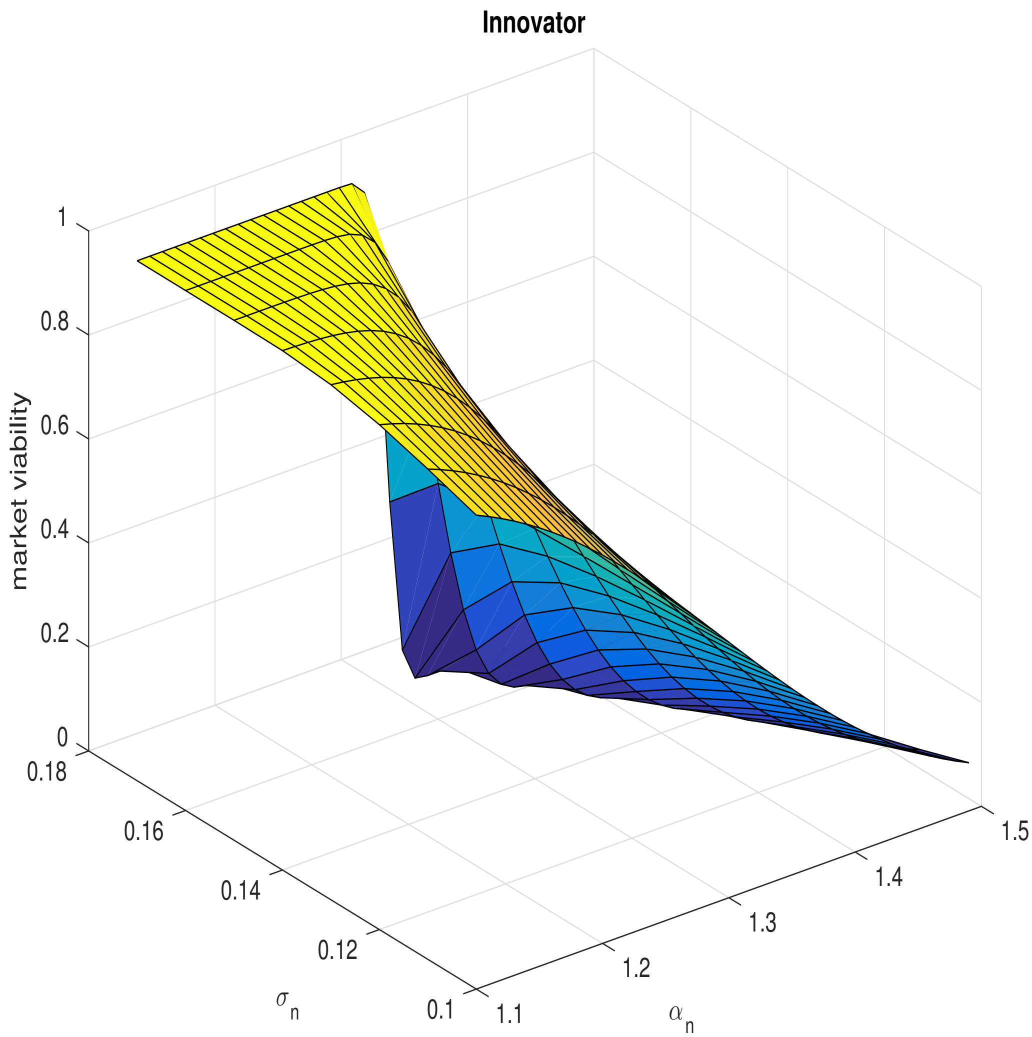


Figure 3.1: Probability of Market Viability at time $t = 1$ for the innovator
This three dimensional figure shows the probability of the market viability with respect to the expected mean α_n and and volatility σ_n of the innovator. The parameters are $\alpha_v = 1.5$, $\sigma_v = 0.175$, $\sigma_\eta = 0.1$, $\beta = 0.05$, $a = 1$.

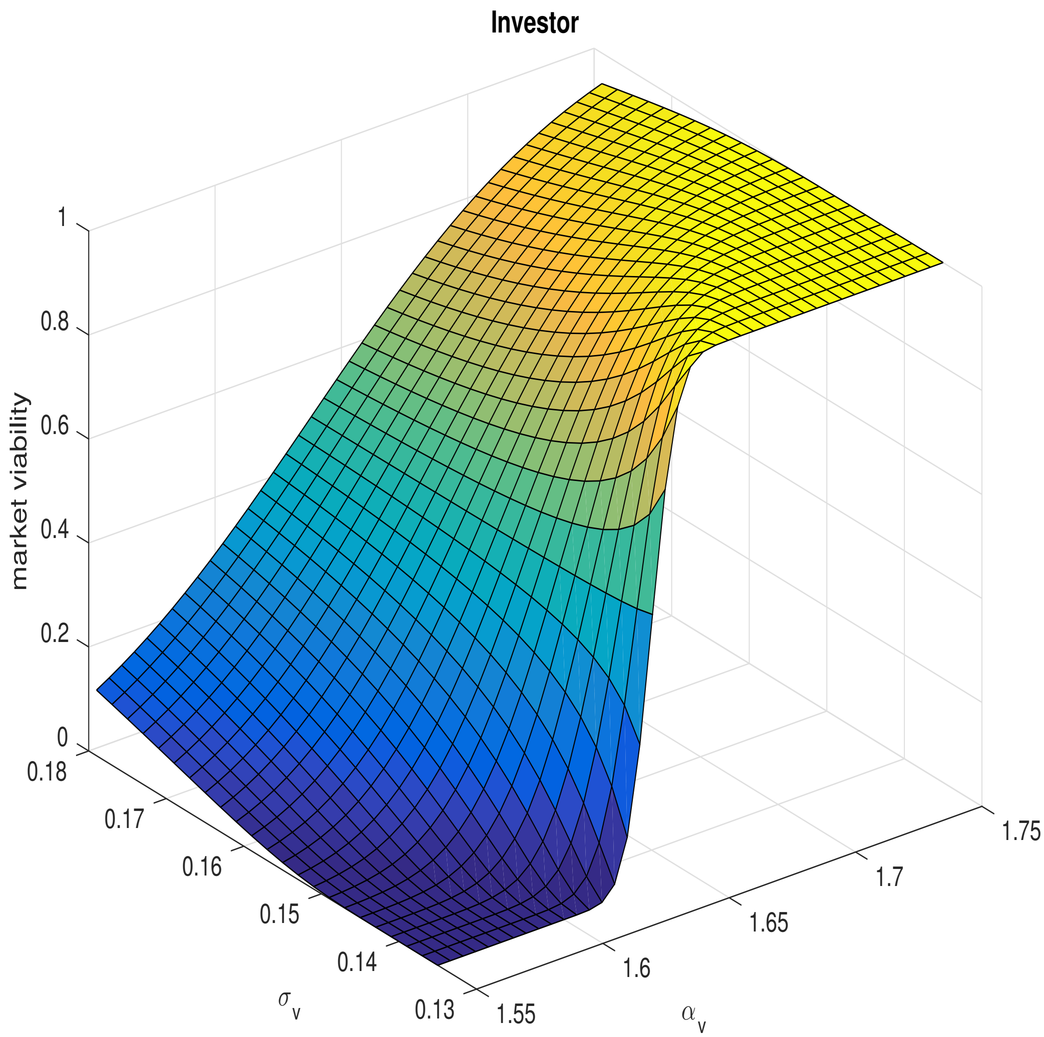


Figure 3.2: Probability of Market Viability at time $t = 1$ for the investor

This three dimensional figure shows the probability of the market viability with respect to the expected mean α_v and volatility σ_v of the investor. The parameters are $\alpha_n = 1.5$, $\sigma_n = 0.13$, $\sigma_\eta = 0.1$, $\beta = 0.05$, $a = 1$.

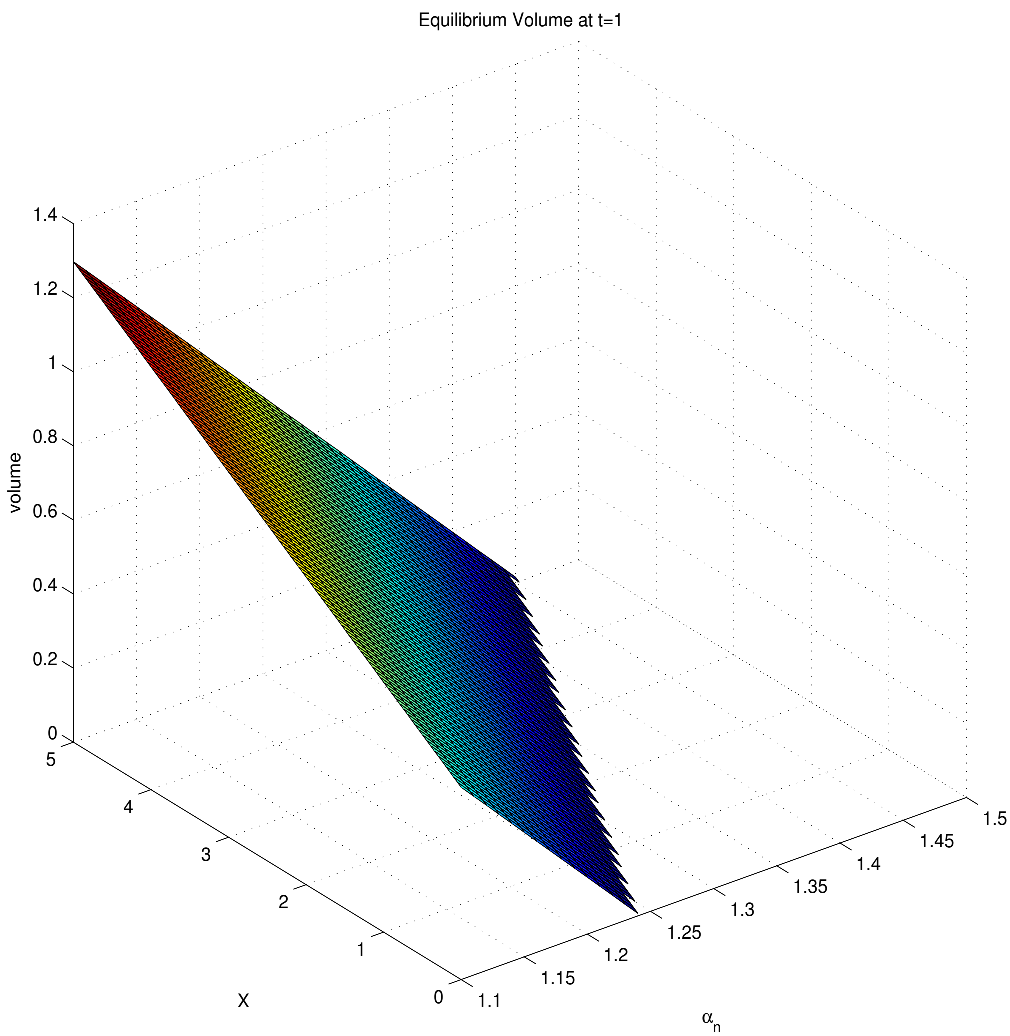


Figure 3.3: Volume at time $t = 1$ for a forward contract

This three dimensional figure shows the market viability of a forward contract at time $t = 1$ with respect to the expected mean α_v of the investor and the price of the underlying state variable X. The parameters are $\alpha_v = 1.5$, $\sigma_v = 0.175$, $\sigma_n = 0.17$, $\sigma_\eta = 0.1$, $\beta = 0.05$, $a = 1$, $\gamma = 2$.

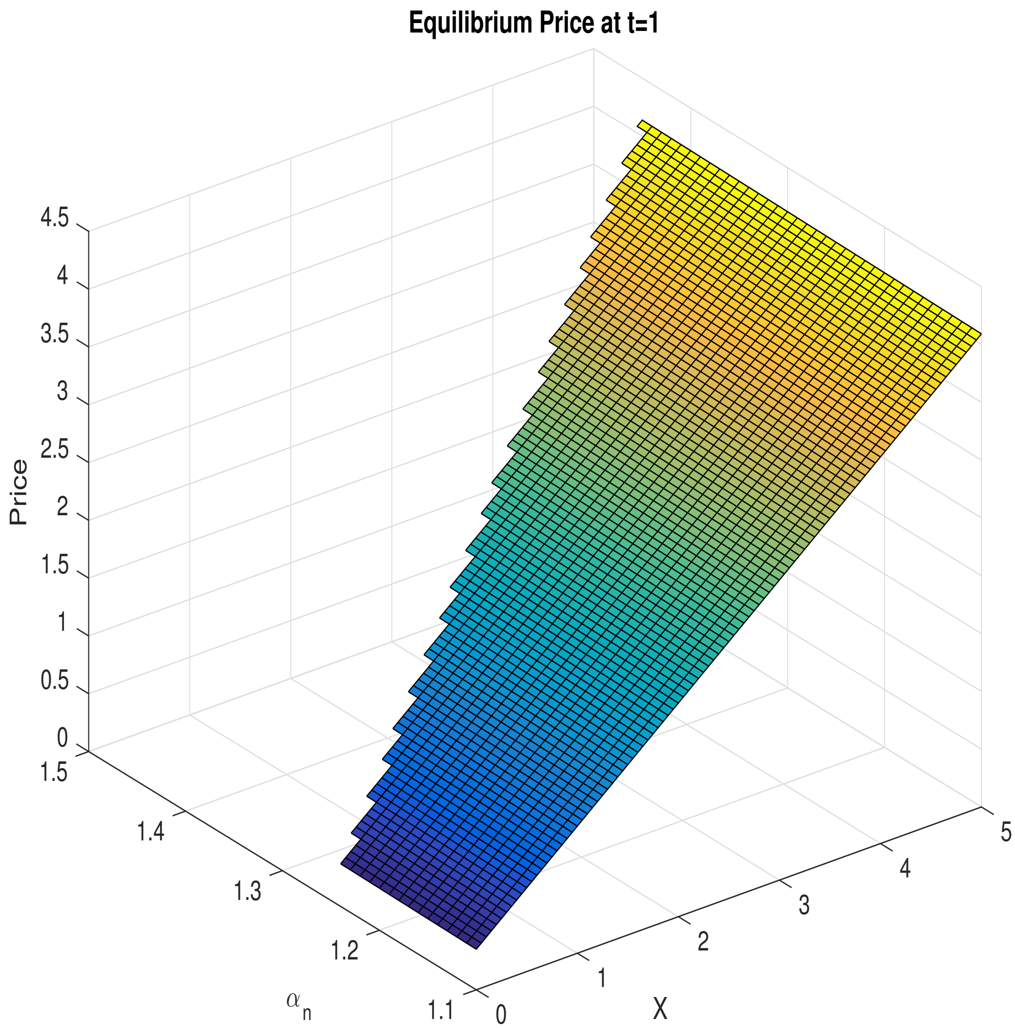


Figure 3.4: Equilibrium Price at time t=1 for a forward contract
This three dimensional figure shows the equilibrium price of a forward contract at time $t = 1$ with respect to the expected mean α_v of the investor and the market price of the underlying state variable X. The parameters are $\alpha_v = 1.5$, $\sigma_v = 0.175$, $\sigma_n = 0.17$ $\sigma_\eta = 0.1$, $\beta = 0.05$, $a = 1$, $\gamma = 2$.

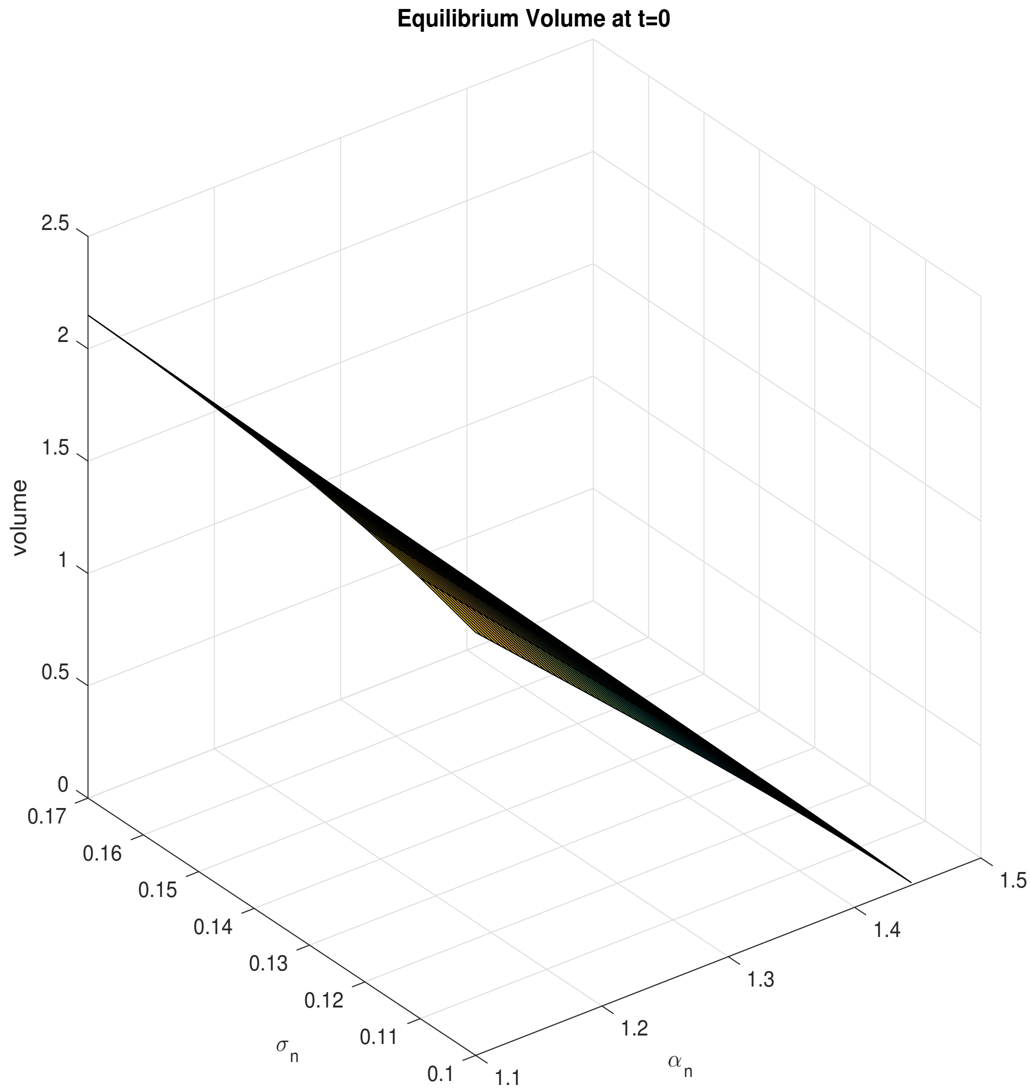


Figure 3.5: Volume at time $t = 0$ for a forward contract

This three dimensional figure shows the market viability of a forward contract at time $t = 0$ with respect to the expected mean α_v and volatility σ_v of the investor. The parameters are $\alpha_v = 1.5$, $\sigma_v = 0.175$, $\sigma_\eta = 0.1$, $\beta = 0.05$, $a = 1$.

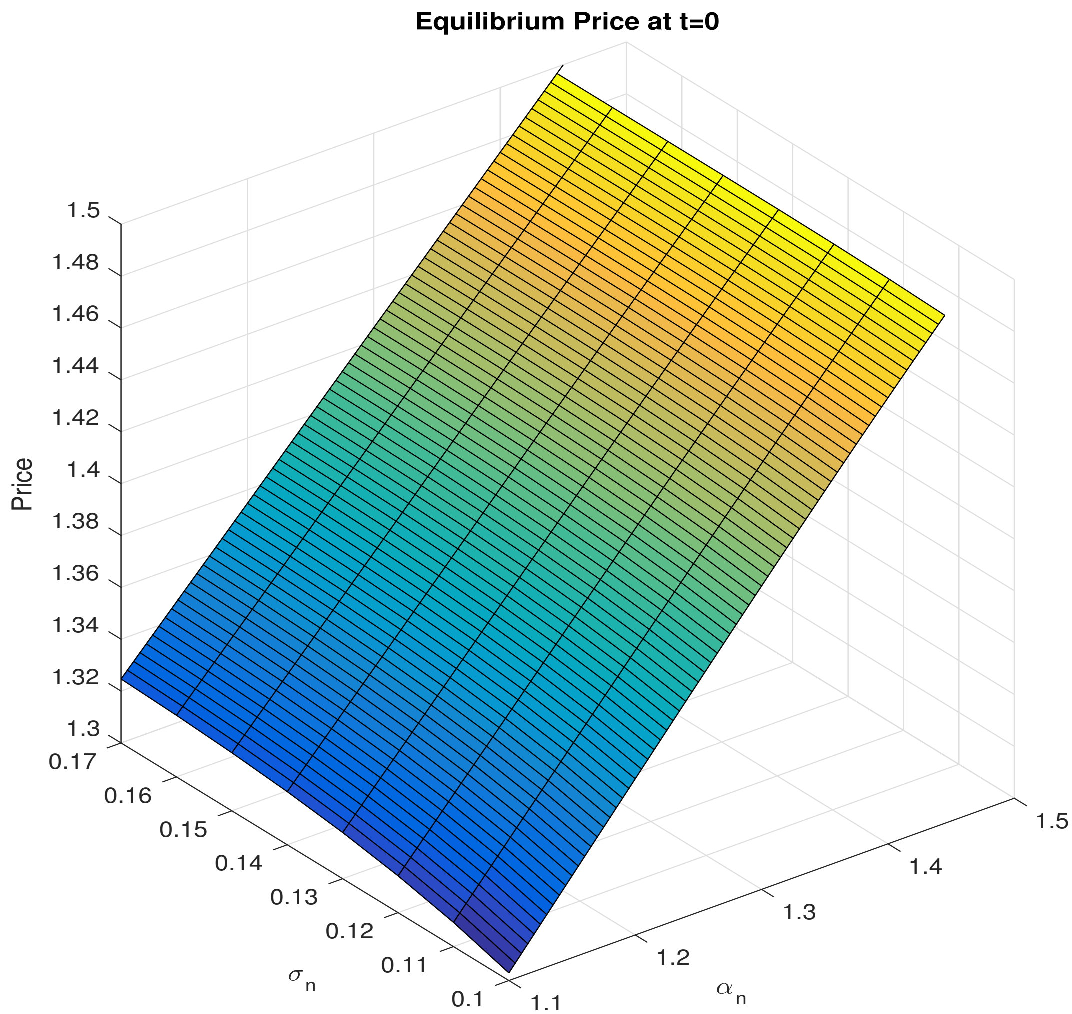


Figure 3.6: Price at time $t = 0$ for a forward contract

This three dimensional figure shows the market price of a forward contract at time $t = 0$ with respect to the expected mean α_v and volatility σ_v of the investor. The parameters are $\alpha_v = 1.5$, $\sigma_v = 0.175$, $\sigma_\eta = 0.1$, $\beta = 0.05$, $a = 1$.

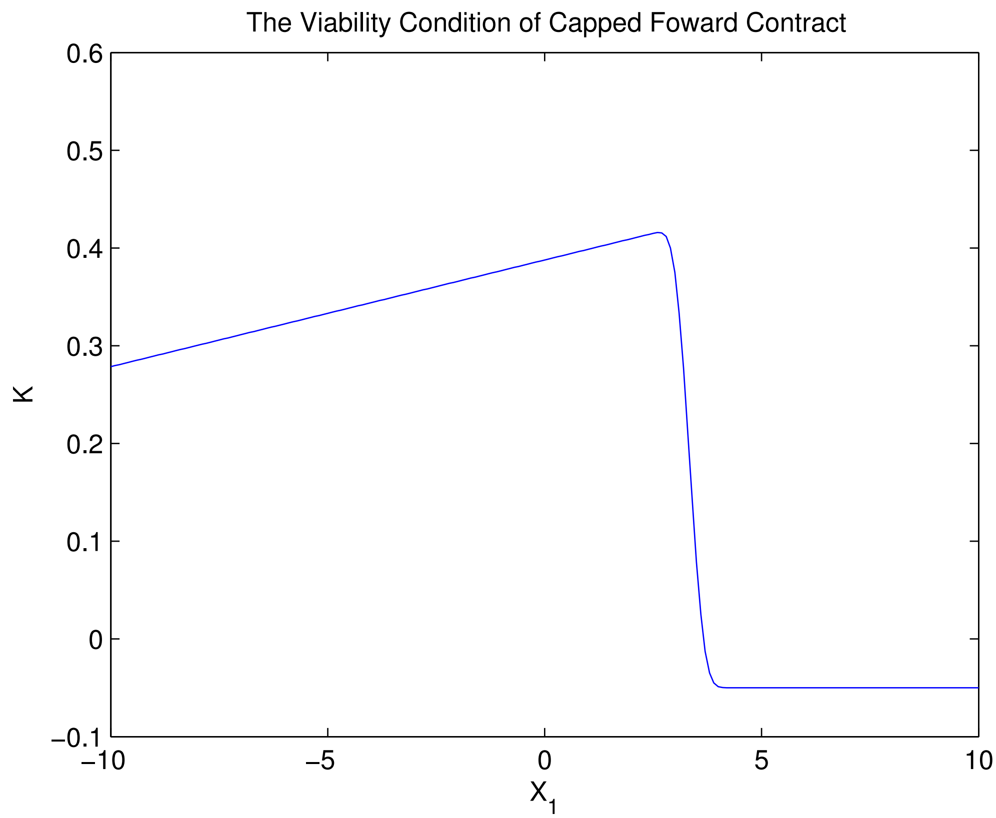


Figure 3.7: Market Viability at time $t = 1$ for a capped forward contract
This figure shows the market viability of an option contract at time $t = 1$ with respect to the the market price of the underlying state variable X. The parameters are $\alpha_n = 5$, $\alpha_v = 7$, $\sigma_n = 0.17$, $\sigma_v = 0.175$, $\sigma_\eta = 0.1$, $\beta = 0.05$, $a = 1$, $K = 1$.

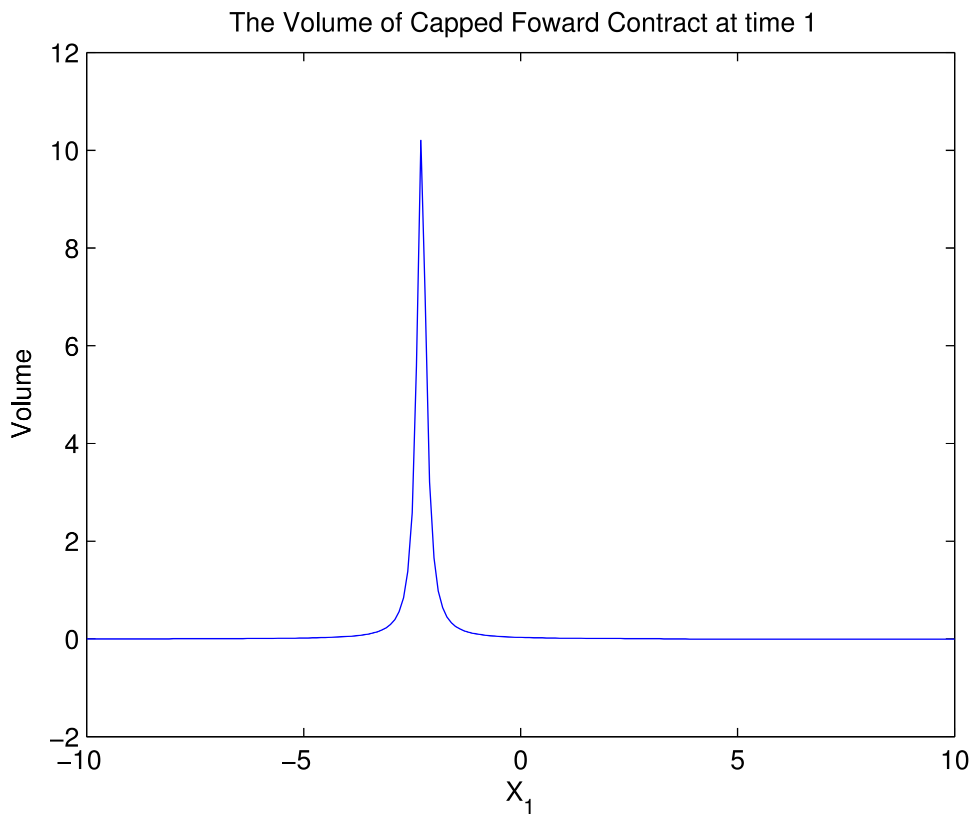


Figure 3.8: Volume at time $t = 1$ for a capped forward contract
This figure shows the market volume of an option contract at time $t = 1$ with respect to the the market price of the underlying state variable X. The parameters are $\alpha_n = 5$, $\alpha_v = 7$, $\sigma_n = 0.17$, $\sigma_v = 0.175$, $\sigma_\eta = 0.1$, $\beta = 0.05$, $a = 1$, $K = 1$.

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