

EFFECTS OF MODIFIED SCHEMA-BASED INSTRUCTION ON REAL-WORLD  
ALGEBRA PROBLEM SOLVING OF STUDENTS WITH AUTISM SPECTRUM  
DISORDER AND MODERATE INTELLECTUAL DISABILITY

by

Jenny Rose Root

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Approved by:

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Dr. Diane M. Browder

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Dr. Ya-yu Lo

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Dr. Fred Spooner

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Dr. Drew Polly

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## ABSTRACT

JENNY ROSE ROOT. Effects of modified schema-based instruction on real-world algebra problem solving of students with autism spectrum disorder and moderate intellectual disability. (Under the direction of DR. DIANE M. BROWDER)

The current study evaluated the effects of modified schema-based instruction (SBI) on the algebra problem solving skills of three middle school students with autism spectrum disorder and moderate intellectual disability (ASD/ID). Participants learned to solve two types of group word problems: missing-whole and missing-part. The themes of the word problems were related to their interests and daily experiences. In addition, participants were taught key mathematics vocabulary terms using constant time delay. Participants were taught how to use an iPad that displayed a task analysis with embedded verbal and specific verbal prompts to complete each step of solving the real-world algebra word problems. This study also examined participant's ability to generalize skills when stimulus supports were faded. Results showed a functional relation between constant time delay and acquisition of mathematics vocabulary terms as well as between modified SBI and mathematical problem solving. Participants were able to successfully solve both types of group problems and had some success with generalizing skills when stimulus supports were faded. The findings of this study provide several implications for practice for using modified SBI to teach mathematical problem solving to students with ASD/ID, and offer suggestions for future research in this area.

## DEDICATION

First, I dedicate this dissertation to the individuals with disabilities and their families who have forever made an impact on my life. It has been through our interactions that I am reminded of why I am in this field. Each of us was created for a purpose, and it is my goal to help equip individuals with disabilities, their teachers, and their families with the tools they need to realize and fulfill that purpose.

Finally, this dissertation is dedicated to my husband Kyle and our son John, who have been my inspiration throughout this past year. You have encouraged, supported, and pushed me through this process. My success and accomplishments are a result of the love you have given me.

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## CHAPTER 1: INTRODUCTION

Mathematical learning is imperative to having a range of career, leisure, and daily living opportunities. According to social cognitive career theory (SCCT; Lent, Brown, & Hackett, 1994), positive science, technology, engineering, and mathematics (STEM) learning and training experiences can enhance career decisions. Students with autism spectrum disorder (ASD) who have positive STEM experiences may not only gain knowledge and skills in these content areas, but also have a broader understanding of how content knowledge is used in real-world settings. A recent analysis of the National Longitudinal Transition Study-2 found the amount of exposure to mathematics courses positively influenced the likelihood an individual with an ASD would choose a STEM major in college (Wei et al., 2015). Similarly, a recent study by Wang (2013) found exposure to mathematics has a stronger effect on choosing a STEM major in college than mathematics achievement. This research underscores the impact exposure to mathematics can have on the vocational decisions of individuals with ASD. For individuals with ASD and an intellectual disability (ID) who may not pursue a formal postsecondary education, high quality mathematics instruction can still positively affect their vocational opportunities and decisions by providing increased skills for job performance and knowledge for job selection.

Mathematics instruction has immediate applications within current and future environments beyond just vocational choices. For example, the concept of cardinality, or

the understanding of the order and magnitude of numbers, is a skill first addressed in kindergarten, and provides the foundation needed for creating sets and comparing quantities. Cardinality can be immediately used to locate a room by watching ascending or descending numbers.

State standards like the Common Core State Standards (CCSS) help educators set targets for the type of mathematical learning experiences individuals will need in daily living and future careers. One domain of mathematics that is emphasized throughout the CCSS is algebra (Common Core State Standards Initiative, 2015). Algebra is closely linked with other domains of mathematics, especially geometry and data analysis, and serves as a way to unify them (National Council of Teachers of Mathematics [NCTM], 2000). The foundations of algebraic reasoning are found in the early elementary grades through the development of early numeracy skills. Early numeracy is sometimes called number sense (NCTM, 2000), and is the umbrella term for the foundational understanding of numbers and their conceptual role within mathematics (e.g., number recognition, patterning, set making and counting, rote counting, symbol use). For example, instruction in recognizing and extending patterns is necessary for the future algebraic concept of functions.

As students build this algebraic foundation with number sense, they also must begin to apply reasoning skills. Reasoning is a defining feature of mathematics and is essential to conceptually understanding mathematics (NCTM, 2000). Reasoning involves developing ideas, exploring phenomena, justifying results, and using mathematical conjectures or informed guesses (NCTM, 2000). Students are expected to use reasoning and to demonstrate conceptual and procedural knowledge across all domains of

mathematics, beginning in first grade and continuing throughout upper level secondary mathematics courses (CCSS Initiative, 2015). Students may work with addition and subtraction equations to understand the meaning of the equal sign and determine whether equations are true or false (e.g.,  $6=6$ ,  $6=7-1$ ,  $4+1=6$ ). This evaluation of the equal sign and ability to make judgments requires algebraic reasoning and conceptual knowledge of the relational property of the equal sign, a more advanced skill than simple memorization of mathematical facts. Structured mathematical experiences in earlier grades build the required foundation for algebraic thinking for later success in algebra (Dougherty, Bryant, Darrough, & Pfannenstiel, 2015). For example, relational understanding of the equal sign and evaluation of equations in elementary grades leads to later skill of using properties of operations to generate equivalent expressions (e.g.,  $5(2+2)=20$ ).

Algebraic reasoning is only useful if it can be applied within problem-solving contexts. According to the NCTM (2000), problem solving skills are used when students are confronted with a problem without a prescribed method for a solution. Problem solving experiences in school settings are typically structured in the format of word problems. In the context of word problem solving, stories present situations requiring a mathematical solution (Stein, Kinder, Silbert, & Carnine, 2006). Van de Walle, Karp, and Bay-Williams (2007) indicated that learning to solve story problems is the basis for learning to solve real-world mathematical problems. In elementary grades, students first begin solving algebra problems by using objects and drawings to represent and solve problems. By middle grades, they are expected to use variables to represent numbers and write expressions when solving a real-world algebra problem. Consideration of the “school effects” of mathematics justifies the need for high quality mathematics

instruction, as school is likely to be the only context where students receive instruction in mathematics, unlike literacy or reading (Van de Walle et al., 2007).

Despite its importance, the evidence base for teaching either algebraic reasoning or problem solving to students with moderate/severe developmental disabilities is sparse (Browder, Spooner, Ahlgrim-Dezell, Harris, & Wakeman, 2008). The majority of skills taught to this population fall within the NCTM standard of numbers and operations (Browder et al., 2008; Spooner, Root, Browder, & Saunders, 2016). Within the studies that have taught algebra to students with moderate/severe developmental disabilities, systematic instruction and a task analysis are common evidence-based instructional features (e.g., Browder, Jimenez, & Trela, 2012; Browder, Trela, et al., 2012; Jimenez, Browder, & Courtade, 2008; Karl, Collins, Hager, & Ault, 2013).

One of the limitations of previous studies that taught algebra to students with moderate/severe disability is that students only developed procedural knowledge. They were not required to conceptually understand the problems. For example, Jimenez et al. (2008) taught high school students with moderate intellectual disability (Mod ID) to solve an algebraic equation. Participants had a graphic organizer that depicted an algebraic equation without numbers and a number line from zero to nine. A task analysis was used to teach participants to “count up” to find the final answer. Similarly, Browder, Jimenez, et al. (2012) taught high school students with Mod ID to solve equations when given story problems. However, the intervention only taught students the procedural steps to plug the information from the problem into the graphic organizer (number line and number sentence) and to “count up” to find the final answer. In contrast, Root, Browder, Saunders, and Lo (2015) and Saunders (2014) used schema-based instruction (SBI), a

step-by-step procedure for solving word problems that directly teaches both conceptual and procedural knowledge to students with ASD and Mod ID.

A schema is an outline or a framework for solving a problem that can be represented through pictures, diagrams, number sentences, or equations (Marshall, 1995; Powell, 2011). SBI has three essential components: (a) identification of the problem structure to determine the problem type; (b) use of visual representations that represent the structure of the problem type to organize information from the problem; and (c) explicit instruction on the schema-based problem solving heuristic, with metacognitive strategy knowledge instruction being an optional additional component (Jitendra et al., 2015). In SBI, the student selects or creates a diagram that fits the structure of the word problem, uses the diagram to solve the problem, and completes the number sentence with the solution (Jitendra & Hoff, 1996). Explicit strategy instruction priming the underlying problem structure is an evidence-based practice for individuals with learning disabilities (LD) and individuals at risk for developing mathematical difficulties (Jitendra et al., 2015). Research is emerging on the effectiveness of SBI to teach students with developmental disability (Rockwell, Griffin, & Jones, 2011; Root, Browder, et al., 2015; Saunders, 2014).

The first study to use SBI to teach problem solving to a student with ASD incorporated measures of algebraic reasoning. Rockwell et al. (2011) taught a 10-year-old female student with ASD without comorbid ID to use schematic diagrams to solve three types of word problems (group, change, and compare). The student was taught to use a heuristic and corresponding graphic organizer for each problem type. After the student had mastered solving all three problems types with the unknowns in the final position

(i.e., the whole in group problems, change amount in change problems, and difference in compare problems), one training session was given on generalization to unknowns in the other positions. The student was able to generalize word problem solving with the unknown in the initial and medial positions.

Since Rockwell et al.'s (2011) study, two studies have investigated SBI to teach problem solving to students who have both ASD and ID (ASD/ID) and have demonstrated that it is effective for this population when additional supports are provided (Root, Browder, et al., 2015; Saunders, 2014). Students with ASD/ID may lack the reading skills to independently access the word problem and the computational fluency to solve the problem. Saunders (2014) taught three male elementary students with ASD and Mod ID to solve group and change problems using modified SBI delivered through computer-based video instruction. The modifications made to traditional SBI included (a) a task analysis and chant with hand motions to serve as a heuristic, (b) enhanced visual supports on the graphic organizers, and (c) incorporation of systematic instruction along with explicit instruction. All three students were able to master the group problem type and one student mastered the change problem type. Root, Browder, et al. (2015) taught three male elementary students with ASD and Mod ID to solve compare problems using a similar modified SBI procedure with virtual and concrete graphic organizers and manipulatives. All three students mastered solving compare problems, with an increased rate of learning in the virtual condition for two of the students and a preference for the virtual condition for all three students.

Technology was a component of both the Root, Browder, et al. (2015) and Saunders (2014) studies, exemplifying the role it can play in providing alternate means of



access to the text (e.g., student controlled read-aloud) and learning materials (e.g., electronic task analysis, virtual graphic organizers, and virtual manipulatives) for students with ASD/ID. Technology-aided instruction (TAI) has a range of possible benefits for students with ASD. TAI has the ability to (a) provide several sets of materials concisely and efficiently, (b) allow review by the student as much as he or she feels necessary, (c) provide consistent implementation without treatment drift, and (d) alleviate the burden of modeling tasks in the setting by adults or peers (Ayres, Mechling, & Sansoti, 2013). Several studies have suggested rationale behind the general preference of individuals with ASD to use technology-based treatments (Bernard-Opitz- Sriram, & Nakhoda-Saupan, 2001; Moore & Calvert, 2000). Moore, McGrath, and Thorpe (2000) theorize students with ASD may prefer the multi-sensory features inherent in TAI to other instructional methods. In addition, TAI avoids difficult social situations, thereby mitigating the social deficits that are characteristic of ASD (e.g., Higgins & Boone, 1996; Moore et al., 2000). Finally, research has repeatedly demonstrated students with ASD display a decrease in challenging behaviors during TAI (e.g., Chen & Bernard-Opitz, 1993; Moore & Calvert, 2000) and an increase in appropriate behaviors (e.g., Bosseler & Massaro, 2003; Heimann, Nelson, Tjus, & Gillberg, 1995).

A combination of TAI and modified SBI was successful in the two studies that taught word problem solving to students with ASD and Mod ID. Saunders (2014) embedded prompting within the electronic task analysis during computer-based video instruction, allowing participants the option to control specific verbal prompts for completing each step of the task analysis on the computer. In addition, technology was used to provide self-initiated read-alouds of the word problem and steps of the task

analysis to compensate for the low level of decoding and comprehension of the participants. Both Saunders (2014) and Root, Browder, et al. (2016) used virtual graphic organizers with virtual manipulatives. Within an alternating treatments design, Root, Browder, et al. found that two of the three participants had an increased rate of learning in the virtual manipulatives condition, and for the third participant the success was equal. When given a choice between the two conditions following mastery, all three participants demonstrated consistent preference for the virtual manipulatives. The findings of Root, Browder, et al. give empirical support for the preference of individuals with ASD for TAI over traditional paper-and-pencil formats.

Although both Saunders (2014) and Root, Browder, et al. (2015) directly taught problem solving and had positive findings, the results have several limitations. Participants were provided with a diagram to fill-in for the number sentence as an additional support that was never faded. In addition, the students were not taught to solve for an unknown in the initial or medial position, placing a ceiling on the algebraic reasoning demand and preventing proof of acquisition of relational understanding of the equal sign. There has been one published study with two participants, one of whom had Mod ID to solve change word problems with unknowns in all three positions. Neef, Nelles, Iwata, and Page (2003) taught the precurent behaviors of identifying the component parts of the word problem (initial set, change set, key words to identify the operation, and resulting set) to fill out a number sentence. The participants were given stimulus supports in the form of a diagram that was similar to those used by Root, Browder et al. and Saunders. These supports were never faded. Although one participant in the study had Mod ID, he was able to independently read the word problems and knew

basic math facts, two prerequisite skills the participants in the Root, Browder, et al. and Saunders studies did not have. These skills are not characteristic of all elementary and middle school students with ASD and Mod ID (Browder & Spooner, 2011).

Although there is some evidence to support the use of SBI to teach students with ASD and Mod ID word problem solving, the prior two studies by Root, Browder, et al. (2015) and Saunders (2014) only taught word problems with the unknown in the final position (e.g.,  $2+5 = ?$ ) with stimulus supports for the number sentence. Rockwell et al. (2011) demonstrated students with ASD who did not have ID required minimal (one lesson) instruction in generalization to unknowns in the initial and medial positions (e.g.,  $? + 5 = 7$  or  $2 + ? = 7$ ). The findings of Neef, Nelles, Iwata and Page (2003) suggest individuals with Mod ID, even those with computational fluency, require systematic instruction in solving word problems with equations in standard and nonstandard formats and with the unknown in the final, initial, and medial position. The need exists to investigate a method for teaching students with ASD and Mod ID a way to solve real world algebra problems with the unknown in multiple positions. In addition, research should be conducted to provide evidence that treatment effects can be maintained after stimulus supports are faded.

In summary, more intensive supports and instruction may be needed for students with ASD and Mod ID to demonstrate the algebraic reasoning and relational understanding of the equal sign necessary to solve word problems with the unknown in multiple positions. Technological supports have been shown to be effective in providing access to materials (Bouck et al., 2013; Browder, Root, Wood, & Allison, 2015; Root, Browder, et al., 2016 Saunders, 2014). Prior investigations into SBI for students with

ASD and Mod ID have successfully used pictorial self-instruction as a replacement for the traditional heuristic in SBI (Root, Browder, et al., 2015, Saunders, 2014). The academic vocabulary used in algebraic problem solving may be unfamiliar to the students, and therefore will need to be explicitly taught. Constant time delay has been effective for teaching academic vocabulary to students with ASD (Browder et al., 2015; Knight, Spooner, Browder, Smith, & Wood, 2013; Riggs, Collins, Kleinert, & Knight, 2013).

The purpose of this study was to evaluate the effects of SBI incorporating a technology platform on real-world algebra problem solving in standard and nonstandard formats and with the unknown in all positions for students with ASD and Mod ID. In addition, this study evaluated whether or not additional supports provided can be faded while mathematical problem solving skills maintain. The following research questions were addressed:

1. What is the effect of constant time delay on the identification of mathematics vocabulary definitions by students with ASD/ID?
2. What is the effect of schema-based instruction using a system of least prompts that incorporates a technology platform on the number of steps performed independently correct to solve a word problem by students with ASD/ ID? (primary research question)
3. What is the effect of schema-based instruction using a system of least prompts that incorporates a technology platform on the cumulative number of word problems solved by students with ASD/ID?

4. Are students with ASD/ID able to maintain problem solving skills, demonstrated by the number of steps of a word problem solving task analysis performed independently correct and total number of problems solved, when stimulus supports are faded?
5. What is the effect of instructor modeling use of electronic prompting during least prompting on student's subsequent initiation of use of the electronic feature to self-prompt by students with ASD/ID?
6. What is the effect of modified schema-based instruction on the change in general word problem solving ability before and after intervention by students with ASD/ID?
7. What is the effect of modified schema-based instruction on the perception of word problem solving of students with ASD/ID?
8. What is the effect of modified schema-based instruction on the perception of word problem solving of teachers of students with ASD/ID?

### Significance of the Study

This study contributed to the limited body of research on teaching algebra to students with ASD and Mod ID. This investigation was the first to teach students with ASD and Mod ID to solve group word problems with both a missing whole (final position) or part (initial or medial position). A technology platform was used to present a task analysis of the steps to solve the problem, which served as a student self-instruction sheet. Embedded within the task analysis were verbal and specific verbal prompts. By measuring the use of self-initiated prompting using technological supports, this study

adds to the literature on self-determination, specifically the skill of self-monitoring within academic tasks. This study contributes to the literature on generalization and fading within academic tasks by measuring the ability of participants to solve word problems without the assistance of a stimulus support in the form of a pre-drawn number sentence. Although constant time delay (CTD) is an evidence-based practice to teach vocabulary to students with significant cognitive disabilities (Browder, Ahlgrim-Dezell, Spooner, Mims, & Baker, 2009), this investigation is the first to teach mathematics vocabulary that is then be directly applied to an algebraic problem solving task.

### Delimitations

This study has several delimitations. Students in the district where this study took place had extensive access to technology in the classroom. In elementary grades, students received instruction in how to use a computer as an elective course. In middle school, students took at least one additional keyboarding or computer course. Within the classroom, students had frequent access to SMARTboards, iPads, and desktop computers. This access had an influence on their learning history, but may not be characteristic of all students in this population.

An additional delimitation for this study was the format of the word problems. Although participants were exposed to novel problems in each session, they were highly formulaic. The key stimuli were located in the same place in each problem and there was be no irrelevant information in the word problem. Only numbers one through nine were used, and they were represented as numerals. In addition, only the group problem type was taught. Students found either the missing whole or the missing parts. Additional problem-types (e.g., change or compare) were not addressed in the study.

The final delimitation is the participant characteristics. The participants were selected because they had mastered a set of early numeracy skills including (a) identifying numbers 1-10, (b) counting with 1:1 correspondence, and (c) creating sets to 10. The participants did not have mastery of addition and subtraction mathematics facts with greater than 50% accuracy or the ability to solve one-step word problems.

#### Definitions of Terms

Algebra: Mathematical skills that include (a) understanding patterns, relations, and functions; (b) representing and analyzing mathematical situations and structures using algebraic symbols; (c) using mathematical models to represent and understand quantitative relationships, and (d) analyzing change in various contexts.

(<http://www.nctm.org/Standards-and-Positions/Principles-and-Standards/Algebra/>)

Alternate Assessment aligned with Alternate Achievement Standards (AA-AAS): The primary method through which students with the most significant disabilities participate in measures of educational assessment and school accountability (Quenemon, Rigney, & Thurlow, 2002)

Applied Behavior Analysis: The science in which tactics derived from the principles of behavior are applied to improve socially significant behavior and experimentation is used to identify the variables responsible for the improvement in behavior (Baer, Wolf, & Risley, 1968, 1987; Cooper, Heron, & Heward, 2007).

Autism Spectrum Disorder: Based on the Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition (DSM-V; 2013), in order to be diagnosed with an autism spectrum disorder an individual must meet the following criteria:

A. Persistent deficits in social communication and social interaction across multiple contexts, as manifested by the following, currently or by history:

1. Deficits in social-emotional reciprocity, ranging, for example, for abnormal social approach and failure of normal back-and-forth conversation; to reduced sharing of interests, emotions or affect; to failure to initiate or respond to social interactions.
2. Deficits in nonverbal communicative behaviors used for social interaction, ranging, for example, from poorly integrated verbal and nonverbal communication; to abnormalities in eye contact and body language or deficits in understanding and use of gestures; to a total lack of facial expressions and nonverbal communication.
3. Deficits in developing, maintaining, and understanding relationships, ranging, for example, from difficulties adjusting behavior to suit various social contexts; to difficulties in sharing imaginative play or in making friends; to absence of interest in peers

B. Restricted, repetitive patterns of behavior, interests, or activities, as manifested by at least two of the following, currently or by history:

1. Stereotyped or repetitive motor movements, use of objects, or speech (e.g., simple motor stereotypies, lining up toys or flipping objects, echolalia, idiosyncratic phrases).
2. Insistence on sameness, inflexible adherence to routines, or ritualized patterns or verbal nonverbal behavior (e.g., extreme distress at small changes, difficulties with transitions, rigid thinking patterns, greeting rituals, need to take same route or eat same food every day).
3. Highly restricted, fixated interests that are abnormal in intensity or focus (e.g., strong attachment to or preoccupation with unusual objects, excessively circumscribe or perseverative interest).



4. Hyper-or hypo reactivity to sensory input or unusual interest in sensory aspects of the environment (e.g., apparent indifference to pain/temperature, adverse response to specific sounds or textures, excessive smelling or touching of objects, visual fascination with lights or movement).
- C. Symptoms must be present in the early developmental period (but may not become fully manifest until social demands exceed limited capacities, or may be masked by learned strategies in late life).
- D. Symptoms cause clinically significant impairment in social, occupational, or other important areas of current functioning

These disturbances are not better explained by intellectual disability (intellectual developmental disorder) or global developmental delay. Intellectual disability and autism spectrum disorder frequently co-occur; to make comorbid diagnoses of autism spectrum disorder and intellectual disability, social communication should be below that expected for general developmental level.

Common Core State Standards: A set of high-quality academic standards in mathematics and English language arts/literacy (ELA) that outline what students should know and be able to do by the end of each grade. State-level education leaders and governors created the standards through a state-led effort with the assistance of teachers, school chiefs, administrators, and content area experts (<http://www.corestandards.org>)

Constant Time Delay: Time delay is a procedure where the presentation of the prompt is systematically delayed in time after the presentation of the natural stimuli (Kleinert & Gast, 1982; Snell & Gast, 1981). In constant time delay, several trials are presented where the natural stimuli and the prompt are presented simultaneously (0-s delay). Subsequent trials

have a set interval of time (e.g., 3 or 4 s) between the presentation of the natural stimuli and the prompt (Wolery & Gast, 1984).

**Evidence-based Practice:** An empirically validated practice. Evidence-based refers to the criteria used to evaluate whether a practice has a sufficient number of quality studies to support it (Courtade, Test, & Cooke, 2015).

**Generalization:** One of the seven defining characteristics of ABA (Baer, Wolf, & Risley, 1968), generalization is defined related to three facets: time, settings, and behaviors (Stokes & Baer, 1977). Extent to which a learner continues to perform the target behavior after a portion or all of the intervention responsible for the behavior's initial appearance in the learner's repertoire has been terminated, in a setting or stimulus situation that is different from the instructional setting, or emits untrained responses that are functionally equivalent to the trained target behavior (Cooper et al., 2007).

**Intellectual Disability:** Based on the Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition (DSM-V; 2013), an intellectual disability (intellectual developmental disorder) is a disorder with onset during the developmental period that includes both intellectual and adaptive functioning deficits in conceptual, social, and practical domains. The following three criteria must be met:

- A. Deficits in intellectual functions, such as reasoning, problem-solving, planning, abstract thinking, judgment, academic learning, and learning from experience, and practical understanding confirmed by both clinical assessment and individualized, standardized intelligence testing.
- B. Deficits in adaptive functioning that result in failure to meet developmental and sociocultural standards for personal independence and social responsibility. Without

ongoing support, the adaptive deficits limit functioning in one or more activities of daily life, such as communication, social participation, and independent living, and across multiple environments, such as home, school, work, and recreation. Adaptive functioning should be addressed using both clinical evaluation and individualized, culturally appropriate, psychometrically sound measures.

C. Onset of intellectual and adaptive deficits during the developmental period.

The severity levels for intellectual disability are based on intelligence quotients (IQ): mild (IQ 70-55), moderate (IQ 55-40), severe (IQ 25-40) and profound (<25). IQ measures are less valid in the lower end of the IQ range.

Learning Strategy: Techniques, principles, or rules that enable a student to learn, solve problems, and to compete tasks independently (Deshler & Schumaker, 1984).

Least Intrusive Prompting: Provision of the opportunity to perform the target behavior with the least amount of teacher assistance (natural stimuli) on each trial before presenting increasingly more intrusive prompts (Wolery & Gast, 1984). Other common terms include system of least prompts, less to more direct assistance (Cuvo, Leaf, & Borakove, 1978), and increasing assistance approach (Billingsley & Romer, 1983).

National Council of Teachers of Mathematics (NCTM): The NCTM is the world's largest mathematics education organization. It was founded in 1920 and has the following mission: to be the public voice of mathematics education, supporting teachers to use equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research (NCTM, 2015).

Problem Solving: Any task or activity for which the students have no prescribed answer (Van De Walle, 2004). In application to mathematics, there are three components: (a)

begins where students are, (b) problematic or engaging aspect of problem is related to mathematics students will learn, (c) requires justification and explanations for methods (Van De Walle, 2004)

Schema: In the context of mathematics, an outline or a framework for solving a problem that can be represented through pictures, diagrams, number sentences, or equations (Marshall, 1995; Powell, 2011).

Schema-Based Instruction: An explicit strategy that primes the underlying problem structure whereby the student selects or creates a diagram that fits the structure of the word problem, uses the diagram to solve the problem, and completes the number sentence with the solution (Jitendra & Hoff, 1996; Jitendra et al., 2015).

Severe Disability: An umbrella term for students with moderate and severe developmental disabilities (Browder & Spooner, 2011).

Significant Cognitive Disability: A term that refers to students who participate in alternate assessment based on alternate achievements standards; many, but not all, of these students have moderate to severe developmental disabilities (Browder & Spooner, 2011).

Self-Determination: A combination of skills, knowledge, and beliefs that cause a person to engage in a goal-directed, self-regulated, autonomous behavior (Field, Martin, Miller, Ward, & Wehmeyer, 1998). Self-determined behavior is caused by the person as opposed to being caused by someone or something. People who are self-determined make or cause things to happen in their own lives (Wehmeyer, Shogren, Zager, Smith, & Simpson, 2010).

Self-Instruction: Teaching students to use one or more self-directed instructional strategies to plan, perform, and monitor a task (Agran, 1997), including management strategies such as self-monitoring and self-recording (Wehmeyer, Agran, & Hughes, 2000).

Self-Monitoring: A procedure whereby a person observes his behavior automatically and records the occurrence and nonoccurrence of a target behavior (Cooper et al., 2007)

Systematic Instruction: Use of prompting and fading to promote acquisition of a new chained or discrete response (Browder & Spooner, 2011).

Task Analysis: Process of breaking a skill down into smaller, more manageable components (Collins, 2012; Gold, 1976).

Technology-Aided Instruction: Technology is a central feature of an intervention that supports the goal or outcome for the student (Odom, Thompson, et al., 2014). Technology is defined as any electronic item/equipment/application or virtual network that is used intentionally to increase/maintain, and/or improve daily living, word/productivity, and recreation/leisure capabilities (Odom, Thompson, et al., 2014).

## CHAPTER 2: REVIEW OF LITERATURE

The purpose of this chapter is to provide a foundation for a proposed multi-component treatment package that will overcome barriers to problem solving through targeted interventions, created using evidence-based practices, for the ultimate goal of algebraic problem solving by students with ASD/ID. As shown in the theory of change in Figure 1, there are several barriers to mathematical problem solving instruction for this population. These include language, early literacy and numeracy skills, and executive functioning. Four solutions will be proposed as a method to create bridges to problem solving: mathematics vocabulary instruction, pictorial self-instruction, technology-aided instruction, and strategy instruction. Following will be a review of literature on these four components of the intervention. The theory of self-determination and science of applied behavior analysis provides the overall philosophical foundation for this review (Baer, Wolf, & Risley, 1968; Wehmeyer et al., 2010). To provide background for the intervention, an explanation will be provided of programmed instruction and its basis for technology-aided instruction, as well as how technology-aided instruction has been used to teach mathematics to students with ASD/ID. Finally, schema-based instruction will be introduced as a promising strategy to teach algebra word problem solving to students with ASD/ID.

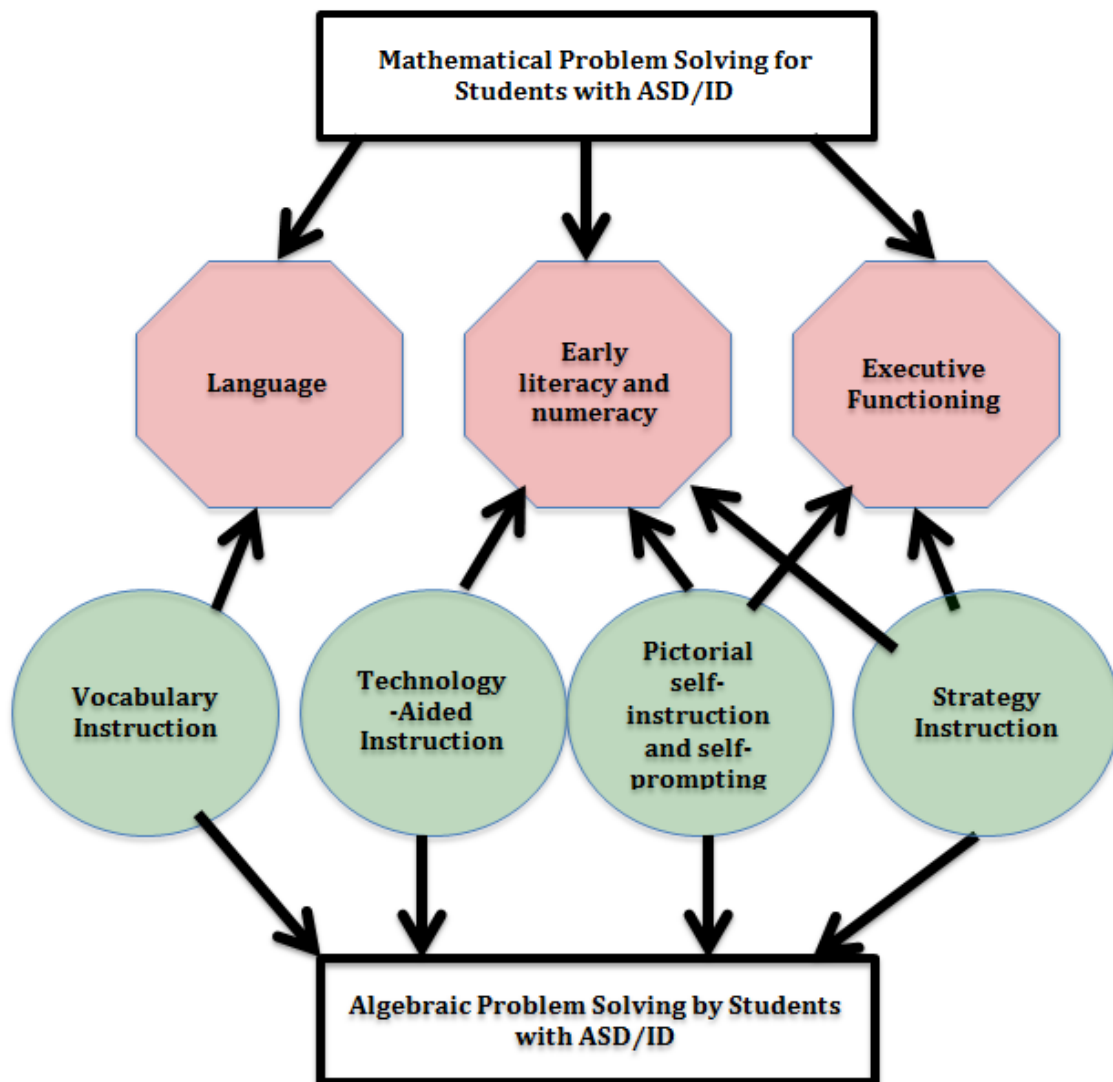


Figure 1: Theory of change

### Mathematical Problem Solving

Mathematical learning is pivotal to having a range of career, leisure, and daily living opportunities. Students with ASD should have the opportunity to learn algebra to (a) gain the basic algebraic thinking skills most adults take for granted, (b) learn methods for solving real life problems, (c) receive the opportunity to achieve what research has shown may be attainable, and (d) have access to the content on which they will be

assessed for school accountability. In a meta-analysis that analyzed instructional components in mathematical intervention studies conducted with students with learning disabilities (LD) published between 1971 and 2007, Gersten et al. (2009) found explicit instruction, use of heuristics, student verbalizations, visuals for teacher and student, sequence or range of problems, teacher feedback, student feedback, and cross-age tutoring had mean effect sizes significantly greater than zero. Some of these established practices for teaching mathematics are gaining popularity and demonstrating effectiveness for students with moderate and severe disabilities, including explicit instruction, use of heuristics, and teacher feedback (Spooner et al., 2016). Progress is slow given the limited scope of skills current mathematics literature addresses and barriers to mathematical problem solving for students with ASD/ID.

#### Mathematics Instruction for Students with ASD and ID

Despite its importance to future functioning, when students with ASD/ID receive mathematics instruction at all, it often focuses on the most basic computation skills. Through memorization and rote learning, students with developmental disabilities can learn the basics of computation or counting money. In contrast, the real world will rarely offer problems condensed to numbers on a page with the operation specified ( $3+2=$ ). The real world typically presents a quantity and an implied relationship; the person has to provide the rest. Problem solving instruction can be used to help students learn how to address a problem by recognizing what they know and need to know about a real life situation.

History of a narrow focus. Literature on teaching mathematics to students with moderate and severe disabilities goes back to the early days of special education and



applied behavior analysis (Borakove & Cuvo, 1977; Bracey, Maggs, & Morath, 1975; Koller & Mulhern, 1977; Trace, Cuvo, & Criswell, 1977). Although the curricular focus for students with severe disabilities has shifted over the past 6 decades, the focus of mathematics instruction has only made meager changes. In their description of the changing emphasis of the curriculum for individuals with severe disabilities, Browder, Spooner, Wakeman, Trela, and Baker (2006) described a movement from a developmental model in the 1960s and 1970s to functional skills following Lou Brown's seminal writings (1979). Into the 1980s, the criterion of ultimate functioning pushed instruction into community settings, and mathematics instruction was related to daily living and community skills (e.g., Gaule, Nietupski, & Certo, 1985; Hastings, Raymond, & McLaughlin, 1989; Matson & Long, 1986). In the 1990s, there was a self-determination movement that advocated for social inclusion, choice making, and goal setting (Browder et al., 2006). Mathematics instruction during this time period continued to reflect a focus on functional applications, such as the one-more-than or next dollar technique (Denny & Test, 1995). After the passage of the landmark reauthorization of the IDEA 1997, all students were required to access the general curriculum. Although the reauthorization mandated general curriculum access for all students with disabilities, including those with significant cognitive disabilities, the focus of the curriculum did not shift to academics until comprehensive federal education reforms included accountability measures.

The mandate to include students with significant cognitive disabilities in Alternate Assessments aligned with Alternate Achievement Standards (AA-AAS) and include the results in measures of adequate yearly progress (AYP) came after the No Child Left Act

in 2001 and the reauthorization of IDEA in 2004. These efforts were the catalyst for the change in curricular focus for students with moderate and severe disabilities. During the early 2000s, teachers commonly taught academics within functional activities (Browder et al., 2007). For example, a middle school teacher might try to address mathematics during personal hygiene routines or science during meal times, if at all. Now, teachers of students with moderate and severe disabilities are realizing that academic instruction and individualized education program (IEP) goal instruction on nonacademic skills are not mutually exclusive. Rather, nonacademic skills can be addressed during naturally occurring times of the day when age-appropriate, and grade-aligned academics can be taught within real-world contexts in a similar routine and schedule as typically developing peers.

The approach of a standards-based IEP helps to balance the needs of students with moderate and severe disabilities based on instructional priorities. A standards-based IEP has goals that link to state standards and others that incorporate life skill needs (Browder, Spooner, & Jimenez, 2011). The advantage of the approach of a standards-based IEP is that it emphasizes individual student needs; functioning as an adult requires both academic and daily living skills (Courtade, Spooner, Browder, & Jimenez, 2012). Browder and Spooner (2015) have proposed that the curricular focus has been further refined in recent years to include a balanced approach of functional and life skills as well as academics that are aligned to grade-level standards. This refinement process has occurred as research efforts to determine how to teach standards-based academics, including mathematics, to students with moderate and severe disabilities (Browder et al., 2008; Browder, Trela, et al., 2012; Creech-Galloway et al., 2013; Jimenez et al., 2008).

The restricted curricular focus of past decades is reflected in mathematics research. In a comprehensive review of literature published between 1975 and 2005, Browder et al. (2008) found 64 studies that met their inclusion criteria of (a) being published in a peer-reviewed journal in English between 1975 and 2005, (b) including at least one participant diagnosed as having a significant cognitive disability, (c) focusing on teaching academic mathematics skills with reports of first-hand data, and (d) using experimental or quasi-experimental design for either group or single subject studies. Across the included studies, 493 individuals with disabilities were represented; 336 with Mod ID, 64 with severe ID, 24 with ASD, 13 with an unspecified developmental disability, and 1 with multiple disabilities. The majority of these studies taught skills in the domains of numbers and operations or measurement (e.g., time and money). Only two studies in their review addressed the NCTM standard of algebra (i.e., Miser, 1985; Neef et al., 2003).

An updated search of the literature that used the same methods as Browder et al. (2008) found 34 studies published between 2005 and May 2015 that taught mathematics to students with moderate and severe disabilities (Spooner et al., 2016). Across the included studies, a total of 170 participants with moderate to severe disabilities were represented; 70 with Mod ID, 12 with severe ID, 5 with a non-specified developmental delay, four with ASD and a mild ID, 31 with ASD and a Mod ID, and 33 had ASD and an unspecified severity of cognitive impairment. Authors compared their findings to those of Browder et al. in terms of NCTM standards and skills. Findings were similar in that the majority of the included studies addressed the NCTM standard of Numbers and Operations ( $n=26$ ); however, eight studies targeted the Algebra standard.

Similar instructional techniques were used to teach a variety of skills within the Algebra standard. Interventions targeted solving equations (Browder, Jimenez, et al., 2012; Browder, Trela et al., 2012; Jimenez, et al., 2008), analyzing patterns (Jimenez & Kemmery, 2013; Jimenez & Staples, 2015) and finding the percent of change within the context of tax or sales price (Collins, Hager, & Galloway, 2011; Hua, Woods-Groves, Kaldenberg, Lucas, & Therrien, 2015; Karl et al., 2013). All of these interventions involved the use of systematic instruction and a task analysis.

Evidence-based practices. The extant literature on teaching mathematics to students with moderate and severe disabilities has provided sufficient quantity and quality of studies to establish evidence-based practices. Courtade, Test, and Cook (2015) describe a two-step process for determining evidence-based practices. The first task is to identify quality studies that use an experimental design to measure the effect of an intervention (outcome) for a target population. Quality indicators for evaluating single-case and group research were developed by R. H. Horner et al. (2005) and Gersten et al. (2005), respectively. The second task is to determine whether or not a sufficient quantity of high quality studies have been found with an intervention of interest. The quality, quantity, and dispersion guidelines outlined by R. H. Horner et al. and Gersten et al. have been interpreted by several research groups, including Browder et al. (2008). There is not a unified standard for identifying an evidence-based practice, which has led to organizations using different criteria (Odom, Collet-Klingenberg, Rogers, & Hatton, 2010). The second part of the term, practice, refers to specific strategies used to change particular behavior/behaviors over a short period of time, or to comprehensive treatment models, which are a collection of practices implemented over a longer period of time to

achieve broad gains (e.g., The Denver Model; Odom et al., 2010). Intervention practices that have been identified as evidence-based for teaching mathematics to students with moderate and severe disabilities, including ASD/ID, include in vivo instruction, systematic instruction, opportunities to respond, technology-aided instruction, use of a graphic organizer or heuristic, manipulatives, and explicit instruction (Browder et al., 2008; Spooner et al., 2016; Wong et al., 2014).

It is important to consider the behavioral requirements of instructional targets. Algebraic problem solving is innately a chained task. This influences the cognitive demands required during problem solving. Professional mathematics organizations have provided guidelines for algebra instruction that take into account the multifaceted cognitive processes that work together during algebraic problem solving tasks (NCTM, 2000). Chained tasks require a specific sequence of discrete responses, each of which results in a stimulus change that serves as a conditioned reinforcer for the response that produced it, and as a discriminative stimulus for the next response in the chain (Cooper et al., 2007). This point can be illustrated with the following word problem: “Jay has 5 trucks. Some are blue and some are red. 3 trucks are red. How many trucks are blue?” A student would need to perform a series of discrete behaviors to arrive at the correct solution. Depending on the learner, this could include: (a) read the problem, (b) determine known and unknown information, (3) determine the operation (subtraction), (c) subtract three from five, and (d) identify the unknown information as two blue trucks. The task could be further broken down; subtracting three from five can be done a variety of ways if the math fact is not memorized, which may create an additional chain of discrete skills (such as using the touch point method or manipulatives).

Chained tasks are basically learned through one of three ways: forward chaining, backward chaining, or total task presentation (Spooner, 1984). Using the example above, the process for solving the word problem could be taught using a forward chaining approach by teaching each step to mastery before moving to the next step. In a forward chaining procedure, the student would not move on to determining the known and unknown information until they were able to read/access the problem without assistance to a set criterion. After mastery of the first step, the student would be taught how to determine known and unknown information (step two). This process would continue until all steps of the chain have been mastered. In contrast, a backward chaining method of instruction would begin with teaching the student to identify the unknown information, or the final step in the chain. Similar to forward chaining, this step would be taught to mastery at a given criterion before backing up to the previous step.

Finally, total task instruction would involve teaching the student all of the steps in each session. In this method of instruction, students have an increased likelihood of making errors that prevent successful completion of the task, in this case solving the problem. Because each step is taught in each session, the cognitive load is increased. Forward and backward chaining rely on mastery of only one of the skills, resulting in a reduced cognitive load and increased likelihood of success. In the case of algebraic problem solving, total task presentation is the only method that will simultaneously address procedural and conceptual knowledge. Alternative strategies will be needed to minimize the overall cognitive load.

## Algebraic Problem Solving

The research on teaching mathematics to students with moderate and severe disabilities, including ASD/ID, has a different content and skill focus from what is recommended by experts in mathematics (Browder et al., 2008, Spooner et al., 2016). Even the most recent research with students with moderate and severe disabilities has an overwhelming focus on Numbers and Operations skills, such as counting-on (Hsu et al., 2014), calculation methods such as dot notation (Fletcher, Boone, & Cihak, 2010), and memorization of basic facts (Rapp et al., 2012). Relatively few studies have combined these skills with algebra (Browder, Jimenez, et al., 2012; Browder, Trela, 2012; Hua et al., 2015; Karl et al., 2013).

In the first study to teach algebra to students with ID, Jimenez, Browder, and Courtade (2008) taught three high school students with Mod ID to solve an algebraic equation. The multicomponent intervention included concrete representation of solving a simple linear equation, task-analytic instruction on the steps to solve the equation in a total task format, multiple trials for learning, and systematic prompting with fading to promote errorless learning. Participants were taught to “count up” through a system of least prompts that included a verbal and physical prompt. A functional relation was found between the multicomponent intervention and solving the algebraic equation.

In two subsequent studies that focused on multiple mathematics standards including algebra, these methods were replicated with middle and high school students with moderate and severe disabilities, including ASD/ID (Browder, Jimenez, et al., 2012; Browder, Trela, et al., 2012). The algebraic equation was used within the context of a story problem featuring familiar themes. An additional feature of the intervention was

that students used a graphic organizer to help them keep track of the steps to solve the problem. In a multiple probe design, Browder, Jimenez et al. (2012) found a functional relation between the multicomponent intervention and solving the algebraic equation. In a group experimental design, Browder, Trela, et al. (2012) found the experimental group outperformed the control group on three of the four math subscales by a statistically significant amount. The algebra performance by the experimental group had a large effect size of 2.11.

A major limitation of these previous algebra studies is that they limited conceptual knowledge. Participants were able to demonstrate “how” to solve the algebraic equations, but not “why” or “when” to use those methods (Saunders, 2014). According to the National Mathematics Advisory Panel (NMAP), preparing students for algebra requires simultaneous emphasis on conceptual understanding, computational fluency, and problem solving skills. The NMAP identified three clusters of skills as most essential for students to learn thoroughly for success in algebra coursework, including fluency with whole numbers, fluency with fractions, and aspects of geometry and measurement. Across these three critical foundations of algebra, emphasis is placed on application in real world situations and problem solving. These guidelines have potential to support further research on developing Numbers and Operations skills through problem solving.

Successful problem solvers are able to combine conceptual and procedural knowledge to solve real-world mathematics problems. Conceptual knowledge reflects knowledge about the relationships or foundational ideas of a topic (Van de Walle et al., 2012), combining the two skills of comprehending the problem and modeling the



problem. Procedural knowledge is demonstrated by fluency in the use of rules and procedures in carrying out mathematical processes and also the symbolism used to represent mathematics (Van de Walle et al., 2012). Two things facilitate conceptual understanding of mathematics, including (a) making mathematics relationships explicit, and (b) engaging students in productive struggle (Bay-Williams, 2010; Hiebert & Grouws, 2007). The instructional strategies and supports provided to students are the keys to giving them access to the critical foundations and grade-aligned standards of algebra (Root, Browder, et al., 2015). In order to be effective mathematical problem solvers, students with moderate and severe disabilities must have high quality, systematic instruction with repeated opportunities for practice (Root, Browder, & Jimenez, 2015).

The learning characteristics of individuals with moderate and severe disabilities present challenges to mathematics problem solving. Theories of problem solving rely on the ability of individuals to use metacognitive strategies to plan and execute problem solving. This requires students to rely on executive functioning and working memory, which are impaired for individuals with a developmental disability. Students with moderate and severe disabilities may be able to learn problem solving if their learning characteristics are taken into account during instruction.

Theories of problem solving. Schroeder and Lester (1989) identified three types of approaches to problem solving which include (a) teaching for problem solving, (b) teaching about problem solving, and (c) teaching through problem solving. When instructors teach for problem solving, the procedural and conceptual knowledge are seen as two separate processes. Procedural fluency, such as using a mathematical formula, is brought to mastery so that students are primed for application to a story problem. When

students are taught about problem solving, processes or strategies are emphasized in isolation and applied to story situations that fit that application. An example of this would be application of the “draw a picture” strategy when given story problems about a teacher passing out 5 cookies to 3 students; all mathematical examples would fit the prescribed strategy. Finally, students learn mathematics through real contexts, problems, situations, and models when they are taught through a problem solving approach (Van de Walle et al., 2012). Procedural and conceptual knowledge are not bifurcated; contexts and models provide meaning as students apply a range of procedures toward a solution. For students with severe disabilities, teaching through problem solving could simultaneously provide access to grade-aligned academics, address the process standard of problem solving, and provide instruction on discrete mathematical skills.

Traditional approaches to problem solving instruction are abstract and require a number of prerequisite metacognitive skills. Polya (1945) first outlined the classic four-step process for solving problems that continues to garner favor and popularity, including (a) understand the problem, (b) devise a plan, (c) carry out the plan, and (d) look back. Mayer (1985) built upon this four-step process in his model of problem solving. According to Mayer, there are four sequential phases to solving a math word problem, including (a) problem translation, (b) problem integration, (c) solution planning, and (d) solution execution. Each phase requires different cognitive skills for successful completion (Krawec, Huang, Montague, Kressler, & Melia de Alba, 2013). At the problem translation phase, semantic language skills are used to construct meaning from the problem. The problem integration phase requires selection of integral parts of the problem and translating them to a mathematical structure. The solution planning and

execution phases involve choosing the correct operations and carrying out those computations to arrive at a correct answer. Each phase is dependent upon successful completion of the previous phase in order for correct execution and ultimate arrival at a correct answer (Jitendra, Griffin, Deatline-Buchman, & Szczesniak, 2007). Errors in any stage of the process will prevent successful demonstration of conceptual and/or procedural knowledge.

Advances in mathematics instruction for students with math difficulties have emerged from multidisciplinary or transdisciplinary endeavors – when researchers in one field face a problem, solutions may arise from related disciplines (Gersten, Clarke, & Mazzocco, 2007). Research from the field of LD may be helpful in pinpointing the performance difficulties experienced by individuals with ASD/ID during math tasks. Kosc (1970) described six categories of performance difficulties in mathematics including (a) verbally designating mathematical terms and relations, (b) manipulating real objects in accordance with the conventions of mathematics, (c) reading mathematical symbols, (d) manipulating mathematical symbols in writing, (e) carrying out mathematical operations, and (f) understanding mathematical ideas and performing mental calculations. Errors in any step in the chain of solving math problems will prevent successful demonstration of conceptual and/or procedural knowledge. Pinpointing the performance difficulties and designing matching interventions may increase success of individuals with ASD/ID in math problem solving tasks.

Problem solving difficulties for students with ASD and comorbid ID. The unique characteristics of students with ASD and other developmental disabilities contribute to the difficulties they face in solving math word problems. Geary and Hoard (2005)

concluded that mathematics learning disabilities are related to the combination of disrupted function of the central executive, which includes attentional control and poor inhibition of irrelevant associations, or difficulties with information representation and manipulation in the language system. Students with ASD/ID also have deficits in these areas. Executive functioning is required for planning, organizing, switching cognitive sets, and working memory (Quill, 2000; Rockwell et al., 2011). Zentall (2007) considers executive functioning a “critical factor in math performance and achievement” (p. 234).

Mathematical problem solving requires a certain level of metacognition, the conscious monitoring and regulation of an individual’s own thought process (Van De Walle et al., 2012). It is important for students to be able to know what they are going to do, how they will go about doing it, and the rationale behind their choices. This iterative cognitive process is used cyclically and students choose a strategy, monitor progress, and make changes as necessary along the way. Metacognition in problem solving may be interrupted for students with moderate and severe disabilities due to a lack of self-monitoring skills. Additionally, working memory deficits can lead to an inability to discriminate relevant events from irrelevant stimuli within word problems, moving a conceptual error in understanding the type of word problem presented into a subsequent procedural error in solving the problem due to the wrong strategies being employed (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007).

An additional barrier for individuals with ASD/ID is the semantic language involved in word problems (Rockwell et al., 2011). Swanson and Beebe-Frankenberger (2004) explain problem solving goes beyond procedural knowledge; in order to understand word problems, students must use the “words, phrases, sentences, and

prepositions to generate a coherent and meaningful interpretation” (p. 471). The semantics of word problem solving can be problematic for students with ASD/ID on numerous levels; it requires an individual to understand both the individual terms as well as to make sense of “what is happening” in the problem. Van de Walle et al. (2012) emphasize the importance of fluency in mathematical language, as students need to use mathematical vocabulary and articulate mathematics concepts in order to learn the language and concepts of mathematics. A barrier at the problem translation phase may prevent further success in the problem solving process for individuals with ASD/ID.

### Summary

There has been a historically narrow focus of mathematical interventions for students with developmental disabilities, including ASD. Therefore, relatively few interventions have addressed mathematical problem solving for individuals with ASD/ID. The barriers to problem solving for students with ASD/ID can be attributed to the vocabulary, self-determination, and meta-cognitive skills it requires. A multi-component intervention that addresses each of these deficits may accommodate problem solving instruction for this population.

### Content Area Vocabulary Instruction

Communication and language skills are one of the three core deficits in individuals with ASD (Quill, 2000). Both expressive and receptive language are likely affected in students with ASD. During academic instruction, these deficits can make it difficult to assess what a student with ASD/ID knows. For example, a student with limited vocal language who is unable to write will always need response options. That

student's expressive language is limited to the options the communicative partner provides. Communication and language are factors in content area instruction as well.

In mathematics, understanding vocabulary of key terms and concepts is essential to developing a conceptual understanding. For example, reciprocal understanding that the equal sign means "same as," not "is" or "equals" is essential for conceptual understanding of algebraic equations. Further, a student needs to understand the concept of "same" and "different," a concept that is a prerequisite for skills across the standards.

In word problem solving, an extra layer of language demands is present; the semantic language of the word problem must be translated into a mathematical representation (Rockwell et al., 2011). There is evidence that students with ASD/ID can learn mathematics problem solving, despite the inherent communication and language demands using evidence-based practices (Rockwell et al., 2011; Root, Browder, et al., 2016; Saunders, 2014).

#### Evidence-Based Practices for Teaching Vocabulary

Although the research on teaching vocabulary to students with ASD/ID has primarily had a functional or basic skills focus (Riggs et al., 2013), the instructional procedures can translate to other types of vocabulary. Sight words, environmental print, and other functional terms are considered tier one vocabulary terms (Beck, McKeown, & Kucan, 2013). This means that these are words that are used in everyday life in a variety of contexts and settings. Alternatively, tier two vocabulary words are those that are used with high frequency across multiple domains, and may be synonyms for tier one words (e.g., equivalent for same). Academic language, especially in general education settings, is made up primarily of tier two vocabulary words (e.g., perimeter). If students do not

have a fluent understanding of these terms, it will be difficult for them to fully access the general curriculum. Finally, tier three words are those that are used with relatively low frequency and only apply to specific domains. An example of a tier three mathematics word would be the geometry term “reflexive.” Although students may need to know these specialized terms in secondary education or employment settings, they most likely do not impede general curriculum access.

One evidence-based practice for teaching vocabulary is constant time delay (CTD; Browder et al., 2009). As an errorless learning procedure, this instructional procedure is effective and efficient. In CTD, two delay intervals are used. First, teachers use a 0-s delay, meaning immediately after the task direction (i.e., “What does add mean?”) a controlling prompt (i.e., “combine”) is delivered. In subsequent rounds, a delay is inserted between the task direction and the controlling prompt (e.g., 3-5-s). If students make an incorrect response, an additional 0-s round is often repeated and the student is told “If you do not know, wait and I will help you.” In its first applied investigation, Touchette (1971) found learning occurred with few to no errors, resulting in it being coined an “errorless learning” procedure.

In a comprehensive literature review, Browder et al. (2009) established CTD as an evidence-based practice for teaching vocabulary to students with moderate and severe disabilities using the R. H. Horner et al. (2005) criteria for single-case design research. A total of 30 experiments met the original inclusion criteria, with 22 experiments meeting all of the R. H. Horner et al.’s quality indicators for high quality studies. Of these 22 experiments, 11 used CTD, far exceeding the required five to establish an evidence-based practice.

## Content Area Vocabulary Instruction for Individuals with ASD and Other Severe Disabilities

Constant time delay is an evidence-based practice that can address one barrier to content area learning for students with ASD/ID: the necessary vocabulary knowledge (Knight, Spooner, et al., 2013). Recently, researchers in the field of general curriculum access have evaluated the effects of CTD to teach content-area vocabulary and its effect on content area conceptual learning. In the area of literacy, CTD has been used to teach WH-rules (Allison, Root, Browder, & Wood, 2016) as well as story elements (Browder, Root, Allison, & Wood, 2015). Science content has also been taught using constant time delay. For example, Knight, Spooner, et al. (2013) taught middle school students with ASD/ID vocabulary related to convection. A functional relation was found between the treatment package, which included CTD, and steps of a task analysis related to knowledge of convection, as well as maintenance of knowledge for two of the participants. In a similar science intervention, Riggs et al. (2013) taught high school students with moderate and severe disabilities concepts of heredity using CTD. Visual examples of dominant traits, recessive traits, and punnett squares were presented on 5x7" laminated cards. One participant demonstrated a functional relation between CTD and knowledge of heredity and all participants demonstrated increased knowledge of at least one concept.

Although these studies addressed vocabulary from a variety of content areas, a common instructional element was the use of CTD. The targeted vocabulary word in each trial was presented in an array with non-examples or distractors. Visual supports were incorporated along with the vocabulary words or definitions in several studies (Allison et



al., 2016; Knight, Spooner, et al., 2013; Riggs et al., 2013). Research with middle school students with ASD/ID has shown that varying levels of visual support that matches the symbolic level of the student is effective in teaching grade-aligned vocabulary (Mims, Lee, Browder, Zakas, & Flynn, 2012). To date, no studies have taught grade-aligned mathematics vocabulary related to problem solving. Visual supports paired with vocabulary words and their definitions may be helpful when teaching mathematics vocabulary using CTD.

### Summary

The vocabulary demands of academic learning targets should be considered in instructional planning. The proposed intervention will require understanding of several tier two vocabulary words to which participants may not have been previously exposed. In order to ensure student errors are not due to a language or vocabulary deficit, these terms will be explicitly taught. The literature on teaching academic vocabulary to students with ASD/ID supports using CTD as well as the incorporation of visual supports.

### Pictorial Self-Instruction and Self-Prompting

People who exhibit self-determination make or cause things to happen in their own lives (Wehmeyer et al., 2010). People acquire skills and attitudes related to self-determination throughout their lives by developing several component skills, including choice-making, decision-making, problem solving skills, goal setting and attainment, self-management, self-advocacy and leadership skills, perceptions of control and efficacy, and self-awareness (Wehmeyer et al., 2010).

Wehmeyer et al. (2010) suggested individuals with ASD have unique difficulties with self-determination component skills due to the nature of their disability. Individuals with ASD are characterized by a triad of deficits in communication, socializations, and interests and activities (Heflin, & Alaimo, 2007). In addition, individuals with ASD have impaired attention, information processing, and social cognition that lie on a continuum of severity (Quill, 2000). Instruction in self-determination component skills may help address deficits associated with characteristics of ASD (Quill, 2000; Wehmeyer et al., 2010).

Self-determined behavior can also be thought of as “self *vs.* other-determined action” and always has a social context (Wehmeyer et al., 2010), which can make problem solving difficult for students with ASD. A problem is “an activity or task for which a solution is not known or readily apparent” (Wehmeyer et al., 2010, p. 478) and solving problems requires flexible thinking, an often-difficult cognitive task for individuals with ASD (Quill, 2000). Individuals with ASD have difficulties with attention and information processing, as well as over-selectivity of attention, which make staying on task difficult and require self-management strategies (Heflin & Alaimo, 2007). Self-monitoring, a component of self-management, consists of self-observation and self-recording (Lee, Simpson, & Shogren, 2007). Deficits in attention for individuals with ASD include atypical responses to sensory stimulation, overselectivity, and difficulty shifting attention between visual and auditory stimuli (Quill, 2000). Challenging behaviors, including time-spent off-task, are barriers to participation in the general education classroom (Carr et al., 1999).

Problem solving is required “in the moment” when individuals face a situation that they must address, but it is also required in a more long term sense to achieve goals when the solution or plan of action is not readily apparent (Agran, Blanchard, Wehmeyer, & Hughes, 2002). The strategies to promote self-determination fall into two categories, which include student involvement in educational planning and direct teaching of skills. When directly teaching self-determination skills, the Self-Determined Learning Model of Instruction (SDLMI) has been shown to be effective (Agran et al., 2002). The SDLMI promotes self-determination in the general curriculum. Self-determination interventions have strong effects on organization skills and productivity in academic assignments. For example, Agran et al. (2002) investigated the effects of self-regulated problem solving instruction on self-set goals for middle school students with ID, including one student with ASD, in the general education classroom. Utilizing the SLDMI and a means-end sequence, participants were taught to solve problems by identifying their goal, determining the action needed to achieve that goal, and modifying their goal or plan along the way as needed. Using a multiple baseline across participants design, the effects of the SDLMI were measured using the Goal Attainment Scale. A functional relation was found between the SDLMI and self-set goal attainment for all participants, including the participant with ASD. As a result, Agran et al. demonstrated students with disabilities, including ASD, can be taught to use a self-regulated problem solving strategy to achieve self-set goals.

Another self-determination skill, self-management, is needed for individuals to be in control of their own behavior instead of relying on others to manage it. Self-management can be difficult for individuals with ASD and often requires explicit

instruction (Lee, et al., 2007). In a meta-analysis of existing research interventions used to increase self-management of learners with autism, Lee et al. (2007) analyzed studies published through October of 2004 and found 11 articles in seven journals which met inclusion criteria, but none were published in any autism-specific journals. Self-management interventions were found to be effective in increasing socially appropriate behaviors using various self-management interventions and with different subjects in varying settings and conditions. There was no statistically significant difference between the effects of the intervention or materials, demonstrating that different iterations of self-management interventions have been shown to be effective in increasing socially appropriate behaviors.

Self-monitoring is a subcomponent of self-management, as it assists an individual in becoming aware of their behavior in order to regulate it. Self-monitoring can transfer the stimulus control from an adult in the environment to an object in the environment or the individual themselves, increasing the independence of the individual with ASD (Taber, Seltzer, Heflin, & Alberto, 1999). For example, Taber et al. (1999) investigated the impact of a self-operated auditory prompting system on the off-task behavior of a student with ASD and Mod ID. The intervention took place in the student's classroom during a writing task and in the cafeteria during a vocational task. The auditory prompting systems played instrumental music the student enjoyed with prompts interrupting the music interspersed at fixed ratios presented at random order of the teacher's voice encouraging him to "keep working," praising him for doing a "good job," or prompting him to "get to work." The auditory prompting system was socially valid and acceptable as people without disabilities frequently wear headphones in work and school

environments. A multiple probe across settings with an embedded withdrawal design was used to measure the effects of the self-operated auditory prompting system, which were measured by the number of the teacher prompts provided when engaged in off-task behaviors. Results of the study found a functional relation between the intervention and decrease in off-task behavior. Results indicate individuals with ASD can be taught self-management skills that may increase independence and decrease dependence on others for prompting in vocational and classroom environments.

Self-monitoring procedures have also been shown to be effective for younger children with ASD. For example, Holifield, Goodman, Hazelkorn, and Heflin (2010) evaluated the effectiveness of a self-monitoring procedure on increasing attending to task and academic accuracy for younger children with ASD. Two male 9 and 10 year old students with ASD who had demonstrated chronic long-term deficits in attending to tasks and had IQs of 39 and 60 were taught a self-monitoring procedure during seatwork for reading and math in a self-contained elementary classroom. During independent seatwork, participants were given a verbal cue of “Attending to task?” which prompted the participants to circle yes or no on a self-monitoring sheet. Using a multiple baseline across participants in two subject areas design, a functional relation was found between self-monitoring and attending to task and academic accuracy. The self-monitoring intervention was found to increase independence, as well as academic performance.

#### Pictorial Self-Instruction for Individuals with ASD and Other Severe Disabilities

For students with moderate and severe disabilities who may have emerging literacy skills, pictorial self-instruction has been effective in teaching self-determination component skills. As recommended by Browder and Shapiro (1985), strategies must be

designed to transfer student responding from teacher delivered to self-delivered stimuli. Self-instruction involves the use of self-talk, printed instructions, or other materials to provide instruction, rather than a teacher (Browder & Shapiro, 1985). The instructions are discriminative stimuli for the target behavior. In pictorial self-instruction, individuals use a task analysis that has pictures (and possibly text) corresponding to each step. The format of the task analyses has varied; for some, they direct an individual through each discrete behavior in a chained task. Another format would be to have a task analysis that depicts each behavior needed to complete a series of chained behavior, or a sort of schedule.

Pictorial self-instruction, in combination with auditory prompting, has been used to teach adolescents with moderate ID to complete chained tasks independently. For example, Mechling and Gast (1997) used a Digivox, an AAC device, to provide pictorial self-instruction and auditory prompts. In each of the windows of the device were photographs of the participant completing each step of the task analysis or photographs demonstrating the step. When a participant pressed the picture, a description of the step (auditory prompt) played. Using an ABAB design, a functional relation was found between using the device to deliver pictorial self-instruction and auditory prompting and completion of the chained tasks across three participants.

More recently, Diegelmann (2015) used a self-monitoring checklist with visual supports as a component of the self-directed IEP. Self-monitoring was used to provide immediate feedback, motivation, and teach the students to self-regulate their learning. In a multiple probe across participants design, a functional relation was found between the

modified self-directed IEP and the number of correct steps of the IEP. Participants were able to generalize to post-intervention mock IEPs using the self-monitoring checklist.

Pictorial self-instruction has also been used in academic contexts with individuals with ASD/ID, including in mathematics. Both Root, Browder, et al. (2015) and Saunders (2014) used pictorial self-instruction as a component of modified SBI in teaching word problem solving to elementary students with ASD/ID. Root, Browder, et al. used a laminated task analysis with pictures and text representing each step. Participants were instructed to “check off” each step as it was completed. Saunders (2014) used a similar task analysis, but it was displayed on a computer screen using SMARTboard© software. Participants were able to click on a step to hear it read aloud (e.g., “Step one says read the problem”) and drag a check mark to each step as it was completed. During training phases, participants were able to click on each step to hear a specific verbal prompt and also view a video model of the step being solved correctly. This application of technology to enhance pictorial self-instruction has promise for increasing independence and reducing dependency on adults for prompting within academic tasks for students with ASD/ID.

## Summary

Self-determination is built by acquiring skills representing each of its components. Important self-determination skills related to academic achievement include problem solving, self-management, and self-awareness. One way to increase self-determination component skills is through pictorial self-instruction. It has been used effectively to teach chained tasks to students with Mod ID and ASD/ID (e.g., Mechling & Gast, 1997; Diegelmann, 2015; Root, Browder, et al., 2015; Saunders, 2014). This

additional support not only transfers control of discriminative stimuli away from a teacher or another skilled learner, but also assists in facilitating executive functioning required in mathematical problem solving.

### Technology-Aided Instruction

For students with moderate and severe developmental disabilities, including ASD/ID, technology can provide critical support for learning and life functioning. Technology has transformed everyday life for many people in the 21<sup>st</sup> century, but for individuals with ASD/ID, it has especially opened doors of opportunity not previously possible. Multiple reviews have demonstrated students with ASD/ID can benefit from technology in learning academic skills (Browder, Saunders, & Root, in press; Knight, McKissick, & Saunders, 2013; Pennington, 2010; Root, Stevenson, Geddes, Ley-Davis, & Test, 2015). From its infancy in the form of teaching machines (Pressey, 1924; Skinner, 1954) to the wide forms and platforms that are ever evolving (Stephenson & Limbrick, 2013), instructors and researchers have evaluated the usefulness of technology as the central feature of an intervention to increase academic skills.

Technology has important instructional implications for individuals with disabilities in the area of mathematics. For example, the provision of alternate forms of information input that have alternate stimulus cues can make systems accessible to a wider range of users. Technology can especially increase opportunities in mathematics for students with moderate and severe disabilities by providing alternate means of access to materials, therefore providing assistance to both students and teachers.



## Foundation of Technology-Aided Instruction

The current application of technology-aided instruction originates from the work of psychologists in the early 20<sup>th</sup> century. Sidney Pressey was the first to experiment with teaching machines. He defined the teaching machine as an automatic or self-controlling device that presents a unit of information, provided some means for the learner to respond to the information, and provided feedback about the correctness of the learner's responses (Pressey, 1924). Approximately 30 years later, B. F. Skinner (1954) published descriptions of his own teaching machines that had some distinct differences from Pressey's machines. Pressey's machine "taught" only by allowing an individual to progress through questions after selecting correct answer. Skinner's machines were capable of requiring constructed responses, such as using levers to create a numerical answer. He believed that it was important to move beyond limited choice formats as they exposed students to the wrong answer. In addition, construction of correct responses represents a higher level of conceptual knowledge than selection of a correct answer from distractors (Benjamin, 1988).

Skinner attributed the lack of professional enthusiasm and adoption of Pressey's machines to cultural inertia, in that there was not a need for instruction to be automated when there was an abundance of teachers. On the other hand, there was a shortage of teachers in the 1950s and 1960s due to the baby boom. The promise of teaching machines was that they provided a solution to the problem of not enough teacher attention to address all students varying needs. It was presumed that the teaching machine could replace some functions the teacher was currently providing. There were, however, criticisms of teaching machines in the late 1960s, including being unnecessary for

presentation of programmed materials, being too expensive, and individuals' doubts about the extent to which the teaching machines could teach (Benjamin, 1988).

The teaching machines developed by Skinner and others, such as the *AutoTutor* (Crowder, 1962), were based on the principles of programmed instruction (Molenda, 2008). Programmed instruction was described by Bijou, Birnbrauer, Kidder, and Tague (1968) as the process of systematically and effectively arranging and rearranging an environment to elicit a change in a targeted behavior. The intent of programmed instruction is to make the teaching-learning process effective and customized to meet individual needs (Molenda, 2008). Skinner's early work in programmed instruction focused on content that was arranged in small chunks of information that built upon each other. In a forward-chaining procedure, the student did not progress forward to a new "chunk" of information until the previous had been mastered. In this form of programmed instruction, learners progressed at an individual pace. The use of reinforcement through knowledge of a correct response increased the likelihood of future correct responding. Programmed instruction was incorporated into textbooks as well, relieving the necessity for hardware. Research using teaching machines to deliver programmed instruction led Skinner to refer to the use of programmed instruction for the application of learning to the practical task of instruction as the *technology of teaching* (Skinner, 1968).

Although the term "teaching machines" is no longer used in modern literature, there are several terms that are used interchangeably to refer to programmed instruction presented through technology. Technology-aided instruction (TAI) is the term that was used by the National Professional Development Center (NPDC) in their review of evidence-based practices (Wong et al., 2014). They defined TAI as instruction where

technology is the central feature to support the goal or outcome for the student. Common examples of TAI would be programs installed on a desktop computer, laptop or notebook computer, tablet computer (e.g., iPad), or mobile device (e.g., iPod, iPhone, Android phones). Another term that is often used is Computer-assisted Instruction (CAI). CAI has a more narrow focus of interventions that use computers as the central feature to support learning, present learning materials, or check learner's knowledge (Anohina, 2005). The difference between TAI and CAI is the specificity of the type of technology that is the focus of the intervention. For the purpose of this dissertation based on the focus of the intervention, TAI will be used and included in the literature review.

#### Technology-Aided Instruction to Teach Students with ASD/ID

The evidence-base of TAI to teach individuals with ASD goes back over 3 decades. In 70s and 80s, the practice of TAI was so promising that Panyan (1984) reviewed the literature published between 1973 and 1984 that used TAI to teach individuals with disabilities in order to identify the unique benefits of computers for individuals with ASD and to make recommendations for the incorporation of computers into educational settings. Though none of the skills targeted in the studies included in the review would be considered "academic" by today's standards (e.g., requesting, social skills) and the studies would not meet current design standards (Gersten et al., 2005; R. H. Horner et al., 2005), the benefits of a computer to provide instruction were evident. Panyan outlined the benefits of TAI as consistency of instruction, an increased ability to promote generalization, the reinforcing capabilities of computers, and the ability to provide preferred sensory stimuli.

Following Panyan's (1984) review, researchers began to compare the effectiveness of teacher-delivered versus technology-delivered instruction. Pleni and Romancyzk (1985) conducted the first study to compare human instruction and TAI for individuals with developmental disabilities was conducted with 17 children, six of whom had ASD. Although there was no overall difference in learning performance between conditions, participants exhibited lower rates of disruptive behaviors and higher rates of compliance with instruction in the technology condition.

As technology has rapidly changed in format and capabilities, the field of special education has followed with empirical evaluations on how this may enhance the lives of individuals with disabilities. Kagahora et al. (2013) conducted a systematic review of the literature on the use of iPods, iPads, and other touch-based devices from 2008 to 2012, with a total of 15 studies meeting their inclusion criteria of intervention using an iPod, iPod Touch, iPod Nano, iPhone, or iPad to include at least one person with a developmental disability. The majority of the studies used touch-based devices to teach communication skills (n=8) and only one focused on academics (Kagohara et al., 2012). In the Kagohara et al. (2012) study, the goal of the intervention was to teach students with ASD to check the spelling of words by following a task analysis and observing a video model on how to use the spell-check function on common word processor programs. The video modeling intervention was delivered via an iPad and was effective for all participants. These were important findings, as they demonstrated both the utility of an iPad to deliver TAI, and the effectiveness of TAI to teach chained academic tasks through the use of a task analysis.

In a literature review on similar technology, Stephenson and Limbrick (2013) focused on the purpose each device served, the success of the device, how individuals with disabilities were taught to use the device, and the level of evidence produced by the studies. They found that despite the popularity and use of touch-based technology, there was limited research on educational software applications. The more successful and rigorous design studies included behavioral instructional strategies along with the touch-based device.

Despite the demonstrated effectiveness of TAI, the use of technology to teach students with ASD has raised concerns in regard to social isolation and limited opportunities for social skills practice when the requirement for interaction with humans is decreased (Bernard-Opitz et al., 1990; Ramdoss et al., 2011). An additional concern is the perseveration on technology and challenging behavior maintained by computer access (Powell, 1996). Despite these concerns, there are numerous benefits of TAI to teach students with ASD, including the ability to adapt to a learner's needs, ability to resemble stimuli and consequences of natural settings, and ability to provide multiple exemplars (Ramdoss et al., 2011).

According to Ramdoss et al. (2011) the use of technology only influences the delivery of instruction; like any intervention, the interventionist, reliability, validity, and learner characteristics influence effectiveness. In their literature review of CAI to teach communication skills to individuals with ASD, Ramdoss et al. found mediums of delivery varied, and commercial programs used in research often become unavailable, as was the case for three out of the four commercial software programs used in studies included in the review. On the other hand, customized programs, such as those created

with PowerPoint®, have varying design features, which make it difficult to pinpoint exact critical instructional features that affect learning.

### Technology-Aided Instruction to Teach Academics

As indicated previously, TAI has been identified as an evidence-based practice for individuals with ASD by the National Professional Development Center; however, the supporting literature varies by skill, content, and format of instruction (Wong et al., 2014). Literature reviews of TAI to specifically teach academics to students with ASD have revealed an increasing trend in publications with a focus on academic applications of technology; however, the range of content area applications of TAI continues to be limited (Knight, McKissick, et al., 2013; Pennington, 2010; Root, Stephenson, et al., 2015) and the quality of the studies often does not meet field standards for consideration as an evidence-based practice (Gersten et al., 2005; R. H. Horner et al., 2005). In the first literature review on the use of CAI to teach academics to students with ASD, Pennington (2010) found 15 articles that (a) were published in a peer-reviewed journal between the years of 1998 and 2008, (b) were based on experimental research, (c) manipulated an independent variable involving the use of CAI, and (d) collected data related to an academic skill. All of the studies that met inclusion criteria addressed literacy skills. Pennington found a total of 15 articles that met inclusion criteria, and concluded that CAI was effective for a limited number of skills, namely literacy skills, but that many of the designs lacked experimental control. Knight, McKissick, and Saunders (2013) had similar findings with 25 out of 25 included studies teaching literacy skills. Inclusion criteria for Knight, McKissick, et al. (2013) included the following: (a) published in a peer reviewed journal between 1973 and 2012; (b) teaching an academic skill; and (c)

inclusion of technology as the intervention or a part of the intervention package, including video modeling. Knight et al., found a total of 25 articles that met inclusion criteria, however the majority of the studies met few quality indicators for single-subject or group design. Similarly, the majority of the studies included in their review targeted literacy skills.

An updated review by Root, Stevenson, et al. (2015) included 29 studies that (a) were published in a peer reviewed journal between 1995 and May 2015, (b) included at least one participant with ASD, (c) used a recognized experimental or quasiexperimental design, (c) had a dependent variable that measured an academic skill, and (d) CAI served as a key component of the independent variable, excluding video modeling. Out of the 29 included studies, 23 taught literacy skills and only two taught mathematics (Bouck et al., 2014; Whalen et al., 2011). The evidence from these three comprehensive reviews indicates the need for research to explore the utility of TAI to teach mathematics skills to students with ASD.

In an analysis of the types of skills addressed through TAI interventions, Browder, Saunders, and Root (in press) reported many studies focused only on discrete skills, such as picture, object, and symbol identification (Bosseler & Massaro, 2003; Chen & Bernard-Opitz, 1993; Whalen et al., 2011). Although these studies were successful in improving the targeted academic skill, Browder et al. point out that little empirical evidence is available to support the use of TAI to teach complex skills to students with ASD and who have more significant support needs, especially in the area of mathematics.

Technology-aided instruction to teach mathematics to students with ASD and other developmental disabilities. The materials used in mathematics may limit access for students with ASD/ID. Technology can increase opportunities in mathematics for students with ASD/ID by providing alternate means of access. Embedded supports such as text to speech, highlighting, symbol writers, magnification, and auditory enhancements can increase independence. Projection and large-scale touch based technology, such as SMARTboards or Promethean boards can provide students an opportunity to interact with materials and benefit from observational learning (Mechling, Gast, & Krupa, 2007). Current applications of TAI in the literature have capitalized on the ability of embedded supports, such as visual cues, to facilitate skill acquisition.

*Purchasing skills.* With the advancement in technology innovation, mobile technology is now able to take TAI into the community and use visual prompts as a type of visual cue. One area of mathematics that has been explored in depth within TAI literature is purchasing skills. Technology has allowed for increased independence and acceptability with incoming innovations. Visual cues have been used to teach individuals with moderate intellectual disability to independently complete chained purchasing tasks. Researchers have studied the effects of various presentations of these cues.

The visual cue delivered through TAI can guide participants through a task analysis for completing a chained behavior. For example, Alberto, Cihak, and Gama (2005) evaluated the use of static picture prompts and video modeling during simulated instruction to teach eight middle school students with Mod ID to use a debit card to withdraw cash from an ATM and purchase two items. Both the static picture prompts and the video modeling were effective in teaching purchasing skills. In addition, Scott,



Collins, Knight, and Kleinert (2013) taught adults with moderate ID to use an ATM using an iPod. A combination of instructional technology was used to create the intervention, including Microsoft® Photo Story 3 to create the video models and record auditory prompts, which were then uploaded to a podcast that individuals used to listen to and watch each step of the task analysis. The combination of video prompting and audio prompting presented on an iPod was effective in teaching the students to use an ATM to withdraw \$20.00 in a community setting.

Visual cues have been shown to be effective in teaching chained mathematics skills related to purchasing. Visual cues displayed using touch-based technology allow for individuals to control the pace and intensity of prompts. Research is warranted to evaluate the effectiveness of visual cues on increased independence within grade-aligned mathematics tasks.

*Grade aligned mathematics.* As mentioned previously, the overall research base on teaching mathematics using TAI to individuals with MSD, including ASD/ID, is sparse. Studies that specifically taught grade-aligned skills are even more limited. Two studies have been published that meet quality indicators for high quality design that taught discrete grade-aligned mathematics skills to students with ASD using TAI (Bouck et al., 2014; Whalen et al., 2011). Whalen et al. (2011) met group design standards (Gersten et al., 2005) in their evaluation of the commercially available software program TeachTown Basics in a between subjects randomized study with 47 young children ages 3 to 6 who attended a preschool program for students with ASD. TeachTown Basics addresses four learning domains, including receptive language, social understanding, life skills, and academic/cognitive skills. In the academic/cognitive skills domain, grade-

aligned mathematics skills such as shapes, comparing quantities, numeral identification, addition, subtraction, number lines, and fractions were addressed. The software program provided massed trials of discrete skills to mastery. Explicit instruction, stimulus fading, and reinforcement of correct responses were incorporated within the software program. Whalen et al. found children in the TeachTown Basics group outperformed the control group across all measures, including academic/cognitive skills that included discrete mathematical tasks.

Researchers have also evaluated the use of TAI to teach discrete mathematical computation skills to elementary aged students with ASD using single-case methodology. Bouck et al. (2014) conducted an alternating treatments design study and investigated the different effects of concrete and virtual manipulatives on acquisition of subtraction skills for three children with ASD. The National Virtual Library of Manipulatives (<http://nlvm.usu.edu>) was used to provide digital base-ten blocks as the virtual manipulatives. Although both the concrete and virtual base ten blocks were effective, participants performed an increased number of steps independently in the virtual condition. The studies by Bouck et al. (2014) and Whalen et al. (2010) suggest that TAI may be effective for teaching discrete computational skills; however, the application of these skills was not addressed within a real-world context.

It is critical that students with ASD are taught how to apply computational skills through problem solving. Real-world problem solving often includes chained tasks. Burton, Anderson, Prater, and Dyches (2013) applied TAI to grade-aligned real-world problem solving. Video self-models displayed on an iPad were used to teach four middle school students with ASD to solve a story problem. Students were given five story

problems with specific price tags, a cash register containing simulated money, and the iPad with a video of each step of the task with the directions to calculate a total cost and give change. The intervention was student-directed; each student controlled the video (rewinding) to re-watch the steps as necessary without teacher facilitation. All four students were able to use the video self-modeling intervention to solve the story problems and generalize skills to novel problems. TAI was used to provide access to the story problem as well as to provide prompting for solving the problem. One limitation of Burton et al. is the repetition of story problems throughout intervention; the students were presented with the same story problems and the same prices several times in intervention and baseline.

TAI has been used to deliver instruction, prompting, and provide alternate access to materials in grade-aligned math problem solving activities with novel problems. Two studies have used TAI and SBI to teach problem solving to students with ASD/ID (Root, Browder, et al., 2015; Saunders, 2014). The findings from these investigations show that TAI can be effective in presenting additional supports for problem solving. While research has specifically evaluated student use of virtual graphic organizers and manipulatives on word problem solving (Root, Browder, et al., 2015), it has not looked at the effect of embedded prompts in a task analysis on word problem solving or self-initiated prompting to solve a word problem for students with ASD/ID.

## Summary

Technology has important instructional implications for individuals with disabilities. From its infancy in the form of teaching machines (Pressey, 1924; Skinner, 1954) to the wide range of forms and platforms that are currently being used (Stephenson

& Limbrick, 2013; Wong et al., 2014), TAI has been successfully used to teach academics to students with ASD and moderate to severe ID (Knight et al., 2013; Root, Stevenson, et al., 2015; Pennington, 2010). There are sufficient high quality studies to demonstrate the effectiveness of TAI to teach mathematics to students with developmental disability and establish TAI as an evidence-based practice to teach grade-aligned mathematics (Spooner et al., 2016). Both conceptual and procedural knowledge has been effectively taught to students with ASD and significant support needs using TAI when combined with systematic instruction. There is a need to focus on more complex problem solving tasks within grade-aligned mathematics to students with ASD and ID.

#### Strategy Instruction for Mathematical Problem Solving

Mathematical problem solving requires a variety of skills with which students with ASD/ID typically struggle, including metacognition, executive functioning, semantic language, and working memory. To attend to the challenges faced by students with ASD in solving math problems, researchers in the field of special education have refined strategy instruction and instructional supports which have been shown to be effective for students with mild disabilities.

#### Foundation of Learning Strategies

Literature dating back more than 4 decades (Fennema, 1972; Peterson, Fennema, & Carpenter, 1989) has emphasized shaping how students think about mathematics in order to design instruction and teach conceptual problem solving. Learning strategies are techniques, principles, or rules that enable a student to learn, solve problems, and to complete tasks independently (Deshler & Schumaker, 1984). The purpose of strategy

instruction is to increase independence in problem solving by giving students the skills to think and act systematically.

### Mathematics Strategy Instruction for Students with ASD/ID

Mathematics learning strategies have been developed to address the multiple steps and processes students must navigate to successfully solve problems. Numerous math strategies have been evaluated for children with learning disabilities (LD) and mathematics difficulties, including visual, generative, and verbal strategies.

One visual strategy that has been used with students with LD and ASD is the concrete-representational-abstract sequence and strategic instruction model (CRA-SIM). This strategy has been used to teach computation strategies by first using manipulatives, and then moving to drawings and pictures once mastery with concrete objects is met. Finally, students learn a strategy that only uses numbers. The CRA-SIM strategy has been used effectively to teach addition, subtraction, and multiplication facts to students with ASD (Flores, Hinton, Strozier, & Terry, 2014; Strozier, Hinton, Flores, & Terry, 2015).

Generative strategies teach students to paraphrase key components of text within a word problem, relying on the research connecting reading comprehension to problem solving (Cornoldi, Drusi, Tencati, Giofre, & Mirandola, 2012; Swanson, Cooney, & Brock, 1993; Swanson, Moran, Lussier, & Fung, 2014). Heuristics are a method or strategy that exemplify a generic approach for solving a problem (Gersten et al., 2009). Heuristics are used to sort and organize information from a problem, but they not necessarily problem-specific. For example, the RUNS heuristic, which stands for “Read

the problem, Underline key information, Number sentence, Solve” can be used for a variety of mathematical operations and standards (Rockwell et al., 2011).

Cognitive strategy instruction (CSI) combines the use of heuristics, verbal, visual, generative strategies. Based on the information processing theory (Sternberg, 1985), CSI addresses the executive functioning skills required by word problem solving. With a emphasis on both cognitive processes and metacognitive skills, the goal of CSI is for students to choose and apply appropriate methods self-monitoring their performance (Montague, 2008). Evidence-based practices of explicit instruction is combined with modeling and verbal rehearsal to help students memorize and internalize a cognitive routine to improve performance (Krawec et al., 2013). The model outlined by Montague (1992) involves seven steps including (a) read the problem, (b) paraphrase and tell the problem in your own words, (c) visualize or draw a diagram, (d) come up with a plan, (e) predict the answer, (f) solve, and (g) check. In addition to the use of generative strategies and heuristics, CSI incorporates visual strategies (drawing a diagram). However, the diagrams are not always prescribed or based on the overall problem type, but rather information presented in the specific problem. Consider the following problem “Jack had four apples. Then 3 apples fell out of his bucket. How many does he have left?” The students would be encouraged to draw a diagram representing exactly what happened in the problem (perhaps four circles for apples, and then crossing out three for being dropped). This visual strategy assisted the student to arrive at the correct answer, but would not necessarily generalize to all problems of that type (change), or even bring awareness to the student of the problem type itself.

Grade-aligned word problems have been taught to students with ASD using CSI. An investigation of the effects of the Solve it! Problem Solving Routine, a commercial CSI program, was successful for adolescent males with high functioning autism who did not have an intellectual disability (Whiby, 2012). Participants were taught to solve multiple step word problems across problem types. In a multiple baseline across participants design, a functional relation was established between the Solve it! Problem solving routine and percent of correctly solved math problems.

Functional word problem solving has also been taught using CSI to individuals with ASD/ID and other developmental disabilities (Hua et al., 2012). A three-step strategy to calculate the tip and total bill was taught to individuals attending a post secondary program at a university for students with intellectual disability and autism and was evaluated using a pretest posttest nonequivalent groups design. The participants in the experimental group were taught a heuristic in the form of a three step mnemonic, namely TIP (Take a look at the bill and enter it into the calculator, Identify the tip by multiplying the total by 15%, and Plus the total and find out how much to pay). Instructional procedures were explicit and followed those recommended by Montague (2003). Results of an ANOVA found the difference between the experimental and control group on both target and generalized items to be statistically significant ( $p < .001$ ). Although there is evidence of effectiveness for using CSI to teach math problem solving to students with ASD (Hua et al., 2012; Whitby, 2012;), the procedures may be too abstract for those with ASD/ID due to low levels of early literacy and early numeracy. This population may need explicit instruction that combines conceptual and procedural knowledge with the use of problem-specific strategies, which can be provided by SBI.

## Schema-based Instruction

Another method of instruction that combines multiple strategies, including generative strategies, heuristics and visual representations is schema-based instruction (SBI). SBI was developed from the schema theory of cognitive psychology (Jitendra & Starr, 2011), and is one type of strategy instruction. A schema is an outline or framework for solving a problem (Marshall, 1995). Schemas can be represented through pictures, diagrams, number sentences, or equations (Powell, 2011). According to Jitendra et al. (2013), the primary focus of SBI is to teach students the underlying mathematical structure of mathematical word problems. Schemas assist students in the problem integration and solution planning phases of problem solving by choosing a schema that matches the problem type and organizing information from the problem onto the schema. To address the translational phase of problem solving, SBI promotes contextual understanding of the word problems through understanding of provided non-mathematical information (Jitendera et al., 2013). In addition, SBI reduces the working memory load of students by concretely grouping informational units into the schema.

There is a general four-step process students are explicitly taught during SBI, which includes (a) reading a word problem, (b) selecting a schematic diagram that fits the word problem, (c) transferring information from schema to mathematical equation, and (d) solving the problem (Powell, 2011). In a seminal study, Jitendra and Hoff (1996) taught three elementary students with LD to solve group (i.e., involving combining two parts of a whole), change (i.e., involving the increase or decrease of a quantity), and compare (i.e., involving the difference between two quantities) word problems. There was a functional relation between SBI and word problem solving of the three problem



types for all participants. Jitendra et al. (2013) demonstrated that SBI is also effective for teaching more complex problems with irrelevant information to students with LD or math difficulties. The SBI package taught one- and two-step word problems with change, group, and compare problem types. The problems were presented in the form of text, graphs, tables, and pictographs that included irrelevant information. In a pretest-intervention-posttest-retention test design, students who received SBI significantly outperformed students who did not on word problem solving posttest.

Research on schema-based instruction with students with ASD. Given the nature of SBI (explicit instruction and incorporation of visual strategies), its efficacy with students with ASD has been recently evaluated. Rockwell et al. (2011) conducted the first investigation of SBI to teach word problem solving to an elementary student with ASD. The participant had prerequisite skills of (a) ability to decode at a second grade level, and perform addition and subtraction computations with digit numbers; (b) no diagnosis of an intellectual disability; (c) ability to communicate verbally; and (d) could attend to one-to-one group instructional sessions lasting 30 min. The SBI included explicit instruction with modeled think-alouds to learn the four-step mnemonic RUN. Students were awarded up to 3 points for each problem solved based on choosing the correct schematic diagram, correctly writing the number sentence, and correctly computing the answer. The participant had difficulty discriminating between the problem types and was given discrimination training. To assist with discriminating between problem types, an essential component of SBI, a sorting activity began each session where she was instructed to identify each problem as “belonging” or “not belonging” to the problem type, and explain her reasoning by identifying the presence or absence of the critical features for each

problem type. The student was able to correctly solve group, change, and compare word problems with finals in the unknown location. In addition, she was able to generalize her problem solving skills to problems with unknowns in the initial or medial location.

In a replication of her first study, Rockwell (2012) taught two students, one of whom was a 12-year-old male with ASD who did not have ID, to solve group, change, and compare problems using similar SBI procedures as Rockwell et al. (2011), including the use of the mnemonic “RUNS”. Both participants were able to decode at a second grade level and perform addition and subtraction computations with two digit numbers without regrouping and were able to communicate verbally. Findings showed that both students solved all three problem types and generalized to unknowns in the initial and medial position.

Research on schema-based instruction with students with ASD/ID. Based on the work of Rockwell et al. (2011) and Rockwell (2012), Saunders (2014) conducted the first investigation of the use of modified SBI to teach math word problem solving to students with both ASD and ID with IQs ranging from 40-55. Three students participated in the study and had prerequisite skills of (a) one-to-one correspondence, (b) identification of numbers one to 10, and (c) the ability to make sets up to 10. One of the students was minimally verbal and English was not spoken in his home. None of the participants were able to solve word problems with single digit numbers prior to the intervention and one of the participants had never been exposed to subtraction. All of the participants received speech therapy for expressive and receptive communication deficits related to their primary diagnosis of ASD. In addition, the reading levels of the participants varied, but

were below grade level and not all of the participants were able to decode the word problems independently.

Using a similar procedures as that by Rockwell et al. (2011) and Rockwell (2012) in combination with the direct instruction concept formation procedure described by Celik and Vuran (2014), participants were explicitly taught to sort word problems by problem-type (i.e., group vs. change) and provide a rationale by stating the “rule” for each problem. Students viewed SBI videos of how to complete each step of a 12-step task analysis, and then were given an opportunity to try each step. Students monitored their progress using a self-instruction checklist with the embedded task analysis steps supported by picture supports and text-to-speech capability. Within training sessions, the students had the opportunity to self-initiate verbal and specific verbal prompts. Modeling as error correction was also provided using the computer-based video instruction. Students used virtual manipulatives on graphic organizers that represented the problem type. All students acquired mathematical word problem solving skills and were able to differentiate between the two problem types.

Most recently, students with ASD/ID have also been taught to solve compare word problems using modified SBI. Root, Browder et al. (2015) evaluated the effects of SBI to teach math word problem solving and compared the effectiveness of concrete and virtual manipulatives, building upon the work of both Saunders (2014) and Bouck et al. (2014). The three elementary students who participated in the study had ASD and Mod ID and had the same prerequisite skills as the participants in the Saunders (2014) study. Using scripted modified SBI lessons, the instructor provided strategy instruction to the students in a one-on-one setting following the steps on the student self-instruction sheet.

The instructor modeled the steps for a 3-day training period, during which no data was taken because the strategy was modeled with active student participation. The participant was taught to follow nine steps of the student self-instruction sheet and check off each step as it was completed, incidentally teaching self-monitoring.

Following the training period, the instructor provided least intrusive prompting (i.e., verbal, specific verbal, and a model prompt) as needed to assist the students in solving the word problems. Generally, the verbal prompt directed the participant to look to his or her self-instruction sheet to see what was next, such as “This step says circle the whats.” The specific verbal prompt directed the participant to the key actions or stimulus of that step (e.g., “This step says circle the whats. Remember, the whats have pictures over them.”). Finally, in the model prompt the instructor would do a full model of the correct action required for the step paired with an explicit think aloud procedure, followed by requesting the participant to repeat the action (e.g., “I am going to circle the whats, the whats have pictures of them. Here is the first what; I am going to circle it. Can you find the second what?”). The instructor would wait for the student to find the what, or do a model prompt to find it, and end with requiring the student to complete the behavior.

Depending on the condition, participants were either provided with a laminated graphic organizer and plastic round manipulatives (i.e., concrete manipulatives) or an iPad 3 with virtual blue circles they dragged onto an identical graphic organizer displayed using the Smartnotebook© application (i.e., virtual manipulatives). The instructor taught the students to use the graphic organizer and manipulatives to represent the number sentence by (a) making a set representing the bigger number in the top ten frame, (b)

making a set representing the smaller number in the bottom ten frame, and (c) finally pushing the counters from the top ten frame that did not have a match in the bottom ten frame into the lower circle on the graphic organizer that was labeled “difference.”

Following mastery, five sessions were conducted where the participants were given a choice between the two conditions. All three participants preferred the virtual condition when they were given a choice between the two and maintained treatment effects. The accelerated rates of independence and preference in the virtual conditions adds to evidence found in past literature reviews and empirical studies related to academic mathematics instruction for students with ASD using TAI (Bouck et al., 2014; Knight, McKissick et al., 2013; Root, Stevenson, et al., 2015; Saunders, 2014). Results of the multiple probe across participants with an embedded alternating treatments design showed a functional relation between modified SBI and word problem solving for the participants.

Students in both the Saunders (2014) and Root, Browder, et al. (2015) studies learned to solve one-step additive word problems independently, despite language deficits and low levels of literacy and numeracy proficiency. This provides evidence that students with ASD/ID benefit from modified SBI that incorporates evidence-based practices for teaching mathematics to students with moderate and severe disabilities, including TAI, systematic instruction, explicit instruction, and graphic organizers or heuristics (Browder et al., 2008; Spooner et al., 2016; Wong et al., 2014). Students in both studies were able to learn to solve one-step additive word problems independently.

One-step algebraic equations have been taught to students with Mod ID. Jimenez et al. (2008) used the number line strategy to teach students to “count up” from the first

known number to the final amount in order to find the unknown variable. Manipulatives on a number line from zero to nine, in conjunction with systematic instruction, and a task analysis were used to teach the students to solve the equation. In a multiple probe across participants design, a functional relation was found between the treatment package and solving the algebraic equation. However, the equation was not presented within the context of a word problem, resulting in a development of only procedural knowledge.

In a series of related studies, Browder, Jimenez, et al. (2012) and Browder, Trela, et al. (2012) taught high school students with Mod ID to solve one-step algebra word problems when given a real-world mathematics story problem. Students were once again provided a number line and a structured equation. Results were positive in both studies, with students acquiring mathematics skills necessary to solve the algebraic equations. One mathematics skill not addressed by these studies was distinguishing between problem types, or knowing what information is known and unknown. For example, all of the problems had missing information in the medial position. Students never developed the ability to discriminate problem type, and therefore demonstrate conceptual understanding.

### Summary

As the curricular focus for students with severe disabilities has progressed in the past 10 years from teaching functional skills to grade-aligned academics, the lack of a research base to support practice has become evident (Browder et al., 2008; Spooner et al., 2016). Evidence-based practices to teach mathematics are continuing to emerge, many of which are the same as those for students with LD (explicit instruction, heuristics, visuals, and feedback; Browder et al., 2008; Gersten et al., 2009; Spooner et al., 2016).

Priming problem structure is also an evidence-based practice for solving mathematics word problems for students with LD (Jitendra et al., 2015). Given the emerging research on modified SBI for students with ASD and ASD/ID (Rockwell, 2012; Rockwell et al., 2011; Root, Browder, et al., 2015; Saunders, 2014), this may be another strategy that is effective across disabilities.

The intent of SBI is to teach the underlying problem structure in order to facilitate conceptual understanding, as well as to provide visual representations of the problem to enhance procedural knowledge for solving the problem. Modified SBI, involving a task analysis to replace a traditional heuristic, enhanced graphic organizers, the use of manipulatives, TAI to provide alternate access to materials, and systematic and explicit instruction, has had positive effects on solving additive word problem solving for students with ASD/ID (Root, Browder, et al., 2015; Saunders, 2014). However, investigations into modified SBI for students with ID and ASD/ID have had limitations that warrant further exploration. Effects on solving additive word problems that require algebraic reasoning have not been explored. For example, all of the word problems in the Root, Browder, et al. and Saunders studies required students to solve for the missing unknown in the final position. Only one study (Neef et al., 2003) has taught students with Mod ID to solve for missing information in all three positions; however, only change type problems were used and the participants knew all basic math facts and were able to read the problem. The one study that taught students with Mod ID to solve an algebraic equation did not provide instruction on use of the strategy within the context of a real-world problem (Jimenez et al., 2008). There is a need to investigate effective and

efficient methods for teaching algebraic problem solving with the unknown in all positions to students with ASD/ID.

### Summary

To address all of the barriers students with ASD/ID face in mathematics problem solving, interventions addressing language, numeracy, and executive functioning may be needed. This could be facilitated through vocabulary instruction, TAI, pictorial self-instruction and self-prompting, and strategy instruction, resulting in successful algebraic problem solving by students with ASD/ID.



## CHAPTER 3: METHODOLOGY

In this study, a single-case multiple probe across participants design was used to evaluate the effects of modified SBI on the word problem solving of students with ASD who also have a moderate ID (ASD/ID). This study was conducted with middle school students with ASD/ID who participated in the AA-AAS. The sections to follow describe the selection criteria for participants and anticipated setting of intervention. The specific methodology of the study, including the research design, measurement of dependent variables, procedures for experimental conditions, methods of data analysis, and potential threats to validity are also explained.

### Participants

Four middle school students classified with ASD/ID were recruited to participate in this study. A purposeful sampling procedure was used to identify participants. Teacher nominations were solicited based on students' mathematical abilities. Participants were selected based on the following inclusion criteria: (a) educational or medical diagnosis of autism, (b) IQ at least three standard deviations below the mean, allowing for standard measurement of error, (c) participation in alternate assessment aligned with alternate achievement standards (AA-AAS), (d) ability to write using a pencil or adaptive writing utensil, and (e) satisfactory performance on mathematical prescreening measure. After parental consent and student assent were obtained, participants were administered a prescreening measure. The prescreening measure assessed the participants' ability to

(a) receptively and expressively identify numerals up to 10, (b) make sets of numbers 10 to 10; (c) count with one-to-one correspondence, (d) copy one and two word phrases, and (e) solve one-step word problems (see Appendix A). The researcher administered this prescreening measure in a one-on-one format. The researcher presented materials to the participant, such as a graphic organizer with a single large oval and manipulatives (e.g., algebra tiles). The researcher modeled how to perform the behavior (e.g., “My turn. I am going to make a set of four. 1, 2, 3, 4”) and then asked the participant to perform a similar behavior (e.g., “Your turn. Make a set of 3”). All skills were modeled first to ensure errors were due to skill deficits, rather than receptive language errors. The prescreening took approximately 5 min to administer per participant. A participant achieved satisfactory performance on the prescreening measure to continue with participation if he or she completed items (a) to (d) with 100% accuracy and item (e) with no more than 25% accuracy.

Anna. Anna was a 14 year-old Caucasian female in the sixth grade with ASD and moderate ID. According to her most recent evaluation data, Anna had a cognitive scale of 53 on the Developmental Ability Scales, 2<sup>nd</sup> edition (DAS-2; Elliot, 2007). Her adaptive behavior composite score was 68 (Vineland Adaptive Behavior Scales, 2<sup>nd</sup> edition; Sparrow, Cicchetti, & Balla, 2005). She was diagnosed with ASD from TEACCH Charlotte at the age of 2.5 years. During a reevaluation in 2012, Anna received a score of 74 on the Gilliam Autism Rating Scale, 2<sup>nd</sup> edition (GARS-2; Gilliam, 2006), which falls within the possible probability of autism range.

Anna was given the Brigance Comprehensive Inventory of Basic Skills, 2<sup>nd</sup> edition (CIBS II; Brigance, 2010) and although no math subtest score was given, the narrative psychological report indicates she was able to solve 8/20 two digit addition and subtraction problems, identify all of the coins by name and give their worth, and answer two out of nine relationship questions using coins and dollar bills. No formal measures of Anna's reading comprehension abilities were provided. On the listening vocabulary comprehension portion of the Brigance, she was able to answer first grade comprehension questions correctly. However, on the listening comprehension subtest she was unable to answer any listening comprehension questions. Anna received a level four (proficient) in both language arts and math on her most recent alternate assessment.

Her most recent IEP reflected that she knew some of her multiplication facts and was able to add fractions with like denominators independently. When solving word problems, Anna could identify the numbers from the problem but has difficulty identifying the operation and is confused by the information presented in the problem. Her goals for the current school year were to solve multi-digit multiplication and subtraction problems with regrouping, add and subtract fractions with unlike denominators, and solve one-step word problems by correctly identifying the operation and ignoring excess information from the problem.

Anna received all of her academic instruction from a special education teacher in a self-contained classroom. She went to lunch, music, and physical education classes with her non-disabled peers. Anna did not receive any related services.

Anna's social and language skills were one of her strengths. She was able to communicate vocally in complete sentences and responded to greetings appropriately.

Anna did have a few preferred topics of conversation, including birthdays and upcoming holidays (i.e., Halloween and Thanksgiving). Although Anna was flexible and went with the interventionist daily, she did not like to leave her classroom when she was in the middle of a task. Also, when the bell rang and she knew she was supposed to go to the next class, she often got agitated and rushed through the worksheets. Anna did not want to engage in iPad games when she finished the worksheets; rather she preferred to just go to class. Toward the end of the intervention, she asked for a specific candy she liked as a reward for finishing the intervention (reaching mastery).

Amanda. Amanda was a 12-year-old Caucasian female in the sixth grade with ASD and moderate ID. Amanda recently transferred from another school out-of-state. According to her evaluation data, Amanda had a full scale IQ of 58 on the Wechsler Intelligence Scale for children – 4<sup>th</sup> edition (WISC-4; Wechsler, 2003). She received a standard score of 66 on the Vineland-II. She had a diagnosis of Pervasive Developmental Disorder-Not Otherwise Specified (PDD-NOS) from a psychiatrist in 2007. Formal educational testing was conducted throughout the course of the study by the school. Amanda was given the Woodcock-Johnson IV (WJ-IV; Schrank, Mather, & McGrew, 2014). Results indicated she scored very low on all subtests (<.01 %ile), including mathematics calculation skills cluster, mathematics problem solving cluster, basic reading skills cluster, and reading comprehension cluster. Amanda's transfer information did not include her alternate assessment scores.

The school developed a comparable IEP while a reevaluation was conducted, as is standard procedure for out-of-state transfers. This comparable IEP indicated Amanda would receive all of her academic instruction from special education teachers in the self-

contained setting. In addition, she received adapted physical education twice per week and speech/language therapy once per week.

Amanda was very compliant and seemed to enjoy working with the interventionist daily. She always willingly left her classroom or task, even if it was a leisure activity. Amanda spoke in complete sentences, although she had a speech impairment that caused her to mispronounce some words with a -th sound (e.g., both, through). Amanda did not appear to engage in any stereotypy or make apparent any preferred activities or topics of conversation. She easily conversed with the interventionist and displayed appropriate social skills. Amanda did not want to play games on the iPad after her math worksheet; rather she preferred to just go back to class.

Stephanie. Stephanie was a 14-year-old Caucasian female in the seventh grade with ASD and moderate ID. According to her evaluation data, Stephanie had a full scale IQ of 50 on the DAS-2. Her adaptive behavior was 70 according to the Vineland-2. She had a diagnosis of ASD according to the GARS-2. Based on the Woodcock-Johnson III (WJ-III; Woodcock, McGrew, & Mather, 2007), Stephanie's standard scores in basic reading (28), math calculation (1) and math reasoning (29) were significantly below the average range for her age. At the end of the previous school year, she scored a level two (not proficient) on her alternate assessment in mathematics but a level four (proficient) on her alternate assessment in language arts.

Based on her most recent IEP, Stephanie was able to solve addition and subtraction computation problems with sums or differences up to ten when using manipulatives. Her current IEP goals were related to adding and subtracting fractions with like denominators, comparing part-to-part relationships, writing ratios to represent

relationships between two quantities, and solving one-step word problems by writing an equation and using a graphic organizer. Stephanie received specially designed instruction in all academic areas from a special education teacher in a self-contained setting. She ate lunch and attended physical education class with her non-disabled peers.

Stephanie was able to speak in sentences composed of two to four words, although these vocalizations were often very quiet and difficult to understand. She did engage in appropriate social interactions, including conversing about upcoming holidays or weekend events. When Stephanie saw the interventionist in the hall in other areas of the building, she smiled and said hello. Stephanie walked with an altered gait, often toe-walking. She also rocked and fidgeted in her seat during instruction. She did not display any other signs of stereotypy and did make preferred activities or topics apparent. Stephanie did not want to engage in an iPad game at the end of her worksheets, instead preferring to go back to class.

### Setting

This study took place in a public middle school in an urban school district in the southeast United States. The school served 1440 students in grades six to eight, which was more than double the state average. Approximately 61.7% of the students enrolled were White, 26% were African American, 7.1% were Hispanic, 2.1% were Asian and 33.7% were economically disadvantaged, meaning they qualified for free/reduced lunch.

Each participant received all of her academic instruction in core content areas from a special education teacher. Intervention sessions were conducted one-on-one with each participant daily in an alcove at the end of a hallway near the participant's classrooms. The alcove had two long tables and several chairs and was free of auditory

and visual distractions. A doctoral candidate (author) in special education implemented all sessions. The student was a board certified behavior analyst (BCBA) with over 9 years of experience working with school-age students with moderate and severe developmental disabilities, including ASD/ID, and held a valid teaching license in both special education and general education middle grades mathematics.

### Research Design

A multiple probe across participants design was used (Gast & Ledford, 2014; R. D. Horner & Baer, 1978; Kratochwill et al., 2010). In a multiple probe design, a variation of multiple baseline, multiple AB data series are compared while the introduction of the intervention is staggered across time to allow for valid causal inferences. In this investigation, the intervention was introduced to three participants. Each participant's data had two phases: baseline (A) and intervention (B). The participants entered baseline simultaneously. Phase change decisions were based on student performance data on the primary dependent variable, namely mathematical WORD PROBLEM SOLVING. A minimum of five baseline data points and one generalization baseline data point were obtained that demonstrated a stable pattern of responding before the first participant entered intervention. Once the first participant showed a stable accelerating trend in mathematical WORD PROBLEM SOLVING, the second participant with stable baseline data pattern entered intervention. This systematic introduction of participants to the intervention continued until all participants were introduced to the intervention.

Participants in baseline were probed a minimum of every eight sessions to facilitate verification and prediction, necessary components of visual analysis of single-case designs (Cooper et al., 2007; Kratochwill et al., 2010). A generalization probe was

conducted every four sessions during intervention for a minimum of two total intervention generalization data points. Criteria for mastery was performing nine of the 10 steps correctly for three out of four problems for two consecutive sessions. In order for a problem to be considered “solved correctly”, steps 3 (label), 5 (fill in equation), 7 (+ or -), and 10 (write answer) had to be completed independently correct. Successful completion of these four steps resulted in a correct equation and answer sentence. These were considered critical steps (Test & Spooner, 1996; Weng & Bouck, 2014).

#### Dependent Variables

There were seven dependent variables measured throughout the course of this study. They include MATHEMATICS VOCABULARY, WORD PROBLEM SOLVING, TOTAL PROBLEMS SOLVED, GENERALIZATION OF WORD PROBLEM SOLVING, SELF-INITIATED PROMPTING, GLOBAL MATHEMATICS ABILITY, and PERCEPTION OF WORD PROBLEM SOLVING.

Mathematics vocabulary. MATH VOCABULARY was the first dependent variable and was measured by the number of correct pairings of math vocabulary words to definitions (e.g., equal/ same as). Responses were scored as independent correct if the participant touches or says the correct answer within 4 s without a prompt from the interventionist. Five vocabulary words were taught, including add, subtract, equal, equation, and label. Each of the five vocabulary words were presented twice in random order for a total of 10 points available in each session.

Word problem solving. The second dependent variable, mathematics WORD PROBLEM SOLVING, was the primary dependent variable and was measured by the total number of points a participant received by independently performing the 10 steps of



the task analysis. The steps of the task analysis are: (1) read the problem, (2) circle the groups, (3) label equation, (4) circle the numbers, (5) fill in equation, (6) use my rule, (7) + or -, (8) make sets, (9) solve, and (10) write answer. Each step is worth one point for a total of 10 possible points per problem and a total of 40 points possible per session. During each data collection session, a participant solved four problems; two of the problems had the unknown in the final position, requiring students to find the whole or “big group,” and two had the unknown in the initial or medial position, requiring participants to find a part or “small group.”

Total problems solved. The third dependent variable was TOTAL PROBLEMS SOLVED, measured by the cumulative number of word problems that received points for the critical steps (3, 5, 7, and 10). All four critical steps must be completed independently correct in order for the problem to be considered “solved correctly”. The result of performing these steps of the task analysis was an accurate equation and answer sentence. Four problems were solved each session; therefore four possible points were available each session.

Generalization of problem solving. The fourth dependent variable measured GENERALIZATION OF PROBLEM SOLVING. Setting and situation generalization was measured by removing the stimulus supports (equation and answer sentence). Generalization was measured once during baseline, and every four sessions in intervention (or at least twice). This measured the generalization of the WORD PROBLEM SOLVING to different materials with less visual supports.

Self-initiated prompting. SELF-INITIATED PROMPTING was the fifth dependent variable and was measured by the number of self-initiated verbal and specific

verbal prompts using the embedded prompts on the iPad. The number of student-initiated verbal (what to do) and specific verbal (how to do) prompts were recorded each session.

Global mathematics ability. The sixth dependent variable was GLOBAL MATH ABILITY. This distal measure was composed of the participant's pre-intervention and post-intervention scores on the Test of Mathematical Abilities, Third Edition (TOMA-2; Brown, Cronin, & McEntire, 1994). The TOMA-2 has four core subtests, including (a) vocabulary, (b) computation, (c) general information, and (d) story problems. The results of the core subtests were combined to form an overall Mathematical Ability Index pre- and post-intervention. The standardization sample of the TOMA-2 had characteristics similar to those reported in the 1990 Statistical Abstract of the United States (U.S. Census Bureau, 1990). The TOMA-2 provides adequate reliability and validity with a coefficient alpha above .80.

Social validity. The PERCEPTION OF PROBLEM SOLVING was measured by social validity questionnaires and interviews pre-intervention and post-intervention. Student social validity data were gathered through a questionnaire with statements related to the procedures and outcomes of the intervention. Students were able to indicate their answers verbally or using response options (yes/no). Teacher social validity data was gathered related to the procedures, goals, and outcomes – specifically the importance of math problem solving skills and instruction for this population. This information was obtained from a survey that included questions with a dichotomous (yes/no) response.

#### Data Collection

Schedule. Prior to the beginning of baseline, students and their teachers were given a social validity questionnaire to obtain their pre-intervention PERCEPTION OF

PROBLEM SOLVING. During baseline, a measure of GLOBAL MATH ABILITY of each participant was obtained using the TOMA-2. In each baseline and intervention session, data was collected related to MATH VOCABULARY, WORD PROBLEM SOLVING, TOTAL PROBLEMS SOLVED, and SELF-INITIATED PROMPTING. Once during baseline, a generalization probe measured GENERALIZATION OF PROBLEM SOLVING by providing a worksheet without stimulus supports. In intervention, generalization probes were conducted every four sessions, or at least twice per participant. Once participants met mastery and had at least two generalization probes, the TOMA-2 was administered to obtain a post-test measure of GLOBAL MATH ABILITY. The social validity questionnaire was administered again to students to measure any changes in PERCEPTION OF PROBLEM SOLVING after the post-test administration of the TOMA-2. After all participants completed the post-test measure of GLOBAL MATH ABILITY, the special education teacher was asked to complete the social validity questionnaire again to measure PERCEPTION OF PROBLEM SOLVING.

Interobserver agreement. Interobserver agreement (IOA) on MATHEMATICS VOCABULARY, WORD PROBLEM SOLVING and, TOTAL PROBLEMS SOLVED was collected for a minimum of 30% of the sessions in each phase for each participant, either in vivo or via permanent product (video) observations. A second trained observer used the same data collection instrument to score student responses independent of the primary researcher's score. IOA was evaluated using an item-by-item method and calculated by dividing the total agreed items by the total agreed and disagreed items and multiplied by 100 (Kazdin, 1982).

Procedural fidelity. A second observer used the data collection instruments (Appendix B) to collect procedural fidelity and document the degree to which the intervention was implemented consistently as designed. Procedural fidelity was collected for a minimum of 30% of the sessions of each phase for each participant. To calculate procedural fidelity, the number of elements correctly implemented was divided by the total number of procedural elements then multiplied by 100 (Billingsley, White, & Munson, 1980).

### Materials

Worksheets consisting of four real world word problems related to a theme (e.g., school dance, working in a restaurant, video game store) were used in all data collection and intervention sessions. The word problems were written collaboratively between the interventionist and elementary and middle school special education teachers during the first year of The Solutions Project. The themes of the word problems represented high-interest topics and scenarios students may encounter in current and future environments. The word problems were written to be free of cultural or gender bias. The word problems were evaluated by an elementary mathematics expert for content validity.

There are three types of additive word problems: group, change, and compare. In group problems (the problem type of interest for the current study), two small groups of different things are combined to make one large group. This type of problem demonstrates a part/whole relationship and is solved through addition when both small groups are known and the whole group is unknown. For example, three apples and two bananas can be combined to make a large group of five fruits. The second problem type is change, which involves an increase or decrease in the amount of one thing. These

problems are dynamic, as opposed to group and compare problems, which are static. An example of a change problem would be “John has three apples. He ate one. How many apples does he have left?” In this problem, subtraction would be used. However, addition would be used to solve the problem “John has three apples. He picked one more apple. How many apples does he have now?” Finally, a compare problem analyzes the difference in quantities, either two different quantities of the same thing or of different things. For example, a compare problem could be “John has two apples and Anna has three bananas. How many more apples does Anna have than John?” Another compare problem would be “John has two green apples and three red apples. How many fewer green apples than red apples does he have?” Compare problems are solved with subtraction. This study only addressed group problems.

For the worksheets, one group word problem was printed on each side of two A4-size pages. Beneath each word problem, there was an equation in the form of boxes for the numerals, a circle for the mathematical symbol, and an equal sign. An answer sentence had an x and an equal sign with an empty box and line for the answer and label of the answer. See Figure 2 for an example of a worksheet with a word problem (missing whole, or “big group”). Of the four problems presented to the participant, two of the problems required the student to solve for the whole when given two parts, and two problems required the student to solve for one of the parts when given the whole and one of the parts. The order of the problems was randomized. Students were not given the same word problem or the same worksheet more than one time.

An iPad with the SMARTnotebook application was used to display the student self-instruction checklist (see Figure 3). The electronic student self-instruction sheet had

embedded verbal and specific verbal prompts. If a participant touched the number for a step, the verbal prompt was activated (e.g., “Step one says read the problem). If a participant touched the “?” to the right of a step, the specific verbal prompt was activated (e.g., “Ask for help reading the problem”).

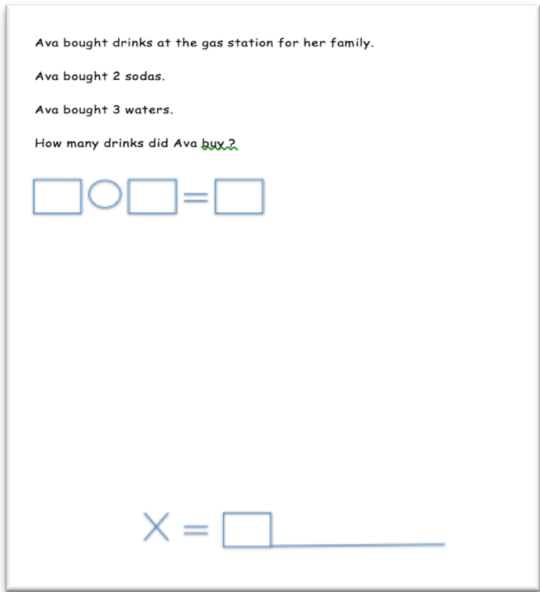


Figure 2: Example worksheet

1.		Read the problem
2.		Circle the nouns
3.		Label number sentence
4.		Circle the numbers
5.		Fill-in equation
6.		Use my rule
7.		+ or -
8.		Make Sets
9.		Solve
10.		Write answer


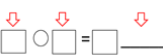
Figure 3: Student self-instruction sheet

## Procedures

Baseline. During baseline, students received typical mathematics instruction on grade-level standards. Classroom mathematics instruction incorporated a variety of formats, including whole-group, small-group, and individual arrangements. In addition, participants frequently completed worksheets or file-folders independently or practiced mathematics skills on the computer. The teacher followed the NC Extensions to the Common Core (<http://www.dpi.state.nc.us/docs/acre/standards/extended/math/6-8.pdf>). The classroom teacher frequently used materials from Unique Learning Systems (N2y, 2014), a curriculum prescribed by the district.

In baseline sessions, the instructor presented the instructional cue “touch the word that means \_\_\_\_\_” and provided the definition, such as “combine groups” for the word “add.” The vocabulary terms were displayed in an array of five. Each term was paired with a symbol. See Table 1 for a list of each term, the definition, and symbol that was paired with it. Each vocabulary word was presented twice.

TABLE 1: Mathematics vocabulary definitions and symbols

Vocabulary Term	Definition	Symbol
Add	Combine	+
Subtract	Take away	-
Equal	Same as	=
Equation	Statement says things are equal	
Label	Name of group	

Next, the participant was given the worksheet, a pencil with eraser, and the iPad 3 that displayed the student self-instruction checklist. The instructor said “Show me how to solve these word problems.” The instructor read the word problem aloud if asked by the

participant. Praise for on-task behavior was given but no error correction or reinforcement for correct answers was provided. This procedure continued until the participant attempted all four problems. If a student did not attempt the problem or stopped working on the problem for 10-s, the problem was removed and the next problem was presented. If the student asked for help (other than to read), the instructor replied “Do your best,” or another similar affirming statement that did not provide any specific feedback or prompting. Following completion of solving the problems, the participant was given the option to play appropriate games on the iPad that were not related to math to reinforce completion of the task for 3-min or return to class.

Modified schema-based instruction (SBI). Each intervention session began with the math vocabulary task. In the first trial, the instructor used 0-s constant time delay to systematically teach the math vocabulary definitions to the participant. Definitions were displayed in a random array of three with one correct answer and two distractors. To model in the 0-s trials, the interventionist read the vocabulary word and touched the correct answer while reading it aloud, then asked the student to do the same. Each vocabulary word definition was taught during the 0-s round one time. The definitions were shuffled between each trial and displayed in a different array of three choices. In the next two trials, 4-s will were inserted between the presentation of the math vocabulary word and definition and the model prompt. If the participant was unable to make an independent correct response, the interventionist provided a model-prompt, as was provided in the 0-s trial, and then asked the student to touch the correct definition. Using a similar procedure as the 0-s round, the words were presented in random order with random distractors. Two 4-s trials were conducted each session. If the participant made



two errors in a row, the interventionist returned to 0-s trials and repeated the missed vocabulary. The words were then represented at 4-s delay. Data was only taken during 4-s delay rounds.

The instructor provided 2 days of strategy instruction to the participant following the sequence on the student self-instruction sheet to model solving the four problems on the worksheet. During these training days, the interventionist modeled how to solve the problems with active student participation (e.g., “My turn. I found the label of the big group. Your turn. Can you circle the label of the big group?”). The participants were taught to follow the student self-instruction sheet and check off each step on the task analysis as it was completed.

If participants were unable to read the problem (Step 1), they were taught to ask the interventionist to read it for them. To circle the “groups” (Step 2), participants found the big group and small groups in the word problem. The “big group” or whole was always be identified in the first sentence, and if it was one of the known variables, again in the second sentence. The “small groups” or parts were in the second and third sentence if they were known, or just the third sentence if only one was unknown. To label their equation (Step 3), participants (a) wrote the “big group” or whole above the third box, (b) wrote the first “small group” or part above the first box, and (c) wrote the second “small group” or part above the second box.

Next, participants circled the numbers in the word problem (Step 4) and filled in the equation (Step 5). Participants put an “x” in the equation to indicate which value was the unknown. Participants then said the rule (Step 7), or a verbal chant. They used information from the chant and equation to create their sets (Step 8). If the “big group” or

whole was the unknown, participants made a set under each of the “small groups” or parts in the equation. If a “small group” or part was unknown, participants made a set to represent the known “small group” or part. To solve the problem (Step 9), they counted all of the manipulatives in the two sets if the “big group” or whole was unknown. If the “small group” or part was unknown, they used a counting-on strategy to make the set for the unknown based on the quantity found in the “big group” or whole. For example, if the equation read  $5+x=7$ , participants made a set of five, and then made a set of two under the  $x$  counting “6, 7.” They then wrote the quantity they counted (Step 10) in the answer sentence ( $x=$  \_\_ label).

Following 2 days of explicit instruction, the interventionist provided least intrusive prompting if the participant failed to make a response. The least intrusive prompting hierarchy consisted of three levels, including a verbal prompt, specific verbal prompt, and a model prompt. See Table 2 below for a description of the verbal, specific verbal, and model prompts for each step of the task analysis (words in bold are what was said by interventionist). All verbal and specific verbal prompts were embedded within the student self-instruction sheet on the iPad as previously described. The instructor provided the model prompt. The participant was given 5-s before each prompt if it appeared they were stuck or not working on a step. If the participant made an error, the interventionist went directly to a model prompt and required the student to repeat the behavior. The interventionist used behavior specific praise after each correct response (prompted or unprompted). As participants demonstrated proficiency on steps of the task analysis, behavior specific praise was faded.

TABLE 2: Prompting hierarchy by step of task analysis

Step	Correct Response	Verbal Prompt	Specific Verbal prompt	Model Prompt
1. Read the problem	No feedback unless the child needs reinforcement for asking appropriately.	<b>Step 1</b> says read the problem	<b>Ask me to read the problem.</b> Wait for students to ask.	<b>Say “read the problem please.”</b> Wait for student to ask, and then read.
2. Circle the groups	<b>Great job! The groups are (small group), (small group), and (big group)</b>	<b>Step 2</b> says circle the groups	<b>Find the groups in the problem and circle them.</b>	<b>My turn. I am going to find the big group in the first sentence.</b> Point to the whole or big group. <b>Your turn. Circle the (label of big group).</b> Wait for student to respond. <b>Now I am going to find the small groups.</b> Point to the small groups. <b>Your turn. Circle the (label of first small group) and (label of second small group).</b> They are the small groups. Wait for the student to circle the groups.
3. Label equation	<b>Excellent job! The small groups are _ and _ and the big group is _.</b> The question asks (read question) and you labeled the answer sentence because we are trying to find how many (label of unknown)	<b>Step 3</b> says label equation	<b>Label the number and answer sentence with the groups you circled.</b>	<b>My turn. I am going find the label of big group.</b> Point to the whole, or “big group” in the problem and point above the third box. <b>Your turn. Write (label of big group) above the third box.</b> Wait for student response. <b>My turn. I am going to find the label of the small groups.</b> Point to the first small group. <b>Your turn. Write (label of first small group) above the first box.</b> Wait for student

response. **My turn, I am going to find the label of the second small group.** Point to the second small group. **Your turn. Write (label of second small group) above the second box.** Wait for student response. **My Turn. Now I am going to find the label for my answer by finding the word after “how many.”** Re-read the question sentence and point to the word after how many. **Your turn. Write (label of unknown) on the answer sentence.** Wait for student to respond.

4. Circle the numbers

**Nice work!** For feedback, restate summary of sentences with numbers in them (e.g., “He had 5 drinks. 2 were soda and some were water)

**Step 4**  
**says circle the numbers**

**Circle the numbers next to the groups in the word problem.**

**My turn. I am going to find the number of (label of first known amount).** Point to the number of the first known group. **Your turn. Circle (#) next to (label of first known amount).** Wait for student response. **My turn. I am going to find the number of (label of second known amount).** Point to the number of second known group. **Your turn. Circle (#) next to (label of second known amount).** Wait for student response.

5. Fill-in equation

**Nice work filling in the equation. You filled in an X for (label of unknown)**

**Step 5**  
**says fill in equation**

**Fill in the equation using the numbers you circled in the word**

Point to first number (#) in word problem next to first group in problem and then point to the box that has that label. **Write (#) here for the number of**

	because that is what we are solving for.		problem. Use an x for what you need to solve for.	(label of first known amount). Point to the 2 <sup>nd</sup> number in the word problem (#), and then point to the box that has that label. <b>Write (#) here for the number of (label of second known amount).</b> Wait for student response. <b>My turn, I am going to find the word “how many” to know where to put my x in the equation.</b> Point to the word “how many.. <b>Your turn. Put an x in the box under (label of unknown) to show what we are solving for.</b> Wait for student response.
6. + or -	<b>Yes, add. In group problems we join two small groups to make one BIG group, so we add.</b>	<b>Step 6 says plus or minus</b>	<b>Do we add or subtract to solve group problems?</b>	<b>In group problems we join two small groups to make one BIG group, so we add. Write a plus in the circle.</b> Wait for student to respond.
7. Use my rule	<b>You got it!</b>	<b>Step 7 says use my rule</b>	<b>Use the rule for group problems.</b>	<b>The rule for group problems is (state rule). Your turn, say the rule for group problems.</b> Wait for student response.
8. Make sets	<b>Awesome job!</b> <u>MISSING WHOLE:</u> You made a set of (#) because there are (#, label) and you made a set of (#) because there are (#, label) and we	<b>Step 8 says make sets</b>	<u>MISSING WHOLE:</u> Make sets to represent how many there are in each small group <u>MISSING PART:</u> Make a set to	<u>MISSING WHOLE:</u> We need to make a set of (#) in the first small group to represent the number of (label). Make a set of (#) here. Point to below the first small group. Wait for student response. <u>MISSING PART:</u> Now, we need to make a set of (#) to represent the

need to find out how many (label big group) there are

MISSING PART:

You made a set of (#) because there are (#, label known small group) and we need to find how many (label small group) there are.

represent how many there are in the first small group

number of (label of known small group or part). Make a set of (#) here. Point to below the second small group. Wait for student response.

9. Solve

MISSING WHOLE:

Nice work! You moved your small group sets to the big group to find out how many (label big group).

MISSING PART:

Nice work! You counted up from the number of (label of known small group) to the number of (label of big group) to find the number of (label of unknown small group).

Step 9  
says solve

MISSING WHOLE:

Move your small groups to the big group and count to find your answer. Wait for student to move sets into end and count.

MISSING PART: Make a set under (label of unknown) by counting up from (# known small group) to (# big group)

MISSING WHOLE:

Watch me first. I am going to move my small groups to the big group and count to find the answer. Combine sets into the end and count. Be sure to say answer with label. Move sets back to original. **Your turn.** Wait for student to repeat.

MISSING PART: Watch me first. I am going to make a set under (label of unknown) by counting up from (# known small group) to (# big group). Make a set and model counting up by saying numbers aloud. Then count the number in the set. Be sure to say answer with label. Move sets back to original. **Your turn.** Wait for student to repeat.

10. Write answer

Nice work!  
State numeral

Step 10  
says write

MISSING WHOLE:

Watch me. I am going to count the number of

and label (e.g., 3      answer  
waters)

**Count the  
number of  
(label of big  
group) and  
write your  
answer in the  
equation.  
MISSING  
PART:  
Count the  
number of  
(label of  
unknown  
small group)  
and write  
your answer  
in the answer  
sentence.**

**(label of unknown).**  
Count the number of  
manipulatives under the  
equation aloud. **Your  
turn. Count the number  
of (label of unknown).**  
Wait for student to repeat.  
**Now write (# answer) in  
the answer sentence.**  
Wait for student to  
respond

The interventionist took data on the number of steps the participant was able to complete independently (WORD PROBLEM SOLVING) as well as prompt levels (SELF-INITIATED PROMPTING). This method of assessment was a type of multiple opportunity probe, in that the participant was given the opportunity to perform a step without help for purposes of data collection and then given prompting as needed to complete the step to set up the next response. Due to the chained nature of solving a word problem, each step was dependent on the correct execution of the one before, and so the interventionist must either prompt or set up each step to determine if later responses in the chain have been mastered. Based on the WORD PROBLEM SOLVING data, the interventionist was able to calculate the TOTAL PROBLEMS SOLVED in each session.

As in baseline, reinforcement for on-task behavior was provided. Participants were given access to iPad games to reinforce task completion after solving all word problems. If a student demonstrated fatigue or behavior that was incompatible with

successfully focusing on the word problems (e.g., anxiety from a fire drill), the interventionist made a determination whether or not to complete the session or finish it the next day. If a student was not able to complete two problems, the data from that day was discarded. If a student completed two problems, but no more, the data was saved and the student completed the session (remainder of worksheet) the next day.

Generalization. The purpose of this measure was to assess whether students were able to generalize WORD PROBLEM SOLVING when stimulus supports (symbols and equation structure) were removed. It was anticipated participants would provide their own supports by drawing in the equation, or they would be able to write and label the number and answer sentences and solve the problem correctly without providing the diagram. During the generalization sessions, the interventionist did not provide prompting or feedback beyond the instructional cue “Show me how to solve these problems.” Three generalization sessions were conducted (one in baseline, two in intervention). The procedures for reinforcement of on-task behavior and task completion continued from baseline and intervention. Similarly, any individual needs for redirection of off-task behavior remained in place.

#### Data Analysis

Visual analysis was used to analyze and interpret the data on MATHEMATICS VOCABULARY and MATHEMATICAL PROBLEM SOLVING, and GENERALIZATION OF PROBLEM SOLVING. Results from each of these variables are displayed graphically. Effective graphs allow for the data to be presented accurately, completely, and clearly which facilitates the viewer’s task of understanding the data (Cooper et al., 2007; Johnston & Pennypacker, 1980). Based on recommendations by



Cooper et al. (2007), two questions were asked during visual analysis: (a) Did the behavior change in a meaningful way; and (b) if so, to what extent can that change in behavior be attributed to the manipulation of the independent variable. These questions were answered through an analysis of the extent and type of variability in the data, level and trends of the data, immediacy of effect, overlap, and consistency of data patterns across similar phases (Kratohwill et al., 2010). Visual analysis was used to determine the presence or absence of a functional relation, which indicates that the occurrence of the phenomena under study is a function of the operation of one or more specified and controlled variables in the experiment, demonstrated by a specific change in the dependent variable produced by manipulating the independent variable and that the change in the dependent variable was unlikely the result of other factors (Cooper et al., 2007).

A table was used to display the number of problems solved out of four during each generalization session to visually display data from GENERALIZATION OF PROBLEM SOLVING. The steps of the task analysis solved independently correct over the four problems were graphed to allow for visual analysis.

A cumulative graph was constructed to visually represent the TOTAL PROBLEMS SOLVED by each participant. The slope of responses in baseline and intervention for each participant was calculated, as well as the difference between the two. Steeper slopes indicate higher response rates. A table was used to display the overall response rate (ORR) and slope by phase for each participant.

The raw scores from four subtests of the TOMA-2 were converted to percentiles and standard scores. These measures were used to analyze the effect of the treatment

package on GLOBAL PROBLEM SOLVING ABILITY. The TOMA-2 was administered pre- and post-intervention. Finally, the results from the social validity questionnaires were analyzed. The difference between ratings for the teacher and student survey pre and post-intervention were interpreted regarding the impact of the intervention of PERCEPTION OF PROBLEM SOLVING and feasibility of the intervention.

#### Potential Threats to Validity

This study demonstrated experimental control based on the recommendations of R. D. Horner and Baer (1978) and Cooper et al. (2007). Experimental control was demonstrated through the visual inspection of change occurring where the intervention was applied, but not where there was no intervention (baseline). Demonstrating experimental control began by collecting baseline data for each participant simultaneously across a minimum of five sessions. Then, each participant was introduced to the intervention in a staggered fashion, only after the previously introduced participant had demonstrated a positive change in level and trend. Staggering the introduction of the intervention across tiers controlled for threats to internal validity due to history, maturation, and testing. Threats due to instrumentation were controlled for by IOA and procedural fidelity data. Gathering baseline data on four participants helped to control for the threat to mortality (attrition) to ensure that there were at least three demonstrations of effect at three different points in time. Threats due to testing were controlled for in several ways. First, only the minimum number of baseline sessions required to meet design standards were conducted with each participant (Kratochwill et al., 2010). Although worksheets administered during each of the sessions in baseline and intervention followed a similar format, they varied by story, numbers chosen, and order

of problems within each worksheet. No worksheet was presented more than once to a participant across baseline, intervention, or generalization sessions. There was a threat of multiple treatment interference in the proposed study due to the nature of the intervention.

Several threats to external validity were controlled for. The setting for the proposed research was an environment with which the participants familiar. Although the interventionist was a doctoral student and not the participants' classroom teacher, one-on-one instruction was a common context for learning for the participants. A common limitation of single-case research is the small sample size. This limits generalization of effects and is a delimitation of the current study. Future replications of the study are needed to control for this. The participants in the study had to meet a defined set of prerequisite mathematics skills. Although the presence of these skills limits generalization to all middle school students with ASD/ID, the explicit information provided on these prerequisite skills will assist in generalization to others who have the same set of skills.

#### External support

This dissertation was conducted as a component of The Solutions Project (IES award R324A13001). The Solutions Project is a goal two development grant funded by the Institute of Education Sciences. The Principal Investigator (PI) of The Solutions Project is Dr. Diane Browder, with Co-PIs being Dr. Fred Spooner and Dr. Ya-yu Lo. The primary purpose of The Solutions Project was to develop a mathematics problem solving curriculum for students with moderate and severe disabilities. This study specifically looked at the application of the curriculum to standards (i.e., algebra), as well as fading of instructional supports. For this dissertation, the resources included graduate

research assistant effort and pre-developed mathematical problems. See Appendix D for a table distinguishing the current study from materials and procedures used in The Solutions Project.

## CHAPTER 4: RESULTS

### Interobserver Agreement

Interobserver agreement (IOA) on MATHEMATICS VOCABULARY, WORD PROBLEM SOLVING, GENERALIZATION OF WORD PROBLEM SOLVING, TOTAL PROBLEMS SOLVED, and SELF-INITIATED PROMPTING was collected for a minimum of 30% of the sessions in each phase for each participant, either in vivo or via permanent product (video) observations. IOA was collected on GLOBAL MATH ABILITY during both the pretest and posttest for each participant. IOA was evaluated using an item-by-item method and calculated by dividing the total agreed items by the total agreed and disagreed items and multiplied by 100 (Kazdin, 1982).

The second observer collected IOA data during baseline for 33% of baseline sessions for Anna (2 out of 6 sessions), 33% of baseline sessions for Amanda (3 out of 9 sessions), and 44% of baseline sessions for Stephanie (4 out of 9 sessions). The agreement was 100% for all three participants during baseline. The second observer collected IOA during intervention for 45% of intervention sessions for Anna (5 out of 11 sessions), 55% of intervention sessions for Amanda (5 out of 9 sessions), and 30% of intervention sessions for Stephanie (3 out of 10 sessions).

For Anna, the mean agreement in intervention was 100% for MATHEMATICS VOCABULARY, 95.4% (range 86 to 100) for WORD PROBLEM SOLVING and 100%

for TOTAL PROBLEMS SOLVED. For Amanda, the mean agreement in intervention was 100% for MATHEMATICS VOCABULARY, 98.8% (range 97-100) or WORD PROBLEM SOLVING, and 100% for TOTAL PROBLEMS SOLVED. For Stephanie, the mean agreement in intervention was 100% for MATHEMATICS VOCABULARY, 97% (range 96 to 100) for WORD PROBLEM SOLVING, and 100% for TOTAL PROBLEMS SOLVED.

#### Procedural Fidelity

A second observer assessed procedural fidelity in order to verify the degree to which prompting was implemented consistently as designed. Fidelity was taken on the data collection instruments (APPENDIX B). Procedural fidelity was collected for a minimum of 30% of the sessions of each phase for each participant. To calculate procedural fidelity, the number of elements correctly implemented were divided by the total number of procedural elements then multiplied by 100 (Billingsley, White, & Munson, 1980).

The second observer collected procedural fidelity data during baseline for 33% of baseline sessions for Anna (2 out of 6 sessions), 33% of baseline sessions for Amanda (3 out of 9 sessions), and 44% of baseline sessions for Stephanie (4 out of 9 sessions). Mean procedural fidelity was 100% for all three participants during baseline. The second observer collected fidelity during intervention for 45% of intervention sessions for Anna (5 out of 11 sessions), 55% of intervention sessions for Amanda (5 out of 9 sessions), and 30% of intervention sessions for Stephanie (3 out of 10 sessions). The mean procedural fidelity in intervention was 99% for Anna (range 97-100), 100% for Stephanie, and 98% for Stephanie (range 96-100).

Results for Question 1: What is the effect of constant time delay on the identification of mathematics vocabulary definitions by students with ASD/ID?

Figure 4 shows the effects of constant time delay on the identification of mathematics vocabulary definitions. The graph shows the number of correct identifications of mathematics vocabulary terms performed by each participant. During baseline all participants had a stable baseline. During intervention all three participants showed a change in level or an increasing trend, with no overlapping data with baseline performance. Visual analysis of the graph indicated a functional relation between constant time delay and identification of mathematics definitions.

Anna. During baseline probes, Anna correctly identified three mathematics symbols on both trials when given the definition for a total of six points in each baseline session. She was able to identify the addition symbol, subtraction symbol, and equal signs when given a definition. During intervention probes, she was able to immediately identify all five mathematics vocabulary symbols on both trials when given the definition for a total of ten points in each intervention session. She reached mastery in three trials.

Amanda. Amanda had similar performance to Anna during baseline, as she was able to consistently identify the symbols for addition, subtraction, and equals when given the definition in both trials in each of her seven baseline sessions. During intervention, Amanda was able to make correct identifications on nine out of ten opportunities (missing X) on the first session. In subsequent sessions she was able to identify all five symbols when given the definition across both trials. She reached mastery in four trials with an average rate of correct responding of 9.75 (range 9-10).

Stephanie. Stephanie was able to consistently identify the symbols for addition and subtraction when given the definition during both trials in each of her eight baseline sessions. During intervention, she increased the number of correct identifications from eight out of ten on the first session, to nine out of ten on the second. During subsequent sessions to identified all five mathematics symbols in both trials. She reached mastery in five trials, with an average rate of correct responding of 9.4 (range 8-10).

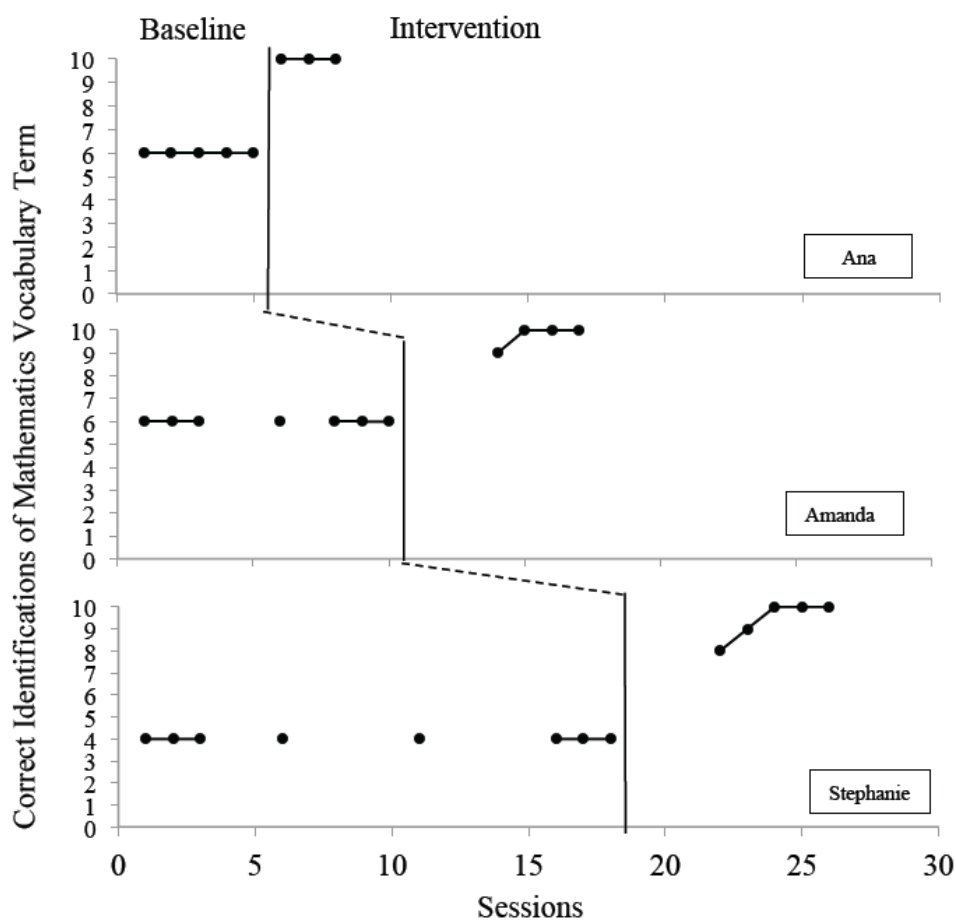


Figure 4: Graph of independent correct identifications of mathematics vocabulary terms when given the symbol and definition.



Results for Question 2: What is the effect of schema-based instruction using a system of least prompts that incorporates a technology platform on the number of steps performed independently correct to solve a word problem by students with ASD/ ID?

Figure 5 shows the effects of schema-based instruction using a system of least prompts that incorporates a technology platform on the number of steps performed independently correct to solve a word problem. The graph shows the number of steps of the task analysis performed independently correct across four word problems (ten steps per problem for a total of forty points). During baseline all participants had a stable pattern of responding. During intervention all three participants showed a change in level or an increasing trend, with no overlapping data with baseline performance. Visual analysis of the graph indicated a functional relation between schema-based instruction using a system of least prompts that incorporates a technology platform on the number of steps performed independently correct to solve a word problem.

Anna. Anna received an average of 4.4 points across the four problems during the five baseline sessions (range 3-7). During baseline, Anna consistently filled in the number sentence with the numbers from the word problem in the order they appeared and randomly chose either addition or subtraction. She generally completed the computation correctly because she has memorized many of her addition and subtraction math facts. However, Anna never labeled her answer (e.g., 5 cucumbers). After three sessions of modeling, she quickly jumped to 28 points across four problems in the first intervention session and maintained an increased level and ascending trend. The step Anna struggled with the most during intervention was Step 3 (label). She was most successful across all steps in word problems that required solving for the missing whole. When she had to

solve for the missing part, she required error corrections in Step 9 (solve) to assist her in counting up. Anna reached mastery of MATHEMATICS PROBLEM SOLVING after nine intervention sessions. She received an average of 34.5 points across the four problems during nine intervention sessions (range 28 to 39).

Amanda. Amanda received an average of 7.14 points across the four problems during seven baseline sessions (range 7-8). During baseline, Amanda consistently filled in the number sentence with numbers from the word problem in the order they appeared and consistently chose addition as the operation. Amanda used a variety of strategies to compute the answer, including touch math and finger counting, if she did not have the math fact memorized. She never used the manipulatives to assist her in solving the problem. After three sessions of modeling, she quickly jumped to 31 points across the four problems and maintained an ascending trend. During intervention, Amanda did not consistently respond with the rule and requested assistance on that step during almost every word problem. Similarly to Anna, she also had difficulty remembering the difference in how to solve (step 9) when presented with a missing part (count up) or missing whole (make sets and combine) problem. Amanda reached mastery of MATHEMATICS PROBLEM SOLVING after seven intervention sessions. She received an average of 35.7 points across the four problems during seven intervention sessions (range 31-40).

Stephanie. Stephanie received an average of 2.75 points across the four problems during eight baseline sessions (range 2-4). During baseline, Stephanie would write random numbers or symbols into the number sentence. She never attempted to read the problem or ask for assistance. After three sessions of modeling, she quickly jumped to 24

points across the four problems for an increase in level and continued with an overall positive trend. On the second intervention session, Stephanie only received 20 points across the four problems. During this session she demonstrated a high rate of off-task behavior that involved the iPad, including repeatedly making marks and erasing them and drawing shapes over the task analysis. The session was discontinued after two problems and resumed the next day. On the first attempt at the fourth intervention session, Stephanie refused to complete the problem or follow directions, so the session was discontinued. When the fourth intervention session was resumed the next day, a reinforcement system was put in place to encourage on-task behavior and task completion. Stephanie was provided a check list with the numbers 1-4 that she would cross out after completing that problem. Stephanie chose her reinforcer from a menu that included playing an iPad game, a small tangible item (e.g., bouncy ball, slinky) or an edible (e.g., smarties or gummy bears) prior to beginning the session and the chosen item remained on the table. Stephanie always chose a tangible item. Following the introduction of the reinforcement system, Stephanie maintained an upward trend in responding and quickly reached mastery of MATHEMATICAL PROBLEM SOLVING after eight intervention sessions. She received an average of 31 points across eight intervention sessions.

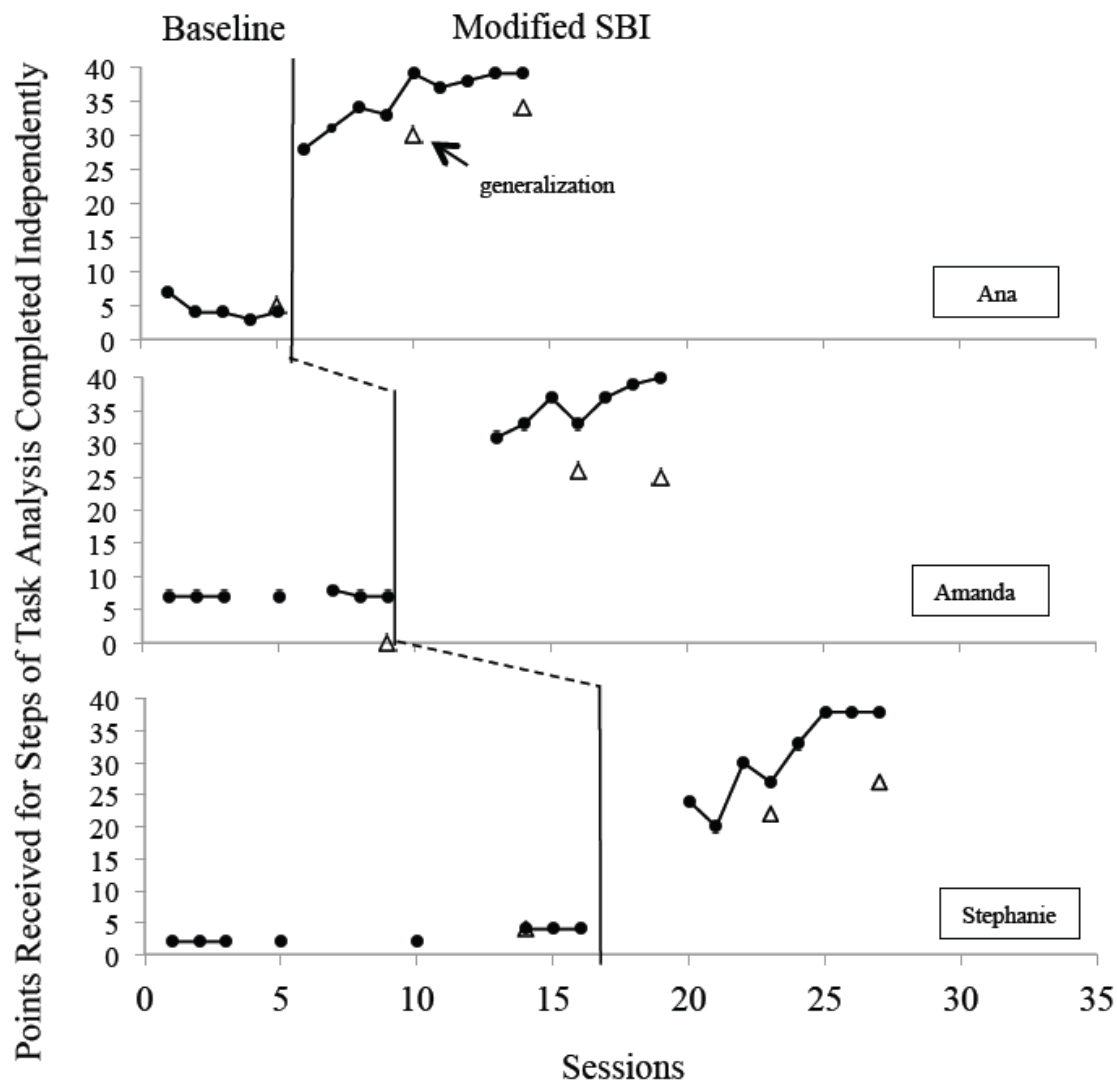


Figure 5: Graph of number of points received from independently completing steps of task analysis by participant.

Results for Question 3: What is the effect of schema-based instruction using a system of least prompts that incorporates a technology platform on the cumulative number of word problems solved by students with ASD/ID?

Figure 6 shows the effects of schema-based instruction using a system of least prompts that incorporates a technology platform on the total number of word problems

solved. This measure was derived from the number of problems in which the participant completed steps 3 (label), 5 (fill in equation), 6 (+ or -) and 10 (write answer). The graph shows the cumulative number of independent responses (i.e., problems solved) each participant performed in order to see the rate of change over time (Cooper et al., 2007; Ferster & Skinner, 1957). An overall response rate (ORR) was calculated for baseline, intervention (SBI), generalization baseline, and generalization intervention (SBI) by dividing the total number of responses (correct problems) recorded during each condition or phase by the number of collection sessions (Cooper et al., 2007). The higher the overall response rate, the greater the effect. In addition, the slope (i.e., rate of change) was calculated for baseline and intervention (MSBI) by dividing the vertical change ( $y_2 - y_1$ ) by the horizontal change ( $x_2 - x_1$ ) on a connected line. Table 3 shows both the overall response rate and slope per phase for each participant.

All three participants had an ORR and slope of 0 during baseline, representing no problems solved. Anna and Amanda did calculate the correct numerical answer, but did not incorporate the other critical steps, specifically labeling their answer (e.g., 5 calculators instead of just 5). All participants had an increase in ORR and slope during intervention, demonstrating their ability to correctly solve the word problems.

Anna. Anna did not solve any word problems correct during baseline or her first intervention session. She was able to quickly begin solving problems, for a total of 22 problems solved independently correct across the intervention sessions. The ORR and Slope intervention was 2.44 problems.

Amanda. Amanda did not solve any word problems correct during baseline. In her first intervention sessions he was able to correctly solve one problem, and quickly began

to increase the number of problems she solved correctly. Amanda regressed following the generalization probe and a break from instruction due to conference travel for the interventionist, only solving 1 problem independently correct. She quickly recovered with daily instruction and solved a total of 18 problems across the intervention sessions. The ORR in intervention was 2.57 and slope was 2.42.

Stephanie. Stephanie did not solve any word problems correct during baseline or the first two intervention sessions. She did quickly increase the number of problems solved across intervention sessions to a total of 16. The ORR and slope in intervention was 2.

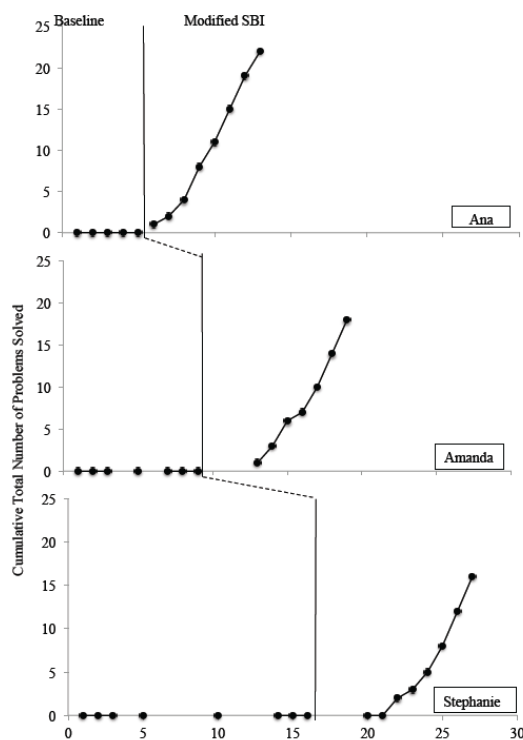


Figure 6: Graph of cumulative number of problems solved by participant across phases.

TABLE 3. Overall response rate and slope per phase for each participant

Condition/Phase	Anna		Amanda		Stephanie	
	ORR	Slope	ORR	Slope	ORR	Slope
Baseline	0	0	0	0	0	0
MSBI	2.44	2.44	2.57	2.42	2	2

*Note.* ORR = Overall response rate (i.e., average rate of response over a given time period), calculated by dividing the total number of responses recorded during a period by the number of observation periods; Slope (i.e., rate of change), calculated by dividing the vertical change ( $y_2 - y_1$ ) by the horizontal change ( $x_2 - x_1$ ) on a connected line.

Results for Question 4: Are students with ASD/ID able to maintain problem solving skills, demonstrated by the number of steps of a word problem solving task analysis performed independently correct and total number of problems solved, when stimulus supports are faded?

Figure , Figure 7 and Table 4 demonstrate the generalization performance of each participant. Each participant was given one generalization probe during baseline and two during intervention. The graph shows the number of steps of the task analysis performed independently correct across four word problems (ten steps per problem) during generalization probes, when the worksheets did not contain stimulus supports. During baseline, two participants demonstrated similar proficiency on the generalization problem and one participant received fewer points than other baseline probes. During intervention participants demonstrated an increased level of responding over baseline generalization

probes, however all generalization probes during intervention demonstrated a decrease rate of independence.

Anna. During the baseline generalization probe, Anna received five points across the four problems; however she did not solve any of the problems correctly. This was consistent with her responding during other baseline probes. Her first intervention generalization probe was conducted after four intervention sessions and she received 30 points. This was a large increase over baseline; however it was below the previous intervention data point of 33 points. She was able to solve missing whole problems correctly, but did not label or fill in the number sentence correctly for missing part problems. Her second generalization point took place after eight intervention sessions. She increased her independent responding over the previous generalization point by receiving 34 points. However, this was a decrease from the eighth intervention point, on which she received 39 points. She was very successful with the missing whole problems and was able to solve one missing part problem correctly. Similarly to her first intervention generalization session, she did not label or fill in an equation correctly for the other missing part problem.

Amanda. During the baseline generalization probe, Amanda received 0 points and therefore did not solve the problem correctly. Without the stimulus supports available, she stated the number sentence aloud (“e.g., 8 plus 4 equals 9”). This was different from her other baseline probes, during which she wrote the numbers from the problem into the number sentence in the order they appeared and added them. Her first intervention generalization session took place after the third intervention session and she received 26 points. This was a large increase from baseline, however it was below the previous data



point of 37 during the third intervention session. She was more successful with missing whole problems than missing part. On the two missing part word problems, she did not label or write a correct number sentence, which prevented her from getting points for solving the problem correctly. Her second generalization probe took place after the seventh intervention session, at which point she had already met mastery. She received 25 points, which was less than the 40 received during the seventh intervention session. It should be noted that a total of seven days passed between the seventh intervention session and the generalization session, and during generalization she did not receive any prompting or feedback. During this last session, she did not set up the missing part equations correctly, which prevented her from solving them correctly. She did not add the label to the answer for the missing whole equations, which prevented them from being considered “correct”.

Stephanie. During the baseline generalization probe, Stephanie received 4 points. This was an increase by 2 points from the previous baseline points, but was maintained in the subsequent three baseline points prior to entering intervention. During this probe she wrote a number sentence with made up numbers but always with a plus, giving her one point with each of the problems. Her first intervention generalization probe took place after the third intervention session and she received 22 points. This was an increase from baseline but a decrease from the third intervention session, during which she received 30 points. During this probe she did not set up the missing part equations correctly. Her final generalization probe took place after the eighth intervention session, and she had already demonstrated mastery. In this generalization probe she received 27 points and demonstrated the same errors in setting up the problem with missing parts.

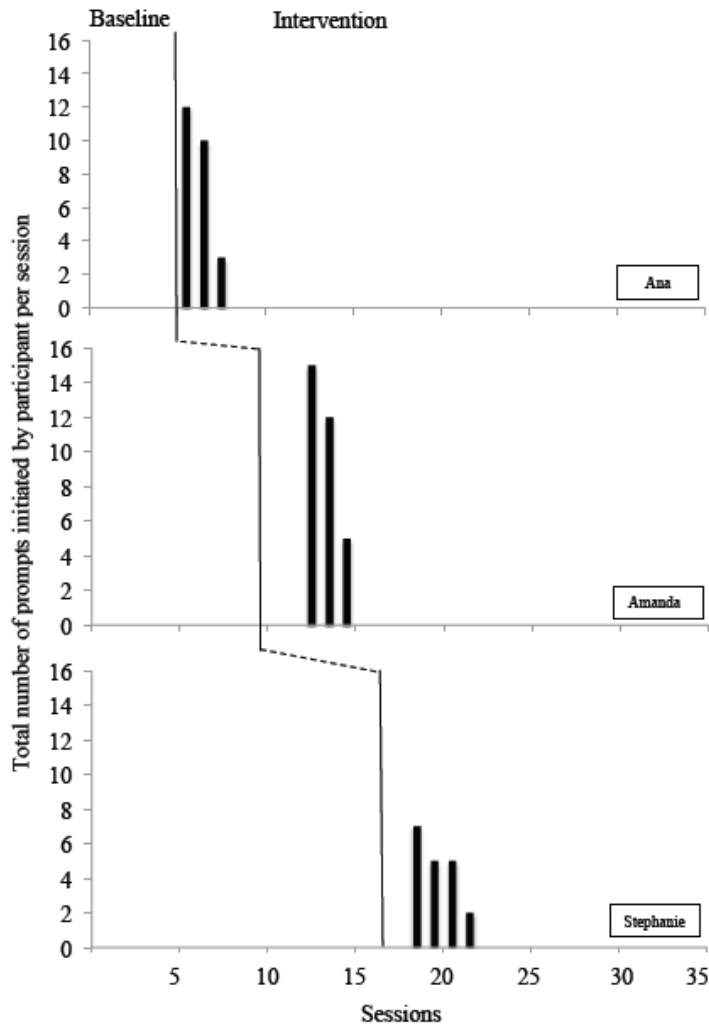


Figure 7: Graph of prompts initiated by participants across phases.

TABLE 4. Number of points and problems solved in generalization probes

	Baseline Generalization		Intervention Generalization # 1		Intervention Generalization # 2	
	Points	# Solved	Points	# Solved	Points	# Solved
Anna	5	0	30	1	34	3
Amanda	0	0	26	2	25	0
Stephanie	4	0	22	1	27	2

Results for Question 5: What is the effect of instructor modeling use of electronic prompting during least prompting on student's subsequent initiation of use of the electronic feature to self-prompt by students with ASD/ID?

Table 5 shows the number of self-prompts initiated by each participant by phase. During both baseline and intervention the task analysis with embedded prompts was provided. None of the participants initiated any self-prompts during the baseline phase. They had not been shown the features of the task analysis and embedded prompts. Following baseline, each participant was shown how to activate the embedded prompts and given instruction on the steps of the task analysis. During intervention, each of the participants initiated the self-prompts. The primary prompts used were verbal prompts (re-reading the steps). Each of the participants initiated the self-prompts frequently in the first few intervention sessions, and then faded their use of both the self-prompts and general attention to the task analysis.

TABLE 5: Number of Self-prompts Initiated by Participants per Phase

Participant	Baseline Self-Prompts	Intervention Self-Prompts
Anna	0	25
Amanda	0	32
Stephanie	0	19

Results for Question 6: What is the effect of modified schema-based instruction on the change in global word problem solving ability before and after intervention by students with ASD/ID?

The results of the global measure of word problem solving before and after intervention (TOMA-2) can be seen in Table 6 (Anna), Table 7 (Amanda), and Table 8

(Stephanie). Overall, the global word problem solving ability measures did not reflect the large change observed in the primary dependent variable. All participants did demonstrate increases in some subtests from pre to posttest, and no participants decreased scores in any subtest from pre to posttest.

Anna performed the same on the pre and posttest in three measures (vocabulary, general information, and story problems). She scored in the 37%ile for vocabulary, <1%ile for computation, and 2%ile for story problems. Her standard score on the computation posttest increased from 2 to 3, or from the <1%ile to 1%ile.

TABLE 6: Results of TOMA-2 pre and posttest for Anna

Subtest	Raw Score		Percentile		Standard Score	
	Pre	Post	Pre	Post	Pre	Post
Vocabulary	4	4	37	37	9	9
Computation	6	7	<1	1	2	3
General Information	1	1	<1	<1	1	1
Story Problems	3	3	2	2	4	4

Amanda scored the same on the pre and posttest for two measures (computation and story problems). She scored in the 1%ile for both computation and story problems. Her standard score on the vocabulary subtest increased from 10 to 11, or from the 50%ile to 63%ile. She also increased her standard score in the general information posttest from a standard score of 2 to 3, or <1%ile to 1%ile.

TABLE 7: Results of TOMA-2 pre and posttest for Amanda.

Subtest	Raw Score		Percentile		Standard Score	
	Pre	Post	Pre	Post	Pre	Post
Vocabulary	4	5	50	63	10	11
Computation	5	5	1	1	3	3
General Information	1	2	<1	1	2	3
Story Problems	2	2	1	1	3	3

Stephanie also performed similarly on the pre and posttest in vocabulary and general information. Her standard score in vocabulary was 6, or the 9%ile and in the general information subtest her standard score was 1, or <1%ile. She improved from pre to post test on computation based on her raw score, but this change was not reflected in the standard score or percentile, which remained at 1 and <1. Similarly, her increase in raw score from pre to posttest on the story problems subtest did not lift her above a standard score of 1 or <1%ile.

TABLE 8: Results of TOMA-2 pre and post-test for Stephanie

Subtest	Raw Score		Percentile		Standard Score	
	Pre	Post	Pre	Post	Pre	Post
Vocabulary	1	1	9	9	6	6
Computation	2	3	<1	<1	1	1
General Information	1	1	<1	<1	1	1
Story Problems	0	1	<1	<1	1	1

Results for Question 7: What is the effect of modified schema-based instruction on the perception of word problem solving of students with ASD/ID?

The results of participant social validity surveys can be seen in Table 9. Each participant completed the same social validity questionnaire pre and post intervention.

Overall, participants responded positively to the pre-intervention survey. Anna responded “yes” to all questions on the pre and post intervention survey. Amanda answered “yes” to all questions except “I am good at math” and “I am good at solving word problems” in the pre-intervention survey but answered “yes” to all questions on the post-intervention survey. Stephanie answered “yes” to four of the pre-intervention survey questions and “no” to three, including “I am good at solving word problems”, “I like to use manipulatives...”, and “I know how to ask for help”. On the post-intervention survey, she responded “yes” to all questions.

TABLE 9: Results from student perception of word problem solving pre and post surveys

	Anna Pre	Anna Post	Amanda Pre	Amanda Post	Stephanie Pre	Stephanie Post
1. I like doing math.	Y	Y	Y	Y	Y	Y
2. I like solving word problems.	Y	Y	Y	Y	Y	Y
3. I like to use technology (like an iPad) to help me learn.	Y	Y	Y	Y	Y	Y
4. I am good at math.	Y	Y	N	Y	Y	Y
5. I am good at solving word problems.	Y	Y	N	Y	N	Y
6. I like to use manipulatives (like blocks or tiles) to solve math problems.	Y	Y	Y	Y	N	Y
7. I know how to ask for help with my schoolwork.	Y	Y	Y	Y	N	Y
Totals	7	7	5	7	4	7

Results for Question 8: What is the effect of modified schema-based instruction on the perception of word problem solving of teachers of students with ASD/ID?

The results of the teacher social validity survey can be seen in Table 10. The students' math teacher completed the same questionnaire pre and post intervention. Overall, the teacher's perception did not change pre to post intervention for most items across the three participants. The teacher answered "yes" to all questions except "My student is currently able to solve one-step addition word problems" for all participants in the pre-intervention survey. Her responses on the post-intervention survey were identical, including her rating of "no" regarding the students' ability to solve algebraic equations.

TABLE 10: Results from teacher perception of word problem solving pre and post surveys

	Anna Pre	Anna Post	Amanda Pre	Amanda Post	Stephanie Pre	Stephanie Post
1. My student likes math	Y	Y	Y	Y	Y	Y
2. My student is currently able to solve one-step addition word problems.	Y	Y	Y	Y	Y	Y
3. My student is currently able to solve algebraic equations.	N	N	N	N	N	N
4. My student likes to use the iPad to learn.	Y	Y	Y	Y	Y	Y
5. My student is able to work independently.	Y	Y	Y	Y	Y	Y
6. My student knows how to ask for assistance.	Y	Y	Y	Y	Y	Y
7. My student is able to use manipulatives appropriately to solve math problems.	Y	Y	Y	Y	Y	Y
Totals	6	6	6	6	6	6

## CHAPTER 5: DISCUSSION

The purpose of this investigation was to evaluate the effects of modified schema based instruction on the mathematics problem solving skills of middle school students with ASD/ID using a multiple probe across participants design. The effects of the intervention on mathematics word problem solving were measured by assessing the steps participants followed to solve a word problem, the total word problems solved, and a global measure of problem solving (TOMA-2). Effects of the components of the intervention were also measured, including (a) the effects of teaching mathematics vocabulary through constant time delay on the identification of definitions and examples of mathematics vocabulary terms and (b) effects of the embedded prompts within the task analysis on the number and type of self-initiated prompts. Participants and their teachers were interviewed before and after the intervention to determine their perception of word problem solving and whether this intervention had an effect on these feelings. In this chapter, outcomes will be discussed by research question with themes that emerged from the outcomes of the intervention explored in relation to the conceptual underpinnings of the components of the intervention. Finally, limitations will be discussed and implications for future research and practice will be provided.

### Mathematics Vocabulary Outcomes

A visual analysis concludes there is a functional relation between constant time delay and identification of mathematics symbols. All participants had baseline responding above



zero, indicating they were already familiar with some terms / symbols. All three participants had knowledge in baseline of the addition symbol (combine groups) and subtraction symbol (take away), while Anna and Amanda were also already familiar with equal (same as). Time delay was effective in teaching the terms equation (statement that says things are equal) and label (name of group).

Vocabulary knowledge and fluency play a crucial role in mathematics, but especially in problem solving. Van de Walle et al. (2012) emphasize the importance of having fluency in mathematical language so that participants are able to both receptively understand the task as well as expressively communicate about mathematics. In the current study, vocabulary terms were chosen because they played a key role in participant understanding of strategy instruction. Although not addressed in previous mathematics problem solving studies, one potential barrier to mathematical problem solving for students with ASD/ID is communication and language. The baseline data from this study showed the participants did not have the concept of equal, equation, or label. Given the status of constant time delay as an evidence-based practice to teach vocabulary to students with moderate and severe disabilities (Browder et al., 2008), this was not a surprising finding. These results support similar findings that used constant time delay to teach academic vocabulary words that were required within tasks (Allison et al., 2016; Browder, Root, Wood, & Allison, 2015; Mims, Hudson, & Browder 2012). This is the first application of constant time delay to teach mathematics vocabulary concepts. The participants in this study quickly learned the targeted mathematical vocabulary terms with this effective method of repeated trials with constant time delay.

## Mathematics Problem Solving and Generalization Outcomes

A functional relation was found between modified SBI and MATHEMATICS PROBLEM SOLVING, based on a visual analysis of the data. All three participants had a steady state of responding in baseline and were able to quickly increase the level and trend of their independent responding to each step of the task analysis following three sessions of modeling.

Experts in the area of mathematics have long emphasized the importance of instruction that imparts both conceptual and procedural mathematical knowledge (Fennema, 1972; Peterson et al., 1989). SBI is one learning strategy that emphasizes both conceptual and procedural understanding of mathematical word problems, but it is not yet an evidence-based practice for students with ASD/ID. In the absence of an established evidence-based practice, Whalon et al. (2009) suggest looking to other disability groups when designing instruction for students with moderate/severe disabilities and determining what supports can be added. SBI is an evidence-based practice for students with learning disabilities (Jitendra et al., 2015). The modified SBI in this study provided strategy instruction as well as instructional supports including systematic instruction, visual supports, and a task analysis, that are evidence-based for students with moderate/severe disabilities.

The modified SBI used in this study contained all essential components of SBI as outlined by Jitendra et al. (2015): (a) identification of the problem structure to determine the problem type, (b) use of visual representations of the structure to determine problem type and to organize information from the problem, and (c) explicit instruction on the problem solving method. The modifications, or enhancements, provided in this study

addressed the barriers students with ASD/ID face in solving mathematics word problems, including semantic language, executive functioning, and metacognition.

Semantic language barriers. Semantic language is a barrier to mathematics problem solving for students with ASD/ID because they have to determine “what is happening” in the problem. Key steps of the task analysis assisted students in systematically answering the question “What is happening?” First, in step 2 (circle the groups), participants identified the key groups in the problem, including the big group (whole) and two small groups (parts). The structured formula each word problem followed assisted in decreasing the difficulty of this task. The big group (whole) was always found in the first sentence. The first small group (part) was always found in the second sentence, and the second small group (part) was found in the second sentence if it was the “unknown”, or in the third sentence if the big group (whole) was unknown. The next step (3; label equation) required participants to reconstruct what the problem was about. The participants used the number sentence to create a graphic organizer, labeling each of the boxes with the first letter of that group, always following the format small group (part) + small group (part) = big group (whole). Finally, when participants stated the rule (step 6), they stated the relationship among the groups in the problem, (e.g., “small group plus small group equals big group; boys plus girls equals friends”).

The use of visual representations to represent the structure of the problem type and to organize information from the problem is a key component of SBI (Jitendra et al., 2015) and assists in overcoming semantic language barriers. According to the National Professional Development Center on ASD (NPDC), visual supports are commonly used in teaching adaptive skills (e.g., engagement, transitions, social skills) and typically are

seen in the form of pictures, written words, objects within the environment, arrangement of the environment or visual boundaries, schedules, maps, labels, timelines, and scripts (NPDC, 2010). Knight and Sartini (2015) found visual supports in the form of graphic organizers, visual diagrams, picture symbols, and visuals of key phrases to be an evidence-based practice for teaching text-based comprehension to students with ASD.

Although none of the studies in the Knight and Sartini review applied visual supports in a mathematical learning task, their rationale for the effectiveness of visual supports remains relevant, as mathematical word problem solving requires text-based comprehension.

Given the findings from neuropsychology that individuals with ASD may have strengths in visual over verbal processing (Roth, Muchnik, Shabtai, Hildesheimer, & Henkin, 2012), visual supports assist students make a permanent written connection and assist in organization of text. The modifications made to SBI in this study related to visual supports included having the participants use a pre-drawn number sentence as a graphic organizer. There is limited discussion in the literature on visual supports regarding fading their use (Knight & Sartini, 2015; Wong et al., 2014). The results of this study support the use of visual supports with students with ASD/ID, however the question remains about whether they can, or should, be faded to promote generalization.

All three participants had difficulty solving the word problems on generalization tasks when the number sentence was faded. Specifically, they had more success with missing-whole problems than problems that presented a missing part. This raises the issue of when and how to fade supports. Perhaps the fading of the number sentence should have been more gradual, such as element-by-element. The participants needed explicit training in generalization (Baer, Wolf, & Risley, 1968).

Generalization is the occurrence of related behavior under conditions that differ from those during training (Stokes & Baer, 1977). According to Stokes and Bear (1977), generalization only occurs when training and generalization settings have sufficiently similar stimulus components. This study used the “train and hope” approach to generalization (Stokes & Baer, 1977), meaning generalization was documented through probes, but no planning or instruction took place to increase the likelihood of its occurrence. The results of the participants on the missing-part problems indicates that another generalization strategy was needed. Sequential modification in the form of gradual fading of stimulus prompts may have been effective. For example, first the answer prompt at the bottom of the worksheet could have been removed, followed by the circle in the number sentence (where the participants wrote “+”), and then the boxes would be faded one by one.

A previous study using modified SBI programmed common stimuli in both the training and generalization conditions. Saunders (2014) trained students using CBVI and tested for generalization in a paper-pencil condition. The salient stimuli for solving the problem were present in both conditions, with only the format of the presentation changing. This strategy proved to be successful, as all participants were able to successfully generalize problem solving to the untrained (paper-pencil) condition. In the case of the current study, the stimulus supports from the training condition were not present in the generalization condition; the purpose of the generalization condition was to determine whether or not participants could perform the behavior without those supports. Therefore, programming common exemplars have been a viable strategy for the current study.

Executive functioning barriers. Besides semantics, a second barrier that students with ASD/ID face in solving mathematical word problems is the executive functioning that is required to sustain attention to the task and complete each step correctly.

Executive functioning is a critical factor in mathematics (Zentall, 2007) and is often the source of difficulty for students with learning disabilities (Geary & Hoard, 2005).

Modifications were made to the instructional techniques and materials or supports typically used in SBI for students with LD with these executive functioning and working memory deficits in mind. A written task analysis replaced the mnemonics typically used in SBI (Jitendra & Hoff, 1997; Rockwell, Griffin, & Jones, 2011). The task analysis allowed for students to attend to each step in the chain and self-monitor progress. Picture supports were added to the written task analysis to compensate for deficits in reading. An additional support for reading deficits was a read-aloud function for each step of the task analysis.

The task analysis also served to address the barrier of metacognition.

Metacognition is required in mathematical word problem solving and involves a student choosing a strategy, monitoring progress, and making changes as necessary (Van De Walle et al., 2012). There is a cyclical relationship between metacognitive and executive functioning difficulties. The task analysis led participants through steps to choose a strategy by directing the order of choices and also ensured that the steps were carried out logically. The task analysis also allowed for self-corrections and error corrections at each small step of the problem, making the corrections efficient and allowing for maximum independence.

The results of this study are similar to the findings from previous studies that also used steps of a task analysis to teach and measure mathematics problem solving (Root, Browder, Saunders, & Lo, 2015; Saunders, Lo, & Browder, 2014). Although Saunders et al. (2014) broke the intervention into two phases, the first focusing on conceptual knowledge and the second on procedural, the participants all demonstrated a quick acquisition of skills as measured by steps of a task analysis completed independently correct. The instructional procedure from Root et al. (2015) is similar to the current study, in that a total task procedure was used to teach the steps of the task analysis. The results of this study support the total task procedure. In a total task procedure, an individual is taught all of the steps in the chain from beginning to end in each teaching trial. The task analysis assists in overcoming executive functioning barriers to mathematical problem solving because it makes each step in the process explicit and is a permanent visual referent.

#### Cumulative Problems Solved Outcomes

The cumulative graph of problems solved by each participant across phases (Figure 6) allows for visual representation of the increase in problems solved once participants entered intervention. All three participants were unable to correctly solve any problems in baseline, resulting in an ORR and Slope of 0 for each participant during that phase. While Anna and Amanda sometimes calculated the correct answer in missing whole problems, they did not label it (e.g., “5 cups” instead of just “5”). All participants were able to quickly begin solving the word problems, although Anna did not solve any problems until the second session and Stephanie did not solve any problems until the

third. Analysis of the ORR and slope during intervention (MSBI) demonstrates an increase for each participant.

These results support the conclusion that MSBI is effective in teaching students to solve mathematical word problems. It is a more conservative measure of mathematical problem solving than steps of a task analysis, as it requires an entire series of behaviors to be correctly, rather than separate discrete behaviors. The word problems were considered “solved” if the participant was able to label the number sentence, fill in the equation with the numbers and operation, and write the final answer with a label. These problem solving steps were considered “critical” (Test & Spooner, 1996; Weng & Bouck, 2014). The rest of the steps of the TA involved the participant showing their process for reaching their answer. Previous studies that used MSBI to teach mathematical problem solving to students with ASD/ID have had similar results. Saunders (2014) found that students were able to increase the steps of the task analysis they were able to correctly complete independently (the process to solve) before the number of problems solved increased (correct answers to problem).

Measuring steps completed of a task analysis allows for analysis of progress in each step in the chained task, provides evidence of skill growth, and helps diagnose how students are solving the problem. This can be especially important for students with cognitive disabilities or for whom it is anticipated that mastery of the entire skill (i.e., solving the problem) will not be immediate upon entering intervention. Therefore, it is a good primary dependent variable, as it shows immediate progress toward mastery and allows for clear interpretation of student error. A measure that collapses the discrete behaviors of solving a problem may not allow for a clear immediate demonstration of



effect when using visual analysis of single-case designs with participants who have cognitive disabilities. In addition, when using such a conservative measure, the scale would only be four, which would increase the variability of the data and contribute to a lag in demonstration of effect. Although the ultimate goal of this study was to teach participants to solve the mathematics word problems, it is important to analyze both their ability to complete the critical steps within the chain (total problems solved) as well as progress on each step (steps of the task analysis). There is an emphasis in general education for students to “show their work”. The task analysis provided an explicit strategy for solving the problem that required participants to do this.

The need to measure both total problems solved and steps of the task analysis is exemplified by the differences between the current study and that by Neef et al. (2003). In the first study to teach mathematics problem solving to students with moderate ID, Neef et al., (2003) taught two adult participants, one of whom was 19 years old and had an IQ of 46, to solve story problems that represented both the change problem type. Although the current study taught only the group problem type, story problems in both studies had the unknown in both the final ( $A+B=?$ ) and initial/medial positions ( $A+?=C$  and  $?+B=C$ ) and the stimulus cues used as a pre-drawn number sentence were identical in the two studies. Neef et al. used precurrent behavior instruction to teach each of the steps for solving the story problems; each component was taught to mastery in a forward chaining procedure (initial set, change set, operation, resulting set, solution) using three phases of training: (1) targeted component known, (2) targeted component unknown, and (3) mixture of known and unknown. In this manner, the authors first taught each “problem type” (i.e., targeted component known or unknown) to mastery and then

explicitly taught discrimination. This is different from the approach taken in the current study, in which participants were taught the steps of solving the problem in a total task procedure. All ten steps were taught in each modeling/intervention session.

The difference between the Neef et al. (2003) investigation and the current study is not only made evident in the chaining procedure, but also in the research design and measurement of the dependent variable. Neef et al. used a multiple probe across behaviors design (Baer, Wolf, & Risley, 1968), with the behaviors being identification of each component of solving the problem (i.e., initial set, change set, operation, resulting set, solution). Neef et al. only provided measures of correct responses to identifying each component of solving the problem, not problems solved. However, it can be inferred from the graphs that participants were not able to solve the problem (identify the resulting set or solution) until after training occurred. In the current study, measures of both discrete behaviors within the chained task (steps of the task analysis) as well as total problems solved were provided in order to make results and ultimate impact of the intervention on mathematical problem solving ability clear. It is important to have a measure of problems solved from the onset because some students may be able to solve and have a correct answer without being able to show how they arrived at it.

#### Self-initiated Prompting Outcomes

All participants used the embedded prompts to some degree following intervention as shown in Table 5. During baseline when participants had not received instruction on the steps of the task analysis or the embedded self-prompting features, they did not attend to the task analysis or self-prompt. Immediately after the first problem was modeled, participants quickly began to use the task analysis and engaged in self-

monitoring by checking off each step. Although the exact behavior of each participant regarding self-initiated prompting varied, all three participants demonstrated a similar pattern of behavior regarding its use. All participants had an increased dependence on the task analysis in the first few sessions of least intrusive prompting, and all participants faded dependence on both the embedded prompts and faded or chunked their reference to the task analysis while solving the problem.

Self-monitoring skills were evident as participants increased proficiency in solving the word problems. As participants began to demonstrate an increased level of independence completing the steps of the task analysis, they all began to “chunk” steps together and faded their dependence on referring to it after completing each step. They gradually stopped checking off the steps and instead just glanced at them, usually after the first problem. Anna even stopped referencing the task analysis completely by the last two intervention sessions, while the Amanda still said steps aloud to herself and said “check” when she finished them. Stephanie continued to glance at the task analysis before beginning the problem and again when it was time for her to say the rule.

While previous studies teaching chained academic tasks to students with ASD/ID have incorporated means of self-initiated prompting (Browder et al., 2015; Saunders, 2014), the use of these features was not measured. Anecdotally, the authors did note that participants faded the reliance on the prompts as they increased independence and reached mastery of targeted skills. The current study empirically supports those anecdotal findings. As participants increased familiarity with the task analysis and required steps, they faded reliance on the task analysis and embedded prompts.

### Global Problem Solving Ability Outcomes

Each of the participants demonstrated an increase in their raw score on at least one subtest from pre to posttest of the TOMA-2. Amanda demonstrated a small increase in her vocabulary raw score, Anna and Stephanie demonstrated a small increase in their computation raw scores, Amanda increased her raw score on the general information subtest, and Stephanie increased her raw score on the story problems subtest. Overall the pretest scores for each of the participants were very low, ranging from a standard score of 1 to 11 and percentiles from <1 to 63, with most percentiles being at the much lower end. All participants had at least one subtest posttest score in the 1%ile or <1%ile. Overall, this global measure did not seem to reflect the change in mathematical problem solving ability seen in the MATHEMATICS VOCABULARY, MATHEMATICS PROBLEM SOLVING, GENERALIZATION OF PROBLEM SOLVING, and TOTAL PROBLEMS SOLVED dependent measures. Although the TOMA-2 was normed with same-age peers (12 and 14 years old), females (50.2% of sample), and with students with disabilities (5% of sample with learning disabilities, 2% of the sample with students with other handicapping conditions), it is unknown whether the students in the normative sample had autism or other developmental disabilities. For this population, progress is typically only seen on behaviors that were directly taught. Standardized mathematics assessments, such as the TOMA-2 address a much wider sample of mathematics than the scope of this study. The targeted behaviors and limited duration of this study likely influenced the marginal gains seen from pre to posttest.

### Perception of Problem Solving Outcomes

Overall, the perception of problem solving questionnaires indicated the participants had a positive response to the intervention. All participants answered “yes” to each of the questions in the post-intervention survey. This final result should be interpreted with the pre-intervention survey in mind. All three participants had mostly favorable (“yes”) responses on that measure as well, with one participant answering yes to every question on both surveys. The participant may not have understood what was being asked or wanted to be compliant or pleasing. Numerous studies only use post-surveys and therefore may only get this final result.

The results of the teacher pre and post surveys also had identical positive answers with a common negative across all participants (“my student is able to solve algebraic equations”). Overall there was no change in her perception of her student’s word problem solving abilities from pre to post intervention. However, the results from this study do not support her perception; all three of the participants were able to solve algebraic equations. Several possibilities exist to explain this discrepancy. First, “solving algebraic equations” was not defined; therefore it is possible that the researcher and teacher were not thinking of the same skill. Second, the word problems used in this study followed a specific formula and only required addition. They did not represent the sample of word problems that may be presented in the teacher’s mathematics course. Most importantly, the teacher never observed the students during the intervention. She was blind to the study’s methods and procedures and only knew that the goal was to work on mathematical word problem solving. Perhaps the teacher would have had a different perspective on the effects of the

intervention if she had been made aware of the methods and materials or observed her students during the course of the intervention.

Although the participants and teacher indicated on the pre-intervention survey that they generally had positive attitudes toward their problem solving ability and the teacher indicated the participants were interested in and able to use the materials (iPad and manipulatives) to solve addition word problems, the results from baseline indicate otherwise. However, the teacher perception that the students were not able to solve algebraic equations did not change from pre to post intervention, the data indicate otherwise. As previously stated, there are several possible reasons for this discrepancy, including a lack of operational definitions and her blindness to the intervention methods and results.

Further analysis of social validity. Perhaps instead of asking whether the intervention had a positive effect, the question should instead be whether or not questionnaire accurately or adequately measures the impact of the intervention on the participants. A different instrument that addressed the social validity of the intervention, either the goals, procedures, or outcomes, may have given a better description of the impact of the intervention from the perspective of the participants and teacher.

Applied behavior analysis is concerned with producing predictable and replicable improvements in “socially important” behavior. The emphasis of the field of applied behavior analysis on producing predictable and replicable improvements in socially important behavior began with the first issue of the Journal of Applied Behavior Analysis, when the mission of the journal was set as “for the publication of applications of analysis of behavior to problems of social importance” (Wolf, 1978). Guidelines on

how to select target behaviors to ensure their social importance are well established (e.g., Ayllon & Azrin, 1968; Rosales-Ruiz & Baer, 1997), but as Cooper et al. (2007) point out, the ultimate question to be asked is whether or not the behavior change will improve the life experience of the individual. The relevance of behavior rule (Ayllon & Azrin, 1968) requires a target behavior to only be selected when it is likely to produce reinforcement in the natural environment. Cooper et al. (2007) suggest targeting behavioral cusps and pivotal behaviors when prioritizing instruction.

In this study, the primary targeted behavior was mathematical problem solving, which is a behavioral cusp. According to Rosales-Ruiz and Baer (1997), a behavioral cusp is “a behavior change that has consequences for the organism beyond the change itself, some of which may be considered important” (p. 537). Bosch and Fuqua (2001) suggested five criteria for determining behavioral cusps: (a) access to new reinforcers, contingencies, and environments; (b) social validity; (c) generativeness; (d) competition with inappropriate responses; and (d) number and relative importance of people affected. Bosch and Fuqua emphasize the consideration of the long term consequences of acquiring a behavior, rather than simply the potential for change in the immediate environment. Mathematical problem solving is emphasized throughout the mathematics curriculum for middle school students and is required for numerous leisure, vocational, and daily living skills. Acquiring mathematical problem solving skills gives individuals with ASD/ID opportunities for reinforcement in current and future environments and activities by increasing both independence and therefore opportunities.

In terms of generativeness, mathematical problem solving is a complex ability or process. The skills taught and targeted in this intervention are building blocks toward the

general sample of mathematical problem solving skills required in grade-aligned mathematics tasks. For example, proficiency in solving one-step algebraic word problems of the group problem type enables the participants with ASD/ID in this study to move on to other additive problem types (i.e., change and compare) as well as more complex group problems, such as those that require multiple steps, have larger numbers, or contain irrelevant information.

In order for mathematical problem solving as a competing behavior, the manifestation of the “inappropriate” behavior that it is replacing must be explored. Problem solving requires persistence when a solution is unknown and requires flexible thinking (Wehmeyer et al., 2010). In baseline, participants demonstrated inflexible thinking and a lack of problem solving by approaching all problems in the same manner (adding numbers in the order provided), if any strategy was used at all. Conversely, following intervention on the target behavior, the participants were able to select and execute an appropriate strategy based on the problem type (i.e., demonstrated problem solving).

Finally, mathematical problem solving of individuals with ASD/ID has an impact on those who control reinforcers and punishers in the specific school environment (i.e., teachers). The acquisition of this skill will influence subsequent skills selected for instruction. In future environments, demonstration of mathematical problem solving skills by individuals with ASD/ID will also impact employers, co-workers, and family members, as mathematical problem solving is required for numerous life activities and increases the independence of the individual while decreasing their reliance on others.



Self-monitoring through self-initiated prompting was the secondary targeted behavior. Self-monitoring as a component of self-management and therefore self-determination is a pivotal behavior, in that once learned, it produces corresponding modifications or covariations in other adaptive untrained behaviors (Cooper et al., 2007, p. 59). Changes in pivotal behaviors produce large accompanying improvements in other areas. Improvements in pivotal behaviors are able to shorten the length of time it takes to acquire related skills, giving the individual with ASD/ID access to affiliated skills in an efficient time frame. Self-management, and specifically self-monitoring, are pivotal behaviors because they increase independence and decrease reliance on outside agents to impact change. The pictorial self-instruction in this study increased not only mathematical problem solving skills but also self-monitoring skills. Participants learned to follow the steps of a pictorial self-instruction checklist (task analysis) to independently perform a chained academic task. This procedure of using a task analysis can be replicated within other chained academic tasks, whether it is solving another type of mathematics problem, or applied to other content areas such as writing or a science.

#### Themes Derived from Outcomes

Mayer's theory of problem solving. The MSBI provided in this study gave explicit instruction to participants through each of Mayer's (1985) phases of problem solving. The student self-instruction sheet provided support in students' executive functioning; it guided participants through the four steps of problem solving and broke the steps into measurable, observable behaviors. The problem translation phase, when semantic language skills construct meaning from the problem, was represented by the steps 1 (read the problem), 2 (circle the groups), 3 (label equation), 5 (fill-in equation)

and 6 (use my rule) in which participants made meaning of the word problem. In these steps, the students systematically selected the integral parts of the problem and translated them into a math sentence. Finally, the solution planning and execution stages of problem solving occurred in steps eight, nine, and 10.

The chained nature of word problem solving makes each step dependent on the successful completion of the prior steps (Jitendra et al., 2007). Similarly, Mayer's theory of problem solving states that errors in one stage can prevent successful demonstration of conceptual and/or procedural knowledge. This dependence of one phase upon another was evident when analyzing student errors in generalization probes when feedback and prompting was not provided. Participants had the most difficulty with correctly labeling and filling in the equation for missing part word problems when the stimulus supports were removed. By failing to correctly translate the semantic language of the problem into a math equation, the strategy for finding a solution was incorrect.

Evidence-based practices. This study incorporated several established evidence-based practices for students with moderate and severe disabilities, including those with ASD/ID (Browder et al., 2008; Spooner et al., 2016). Systematic instruction, specifically a system of least prompts and a chaining procedure, were used to teach participants the steps to solving a word problem. Although several previous studies have used a combination of these systematic instruction techniques to teach a chained mathematical task to students with moderate and severe developmental disabilities (e.g., Browder, Jimenez et al., 2012; Browder, Root, et al., 2016; Browder, Trela et al., 2012; Creech-Galloway et al., 2013; Saunders, 2014), this was the first to present the task analysis on an iPad and embed the first two levels in the system of least prompts within the iPad. In

doing so, the current study combined two evidence-based practices: systematic instruction and technology aided instruction.

An additional evidence-based instructional procedure used in the current study was explicit instruction. Explicit instruction is an essential component of traditional SBI, which is an evidence-based practice itself for students with learning disabilities (Jitendra et al., 2015). Gersten et al. (2009) defined explicit instruction within a mathematical task as having three components: (a) the teacher demonstrates a step-by-step plan (strategy) for solving the problem, (b) this step-by-step plan is specific for a set of problems, and (c) students use the same procedure/steps demonstrated by the teacher to solve the problem. In the current study, the instructor modeled the strategy for several sessions with active student participation. Following sessions used a system of least prompts to provide support to participants during guided practice through the word problems. An important component of explicit instruction is providing multiple opportunities to respond, which is also an evidence-based practice for teaching mathematics to students with moderate and severe disabilities (Archer & Hughes, 2011). Within the current study, participants were given four opportunities to practice the strategy within each session.

Finally, this study contributes to the literature on using manipulatives, a recently established evidence-based practice for teaching mathematics to students with moderate and severe disabilities (Spooner et al., 2016). In the current study, manipulatives supplemented the participants' current level of procedural knowledge and enhanced problem solving skills given their lack of fluency in mathematical facts.

Verbal behavior. Mathematics vocabulary terms were taught to participants at the beginning of the intervention to ensure they understood the terms and concepts used in

the mathematical problem solving task. This direct teaching of vocabulary influenced their verbal behavior repertoires. Verbal behavior is behavior that is reinforced through the mediation of another person's behavior (Skinner, 1957). In applied behavior analysis, it is more important to consider behavior in terms of its function rather than its form. Verbal behavior is not synonymous with vocal behavior (i.e., talking) – rather it is behavior that involves a social interaction between a speaker and listener. The field of applied behavior analysis is concerned with socially significant behavior, and according to Cooper et al. (2007), most socially significant behaviors involve verbal behavior. Consequently, behaviors related to learning such as asking and answering questions, reading, and writing, require verbal behavior. During the modified SBI, participants needed had to respond to questions (intraverbals) that included mathematics vocabulary terms. An incorrect response to a question didn't necessarily mean that participants did not know the answer, but rather it was possible that they did not understand the words in the question. Directly teaching the mathematics vocabulary terms and ensuring they were able to tact (i.e., identify symbol when given the definition) added to their verbal behavior repertoire influenced the interpretation of intraverbal responses.

Self-determination. The intervention used in this study taught the self-management skill of self-monitoring. Self-monitoring allows for individuals to be in control of their own behavior instead of relying on others to manage it. The development of self-management, and specifically self-monitoring, contributes to the development of self-determination by giving individuals the skills needed to control what happens in their own lives (Wehmeyer et al., 2010). Component skills that improve the self-determined behavior of individuals, such as self-monitoring, are pivotal behaviors. A positive change

in self-monitoring within the context of mathematical problem solving imparts a positive change on the ability of the individual to self-monitor in a similar way within alternate contexts or tasks.

In mathematical problem solving, self-management is needed to regulate behavior and actions through a series of discrete behaviors, each of which are dependent on the correct execution of the preceding step. This intervention facilitated self-management through the steps of the task analysis for solving word problems by teaching students to attend to the picture and words of each step and check off each step as it was completed. While the use of a task analysis to teach a chained behavior to individuals with disabilities is a common instructional strategy, pictorial self-instruction promotes self-management as it transfers control of discriminative stimuli away from a teacher or another skilled learner to the individual. The results of this study add to the literature supporting the effectiveness of pictorial self-instruction to complete chained tasks for individuals with Mod ID and ASD/ID (e.g., Mechling & Gast, 1997; Diegelmann, 2015; Root, Browder, et al., 2015; Saunders, 2014).

Contribution to current knowledge of how to teach mathematical problem solving. The results of the current study add to the limited research base on how to teach mathematical problem solving to students with ASD/ID. Several studies have used a task analysis and systematic instruction to teach students *how* to solve problems (e.g., Browder, Jimenez, et al., 2012; Browder, Trela, et al., 2012; Jimenez et al., 2008), however only two have provided conceptual strategy instruction that taught students with ASD/ID *when* and *why* to use a strategy to solve problems. For example, a number line has been used to teach students with ASD/ID to “count up” from the first known number

to the final amount to find the unknown variable (Browder, Jimenez, et al., 2012; Browder, Trela, et al., 2012). Although the equation was presented within the context of a word problem, students were only required to develop procedural knowledge of how to use the number line strategy.

In contrast, recent studies have used manipulatives and systematic instruction to facilitate the procedural component of problem solving, but used modified SBI to teach the conceptual process of problem solving to students with ASD/ID. Saunders (2014) taught elementary students with ASD/ID to solve both group and change problems using modified SBI presented using CBVI. Root et al. (2016) used modified SBI to teach elementary students with ASD/ID to solve compare problems. The current study contributes to this emerging research base on the effectiveness of modified SBI to teach conceptual and procedural knowledge of mathematical problem solving to students with ASD/ID. However, this study showed that students can still be successful with less supports and materials that are similar to their grade-level peers (i.e., worksheets) and are able to solve algebraic word problems.

#### Limitations of the Current Study

This study had several limitations. First, students who participated in the study were very familiar with using technology. The SMARTNotebook © app used on the iPad to display the task analysis was very similar to the SMARTboards the students routinely used in their classrooms. This access influenced the participants' learning history, but may not be characteristic of all students in this population.

The materials used in the study were an additional limitation. The word problems all followed a predictable and structured format. Despite each session using novel

worksheets with new word problems, they followed the same formula depending on the problem type with the big group found in the first sentence, small groups found in second/third sentences, and no irrelevant information. Students were only exposed to problems that contained quantities of one through nine. Only the group problem type was taught, so students only discriminated between missing part and missing whole. The technology used in the study (i.e., SMARTnotebook application) adds additional limitations due to the frequency with which the embedded prompting did not work as designed or the application crashed, requiring the iPad to be restarted. This likely had an impact on the frequency with which the participants used the self-prompting features.

A third limitation of the current study was the lack of generalization for missing part problems when stimulus supports were removed. The train and hope approach to generalization (Stokes & Baer, 1977) was not adequate for the target behavior to maintain in that condition. Participants were successful in generalizing missing whole problems when stimulus supports were removed. The results of the current study do not support the assumption the participants would be able to solve missing part problems without the stimulus supports provided during training.

Participant characteristics were an additional limitation. The participants were selected because they demonstrated adequate early numeracy skills on the prescreening measure but were unable to solve missing whole and missing part word problems. These characteristics may not be representative of all middle school students with ASD/ID and should be considered when making generalizations regarding the results.

A final limitation was the limited duration of the current study. The participants did not demonstrate generalization of both problem types. Perhaps explicit training on

generalization would have increased their success on these tasks. It is unknown whether the visual supports could have been completely faded and participants still maintained mastery of both problem types.

#### Recommendations for Future Research

The results of this study lead to several recommendations for future research, including technology-based instruction, modified SBI across additional domains of mathematics, training for generalization, the format of instruction, and social validity measures. The results of this study add to the body of literature citing the effectiveness of constant time delay to teach vocabulary, including academic vocabulary (Browder et al., 2009). Future research with this population should continue to emphasize simultaneously teaching targeted task (e.g., problem solving, inquiry based science, reading comprehension) and the academic vocabulary required for students to have a full understanding of the concepts and procedures.

A second recommendation for future research would be an evaluation of a comprehensive technology-based mathematical problem solving intervention, with emphasis on both mathematical problem solving outcomes as well as self-monitoring and self-instruction. There is a growing body of evidence that modified SBI incorporating a technology platform can increase independence (Root, Browder, et al., 2015) and impart mathematical problem solving skills (Saunders, 2014). While the current study aimed to measure self-initiated prompting, a component of self-monitoring, the technology limited efficacy. Future research that blends teacher-delivered instruction, embedded prompts (including video models) in a technology platform may provide more useful information



on how to teach self-monitoring and increase independence in mathematical problem solving.

A third recommendation for future research would be to expand the targeted domains of mathematics using modified SBI. This study was the first to teach algebraic problems to students with ASD/ID using modified SBI, however all problems were of the group problem type. Future research into teaching students with ASD/ID how to solve change and compare problems with unknown information in the initial or medial position is needed. A recent comprehensive literature review by Spooner et al. (2016) has highlighted the need to also address problem solving related to the data analysis. Future investigations could use the methods outlined in this study to teach students with ASD/ID how to solve word problems based on information provided in charts or graphs.

A fourth recommendation for future research is to plan for generalization using one of the nine methods outlined by Stokes and Baer (1977). The train and hope method, which has been cited as both the most common and least effective (Collins, 2012; Stokes & Baer, 1977), did not prove successful for the participants in this study for the missing part problem type. Future research should consider the use of sequential modification when fading the stimulus supports use modified SBI. This planned fading of supports would allow participants to attend to one component at a time and future research may show that this is a successful approach (Collins, 2012).

A fifth recommendation for future research would be to expand the format and setting of instruction. The current study took place in a one-on-one setting outside of the participants' classroom. Future investigations should explore the use of a small group

format. In addition, the setting of future research could be more inclusive, such as in a general education mathematics class or with same-age peer tutors.

A final recommendation for future research is to refine methods of obtaining social validity data from participants with ASD/ID. The aim of applied behavior analysis is to impart socially significant change. Mathematics problem solving is a socially significant behavior, as it is both a behavioral cusp and requires self-management, a pivotal behavior. It affects both participants and other individuals in their communities (Wolf, 1978). The participants should value intervention's methods and goals as well. Future research should focus on methods to obtain valuable social validity data from participants with ASD/ID that considers their learning histories and communication abilities.

#### Recommendations for Practice

The findings of this study provide several areas of practical implications related to vocabulary, use of student-accessible task analyses, and modified SBI. Practitioners should consider the vocabulary demands of mathematical problem solving tasks. In order for students to develop conceptual understanding and be able to understand the procedural directions provided by the instructor, they need to understand the vocabulary terms used. Constant time delay is an evidence-based practice for teaching vocabulary (Browder et al., 2009). Based on the results of this study, instructors should use this instructional technique to teach mathematics vocabulary terms. While it is important for students to understand vocabulary used in mathematics problem solving tasks, it is not necessarily that they demonstrate mastery of these terms prior to beginning instruction in

problem solving. Rather, as this study demonstrated, it is effective to teach the vocabulary and problem solving simultaneously.

A second implication for practice is that providing students with accessible task analyses can improve self-monitoring and independence. Many academic tasks are chained, meaning that in order for them to be executed correctly, students must correctly complete a series of discrete skills in a prescribed order. Mathematics problem solving is a chained task. Task analytic instruction has a history of success in teaching students to complete mathematical problem solving tasks (Neef et al., 2013; Root, Browder, et al., 2015; Saunders, 2014). Breaking down the steps of solving a word problem into a task analysis that is provided to the student in an accessible format, for example using pictures paired with words and using technology to provide read alouds, is one way that practitioners can encourage self-instruction. Making the steps of the problem available to the student, increases independence, as the student can self-manage behaviors and even self-correct when a mistake has been made. The process of evaluating a plan and making corrections along the way is essential to developing problem solving skills (Van de Walle et al., 2012). Practitioners should incorporate the use of task analyses in accessible formats into mathematics instruction. Students will not only make gains in solving targeted mathematics problems, but also will also increase independence and learn self-monitoring skills that may transfer to other areas. However, the successful use of task analysis and chaining as an instructional procedure goes beyond providing the student with the task analysis, as the method of instructing students through each step must be considered.

This leads to a third implication for practice, which is that modified SBI can provide students with both the conceptual and procedural knowledge necessary to complete mathematics problem solving tasks. Modified schema-based instruction is a strategy with emerging evidence of promise for individuals with ASD/ID (Root, Browder, et al., 2015; Saunders, 2014). In modified SBI, established evidence-based practices such as systematic instruction (e.g., system of least prompts and a chaining procedure) and use of manipulatives, are added to explicit instruction to teach the students how to complete the steps of the task analysis and solve the word problem. To induct the procedure, the instructor provides modeling with numerous opportunities for student responding. After several modeling sessions, the instructor then uses a system of least prompts to assist students when they are unsure of what to do. The system of least prompts involves a hierarchy of three levels: verbal prompt (what to do), specific verbal prompt (how to do), and modeling (show). However, unlike other studies that used a system of least prompts in a chained academic task (e.g., Browder et al., 2015; Jimenez, Browder, & Courtade, 2008), if a student made an error the instructor went straight to a model prompt. Practitioners can use modified SBI to teach mathematical problem solving tasks to students with ASD/ID by adding evidence-based practices (e.g., enhanced visual cues, systematic instruction, task analysis) to essential components of SBI (i.e., identification of problem structure, use of visual representations that represent structure of problem, and explicit instruction). The results of this study even show that students may be able to use a number sentence as a graphic organizer, although explicit instruction in generalization will likely be needed before the number sentence can be successfully faded away.

## Summary

This study evaluated the effects of modified SBI on the real world algebraic problem solving skills of middle school students with ASD/ID. Participants were successfully taught mathematics vocabulary terms that were used within the intervention using constant time delay. Modified SBI was used to teach students to solve word problems that depicted real world situations. The modified SBI included (a) a task analysis paired with pictures in each written step presented on an iPad with embedded verbal and specific verbal prompts as a heuristic for solving the problem, (b) a pre-drawn number sentence that served as a graphic organizer with manipulatives, and (c) explicit and systematic prompting to teach each step of the task analysis, addressing both conceptual and procedural knowledge of word problem solving.

The components of the intervention showed effective in teaching MATHEMATICAL VOCABULARY, MATHEMATICAL PROBLEM SOLVING, and increasing the TOTAL PROBLEMS SOLVED. A functional relation was found between CTD and identification of mathematics vocabulary terms and between modified SBI and independently completing steps of the task analysis for solving word problems. In addition, all three participants were able to solve word problems. Participants were able to generalize their problem solving skills when some of the stimulus supports (i.e., pre-drawn number sentence) were faded for missing-whole problems, but demonstrated additional instruction may be necessary to successfully generalize to missing-part problems.

The participants in the current study reaped several benefits from their involvement. The targeted skills, including mathematics vocabulary, mathematical

problem solving, and self-monitoring through self-initiated prompting, have benefits to the students that go beyond the immediate study. Mastery of the targeted vocabulary terms will benefit students across their mathematics instruction, as these are common terms used across domains of mathematics. Mathematical problem solving is a behavioral cusp (Bosch & Fuqua, 2001; Rosales-Ruiz and Baer, 1997) as it has immediate benefits to current environments as well as potential to influence future opportunities. Finally, self-monitoring through self-initiate prompting is a pivotal behavior that can be potentially applied to multiple contexts and positively influence subsequent learning and behavior.

High quality mathematics instruction has an impact on opportunities and choices of individuals with ASD (Wang, 2013; Wei et al., 2015). The “school effects” of mathematics differentiate problem solving from other skills targeted in the school setting, such as literacy, particularly for students with ASD/ID. Algebraic reasoning and problem solving skills are critical to success in later mathematics, and therefore should be addressed early and often throughout the educational career of students with ASD/ID. The results of this study add to the evidence base of instructional practices for teaching mathematics to students with ASD/ID. Considering the current limited number of high quality studies that have taught algebra or problem solving to this population (Browder et al., 2008; Spooner et al., 2016), results could have significant implications for future mathematical instruction for students with ASD/ID, and therefore their future opportunities and choices.

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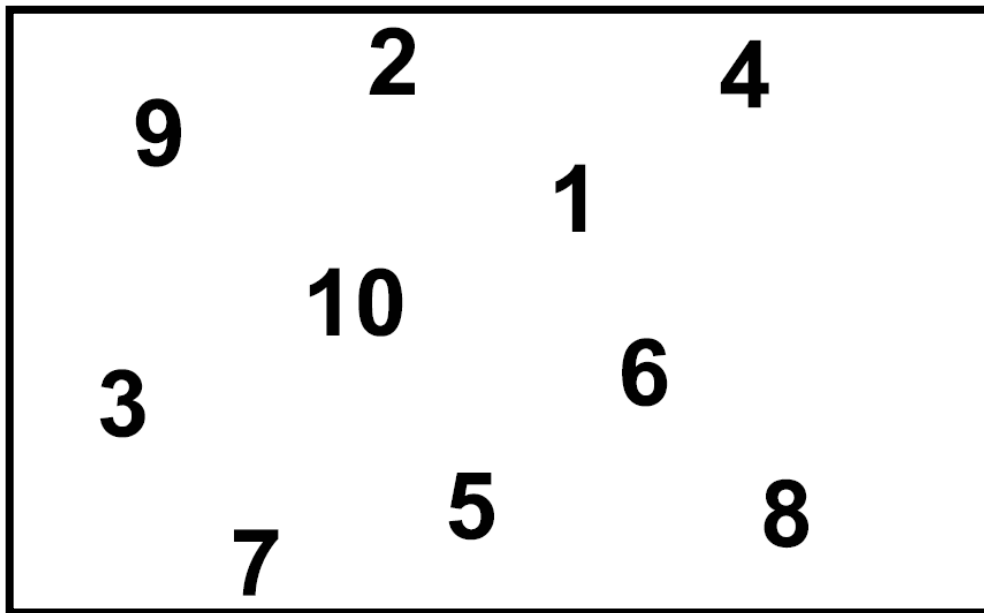
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## APPENDIX A: PRESCREENING TOOL

Receptive first: "Show me \_\_\_\_." State numerals in random order.

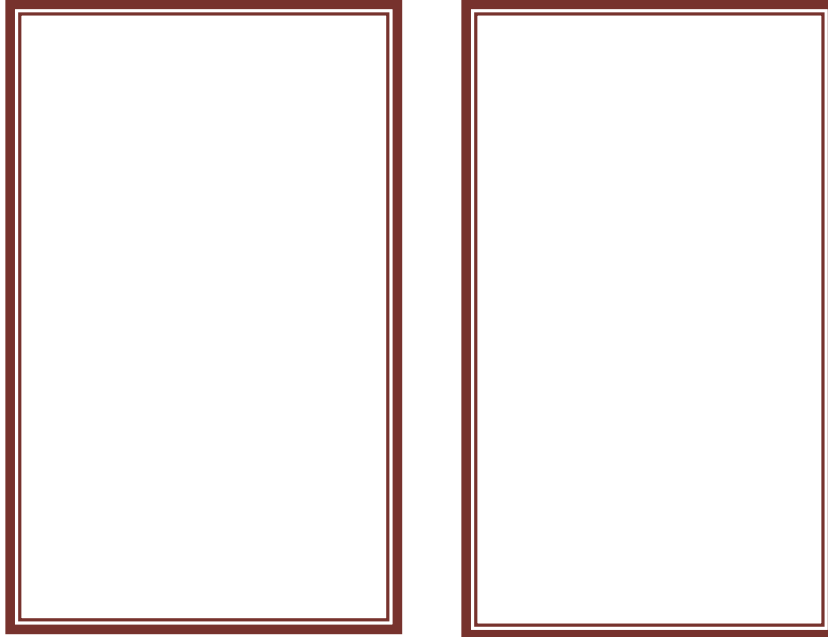
Expressive: Let S read all numbers aloud independently. If S needs a starter prompt, say "when I point to a number, say its name."



Put manipulatives in each box. R side should have more.

**"Point to the box that has more."**

Remember to praise students for answering. Record response.

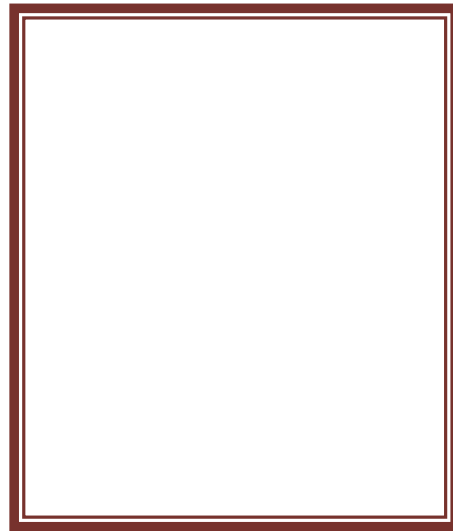
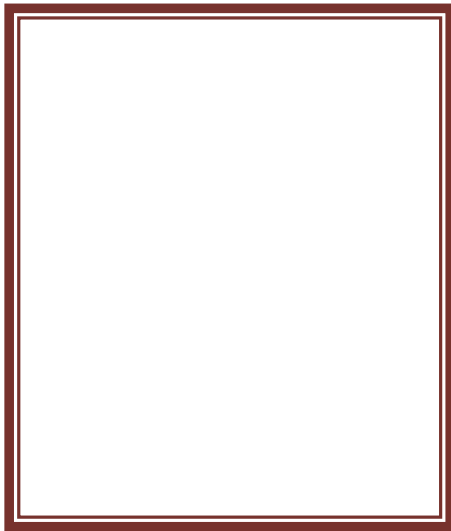




Put manipulatives in each box. R side should have more, L side has fewer.

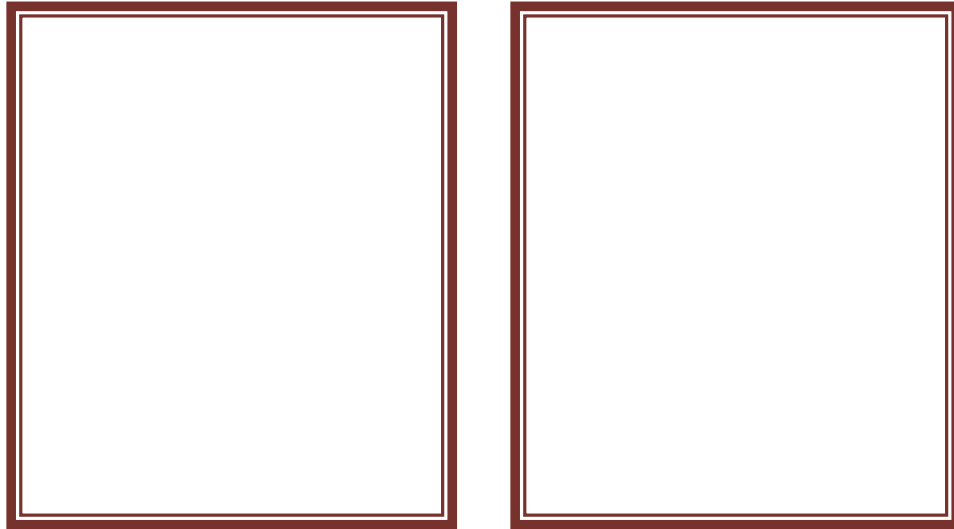
**“Point to the box that has fewer.”**

Remember to praise students for answering. Record response

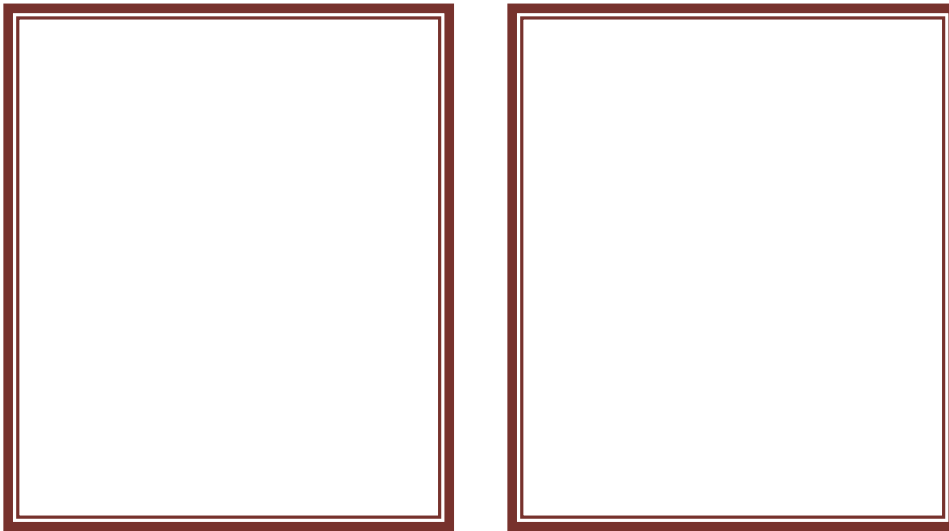


Draw (nonmoveable) circles in each box with fewer in L.  
Point to the first box and ask, “Does this box have more or fewer?”

Remember to praise students for answering. Record response.



Draw (nonmoveable) circles in each box with more in R.  
Point to the second box and ask, “Does this box have more or fewer?”  
Remember to praise students for answering. Record response.



Point to the last line of the problem. Say “Can you write “rainy days” in the box?”

Lee is graphing the rainy days for this month.



There have been \_\_\_\_ rainy days this month.

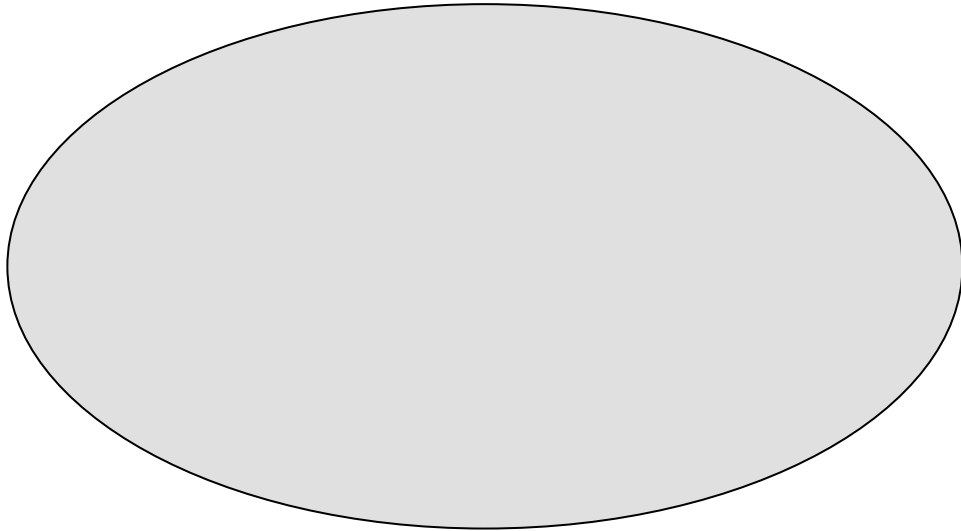


This week there were \_\_\_\_ more rainy days.

How many rainy days have there been in all?

--

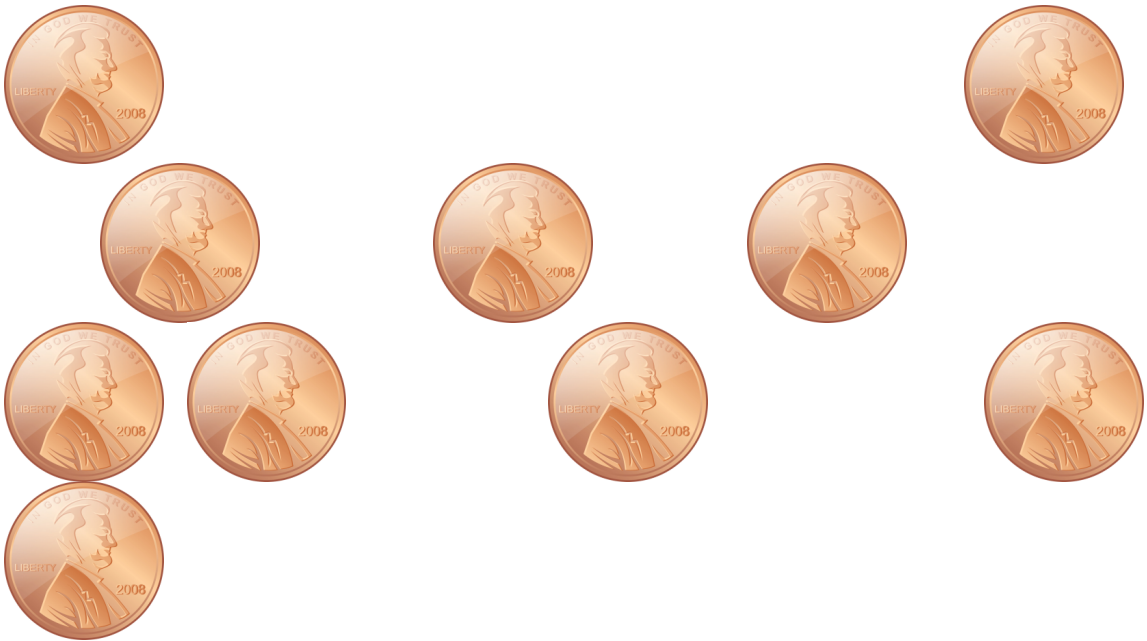
Place manipulatives in set (three, nine). Say, "Count."



**“Count the pennies.”** Remember to praise students for answering.

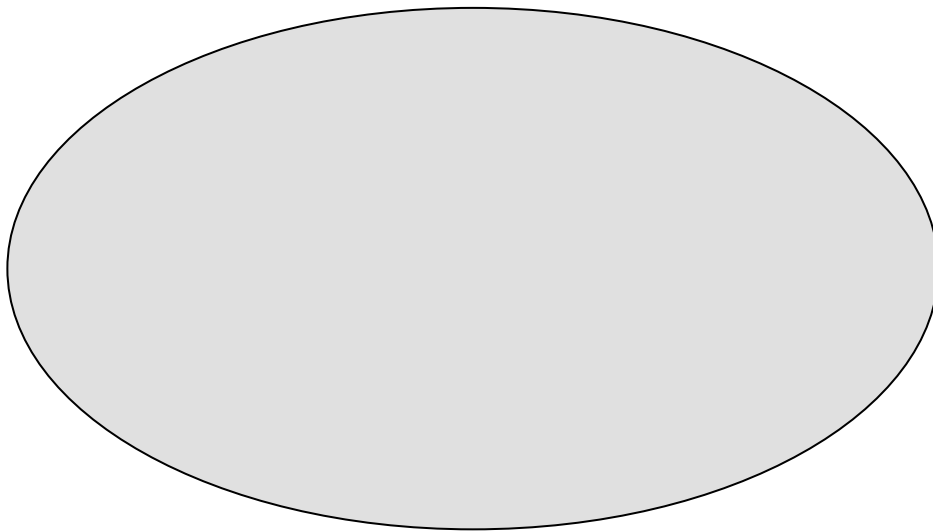


**“Count the pennies.”** Remember to praise students for answering.



*Model:* Place 11 counters in front of the student. Say, “**Watch me make a set of three in the circle.**” Drag from pile and count with 1:1 correspondence. Clear set maker and then say, “**Your turn. Make a set of five.**”

Clear set maker and then say, “**Now make a set of seven.**” Clear set maker and then say “**Now make a set of ten.**”





Cover top/bottom equation. Say, **“Read this equation.”** Move to next equation and cover first.

$$4 + 1 = \boxed{5}$$

$$4 - 1 = \boxed{3}$$

*Model:* Lay out 11 counters. Say, **“Watch me solve the problem.”** Point to five, say **“5,”** then make 1st set counting aloud with 1:1 correspondence. Point to plus and three while reading **“plus three.”** Then make 2<sup>nd</sup> set counting aloud with 1:1 correspondence. Point to equals, say **“equals,”** then slide all counters to last circle and count with 1:1 correspondence. Then read number sentence, **“five plus three equals (write answer) eight.”**

$$5 + 3 = \square$$

$$+ =$$

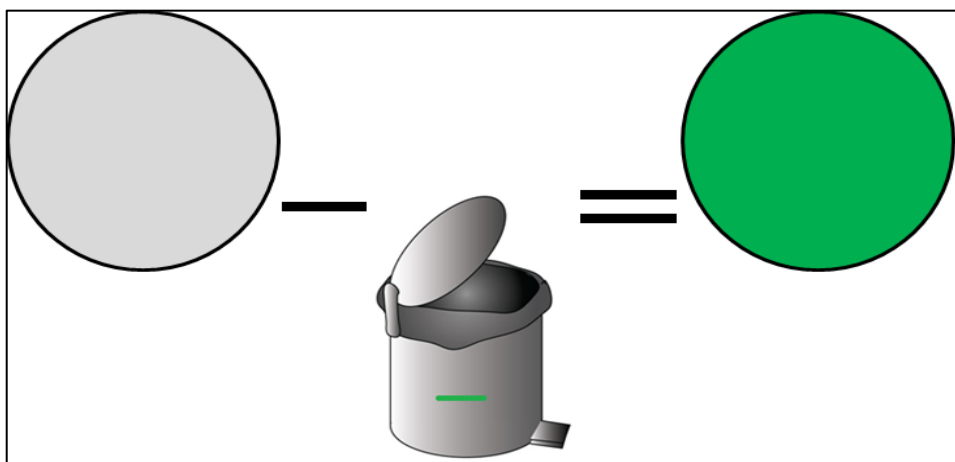
Give students 11 counters. Say, **"Your turn. Solve the problem."**

$$3 + 4 = \square$$

$$+ =$$

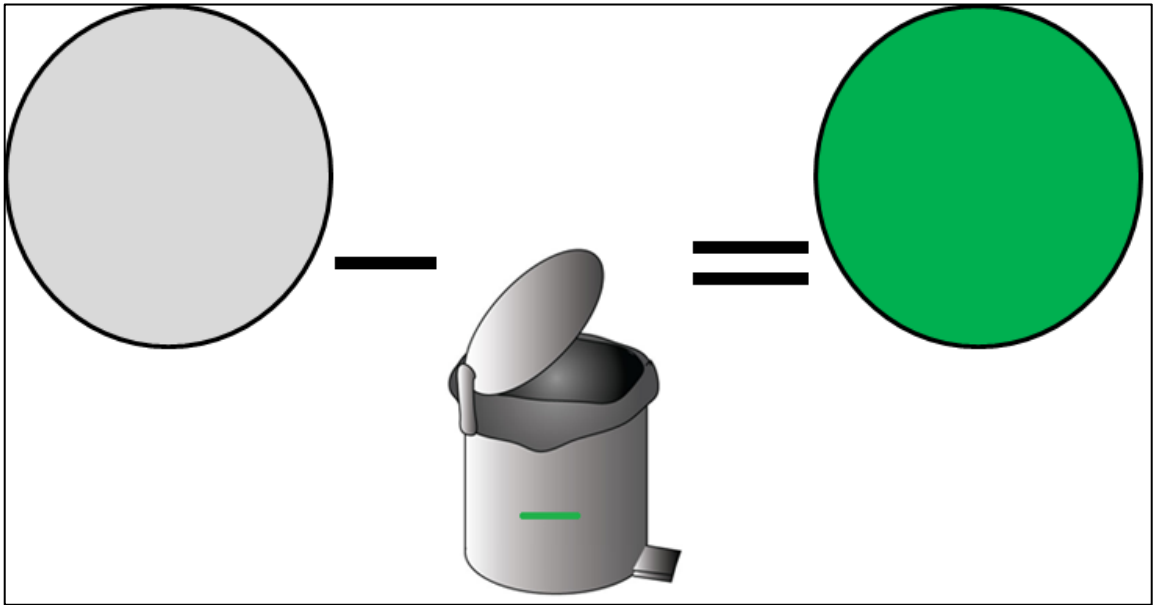
*Model:* Lay out 11 counters. Say, **“Watch me solve this problem.”** Point to eight, say **“8,”** then make 1st set counting aloud with 1:1 correspondence. Point to minus and three while reading **“minus three.”** Then remove counters from first set to trashcan counting with 1:1 correspondence. Point to equals, say **“equals,”** then cover counters in trashcan with right hand and slide all counters to last circle using left hand, then count with 1:1 correspondence. Then read number sentence, **“eight minus three equals (write answer) five.”**

$$8 - 3 = \square$$

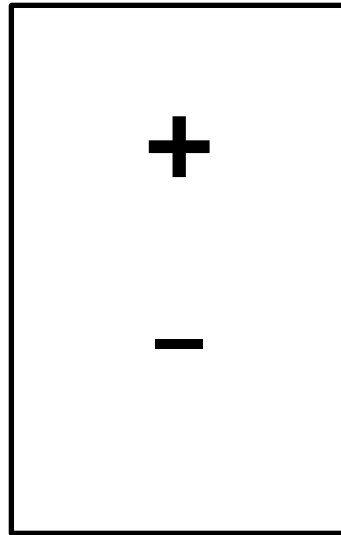


Give students 11 or more counters. Say, **"Your turn. Solve this problem."**

$$7 - 5 = \square$$



Point to the box on the right. Say, “**Show me plus.**” Repeat for “**minus.**” If no response, say “**take away**”.



Cover bottom equation. Point to the circle. "Look! The math sign is missing missing." Read number sentence. "Two plus two equals four. The plus sign is missing. Can you draw a plus?"

$$\boxed{2} \bigcirc \boxed{2} = \boxed{4}$$

Cover top equation. Point to the circle. "Look! The math sign is missing here too." Read number sentence. "Three minus two equals one. The minus sign is missing. Can you draw a minus?" If student only knew "take away," replace with "take away."

$$\boxed{3} \bigcirc \boxed{2} = \boxed{1}$$

Say, "I am going to read some word problems and ask you to solve them. If you cannot solve them, it is ok. Just try your best." Read problem, and restate instructional cue if need be "Solve this problem."

Andy went to the school dance with his friends.

The DJ played 5 country music songs.

The DJ played 3 rap songs.

How many songs did the DJ play?



Say, "I am going to read some word problems and ask you to solve them. If you cannot solve them, it is ok. Just try your best." Read problem, and restate instructional cue if need be "Solve this problem."

Andy danced with 8 people at the school dance.

Andy danced with 3 girls and some boys.

How many boys did Andy dance with?

Say, "I am going to read some word problems and ask you to solve them. If you cannot solve them, it is ok. Just try your best." Read problem, and restate instructional cue if need be "Solve this problem."

Andy bought snacks for friends at the basketball game.

He bought 7 buckets of popcorn.

He bought 2 candy bars.

How many snacks did Andy buy?

Say, "I am going to read some word problems and ask you to solve them. If you cannot solve them, it is ok. Just try your best." Read problem, and restate instructional cue if need be "Solve this problem."

Andy packed 9 drinks for his soccer team.

Andy packed 4 waters and some juice boxes.

How many juice boxes did Andy pack?



## APPENDIX B: DATA COLLECTION INSTRUMENTS

Date:              Data Collector:              Phase: BL IV M      IOA: Yes / No      Video: Yes / No

Student: 1 2 3 4      Worksheet #:

Step	Behavior	IC	V	SV	M	EC	PF
10. Write answer	Writes numeral in box with label	10	10	10	10	10	
9. Solve	Missing whole; puts two sets together and counts, Missing Part; counts up from first small group to whole amount	9	9	9	9	9	
8. Make sets	Missing whole; makes a set under each small group, Missing Part; makes a set under the first small group	8	8	8	8	8	
7. + or -	Writes/selects "+" in # sentence	7	7	7	7	7	
6. Use my rule	States rule	6	6	6	6	6	
5. Fill-in equation	Writes numerals in equation	5	5	5	5	5	
4. Circle the numbers	Circles #s in word problem	4	4	4	4	4	
3. Label equation	Labels b, s, and s	3	3	3	3	3	
2. Circle the groups	Circles nouns	2	2	2	2	2	
1. Read the problem	S has T read problem	1	1	1	1	1	
___ Missing part	TOTAL						
___ Missing whole	CORRECT:						
	TOTAL PROMPTS (SELF/TEACHER)						
Step	Behavior	IC	V	SV	M	EC	
10. Write answer	Writes numeral in box with label	10	10	10	10	10	
9. Solve	Missing whole; puts two sets together and counts, Missing Part; counts up from first small group to whole amount	9	9	9	9	9	
8. Make sets	Missing whole; makes a set under each small group, Missing Part; makes a set under the first small group	8	8	8	8	8	
7. + or -	Writes/selects "+" in # sentence	7	7	7	7	7	
6. Use my rule	States rule	6	6	6	6	6	
5. Fill-in equation	Writes numerals in equation	5	5	5	5	5	
4. Circle the numbers	Circles #s in word problem	4	4	4	4	4	
3. Label equation	Labels b, s, and s	3	3	3	3	3	
2. Circle the groups	Circles nouns	2	2	2	2	2	
1. Read the problem	S has T read problem	1	1	1	1	1	
___ Missing part	TOTAL						
___ Missing whole	CORRECT:						
	TOTAL PROMPTS (SELF/TEACHER)						

Key: / teacher provided prompts, o self-initiated prompts

Step	Behavior	IC	V	SV	M	EC	PF
10. Write answer	Writes numeral in box with label	10	10	10	10	10	
9. Solve	<u>Missing whole</u> : puts two sets together and counts, <u>Missing Part</u> : counts up from first small group to whole amount	9	9	9	9	9	
8. Make sets	<u>Missing whole</u> : makes a set under each small group, <u>Missing Part</u> : makes a set under the first small group	8	8	8	8	8	
7. + or -	Writes/selects "+" in # sentence	7	7	7	7	7	
6. Use my rule	States rule	6	6	6	6	6	
5. Fill-in equation	Writes numerals in equation	5	5	5	5	5	
4. Circle the numbers	Circles #s in word problem	4	4	4	4	4	
3. Label equation	Labels b, s, and s	3	3	3	3	3	
2. Circle the groups	Circles nouns	2	2	2	2	2	
1. Read the problem	S has T read problem	1	1	1	1	1	
___ Missing part ___ Missing whole	TOTAL CORRECT:						
	TOTAL PROMPTS (SELF/TEACHER)						
Step	Behavior	IC	V	SV	M	EC	PF
10. Write answer	Writes numeral in box with label	10	10	10	10	10	
9. Solve	<u>Missing whole</u> : puts two sets together and counts, <u>Missing Part</u> : counts up from first small group to whole amount	9	9	9	9	9	
8. Make sets	<u>Missing whole</u> : makes a set under each small group, <u>Missing Part</u> : makes a set under the first small group	8	8	8	8	8	
7. + or -	Writes/selects "+" in # sentence	7	7	7	7	7	
6. Use my rule	States rule	6	6	6	6	6	
5. Fill-in equation	Writes numerals in equation	5	5	5	5	5	
4. Circle the numbers	Circles #s in word problem	4	4	4	4	4	
3. Label equation	Labels b, s, and s	3	3	3	3	3	
2. Circle the groups	Circles nouns	2	2	2	2	2	
1. Read the problem	S has T read problem	1	1	1	1	1	
___ Missing part ___ Missing whole	TOTAL CORRECT:						
	TOTAL PROMPTS (SELF/TEACHER)						
Total steps independent correct:							
Total self-initiated prompts:							
Total teacher prompts:							
Total word problems solved:							

## Math Vocabulary Data Sheet

Participant: 1 2 3 4

Date:															
Data Collector:															
Video:															
IOA:															
Pf:															
Phase:	Base / Int		Base / Int		Base / Int		Base / Int		Base / Int		Base / Int		Base / Int		
	trial 1	trial 2	trial 1	trial 2	trial 1	trial 2	trial 1	trial 2	trial 1	trial 2	trial 1	trial 2	trial 1	trial 2	
Add															
Subtract															
Equal															
Number s.															
Label															
TOTAL IC:	/ 10		/ 10		/ 10		/ 10		/ 10		/ 10		/ 10		
+ = Independent correct    - = incorrect    p = prompted correct (waited)															

Date:														
Data Collector:														
Video:														
IOA:														
PF:														
Phase:	Base / Int		Base / Int		Base / Int		Base / Int		Base / Int		Base / Int		Base / Int	
	trial 1	trial 2	trial 1	trial 2	trial 1	trial 2	trial 1	trial 2	trial 1	trial 2	trial 1	trial 2	trial 1	trial 2
Add														
Subtract														
Equal														
Number s.														
Label														
TOTAL IC:	/ 10		/ 10		/ 10		/ 10		/ 10		/ 10		/ 10	
+= Independent correct    - = incorrect    p = prompted correct (waited)														

## APPENDIX C: SOCIAL VALIDITY QUESTIONNAIRES

### Student Perception of Problem Solving

Student: \_\_\_\_\_ Interviewer: \_\_\_\_\_ Date: \_\_\_\_\_

“I have some questions to ask you. I want to see how you felt about the math we did together.”

- |    |   |     |    |
|----|---|-----|----|
| 1. | I like doing math.  | Yes | No |
| 2. | I like solving word problems.   | Yes | No |
| 3. | I like to use technology (like an iPad) to help me learn                  | Yes | No |
| 4. | I am good at math.  | Yes | No |
| 5. | I am good at solving word problems.                                       | Yes | No |
| 6. | I like to use manipulatives (like blocks or tiles) to solve math problems | Yes | No |
| 7. | I know how to ask for help with my school work.                           | Yes | No |



## Teacher Perception of Problem Solving

Teacher: \_\_\_\_\_ Interviewer: \_\_\_\_\_ Date: \_\_\_\_\_

- |    |  |     |    |
|----|--|-----|----|
| 1. | My student likes math.   | Yes | No |
| 2. | My student is currently able to solve one-step addition word problems.       | Yes | No |
| 3. | My student is currently able to solve algebraic equations.                   | Yes | No |
| 4. | My student likes to use the iPad to learn.                                   | Yes | No |
| 5. | My student is able to work independently.                                    | Yes | No |
| 6. | My student knows how to ask for assistance.                                  | Yes | No |
| 7. | My student is able to use manipulatives appropriately to solve math problems | Yes | No |

APPENDIX D: DISTINCTIONS BETWEEN CURRENT STUDY AND WORK FROM  
THE SOLUTIONS PROJECT

Component	Current Study	The Solutions Project
Steps of task analysis	<ol style="list-style-type: none"> <li>1. Read the problem</li> <li>2. Circle the groups</li> <li>3. Label equation</li> <li>4. Circle the numbers</li> <li>5. Fill-in equation</li> <li>6. Use my rule</li> <li>7. + or –</li> <li>8. Make sets</li> <li>9. Solve</li> <li>10. Write answer</li> </ol>	<ol style="list-style-type: none"> <li>1. Read the problem</li> <li>2. Circle the “what”</li> <li>3. Find label in question</li> <li>4. Same, different, more/fewer</li> <li>5. Use my rule</li> <li>6. Choose GO</li> <li>7. Circle the numbers</li> <li>8. Fill-in number sentence</li> <li>9. + or –</li> <li>10. Make Sets</li> <li>11. Solve and write answer</li> </ol>
Format of task analysis	<ul style="list-style-type: none"> <li>-Presented on an iPad 3 using SMARTNotebook® application</li> <li>-Students “check off” each step by dragging a check onto box</li> </ul>	<ul style="list-style-type: none"> <li>-Laminated paper</li> <li>-Students “check off” each step by using a dry erase marker</li> </ul>
Student materials	<ul style="list-style-type: none"> <li>-Worksheet containing word problems with a structured equation available as a stimulus prompt</li> </ul>	<ul style="list-style-type: none"> <li>-Laminated word problems</li> <li>-“Problem solving mat” consisting of area to put word problem, structured number sentence, area to circle “same, different, more/fewer”, and designation for laminated graphic organizer</li> </ul>
Graphic organizer	<ul style="list-style-type: none"> <li>-Students use space under structured equation to create sets, using it as a graphic organizer or stimulus prompt</li> </ul>	<ul style="list-style-type: none"> <li>-Pre-drawn graphic organizer with visual supports</li> </ul>
Instruction	<ul style="list-style-type: none"> <li>-3 sessions of modeling using active student responding</li> <li>-System of least prompts embedded within electronic task analysis provided by instructor or student</li> <li>-Model prompt by instructor when error occurs</li> </ul>	<ul style="list-style-type: none"> <li>-3 sessions of modeling using active student responding</li> <li>-System of least prompts provided by instructor</li> <li>-Model prompt by instructor when error occurs</li> </ul>
Targeted skills	<ul style="list-style-type: none"> <li>-Solving group problems with unknown in final or medial position</li> </ul>	<ul style="list-style-type: none"> <li>-Solving group, change, and compare problems with unknown always in the final position</li> </ul>
Word problems	<ul style="list-style-type: none"> <li>-Group word problems with</li> </ul>	<ul style="list-style-type: none"> <li>-Group, change, and compare</li> </ul>

	the unknown in both medial and final position. -No pictures	problems with unknown in final position -Pictures above “whats” or key nouns
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