DEVELOPING A FRAMEWORK FOR THRESHOLD CONCEPTS IN ELEMENTARY MATHEMATICS METHODS COURSES

by

Courtney Glavich Mayakis

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Approved by:

____
Dr. David Pugalee

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Dr. Michelle Stephan

____
Dr. Luke Reinke

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Dr. Bettie Ray Butler

____
Dr. Amy Good

ABSTRACT

COURTNEY GLAVICH MAYAKIS. Developing a framework for threshold concepts in elementary mathematics methods courses.
(Under the co- direction of DR. DAVID K. PUGALEE and DR. MICHELLE STEPHAN)

Teacher education programs have long sought to increase preservice teachers' competency through coursework that promotes content, pedagogical knowledge, and specialized forms of knowledge that interweave the two. Still, newly licensed teachers are entering K-12 institutions ill-equipped to provide mathematics instruction that is both impactful and meaningful. This problem has persisted consistently throughout elementary education as teacher candidates enter the classroom without the conceptual knowledge to teach mathematics effectively (National Research Council, 2003; Ryan & Williams, 2007; Stacey, et al., 2001). This dissertation aims to understand what standards and frameworks are available for teacher educators to utilize in the planning of their instruction, how teacher education programs in the primary grades are structured, and what elementary professors believe to be key concepts and tasks in the development of teacher candidates. From this information, along with a quantitative study that explicated teacher effectiveness from the various public institutions in this particular state in elementary mathematics, a case study was conducted to understand how one particular professor was developing teacher candidates. Based on this research, a conceptual framework was developed to illustrate what content, pedagogical content knowledge, instructional tasks and best practices this professor utilizes throughout her teaching. This conceptual framework illustrates the idea of developing teacher candidates' knowledge of an overarching mathematical concept to allow application of this knowledge to other singular units of mathematical content and pedagogical knowledge. This new approach of developing threshold concepts opens mathematics education to overarching key concepts in elementary teacher education, or what is referred to throughout this dissertation as threshold concepts. These threshold concepts connect singular units of curriculum to a larger, overarching concept, that once internalized, would allow teacher candidates the ability for effective application of these concepts.

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CHAPTER I: INTRODUCTION

A Nation at Risk report (National Commission on Excellence in Education [NCEE], 1983), written over 30 years ago, alarmed the general public, educational scholars, practitioners, and even policy makers, and has been a catalyst for educational reform and changes in policies regarding the United States' educational system. This report highlighted the rapid decline of student achievement and the failure of the United States to adequately prepare students to enter a changing workforce. It also asserted that the quality of teachers entering the profession in the United States was inadequate in comparison to that of other advanced nations (NCEE, 1983). However, the efforts by stakeholders to change the narrative of American education in response to this report has not worked. Reforms have increased accountability, such as standardized testing and teacher evaluations, but have not substantially increased the quality of teachers and the rigor of curriculum (Darling-Hammond, 2010). Some even argue that these reforms have had a detrimental impact on our present-day education system (Darling-Hammond, 2010).

According to data from the U.S Department of Education (2014), American high school seniors perform at proficient levels in mathematics but lack the interest to pursue careers related in this field, consequently preventing them from entering college in areas of study associated with the discipline (National Academies of Science, 2006). Darling-Hammond (2010) has said, "We cannot just bail ourselves out of this crisis. We must teach our way out" (p. 3), and in order to be proactive this teaching must begin at the elementary level in order to prepare students for the 21st century (American Association of Colleges for Teacher Education [AACTE], 2007; National Academies of Science, 2006; National Research Council [NRC], 2006). National and international assessments continue to illustrate that our country is not providing rigorous mathematics preparation in K-12 schools.

The root of this issue begins with the mathematical knowledge obtained at the elementary level, and therefore the level of mathematics instruction that is afforded to elementary students. With the knowledge that teacher effectiveness is linked to student achievement (Darling-Hammond, 2000; Darling-Hammond & Youngs, 2002; Hill, Rowan & Ball, 2005; Wayne & Youngs, 2003), we must ensure that teachers are adequately equipped with the content and pedagogical content knowledge (PCK) to effectively teach mathematics in the primary grades. This starts with elementary education teacher preparation programs. However, with these programs focusing on all content areas, it is often difficult to warrant the time needed to teach all elementary mathematical content, PCK and instructional practices. With these programs utilizing disconnected standards that lack the cohesive relationships between content, PCK, and instructional practices, it is difficult to ensure that elementary preservice teachers (PTs) are obtaining the conceptual depth linked to these understandings in their undergraduate programs in order to become successful K-5 mathematics teachers (Ball & Bass, 2000).

Statement of the Problem

Some policies and newly constructed standards have attempted to enhance educational programs to ensure proficiency in K-12 mathematics, including No Child Left Behind (NCLB) and the Common Core State Standards (Hanushek, Peterson, & Woessmann, 2014). However, very few policies, frameworks, and/or standards have addressed mathematics education for preservice teachers in the elementary grades that links content and pedagogy. The Council for the Accreditation of Educator Preparation (CAEP) K-5 Elementary Teacher Standards is the leading set of K-5 teacher educator standards used currently (general standards). Standard 2 within the CAEP standards addresses content in all four subject areas. Below is the written standard for mathematics for teacher educators:

> Candidates demonstrate and apply understandings of major mathematics concepts, algorithms, procedures, applications and mathematical practices in varied contexts, and connections within and among mathematical domains (Number and Operations, Operations and Algebraic Thinking, Measurement and Data [both Statistics and Probability] and Geometry) as presented in the rationale for the CAEP Mathematics Content for Elementary (K-5) Teachers. (CAEP, 2015, p. 2)

Even though this CAEP standard lists the four domains associated with mathematics in the primary grades, the specific concepts and ideas under each domain are unspecified and fail to link these content domains with the pedagogical knowledge necessary for each. Even though the Association of Mathematics Teacher Educators (AMTE) has recently introduced a new framework coupled with standards and instructional practices to elementary mathematics teacher education, it is not necessary for programs to utilize them, unlike CAEP. However, even with AMTE addressing these concepts, we are tasked with teaching multiple content areas to elementary education teachers. If we do not have the time to teach everything in K-5 mathematics, then what do we teach? What are the high impact mathematical topics?

Purpose

The purpose of this study is to identify critical foundational mathematical topics that combine content, pedagogical, and instructional practices, which when understood conceptually, can advance a preservice teacher's thinking. These critical foundational topics should link smaller units of interrelated curriculum in order for PTs to gain optimal understanding of the connectedness of these units of curriculum.

Student Achievement and Teacher Quality

There are several variables that correlate with K-12 student achievement in mathematics, including teacher quality and consequently teacher preparation (Darling-Hammond, 2000; 2015; Hill, Rowan & Ball, 2005). It is well documented that elementary mathematics teachers in the United States often lack the content and pedagogical knowledge to be successful in the classroom (Mewborn, 2001; Ma, 1999), hindering their students' academic achievement. Creating a guiding framework that focuses equally on content, pedagogy, and instructional supports and provides a connection between these types of knowledge could assist in the lack of conceptual knowledge that many elementary mathematics teachers often experience when entering the classroom and could result in an efficient and fundamental tool for teacher educators. In elementary preservice education, there are many content areas and aspects of teaching that are required within preservice teacher programs. Due to this, many universities only have time to provide instruction related tone or two elementary mathematics content and pedagogical course(s). This tool could be utilized by teacher educators to discern key concepts from areas of content that are not as essential for PTs to understand and be effective in the classroom

The Research Questions

The three manuscripts and subsequently the conclusion compiled in this dissertation (Chapters 2-5) will answer the overarching research questions:

1. What key mathematical concepts do K-5 teacher educators focus on in their

practice?

2. What are the key instructional supports that assist in the development of the above mathematical concepts?

These questions will be answered through the use of the research questions in each individual manuscript.

In the first manuscript (Chapter 2) entitled *What Should We Teach in Elementary Mathematics Methods Courses? The Argument for Core Pedagogical Content Knowledge in K-5 Mathematics Teacher Preparation* gives an overview of the available frameworks and standards that exist as a guide to elementary education teacher educators to assist them in the development of teacher candidates. In this manuscript we argue that core concepts in elementary mathematics teacher preparation should be used in order to assist in a cohesive framework for teacher educators.

In the second manuscript (Chapter 3) entitled *Elementary Teacher Preparation Programs: An Exploration of Components that Matter*, we utilize key elementary mathematics preservice teacher assignments by creating a general overview of these assignments collected from current teacher educators in a particular state, and analyze these assignments through the lens of practice-based teacher education, as well as utilize self-assessment questions answered by the professors as to why the specific assignments were chosen. This aspect of the study will be guided by the following research questions:

1. What do teacher educators believe best prepare teacher candidates for the classroom?
2. How do teacher educators best prepare teacher candidates?

In the third manuscript (Chapter 4) entitled *How Should We Develop Teacher Candidates? An Exemplar from a K-2 Mathematics Methods Course*, one professor was selected from a data set utilizing quantitative data that demonstrates that the students whose teachers graduate from this professor's particular program show significantly higher growth on their end of grade assessments than their peers in this particular state in elementary mathematics (i.e. exceeded growth as a K-5 student). Through interviews and observations, we identified and analyzed how this professor instructs PTs, as well as how the professor's instruction integrates content, pedagogy, instructional tasks, and other concepts and frameworks. The research questions for this component of the study are:

1. How is the integration of content, pedagogy, instructional tasks and other concepts and frameworks displayed in a professor's mathematics instruction?
2. What key concepts are utilized in the professor's teaching?

Finally, to conclude the studies in manuscripts two and three, we introduce the idea of threshold concepts. Threshold concepts, a new idea to mathematics education, can be linked to similar conceptual ideas about learning and teaching that currently exist. This particular conceptual framework seeks to link the often-disconnected relationship between pedagogy and content in teacher education, and answers the following research question:

1. How can the relationships between pedagogical content knowledge, instructional tasks and other key concepts in teacher education result in in-depth understanding of a threshold concept?

This chapter accomplishes this by detailing a framework that teacher educators can utilize for planning mathematics instruction by highlighting the relationship between singular units of pedagogical content knowledge to a larger overarching concept (the threshold concept) and how this knowledge can be supported through instructional tasks and other guiding concepts. It introduces the need for threshold concepts by highlighting the disconnect between important types of teacher knowledge and contextualizes threshold concepts through the lens of base 10 in order to demonstrate how the notion of linking pedagogy, content, and meaningful instructional tasks could assist elementary mathematics PTs in understanding and practicing the mathematics they are charged with teaching.

Setting

In this section I describe the participating teacher educators and the professional development setting in which the initial data were collected.

Participating Teacher Educators

The participating teacher educators in this particular study were current practicing teacher educators and were recruited from one particular state. These teacher educators were selected to participate on the following criteria:

1. Employed at a public university in one particular state.
2. Currently teach a mathematics methods or content course(s) associated with the elementary education program at their respective universities.

These teacher educators were solicited for their participation from the Dean at each of their colleges, as well as the Dean from my university. This data was collected from a smaller subset of data from Project ATOMS (Accomplished Elementary Teachers of Mathematics and Science Project) in conjunction with North Carolina State University. A total of 17 teacher educators participated in the study from 14 unique public universities throughout the state.

Professional Development

The initial data sets were collected in a professional development setting. Teacher educators from the state were invited to attend one of two professional development settings, depending on the convenience of the location. These two professional development settings were held at two different universities in order to ensure maximum participation. This professional development was a full-day session in which teacher educators discussed the key assignments they submitted, as well as the key assignments of other teacher educators.

Data Set

Data collection took place over the course of a semester, and consisted of document collection and analysis, qualitative self-evaluations, observations, and interviews.

Document Analysis

I collected and analyzed a total of 19 mathematics elementary methods and mathematics content course assignments utilized by teacher educators throughout these respective universities. These assignments were given by professors from the universities and were deemed key assignments in the development of their teacher candidates (*note:* some assignments collected were from joint mathematics and science courses and were utilized in this data set).

Qualitative Self-Evaluations

During the professional development, teacher educators were asked specific short answer questions through the use of a Google survey regarding their perception of the assignment they submitted as their *key* assignment in developing mathematics elementary teachers. Teacher educators were asked a series of seven questions, and these responses were analyzed in conjunction with the document analysis of their key assignments. A total of 16 self-evaluations were collected and analyzed.

Interviews

The interviews were completed after the conclusion of the professional development session. One mathematics elementary methods professor, selected from the data set based on quantitative data from a previous study suggesting that that the teacher candidates who graduate from her university produce high growth measures for their K-5 students, was interviewed before and after observations of her classroom teaching. These interviews aimed to understand what methods were utilized in her classroom to intertwine content, pedagogy, and instructional tasks.

Observations

The five observations occurred after the conclusion of the professional development session and were completed in conjunction with the interviews described above with the same professor. The observations of her instruction were documented using a protocol and recorded for fidelity. These observations aimed to support the interviews in understanding how content, pedagogy, and instructional tasks were intertwined in her classroom to support the development of teacher candidates. A protocol was used and developed from the goals of the study to narrow the focus of the observations.

Summary

The aim of this study was to identify an overarching mathematics concept that utilized mathematics pedagogical content in elementary mathematics teacher education, along with the supports needed to develop this concept effectively. I analyzed teacher educators' current assignments given to PTs in elementary mathematics to gain an understanding on what mathematical concepts teacher educators view as important. I administered self-evaluations to reveal the perceptions of teacher educators in conjunction with their key assignments. Finally, one professor from the data sets was selected to interview to gain insight on what she views to be threshold concepts in elementary mathematics preservice teacher education and her instruction was observed to see how these threshold concepts are reflected in teaching practices. The three manuscripts and the conclusion that follows aim to introduce a conceptual framework that will utilize content, pedagogy, and instructional supports effectively to produce a deep conceptual understanding of elementary mathematics concepts,and identify these overarching critical concepts that will enable teacher educators to instruct their PTs in an effective and cohesive manner.

CHAPTER II: WHAT SHOULD WE TEACH IN ELEMENTARY MATHEMATICS METHODS COURSES? THE ARGUMENT FOR CORE PEDAGOGICAL CONTENT KNOWLEDGE IN K-5 MATHEMATICS TEACHER PREPARATION

Courtney Glavich Mayakis & John Williams

Elementary education teachers graduating without in-depth content knowledge as well as the tools and instructional strategies to teach K-5 mathematics effectively has been discussed as an issue in teacher preparation today (National Research Council, 2003; Ryan & Williams, 2007; Stacey, et al., 2001). Even though many teacher education programs face challenges regarding various aspects of their programs, elementary teacher education presents unique challenges to teacher educators. Since elementary teachers are tasked with teaching multiple content areas, teacher education programs serving elementary preservice teachers (PTs) must ensure that they are gaining all the knowledge necessary to be successful teachers. This requires teacher educators to carefully select what they teach PTs in their methods courses, often forcing teacher educators to “cut-out” vital aspects of pedagogical content knowledge. This can have detrimental effects on teacher effectiveness especially as it relates to mathematics.

However, Mewborn (2001) notes that various studies (i.e. Ball & Wilson, 1990; Fuller, 1997; Ma, 1999) that have employed qualitative data methods to understand the complexities of content and teaching practices found that elementary education teachers who were strong in mathematics content were not always strong in conceptual understanding, dismissing the idea that strong content background equates to a strong conceptual background. Fennema and Franke (1992) concluded ‘‘that when a teacher has a conceptual understanding of mathematics, it influences classroom instruction in a positive way’’ (p. 151). Mewborn (2001) asserts that various research studies (i.e. Ball &

Wilson, 1990; Fuller, 1997; Ma, 1999) have suggested that even though elementary education teachers may have an understanding of the algorithms and procedures that are utilized in their curriculum, they do not always have a conceptual understanding of the mathematics and that "their knowledge tends to be compartmentalized and fragmented and, therefore, not easily transferable from one domain to another" (Mewborn, 2001).

However, if a strong background in content knowledge is not enough, how are we developing PTs into effective mathematics teachers who are not only able to teach their content, but also able to understand concepts related to their mathematics curriculum? One thought is that elementary teacher education programs are not providing enough coursework surrounding mathematics in their preservice programs. The American Council on Education (1990) has suggested that more mathematics coursework can be related to increased student learning. However, Monk (1994) discovered that this was true only up to a total of five courses, after which student achievement becomes diminished. Wilkins (2008) suggests that "being good" in mathematics or simply taking more courses in mathematics may not provide PTs with the sufficient background to teach mathematics in their classroom effectively and argues that there may be other factors that influence this. These factors, or the effective teaching practices that elementary education programs should focus on, can be examined through the lens of Shulman's (1986) three types of teaching knowledge.

Schulman's Three Types of Teaching Knowledge

According to Shulman (1986) there are three types of knowledge that teachers must understand in order to be successful:

(1) *Content Knowledge*: Refers to not only the facts and knowledge of a specific

discipline but also the structures of this discipline.

(2) *Pedagogical Content Knowledge*: Refers to the content knowledge and strategies which are specific to teaching that particular content.

(3) *Curricular Content Knowledge*: Refers to the knowledge of a program or set of instructional materials that is required for a specific content for a certain grade level.

The significance of content and pedagogical knowledge in K-5 mathematics is vital to the effectiveness of classroom teachers (Belge, 1979; Cochran, DeRuiter & King, 1993; Kahan, Cooper, & Bethea, 2003). Hill, Ball, and Schilling (2008) contextualize Schulman's three types of teaching knowledge through the lens of effective mathematics instruction. These authors propose that not only do mathematics teachers possess content subject matter, but also have different types of knowledge that are useful for effective mathematics instruction.

Hill et al. (2008), define this other knowledge as KWS or "content knowledge intertwined with knowledge of how students think about, know or learn this particular content" (p. 375). This type of knowledge concentrates on how students learn through a certain topic, and what misconceptions that students may typically have when learning a certain concept. The authors argue that KWS is an extension of Schulman's (1986) PCK or

> an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. (p. 9)

Schulman (1986) supports Hill et al. (2008) by highlighting that understanding student thinking is a key component in pedagogical content knowledge. Ball and Bass (2000) define pedagogical content knowledge as the intertwining of content and pedagogical knowledge and argue that this key component of teacher knowledge is often disconnected throughout instruction and practice. Another component to his knowledge is specialized content knowledge (SCK), or the understanding of the mathematical operations necessary to reach the correct answer for a problem (Ball, Thames, & Phelps, 2008; Hill et al., 2008). With these types of teacher knowledge being crucial in teacher development, are the standards and frameworks used to guide teacher preparation addressing these adequately?

Guiding Standards for Elementary Mathematics Teacher Education

Stakeholders in teacher quality and accreditation of teacher education programs have developed mathematical frameworks and general standards for K-12 teacher education to ensure that PTs are learning critical teaching practices. These mathematical frameworks for initial teacher licensure are based on research and best practices for mathematics education (Association of Mathematics Teacher Educators, 2013; Council for the Accreditation of Educator Preparation, 2015 & National Council of Teachers of Mathematics, 2012). These organizations created reformed mathematical standards for K-12 students that focus on student-centered instruction, shifting from a teacher-oriented instructional environment (Wilson, Mojica & Confrey, 2013). With most of these organizations focusing on K-12 classroom mathematical standards, some are now starting to develop or refine K-12 teacher education standards for mathematics.

Mathematical frameworks for initial teacher preparation were not readily discussed or developed until the 1980s when the National Council for Accreditation of Teacher Education (NCATE), in conjunction with the National Council of Teachers of Mathematics (NCTM), developed the first set of scholarly mathematics teacher preparation standards (NCTM, 1998). According to these program standards, institutions of higher education should respond to these standards if their program includes the mathematics teacher preparation of future mathematics teachers' in

> (1) middle school, (2) junior high school mathematics, and/or (3) senior high mathematics education. A program review for elementary education must be prepared if the program offers an area of concentration in mathematics or an emphasis in mathematics in the preparation of K-4 teachers. (NCTM, 1998, p. 2)

These first set of program/teacher educator standards created by NCTM in 1998, has evolved from a standard checklist, to a set of indicators and standards from several entities that embody rigorous themes related to mathematics instruction and content in teacher education programs (AMTE, 2013; CAEP, 2015; NCTM, 2003).

Several of these entities (AMTE, 2013; CAEP, 2015; NCTM, 2003) capture mathematical frameworks situated in secondary education, as well as some in elementary education mathematics programs. There are two sets of mathematical frameworks or mathematical recommendations that will be synthesized in this review of the literature: The Council for the Accreditation of Educator Preparation (CAEP) and the Association of Mathematics Teacher Educators (AMTE). The Council for the Accreditation of Educator Preparation's (CAEP) K-5 Elementary Teacher Standards is the leading set of K-5 teacher educator standards used currently (general standards). The Association of Mathematics Teacher Educators (AMTE) recently released program standards for teacher preparation for elementary mathematics that allows for a specific outline of essential mathematical standards for elementary PTs.

Council for the Accreditation of Educator Preparation (CAEP) K-5 Elementary Teacher Standards

In an effort to strengthen teacher education programs and restructure the accreditation process, CAEP set forth a new set of standards for K-5 elementary teachers. The standards are intended to trim down the accreditation process and allow programs to focus more on improving their graduates' impact on student achievement (Heafner, McIntyre, & Spooner, 2014). The CAEP K-5 elementary teacher standards consist of five cohesive standards supported by key elements to guide teacher educators on how to implement these standards in their instruction.

Standard 1 consists of understanding and addressing each child's developmental and learning needs (CAEP, 2015). Standard 2 discusses the ability to understand and apply content and curricular knowledge for teaching (CAEP, 2015). Standard 2 is comprised of five key elements as more specific goals within this particular standard. Most of these key elements are aligned with specific content areas. Four of the key elements discuss candidates understanding key components of literacy, mathematics, social studies, and science/engineering practices. In examining the mathematics curricular standards addressed in Standard 2, it is evident that one key element is focused on the major mathematical domains in elementary teaching:

> Candidates demonstrate and apply understandings of major mathematics concepts, algorithms, procedures, applications and mathematical practices in varied contexts, and connections within and among mathematical domains (Number and Operations, Operations and Algebraic Thinking, Measurement and Data [both Statistics and Probability] and Geometry) as presented in the rationale for the CAEP Mathematics Content for Elementary (K-5) Teachers. (CAEP, 2015, p. 2)

However, this is the only portion of the CAEP standards in which mathematical practices and content are explicitly named. The last key element of this standard discusses how candidates should understand,

> developmental and differentiated learning, curricular standards, practices, the language of the disciplines, assessment, and learning progressions as they relate and connect to content knowledge for teaching (CAEP, 2015, p. 2).

Standard 3 explains the need for candidates to develop and adapt meaningful engagement strategies for all learners. Standard 4 contains seven key elements that focus on the candidate supporting the child's learning. Standard 5 particularly lends itself to addressing the professional development of candidates as K-5 elementary teachers. Although the CAEP standards for K-5 elementary teachers are rooted in NCATE philosophies, there is little data demonstrating the extent to which these new standards are impacting teacher education, since these are newly created.

Association of Mathematics Teacher Educators (AMTE)

The Association of Mathematics Teacher Educators (AMTE) is another professional organization that addresses mathematical standards with a focus on recommendations for teacher candidates. AMTE has recently released a comprehensive set of standards used to guide the instruction of all P-12 teacher education programs, as well as minimum requirements regarding coursework. AMTE suggests that 12-semester hours of mathematical education of teachers as well as a minimum of six weeks of statistical education of teachers should be included in P-5 mathematics teacher education (AMTE, 2017). AMTE groups these mathematical standards, for the primary grades, into two sections: early childhood mathematics, and upper elementary preparation.

AMTE distinguishes two separate parts of the early childhood standards: elaborations of the knowledge, skills and dispositions needed by well-prepared beginning early childhood teachers of mathematics (part 1); and elaborations of the characteristics needed by effective programs preparing early childhood teachers of mathematics (part 2). In part 1, AMTE describes four standards that are directly applicable to content, pedagogy, and pedagogical content knowledge: deep understanding of early mathematics (EC.1), positive attitudes towards mathematics and productive dispositions toward teaching mathematics (EC.2), mathematics learning trajectories: paths for excellence and equity (EC.3), and tools, tasks, and talk as essential pedagogies for meaningful mathematics (EC.4). Similarly, to early childhood standards, AMTE also distinguishes between two distinctive parts in upper elementary standards: elaborations of the knowledge, skills, and dispositions needed by well-prepared beginning teachers of mathematics in the upper elementary grades (part 1); and elaborations of the characteristics needed by effective programs preparing mathematics teachers for upper elementary (part 2). Within part 1 of the upper elementary standards, mathematics concepts and connections to mathematical practices (UE.1.), pedagogical knowledge and teaching practices (UE.2.) and tools to build student understanding (UE.3.) are the most closely linked to content, pedagogy, and pedagogical content knowledge in AMTE’s upper elementary standards. Within this set of standards, ATME suggests that content, methods, and clinical experiences are the three characteristics that must be a focus in order for teacher education programs to generate effective teachers.

With this framework, AMTE provides elementary teacher educators with the specific, in-depth standards that the CAEP standards do not provide teacher educators. Even though ATME separates the content and pedagogical knowledge needed for PTs to be successful in early childhood and upper elementary mathematics, they give content specific examples of how these two knowledges are intertwined. In part 1, standards for early childhood and upper elementary education, AMTE details the content that is essential to preparing elementary PTs. Even though they provide specific content examples, they do not discuss how these concepts can be connected to PCK under a broader conceptual idea. They argue that these content exemplars are "specific but not exhaustive examples" (AMTE, 2017, p. 49). AMTE (2017) details a cohesive learning trajectory for subitizing in the early childhood mathematics but does not give an example developmental learning trajectory (LTs) for upper elementary and does not discuss how LTs can be utilized in these grades. AMTE describes tasks in part 2 (programmatic features) that teacher educators could utilize with PTs in regards to PCK in both early childhood and upper elementary mathematics. However, these tasks are not interconnected under overarching frameworks, and do not link to other singular units of key pedagogical content knowledge. Even though these tasks could be helpful to teacher educators, it does not detail how these particular tasks could translate into teacher educators' classrooms (i.e. assignments that would be given, assessment, teaching techniques, etc.), or what successful instruction of these interconnected knowledges (content and pedagogy) would entail.

Guiding Frameworks for Elementary Mathematics Teacher Education

One might raise a question as to whether these standards created for teacher educators are enough to support the learning of PTs mathematical conceptual knowledge in elementary education. In order for PTs to understand and deliver effective mathematics instruction (outputs), effective mathematical knowledge to teach (inputs) must be provided. Even though CAEP and AMTE offer standards, recommendations, and characteristics that can guide the instruction of PTs, instructional frameworks, such as learning trajectories and cognitive guided instruction, also guide the way that teacher educators instruct their PTs. These frameworks are focused on best practices and student thinking and learning and help create a context in which to deliver mathematical standards.

Learning Trajectories

Timmermans (2010) explains, “the most significant aspect of learning lies not in the outcomes of learning, but in the process of learning” (p. 3). However, to capture the efficiency and rigor of the learning process in respective curricula fully, one must understand the key concepts in their curricular specific learning processes or these “trajectories of learning” (Timmermans, 2010, p. 3). Perkins (2007) explains that LT’s are “especially pivotal to a stage-like advance in understanding a discipline” (p. 36). Research shows that knowing and understanding mathematical learning trajectories can assist teachers in better sequencing activities and instructional tasks to fit the needs of their students better (Clements & Sarama, 2004; Daro, Mosher, & Corcoran, 2011; Sztajn, Confrey, Wilson, & Edgington, 2012).

Simon (1995) stated that a "hypothetical learning trajectory" included "the learning goal, the learning activities, and the thinking and learning in which the students might engage" (p. 133). Clements and Samara (2004) mirror this definition by arguing that a learning trajectory consists of three components: the actual learning goal or objective, the developmental progressions of thinking and learning, and sequence of instructional tasks to accomplish this objective. Confrey et al. (2009) states:

> a researcher-conjectured, empirically-supported description of the ordered network of constructs a student encounters through instruction (i.e., activities, tasks, tools, forms of interaction and methods of evaluation), in order to move from informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time. (p. 347)

However, Battista (2004) argues that learning trajectories are sections or layers of conceptual thinking in which students pass from pre-instructional concepts, progressing through various other, inherently more sophisticated concepts, concluding their understanding at a recognized mathematical concept. However, the inherent goal of learning trajectories is to study developmental progressions in conjunction with a sequence of instructional tasks to create an optimal learning environment in order for students to absorb an instructional goal. Learning trajectories, or LTs, once developed will eventually lead students to transform their conceptual thinking of mathematical concepts. They produce concepts from trajectories of learning that can transform a way a student thinks about a particular concept.

Cognitive Guided Instruction

According to Carpenter and Fennema (1991), cognitive guided instruction (CGI) is the idea that "teachers use knowledge from cognitive science to make their own instructional decisions" (p. 10). Vacc and Bright (1999) assert that, through this type of instruction, teachers use their research-based knowledge regarding student learning to plan and lead the design of their instruction, and therefore seek to understand an individual student's thinking. Teachers who utilize CGI principles believe eliciting and understanding students' thinking is a key component in their instructional planning. They lead their students in problem solving and exploring of concepts, utilize this problem solving in determining their students' conceptions by listening and responding with questions until their thinking becomes clear, and then proceed to make instructional mathematical decisions that fit the needs of their students (Fennema, et al., 1989).

With theoretical underpinnings in Vygotsky (1962) CGI, "provides a basis for teachers to interpret, transform, and reframe their informal or spontaneous knowledge about students' mathematical thinking" (p. 5). Due to these theoretical underpinnings, a large factor of how well a teacher implements CGI principles rely on their beliefs about mathematics instruction and learning (Fennema et al., 1996). The CGI framework enables elementary education mathematics teachers to establish and understand students' thinking, while creating instructional strategies that foster students' conceptual understanding on the basis of their current conceptual thinking.

These frameworks and standards guide teacher educators' instruction of PTs. Even though these standards and frameworks focus on a variety of aspects of teaching and learning, such as content, instructional strategies, student thinking and learning, and students' progressions of thinking, they lack a cohesive interweaving of content and pedagogical knowledge, and therefore fail to focus on pedagogical content knowledge. However, the significance of content and pedagogy, and the conceptual power of bridging these knowledges to form pedagogical content knowledge, are of great importance to effective teaching.

Significance of Content Knowledge

When discussing K-5 mathematics student learning, it is often heard that students would learn more math if their teachers knew more math (Kahan, Cooper & Bethea, 2003). In a study conducted on teachers' knowledge of mathematics, Belge (1979) concludes that the importance of profound understanding and knowing one's content knowledge is frequently dismissed in elementary teacher education. We know that the mathematics content knowledge that PTs possess is vital to their teaching success (Marks, 1990), and has been consistently deemed important in the field of teaching. Since the No Child Left Behind Act, signed into office in 2002, the idea of "highly qualified teachers" has been in the forefront. Highly qualified teachers is defined in this act as a teacher that has: 1) earned a bachelor's degree, 2) obtained full state certification or licensure, and 3) proven that they know each subject they teach (No Child Left Behind, 2002). What encompasses proving they know each subject they teach can depend on the measure set forth by each state. For example, in the state of North Carolina, K-5 PTs acquiring a teaching license must pass A Core Academic Skills for Educators for writing, reading and mathematics (Educational Testing Services, 2016). The mathematics test consists of a total of 56 questions, with number and quantity consisting of 30% of the test, algebra and functions also consisting of 30% of the tests, geometry and statistics consisting of 20% of the test with probability consisting of 20% of the test as well.

These tests consist of content that teachers should know in order to be successful in teaching in their field. Due to the rigorous content on many of these mathematics tests, some states are finding it difficult for teacher candidates to pass their required tests. States such as South Dakota and Michigan have written locally about the problem that their aspiring teachers are facing (Anderson, 2015 & French, 2013). As Hill, Rowan, and Ball (2005) explain, teacher knowledge has typically been measured by coursework or degrees attained. However, there has been a recent development in the literature that has looked closer at teachers' knowledge in terms of the "ability to understand and use subject-matter knowledge to carry out the tasks of teaching" (Hill et al., 2005, p. 372), originally described by Shulman (1986).

However, are these important concepts (content knowledge and pedagogical knowledge) being integrated in initial licensure standards and/or instruction? For example, the CAEP standard that singularly addresses mathematics concepts, algorithms, procedures and practices, and connections throughout mathematics domains, is limited in scope. It does not address the type of advanced content knowledge that is required to increase learners' understanding of the different connections between mathematical domains. Specifically looking at content, there have been numerous studies that look closely at different content specific aspects of each domain that are vital to student learning in K-5 mathematics. For example, the development of fractional concepts is readily discussed in the literature, as well as an emphasis on measurement, patterns, number sense, and the development of algebraic thinking, just to name a few of the underlying key conceptual practices in early mathematics (Cai et al., 2005; Clements & Stephan, 2004; Empson, 1999; Mulligan & Mitchelmore, 2009; Reys, 1991; Van de Walle, Karp, & Williams, 2007). If these specific mathematical concepts change teacher thinking and practice, and consequently student learning, are not being addressed interconnectedly in the current standards, how are we ensuring that these are the concepts that are being taught in universities?

Significance of Pedagogical Knowledge

It is vital that teachers are able to become experts in all three areas of knowledge as it relates to their curriculum. However, there has been a special interest on pedagogical knowledge. The word pedagogy was derived from the Greek word “peadagogue” which means a keeper, rather than a teacher (Hall, 1905). It has recently evolved to mean the art or science of teaching. In terms of teacher knowledge, many argue whether prospective teachers should major in education or if they should major in a discipline (Ball & Bass, 2000).

Dewey (1904) classified good teachers as teachers who generate “genuine intellectual activity” in their students. He also believed that the “methods” on how to achieve this were inevitably tied to disciplines. “Despite these prescient ideas, teacher education throughout the 20th century has consistently been structured across a persistent divide between subject matter and pedagogy” (Ball & Bass, 2000, p. 242). Ball and Bass (2000) explain how this divide is often discussed within debates of equity and reaching all students. As Hooks (2014) explains, engaged pedagogy is a way to make the learning process more effective for our students. We should not only look at teaching as a way to share information but a way to foster our students' curiosity and understanding. Given the research and conceptualization of what effective pedagogy can do for students in the classroom, it is inevitable that effective teaching and learning of pedagogy by prospective teachers could support equitable outcomes in school. Previous literature has shown that pedagogical techniques in a K-5 mathematics setting, or any setting for that matter, would result in effective learning and understanding for students. Teachers who are able to explain more than one strategy for solving mathematical problems will have a better chance of reaching students who might not otherwise understand the one method that a teacher without a strong pedagogical background might teach.

According to Ball, Thames, and Phelps (2008), "Although the term pedagogical content knowledge is widely used, its potential has been thinly developed" (p. 389). It has also lacked empirical evidence and definition that has limited its usefulness in the teaching world. Empirical studies scarcely define pedagogical knowledge. Besides Shulman's original work, there have been little efforts to create well-specified accounts of what teacher candidates know in this area (Cochran et al., 1993). Without empirical proof of what teachers need to know in order to be successful, these claims have a minimal impact on furthering the field of education (Foss & Kleinsasser, 1996).

The Case for Intertwining Content and Pedagogy: Strengthening Pedagogical Content Knowledge

Spearheaded by Schulman's three types of teacher knowledge (1986), a growing body of literature has "begun to conceptualize teachers' knowledge for teaching differently, arguing that teacher effects on student achievement are driven by teachers' ability to understand and use subject-matter knowledge to carry out the tasks of teaching" (Hill et al., 2005, p. 372), instead of conceptualizing teachers' knowledge using measures outputs such as courses taken, grades in courses, or scores on skills test (Kleickmann et al., 2013). In other words, this growing argument specifies that student achievement is specifically directed to *what* teachers know and *how* they use it to instruct. Therefore, we argue that content (what) and pedagogy (how) should be focused on equitability to ensure that there is an undeniable focus on both as equivalent entities to the importance of primary mathematics teaching. The current standards in usage for K-5 mathematics teacher educators lack a core focus on pedagogical practices and content in breadth and depth of core ideas and understandings related to K-5 mathematics teaching. Historically, content has been the main focus of teacher education courses and programs. In order to increase teacher effectiveness in the K-5 mathematics classroom both pedagogy and content need to be addressed.

Since Shulman (1986) stressed the importance of teachers being immersed in pedagogy and content, teacher education programs have sought to infuse them within their programs, and mathematical accrediting organizations have strived to align their standards to meet this challenge. Still, despite studies linking teacher pedagogical effectiveness in math to student achievement (Campbell et al., 2014; Clotfelter, Ladd, & Vigdor, 2010), current teacher mathematical standards lack frameworks that address pedagogy and content *together*. Even though AMTE has recently written standards that embed pedagogy into content, and describes some of these notions as well as gives example tasks for preservice teachers, they are not exhaustive or extensively interconnected ideas of pedagogy and content that link to overarching key mathematical concepts. Linking overarching key ideas to singular pedagogical and content knowledge can allow preservice teachers to understand mathematics in a conceptual way in primary mathematics. In a study of international teacher preparation programs and mathematical standards, Tatto, Lerman, and Novotna (2010) discovered a low emphasis on instructing primary mathematics teachers on pedagogy in the U.S. Among the countries which place a low emphasis on teacher education in mathematics, their study determined that teachers found themselves resembling general education teachers, rather than mathematics teachers (Tatto, Lerman, & Novotna, 2010). The lack of integrated mathematical content and pedagogical knowledge within the standards forces teachers to develop on their own, that may or may not be effective solutions to their own deficiency (Daro et al., 2011). This effort tends to have teachers relying on ineffective methods, instead of developing more efficient methods of instruction.

A Case for Core Pedagogical Concept Knowledge in K-5 Mathematics Teacher Education

In conclusion, it is evident throughout the literature that good teacher education can be directly related to effective teaching. However, it is also apparent that elementary mathematics teachers graduate without the essential conceptual knowledge to instruct their students effectively (Campbell et al., 2014; Clotfelter et al., 2010; Mewborn, 2001; Ball & Bass, 2000). Many relate this lack of conceptual knowledge and teacher readiness to the inherent separation of content and pedagogy (Campbell et al., 2014; Clotfelter et al., 2010; Ball & Bass, 2000). As Schulman (1986) discusses, pedagogical content knowledge is a key component in the delivering of instruction and contextualizing content. Yet, the interweaving of content and pedagogy, or pedagogical content knowledge, is lacking in current teacher education programs and practicums (Ball & Bass, 2000), even though this connection could strengthen knowledge and instruction (Dewey, 1904/1964).

Therefore, we seek to understand how these special forms of knowledge can be taught through the interconnection of singular units of pedagogical content knowledge that link to an overarching mathematical concept that can span domains and grade levels. We argue that the development of threshold concepts in elementary mathematics teacher education can assist in this connection. A threshold concept is a key idea that can open a new and previously unreachable way of thinking about something once a student has internalized this idea (Meyer & Land, 2003).

> Students who have not yet internalized a threshold concept have little option but to attempt to learn new ideas in a more fragmented fashion. On acquiring a threshold concept, a student is able to transform their use of the ideas of a subject because they are now able to integrate them in their thinking. (Meyer & Land, p. 53)

Additionally, this gap underscores a key area where threshold concepts can inform practice related to teachers' ability to apply and effectively teach specific content. Their ineffectiveness to deliver inadequate instruction to students is partly due to their inability to evolve past their current procedural understanding of concepts. Using threshold concepts in mathematics would serve to change the conceptualization process of preservice teachers, ultimately fostering effective mathematics instruction once they are classroom teachers. By identifying these threshold concepts in K-5 mathematics education, teacher educators can have frameworks to utilize in order to ensure that preservice teachers are learning the content and pedagogical knowledge vital to successful mathematics instruction for K-5 students.

With many universities only having time to instruct one or two elementary mathematics content and pedagogical course(s), the insertion of SCK would be useful as a guide for teacher educators to discern key interconnected concepts that are essential for preservice teachers to understand, and how these overarching concepts (PCK, CK and SCK) can relate to smaller units of curriculum and specific teaching techniques. However, we urge further research and discussion regarding *what* should be included with describing these frameworks, ultimately leading to the discovery and use of threshold concepts in K-5 mathematics teacher education.

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CHAPTER III: ELEMENTARY TEACHER PREPARATION PROGRAMS: AN EXPLORATION OF COMPONENTS THAT MATTER

Courtney G. Mayakis, Temple Walkowiak & Ellen McIntyre

Recently, teacher education has been in the forefront of criticism (Darling-Hammond, 2000; Forzani, 2014). This fact, coupled with research that states teachers are closely linked if not the most important factor to student achievement (Darling-Hammond, 2010; Cochran-Smith & Zeichner, 2009), has spurred intense teacher initiatives. From the 1980s Carnegie Task Force on Teaching as a Profession to more recent research reforms such as the new accreditation standards for preservice teacher programs, Council for the Accreditation of Educator Preparation (CAEP, 2015), the focus has been on preparing more effective classroom teachers. These current sets of initiatives were launched to improve accreditation, provide professional standards for teacher educators, and to utilize "a collection of analysts, policy makers, and practitioners of teaching and teacher education" which "argued for the centrality of expertise to effective practice" (Darling-Hammond, 2010, p. 36). These reforms include a stronger connection to theory in the university and practice in the classroom aligning with practice-based teacher education, a focus on methods courses, student learning and thinking, as well as pedagogical content knowledge, and solidification of coursework around assessment and data (Boyd, Grossman, Lankford, Loeb, & Wyckoff, 2008; Darling-Hammond, 2006; Forzani, 2014). A growing body of research has suggested that teacher education programs who have actively responded to these initiatives have reported graduating preservice teachers who have students with higher value-added scores, often receive higher scores on their administrator observations, and report feeling more prepared to teach (Boyd et al., 2008; Darling-Hammond, 2006).

Even with the prominent strides within teacher education programs, there have been many critiques of traditional preparation programs (Darling-Hammond, 2000; Grossman, 2008; Valli, 1992). These critiques include lack of a consistent and useful common language for addressing teacher education and the absence of practice-based teacher education (Grossman, 2008; Darling-Hammond, 2000; Valli, 1992). These critiques have also argued that teacher education programs can make a difference and can prepare teachers successfully if created and implemented effectively (Darling-Hammond, 2010; Darling-Hammond et al., 2005). Within teacher education lies subsets of various programs, one of these subsets being elementary education. Elementary teacher education presents its own unique challenges since K-5 teachers are required to be certified to teach multiple content areas. This situation leads to the "crowding" out of methods and content courses in some disciplines within elementary teacher education programs.

One area that has presented great concern in ineffective preparation in elementary education programs has been in the field of mathematics. Many argue that elementary education teachers are not being prepared to effectively plan and instruct mathematics in the primary grades (Ball & Bass, 2000; Ball & Wilson, 1990; Hill, Ball, & Schilling, 2008; Mewborn, 2000). Part of this has been attributed to the lack of consistency in what is deemed "important" for teachers to know and be able to do in elementary mathematics, as well as the absence of practice in teacher preparation programs (Darling-Hammond et al., 2005; Forzani, 2014). In order to understand the lack of consistency within these elementary programs, we must first understand how these programs are structured, what barriers prevent the unity of these characteristics, and what teacher educators value in the preparation of teacher candidates. To help us understand the structure of elementary programs and the barriers, the study reported here describes qualitative data collected and analyzed from elementary education programs in one particular state from public 4-year institutions, with a focus on their mathematics content and methods courses, along with key features of their programs.

Purpose of the Study

A data set utilizing a value-added approach that compares the average learning gains for students whose teachers graduated from different teacher preparation programs in North Carolina was created to understand how effectively teacher preparation programs were preparing teachers (Bastian, Patterson & Pan, 2015). This quantitative analysis found that one particular university's graduates produce higher average learning gains for students in elementary mathematics as compared to their peers graduating from different teacher preparation programs (Bastian, Patterson & Pan, 2015). Therefore, the purpose of this study is to foster an understanding of the structures of elementary education programs and their mathematics methods and content courses across public universities within one particular state. We aim to understand what teacher educators do at this institution in their current mathematics methods and content courses, and what this particular university does to graduate teacher candidates who produce higher average learning gains than their peers at neighboring institutions. These programmatic features and professor actions will be understood through the implementation of descriptive, qualitative research. We collected and analyzed syllabi, key assignments, and qualitative self-assessments regarding key assignments, and enlisted the expertise of two specialists in the field to create a rubric to score the overall quality of key assignments. We also implemented and analyzed focus groups of the elementary education program coordinators and mathematics content and methods professors. The data assisted in answering the following questions:

(1) What do teacher educators believe best prepare teacher candidates for the classroom?

(2) How do teacher educators prepare teacher candidates?

Literature Review

Teacher Preparation Programs

Traditional teacher education (i.e. a 4-year degree that leads to teacher certification) has a powerful impact on student achievement and teacher effectiveness (Darling Hammond, 2010; Ferguson, 1991). Ferguson (1991) studied 900 districts in Texas and the teacher quality within those school systems, and found that "teacher expertise," defined by a teacher's education, experience, and their scores on the licensure exam, was the most influential factor in determining student achievement, even outscoring socioeconomic status as a student achievement factor. However, not all teacher education programs are created equal. In a study conducted by Boyd, Grossman, Lankford, Loeb, and Wyckoff (2008) of teacher programs in New York City, teacher education programs who graduated teachers that produced higher-value added scores on student assessments had several commonalities. First, these programs prioritized student teaching experiences by matching the context of their preservice teacher's student teaching experiences to that of their later teaching assignments and ensuring that these were quality field placements. These programs had a heavy focus on coursework related to reading and mathematics content and methods and focused on exposing candidates to specific practices and tools to apply to their clinical experiences. Preservice teachers (PTs) at these institutions also had opportunities to immerse themselves in the local district curriculum and had a higher percentage of tenured faculty as their methods and content professors. They were also required to present a capstone project at the end of their student teaching.

Even though some teacher education programs have altered their structures to respond to current criticisms, teacher education programs still have many obstacles to overcome. As Darling-Hammond (2010) asserts:

> While there has been some progress in teaching, many universities still struggle with constraints that haven't been fully resolved in the 50 years since normal schools were brought into universities: the loss of tight connections to the field, issues of status within the university, problems with the qualifications of those who teach in teacher education—in particular their knowledge of how disciplinary principles translate into good teaching—fragmentation of courses, and treatment as a cash cow. (p. 39)

These issues can be seen throughout the policies and politics of the culture of universities, as well as the climate and stigma of teaching in the United States. For example, the use of standards in elementary programs focus on a generalist approach since elementary teachers are expected to be experts in all four content areas. Preservice teachers are expected to graduate from their elementary programs with a strong understanding of the mathematics vital to teaching their K-5 students, but what are they actually being taught in their programs? There has been an abundance of research to suggest that teachers are not graduating with the conceptual knowledge to teach mathematics effectively (Ball, 1990; Ball, Hill & Bass; 2005; Mewborn, 2001). Yet, these teachers are "graduates of the very system we seek to improve" (Ball, Hill & Bass, 2005, p. 14), and therefore their teacher education must aid in supporting teacher candidates' mathematical understandings that were not developed during their K-12 schooling. This is why the implementation of K-5 mathematics standards are critical in guiding the work of teacher educators.

Guiding Standards in Elementary Mathematics

Most of the focus of reforms in recent years surrounding mathematics has been on standards and curriculum in K-12 districts and schools, with a lesser emphasis on teacher education, especially at the elementary level. However, entities such as the Council for the Accreditation of Educator Preparation (CAEP) and the Association of Mathematics Teacher Educators (AMTE) have established a guiding set of standards for elementary teacher preparation, with AMTE's standards focusing specifically on the development of elementary mathematics teachers.

The Council for the Accreditation of Educator Preparation (CAEP). The Council for the Accreditation of Educator Preparation (CAEP) developed the leading set of standards in elementary education currently used in teacher education programs. These standards are broad in scope and discuss many aspects of teaching; they are not entirely focused on content. In examining the mathematics curricular standard addressed in CAEP, Standard 2 includes a singular substandard specifically addressing mathematics:

> Candidates demonstrate and apply understandings of major mathematics concepts, algorithms, procedures, applications and mathematical practices in varied contexts, and connections within and among mathematical domains (Number and Operations, Operations and Algebraic Thinking, Measurement and Data [both Statistics and Probability] and Geometry) as presented in the rationale for the CAEP Mathematics Content for Elementary (K-5) Teachers, (CAEP, 2015, p. 2)

However, this is the only mention of mathematical practices and content with this set of standards and does not specifically outline content within the domains of mathematics. Standard 2 encompasses general statements regarding curriculum for all four content areas. CAEP does not give enough guidance in how to develop strong mathematics teacher educators through the use of their standards. The Association of Mathematics Teacher Educator's (AMTE) set of elementary education mathematics standards for teacher candidates is currently the only set of standards that addresses mathematics in depth for initial teacher licensure in the primary grades.

The Association of Mathematics Teacher Educators (AMTE). The Association of Mathematics Teacher Educators (AMTE) has developed in-depth standards for the initial licensure of teacher candidates in elementary mathematics, grouped into two separate categories (2017): early childhood mathematics, and upper elementary preparation. Both early childhood mathematics and upper elementary preparation are divided into two parts: elaborations of the knowledge, skills and dispositions needed by well-prepared teachers of mathematics (part 1); and elaborations of the characteristics of effective programs preparing teachers of mathematics (part 2). Even though they have similar parts incorporated within early childhood and upper elementary preparation, different content, pedagogy, and pedagogical content knowledge is emphasized in each to demonstrate the differing levels of content and child development. In part 1 of both early childhood and upper elementary mathematics, AMTE defines standards that are pertinent to pedagogy, content, and examples of pedagogical content knowledge. These standards separate content and pedagogy to emphasize importance on each of these teacher knowledges. However, AMTE provides specific examples of how content and pedagogy can be intertwined, and how this pedagogical content knowledge can be taught in mathematics teacher education courses. AMTE also intertwines the use of learning trajectories within some of their standards.

Not only do these standards encompass specific content that should be learned during a teacher candidate's preparation but also programmatic suggestions for these programs. For example, AMTE suggests that 12-semester hours of mathematics content and pedagogy courses, and a minimum of six weeks of statistical education should be contained within in the program requirements of P-5 mathematics teacher education (AMTE, 2017). Even though these standards are based on best-practices in elementary mathematics, teacher education programs are not required to respond to these standards within their programs through the use of evidences, portfolios or otherwise since they are not required for accreditation. With the CAEP standards being the only set of standards that most elementary education programs in the country are responsible to adhere to, how do we know what specific mathematical concepts are being taught in methods and content courses? If we do have a set of standards that embody best practices and foundational content, how do we utilize these, and more importantly, how do we teach preservice teachers to utilize them?

Conceptual Framework: Practice-Based Teacher Education

In recent years, the growing criticism of teacher education has sparked reforms that argue for the focus of practice-based teacher education (Forzani, 2014). Even though there are differing perspectives on what practice-based teacher education means for teacher candidates and teacher preparation programs, many agree that it encompasses training around the profession, centered on practices involving teaching through the use of practice in an instructional setting (Ball & Cohen, 1999; Forzani, 2014; Matsko & Hammerness, 2014). Many have argued that the theory taught in university settings and the practice of teaching, are often disconnected and leave teacher candidates unprepared when entering the classroom (Ball & Cohen, 1999; Darling-Hammond, 2000, 2010; Darling-Hammond, Hammerness, Grossman, Rust, & Shulman, 2005; Grossman, 2008). This current criticism has resulted in reforms with a focus on practice-based teaching (Ball & Forzani, 2009; Darling-Hammond, 2010; Deans for Impact, 2016; Grossman & McDonald, 2008; Zeichner, 2010). Zeichner (2010) writes:

> This work in creating hybrid spaces in teacher education where academic and practitioner knowledge and knowledge that exists in communities come together in new less hierarchical ways in the service of teacher learning represents a paradigm shift in the epistemology of teacher education programs, (p.480)

This epistemological shift has moved programs towards an intertwining of theory and practice and offers teacher candidates spaces for deliberate practice of teaching and immediate feedback of this practice (Deans for Impact, 2016). This shift has warranted the discussion of what skills teacher candidates should practice and what is important for them to know and be able to do in order to be an effective teacher. Often defined as "core" practices, these practices are research-based instructional techniques that novice teachers should master in order to become effective beginning teachers (Grossman & McDonald, 2008; Ball & Forzani, 2011). In order to master instructional techniques, teacher candidates must be afforded the opportunity to practice such techniques. This practice must develop from teacher education programs that imbed this type of practice-based teacher education within their course requirements, assignments, and field placements.

Characteristics of Practice-Based Teacher Education

In the article *Combining the Development of Practice and the Practice of Development in Teacher Education*, Ball, Sleep, Boerst and Bass (2009) discuss their development of a curriculum focused on the practice of teaching, specifically around the work of K-5 mathematics. They describe several facets of the development of this curriculum:

Articulating the Work of Teaching Mathematics. This addresses the "core domains of teaching" —"for example, planning, choosing and using representations, conducting discussions of mathematics problems" (Ball et al., 2009, p. 460), and analyzing how to translate these into teachable components.

Identifying and Choosing High-Leverage Practices. High-leverage practices, or "teaching practices in which the proficient enactment by a teacher is likely to lead to comparatively large advances in student learning" (Ball et al., 2009, p. 460), have been developed around the practice of teaching. These 19 high leverage practices were developed by researchers at the University of Michigan (Teaching Works, 2017), and include practices such as leading a group discussion, explaining and modeling content, practices, and strategies, and eliciting and interpreting individual students' thinking.

Mathematical Knowledge for Teaching. Ball et al. (2009) describes mathematical knowledge for teaching as:

> Skilled mathematics teaching requires more than simply learning how to enact particular pedagogical tasks. It also requires knowing and using mathematics in ways that are distinct from simply doing math oneself. For example, in preparing a mathematics lesson, teachers must identify and

> understand the mathematical goals of particular activities, anticipate the varied ways students might respond, and prepare mathematically for what might happen as a lesson unfolds. Teachers must be able to "unpack" mathematical ideas and be able to scaffold them for pupils' learning.

This specific type of teaching requires teachers to have the knowledge of not just mathematics content, curriculum, and pedagogy but also the knowledge of how students will respond to this mathematical knowledge. Time and space to practice these are also key in practice-based teacher education (Forzani, 2014), and is important in the reflection process of teacher educators and preservice teachers. These characteristics will be utilized throughout the analysis of data collected from this study.

Method

Because we aimed to understand the practices of all state institutions, a descriptive study was used to collect information on programmatic features of each unique elementary education program and what professors do in these elementary mathematics methods and content courses. According to Dulock (1993), the purpose of descriptive research is to accurately describe a situation or group, provide a truthful portrayal or account of characteristics of this particular situation or group, and answer questions about ongoing events. This descriptive qualitative research will allow us to comprehend what is occurring in elementary education programs within these respective universities.

Participant Information

The 17 participating teacher educators in this study were current practicing teacher educators and were recruited from one state throughout public universities. These teacher educators were selected to participate based on the following criteria: a) employed at a public university in one particular state and b) currently teaching a mathematics methods or content course(s) embedded in their elementary education program at their particular university. A total of 17 teacher educators participated in the study from 14 unique public universities throughout the state. These teacher educators were invited to participate by the deans at each of their colleges.

Data Collection and Analysis

Most data were collected from two separate sessions in which the same activities were utilized. These activities consisted of qualitative self-evaluations of key assignments submitted by professors and focus groups. This meeting was a full-day session in which teacher educators discussed the key assignments they submitted and the key assignments of other teacher educators through the lens of the rubric utilized in this study. They also participated in focus group sessions. Key assignments, syllabi of courses, and program degree sheets were collect via email prior to the two meetings.

Document Analysis

These assignments were given to us by teacher educators as key assignments in their respective mathematics methods and content courses that they thought were crucial in developing teacher candidates. We collected a total of 19 mathematics elementary methods and content assignments utilized by teacher educators throughout these respective universities. Most assignments were from courses focused solely on mathematics methods or content, although some were from joint mathematics and science courses. Using an open-ended response survey, we asked teacher educators why they identified that particular assignment as their key assignment (see Appendix C). Syllabi and program degree sheets were also collected from all participating public universities.

Each program degree sheet was analyzed for the number of elementary mathematics methods and content courses required for each elementary education program.

Qualitative Self-Evaluations

We provided teacher educators specific questions through the use of an open-ended, short answer survey (see Appendix C) regarding the key assignments submitted. Teacher educators were asked a series of seven questions, and these responses were summarized accordingly. The series of questions were developed to understand why teacher educators believed this assignment to be key in developing mathematics elementary teachers. A total of 16 self-evaluations were collected and analyzed. The analysis consisted of a short description of teacher educators' reasoning, in order to triangulate data amid the rubric evaluations and document analysis.

Focus Groups

At the culmination of the meetings, mathematics professors and program coordinators of these elementary education programs were placed in two separate groups consisting of approximately 5-10 teacher educators each. Between the two meetings there were two groups of program coordinators and two groups of mathematics teacher educators. Participants were asked a series of questions regarding their programs, assignments, and field placements (see Appendix C). These focus groups interviews were conducted by the research team and each group consisted of a note taker who recorded the responses of teacher educators. We strategically placed the focus group sessions at the conclusion of the meeting in order to heighten dialogue on assignments, since a large portion of the meeting involved analysis and discussion of these assignments. The answers to these focus group questions were analyzed using constant comparative analysis (Glaser, 1965). We first coded focus group transcripts line by line using open coding, and then connected these categories by identifying overarching concepts. From these concepts, themes emerged, and we compared these themes to the initial codes to ensure that they reflected the data. Throughout the data analysis process, the research team discussed the findings, the data collection process, and analysis as a group (Shenton, 2004).

Findings

Number of Content and Methods Courses Across Mathematics Classes in Programs

After analyzing 14 degree program sheets from the participating public universities, a summary of the number of credits required for content and mathematics methods courses in the elementary education programs was recorded. Most programs required their students to enroll in 6 content credit hours and 6 methods credit hours, with 12 content credit hours being the maximum and 3 content credit hours being the minimum content credit hours. Some of these institutions required general education mathematics courses, while others required content courses specific to elementary education mathematics. The maximum mathematics methods credit hours required for teacher candidates were 6 credit hours, with 1.5 credit hours (or ½ of a methods course) being the minimum required. There was one university in particular that required teacher candidates to complete the maximum in both of these categories, with 12 content credit hours and 6 methods credit hours. The remaining institutions had a mix between the 3-6 content credit hours and 1.5-6 methods credit hours.

Features of Key Assignments and the Practice of Teaching

We analyzed key assignments through the lens of practice-based teacher education to identify whether teacher educators in elementary education programs thought of practice as a key task in developing elementary education candidates in mathematics. It was clear that there were similarities and differences in the assignments received. For example, many key assignments involved one-on-one interviews with students or peers in their program regarding mathematical concepts. However, the degree as to which teacher candidates were required to rehearse the task varied. For example, in assignment E.1 (see Appendix D for brief description of assignments), teacher candidates were required to conduct an interview with another teacher candidate and videotape this interview. These interviews consisted of a set of questions that teacher candidates could potentially ask their K-2 students regarding subtraction problems involving numbers to 20. After both teacher candidates conducted their interviews, they analyzed the video of themselves, and were required to reflect on these videos. Teacher candidates had a "before video," a "midterm video," and an "after video." After each video, teacher candidates were given a list of interventions and assessments to use in order to analyze their video. Out of the 19 assignments analyzed, this particular assignment was the only assignment that required students to rehearse the same instructional task, after a set intervention, to further their practice. This task utilized two main components of practice-based teacher education such as: (1) giving students time and place to practice, and (2) working on the mathematical knowledge for teaching.

Other assignments encompassed student interviews as well, usually consisting of problem-based tasks. Assignments such as B.1, H.2, K.1, and B.2 (descriptions labeled in

Appendix B) offered tasks that required practice either one-on-one or in small group settings with peers and/or K-5 students. These tasks required a main component of practice-based teacher education requiring preservice teachers to focus on high leverage practices such as eliciting student thinking and small group instruction. In assignment B.1, teacher candidates were required to take notes of student responses and observations as another teacher candidate interviewed a K-5 student one-on-one from a list of problem-based tasks given to teacher candidates. They were then required to switch places and repeat the process. Teacher candidates were then able to confer with their partner and generate a report of their analysis of the student's understanding of the concept. With this assignment, preservice teachers were able to practice mathematical knowledge for teaching and were given time and space to practice. Even though these assignments had some component of reflection and practice in their project description, they were lacking the chance for teacher candidates to apply their reflection to the same instructional task to improve their teaching.

An additional overarching theme of many assignments was the planning and teaching of a lesson to either K-5 students or peers in their program involving a mathematics topic. This idea of microteaching, or teaching a lesson to other teacher candidates, involves all three components of practice-based teacher education: articulating the work of teaching mathematics, high leverage practices, and mathematics knowledge for teaching through the development of lessons and the thought process that is involved with creating a sound mathematical lesson. Assignments M.1, L.1, J.1, G.2, F.1, C.1, G.1 and G.2 required teacher candidates to plan and teach a lesson to a group of peers and/or K-5 students.

Even though these lessons encompassed key characteristics of practice-based teacher education, rigor regarding content and planning of each lesson or unit varied. For example, assignment J.1 required teacher candidates to select a chapter out of a mathematics textbook and create a lesson based on the chapter. After the creation of the lesson they were required to teach the lesson to their peers and discuss how the lesson could be altered based on the needs of students in a K-5 classroom. Assignment L.1 gave teacher candidates a variety of instructional contexts and content required in the structuring of their lessons. Teacher candidates were tasked with creating three different math lessons to present to their peers. Each math lesson was planned utilizing a different instructional setting (whole-group, small-group, one-on-one) and focused on three unique domains of mathematics. Other assignments such as H.2, H.1, and G.2 required students to observe a mathematics classroom, analyze student work, and/or conduct one-on-one interviews in order to utilize data to assist in the creation of a mathematics lesson plan.

Some of these assignments were accompanied by a clear description of what teaching techniques to use and the specific content to cover. Yet, a handful of these assignments were lacking clear directions on how to implement the lesson plans or the tasks/topics that professors desired teacher candidates to utilize for their interviews. Some of these assignments specifically detailed that teacher candidates would choose their own tasks, topics and/or teaching techniques to use, instead of professor selecting these beforehand. Understandably, due to teacher candidate’s placement with clinical educators (mentor teachers in the field) in K-5 classrooms, some teacher educators expressed that it was difficult to choose a task for teacher candidates to complete because many clinical educators had a pacing guide with certain material they were required to cover while mentoring the teacher candidates. However, not all of key assignments analyzed occurred during teacher candidate's student teaching year, and it was unclear as to whether teacher educators conferred with preservice teachers on their topics prior to implementation of the assignment.

Assignment K.2 required a community component in which teacher candidates would lead and plan for a math night at the school, an after-school event where guardians and K-5 students were invited to participate. This assignment had components of practice-based teacher education, an opportunity for teacher candidates to practice communication with guardians and students, and engaging in the community within and outside of the school environment, which is also a high-leverage practice. Even though assignment K.2 did not display characteristics of practice, it required teacher candidates to examine curriculum materials and/or lesson plans critically, and modify these materials based on research and best practices, which aligns with articulating the work of teaching mathematics as well as the mathematical knowledge of teaching. Teacher candidates were required to develop rationales for their modifications. Assignment B.3 had teacher candidates practicing teaching, but the focus was on the development of a pre-and post-assessment for the purposes of analyzing data from students' understanding of that particular lesson. This idea of analyzing student data is also an example of a high-leverage practice.

Other key assignments included reading peer reviewed articles and participating in a data investigation. In conclusion, many key assignments focused on the practice of teaching through the use of microteaching (teacher candidates teaching a K-5 lesson to other teacher candidates) or the instruction of K-5 students. However, many assignments were not specific with the types of tasks and/or content that teacher candidates were to use in the planning of their lessons or interview questions. All assignments but one (which focused on the practice of teaching) only required teacher candidates to practice the task once. Even though many of these assignments had a reflective component embedded within the project, this reflection was left unused in the planning of a second attempt of the task. Therefore, teacher candidates were given very few chances to alter their instruction based on reflection.

Focus Group Discussions

There were several topics consistently discussed throughout the focus group interviews regarding the elementary education programs, mathematics content and methods courses, the types of assignments given to teacher candidates, as well as the intricacies of field placements. It was evident that there were overlapping themes across data that emerged as to what teacher educators valued as key components in developing teacher candidates.

Cohesion of Knowledge, Quality of Field Placements, and the Gap Between Theory and Practice. Teacher educators discussed barriers and constraints within school districts regarding access for student teaching opportunities, sometimes making it difficult to provide preservice teachers with an abundance of practice within contexts. Many argued that there was a disconnect between methods/content mathematics courses in the elementary education program, and what is being learned in field placements, since most student teacher liaisons are not their professors. However, many teacher educators agreed that field experiences were very important in the development of teacher candidates.

> *We are unanimous that field experience is important (interaction with students) and quality and time of field experience is important. And specifically, in the*

> *"math" time -- more narrow part of time. Who is supervising those practicums? You could have practicum supervisors and teachers that are not always good role models in math. The norm that is out there is not what we are shooting for.*

In this quote it is evident that the teacher educator is concerned about who is supervising practicums, and therefore what type of feedback a student teacher is receiving after their practice in the field. General comments were made about the concerns that professors had regarding their teacher candidates lack of connections between content and methods courses, and the absence of cohesion and consensus on what is taught in methods courses for teacher candidates to learn and utilize in the field.

> *There are different focuses: CGI, pedagogy for social justice, a lot of different perspectives across different programs. What is worrisome is when we decided to define methods courses, even though we are talking about math, what is worrisome is that we all do not define the methods courses either. What is methods? There is not any kind of a consensus across methods classes.*

The use of adjunct faculty in some courses leads to inconsistency in what is focused on throughout these courses, and consequently the knowledge teacher candidates take with them into the field.

> *We have a ton of students in our college. We have 87 teacher supervisors. And there is an inconsistency with the fact that we have different teacher supervisors and different lecturers teaching the same courses. I'm not in the elementary education department so I don't have control over their placements.*

This excerpt demonstrates how teacher preparation programs struggle with cohesion and uniformity in field placements due to the number of teacher candidates and the concern with placing them in quality field placements. Cohesion of what is taught within content and methods courses, and how this knowledge is applied in placements was an ongoing concern of teacher educators and program coordinators. With teacher educators unanimously agreeing on the power of field placements, their lack of unity and consistency was limiting for the development of teacher candidates.

Teacher Candidates' Ability to Choose Key Tasks for Teaching. There was a large concern with allowing teacher candidates the flexibility to choose their own tasks and content for lesson plans or small group instruction or having teacher educators review candidates' choices rather than teacher educators choosing their tasks and content for them. Teacher candidates may not be fully aware of key concepts or tasks that are essential to the deep understanding of mathematics and are sometimes more concerned with making a lesson "cute" instead of creating a rigorous mathematical task or lesson. However, the ability for preservice teachers to choose their own tasks is a key component in the work of practice-based teacher education and high leverage practices. Some professors argued that giving students time to perfect a few key tasks was more important than choice of task.

> *This is the task that we are going to all teach. The students are not curriculum designers. This is it, this is what you need to know how to do really well. So I am more controlling on what I think students need to be able to do with minimal options. Just because I don't give you a lot of options does not mean I am not developing teachers very well. In 3-5 there are pillar things that preservice teachers need to know and you need to rehearse this in the safe space of your classroom.*

In this excerpt, the teacher educator is more concerned with teaching teacher candidates how to instruct certain tasks well, instead of allowing teacher candidates to choose their own tasks. Most teacher educators reasoned that assigning teacher candidates a specific task, key to the teaching of mathematics content, would allow teacher candidates to become experts at crucial content, and assist in the development of effective teachers.

Discussion: Implications for Teacher Preparation

From the data collected, many professors and faculty in the elementary education programs across this state's universities have addressed areas of improvement throughout their programs as well as elementary education teacher preparation in general. Many of these stakeholders are worried with what is being taught in K-5 mathematics content and methods courses, how it is being taught, and how authentic experiences can shape this instruction in the field. Throughout the data discussed several common themes were discovered—many of which have implications for elementary education teacher preparation.

Reflection in Practice and Task Choice

Reflection on practice was a consistent theme within focus group sessions; however, there were many approaches to utilize reflection within key assignments. Research has indicated that the videotaping of instruction can have a positive impact on teaching; however, task choice and learning goals must be carefully selected in order to have a high impact on the fostering of reflection (Blomberg et al., 2014; Seidel et al., 2011). Yet, many tasks that were videotaped within key assignments for this study were not tasks that were selected for teacher candidates by teacher educators, and there was no evidence that suggested that teacher candidates conferred with their teacher educators before implementing the task. Choosing specific and meaningful mathematical tasks for teacher candidates to complete in whole group and small group lessons, or one-on-one interviews, or confer with teacher candidates regarding their tasks, instead of allowing teacher candidates to choose their own tasks or content to teach without feedback was seen as crucial in the development of teacher candidates throughout the focus groups. As such, this skill weaves through all characteristics of practice-based teacher education. Teaching specific and pinpointed reflection to teacher candidates through the use of videotaped instruction would allow teacher candidates to reflect on various parts of their teaching such as: their content knowledge, their pedagogical content knowledge, and how they interact with their students. Teacher educators agreed that it was important for teacher candidates to understand how to use reflection to alter their instruction. Analyzing instruction for the use of improving it is considered a high-leverage practice and is essential to practice-based teacher education. However, after analyzing all key assignments, there was only one assignment in which teacher candidates were actually required to enact this reflection to perfect a particular task.

Rigor of Mathematics Methods and Content Courses

Rigor in mathematics content and methods courses is important. There is debate as to whether mathematics content courses for elementary education should be derived and created from the mathematics department (Ball & McDiarmid, 1989). Even though these professors are experts in content, it is not clear as to whether they are experts in what elementary education teachers need to know in order to be successful in the classroom (Ball & McDiarmid, 1989). This type of mathematical knowledge for teaching is a key characteristic in practice-based teacher education. From a review of the documents, some universities had mathematics content courses geared towards elementary education teacher candidates, while others had generic mathematics content courses. However, teacher educators through this study strongly emphasized the importance of elementary education professors and mathematics professors collaborating to ensure that teacher candidates are gaining relevant content to their practice throughout these courses.

Cohesion in Methods, Content and Field Placements: The Gap Between Theory and Practice

An underlying common concern through focus groups as well as examining key assignments was the lack of consistency through mathematics elementary education courses, specifically through methods courses in the types of tasks given and the content taught. Consistency throughout methods courses is essential to the development of mathematical teaching in K-5 teacher candidates, especially when future teachers graduating from different programs will be expected to teach the same content and curriculum at a consistent high level. This lack of cohesion trickles down to what knowledge teacher candidates utilize in the field. Darling-Hammond (2009) writes:

> the clinical side of teacher education has been fairly haphazard, depending on the idiosyncrasies of loosely selected placements with little guidance about what happens in them and little connection to university work. (p. 11)

The gap between theory and practice, and what is taught within these methods courses was an ongoing theme throughout focus groups and a concern of teacher educators. It was evident through the review of assignments that many professors have different opinions about what should be taught in methods courses. However, which of these tasks has the largest impact? For universities that are only able to teach one joint mathematics and science methods course to their teacher candidates, it is crucial that they choose the most meaningful tasks and/concepts. However, what are these tasks and/or concepts? Further research is needed to investigate these tasks and concepts.

Limitations

A limitation of this study was the constraint of the location of participants. The teacher educators participated in this study were selected from one particular state from only public universities. Even though this unites these teacher educators with unique and common challenges particular to this one state, it also limits the scope of data collected. Another limitation of the study was the submission of only one key assignment for document analysis. It is difficult to gain an in-depth insight of what teacher educators teach through the use of only one assignment.

Future Research

Future research should be conducted on which tasks and concepts are the most effective to implement in elementary mathematics content and methods courses. More specifically, it is important to create cohesion in what is taught in elementary education programs, but also to ensure that the tasks, concepts, and content being introduced to elementary mathematics teacher candidates are aligned with K-5 standards, since these are the standards that future teachers are expected to understand and know when graduating from their elementary education programs. With a cohesive definition and practice of methods in elementary mathematics teacher education, field placements would have a consistent framework of what teacher candidates should be practicing in the field. Research conducted on the most meaningful content, tasks, and teacher knowledge for elementary mathematics would benefit teacher preparation programs that require few mathematics methods and content courses for elementary teacher candidates. A framework detailing this meaningful knowledge would allow teacher educators to instruct more efficiently and effectively. With teacher candidates developing expertise in multiple subject areas, this is important for the future of elementary mathematics teacher education.

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Appendix A
Key Assignment Questionnaire

AT TOP OF GOOGLE FORM

You were asked to provide “a description of a *key* assignment that you believe has the biggest impact on your students’ development as teachers” (e-mail from Dr. Ellen McIntyre, October 2015). You were also asked to provide the assessment rubric.

MATH

1) Your name
2) University
3) Course title
4) Brief description of course (e.g., “math methods course for grades K-6”; “math methods course for K-2”)
5) Title of assignment
6) Explain why you provided THIS assignment as a *key* assignment in the development of your teacher candidates in your program.
7) Provide specific reasons as to how this assignment develops your teacher candidates’ *pedagogical skills and strategies* as it relates to the teaching and learning of mathematics, IF IT DOES.
8) Provide specific reasons as to how this assignment develops your teacher candidates’ *content knowledge* as it relates to the teaching and learning of mathematics, IF IT DOES.
9) Describe how your assessment rubric is utilized to evaluate “your students’ development of teachers,” with specific attention to the *pedagogical skills and strategies* and *content knowledge* that you outlined above.

Appendix B
Descriptions of Key Assignments

#1 Assignment B.1

Teacher candidates will conduct 20-minute interviews with a partner. One partner will administer the interview while the other partner will take as many notes as possible focusing on questions that are asked and responses given. With the next student, partners will switch roles. After the interview, partners will discuss questions regarding the students' mathematics conceptions and understanding. Each pair will submit a description of the problems posed, the questions asked and the responses from the child and include student work samples (*Note:* the interview problems and tasks were given to teacher candidates in this assignment, but students were able to choose from those listed).

#2 Assignment B.2

Teacher candidates will prepare an activity for a small group of students on the topic of fractions or addition and subtraction. Teacher candidates will choose a topic, find the corresponding Common Core standard, research how it can be taught, and create a handout that includes practice problems for the small group.

#3 Assignment B.3

The student *work sampling project* requires teacher candidates to develop a pre/post assessment for a lesson from a unit design that teacher candidates will teach during the semester (whether created from another class, or given to them to teach by their mentor teacher). Teacher candidates must create the pre/post assessment. Teacher candidates will administer these before and after the teaching of their unit, collect student samples, analyze and reflect on the student work and their teaching. Teacher candidates will be required to analyze their students' gains in growth from the pre-to the post-assessment, and interpret these results through the use of a written narrative.

#4 Assignment C.1

The goal of this project is to fully implement a whole-class discussion-based mathematics lesson is grades 3-6. The four project components are briefly described here. Candidates work in groups and individually throughout the semester to complete the tasks which are 1) pre- assessment/identifying learning goals, 2) lesson plans, 3) teaching the lesson and video-taping lesson, and 4) reflection and analysis.

#5 Assignment D.1

No description included with assignment – only a rubric was provided.

#6 Assignment D.2

Teacher candidates will conduct a data investigation and create a question that may be an issue related to education and teacher candidates. The investigation must involve numerical data. Example that is given is "What grade level do you want to teach?" Teacher candidates must formulate questions, collect data, analyze data, and interpret results based on the collected data.

#7 Assignment E.1

For this assignment teacher candidates, will conduct an initial video at the start of the semester demonstrating how they would address, as future teachers, subtraction problems within 20 (for example, 15 minus 8). Exemplars of this assignment will be viewed by teacher candidates that past teacher candidates have completed. A second teacher candidate will play the role of the elementary student in a role-play for the video. These videos reflect the teacher candidates' initial thinking about the topic of K-2 mathematics. This initial video demonstrates where teacher candidates begin the class. After a few weeks, teacher candidates are given the opportunity to reflect on this initial video given their learning up to that point. Teacher candidates are required to complete this video again at the middle of this math methods course and again at the end to show their growth in understanding and teaching early elementary mathematics.

#8 Assignment E.2

Teacher candidates and their field placement partner (another teacher candidate) will critically examine a specific lesson from existing curriculum materials used in field placement. After the critical examination, teacher candidates and their field placement partners will modify the lesson and provide rationales for the modification. The teacher candidate and their partners will teach and video tape each version of the lesson and reflect on both implementations.

#9 Assignment F.1

For this assignment teacher candidates will teach a math lesson leading a whole class discussion focused on math in students' field placements. Teacher candidates will need to include the "talk moves" discussed in class during the semester (*Note:* the "talk moves" were not described in the lesson description). The lesson should involve students in problem solving related to developing number sense in some way. The teacher candidate will need to prepare a lesson plan using the guidelines as appropriate in the Practicum Manual. Teacher candidates are asked to video record themselves teaching the lesson, and analyze their teaching afterwards.

#10 Assignment G.1

Teacher candidates are to observe a mathematics classroom. Then they are to create a lesson, teach it to the whole class, and analyze the student learning as a whole class. Teacher candidates will then focus on one student to analyze work from, as well as analyze their teaching of the lesson.

#11 Assignment G.2

Students will write a plan for and teach a lesson in class. The purposes of this assignment include practice and demonstration of teaching mathematics concepts and the ability to evaluate mathematics lessons (*Note:* the audience of the lesson is not clear from the assignment description).

#12 Assignment G.3

Teacher candidates will be responsible for completing an annotated bibliography of peer reviewed articles in the field of education, focusing on practitioner articles. This annotated bibliography will include a summative component (summary of the article) and an evaluative component (an evaluation of its usefulness to the student as a future teacher). Articles should be relatively current (newer than 1990) unless it is a seminal work. For each source, there should be an introduction of the work, along with the summative and evaluative components.

#13 Assignment H.1

This assignment is a two-part assignment. The first part requires teacher candidates to complete a chart by observing classroom features, and how these features are displayed by the teacher and the student's actions (an example classroom feature is, "Students share their ideas"). However, if these specific classroom features are not displayed within the observation, students are required to write down what they saw in place of that classroom feature. There is no direct pacing but it is expected that the prompts be completed over the course of 3 visits. The second component is a task-based interview with two students from the teacher candidates' field placement. The teacher candidate must choose a handful of tasks to complete with each student, and use follow-up questions to understand their mathematical conceptions and thinking (*Note:* example follow-up questions are given in the assignment; however, no example tasks are present).

#14 Assignment H.2

Teacher candidates will participate in three major activities during their field placement: 1) Interview two teachers, 2) Complete observations of two different math lessons from different classes at different grade levels, and 3) teach a small group of 2-3 students and conduct a pre- assessment, two lessons and a post-assessment. For the latter part, the teacher candidates will create a lesson plan and analyze one student's progress.

#15 Assignment J.1

Teacher candidates will create a chapter presentation to demonstrate content mastery of the assigned topic of the mathematics chapter assigned, demonstrate skill in developing a lesson plan and presenting a lesson, demonstrate understanding of and sensitivity to teaching all children, demonstrate understanding of assessment, and to review or re-teach the chosen material to the remainder of the class. Presentations should be in the form of a lesson for our class (not an elementary school class). Each lesson presented should include 1) assessment of the material covered in the lesson (but the assessment will be in the report and will not be administered), 2) discussion about how the specific lesson provided could be differentiated to meet the needs of students with ADHD or other disabilities, include content to meet the needs of culturally diverse students, and 3) activities that involves the class, including handouts as needed for each member of the class.

#16 Assignment K.1 [joint methods/math & science

During this math and science methods course, teacher candidates will choose one type of conservation of substance, of liquid volume, of number, of weight or of area and develop a task to evaluate a child for that conservation. The students will then use the task which they have developed to interview four children under the age of twelve. The interviews will be documented on video and a written report will be submitted which includes, a description of the conservation task developed, a description of each child, a summary of the interview responses, the teacher candidates' analysis of the responses and an explanation of what the teacher candidate discovered as a result of the project.

#17 Assignment K.2

During their mathematics methods course candidates will work together to plan, promote, and present a family math night program at a local elementary school. The math night will consist of a rotation of hands-on mathematics activities lead by the candidates, designed to improve student test scores and increase parental understanding of the NC Core Curriculum for mathematics. Candidates will also provide strategies to parents and other community members for helping students improve their mathematics skills.

#18 Assignment L.1

Teacher candidates will create and present three short (20 minute) math projects. Each project (a teaching lesson) will focus on a different content standard and will require different types of activities and methods of instruction (whole group, small group and one-on-one instruction). The content strands are number and operations – fractions, operations and algebraic thinking, and geometry.

#19 Assignment M.1 (joint math and science methods)

Students will select a topic and plan a lesson based on the lesson plan given (*Note*: no other information is given regarding type of topic or audience).

Appendix C
Focus Group Questions

1. You just spent much of your day reviewing assignments that teacher educators have identified as their key assignment that helps develop candidates into elementary teachers of mathematics. From these analyses (no need to identify any one assignment unless you want to), what stands out for you about tasks/assignments that *best* develop candidates into elementary mathematics teachers? Why?

   (Long pause; collect responses, follow up, with "What else?" as much as you can. Push their thinking on this question and other questions that follow.)

2. What other specific, individual tasks, assignments, experiences, readings, etc. do you believe are essential for the development of elementary mathematics teachers? (pause and let them to respond to math teachers first.)

3. Now, let's step back from assignments. What aspects/features of elementary teacher preparation programs *best* prepare candidates to teach? Why do you think this?

4. What is often left out of teacher preparation programs, and why? (Pause and let them respond.)

   a. What gets "crowded out" of your program that you believe is important, simply because there is no time?

5. What happens in programs (yours or others) that exists but probably is a waste of time, or at least not worth the time?

6. Now step back from programs, what experiences (in general) best prepare candidates to teach? Why do you believe this?

CHAPTER IV: HOW SHOULD WE DEVELOP TEACHER CANDIDATES? AN EXEMPLAR FROM A K-2 MATHEMATICS METHODS COURSE

Courtney G. Mayakis

Shulman's (1986) work has been a guiding force in teacher education discourse and research since its dissemination. Shulman suggested that within the profession of teaching, there were specific professional knowledges that teachers should know in order to be effective in the classroom. He argued that there were three types of professional knowledge: (i) content knowledge, or the deep conceptual knowledge that a teacher possesses about the content they teach beyond procedures and facts; (ii) pedagogical content knowledge, or the integration of subject expertise and content knowledge needed to teach in a particular subject area; and (iii) curricular knowledge, or the curriculum and tools utilized in that particular subject and/or grade level. Shulman (1986) referred to the content taught and questions asked to elicit student thinking as the "missing paradigm" in teacher education. Within this lies the idea of Shulman's pedagogical content knowledge (PCK) along with conceptual understanding of content and students' thinking. In past years researchers and practitioners have argued that mathematics educators must possess a type of teaching knowledge specific only to the teaching of mathematics that encompass these knowledges (Ball, Thames, & Phelps, 2008; Hoover, Mosvold, Ball & Lai, 2016). The guiding conceptual framework for this study is Shulman's (1986) original "missing paradigm" of pedagogical content knowledge, combined with Hill, Ball, and Schilling's (2008) extension of Schulman's pedagogical content knowledge for mathematics teaching. They define this knowledge as KWS and describe it as the combination of content knowledge and how students think and learn about the content.

Purpose

The purpose of this study is to understand how one teacher educator instructs preservice teachers (PTs) and develops them to grapple with the mathematical knowledge of teaching, as well as key PCK components in elementary mathematics. The professor chosen for this particular study was selected based on third party quantitative data (Bastian, Patterson & Pan, 2015; described in Chapter 3) which suggests that teachers who graduate from this elementary education program, in which the professor instructs mathematics methods, graduates teachers with value-added scores surpassing their peers from different public institutions within the same state. With this university's PTs taking the same number of methods courses as that of some of their peer institutions, an in-depth study of *what* is occurring in a methods course and how the professor taught was studied in order to understand what is occurring throughout this professors' instruction. The interviews used the themes of the observations that were conducted throughout this study. The interviews sought to specify the interlocking pedagogical and content concepts that this professor (pseudonym Veronica for the purposes of this study) believe to be the most important when developing elementary mathematics teachers. The observations aimed to verify and understand how Veronica implemented these concepts throughout her instruction. The research questions that will guide the study are as follows:

1. How is the integration of content, pedagogy, instructional tasks, and other concepts and frameworks displayed in a professor's mathematics instruction?
2. What key mathematics concepts are utilized in Veronica's teaching?

Method

Research Design

Even though it is evident that there is mathematical knowledge for teaching that exists, it is still unclear as to what specific content, pedagogy, instructional task, and concepts encompass this mathematical knowledge. Due to this, a grounded theory approach was used as the guiding structure for the design of this study and highlights the systematic creation of an emerging theory from data presented, rather than validating a theory through the use of data (Glaser & Strauss, 1967). Charmaz (2014) surmises that there are "telling distinctions about what stands as a grounded theory" (p. 15) that exist in the researcher's actions. These are:

(1) Conduct data collection and analysis simultaneously in an integrative process
(2) Analyze actions and processes rather than themes and structure
(3) Use comparative analysis
(4) Draw on data (e.g. narratives and descriptions) in service of developing new conceptual categories.
(5) Develop inductive abstract analytic categories through systematic data analysis.
(6) Emphasize theory construction rather than description or application of current theories.

These analytical practices were essential to utilizing a grounded theory framework for this study.

Participant Information

One professor was selected from the data set utilized in the second manuscript (Chapter 3). This selection was based on the quantitative data discussed in the above section (Bastian, Patterson & Pan, 2015). This professor, Veronica, teachers K-2 methods at a public institution in North Carolina. She is currently a teaching assistant professor and has worked at this particular university since the beginning of her career as a teacher educator.

Data Collection and Analysis

All data were collected from one institution from a single mathematics methods course in one state. This course focuses on the methods and content associated with early elementary (K-2) mathematics education. The data and participant were an extension of the study conducted in the second manuscript that focused on the structure of elementary education programs and the key assignments focused on the development of teacher candidates in elementary mathematics. The data collection process occurred over the span of six months in conjunction with the dates of the course.

Interviews

An interview was conducted with Veronica prior to the beginning of the instructional observations (see Appendix A) to discuss how she planned for instruction and her teaching style. Through the course of five months there were approximately 10 other interviews conducted, including interviews before instructional observations through the form a pre-conference, and interviews conducted after the observations at the professor's institution. Each pre- and post-observation interview consisted of 3-5 open-ended questions and were explicit questions surrounding Veronica's instructional views on key content, PCK, and instructional practices and supports. The post-observation interview questions were created based on “look fors” from the observation of instruction (see Appendix A). These interviews, as Charmaz (2014) explains in *Constructing Grounded Theory*, were intensive interviews that focus “the topic while providing the interactive space a time to enable the research participants’ views and insights to emerge” (Charmaz, 2014, p. 85). This type of interviewing allows for the researcher to pursue ideas that emerge within the interview and provides for an open-ended structure of interviewing. These interviews were recorded, transcribed, and coded for emerging themes.

Utilizing a constructivist grounded theory approach, I focused on “language and discourse” which “fosters encouraging participants to reflect upon their experiences during the interview in fruitful ways for advancing theory construction” (Charmaz, 2014, p. 95). The interview analysis process was intertwined with that of the data collection process. During the analysis process, line-by-line coding was utilized, and the initial coding represented the data as closely as possible.

These initial codes lead to focused coding or the comparisons that are created with and between the initial codes, which then lead to the emergence of a theme or theory (Charmaz, 2014). Constant comparative analysis was used after the initial coding (Glaser & Strauss, 1967). That is, the interviews were compared to earlier and later interviews, earlier and later observations, as well as vivo codes to establish “analytical distinctions” (Charmaz, 2014, p. 132).

Observations

Observations of Veronica's instruction throughout her mathematics methods course was documented utilizing field notes, "look fors," and memoing (see Appendix A for "look fors"). Observations were videotaped for utilization in the constant comparative analysis process. There were a total of five observations, with each observation spanning the length of the course. Veronica's course was a 3-hour methods section that met once a week for teacher candidates in the fall of their junior year of an elementary education program that focused on STEM methods. This course in particular focused on K-2 mathematics methods and learning how to elicit and respond to the thinking of one student versus focusing on classroom instruction. Students are also required to take another mathematics course that focuses on 3-5 methods and content with a shift in focus to planning and implementation of whole group instruction. However, for the purposes of this study, only the K-2 methods course will be analyzed and discussed. The observations of Veronica's instruction aimed to understand what PCK elementary mathematics content is covered throughout her methods class, and *how* she instructs PTs to develop the mathematical knowledges of teachings.

The field notes along with the "look fors" for these observations were coded line-by-line. Parts of the videotapes were transcribed and coded line-by-line as well to assist in the data analysis process. Memoing was also used to help form the meaning of themes throughout the observations. This line-by-line coding with observations and interviews also highlighted the gaps and overlapping themes in the data (Charmaz, 2014).

Findings

Throughout the data analysis several themes emerged surrounding the PCK, instructional tasks, and foundational concepts taught and infused in Veronica's mathematics methods course.

Theme 1: Building Empathy and the Case for Equity

Throughout her interviews, Veronica indicated that she wanted to instill an innate since of empathy in her teacher candidates. However, this idea of empathy can be seen throughout the tasks and assignments integrated within her course. In an interview, Veronica discussed why it was important for her to build this empathy.

> *So how do we as a whole try to help them make sure they will grow up in the sense of understanding a little bit more what it means to teach? But if I don't teach you empathy and you're going to public schools and you're in a place that has racist structures and problems like that, it's really easy for young teachers to see through the lens of the structures already in place. So I think it's critical to make sure that I teach my students in a way that they understand that looking at a kid is not a proxy for actually understanding the student's abilities*.

Veronica supported this goal in her classroom by introducing her students to an activity in the first class of the semester called "Base 6." She started the lesson by explaining to her PTs that they were going to add, subtract, and decompose numbers in a new base system called "Base 6." In Base 6 numbers were counted as 1, 2, 3, 4, 5, 10, then 11, 12, 13, 14, 15, 20, then 21, 22, 23, 24, 25, 30, and so on. Veronica gave her PTs time to grapple with the problems, although it was evident that some were frustrated in attempting to add, subtract, and decompose within this new base system. After giving her PTs time to attempt to solve these problems in partners she brought them back as whole class to explain her rationale for the activity.

> *This is the process of what it feels like for your elementary students to learn a base system. Some of you picked it up quicker—so what? It is not your job to*

> *identify the "smart" kids in your classroom. It is not your job to try to label your students abilities.*

Veronica specifically concentrates on empathy in the beginning of her course where she has students read articles, and watch lectures and videos focused on the idea of tracking in schools as well as issues of equity in mathematics. On the issues of tracking Veronica stated:

> *It's important to me that they become offended by the idea that it's important to decide which six-year-olds are smart and which six-year-olds aren't smart. I want to teach them how to teach math and teach them how to think about humanity.*
>
> *I'm really trying to get them to understand it is not their job to identify the strong students and the weak students. It's their job to provide rigorous content to all students.*

Veronica's emphasis on building this innate sense of equity into her PTs can be seen prominently interwoven between the next two themes discovered throughout her instruction: changing beliefs about mathematics teaching and learning, and using learning trajectories coupled with asset language.

Theme 2: Changing Beliefs About Mathematics Teaching and Learning

As teacher educators, we often forget that our teacher candidates are products of the same system that we are trying to change (Darling-Hammond, 2010). Changing teacher beliefs can be difficult but without confronting what teachers believe regarding mathematics and teaching, it is tough to build new constructs. Pajares (1992) explains how preservice teachers are "insiders in a strange land." Since teacher candidates have had at least a decade of schooling in a setting similar to that they are training to teach in, they have strong preconceived notions of school. Changing perceptions and beliefs for insiders can be extremely taxing and difficult to do (Pajares, 1992). However, it is hard to create space for new beliefs when teacher candidates are attached to their previous ones.

Veronica attends to this concern in her instruction by layering opportunities within assignments, tasks, and classroom instruction to alter the beliefs that teacher candidates have about teaching and learning mathematics, and how much of an impact these beliefs can have on their future students.

> *I want them to leave understanding that issues of equity are intrinsically related to how we understand student's content knowledge. So for instance, if we believe that math is about memorization, we can convince ourselves that the high, medium, low thing is useful right? Once we start to understand that, you know, once you understand these problems with tracking and how part of those problems is because of how we misunderstand what math is. So the first objective is that they can on some level take in or articulate that how understanding the inequities that are built into tracking are connected to our superficial understanding how what it means to teach math.*

Veronica discusses with students how this idea of tracking and how the implementation of it when they were in school had an impact on them.

> *A couple kids were confessional and said things like, yeah, I missed out by the highest group by a point and I think that affected me or, you know, I was in the highest track but I didn't understand anything. So I'm make sure to create a little bit of room for that this morning from them to talk about. Well, how did tracking affect you?*

Veronica works with her teacher candidates to change their beliefs about teaching mathematics and more importantly their beliefs about students in a systematic fashion.

Theme 3: Using Learning Trajectories with Asset Language

Along with working with her teacher candidates to change their beliefs and infuse empathy and discussions of equity within their instructional practices, Veronica instills in her teacher candidates the importance of portraying and thinking about their students through an asset-based lens. She uses learning trajectories as the foundation for this asset language so that they are able to discuss what their future students can do, and what her teacher candidates can do as teachers to build on that particular knowledge. Veronica uses videos of past teacher candidates completing specific tasks with K-2 students to show her current teacher candidates, to create space for them to practice discussion on what their future K-2 students *can* do. She describes an example of a past teacher candidate's video working with a student on subitizing and how she wants her teacher candidates to think about the student in the video.

> *Oh, wait a minute, what can he do? I need to get them in the habit of looking at their learning trajectories; what can he do? Because that's the habit I want them to have for their own kid and it's a problem they sometimes have in student support meeting—they really want to get at what their kid can't do. So, what can he do? I know you want this but while he was doing this unsuccessfully look at all the other things he showed you.*

Learning trajectories offer a tool for teacher candidates to use to reach this attainment of asset-based language while discussing their students. Sarama and Clements (2009) explain mathematical learning trajectories as consisting of three parts:

> …. a mathematical goal, a developmental path along which children's math knowledge grows to reach that goal, and a set of instructional tasks, or activities, for each level of children's understanding along that path to help them become proficient in that level before moving on to the next level. (p. 64)

The mathematical goal is the objective that a teacher seeks to teach their students. The developmental path is the natural progression of how a child would learn that particular goal, or the "levels of thinking, each more sophisticated than the last, leading to achieving the mathematical goal" (Sarama & Clements, 2009). The last crucial part of learning trajectories, the activities and tasks that help students move from one level to the next, inform teachers of what they can do to move a student's learning progression. Veronica interweaves these three parts of mathematical learning trajectories within various aspects of the pedagogical content knowledge that she focuses on in her classroom and incorporates these into facets of her instructional tasks while utilizing a practice-based approach to develop her teacher candidates.

Theme 4: The Use of Instructional Tasks and The Big Four

Veronica focuses on what she calls "the big four" of instructional tasks and assignments in her methods course. She explains her rationale for only focusing on four assignments in-depth in the following quote:

> *These are what I'm calling the big four. So basically, what are something things that I can teach that are relatively accessible to a young teacher but have high impact. It's not that I only have four things to say. It's that I know you can't learn everything in three and a half months about teaching K-2. Would I love to teach everything? Would I love to teach them about how to have an amazing discussion with kindergarteners? Yes. Would I love to teach them everything about geometry? Yes. But I can't, so I have to decide what are the high impact skills and what can I do in three and a half months. And I decide from my experience both with young teachers and in-service teacher and what they tell me, because I work with in-service teachers all over the state. They tell me what is valuable and what's helped them and what they wished they had learned in college.*

Veronica believes that focusing on these four assignments and teaching her preservice teachers how to instruct and administer these four tasks well to their K-2 students, will extensively build their students' number sense in their future classrooms and contribute to their abstract understanding of base ten. These tasks support foundational number sense for students in early elementary mathematics. Each of the "big four" is administered to the same student in the teacher candidate's K-2 field placement. This student is selected by the teacher candidate's cooperating teacher, and is often a "struggling" mathematics student. They use the information they gather about the student's mathematical knowledge and processes from administering the tasks to discuss the student's mathematical process at a mock support meeting, which is the culminating assignment for this course. Her "big four" include: teaching her candidates how to subitize, how to administer and gather relevant mathematical information about her students on the number knowledge test, how to accurately teach their students to play the vertical number line game to 10, and the circuit number line game.

Learning How to Subitize. Subitizing is the ability to instantly see "how many" (i.e. the ability to instantly see how many dots on a flashcard instead of counting the dots), and "is the direct perceptual apprehension of the numerosity of a group" (Clements, 1999, p. 400). According to Clements (1999) there are two types of subitizing: perceptual subitizing and conceptual subitizing. Perceptual subitizing is being able to identify a number without using other types of mathematical knowledge. Conceptual subitizing involves other types of mathematical knowledge. For example, perceptual subitizers will be able to see 3 dots on a domino without the use of other mathematical processes. However, conceptual subitizers can see two groups of 3 dots on a domino and add those 2 groups of 3 dots together to get a total of 6. This type of subitizing involves a mathematical process because the human brain cannot "see" numbers larger than 5 (Dehaene, 1997).

Subitizing is a foundational skill in the building of number sense in early elementary students. According to Clements (1999):

> Students can use pattern recognition to discover essential properties of number, such as conservation and compensation. They can develop such capabilities as unitizing, counting on, and composing and decomposing numbers, as well as their understanding of arithmetic and place value-all valuable components of number sense. (p. 405)

Veronica also believes that subitizing is a foundational skill and leads to the development of other mathematical processes in elementary education students. She believes that practicing subitizing can assist in the teaching and understanding of the above skills that

Clements (1999) discusses above. Therefore, one of the tasks that Veronica gives her teacher candidates to complete during the course of the semester is to create a set of flash cards that either focus on perceptual or conceptual subitizing. After practicing how to administer and facilitate discussion around the subitizing cards with a classmate, teacher candidates then videotape themselves completing the activity with their student in their field placement. Veronica watches these videos with her teacher candidates to give feedback on the administering of the subitizing task to students (i.e. they held the card up for too long, so students may be counting instead of subitizing).

Number Knowledge Test. This particular task, developed by Sharon Griffin, is used for the assessment of early number sense in elementary education students. Teachers administer the assessment to test for mathematical knowledge in students such as counting and cardinality, magnitude, and quantity discrimination (Griffin, 2004). The Number Knowledge Test (NKT) consists of Levels 0 to 3, and each developmental level is associated with an age (Griffin, 2004). Once a student receives a passing score in one level, the teacher then asks the student questions from the next level and adds up that score. The questions are asked verbally by the teacher to the student, and some questions are accompanied by visual representations. After administering the NKT, the developmental level in which the student scores in will allow teachers to understand where a student is in certain number sense component and developmentally where they need to be. Veronica argues that the NKT is an essential assessment for teacher candidates to not only know how to assess, but how to use the assessment to guide the instruction of their students. She *emphasizes* this importance in the following quote:

> *I want to know does this this kid has an internalized sense of the number line to 10, to a 100? Is this kid using base-10 strategies? How do you know which ones?*

> *I just need to know that first. And so, teaching them the assessment and the purpose of the assessment is surprisingly hard. I cannot tell you how often I saw that the number knowledge test tests two things: internalized sense of the number line and whether or not you're using base-10 strategies.*

Veronica spends time instructing her teacher candidates how to administer this test in her methods course and has them practice with classmates prior to administering it to the students they have been working with in their field placements. They gather information about their student through the administration of this test and utilize this information in their culmination assignment.

The Vertical Number Line Game to 10 and The Circuit Number Line Game. Veronica instructs her teacher candidates how to play the vertical number line game to 10 and the circuit number line game, also developed by Griffin, to build number sense, knowledge of mathematical facts, cardinality and magnitude in their future K-2 students (see Appendix B and Appendix C for directions to both games and example questions that could be asked to elementary student playing the games). The vertical number line game to 10 hones in on mathematical understanding, processes and facts within 10, while the circuit number line game focuses on these processes beyond 10. Veronica explains her rationale for using the vertical number line game to 10 as one of her "big four" tasks:

> *What's really nice about the number line game to 10 is it's a very discrete task but there's so many things I can learn from watching a kid do this and asking the right questions. Its like so, you're on five and they're on seven, who's winning? She is. By how many? I don't know, seven. Okay. They don't know how to match, compare. They don't know how to decompose a seven into five up to which we're even on the two extra. I can practice my facts to 10 without torturing a kid with flashcards, right?*

This quote characterizes the type of mathematical knowledge that Veronica is attempting to develop within her teacher candidates, which can be obtained from correctly playing the game with their K-2 students and asking the right questions to elicit student thinking.

Veronica discusses how the circuit number line game starts to develop the recursive pattern of base ten, or the idea that our numeration system has a distinct pattern that allows us to calculate and understand value.

> *And I believe that the circuit number line game goes a long way to get the kids to understand how the Hindu-Arabic numeration system works, right? I think that the most sophisticated of the big four is that circuit number line game.*

Through this dialogue with Veronica, it is evident that she utilizes the circuit number line game as a more sophisticated way of understanding how base 10 works and to practice the mathematical processes within this system.

Similar to the other tasks, Veronica spends time instructing her teacher candidates how to play both games in her methods course. She then has them rehearse with classmates prior to playing them with the student they have been working with in their field placements and spends time practicing what types of questions they could ask and what type of information they could learn about their K-2 students' mathematical knowledge from these questions. Teacher candidates are also required to videotape themselves playing the games with their chosen student along with other students in their field placement for Veronica to watch during the culmination assignment.

The Culminating Assignment: Mock Parent Support Meeting. During the semester, teacher candidates are required to complete the "big four" with the same K-2 student in their field placement. After these assignments are completed, teacher candidates compile information regarding their student's progress in mathematics, learned from completing these tasks with them, and attempt to utilize learning trajectories to discuss their student through an asset-based lens. This is completed at the culmination of the semester, and is what Veronica refers to as the mock parent support meeting. Each parent support meeting is completed by the teacher candidates individually and three of their peers are present to observe and learn from each other during the meetings. Veronica plays the role of the parent and the teacher candidates play the role of the teacher. During this meeting, Veronica also watches the videos of teacher candidates subitizing and playing the number line games with their selected student and gives teacher candidates individualized feedback regarding the implementation of their tasks. The goal of this assignment is to practice discussing what their students can do, and what teacher candidates can do as teachers in order to assist their students in shifting them into the next developmental progression of particular mathematical concepts. Veronica explains how the "big four" play a vital role in teacher candidates' discussion of their student and being able to articulate how early elementary concepts can lead to higher order thinking in mathematics when conferring with parents in the future.

> *Tell them look, I'm getting them ready for algebra. Look, let me show you how. Look, we need kids to not be in one's world. We need the to be thinking in clumps. Look, it's the big part of the Common Core Standards that numbers are made of other numbers. So, I teach them how to explain to a parent why this activity that seems so simple is actually quite lush.*

One of the most important and complex aspects of teaching is the ability to collect information about a student from all instructional tasks given and then utilizing this data to inform decisions on instructional choices. This assignment allowed Veronica's teacher candidates to experience and practice how they would discuss this information, as well as their instructional choices, with a parent. I was present for most of these mock parent teacher meetings, and it was evident that the teacher candidates had deep conceptual knowledge regarding their student's understanding of mathematical processes and could discuss their strengths in detail. As Veronica, acting as the parent, asked questions regarding the information that the teacher candidates were conveying to her about the student, and she often corrected their language to conform to a more asset-based approach. All of the teacher candidates' instructional tasks completed during Veronica's methods course, including the culminating assignment, focused on teacher candidates practicing and "doing" these tasks in an instructional setting.

Theme 5: Infusing Practice-Based Education

Practice-based education encompasses training around the work of teaching, centered on practices involving the profession, through the use of rehearsing these skills in an instructional setting (Ball & Cohen, 1999; Forzani, 2014; Matsko & Hammerness, 2014). The criticism that theory taught in university settings and the actual practice of teaching are often disconnected and leave teacher candidates unprepared when entering the classroom has triggered reforms that focus on practice (Ball & Forzani, 2009; Darling-Hammond, 2010; Deans for Impact, 2016; Grossman & McDonald, 2008; Zeichner, 2010). Zeichner (2010) asserts:

> This work in creating hybrid spaces in teacher education where academic and practitioner knowledge and knowledge that exists in communities come together in new less hierarchical ways in the service of teacher learning represents a paradigm shift in the epistemology of teacher education programs. (p. 480)

This shift in creating more spaces for practice has initiated a discussion of what is important for teacher candidates to know and be able to do in order to be an effective teacher, and the skills that are important for them to practice. Veronica echoes the epistemological shift of practicing the skills needed for the work of teaching with the following statement:

> *You remember doing that way more than you remember a paper because that paper is- the skill that it takes to write a paper is totally different skill than it takes to talk to a parent in a meeting, right? And to be able to explain what you saw a child do in a cogent manner, totally different skills. So, I plan thinking what I want them to be able to do, not what I want them to know.*

Reverting back to the instructional tasks that Veronica incorporates into her methods course, they all embody aspects of practice. Teacher candidates first practice administering the tasks to their classmates before completing the task with their student in their field placement. They videotape themselves implementing the task to their student in their field placement to receive feedback from Veronica. They use knowledge of their student through these instructional tasks to utilize during their mock parent meeting. However, in order to understand the intricacies of their students' mathematical knowledge, teacher candidates must first recognize how these singular units of pedagogical content knowledge are connected to each other.

Theme 6: Interconnecting Pedagogical Content Knowledge

Veronica's instruction is heavily infused with the PCK connections involved in the instructional tasks given to teacher candidates. Not only does she emphasize the connections between PCK throughout instructional tasks but how this PCK connects to content standards.

> *There are so many connections, and so then I have to teach them okay, this is not just an activity. How does this connect? What does this connect to? Okay, let's look at our content standards. What does this connect to? What are the content standards connected to?*

Veronica focuses on additive reasoning in her K-2 methods course. Within the scope of additive reasoning she leverages PCK concepts that she believes will have a high impact effect on the mathematical knowledge of her teacher candidates. These concepts include subitizing, the recursive pattern of ten, break apart to make ten and building an internalized sense of the number line. These PCK concepts are infused into each of the instructional tasks that Veronica employs in her methods course, and each concept connects to complex idea of base 10.

Veronica's instructional task of the subitizing cards prepares teacher candidates to be able to connect the idea of "seeing" a number as a perceptual subitizer to being able to break apart to make ten as a conceptual subitizer. For example, if a K-2 student can "see" a group of 5, a group of 4, and a group of 3, they will be able to "see" the group of 5 and 4 as 9, and the take 1 from the 3 to create a group of 10 and have 2 left over. The vertical line game to 10 and the circuit number line game builds an internalized sense of the number line for K-2 students and enables a strong foundation of number sense. The circuit number line in particular is sophisticated in nature because it fosters an understanding of the recursive nature of base 10, of the idea that the Arabic numeration system as a distinctive pattern in quantity and magnitude.

Limitations

During the employment of grounded theory, even though I checked the meanings of the data with a supervising faculty member, the observations and interviews were only conducted by me, a single researcher, as opposed to having a team of researchers for data analysis comparison in real time (Charmaz, 2014). Even though Veronica was chosen based on quantitative data that suggests her instruction is effective in elementary mathematics (Bastian, Patterson & Pan, 2015), the data and emergence of themes and theories were created utilizing a small subset of a larger group of teacher educators.

Implications and Further Research

Veronica plans her instruction around what she wants her teacher candidates to be able to do as teachers, and not necessarily what she wants them to know, through a practice-based lens. She emphasizes and leverages a few instructional tasks to ensure that her teacher candidates can do these well and considers these to be high-impact skills in K-2 elementary mathematics. Along with these practices she builds foundational knowledge of PCK within her teacher candidates and connects singular units of PCK to overarching mathematical concepts through the understanding of learning trajectories. Throughout these practices Veronica intertwines a sense of empathy and equity within her teacher candidates and creates a space to confront prior beliefs about teaching and learning mathematics.

The purpose of this study was to see how a particular professor instructs her teacher candidates in a methods course, and what PCK concepts and instructional tasks she utilizes throughout this course. The reasoning for this study stemmed from qualitative data that suggests the teachers who graduate from the program in which this particular professor teaches has students who produce higher value-added scores than that of their peers graduating from public neighboring institutions within the one state. The construction of a conceptual framework linking these PCK and instructional tasks together would be useful in the recreation of Veronica's K-2 methods course for use in other methods courses.

Even though the interviews with Veronica discussed other facets of elementary mathematics, the observations conducted were that of early elementary mathematics (K-2) and did not span to upper elementary (3-5) mathematics. Further research is needed to determine how these early elementary PCK concepts and instructional tasks link to those in upper elementary mathematics and not only how this might look as concept map, but also how these two courses combined can be viewed as one methods course for institutions that require only one in their elementary programs.

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Appendix A

Interview and Observation Protocol

Interview Questions Prior to 1st Visit

(1) Describe how you plan for instruction.
Sub Questions
a. What standards or frameworks do you use to assist in your planning?
b. How do you decide what to mathematics content to include?
(2) What preservice teacher (PT) mathematics (i.e. content) do you focus on during your planning and instruction of your methods courses?
(3) What children's mathematics do you focus on during your planning and instruction of your methods courses?
(4) *Question about tasks on Canvas page:* What mathematical knowledge do you think preservice teachers will understand after completing this task? What will they be able to do after completing this task?
(5) Describe the 5 main learning goals, associated with mathematics pedagogical content knowledge or children's mathematics, that you would argue are the most important in K-5 mathematics?

Definitions

Preservice Teacher Mathematics: The content taught to ensure that preservice teachers understand the content they are required to teach their students.

Children's Mathematics: The instructional strategies associated with content that are taught to preservice teachers so that they are able to instruct their K-5 students' successfully. These instructional strategies should encompass how students learn and strategies to help understand K-5 students' thinking.

Pre-Observation Questions

(1) What are the learning goals for this particular lesson?
a) Learning goals specific to preservice teacher (PT) mathematics (or content)?
b) Learning goals specific to children's mathematics (or PCK)?
c) Learning goals specific to instructional strategies?
(2) What are the instructional activities that you will be utilizing for this lesson?
(3) How will you know if your learning goals are met?

During Observation – "Look Fors"

(1) PCK – in relation to all learning goals; big learning goals
(2) The way in which learning goals are supported
(3) Enactment of instructional activities

Post-Observation Questions

3-4 Post-observation questions will be asked from what is recorded and observed during the observation surrounding the pre-observation questions and the during observation "look fors." These will be more for clarification and elaboration.

EXAMPLE
OBSERVATION "LOOK FORS"

(1) **PCK.** Utilizing an additive strategy: decomposing two, three-digit numbers to add.
(2) **Learning Goal Supports.** Using K-5 student work, the use of Base-10 blocks, specific tasks given and student work.
(3) **Enactment of Activities.** A review of preservice teacher mathematics (content), then using a task for PTs to work through, then have PTs look at the same task in which K-5 students have worked through the task to analyze student work (essentially the instructional sequence and the grouping of the class).

POST-OBSERVATION QUESTIONS

(1) I saw that you focused your instruction around the decomposition of three-digit numbers to add. If PTs understand this concept, what else could they apply this knowledge too? What larger PCK concept would this be connected to?
(2) I saw that you gave students a task in which they had to analyze student work of adding two, three-digit numbers utilizing decomposition. What was your intention behind using this particular task? How does this support your original learning goal of ____.
(3) I saw that you first reviewed the content (PT mathematics) of this particular PCK concept, then used a task for PTs to work through, then had PTs look at the same task in which K-5 students have worked through, to analyze their work. Why did you choose this particular instructional sequence? How does this support your original learning goal of ____.
(4) Do you think the lessons learning goals were met? What do you expect PTs to be able to do after this lesson?

Appendix B

Vertical Number Line Game to 10 Instructions
Adapted from Sharon Griffin/Number Worlds/SRA
(Taken from Veronica's Course Pack)

- One Game Board for each two players (each player uses one vertical number line for their ascent to ten)
- One Pawn game piece per player
- 10 colored counters per player
- One die per game with pips from 0-5.

1. Students will take turns rolling the die. They will count the number of dots, and then count out the same number of counters. Ex. Die lands on 4. Count out 4 counters.
2. The student will place the counters one at a time onto their number line, starting one place higher than where the pawn is located.
    a. Encourage the students to count aloud as they place the counters on the number line so that the other students can make sure that it is being done correctly.
3. Once the counters are one the number line, the student will move the pawn from the starting position, touching each counter with the pawn and once again, counting up to the last counter where the pawn will now rest.
4. Winner: The student who lands EXACTLY on 10. If, for instance, a student is on 8 and rolls a two. If a student is on 8 and rolls a 5 they don't move. If they roll a 1 they would proceed as before and go to 9. But from there they need a 1 to win.
    - Key Questions: Who is winning? How do you know? Where will you land if you roll a ____? How many more do you need to land exactly on 10? How many over 10 would you be if you rolled a _____?
    - Once Students are adept at the vertical number line game UP, then you can start on 10 and see who can get down to 0/ground level. This will help them with counting back from 10 and subtraction.

Appendix C

Circuit Number Line Game Instructions
Adapted from Sharon Griffin/Number Worlds/SRA
(Taken from Veronica's Course Pack)

LEVEL 1 (K)

- One Circuit Board (a board that is a circular number line from 0-9)
- Two Game Pieces
  Award Cards

1. The students will use a die to determine how far they move around the circuit.
    - You can use 1-6 numbered die or 0-5 numbered die
2. With each complete circuit, a student can pick up an "Award Card"
3. Who Wins?
    a. First student to reach X Award cards
    b. Who has the most Award Cards in X minutes (allotted time to play)

LEVEL 2 (1st-2nd Grade)

- One Circuit Board
- Two Game Pieces
- Ten Cards

1. The students will use a die to determine how far they move around the circuit.
    - You can use 1-6 numbered dice or 0-5 numbered
2. With each complete circuit, a student can pick up a "Ten Card"
    - Use this time to make sure students understand that 10 ones = 1 Ten
3. Who Wins?
    a. First student to reach X "Ten" cards
    b. Who has the most "Ten" Cards in X minutes (allotted time to play)

    A key question is "who is winning…how do you know" because this is the inroad into understanding that some things have greater value than other things. 1 award card is worth more than 1 on the circuit board. Why? What does the 1 award card represent? What does the 1 on the circuit board represent?

CHAPTER V: CONCLUSION

Courtney Glavich Mayakis & John Williams

It has been increasingly important in the 21st century to provide a teaching environment that supports learners to be competitive in our innovative society. In schools today, most students are being educated to perform jobs and solve problems that have not yet been created (Darling-Hammond, 2015). However, on international assessments it is clear that the United States consistently falls behind other developed countries in educating students compared to peer countries. For example, in 2009 the United States ranked 25th in mathematics on the Programme for International Student Assessment (PISA), and fell to 39th in mathematics in 2012. Therefore, a growing concern remains how to achieve and demonstrate growth in mathematics education, as we consistently lag behind other countries in this key area (Battista, 2001; National Center for Educational Statistics, 2015; National Commission on Mathematics and Science Teaching for the 21st Century, 2000). Changing the trajectory of the U.S. in mathematics rankings will require a restructuring of mathematics instruction from teachers.

Several variables correlate with K-12 student achievement in mathematics including teacher quality and teacher preparation (Darling-Hammond, 2000, 2015; Hill, Rowan & Ball, 2005). In a study conducted by Campbell et al. (2014), the researchers identified a significant relationship between students' mathematical achievement and their teachers' mathematical knowledge. Other research has also shown a significant correlation between teachers' mathematical knowledge, student achievement, and gain scores on standardized tests (Baumert et al., 2010; Hill et al., 2005). Research has also suggested that other factors such as beliefs and expectations of teachers of "who can and cannot learn mathematics," as well as the practice of teaching mathematics prior to entering a classroom, can all have a meaningful impact on how a teacher delivers instruction, therefore significantly impacting student achievement (Ball, 1996; Beswick, 2006; Hill et al., 2005). Research has shown that there is a clear connection between K-5 students not doing well in mathematics and K-5 teachers struggling to teach mathematics effectively. With the responsibility to prepare elementary teachers falling on that of preservice programs along with the burden to prepare elementary teachers to be experts in each subject, if we cannot teach everything to our preservice teachers (PTs) within the realm of elementary mathematics, what do we teach? And how do we know if it is effective?

We will discuss and recommend the creation of a cohesive framework for mathematical concepts in elementary teacher education, focused on early elementary (K-2) that are deemed conceptual gateways in order to ensure the effective preparation of these future teachers. These conceptual gateways are overarching concepts that enable in-depth understanding of content interwoven with pedagogical knowledge. We associate these conceptual gateways to the idea of threshold concepts (TCs), which according to Meyer and Land (2003):

> …. Can be considered as akin to a portal, opening up a new and previously inaccessible way of thinking about something. It represents a transformed way of understanding, or interpreting, or viewing something without which the learner cannot progress. As a consequence of comprehending a threshold concept there may thus be a transformed internal view of subject matter, subject landscape, or even worldview. (p. 1)

By identifying these TCs in primary mathematics education, teacher educators can have frameworks to utilize in order to ensure that preservice teachers are learning the content and pedagogical knowledge that is vital to successful mathematics instruction for elementary students. Along with these threshold concepts will be foundational concepts regarding teacher's beliefs and instructional tasks or supports to assist in the development of these threshold concepts. We will set the context for the progression of how this conceptual framework was created, along with an explanation of the key components of the conceptual maps that are used to support Base 10 as a threshold concept (see Appendix A). Then we will explain further research needed to identify more threshold concepts in elementary mathematics to assist in the development of teacher candidates.

Context of Study

The creation of these conceptual frameworks was derived from a larger study that explored how elementary education professors in a particular state were developing preservice elementary teachers throughout their respective universities. In 2015, data was released from a study by Bastian, Patterson, and Pan that matched new teachers to their students' achievement and average gain scores in elementary mathematics and various other subjects, and paired them to their graduating institutions in this particular state. From this data, a qualitative study was employed to gain an understanding of the structures of these elementary education programs, as well as what professors in these programs believed to be key in developing teacher candidates. We collected and analyzed key assignments, syllabi, and qualitative self-assessments regarding key assignments, as well as implemented and analyzed focus groups of the elementary education program coordinators and mathematics content and methods professors.

However, the data collected and analyzed in the qualitative study depicted only what teacher educators said and not necessarily what they did to develop teacher candidates. In order to gain a clear picture of best practices in the development of elementary mathematics teacher candidates, a professor of the university whose graduates produced the highest achievement gains and growth scores for students in elementary mathematics in Batisan, Patterson, and Pan’s study (2015), as compared to peers graduating from different teacher preparation programs in this state, was studied carefully. Qualitative data was collected from the one professor teaching the university’s elementary mathematics K-2 methods in the form of observations, video recordings, and interviews to discover how the professor was preparing teacher educators. These data assisted in the creation of a threshold concept, base 10, as well as the supports needed teach this concept effectively in a K-2 methods course.

Base 10 as a Threshold Concept

According to Land, Cousin, Meyer, and Davies (2006), “It has long been a matter of concern to teachers in higher education why certain students ‘get stuck’ at particular points in the curriculum while others grasp concepts with comparative ease” (p. 53). Therefore, the idea of TCs was developed to help cultivate a solution to this problem. According to Meyer and Land (2003) a threshold concept is often considered a “core concept” in that discipline. According to Cousin (2006), it is frequent that instructors at universities are expected and therefore teach an immense amount of content in their courses. In turn, they expect their students to regurgitate this vast amount of information taught in their university courses. In contrast, recognizing TCs in respective disciplines would allow instructors to teach gateway concepts that are vital to their disciplines instead of teaching a wide array of often disjointed content associated to that discipline or course. Land et al. (2006) explain the importance of TCs in disciplines:

> Once a student has internalized a threshold concept they are more able to integrate different aspects of a subject in their analysis of problems.

> Students who have not yet internalized a threshold concept have little option but to attempt to learn new ideas in a more fragmented fashion. On acquiring a threshold concept, a student is able to transform their use of the ideas of a subject because they are now able to integrate them in their thinking (p. 53).

Base 10 will be utilized as a threshold concept prototype to conceptualize the idea of K-5 mathematics threshold concept. We argue that this key concept is essential in K-5 mathematics teacher education and subsequently (Cooper & Tomayko, 2011; Ponce, 2015), K-5 student learning, and is undoubtedly a threshold concept meeting the five characteristics of TCs outlined by Meyer and Land (2003):

(1) **Transformative:** That is, once a threshold concept is understood, it has the ability to change the way one perceives the subject matter. It has the power to shift values, evoke emotions or change attitudes.

    Base 10 Example. Once a preservice teacher fully understands the concept of Base 10, it will transform the way that they think and in turn the way they teach about number. For example, when teaching the traditional algorithm for subtraction, instead of using the terminology such as "borrowing from a neighbor," preservice teachers with an in-depth knowledge of Base 10 and how it applies to operations, would instead explain to students the conceptual idea of regrouping instead of the procedural idea of "borrowing."

(2) **Irreversible**: This refers to the idea that a threshold concept is not easily if ever forgotten.

    Base 10 Example: Following the regrouping example under the concept of Base 10, it would be difficult for preservice teachers to forget that instead of “borrowing from a neighbor,” their students are in fact having to decompose and change the representation of the tens or hundreds in the number in order to be able to subtract. With this knowledge, preservice teachers would be more inclined to teach the concept, and not just the algorithm.

(3) **Integrative**: Refers to the idea that a threshold concept can expose the interrelatedness with other concepts that were once seemingly unrelated. ***Base 10 Example:*** The concepts related under Base 10 are extensive and are interconnected. For example, subitizing is a basic skill but is related to more advanced skills such as accessing numbers in groups other than one, that leads to skip counting, which leads to breaking apart a multi-digit number to make 10’s, which is decomposing numbers using the Base 10

(4) **Troublesome**: These TCs can be difficult for some learners. ***Base 10 Example:*** Some preservice teachers will find the idea of using Base 10 to teach conceptual ideas instead of procedures undoubtedly troublesome. Many preservice teachers enter the teaching profession understanding only procedures, but not the way in which these procedures work and why they work, and therefore cannot articulate it in their practice. When confronting an individual with a new idea, it is disruptive to their existing schema, and requires practice and conversation to ensure that these key concepts are not overpowered by their already existing conceptions and misconceptions.

(5) **Bounded**: Certain concepts are restricted by their disciplines, and when crossing the threshold of boundedness, could cross that of the discipline line.

Base 10 Example. Base 10 is bounded in that it only refers to the mathematical disciplines, even though skills of Base 10 can cross over grade levels. Base 10 is not an interdisciplinary concept that can be utilized interchangeably. When teaching Base 10 in mathematics, teachers must inform students of the specific context behind its usage, which may conflict with how the concept is used outside the confines of mathematics. It is incumbent of the teacher to explain how Base 10 specifically operates within the confines of mathematics.

However, it is not enough to know threshold concepts, but it is as equally important to understand how to develop teachers to obtain an in-depth understanding of the concept, and what supports are needed to result this in-depth understanding. We argue that the smaller units of pedagogical content knowledge that form base 10, the emphasis on changing beliefs about components of teaching, as well as the significance of practice-based teacher education, are invaluable to the development of Base 10. In the below sections we will explain the importance of each of the concepts associated with the concept map for Base 10 (see Appendix A for concept map).

Changing Beliefs About Components of Teaching

Before content and instructional tasks can be addressed it is important to confront the existing beliefs that PTs have about teaching mathematics, how their students learn mathematics, and how they view and talk about their students.

Empathy

Despite the standardization of teacher education programs, and their efforts to ensure that all preservice teachers are gaining content, subject, and pedagogical knowledge, a key aspect that is often overlooked is teacher's willingness to develop a sense of empathy. Prior to entering the teaching profession, candidates enter teacher education programs with predefined sets of beliefs, which puts them at odds with the lives, backgrounds, and experiences of their students. A few studies have noted that teachers struggle with comprehending what empathy is, and how to effectively use empathy to strengthen the learning opportunities for their students (Berlak, 2004; Winans, 2010, 2012).

Empathy allows an individual to understand differences and value alternative perspectives. The notion of empathy requires individuals to distinguish their feelings and experiences and understand certain circumstances from the individuals who are to receive the empathy (Decety & Lamm, 2006). Teachers employing empathy seek to discover the perspective of their students, investigate the context with those perspectives (Baston, Early, & Salvarani, 1997), and utilize their student's experiences as tools to enrich the learning environment (Halpern & Weinstein, 2004). Teachers' use of empathy increases their receptiveness towards their students, opening future possibilities for students to express constructive, critical feedback.

Debunking How Preservice Teachers Were Taught

Employing learning trajectories requires a fundamental understanding of what empirically works in mathematics instruction, which is often at odds with what preservice teacher were taught in the K-12 experiences or during their university coursework. For example, preservice teachers are often trained to view mathematical subjects, content, and the practice of mathematics as separate items. Critical to the instruction of mathematics is the debunking of thoughts and beliefs along these ill-informed lines (Ball, 1996), and preparing preservice teachers to instruct content over multiple and various opportunities (Common Core State Standards Initiative, 2010). When preservice teachers hold on to misconceptions and myths about mathematics that they either learned during their coursework, or during their own K-12 learning experiences, they tend to enter the field with anxieties with how to properly teach mathematics (Kogelman & Warren, 1978). This has a negative effect on their students' learning opportunities, which reduces their academic performance in mathematics because they are taught to learn correct answers, instead of being supported in their quest towards mathematics conceptual development (Beswick, 2006).

Correcting preservice teachers' misconceptions about mathematics is no easy task, however, Arvold and Albright (1995) determined in their study of secondary mathematics teachers that in order to create a dualistic ideology amongst preservice teachers, they must be provided with an alternative theoretical framework that is both plausible and implementable in the classroom. This is important because typically when students have difficulties in learning mathematics, preservice teachers normally revert to their own difficulties with mathematics. By creating a positive and equitable learning environment for preservice teachers, it is possible to reform their understanding of mathematics, reduce their internal anxiety of teaching mathematics, and strengthen their capability to relay information to students once they enter the profession (Belbase, 2010; Miller & Mitchell, 1994).

Asset Language

Essential to a teacher's use of genuine empathy in their instruction is the application of asset-based language. Traditional modes of instruction do not value the funds of knowledge that reside in students, their culture, or their communities. This deficit-based style seeks to reaffirm dominant power ideologies in the classroom, which often stigmatizes students if their capabilities do not afford the teacher the opportunity to instruct with ease (Freire, 1970). Asset-based language is the direct opposite, as this approach emphasizes the use of local knowledge in the classroom.

Teachers who incorporate asset-based language value the art of teaching in addition to the limitless possibility for students to learn (Freire, 1970; 1988; Sharkey, Clavijo, & Olante, 2012). All participants in the classroom are deemed valuable within this approach. Student contribution is valued, and teachers affirm their students in a manner that allows them to co-construct the curriculum, which optimizes the teacher's ability to effectively infuse the student's perspective and their own into the subject matter. Researchers have found that successful teachers incorporate asset-based language as way of life, rather than just using it sporadically with random lessons (Freire, 1970; 1988; Sharkey, Clavijo, & Olante, 2012). The inclusive usage of this concept speaks to the way teachers view reality, and how they can create new supportive opportunities for students to engage in learning.

Pedagogical Content Knowledge (PCK) and Their Links to Instructional Tasks

This conceptual framework of Base 10 (Appendix A) focuses on the development of number sense. As Griffin explains (2004a):

> The discipline of mathematics comprises three worlds: the actual quantities that exist in space and time; the counting numbers in spoken language and formal symbols, such as written numerals and operation signs. Number sense requires the construction of a rich set of relationships among these worlds. Students must first link the real quantities with counting numbers. Only then can student connect this integrated knowledge to the world of formal symbols and gain an understanding of their meaning. (p. 40)

The PCK and instructional tasks that help develop this PCK described below are imperative in the development of number sense and base 10. These instructional tasks and the PCK highlighted through these tasks would allow K-2 students to connect the meaning behind digits to formal symbols and the construction of the quantity and magnitude behind these symbols.

Subitizing

Subitizing, or the ability "to instantly see how many," is a foundation mathematics skill for students in early elementary (Clements, 1999). The brain "sees" numbers through an analogue processor, but only has the ability to do this in small quantities up to five or six (Dehaene, 1997). Perceptual subitizers have the ability to see these quantities up to five or six, but conceptual subitizers have the ability to piece together the quantities they can "see" (Clements, 1999). For example, if a child is looking at a flash card that consists of eight dots, they will be able to "see" these dots as two groups of four or a group of five and a group of three.

Hurst and Hurrell (2014) attribute the foundational skill of practicing and perfecting subitizing to the idea of "trusting the count." When children "trust the count" they are able to think in entities of base 10 and can start to apply this knowledge to multiplicative thinking. Embedding meaning into digits for early elementary children is extremely important in developing their number sense. Cain and Faulkner (2010) explain this idea with the question “Is the word C-A-T a cat?” The meaning of cat is communicated through the use of words, similarly to how the quantity of five is communicated through the number five. Subitizing helps to foster these foundational meanings of what digits represent to children, often leaving to a more complex way of thinking and understanding magnitude and quantity.

The instructional task of *subitizing cards* is the idea of utilizing a platform for teacher candidates to practice conceptual and perceptual subitizing with their K-2 students. These cards (see Appendix B for an example) are flashcards that use different groups of dots spanning from perceptual to conceptual subitizing to allows PTs to practice how to subitizing with their K-2 students so they can perfect this skill prior to teaching. Understanding the larger PCK and mathematical knowledge that links subitizing to the more complex knowledges of base 10 is important to building the mathematical foundation of PTs so they are able to support this complex thinking in their classrooms.

Break Apart to Make Ten

Perceptual subitizing allows for students to “see” numbers up to five or six, but conceptual subitizers can use their perceptual knowledge to break apart numbers to create units of 10. For example, if a student sees a group of six and then a group of five, they can break the five apart into two groups: one group with 4 ones and one group with one. Then they could make ten from the group of six and the group of four to create a ten with one leftover. This level of mathematical knowledge allows students to enable their knowledge of number up to and beyond 10. The other unit of PCK and instructional tasks described below allows for the growth and development of students' strategies to break apart quantities to make tens.

Recursive Pattern of Base Ten

A recursive pattern is a rule that tells you a start number of a pattern and how the pattern continues. A recursive pattern of base ten describes how the Arabic numeration system has an innate pattern in which we count and decipher quantity utilizing. For example, we know that digits in base 10 start with 0 and we can count from 0 to 9, but once we reach 10, we use two digits to represent this quantity and symbolizes a group of 10 ones as 1 ten; hence the 1 in the tens place value. The *circuit number line* enables students to practice and develop a deep understanding of this recursive pattern (Griffin, 2004b).

The *circuit number line* is a game board set up in a circle, ranging from digits 0 to 9. K-2 students start on the "0" place, and take turns rolling the dice to move that number of spaces. After a student passes or lands on 0, they receive a trophy card. Students then start to understand that the value of the trophy card is worth one ten. This game is sophisticated in nature because it forces students to think about the meaning of the trophy card and allow teachers the opportunity to ask questions regarding magnitude and quantity. For example, if Student A is on 1 but has passed 0 three times, and Student B is on 8, but has only passed 0 two times, a student would have to conceptually understand the meaning of the trophy card to be able to internalize that Student A has the larger quantity in the game. Asking students questions such as "Who is winning? By how many? Who has more? Who has less?" can aid teachers in the understanding of their students' knowledge of magnitude and assist its development. The practice of instructional tasks such as this will develop teacher candidates who effectively elicit student thinking around the ideas of digits, quantity, and magnitude, and can enhance their knowledge as classroom teachers to develop this number sense, accompanied with an internalized sense of the number line within their K-2 students.

Internalized Sense of the Number Line

Griffin (2004a) explains that by the age of four, "children have constructed two schemas; one for making global quantity comparisons, and another for counting" (p. 40). By age five or six children's thought about these schemas transform in a single "conceptual structure for number" (Griffin, 2004a). According to Griffin (2004a) this new conceptual structure enables children to use numbers without relying on objects or a number line to count. When students develop this, they are internalizing the number line and are able to justify magnitudes and quantities without the use of other concrete supports (Griffin, 2004a).

The game discussed above, the *circuit number line*, as well as the game *race to ten* assists in this transformation of a single "conceptual structure for number" and an internalized sense of the number line. The *race to ten* is a vertical number line game that spans from 0 to 10. Similarly, to the *circuit number line* students will roll a dice to see how many spaces they move from 0. However, this game only focuses on counting to 10 and not beyond. If a student is on 8 and rolls a 3, they are unable to move. They must roll the exact number of spaces to land on 10. This game focuses on magnitude and quantity, as well as internalized sense of the number line within 10, while the *circuit number line* focuses on this knowledge beyond 10.

Number Knowledge Test

The *number knowledge test* (NKT) developed by Sharon Griffin (2009), tests early elementary students for deep conceptual knowledge of number and number sense. Griffin (2009) explains the development and use of the NKT:

> (a) test questions were designed to be novel tasks to minimize the extent to which performance would be influenced by previous instruction and (b) the test was designed to be an oral test to tap conceptual understanding and working memory capacity and to minimize the procedural supports that can be obtained through the use of paper and pencil. (p. 99)

There are four levels, ranging from Level 0 to Level 3 (see Appendix C for test questions) that test for this conceptual knowledge.

Level 0 tests the ability to count and that ability to understand global quantity. This level utilizes concrete manipulatives in order to assist in this level (Sharon, 2009). No other levels use manipulatives. Level 1 test items,

> ... included at this level assess children's understanding of the single-digit number sequence (items 2, 3, 6, and 9), their ability to make relative size judgments (items 4 and 5), and their ability to use their knowledge of counting to add and subtract small sets (items 1, 7, and 8). (Griffin, 2009, p. 99)

Level 2 mimics that of Level 1 but at a more complex level, especially as it relates to magnitude. Questions on this level address students' understanding of the tens place value versus that of the ones place value. Students are asked questions such as *Which is bigger: 69 or 71?* (see Appendix C). This require students to understand that the 7 in 71 has a higher value than the 6 in 69. If students are only looking at the ones place to determine value they would think that 69 is larger than 71 because of the 9 ones in the ones place in 69. Level 3 test items are similar to that of Level 2 but require a deeper understanding of magnitude and quantity to answer. All of the PCK and the instructional tasks that are used to support these concepts build the foundation for the mock parent teacher conference.

Developmental Progressions and Best Practices

Practice-Based Education

In recent decades, there has been a considerable amount of literature published on importance of embedding practice-based education in preservice learning (McDonald, Kazemi, & Kavanagh, 2013). Practice-based learning is defined as the concentrated effort to immerse preservice teachers in the work of teaching, rather than in ambiguous theoretical concepts presented in their courses (Forzani, 2014). These efforts align preservice teachers with the core basics of teaching, rather than instructing them to become just spectators of what works and what does not work in a classroom.

The benefit of adding practice-based education to the development of preservice mathematics teachers is that it provides them with practical insight into how instruction is provided for students for specific content areas. This simplified approach to observing instruction allows for university instructors and clinical educators to hone in on specific practices that preservice teachers can enact on day one for certain lessons. Preservice mathematics educators are then able take local, salient issues within the classroom, and create measures to solve future similar problems based on the actual practices they were able to take partake in (Higgins & Eden, 2015).

Learning Trajectories

As indicated by Simons (1995), learning trajectories in mathematics exemplify a cyclical process that analyzes the strengths of students and uses the data from this analysis to develop a progression of thinking and reasoning by the student. This progression or "trajectory" is a cycle situated in engaging students in learning opportunities, combined with the refinement of their learning opportunities based on empirical and theoretical evidence from prior lessons. This refinement by teachers seeks to identify the goals first, and then reverse engineer the specific tasks that will assist the student to obtain that goal (Stevens, Shin, & Krajcik, 2009). There is a deep, explainable connection between what is being learned by the student, and the tasks employed to help them conceptualize and operationalize the mathematical knowledge on which they were instructed (Clements & Samara, 2004). Each task is specifically connected to a point, or several points on the trajectory, and subsequent tasks on the trajectory build upon previous lessons and tasks. When utilizing learning trajectories, teachers are constructing an affirmative hypothesis, which is often rooted in asset-based language, asserting that a student can achieve a specified goal, if provided the correct content and tasks that embolden them to evolve through a logical set of mental and physical actions (Gravemeijer, 1999).

Mock Parent-Teacher Conference: The Culminating Assignment

The mock parent-teacher conference showed in the Base 10 framework is the culmination of the PCK and instructional tasks that teacher candidates use in order to discuss the progress of a K-2 student utilizing all practices discussed in the previous sections of this paper. After completing the games and assessments discussed in the

Pedagogical Content Knowledge (PCK) and Their Links to Instructional Tasks section with one student from their field placements, teacher candidates gather this information from the NKT, circuit number line game, race to ten vertical number line and the subitizing cards to discuss their students' mathematical knowledge in a cogent manner using asset-based language. Teacher candidates use their knowledge of learning trajectories to discuss what their student is excelling at, and what supports they will put into place to advance their student to the next developmental progression. These mock parent-teacher conferences are done by the teacher candidates themselves with the professor assuming the role of the parent, asking questions throughout the meeting to probe the teacher candidate's knowledge of their students. This culminating assignment embodies practice-based teacher education as teacher candidates must hold the mock parent-teacher conference as if they were consulting with their student's parent. This practice solidifies the teacher candidates' knowledge regarding the PCK concepts within base 10 that are focused on during this particular K-2 methods course.

Supporting Threshold Concepts: Linking Content, Pedagogy, Instructional Tasks and Best Practices

These facets of Veronica's instruction create a foundation for her teacher candidates to develop their conceptual understanding of base 10. Veronica uses instructional supports, tools and frameworks with a focus on specific pedagogical content knowledges to support facilitate conceptual understanding of digits and how these related to quantity and magnitude to enable a strong conceptual understanding of base 10 in her teacher candidates (see Appendix A). We argue conceptual maps of threshold concepts (TCs) such as the base 10 map outlined in Appendix A will enable preservice teachers to grasp a deeper understanding of pedagogical knowledge. In this respect, we are linking a profound understanding of essential content knowledge to the ability to utilize a variety of teaching techniques, instructional tasks, and extensive pedagogical practices. Even though we are suggesting that TCs should encompass overarching concepts of content knowledge, these concepts, when understood fervently, will result in the heightened ability to demonstrate pedagogical techniques. A strong understanding of content, pedagogical practices, and how teacher candidates to understanding how specific instructional tasks can support these practices, allows teacher candidates and subsequently their elementary students to understand the *why* and *how* behind the mathematical content being taught and the formation of a deep conceptual understanding of number sense.

The creation of more frameworks that are able to address TCs (i.e. transformative key concepts) in K-5 mathematics education would be beneficial to teacher educators and ensure that preservice teachers are learning key concepts that will attribute to their effectiveness as not only a K-5 teacher but a K-5 mathematics teacher. With K-5 teacher education struggling to provide sufficient mathematics instruction to their teacher candidates, it is compelling that we reform pre-service teacher mathematics education in the primary grades. We recommend the creation of these conceptual maps, focusing on threshold concepts, for the improvement of connecting pedagogical content knowledge, instructional tasks and practices that are important for the development of conceptual knowledge of K-5 mathematics preservice teachers.

In conclusion, we argue that in order for pre-service teachers to morph into effective K-5 mathematics educators in the classroom, a set of frameworks should be developed in order to ensure that pre-service teachers are learning and understanding concepts that are vital to efficient, effective, and multifaceted mathematics teaching. The question remains: what are these key concepts and instructional tasks that will inherently transform the thinking of these future educators? An example of what a possible threshold concept could look like is denoted in Appendix A.

The creation of these TCs could result in an efficient and fundamental tool for teacher educators. It is evident that, in elementary preservice education, many facets of teaching are required in order for teacher candidates to be effective elementary teachers. With many universities only having time to instruct one or two elementary mathematics content and pedagogical course(s), this type of tool would be useful as a guide for teacher educators to discern key interconnected concepts that are essential for preservice teachers to understand, and how these overarching concepts can relate to smaller units of curriculum, instructional tasks, and specific teaching techniques. However, we urge that further research and discussion regarding *what* should be included within the creation of these frameworks is needed to the discovery of future TCs in K-5 mathematics teacher education.

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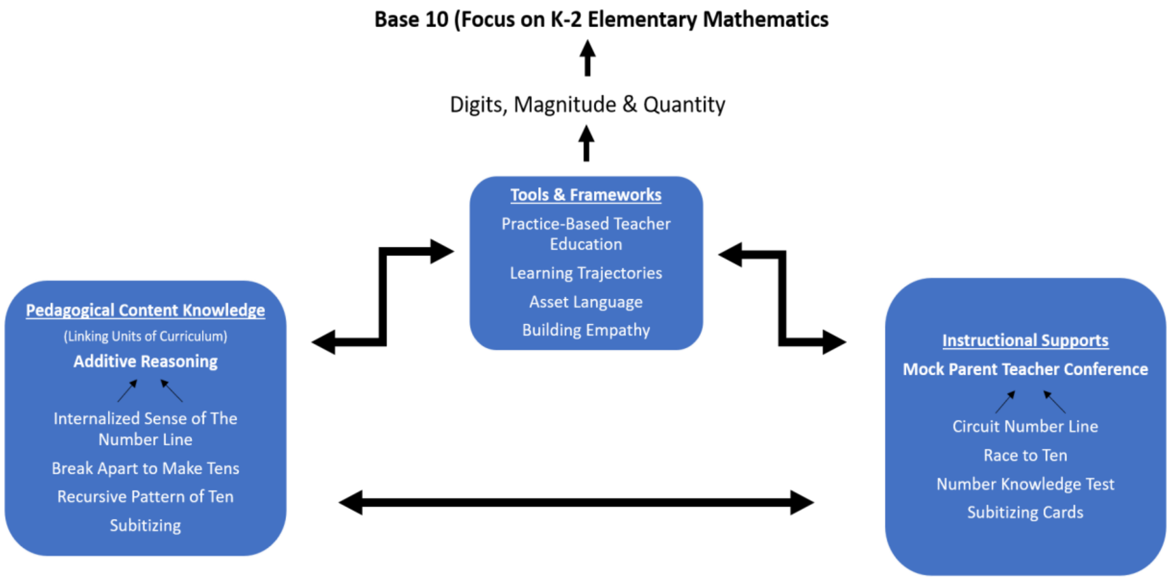
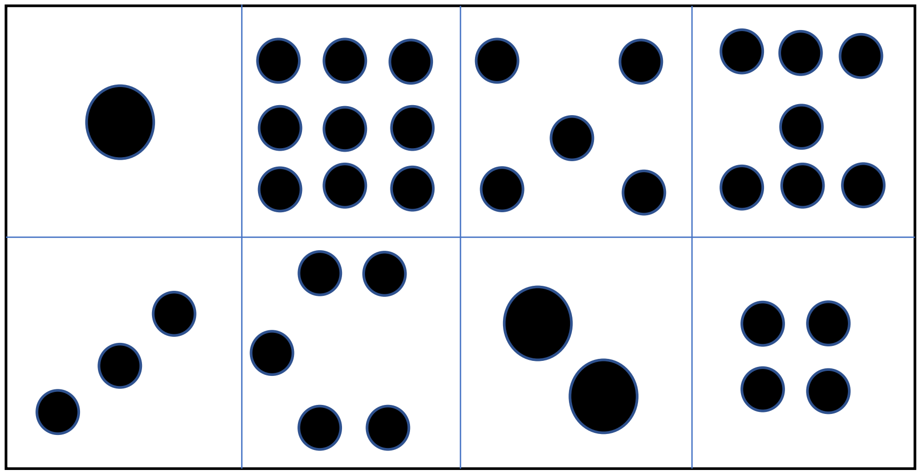
Appendix A

Threshold Concept



Appendix B

Example of Subitizing Cards



Appendix C

Number Knowledge Test Questions: Adapted from Sharon (2009)

Level 0 (4-year-old level): Go to level 1 if 3 or more correct
1. Can you count these chips and tell me how many there are? (Place 3 counting chips in front of child in a row)
2a. (Show stacks of chips, 5 vs. 2, same color). Which pile has more?
2b. (Show stacks of chips, 3 vs. 7, same color). Which pile has more?
3a. This time I'm going to ask you which pile has less.
(Show stacks of chips, 2 vs. 6, same color). Which pile has less?
3b. (Show stacks of chips, 8 vs. 3, same color). Which pile has less?
4. I'm going to show you some counting chips (Show a line of 3 red and 4 yellow chips in a row, as follows: R Y R Y R Y Y). Count just the yellow chips and tell me how many there are.
5. Pick up all chips from the previous question. Then say: Here are some more counting chips (Show mixed array [not in a row] of 7 yellow and 8 red chips). Count just the red chips and tell me how many there are.

Level 1 (6-year-old level): Go to level 2 if 5 or more correct
1. If you had 4 chocolates and someone gave you 3 more, how many chocolates would you have altogether?
2. What number comes right after 7?
3. What number comes two numbers after 7?
4a. Which is bigger: 5 or 4?
4b. Which is bigger: 7 or 9?
5a. This time, I'm going to ask you about smaller numbers. Which is smaller: 8 or 6?
5b. Which is smaller: 5 or 7?
6a. Which number is closer to 5: 6 or 2? (Show visual array after asking the question)
6b. Which number is closer to 7: 4 or 9? (Show visual array after asking the question)
7. How much is 2 + 4? (OK to use fingers for counting)
8. How much is 8 take away 6? (OK to use fingers for counting)
9a. (Show visual array −8 5 2 6—and ask child to point to and name each numeral). When you are counting, which of these numbers do you say first?
9b. When you are counting, which of these numbers do you say last?

Level 2 (8-year-old level): Go to level 3 if 5 or more correct
1. What number comes 5 numbers after 49?
2. What number comes 4 numbers before 60?
3a. Which is bigger: 69 or 71?
3b. Which is bigger: 32 or 28?
4a. This time I'm going to ask you about smaller numbers. Which is smaller: 27 or 32?
4b. Which is smaller: 51 or 39?
5a. Which number is closer to 21: 25 or 18? (Show visual array after asking the question)
5b. Which number is closer to 28: 31 or 24? (Show visual array after asking the question)
6. How many numbers are there in between 2 and 6? (Accept either 3 or 4)

7. How many numbers are there in between 7 and 9? (Accept either 1 or 2)
8. (Show card 12 54) How much is 12 + 54?
9. (Show card 47 21) How much is 47 take away 21?

Level 3 (10-year-old level): Go to level 4 if 4 of more correct

1. What number comes 10 numbers after 99?
2. What number comes 9 numbers after 999?

3a. Which difference is bigger: the difference between 9 and 6 or the difference between 8 and 3?

3b. Which difference is bigger: the difference between 6 and 2 or the difference between 8 and 5?

4a. Which difference is smaller: the difference between 99 and 92 or the difference between 25 and 11?

4b. Which difference is smaller: the difference between 48 and 36 or the difference between 84 and 73?

5. (Show card, ‘‘13, 39’’) How much is 13 + 39?
6. (Show card, ‘‘36, 18’’) How much is 36 − 18?
7. How much is 301 take away 7?