NAVIGATING THE EPISTEMOLOGICAL ROCKY WATERS OF MATHEMATICS EDUCATION: AN INSTRUMENTAL MULTIPLE CASE STUDY

by

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A dissertation submitted to the faculty of
The University of North Carolina at Charlotte
in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in
Curriculum and Instruction

Charlotte

2009

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ABSTRACT

AMÉLIE G. SCHINCK. Navigating the epistemological rocky waters of mathematics education: An instrumental multiple case study. (Under the direction of DR. DAVID K. PUGALEE)

The various beliefs about mathematics and its role in society led to differing beliefs about what it means to learn mathematics and how best to teach it. Mathematics education research and organizations such as the National Council of Teachers of Mathematics (NCTM) champion a combination of constructivist and sociocultural approaches to mathematics teaching and learning. Within this paradigm, mathematics teachers are cast as guides in their students' individual meaning-making experience. However, since mathematics is at the heart of science and technology, which are in turn the basis of the new global, knowledge-based economy, mathematics achievement scores on standardized tests are inexorably tied to national policy discourses of global competitiveness. Meeting the "global challenge" has resulted in government policies which reflect a view of effective mathematics teaching focused on accountability and measured outcomes. Underlying such policies is a positivist view of mathematics as a fixed body of facts and procedures which students need to internalize. Mathematics teachers function at the nexus of these differing beliefs about mathematics teaching and learning. This dissertation offers a case study analysis of three high school mathematics teachers as they navigate different belief systems while making professional decisions related to their work. This study examines the periods of tension, conflict, reflection, and resolution teachers experience while managing competing goals. The study concludes with implications for teacher education, as well as recommendations for future research.

ACKNOWLEDGEMENTS

I would like to express my gratitude to the many people who have supported and encouraged me during the process of writing this dissertation. I wish to first acknowledge Dr. David K. Pugalee, to whom I am forever indebted for providing constant support, inspiration and advice. I also would like to recognize and thank my committee members for providing the feedback and guidance needed throughout the writing process. My thanks to the teacher participants of this study for patiently sharing their experiences with me. I learned so much from you.

I would also like to acknowledge my family and friends whose love and support made the writing of this dissertation a beautiful and satisfying experience. Thank you to all my friends – Audrey, Jeff, and the Saccos – who knew not to ask a follow up question once "How's the dissertation going?" was met with silence. You indulged me while I immersed myself in my writing, and recognized when I needed to be rescued from it. To Jen, a special thank you for accompanying me along the incredibly arduous and rewarding journey of earning a doctorate. I will forever cherish our friendship.

Lastly, I am eternally grateful to my parents, Nicole and Robert Schinck, for always supporting my endeavors, and for the many long distance phone conversations which always brought me to tears with laughter. Merçi de tout mon cœur.

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CHAPTER 1: INTRODUCTION

Background

The various beliefs about mathematics and its role in society led to differing beliefs about what it means to learn mathematics and how best to teach it. Researchers and faculty within mathematics education departments and organizations, such as the National Council of Teachers of Mathematics (NCTM) and the Association of Mathematics Teacher Educators (AMTE), promote a combination of constructivist and sociocultural approaches to mathematics teaching and learning (*e.g*. Chapman, 2002; Cobb, 1994; Cooney & Shealy, 1997; NCTM, 1980, 1989, 2000).

Within this framework, teachers are cast as guides in their students' mathematical meaning-making experience. The learning of mathematics is seen as an active, constructive process in which problem solving, reasoning and proof, communication, connections and representations are of fundamental importance (NCTM, 1989, 2000). The learning *process* is valued over the *result* of a mathematical activity. Mathematics is believed to be a part of human experience, not apart from it.

Much time and effort is exerted in mathematics methods classes and professional development workshops to expose, and perhaps convert, pre-service and in-service teachers to the above set of beliefs about mathematics, its teaching and its learning. Curcio and Artzt (2005) stated that one of the most important and challenging jobs for teacher educators is to prepare pre-service and in-service teachers to enact the principles and standards set forth by NCTM (1989, 2000).

A look at textbook selections for mathematics methods classes sheds light on the belief system underlying these classes. For instance, textbooks selected for elementary and middle school mathematics methods classes, such as Van de Walle's (2006) *Elementary and middle school mathematics: Teaching developmentally*, reflect the NCTM Principles and Standards for School Mathematics (2000), and discuss at length the benefits of constructivist and student-centered mathematics instruction. The back cover for Van de Walle's (2006) book has the following message from the author:

> Research in mathematics education has consistently found that understanding and skills are best developed when students are allowed to wrestle with new ideas, to create and defend solutions to problems, and to participate in a mathematical community of learners.

This approach is the central theme of Van de Walle's book – one of the leading K-8 mathematics methods text.

Similarly, textbooks selected for secondary mathematics methods classes, such as *Teaching secondary mathematics: Techniques and enrichment units* (Posamentier & Stepelman, 2006), *Mathematics classrooms that promote understanding* (Fennema & Romberg, 1999) and *Windows on teaching math* (Merseth, 2003), make pedagogical recommendations to teachers based on a synthesis of mathematics education research. The motivation is to respond to the ongoing call by the National Council of Teachers of Mathematics to change both the content and practice of teaching mathematics to be more in line with research findings.

In general, mathematics methods classes, responsible for training the next wave of mathematics teachers, are taught by mathematics education specialists who are well versed in mathematics education research; they thus have a very clear idea of what a mathematics classroom *should* look like. As a consequence, these classes tend to promote the constructivist and sociocultural approaches to the teaching of mathematics found in the mathematics education literature.

Mathematics content classes aimed at future teachers are taught by a combination of mathematics specialists as well as mathematics education specialists. Textbook selections for these classes cite the Principles and Standards of School Mathematics (NCTM, 2000) liberally, and often advocate a problem-solving approach (e.g. Beckmann, 2004; Billstein, Libeskind, & Lott, 2007; Masingila, Lester, & Raymond, 2002).

Moreover, much professional development material aimed at K-12 pre-service and in-service mathematics teachers base their recommendations and strategies on mathematics education research, encouraging teachers to develop learning through problem solving and to consider the sociocultural aspects of the mathematics classroom (e.g. Posamentier & Jaye, 2006; Richardson, 1999; Wall & Posamentier, 2007). Online professional development workshops by the National Council of Teachers of Mathematics (NCTM e-workshops) and the Annenberg Foundation (www.learner.org) also encourage mathematics teachers to adopt this approach to the teaching of mathematics.

However, since mathematics is at the heart of science and technology, which are in turn the basis of the new global, knowledge-based economy, mathematics education has attracted much government and public interest. Mathematics achievement scores on standardized tests are inexorably tied to national policy discourses of global competitiveness emphasizing a strong causal relationship between mathematics achievement and economic prosperity (e.g. National Science Foundation, 2001; OECD, 2007; TIMMS, 2003). One illustrative example of the use of the global competitiveness discourse to promote specific educational goals, is the letter written in January of 2009 by The National Science Board (N.S.B., 2009) to the Obama administration. The letter contained recommendations to advance STEM (science, technology, engineering, and mathematics) education by guaranteeing “that all American students are provided the educational resources and tools needed to participate fully in the science and technology based economy of the 21st century” in order to ensure “the long-term economic prosperity of the Nation” (p. 1).

The political dimension of mathematics education is not a recent development. For instance, the 1983 report *A Nation at Risk: The Imperative for Educational Reform* by the National Commission on Excellence in Education, greatly influenced the direction of mathematics education in the United States (Klein, 2003). *A Nation at Risk* harshly criticized the United States’ educational practice, and contributed to the public perception, present to this day, that American public schools fail to meet the nation’s need for a highly-skilled, globally competitive workforce. The report begins with:

> Our Nation is at risk. Our once unchallenged preeminence in commerce, industry, science, and technological innovation is being overtaken by competitors throughout the world…the educational foundations of our society are presently being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people” (National Commission on Excellence in Education, 1983, p. 5).

Among other things, the commission was charged with assessing the quality of the American education system as it compares to other advanced nations. It found, for instance, that approximately one-third of high school graduates in the United States could not solve a mathematics problem requiring several steps. Recommendations included that students should have a certain level of proficiency in basic skills and that they be guided in the development of deep conceptual understanding in order for them to become skilled problem-solvers.

Still, globalization has reshaped the debate and amplified the urgency of the discussion. Meeting the "global challenge" has meant meeting business demands and the commodification and vocationalization of disciplines such as mathematics (Jordan & Yeomans, 2003). In fact, Morrow & Alberto Torres (2000) identify the commodification of education as the "crucial, pervasive structural effect that defines the specific, neoliberal form of globalization taking place" (p. 39). Education in general, and mathematics education in particular, has come to be perceived as a "personal good rather than public service" (Fitszimons, 2002, p. 60) due to the forces of globalization.

The application of the corporate model resulting in the era of effectiveness focused on measured outcomes and accountability is now present in a great number of countries, including England, Australia, New Zealand, the U.S., and Japan (Astiz, Wiseman, & Baker, 2002; Atweh & Clarkson, 2002b; Hill, 2008; Jordan & Yeomans, 2003). The standards and accountability movement reached a new level in the United States in 2001 with the enactment of the *No Child Left Behind Act* [NCLB] (U. S. Congress, 2001), significantly expanding the role of the federal government in school reform by mandating that standards be adopted by states for which they would be held accountable.

Twenty five years after a *Nation at Risk* was published, the U.S. Department of Education published *A Nation Accountable* (U.S Department of Education, 2008), highlighting this new era of educational accountability. The executive summary for *A Nation Accountable* states that “we remain a nation at risk but are also now a nation informed, a nation accountable, and a nation that recognizes there is much work to be done” (U.S. Department of Education, 2008, p. 1).

Corporate managerialism has entered and permeated the structure of education in what Bourdieu (1998) called a heteronomic process; rules from one field entering another and causing a loss of autonomy in the field being entered. This approach to education has immense pedagogical implications:

> Continuous monitoring implies that learning takes place in measurable increments and that constant testing somehow contributes to enhanced performance. Whether it does or not, it reinforces educational practice which has no space for conversation, exploration, or the personalization of learning (Kohl, 2009, p. 2).

Within this framework, teachers are asked to help students memorize facts and practice skills through repetitive exercises. They are inundated by the focus on student achievement on high-stakes tests (NCLB, 2001).

A test is termed "high-stakes" when the results are used to make important decisions that affect students, teachers, administrators, or schools. The market orientation of education as a result of neoliberal philosophy, has resulted in a rhetoric of competition and accountability to show a product for tax payers’ money—higher scores.

In a metasynthesis analyzing 49 qualitative studies on the effects of high-stakes testing on curriculum, Au (2007) found that one main effect was "an increase in teacher-centered instruction associated with lecturing and the direct transmission of test-related facts" (p. 263). More specifically, the metasynthesis suggested that high-stakes tests encouraged "curricular alignment to the test themselves" (p. 263). Content was found to be narrowed and delivered in isolated and disconnected pieces.

Newman & Beck (2000) claim that the existence of high content and performance standards without the existence of high opportunity to learn standards does not allow teachers the flexibility to meet students where they are. Teachers are pressed to push students along the predetermined, designated academic path. Howe (1995) sardonically stated that “better educational standards can eliminate low achievement…no more effectively than better nutritional standards can eliminate hunger under famine conditions” (p. 22).

Underlying such policies is the view of school subjects, including mathematics, as *a priori* knowledge, which students need to internalize. The underpinning assumption by policy-makers and employers is one of mathematics as a static tool kit with an identifiable content and stable structure that are both teachable and, most importantly, testable (Apple, 1979; Ellis & Berry III, 2005). The forces of globalization have thus reinforced a positivist, pragmatic, and utilitarian approach to mathematics education.

While mathematics education research, teacher education, and recommendations from organizations such as the NCTM have been increasingly influenced by sociocultural and constructivist perspectives, critical theorists such as Keitel, Kotzmann, and Skovsmose (1993) write of their concern that the mathematics curriculum implicitly supports a positivist, absolutist philosophical stance that socializes students for routine work in the technologically advanced workplace. These authors note that much of the standardized curricula that are being implemented around the globe are designed to meet the needs of employers, developing in students “an ideological base for a servility towards technology” (Keitel, Kotzmann & Skovsmose, 1993, p. 269). Hill (2008) agrees with Keitel, Kotzmann, and Skovmose’s (1993) assessment, stating that "schooling and other education services train people for the market, developing 'appropriate' sets of skills, knowledges and personality dispositions suited to various niches in the labour market" (p. 7).

This positivist approach to mathematics education is discordant with the wide spread acceptance of epistemological stances such as constructivism in mathematics education research. These incongruous dynamics are colliding and proving to be most challenging for classroom mathematics teachers who are asked to function at the nexus of the differing and overlapping beliefs about mathematics teaching and learning. This is especially true for teacher-scholars, well versed in mathematics education theories, who find themselves teaching in a context in which effective mathematics teaching is defined and measured differently by the various stakeholders.

The Issue

Mathematics teachers are charged with simultaneously being guides in the personal construction of students’ creation of *their* mathematical reality—preparing students to be productive members in the global knowledge society—while also producing high test scores and passing rates. They are told to use “differentiated instruction,” be “student-centered,” and to develop concepts through “problem-solving,” while these notions are viewed in vastly different ways by their school administrators and their mathematics education professors. Teachers must navigate what the constructivist and sociocultural perspectives have taught them about mathematics education with the government policies which reflect a view of effective mathematics teaching focused on measured outcomes, accountability and standardization.

Significance of the Study

The effects of globalization on mathematics education have only just begun to be a topic of study in mathematics education research (Atweh & Clarkson, 2001; Atweh & Clarkson, 2002a, b; Keitel, Kotzmann, & Skovsmose, 1993; Thomas, 2001). As will be seen in Chapter 2, the effects of globalization on educational institutions are intimately tied to the discourse about the knowledge society, where the forces of globalization are seen as privileging a particular type of mathematical knowledge. Forstorp (2008) stated: “The thesis of the knowledge society is based on a theory of knowledge in its instrumentalized form. This form of knowledge is linked to a contemporary trajectory of work that is virtuous and explicitly placed at the very top of a production process” (p. 233). This makes explorations into questions of epistemology that much more pressing. Apple (1997) claimed that “too little focus has been placed on the political economy of what knowledge is considered high status in this and similar societies”
(p. 598).

Much of the literature on the topic of varying epistemologies in mathematics education focuses on changing teacher beliefs about the nature of mathematics learning and teaching to more constructivist and sociocultural views (Chapman, 2002; Cooney & Shealy, 1997; Wilson & Cooney, 2002). This approach to teacher belief research often disregards the socio-political milieu in which educators operate. In addition, the work done on the effects of political forces on mathematics education tends to take a macro level approach, or focus on the effects of one specific mandate such as the *No Child Left Behind Act* of 2001. No previous work has examined how clashing epistemologies are navigated by individual mathematics educators who have been strongly influenced by mathematics education research's perspective, yet must work in an educational system shaped by the demands of globalization.

Dissertation Title

The imagery offered by Thagard (2000) of a system of beliefs as a raft floating on a sea is part of the inspiration behind the title for this dissertation: *Navigating the Epistemological Rocky Waters of Mathematics Education.* According to Thagard (2000), a system of beliefs:

> … is not like a house that sits on a foundation of bricks that have to be solid, but more like a raft that floats on the sea with all the pieces of the raft fitting together and supporting each other. A belief is justified, not because it is indubitable or is derived from some other indubitable beliefs, but because it coheres with other beliefs that jointly support each other (p. 5).

Kenny (2007) referred to “the leaking constructivist boat adrift in an ocean of realism” (p. 58), further inspiring the imagery used throughout this dissertation of the “epistemological rocky waters of mathematics education.”

The Researcher

In order to better understand the purpose of this study, it will help the reader to get better acquainted with the researcher. I obtained a Bachelors of Science with a

Specialization in Mathematics and a Masters of Science with a concentration in Number Theory from Concordia University in Montréal, Canada. During this time, I also tutored and taught mathematics at local schools at various levels, in both French and English.

I then took a position as a mathematics instructor at a 2-year college in a rural/suburban area of the Southeast of the United States, where I had the privilege to teach most of the mathematics courses offered in the first two years of college, such as Precalculus, Calculus I-III, College Algebra, Statistics, and many more. After four years of teaching full time, I began my Ph.D. in Curriculum and Instruction in Mathematics Education. I have been employed as a graduate research assistant at the University of North Carolina at Charlotte for the past two years, during which time I have worked on a number of research projects in the field of mathematics education.

As a research assistant, I took advantage of the numerous opportunities offered to me to extend my scholarship. I have collaborated with professors to research, present and publish on a variety of research subjects, such as, writing in the mathematics classroom (Pugalee & Schinck, 2007), contemporary metaphor theory as a tool to reveal student beliefs about mathematics and as a catalyst for student reflection (Schinck, Neale, Pugalee, & Cifarelli, 2008), problem solving as a means to develop conceptual competence and adolescent girls' experience with mathematics (Lim, Schinck, & Chae, 2009). I have also had occasion to work as an editor and reviewer on a number of projects (e.g. Pugalee, Rogerson, & Schinck, 2007). These opportunities have provided me with knowledge in a wide variety of important topics in mathematics education.

Having these different experiences—coming from a pure mathematics background, teaching at different grade levels, in rural and urban areas, in different countries, in different languages, and becoming familiar with mathematics education research—made me cognizant of the fact that various beliefs about mathematics and its teaching exist, and deeply influence teachers' work. Furthermore, it made me aware of the powerful effect of the socio-political milieu on mathematics education. Although students and teachers are often more likely to focus on the local implications of education such as employment, and achievement scores, the socio-political milieu nonetheless undeniably affect their work. The following quote by Thomas (2001) profoundly impacted the direction of my study:

> Mathematics education is political…to ignore the effects of this in an economically globalized and changing world has the potential to constrain or render ineffectual much mathematics education research (p. 96).

Purpose of the Study

The purpose of this study is to provide a thick description of the lived experiences of three mathematics educators as they navigate different belief systems when making professional decisions related to their work as teachers. This dissertation endeavors to examine the tensions involved and the way in which these three mathematics educators manage competing goals. This study has been developed to follow the tradition of the instrumental, multiple case study in which an issue or phenomenon is explored by performing several case studies (Stake, 1995). The research design also borrows some features from phenomenology, which aims to explain how social phenomena are experienced and constructed by individuals or groups of individuals.

This dissertation aspires to be a first step in answering the call for "richly elaborated case studies that focus on the long-term struggles of classroom teachers who manage the intellectual challenges, pedagogical renewals, cultural transformations, and political upheavals of transforming one's practice" (Windschitl, 2002, p. 162), with associated implications for teacher education.

Research Questions

The specific research questions guiding the study are:

1. How do individual, successful teachers navigate the beliefs shaped by mathematics education research, workshops, methods classes and the discourse of preparing students to be competitive in the global economy?
2. How do mathematics educators experience the periods of conflict, reflection and resolution between the different belief systems to which they have been exposed?

Summary and Introduction to Chapter 2

Chapter 1 provided an introduction to this research study. The introduction gave a brief description of the issue that motivated the study: The clashing epistemologies about mathematics and its teaching that classroom mathematics teachers are asked to navigate. The introduction also presented to the reader the inspiration behind the title of this dissertation—*Navigating the Epistemological Rocky Waters of Mathematics Education*—as well as the personal journey of the researcher to the topic, and the overarching purpose of the study, including the two guiding research questions.

The following chapter will describe the literature relevant to the study. This includes an exploration of the literature on the paradigm shift which occurred in mathematics education, on teacher beliefs, on the current socio-political milieu, and on mathematics teachers' experiences within this milieu.

CHAPTER 2: REVIEW OF THE LITERATURE

Introduction: Need for a Literature Review

In order to better understand the rocky waters of beliefs about mathematics and its teaching in which teachers operate, this literature review will first focus on the paradigm shift from positivist to constructivist and sociocultural perspectives, which began in mathematics education in the 1970's. This historical discussion leads me to explore the recent research on teacher beliefs. I then discuss the current socio-political milieu, especially the associated discourses of globalization and the knowledge society, as they relate to the epistemological foundations of mathematics education.

Ultimately, the purpose of the study guiding this review — to provide a thick description of the lived experience of three mathematics educators as they navigate different belief systems when making professional decisions related to their work as teachers—calls for a presentation of relevant literature on teacher experience as they teach mathematics in the knowledge society and mediate various belief systems. Since this study is situated within a socioconstructionist theoretical position, I also present a critical perspective of the literature in an effort to illustrate how the various belief systems involved have influenced classroom practices in the teaching of mathematics.

A Review of the Literature

Paradigm Shift in Mathematics Education

Mathematics is related to our understanding of the world. The effect various beliefs have on mathematics education shapes the way students will eventually view and interact with the world. Niss (1994) discerned four perspectives on the discipline of mathematics: as a pure or applied science, a system of instruments, a field of aesthetics, and as a teaching subject.

First, mathematics can be viewed as a *pure or applied science*, where pure mathematics is directed at mathematical objects and applied mathematics is directed towards extra-mathematical objects such as physics. An example of pure mathematics is number theory, which is principally concerned with the properties of numbers. Applied mathematics enables the answering of questions about the physical world; the revelation of order from chaos (Kline, 1979).

Secondly, mathematics can be seen as a *system of instruments* which can assist in decision-making, through statistical analysis for instance. Mathematics as an instrument is highly valued by industry: “It is only when mathematical theories such as linear optimization or graph theory are converted into computer programs, for example, that they can be applied to industrial or other sites in the economic system to maximize productivity” (Fitzsimons, 2002, p. 23).

Thirdly, mathematics can be described as a *field of aesthetics*, which is beautiful on par with great art. Although inconceivable to some, as much pleasure can be derived from Euclid’s proof of the infinitude of primes as from Van Gough’s masterpiece *Starry Night.*

Finally, mathematics can be viewed as a *teaching subject,* a principal concern of mathematics education research.

Mathematics education research is a relatively young field; only two centuries old and only burgeoning in the past forty years (Kilpatrick, 1992). Mathematics education encompasses the study of mathematics teaching, as mentioned above, as well as the study of mathematics learning, problems about the nature of mathematics, mathematical knowledge and the social contexts of mathematics learning and teaching (Ernest, 1998). It is a field of knowledge in its own right, with its own terms, concepts, problems, and theories.

A paradigm shift occurred in mathematics education in the 1970's as a result of revolutions in perspectives within the disciplines of mathematics and mathematics education. A paradigm refers to shared assumptions about the nature of reality (ontology), and knowledge (epistemology), and about the best way to investigate reality (methodology) (Kuhn, 1962). The three elements of a paradigm are undergrid by a system of basic beliefs.

Adherents of a certain paradigm will generally work on similar problems using epistemologically compatible methodologies. Cooney and Shealy (1997) document the shift from the traditional view of mathematics as a well-defined field of inquiry that is timeless and unchanging to the view of mathematics as a "way of thinking about the external world, a category of constructing meaning" (Cooney & Shealy, 1997, p. 89). Tymoczko (1979) suggested that the shift in paradigm in mathematics was in part triggered by the proof of the Four-Color Theorem by Appel & Haken in 1977. The proof of the Four-Color Theorem was based entirely on a computer experiment. This shook the foundation of mathematics at the time, which had been seen as an "a priori activity in which the mathematician chooses an axiomatic system and then, working in isolation, follows formal deductive methods to produce irrefutable conclusions" (Cooney & Shealy, 1997, p. 88).

Wilson and Cooney (2002) suggest that the movement away from an ontological perspective toward individual sense-making was also in part spurred by Kuhn's (1962) classic work *The Structure of Scientific Revolutions*. In this groundbreaking work, Kuhn questioned the then unexamined ontological approach to mathematics, science and their education. Kuhn (1962) argued that the evolution of scientific thought does not emerge from the accumulation of facts, but from drastic, paradigm shifts.

An example of such a paradigm shift in the mathematical domain is the shift from the monopoly of Euclidean geometry with the introduction of non-Euclidean geometry. In order for such a drastic change to occur, Euclid's fifth postulate, which defines parallel lines in a way equivalent to saying that the sum of the angles in a triangle add up to 180 degrees, needed to be questioned. Euclid's fifth postulate had been accepted truth for millennia. As Dossey (1992) states, "the establishment of the consistency of non-Euclidean geometry…finally freed mathematics from the restrictive yoke of a single set of axioms thought to be the only model for the external world" (p. 40). It is important to note that Euclidean geometry was not shown to be wrong, but simply not the only way to see the world.

Mathematics education was also experiencing a reexamining of traditional views during this period due to emerging theories from cognitive science and the surfacing of the sociocultural perspective influenced in large part by the work of two psychologists: Jean Piaget and Lev Vygotsky.

Piaget greatly influenced constructivist thought in mathematics education. Before Piaget’s contribution, mathematics was regarded as mind-independent. One would discover the fundamental mathematical structures through rationality. To know mathematics was to be able to identify concepts and perform procedures correctly. As Steffe & Kieren (1994) write, one of the long-lasting effects of Piaget’s work is that mathematics education research began to include, along side empirical studies of mathematics “best practices,” studies where researchers observed and described the mechanisms that mathematics learners use to build up mathematical knowledge in a particular learning space (Capraro, Capraro, & Cifarelli, 2007; Steffe & Wiegel, 1994; Thompson, 1994).

One of the tenets of using constructivism in the mathematics classroom, as explained by Steffe (1991), is choosing to focus on a learner's mathematical activity in learning environments as opposed to focusing on the results or product of their activity. As Tom Kieren, in an interview with Carolyn Kieran about the changes that occurred in mathematic education (Kieran, 1994) stated, the field, in its infancy, was “linearizing learning, and learning isn’t linear” (p. 587).

The 1980’s were a time of constructivist revolution which caused a drastic shift in the way mathematical knowledge, its learning and its teaching were understood in mathematics education research. The first time the *Journal for Research in Mathematics Education* used “constructivist” in an article title was 1983 (Steffe & Cobb, 1983). The work of von Glasersfeld (1981, 1984, 1987, 1989), and Steffe & Cobb (1983, 1988) provided a different theoretical framework for mathematics education. The key idea that emerged from these writings was that individuals create their own reality through actions and reflections within the space of their experience. This type of constructivism has come to be known as *cognitive constructivism*. The constructivist framework of mathematical knowledge called for mathematics educators to be recast as guides in the construction of students' creation of *their* mathematical reality.

Of fundamental importance to Piaget's theory, is the concept of *viable knowledge* and its construction. An individual makes sense of an experience using a *scheme*, that is, a certain way of organizing experience. An experience is assimilated by fitting into a pre-existing conceptual structure. For Piaget, knowledge is a form of adaptation, where *equilibrium* is the goal. Equilibrium, in this context, can be understood as a *satisfactory organization* (Von Glasersfeld, 1987). The often-cited quote by Piaget states this concisely: "The mind organises the world by organising itself (Piaget, 1937, p. 311 qtd. by Von Glasersfeld, 1995). Learning occurs when a certain scheme encounters a perturbation and an individual must *accommodate* to maintain or return to equilibrium. A clash with a scheme means that the scheme did not work and should be reconsidered.

Yet, when a scheme works, it is simply *viable* and "no inference about a real world can be drawn from this viability, because a countless number of other schemes might have worked as well" (Von Glasersfeld, 1995, p. 90). Learning occurs in this instance as well. Yet, one must keep in mind that facts remain viable only as long as they do not clash with what we expect them to do. In this model, social interaction is presumed to promote opportunities for the learner to come face to face with discrepancies between their views and the world, which would possibly lead to an accommodation of their scheme: “In a sense, Piagetians are saying that what children are able to observe about the world is more dependent on what they already know – that is, on their own special system of thinking – than it is on what actually exists” (Gallagher & Reid, 2002, p. 1).

The work of Vygotsky also greatly changed the landscape of mathematics education, by adding a social-interactionist orientation to the Piagetian individual-cognitive theoretical framework (Kieran, 1994). Vygotsky (e.g. 1978, 1986) argued that an individual's development of higher-order functions can only really be understood when taking into account the external social world in which the individual developed. Knowledge was still seen as constructed, but the process of construction was socially and culturally situated.

Vygotsky claimed that higher mental functions appear first on a social level, through interaction with a more capable individual. This is stage I of what Vygotsky termed the *Zone of Proximal Development* (ZPD). The ZPD is defined as the range of problems that are too complex for a learner to solve on their own, but that could be solved with the assistance of a more capable partner. In this stage, the novice still relies on socializing to assist their performance. Only later can the individual begin to interiorize the concept.

The example given by Tharp and Gallimore (1991) is that of a father gently questioning a child as to the possible whereabouts of a lost toy. Through this interaction, the child is guided to recall the location of the lost item. With repeated similar interactions, it is implied that this will eventually develop to a higher stage in the ZPD, self-directed performance. The "assisted performance of apprentices in joint activity with experts becomes the vehicle through which interactions of society are internalized and become mind" (Tharp and Gallimore, 1991, p. 8). Although Vygotsky's work mostly concerned children, self-directed speech also has been shown to assist adults in the ZPD.

Thus, for Vygotsky, development is from the social to the individual. Piaget saw cognitive development working in the opposite direction: from the individual to social. These different views of the social aspect of the mind is summed up in the following quote by Rogoff (1990): "The two theories are based on different perspectives: Vygotsky focuses on the social basis of mind, while Piaget focuses on the individual as starting point" (p. 140). In fact, one of the main critiques of Piaget was that he significantly underestimated the role of the social context in the development of the individual.

Piaget and Vygotsky came to different conclusions about many other aspects of human cognitive development. As stated above, Vygotsky promoted the idea of a novice interacting with a "more capable other" in order to be assisted in the zone of proximal development. In this model, the more capable other assists the learner by providing hints, questions, etc. This process is called scaffolding. In practice, this has resulted in the apprenticeship model, and to the cooperative learning movement. Vygotsky’s legacy also includes research focused on the culture of the mathematics classroom. Piaget, on the other hand, believed that to be truly fruitful, social interaction needed to be amongst equals trying to understand each other's alternative views (Rogoff, 1990).

The different points of view of Piaget and Vygotsky led to the two main perspectives in mathematics education: The constructivist and sociocultural perspectives. Maturana’s (1978) notion of *autopoiesis* extended Piaget’s concept of the mind as a self-organizing system to include the sociological alongside the psychological in the constructive process. In other words, much of what an individual learns about mathematics and its teaching is through the interactions within different communities. Cobb (1994) makes a convincing case for the complementary nature of these two perspectives. Cobb (1994) argues that the constructivist perspective has much to say about the processes by which students learn, while the sociocultural perspective, which emphasizes the social and cultural nature of mathematics, its teaching, and its learning, can "inform theories of the conditions for the possibility of learning" (p. 13).

The dispute, as related by Cobb, is whether "mathematical learning is primarily a process of active cognitive reorganization or a process of enculturation into a community of practice" (p. 13). Cobb (1994) is sensitive to the fact that "constructivist theories fail to account for the production and reproduction of the practices of schooling and the social order" (Cobb, 1994, p. 18). However, Cobb (1994) convincingly explains his view that "mathematical learning should be viewed as both a process of active individual construction and a process of enculturation into the mathematical practices of a wider society." Cobb (1994) highlights the conflicts that teachers face when trying to navigate these two theories by citing Ball (1993):

> How do I create experiences for my students that connect with what they now know and care about but that also transcend the present? How do I value their interests and also connect them to ideas and traditions growing out of centuries of mathematical exploration and invention? (p. 375)

One could add to the list of questions put forth by Ball (1993): How do I resolve what the constructivist and sociocultural perspectives have taught me about mathematics education with the government policies which reflect a view of effective mathematics teaching focused on measured outcomes, accountability and standardization? Furthermore, what exactly is meant of me as a mathematics teacher when I am asked to prepare students to be competitive in the global economy? What mathematical knowledge should I help my students gain in order to prepare them to be global citizens of the knowledge society?

Cobb (1994) states that teachers are asked to mediate between students' personal meanings and culturally established mathematical meanings. In the sociocultural perspective, the culture under question is often the classroom culture, or school culture. Unfortunately, very rarely in the mathematics education literature does the word *culture* include forces that inevitably influence school and classroom culture such as the effects that the discourse of globalization has had on mathematics education. This lies at the heart of effective teaching as "claims about what counts as improvement reflect beliefs and values about what it ought to mean to know and do mathematics in school (Cobb, 1994, p. 19).

Ellis and Berry III's (2005) historical account of mathematics education reform movements, documents a shift from what the authors termed the *procedural-formalist paradigm* (PFP) to the *cognitive-cultural paradigm* (CCP). In their article, Ellis & Berry III (2005) argue that previous reform movements such as the Progressive Movement, the New Math Movement, and the Back-to-Basics Movement were all firmly situated in the PFP paradigm.

Within the framework of the PFP, mathematics is outside of human experience and mathematics education's focus is to help students "internalize a fundamental body of basic mathematical knowledge" (Ellis and Berry III, 2005, p. 11). An effective teacher is one that delivers well-organized lectures and then asks students to memorize facts and practice skills through repetitive exercises. The authors claim that these reform movements failed because they stayed within an obsolete paradigm.

The CCP, on the other hand, "takes mathematics to be a set of logically organized and interconnected concepts that come out of human experience, thought and interaction" (Ellis & Berry III, 2005, p. 12). Within the frame of the CCP, a teacher is asked to guide students' active role in developing mathematical concepts. Ellis and Berry III (2005) are optimistic about the move from PFP to CCP.

The National Council of Teachers of Mathematics (NCTM, 1989, 2000) reflects the shift to the cognitive-cultural paradigm in past and current recommendations. NCTM (1989, 2000) recommendations are founded on the notion that learning is an active, social process in which students construct new ideas or concepts based on their current knowledge. NCTM’s Principles and Standards for School Mathematics (2000) emphasizes the need for teachers to create a culture of learning in their classroom in which students learn with understanding and construct conceptual mathematical meaning through a problem-solving approach. Recommendations by the NCTM (1980, 1989, & 2000) have included a call for a focus on problem solving by teachers, positioning problem solving ability as the overarching goal of mathematics education. For instance, NCTM's 1980 recommendations stated that:

> The development of problem solving ability should direct the efforts of mathematics educators through the next decade. Performance in problem solving will measure the effectiveness of our personal and national possession of mathematical competence (p. 2).

Implementing a program of problem solving that is consistent with the NCTM Principles and Standards for School Mathematics (2000) for teaching mathematics means using problem solving as an *approach* to constructing new mathematical knowledge:

> Problem solving means engaging in a task for which the solution is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understanding. Solving problems is not only a goal of learning mathematics but also a major means of doing so (NCTM, 2000, p. 51).

Student understanding is connected to open-ended questions and an inductive teaching style. It is worth noting that the NCTM recommendations are interpreted by some as a reflection of what von Glasersfeld (1989) termed “trivial constructivism,” a form of constructivism that asserts that learners build up their mathematical cognitive structures while holding on to the notion that the cognitive structure being built up are reflections of an ontological reality. Indeed, von Glasersfeld (1995) has, in recent years, tried to avoid the use of the term “epistemology” and has instead written about human “knowing.” Von Glasersfeld (1995) reasons that:

> Though I have used them in the past, I now try to avoid the term ‘epistemology’ or ‘theory of knowledge’ for constructivism, because they tend to imply the traditional scenario according to which novice subjects are born into a ready-made world, which they must try to discover and ‘represent to themselves’ (pp. 1-2).

Nevertheless, NCTM recommendations can be said to be a departure from the previously held views of mathematics learning and teaching. This is in contrast to the type of objectives found in the NC Standard Course of Study, for instance. Competency goals for Algebra I in North Carolina, U.S., are phrased as follows (PSONC, 2003):

- The learner will perform operations with numbers and expressions to solve problems.
- The learner will use relations and functions to solve problems.

Problems are not to be used to *develop* the concept of a relation or function. It is implied that the concepts of number, expression, relation and function will be taught by the teacher, formulas will be given, and then and only then will problems, most likely exercises, be introduced.

Although Ellis & Berry III (2005) are hopeful about the educational consequences of the change of paradigm, Cooney & Shealy (1997) point out that the shift did not reach very far outside of academia. Parents, students, school administrators, employers and government policy makers still hold a view of mathematics as a fixed and unchanging body of facts and procedures. To do mathematics is to calculate answers to a set of problems. The method of assessment within this framework is naturally performance oriented.

Such differing beliefs about mathematics and its teaching by the various stakeholders in mathematics education result in conflict, frustration, etc. This conflict, unresolved to this day, is illustrated by the debate between Brophy and Confrey in a 1986 issue of *JRME* (Brophy, 1986). Brophy criticized Confrey's constructivist research in the following manner:

> If it is to be of much practical use, however, such input will have to become much more specific, prescriptive, and empirically based…it will have to come to grips

> with the challenge facing the typical K-12 teacher (teach 20 to 40 students to preset curriculum objectives while working within time and resource constraints), and it will have to include process-outcome data that allow for a scientific assessment of the hypothesized effects of recommended procedures (p. 363).

Although Brophy's point of view is considered somewhat outdated in mathematics education research circles, these concerns are very much at the forefront of teachers' day-to-day decision-making process.

The change in beliefs about what constitutes mathematics, its learning and, consequently, its teaching, meant different emphases in mathematics education research and different expectations of teachers. Teachers were called upon to teach and *think* differently. They were asked to consider the epistemology underpinning constructivism in which there are no objective truths independent of human perception and construction. This resulted in an explosion of literature on teachers' beliefs.

Teacher Beliefs

The paradigm shift in mathematics education suggested that not only well-defined and quantifiable variables impacting mathematics teaching were worthy of study. Understanding and changing teacher beliefs was (and is) seen as vital to reforming mathematics education to be consistent with findings from mathematics education research and recommendations from organizations such as the National Council of Teachers of Mathematics.

This new focus on teacher beliefs by educational researchers has undeniably impacted teacher education programs (e.g. Beck & Kosnick, 2006). Teachers are asked not only to transform the problems they pose and make use of small-group learning, but to change their basic epistemological perspective about mathematics and its education (Cole, 1997; Fang, 1996; Franke, Fennema & Carpenter, 1997; Hofer & Pintrich, 1997; Leder, Pehkonen & Torner, 2002; Windschitl, 2002).

The word *belief* has many meanings in the mathematics education literature. Still, underlying all studies on educational beliefs is the notion that epistemological beliefs influence knowledge acquisition and interpretation, task definition and selection, interpretation of course content, and comprehension monitoring (e.g. Leder, Pehkonen, & Torner, 2002; Pajares, 1992). Beliefs act as cognitive and affective filters through which new knowledge will pass.

Teacher beliefs refer to teachers' integrated system of personalized assumptions about students and the learning process, about the role of schools in society, about the nature of mathematics, knowledge (epistemology) and its acquisition. Also included are views about teachers themselves such as teacher efficacy, curriculum, and pedagogy. For instance, teachers may believe that learning mathematics is a function of drill and recall, that students in Algebra II should not be allowed to use calculators, that a multiple-choice test measures mathematical understanding, or that the use of writing in the mathematics classroom promotes students' conceptual understanding.

The literature on teacher beliefs suggests a strong link between beliefs and practice (Chapman, 2002; Fang, 1996; Franke, Fennema & Carpenter, 1997; Thompson, 1992; Wilson & Cooney, 2002). Research findings in this field of study often take the form of categorization of beliefs. Some researchers have used Ernest's (1989) three views on mathematics to sort teacher beliefs into problem solving, Platonist, and instrumentalist. Within this framework, teachers who hold the problem solving view see mathematics as a “continually expanding field of human creation and invention” (Ernest, 1989, p. 93), Platonists view mathematics as a static body of knowledge to be discovered, whereas instrumentalists view it as a collection of useful skills and procedures.

Teacher beliefs about mathematics teaching have also been categorized. An example is Kuhs and Ball’s (1986) summarization of teachers’ views into four categories: “learner-focused” which emphasizes the learner’s construction of mathematical knowledge, “content-focused with an emphasis on conceptual understanding,” “content-focused with an emphasis on performance” and “classroom-focused,” where classroom management and discipline is central.

Science education researchers also followed suit with such categorization approaches to teacher beliefs. For instance, Luft and Roehrig (2007) classified science teacher beliefs into five categories: traditional, instructive, transitional, responsive, and reform-based. The obvious implications of the above findings are that teacher beliefs can be neatly sorted into discrete categories and that, furthermore, a hierarchy of beliefs exists: problem solving is more desirable than instrumentalist, learner-focused is more desirable than classroom-focused and reform-based is considered more desirable than traditional.

Much of the literature in this field is thus concerned with how teachers’ basic epistemological perspectives can be changed to be more in line with the recommendations coming out of the field of mathematics education in order to improve mathematics teaching (Cobb, Wood, Yackel, & McNeal, 1992; Franke, Fennema, & Carpenter, 1997; Leder, Pehkonen, & Torner, 2002). The reasoning is if teacher beliefs are changed through exposure to mathematics education research findings in methods classes and in-service professional development, you change the way teachers teach. Mathematics teacher educators are asked to "move teachers away from mathematics as teachers have most likely experienced it as students for over a decade and guide teachers toward a view of mathematics that is more consistent with the NCTM standards" (Taylor, 2002, p. 138).

However, a number of studies have shown that the relationship between beliefs and practice is much more complex than a simple causal-relationship (Ball, Lubiensky & Mewborn, 2005; Gates, 2006; Remillard, 2005; Speer, 2005). Many studies document the disconnect between what teachers say they believe and what they do once in the classroom; the gap between teachers' professed beliefs and attributed beliefs (e.g. Cooney, 1985; Simon & Schifter, 1991; Speer, 2005).

Some authors (e.g. Wilson & Cooney, 2002) see a cyclical relationship between changing beliefs and practice whereas others see teacher content knowledge as the missing piece in truly changing teacher practice in the classroom. Ball, Lubiensky & Mewborn (2001), for instance, found that changing beliefs is not sufficient in changing practice if content knowledge is lacking. Remillard's (2005) analysis of twenty-five years of curriculum research concluded that curriculum implementation was substantially driven by teachers' content knowledge, their knowledge required for teaching specific content, and their beliefs about teaching and learning.

Simon and Schifter (1991) assessed the instruction and evolving beliefs of teachers involved in a Summer Math for Teachers project. The project is based on a constructivist epistemology. The researchers found that teachers could easily change the implementation of a particular strategy but that the underlying views about learning were much more resistant to change.

Speer (2005) attributes much of the inconsistencies between teachers' professed and attributed beliefs to methodological reasons: "artifacts of the methods used to collect and analyze relevant data and the particular conceptualizations of beliefs implicit in the research designs" (p. 361). She also contends that the emphasis on categorizing teacher beliefs may be keeping the field from attaining a more comprehensive or holistic understanding of teacher's beliefs.

Leatham's (2006) view of mathematics teachers' beliefs as "sensible systems" based on previous work by Cooney (Cooney, 1995; Cooney & Shealy, 1997; Cooney, Shealy & Arvold, 1998) is a viable alternative to the discrete categorization approaches to teacher belief research.

A key component of Leatham's framework for seeing mathematics teachers' beliefs is that beliefs tend to be clustered in isolation from other beliefs. This beliefs clustering allows for a person to believe one thing in one context and another, perhaps contradictory belief, in another context: "Consequently, seemingly contradictory beliefs may exist in different belief clusters with no explicit or delineation of context" (Leatham, 2006, p. 95).

If one accepts Leatham's premise of beliefs as "sensible systems," then the concept of inconsistencies in a teacher's belief system and the gap between belief and practice vanish, as certain beliefs simply have more influence over certain actions *in certain contexts*. In educational research, such flexibility in choosing different components of ideologies in specific domains has been referred to as *bricolage*, where a viewpoint is constructed and created from a diverse range of ideologies which happen to be available. This helps avoid the reductionism of much of the research on teacher belief centered on categorization. Pachler et al. (2008) put forth:

> The point to be made here is that there is no, and has probably never been an ideological homogeneity in the field of education, and teachers move through different aspects of their jobs using very different ideological tools and orientating-often simultaneously-toward different ideological ‘centers’: themselves, their colleagues, their groups of learners, the head teacher, the school as an institution with a tradition, the education system, the curriculum, the government, society-at-large, and so on. Their discourses reveal traces of such multiplicity and layering (p. 440).

Other authors (e.g. Cuban, 1984; Ellis, 2005; Gates, 2006) have highlighted the stubbornness of beliefs and educational practice and have claimed that, perhaps, it is the “hegemonic nature” of education traditions and beliefs that may be responsible for the “widespread failure of the history of reform in mathematics education” (Gates, 2006, p. 349).

In his 2005 doctoral dissertation, Ellis (2005) investigated, through an analysis of historical data, “how the discourse of school mathematics took shape in the United States” (p. 5) in such a way that:

> Despite the reform efforts of the past hundred years school mathematics practice in the United States today and the outcomes produced remain stubbornly similar to those of over a century ago. Teachers are still the center of authority; students are still assessed primarily on the basis of their success with times tests of

> mathematical procedures and skills; classes are still tracked as students move into higher levels of mathematical study … (p. 16).

The literature supports the contention that educational discourse has been historically stable and unyielding to change (e.g. Bourdieu, 1972; Brophy & Good, 1974; Cobb et al, 1992; Cuban, 1984). Bourdieu (1972) refers to the truths taken for granted or "the universe of possible discourse" (p. 164) in any social realm, including the realm of education, as the *doxa.* The *doxa* of mathematics education is revealed through the description of the traditional mathematics teacher offered by Stodolsky (1988):

> Math instruction places all but the exceptional student in a position of almost total dependence on the teacher for progress through a course. In essence, the traditional math classes contain only one route to learning: teacher presentation of concepts followed by independent student practice (pp. 122-123).

This situation has led researchers to argue that studies of educational beliefs, and mathematics education in general, need to consider the impact that the greater social, cultural and political context has on mathematics education (Atweh & Clarkson, 2002a, b; Barkatsas & Malone, 2005; Gates, 2006; Pajares, 1992). Mathematics educators do not operate in a vacuum. Their beliefs and consequent practice are influenced by their experience as a student, by the school's environment, by their mathematics education training, and by the greater socio-political milieu. As Gates (2006) points out:

> To claim that studies of mathematics and mathematics teachers can only reside within mathematics itself will fail to address the very foundations upon which much mathematics and many teacher beliefs rest. This is consistent with the

> position that mathematics itself is a social construct constituted by social forces and social needs and conventions (p. 347).

Gates (2006) goes beyond the cognitive paradigm usually employed in mathematics education research to investigate teacher belief, choosing, instead, to develop a socially situated understanding of beliefs. Highly influenced by Bourdieu's (1972) claim that educational institutions play a central role in social reproduction, Gates (2006) tackles the enduring problems of mathematics teacher change taking the social aspect of education into account: Why are teacher behaviors and beliefs so difficult to affect?

Cole (1997) noted that teachers face many impediments to reflective practice that renders difficult the actualization of their beliefs into practice, resulting in the theory-practice rift observed by educational researchers. Impediments cited by Cole (1997) include external structures imposed by schools, the profession, the government and the public at large. Consequently, teachers are "in danger of being socialized to adopt norms of compliance and conformity" and "those who resist such practices and strive to uphold their beliefs ... do so usually at great cost" (p. 15). These impediments, Cole (1997) argued, should be explicitly addressed in teachers' professional education.

Windschitl (2002) developed a framework of dilemmas that teachers experience when trying to implement constructivist instruction, which included political dilemmas alongside conceptual, pedagogical and cultural dilemmas, in order to probe in depth the "full scope of challenges faced by teachers" (p. 131). Political dilemmas involved confronting issues of accountability with various stakeholders. Representative questions of concern in this category of dilemma included: "How can diverse problem-based experiences help students meet specific state and local standards?" and "Will constructivist approaches adequately prepare my students for high-stakes testing …?" (Windschitl, 2002, p. 133). Thomas (2001) argued that mathematics educators cannot continue to ignore the political forces that influence their discipline:

> Mathematics education is political…to ignore the effects of this in an economically globalized and changing world has the potential to constrain or render ineffectual much mathematics education research. In spite of the attention given to social context, the politics of mathematics education research is given little attention, and many mathematics educators tend to ignore the political milieu in which their work is situated (p. 96).

In reply to this criticism of mathematics education research, let us now turn our attention to a review of the literature on the socio-political milieu in which mathematics teachers' work is situated.

Socio-Political Milieu

In this section, I begin by examining some useful definitions of globalization. The classic economic definition is visited and the concept of globalization as shared cognitive space is examined. The conversation then progresses to the effects of globalization on education. In particular, the translation of the business model to education institutions is considered. This discussion also includes a study of the concept of the knowledge society, also known as the knowledge economy or learning-community, which critically examines the question, "What knowledge is of value in such a society?" Finally, a lengthy discussion about the effects on mathematics education of the discourses of global competitiveness and the knowledge society is offered. The role of mathematics teachers in this context is also considered.

Globalization

The term globalization has numerous meanings in the literature. This is to be expected as globalization is multifaceted and its progression ongoing. Meyer (2007) distinguishes between *globalization as exchange* and *globalization as cultural and institutional*. A brief discussion of globalization as exchange will first be presented, followed by a lengthier survey of the literature on globalization as cultural and institutional, especially as it relates to educational institutions.

Globalization as exchange focuses on the expanded economic exchange between nation states, "including processes of production, consumption, trade, capital flow, and monetary interdependence" (Burbules & Alberto Torres, 2000, p. 2). Authors agree that globalization has meant a decline of state intervention in national economies in favor of neoliberal free-market ideology, a weakening of the nation state, an increase in international integration of national economies and flows of technology, labors and workers (Stromquist & Monkman, 2000; Jones, 2000; Levin, 2002). Stromquist and Monkman (2000) refer to globalization as the "strong, and perhaps irreversible, changes in the economy, labor forces, technologies, communication, cultural patterns, and political alliances … imposing on every nation" (p. 3).

The liberal economic ideology started with Adam Smith's 1776 book *The Wealth of Nations*. Smith believed that markets free of government interventions had their own rationality, seemingly guided by an "invisible hand." Yet, when such *laissez-faire* proved to foster great inequalities in the distribution of wealth, economic theories that trumpeted the role of government in economic affairs became popular.

John Maynard Keynes's theory, founded on the positive effect governments could have on investor confidence, was welcomed in the post-war era (Peet et al., 2003, p. 7). Influential organizations such as the International Monetary Fund were founded on Keynesian principles. Friedrich von Hayek, 1974 Nobel Prize winner in economic science, was influential in the rebirth of Smith's austere liberal ideal in the form of *neoliberalism*.

Von Hayek, believed that "collectivist economic planning leads inevitably to totalitarian tyranny" (Peet et al., 2003, p. 10). Von Hayek theorized that free markets were the best manner in which to regulate price and production of goods and services - not government bureaucracies. He also argued that traditional government services, such as schools and hospitals, should be privatized in order for them to be controlled by market forces.

Neoliberalism brings us back to Smith's time of complete faith in self-regulating markets and ideas about personal freedoms. Von Hayek's view would ultimately win the day on the world stage. Peet et al. (2003) explains that neoliberalism is "founded on right-wing, but not conservative, ideas about individual freedom, political democracy, self-regulating markets and entrepreneurship" (p. 8).

Some social scientists (Jordan & Yeomans, 2003; Peet et. al., 2003) have proposed that globalization may simply be a discursive construction by richer nations in order to facilate economic collaboration and to impose their economic ideology onto developing nations.

Robertson (1992, qtd. in Henry & Taylor, 1997) defines globalization as referring to "both the compression of the world and the intensification of consciousness of the world as whole” (p. 46), while Harvey (qtd. in Stromquist & Monkman, 2000, p. 3) called it a “time-space compression” due, in large part, to electronic media. This results in a shared cognitive space. The definition of globalization as a shared space is put in economic terms by Stromquist and Monkman (2000) in the following way:

> A set of processes by which the world is rapidly being integrated into production and financial markets, the international trade, the internationalization of production and financial markets, the internationalization of a commodity culture promoted by an increasingly networked global telecommunications system (p. 4).

Globalization and Education

Although the literature is in relative agreement on the economic definition of globalization, the effect of globalization on culture and social institutions, such as educational institutions, is a much more difficult phenomenon to comprehend. As mentioned in Burbules & Alberto Torres (2000), the economic effects of globalization as described above "force national educational policies into a neoliberal framework" (p. 20) which among other things, tends to emphasize rational management of school organizations and performance assessment" (i.e. high-stakes testing).

According to Spring (2008), the major global educational discourses are neoliberalism, the knowledge society or economy, lifelong learning (learning community) and brain circulation. I will first expand on the global education discourse of neoliberalism, and then nuance the discussion by introducing the knowledge society and the related concept of lifelong learning.

Neoliberalism and education. Educational policies reflect what counts as legitimate or valued knowledge, and, furthermore, what is considered good teaching and learning. Policy-makers see a strong role for the government in shaping the educational policies that will produce the skilled “knowledge-workers” required to compete in the global economy (Fitzsimons, 2002). Underlying this philosophy is the belief that “educational effectiveness comes from central control and formal accountability” (Thomas, 2001, p. 101).

In this framework, education serves neoliberal goals by producing individuals that are enterprising and competitive. Hopper (2000) believes that neoliberalism has a certain slant on the foundations of truth and the nature of knowledge. To Hopper (2000), neoliberalism has an undeniably positivist epistemology, with a strong faith in scientific truths above other forms of knowledge. Technologically advanced countries thus have control over knowledge with the power to “construct the parameters of meaning” (Hopper, 2000, p. 103). Authors such as Burbules & Alberto Torres (2000), Labbert & Hattingh (2006) and Keitel, Kotzmann, and Skovsmose (1993) have also written at length about the pragmatist beliefs and positivist certainty of reality underlying much educational policy.

It is well documented that many schools around the word and at all levels have adopted a corporate model of education in response to global forces (Currie & Subotzky, 2000; Levin, 2002; Olssen & Peters; 2005). In practical terms, this has meant a managerial emphasis on measured outcomes and efficiency, a vocationalization of education, and a shift away from the liberal arts towards applied sciences, technology and other disciples with strong ties to the market (Abue, 2002).

Olssen & Peters (2005) discuss the influence the rise neoliberalism has had on higher education. They argue that a mimicking of the business model by universities has led to an abandonment of academics as independent professionals for an emphasis on measured outcomes. A higher value is placed on disciplines that garner grant money and grants are generally given to research areas that can demonstrate relevance to the labor market. Disciplines that have, at best, weak links to the market economy, such as philosophy or pure sciences, have had to redefine and even justify their existence.

Since this is now the prevailing view in schools, it is no surprise that **e**ducational goals must be spelled out in concrete terms in order to measure effectiveness. For instance, syllabi must have goal-oriented objectives clearly spelled-out: “Upon completion, students will be able to solve quadratic equations.” Goals such as: “Students will gain a better appreciation of the beauty of mathematics” or “students will learn how to creatively and collaboratively problem solve” would be considered laughable and useless. How would we measure creativity? How would appreciating the beauty of mathematics be relevant to current or future labor market conditions?

Knowledge society. The effects of globalization on education institutions are intimately tied to the discourse about the knowledge society. This connection makes investigations about epistemology and what is valued knowledge in a global society, including the mathematics that is valued, that much more pressing. It also complicates and nuances the discussion of the effects of globalization on education, including mathematics education.

On the one hand, the above discussion exposes how globalization and its discourse have meant the adoption of the corporate model in schools around the world and at all levels. This has brought about an era of performativity, measured outcomes, high stakes testing and its consequent pedagogy with its underlying post-positivist and pragmatist beliefs about the nature of mathematics and its teaching.

However, the concept of the *knowledge society* somewhat complicates this discussion. There is the claim that globalization has brought about an era of postmodernity in which the speed of change of knowledge has increased so rapidly that it has helped call into question matters of 'truth' and 'reality' as relative (e.g. Jarvis, 1999). Proponents of this position cite the quickness at which knowledge becomes obsolete (temporary knowledge) and the fact that schools are no longer the only providers of knowledge and truth. The mass media, the World Wide Web, businesses, etc. are now also purveyors of information.

As Levin (2002) points out “synchronous global television broadcasts and asynchronous world wide web postings are among the symbols of a global culture” (p. 122). Interestingly, these are exactly the symbols used by postmodernists to explain their skepticism of a positivist certainty of reality. Postmodernists question the very foundations of reality and reject the possibility of certainty. The postmodernist view of the role of technology can best be summed up by Baudrillard (qtd. in Gubrium & Holstein, 1997):

> Through television, we are taken instantaneously to distant and disparate places. Space in terms of distance doesn’t seem to matter. In seconds, contrasting images are juxtaposed, jarring a modern sensibility built on things that are separate and distinct from one another … Reality or modern time and space, are “cranked up” to the point where the objects normally associated with the real no longer apply (p. 78).

Thus, while electronic media has allowed for the spread of the neoliberal positivistic idea of knowledge, it is also responsible for many scholars becoming uncomfortable with this approach to truth (epistemology). In the knowledge society, the new, “informational mode of development, the source of productivity lies in technology of knowledge generation, information processing, and symbol communication” (Castells, 1996, p. 17).

The term *knowledge society* (sociology) is often used synonymously with the notions of (global) *knowledge economy* (economy*)*, *information society* (computer science and information technology) and *learning society* (education). These notions have been written about at great length.

A Google search (in January 2009) returned approximately 779,000 hits for *knowledge society*. An ERIC database search in education for “knowledge society” returned approximately 170 publications on the topic and 420 which prominently featured the notion of the learning society. In fact, the journal *International Journal of Lifelong Education* was founded in 1992 “with the expectation that we would soon be encountering lifelong education on an international scale” (Jarvis, 2006, p. 201).

The knowledge society is often depicted as being driven by creativity and ingenuity where the primacy of manufacturing is replaced by knowledge. The World Bank (2003) defined it this way: “A knowledge-based economy relies primarily on the use of ideas rather than physical abilities and on the application of technology” (p. xvii).

Members of the knowledge society must become *life-long learners* in order to remain abreast of the ever-changing valued knowledge and remain productive members of the society. In other words, in the knowledge society it is knowledge, not goods and services, that is the primary exploitable commodity. Within this highly ideological discourse, knowledge is marketed as the main survival tool in a globalized world.

Specifically, the knowledge of value in the knowledge society is scientific and technological knowledge. This is a society in which pragmatism rules as a theoretical framework, in which “knowledge, as such, has no intrinsic value; it is only its use-value as a scarce resource which is significant. Hence research and development are at the heart of the productive processes and knowledge has to be practical” (Jarvis, 2006, p. 203).

In this vision of a globalized, ever-changing knowledge-driven world, teachers are asked to prepare students to become life-long learners in order for them to stay abreast of all innovations in their field of employment created by quickly changing technologies and knowledge. Spring (2008) discussed the role of K-12 educators given the discourse of the knowledge society and of life-long learning: “In this context, primary and secondary education becomes preparation for the lifelong learning required by the rapidly changing technology of the knowledge economy” (p. 9). The research centered on changing teacher beliefs about mathematics and its education documented above is partly motivated by the drive to encourage teachers to move toward a view of mathematics that will help them guide their students to engage in the complexities of the 21st century (Cooney & Shealy, 1997).

School teachers are charged with equipping students with the skills and deep cognitive abilities that are at a premium in such a vision of society: creative thinking, capability to reskill (life-long learners) and relocate as the economy shifts around them, ability to work collaboratively, identify and solve problems. In other words, the crucial skill that teachers are to be imparting to their students in order for them to be ready to participate in the knowledge society is *learning how to learn* (Valimaa & Hoffman, 2008).

Morrow & Alberto Torres (2000) have said that the overall effect of neoliberal policies on education, as described above, has been a shift toward competence-based skills to the detriment of fundamental and critical competences an individual would need for autonomous learning key to life-long learning skills essential to participation in the knowledge society. Hargreaves (2003) agrees with Morrow & Alberto Torres (2000):

> In many parts of the world, the rightful quest for higher educational standards has degenerated into a compulsive obsession with standardization. By and large, our schools are preparing young people neither to work well in the knowledge economy nor to live well in a strong civil society" (p. 2).

Lee (2005) points out that this emphasis on education for work and the consequent focus on facts and procedures in school mathematics will in fact prepare students for jobs that won't exist by the time they enter the job market. To truly prepare students for tomorrow's world, Lee (2005) proposes there should be an emphasis in the mathematics classroom on thinking skills and affective education in order to equip students with "skills to handle a world of non-linear change" (p. 172).

In an education speech given September 9, 2008 in Ohio (Obama, 2008), then presidential candidate Barak Obama used the knowledge economy discourse to explain his position on his proposed reform of No Child Left Behind (2001):

> And don't tell us that the only way to teach a child is to spend most of the year preparing him to fill in a few bubbles on a standardized test. (Cheers, applause.) I don't want teachers to the -- teaching to the test. I don't want them uninspired and

> I don't want our students uninspired. (Applause.) So what I've said is we will measure and hold accountable performance, but let's help our teachers and our principals develop a curriculum and assessments that teach our kids to become not just good test-takers. We need assessments that can improve achievement by including the kinds of research and scientific investigation and problem-solving that our children will need to compete in a 21st century knowledge economy.

Globalization, Mathematics Education and the Mathematics Teacher

Mathematics is related to our understanding of the world, and the effect globalization has on this discipline’s education shapes the way students will eventually view and interact with the world. Since mathematics is at the heart of science and technology, which are in turn the basis of the new global, knowledge-based economy, mathematics education is highly valued by governments around the world, including in the U.S. It is particularly valued for its perceived economic importance not its intrinsic value or as an important facet of human culture. The issues described above in the “globalization and education” section are thus particularly relevant to mathematics education.

The Partnership for 21^{st} Century Skills (Partnership for 21^{st} Century Skills, 2007) lists mathematics as one of the core subjects and 21^{st} century themes considered essential for students in the 21^{st} century, and advocates moving “beyond a focus on basic competency in core subjects to promoting understanding of academic content at much higher levels” by weaving global awareness, entrepreneurial, civic and health literacy into the teaching of mathematics.

The essential Life and Career Skills identified by The Partnership for 21st Century Skills are: flexibility and adaptability, initiative and self-direction, social and cross-cultural skills, productivity and accountability, and leadership and responsibility. In particular, students need be prepared to “work effectively in a climate of ambiguity and changing priorities” as well as “collaborate and cooperate effectively with teams” and “use problem-solving skills to influence and guide others toward a goal” (Partnership for 21st Century Skills, 2007).

Mathematics achievement scores on standardized tests have become inexorably tied to national policy discourses of global competitiveness emphasizing a strong causal relationship between mathematics achievement and economic prosperity (e.g. National Science Foundation, 2001, No Child Left Behind, 2001; OECD, 2007; TIMMS, 2003). Consequently, mathematics curriculum development is underpinned by the “human capital” discourse, in which quality mathematics education play a central role in providing individuals with the set of skills and knowledge to produce economic value.

Because adequate preparation in mathematics is rapidly becoming a requisite for workplace entry and mobility in today’s information, knowledge society (Flowers & Moore, 2003), questions of unequal access to quality mathematics education are inexorably tied to social inequities. Namukasa (2004) is very critical of the privileged role of mathematics in our increasingly globalized world, describing it as a "cultural homogenizing force, a critical filter for status, a perpetuator of mistaken illusion of certainty and, an instrument of power" (pp. 2-3). Sriraman & Steinthorsdottir (2007) agree with Namukasa (2004) in that mathematics serves as a gatekeeper to other areas of study, pointing to the calculus sequence as a prime example of a way to filter out students. Gates (2000, p. 14) contends that "mathematics plays a significant role in organizing the segregation of our society."

It is worth noting that although some studies support a causal link between mathematics and science achievement and a nation's economic growth (e.g. Hanushek & Kimko, 2000; Drori et al. 2003), others (e.g. Ramirez, Luo, Schofer, & Meyer, 2006) have called this connection into question. The assumed causal links that tie mathematics and science achievement to national economic vitality are as follows: national curricular and pedagogical modernization leads to math and science interest and achievement, which in turn leads to scientists and engineers in higher education and in the labor force, which results in national economic development.

Each step of this reasoning has its critics. For example, Ramirez, Luo, Schofer, and Meyer (2006) found that the relationship between a nation's mathematics and science achievement and national economic growth was time and case sensitive. Although they found that countries with high science and mathematics achievement grow more rapidly, the effect was considerably reduced when the "Asian Tigers" were removed from the statistical analysis (i.e. Hong Kong, Singapore, South Korea and Taiwan between 1980 and 2000).

Furthermore, this study also found that moving from mid-achievement scores to top achievement scores had virtually no economic consequences. The only causal link between achievement and economic growth was found when a very low performing country increased its achievement scores in the mathematics and science fields to mid-level performance. As the authors stated:

> This is a striking finding that calls into question the disproportionate attention (and envy) focused on those few countries with the very highest achievement scores. Such countries do not experience substantially greater economic growth than countries that are merely average in terms of achievement (Ramirez, Luo, Schofer, & Meyer, 2006, p. 14).

Nevertheless, policy discourse in the U.S. and in many countries around the world continues to emphasize a strong causal relationship between mathematics achievement and economic prosperity. The inordinate amount of interest about PISA 2006 (OECD, 2007) and TIMMS (2003) results are part of this discourse. These international comparison data are taken very seriously by governments as a key measure of performance of their educational system. Poor mathematics scores on international standardized tests have embarrassed the U.S. and have led to calls for making schools more accountable, efficient and responsive to the public sector (e.g. Rimer, 2008; Rothstein, 2000).

There has been a convergence of school mathematics curricula including content sequencing. The content taught is very similar worldwide (Atweh, & Clarkson, 2002a). This has not been alarming until recently because many mathematics educators felt that this was simply due to the *nature* of mathematics itself. If one takes the view that mathematics and science are objective and acultural activities, homogeneous, unanimously agreed upon curricula are only natural.

Mathematics education researchers and science historians have a different viewpoint. For instance, calculus was introduced in American schools in the late 50's in response to the launch of Sputnik by the Soviet Union and the U.S.'s fear of falling behind in science and technological development. This crisis led to educational reforms, especially in the areas of mathematics, science and technology (Fitzsimons, 2002).

Historically, countries have chosen to include or exclude a particular subject from school mathematics because of political and military concerns, rather than economic goals as we are seeing more recently (Ramirez, Luo, Schofer & Meyer, 2006). The study of mathematics in the cultural context in which it arises is called ethnomathematics. Ethnomathematics has gained popularity in mathematics education circles in recent decades and has brought to the forefront the notion of mathematics as a culturally embedded knowledge (D'Ambrosio, 2001; Ruthven, 2001). So has the history of mathematics, which shows the variability of mathematical approaches and traditions of various countries and civilizations. Ironically, while researchers have been increasingly interested in the pedagogical needs of diverse learners and the effect of culture on learning, standardized/globalized curricula are being implemented in order to meet the needs of employers rather than students.

Internationalization and globalization in mathematics education. International collaboration is not new to the mathematics community. *Internationalization* of the discipline refers to any academic activity that involves a cross-cultural collaboration (Atweh & Clarkson, 2001). Mathematics education is probably one of the most international subjects in higher education (Robitaille & Travers, 1992). Mathematics educators, for instance, welcome the increase in international students, and truly value the abundance and quality of international conferences and journals (Atweh & Clarkson, 2002b).

The dramatic enhancement of communication technologies has facilitated the possibility of personal exchange and mutual understanding. Some examples of international organizations and coordinating bodies charged with the internationalization of mathematics education are: the International Group of Psychology of Mathematics Education, the International Congress in Mathematical Education (ICME), the Comité Interamericano de Educación Matemática, the African Mathematics Union, Mathematics Education Research Group of Australasia, the Congress of the European Society for Research in Mathematics Education (CERME) and the International Organization of Women and Mathematics Education (IOWME), to name but a few. Many of these organizations hold annual conferences and organize study projects, bringing together mathematics education researchers and educators from many countries.

However, Atweh and Clarkson (2002a) argued that internationalization and globalization of mathematics education differ on the degree of autonomy they allow for participating nations and, consequently, participating educators: “While international collaborations tend to be transparent and enjoy a degree of autonomy in participation, globalization processes are often associated with forces that are impersonal and beyond the control and intentions of any individual or groups of individuals” (p. 23). Globalization is seen as “compelling rather than invitational” by mathematics educators and researchers around the world (McGinn, 1995, qtd. in Atweh & Clarkson, 2001, p. 80).

Astiz, Wiseman & Baker (2002) would disagree with Atweh and Clarkson (2002a). These authors assert that choices have been made by governments, corporations, communities and individuals, to participate in the process of globalization, at each level up to the transnational.

Currie and Subotzky (2000) suggest that the emergence of the “entrepreneurial university” for instance, is not an inevitability, and that countries such as France, Norway and South Africa have found alternative practices to the managerial model. The point made by Currie and Subotzky (2000) is that, while countries can no longer “opt out” of the global economy, they have a say in the degree in which they want to embrace aspects of the free-market ideology and how their decision will affect the face of education. For instance, both France and Norway’s governments have made a concerted effort to maintain the welfare state while welcoming privatization.

The compromise between unfettered market capitalism and government interventions is referred to as the “Third Way” by Currie and Subotzky (2000). Though it is true that some countries have successfully integrated some aspects of the managerial approach to education and rejected others, it is clear that countries with little economic power will find it difficult to take that “Third Way.” This alternative is a luxury that many countries will not be able to afford.

Official documents from government science and mathematics agencies such as the National Science Foundation (NSF) often refer to globalization in positive terms. In NSF’s 2001 task force document, globalization is referred to as: “The worldwide integration of nations through trade, capital flows, diffusion of information, movements of people, and operational linkages among firms and other organizations” where science and engineering will be “key determinants of economic growth, quality of life, and the health and security of our planet in the 21st century” (NSF, 2001, p. 19). This document emphasizes the value of international collaboration such as the concerted effort needed to reign in infectious diseases like AIDS, ebola, and tuberculosis (NSF, 2001, p. 25).

Atweh and Clarkson (2002b) point out that many international gatherings are dominated by discussion of the UK and the US reforms such as the Cockroft report and the Standards documents published by NCTM. Further, keynote speakers from the United States tend to only refer to literature from the United States. A quick look at the reference list of most American mathematics education articles will convince any reader that U.S. researchers often omit work done in other countries.

This has resulted in the fact that many countries do not have their own national identity in mathematics education and defer to the standards established by the West. For instance, Atweh and Clarkson (2002a) report the story of the induction ceremony of a group of Ph.D. students in Columbia at which a professor gave a lecture on the main developments of educational thought during this century. Although he cited Piaget, Von Glaserfeld, Kuhn and other familiar educational theorists, he did not make reference to Paulo Freire of Brazil or Orlando Fals Salvador of Columbia.

Many mathematics educators and researchers feel uneasy about the uncritical globalization of issues and schools of thought in mathematics education and many have spoken up. For instance, at the 1992 International Congress on Mathematics Education (ICME) regional conference held in Canada, Usiskin (1992) mentioned, “the extent to which countries have become close in how they think about their problems and, as a consequence, what they are doing in mathematics education.” At this same conference, Rogers (qtd. in Atweh & Clarkson, 2002a) noted that:

> All our theories about learning are founded in a model of the European Rational Man, and that this starting point might well be inappropriate when applied to other cultures…the assumptions that mathematics is a universal language, and is thus universally the same in all cultures cannot be justified. Likewise, the assumption that our solutions to local problems…will have universal applications is even further from the truth (p. 86).

Interestingly, this concern is not felt by many developing countries. In fact, many mathematics educators in developing countries warn against an overemphasis on ethnomathematics. For instance the president of the African Mathematical Union in 1995 posited that such an emphasis “may be at the expense of actual progress in the mathematics education of the students” (Kuku, 1995, p. 407).

Some trends in mathematics education attributed to globalization in the literature include: similarity in research questions and methodologies, convergence of school curricula including content sequencing, standards of research and wide spread acceptance of epistemological stance (Atweh & Clarkson, 2002a). Many educators see the fact that “the value of an American-style education transverses the world as entire courses, complete with syllabus and textbooks” (Atweh & Clarkson, 2002a, p. 165) as a form of colonialism contradictory to the scientific philosophy of debate and free exchange of ideas. As a Columbian educator interviewed by Atweh and Philip (2002b) said, an *uncritical* adoption of concepts from overseas is a form of colonialism. She cites the worldwide adoption of calculus textbooks from the United States as an example of academic colonialism.

Just as pervasive, is the use of foreign educational consultants by many developing countries. For instance, a developing country in Latin America may call upon consultants in the U.S. or U.K. to help them with educational reforms. Often, developing countries see the use of imported mathematics curricula and even educational philosophies as their best chance for their country's economic development. Thus, global collaboration runs the risk of being a subtler form of neo-colonialism. As one educator in an Australian Focus group as part of Atweh and Clarkson's (2002b) study stated:

> The critical part is that there is an ideology out there, that if you take a western view, whatever that may be, and I think that even within the Western view, it's a narrow American view, that if they take that view of the world then that's what is going to give them access to power…That is the way they're going to get out of their, in a sense, oppressed state, by adopting the American curriculum (p. 9).

The mathematics educator. As governmental control has increased and educational *theory* has become subservient to educational *policy*, teachers and educational researchers have lost much of their autonomy (Hargreaves, 2003; Lewis, 1998). Educators at all levels have been transformed into functionaries of the state. Mathematics educators lament the commodification and standardization of the curricula in an era when mathematics education research has concentrated on the pedagogical needs of the individual, diverse learners, social context and constructivism (Atweh & Clarkson, 2002a, b). As Lewis (1998) concluded:

> The intensified exploitation of teachers which has been exacted through the operationalization of source-book-curricula has pre-empted debate over the pedagogical impasse they present for educators. Indeed the worse excesses of

> commodified curricula have seen teachers' roles completely redefined and their professional autonomy undermined (p. 4).

Kohl (2009) referred to the ambiance created in schools by scripted curriculum, teacher accountability, continuous monitoring of student performance, high stakes testing and punishment for not meeting standards as an "educational panopticon." A panopticon is a type of prison designed in the 18th century in which control was asserted over prisoners by creating an environment in which inmates "would internalize and accept the idea of total and continuous surveillance whether or not it was actually happening" (Kohl, 2009, p. 2).

The managerial approach to education is underpinned by the belief that "educational effectiveness comes from central control and formal accountability" (Thomas, 2001, p. 101). Further, when mathematicians and mathematics educators have spoken out against the commodification and vocationalization of mathematics, they are dismissed as elitists. To raise questions about a "standardized curriculum" has become raising questions about "education for all" and has effectively silenced most critics. The loss of control over their work results in teachers feeling anxious and conflicted as "they are forced to teach in ways that do not measure up to their personal standards of the way things should be" (Cole, 1997, p. 15).

Although mathematics education has long been asked to meet industry's needs and to be at the forefront of national economic competitiveness, political rhetoric was mediated by a powerful ideology of liberal education and a strong tradition and respect for educators' and researchers' autonomy and professionalism.

However, educators have seen their “values and expertise breached so that their work is now opened up to reconstruction as constituents of market-driven business enterprise" (Singh, Kenway, & Apple, 2005, p. 14). The cultural meaning of school has changed to become places where management authority, not collegial culture and humanistic values, establishes the ethos of the school.

As teachers and educational researchers have been stripped of their professionalism, they have been simultaneously stripped of their agency to affect educational policy. According to Holland et al. (1998), agency is the ability to act and understand one’s power. A person with agency is a knowledgeable and committed participant. They have control over their own behavior and the ability to act on their world. Thomas (2001) surmises that:

> The mathematical scientists – teachers, mathematics educators, and discipline specialists – are more remote from decision making that is increasingly influenced by bureaucrats and global economic considerations rather than educational and equity considerations (p. 97).

The research reflects a disconnect between the constructivist and sociocultural perspectives on teaching and learning advocated in mathematics education research and the positivist belief system espoused by mathematics education policy. Although students and teachers alike tend to pay more attention to the local implications of education (e.g. employment, graduation rates, etc.) the discussion above shows that the greater socio-political milieu unequivocally affects their work.

Surrounded by the rocky waters of mathematics education, teachers are shown as navigating the different belief systems in their day-to-day decision-making. If mathematics education research suggests that one should incorporate writing in mathematics classes because it will help students gain meaningful understanding but "meaningful understanding" is not easily measured, what does one do and why? What is the decision making process behind whatever the decision might be? How do individual mathematics teacher interpret and mediate the major global educational discourses of neoliberalism and the knowledge society? What do teachers experience when faced with these decisions day in and day out?

Summary and Introduction to Chapter 3

This chapter reviewed the literature relevant to this study. It provided a survey of the literature on the paradigm shift from positivist to constructivist and sociocultural perspectives which occurred in mathematics education. An overview of the literature on teacher beliefs followed. An extensive discussion of the major political forces influencing education, including mathematics education, was then presented detailing the effects of the two main global educational discourses: Neoliberalism and the knowledge society. The role of the mathematics teacher in such a context was also explored.

The following chapter will explain the method chosen to study the phenomenon of mathematics teachers navigating differing epistemologies – an instrumental multiple case study – as well as the theoretical framework which helped organize and explain the data. The way the data was collected and analyzed is also discussed.

CHAPTER 3: METHODOLOGY AND THEORETICAL FRAMEWORK

As researchers are always informed by a theoretical perspective in their attempt to better understand the world, this chapter begins with a discussion of my theoretical framework. After having established the theoretical framework, I will then discuss the research methodology selected for this study, which was qualitative, instrumental, multiple case study employing components of phenomenology. This discussion includes a description of the participant selection, time line of the study and methods of data collection and analysis.

Theoretical Framework

The development of a good theoretical framework is vital to conducting high-quality case study research. It is the lens used for data collection and it is the vehicle for generalizing particular results to a broader theory. It is the way that I, as the researcher, have chosen to know the world.

I am using a social constructionist lens, derived from the work of Vygotsky and made prominent in the U.S. by Berger and Luckmann's 1966 book *The Social Construction of Reality*. Vygotsky argued that learning is a social activity and that an individual can only really be understood when taking into account the external social world in which the individual developed. That is, "the behaviour of man is the product of development of a broader system of social ties and relations, collective forms of behaviour and social co-operation" (Vygotsky & Luria, 1994, p. 138).

According to Vygotsky (1986), whose work social constructionism and sociocultural perspectives often rests, “the true direction of the development of thinking is not from the individual to the social, but from the social to the individual” (p. 36). Therefore, from a socio-constructionist perspective, when attempting to understand how an individual develops a concept, such as mathematics and its teaching, one must acknowledge what social constructs are informing that concept. To understand the phenomenon of an individual, their thinking and the development of that thinking, one must also understand the social constructs affecting and influencing the individual and, thus, the phenomenon, in their social and historical contexts.

Since one of the main purposes of this study was to analyze how teachers of mathematics navigate the different beliefs about mathematics, I examine the social constructs influencing the formation of these beliefs. My research questions imply the political underpinnings of the participants’ concepts of mathematics and how they enact those concepts; I approach the data analysis stage of the study with a critical eye as a means to interpret and explain what is observed during data collection. Understanding these influences provides a greater depth of knowledge about how teachers come to conceptualize what mathematics is, which in turn informs their teaching practices.

Underpinned by Vygotsky’s notion of multiple, socially constructed truths and experiences, this study is not centered on providing a version of correct teacher behavior or best practices as it is. Instead, the instrumental multiple case study approach, influenced by phenomenological research, is used to capture teachers’ lived-experiences as they mediate the beliefs in part shaped by mathematics education research and the external discourse of "effective teaching.”

Leatham’s (2006) view of mathematics teachers’ beliefs as “sensible systems,” discussed earlier in Chapter 2, provides a useful lens through which to see teacher beliefs and practice while circumventing value judgments about best practices and inconsistencies in attributed and professed teacher beliefs.

A key component of Leatham’s framework for describing teachers’ lived-experiences as they mediate different belief systems is the very important notion that beliefs tend to be clustered in isolation from other beliefs, and that this belief clustering allows for a person to believe one thing in one context and another, perhaps contradictory belief, in another context. As seen in Chapter 2, this allows for seemingly contradictory beliefs to exist in different belief clusters with no explicit or delineation of context (Leatham, 2006). Within Leatham’s framework, inconsistencies in a teacher’s belief system and the gap between belief and practice vanish as certain beliefs simply have more influence over certain actions in certain contexts.

The concept of *bricolage* (see Chapter 2), where a viewpoint is constructed and created from a diverse range of ideologies which happen to be available, will also inform the way I will write about teacher beliefs. This helps avoid the reductionism of much of the research on teacher belief centered on categorization.

I am also using aspects of Gates' (2000) framework for the sociological perspective on teacher beliefs developed in his unpublished Ph.D. dissertation. Gates’ framework is shaped by French sociologist Bourdieu’s (1972) claim that educational institutions play a central role in social reproduction. The resulting framework is comprised of three key components: *habitus, ideology* and *discourse*. The term *habitus*, as described by Bourdieu (1972), is the set of dispositions which come largely from our up-bringing. In practice, *habitus* is “history turned into second nature. It is through *habitus* that objective structures and relations of domination reproduce themselves within us” (Bourdieu, 1972, paraphrased by Gates, 2006, p. 352). In essence, *habitus* comprises the dispositions, values, skills and understandings individuals accumulate across their lifetimes. Bourdieu states that *habitus* is a consequence of socialization within specific contexts through which “one develops distinctive...ways of ‘seeing’, ‘being’, ‘occupying space’, and ‘participating in history’” (Carrington & Luke, 1997, p. 101).

The formation of the *habitus* is deeply embedded in social and cultural influences and is significant in individuals’ sense of themselves and others, of the nature of interactions, and of the meanings made through individual’s interactions in specific social and cultural contexts. Bourdieu argues that through “*habitus*, we have a world of common sense, a world that seems self-evident” (Bourdieu, 1989, p. 19). According to Bourdieu, one’s *habitus* is in a constant state of ebb and flow with the world. The work of Bourdieu and the concept of *habitus* became an important addition to my theoretical framework.

The operational definition offered by Gates (2006) for *ideology* is (a) the structure of ideas about the relations between individuals and (b) ideas about how power is used at a practical level in order for society to function. Teachers’ ideological framework includes their views on the role of education in society, the role of the teacher in the classroom and about learning.

Discourses are the “interactional means whereby we live out and act out our ideological framework” (Gates, 2006, p. 354).

Also of importance to my study is Gates' (2006) concept of *mediating relations* described as the:

> Level of operation between one's values and one's engagement in external discourses. The terminology is intended to suggest that these relationships mediate, in the sense of working to bring about an agreement between one's values and one's position within and towards external discourses (p. 345).

Gates (2006) goes on to say that "these relations form a conduit between agency and structure, between disposition and discourse" (p. 345) and that the amount of flexibility and tension an individual can manage will be a function of one's *habitus*.

Methodology

Why Qualitative?

The main goal of qualitative research is to "describe and clarify experience as it is lived and constituted in awareness" (Polkinghorne, 2005, p. 138).

Given that the purpose of this study is to provide a thick description of the lived experience of three mathematics educators as they mediate the different belief systems shaped by mathematics education research, recommendations by organizations such as the National Council of Teachers of Mathematics, workshops, mathematics methods classes and the major global educational discourses, it is crucial that I "describe and clarify" these ideas within their lived context. In order to describe these lived experiences, I draw upon case study methodology while also using phenonemenological techniques of interview (van Manen, 2001).

Research Design: Instrumental Multiple Case Study

The current research was a semester long, instrumental multiple case study of three high school mathematics teachers as they navigate different belief systems when making professional decisions related to their work as teachers. The case study methodology was selected in order to relate a detailed description of the uniqueness and commonalities of the lived experiences of these three mathematics educators (Stake, 1995). Yin (2003) defines a case study as "an empirical inquiry that investigates a contemporary phenomenon within its real-life context" (p. 13).

In this study, the “real-life context” is situated within the socio-political context described in the review of the literature (see Chapter 2). As Merriam (1998) explains further:

> A case study design is employed to gain an in-depth understanding of the situation and meaning for those involved. The interest is in process rather than outcomes, in context rather than a specific variable, in discovery rather than confirmation (p. 19).

A multiple case design was chosen for this study as it is considered less vulnerable than single-case design and because "the analytic benefits from having two (or more) cases may be substantial" (Yin, 2003, p. 53). For one, studying multiple cases instead of one case helps blunt possible criticisms about the uniqueness of a particular single case. Analytic conclusions arising from multiple cases are more powerful than those coming from a single case since the generalizability of the findings are expanded. Multiple case design makes use of the same procedures for each case.

Generalization is always a concern with case study research. Yin (2003) argues that case studies "are generalizable to theoretical propositions and not to populations or universes” (p. 10). He continues to state that, in contrast to survey research, which can make generalization on the basis of statistical tests, case study research relies on analytical generalization. This entails the researcher generalize particular results to a broader theory. This is accomplished by discussing broader theoretical issues along side the thick description of the lived experiences of the participants.

The presented case studies are *instrumental* as I aspire to gain a greater understanding of the effects of a bigger, socio-political phenomenon through the study of these three cases. This is done in an effort to provide insight into an issue in order to produce better theorizing (Stake, 1995). An instrumental case study approach is in contrast to an *intrinsic* case study approach in which the cases are situated within a unique, unrepeatable sociocultural phenomenon, which results in an intrinsic and primary interest in the case itself.

The present study also makes use of some components of phenomenology, primarily a reliance on in-depth, open-ended interviews as the primary data (Creswell, William, Hanson, Plano & Morales, 2007).

The chief aim of phenomenological research is "to 'borrow' other people's experiences in order to better be able to come to an understanding of the deeper meaning or significance of an aspect of human experience, in the context of the whole of human experience" (van Manen, 2001, p. 62). Consequently, to collect data for each case, I used interviews to allow for a narrative description of teachers lived-experiences to emerge.

In order to truly understand how any social phenomenon is experienced by individuals or groups of individuals, a researcher must strive to convey the individual's words, thoughts, feelings and reactions as that individual is experiencing the phenomenon. The phenomenological perspective strives to do so by utilizing a participant's own words or by portraying the experience from the participant's perspective as accurately as possibly.

The primary data source in phenomenological research, the interview, is also highly prized in case study research. The power of the interview was described as follows by Stake (1995) in his book on case study research:

> Much of what we cannot observe for ourselves has been or is being observed by others. Two principal uses of case study are to obtain the descriptions and interpretations of others. The case will not be seen the same by everyone. Qualitative researchers take pride in discovering and portraying the multiple views of the case. The interview is the main road to multiple realities (p. 64).

The interview, as a data collection tool, allows the researcher to explore aspects of a teacher's thinking that cannot be easily captured through written self-report or observations. Much more will be said about the manner in which interviews were conducted in the "data collection" section below.

Participant Selection

The participant selection followed criteria-based sampling guidelines (Creswell, 2005). As the purpose of the study is to explore the experiences of teachers mediating various belief systems, including the beliefs found in much of mathematics education literature and mathematics education methods classes, it was essential that my participants have a strong background in mathematics education and that they be reflective and able to articulate their experiences.

The initial pool of potential participants included seventeen high school mathematics teachers who, at a minimum, had taken three or more mathematics education classes. Preference in selection was given to teachers who were pursuing or held a Masters in Mathematics Education, or were currently pursuing a Ph.D. in Mathematics Education, as these teachers had a demonstrably strong knowledge of mathematics education.

This initial list of seventeen names was obtained through various sources, including recommendations from mathematics education professors, as well as principals, assistant principals and teachers at local high schools. Once this list was compiled, an e-mail invitation to participate in the study was sent to all seventeen teachers (see APPENDIX A for example of invitation e-mail). Out of the seventeen initial potential participants, eight volunteered to be part of my study.

Again, as I am attempting to gain an understanding and describe the lived experiences of teachers mediating clashing epistemologies, it was vital that my participants had lived this experience and be able to articulate it. I thus informally interviewed these 8 potential candidates prior to formally inviting them to participate in the study in order to establish whether they have lived the phenomenon I am studying. This interview included questions about their educational background, the beliefs about teaching of mathematics they espouse, their current teaching context and whether they had encountered impediments to the actualization of their beliefs in their current context.

After these informal interviews were completed, a decision was made to collect data on six of the eight volunteering teachers (four males and two females). I thus approached these six teachers formally asking them to be part of my study. After explaining the research in greater detail, I asked them to read the rights and requirements of participation in the study and sign consent forms.

The decision to report on only three teachers in this dissertation was taken after the data analysis process showed that these three cases were of singular interest due to the particular combination of commonalities and uniqueness of the themes that emerged from these three participants' experiences.

These three teachers had particularly well-developed knowledge in the constructivist and sociocultural epistemological foundations of mathematics education research, had strong mathematical content knowledge, had idealized images of themselves as teachers and were experiencing some impediments to the actualization of these beliefs in the reality of their current school context. The similarity in their experiences was a factor in the decision to report on these three particular cases, whereas the differences in their experiences provided a fuller narrative of the ways teachers mediate clashing epistemologies.

The decision to concentrate on three cases was not made lightly. My decision to focus on only three cases was guided by Patton's (2002) definition of *information-rich* cases: "Information-rich cases are those from which one can learn a great deal about issues of central importance to the purpose of the research…" (p. 46). The analysis showed that the cases for three participants, Martin, Lea and Martha, were not information-rich in that they did not provide the level of insight necessary for me to learn about issues of central importance to the purpose of my research.

For one, Lea and Martha had difficulty in articulating mathematics education research, their teaching philosophy and/or beliefs and, thus, could not express their epistemological struggle in rich detail. For instance, although one participant (i.e. Lea) was conflicted about her role as a teacher, stating: “I feel kinda bad cramming it in their head, you have to do this and you have to learn this,” she could not answer the question: “How do you know when a student understands?” further than “they pass the test.” It is worth noting that Lea had considerable exposure to mathematics education research findings and recommendations, both from mathematics education classes and in-service professional development workshops. She also had very advanced mathematics content knowledge, having taken courses such as topology and non-Euclidean geometry. However, much of her answers centered on procedural knowledge acquisition. The following interview excerpt is representative of the data collected from Lea during interviews (R denotes the researcher’s part and L denotes Lea’s part):

R: Can you expand on what you mean when you say: they need to be able to think? What does that mean to be able to think mathematically?

L: Be able to apply your knowledge not just work out a problem that somebody has already given you. Use it. We're doing like now, we're starting the exponentials. If you put money in the bank, how long before you have $400…

One teacher-participant (i.e. Martha), showed little willingness to provide course artifacts. Furthermore, interviews with this participant were much less in-depth than with the other five participants due to her very busy schedule. For this reason, data was collected fully for only five out of the six original participants.

Another participant (i.e. Martin) had extremely advanced content knowledge in a specialized area of mathematics (i.e. Rubik's cubes and associated algorithms and group theory). While this participant had considerable familiarity with mathematics education research, extremely sophisticated mathematical content knowledge and did face impediments to the actualization of his beliefs, this individual's focus during interviews was primarily on his specialized area of interest. A simple word search of Martin's interviews resulted in 30 mentions of "Rubik's cube" and associated concepts. For instance, when asked: "What do you like about teaching mathematics?," Martin answered: "You know, every time you teach a kid how to solve a Rubik's cube, that's a little victory." When asked: "Tell me about activities that you perform as a teacher that you find meaningful." Martin's response was: "I can get my kids excited by doing a blindfold Rubik's cube demo, but it doesn't carry over." Although fascinating, this topic did not easily connect with a discussion of the greater socio-political issues.

Data Collection

Before beginning the data collection, I first obtained approval from the Institutional Review Board at the university at which the research was conducted. The participants all signed forms indicating their consent to be part of the study. See APPENDIX B for the participants' consent form.

The data collection period began in August, 2008 and ended in December, 2008. The data collection included multiple sources of data upon which to draw in order to write a "thick description" (Geertz, 1973) of these three teachers personal understanding of their experiences. Thick description includes a description of the context as well as the behavior or phenomenon under study. Interview and artifacts were the primary methods of data collection. Observations were also conducted.

Interviews

A series of in-depth, open-ended interviews were the primary data source. Three interviews with each case study participant were performed. Each interview lasted no less than one hour and no more than two hours. The first round of interviews were conducted at the end of September, 2008, the second round at the beginning of November, 2008 and the third round at the beginning of December, 2008. Participants and I met in many locations, including on the university campus, their school/classroom, and local cafes and restaurants. All interviews were recorded using a digital voice recorder and transcribed verbatim. After transcription was completed, participants were encouraged to review each interview transcripts for accuracy.

Interviews were used with the express purpose of gaining in-depth knowledge from participants about their experiences (deMarrais & Lapan, 2004). In keeping with the aim of the study, the guiding questions underpinning our conversations were open ended, and designed to invite participants to reflect about and share their experiences. The interview protocol used for this study can be found in APPENDIX C.

I agree with Taylor and Bogdan's (1998) view of the interview as "a form of social interaction" (p. 98). I thus constructed and conducted my interviews as a focused conversation. Although I made an effort to follow the structure of the interview protocol, I also allowed the participant to deviate as needed. A successful interview is described by Bogdan and Biklen (1992) as one "in which the subjects are at ease and talk freely about their points of view" (p. 97) and which simultaneously yields useable and rich data that reveals the views of the participants.

Before the first interview, I had already begun to gather and read course syllabi, teacher websites and policy documents from the target institutions. The first interview conducted primarily focused on each participant's educational experiences, including their familiarity with mathematics education research, their teaching experiences and their beliefs. I also began to delve into each teacher's school context, especially as it relates to the discourse about global competitiveness and how it manifests itself in practice.

The interview protocol (APPENDIX C) is a representative, but not exhaustive list of interview questions. Multiple interviews were necessary to pose these questions. In general, probing questions were constructed based on a participant's response. The participants were also encouraged to discuss relevant issues with the investigator, taking the interview in unforeseen directions. For instance, an e-mail communication from the school district's superintendant received by two of the three participants prompted much discussion in the first interview about the meaning of "preparing students for the global economy." Although this line of questioning had not been anticipated or planned by the researcher for the initial interview, it was more than welcomed and explored, as it ties directly to the research questions of this dissertation study.

An interview excerpt that illustrates how I respected the above mentioned principles of interviewing can be found in APPENDIX D. After discussing what it means for this participant to be a "good teacher," I wanted to delve a little deeper into this teacher's beliefs about mathematics and its teaching. The excerpt presented in

APPENDIX D demonstrates how interviews were conducted in a conversational tone, where the participant's answers inform the next interview questions, while still being mindful of the overall research focus. Much of the later questions in the interview revisited the realization by this teacher that he didn't feel like he had a cohesive teaching philosophy. He felt confused by what he had learned as a student of mathematics, as a student of mathematics education and the expectations of him as a teacher communicated by his school.

Interview questions were organized in several themes: background, beliefs about the teaching of mathematics, epistemology, mathematics education research, broader sociocultural dimensions and standards, and decision and change. These themes emerged from the research questions I am endeavoring to answer in this study: "How do individual, successful teachers navigate the beliefs shaped by mathematics education research, workshops, methods classes and the discourse of preparing students to be competitive in the global economy?" and "How do mathematics educators experience the periods of conflict, reflection and resolution between the different belief systems to which they have been exposed?" It was thus important to construct questions in order to ascertain my participant-teachers background, school context, beliefs and struggles.

The aim of the first interview was to build a level of rapport and comfort between the teacher and I, as well as to lay down the broad strokes of a portrait of them as teachers by gathering background information and discussing their beliefs about mathematics and its teaching. Questions about the mathematics education literature that had influenced them were also raised. I wanted to learn about their journey into high school mathematics teaching. Examples of questions asked during the first interview are: "How long have you been teaching?," "What mathematics classes have you taught?," "What does it mean to you to be a good mathematics teacher?," "What does it mean for someone to know or understand a mathematical concept?" and "What are some essential skills you are trying to foster in your students?" Other areas of questioning such as 'context and standards' and 'decision and change' (see interview protocol APPENDIX C) were introduced briefly in the first interview, but were reserved as central themes for the next interviews.

Subsequent interviews always began with the question: "What has been on your mind since we last talked?" This question often yielded topics which could be mined later on in the interview. It also reinforced that I cared about their day-to-day struggles as teachers and that I was there to listen as much as I was there to ask a predetermined set of questions.

The second interview primarily focused on the broader sociocultural dimensions of their experiences as teachers, from their school context to the role of mathematics in society. Questions asked were: "Describe the atmosphere in your school.," "What are some of the missions and goals?," "What is the role of testing in your classroom?," "How do EOC tests affect your teaching method?," "Describe the job market for which you are preparing your students?," and "Your school has as its mission to prepare students to be competitive in a global economy. What does that mean to you? What does that mean to your administration?" Questions about "decision and change" (see APPENDIX C) were also raised in the second interview, and were the primary topic of the third interview.

The purpose of the third interview was to discuss at great length some of the struggles, mediation, conflicts, decision and change that these teachers have faced in their mathematics teaching career. Questions asked included: "What are some major changes that you see in your teaching since you started?," "To what do you attribute these changes?" and "Please describe a moment when you felt very far from being the ideal that you have of yourself as a teacher." Questions also stemmed from statements made in previous interviews which needed clarification or further discussion, from e-mail communications or other artifacts.

Artifacts

Initially, participants were also asked to keep an informal journal, chronicling any event or thoughts they considered a good discussion point for the next interview. Key journal entries were to be used in guiding future interviews. In actuality, this request morphed into an electronic journal of sorts, with participants detailing certain events or thoughts through e-mail communication. The content of these e-mail exchanges were often used as fodder for future interviews.

For instance, on December 1st, 2008, one of the teacher participants (i.e. Michael) sent me an e-mail which contained the following: "Last week, I was put on an action plan, i.e., my performance is below standards in class management and teacher effectiveness." Another e-mail followed that day which provided details of Michael's "action plan," which included a "return to the traditional format" and strict adherence to the syllabus. Michael was asked to employ a protocol to conduct his classes (see p. 133). A subsequent interview with Michael focused on this new development, especially on the topic of the "traditional approach" to the teaching and learning of mathematics. See Chapter 4 for a lengthy discussion of Michael's case.

Participants were also asked to forward e-mails from their school or school district relevant to this study to me. Furthermore, I gathered artifacts either mentioned by participants in the course of an interview, or documents that might help understand the teacher participant's school. Artifacts included in this category are: textbooks, research literature referred to by participant teachers, information about the research sites and mathematics department publicly available (e.g. memos and meeting minutes/agendas, AYP, institutional demographic data), course syllabi, action plan, pilot study information, etc. Table 1 and 2 (see pp. 78-79) detail the artifacts collected from five teacher participants: John, Michael, Sergen, Martin and Lea.

Artifacts such as school report cards, school improvement plans, principals' messages, Algebra I common plans and school websites were assembled in order to better establish the school context of each participant. For instance, obtaining Alleny Academy's (i.e. Sergen's school) 2006-2007 report card and school improvement plan helped explain the intense focus by the school administration on raising End-of-Course (EOC) proficiency results in Algebra I and Algebra II. It also provided some insight as to the pressure that mathematics teachers at Alleny Academy might be under to raise said scores.

Artifacts such as course syllabi, teacher WebPages, projects, grading rubrics, review materials and participant created drawings and diagrams were collected in an effort to illustrate aspects of participants' beliefs about the teaching of mathematics and their pedagogy identified during interviews. For instance, Sergen's syllabus for AP Calculus BC contained a section about his teaching philosophy (see p. 155). These artifacts also helped inform future interviews.

Artifacts were often used to shape future interview questions. For instance, Lea Torres' syllabus for Algebra II, which I obtained before our first interview on September

23rd, 2008, included a section entitled "method for mastery" corresponding to each stated curricular goal. A portion of this artifact is reproduced in Figure 1 below:

Curricular Goal	Time-line	Method for mastery
1. Students will be able to graph, analyze and solve problems using absolute value, quadratic, exponential, cubic and linear equations and inequalities. A special emphasis is on quadratic equations.	Weeks 2 – 15	❖ Daily warm-ups ❖ Weekly assessments in the form of tests or small projects ❖ Graphing equations on the calculator and on paper
2. Students will be expected to solve systems of equations in context of a word problem. These systems may be linear, exponential or conic in nature.	Weeks 15 - 20	❖ Daily warm-ups ❖ Weekly assessments in the form of tests or in-class group work ❖ Graphing and solving systems (if possible) on the calculator
3. Students will be able to use matrices to solve problems. Especially to solve linear systems of equations.	Weeks 12 - 20	❖ Daily warm-ups ❖ Weekly assessments in the form of tests or in-class group work

Figure 1. Portion of Lea Torres' Algebra II syllabus (course artifact, 09/09/2008).

The mention of "method for mastery" present in this course artifact led to questions about Lea's beliefs about "mastery" in our first in-depth interview (R denotes the researcher's part and L denotes Lea's part):

R: So can you help me understand what mastery means to you?

L: (sigh)...You want to say it's not test scores...I am excited when my final exam scores are good. But for the kids to be able to do it, for them to help other people be able to do it. I do try to do a lot of group work. I might give 15 minutes of a test to just do a review. Five kids will gravitate toward the smart kid in class, and he'll explain it, and I can hear two people debating you do this no you do this then you do this...

R: Teach to mastery, that comes from your department that has decided to do that, or your whole school?

L: The whole school decided. It was decided for us (laughs).

R: It was decided for you?

L: Yep …

During this interview, Lea admitted that "mastery…it comes down to grades, which you hate to…It comes down to grades because that's what we have to report." Without this course artifact, questions about "mastery" most probably would not have been addressed during our first interview.

Finally, artifacts such as midyear reviews, Mr. Gilbert's action plan, Mr. Gilbert's reply to his midyear review and e-mail communications were collected with the aim to help chronicle the particular struggles and impediments teacher-participants' faced in the actualization of their beliefs. For instance, Michael Gilbert's reply to his scathing midyear review, reproduced in Chapter 4, illustrates Michael's frustration, dilemmas, conflicts and struggle.

Table 1. Artifacts collected from John Winslow, Michael Gilbert, and Sergen Manzik.

John Winslow	Michael Gilbert	Sergen Mansik
1. Southbrook HS 2006-2008 EOC	1. Gregory HS school progress report 2007-2008	1. Alleny Academy Report Card 2006-2007
2. 2007-2008 EOC prelim results	2. Gregory HS School improvement Plan	2. Alleny Academy School Improvement Plan
3. Southbrook HS graduation rate (2006-2008)	3. E-mail by superintendant	3. Alleny Academy website
4. 2007-2008 school report card	4. Algebra Alliance meetings notes	4. Alleny Academy Principal's message to families
5. S. B. HS Performance & Dem.	5. Homeroom Teachers (e-mail)	5. Mr. Mansik's webpage
6. Southbrook HS website	6. Attendance Procedures	6. E-mail by superintendant
7. Mr. Winslow's online class page	7. Team building committee	7. Tardy policy sign
8. Mr. Winslow Resume	8. EOC Common Plan	8. Minutes for Algebra I meeting
9. AP stats project #1-3	9. Algebra I Plan	9. Attendance Policy
10. AP stats project grading rubric	10. Midyear Review	10. Mr. Manzik's midyear review
11. AP stats project format	11. Mr. Gilbert Resume	11. AP council meeting minutes 9-4-08
12. AP stats syllabus	12. Action Plan	12. AP Calculus BC Syllabus
13. Mr. Winslow's midyear review	13. Mr. Gilbert's reply to midyear review	13. Precalculus Syllabus
14. AP stats example 1 & 2 of test	14. Mr. Gilbert website	14. Homework examples for AP Calculus BC
15. Precal Project 1 &2	15. Review material for AFM	15. Homework example for Precalculus
16. Precal Project Grading Rubric	16. AFM warm-ups 1-19	16. Teaching Philosophy
17. KWL chart	17. AFM web search project	17. E-mail communications
18. Precal Project Format	18. AFM linear financial model	
19. Algebra I & II, precal syllabi	19. Traditional teacher diagram	
20. The world is flat/Growing up digital	20. AFM syllabus v1	
21. Did you Know?	21. AFM syllabus v2	
22. E-mail communications	22. Algebra I support material	
	23. Algebra I warm-up 1-6	
	24. Algebra I practice test	
	25. E-mail communications	

Table 2. Artifacts collected from Martin Middleton and Lea Torres.

Martin Middleton	Lea Torres
1. Alleny Academy Report Card 2006-2007	1. Vale HS school progress report 2007-2008
2. Alleny Academy School Improvement Plan	2. Vale HS School improvement Plan
3. Article about	3. E-mail by superintendant
4. Alleny Academy website	4. Attendance Policy
5. Alleny Academy Principal's message to families	5. EOC Planning minutes
6. Mr. Middleton's webpage	6. Algebra II Planning minutes
7. E-mail by superintendant	7. Ms. Torres Resume
8. Tardy policy sign	8. Ms. Torres website
9. Martin's response paper "an instance of questionable scholarship"	9. Review material for Algebra I & II
10. Martin's resignation letter	10. Algebra I & II support material
11. Discrete Math Syllabus	11. Algebra I & II warm-up
12. Discrete Math Homework	12. Algebra I & II practice tests
13. Principal message	13. Algebra I & II syllabi
14. E-mail communication	14. Algebra I & II 1st quarter objectives
15. Hosiers	15. Algebra I & II 2nd quarter objectives
16. Book about Rubik's cube	16. Algebra I & II pacing guides
17. E-mail communications	17. E-mail communications

Observations. Observations were conducted at the invitation of the teacher-participants to experience the participants' school context and to capture moments that might inform the interview protocol for a subsequent interview. Two observations for each participant took place. The first observation took place between the first and second interview (i.e. in October, 2008) and the second observation took place between the second and third interview (i.e. end of November, 2008).

Observations lasted approximately half a day each. Field notes were taken and reviewed in order to plan the next interview protocol. It is important to note that the purpose behind these observations was not to render value judgments about best practices or to chronicle discontinuities in teacher attributed and professed beliefs. Stake (1995) noted that: "During observations, the qualitative case study researcher … lets the occasion tell its story, the situation, the problem, resolution or irresolution of the problem (p. 62). As I am attempting to portray teachers' experiences from *their* perspective, the primary aim of observations was to direct future interviews. With that in mind, the great majority of field notes were in the form of questions. For instance, on a visit to Alleny Academy (Martin's school), I observed a huge banner hanging above the front entrance of the school featuring a photograph of Earth from space which claimed: *Reach Further. Global competitiveness starts here.* This observation led me to ask Martin in a subsequent interview about the presence of the global competiveness discourse in his school. Incidentally, this same banner was observed above the entrance of Michael's school entrance.

This array of sources of evidence all focused on my research questions. Using various sources of data is a staple of quality case study research. Yin (2003) states that: "With data triangulation, the potential problems of *construct validity* also can be addressed because the multiple sources of evidence essentially provide multiple measures of the same phenomenon" (p. 99). In addition, participants were asked to review drafts of their interviews and of their case study reports for possible inaccuracies.

A case study database containing raw data such as interview transcripts, e-mail communications and scanned documents was kept and available for independent inspection, increasing the reliability of the study (Yin, 2003). Pseudonyms were used throughout the database and identifying details were changed to preserve confidentiality and anonymity.

Data Analysis

Data collection and data analysis are not mutually exclusive phases in case study research; in fact they are simultaneous activities in quality qualitative research (Merriam, 1998). A good case study investigator will "create a rich dialogue with the evidence" (Yin, 2003, p. 59) by:

> ... pondering the possibilities gained from deep familiarity with some aspect of the world, systemizing those ideas in relation to kinds of information one might gather, checking the ideas in the light of that information, dealing with the inevitable discrepancies between what was expected and what was found by rethinking the possibilities of getting more data, and so on. (Becker, 1998, p. 66)

For that reason, a recursive and inferential data analytical process was adopted. Hence, a review and analysis of data collected in the first phase (first interviews plus related documents) was performed by adopting coding and category-building guidelines set forth by Merriam (1998), producing a set of emerging themes which helped focus the study. By theme I mean "a statement of meaning that (1) runs through all or most of the pertinent data, or (2) one in the minority that carries heavy emotional or factual impact" (Ely, Vinz, Downing & Anzul, 1997, p. 206).

According to Merriam (1998), categories are to be created based on the following axioms: the categories should *reflect the purpose of the research;* they should be *exhaustive* in as much as all relevant data unit belongs to one category or another; they should be *mutually exclusive;* they should be *sensitizing* by yielding a meaningful picture of the phenomenon; and they should be *conceptually congruent*.

This initial analysis required an expansion and rethinking of the review of the literature, thus influencing the second round of data collection (i.e. second interviews and related documents). This process was repeated for each phase of the data collection.

Once the data collection was completed for five teacher-participants, transcripts and archival material were once again coded using open coding and themes were identified for individual participants. ATLAS-ti was used to code the interview transcripts and categories were constructed from the codes using the above mentioned guidelines for category-building.

Examples of codes include: two worlds (2w), adapt, agency, assessment, beliefs about mathematics (b. math), beliefs about the teaching of mathematics (b. teach.), balance, battle, background (bckg), bricolage (bric), change, cluster, communicate, community, conceptual, confusion, control, culture, decisions, differentiated instruction (diff. instruct.), diplomat, discussion, empower, EOC, epistemological beliefs (epist), example of activity (Ex. Act.), fight, global discourse (glob disc), habitus, incorporate, global discourse/knowledge society (kno. soc.), linear (lin), mathematics education background (math. ed. bckg), No Child Left Behind (NCLB), global discourse/neoliberal (neolib), reflect, respect, standards, student-centered (s-c), school context (school ctx), struggle, technology (tech), traditional, understanding, urban, writing.

To illustrate the coding process, Table 3 below provides examples of participant quotes and the corresponding assigned codes.

Table 3. Example of Coding

Participant quote	Codes
Helping them learn how to read which I thought would be good for all their subjects. Not part of the math curriculum	change, struggle, writing

Table 3. (continued)

They wanted the kids to have an answer so they could have a feeling of getting it right to which I said "that wasn't my goal!"	battle, b. teach
It is difficult to kind of see how conceptual understanding can improve procedurally based EOC scores	conceptual, EOC, struggle
…although we've been taught that alternative forms of assessment are good that doesn't change the fact that you go out and you're having to teach to the test.	adapt, math. ed. bckg., neolib., school context.
"One of the things I took from the World is Flat is that the future of any jobs in America is going to be for those who can take information and use it beneficially."	glob. disc.
"I like and I express to my students that I like messy answers because those are what we have and then we need to work on those and reflect on those and develop strong concepts."	b. teaching, conceptual, reflect.
"So that contradictory language in my interviews is because I exist in both worlds right now. And I feel comfortable in both worlds."	2w
"…a little skepticism and critical thinking. Is that really math? I'm not sure. It's not English, we know that. But it might be math. Am I putting that into my topics? Is that a world-wide skill? Yes."	b. math., b. teaching, decisions, glob. disc., writing.

Merriam's (1998) guidelines for category-building, as explained above, led me to the creation of the following categories from the above codes: background, beliefs, social context, global discourse, mediation/navigation/decision.

Themes for the five individual participants were identified and a decision was made to report on three teachers in this dissertation after the data analysis showed that these cases exhibited a particular combination of commonalities and uniqueness of emergent themes, as explained in the *participant selection* section above. Individual case study reports were written for these three teachers and are reported in Chapter 4.

Data were then analyzed across these three cases, examining within themes for similarities and differences (Stake, 2000). Findings of the cross-case analysis are visited in Chapter 5, leading to a discussion of conclusions, including implications for teacher education and possibilities for future research.

Summary and Introduction to Chapter 4

In this chapter, I discussed the social constructivist framework used for this study, and the components of Leatham's (2006) and Gates' (2006) frameworks for studying teachers' beliefs utilized in this study were mentioned. Chapter 3 also explained the instrumental multiple case study methodology selected for this study, including the manner in which participants were selected, the data collected and analyzed. This discussion also covered the aspects of phenomenology adopted together with the particular approach to interviews used by the researcher.

The next chapter will present individual case study reports for the three teacher-participants selected: John, Michael, and Sergen.

CHAPTER 4: FINDINGS

Chapter 4 will present individual case studies for *John the Commuter*, *Michael the Boxer* and *Sergen the Diplomat*. The metaphorical titles given to each case study is meant to reflect the distinct lived experiences of these three mathematics educators as they navigate different belief systems about mathematics when making professional decisions related to their work as teachers. In Latin, *metaphora* refers to something that is carried somewhere else. Metaphors allow a mapping between two conceptual domains, and provide a window for understanding a relatively abstract subject matter in terms of a more concrete subject matter (e.g. Lakoff, 1993; Schinck, Neale, Pugalee, & Cifarelli, 2008). Metaphors link two meanings by transporting the meaning from one semantic sphere to another. According to Lakoff and Johnson (1980), metaphor is of fundamental importance to meaning making—how we think is fundamentally metaphorical. The creation of metaphors provides structure to our experiences. They are thus helpful in providing the reader some understanding of the teacher-participants' lived-experiences and belief systems through the more concrete and familiar concepts of a commuter, a boxer and a diplomat.

The choice of the image of a *commuter* to illustrate John's case, *boxer* for Michael and *diplomat* for Sergen, will be made explicitly clear during the reading of each case. Pseudonyms are used throughout and identifying details have been changed to preserve confidentiality and anonymity.

All three participants had idealized images of themselves as teachers that they were finding difficult to actualize in their school context as their reality was often at odds with ideals and beliefs developed during exposure to mathematic education classes.

Although John, Michael and Sergen encountered similar struggles during the studied semester, the manner in which they each navigate and mediate the different belief systems about mathematics was often times different, providing a fuller picture of how mathematics teachers live the periods of conflict, reflection and resolution between the different belief systems to which they have been exposed.

CASE 1: JOHN THE COMMUTER

John Winslow's case reveals a complicated world in which the high school mathematics teacher must navigate the various discourses about “effective” mathematics teaching. John’s beliefs about mathematics, its learning and its teaching are complex, shaped as they are by his management experience in the textile industry and his graduate studies in mathematics and mathematics education. John, as a mathematics teacher, embodies the two main global educational discourses—the knowledge society discourse and neoliberalism—which, on the one hand, claims that education must prepare students to *learn how to learn,* and on the other, has meant a reinforcement of a positivistic and pragmatic perspective on mathematics education. In this case study, John will be shown as “commuting” between “worlds”, traveling between clusters of beliefs (Leatham, 2006) about mathematics and its teaching.

Introduction to John

John Winslow is a Caucasian male in his early forties who began teaching mathematics at South Brook High School in 2000. South Brook High School will be described in more detail below. John is a dedicated mathematics teacher and scholar. During the time of this study, John was working full time as a high school mathematics teacher and pursuing his Ph.D. in mathematics education part time, all the while taking the time to obtain his National Board certification and becoming qualified to teach classes online with his state’s Virtual Public Schools System. John often attends research conferences and professional development workshops and frequently reads about mathematics education and education in general.

John’s previous career was as a manager in a textile factory (12 years). When the company for which he worked shut down, John decided it was time to make a change. Although interested in mathematics since high school, John stated in his first interview that he initially chose to teach mathematics "because that's where all the jobs were. They didn't want social studies teachers because they had a glut of those. They didn't want English teachers because they had a glut of those. They wanted math."

After substitute teaching and tutoring algebra for a semester, John was hired at South Brook High School as a beginning teacher in 2000. During his eight year teaching career, John has taught Geometry, Algebra I and II (EOC classes), statistics and precalculus (non-EOC classes). At the time of the study, John was teaching precalculus and statistics and was now the Chair of the Mathematics Department at South Brook High School.

John's School Context: South Brook High School

South Brook High School is a grades 9-12 public high school, located in a suburban area of the South East of the United States. In the 2008-2009 school year, South Brook High School employed 90 faculty members, nine of which are National Board certified and served approximately 1300 students. The demographic composition of South Brook High's student population is primarily Caucasian (79%), with 18% African American students and less than 3% Hispanic, Asian and American Indian students, where 20% of students have been designated Economically Disadvantaged. The demographic information for South Brook High School has been summarized in Table 4 below.

Table 4. Demographic Summary of South Brook High School.

System Type	*Grades Serviced*	*Student Population*	*Students Eligible for Free or Reduced Lunch*	*Racial Background*
Suburban	9-12	1293	20%	Caucasian – 79% African American – 18% Hispanic – 2% Asian/Pacific Islander – 1% American Indian/Alaskan Native – <1%

South Brook High has a tight community of dedicated teachers with almost 60% of teachers with 10 or more years of experience and a low teacher turnover rate (approx. 20%). When asked to describe his school, John replied that the aspect that sets his school apart is the very high expectations for student behavior, student involvement and student achievement. The expectations of passing rates, graduation rates and general student EOC proficiency is ever-present in conversations with administration and amongst teachers.

One impetus for the performance-oriented culture now present at South Brook High School was the significant drop in Algebra I and Algebra II scores from the 2005-2006 school year to 2006-2007 (see Table 5 below). During this period, the percentage of students on grade level or above plummeted from 90.1% to 67.8% in Algebra I and from 72.1% to 48.3% in Algebra II. The End-of-Course proficiency results for geometry remained comparatively stable, although still significantly below the school target of 80% students on grade level or above.

Table 5. EOC Results for Algebra I, Geometry and Algebra II at South Brook HS.

	Algebra I	*Geometry*	*Algebra II*
2005-2006	90.1%	69.4%	72.1%
2006-2007	67.8%	62.7%	48.3%
2007-2008	68.3%	60.4%	53.6%
Target	80%	80%	80%

South Brook High School did not meet its Annual Yearly Progress (AYP) in the 2007-2008 academic year. AYP is a federal standard set for schools by the No Child Left Behind Act (2001). State test results are used to set targets, including attendance and graduation rates. For a school to meet its Annual Yearly Progress, all targets must be met. South Brook High school did not meet AYP in 2007-2008 since it did not meet all of its 17 target goals.

Mathematics teachers at South Brook High are under considerable pressure to raise the percentage of students on grade level or above to the school target of 80%. John stated that:

> Our administrative goal is to be number one or number two in the county in every category. So all EOC scores: US history, English, Civics and Economics, and all the mathematics. We want to be number one or number two.

According to John, the expectation to be number one or number two in the county: "Flows from the administration to the teachers, and from the teachers to the students."

John's Professional Background

Between 2001 and 2004, while working full time at South Brook High School as a mathematics teacher, John obtained his license in secondary mathematics through a program at a local research university, Blue Coast University, designed specifically for the lateral-entry secondary mathematics teacher. At the end of this process, John was considered "highly qualified" under the No Child Left Behind Act of 2001 in his state; John had passed the state teacher licensing examination, had earned a bachelor's degree and had demonstrated content knowledge in mathematics, a core academic subject.

Having developed a strong interest in mathematics education during the licensure process and his experience in the classroom, John decided to continue on to the Masters of Art in Mathematics Education at the same university in 2004. John described the M.A. in Mathematics Education at Blue Coast University as a “hybrid” because it required him to take six graduate level mathematics classes, as well as a combination of mathematics and general education classes. This combination at once licensed him to be a state public school mathematics teacher with graduate level pay while also qualifying him to teach mathematics at a community college.

John admits being frustrated at first with the advanced mathematics classes offered in the program as they were more abstract and proof-oriented than any of the mathematics classes to which he had previously been exposed: "I can remember talking about the epsilon-delta definition of a limit, and I can, in my mind, I can picture what that looks like, as something approaches something, it makes sense to me. But to use mathematical induction to prove it, that was difficult for me. So I was frustrated a lot."

Although frustrated at the time of his first exposure to advanced mathematics, John admits that these courses changed his philosophy of teaching, and enabled him to see the beauty of mathematics, particularly the beauty of “empirical studies and quantifying phenomena mathematically.” John especially enjoyed his classes in Abstract Algebra and non-Euclidean geometry as it opened up his world about what mathematics could be. He "flavors" his classes with some of the more interesting concepts he learned in these classes with the expressed purpose of exposing his students to alternative approaches to algebra and geometry. In his first interview, John claimed that these advanced mathematics classes were very influential in changing his perception and beliefs about mathematics from an instrumental belief, where mathematics is viewed as a collection of skills and procedures, to a way of looking and understanding the world around us.

After five years in the classroom and equipped with the new knowledge and belief system he gained from his licensure and Masters of Mathematics Education, John had become increasingly fascinated by public schooling in general and mathematics education specifically. He thus decided to pursue a Ph.D. in Curriculum and Instruction with a specialization in Mathematics Education at the same university at which he had obtained his licensure and his Masters—Blue Coast University. At the time of the study, John had been pursuing his Ph.D. part time for three years. During this time, he was exposed to a vast amount of research in mathematics education.

John especially remembers struggling with the constructivist and sociocultural epistemological foundations of much of mathematics education research which took him far away from his set of dispositions—his *habitus*. During his graduate studies, John studied at great lengths the work of Piaget, Steffe, Cobb and Von Glasersfeld, amongst others. Through these readings, John’s ideology and discourse have been (in part) transformed. John has come to believe that learning should be built upon a student’s schema, that is, knowledge that a student already possesses.

That being said, John recalls a failed attempt at incorporating a constructivist form of assessment in his teaching—the KWL(H) chart (WHAT we know, What we WANT to know, What we have LEARNED, HOW we know it.) The KWL(H) chart is meant to provide teachers a structure with which to guide and support their students' construction of knowledge. In his account, John mentions the use of foldables™. An example of a foldable™ has been provided for the reader in Figure 2 below:

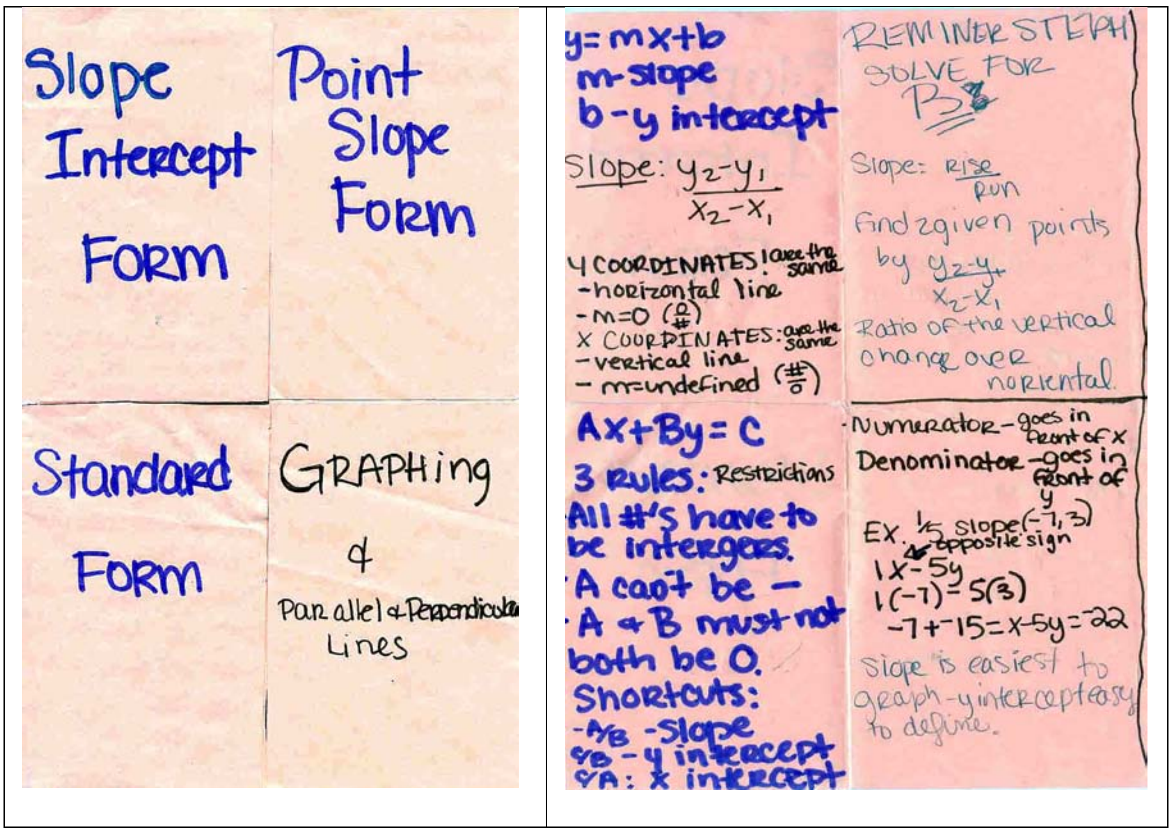

Figure 2. Product of Role, Audience, Format, Topic Activity.

Source: Pugalee (2007).

John recalled:

> So I was teaching geometry one time, and I made my kids do these foldables, which is a graphic organizer. So I did that, and then I had them do the KWL chart

> and uhm, I really just expected them to use this and pick it up and do something with it, and you know, I don't see where it enabled students to learn.

This incident left John feeling uncertain about the viability of much of mathematics education research in the classroom, and made him cautious to use further activities suggested by the research "lock, stock and barrel."

John and the Global Competitiveness Discourse

John credits the advanced mathematics classes he took during his graduate studies for one of his main goals as a teacher: to develop his students' analytical mind:

> One day they're going to be paid for the decisions that they make. The way to make right decisions is to be able to take different sources of data and be able to successfully analyze them, and that requires an analytical ability. That's the benefit of higher-level math. It teaches one to think analytically.

John's desire to hone his students' data analytical skills is inexorably tied to his view of the society for which he is preparing his students: a technologically-driven, knowledge society. John admits that although the global competitiveness discourse is present in his school, that the epistemological or pedagogical implications of this discourse have not been discussed either in his school context or in his extensive mathematics education studies. John has thus had to decipher this discourse on his own. John has surmised that collaboration, capability to adapt, data analysis and technological fluency are the main skills he needs to foster in his students to prepare them for such a world.

John believes that the future belongs to individuals who can gather, synthesize and analyze great volumes of information. John's beliefs are shaped by his time in industry, as well as his readings of books such as Friedman's 2005 National Bestseller

The World is Flat: A Brief History of the 21st Century and Tapscott's 1997 book *Growing up Digital*. Both books discuss the centrality of technology in the globalization movement and the qualities that American teachers need to cultivate in students so that the will be able to thrive in the new digital, knowledge society.

John uses the video *Did You Know?* based on *The World is Flat* in his classes to open up discussions about globalization and its effects. Talking about this video and its place in his classroom, John stated that:

> It gives a lot of information. It just keeps throwing facts up there about India and China and their numbers of Ph.D.s and how information is gaining momentum, and it's got music in the background, and the point to it is that the future that high school students face today is different than the future that I faced, and it's different from the future my parents faced, and their parents, etc. The future that these kids face is more uncertain, but that's not necessarily a bad thing. For the future they face, they're going to have to learn to adapt to a lot of different things technologically. And their ability to adapt, their ability to harness and gather information and to use it to their advantage is going to be what gets them into the higher paid positions and into the better life than others. So with that in mind, I try to integrate how does math fit into that. Well generating information is the empirical part of things. That's data collection.

More will be said below about John's views of his role as a mathematics teacher preparing students to be competitive in a global society.

Themes for John the Commuter

John's case reveals the complex world in which the high school mathematics teacher must navigate the various beliefs about mathematical knowledge, its acquisition and its role in society. In what follows, John the Commuter's main themes will be explored: two worlds, the real world and decisions. John will be shown as making decisions and "commuting" between "worlds"—between clusters of beliefs about mathematics and its teaching.

Two Worlds

While pursuing his Ph.D., John began to feel like he inhabited two worlds which understood mathematics, its learning and its teaching vastly differently. In our third in-depth interview together, John admitted that he held, and acted on, a "collision of beliefs" about mathematics and it teaching:

> I'm in the classroom every day, with my high school students. I'm in the parent meetings, I'm in the principal meetings, I'm in the county meetings. I read the paper about the school that I teach at and about our school system. I see the stuff that every classroom teacher is going to see on a day-to-day basis. But then, I leave there in the afternoon, and I go to a university, and I sit in classes, and I hear the advanced theories…right now all I see is a collision between the two, and I want to see if there's a road or a bridge that can be built between the two, so that student success can be enhanced … So that contradictory language in my interviews is because I exist in both worlds right now. And I feel comfortable in both worlds.

The complaint about the disconnect between education theory and practice has long been explored in the education literature (e.g. Cobb, 1988; 1947; Jones, Reid, & Bevins, 1997; Lerman, 1994; Malara & Zan, 2003). One criticism that continues to be raised is that teachers feel they are entering a world which was never discussed in the university setting when stepping into the classroom. They often state that there was too much theory in their preparation.

John personifies Leatham's (2006) view of teacher beliefs as sensible systems in which seemingly contradictory beliefs may coexist peacefully in isolated clusters. John has internalized and uses different belief clusters depending on context in order to function successfully as a high school mathematics teacher. The clustering of beliefs allows him to be comfortable in both worlds.

John is seen as "commuting" between these isolated worlds. John was termed "the commuter" because, although isolated from one another, these worlds are relatively close to each other for John, and a road or a bridge could exist between them. On occasion, John brings back a souvenir or an idea from one world to the other, such as the KWL chart, but these worlds, on the whole, remain intact, distinct and separate. John's background in administration affords him the opportunity to understand "administrative talk and thinking," while John's significant background in mathematics education allows him to understand the "talk and thinking" of that world.

Although he sometimes feels like a "sell-out," John is as comfortable writing a reflection about radical constructivism as he is brainstorming ideas for raising Algebra I EOC scores with his school administration. End-of-Course proficiency results are not discussed in the paper on radical constructivism, and Steffe or Von Glasersfeld are not mentioned by John when he is in his principal's office having a conversation about EOC results. John is ever-mindful of the differing languages that will be valued and understood in these different worlds.

The beliefs espoused by the two worlds—the world of the high school classroom teacher and the world of mathematics education doctoral student—collide particularly violently for John on the concept of "student-centered." Much of the literature on cognitive constructivism to which John was exposed during his graduate studies is focused on observing and describing the mechanisms that an individual mathematics learner uses to build up mathematical knowledge in a particular learning space (e.g. Thompson, 1994; Steffe & Wiegel, 1994).

Within this paradigm, mathematics teachers are guides in the construction of a student's creation of *their* mathematical reality. However, as John exclaimed in frustration during our second interview together: "I don't teach *one* student. I teach a *group* of students!"

John often talked about his frustration about the focus in mathematics education classes and methods classes on "student-centered" instruction, as he interprets this to mean "*one* student-centered" instruction. For John, doing what is best for the group, his school, is equal to doing what is best for individual students: "I don't see why it's wrong to claim that good test scores (on standardized tests) are benefiting the students."

Interestingly, this last statement was verbalized by John within minutes of voicing his uncertainty about whether or not these tests help students become better learners. It is also of interest to note, as we will see below, that much of the assessment and assignment in John's classes are far removed from standardized, multiple-choice tests.

Although South Brook High School, like most high schools, proclaims to be student-centered, the measurements of student success are invariably based on aggregate statistics such as EOC pass rates in Algebra I, Algebra II and Geometry, as is required from the No Child Left Behind Act. These statistics, by definition, rely on a standardization of the mathematics curriculum and its teaching, where students are evaluated *as a group*.

John struggles to find a way to keep the wonderful ideas that he learned from reading Piaget and Steffe alive in such a context. The related concept of “differentiated instruction” is also a sore point for John:

> Differentiated instruction comes across as I gotta teach this way to Johnny, and this way to Sally, and this way to Juan. And quite frankly, I can’t teach twenty-five different ways to twenty-five different people. I do love the classroom setting because it is such a diverse group of personalities. But to harness all those personalities and pull them together and get them ALL to meet a certain baseline…”

The fact that John struggles with seemingly discordant, even contradictory or colliding beliefs about mathematics teaching is also evident in his discussion about the concept of *standards*. John's language in our interviews reflects the two main ways standards are discussed in the literature: State and federally mandated standards (NCLB, 2001 - e.g. EOC, AYP, etc.) and NCTM standards (and principles).

State mandated standards are the standards of one world; the world in which John is in his classroom every day, with his high school students, in parent meetings, the principal’s office, in the county meetings, reading the paper about his school and his school system. These standards are grounded in the belief that there are a set of grade-appropriate, basic mathematics skills that students must internalize through directive teaching. If one takes this belief about mathematics for granted, then measuring students' abilities and proficiencies through standardized, high-stakes tests is consistent.

John's administrative background in the textile industry has given him an appreciation for standards, or what John refers to as industrial standards, for which students, teachers, schools and the education system in general are held accountable under the No Child Left Behind Act (2001): "Pretty much everywhere you go, no matter where you work, there's always basic rules, procedures and guidelines to follow. Being in textiles gave me an appreciation of that."

John believes that people tend to do better when there are clear, measurable goals defined for which they are accountable as it gives everyone a focus, "something to shoot for." John relates that this mind-set, as he calls it, was formed in the industrial setting and that it naturally transferred to the educational setting.

As a consequence, John believes in a linear progression in his teaching of mathematical concepts, which he likens to a product's progression in a textile factory and calls the "business of learning":

> I'm still fairly linear in my progression, in my thinking and with my students. So what I do when I try to teach, I rarely try to jump from point A, to point C, to point F, to point Q. I generally try to go A to B to C to D. I think students have a comfort in knowing that there's a progression to this stuff. There's a logic. That came from industry. Because you know, in a textile factory there's a start and a stop, and in between, there's a progression.

John's belief in the linearity of the mathematics teaching and learning enterprise means that he likes to "start with a concept and move that concept through more and more advanced progressions."

Given John's belief in the linearity of mathematics teaching and learning, John approaches trigonometry in his precalculus class by first spending a considerable amount of time on the concept of angle formation. What it truly means to draw an angle "because you're taught one way in geometry and then in trigonometry you have this whole issue of this angle making circles. So I really spend a lot of time trying to ground them in that before I move into right triangle trigonometry and the unit circle and showing them the link between that."

The belief in the industrial standard affords John a high level of comfort while navigating his school context, which places a high premium, as discussed above, on student achievement: EOC pass rates, percentage drop out, graduation rates, etc. The fact that success in his education setting is measured numerically is consistent with John's belief in measurable, baseline standards. The application of the corporate model to education, or the actualization of the neoliberal global education discourse, is in line with John's experience in industry.

For instance, John appreciates the mathematics education research which argues that standardized multiple-choice tests facts and not process (Herman & Golan, 2005; Alfie, 2000) but he is easily swayed by the argument that multiple-choice tests are an efficient and economical way to assess students' baseline level of conceptual understanding.

John loves his school, and his desire to do what is efficient and economical while producing competitive scores on high-stakes tests (e.g. EOC tests) and graduation rates is intimately tied to John's aspiration to do what is best for his school in general, not only individual students. As was previously mentioned, a test is termed "high-stakes" when the results are used to make important decisions that affect students, teachers, administrators, or schools.

John at once understands and reports high-stakes testing results (EOC) while musing and enacting the belief that a more genuine assessment would come in the form of portfolios which would involve writing and projects. An example of an assignment given in John’s class which requires writing can be found in APPENDIX E.

The above discussed one way that John understands the concept of educational standards and would seem to reflect the mathematics teacher and mathematics classroom described by Keitel, Kotzmann and Skovsmose (1993) in which a utilitarian approach to mathematics education has been adopted with the underpinning belief about mathematics as a static tool kit with an identifiable content and stable structure that are both teachable and testable.

On the other hand, John studied the Principles and Standards for School Mathematics set by the National Council of Teachers of Mathematics (2000) during a course on Issues in Secondary Mathematics taken during his Masters in Mathematics Education at Blue Coast University.

NCTM (2000) emphasizes the need for teachers to create a culture of learning in their classroom in which students “learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (NCTM, 2000, p. 20).

The advocated manner of helping students construct conceptual mathematical meaning is through problem solving.

John delved deeper in his study of problem solving as a means of developing mathematical competency in students during his Ph.D. The NCTM Principles and Standards for School Mathematics (2000) changed John's beliefs about the teaching of mathematics, giving him his first glimpse at the time at a (constructivist) problem-solving approach to teaching.

John’s teaching was also greatly influenced by the Data Analysis and Probability Standard for Grades 9-12, which states that teachers should enable students to formulate and answer questions that can be addressed with data analysis. John interprets this standard as refocusing the teacher on "how a formula comes about" as opposed to the formula itself: "Math is based on data analysis; going out and observing something and quantifying it mathematically. That is something we lose sight of in our teaching." Although John believes in both types of standards he recognizes the fundamental differences involved in their application:

> I don’t think the intent of the state goals and the NCTM goals are different. I think the intent is to raise the bar for everyone mathematically. That's not what actually happens at the school level. It's a target, and so I'm going to shoot for that target, and that's all I'm going to do. It's not because I'm lazy or anything like that. It's just that my focus is on hitting that target, and so that's what I'm going to do. I'm going to hit that target.

The Real World

For John, neither the corporate managerialism present in his school context nor the findings and recommendations from the field of mathematics education correspond to the “real” world. Strongly influenced by his years in a dying industry (textile), John is acutely aware that he is preparing his students for an uncertain world. As seen above, his graduate studies in mathematics education have given him insight into alternative learning theories and epistemological foundations for teaching.

However, his management experience also gave him an appreciation for the corporate model of efficiency and accountability. As a result, John, personifies the nuanced discourse of globalization's effect on mathematics education which - on the one hand, claims that education must prepare students to *learn how to learn* in order to remain competitive in the global knowledge society – and on the other, that globalization has meant a reinforcement of the rational management of school, as well as a positivistic perspective on mathematics education.

It cannot be overstated that the fact that the textile industry was "solid for a long time but then became unsolid" has made John intensely aware that the world and, more specifically, the job market for which he is preparing his students is uncertain, knowledge-based, and that students will, above all else, need to be able to learn and adapt quickly:

> I think that most cases what students are going to be entering into, is a job market that, here's the job today. Tomorrow it may not be there. Therefore you need to be able to jump into the next job and have a fluid transition. The link between all of that is technology. So in my teaching, I'm looking at what can I use technology-

> wise specifically, but otherwise as well, to adapt to students new learning styles, but also to give them those skills that they can use so they know how to readily adapt to other things too. So in my teaching, what I want to do is teach them to learn how to think and adapt quickly and move from here to here to here.

Although, as mentioned above, John professed to me his belief in a traditional, linear lecture approach to teaching, John's activities and assessment are often times project-based, requiring writing and collaboration. For instance, because he believes collaboration to be a skill his students will need in the knowledge society, John has focused on small-group learning in both his research and his teaching.

Small Group Learning. Influenced by mathematics education research, John strongly believes in the cognitive and social benefits of small-group or cooperative learning. In his mathematics education studies, John became familiar with the work of Vygotsky who promoted the idea of a novice interacting with a "more capable other" in order to be assisted in the zone of proximal development. In this model, the more capable other assists the learner by providing hints and questions (scaffolding). As seen in Chapter 2, one of Vygotsky's legacies is the apprenticeship model and the cooperative learning movement.

Many educators and researchers in mathematics education advocate using small-group learning, also called cooperative learning, as an alternative to the lecture method at all levels as it involves active learning, thinking and interpersonal communications (e.g. Wahlberg, 1997). They argue that the lecture method promotes passive learning and rote memorization and that students who survive courses taught with the lecture method develop a very skewed perception of mathematics as a collection of skills with no connection to critical reasoning.

Still, for many teachers, putting aside the comfort and power that is inherent in the lecture method is not an easy task as is humorously pointed out by Jensen and Davidson in their 1997 article entitled *12-Step Recovery Program for Lectureholics*. John admits that:

> I prefer to be the center of attention. They all listen to me, I talk about it. I can explain it to them. I want to stand in the middle and talk and have them listen to me and then they work on it a little bit and then I talk some more. That's how I would prefer to do it. But I know that is not the best way to get a group to learn.

For some time, collaboration in the classroom took a back seat to competition as business interests advocated competition among students (Johnson & Johnson, 1994). However, as companies increasingly rely more heavily on teamwork to improve productivity, industry representatives are asking for graduates that have collaborative skills and can effectively communicate mathematics and technology. Because of the demand for such skills, most mathematics courses now have a collaborative work element as a course objective.

The term cooperative learning has been used to describe a variety of learning environments. It seems that each researcher in the area has her own definition. Nevertheless, it is agreed that cooperative learning is a structured, systematic instructional strategy in which small groups work together toward a common goal (Reynolds et al., 1995). Putting students in groups has been recognized as insufficient to realizing the benefits of cooperative learning. Structured learning tasks need to ensure interaction among students and classroom norms need to be established that support that interaction (Reynolds et al., 1995).

A recovering and occasionally relapsing *lectureholic*, John has worked very hard to become more adept at creating groups in which: a significant amount of the course work is done in groups, there is a positive interdependence among team members, all members participate, and the assessment process includes group work. These are some of the essential elements of cooperative learning according to leading researchers in the subject (Johnson & Johnson, 1994; Leikin & Zaslavsky, 1999; Reynolds, et al., 1995).

To welcome small-group learning, John had to relinquish some of his traditional notions of accountability in which assessment and measurement of individual understanding is paramount. John's focus on small-group learning in his classroom is particularly interesting given his belief in the importance of individual performance assessments such as End-of-Course exams.

Decisions

As all teachers, John must make decisions daily that will fundamentally affect his students' learning. John bases each decision on the belief cluster that is appropriate within the given context. Two decisions exemplify the complexity of this process for John. During our second interview, I asked John: "In the school setting, how do you decide what to teach and what not to teach?" John recounted one particularly difficult decision he had had to make a few years prior to the study, while he was teaching Algebra I, an EOC course. As described above, South Brook High School has a set target of 80% students on grade level or above in all EOC courses, including Algebra I. Algebra I teachers are given a pacing guide, geared toward the selected Algebra 1 textbook, which states what topics are to be taught, how much time is to be allocated for this topic, and the corresponding section in the textbook.

For instance, the following is an example of a typical portion of an Algebra I syllabus (Fig. 3):

Wednesday 11/5	3-6 Ratios and Proportions	HW 3-6 #1-23 odd
Thursday 11/6	3-7 Solving Equations and Formulas / "Literal Equations"	HW 3-7 #2-18 even

Figure 3. Selected portion of Algebra I syllabus (Course artifact, 10/12/2008).

This particular section of the syllabus was selected as it highlights the decisions that a teacher with considerable background in mathematics education must make on a day-to-day basis. The National Council of Teachers of Mathematics declared that proportional reasoning "is of such great importance that it merits whatever time and effort must be expended to assure its careful development" (NCTM, 1989, p. 82).

Furthermore NCTM (2000) identified proportional reasoning as a unifying thread that brings much of school mathematics together into a coherent whole. The attainment of proportional reasoning is considered a milestone in students' cognitive development. Many mathematical concepts rely on a solid understanding of proportional reasoning: slope of a line, percentages, similarity, trigonometry, etc.

However, as one can observe in Figure 3 above, "Ratios and Proportions" are allotted one class period: Wednesday 11/5. According to the syllabus, the teacher should move on to the topic "solving equations and formulas" (i.e. section 3-7), by Thursday 11/6. John admits that, as he was teaching Algebra 1: "About three-quarters of the way through, I realized uh oh, I spent too much time on this, and this. Then I started to think, how is this going to affect my passing rate?"

John had to make a decision—to stop three-quarters of the way through the curriculum and start reviewing to give his students a better chance at giving correct answers on the End-of-Course test on the topics that had been covered, or finish teaching all topics. John decided to “re-teach a bunch of stuff.” His thought was “I want 80% to pass.”

> When I hear a lot of people talk about skill and drill and they talk about it negatively, I understand the negative talk. But teach an EOC class and see if you don’t go back to skill and drill. See if you don’t do that. Because when it comes down to it, you want that passing rate, and the way to do it is practice.

John’s decision to focus on tested material and to use “skill and drill” in this context is consistent with Au’s (2007) findings that the main effect of high-stakes tests was a narrowing of content and "an increase in teacher-centered instruction associated with lecturing and the direct transmission of test-related facts" (p .263).

A compilation of teacher survey responses by Sunderman, Tracey, Kim, & Orfield (2004) also shows that as a result of the accountability system found in schools (NCLB, 2001) teachers “ignored important aspects of the curriculum, de-emphasized or neglected untested topics, and focused instruction on the tested subjects, probably excessively (p. 4).

It is important to note that “the people who talk negatively about skill and drill” to whom John is referring in the above quote are his mathematics education professors, and the research literature. The fact that John distinctly, and emotionally, remembered this decision years after the fact, indicates that it was not made lightly. It is crucial to keep in mind that even though John has been exposed to and partially converted to the belief system about mathematics and its teaching promoted by the research, and has sufficient content knowledge to actualize these beliefs, this knowledge and change in belief system was not enough to change John's practice in *every* context.

In the situation described above, John's belief that doing what is best for the school as a whole is also doing what is best for individual students was favored. The administrative goal to "be number one or number two in the county in every category" and the related goal to reach the school target of 80% students on grade level or above in all EOC courses took priority over John's understanding and beliefs about the importance of concepts such as proportional reasoning.

Interestingly, when faced with a similar situation, John chose differently. John related to me the main complaint from many mathematics teachers in the department, including him and the decision-making process behind his choice:

> I would love to teach students how to think outside the box and all that, but I can't because they have to learn the basics. I've got to spend three weeks teaching them how to factor instead of three days. I understand that argument – I've made it myself. I just decided one day, I'm still going to teach that other stuff and see what happens. And I see a lot of students that are ready to look at some open-ended, advanced stuff, without necessarily knowing how to factor. Alright they can't factor. That doesn't mean we can't move on.

In this case, John decided to let go of his belief in the linearity of learning and teaching in order to go "beyond the basics."

Conclusion

Throughout my work with John, it became clear that he navigates the different belief systems about mathematics, and the pedagogical practices they imply, by a combination of clustering and *bricolage*. The clustering of beliefs allows John to be comfortable in many worlds. John feels a twinge of guilt that he is sometimes short-changing his students, however, he is accepting of the importance of pass rates as an overriding goal. John also tries to bridge the many worlds by “picking and choosing” (i.e. *bricolage*) beliefs and practices from each.

One example of *bricolage*, is John’s attempts at bringing in research that may help in raising achievement scores into the classroom. Another example is John’s selection of research relevant to the skills that John believes his students will need to be competitive in the ever-changing knowledge economy: adaptability, data-driven decision-making, collaborative and communication skills. Another example is John’s easy and quick removal of the KWL (H) chart as a form of assessment - it did not meet John’s immediate goals and was thus summarily dismissed

Let us now turn our attention to Michael’s case. Although Michael experiences a similar disconnect between worlds or beliefs about mathematics and its teaching as John, he chooses to navigate them in a markedly different way than John.

CASE 2: MICHAEL THE BOXER

During the course of this study, Michael, a self-proclaimed non-traditional mathematics teacher and somewhat of an iconoclast, continually struggled to find a balance between actualizing his beliefs of effective teaching - primarily influenced by his time in industry, and his experience with mathematics education research - and the demands placed on him by his school. Michael sees his role as a mathematics teacher holistically, that is, he believes he has a responsibility to help students understand the mathematics curriculum with which he is charged, as well as help them develop writing, problem solving, critical thinking and general life skills. He describes his role as an *educator-mentor*.

Throughout the academic year, Michael was chastised during his reviews, and put on a three-month action plan by his school in November. Michael’s interviews contained the words “battle,” “balance,” “struggle” and “tenacity” frequently, conjuring up the image of a boxer in the ring. Michael was termed the boxer because of his resilience and sense of agency in the face of impediments to the actualization of his beliefs about the teaching of mathematics.

Introduction to Michael

Michael Gilford is a Caucasian male in his late fifties who, like John, came to high school mathematics teaching as a second career. Michael had previously worked 35 years in the Internet Technology (IT) field, was outsourced, and became a teacher with the express purpose to make a difference in young adolescents' lives. One of Michael’s main aspirations as a mathematics teacher is to develop his students' mathematics literacy, which he describes on his website as "the ability to communicate and use concepts best described with analytical or numerical logic."

At the time of this study, Michael is in his third year of teaching at Gregory High School (see below for description of Gregory HS). Michael has had experience teaching various mathematics courses. In his first year of teaching, Michael taught Algebra II and Discrete Mathematics. In his second year, he taught Discrete Mathematics and Advanced Functions and Modeling (AFM). In his third year, the year in which this study is being conducted, Michael is teaching Algebra I and Advanced Functions and Modeling (AFM).

Michael's School Context: Gregory High School

Gregory High School is a large, grades 9-12, comprehensive, public high school in the DEC School District, located in a large urban center in the South East of the United States. In the year of this study, Gregory High School employed 240 faculty members and served over 2000 students. Gregory High School has a diverse student population composed of 49% African American, 29% Caucasian, 15% Hispanic, 5% Asian and approximately 2% of American Indian and Multi-Racial/Other. In addition, 43% of Gregory High's population is receiving Free or Reduced Lunch. See Table 6 below for a summary of Gregory High School's demographics.

The Fifth Annual Report of the DEC Equity Committee provides statistics about the many ongoing equity disparities within DEC. This report is sobering, as it relates disparities in achievement according to geography, socioeconomic status and race. For instance, the equity committee reports that out of DECs 17 high schools, only four got a grade above D, which means that only four high schools had 60% or more students at grade level.

Table 6. Demographic Summary of Gregory High School.

System Type	*Grades Serviced*	*Student Population*	*Students Eligible for Free or Reduced Lunch*	*Racial Background*
Urban	9-12	2155	43%	Caucasian – 29% African American – 49% Hispanic – 15% Asian/Pacific Islander – 5% American Indian/Alaskan Native – 2%

The 2007-2008 School Progress Report for Gregory High School reports that it did not make its Adequate Yearly Progress (AYP) in the 2007-2008 academic year. Out of 27 targets Gregory High had set, they met 23. According to Gregory High School's School Improvement Plan, their number one goal is "attaining high academic achievement for all students" which includes targets for students on grade level or above on EOC classes including Algebra I, Algebra II and Geometry.

Across the board, for all ethnicities and for all EOC classes, the target for students on grade level or above is 90%, surpassing overall district objectives. The actual percentage of students on grade level or above in Algebra I at Gregory High in the 2006-2007 school year was 75% in Algebra I, 49% in Geometry and 49% in Algebra II, well below the 90% target. Improving Algebra I EOC scores has been a goal for Gregory High School since the poor performance of the 2004-2005 school year (40% of students’ scores at or above grade level for Algebra I).

Algebra I. According to the North Carolina Standard Course of Study (PSONC, 2003) for Algebra I, students are expected to gain knowledge in algebraic concepts such as operations with polynomials and matrices, creation and application of linear functions, nonlinear functions such as exponential functions and algebraic representations of geometric relationships. Furthermore, "students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relations and use those representations to solve problems.”

As Table 7 below shows, the EOC proficiency results for Algebra I rose from 40% in 2004-2005 to 76% in 2005-2006 at Gregory High School. The main strategies implemented in order to increase student achievement have been to put into practice more assessments in order to analyze "student mastery on EOC tests" and to offer EOC blitzes toward the end of each semester.

Table 7. EOC Results for Algebra I, Geometry and Algebra II at Gregory High School.

	Algebra I	*Geometry*	*Algebra II*
2004-2005	40%	44%	48%
2005-2006	76%	46%	74%
2006-2007	75%	49%	49%
2007-2008	72%	-----	-----
Target	90%	90%	90%

Michael characterized the school context about Algebra I as follows: "They made no bones about it, there are no excuses. Anybody in the class must be passing the EOC exam." Michael defined Algebra I as first and foremost an EOC course for which the school and teachers are under tremendous pressure to show improved student achievement since schools are evaluated on the basis of test performance in EOC classes. Further focus on performance in Algebra I was added in the 2008-2009 academic year, as a five-year pilot program aimed at improving student achievement in Algebra I was launched in Gregory High, as well as two other local high schools.

As Algebra I is generally the first course in the high school mathematics sequence, is an EOC course and is required to graduate high school in North Carolina, the main focus for Michael in this class is the preparation of students for the EOC exam. His second goal - as expressed on his teacher website and in his interviews and as is evident in his teaching practice - is to develop math literacy. For Michael, this means that his Algebra I students are expected to gain knowledge in problem-solving skills, mathematical techniques and notation and English vocabulary in order to increase reasoning and literacy.

During my interviews with Michael, he acknowledges that despite his best efforts, his Algebra I classroom is still primarily influenced by the EOC standardized test. Michael found that the culture of his school changed recently from “do the right thing for the kids” to “do the right thing for the grades.”

As indicated by Michael, the success in raising Algebra I EOC proficiency results from 40% in 2004-2005 to 76% in 2005-2006 was a double-edged sword. Gregory High was praised in the local newspapers as improving, changing the public’s perception of the school for the better. However, the techniques that had shown success in raising student achievement - traditional teaching, frequent testing, after school test preparation blitzes - are now more entrenched than ever in the Gregory High School culture. As Michael describes:

> You could be a different kind of teacher if your school had not succeeded, in which case there were no risks. If you are coming from a school where the test

> results are poor, you can then change procedures and take advantage of new educational logic, take advantage of new research. But if your school has succeeded, the risk of change is very high.

Michael is uncomfortable with this version of success, but he knows that “the community understands a school’s success based on newspaper accounts and that “the newspaper reports of success are measurable with alignment to No Child Left Behind.” Michael understands that there are cultural and social norms and constructions related to the profession of teaching. In other words, teaching is a social practice and is, as Shulman (1993) argues, the property of the community in which it takes place. Michael knows and accepts the fact that this opens teachers to criticism and public scrutiny.

Michael loves his school and understands that having measurable outcomes, such as the ones reported on the school's progress reports, is of paramount importance to a school's success, as it is tied to the community's perception of the school. Michael professes that even though he does not agree with much of the decisions engendered by what he termed the “No Child Left Behind Pedagogy,” doing what he can to help students perform better on tests that will be reported back to the community and be part of the school's evaluation, makes him feel proud and "part of the school.”

Michael’s Professional Background

After being outsourced from the field of Internet Technology (IT), and having decided that he wanted to be a mathematics teacher, Michael went back to school to obtain his Bachelors of Arts with a major in mathematics as well as his teacher certification at Blue Coast University. During this time, Michael took methods classes in which he was exposed to mathematics education research. In these classes, Michael remembers reading about teaching mathematics to adolescents, cooperative learning, differentiated instruction and the work of Piaget and other constructivist scholars. When asked by the researcher, Michael stated that the political aspects of mathematics education were not discussed in these classes.

Michael's beliefs about the teaching of mathematics were also greatly influenced by workshops in the PEAK Teaching for Excellence Model™ (PEAK). According to the PEAK Learning Systems Inc.'s website, PEAK was developed by Spence Rogers in order to build "high performance classrooms and learning environments" through "enhanced motivation; aligned curriculum; effective assessment; and research-supported, brain-compatible instruction." (PEAK, 2009).

Michael attended these workshops at the direction of the Gregory High School Assistant Principal of Instruction during his first year as a mathematics teacher. Throughout his interviews, Michael mentioned a strong belief in the following components of PEAK:

1. Differentiated instruction matched to the needs of each learner
2. Value placed on quality of learning rather than speed of learning
3. Forgiveness and coaching until standards are met.

Because Michael believes in quality learning rather than its timing and in forgiveness when standards are not met (PEAK item 2 and 3 above), he is frustrated at the fact that his students are "not allowed to make mistakes. Great way to teach somebody something! You only get one shot. There's no time for mistakes because we've got something else to do."

Michael and the Global Competitiveness Discourse

Gregory High School is part of the DEC school district whose motto, written on the top of web pages, on many e-mails and hardcopy communications with faculty and staff and, in some cases, on big banners exhibited at the entrance of schools is: *Reach Further. Global competitiveness starts here.* Gregory High also uses the discourse of global competitiveness freely to motivate the goal of increasing student achievement as part of their vision and strategy to prepare students to "compete in an ever-growing global community" (Gregory High's School Progress Report 2007-2008).

As discussed at length in the literature review (see Chapter 2), mathematics education is tied to notions of national economic competitiveness, which has resulted in a standardization of the mathematics curriculum emphasizing skills, procedures and a focus on pragmatic knowledge. One school of thought thus perceives the effects of globalization on mathematics education as a reinforcement of the positivistic perspective on mathematics and science.

The result is an educational discourse of competition, as seen in DEC school district and Gregory High School, in which high-stakes tests are the primary (valued) means of assessing knowledge and in which students, teachers, schools, school districts and countries are in a competition over test scores.

Like all other teachers employed by the DEC school district in the 2008-2009 school year, Michael received an e-mail from DEC's superintended the week before school started, welcoming teachers back to school and reiterating the main mission of the district: "Educating children for the global economy that they'll be joining as adults."

Although Michael stated that the DEC school district had “fallen in love with the global economy talk,” he also admits that the language of global competitiveness does not make it very far past the web pages and banners. Michael does not recall a meeting with teachers and/or administrators in which details were given about what exactly “preparing students to compete in an ever-growing global community” would entail for a mathematics teacher at Gregory High School.

When asked by the researcher in our first in-depth interview about what such a discourse means to him as a mathematics teacher, Michael sighed, and paused for a long time to gather his thoughts. The following interview excerpt followed: (R denotes the researcher’s part and M denotes Michael’s):

M: (sigh)...Adolescents at this age, have a view of the world that extends only a little bit farther than their car can drive. One of the things I'm consciously trying to extend is their idea of the geography of where we are in terms of what's happened, and that's probably the only global economy were talking about. I did more with discrete when we talked about voting patterns, in AFM we're just trying to get them to see beyond their own experience level...and in Algebra I. What we're really doing, and this goes back to Gregory High. Gregory High has a motto that says every eagle succeeds.

R: Yeah.

M: And the idea is we're trying to get the most out of every kid. And...it's not...it's a much more smaller goal. I've aligned myself with that one.

[…]

R: A general question: As a math teacher, what kind of skills do you think that someone needs to be competitive in the global economy?

M: For me that's easy. Did you see the title on my webpage? It has not changed. It's math literacy. My goal in all my classes is to teach math literacy, which means they should be as comfortable with numbers and numeric reasoning as they are with language and language reasoning. And that's really what I'm trying to do for all my students.

In further interviews, Michael specified other skills that he is trying to cultivate in his students as a mathematics teacher in order to prepare them for membership in the knowledge society. In particular, Michael adamantly believes it is important for his students to develop critical thinking (skepticism) and the ability to cope with an ambiguous and non-linear world, in which issues are not black and white and answers are not right or wrong. These particular beliefs, and Michael's attempts at their actualization, was a major source of conflict and frustration for Michael throughout the studied semester, as will be explored below.

Themes for Michael the Boxer

The beliefs Michael wants to enact in his mathematics classroom are in constant competition with the standardization of mathematics courses such as Algebra I. As mentioned in the introduction to this case study, the words "battle," "balance," "struggle" and "tenacity" were used quite frequently by Michael during interviews. The main themes that emerged from Michael's lived experience as a mathematics teacher are the balancing act and battle—two important components of boxing. As we will see below, Michael’s case highlights that educators who resist institutional norms and strive to uphold their beliefs often do so at great personal cost.

Balancing Act

Michael's beliefs about mathematics teaching is influenced by four main sources: his time in industry, the mathematics literature he has been exposed to such as constructivist theories, his PEAK methodology training, and his desire to do what's best for the school as a whole.

When I asked him about what he feels or thinks about before walking into his classroom, he described a balancing act in which he must balance the syllabus and pacing guide requirements set out by Algebra I weekly meeting with his desire to help his students acquire "mathematics that has legs outside the classroom…outside of just pushing buttons on a calculator.”

Michael has a well-developed and well-articulated belief about what it means for his students to *understand*, although he has had to adapt this teaching to be in line with the current culture of measured outcomes as evidence of student understanding. For Michael, understanding requires a synthesis of information, putting together facts that haven't been put together before. It also requires time to process information out of short-term memory and into long-term understanding.

> My theorem that I have to change is people need time to absorb and reflect. Time to make a fast answer doesn't indicate knowledge. It indicates parrothood. They can speak back what they've just been told and that's nothing to do with understanding. That has to do with short-term retention. The argument being that

> if you do short-term retention multiple times, you get to understanding. My argument is you don't, you get to short-term retention.

Although his success as an Algebra I teacher is primarily measured by his students' test scores, Michael is also concerned with teaching his students "mathematics that has legs outside the Algebra I classroom." Although Michael does not believe that EOC tests measure synthesis skills or long-term understanding, he continues to challenge his students to go beyond rote memorization. Careful not to ask compound or ambiguous questions in testing situations, Michael often asks compound questions or questions that fall outside the curriculum in class or homework situations. Balance.

For instance, after having introduced quadratic equations to his students, Michael asked more in depth question such as: "If you have a second degree equation, a parabola, with a positive coefficient, what's the maximum?" Michael reports that most of his students defaulted to giving the formula for calculating the vertex or the zeroes, which to Michael, meant that understanding had not been achieved. However, Michael concedes that his belief in what constitutes understanding is time consuming and perhaps not feasible in his current context.

Because of his PEAK training and his familiarity with mathematics education literature through his methods classes, Michael also believes in differentiated instruction (PEAK item 1 above). Michael's concept of "differentiated instruction" is in line with Tomlinson (2004) who defined the term as follows: "Ensuring that what a student learns, how he/she learns it, and how the student demonstrates what he/she has learned is a match for what that student's readiness, interests, and preferred mode of learning are" (p. 188).

Although the 2007-2008 School Progress Report for Gregory High School discusses with pride the overarching goal of using differentiated instruction in order to adapt the curriculum to individual learning needs, Michael found that the term "differentiated instruction" at Gregory High School means "teaching three or four different ways to do the same thing—not teaching at quantitatively different levels." Michael is frustrated on this point as well: "Piaget has died in DEC! Everybody is the same! We've got them all jumping through that hoop on the same day!"

Differentiated instruction has become synonymous at Gregory High with teaching to different learning styles according to Fleming's VARK model: Visual, Auditory, Reading and Kinesthetics. Although learning style theories have been criticized by many educational psychologists/researchers (e.g. Stahl, 2002), Michael has been instructed to teach so that visual learners have access to visual aids such as slides, graphical representations and diagrams, auditory learners have lectures, readers are provided handouts and kinesthetic learners are afforded some hands on activity.

Because of the "push" at Gregory High to cater to students' learning styles, Michael found balance between these demands and his beliefs by bringing his students in the school library to take two different online learning style assessments. In doing so, Michael complied with "the powers that be."

However, Michael took this opportunity to begin a discussion about test validity, margins of error and skepticism/critical thinking: "Kids need to understand that when somebody gives you a test that says this test proves you're exactly an auditory learner, and you have to do all these things, that's o.k. There might be another test that says that you're not that…"

Battle

Recall that Michael sees his role as a mathematics teacher as a mentor-educator who believes it is part of his responsibility to guide students through the development of their writing, problem solving, critical thinking and general life skills. Michael stated that he quickly realized his students in his Algebra I and AFM classes had difficulty with word problems because they had difficulty with words as well as synthesizing thoughts:

> It turns out, they have problems with words like *from* and *of*. You subtract *from*. They also have problems when there are two sentences in a question, and they only answer the first one.

To help his students with their difficulty with words, with the synthesis of thoughts, and to encourage students to share their beliefs about mathematics, Michael began including writing in his mathematics classroom.

Writing in the Mathematics Classroom. Writing is an important mode of communication that has potential as a learning tool, which can assist students in the development of conceptual understanding. The mathematics education literature supports the premise that writing promotes a deeper understanding of mathematics as students extend their metacognitive and critical thinking abilities (Artzt & Armour-Thomas, 1992; Carr & Biddlecomb, 1998; Powell, 1997; Pugalee, 2004, 2001). The importance of communication in learning mathematics is emphasized by the National Council of Teachers of Mathematics (2000), who includes communication among the five process standards (communication, problem solving, reasoning and proof, connections and representation).

Writing, in general, is viewed as a generative act requiring a deliberate analytical action from the composer (Vygotsky, 1986) as association between current and new knowledge become part of deliberate web of meaning. Writing in mathematics can provide a tool for students to reflect and verbalize about important mathematical concepts and ideas, thus supporting analysis, comparison of facts and ideas, and synthesis (Farrell, 1978). Such activities have been shown to promote ownership of mathematical learning and provide a strong foundation for continued growth of mathematical skills and understanding.

One writing activity that Michael was particularly proud of including in his Algebra I and AFM classes was an activity he termed the *cool down*. This is an activity he adopted from PEAK and some of the research mentioned above on writing in the classroom. In this activity, Michael asked students to write a paragraph response to questions such as:

- Why do people learn mathematics?
- What is the hardest part of mathematics for others?

Some of Michael's cool down questions were not specifically about mathematics, but they reflect Michael's desire to encourage his students to think about ambiguous concepts and to synthesize their thoughts. Examples of these types of cool down questions are:

- I've heard it said that a good start means a good end. What do you think?
- Some people say first impressions are most important. Other people say a tiger can't change its stripes. What do you think about it?"

Michael took great care to formulate his questions in such a way that there were no right answers and that two thoughts, which didn't naturally match, were being forced together. Michael also asked his students to create their own cool down questions. Michael's goals with these activities are complex. He wants to give a voice to his students, "take the temperature of the room" sort of speak. Also, having spent thirty-five years in an ever-changing industry (IT) to finally be outsourced left Michael, like John, sensitive to the uncertain world for which he is preparing his students. Michael argues that an emphasis on education for work and the consequent focus on facts and procedures in school mathematics prepare students for jobs that won't exist by the time they enter the job market.

With that in mind, Michael believes it is part of his responsibility as a *mentor-educator* to place an emphasis in the mathematics classroom on thinking skills and affective education in order to equip students with "skills to handle a world of non-linear change" (Lee, 2005, p. 172). Michael thus believes it is important for his students to learn to process ambiguous statements, statements that do not have a correct answer, unlike the multiple-choice questions students are asked to answer in EOC tests. He also wants to encourage his students to write and create a culture in which writing is part of a mathematics classroom.

During his review in November, Michael was told that his cool down questions were a "very bad idea because it's not part of the math curriculum." Michael reasons that:

> I think that in education, I'll generalize to education, there are many, many linear thinkers…and for linear thinkers, the curriculum should be a box. The idea of a math teacher teaching English or worried about it seems to them not a plus but somewhere he's loosing out on what his role is. Which is less than I see my role.

Michael was promptly asked to remove his cool down questions that were not "mathematically oriented" and did not have a specific answer: “They wanted the kids to have an answer so they could have a feeling of getting it right. To which I said: That wasn't my goal!”

Michael, being a relatively new teacher, did as he was told—he removed his cool down questions. However, Michael has a great sense of agency in the face of impediments to the actualization of his beliefs. His review also asked Michael to do more reviewing of concepts. Michael decided to substitute his cool down questions with a review in which he gives students a bullet point form summary of every lesson to reinforce concepts, and asked his students to translate each bullet point into sentences in order for them for formulate a coherent written narrative of the lesson.

In this instance, Michael skillfully navigated the different beliefs about the place of writing in the mathematics classroom. As Michael stated “there are a few battles I can win in Algebra I, but they’ll be small.” He is committed to his beliefs about writing's place in the mathematics classroom, while being equally committed to his school's mission and demands. Feeling like he had won a small victory, Michael quipped:

> The comedy of this routine is that the school is driving now at a curriculum level towards more word problems which makes me feel validated that I was doing things right. Do I feel understood? NO. Did I give up on that? I'm in my third year. I salute. I do whatever they tell me joyously.

The poor performances on his November review made Michael rethink his teaching to be more in line with what he terms “traditional teaching.”

Traditional Teacher. Michael contrasts himself to traditional teachers often in his statements. In order to better understand Michael’s beliefs about his role as a mathematics teacher, it is important to understand what Michael means by the term *traditional teacher* as it is what he is actively rejecting. Michael also often referred to traditional pedagogy as the “No Child Left Behind Pedagogy”:

> We've all adopted the No Child Left Behind Pedagogy that says that whatever is necessary to pass the test must be the right thing to do, and since it has to be done now, we don’t have to worry about later.

When asked to explain what he meant by traditional teaching, Michael drew a diagram which I have reproduced below in Figure 4. As he was drawing the diagram, Michael was describing that:

> I call it a traditional approach, which is introduction of a topic, homework on a topic, topic is complete. Add a second topic that's associated with it, that uses it possibly. Once you get a collection of those, have a review day where everything is brought together. Upon completion of the review day, there's an exam. And that's pretty much how I characterize the traditional approach as I've seen it practiced.

Of the effect on students of traditional instruction, Michael believes that:

> Overall my view of traditional teaching methods is that it makes kids passive. They don’t learn critical thinking or knowledge. They learn how to rote respond. A lot of my students are very uncomfortable with me when they have open-ended questions ‘cause they would like to give me the right answer, and they don’t know what it is, and I haven’t told them what it is.

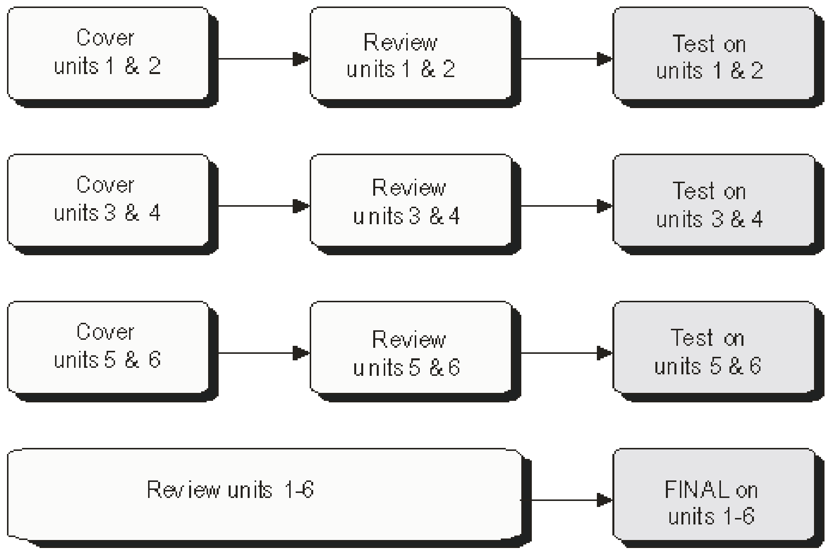

Figure 4. Michael's diagram of traditional teaching.

Because of his poor November review, Michael reluctantly adopted a teaching approach more in line with what he understands to be traditional teaching. Apart from the suggested changes to his teaching approach mentioned above, Michael was asked to drastically change the way he taught Advanced Functions and Models—to, in essence, dePEAK it. As the reader can see from the change in Michael's AFM syllabus from before his November Review (APPENDIX G) to after his November Review (APPENDIX H):

1. Resources external to the textbook have been removed. For instance, all references to activities and modules found on the North Carolina School of Science and Mathematics are no longer present in the "AFM Syllabus Version 2."
2. The hierarchical/linear approach to mathematics teaching has been reestablished. Covering section 6-3 is followed by 6-4 which is followed by 6-5, and so on.
3. More advanced topics, such as "Probability" and "Decisions through Data" have been eliminated in favor of review in preparation for the final exam.

Despite these concessions to school demands by Michael, his mid-year review was also quite poor (see APPENDIX F). Michael's reply to this review, indicative of Michael's sense of agency and resiliency, is reproduced in Figure 5 below:

From: M. Gilford

To: File

RE: Response to Mid Year Appraisal 2/10/09

At the direction of Gregory High administration, my classroom management practices, pacing guide and assessment methods were changed from 'best practices' to 'old-school' practices during the Fall 2008. The elimination of teaching using best practices was the major cause of this poor review.

These best practices were learned during PEAK training. I attended PEAK at the direction of the Gregory High School Assistant Principal of Instruction my first year. My second year I attended Saturday sessions. Following this training, I had much success in the classroom (teaching Discrete Math and AFM) using best practices techniques. Administration at Gregory High School acknowledges that they do not understand PEAK encapsulated best practices, nor are they interested in using them at this time.

Three years ago, I came to Gregory High school because of the school's reputation and the principal's ability to support his teachers and gain outstanding performance. My first year assessment was done by a math teacher who went on to become a DEC assistant principal. My second year assessment was done by the Math Department chairman. Both evaluations were satisfactory.

This appraisal reflects a discord between encouraging teachers to move to ‘best practices’ while assessment is done using models developed in the 1970’s and 1980’s. This year I had expected to continue to improve my teaching skills and took a summer course to enhance both AFM and Discrete Math instruction. However I was assigned to the Algebra I pilot project and told mid-semester to eliminate best practices from my AFM class. The Pilot Project at Gregory High School was lead by teachers with many years of Algebra I teaching experience who choose to repeat their traditional methods with minor modifications.

As I have been directed to <u>not</u> use ‘best practices’, I need to continue to regear my classroom management to ‘old school’ techniques (silent classroom, desks in neat rows, assessment practices without retesting, train-track pacing guides with review days to catch-up, etc.) I have requested support from administration to reach a satisfactory review in order continue teaching within DEC and achieve organizational cohesion at GH. However, there is a question whether a fair evaluation is possible when my preference towards ‘best practices’ is so well known.

I am challenging this appraisal. It is my understanding that best practices are supported by DEC. Further I cite my prior appraisals and a current observation

(12/1/08) by the Curriculum Resource Teacher that my classroom management is within standard performance measures.

Figure 5. Michael's Response to Mid Year Review (artifact, 2/10/09).

For Michael, being continuously monitored and evaluated limited the time he had for imagining and enacting transformative pedagogies. At the end of November, 2008, Michael was put on an "action plan" which included a return to the traditional format and strict adherence to the syllabus. Michael was asked to employ the following protocol to conduct his classes:

1. Procedures review and adjust.
2. Warm-up (on the overhead when the students arrive).
3. Review of homework.
4. Lesson (new material).
5. Group activity (assessment/adjustment of new material).
6. Homework assignment (reinforce new material).
7. Summary review.

The poor reviews in combination with the culture of high-stakes testing which exists at Gregory High School turned his school into a place of disenchantment. As Sumsion (2002) noted, the process of becoming a teacher is not "a solitary or self-contained process-it occurs in a time and space where others, some much more powerful than yourself, also are bent on constructing 'you' in an image they value" (p. 874). Because of this tension, Michael began to feel frustrated by the unreceptiveness of colleagues, and retreated somewhat to the personal landscape of the classroom which offered him the autonomy to pursue his vision and enact his epistemological beliefs.

Michael asserted that: “While there is absolute consensus on the test and the syllabus curriculum, what goes on in my classroom, I have control over!” Clandinin and Connely (1995) remarked that: "The privacy of the classroom is a safe place where teachers are free to live their stories of practice" (p. 13). Michael the Boxer found the need to retreat into his corner. Throughout the semester, Michael increasingly felt “managed” and stripped of his agency. The scripted curriculum, teacher accountability, continuous monitoring of student performance, high stakes testing and punishment for not meeting standards at Gregory High School mirrors what Khol (2009) described as an “educational panopticon,” frustrating and, often times, completely impeding much of Michael’s efforts at the actualization of his beliefs. In February, 2009, Michael sent me the following e-mail (Fig. 6):

> Next year will probably not be at Gregory HS. Read the mid-year reply first. If you have contact who can help me finding a position that supports new methods to motivate students let me know.
>
> Michael Gilford
> Math Teacher, Gregory High School

Figure 6. Michael e-mail communication (artifact, 02/19/09).

Conclusion

Michael the Boxer was shown to have a strong sense of agency in the face of impediments to the actualization of his beliefs about the teaching of mathematics. Michael sees himself as an educator-mentor who has a responsibility to help students understand the mathematics curriculum with which he is charged, as well as help them develop writing, problem solving, critical thinking and general life skills.

The manner in which Michael chose to mediate the external discourses, often at odds with his set of dispositions (*habitus*) and his ideology was through a combination of *balance* and *battle*.

One example of balance is Michael's attempt at both fulfilling the "differentiated learning" requirements of his school by administering "learning styles" assessments, while satisfying his need to instill of skepticism about such tests in his students.

Often times, Michael struggled to find a balance between his beliefs of effective teaching and the demands placed on him by his school. He found himself fighting the lure of the "traditional" teaching method and the reign of absolute mathematical truths, accountability and measured outcomes, in order to teach in a way which makes room for ambiguities and creativity.

In certain instances, Michael would choose to wage battle. As Michael strongly believes that writing has its place in the mathematics classroom, he chose to keep writing in a form that would be more palatable to his administration than his cool down questions. Michael's tough response to his poor mid year appraisal in another example of battle.

Let us now explore the case of Sergen who will be shown to have fascinating areas of commonality and uniqueness with John and Michael in the manner with which he navigates the epistemological rocky waters of mathematics education.

CASE 3: SERGEN THE DIPLOMAT

Sergen Manzik's case reveals a well-balanced and relatively smooth navigation of the epistemological rocky waters of mathematics education by a successful high school mathematics teacher. Sergen was termed *the Diplomat* because he is seen as dealing tactfully and skillfully with others—engaging in negotiations and mediations about belief systems and often acting as an intermediary between his school administration and other mathematics teachers. Sergen's interviews contained a high frequency of the words "reflect," "discuss," "understand," "empower" and "respect."

Sergen, consciously and adamantly rejected the use of the word "struggle" in his interviews to describe his experiences: "It's not really a struggle. It's conflicting views that I'm happy, you know, to incorporate." More will be said below about the manner in which Sergen incorporates different beliefs about mathematics and its teaching in his practice.

As we will see, much of Sergen's diplomatic acumen comes from his broad view of his role as a teacher, as well as his beliefs about the nature of knowledge and knowing. For Sergen, "there are all sorts of different types of knowing," and these should be respected and incorporated.

Sergen believes it is his role as a teacher to provide his students with basic mathematics skills so they can "survive in different countries, different institutions, with different teachers." He also wants to help his students' self-esteem by empowering them to become intellectuals, prepare students to be competitive in the knowledge society and help improve the reputation of his school in the broader community by producing high achievement scores on End-of-Course tests.

Introduction to Sergen

Sergen Manzik is a male in his early thirties of Turkish origin. Sergen began his academic career in his country of origin in mechanical engineering. During the second year of his Masters of engineering, Sergen decided that he "didn't want to do anything like what engineers do." He thus decided to take an opportunity that was offered to high-achieving graduate students in his country to study abroad in the field of education. He obtained a Masters and began a Ph.D. in Mathematics Education at East State University. After taking much of the required courses towards his Ph.D., Sergen decided to transfer to Blue Coast University, where both John and Michael also studied, to complete his Ph.D.

At the time of this study, Sergen was in the second year of his doctoral program at Blue Coast University. The decision to transfer from East State to Blue Coast University was precipitated by two main factors: (1) Sergen wanted in-class teaching experience at the high school level. Blue Coast University allowed him to be a part-time doctoral student and thus be a teacher during the day. (2) Furthermore, Sergen found that East State University's Mathematics Education program was primarily focused on the use of technology in the mathematics classroom, while he wanted to study a broader selection of mathematics education topics.

Sergen began teaching mathematics at Alleny Academy in 2005; that is, Sergen was in his fourth year of teaching at the time of the study. Sergen's school context will be described in further detail below. During his teaching career, Sergen has taught Precalculus, Calculus, SAT Prep, Discrete Mathematics and Technical Mathematics. At the time of this study, Sergen was teaching Calculus BC and Precalculus, and was now the Chair of the Mathematics Department at Alleny Academy. Sergen described his role as chair as, in part, being a spokesperson “taking the word from the administration and disseminating to math teachers, related to their classes and such.”

Sergen’s overarching goal as a mathematics teacher is to help his students become intellectuals, equipped with as many “intellectual tools” as they will need to reason and analyze a variety of situations, in a variety of ways, depending on context.

> I want my students when they go to college or when they go to different venues or workplaces – I want them to be intellectuals. I want them to be able to look at problems and identify problems and have a clear sense of what the problem is and what a solution might look like … I want them to be aware of themselves … I want them to be critical about what other people are saying and what they’re saying.

Sergen provided me with a variety of examples to illustrate what he means by “intellectual tools.” For instance, understanding how to prove or disprove a universal statement such as *even* + *even* = *even* is an “intellectual tool” because it is useful in mathematics, as well as in analyzing other “for all” statements in other contexts. As can be seen in the quote above, Sergen wants to equip his students with critical thinking skills, which involves having many intellectual tools at their disposal with which to analyze a problem, a situation, or a statement.

Sergen’s School Context: Alleny Academy.

Alleny Academy is a grades 9-12 public high school located in an urban area of the South East of the United States. Like Gregory High School, it is part of the DEC school district. In the year of the study, Alleny Academy employed 82 teachers, 3 of which are National Board certified and served 975 students. Alleny Academy's student population is composed of 82.5% African American, 7.8% Caucasian, 6.5% Hispanic and 3.3% other, where 60% of the student population qualifies for Free or Reduced Lunch (see Table 8 below).

Table 8. Demographic Summary of Alleny Academy.

System Type	*Grades Serviced*	*Student Population*	*Students Eligible for Free or Reduced Lunch*	*Racial Background*
Urban	9-12	975	60%	African American – 82.5% Caucasian – 7.8% Hispanic – 6.5% Other – 3.3%

Alleny Academy did not meet its Annual Yearly Progress in the 2007-2008 academic year, as it met only 9 out of its 13 target goals. Alleny Academy was depicted by Sergen as an "urban school." Sergen characterized the student population at Alleny Academy as such:

> 90% of our student population is first-generation college bound kids, and 65-70% are getting free lunch. So they're students of poverty. They're coming from lower socio-economic backgrounds. But we are trying…those inputs, those outside effects, not to let them bother our education. We want those outside factors to have no effects at all. We want to maximize teacher effect and school effect and then mobilize kids and give them good math and science education.

As the Ph.D. in Curriculum and Instruction with a specialization in Mathematics Education at Blue Coast University is under the Urban Education umbrella, Sergen feels that he has gained a profound understanding of some of the sociocultural aspects of the educational enterprise. In particular, Sergen is familiar with the critical issues and characteristics of American urban centers, and how these issues impact urban schools, and this knowledge has affected how he views his role as a mathematics teacher.

Although both John and Michael were also exposed to these issues during their education studies, and Michael's school qualifies as an urban school, neither of the above participants mentioned this as a significant factor in their decision-making process as teachers. However, Sergen made mention of this fact about Alleny Academy and its population on numerous occasions during our various interactions, especially as it relates to his beliefs about the teaching of mathematics. It is thus important to take pause and briefly explore the issue in more depth here.

Some characteristics usually associated with urban centers are high levels of poverty, single-parent families, English as a second language, crime and drug abuse and resegregation according to social economic status (Kozol, 2000, 2005; Wegmann, 1994). In general, the term urban has become "a signifier for poverty, nonwhite violence, narcotics, bad neighborhoods, an absence of family values, crumbling housing and failing schools" (Steinberg & Kincheloe, 2004, p. 2). The characteristics of urban centers discussed above contribute to inequalities in urban schools and result in consequences such as the under-preparedness of students for the next level of schooling, discipline problems, high drop out and teacher turn-over rates (Anyon, 1997, Spring, 2007).

Describing how his particular school context affects his beliefs about the teaching of mathematics, Sergen indicated that:

> We face different issues … we have a lower income school and population where we are facing all sorts of issues that need to be taken care of. If you treat our students and be insensitive to students, and treat them middle class students, and try to teach them like without having relationships, without helping them develop aspirations to go to college, and take the time…without supporting, building, reaffirming students. Boosting their self-esteem. If you don't do those, then by just offering them plain mathematics education, just math concepts and procedures and relationships, they won't grow I think, because they cannot. They have other issues that need to be addressed. So my job is to be sensitive to all the issues and in the mean time, give as much as I can in terms of conceptual, a little bit of this, a little bit of that, and in the mean time cover as much as possible in the curriculum.

More than a third of the teachers at Alleny Academy have less than 3 years of experience. The teacher turnover rate is very high (approx. 36% in 2007-2008). This explains one reason why Sergen, after only three years of experience at Alleny, is Chair of the Mathematics Department. The high amount of teacher turn over in urban schools has been attributed in the literature to many factors: Low teacher salaries and under preparedness of teachers to work in an urban school environment (Anyon, 1997, Spring, 2007). Better, more experienced teachers are generally rewarded with teaching posts in higher-status schools where salaries are higher and working conditions are better.

Borman and Overman (2004) found that teachers in urban schools are often directed to focus on the basics that will produce higher achievement scores on End-of-Course proficiency tests and often report a lack of support for mathematics teaching focused on conceptual understanding. Sergen also points to the “new movement towards testing and such” as one reason for low retention rates in his school. Discussing the accountability movement present in his school and others:

> Because if we hold people accountable then, first of all we impose some external control factors onto dedicated artists, like teachers who are there for the kids who are there for their growth. It would impose some external factors that’s kind of disrespecting their art and their craft. And then you would kill the spirit of teacherhood, and you would produce business like adults who are producing those kinds of people who are unconsciously following here is what you got to do and here is the test and this is how we do it, and then the kid does well on the test. And then we look successful like we improved ten scores in our composite EOC scores. But look at our people. We have nobody there. We have all the building is all sleeping. All one thousand students are not thinking. And look at our people. Look at our teachers. They're not content. They're burned out and they're not happy with administrator e-mailing them you have to do this you have to that.

Sergen describes his school as being in transition. A new principal with strong leadership skills and educational research background has meant a renewed focus on improving the reputation of Alleny Academy in the public forum. The drive is to attract better qualified teachers and lower the teacher turnover.

As part of the school's strategy to improve their image, there has been a strong emphasis on raising EOC scores in order to meet AYP and improve their School Progress Reports and state School Report Card. In particular, raising EOC scores in Algebra I and Algebra II is of great importance to the school administration.

As Table 9 below shows, the EOC proficiency results for Algebra I, Algebra II and Geometry have consistently remained well below the 80% target set by Alleny Academy and the DEC school district.

Table 9. EOC Results for Algebra I, Geometry and Algebra II at Alleny Academy.

	Algebra I	*Geometry*	*Algebra II*
2005-2006	66%	24%	50%
2006-2007	72%	40%	23%
2007-2008	62.9%	NA	NA
Target	80%	80%	80%

Sergen recalls the particular embarrassment the 2006-2007 Algebra II scores caused for his school (i.e. only 23% of students' scores at or above grade level for Algebra II). Sergen is dedicated to his school, the mathematics department and their mission and reputation in the broader community.

Sergen's Professional Background

Sergen feels that his mathematics education background has primarily given him an awareness of "different ideas like conceptual understanding and how to tap into students' analytic thinking." His mathematics education research knowledge and beliefs guide much of his actions. All lessons taught in Sergen's classroom are geared toward conceptual understanding. Students are encouraged to explore concepts and to probe multiple solution paths.

Sergen related to me a revelation that he had during his study of mathematics education at East State University which drastically changed his beliefs about mathematics and the *knowing* of mathematics. Sergen reported being an excellent mathematics student throughout his academic career - in high school and in his engineering program. In the second year of his doctoral program in mathematics education at East State University, Sergen became really and truly conscious, for the first time, of certain very basic concepts in mathematics, such as the difference between the function f and $f(x)$ at a specific value of x, seeing the expression x+1 as *one* object and proportional thinking.

When talking about these revelations, Sergen is manifestly recalling a very profound moment in his life: "I never thought x+1 was one thing, you know. I never thought! … So this humbled me, I think, in terms of what do I know about math. What it means to know. What it means to not know."

This humbling experience changed Sergen's epistemological beliefs. It helped him "reorganize" his "understanding of all of mathematics." Sergen reached the conclusion that "there are all sorts of different types of knowing." To illustrate this point, Sergen discussed the different ways one can understand fractions. For instance, as 2/3 being two integers that are divided, versus memorizing the definition of a rational number, or seeing 2/3 as a symbol to be manipulated within the given mathematics problem. If a student can add 2/3 and 4/5 together, do they know about and understand fractions? In Sergen's view, they do, on one level – the level at which symbol manipulation means *knowing*.

Sergen’s beliefs were greatly shaped by the constructivist and sociocultural approaches to mathematics teaching and learning found in the mathematics education literature. In his interviews, Sergen reflects a (radical) constructivist point of view, in which individuals create their own reality through actions and reflections. This individually constructed reality is not necessarily a reflection of an ontological reality. Sergen claims:

> I don't feel like I know anything. So I don't have strong convictions about things and meanings. So, in my mind a happy balance is a happy world and the confusion, the unknown. Because if I were to know something, if I said to myself that's what it is, then I would create a kind of illusion about reality and about the world.

Sergen applies a constructivist perspective in his classroom by connecting open-ended questions and an inductive teaching style to student understanding. Sergen has also internalized some aspects of the sociocultural perspectives on mathematics education (e.g. Vygotsky & Luria, 1994), in which knowledge is socially and culturally situated: “I guess if the beliefs are consistent with or in concert with the community or the society's and other people's beliefs then they become knowledge you know.”

Sergen and the Global Competitiveness Discourse

Like Michael’s school – Gregory High School – Alleny Academy is part of the DEC school district whose motto is: *Reach further. Global competitiveness starts here.* Like all other teachers employed by the DEC school district in the 2008-2009 school year, including Michael, Sergen received an e-mail from DEC’s superintended the week before school started, welcoming teachers back to school and reiterating the main mission of the district: "Educating children for the global economy that they'll be joining as adults." Alleny Academy also uses the discourse of global competitiveness to underline their mission. For instance, the principal’s welcome page includes the statement that “in an ever changing society, the expectation is that our students are prepared to compete on a global scale.”

Alleny Academy’s mission, as stated on the school official website, is to offer a quality education which includes equipping students with the capability to “think creatively, make data-driven decisions, solve problems, visualize, and know how to learn and to reason.” This description is unmistakably designed to mirror the literature discussing the central role of K-12 education in preparing students to be competitive members of the knowledge society by imparting crucial skills such as learning how to learn, creative thinking and problem solving.

Like John and Michael, Sergen admits that the epistemological and pedagogical implications of this discourse have not been made explicit in his school context (nor in his extensive study of mathematics education). Sergen does not recall of a meeting with teachers or administrators in which details were given about what exactly preparing students “to compete on a global scale” would require of a mathematics teacher at Alleny Academy.

When asked directly by the researcher about what such a discourse means to him as a mathematics teacher, Sergen is forced to think about the issue, clearly for the first time (R denotes the researcher’s part and S denotes Sergen’s):

R: This is on principal (name withheld)’s webpage: "In an ever changing society the expectation is that our students are prepared to compete on a global scale.”

S: Yeah…

R: "As we prepare our students to meet these global demands it is imperative that we facilitate learning through ..." So what I would like to know, for you, what does that mean, as a math teacher? When you're teaching your classes, what does it mean to be preparing students for the...

S: …global economy...

R: To compete on a global scale...yes.

S: These are developed by administration mainly, but with the administration lead, with the input of all the parents and committee members. There's also committee members...What it means is we should have high expectations.

R: What does that mean?

S: High expectations...uh...

R: For grades? For ...?

S: uh…teach a very rigorous math course, meaning that attend every class, as a teacher attend every class, be with your students every day. Attend. Be with your students, and don't waste any time during 90 minute period, meaning design lessons where students are constantly actively learning math, and also learning math content that is equivalent to what higher achieving schools learn in equivalent classes. Classes across the United States. For example when my student graduates from my class, passes my class and then goes to Ohio somewhere and then transfers to a high school and take calculus, AP calculus AB lets say there during senior year…they should be ready and should be just fine.

> Because my student should have learned all the necessary requirements for high achieving high school in say Cleveland, Ohio. That's how I read it.

The fact that Sergen begins his answer with “these are developed by administration” reflects his belief that much of the global competitiveness discourse found in his school is what he termed “lip-service,” detached from his role as a mathematics teacher. This is in great part due to the overall lack of discussion on the subject in his school context, or in his graduate studies:

> I said lip-service because in order for it to become an issue, in order for me to take what they say seriously, and for me to listen to them, to listen to what the politicians are saying or what people who talk about we need to be globally competitive and such and such…they at first have to lay out *what that means* (italics added for emphasis).

This interview excerpt also shows how the concept of standardization has become inexorably tied to the discourse of preparing students for a global society. The argument “to be competitive globally, students need to be equipped with the same skills as all other students” was made by John, Michael and Sergen—seeming to be the default justification for the high focus on achievement on standardized test as part of the global competitiveness discourse.

Themes for Sergen the Diplomat

Sergen tries to “get along with everybody” by being “nice and respectful” toward other views about mathematics and its teaching: “Cause the reason I don’t complain is I don’t see the world in terms of these are the things that you gotta do otherwise the worst things will happen to you. Like that kind of world view.” However, Sergen does have a sense of agency and uses it sparingly, like political capital, when it is important to him and he feels like he could actually make a difference in this particular arena.

In what follows, the main themes that emerged from Sergen the Diplomat's lived experience as a high school mathematics teacher will be examined: different types of knowing and reflection & discussion.

Different Types of Knowing

The fact that Sergen believes in "different types of knowing" is key to understanding his success at navigating the epistemological rocky waters of mathematics education. It also had profound implications for his approach to the teaching of mathematics: "As a teacher, I started expecting problems – expecting weaknesses. As opposed to thinking they should know this, this is very basic, and they should know this and they should know that." Sergen thus tries to incorporate and nurture in his teaching, different types of knowing about mathematics: from pure symbol manipulation to helping his students grasp the conceptual underpinnings of such manipulations.

For instance, Sergen uses many procedural problems from the book in his precalculus class, but he supplements these "with little concept discussions." For example, when a problem requires the use of the Pythagorean Theorem, Sergen is not satisfied with students "parroting" $a^2+b^2=c^2$. He wants them to have "played with the concepts"—not to rely on rote memorization of formulas: "Every now and then one student will say yeah it depends if you call your hypotenuse *b*, then its $a^2+c^2= b^2$. There you go that's right!"

Sergen admits that it is sometimes difficult to see how nurturing conceptual understanding can quickly and directly improve "procedurally based EOC scores":

> I haven't heard anything about the clear path or the clear map of helping students to score well on the EOC. The problem with the EOCs is there are lots of topics. There are like hundreds, two hundred topics that need to be covered, if you explore ideas you will fall behind. You want to cover 200 you will cover 150. And those 50 they will miss. That's the only thing. Like if they can cover all 200 topics AND teach conceptually, that would be great. But we don’t know, we don’t see the course of action, like the plan. We don't know how to squeeze in those 200 topics into a conceptual plan of teaching and learning path. So we don’t know that. We don’t have that.

Although Sergen wants to promote conceptual understanding in his students, he is sympathetic to the demands of the social contract and obligations that the school has with the community of parents in which success is measured by EOC scores:

> How could parents assess the success of schools by the number of teachers that provide conceptual understanding?” In the end, “no parent is demanding that their teacher is getting conceptual underpinnings of logarithms. No parent demands that. The only way, the only understanding they have about good school is the school’s reputation and the only measure of a good reputation is good EOC scores.

Sergen’s quandary is reminiscent of the ones highlighted by Windschitl (2002) in his framework of dilemmas that teachers experience when trying to implement constructivist instruction. Recall from Chapter 2 that representative questions of concern in this category of dilemma included: How can diverse problem-based experiences help students meet specific state and local standards?," and "Will constructivist approaches adequately prepare my students for high-stakes testing …?"

However, Sergen heavily relies on his readings of mathematics education research which suggests that conceptual understanding leads to higher achievement in procedural mathematics as well to help mediate these competing goals.

Reflection and Discussion

Sergen is a committed and reflective mathematics teacher – often spending ten hours per day at school and continuously thinking and rethinking about his teaching. This continuous reflection is part of the reason why Sergen can navigate different belief systems without much struggle or frustration:

> I like to reflect on things in my practice and my beliefs, constantly. Like a constant stream of reflective experience. *So in that reflection experience, in the experience of reflecting all the time, I don't stop anywhere and say, this is it. This is what it is* [italics added]. But instead I go and reflect and reflect and then when I do certain things…like what I do is a kind of product of my earlier reflection sure. And then there is learning and connections that I developed and created but it's a kind of learning process…

Sergen feels that his precalculus students, fresh out of Algebra II (an EOC course), have developed a "culture about the teaching and learning of mathematics." Sergen compared his students to racehorses who have been discouraged from reflection and are thus unaware of the purpose of most of the mathematical symbol manipulation they are responsible in reproducing successfully on tests: "They're unconsciously multiplying things and adding them and moving things around, manipulating things."

Within this culture, there is no room for reflection and the role of the teacher is to tell students whether they are right or wrong quickly. If they are wrong, the teacher's role is then to give them step by step instructions on how to correct their solution: "They think that good learning means like clear, open and shut case. Like this is it, I got it, no I didn't get it. They're used to those affirmations with their answers being correct or rejections of their answers." When I asked why he chose the term "culture," Sergen said that it was a culture in that it is shared by the majority of students (and many teachers), it is expected and accepted behavior, and is communicated constantly verbally or through action.

Sergen began his second interview in November, stating that he was fighting this culture: "Now, what I want is, I want to tell them that no, that's not how it works, that's not how we learn or teach math. Like here is how we do, step-by-step. How we solve these types of problems, and then you emulate that and then produce the same steps and then you will get an A." Sergen reports that overall, his students are very resistant and uncomfortable with ambiguity and confusion.

Although Sergen, like Michael, used the word "fight," the manner in which he "fights" did not conjure up the image of a boxer standing his ground, but of a diplomat that is willing to compromise on many points, but not this one. For Sergen, this is not how he sees his role as a teacher and he is unwilling to make concessions on that one issue.

Sergen fights this culture by slowing the process down, incorporating many discussions about ideas and concepts and promoting confusion. As mentioned above, Sergen's beliefs about mathematics and its teaching were greatly influenced by the sociocultural and constructivist perspectives found in the mathematics education literature. As such, Sergen often chooses to focus on his students' mathematical activity in the classroom learning environment, as opposed to focusing on the results or product of their activity. Sergen's teaching practices are rich in conversation as he believes that learners construct knowledge through interaction with their environment, including interaction with others. He thus often invites students into mathematical discussions in which they must explain and defend their ideas. On the whole, Sergen views understanding as:

> very dirty, and very ugly, and very laborsom, and multi-layered, and rich, and involves different functions of the mind. It's kind of messy, the whole process of mind, and there will be lots of confusion. Like for example, in my class discussions, I promote confusions. I promote an environment where there will be a lot of "what?," "what is that?," "I don't understand," "what do you mean?"

Sergen is extremely well versed in constructivist theories and their pedagogical implications. For instance, Sergen referred to Piaget's schema theory several times during our interactions. As mentioned in Chapter 2, schema theory claims that individuals make sense of an experience using a *scheme*, which is a way of organizing experience. An experience is assimilated by fitting into a pre-existing conceptual structure. However, he is also realistic about the time constraints he is under and the other goals he must tend to. Nevertheless, as can be seen from the quote below, Sergen has created a compromise between using constructivist theories as described in the literature and giving up entirely on the this particular approach to teaching:

> I don't have a lot of time to design curricular material or worksheets and problem situations where they use their own existing preconceptions and then through

> acting on certain problem situations and reflecting on their work and then developing more powerful concepts. That would be wonderful but I don't have time … but what I do I kind as sometimes a generic student who makes mistakes and I expose them to that students' mistakes and I leave to them to kind of see and assimilate because they can assimilate they do have similar conceptions. I make mistakes and then facilitate discussion and the students have access to it.

Sergen described a particular "misconception" he brought up in class in order to facilitate discussion. Given $(f(x)+g(x))/f(x)$, he cancelled the $f(x)$s out to simplify the expression to $g(x)$. This initiated a long discussion with his students about this action: "I don't say it's right or wrong. I don't say no you can not cancel there is no multiplication between f(x) and g(x) it's an addition." Instead, Sergen asked his students to consider (2+3)/2:

> Let's add the numerator. 2+3=5 divided by 2 is 2.5. Alright. Let's do the other way, (2+3)/2, 2's cancel so the answer is 3. So which is correct? Students say well no it should be 2.5. And another one says no it's 3. O.K. so why do we get two different answers. Two different ways we should get the same answer right. They say yes, we should get the same answer. So how does that work? And then we start talking about the identity element of addition and subtraction, and the identity element of multiplication and division…

In line with NCTM (2000) recommendations for teachers to create a culture of learning in their classroom in which students construct conceptual mathematical meaning through problem-solving and communication, Sergen wants to create a community of learners in his classroom by encouraging students to struggle with new concepts and to create and communicate solutions to problems. Sergen's syllabus for AP Calculus BC reveals some of his beliefs about the teaching of mathematics. In this document, Sergen explains how he probes student thinking by asking “what if” questions in order to stimulate discussions of concepts. He also includes a detailed description of his use of small group discussions in which students are grouped into “expert groups” and “home groups”:

> Within the expert groupings, student work with a small group of students to solve and fully understand a set of problems. They then return to their home group, and serve as the facilitator for their home group to solve a variety of problems. In this manner, each student is an “expert” for their questions, while being coached to successful solutions to all problems by other members of their “home” groups. This strategy is effective as it improves student involvement and ownership of achieving mastery in the desired content areas. Having to discuss and explain solutions verbally strengthens their understanding of the solution rationale.

Much can be learned about Sergen as a teacher from the above artifact: his use of cooperative learning, his belief in empowering students by giving them ownership of their learning process, and his belief in verbal communication as a means to promote deep conceptual understanding.

Like John, Sergen is rarely “on pace” and like Michael, he is not a linear teacher. Although Sergen experiences much resistance from his students when introducing confusion into his teaching, he believes it is the only way to reach a deep, conceptual understanding of mathematics: “I'm always behind the pacing guide because the pacing guide asks me to perform a laundry list of topic that needs to be thrown at students. So, I feel bad when I cover the curriculum because I'm not going to equip students with strong understandings.”

Not being "on pace" was a source of struggle and frustration for both John and Michael. I asked Sergen how he manages to navigate different goals without frustration. Sergen described his approach to tasks and objectives as "amorphous." Instead of focusing on strict objectives that are related to a section in the textbook and given a time frame, Sergen looks at the pacing guide more in terms of concepts:

> I look at it very globally. O.K. we need to teach the function concept here. Second goal we need to look at polynomials. Sure. The third goal we need to study exponential and logarithmic functions. After I look at that, I never look back and say did I make sure that objective 3.3. Logarithmic such and such is accomplished. I don't care about that.

Being a reflective practioner himself, Sergen cultivates reflection in his students as he wants them to be aware of themselves as active participants in their mathematics learning and as intellectual beings. "I try to empower students with some powerful ideas and then having them enjoy and master and solve problems so they feel empowered to think."

He does this by respecting and incorporating different types of knowing and by creating a classroom environment in which discussions about conceptual underpinnings are daily occurrences. An example of a classroom interaction between Sergen and his students will illustrate how he "covers" procedural and conceptual understanding as well as empower his students as intellectuals. On the board: $f(x) = \sqrt{-x}$ and its corresponding graphical representation:

Student 1: You can't have a negative inside the radical!

Sergen: Did you see me writing negative on the board inside a radical. I'd never do that! I would never do. And I've never done that. Anybody see a negative number inside the radical?

Student 2: Yeah, there's a negative there isn't there?

Sergen: No there's no negative number there. Oh I see. Are you talking about this $-x$?

Student 2: Yeah.

Sergen: That's not negative that's positive isn't it?

Student 3: Yeah, yeah, it's positive because *x*'s are negative! Look at the graph, the graph is facing towards negative infinity and *x* is all from 0 towards negative infinity. When *x*'s are negative and you multiply by a negative it becomes positive.

Sergen: Oh, alright, that's interesting. So let's try to understand that what he said. Can somebody else paraphrase that?

Another way that Sergen promotes reflection in his students is by encouraging "messy answers" in which students do not erase any part of their thinking, but leave everything on the page, perhaps with some parts crossed out, in order to better reflect on their process:

> I like and I express to my students that I like messy answers because those are what we have and then we need to work on those and reflect on those and develop stronger concepts but at the same time, I'm kind of shaped or molded into a kind of a curricular spectrum sort of speak, curricular mind set, where accuracy and rigor and procedural math problem solving focused on, you know, answers, are welcome and are being tested on the EOC test on standardized test.

Conclusion

Sergen's viewpoint is constructed from a diverse range of ideologies (i.e. *bricolage*) which are woven together relatively harmoniously. Sergen was shown to navigate the epistemological rocky waters of mathematics education by choosing to view conceptual understanding as the connective tissue between his various goals as a mathematics teacher: helping his school's reputation in the broader community by producing high EOC scores, empowering his students as intellectuals and preparing students to be competitive in the knowledge society. Although Sergen wants to prepare his students to be intellectuals who can participate in the knowledge economy, he also feels that much of the global competitiveness discourse present in his school and school district is just "lip-service" because the meaning and pedagogical implications behind the rhetoric have not been discussed. Like John and Michael, Sergen was left to individually decipher what is meant of him as a mathematics teacher when he is asked to prepare students to be competitive in the global economy.

While Sergen must make decisions that affect student learning daily, frustration and struggle did not appear as prominent themes in Sergen's case, as he could often find a suitable compromise. For instance, although Sergen's beliefs were greatly shaped by the constructivist and sociocultural approaches to mathematics teaching and learning found in the mathematics education literature, he does not get frustrated in trying to incorporate constructivist ideas in his classroom despite the time and epistemological constraints.

Instead, Sergen mediates the tension between the external discourse and his *habitus* and ideology by consciously making well-planned "mistakes" and then facilitating discussions in order for students to “reflect, see, and assimilate, because they can assimilate. They do have similar conceptions.” Furthermore, the “amorphous” manner in which Sergen views objectives on the pacing guide helps dull the possible frustration he might feel at never being “on pace” and not completely actualizing his beliefs.

Summary and Introduction to Chapter 5

In Chapter 4, individual case study reports for John the Commuter, Michael the Boxer and Sergen the Diplomat were presented. These case study reports offered a discussion of each participant’s school context, their experience as high school mathematics teachers with the global competitiveness discourse, and the personal journey that has shaped their current beliefs about the nature of mathematics, its role in society, and its teaching and learning. The main themes for John the Commuter (two worlds, the real world and decisions), Michael the Boxer (balancing act and battle), and Sergen the Diplomat (incorporating different types of knowing and reflection & discussion) were discussed at length in an effort to examine how clashing epistemologies are navigated by individual mathematics educators who have been strongly influenced by mathematics education research's perspective, yet must work in an educational system shaped by the demands of globalization.

I now move to present the conclusions of this research. In Chapter 5, the major themes that emerged through a cross-case thematic analysis of the three individual case studies will be discussed, and implications for teacher education and areas of future research will be suggested.

CHAPTER 5: CONCLUSION

This qualitative study was designed to explore the manner in which mathematics educators navigate different beliefs about mathematics, its role in society, its teaching and its learning. One of the aims was to better understand how the greater socio-political milieu, as described in Chapter 2, affects mathematics education by providing a thick description of mathematics teachers' lived experience within this milieu.

In this chapter, I begin with the limitations of the study. Next, I examine in greater detail two major themes that emerged through a cross-case analysis of the three individual case studies reported in Chapter 4: "Navigating rocky waters" and the "Global discourse." This discussion will be focused on answering the research questions posed in Chapter 1:

1. How do individual, successful teachers navigate the beliefs shaped by mathematics education research, workshops, methods classes and the discourse of preparing students to be competitive in the global economy?
2. How do mathematics educators experience the periods of conflict, reflection and resolution between the different belief systems to which they have been exposed?

Then, implications for teacher education are discussed, as well as areas of further research that would enhance this study.

Limitations of the Study

Prior to discussing the conclusions of this study, it is necessary to highlight some limitations. It is not possible to generalize from the experiences of these three mathematics teachers in order to make prescriptive suggestions as to a clear course of action. The purpose of this qualitative study was to chronicle the results of the exploration of a particular phenomenon—the lived experience of three mathematics educators as they navigate different belief systems when making professional decisions related to their work as teachers. As discussed in Chapter 3, although generalization is always a concern with case study research, Yin (2003) has argued that case studies "are generalizable to theoretical propositions and not to populations or universes" (p. 10). This is accomplished below by discussing broader theoretical issues alongside the major themes which emerged from a comparison of all three cases.

It is my hope that an audience of mathematics educators will be able to relate to this study, to these three teachers' experiences depicted through thick descriptions, and that this could help influence the pedagogical practices of mathematics teachers and mathematics teacher educators and motivate future studies in this area.

Cross-Case Analysis

In what follows, the findings of a cross-case analysis of two major themes—navigating rocky waters and global discourse—are discussed.

Navigating Rocky Waters

The three individual in-depth case studies for John, Michael and Sergen presented in Chapter 4 document the epistemological rocky waters that the successful classroom mathematics teachers must navigate.

Beliefs. John, Michael and Sergen have worked arduously to change their practice and beliefs to include some mathematics education research recommendations and findings – taking undergraduate and graduate courses in mathematics and mathematics education; familiarizing themselves with research findings; reflecting upon their practice and reorganizing their belief system about mathematics and its teaching. They are passionate, dedicated, reflective practioners.

These three high school mathematics teachers are well versed in mathematics education theories. They are familiar with the epistemological underpinnings of constructivism and have incorporated, to varying degrees, the work of Piaget, Vygotsky, Von Glasersfeld, Steffe, Cobb, etc. into their beliefs about mathematics and its teaching. They have read research articles about small-group learning, writing, alternative assessments, differentiated instruction, etc. They are familiar with recommendations by the National Council of Teachers of Mathematics (NCTM, 2000) encouraging mathematics teachers to develop meaningful, conceptual learning through problem solving.

In particular, John was shown to have moved from a strictly instrumental belief system about mathematics, mostly shaped by his time in the textile industry, to a belief system that incorporates elements of constructivism, such as scheme theory. According to John, his studies in mathematics and mathematics education bestowed him with a "maturity" as a teacher, which he described as a change in beliefs from "what's the best way to teach this to what's the best way to get the students to learn this. There's a difference between the two. I'm not sure that I recognized those differences early on."

John, who admittedly feels most comfortable (*habitus*) in a traditional, lecture, linear mode of teaching, and believes in the industrial standard, was shown to have a willingness to relinquish some of his control as the possessor of an absolute mathematical ontological reality in order to help his students' individual sense making. John has incorporated small group learning, writing and attempts at constructivist activities in his practice.

Michael has also been shown to have internalized many of the beliefs promoted by the research through his mathematics education studies and his participation in workshops in the PEAK Teaching for Excellence Model ™. He believes in guiding the development of his students' critical thinking skills through open-ended and ambiguous problems, affective education (*e.g*. his cool down questions) and differentiated instruction as defined in the research (as opposed to learning styles). Michael considers himself a "standards" driven teacher, that is, he has adopted the principles and standards set forth by NCTM (1989, 2000).

Sergen's beliefs about mathematics and its teaching were greatly influenced by the constructivist and sociocultural perspectives in mathematics education. His understanding of what constitutes mathematical knowledge was drastically changed by his humbling experience during his graduate studies in mathematics education, as recounted in Chapter 4. Sergen allows for "different types of knowing" in his classroom, while encouraging the development of deeper conceptual understanding in his students through reflection, discussions and confusion.

Furthermore, all three teacher-participants have sophisticated mathematical content knowledge, having, at a minimum, the equivalent of a Bachelors with a major in

Mathematics. These three participant teachers have taken such advanced courses as non-Euclidean geometry, abstract algebra, differential equations and analysis.

In sum, John, Michael and Sergen have changed their basic epistemological perspective about the nature of mathematics and its acquisition to be more in line with the recommendations coming out of the field of mathematics education, and have the sufficient content knowledge to put these new beliefs into practice. Nevertheless, as was seen in their individual case studies, these three participant-teachers still struggled to find a balance between the various, sometimes overlapping and sometimes clashing beliefs about effective mathematics teaching.

Navigate. Although their lived experiences shared many similarities, the manner in which John the Commuter, Michael the Boxer and Sergen the Diplomat chose to navigate the different belief systems about mathematics, and the pedagogical practices they imply, also presented some marked differences. These differences provides a more complete portrait of the various approaches individual, successful teachers find to mediate the beliefs shaped by mathematics education research, workshops, methods classes and the discourse of preparing students to be competitive in the global economy. It also provided depth in understanding how mathematics educators experience the periods of conflict, reflection and resolution between the different belief systems to which they have been exposed.

Leatham’s (2006) framework for viewing mathematics teachers’ beliefs as sensible systems in which belief clustering allows for different sometimes contradictory beliefs to be accessed depending on context is especially enlightening when attempting to understand why John, for instance, who was pursuing a Ph.D. in mathematics education at the time of the study, would revert back to lecture and skill and drill in certain contexts. Although John's *habitus*, his set of dispositions, are in line with the application of the corporate model as applied to education (i.e. linear, traditional teaching, with measurable, testable outcomes), John's ideology and discourses have been intensely reshaped by his studies in mathematics education. John mediates between his dispositions and discourses by a combination of clustering and *bricolage* – requiring him to commute between worlds. John has internalized different beliefs in clusters and he calls upon the appropriate cluster depending on context in order to function successfully as a high school mathematics teacher. Although this provides him with a high level of comfort while inhabiting and working within different worlds, it also diminishes his potential to affect change. The world of the classroom with its traditionally positivist, absolutist beliefs about the nature of mathematics and its teaching remains fairly untouched by the constructivist and sociocultural perspectives on knowledge with which John has become familiar during his mathematics education studies. Research findings are allowed to visit the world of the classroom if they are proven to raise achievement scores.

Michael, whose beliefs are in constant competition with the standardization and positivist beliefs about mathematics and its teaching espoused by his school, chose to wage battle and, occasionally, seek balance. Michael's priorities as a mathematics teacher – mathematics literacy, communication and ambiguity – are certainly in line with recommendations by organizations such as The Partnership for the 21st Century Skills (2007). This organization identified the ability to "work effectively in a climate of ambiguity and changing priorities" as well as the capacity to "collaborate and cooperate effectively with teams" as crucial skills with which to equip students in order for them to fully participate in the knowledge society.

Yet, Michael's beliefs and goals as a mathematics teacher frequently collided with the demands placed on him by his school context in which mathematical knowledge has been interpreted through a positivistic epistemological paradigm. What is revealed in Michael's case is the rocky waters that a teacher must navigate as he constantly chooses between his inherit desires to teach in a way in which ambiguities exists in order to prepare his students for critical and non-linear thinking, and the absolutes that must exist in a mathematics classroom reigned by accountability and measurable outcomes. Although Michael the Boxer was shown to have a strong sense of agency in the face of impediments to the actualization of his beliefs about the teaching of mathematics, his case also reveals the great personal cost that a teacher might pay when choosing to mediate the different beliefs through battle.

Sergen's case was in stark contrast to John the Commuter and Michael the Boxer who struggled or battled to balance the pedagogical implications of the different epistemologies available to them. Although Sergen must make decisions, and mediate between different belief systems daily, frustration and struggle did not appear as salient themes in his case. The radical shift in Sergen's personal epistemology which occurred during his mathematics education graduate studies made him mindful of the various beliefs about mathematics, its learning, its teaching and its role in society. Sergen believes that there are many different kinds of knowing and he views it as his role as a mathematics teacher to successfully accommodate all types of mathematical knowledge.

Although for John, raising EOC scores and using his knowledge of constructivist and sociocultural theories to advance conceptual understanding were clustered into two different worlds between which he had to commute, and for Michael competing goals for which he had to stand his ground, find balance and fight, Sergen views fostering conceptual understanding as both the tool to potentially increase achievement scores on procedural tests and empower students as intellectuals. Because of this connection in his belief system, Sergen navigates the rocky waters of mathematics education without much frustration or inner conflict.

This dissertation study helps explain why changing teacher beliefs is not sufficient to completely change teacher practice, even if the adequate mathematical content knowledge has been acquired by the teachers. The lived experiences of these three mathematics teachers could only be understood when taking the greater socio-political milieu in which teachers' work is situated into account.

Global Discourse

During the course of exploring how mathematics teachers navigate different belief systems related to their work as teachers, a sub research question suggested itself: How do individual mathematics teacher interpret and mediate the major global educational discourses of neoliberalism and the knowledge society?

John, Michael and Sergen all discussed the presence of the global discourse of competitiveness in their school context and in their lives as mathematics teachers during their interviews. Both Michael and Sergen received an e-mail from the superintendant of the DEC school district at the beginning of the 2008-2009 school year, reminding them of the overarching mission of the district: "educating children for the global economy that they'll be joining as adults." The global discourse is part of the landscape of their experience as high school mathematics teachers, the motto "Reach Further. Global competitiveness starts here" ever-present in the form of banners at the entrance of their school, in e-mail communications and on top of their teacher websites. In Michael's words, schools have "fallen in love with the global economy talk."

The focus on raising mathematics achievement scores, especially in the EOC courses Algebra I, Algebra II and Geometry, is inexorably tied in their school context to the goal of preparing students to compete in a knowledge-driven, ever-changing global economy. In some cases, such as Sergen's school Alleny Academy, general guidelines as to the skills teachers should be imparting to their students are also provided: the ability to be creative, identify and solve problems and to make data-driven decisions. As mentioned in Chapter 4, this description is designed to mirror the literature which argues that the role of K-12 educators in the preparation of students to be competitive members of the knowledge society is to help them learn how to learn, think creatively and problem solve (Cooney & Shealy, 1997; Lee, 2005; Spring, 2008; Valimaa & Hoffman, 2008). Of course, these guidelines are vague and were of very little help to Sergen in guiding his practice as a mathematics teacher.

All three participants related being aware, to varying degrees, of newspaper reports tying mathematics achievement scores on standardized tests to national policy discourses of global competitiveness emphasizing a strong causal relationship between mathematics achievement scores and economic prosperity. As discussed at length in the literature review (see Chapter 2), mathematics education is often tied to notions of national economic competitiveness. The result is an educational discourse of competition and a focus on pragmatic knowledge, as seen in the school contexts of all three participants, in which high-stakes tests are the valued means of assessing knowledge and the success of a mathematics teacher. All three participants mentioned the desire to do what was best for their school as a whole. To all three teacher-participants, this meant raising the school's reputation in the wider community, and "the measure of a good reputation is good EOC scores" (Sergen, Interview 2, 11/02/08).

However, the concrete epistemological and pedagogical implications of such a discourse were not discussed in the school contexts or in the mathematics education classes of all three teacher-participants. Neither John, Michael, nor Sergen recalled a meeting with teachers or administrators in which details were given about what exactly preparing students "to compete on a global scale" would require of a mathematics teacher. Apart from brief (and alarmist) mentions of NAEP, TIMMS and PISA results, none of them could point to an instance when the greater socio-political context was examined in their mathematics education research classes or in their methods classes. The superficial use of the global competitiveness discourse left these teacher-participants, especially Michael and Sergen, feeling like it was "lip-service" or "talk," detached from their roles as mathematics teachers. As explicitly mentioned by Sergen, for him to take this discourse into consideration "they first have to lay out what that means."

John, Michael and Sergen were thus forced to make sense of their role in the preparation of students for the global economy/knowledge society, without the help or guidance or their school or mathematics educators in their mathematics education programs. Not surprisingly, each participant interpreted this discourse in somewhat differing ways—John choose to focus on the concept of the industrial standard, data analysis, collaboration and adaptability; Michael on mathematics literacy, communication (i.e. writing) and ability to cope with ambiguity; and Sergen on conceptual understanding (through much confusion), communication and collaboration (i.e. discussions) and on empowering his students to become intellectuals.

Out of the three teacher-participants in this study, John had done the most outside readings on globalization and its implications for education, reading books such as *The World is Flat: A Brief History of the 21st Century* (Friedman, 2005) and *Growing up Digital (*Tapscott, 1997). This heightened awareness resulted in a clearer view for John of the skills he felt should be cultivated in students in order for them to thrive in the new digital, knowledge society. John makes explicit mention of globalization in his classroom, showing the video *Did You Know?* based on *The World is Flat* in order to promote discussions. He also has chosen to focus in his class on skills, such as collaboration and data analysis, which are directly related to his belief about the society for which he is preparing his students.

Although each participant had different approaches, there was surprising agreement from all three teacher participants that students needed to be able to adapt, collaborate, cope with uncertainty, and gain a deeper knowledge of mathematics through a focus on analytical or conceptual understanding in order to be competitive in the knowledge society. That these goals were epistemologically opposite to much of the demands of the corporate managerialism which has entered the structure of education, such as high stakes testing and continuous monitoring, was also present in their discussions.

As both John and Michael had considerable industry experience previous to their teaching careers, and that both were in industries that have seen drastic changes in the past decades (textile and IT) they were particularly sensitive to the fact that students need to be able to adapt and deal with an uncertain "world of non-linear change" (Lee, 2005, p. 172). For John, equipping students with *adaptability* is mostly centered on technological savviness and data analytical skills. Michael, on the other hand, chose to infuse adaptability and ambiguity in the form of open-ended problems and "cool down" writing prompts which had "no right or wrong answer." Although Sergen did not mention the fact that he was preparing students for an uncertain world for which they would need *adaptability*, Sergen creates a classroom environment in which *confusion* is welcomed and in which he fights the culture about the teaching and learning of mathematics in which the role of the teacher is to "tell students whether they are right or wrong quickly." Although all three participant-teachers incorporate the "ambiguity," "confusion," and "uncertainty" in their teaching practice, John and Michael do so because they feel they are preparing students for an uncertain world. Whereas Sergen incorporates confusion in his teaching because it is tied to his beliefs about "understanding" which is "messy" and involves "lots of confusion."

See Table 10 below for a summary of John, Michael and Sergen's interpretation of the global discourse present in their school, the pedagogical implications as interpreted by each participant and representative quotes from their interviews.

Table 10. John, Michael, and Sergen’s interpretations of the global educational discourse.

	Skills	Pedagogical Implications	Representative Quote(s)
John	Industrial standard	Linear, traditional, lecture focused on testable items.	- The business of learning - I teach like a textile factory. A linear progression. - I want to be the center of attention, so I want to stand in the middle and talk, and have all them listen to me.
	Data Analysis	Focus on problems solvable through statistics and modeling	The future of any jobs in America is going to be for those who can take information and use it beneficially.
	Collaboration	Small group learning	Well the small group learning thing focuses more on how can the student learn rather than how can the teacher teach.
	Adaptability	Technological fluency and problem solving	You need to be able to jump into the next job and have a fluid transition. The link between all of that is technology.
Michael	Math Literacy	Focus on arithmetic and “real life” applications (e.g. compound interest)	My goal in all my classes is to teach math literacy
	Communication	Cool down/writing	When somebody says to me "where is it in the math curriculum?,” and I’m going it’s reading, I'm in high school, I think it’ll help. And they’re saying "Well shouldn't that be taught in English?"
	Ambiguity	Cool down/ambiguous or open-ended questions	I tried to have ambiguous statements more than multiple questions. There would be two thoughts that would come together. There is no right answer.

Table 10. (continued)

Sergen	Conceptual Understanding	Create a community of learners – students wrestle with ideas, create and defend solutions to problems.	I like and I express to my students that I like messy answers because those are what we have and then we need to work on those and reflect on those and develop stronger concepts.
	Communication and Collaboration	In-class discussions where confusion is created and student reflection promoted.	- In my class discussions, I promote confusions. I promote an environment, try to create an environment there will be a lot of confusion. A lot of "what?" "What is that?," "I don't understand," "What do you mean?" - That global competitiveness!! That's a skill (communicate mathematically) that they learn and I want them to be more aware of themselves intellectually.
	Empowering/Intellectuals	Through discussion and promoting conceptual understanding.	I try to empower students by not requiring them, here's what you gotta do. But instead exposing them to some powerful ideas and then having them enjoy and master and solve problems so they feel empowered to think.

Implications for Teacher Education

> The paradox of education is precisely this - that as one begins to become conscious one begins to examine the society in which he is being educated. The purpose of education, finally, is to create in a person the ability to look at the world for himself, to make his own decisions, to say to himself this is black or this is white, to decide for himself whether there is a God in heaven or not. To ask questions of the universe, and then learn to live with those questions, is the way he achieves his own identity. But no society is really anxious to have that kind of person around. What societies really, ideally, want is a citizenry which will simply obey the rules of society. If society succeeds in this, that society is about to perish (Baldwin, 1988, p. 4).

Though one can not generalize from three cases to offer prescriptive suggestions, the findings of this dissertation study imply three starting points of conversations for mathematics teacher educators (MTEs). The first implication is:

1. The effects of the greater socio-political milieu can not continue to be ignored in mathematics methods classes, mathematics education classes, mathematics content classes for future teachers, and in professional development.

As educational policy has become dominant over educational theory, mathematics teachers have lost much of their autonomy, their professionalism, and their agency to affect change (Kohl, 2009; Lewis, 1998; Thomas, 2001). Mathematics teacher educators can help teachers regain some of their agency and professionalism by inviting them into a conversation of the political components of their chosen profession.

As was seen, the global discourse of competitiveness is very much present in teachers' school contexts and in their lives as mathematics teachers. However, as was also seen, the epistemological or pedagogical implications of this discourse are not discussed in teachers' school contexts or in their mathematics education studies. If mathematics teachers are to be charged with equipping students with the skills and deep cognitive abilities they will need to participate in the global knowledge society (Valimaa & Hoffman, 2008), an in-depth discussion of this discourse needs to be present in teacher education programs. MTEs can help individual mathematics teacher interpret and mediate the major global educational discourses of neoliberalism and the knowledge society. There is an urgent need for MTEs to facilitate a critical dialogue with teachers centered on questions such as:

- What mathematical knowledge should I help my students gain in order to prepare them to be global citizens of the knowledge society?
- How do I resolve what the constructivist and sociocultural perspectives have taught me about mathematics education with the government policies which reflect a view of effective mathematics teaching focused on measured outcomes, accountability and standardization?

This conversation does not preclude the traditional focus in teacher education on enhancing teacher mathematical content and pedagogical knowledge. It is meant to be complementary. Relevant and effective teacher education must enable teachers to examine the nature of teachers' work by making explicit mention of the socio-political context of education. As McLaren and Baltodano (2000) noted, teacher education should "locate the schooling process in both local and global socio-economic and political contexts, while exploring the relations between them" (p. 43).

The second implication is:

2. Given the era of effectiveness focused on measured outcomes and accountability now present in schools (Ellis, 2005; Hill, 2008; Kohl, 2009; U.S. Department of Education, 2008), mathematics teacher educators can help teachers develop ways to cope with the conflicts and dilemmas they will surely face when trying to actualize the beliefs about mathematics learning and teaching they have acquired through their exposure to mathematics education research.

It is not sufficient, for instance, to discuss the ways that writing can be used in the mathematics classroom to promote students' conceptual understanding, without mentioning the probable impediments one will face in incorporating writing in one's practice. Changing teachers' beliefs about writing's place in the mathematics classroom is inadequate in transforming practice if a teacher works in a context which views writing as "not part of the math curriculum." Such impediments to the actualization of teacher beliefs into practice should be explicitly addressed in teacher professional education (Cole, 1997).

Lastly, as was seen in Chapter 2, understanding and changing teacher beliefs is vital to reforming mathematics education (Beck & Kosnick, 2006; Leder, Pehkonen & Torner, 2002). The fact that three of the original six participant-teachers (i.e. Martin, Martha and Lea), all with extensive mathematics education backgrounds, had considerable difficulty in articulating their beliefs about mathematics and its education suggests a third implication for MTEs:

3. Mathematics teacher educators should provide opportunities for teachers to formulate and dialogue about their beliefs.

This conversation could touch upon teacher beliefs about the nature of mathematics, about its learning and its teaching. Furthermore, teachers should be encouraged to reflect upon their beliefs about the role of school and mathematics in society.

Areas for Future Research

In bringing this dissertation study to a close, I suggest four main avenues for future research and conversation.

Firstly, this study focused on three special cases. Further study needs to be done to describe the process by which mathematics educators *successfully* mediate between the constructivist and sociocultural perspectives on mathematics education and the view of effective mathematics teaching focused on measured outcomes, accountability and standardization present in their school contexts. Incorporating a greater number and diversity of teachers in future studies would broaden the scope of the findings. Future studies on this topic would enhance the in-depth description provided herein of the struggles, mediation, conflicts, resolutions and decisions that classroom mathematics teachers face when navigating differing beliefs about mathematics teaching and learning.

Secondly, studies on teacher retention are also suggested by the findings of this dissertation, especially the case of Michael the Boxer. How many qualified, passionate and dedicated mathematics teachers leave the teaching profession when they are prevented by their school context to move from an emphasis on procedural knowledge to conceptual knowledge?

Thirdly, this dissertation highlights the need for studies of mathematics teacher beliefs which resist the reductionism of categorization, and which take into account the greater socio-political milieu in which teachers' work is situated. Gates' (2006) admonition of much mathematics education research, quoted in Chapter 2, bares repeating here: "To claim that studies of mathematics and mathematics teachers can only reside within mathematics itself will fail to address the very foundations upon which much mathematics and many teacher beliefs rest" (p. 347).

Fourthly, though this study focused on the lived experiences of three specific high school mathematics teachers, these case studies were meant to be *instrumental* in gaining a greater understanding of the effects of a bigger, socio-political phenomenon—the effects of globalization and its associated discourses on mathematics education. Future research focused on producing a theoretical model of the effects of the greater socio-political milieu on mathematics education, including the effects on mathematics teachers, would be an immensely significant contribution to scholarship.

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APPENDIX A: INVITATION E-MAIL TO PARTICIPATE IN STUDY

Dear (name withheld),

My name is Amelie Schinck. I am a doctoral student at UNC Charlotte, preparing to do my dissertation study, and I am inviting you to be a participant. I am contacting you on the recommendation of (name) who spoke to you about my topic and told me you had shown some interest in being a participant. My study explores how mathematics teachers, like you, navigate the different belief systems about the teaching of mathematics.

If you are interested in participating, I will want to interview you at least three times during the Fall of 08, starting the third week of September. Each interview will last between 1 and 2 hours. We can schedule these interviews at a time and location that is convenient for you. You may benefit from participation in this study because it will provide a space for you to reflect on your practice as a teacher. A pseudonym for you will be used in my writing.

If you would like more information, or if you feel you would be interested in participating in my study, please contact me at agschinc@uncc.edu. You are also welcomed to call me at 704-345-7014.

Thank you for your time and have a great day,

Amelie Schinck
Doctoral Candidate in Curriculum and Instruction
Mathematics Education
University of North Carolina at Charlotte
9201 University City Blvd.
Charlotte, NC 28223-0001
704.345.7014
agschinc@uncc.edu

APPENDIX B: INFORMED CONSENT FORM

College of Education
Department of Middle, Secondary,
and K-12 Education
704-687-8875

Project Title and Purpose:

Navigating the Epistemological Rocky Waters; A Multiple Case Study will be a qualitative study involving four high school teachers. The purpose of the proposed study is to provide a thick description of the lived experience of four mathematics educators as they navigate different belief systems about mathematics and its teaching when making professional decisions.

Investigator:

This study is being conducted by Amélie G. Schinck, B.Sc.; M.Sc. and Ph.D. Candidate at the University of North Carolina at Charlotte. The responsible faculty member is Dr. David K. Pugalee in the Department of Middle, Secondary, & K12 Education at UNC Charlotte.

Description of Participation & Length of Participation:

Case study methodology will be used in this study. You will be asked to participate in interviews, keep a journal, and keep relevant archival data such as memos and minutes of meetings. Initial interviews will be conducted in the beginning of August 2008. A minimum of three in-depth interviews will be conducted with each participant. Final interviews will be conducted between November and December of 2008. All interviews will be tape-recorded. Each interview will last approximately 1 to 2 hours.

Risk and Benefits of Participation:
There are no known risks to participate in this study. However, there may be risks which are currently unforeseeable. You may benefit from your participation in this study because it will provide a space for you to reflect on your practice as a teacher.

Volunteer Statement:

You are a volunteer. The decision to participate in this study is completely up to you. If you decide to be in the study, you may stop at any time. You will not be treated any differently if you decide not to participate in the study or if you stop once you have started.

Confidentiality:

Any information about your participation, including your identity, is completely confidential. The following steps will be taken to ensure this confidentiality:

1) You will choose a pseudonym, first and last name that will be used in all transcripts and publications.
2) In any publication of the research results, your school's identity will be masked.
3) All tapes will be labeled with your pseudonym.
4) Only people directly involved with analysis of the data will have access to the tapes and transcripts.
5) All interview tapes will be destroyed before the expiration of the study.
6) All records will be stored in a locked file cabinet, only available to the principle investigator.

Fair Treatment and Respect:

UNC Charlotte wants to make sure that you are treated in a fair and respectful manner. Contact the university's Research Compliance Office (704.687.3309) if you have questions about how you are treated as a study participant. If you have any questions about the actual project or study, please contact Ms. Amélie G. Schinck (704.345.7014; agschinc@uncc.edu) or Dr. David Pugalee (704.687.8887; dkpugale@uncc.edu).

Approval Date:

This form was approved for use on 02/18/2008 for use for one year.

Participant Consent *(for participants who are at least 18 years of age)*

I have read the information in this consent form. I have had the chance to ask questions about this study, and those questions have been answered to my satisfaction. I am at least 18 years of age, and I agree to participate in this research project. I understand that I will receive a copy of this form after it has been signed by me and the principal investigator of this research study.

____ _________
Participant Name (PRINT) DATE

____
Participant Signature

____ _________
Investigator Signature DATE

APPENDIX C: INTERVIEW PROTOCOL

This is a representative, but not exhaustive list of interview questions. Multiple interviews were necessary to pose these questions. In general, probing questions were constructed based on a participant's response. The participant was encouraged to discuss relevant issues with the investigator, taking the interview in unforeseen directions. The goal of the first interview was be to establish rapport with the participant, gather background information, and discuss their beliefs about the teaching of mathematics, including how research in mathematics education has influenced them. Other areas of questioning such as 'context and standards' and 'decision and change' were introduced briefly in the first interview, but were central to the subsequent interviews.

1. Background

- Tell me a bit about yourself...
  - How old are you?
  - Where are you from?
  - Where do you teach?

- Describe your educational experiences so far in your life.
  - Where did you study?
  - What did you study?
  - What degrees do you hold?

- Describe your teaching experiences so far in your life.
  - How long have you been teaching?
  - Where have you taught?
  - What subjects have you taught?
  - What subjects are you currently teaching?
  - What grade level have you taught?
  - Why did you become a mathematics teacher?

2. Beliefs about the teaching of mathematics

- Describe what it means to you to be a good mathematics teacher.

- Describe what you think it means for your school/administrator/math department to be a good math teacher.

- There are many different types of mathematics teacher. What type of teacher do you think you are?
  - What are your priorities as a mathematics teacher?
  - What do you think it is important for your students to know?
  - List three adjectives that best describe you in the classroom.

- Describe what you think it means to your students to have a good math teacher.
    - How do your students view you? How would your students describe you?

- What do you like about teaching mathematics?

- How do you maximize student learning in your classroom?

- How do you describe your role as a teacher?

- How do you know when your students understand?

- How do you know when learning is occurring in your classroom?

- What do you dislike about teaching mathematics?

- List three adjectives that best describe you at school outside of the classroom.

- Give me a picture of your mathematics class. What does it look like?

- How do you feel when you are on your way to class?

- Tell me about the ideal teacher. The teacher you would be if you worked in a perfect environment.

- Tell me about activities that you perform as a teacher that you find meaningful. Meaningless.

- What are some of the essential skills you are trying to foster in your mathematics students (creativity, problem solving, real life applications...)
    - What is the role of mathematics in society?
    - What are you preparing students for in your class?

3. Epistemology

- How do you think people learn mathematics?

- What does it mean for someone to 'know' something (in math)...to understand something?

- How does a student show you he/she 'knows' or understands a math concept.

- What comes to mind when you think of mathematics (metaphor)

4. Mathematics Education Research

- Describe some of the mathematics education research that influenced you.
  - Who are some of the authors that inspired you?
  - What school of thought on mathematics education do you most identify with?

- What is the best way to teach mathematics? Is that how you teach? Why or why not?

- Describe some changes you have made in your overall approach to teaching because of mathematics education research.

- Did you ever take any methods classes? What did you learn about the teaching of mathematics in these classes?

- You said you were a constructivist (or alternate answer). What does that mean to you? Give me an example where you used that belief system in your classroom.

5. Broader Sociocultural Dimensions and Standards

- Describe the atmosphere of your school. What are some of the missions or goals?

  - What about the math department?
  - Describe a math department meeting that stuck in your mind? Positive/Negative.

- How do you get students ready for tests? Do you change your teaching method?
- Have you changed anything about your teaching because another teacher or a superior told you to?

- Tell me what 'math achievement' means to you?
  - How important is achievement in your school/department?
  - How do you measure 'math achievement'?
  - How would you measure achievement differently, if given the choice?

- What is the role of testing in your classroom? What about EOC tests? How do they affect your teaching method?

- How do you feel about EOC tests?

- What type of work do you ask students to do in the classroom? Outside the classroom?

- How do you use recommendations by NCTM in your classroom?

- Tell me about a moment recently where you took the 'easy way out' pedagogically. Where you knew what would be the best thing to do, but didn't do it. Why did you make that choice?

- How do you think your department/administration sees mathematics?

- What is a good result in your math classroom?
    - Tell me about a recent 'success' in your math classroom.

- Describe the atmosphere in your math department. Describe your role. Describe how you act in a departmental meeting.

- In the school setting, how do you decide what to teach and what not to teach?

- Who do you feel has control over what you teach in the classroom?

- Who do you feel has control over how you teach?

- How do you decide when to move on to a new topic in your classroom?

- Have you ever been asked to do things differently in your classroom by a superior? Describe this event.

- Do you feel the math curriculum that you teach supports learner/student-centered instruction? Explain. Give me a specific example.

- Question about satisfying state/national standards

- How is your effectiveness as a teacher assessed?
    - Student grades
    - Student ratings
    - Open-ended student comments
    - Classroom observations
    - other

- Have you attended workshops/professional development offered by your school? What did you learn about teaching math/expectations of teaching math from these professional development opportunities?

- What kind of jobs/world do you feel you are preparing your students for?

- I saw form your school's website that their mission is to prepare students for the global economy. What does that mean to you as a mathematics teacher? Is this a topic of discussion in mathematics department meetings?

6. Decision and change

- Can you give me an example of a pedagogical decision that you made recently where you relied on math ed. research?

- Give me an example of a decision that you made pedagogically where you based decision on 'experience' as opposed to research.

- What do you see as a major change in your mathematics teaching since you have started? What do you think caused this/these change(s)?

- As a math teacher, is there something that you do that is hard, that students don't like, that the administration doesn't like, but that you 'know' is the 'best method'?

- Give me an example of a view that you hold that is contrary to mathematics education research?

- What different ideas about math and its teaching do you see circulating in your classroom?

- Tell me about the development of your belief system. Literature, textbook, how you learned math.

- How have your beliefs about mathematics changed in your life?

- How have your beliefs about the teaching of mathematics changed in your life?

- Please describe a moment when you felt you were being the teacher you want to be. When you got close to your ideal of yourself as a teacher.

- Please describe a moment when you felt very far from being the ideal that you have of yourself as a teacher.

- Have you tried to implement some pedagogical choice based on research but were met with resistance? What did you do?

7. Good follow up prompts

- What has been on your mind since we last talked?
- Questions stemming from journal entries

APPENDIX D: MARTIN MIDDLETON INTERVIEW EXCERPT

After discussing what it means for this participant to be a "good teacher," I wanted to delve a little deeper into this teacher's beliefs about mathematics and its teaching. R denotes the researcher's part, and M denotes the teacher-participant's (i.e. Martin Middleton):

R: You gave me a metaphor, an image, for what mathematics is to you. Could you do the same for what being a mathematics teacher is?

M: (sigh) First thing that came to my mind was a short order cook.

R: O.K. Can you explain that?

M: I don't know why it came, it's just literally what happened. I was thinking of the guy you know, when you go to a restaurant, the guy back there doing the orders. You know someone hands him an order and he cooks whatever it is and hands it out. What I did yesterday on the last day, I had kids that hadn't been overly concerned about learning their math and aren't getting the grades they wanted, kids that are realizing that they need to take one last chance to pass for the quarter. And I had all sorts of kids in my class taking tests after school yesterday.

R: So you as the short order cook, what are you pushing out there?

M: Yesterday I was pushing tests to retake. So much of it is having a bag of tricks, things you know to do when you get into a particular situation. Uh, being able to teach well, and this is part of why you can't totally learn at a college setting how to be a good teacher is knowing o.k. whenever a kid does this here's your best reply to it. Having something at the ready.

Knowing the answer to your kids' off the wall questions. Knowing how to answer it.

R: The short order cook, that was because of the testing right? You're pushing out tests.

J: Well part that, but you never know on a day-to-day basis what you're going to deal with and knowing what you're going to do if you suddenly found out one of your students is pregnant. You don't on a day-to-day basis have an overall strategy. You may have one but, on a day to day you can't really tell so long as you've got reasonable responses to whatever goes on. In some ways, it's kind of like working at an ER. Person comes in with a particular type of an injury and you have to know what to do with it.

R: That's interesting, because it's very different from what you described when I asked you about the meaningful moments as a teacher. You told me about beautiful one-on-one moments where you...

J: I know!

R: You would see the light bulb...How would you paint an image of those moments as a teacher? That's not a short order cook or an ER doctor.

J: NO! That's true. And if you're getting, if you're having difficulty understanding what teaching is to me, it's probably because I don't know either.

APPENDIX E: EXAMPLE OF ASSIGNMENT GIVEN BY JOHN

Are SAT Scores Linked?

In this project you will use data given below on SAT-Verbal and SAT-Math scores to investigate the association between them. These are the actual scores of a group of students who took the exam in 2003. Your overall report should be a thorough and complete argument regarding your findings. In your report you should address the correlation coefficient, how it is derived, and its meaning within the context of the problem. Also tell which variable is an explanatory one and which is a response and why. Investigate in both ways – SAT Math vs. SAT Verbal, SAT Verbal vs. SAT Math and discuss the correlation changes, if at all? Furthermore, you should investigate how well one variable serves as a predictor for another. In this investigation discuss how the correlation coefficient influences the slope of the LSRL. Not only should you use the knowledge you have gained from chapter three about scatterplots, regression, analysis, and the appropriateness of the regression model but you may also want to consider using the display analysis knowledge you have gained from chapter one (i.e., boxplots or histograms with numerical summaries). The more substance you place in your argument the better your chance for a perfect score.

The report will summarize your investigation and must follow all guidelines of the AP Stat Project Format document located on the class webpage. This project counts as a test grade and will be scored according to the rubric located on the class webpage. The due date for the project is Friday, February 27, 2009 at the beginning of the class period.

SAT-Math	SAT-Verbal	SAT-Math	SAT-Verbal
680	780	570	500
450	570	600	510
440	550	700	680
610	500	720	770
730	720	650	800
530	570	670	660
700	600	800	590
640	530	800	800
740	800	570	580
650	740	590	600
580	550	610	680
520	590	580	640
620	580	690	600
700	740	660	580
640	560	620	670
710	660	750	560
700	730	610	610
580	610	540	500
520	480	500	470

APPENDIX F: MICHAEL GILFORD'S MIDYEAR REVIEW

Teacher: Michael Gilford
School: Gregory High School
Observation Date: December 1, 2008
Observer: Curriculum Resource Teacher

Management of Instructional Time:
Students were all seated and taking notes at their desks. The class was already started when I entered the room. The students were given a warm up on the coordinate plane and we were taking notes on the lesson for this topic. There were no materials needed for this lesson other than overheads and student notebooks. The teacher was using the book and a three ring binder to refer to when working through the lesson. The students were on task as far as taking notes quietly at their desks. The material presented was not an algebra 1 objective but a prerequisite skill taken from 8^{th} grade. The topic of study followed the East Mecklenburg Algebra 1a pacing guide. Due to the low level of material there were no higher level thinking skills being used.

Management of Student Behavior:
The students were aware of what they were expected to do. There was no inappropriate behavior noted other than a few students sitting in the front/right-hand of the classroom (looking at the classroom from the back) who were talking during instruction. This was not a disruption to the other class members and the behavior was ignored by the teacher. A few students in the room called out answers.

Instructional Presentation:
The objective on the board stated 4 -1 with the essential question "write coordinates on the graph." The instruction was teacher directed. The students only participated in quick one word answers through out the lesson. There was no room for critical thinking due to the level of the material. The students were not engaged in ongoing problem solving, rather copying material from the overhead. The teacher spoke fluently and had the assignment ready to hand out.

Instructional Monitoring:
The teacher moved throughout the desks one or two times during the presentation. The questions that were used were often followed by the teacher answering for the students. The students were not called on. One student in particular (red sweatshirt in the front of the room) answered 8 of the questions during the period I was observing. The teacher made reference to independent and dependent variables, a student asked what that meant and the teacher used a very relevant example to explain the terms using pulse rate and time. The teacher was very clear in describing these terms.

Instructional Feedback:
There was very little wait time for any instructional feedback. The students were copying notes and then asked to complete an assignment. Most questions were answered with one

word coming from random students. Questions were asked such as “which axis?,” “are you sure?,” “what quadrant?,” “you ok with this?.”

Notes from Observer:
The students were very well behaved in this classroom and the teacher seemed to have a good rapport with the students. The teacher was following the East Mecklenburg Algebra 1 pacing guide. The topic of study was very general and could have been made much more challenging for the students.

Suggestions:
Due to the nature of the topic an anticipatory set to see what the students already knew would have been appropriate. It is a shame to spend a whole class period on coordinate planes if the students already knew the material. There are several questions that can be asked to heighten the level of critical thinking during this lesson. The teacher could have used things as “If the x value is greater than zero and the y value is less then zero, what quadrant would you be in?.” I would suggest using questioning strategies to check for student understanding. Each student should be called on and answers should come from the students instead of the teacher. During “their turn” students could have been called up to the board or given ample time to finish the assignment. A lesson plan with terms and questions should be developed instead of having to go back and forth with the book. I suggest to the whole team to incorporate activities into the lessons that involve student participation.

APPENDIX G: MICHAEL GILFORD'S AFM I SYLLABUS V1

Day	**Date**	**Book / Section**	**Notes**	**Homework**
Monday	**8/25**		Hand out books, Review course scope.	Complete paperwork
Tuesday	**8/26**	(G) 1 – 3	Graphing Linear Equations	Chpt 1.3 Summarize, Examples 1, 2, 3, 4 Return paperwork
Wednesday	**8/27**	(G) 1 – 4	Writing Linear Equations	(G) 1.4 Summarize examples 1,2,3 (G) 1.3: 9,11,25-35 odd
Thursday	**8/28**	(G) 1.6	Modeling: constant change	(G) 1.4: 7-23 odd (G) 1.6 Summarize examples 1,2
Friday	**8/29**	(G) 1 **-** 7	Piece – wise functions	(G) 1.6: 7,15 odd (G) 1.7 Summarize examples 1,4,5
Tuesday	**9/2**	Section 5 (NC Dept of Public Instruction)	Class work Modeling with Piecewise functions	(G) 1.7: 1, 5,11,17,19
Wednesday	**9/3**	Section 5 (NC Dept of Public Instruction)	Class work Modeling with Piecewise functions	(G) pg 59 -60 31-51
Thursday	**9/4**	Section 5 (NC Dept of Public Instruction)	Class work Modeling with Piecewise functions	(G) pg 60-61: 53-69
Friday	**9/5**	**TBD**	**TBD**	
Monday	**9/8**	(G) 4 – 1	Polynomial Functions Quadratic Forms	Summarize 4-1 examples 1-5
Tuesday	**9/9***	(G) 4 – 2	Solving Quadratics graphing and Factoring	(G) 4.1: 1, 5-27 odd (G) 4-2 Summarize examples 1-5
Wednesday	**9/10**	(G) 4 – 2	Solving Quadratics completing the square and graphs	(G) 4-2 3- 15 odd
Thursday	**9/11**	(G) 4 - .3	Remainder and Factor Theorems	(G) 4-3 Summarize examples 1-5
Friday	**9/12**	(MMA) 4.4	Class work Homework 4.4 3-8	(G) 4-2 Summarize examples 1-5
Monday	**9/15**	(G) 4 – 2	Quadratic Equations: Completing the square	Chpt.4.2 1-19
Tuesday	**9/16***	(MMA) 4.5	Quadratics – TranslationClass work 1 – 7	Homework 1-8 handout.

Wednesday	**9/17**	(MMA) 4.5	Quadratic translation Class work 1-7 Homework 1-8	(G) 4.8 Summarize Example 1-3
Thursday	**9/18***	**TBD**	**TBD**	
Friday	**9/19**	(G) 4.8	Modeling Quadratics	(G) 4.8 1-25 odd (G) 3.4 Summarize Examples 1-5
Monday	**9/22***	(G) 3 – 4	Inverse Functions	(G) 3.4 1-13 odd (G) 11.2 Summarize Examples 1-5
Tuesday	**9/23**	(G) 11 – 2	Exponential Functions	(G) 3.4 15, 17, 21-33 odd 39 (G) 11.3 Summarize Examples 1-2
Wednesday	**9/24***	**TBD**	**TBD**	
Thursday	**9/25**	(G) 11 – 3	The number "**e**"	(G) 11.2: 9,19-20,23, 25,27
Friday	**9/26**	(G) 11 – 4	Logarithmic Functions	11-1, Exercises 19-35
Monday	**9/29***	(MMA) 7.3	Compounding and Logs	
Wednesday	**10/1**	**TBD**	**TBD**	
Thursday	**10/2**			
Friday	**10/3**	(G) 11 – 5	Common Logs	
Monday	**10/6***	(G) 11 – 6	Natural Logs	
Tuesday	**10/7**	(G) 11 – 7	Modeling Logs	
Wednesday	**10/8**		Modeling Project	
Thursday	**10/9**		Modeling project	
Friday	**10/10**		Modeling Project	
Monday	**10/13**	(G) 5 – 2	Trig Ratios	
Tuesday	**10/14**	(G) 5 – 2	ATrig Ratios	
Wednesday	**10//15**	**TBD**	**TBD**	
Thursday	**10/16**	(G) 5 – 5	Solving Right Triangles	
Friday	**10/17**	(G) 5 – 4	Applying Trig Ratios	
Monday	**10/20**	(G) 5 – 5	Solving Right Triangles	
Tuesday	**10/21**			
Wednesday	**10/22**			
Thursday	**10/23**			
Friday	**10/24**			
Monday-Friday	**10/27-10/31**	**MID TERM**	**EXAMS**	

End of 1st semester

Day	**Date**	**Book / Section**	**Notes**	**Homework**
Wednesday	**11/5**	(G) 5 – 6	Law of Sines	
Thursday	**11/6**	(G) 5 – 8	Law of Cosines	
Friday	**11/7**		Review	

Monday	**11/10**		**TEST**	
Wednesday	**11/12**	(G) 5 – 1	Angles and Degrees	***UNC Module 7 • Resources (Trigonometric Functions) from **NCSSM** (http://www.dlt.ncssm.edu/afm/topic.htm)
Thursday	**11/13**	(G) 6 – 1	Unit Circle	
Friday	**11/14**	(G) 6 – 1	Angles and Radians	
Monday	**11/17**	(G) 6 – 3	Graphing Sine and Cosine Functions	
Tuesday	**11/18**	(G) 6 – 4	Amplitude and period	
Wednesday	**11/19**	(G) 6 – 5	Translations	
Thursday	**11/20**	(G) 6 – 6	Modeling with Sinusoidal Functions	
Friday	**11/21**	TBD		
Monday	**11/24**	TEST	**EMPT TEST**	
Tuesday	**11/25**			
Monday	**12/1**	(G) 14 - 1 (MMA) 2.2	Decisions through Data Videos 1 & 2	**Decisions Through Data** Lessons 1-8 • ***UNC Module 2 • Resources (Univariate Data) from **NCSSM** (http://www.dlt.ncssm.edu/afm/topic.htm)
Tuesday	**12/2**	(G) 14 - 2 (MMA) 2.3	Decisions through Data Videos 3 & 4	
Wednesday	**12/3**	(G) 14 – 3	Decisions through Data Videos 5 & 6	
Thursday	**12/4**	(G) 14 – 4	Decisions through Data Videos 7 & 8	
Friday	**12/5**	Flex Day	Project (?)	
Monday	**12/8**	REVIEW		
Tuesday	**12/9**	**Tbd**		
Wednesday	**12/10**	(G) 13 – 1 & 13 - 2	Permutations and Combinations No circular permutations	[Omit circular permutations] • ***UNC Module 3 • "Basketball: With the game on the line …" and extensions, ***Resources for Algebra***. • Modules (Frankfurter High, Torn Shirts) from ***Does This Line Ever Move?*** (Key Curriculum) or **HS Operations Research** website (www.hsor.org) • Resources (Probability) from **NCSSM** (http://www.dlt.ncssm.edu/afm/topic.htm)
Thursday	**12/11**	(G) 13 - 3	Probability no odds	
Friday	**12/12**	(G) 13 – 4	Probability of Compound events	
Monday	**12/15**	(G) 13 – 5	Conditional probablility	
Tuesday	**12/16**	(G) 13 – 6	Binomial Theorem	
Wednesday	**12/17**	Flex Day	(MMA) 4.1 thru 4.3	
Thursday	**12/18**	REVIEW		
Friday	**12/19**	**Tdb**		
Monday	**1/5**	(G) 12 – 1 (MMA) 7.1	Arithmetic Series and Sequences	

<table>
<tr><td>Tuesday</td><td>1/6</td><td>(G) 12 – 2
(MMA) 7.2</td><td>Geometric Series and Sequences</td><td rowspan="7">• ***UNC Module 8
• Resources (Recursion) from NCSSM (http://www.dlt.ncssm.edu/afm/topic.htm)</td></tr>
<tr><td>Wednesday</td><td>1/7</td><td>(G) 12 – 3
(MMA) 7.4</td><td>Infinite Series and Sequences</td></tr>
<tr><td>Thursday</td><td>1/8</td><td></td><td>Flex Day</td></tr>
<tr><td>Friday</td><td>1/9</td><td>(G) 12 – 5</td><td>Sigma notation</td></tr>
<tr><td>Monday</td><td>1/12</td><td>(G) 12 - 6</td><td>Binomial Theorem</td></tr>
<tr><td>Tuesday</td><td>1/13</td><td>(G) 12 – 8</td><td>Sequences and Iteration</td></tr>
<tr><td>Wednesday</td><td>1/14</td><td>(MMA) 7.5</td><td></td></tr>
<tr><td>Thursday</td><td>1/15</td><td>Test</td><td></td><td></td></tr>
<tr><td>Friday</td><td>1/16</td><td>TEST</td><td></td><td></td></tr>
<tr><td>Tuesday-Thursday</td><td>1/19 – 1/22</td><td>Test</td><td></td><td></td></tr>
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APPENDIX H: MICHAEL GILFORD'S AFM SYLLABUS VERSION 2

Monday	**11/3**		Holiday	
Tuesday	**11/4**		Holiday	
Wednesday	**11/5***		Media Center	Using Excel, build a model
Thursday	**11/6***		Media Center	Sensitivity analysis.
Friday	**11/7**	5-6 and 5-7	Review Law of Sines	5.7: 19-29 odd 5.7: 31-33-35 odd;
Monday	**11/10**	5 – 8	Law of Cosines	UNIT Circle Quiz 5.8: 11-17 odd, 27, 29, 31 odd;
Tuesday	**11/11***		HOLIDAY	
Wednesday	**11/12***	Chapters 5.2 – 5.8	Review “Solving Triangles”	Study for test: Pg 336- 338: 23, 33, 37, 43, 45, 47, 53, and -55
Thursday	**11/13**	Test	Solving different kinds of triangles	
Friday	**11/14***	6 – 1	Conversion Degrees and Radians; reference Angles	6.1: 7- 15odd; 49-53 odd;
Monday	**11/17**	Review	Unit circle special cases 30, 60 90	
Tuesday	**11/18**		Sine = y, cosine = x	
Wednesday	**11/19***		Tan Cot, Sec	
Thursday	**11/20**	Quiz 7- 5	Unit circle Solving Trig Equations	7.5: 17, 18, 21, 23, 25, 26, 30, 31, 37, 39
Friday	**11/21***	Review		
Monday	**11/24**	Practice Placement	Ncempt.org	
Tuesday	**11/25**	Test	Unit circle, 6 trig functions, solving equations.	
	11/26 – 11/28	Thanksgiving		
Monday	**12/1**	6 – 3 6 - 4	Graphing Sine and Cosine Functions; Amplitude and period Using 5 points	6.3: 9-17 odd 6.3: 27, 29 6.4: 9-51 odd;
Tuesday	**12/2**	6 – 5	Translations of the Sine and Cosine	6.5: 9-13, 27-33 odd;
Wednesday	**12/3**	6 – 6	Modeling Sinusoidal Functions with a T table	6.6:1-15 odd
Thursday	**12/4**	6 – 7	Graphing Other Trigonometric Functions: Sec, Csc	6.7: 1-7 odd; 21-33 odd:

Friday	**12/5**	6 – 7	Graphing Other Trigonometric Functions: Tan and Cot	6.7: 37-43 odd
Monday	**12/8**	Review	Review 6-1 and 6-3 – 6-7	Pg 415 – 416: 11, 12, 17,19, 31, 33, 37, 39 and 41
Tuesday	**12/9**	Test		
Wednesday	**12/10**	12 – 1 12 - 5	Introduction to Series and Sequence Σ (Sigma) notation	12.1: 1-5 odd; 12.5: 1-5 odd;
Thursday	**12/11**	12 – 1,	Arithmetic Series and Sequences	12.1: 9-15 odd 12.1: 21-31 odd;
Friday	**12/12**	12 – 2	Geometric Series and Sequences	12.2: 1-5 odd; 12.2: 7-15 odd;
Monday	**12/15**	12 – 2	Geometric Series and Sequences	12.2: 27-35 odd;
Tuesday	**12/16**		Mixed problems	Pg 830 – 832 11 – 21 and 31, 33
Wednesday	**12/17**	Review		
Thursday	**12/18**	Test		
Friday	**12/19**			
	12/22 – 1/1/09	Christmas Holiday		
Monday	**1/5/09**	12 - 6	Binomial Theorem	12.6: 1-5 odd; 12.6: 17-31 every other odd
Tuesday	**1/06**	12 - 8	Sequence and Iteration	12.8: 1-9 odd;
Wednesday	**1/07**	Mindset	Chapter 1	Chapter 1 Pg 1- 25
Thursday	**1/08**		Chapter 1t	
Friday	**1/09**		Chapter 2	Chapter 2 pg 22 and 23
Monday	**1/12**		Chapter 2	
Tuesday	**1/13**		Review	
Wednesday	**1/14**	Review		
Thursday	**1/15**	Review		
Friday	**1/16**	Exams		
Tuesday	**6/3**	Exams		

End of 2rd semester