OPTIMAL GROUP PURCHASING DECISIONS UNDER SUPPLY CHAIN CONTRACTS AND COMPETITION

by

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ABSTRACT

ABDOLLAH MOHAMMADI. Optimal Group Purchasing Decisions under Supply Chain Competition and Contracts. (Under the direction of DR. ERTUNGA C. OZELKAN)

Group purchasing (GP) is a procurement strategy by which the retailers can negotiate better prices by increasing their negotiation power through collaboration with each other. GP problem can be modeled as a generalized newsvendor problem, although it is more realistic to model this problem with stochastic demand, current literature on GP is mostly focused on problems with deterministic demand. Comparing the single retailer newsvendor vs. a newsvendor problem with multiple retailers, there has been more attention paid to the newsvendor problem with single retailer. When there are multiple retailers, competition would be another important aspect to consider, which is lacking in parts of the literature and will be considered in this research. Different contracting scenarios such as revenue-sharing and buyback contracts are other aspects which can be considered in the GP problem which has not been studied so far. Given that; four research questions are defined to investigate in this study: 1) the first question investigates the newsvendor problem with quantity discount pricing from supplier by exploring an analytical approach to solve this problem building on existing solutions from the literature; next a second novel solution approach is proposed which solves the problem in fewer steps; answering this question makes the foundation for our subsequent research questions. 2) the second research question studies the GP problem with multiple symmetric retailers; this research question is an extension of the first research question which investigates the GP supply chain consisting of multiple symmetric retailers. 3) third research question explores

the solution to GP with multiple asymmetric retailers and suppliers; since this problem is complex to solve, the GP problem is divided into two sub-problems, retailers' problem, and suppliers' problem which are solved separately and then brought together to provide an answer to the overall GP problem, and 4) finally, fourth research question introduces different supply chain contracts to the GP problem and investigates studying the effect of these contracts on the retailers' profit. Mathematical results as well as managerial insights are provided for each model through sensitivity analysis and numerical experiments.

Chapter 3 addresses the newsvendor problem with supplier's quantity pricing, after proposing two solution approaches to solve the problem using a simulation-optimization approach, a full factorial analysis is done for five factors including demand parameters as well as pricing parameters. Based on the analysis, all the single factors have significant impact on the response factor in all cases, but it is not the same for the two-way interaction of the parameters. The symmetric multi-retailer problem is solved next; where a proposition is proposed which assists to extend the approaches that are developed for the newsvendor problem to the GP problem with symmetric retailers.

The asymmetric retailers' problem is addressed in Chapter 4 and a general approach is proposed to solve the multi-retailer and multi-supplier problem. An analytical solution is provided to the problem that can be used to solve the problem with any number of retailers and suppliers, but the solution grows significantly with each additional retailer/ supplier. Thus, the solution to the two retailer and two supplier problems as well as the three retailer and three supplier problem is displayed in this research. Numerical analysis is provided for the two retailer and two supplier problem with 12 input factors.

Supply chain contracts including buyback and revenue-sharing contracts are introduced to the GP problem in the chapter 5; and a numerical analysis is provided with the addition of these contacts to the GP problem which increases the input parameters to 13 input factors for a two retailer and two supplier problem.

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LIST OF ABBREVIATIONS

Abbreviation	Definition
GP	Group Purchasing
B2B	Business-to-Business
B2C	Business-to-Consumer
GPO	Group Purchasing Organization
QDF	Quantity Discount Function
IP	Individual Purchasing
MINLP	Mixed Integer Non-Linear Programming
PB	Plackett-Burman
BB	Buyback
RS	Revenue-Sharing

1 INTRODUCTION

Procurement costs typically account for a significant share of the total expense of a company, corresponding to 25% to 63.5% of the total costs, depending on the industry (Ellram 1995). For example, in the healthcare industry, procurements costs are around 40% of the total expenses (Jayaraman, Taha et al. 2014). Saving on purchasing costs translate directly into increased profits which is significant especially in low profit margin industries such as retail (Handfield and Nichols 2002). One of the strategies for saving in procurement cost is GP. In GP, different companies aggregate their demand to increase the overall negotiation power to get the best price for their commodities (Hsieh 2009). GP has been around in the business market for several decades, in healthcare industry it has been around since late 1800s (Hu, Schwarz et al. 2012). It has been applied in business-to-business (B2B) as well as in business-to-consumer (B2C) environments (Anand and Aron 2003). In a B2B setting, GP is used in industries such as healthcare, manufacturing, automotive, logistics, and grocery, while in a B2C setting, one can buy almost anything from GP websites such as Groupon or LivingSocial. Davenport and Kalagnanam (2002) categorize an auction mechanism similar to GP for price negotiations into two different types of GP in a B2B setting: combinatorial auctions and volume discount auctions. Combinatorial auctions which are used for procurement of a set of products, have been applied in selecting carriers, for contracting bus routes and even for selecting projects for the space shuttle (Banks, Ledyard et al. 1989, Ledyard, Olson et al. 2002, Vries and Vohra 2003); and volume discount auctions are used for procurement of a single item.

Some of the companies deploying GP in different industries include Novation and Premier in healthcare, Mfrall in manufacturing, Covinist in automotive, Polysort in plastic, and Transplace in logistics (Keskinocak and Savaşaneril 2008). In a GP environment, there are also Group Purchasing Organizations (GPOs), which act as a mediator between retailers and suppliers, allowing retailers save in procurement costs through the aggregation of demand (Figure 1). GPO connects the retailers to the suppliers and vice versa, where the retailers give their requirements to GPO, and the GPO negotiates with the suppliers over the price and quality to get the best deal for the retailers. An example for a GPO in manufacturing is "Prime Advantage", which has hundreds of suppliers and retailers in its buying network claims to bring 8% to 40% in cost saving for their members depending on the category (Partners 2018). Among all applications, healthcare seems to be the industry which has the most noticeable application of GP. According to Schneller (2009) GPOs brought in $36 billion in direct saving for the healthcare industry in 2009.

From a modeling perspective, GP can be considered as a volume-discounted, multi-retailer, multi-supplier newsvendor problem with retail competition. This problem can be

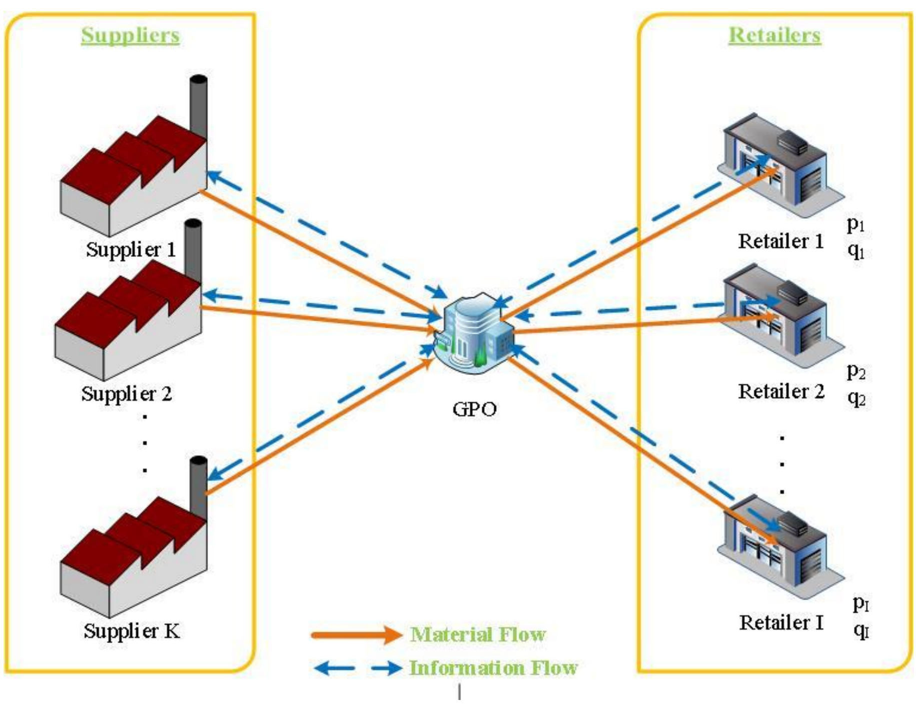

Figure 1. Layout of a group purchasing supply chain, material and information flow

related to Newsvendor Problem and Contracts in supply chain as well as Group Purchasing literature.

Many of the previous work has studied GP in a deterministic environment (Chen and Roma 2011, Zhou and Xie 2014), in this research, an stochastic demand scenario will be considered by developing GP based on newsvendor problem (Luo and Wang 2015, Karabağ and Tan 2017). A significant number of research has been done on the contracts in the supply chain, but this has not been considered in the GP problem; contracts in the GP problem will be modeled and solved in presence of buy-back and revenue-sharing contracts to study the effect they have on the decision making and profit levels (Ping, Shen et al. 2015, Zhou, Dan et al. 2017). Another aspect of the GP literature is that many of the papers have focused on the retailer's problem and assume that there is only one supplier in the supply chain, in this research the multi-supplier case will be modeled to find optimal assignment of orders to each supplier based on their quantity discount function (QDF) to minimize the total purchasing cost (Karabağ and Tan 2017).

This research will address several gaps in the literature answering the following research questions:

As it will be formally elaborated under the Literature Review Section, although from a modeling perspective it is more realistic to consider problems with stochastic demand, current literature on GP is mostly focused on problems with deterministic demand. Also, while there is a rich literature on the newsvendor style problems with a single retailer, there are not many papers modeling the multi-retailer cases. Competition is another important aspect in GP which is lacking in the literature and will be considered in this research. Different contracting scenarios such as revenue-sharing and buyback contracts are other aspects which can be considered in the GP problem which has not been studied so far. To evaluate the existing research and to lead our research path, five parameters are defined with varying levels for defining GP problems; these parameters include number of suppliers and retailers, existence of competition between retailers, availability of contract between suppliers and retailers. These factors and their levels are presented in Table 1. After studying the literature, the following research questions are proposed for this study:

1. What is the optimality condition for the price and order quantity in a newsvendor style GP with stochastic demand?
2. What is the effect of competition between the retailers on the optimal GP results?
3. How do the asymmetric retailers compare with the symmetric retailers in terms of their decisions?
4. What is the effect of different supply chain contracts on the profit levels of the retailers?

Table 1. Different factors and their levels in a GP problem

Factors	# of Levels	Levels
# of Suppliers	3	1, 2, Multi
# of Retailers	2	2, Multi
Retailer competition	2	Yes, No
Contract	3	No contract, Revenue-sharing, Buyback
Demand type	2	Deterministic, Stochastic

In this research, these research questions will be answered by modeling the expected profit levels and develop analytical and numerical methods to solve each of the problems. The optimality condition will be developed and validated by numerical examples and sensitivity analysis.

2 LITERATURE REVIEW

This research can be rooted back to group purchasing, newsvendor problem and contract management in supply chains. Accordingly, the literature is reviewed under each of the mentioned domains in the following sections.

2.1 Group Purchasing

Group Purchasing is procurement strategy that leverages the power of aggregated demand to get lower pricing from the suppliers. U.S healthcare industry saves an estimated $36B through the practice of GP (Schneller 2009), small and medium size firms can save between 7.4 to 12.5% in purchasing price by practicing GP (Ghaderi and Leman 2013), in Poland cities can save up to 15.4% on electricity though GP (Piorunowska-Kokoszko 2015). GP history can be traced back to 1900s with the formation of first GP Organization in hospital industry known as the Hospital Bureau of New York (Sethi 2009). Online GP was introduced to business world as a B2C model in 1999, GP has been deployed online by several pioneers such as Mercata.com, Accompany.com, actBIG.com, CoShopper.com, C-Tribe.com, DemandLine.com, Let’s Buy It, OnlineChoice.com, PointSpeed.com,

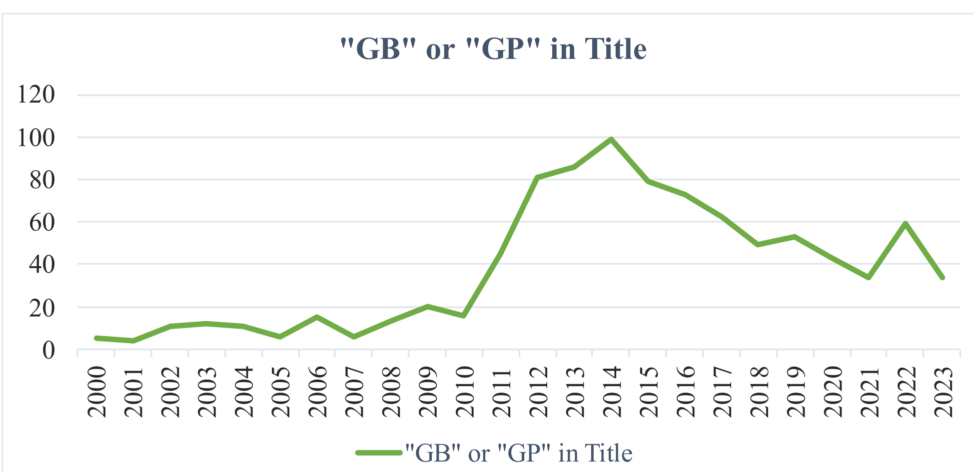

Figure 2. Trend of group purchasing publication since year 2000

Shop2gether, VolumeBuy and Zwirl.com (Kauffman and Wang 2002), these platforms may target B2B customers as well as B2C customers; this research studies GP in supply chain in a B2B environment where retailers cooperate in procurement while potentially competing for the end consumer. Figure 2 shows the trend of number of publications with the word “group buying” or “group purchasing” in their title since 2000 till 2023 appeared on google scholar. It is seen that GP has taken a significant increased attention from researchers between 2010-2015, then it took a downturn from 2016-2020 and it is trending up again since 2021. Here, the relevant papers in the context of a supply chain will be review.

Chen and Roma (2011) has studied comparison of GP with Individual Purchasing (IP) in a two-level distribution channel with competing retailers. They found out that the symmetric retailers always benefit from GP, while in the asymmetric case, GP benefits the weak player. Zhou and Xie (2014) developed their model based on Chen and Roma (2011), where the supplier is an active member and acts as the Stackelberg leader. They found out that in this case, GP can be potentially detrimental to all members of the supply chain except when there are economies of scale.

Luo and Wang (2015) studied the GP in a newsvendor framework between a single retailer and single customer to find optimal discount, order quantity and minimum order quantity with fixed retail price. Through numerical results they found out that GP can bring more benefit when demand changes with price. Karabağ and Tan (2015) studied a two-period GP supply chain consisting of suppliers, GPO, and retailers. Numerical results of their model revealed that all suppliers and retailers could benefit from GPO.

One of the noticeable applications of GP in US is in healthcare industry, hence there are many research studies in this field. Weinstein (2006) did comprehensive research about GP in healthcare about where it started from, how it works and how GPOs can be harmful by forming an oligopoly in the industry besides having some benefits to the hospitals by providing lower prices and technical assistance. Similarly (Zhou and Xie 2014) discussed how GPOs can hurt all the member of the supply chain, the results in this research contradicts with these cases which could be because the models that are discussed here don't account for suppliers' active pricing based on retailers' practicing GP or IP; also the models here are not specific to the healthcare industry which could be another reason for this difference. Jayaraman, Taha et al. (2014) reviewed the healthcare supply chain in the framework of GP and the services GPO offers to their members such as contracts, governance, value analysis, and spend analysis. Saha, Seidmann et al. (2010) reviewed the economies of healthcare procurement with GP and the benefits they provide to their members, as well as the issues raised around how GPOs work and what their real contribution is to the industry. They also presented the new roles of GPOs as an information and consulting service provider on spend and revenue management tools, optimization , statistical and contract management tools, which help hospitals to reduce procurement and transaction costs (Saha, Seidmann et al. 2010). GPOs also provide information-based tools that can benefit the sellers and the whole supply chain. One of the important questions about GPOs is about whether to allow them to collect fees or not, under the Social Security Act of 1987, GPOs are allowed to collect a share of %3 of the purchase; Hu, Schwarz et al. (2012) examine the effect of contract administration fees (CAF) in a GP supply chain, where all members are profit seeking. They used a game theoretic model and concluded that CAF does not increase total purchasing cost of the members, though it might increase unit purchasing cost. Hu and Schwarz (2011) study the effect of forming GPO on health providers as well as manufacturers by a hoteling duopoly model and conclude that eliminating the CAF will not benefit nor harm any of the players and forming GPO can increase competition and lower provider's cost. Lee, Langdo et al. (2023) studied the impact of GPOs on 6,251 hospitals' efficiency and profitability using a data envelopment analysis (DEA) approach; their analysis indicated that GPO results in increased efficiency for hospitals, but it does not have a significant impact on profitability. Ahmadi, Pishvaee et al. (2018) and Safaei, Heidarpoor et al. (2017) studied the application of GP in a healthcare supply chain using mathematical modeling; through numerical examples they concluded that the GP can benefit all members of the supply chain.

Regarding the solution approach to GP, most of the research related to this work looked at the problem from a game theoretic point of view and try to find equilibrium solutions and optimality condition (Keskinocak and Savaşaneril 2008, Chen and Roma 2011, Hu and Schwarz 2011, Hu, Schwarz et al. 2012, Karabağ and Tan 2015, Luo and Wang 2015, Ping, Shen et al. 2015, Karabağ and Tan 2017, Zhou, Dan et al. 2017); on the other side there are some papers that have employed the mathematical modeling and numerical experimenting approach to solve a range of GP problems (Ozelkan E. C., Geismar et al. 2003, Ozelkan 2006, Mohammadi and Ozelkan 2015, Mohammadi and Ozelkan 2016, Ahmadi, Pishvaee et al. 2018, Safaei, Heidarpoor et al. 2018, Mohammdi and Ozelkan 2022). The approach used in this research is a combination of analytical and numerical methods to solve the problems, which will be discussed in detail in the next chapters.

Table **2** displays how this research compares to the related literature in GP in terms of the factors that are considered in this research.

Table 2. Table of references related to this research

Reference	# of Suppliers	# of Retailers	Retailer Competition	Contract	Demand Type
Petruzzi and Dada (1999)	Single	Single	N	N	S
(Keskinocak and Savaşaneril 2008)	Single	Multiple	Y	N	D
Chen and Roma (2011)	Single	Two-Multiple	Y	N	D
Hu and Schwarz (2011)	Two	Multiple	N	N	D
Hu, Schwarz et al. (2012)	Single	Multi	N	N	D
Zhou and Xie (2014)	Single	Two-Multiple	Y	N	D
Karabağ and Tan (2015)	Multi	Multi	N	N	S
Luo and Wang (2015)	Single	Single	N	N	S
Ping, Shen et al. (2015)	Single	Single	N	Y	S
(Karabağ and Tan 2017)	Multiple	Multiple	N	N	S
(Zhou, Dan et al. 2017)	N/A	Two	Y	Y	S
(Ahmadi, Pishvaee et al. 2018)	Multiple	Multiple	N	N	D
(Safaei, Heidarpoor et al. 2018)	Multiple	Multiple	N	N	D
This research	Single-Multiple	Single-Multiple	Y	Y	S

D: deterministic
S: stochastic

2.2 Newsvendor Problem

Many of the literature in GP consider a deterministic demand case, while a stochastic market demand is closer to reality. Newsvendor problem is the closest classic problem to GP if we want to consider stochastic demand and we can extend the newsvendor problem to GP by expanding the number of retailers (Karabağ and Tan 2015, Luo and Wang 2015). The literature in newsvendor problem is rich and can give us great insight for solving GP problems.

Edgeworth's research on inventory theory (Edgeworth 1888) applied in banking industry is considered as the foundation of what we know today as "newsvendor problem"; but the term was not used actually until 1951 that the current newsvendor problem was mentioned in the literature (Arrow, Harris et al. 1951, Morse and Kimball 1951). One of the seminal research in newsvendor problem is presented by Petruzzi and Dada (1999), who studied the newsvendor problem under joint replenishment and pricing decisions with additive and multiplicative demand cases, they found out that the optimal price in additive case is always lower than the riskless profit, while in the multiplicative case this is reverse.

Arcelus, Kumar et al. (2012) studied the impact of rebate provider in a newsvendor framework on profitability of the channel and provided analytical condition for three scenarios i.e., supplier only, retailer only or both offering rebate.

Salinger and Ampudia (2011) took an innovative approach to get insight to newsvendor decisions by using the Lerner rule. They state that multiplicative demand causes an increased optimal price compared to additive demand through generalizing the Lerner relationship to the newsvendor problem.

Supplier and retailer coordination is a remedy to confront the random demand, especially in a market with short product life cycle. Weng (2004) developed a general newsvendor model where the supplier induces the retailer to coordinate its ordering quantity through offering an all-unit quantity discount in a two-period ordering policy. This research revealed that though coordination does not always lead to significant profit increase, it can help reduce the operating costs. Jadidi, Taghipour et al. (2016) also modeled the newsvendor problem in a two-period model with price and time-sensitive demand.

Since the newsvendor always sells the remaining items at a marked down price, the consumers can be strategic and wait until the end of the selling season to buy at a lower price, Ye and Sun (2016) has considered a case where the consumers have an expectation of probability of snatching the product at salvage price and the retailer has an expectation on the reserved price of the customers; they showed that this strategic behavior of the consumers can even benefit the retailer through increased profit in some cases.

2.3 Contracts in Supply Chain

Contracts are mechanisms to coordinate the supply chain and to incentivize the supply chain members to stay in the contract through transfer payments such that optimal decision for each member of supply chain also optimizes total supply chain objective (Cachon 2003).

Cachon (2003) has done a comprehensive study on the supply chain coordination with contracts based on the newsvendor problem and expanding it to more complex scenarios. Wang and Chen (2015) studied the newsvendor problem with options contract where the retailer can practice single ordering or mixed ordering; even though optimal and unique solution exists in both cases, they found out that mixed ordering is the optimal ordering strategy for the retailer. Wang and Choi (2014) also studied a single-period supply chain in a newsvendor setting under buy-back contract, where the supplier leads the game by deciding the wholesale price and the retailer follows by deciding on retail price and order quantity; they provided analytical solution to the problem and found out that the buy-back contract cannot coordinate the problem, thus they analyzed pareto-improvement of the system and found unique pareto-equilibrium of the supply chain.

Gerchak and Wang (2004) examined the revenue-sharing and wholesale-price contract in the context of an assembly operation between an assembler and its suppliers, where the assembler/retailer makes contracting decisions to optimize its own performance. They showed that the revenue-sharing plus a surplus subsidy outperforms the wholesale price contract in this problem. Yao, Leung et al. (2008) also studied the revenue-sharing contract in a supply chain with one supplier and two competing retailers where the supplier is the leader and retailers are followers. They provided analytical and numerical solution to the model; using thorough experimental analysis found out that the revenue-sharing offers better performance and flexibility to the supply chain.

In the context of GP supply chain, Ping, Shen et al. (2015) studied a two tier newsvendor style supply chain to design coordinating buyback contract in a B2C environment. The supply chain consists of a single supplier, single retailer and customers who can choose to do GP; the retailer offers different prices for IP and GP case, the demand function also changes with respect to the pricing method.

As mentioned before, this research is trying to address GP utilizing ideas from newsvendor problem and contracts in supply chain as well as GP literature. It relates to the newsvendor literature because newsvendor problem can be considered as a simplified GP problem, and we use this idea to develop the methods and ideas that are used in newsvendor research to solve more complex GP problems. This research roots to the work by Petruzzi and Dada (1999) and employs the ideas from this research to solve GP in a simplified setting and expands their method to address more general problems. The literature in contract is used to model the contracts in the GP problem and study the contract's effect on the retailers and suppliers. The work of Cachon (2003) on contracts was employed to model the profit functions of retailers and suppliers under contract and study its effect on them. In the GP literature, the research that is most related to our work is the study done by Chen and Roma (2011) which we based our work on their research and expanded upon their model. The contributions of this research can be described in four points which relate to four research questions mentioned before:

- Modeling GP as a newsvendor problem facing QDF and relating it to a GP with multiple symmetric retailers.
- Studying the effect of competition between retailers on the GP and provided solution and analysis to this problem.
- Modeled the GP with asymmetric retailers and Suppliers and developed new analytical solution approach to address this problem.
- Introducing the contracts to the GP problem and studied the effect of these contracts on the retailers in a GP context.

These contributions are addressed in more detail in the next sections. Chapter 3 delivers the first and second contribution for symmetric retailers' case. Chapter 4 details the third contribution for the GP with asymmetric retailers and suppliers. Finally, chapter

5 discloses the last contribution by studying the effect of buy-back and revenue-sharing contracts on the decision parameters.

3 NEWSVENDOR and GROUP PURCHASING PROBLEM WITH SYMMETRIC RETAILERS

As mentioned before, the GP is related to the price-setting newsvendor problem in a way that the retailers make price and order quantity decision given wholesale price information. In this chapter, first, a special case of GP will be studied, where a newsvendor faces a quantity discount pricing from the supplier instead of the common fixed wholesale price. Next, a GP with symmetric retailer will be studied, where the retailers cooperate in procurement given QDF from the supplier. Figure 3 displays the schematic of the supply chain for the latter problem.

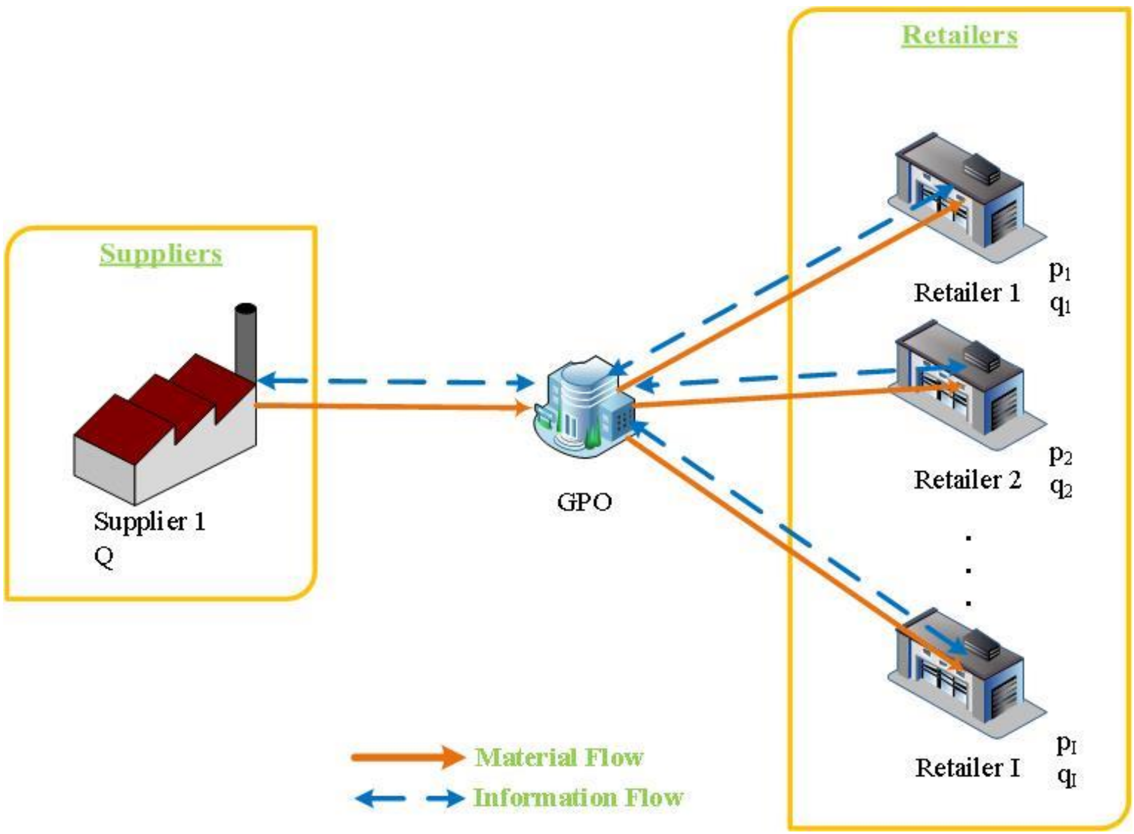

Figure 3. Schematic of the supply chain for group purchasing with symmetric retailers

As shown in the figure, the GP with symmetric retailers consists of $i = 1, \dots, I$ retailers, and one supplier. The retailers face stochastic demand and try to maximize their own profit by optimizing retail price, p_i and ordering quantity, q_i. The retailers aggregate the order quantities to $Q = \sum q_i$ and send it to the GP which passes the quantity to the supplier to maximize the discount based on the QDF. The details of the profit functions for each problem will be discussed in their relative chapters.

This chapter start with a special case of GP with symmetric retailers as a newsvendor problem and then extend the model to cover features like multiple non-competing symmetric retailers, competing retailers, asymmetric retailers, multiple suppliers, and contracts. Table 3 summarizes all parameters and variables that are used in this research.

Table 3. Description of the parameters, functions and decision variables

	Notation	**Description**
Indices	$i, j = 1, \dots, I$	Retailers
	$k = 1, \dots, K$	Suppliers
Input Parameters/ Function	c_i	Wholesale cost form supplier to retailer i
	$D_i(p_i, p_j, \epsilon)$	Demand function of retailer i $D_i(p_i, p_j, \epsilon_i)$=$a_i - (b_i + \gamma_i)p_i + \frac{\gamma_i}{I-1}\sum_{j \neq i} p_j + \epsilon_i$ (in simple form $D(p, \epsilon) = y(p) + \epsilon$)
	a_i	Base demand for retailer i ($a_i > 0$)
	b_i	Price elasticity factor for retailer i ($b_i > 0$)
	ϵ	Random factor
	A	Lower bound of $\boldsymbol{\epsilon}$ ($A > -a$ in additive demand form)
	B	Upper bound of $\boldsymbol{\epsilon}$
	$f(.)$	Density function of random factor
	$F(.)$	Cumulative distribution function of random factor
	s_i	Shortage cost for retailer i
	v_i	Salvage price for retailer i
	γ_i	Competition factor for retailer i $\gamma_i < b_i$
	$w_k(o_k)$	QDF Function of supplier k $w_k(o_k) = m_k + \frac{d_k}{o_k^{e_k}}$ (in simple form $m + {d}/{o^e}$)

	m_k	Base price for supplier k
	d_k	Discount rate for supplier k
	e_k	Steepness factor for supplier k $(-1.00 \leq e \leq 1))$ $de > 0$
	z_i	Transformation parameter $z_i = q_i - y(p_i)$
	$r(.)$	Hazard rate $r(.) = {f(.)}/{1 - F(.)}$
	t_i	Transformation parameter $t_i = a_i + (b_i + \gamma_i)c_i + \mu$
	W_k	Capacity of supplier k
	$V_i(q_i)$	expected salvage count calculated for each retailer in buyback contract
	v	expected salvage count per unit of product in buyback contract $v < w(q)$
	$S_i(q_i)$	share of the retailers' revenue which is transferred to the supplier
	s	expected transfer payment to supplier is for each unit sold in Revenue Sharing Contract
	h_k	Unit cost of product for supplier k
Decision Variables	p_i	Retailer price of retailer i
	q_i	Order quantity of retailer i
	o_k	Order quantity assigned to supplier k
	O	Total order Quantity to suppliers $O = \sum o_k$
	l	Slack variable

3.1 Newsvendor Problem with Quantity Discount

One can model a simplified version of GP as a price-setting newsvendor problem, where there is one retailer and one supplier with QDF pricing and stochastic demand, and the retailer decides the optimal order quantity and retail price. An analytical approach is developed here which will be the base to develop more challenging extensions with multiple retailers, competition and contracts in the next chapters. Petruzzi and Dada (1999) studied the optimal joint order and pricing decisions in a newsvendor problem setting and provided conditions for existence of a unique optimal price and order quantity. The basic model here is based on their model except that here the wholesale price is not fixed, but based on a QDF pricing. Two approaches have been developed to solve this problem; the first one is based on relaxing the QDF by replacing it with a fixed price c and using the method developed by Petruzzi and Dada (1999) to solve the problem by proposing a 2-step simulation optimization algorithm to include the QDF from supplier in the problem to find the numerical optimal solution. A second method is also developed which uses a similar approach, but without fixing the QDF and solves the problem in a 1-step algorithm. It turns out that the 2-step method is an approximation of the 1-step method, and it yields close to optimal results.

Let us consider a classic newsvendor problem, where there is a single retailer (newsvendor), facing a random and price sensitive demand from consumers. The demand function can be presented in different forms e.g. "additive" or "multiplicative" forms (Petruzzi and Dada 1999). This research will consider only additive demand function as described next.[1]

In "additive" demand form; $D(p,\epsilon) = y(p) + \epsilon$ (Mills 1959), the first part is an additive price sensitive demand, decreasing in price: $y(p) = a - bp$; where $a > 0$ is the base demand and $b > 0$ is the price elasticity factor. This type of linearly decreasing demand function is common in the economic literature (Keskinocak and Savaşaneril 2008, Yao, Leung et al. 2008, Wang and Chen 2015).

Price elasticity is defined as the change in demand in response to change in price of a product; the higher elasticity factor means the demand changes more with the change in price and the lower elasticity factor means the demand is less sensitive to the price

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[1] Since there is only one supplier and one retailer, no indexing is utilized in this section.

change. Elasticity factor b is closer to zero (between 0 and 0.6) for "inelastic" products; examples of inelastic products are essential products with less substitutes such as salt, gasoline (short run), tobacco and coffee. Products with elasticity factor value of close to one (between 0.5 and 1.5) are called "unitary elastic", most consumer goods fall under this category; examples are gasoline (long-run), movies, housing, and tires. Luxury products such as restaurant meals, foreign travel and automobiles have the highest elasticity of above 1.5; these products have "elastic" demand (Anderson, McLellan et al. 1997).

The last part of the demand function, ϵ captures the random factor with probability density function $f(.)$ and cumulative distribution function $F(.)$ On the range $[A, B]$. To make sure that $D(p, \epsilon)$ is positive for some range of p, we assume $A > -a$ in additive demand form (Petruzzi and Dada 1999).

Beside the consumer side, on the other side, the newsvendor faces a supplier who sets the price based on a QDF, which is a decreasing function of order quantity q. QDF can be formulated as $w(q) = m + {}^{d}/_{q^e}$ [2]; where m is the base price, d is discount rate, e is the steepness and q is the order assigned to the supplier (Schotanus, Telgen et al. 2009). Schotanus, Telgen et al. (2009) showed that this discount format matches very well with 66 discount schedules in practice. To make sure $w(q)$ is a decreasing function of q we should set $de > 0$. The higher the absolute value of d the higher the effect of demand aggregation; as well, in order to have a convex QDF and concave total cost we should limit $e \in [-1,1]$ (Chen and Roma 2011).

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[2] In the newsvendor problem $q = o$, so in this chapter o is replaced with q.

Given the above parameters, the sequence of procurement process steps for the newsvendor with QDF progresses as follows: given the data about the consumer demand and supplier's pricing, the newsvendor should decide about the order quantity q and the retail price p at the start of a selling period. Due to the stochastic nature of the demand two scenarios can happen: either observed demand exceeds q, in which case the newsvendor will face a shortage cost s; or the observed demand does not surpass q where the newsvendor may be hit by overage cost for having excess inventory, which would be salvaged at a price/cost v[3]. The profit function of the newsvendor can be written as:

$$\pi(p,q) = \begin{cases} pD(p,\epsilon) - w(q)q + v[q - D(p,\epsilon)], & D(p,\epsilon) \le q \\ pq - w(q)q - s[D(p,\epsilon) - q], & D(p,\epsilon) > q \end{cases} \qquad 3.1$$

Where the goal is to maximize the profit function, while the retail price p and order quantity q are decision variables of the retailer. Due additive demand, $D(p,\epsilon) = y(p) + \epsilon$ can be substituted in 3.1. It is more convenient to do the analysis on the profit function if a variable transformation is applied by replacing $y(p)$ with a new variable $z = q - y(p)$ (Thowsen 1975):

$$\pi(p,\mathrm{z}) = \begin{cases} p[y(p) + \epsilon] - w(q)[y(p) + z] + v[z - \epsilon], & \epsilon \le z \\ p[y(p) + z] - w(q)[y(p) + z] - s[\epsilon - z], & \epsilon > z \end{cases} \qquad 3.2$$

---

[3] The salvage price/cost, v must always be lower than per unit purchasing cost $w(q)$, and since $w(q)$ is dependent on the order quantity, the value of $w(q)$ is not known before solving the problem. To set an upper limit for v, a lower limit can be found for $w(q)$. For positive values of e, $w(q) \ge m$, thus $v \le m$ satisfies the requirement for $v \le w(q)$. For negative values of e, from Schotanus, F., J. Telgen and L. de Boer (2009). "Unraveling quantity discounts." Omega **37**(3): 510-521. ; it is known that $q \le \left((-1+e)\,{}^{d}/_{m}\right)^{1/e}$; so, $w(q) \ge m + d/\left[\left((-1+e)\,{}^{d}/_{m}\right)^{1/e}\right]^{e} = em/(-1+e)$, thus $v \le em/(-1+e)$ satisfies the requirement for $v \le w(q)$.

Two approaches are proposed to handle QDF; in the first method the QDF is fixed so that available solution from literature (Petruzzi and Dada 1999) can be used, in the second approach an optimal method is developed while the QDF is kept as a variable which helps solving the problem faster and simpler.

3.1.1 Two-Step Heuristic Approach

As mentioned earlier, to use the available solution method in the literature, initially the QDF is fixed by replacing $w(q)$ with a fixed supplier price c. After these substitutions, 3.2 will be:

$$\pi(p,z) = \begin{cases} p[y(p)+\epsilon] - c[y(p)+z] + v[z-\epsilon], & \epsilon \le z \\ p[y(p)+z] - c[y(p)+z] - s[\epsilon - z], & \epsilon > z \end{cases} \quad 3.3$$

The interpretation to the profit function is that if the value of z is greater than the realized value of ϵ there will be overage, while the retailer will face shortage if z is less than the realized value of ϵ. The goal is to find the best set of decision for p and z to maximize the expected profit. Expected profit function is:

$$\begin{aligned} E[\pi(\mathrm{p},\mathrm{z})] &= \int_A^z (p[y(p)+u] + v[z-u])f(u)du \\ &\quad + \int_z^B (p[y(p)+z] - s[u-z])f(u)du - c[y(p)+z] \\ &= \Psi(p) - L(z,p) \end{aligned} \quad 3.4$$

Where $\Psi(p) = (p-c)[y(p)+\mu]$ represents the riskless profit function in the deterministic case of the problem (Mills 1959) and $L(z,p) = (c-\mathrm{v})\Lambda(z) + (p+s-c)\Theta(z)$ represents the loss function when uncertainty is added to the problem (Silver and Peterson 1985); $\Theta(z) = \int_z^B (u-z)f(u)du$ is the expected shortage and $\Lambda(z) = \int_A^z (z-u)f(u)du$ is the expected overage, the details of the restructuring of the expected profit function is provided in Appendix I. From Petruzzi and Dada (1999) (Theorem 1. (b)) it is known that if $F(.)$ satisfies the condition $2r(z)^2 + \frac{dr(z)}{dz} > 0$ for $A \leq z \leq B$; where $F(.)$ is the distribution function and $r(.) = \frac{f(.)}{[1-F(.)]}$, then z^* is the largest z in the region $[A, B]$ that satisfies $\frac{\partial \mathrm{E}[\Pi(z,p)]}{\partial p} = 0$. Assuming ϵ has a uniform distribution, one can prove that it adheres to this theorem.

Corollary 1: Optimal order and price in the newsvendor problem with additive demand and uniform distribution is to order $q^* = y(p^*) + z^*$ where z^* is the largest z in the range $[A, B]$ that satisfies $R(z) = \frac{\mathrm{dE}[\pi(z,p(z))]}{dz} = 0$ where $p^* = p(z) = p^0 - \frac{\Theta(z)}{2b}$ and $p^0 = \frac{a+bc+\mu}{2b}$.

Proof: See Appendix II.

Using Corollary 1, a search algorithm can be developed to find z^* in the region $[A, B]$. For faster convergence and accurate result, Corollary 1 is combined with Newton's method to find z^* for a fixed c. After finding p^* and q^* for a fixed c, the QDF pricing should be addressed. In order to solve this problem a simulation optimization algorithm is developed, the algorithm initiates with a starting point for $w(q) = c^0$ then solves the problem and finds z^*, p^*, q^* for c^0; next the QDF engages in by taking q^* and returning updated $c = w(q^*)$ which is going to be used to find another set of optimal z^*, p^*, q^*; since the expected profit function is concave in each loop, the local optimum point in each loop is the global optimum. The loop will be repeated until c converges and the change in each loop is less than σ. Figure 4 shows the process of finding equilibrium parameter values for newsvendor problem with QDF.

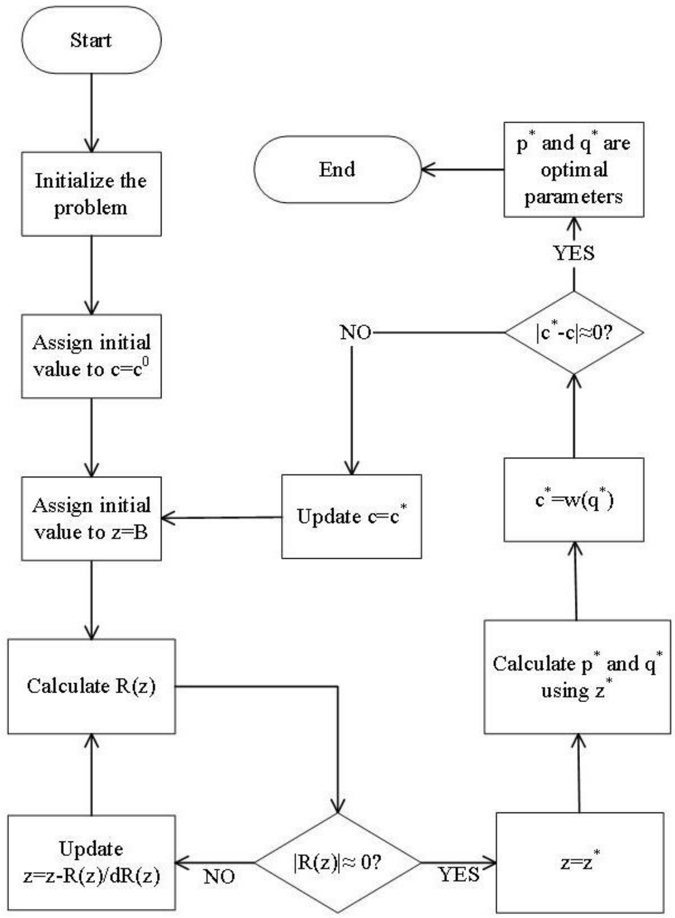

Figure 4. Process chart for finding optimal price and order quantity for newsvendor problem with QDF

To demonstrate the applicability of the proposed method and display the path to optimal parameters, consider a problem where the random parameter ϵ has a uniform distribution ($\epsilon \sim U[A, B]$) with parameters $A = 0$ and $B = 10$, the base demand $a = 20$, the base purchasing price $m = 10$, salvage and shortage price are $v = 1$ and $s = 1$ respectively, the elasticity factor b is 0.5, the discount rate d is 5 and $e = 1$ is steepness. The optimal result for this case is to order $q_i = 11.97$ product and sell them for $p_i = 29.70$ which will bring in a profit of 163.56 for the newsvendor, the optimal value of z is 6.83.

Figure 5 displays how the proposed algorithm converges towards optimal values of z and p in each iteration; in each iteration of the algorithm, optimal p and q will be used to update supplier's price; then next iteration will be executed from the last optimal point of z until the supplier's price converges. In the next section, a different approach is proposed to solve this problem in a more efficient manner.

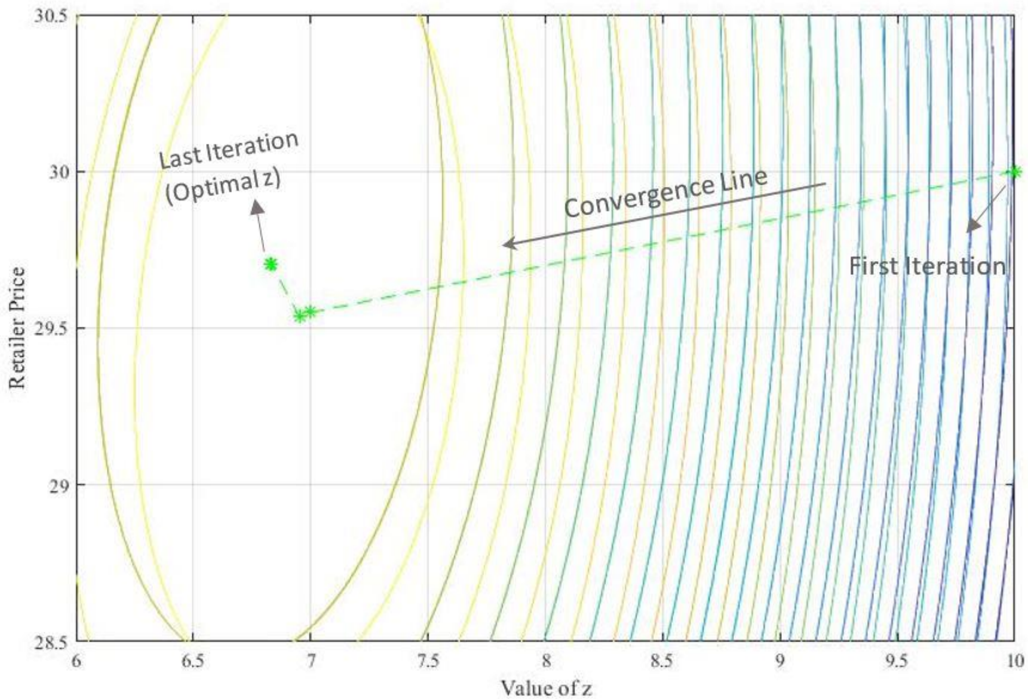

Figure 5. Retailer's profit contour in different steps of the algorithm and convergence line in the newsvendor setting with quantity discount- 2-step approach

3.1.2 One-Step Optimal Approach

Unlike the heuristic approach in the previous section, if the QDF is kept in the profit function as-is without fixing it, the problem will be more complex in terms of first and second derivative calculation and difficult to prove concavity in the general case; but it can be simplified for special cases of QDF. In this section, the concavity condition will be studied first in the general form, but since concavity cannot be proved for the general case of the problem, proof is provided for special cases of steepness factor e. The variable transformation of replacing $y(p)$ with a new variable $z = q - y(p)$ which was applied to 3.1, now needs to be applied to $w(q)$ in 3.2, which results in $w(p,z) = m + d/[y(p)+z]^e$, doing so, the profit function will look like:

$$\pi(p,z) = \begin{cases} p[y(p)+\epsilon] - [m + d/[y(p)+z]^e][y(p)+z] + v[z-\epsilon], & \epsilon \le z \\ p[y(p)+z] - [m + d/[y(p)+z]^e][y(p)+z] - s[\epsilon - z], & \epsilon > z \end{cases} \qquad 3.5$$

Given the above profit function the expected profit can be displayed as:

$$\begin{aligned} E[\pi(\mathrm{p},\mathrm{z})] = (\mathrm{p}-\mathrm{m}).[y(p)+\mu] - d[y(p)+z]^{1-e} \\ - (\mathrm{m}-v)\int_A^{\mathrm{z}} (\mathrm{z}-u)f(u)du - (\mathrm{p}+\mathrm{s}-\mathrm{m})\int_{\mathrm{z}}^{B} (u-\mathrm{z})f(u)du \qquad 3.6 \\ = \Psi(p,z) - L(p,z) \end{aligned}$$

Where $\Psi(p,z) = (\mathrm{p}-\mathrm{m}).[y(p)+\mu] - d[y(p)+z]^{1-e}$ and $L(p,z) = (\mathrm{m} - v)\Lambda(\mathrm{z}) + (\mathrm{p}+\mathrm{s}-\mathrm{m})\Theta(z)$ are similar to the previous section, except that m has replaced c in both Ψ and L and an added term $\langle -d[y(p)+z]^{1-e}\rangle$ to Ψ, which results in both Ψ and L to be a function of p and z. To maximize the expected profit, let's look at the first and second derivatives of 3.6 with respect to p and z:

$$\frac{\partial \mathrm{E}[\pi(\mathrm{p},\mathrm{z})]}{\partial z} = -(m - v) - (p + s - v)[F(z) - 1] - d(1 - e)[y(p) + z]^{-e} \qquad 3.7$$

$$\frac{\partial^2 \mathrm{E}[\pi(\mathrm{p},\mathrm{z})]}{\partial z^2} = -(p + s - v)f(z) + de(1 - e)[y(p) + z]^{-e-1} \qquad 3.8$$

$$\frac{\partial \mathrm{E}[\pi(\mathrm{p},\mathrm{z})]}{\partial p} = 2b(p^0 - p) - \Theta(z) + db(1 - e)[y(p) + z]^{-e} \qquad 3.9$$

Where $p^0 = \frac{a+bm+\mu}{2b}$ 3.10

$$\frac{\partial^2 E[\pi(\mathrm{p},\mathrm{z})]}{\partial p^2} = -2b + db^2e(1 - e)[y(p) + z]^{-e-1} \qquad 3.11$$

First, let us examine 3.8 to check how is the concavity situation of the profit with respect to z, the first part of 3.8 is always negative:

$$-(p + s - v)f(z) \leq 0 \because\ p \geq w(q)\ \wedge v \leq w(q) \therefore p \geq v \therefore (p + s - v) \geq 0 \qquad 3.12$$

Looking at the second part of 3.8, it is always positive for $e \in [-1,1]$ thus 3.8 can be negative only if the second term is smaller than the absolute value of the first term:

$$de(1 - e)[y(p) + z]^{-e-1} \leq (p + s - v)f(z) \qquad 3.13$$

Proposition 1: The profit function for the newsvendor problem with QDF is concave with respect to z only if 3.13 holds true.

Let us look at 3.13 for several special cases of e:

$$e = 1 \Rightarrow\ de(1 - e)[y(p) + z]^{-e-1} = 0 \leq (p + s - v)f(z) \qquad 3.14$$

$$e = 0 \Rightarrow\ de(1 - e)[y(p) + z]^{-e-1} = 0 \leq (p + s - v)f(z) \qquad 3.15$$

$$e = -1 \Rightarrow\ de(1 - e)[y(p) + z]^{-e-1} = -2d \leq\geq (p + s - v)f(z) \qquad 3.16$$

For $e = -1$ since $de > 0$, thus $d < 0$ and $-2d > 0$; so, it is not possible to prove if inequality 3.163.13 holds true. So far, it was found that the profit function is concave in z for $e = 1$ and $e = 0$, for other values of e, the concavity could not be proved or disproved in the general case.

Corollary 2: The profit function for the newsvendor problem with QDF is concave with respect to z for $e = 1$ and $e = 0$.

The same approach can be applied to 3.11 to check the concavity of the profit function with respect to p, the first part is always negative:

$$-2b \leq 0 \because b \geq 0 \tag{3.17}$$

The second part of 3.11 is always non-negative for $e \in [-1,1]$ thus 3.11 can be negative only if:

$$db^2 e(1-e)[y(p)+z]^{-e-1} \leq \ 2b \tag{3.18}$$

Proposition 2: The profit function for the newsvendor problem with QDF is concave with respect to p only if 3.18 holds true.

Obviously for $e = 1$ and $e = 0$ the left-hand side of 3.18 becomes 0 and thus the inequality 3.18 holds. For $e = -1$:

$$db^2 e(1-e)[y(p)+z]^{-e-1} = \ -2db^2 \leq 2b \ \Rightarrow db \geq -1 \tag{3.19}$$

Therefore, the inequality 3.18 holds for $e = -1$ only if $db \geq -1$. As a result, talking concavity terms it can be said that the profit function is concave in p for $e = 1$ and $e = 0$, for $e = -1$ it is concave if $db \geq -1$.

Corollary 3: The profit function for the newsvendor problem with QDF is concave with respect to p for $e = 1$ and $e = 0$; for $e = -1$ it is concave if $db \geq -1$.

Since $E[\pi(\mathrm{p,z})]$ is concave with respect to p for several values of e; the optimization problem can be reduced to a single variable z by solving it for the optimal value of p as a function of z where it is concave. Given 3.9 and Corollary 3, results in Lemma 1:

Lemma 1: For a fixed z, at concave points of the objective function with respect to p, the optimal price can be found as a function of z:

$$\begin{cases} e = 1 \Rightarrow p^* = p(z) = p^0 - \dfrac{\Theta(z)}{2b} \\ e = 0 \Rightarrow p^* = p(z) = p^0 - \dfrac{[\Theta(z) - db]}{2b} \\ e = -1 \wedge db \geq -1 \Rightarrow p^* = p(z) = \dfrac{p^0 + d(a+z) - \Theta(z)/2b}{1 + db} \end{cases} \qquad 3.20$$

Substituting p^* into $E[\pi(\mathrm{p,z})]$ for each of the scenarios above converts the optimization problem to a single variable optimization over z. Next, the optimality condition of $E[\Pi(\mathrm{p(z),z})]$ for each case of e mentioned in Lemma 1 can be analyzed.

Theorem 1: The optimal order and pricing policy in the newsvendor problem with a QDF and $e = 1$ is to order $q^* = y(p^*) + z^*$ units and sell at the unit price p^*, where p^* is determined using Lemma 1 and z^* is defined based on the following:

a) If $F(.)$ is a random distribution function, then a complete search over the range $[A, B]$ will determine z^*.

b) If $(m - v) > 0$ and $F(.)$ satisfies the condition $2r(z)^2 + {dr(z)}/{dz} > 0$ for $A \leq z \leq B$ and $r(.) = {f(.)}/{1 - F(.)}$; then $z^*$is the largest z in the range $[A, B]$ that satisfies $\frac{dE[\pi(z,p(z))]}{dz} = 0$

c) If condition b is met and $a - b(m - 2s) + A > 0$, then z^* is the unique z in the range $[A, B]$ that satisfies ${dE[\pi(z, p(z))]}/{dz} = 0$.

Proof: See Appendix III.

When $e = 1$ is replaced in $E[\pi(p, z)]$, it becomes very similar to the profit function with fixed purchase price c. The only different is that c is replaced with m and the profit function has an added term $(-d)$. Thus, the structure of the derivatives are very similar to the problem with fixed purchasing price in (Petruzzi and Dada 1999). If condition b in Theorem 1 satisfies it means that $E[\pi(p(z), z)]$ has at most two extreme points and the larger one of those is maximum. Condition c assures that the function has only one extreme point which is the z^* we are looking for.

Theorem 2: The optimal order and pricing policy in the newsvendor problem with a QDF and $e = 0$ is to order $q^* = y(p^*) + z^*$ units and sell at the unit price p^*, where p^* is determined using Lemma 1 and z^* is defined based on the following:

a) If $F(.)$ is a random distribution function, then a complete search over the range $[A, B]$ will determine z^*.

b) If $(m + d - v) > 0$ and $F(.)$ satisfies the condition $2r(z)^2 + {dr(z)}/{dz} > 0$ for $A \leq z \leq B$ and $r(.) = {f(.)}/{1 - F(.)}$; then $z^*$is the largest z in the range $[A, B]$ that satisfies ${dE[\pi(z, p(z))]}/{dz} = 0$:

c) If condition b is met and $a - b(m + d - 2s) + A > 0$, then z^* is the unique z in the range $[A, B]$ that satisfies ${dE[\pi(z, p(z))]}/{dz} = 0$:

Proof: See Appendix IV.

With $e = 0$, the expected profit function $E[\pi(p, z)]$ is different from the profit function with fixed purchase price c, where c is replaced with m and the profit function has an added term $(-d[y(p) + z])$. Unlike the scenario with $e = 1$, since the added term in this case is multiplied by a function of p and z, it will not eliminate from the derivatives of the profit function, the structure of the derivatives will be similar to the problem with fixed purchasing price in (Petruzzi and Dada 1999) with added terms. If condition b in Theorem 2 satisfies it means that $E[\pi(p(z), z)]$ has at most two extreme points and the larger one of those is maximum. Condition c assures that the function has only one extreme point which is the z^* we are looking for.

Theorem 3: The optimal order and pricing policy in the newsvendor problem with a QDF and $e = -1$ is to order $q^* = y(p^*) + z^*$ units and sell at the unit price p^*, where p^* is determined using Lemma 1 and z^* is defined based on the following:

a) If $F(.)$ is a random distribution function, then a complete search over the range $[A, B]$ will determine z^*.

b) If $-(m-v)-2d\left[{a+B-bp^0}\big/{1+bd}\right]<0$ and $F(.)$ satisfies the condition 3.21 for $A \le z \le B$ and $r(.)={f(.)}\big/{1-F(.)}$; then $z^*$is the largest z in the range $[A,B]$ that satisfies ${dE\left[\pi\left(z,p(z)\right)\right]}\big/{dz}=0$:

$$2bd\left[f(z).r(z)-\frac{2.df(z)/dz}{[1-F(z)]}\right]+2f(z)^2$$
$$+[1-F(z)]\big[2bd+\ [1-F(z)]\big].dr(z)/dz>0 \qquad 3.21$$

c) If condition b is met and condition 3.22 holds, then z^* is the unique z in the range $[A,B]$ that satisfies ${dE\left[\pi\left(z,p(z)\right)\right]}\big/{dz}=0$:

$$-(1+bd)(m-s)+p^0(2bd+1)-d(a+A)$$
$$+(2bd-1)\,(\mu-A)/2b>0 \qquad 3.22$$

Proof: See Appendix V.

The case with $e=-1$ is more complex compared to the previous two cases, the expected profit function $E[\pi(p,z)]$ has an added term which is squared $(-d[y(p)+z]^2)$ as well as replacing parameter c with m, this makes the first and second derivative much more complicated compared to other cases. For the same reason, the condition b and c for this theorem are not as simple as previous ones but they serve the same goal; satisfying condition b in means that $E[\pi(p(z),z)]$ has at most two extreme points and the larger one of those is maximum, while condition c assures that the function has only one extreme point.

Using Theorems 1-3, an optimization algorithm is developed to solve numerical problems, compared to the algorithm developed in the previous section, it does not have to adjust itself for an updated purchasing price in each step since it is already considered in the analytical part of the solution. Figure 6 displays the process of finding equilibrium parameter values for the problem in the 1-step approach, this method is simpler than the 2-step method and is superior in terms of considering the complexity of the problem.

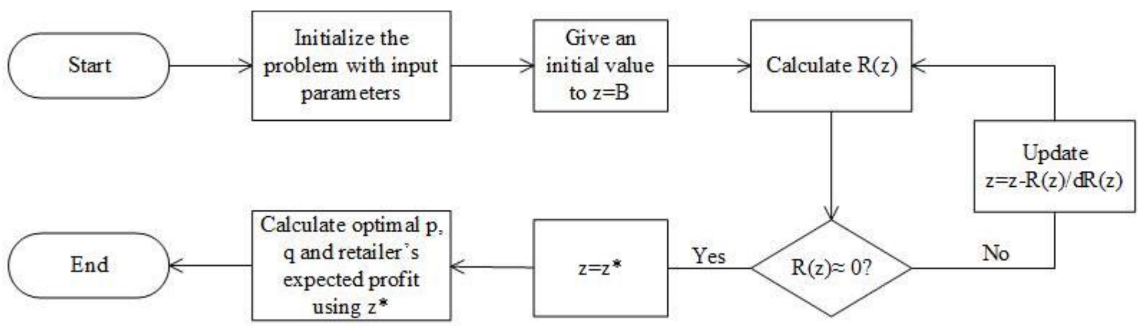

Figure 6. Process chart for finding optimal price and order quantity for newsvendor problem with QDF and additive Demand using the 1-step approach

To compare the solutions, consider the same problem provided in the previous section, where the random parameter ϵ has a uniform distribution ($\epsilon \sim U[A, B]$) with parameters $A = 0$ and $B = 10$, the base demand $a = 20$, the base purchasing price $m = 10$, salvage and shortage price are $v = 1$ and $s = 1$ respectively, the elasticity factor b is 0.5, the discount rate d is 5 and $e = 1$ is steepness. Since $e = 1$, let's check how this problem matches with Theorem 1. If we replace the parameters in condition b, the problem matches with this condition since $-(m - v) = -(9) < 0$ and $2r(z)^2 + dr(z)/dz = 3/(B - z)^2 > 0$, thus z^* is the largest z in the range $[A, B]$ that satisifies $dE[\pi(z, p(z))]/dz = 0$. Also, upon inspecting further we find out that the problem also matches condition c since $[a - b(m - 2s) + A] = 16 > 0$, thus we know that z^* is unique for this problem. Knowing this helps us find z^* faster and with higher confidence. The optimal result for this case is to order $q_i = 12.18$ product and sell them for $p_i = 29.54$ which will bring in a profit of 163.60 for the newsvendor, the optimal value of z is 6.95. Figure 7 displays how the proposed algorithm converges towards optimal values of z and p in each iteration; in each iteration of the algorithm, the z is updated until the condition $R(z) \approx 0$ is satisfied.

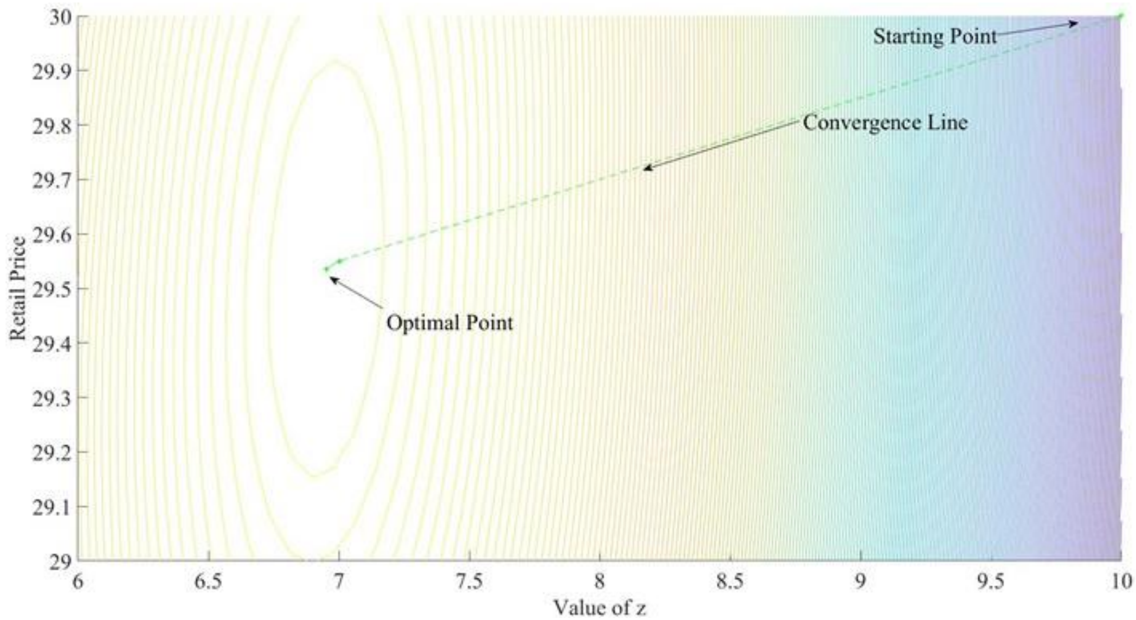

Figure 7. Retailer's profit contour in different steps of the algorithm and convergence line in the newsvendor setting with quantity discount- 1-step approach

3.1.3 Test Cases and Data Collection

This section is dedicated to describing the data used for experimental analysis. To have a more meaningful analysis, three industry cases are collected from related literature and one company which will be used to demonstrate the applicability of the developed methods in the consequent chapters to solve GP problem in different industries and to draw insights with regards to usefulness of GP in these industries. The parameters derived from these sources are used to generate test cases for the experiments. First, each industry case will be discussed below and then all the parameters used in this research will be presented.

Case 1 (C1): The first industry case is one of the clothing companies in the US, as the manufacturing plants provide different pricing depending on the order quantity of any specific garment, the clothing company needs to decide about an order quantity that maximizes its expected profit based on the expected demand of the product, due to high lead times (between 4-9 months depending on the product) they have to estimate the demand over that period. The GP problem can be considered in this context as a newsvendor facing QDF or as a multi retailer GP problem.

Case 2 (H1): The second case which is from the literature is the application of GP in healthcare industry by Ahmadi, Pishvaee et al. (2018) where they provide a mixed integer non-linear programming (MINLP) model to solve a GP problem consisting of several healthcare providers, a profit seeking GPO and several retailers and study the problem by modeling it as a mixed integer non-linear programming model and providing numerical analysis.

Case 3 (H2): The third literature case is the application of GP for pharmacies where the pharmacies group together based on various factors to purchase a set of medical products; a goal programming approach is used to solve a multi-objective mathematical model to decide on which pharmacies are clustered together and the ordering strategy (Safaei, Heidarpoor et al. 2017).

Case 4 (G1): The last case is in the chain store industry for procurement of dairy products from a set of suppliers to a group of grocery stores given the demand and supply restriction. They model the problem as a multi-level problem to determine optimal decision making for GPOs and retailers (Ahmadi, Heydari et al. 2021).

The parameter ranges presented in Table **4** from the mentioned industry cases are used to develop test cases which will be used for experimental analysis in this research. Presented below are the ranges for all the input parameters used in this research.

Table 4. Parameter levels for test cases

Parameter/Case	**Case 1**	**Case 2**	**Case 3**	**Case 4**
$\boldsymbol{a_i}$	[4320,5940]	[2918,4377]	[629,944]	[130,196]
$\boldsymbol{b_i}$	[154.22,231.33]*	[0.55,0.82]*	[0.010,0.015]*	[0.67,1.0]*
$\boldsymbol{\gamma_i}$	[78.96,123.37]*	[0.28,0.44]*	[0.005,0.008]*	[0.34,0.53]*
$\boldsymbol{A}$	-400	-500*	-200*	-50*
$\boldsymbol{B}$	2000	1500*	1000*	500*
$\boldsymbol{s_i}$	6	3000*	8400	60*
$\boldsymbol{v_i}$	3.05	350*	3000*	27*
$\boldsymbol{m_k}$	[7.69, 8.29]	[1523.82, 1831.54]	[21607.20, 21942.23]	[59.84, 72.00]
$\boldsymbol{d_k}$	[-0.000226, -0.000144]	[-0.068872, -0.010640]	[-1.089665, -1.087204]	[-0.020686, -0.005361
$\boldsymbol{e_k}$	-1	-1	-1	-1
$\boldsymbol{W_k}$	[15000, 30000]	[4000, 4500]	[8000, 10000]	[360,780]

*These parameters are generated.

It should be noted that the not all the parameters were readily available in all the cases which were generated to fit to each industry based on available data. To streamline the test problem generation, all the parameters that will be used repetitively in the next chapters are displayed in Table **5**, for each of the parameters three levels are created and, in each chapter, the applicable parameters will be selected from this table and a full factorial analysis will be ran to study the effects of the parameters on the decision factors. These parameters were selected as the representative of the market demand/ retailer parameters as well as supplier related parameters. Considering all the variations, there can be a minimum of 729 test cases generated for each industry case in a full factorial analysis.

Table 5. Parameters Levels per industry case

Industry	**Level**	a_i	b_i	γ_i	m_k	d_k	W_k
1	1	4,320	154.22	78.96	7.69	(0.000226)	15,000
1	2	5,400	192.77	98.70	7.97	(0.000188)	20,000
1	3	5,940	231.33	123.37	8.29	(0.000144)	30,000
2	1	2,918.13	0.55	0.28	1,523.82	(0.068873)	4,000
2	2	3,647.667	0.682	0.35	1,771.09	(0.045484)	4,250
2	3	4,377.20	0.82	0.44	1,831.54	(0.010640)	4,500
3	1	629.33	0.010	0.005	21,607.20	(1.089665)	8,000
3	2	786.67	0.013	0.007	21,891.69	(1.089223)	9,000
3	3	944.00	0.015	0.008	21,942.23	(1.087204)	10,000
4	1	130.67	0.67	0.34	59.84	(0.020686)	360
4	2	163	0.83	0.43	67.38	(0.017777)	540
4	3	196.00	1.00	0.53	72.00	(0.005361)	780

3.1.4 Experimental Results

To study the effect of different input parameters on the problem in the provided cases, a set of test cases are generated based on the given ranges and the problem was solved for all the different variations of the selected parameters. The factors selected to perform the sensitivity analysis are base demand a_i, the elasticity factor b_i, base price m_k and the discount scale d_k; the rest of the parameters are fixed in each industry case. The levels for each parameter are selected from Table **5**, since there are 4 parameters and three levels for each parameter, there will be 81 test cases for each industry.

First, let's look at the ANOVA table for each industry case shown in Table 6- Table 9, for all the cases the one-way and two-way interactions are significant, except for case 3 where the two-way interaction of $b_i * d_k$ and $m_k * d_k$ does not have a significant effect on the profit level.

Table 6. ANOVA Table for profit function - Case 1

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Model	32	2.34855E+11	7339220066	1799688.09	0.000
Linear	8	2.29303E+11	28662848143	7028565.15	0.000
a_i	2	1.41612E+11	70805756959	17362645.66	0.000
b_i	2	84090568435	42045284217	10310141.48	0.000
m_k	2	813335107	406667553	99721.05	0.000
d_k	2	2787367681	1393683841	341752.42	0.000
2-Way Interactions	24	5552256980	231344041	56729.07	0.000
a_i*b_i	4	5260216984	1315054246	322471.25	0.000
a_i*m_k	4	21454396	5363599	1315.24	0.000
a_i*d_k	4	262733517	65683379	16106.56	0.000
b_i*m_k	4	469926	117481	28.81	0.000
b_i*d_k	4	5680390	1420097	348.23	0.000
m_k*d_k	4	1701767	425442	104.32	0.000
Error	48	195746	4078		
Total	80	2.34855E+11			

Table 7. ANOVA Table for profit function - Case 2

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Model	32	8.05908E+14	2.51846E+13	430159.80	0.000
Linear	8	7.80170E+14	9.75212E+13	1665686.82	0.000
a_i	2	5.00838E+14	2.50419E+14	4277223.07	0.000
b_i	2	2.52144E+14	1.26072E+14	2153340.74	0.000
m_k	2	1.90630E+13	9.53148E+12	162800.11	0.000
d_k	2	8.12440E+12	4.06220E+12	69383.37	0.000
2-Way Interactions	24	2.57384E+13	1.07243E+12	18317.46	0.000
a_i*b_i	4	2.42117E+13	6.05291E+12	103385.28	0.000
a_i*m_k	4	5.15205E+11	1.28801E+11	2199.96	0.000
a_i*d_k	4	8.47210E+11	2.11802E+11	3617.64	0.000
b_i*m_k	4	43602152802	10900538200	186.18	0.000
b_i*d_k	4	77483563261	19370890815	330.86	0.000
m_k*d_k	4	43285566554	10821391639	184.83	0.000

Error	48	2810263057	58547147
Total	80	8.05911E+14	

Table 8. ANOVA Table for profit function - Case 3

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Model	32	4.03291E+16	1.26029E+15	4.70811E+08	0.000
Linear	8	3.89661E+16	4.87077E+15	1.81960E+09	0.000
a_i	2	2.60507E+16	1.30254E+16	4.86595E+09	0.000
b_i	2	1.29090E+16	6.45450E+15	2.41124E+09	0.000
m_k	2	6.41392E+12	3.20696E+12	1198039.51	0.000
d_k	2	1346309609	673154804	251.47	0.000
2-Way Interactions	24	1.36301E+15	5.67919E+13	21216031.25	0.000
a_i*b_i	4	1.36270E+15	3.40674E+14	1.27267E+08	0.000
a_i*m_k	4	2.75383E+11	68845664710	25719.01	0.000
a_i*d_k	4	210719515	52679879	19.68	0.000
b_i*m_k	4	32685314743	8171328686	3052.60	0.000
b_i*d_k	4	24550883	6137721	2.29	0.073
m_k*d_k	4	81323	20331	0.01	1.000
Error	48	128488302	2676840		
Total	80	4.03291E+16			

Table 9. ANOVA Table for profit function - Case 4

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Model	32	40681094558	1271284205	258421.99	0.000
Linear	8	39511139829	4938892479	1003959.95	0.000
a_i	2	24168770590	12084385295	2456469.53	0.000
b_i	2	13153941335	6576970668	1336942.48	0.000
m_k	2	1147825224	573912612	116662.85	0.000
d_k	2	1040602680	520301340	105764.95	0.000
2-Way Interactions	24	1169954729	48748114	9909.34	0.000
a_i*b_i	4	1043187740	260796935	53013.84	0.000
a_i*m_k	4	24075450	6018862	1223.49	0.000
a_i*d_k	4	86072165	21518041	4374.11	0.000
b_i*m_k	4	1629672	407418	82.82	0.000
b_i*d_k	4	7307479	1826870	371.36	0.000
m_k*d_k	4	7682224	1920556	390.40	0.000
Error	48	236132	4919		
Total	80	40681330690			

Also, looking at the Main Effects plot for the retailer profit in Figure 8- Figure 11, it can be concluded that the demand parameters have the highest effect on the profit function, also in Case 3, the effect of m_k and d_k is low which could be due to the levels of these factors in this industry.

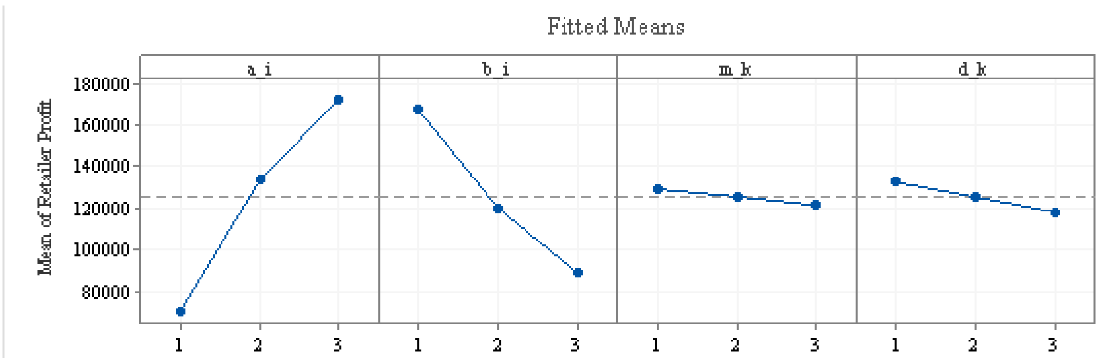

Figure 8: Main effects plot for retailer profit – Case 1

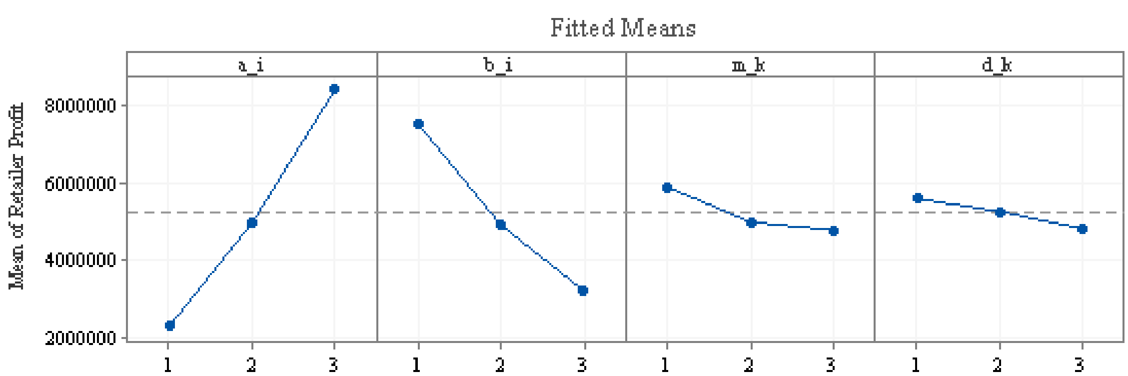

Figure 9: Main effects plot for retailer profit – Case 2

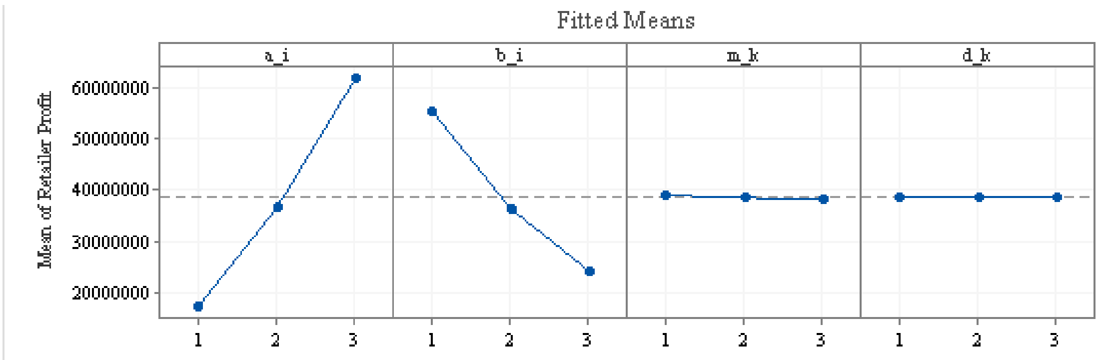

Figure 10: Main effects plot for retailer profit – Case 3

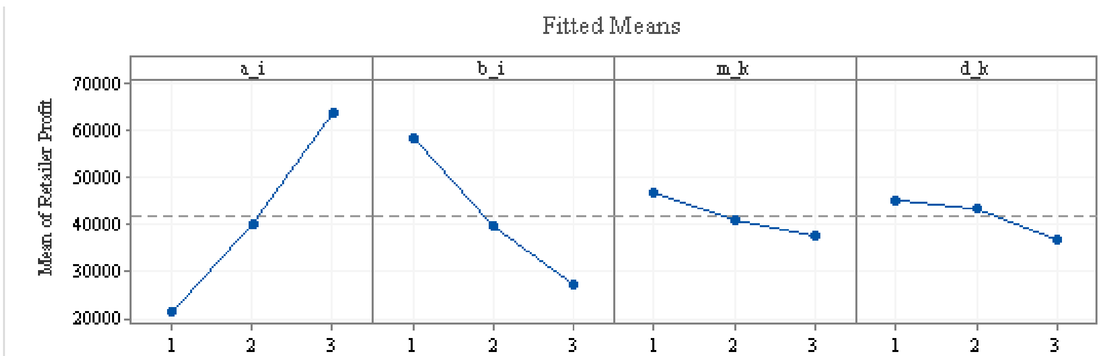

Figure 11: Main effects plot for retailer profit – Case 4

3.2 Group Purchasing Problem of Symmetric Retailers

This section takes the newsvendor's problem with QDF one step further by analyzing it for GP with any number of symmetric retailers. While previously, the problem had only been examined with a single retailer (newsvendor). It is possible to think of the GP problem with multiple symmetric retailers as an extension of the newsvendor problem with QDF, in which the retailers place their orders as a group. This problem will be addressed as a competing problem, but as it will be discussed later, the competition does not affect the solution in the symmetric form.

In "additive" demand form; the demand function for multiple competing retailers can be represented as below:

$$D_i(p_i, p_j, \epsilon_i) = a_i - (b_i + \gamma_i)p_i + \frac{\gamma_i}{I-1}\sum_{j \neq i} p_j + \epsilon \qquad 3.23$$

Where 3.23 is a function of retailer i's price as well as other retailers' price. This model has been deployed widely in the literature with retailer competition (Ingene and Parry 1995, Padmanabhan and Png 1997, Yao, Leung et al. 2008, Yushuang, Pengfei et al. 2012). In this model a_i is the base demand of each retailer which represents the size of the market base accessible to retailer i; b_i is the price elasticity of the customers to retailer i's price and γ_i is price elasticity of customers to retailer i's competitors. To ensure the response function has negative slope we assume $\gamma_i < b_i$. The random factor in the demand function is modelled by ϵ which is assume to have a uniform distribution; $\epsilon \sim U[A, B)]$.While this demand function is based on Zhou and Xie (2014) with addition of the demand elasticity; similar demand functions are used by Yao, Leung et al. (2008) and Yushuang, Pengfei et al. (2012) for special two-retailer case, but the author of this research is critic of their demand function which is the form below:

$$D_i(p_i, p_j, \epsilon_i) = a_i - b_i p_i + \gamma_i p_j + \epsilon_i \qquad 3.24$$

Based on the initial experiments with this demand function, it produces questionable results in terms of their effect on the decision parameters and profit levels. Research into different demand functions could be the topic of a separate research.

Given the above description, the sequence of procurement steps for the retailers progresses as follows: given the data about the consumer demand and supplier's pricing, at the start of a selling period, the order quantity q_i and the retail price p_i should be decided for each retailer. Next, the retailers will aggregate the order quantity and purchase quantity

$Q = \sum_{i=1}^{I} q_i$ from the supplier. Due to the stochastic nature of the demand, two scenarios can happen for each of the retailers: either observed demand exceeds q_i, in which case the retailer will face a shortage cost s_i; or the observed demand subceed q_i, where the retailer will be hit by overage cost for having excess inventory, which will be salvaged at a price/cost v_i. Thus, the profit function of each retailer i can be written as:

$$\pi_i(p_i, p_j, q_i) = \begin{cases} p_i D_i(p_i, p_j, \epsilon) - w(Q) q_i + v_i[q_i - D_i(p_i, p_j, \epsilon)], & D_i(p_i, p_j, \epsilon) \le q_i \\ p_i q_i - w(Q) q_i - s_i[D_i(p_i, p_j, \epsilon) - q_i], & D_i(p_i, p_j, \epsilon) > q_i \end{cases} \quad 3.25$$

Where the goal is to maximize the profit function for each retailer while the retail price p_i and order quantity q_i are decision variables. For symmetric retailers, one important statement can be made which will help tackle the problem:

Proposition 3: If all the parameters for the retailers are identical, one can conclude that the optimal values for the retail price p_i and order quantity q_i will be similar as well; in other words, $\begin{cases} p_i^* = p_j^* = p^* \\ q_i^* = q_j^* = q^* \end{cases} i \neq j = 1, \ldots, I$. This statement is vital in solving this problem. A secondary derivative of this statement is that $\pi_i^* = \pi_j^* = \pi^*, i \neq j = 1, \ldots, I$.

Applying Proposition 3 to the demand function, it will simplify to the below function:

$$D_i(p_i, p_j, \epsilon_i) = a_i - b_i p_i + \epsilon \quad 3.26$$

Which is exactly the demand function for the newsvendor problem, due to the symmetricity of the retailers, even the competition factor does not have any effect on the final demand function. Another implication of Proposition 3 is that once can solve the problem for one retailer and extend it to any number of symmetric retailers, doing so as well as applying the variable transformation $z_i = q_i - y(p_i)$; 3.25 can be written for one retailer as:

$$\pi_i(p_i, z_i) = \begin{cases} p_i[y(p_i) + \epsilon] - w(Q)[y(p_i) + z_i] + v_i[z_i - \epsilon], & \epsilon \leq z_i \\ p_i[y(p_i) + z_i] - w(Q)[y(p_i) + z_i] - s_i[\epsilon - z_i], & \epsilon > z_i \end{cases} \quad 3.27$$

In the next sections, the heuristic and optimal approaches that were developed for the newsvendor problem will be extended to solve the GP with symmetric retailers' case.

3.2.1 Two-Step Heuristic Approach

Provided the description above, one can use the heuristic approach developed for the single retailer problem and extend it to the several symmetric retailer GP. The formulations that are provided for the newsvendor case are very similar to the multiple symmetric case, thus they are not repeated here. Using Corollary 1, the search algorithm developed in 3.1.1 can be applied to this case after adjusting it for $w(Q)$ instead of $w(q)$. Figure 12 displays the updated process for finding equilibrium parameter values for GP with symmetric retailers.

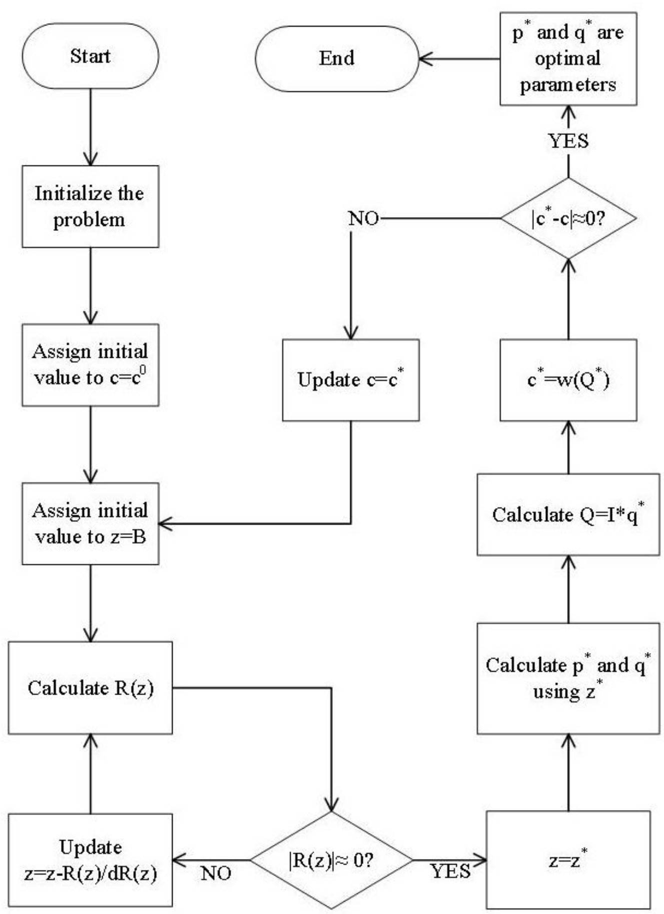

Figure 12. Process chart for finding optimal price and order quantity for group purchasing with symmetric retailers

To display the applicability of the proposed method and the path it takes to optimal parameters, consider the test case that was provided in 3.1.1 but with 5 symmetric retailers, each retailer will have the same parameters, but cooperate in purchasing. The optimal result for this case is to order $q_i = 12.14$ product and sell them for $p_i = 29.3$ which will bring in a profit of 167.6 for each retailer, the optimal value of z is 6.92. Figure 13 displays how the proposed algorithm converges towards optimal values of z and p in each iteration; in each iteration of the algorithm, p^* and q^* is used to update supplier's price; then next iteration is executed from the last optimal point of z until the supplier's price converges. In the next section, the optimal approach developed for newsvendor will be extended to solve this problem.

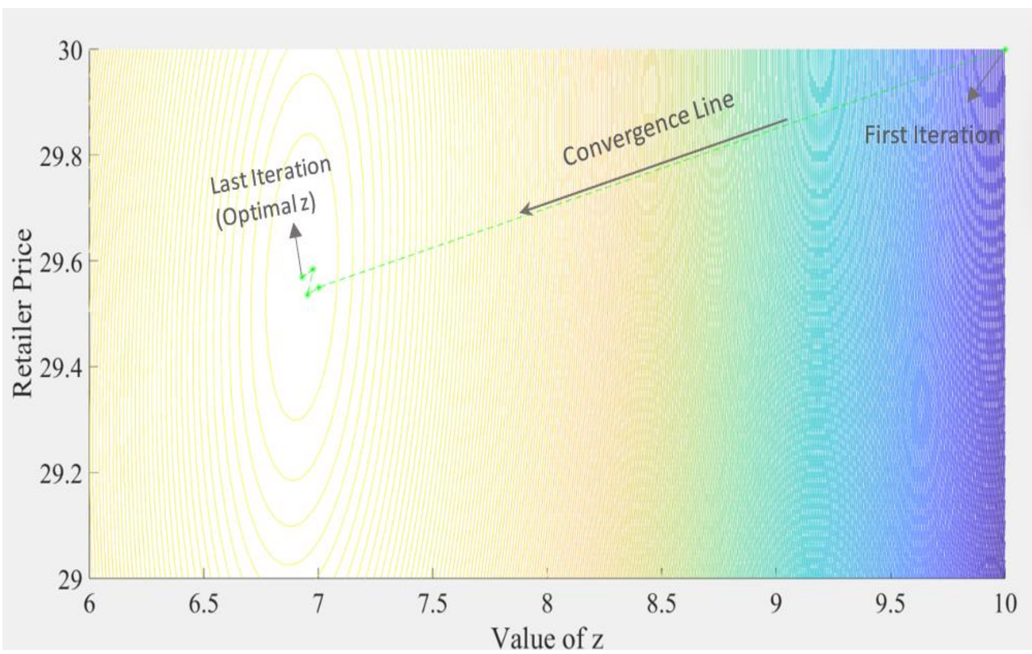

Figure 13. Retailer's profit contour in different steps of the algorithm and convergence line in group purchasing with symmetric retailers- 2-step approach

3.2.2 One-Step Optimal Approach

Like the heuristic method, it is expected that the optimal approach developed in 3.1.1 be expandable to solve GP case with symmetric retailers, which is the topic of this section. The variable transformation of replacing $y(p_i)$ with a new variable $z_i = q_i - y(p_i)$ which was applied to 3.1, now needs to be applied to $w(Q)$ in 3.253.2, which results in $w(p_i, z_i) = m + d/\langle I[y(p) + z]\rangle^e$, doing so, the profit function will look like:

$$\pi(p_i, z_i) = \begin{cases} p_i[y(p_i) + \epsilon] - [m_i + d_i/\langle I[y(p_i) + z_i]\rangle^e][y(p_i) + z_i] + v_i[z_i - \epsilon], \epsilon \le z_i \\ p_i[y(p_i) + z_i] - [m_i + d_i/\langle I[y(p_i) + z_i]\rangle^e][y(p_i) + z_i] - s_i[\epsilon - z_i], \epsilon > z_i \end{cases} \quad 3.28$$

Given the above profit function the expected profit can be displayed as:

$$E[\pi(p_i, z_i)] = (p_i - m_i).[y(p_i) + \mu] - I^{-e}d_i[y(p_i) + z_i]^{1-e} - (m_i - v_i)\int_A^{z_i}(z_i - u)f(u)du - (p_i + s_i - m_i)\int_{z_i}^{B}(u - z_i)f(u)du = \Psi(p_i, z_i) - L(p_i, z_i) \quad 3.29$$

Where $\Psi(p_i, z_i) = (p_i - m_i).[y(p_i) + \mu]$ and $L(p_i, z_i) = (m_i - v_i)\Lambda(z_i) + (p_i + s_i - m_i)\Theta(z_i) + I^{-e}d_i[y(p_i) + z_i]^{1-e}$. To maximize the expected profit, let's look at the first and second derivatives of 3.29 with respect to p_i and z_i:

$$\frac{\partial E[\pi(p_i, z_i)]}{\partial z_i} = -(m_i - v_i) - (p_i + s_i - v_i)[F(z_i) - 1] - I^{-e}d_i(1 - e)[y(p_i) + z_i]^{-e} \quad 3.30$$

$$\frac{\partial^2 E[\pi(p_i, z_i)]}{\partial z_i^2} = -(p_i + s_i - v_i)f(z_i) + I^{-e}d_i e(1 - e)[y(p_i) + z_i]^{-e-1} \quad 3.31$$

$$\frac{\partial E[\pi(p_i, z_i)]}{\partial p_i} = 2b_i(p_i^0 - p_i) - \Theta(z_i) + b_i I^{-e}d_i(1 - e)[y(p_i) + z_i]^{-e} \quad 3.32$$

Where $p_i^0 = \frac{a_i+b_i m_i+\mu}{2b_i}$ 3.33

$$\frac{\partial^2 E[\pi(p_i, z_i)]}{\partial p_i^2} = -2b_i + b_i I^{-e} d_i e(1-e)[y(p_i)+z_i]^{-e-1} \quad 3.34$$

First, let us examine 3.31 to check how is the concavity situation of the profit with respect to z_i, the first part is always negative:

$$-(p_i + s_i - v_i)f(z_i) \leq 0 \because\ p_i \geq w(Q)\ \wedge v_i \leq w(Q) \therefore p_i \geq v_i$$
$$\therefore (p_i + s_i - v_i) \geq 0 \quad 3.35$$

Looking at the second part of 3.31, it is always positive for $e \in [-1,1]$ thus it can be negative only if the second term is smaller than the absolute value of the first term:

$$I^{-e} d_i e(1-e)[y(p_i)+z_i]^{-e-1} \leq (p_i + s_i - v_i)f(z_i) \quad 3.36$$

Proposition 4: The profit function for the newsvendor problem with QDF is concave with respect to z_i only if 3.36 holds true.

Let us look at 3.36 for several special cases of e:

$$e = 1 \Rightarrow\ I^{-e} d_i e(1-e)[y(p_i)+z_i]^{-e-1} = 0 \leq (p_i + s_i - v_i)f(z_i) \quad 3.37$$

$$e = 0 \Rightarrow\ I^{-e} d_i e(1-e)[y(p_i)+z_i]^{-e-1} = 0 \leq (p_i + s_i - v_i)f(z_i) \quad 3.38$$

$$e = -1 \Rightarrow\ I^{-e} d_i e(1-e)[y(p_i)+z_i]^{-e-1} = -2Id_i \leq\geq (p_i + s_i - v_i)f(z_i) \quad 3.39$$

For $e = -1$ since $d_i e > 0$, thus $d_i < 0$ and $-2Id_i > 0$; so, it is not possible to prove if inequality 3.36 holds true. So far, it was found that the profit function is concave in z_i for $e = 1$ and $e = 0$; for other values of e, the concavity could not be proved or disproved.

Corollary 4: The profit function for the GP problem with symmetric retailers is concave with respect to z_i for $e = 1$ and $e = 0$.

The same approach can be applied to 3.34 to check the concavity of the profit function with respect to p_i, the first part is always negative:

$$-2b_i \leq 0 \because b_i \geq 0 \quad 3.40$$

The second part of 3.34 3.33is always non-negative for $e \in [-1,1]$ thus it be negative only if:

$$b_i^2 I^{-e} d_i e(1-e)[y(p_i) + z_i]^{-e-1} \leq \; 2b_i \quad 3.41$$

Proposition 5: The profit function for the newsvendor problem with QDF is concave with respect to p only if 3.41 holds true.

Obviously for $e = 1$ and $e = 0$ the left-hand side of 3.41 becomes 0 and thus the inequality holds. For $e = -1$:

$$d_i I^{-e} b_i^2 e(1-e)[y(p_i) + z_i]^{-e-1} = \; -2d_i I b^2 \leq 2b_i \; \Rightarrow d_i I b_i \geq -1 \quad 3.42$$

Therefore, 3.41 holds for $e = -1$ only if $b_i I d_i \geq -1$. As a result, regarding concavity it can be said that the profit function is concave in p for $e = 1$ and $e = 0$, for $e = -1$ it is concave only if $b_i I d_i \geq -1$.

Corollary 5: The profit function for the newsvendor problem with QDF is concave with respect to p for $e = 1$ and $e = 0$; for $e = -1$ it is concave if $b_i I d_i \geq -1$.

Since $E[\pi(\mathrm{p_i}, \mathrm{z_i})]$ is concave with respect to p_i for several values of e; the optimization problem can be reduced to a single variable z_i by solving it for the optimal value of p_i as a function of z_i where it is concave. Given 3.32 and Corollary 5, results in Lemma 2:

Lemma 2: For a fixed z_i, at concave points of the objective function with respect to p_i, the optimal price can be found as a function of z_i:

$$\begin{cases} e = 1 \Rightarrow p_i^* = p(z_i) = p_i^0 - \dfrac{\Theta(z_i)}{2b_i} \\ e = 0 \Rightarrow p_i^* = p(z_i) = p_i^0 - \dfrac{[\Theta(z_i) - Id_i b_i\,]}{2b_i} \\ e = -1 \ \wedge \ b_i Id_i \geq -1 \ \Rightarrow p_i^* = p(z_i) = \dfrac{p_i^0 + Id_i(a_i + z_i) - \Theta(z_i)/2b_i}{1 + b_i Id_i} \end{cases} \qquad 3.43$$

Comparing p_i^* for different values of e in this problem vs. the newsvendor case discussed in 3.1.2, they line up exactly for all cases of , except for $e = -1$. Substituting the p_i^* for the first two cases in the profit function yields similar values of $\mathrm{dE}[\pi(\mathrm{p_i}(\mathrm{z_i}), \mathrm{z_i})]\big/dz$, as a result Theorem 1 and Theorem 2 applies to the GP with symmetric retailers for these cases of e. For $e = -1$, substituting p_i^* into $E[\pi(\mathrm{p_i}, \mathrm{z_i})]$ converts the optimization problem to a single variable optimization over z_i. Next, the optimality condition of $E[\pi(\mathrm{p_i}(\mathrm{z_i}), \mathrm{z_i})]$ for this case of e can be analyzed.

Theorem 4: The optimal order and pricing policy in the GP problem with symmetric retailers and $e = -1$ is to order $q_i^* = y(p_i^*) + z_i^*$ units and sell at the unit price p_i^*, where p_i^* is determined using Lemma 2 and z_i^* is defined based on the following:

a) If $F(.)$ is a random distribution function, then a complete search over the range $[A, B]$ will determine z_i^*.

b) If $-(m_i - v_i) - 2Id_i \left[{a_i + B - b_i p_i^0}\middle/{1 + b_I Id_i}\right] < 0$ and $F(.)$ satisfies the condition 3.44 for $A \le z \le B$ and $r(.) = {f(.)}/{1 - F(.)}$; then $z_i^*$is the largest z_i in the range $[A, B]$ that satisfies ${dE[\pi(\mathrm{z_i}, \mathrm{p_i}(z_i))]}/{d\mathrm{z_i}} = 0$:

$$2b_i Id_i \left[f(z_i).r(z_i) - \frac{2.d_i f(z_i)/dz_i}{[1 - F(z_i)]}\right] + 2f(z_i)^2 + [1 - F(z_i)]\big[4b_i Id_i + [1 - F(z_i)]\big].dr(z_i)/dz_i > 0 \quad 3.44$$

c) If condition b is met and condition 3.45 holds, then z_i^* is the unique z_i in the range $[A, B]$ that satisfies ${dE[\pi(\mathrm{z_i}, \mathrm{p(z_i)})]}/{d\mathrm{z_i}} = 0$:

$$-(1 + b_i Id_i)(m_i - s_i) + p_i^0(2b_i Id_i + 1) - Id_i(a_i + A) + (2b_i Id_i - 1)\,(\mu - A)/2b_i > 0 \quad 3.45$$

Proof: See Appendix VI.

As mentioned earlier, for $e = 1$ and $e = 0$, Theorem 1 and Theorem 2 can be applied to solve the problem. For the case with $e = -1$, since replacing $p^*(z_i)$ in the profit function yields a different equation, the optimality condition needed to be recalculated. Satisfying condition b in means that $E[\pi(\mathrm{p_i}(\mathrm{z_i}), \mathrm{z_i})]$ has at most two extreme points and the larger one of those is maximum, while condition c assures that the function has only one extreme point.

Using Theorems 1,2 and 4, an optimization algorithm is developed to solve numerical problems for GP with symmetric retailers. The process of finding equilibrium parameter values for the problem in the 1-step approach is like the one described newsvendor problem displayed in Figure 6, this method is simpler than the 2-step method and is superior in terms of considering the complexity of the problem. To display the applicability of the proposed method and the path it takes to optimal parameters, consider the test case that was provided in in the previous section. It was already shown that this problem matches with Theorem 1.b and c in the newsvendor case, thus, it can be shown that it applies to the GP case as well. Thus, z_i^* is the largest z_i in the range $[A, B]$ that satisifies $dE\left[\pi\left(z_i, p_i(z_i)\right)\right]/dz_i = 0$. Using the developed algorithm and Theorem 1.b, optimal result for this case is to order $q_i = 12.18$ product and sell them for $p_i = 29.53$ which will bring in a profit of 167.6 for each retailer, the optimal value of z is 6.95. Figure 14 displays how the proposed algorithm converges towards optimal values of z and p in

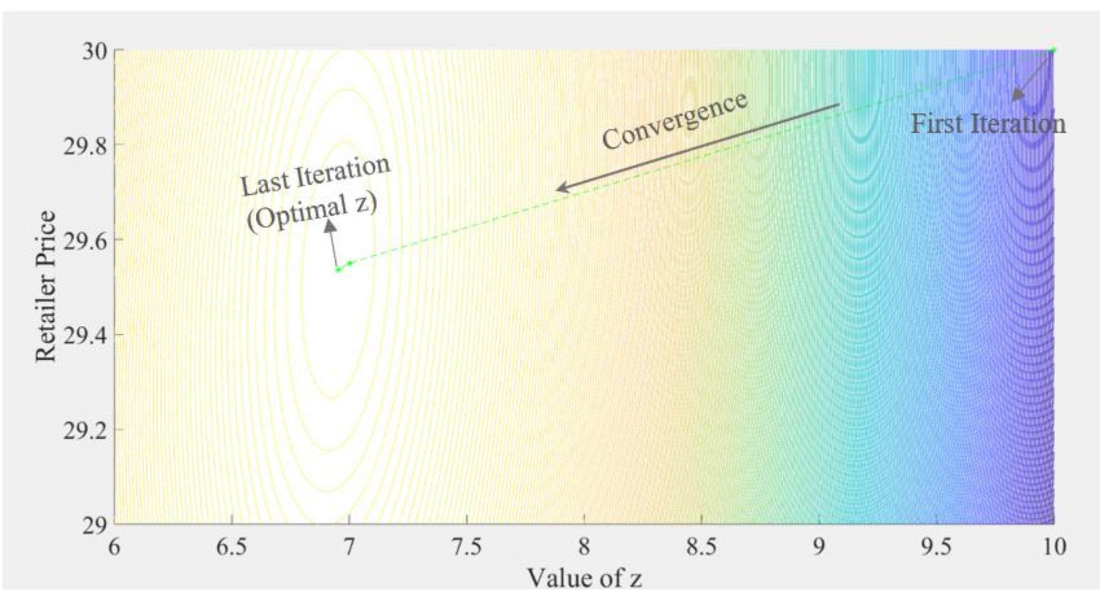

Figure 14. Retailer’s profit contour in different steps of the algorithm and convergence line in group purchasing with symmetric retailers- 1-step approach

each iteration; in each iteration of the algorithm, p^* and q^* is used to update supplier's price; then next iteration is executed from the last optimal point of z until the supplier's price converges.

3.2.3 Experimental Results

Here, the factors selected to perform the sensitivity analysis are the same as the newsvendor case plus an additional factor and it is the number of retailers i.e. base demand a_i, the elasticity factor b_i, base price m_k, the discount scale d_k and the number of Retailers I; the rest of the parameters are fixed in each industry case. The levels for each parameter are selected from Table **5**, since there are 5 parameters and three levels for each parameter, there will be 243 test cases for each industry.

First, let's look at the ANOVA table for each industry case shown in Table 10-Table 13, In case 1 all the single and 2-way interactions are significant except the two-way interactions of $b_i * m_k$, $b_i * d_k$ and $m_k * d_k$. In case 2, only the 2-way interaction if $b_i * m_k$ is not significant. For case 3, the 2-way interactions between $a_i * d_k$, $b_i * d_k$ and $m_k * d_k$ are not significant; while in case 4, only the $b_i * m_k$ interaction is not significant at $p - value = 5\%$.

Table 10. ANOVA analysis for profit function- symmetric retailers- Case 1

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Model	50	43183108277	863662166	16760.39	0.000
Linear	10	41878814986	4187881499	81270.81	0.000
a_i	2	21163686908	10581843454	205353.23	0.000
b_i	2	15335180022	7667590011	148798.68	0.000
m_k	2	194950403	97475201	1891.62	0.000
d_k	2	603025944	301512972	5851.22	0.000
I	2	4581971709	2290985854	44459.30	0.000
2-Way Interactions	40	1304293291	32607332	632.78	0.000
a_i*b_i	4	772635171	193158793	3748.48	0.000
a_i*m_k	4	2445740	611435	11.87	0.000
a_i*d_k	4	25285411	6321353	122.67	0.000
a_i*I	4	210613551	52653388	1021.80	0.000
b_i*m_k	4	103721	25930	0.50	0.733
b_i*d_k	4	297256	74314	1.44	0.222
b_i*I	4	8065443	2016361	39.13	0.000
m_k*d_k	4	231567	57892	1.12	0.347
m_k*I	4	2589734	647433	12.56	0.000
d_k*I	4	282025695	70506424	1368.26	0.000
Error	192	9893752	51530		
Total	242	43193002029			

Table 11. ANOVA analysis for profit function- symmetric retailers- Case 2

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Model	50	1.06805E+15	2.13611E+13	2039.91	0.000
Linear	10	1.00620E+15	1.00620E+14	9608.89	0.000
a_i	2	5.90249E+14	2.95125E+14	28183.43	0.000
b_i	2	2.68086E+14	1.34043E+14	12800.69	0.000
m_k	2	2.75344E+13	1.37672E+13	1314.73	0.000
d_k	2	6.09092E+13	3.04546E+13	2908.32	0.000
I	2	5.94218E+13	2.97109E+13	2837.29	0.000
2-Way Interactions	40	6.18529E+13	1.54632E+12	147.67	0.000
a_i*b_i	4	2.38825E+13	5.97063E+12	570.18	0.000
a_i*m_k	4	5.59908E+11	1.39977E+11	13.37	0.000
a_i*d_k	4	4.67465E+12	1.16866E+12	111.60	0.000
a_i*I	4	4.48246E+12	1.12061E+12	107.02	0.000
b_i*m_k	4	24492643731	6123160933	0.58	0.674
b_i*d_k	4	1.48084E+11	37020936506	3.54	0.008
b_i*I	4	1.12588E+11	28146993407	2.69	0.033
m_k*d_k	4	2.66874E+11	66718498895	6.37	0.000

m_k*I	4	2.49631E+11	62407644471	5.96	0.000
d_k*I	4	2.74517E+13	6.86292E+12	655.39	0.000
Error	192	2.01054E+12	10471560566		
Total	242	1.07006E+15			

Table 12. ANOVA analysis for profit function- symmetric retailers- Case 3

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Model	50	1.13832E+16	2.27664E+14	5241080.40	0.000
Linear	10	1.10776E+16	1.10776E+15	25501873.41	0.000
a_i	2	5.96895E+15	2.98448E+15	68705997.08	0.000
b_i	2	4.71420E+15	2.35710E+15	54263082.81	0.000
m_k	2	3.72914E+12	1.86457E+12	42924.49	0.000
d_k	2	1599785686	799892843	18.41	0.000
I	2	3.90715E+14	1.95357E+14	4497344.27	0.000
2-Way Interactions	40	3.05601E+14	7.64003E+12	175882.15	0.000
a_i*b_i	4	2.76819E+14	6.92046E+13	1593168.69	0.000
a_i*m_k	4	54808070971	13702017743	315.44	0.000
a_i*d_k	4	85719244	21429811	0.49	0.741
a_i*I	4	2.19950E+13	5.49875E+12	126587.51	0.000
b_i*m_k	4	16364097886	4091024471	94.18	0.000
b_i*d_k	4	20499827	5124957	0.12	0.976
b_i*I	4	6.68032E+12	1.67008E+12	38447.09	0.000
m_k*d_k	4	157897	39474	0.00	1.000
m_k*I	4	35392680957	8848170239	203.69	0.000
d_k*I	4	738047598	184511899	4.25	0.003
Error	192	8340165970	43438364		
Total	242	1.13832E+16			

Table 13. ANOVA analysis for profit function- symmetric retailers- Case 4

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Model	50	35661066482	713221330	22160.19	0.000
Linear	10	33829852639	3382985264	105111.25	0.000
a_i	2	8807194610	4403597305	136822.25	0.000
b_i	2	14315947858	7157973929	222402.28	0.000
m_k	2	1145831922	572915961	17800.82	0.000
d_k	2	3868449686	1934224843	60097.45	0.000
I	2	5692428564	2846214282	88433.48	0.000
2-Way Interactions	40	1831213843	45780346	1422.42	0.000
a_i*b_i	4	277467207	69366802	2155.27	0.000
a_i*m_k	4	1528750	382187	11.87	0.000
a_i*d_k	4	18246415	4561604	141.73	0.000
a_i*I	4	26010156	6502539	202.04	0.000

b_i*m_k	4	174420	43605	1.35	0.251
b_i*d_k	4	891441	222860	6.92	0.000
b_i*I	4	1124567	281142	8.74	0.000
m_k*d_k	4	3374026	843507	26.21	0.000
m_k*I	4	4328693	1082173	33.62	0.000
d_k*I	4	1498068169	374517042	11636.46	0.000
Error	192	6179483	32185		
Total	242	35667245965			

Looking at the Main Effects plot for the retailers' profit in Figure 16-Figure 18, it can be said that after the demand parameters, the number of retailers have the highest effect on the profit function, the effect of m_k and d_k is lowest overall, but compared to the newsvendor case, it is higher. Again, here in case 3 the pricing parameters have a very low effect on the profit level.

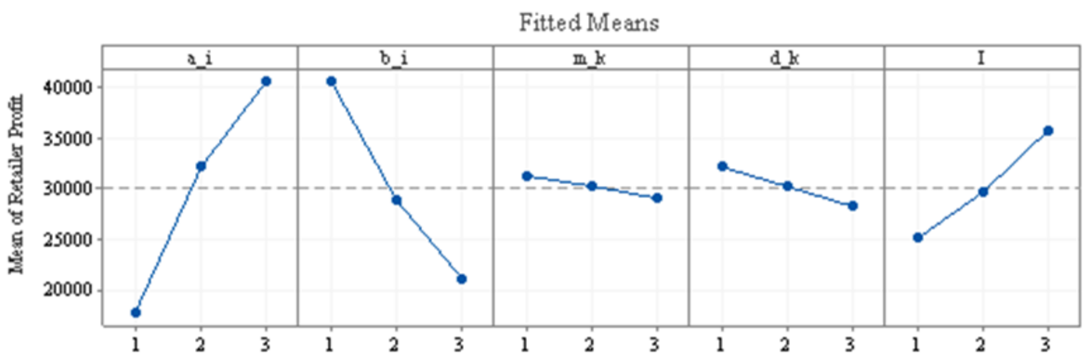

Figure 16: Main effects plot for retailer profit- Symmetric retailers – Case 1

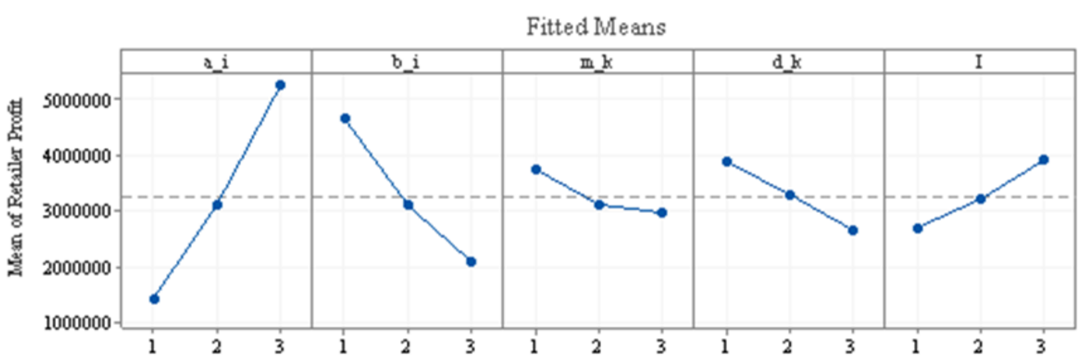

Figure 15: Main effects plot for retailer profit- Symmetric retailers – Case 2

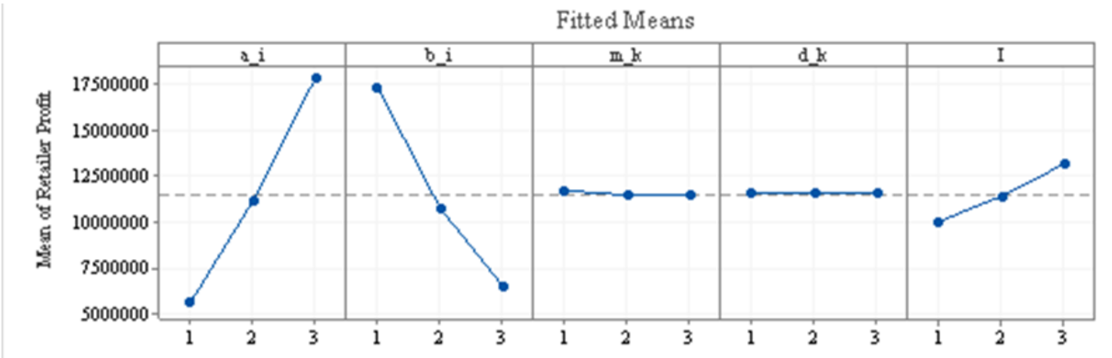

Figure 17: Main effects plot for retailer profit- Symmetric retailers – Case 3

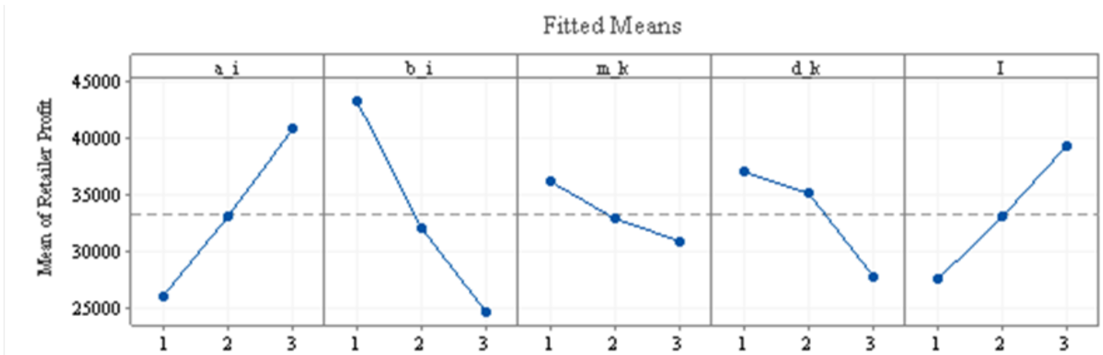

Figure 18: Main effects plot for retailer profit- Symmetric retailers – Case 4

3.3 Summary

In this chapter, the problem of newsvendor problem with QDF as well as GP with symmetric retailers was addressed and analyzed. For the newsvendor problem two approaches were proposed and optimality condition was studied in different scenarios. The first approach called the 2-step heuristic approach is a simulation optimization method that builds on the results of the newsvendor problem from the literature. The second approach is a 1-step analytical optimization method, which studies the optimality condition of the quantity discounted newsvendor problem. Next, these approaches were extended and analyzed for the symmetric GP case. Using the proposed methods, a full-factorial analysis was done for both the newsvendor problem and the GP problem with symmetric retailers for a selected set of input parameters to study their effect on the profit level. For the newsvendor problem a_i, b_i, m_k and d_k were selected as representative of demand and supply side factors. Next, in the GP problem with symmetric retailers an additional parameter I was included to study the effect of increased number of retailers in GP compared to the newsvendor problem. Based on the analysis, all of the main factors emerged as significant, while with regards to the two-way interactions this is not true. Some of the main observations are summarized below:

1. The demand parameters a_i and b_i always have the highest effect on the profit level.
2. The supplier pricing factors m_k and d_k have lower impact on the profit level.
3. In the symmetric retailer case, the number of retailers always has a significant impact on the response factor, even higher than the pricing parameters. Higher number of retailers always result in an increased profit levels for all retailers.

4 GROUP PURCHASING PROBLEMS WITH ASYMMETRIC RETAILERS

Unlike the previous chapter where we considered symmetric retailers, in this chapter, the retailers are considered as asymmetric, meaning the input parameters including the demand function, stochastic factor as well as salvage and shortage cost can take different value for each retailer; also, suppliers' pricing factors vary for each supplier in this problem. Due to the quantity discount pricing dependence, price-sensitive demand, and suppliers' order assignment problem, unlike the symmetric retailers' problem, the asymmetric retailers' problem cannot be simplified or treated similar to the newsvendor case as before. Furthermore, suppliers' problem adds to the complexity of the problem. To tackle this challenging problem with asymmetric retailers; the problem will be divided into two sub-problems, the retailers' problem, and the suppliers' problem. Hence, we will develop solution methods for the retailers' problem and suppliers' problem separately. The retailers' problem addresses the decision parameters that the retailer side of GP deal with i.e., order quantity and retail price, given a purchasing price which is the output of the suppliers' problem. On the other hand, the suppliers' problem deals with the decision parameters about the suppliers, such as procurement quantities from each supplier given the total order quantity from retailers and pricing function from the suppliers. These two problems can be solved sequentially until a convergence point is achieved for the GP problem. Figure 19 displays the general GP supply chain, including multiple suppliers and multiple retailers who cooperate using GP.

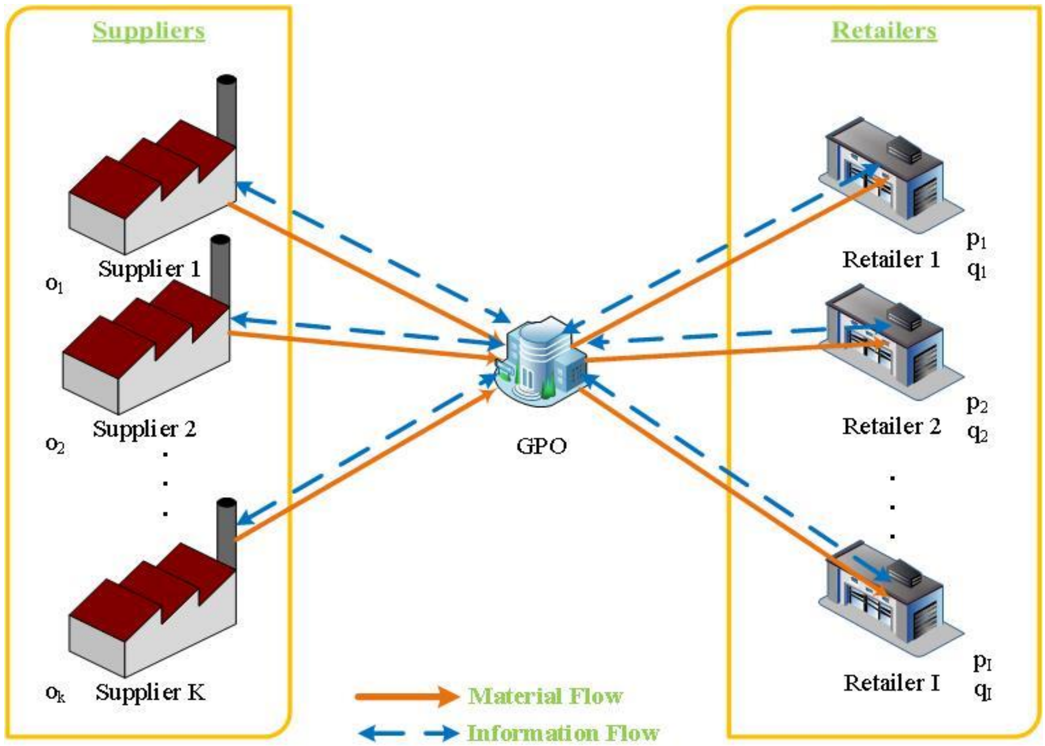

Figure 19. Group purchasing supply chain

4.1 General Asymmetric Retailers' Problem

The retailers' problem is to find the optimal pricing and order quantity given the purchasing price and the demand data for each retailer. First, the general problem and the proposed solution approach for any number of retailers will be presented, and then specific solutions for the two and three-retailer problems will be provided. As shown in Figure 20, the retailers' problem includes I retailers that face stochastic and price-sensitive demand. Each retailer decides about the price and order quantity to maximize its own profit with respect to the demand, competitors' pricing, and the purchasing price. The demand function for each retailer i and competitor retailers j is similar to the one presented in 3.23, thus it is not repeated here.

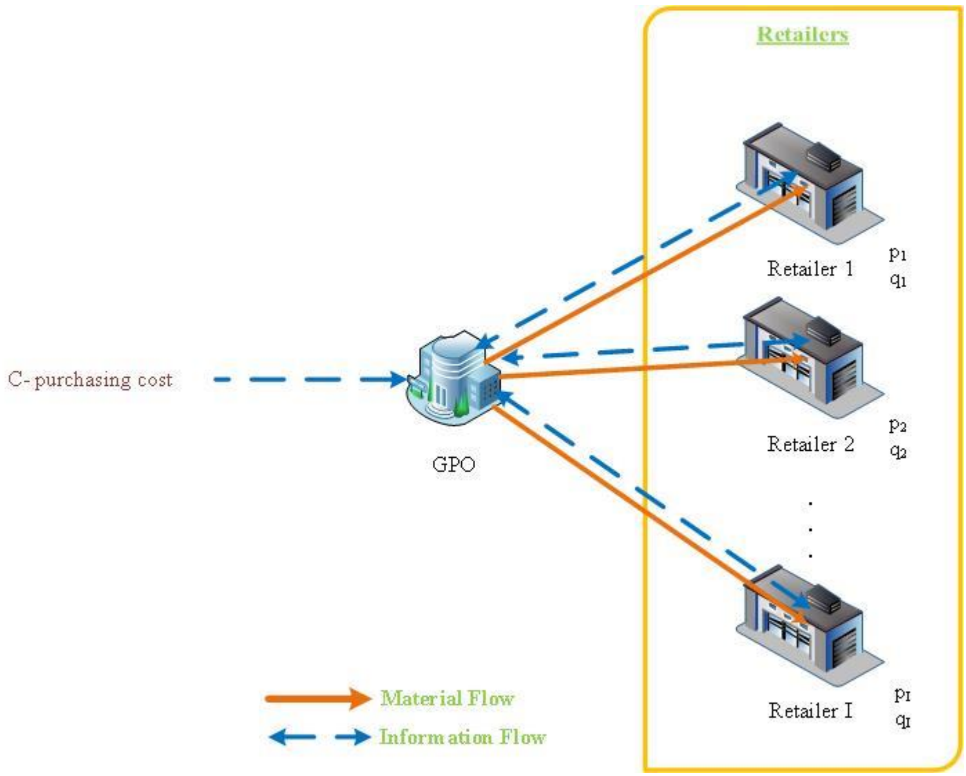

Figure 20. Assymetric retailers' problem

If the retail price and order quantity for retailer i is represented as p_i and q_i; and for competitor retailers j as p_j and q_j; substituting the wholesale price c_i for $w(Q)$ in 3.25, the profit function for each retailer i can be written as:

$$\pi_i(z_i, p_i, p_j) = \begin{cases} p_i D_i(p_i, p_j, \epsilon) - c_i q_i + v_i[q_i - D_i(p_i, p_j, \epsilon)], & D_i(p_i, p_j, \epsilon) \le q_i, j \ne i \\ p_i q_i - c q_i - s_i[D_i(p_i, p_j, \epsilon) - q_i], & D_i(p_i, p_j, \epsilon) > q_i, j \ne i \end{cases} \quad 4.1$$

In the above profit function, the first case happens if the demand is less than the order quantity and the retailer face a salvage cost $(c_i - v_i)$ for each leftover item, while in the second case the retailer faces a shortage cost s_i for each missed demand due to ordering less than demand. The purchasing cost c_i is assumed to be equal across all retailers independent of their order quantity. Substituting $D_i(p_i, p_j, \epsilon)$ with $y_i(p_i, p_j) + \epsilon$ and setting $z_i = q_i - y_i(p_i, p_j)$ will result in a more useful and abstract expression of the profit function:

$$\pi_i(z_i, p_i, p_j) = \begin{cases} p_i[y_i(p_i, p_j) + \epsilon] - c_i[y(p_i, p_j) + z_i] + v_i[z_i - \epsilon], \epsilon \le z_i, j \ne i \\ p_i[y_i(p_i, p_j) + z_i] - c_i[y(p_i, p_j) + z_i] - s_i[\epsilon - z_i], \epsilon > z_i, j \ne i \end{cases} \quad 4.2$$

This expression changes the perspective of the profit function calculation from focusing on q_i to z_i; if the choice of z_i is higher than the observed value of ϵ, leftover occurs; and if z_i is smaller than ϵ, shortage occurs. If p_i^*, p_j^* and z_i^* maximize the expected profit, then the optimal ordering policy would be $q_i^* = y_i(p_i^*, p_j^*) + z_i^*$. If $f(u)$ and $F(u)$ are representative of probability and cumulative density functions and μ is the mean of the random variable ϵ; then expected profit function can be presented as:

$$E[\pi_i(z_i, p_i, p_j)] = \int_A^{z_i} \left(p_i[y_i(p_i, p_j) + u] + v_i[z_i - u]\right) f(u)du + \int_{z_i}^{B} \left(p_i[y_i(p_i, p_j) + z_i] - s_i[u - z_i]\right) f(u)du - c_i[y_i(p_i, p_j) + z_i] = \Psi_i(p_i, p_j) - L_i(z_i, p_i) \quad 4.3$$

Where the first part is:

$$\Psi_i(p_i, p_j) = (p_i - c)[y_i(p_i, p_j) + \mu] \quad 4.4$$

Which can be called risk-less profit function because it calculates the profit function in case the random variable hits its mean value; and the second part is:

$$L_i(z_i, p_i) = (c_i - \mathrm{v_i})\Lambda(z_i) + (p_i + s_i - c_i)\Theta(z_i) \qquad 4.5$$

Analogous to the newsvendor case in 3.1.1, this term calculates the loss due to shortage and surplus.

To maximize the expected profit for each retailer i, let's find the first and second order derivative with respect to z_i and p_i:

$$\frac{\partial E\left[\pi\left(\mathrm{z_i}, \mathrm{p_i}, p_j\right)\right]}{\partial \mathrm{z_i}} = -(c_i - v_i) + (\mathrm{p_i} + s_i - \mathrm{v_i})[1 - F(\mathrm{z_i})] \qquad 4.6$$

$$\frac{\partial^2 E\left[\pi\left(\mathrm{z_i}, \mathrm{p_i}, p_j\right)\right]}{\partial \mathrm{z_i^2}} = -(\mathrm{p_i} + s_i - \mathrm{v_i})f(z_i) \qquad 4.7$$

$$\frac{\partial E\left[\pi_{\mathrm{i}}\left(z_i, p_i, p_j\right)\right]}{\partial p_i} = \left[y_i\left(p_i, p_j\right) + \mu\right] - (\mathrm{b_i} + \gamma_i)(\mathrm{p_i} - \mathrm{c_i}) - \Theta(z_i)$$

$$= [a_i - (b_i + \gamma_i)p_i + \gamma_i \frac{1}{n-1}\sum_{\substack{j=1 \\ j \neq i}}^{n} p_j + \mu] - (b_i + \gamma_i)(p_i - c_i) - \Theta(z_i) \qquad 4.8$$

$$= 2(\mathrm{b_i} + \gamma_i)\left[\frac{\mathrm{a_i} + (\mathrm{b_i} + \gamma_i)c_i + \mu + \gamma_i\, {}^{1}\!/_{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} p_j}{2(\mathrm{b_i} + \gamma_i)} - \mathrm{p_i}\right] - \Theta(\mathrm{z_i}) = 2(\mathrm{b_i} + \gamma_i)(p_i^0 - p_i) - \Theta(z_i)$$

Where $p_i^0 = {\mathrm{a_i} + (\mathrm{b_i} + \gamma_i)c_i + \mu + \gamma_i\, {}^{1}\!/_{n-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} p_j} \Big/ {2(\mathrm{b_i} + \gamma_i)}$ 4.9

$$\frac{\partial^2 E\left[\pi_i\left(z_i, p_i, p_j\right)\right]}{\partial p_i^2} = -2(b_i + \gamma_i) \qquad 4.10$$

The first term of equation 4.8 $[2(b_i + \gamma_i)(p_i^0 - p_i)]$ is the partial derivative of 4.4 with respect to p_i; which is the risk-less profit function. Equating this term to zero and finding the value of p_i that would satisfy this equation yields the optimal risk-less price which is p_i^0. Unlike the newsvendor case in Petruzzi and Dada (1999) were the value of p_i^0 can be found by solving one equation; in this problem -as it can be seen above- solving the single equation will result in a term that is a function of p_j which is also a decision variable. To find the value of the risk-less price in terms of the known parameters the first order condition needs to be satisfied for the risk-less profit function for all the retailers simultaneously. In this problem p_i^0 can be found by solving the below system of equation for all retailers i:

$$\begin{cases} \dfrac{\partial \Psi(p_1, p_j)}{\partial p_1} = 0 & j = 2, \dots, I \\ \dfrac{\partial \Psi(p_2, p_j)}{\partial p_2} = 0 & j = 1, 3, 4, \dots, I \\ \qquad \vdots \\ \dfrac{\partial \Psi(p_i, p_j)}{\partial p_i} = 0 & j = 1, \dots, I, j \neq i \\ \qquad \vdots \\ \dfrac{\partial \Psi(p_I, p_j)}{\partial p_I} = 0 & j = 1, \dots, I-1 \end{cases} \qquad 4.11$$

Assuming there are I retailers in the problem, to solve the system of equations in 4.11, one needs to solve a system of I equations and I variables. If $t_i = a_i + (b_i + \gamma_i)c_i + \mu$; the $I * I$ system of equations can be displayed in matrix form as:

$$\begin{bmatrix} 2(b_1+\gamma_1) & -\frac{\gamma_1}{I-1} & \cdots & -\frac{\gamma_1}{I-1} \\ -\frac{\gamma_2}{I-1} & 2(b_2+\gamma_2) & \cdots & -\frac{\gamma_2}{I-1} \\ \vdots & \vdots & \vdots & \vdots \\ -\frac{\gamma_i}{I-1} & -\frac{\gamma_i}{I-1} & \cdots & -\frac{\gamma_i}{I-1} \\ \vdots & \vdots & \vdots & \vdots \\ -\frac{\gamma_I}{I-1} & -\frac{\gamma_I}{I-1} & \cdots & 2(b_I+\gamma_I) \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_i \\ \vdots \\ p_I \end{bmatrix} = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_i \\ \vdots \\ t_I \end{bmatrix} \quad 4.12$$

As it can be seen in the coefficient matrix, in each row i, column i has a value of $2(b_i + \gamma_i)$ and the rest are $-\frac{\gamma_i}{I-1}$. For a problem with I asymmetric retailers, the profit function is going to be a function of $I + 1$ variables, z_i and p_i^0 for all values of i. To solve this problem, first, the $I * I$ system of equations needs to be solved to find the riskless optimal price, p_i^0. With increasing I, solving the $I * I$ system of equation becomes not only challenging but finding a general compact solution for any I-retailer problem is not possible. Since a closed solution cannot be found it is rather difficult to analyze the problem structure and derive insights for the general problem. After discussing general results for the I-retailer below, the two and three retailer problems will be studied in detail.

Consider the partial derivative 4.8, which is a function of p_i, p_j, and z_i, assuming I is the number of retailers, the problem has $I + 1$ variables. The riskless profit function p_i^0 contains $I - 1$ of those variables; finding the value of p_i^0 by solving 4.12 removes p_j from the equation and reduces the number of variables from $I + 1$ to only 2 variables p_i and z_i. This variable reduction eases finding optimal p_i and z_i, once this transformation is done, the method used in 3.1.1 can be used to solve this problem.

Lemma 3: In the GP problem with asymmetric retailers, the optimal risk-less price $\mathrm{p_i^0}$is unique and converts 4.8 from an $(\mathrm{I}+1)$-variable problem to a 2-variable problem. It is found by solving the below equation for all retailers in an I-variable I-equation system of equations:

$$\frac{\partial \Psi_i(p_i, p_j)}{\partial p_i} = 0 \quad j = 1, \dots, n, j \neq i \qquad 4.13$$

Proof: The riskless profit function Ψ_i is a function of p_i and p_j and the optimal price p_i found through the first order condition is a function of p_j. Satisfying the first order condition for all retailers simultaneously in the form of a system of equation in 4.12 for all p_i will result in finding the optimal price p_i^0 that is not a function of other variables. Replacing this value in 4.8 removes p_j from the equation. ■

Observing the second partial derivative with respect to z_i in 4.7; we can say that $(p_i + s_i - v_i)$ is always positive because p_i and s_i are positive and $v_i \leq p_i$. Thus, the term $-(\mathrm{p_i} + s_i - \mathrm{v_i})f(z_i)$ is always negative, so the profit function is concave downward in z_i. The same case is with the second partial derivative with respect to p_i in 4.10, because $b_i \geq 0$ & $\gamma_i \geq 0$. Therefore, we know that $E[\pi_\mathrm{i}(z_i, p_i, p_j)]$ is concave downward in both p_i and z_i; thus, the optimization problem can be reduced to a simpler problem. Using Zabel's approach (Zabel 1970) one can first find optimal value of p_i as a function of z_i and substitute it in 4.6, then the search can be made over the single variable space to find the optimal z_i.

Lemma 4: For a given z_i the optimal price can be defined as a function of z_i:

$$p_i = p_i^0 - \frac{\Theta(z_i)}{2(b_i + \gamma_i)} \qquad 4.14$$

Proof: By applying the Zabel's approach (Zabel 1970) of first finding $p^*$for a fixed z_i and then searching over the single variable function to maximize the expected profit. Lemma 5 stems by implementing this approach on 4.8 & 4.10. ■

Applying Lemma 4 for all p_i and p_j in the expected profit function changes it to a function of z_i and z_j. Lemma 5 follows by checking the first order condition on the new profit function.

Lemma 5: Even though the converted profit function is a function of z_i and z_j, z_i^* may be found independent of the value of z_j.

Proof: After applying Lemma 4, the first derivative of the profit function with respect to z_i will be a function of only z_i, therefore z_i^* is not a function of z_j. Hence z_i^* will result in the optimal expected profit no matter what the competitors' decision is on z_j. ■

Let's call the first derivative of the profit function with respect to z_i as $R_i(z_i)$. Finding the values of z_i that satisfy the first order condition means finding the roots of $R_i(z_i)$.

$$R_i(z_i) = \frac{\partial E\left[\pi\left(\mathrm{z_i}, \mathrm{p_i}(z_i), p_j(z_j)\right)\right]}{\partial \mathrm{z_i}}$$

$$= -(c - v_i) + \left(\mathrm{p_i^0} + s_i - \mathrm{v_i} - \frac{\Theta(z_i)}{2(b_i + \gamma_i)}\right)[1 - F(\mathrm{z_i})] \qquad 4.15$$

Discovering the values of z_i that satisfy first-order condition is important in finding the optimal profit. Interpreting the shape of the $R_i(z_i)$ based on the problem's parameters is fundamental in finding the optimal z_i; Theorem 5 sheds light on this problem:

Theorem 5: The optimal order and pricing policy in the group buying problem with I competing asymmetric retailers is to order $q_i^* = y_i(p_i^*) + z_i^*$ units and sell at the unit price p_i^*, where p_i^* is determined using Lemma 4 and Lemma 5 and z_i^* is defined based on the following:

a) If $F(.)$ is a random distribution function, then a complete search over the range $[A, B]$ will determine z_i^*.

b) If $F(.)$ satisfies the condition $2r(z_i)^2 + \frac{dr(z_i)}{dz_i} > 0$ for $A \leq z_i \leq B$ and $r(.) = \frac{f(.)}{1-F(.)}$; then $z_i^*$is the largest z_i in the range $[A, B]$ that satisfies $\frac{dE\left[\pi_i\left(z_i, p_i(z_i), p_j(z_j)\right)\right]}{dz_i} = 0$

c) If condition b is met and $a_i - (b_i + \gamma_i)(c_i - 2s_i) + A > 0$, then z_i^* is the unique z in the range $[A, B]$ that satisfies $\frac{dE\left[\pi_i\left(z_i, p_i(z_i), p_j(z_j)\right)\right]}{dz_i} = 0$.

Proof: See Appendix VII.

The general approach of the Theorem 5 is similar to Petruzzi and Dada (1999), but the profit function here different than their case because their problem was a newsvendor problem and here there are multiple asymmetric competing retailer problem. Since the profit function for each retailer i is a function of z_i and z_j, Theorem 5 needs to be applied for all the retailers to find optimal profit for each retailer i.

The first step in finding p_i^* and q_i^* here is solving the system of equation in 4.12 and finding p_i^0 for the I-retailer problem. Even though it is possible to solve it for any number of retailers, starting with two-retailer problem, with each added retailer to the problem, the solution grows very quickly in size, which is shown in the next sections. The answer to each size of the problem can be compacted, but it cannot be generalized for any I number of retailers. Solutions for the two and three retailer problems are presented in the next sections; the reader can find the four and five retailer solutions in the Appendix VIII. Examples for two and three retailer problem will be solved in the next sections to show the effectiveness of the solution.

4.1.1 Two asymmetric retailer problem

This problem is a special case of the general problem for which the solution approach is mentioned in the previous section, the developed approach will be used to solve this problem here. Assigning i and j as the index for any of the retailer 1 & 2 in this case; the partial derivative of the expected profit function with respect to p_i, $\frac{\partial E[\pi_i(z_i,p_i,p_j)]}{\partial p_i}$ is a function of both p_i and p_j, so the riskless profit function p_i^0 will be a function of p_j. To simplify the problem, one can find $\frac{\partial E[\pi_i(z_i,p_i,p_j)]}{\partial p_i}$ for both retailers and solve the 2 variable, 2 equation system of equation for p_i to find the riskless profit independent of p_j. In the two-retailer problem, the partial derivative of the expected profit function with respect to p_1 and p_2 for retailer 1 and 2 is as the following:

$$\begin{aligned}\frac{\partial E[\pi_1(z_1,p_1,p_2)]}{\partial p_1} &= [y_1(p_1,p_2)+\mu] - (\mathrm{b}_1+\gamma_1)(\mathrm{p}_1-\mathrm{c}_1) - \Theta(z_1) \\ &= 2(\mathrm{b}_1+\gamma_1)(p_1^0-p_1) - \Theta(z_1)\end{aligned} \qquad 4.16$$

$$\begin{aligned}\frac{\partial E[\pi_2(z_2,p_2,p_1)]}{\partial p_2} &= [y_1(p_2,p_1)+\mu] - (\mathrm{b}_2+\gamma_2)(\mathrm{p}_2-\mathrm{c}_2) - \Theta(z_2) \\ &= 2(\mathrm{b}_2+\gamma_2)(p_2^0-p_2) - \Theta(z_2)\end{aligned} \qquad 4.17$$

Where $p_i^0 = {{\mathrm{a_i} + (\mathrm{b_i} + \gamma_i)c_i + \mu + \gamma_i p_j} \big/ {2(\mathrm{b_i} + \gamma_i)}}$ which is a function of p_j, for each retailer p_1^0 and p_2^0, can be found by solving the below system of equation for p_1 and p_2:

$$\begin{cases} \dfrac{\partial \Psi(p_1, p_2)}{\partial p_1} = 0 \\ \dfrac{\partial \Psi(p_2, p_1)}{\partial p_2} = 0 \end{cases} = \begin{cases} y_1(p_1, p_2) + \mu - (b_1 + \gamma_1)(p_1 - c) = 0 \\ y_2(p_2, p_1) + \mu - (b_2 + \gamma_2)(p_2 - c) = 0 \end{cases} \qquad 4.18$$

By solving the above system, p_i^0 for retailer 1 and 2 can be found as:

$$p_1^0 = \frac{\gamma_1(a_2 + (b_2 + \gamma_2)c + \mu) + 2(b_2 + \gamma_2)(a_1 + (b_1 + \gamma_1)c + \mu)}{4(b_1 + \gamma_1)(b_2 + \gamma_2) - \gamma_1\gamma_2} \qquad 4.19$$

$$p_2^0 = \frac{\gamma_2(a_1 + (b_1 + \gamma_1)c + \mu) + 2(b_1 + \gamma_1)(a_2 + (b_2 + \gamma_2)c + \mu)}{4(b_1 + \gamma_1)(b_2 + \gamma_2) - \gamma_1\gamma_2} \qquad 4.20$$

After substituting $t_i = a_i + (b_i + \gamma_i)c + \mu$, the results simplify to:

$$p_1^0 = \frac{\gamma_1 t_2 + 2(b_2 + \gamma_2)t_1}{4(b_1 + \gamma_1)(b_2 + \gamma_2) - \gamma_1\gamma_2} \qquad 4.21$$

$$p_2^0 = \frac{\gamma_2 t_1 + 2(b_1 + \gamma_1)t_2}{4(b_1 + \gamma_1)(b_2 + \gamma_2) - \gamma_1\gamma_2} \qquad 4.22$$

By replacing these values in partial derivative of the expected profit function with respect to each p_i and solving it for p_i, it can be converted to a function of z_i which provides the relationship between p_i and z_i at the optimal point shown in Lemma 4. Using Lemma 4 changes the profit function for each retailer as a function of z_1 and z_2:

$$E[\pi_1(z_1, p_1(z_1), p_2(z_2))] = \Psi_1\big(p_1(z_1), p_2(z_2)\big) - L_1\big(z_1, p_1(z_1)\big) = \left(p_1^0 - \frac{\Theta(z_1)}{2b_1} - c\right)\left[a_1 - (b_1+\gamma_1)\left(p_1^0 - \frac{\Theta(z_1)}{2(b_1+\gamma_1)}\right) + \gamma_1\left(p_2^0 - \frac{\Theta(z_2)}{2(b_2+\gamma_2)}\right) + \mu\right] - (c - v_1)\Lambda(z_1)\left(p_1^0 - \frac{\Theta(z_1)}{2(b_1+\gamma_1)} + s_1 - c\right)\Theta(z_1) \quad 4.23$$

$$E[\pi_2(z_2, p_2(z_2), p_1(z_1))] = \Psi_2\big(p_2(z_2), p_1(z_1)\big) - L_2\big(z_2, p_2(z_2)\big) = \left(p_2^0 - \frac{\Theta(z_2)}{2(b_2+\gamma_2)} - c\right)\left[a_2 - (b_2+\gamma_2)\left(p_2^0 - \frac{\Theta(z_2)}{2(b_2+\gamma_2)}\right) + \gamma_2\left(p_1^0 - \frac{\Theta(z_1)}{2(b_1+\gamma_1)}\right) + \mu\right] - (c - v_2)\Lambda(z_2)\left(p_2^0 - \frac{\Theta(z_2)}{2(b_2+\gamma_2)} + s_2 - c\right)\Theta(z_2) \quad 4.24$$

And the partial derivative with respect to z_i for each retailer is:

$$\frac{\partial E[\pi_1(z_1, p_1(z_1), p_2(z_2))]}{\partial z_1} = -(c - v_1) + \big(1 - F(z_1)\big)\left(p_1^0 + s_1 - c - \frac{\Theta(z_1)}{2(b_1+\gamma_1)}\right) \quad 4.25$$

$$\frac{\partial E[\pi_2(z_2, p_2(z_2), p_1(z_1))]}{\partial z_2} = -(c - v_2) + \big(1 - F(z_2)\big)\left(p_2^0 + s_2 - c - \frac{\Theta(z_2)}{2(b_2+\gamma_2)}\right) \quad 4.26$$

Which is only a function of only z_i as mentioned in Lemma 5; finding the roots of this function for both retailers, will help in finding the optimal profit function for each retailer. Using Theorem 5, it can be verified that $F(.)$ satisfies the condition b of Theorem 5 for uniform distribution of ϵ; so z_i^* is the largest z in the range $[A, B]$ that satisfies $\frac{dE\left[\pi_i\left(z_I, p_i(z_i), p_j(z_j)\right)\right]}{dz_i} = 0.$

4.1.2 Three asymmetric retailer problem

Here, the general approach developed in 4.1 will be used to solve the special three-retailer problem case; the solution to the 4.12 will be shown and numerical experiments will be done with the three retailer case to show the usefulness of the developed approach. Assigning i, j and n as the index for retailers 1, 2 & 3; the partial derivative of the expected profit function with respect to p_i, $\frac{\partial E[\pi_i(z_i,p_i,p_j,p_k)]}{\partial p_i}$ is a function of p_i, p_j and p_n, so the riskless profit function p_i^0 will be a function of p_j and p_n. To simplify the problem one can find $\frac{\partial E[\pi(z_i,p_i,p_j,p_k)]}{\partial p_i}$ for all retailers and solve the 3 variable, 3 equation system of equation in 4.12 for p_i to find the riskless profit p_i^0 independent of p_j and p_n. In the three-retailer problem, the partial derivative of the expected profit function with respect to p_1, p_2 and p_3 for retailer 1, 2 and 3 is as the following:

$$\frac{\partial E[\pi_1(z_1,p_1,p_2,p_3)]}{\partial p_1} = [y_1(p_1,p_2,p_3)+\mu] - (b_1+\gamma_1)(p_1 - c) - \Theta(z_1) \qquad 4.27$$

$$= 2(b_1+\gamma_1)(p_1^0 - p_1) - \Theta(z_1)$$

$$\frac{\partial E[\pi_2(z_2,p_2,p_1,p_3)]}{\partial p_2} = [y_2(p_2,p_1,p_3)+\mu] - (b_2+\gamma_2)(p_2 - c) - \Theta(z_2) \qquad 4.28$$

$$= 2(b_2+\gamma_2)(p_2^0 - p_2) - \Theta(z_2)$$

$$\frac{\partial E[\pi_3(z_3,p_3,p_1,p_2)]}{\partial p_3} = [y_3(p_3,p_1,p_2)+\mu] - (b_3+\gamma_3)(p_3 - c) - \Theta(z_3) \qquad 4.29$$

$$= 2(b_3+\gamma_3)(p_3^0 - p_3) - \Theta(z_3)$$

Where p_1^0, p_2^0 and p_3^0 are the optimal riskless price which is found by solving the below system of equation for p_1, p_2 and p_3:

$$\begin{cases} \dfrac{\partial \Psi(p_1, p_2, p_3)}{\partial p_1} = 0 \\ \dfrac{\partial \Psi(p_2, p_1, p_3)}{\partial p_2} = 0 \\ \dfrac{\partial \Psi(p_3, p_1, p_2)}{\partial p_3} = 0 \end{cases} \Rightarrow \begin{cases} y_1(p_1, p_2, p_3) + \mu - (b_1 + \gamma_1)(p_1 - c) = 0 \\ y_2(p_2, p_1, p_3) + \mu - (b_2 + \gamma_2)(p_2 - c) = 0 \\ y_3(p_3, p_1, p_2) + \mu - (b_3 + \gamma_3)(p_3 - c) = 0 \end{cases} \quad 4.30$$

By solving the above system, we find the value of riskless profit for retailer 1, 2 and 3. After substituting $t_i = a_i + (b_i + \gamma_i)c + \mu$, the p_i^0 will be:

$$p_1^0 = \frac{\gamma_1\gamma_2 t_3 + \gamma_1\gamma_3 t_2 - \gamma_2\gamma_3 t_1 + 4\gamma_1 \begin{pmatrix} (b_2 + \gamma_2)t_3 \\ +(b_3 + \gamma_3)t_2 \end{pmatrix} + 16(b_2 + \gamma_2)(b_3 + \gamma_3)t_1}{32(b_1 + \gamma_1)(b_2 + \gamma_2)(b_3 + \gamma_3) - \gamma_1\gamma_2\gamma_3 - 2\gamma_1 \begin{pmatrix} \gamma_2(b_3 + \gamma_3) \\ +\gamma_3(b_2 + \gamma_2) \end{pmatrix} - 2\gamma_2\gamma_3(b_1 + \gamma_1)} \quad 4.31$$

$$p_2^0 = \frac{\gamma_1\gamma_2 t_3 - \gamma_1\gamma_3 t_2 + \gamma_2\gamma_3 t_1 + 4\gamma_2 \begin{pmatrix} (b_1 + \gamma_1)t_3 \\ +(b_3 + \gamma_3)t_1 \end{pmatrix} + 16(b_1 + \gamma_1)(b_3 + \gamma_3)t_2}{32(b_1 + \gamma_1)(b_2 + \gamma_2)(b_3 + \gamma_3) - \gamma_1\gamma_2\gamma_3 - 2\gamma_2 \begin{pmatrix} \gamma_1(b_3 + \gamma_3) \\ +\gamma_3(b_1 + \gamma_1) \end{pmatrix} - 2\gamma_1\gamma_3(b_2 + \gamma_2)} \quad 4.32$$

$$p_3^0 = \frac{-\gamma_1\gamma_2 t_3 + \gamma_1\gamma_3 t_2 + \gamma_2\gamma_3 t_1 + 4\gamma_3 \begin{pmatrix} (b_1 + \gamma_1)t_2 \\ +(b_2 + \gamma_2)t_1 \end{pmatrix} + 16(b_1 + \gamma_1)(b_2 + \gamma_2)t_3}{32(b_1 + \gamma_1)(b_2 + \gamma_2)(b_3 + \gamma_3) - \gamma_1\gamma_2\gamma_3 - 2\gamma_3 \begin{pmatrix} \gamma_1(b_2 + \gamma_2) \\ +\gamma_2(b_1 + \gamma_1) \end{pmatrix} - 2\gamma_1\gamma_2(b_3 + \gamma_3)} \quad 4.33$$

Comparing the values of the p_i^0 for the two-retailer and three-retailer case, the size of the parametric answer grows very fast by adding only one more retailer. Though it is possible to solve larger test problems with numeric values, the solution to the parametric problem is going to be very large to display. Two additional examples for 4 and 5 retailer problem cases are presented in the Appendix VIII to further illustrate the challenges representing the solution in a compact closed formulation for the general I-retailer case.

Replacing these values in partial derivative of the expected profit function with respect to each p_i and solving it for p_i we can convert it to a function of z_i which provides the relationship between p_i and z_i at the optimal point shown in Lemma 4. Using Lemma 4, the profit function for each retailer will change to a function of z_1, z_2 and z_3:

$$\begin{aligned} &E[\pi_1(z_1, \mathrm{p}_1(\mathrm{z}_1), \mathrm{p}_2(\mathrm{z}_2), p_3(z_3))] \\ &= \Psi_1(p_1(z_1), p_2(z_2), p_3(z_3)) - L_1\big(z_1, p_1(z_1)\big) \\ &= \left(p_1^0 - \frac{\Theta(\mathrm{z}_1)}{2(b_1+\gamma_1)} - c\right)\left[a_1 - (b_1+\gamma_1)\left(p_1^0 - \frac{\Theta(\mathrm{z}_1)}{2(b_1+\gamma_1)}\right)\right. \\ &\left. + \gamma_1 \frac{1}{2}\left(p_2^0 - \frac{\Theta(\mathrm{z}_2)}{2(b_2+\gamma_2)} + p_3^0 - \frac{\Theta(z_3)}{2(b_3+\gamma_3)}\right) + \mu\right] \\ &- (c - \mathrm{v}_1)\Lambda(z_1)\left(p_1^0 - \frac{\Theta(\mathrm{z}_1)}{2(b_1+\gamma_1)} + s_1 - c\right)\Theta(z_1) \end{aligned} \quad 4.34$$

$$\begin{aligned} &E[\pi_2(z_2, \mathrm{p}_2(\mathrm{z}_2), \mathrm{p}_1(\mathrm{z}_1), p_3(z_3))] \\ &= \Psi_2\big(p_2(z_2), p_1(z_1), p_3(z_3)\big) - L_2\big(z_2, p_2(z_2)\big) \\ &= \left(p_2^0 - \frac{\Theta(\mathrm{z}_2)}{2(b_2+\gamma_2)} - c\right)\left[a_2 - (b_2+\gamma_2)\left(p_2^0 - \frac{\Theta(\mathrm{z}_2)}{2(b_2+\gamma_2)}\right)\right. \\ &\left. + \gamma_2 \frac{1}{2}\left(p_1^0 - \frac{\Theta(\mathrm{z}_1)}{2(b_1+\gamma_1)} + p_3^0 - \frac{\Theta(z_3)}{2(b_3+\gamma_3)}\right) + \mu\right] \\ &- (c - \mathrm{v}_2)\Lambda(z_2)\left(p_2^0 - \frac{\Theta(\mathrm{z}_2)}{2(b_2+\gamma_2)} + s_2 - c\right)\Theta(z_2) \end{aligned} \quad 4.35$$

$$[\pi_3(z_3, \mathrm{p}_3(\mathrm{z}_3), \mathrm{p}_1(\mathrm{z}_1), p_2(z_2))]$$

$$= \Psi_3\big(p_3(z_3), p_1(z_1), p_2(z_2)\big) - L_3\big(z_3, p_3(z_3)\big)$$

$$= \left(p_3^0 - \frac{\Theta(\mathrm{z}_3)}{2(b_3+\gamma_3)} - c\right)\left[a_3 - (b_3+\gamma_3)\left(p_3^0 - \frac{\Theta(\mathrm{z}_3)}{2(b_3+\gamma_3)}\right)\right. \quad 4.36$$

$$\left. + \gamma_3 \frac{1}{2}\left(p_1^0 - \frac{\Theta(\mathrm{z}_1)}{2(b_1+\gamma_1)} + p_2^0 - \frac{\Theta(z_2)}{2(b_2+\gamma_2)}\right) + \mu\right]$$

$$- (c - \mathrm{v}_2)\Lambda(z_2)\left(p_3^0 - \frac{\Theta(\mathrm{z}_3)}{2(b_3+\gamma_3)} + s_3 - c\right)\Theta(z_3)$$

And the partial derivative with respect to z_i for each retailer is:

$$\frac{\partial E[\pi_1(z_1, \mathrm{p}_1(\mathrm{z}_1), \mathrm{p}_2(\mathrm{z}_2), p_3(z_3))]}{\partial z_1}$$

$$= -(c - v_1) + \big(1 - F(z_1)\big)\left(p_1^0 + s_1 - c - \frac{\Theta(z_1)}{2(b_1+\gamma_1)}\right) \quad 4.37$$

$$\frac{\partial E[\pi_2(z_2, \mathrm{p}_2(\mathrm{z}_2), \mathrm{p}_1(\mathrm{z}_1), p_3(z_3))]}{\partial z_2}$$

$$= -(c - v_2) + \big(1 - F(z_2)\big)\left(p_2^0 + s_2 - c - \frac{\Theta(z_2)}{2(b_2+\gamma_2)}\right) \quad 4.38$$

$$\frac{\partial E[\pi_3(z_3, \mathrm{p}_3(\mathrm{z}_3), \mathrm{p}_1(\mathrm{z}_1), p_2(z_2))]}{\partial z_3}$$

$$= -(c - v_3) + \big(1 - F(z_3)\big)\left(p_3^0 + s_3 - c - \frac{\Theta(z_3)}{2(b_3+\gamma_3)}\right) \quad 4.39$$

Which is a function of only z_i as mentioned in Lemma 5; by finding the roots of this function for all retailers, the optimal profit for each retailer can be found. Using Theorem 5, one can verify that $F(.)$ satisfies the condition b of Theorem 5 for ϵ with uniform distribution; so z_i^* is the largest z in the range $[A, B]$ that satisfies $\frac{dE\left[\pi_i\left(z_i, p_i(z_i), p_j(z_j), p_k(z_k)\right)\right]}{dz_i} = 0.$

4.2 Suppliers' Problem

In the previous section, a solution was proposed to address the retailers' side of the problem, which was about how the retailers would order and price the product to maximize their profit given the demand, competition data as well as purchasing price. Here, the suppliers' problem will be addressed which is about assigning the orders to the suppliers given the total order from the retailers and pricing data from the suppliers such that the total purchasing cost is minimized.

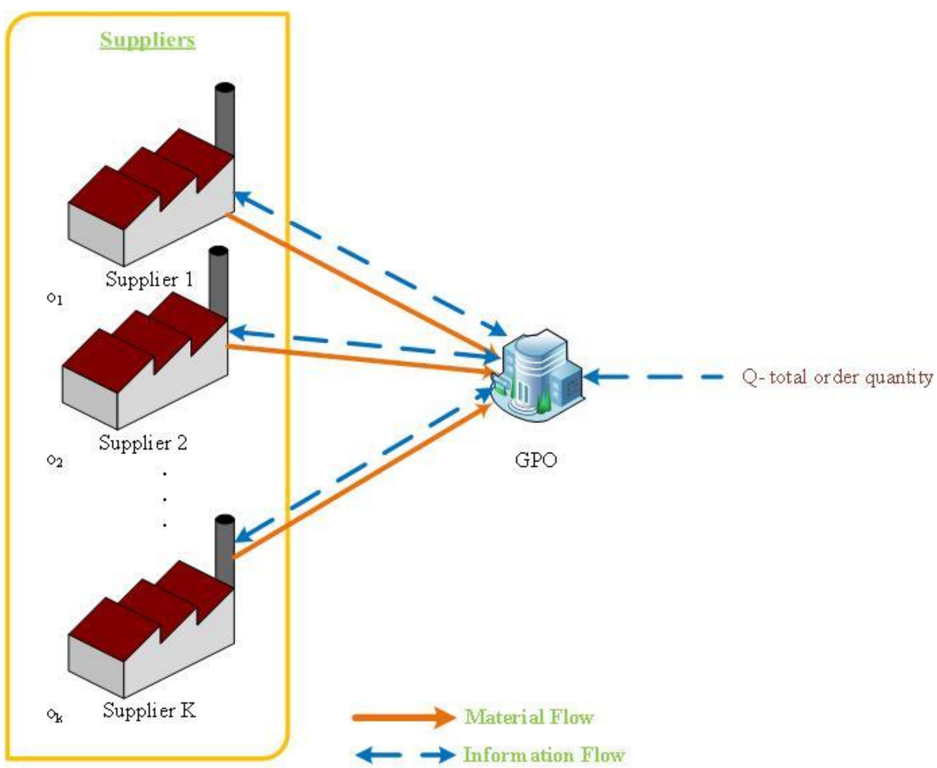

Figure 21. Suppliers' problem

In the general Suppliers' problem there are k suppliers and the goal is to determine the order assignment to each supplier to minimize the purchasing cost. The available information about suppliers is the pricing information which in a GP context will be based on a quantity discount pricing method. The general QDF for each supplier k is in the form below:

$$w_k(o_k) = m_k + \frac{d_k}{o_k^{e_k}} \quad 4.40$$

Where $m_k \geq 0$ is the base price, d_k is the discount scale, e_k $(-1.00 \leq e \leq 1)$ is the steepness and o_k is the order quantity assigned to supplier k. This model is proposed by (Schotanus, Telgen et al. 2009) and is claimed to have flexibility to represent 66 practical discount schedules. To ensure that the purchasing price is decreasing with o_k, $d_k e_k > 0$. Knowing the QDF and capacity W_k for each supplier; and the total order quantity from retailers O; the order assignment problem can be modeled as the below optimization problem:

$$Min \sum_k w_k(o_k).\, o_k \quad 4.41$$

s.t:

$$\sum_k o_k = O \quad 4.42$$

$$o_k \leq W_k \quad \forall k \quad 4.43$$

Where the objective is minimizing the total purchasing cost, the first constraint tries to satisfy all the demand and the second constraint is the capacity requirement for each supplier. The problem of suppliers' order assignment is an optimization problem with k

variables and k+1 parameters. This problem can be solved using a non-linear optimization method. Here an analytical solution based on Lagrange multipliers will be proposed.

Edwards and Penney (2013) define the condition for Lagrangian method such that the gradients of the constraints should be:

(1) nonzero,

(2) non-parallel

(3) equality

The constraints in this problem meet the first and second conditions, to meet the third condition, a slack variable l is introduced to the problem:

$$Min\ f(o_k) = \sum_k w_k(o_k).\,o_k \tag{4.44}$$

s.t:

$$h(o_k) = \sum_k o_k = O \tag{4.45}$$

$$g_k(o_k, l_k) = o_k + l_k^2 = W_k \quad \forall k \tag{4.46}$$

The slack variable is added in squared form to make sure it is non-negative. The Lagrangian for this problem can be written as:

$$\mathcal{L}(o_k, l_k, \lambda, \mu_k) = f(o_k) + \lambda[h(o_k) - O] + \sum_k \mu_k[g_k(o_k, l_k) - W_k] \tag{4.47}$$

Calculating the gradient of $\mathcal{L}$ and setting it equal to zero will include all the equations needed to solve the problem:

$$\nabla\mathcal{L} = 0 \tag{4.48}$$

$$\nabla\mathcal{L} = \begin{bmatrix} \partial\mathcal{L}/\partial o_1 \\ \partial\mathcal{L}/\partial o_2 \\ \vdots \\ \partial\mathcal{L}/\partial o_k \\ \partial\mathcal{L}/\partial l_1 \\ \partial\mathcal{L}/\partial l_2 \\ \vdots \\ \partial\mathcal{L}/\partial l_k \\ \partial\mathcal{L}/\partial \mu_1 \\ \partial\mathcal{L}/\partial \mu_2 \\ \vdots \\ \partial\mathcal{L}/\partial \mu_k \\ \partial\mathcal{L}/\partial \lambda \end{bmatrix} = 0 \tag{4.49}$$

As it can be seen the gradient operates on four sets of variables $(o_k, l_k, \mu_k, \lambda)$. Finding the above partial derivatives will result in a system of equations, which can be solved to obtain the solution to the original optimization problem. For a K-supplier problem there is going to be $3K + 1$ variables and equations. In the next sections the approach will be applied for two and three supplier problem.

4.2.1 Two supplier problem

Using the approach proposed in the previous section, consider a 2-supplier problem, the Lagrangian for this case is:

$$\mathcal{L}(o_1, o_2, l_1, l_2, \mu_1, \mu_2, \lambda)$$

$$= \sum_{k=1}^{2} \left[m_k + \frac{d_k}{o_k^{e_k}} \right] * o_k - \lambda \left[\sum_{k=1}^{2} o_k - O \right] - \sum_{k=1}^{2} \mu_k [o_k + l_k^2 - W_k] \qquad 4.50$$

The gradient of the Lagrangian is:

$$\nabla\mathcal{L} = \begin{bmatrix} \partial\mathcal{L}/\partial o_1 \\ \partial\mathcal{L}/\partial o_2 \\ \partial\mathcal{L}/\partial l_1 \\ \partial\mathcal{L}/\partial l_2 \\ \partial\mathcal{L}/\partial \mu_1 \\ \partial\mathcal{L}/\partial \mu_2 \\ \partial\mathcal{L}/\partial \lambda \end{bmatrix} = \begin{bmatrix} m_1 + d_1(1 - e_1)o_1^{-e_1} - \lambda - \mu_1 \\ m_2 + d_2(1 - e_2)o_2^{-e_2} - \lambda - \mu_2 \\ 2l_1\mu_1 \\ 2l_2\mu_2 \\ o_1 - W_1 + l_1^2 \\ o_2 - W_2 + l_2^2 \\ o_1 + q_2 - O \end{bmatrix} = 0 \qquad 4.51$$

Hence, the seven Lagrangian multiplier equations are as the following:

$$m_1 + d_1(1 - e_1)o_1^{-e_1} = \lambda + \mu_1 \qquad 4.52$$

$$m_2 + d_2(1 - e_2)o_2^{-e_2} = \lambda + \mu_2 \qquad 4.53$$

$$2l_1\mu_1 = 0 \qquad 4.54$$

$$2l_2\mu_2 = 0 \qquad 4.55$$

$$o_1 + l_1^2 = W_1 \qquad 4.56$$

$$o_2 + l_2^2 = W_2 \qquad 4.57$$

$$o_1 + o_2 = O \qquad 4.58$$

Each inequality constraint in the main problem has the potential to be active or inactive. If active, then the associated slack variable is zero while the Lagrangian multiplier will be non-zero. On the opposite side if the inequality is inactive then the slack variable will be non-zero and the Lagrangian multiplier will be zero. So, 4.54 and 4.55 suggest two alternatives:

4.54 offers: either $l_1 \neq 0$ and $\mu_1 = 0$, in which as a result $o_1 = W_1 - l_1^2$, or $l_1 = 0$ and $\mu_1 \neq 0$, in which case $o_1 = W_1$.

4.55 offers: either $l_2 \neq 0$ and $\mu_2 = 0$, in which as a result $o = W_2 - l_2^2$, or $l_2 = 0$ and $\mu_2 \neq 0$, in which case $o_2 = W_2$.

Therefore, according to the 4.52-4.58 one of the following cases will potentially happen:

Case I: $\begin{cases} \mu_1 = 0 \\ \mu_2 = 0 \end{cases} \Longrightarrow \begin{cases} o_1 < W_1 \\ o_2 < W_2 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1 - e_1)o_1^{-e_1} = \lambda \\ m_2 + d_2(1 - e_2)o_2^{-e_2} = \lambda \\ o_1 + o_2 = O \end{cases}$

Case II: $\begin{cases} \mu_1 = 0 \\ \mu_2 \neq 0 \end{cases} \Longrightarrow \begin{cases} o_1 < W_1 \\ o_2 = W_2 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1 - e_1)o_1^{-e_1} = \lambda \\ m_2 + d_2(1 - e_2)o_2^{-e_2} = \lambda + \mu_2 \\ o_1 + o_2 = O \end{cases} \Rightarrow \begin{cases} o_1 = O - o_2 \\ o_2 = W_2 \end{cases}$

Case III: $\begin{cases} \mu_1 \neq 0 \\ \mu_2 = 0 \end{cases} \Longrightarrow \begin{cases} o_1 = W_1 \\ o_2 < W_2 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1 - e_1)o_1^{-e_1} = \lambda + \mu_1 \\ m_2 + d_2(1 - e_2)o_2^{-e_2} = \lambda \\ o_1 + o_2 = O \end{cases} \Longrightarrow \begin{cases} o_1 = W_1 \\ o_2 = O - o_1 \end{cases}$

Case IV: $\begin{cases}\mu_1 \neq 0 \\ \mu_2 \neq 0\end{cases} \Longrightarrow \begin{cases}o_1 = W_1 \\ o_2 = W_2\end{cases} \Rightarrow \begin{cases}m_1 + d_1(1-e_1)o_1^{-e_1} = \lambda + \mu_1 \\ m_2 + d_2(1-e_2)o_2^{-e_2} = \lambda + \mu_2 \\ o_1 + o_2 = O\end{cases}$ ⇒ this case is feasible and optimal if $W_1 + W_2 = O$.

Comparing the four possible cases, it can be said that case IV happens only if the total suppliers' capacity is equal to the total demand, in this case there is no need to optimize the demand assignment; there is only one option and that is assigning the full capacity to each supplier. For case II & III the values of the assigned order to each supplier is known and a comparison between the total cost determines which one is better. The first case is a general assignment case, where there are three equations and three unknowns (o_1, o_2 and λ) which can be found by solving the three variable three equation system of equations.

4.2.2 Three supplier problem

Let's consider a 3-supplier problem, the Lagrangian for this case can be defined as:

$$\mathcal{L}(o_1, o_2, o_3, l_1, l_2, \mathrm{l}_3, \mu_1, \mu_2, \mu_3, \lambda) = \sum_{k=1}^{3}\left[m_k + \frac{d_k}{o_k^{e_k}}\right] * o_k - \lambda\left[\sum_{k=1}^{3} o_k - O\right] - \sum_{k=1}^{3}\mu_k[o_k + l_k^2 - W_k] \qquad 4.59$$

The gradient of the Lagrangian is:

$$\nabla\mathcal{L} = \begin{bmatrix} \partial\mathcal{L}/\partial o_1 \\ \partial\mathcal{L}/\partial o_2 \\ \partial\mathcal{L}/\partial o_3 \\ \partial\mathcal{L}/\partial l_1 \\ \partial\mathcal{L}/\partial l_2 \\ \partial\mathcal{L}/\partial l_3 \\ \partial\mathcal{L}/\partial \mu_1 \\ \partial\mathcal{L}/\partial \mu_2 \\ \partial\mathcal{L}/\partial \mu_3 \\ \partial\mathcal{L}/\partial \lambda \end{bmatrix} = \begin{bmatrix} m_1 + d_1(1-e_1)o_1^{-e_1} - \lambda - \mu_1 \\ m_2 + d_2(1-e_2)o_2^{-e_2} - \lambda - \mu_2 \\ m_3 + d_3(1-e_3)o_3^{-e_3} - \lambda - \mu_3 \\ 2l_1\mu_1 \\ 2l_2\mu_2 \\ 2l_3\mu_3 \\ o_1 - W_1 + l_1^2 \\ o_2 - W_2 + l_2^2 \\ o_3 - W_3 + l_3^2 \\ o_1 + o_2 + o_3 - O \end{bmatrix} = 0 \quad 4.60$$

Hence, the ten Lagrangian multiplier equations are as the following:

$$m_1 + d_1(1-e_1)o_1^{-e_1} = \lambda + \mu_1 \quad 4.61$$

$$m_2 + d_2(1-e_2)o_2^{-e_2} = \lambda + \mu_2 \quad 4.62$$

$$m_3 + d_3(1-e_3)o_3^{-e_3} = \lambda + \mu_3 \quad 4.63$$

$$2l_1\mu_1 = 0 \quad 4.64$$

$$2l_2\mu_2 = 0 \quad 4.65$$

$$2l_3\mu_3 = 0 \quad 4.66$$

$$q_1 + l_1^2 = W_1 \quad 4.67$$

$$q_2 + l_2^2 = W_2 \quad 4.68$$

$$q_3 + l_3^2 = W_3 \quad 4.69$$

$$o_1 + o_2 + o_3 = O \quad 4.70$$

Each inequality constraint in the main problem has the potential to be active or inactive. If active, then the associated slack variable (l_k) is zero while the Lagrangian multiplier (μ_k) will be non-zero. On the opposite side if the inequality is inactive then the slack variable (l_k) will be non-zero and the Lagrangian multiplier (μ_k) will be zero. So, 4.64, 4.65 and 4.66 suggest two alternatives:

4.64 offers: either $l_1 \neq 0$ and $\mu_1 = 0$, in which as a result $o_1 = W_1 - l_1^2$, or $l_1 = 0$ and $\mu_1 \neq 0$, in which case $o_1 = W_1$.

4.65 offers: either $l_2 \neq 0$ and $\mu_2 = 0$, in which as a result $o_2 = W_2 - l_2^2$, or $l_2 = 0$ and $\mu_2 \neq 0$, in which case $o_2 = W_2$.

4.66 offers: either $l_3 \neq 0$ and $\mu_3 = 0$, in which as a result $o_3 = W_3 - l_3^2$, or $l_3 = 0$ and $\mu_3 \neq 0$, in which case $o_3 = W_3$.

Therefore, according to 4.61-4.70 one of the following cases will potentially happen at the optimal point:

$$\text{Case I: } \begin{cases} \mu_1 = 0 \\ \mu_2 = 0 \\ \mu_3 = 0 \end{cases} \Longrightarrow \begin{cases} o_1 < W_1 \\ o_2 < W_2 \\ o_3 < W_3 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1 - e_1)o_1^{-e_1} = \lambda \\ m_2 + d_2(1 - e_2)o_2^{-e_2} = \lambda \\ m_3 + d_3(1 - e_3)o_3^{-e_3} = \lambda \\ o_1 + o_2 + o_3 = O \end{cases}$$

$$\text{Case II: } \begin{cases} \mu_1 \neq 0 \\ \mu_2 = 0 \\ \mu_3 = 0 \end{cases} \Longrightarrow \begin{cases} o_1 = W_1 \\ o_2 < W_2 \\ o_3 < W_3 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1 - e_1)o_1^{-e_1} = \lambda + \mu_1 \\ m_2 + d_2(1 - e_2)o_2^{-e_2} = \lambda \\ m_3 + d_3(1 - e_3)o_3^{-e_3} = \lambda \\ o_1 + o_2 + o_3 = O \end{cases}$$

$$\text{Case III: } \begin{cases} \mu_1 = 0 \\ \mu_2 \neq 0 \\ \mu_3 = 0 \end{cases} \Longrightarrow \begin{cases} o_1 < W_1 \\ o_2 = W_2 \\ o_3 < W_3 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1 - e_1)o_1^{-e_1} = \lambda \\ m_2 + d_2(1 - e_2)o_2^{-e_2} = \lambda + \mu_2 \\ m_3 + d_3(1 - e_3)o_3^{-e_3} = \lambda \\ c_2 + o_1 + o_3 = O \end{cases}$$

Case IV: $\begin{cases} \mu_1 = 0 \\ \mu_2 = 0 \\ \mu_3 \neq 0 \end{cases} \Longrightarrow \begin{cases} q_1 < W_1 \\ q_2 < W_2 \\ q_3 = W_3 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1-e_1)q_1^{-e_1} = \lambda \\ m_2 + d_2(1-e_2)q_2^{-e_2} = \lambda \\ m_3 + d_3(1-e_3)q_3^{-e_3} = \lambda + \mu_3 \\ W_3 + o_1 + o_2 = O \end{cases}$

Case V: $\begin{cases} \mu_1 \neq 0 \\ \mu_2 \neq 0 \\ \mu_3 = 0 \end{cases} \Longrightarrow \begin{cases} o_1 = W_1 \\ o_2 = W_2 \\ o_3 < W_3 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1-e_1)o_1^{-e_1} = \lambda + \mu_1 \\ m_2 + d_2(1-e_2)o_2^{-e_2} = \lambda + \mu_2 \\ m_3 + d_3(1-e_3)o_3^{-e_3} = \lambda \\ W_1 + W_2 + o_3 = O \end{cases} \Rightarrow$

$\begin{cases} o_1 = W_1 \\ o_2 = W_2 \\ o_3 = O - o_1 - o_2 \end{cases}$ ⇒ This case is feasible only if $W_1 + W_2 \leq O$

Case VI: $\begin{cases} \mu_1 = 0 \\ \mu_2 \neq 0 \\ \mu_3 \neq 0 \end{cases} \Longrightarrow \begin{cases} o_1 < W_1 \\ o_2 = W_2 \\ o_3 = W_3 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1-e_1)o_1^{-e_1} = \lambda \\ m_2 + d_2(1-e_2)o_2^{-e_2} = \lambda + \mu_2 \\ m_3 + d_3(1-e_3)o_3^{-e_3} = \lambda + \mu_3 \\ o_1 + W_2 + W_3 = O \end{cases} \Rightarrow$

$\begin{cases} o_1 = O - W_2 - W_3 \\ o_2 = W_2 \\ o_3 = W_3 \end{cases}$ ⇒ This case is feasible inly if $W_2 + W_3 \leq O$

Case VII: $\begin{cases} \mu_1 \neq 0 \\ \mu_2 = 0 \\ \mu_3 \neq 0 \end{cases} \Longrightarrow \begin{cases} o_1 = W_1 \\ o_2 < W_2 \\ o_3 = W_3 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1-e_1)o_1^{-e_1} = \lambda + \mu_1 \\ m_2 + d_2(1-e_2)o_2^{-e_2} = \lambda \\ m_3 + d_3(1-e_3)o_3^{-e_3} = \lambda + \mu_3 \\ W_1 + o_2 + W_3 = O \end{cases} \Rightarrow$

$\begin{cases} o_1 = W_1 \\ o_2 = O - W_1 - W_3 \\ o_3 = W_3 \end{cases}$ ⇒ This case is feasible inly if $W_1 + W_3 \leq O$

Case VIII: $\begin{cases} \mu_1 \neq 0 \\ \mu_2 \neq 0 \\ \mu_3 \neq 0 \end{cases} \Longrightarrow \begin{cases} o_1 = W_1 \\ o_2 = W_2 \\ o_3 = W_3 \end{cases} \Rightarrow \begin{cases} m_1 + d_1(1-e_1)o_1^{-e_1} = \lambda + \mu_1 \\ m_2 + d_2(1-e_2)o_2^{-e_2} = \lambda + \mu_2 \\ m_3 + d_3(1-e_3)o_3^{-e_3} = \lambda + \mu_3 \\ W_1 + W_2 + W_3 = O \end{cases} \Rightarrow$

$\begin{cases} o_1 = W_1 \\ o_2 = W_2 \\ o_3 = W_3 \end{cases} \Longrightarrow$ this case is feasible only if $W_1 + W_2 + W_3 = O$.

Comparing the possible combinations, one can say that cases V-VIII happens only if certain conditions are met, as described under each case. The cases I-IV are general assignment cases and require solving the four variable, four equation system of equation to find the values of o_k, μ_k and λ. A comparison of total cost between all the feasible cases will lead to finding the best combination of assignment.

So far, experiencing with the Lagrangian method it can be said that, for a 2-supplier problem, there are 4 cases to be considered and 8 scenarios for a 3-supplier problem, by generalization it can be concluded that there will be 2^k cases for a k −supplier problem; the complexity of considering these cases will make it more complex to solve the larger problems using the Lagrangian method. For larger problem one may choose to consider other linear or non-linear optimization methods as well.

4.3 Group Purchasing Problem

As mentioned at the beginning of this chapter, first the GP problem was split to two sub-problems, i.e. retailers' problem and suppliers' problem; then each of them were solved separately. In this section these two sub-problems are merger back together into one as shown in Figure 22 to solve the GP problem by solving each sub-problem sequentially until convergence is achieved. Figure 23 displays the steps for the sequential solving procedure to find the optimal answer to GP problem.

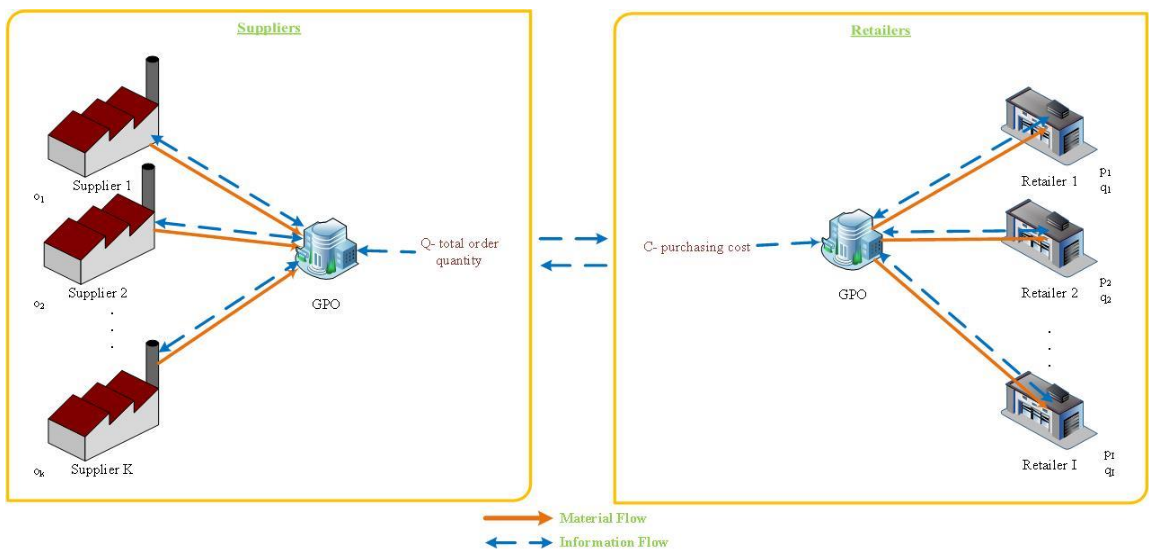

Figure 22. Merging the suppliers' and retailers' problem

The input to the algorithm for GP problem is all the information needed to solve the retailers' problem, as well as the information needed to solve the suppliers' problem. Since this algorithm is a numerical method, we need to set a stopping criteria factor which is the change in suppliers' average optimal price in each step compared to the previous step which is used as a criterion to accept the current result. In the next section the results from this simulation-optimization algorithm will be presented.

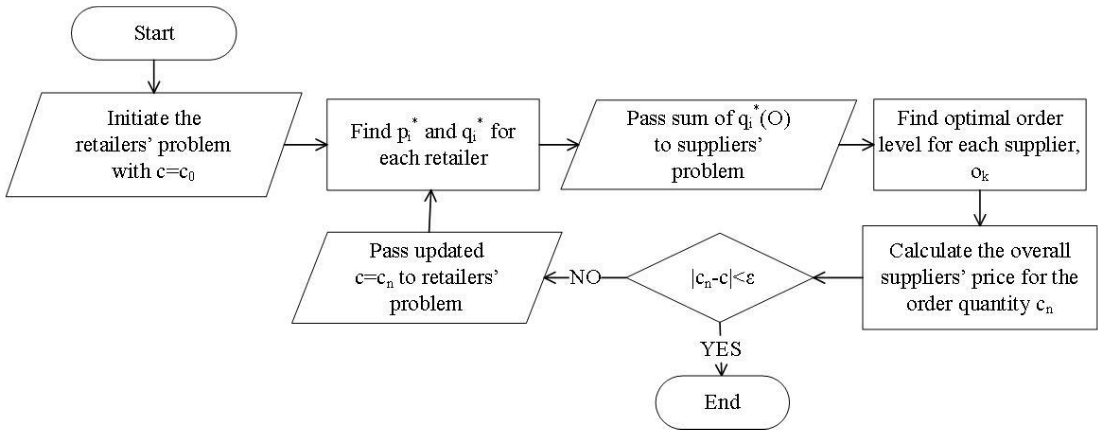

Figure 23. Process chart for finding the optiaml results to the group purchasing problem

4.3.1 Experimental Results

Here, the goal is to show the applicability of the proposed solution method and to study the effect of input parameter on the response factors. To test the effect of input parameters, the total retailer profit is considered the response factor and a set of input parameters for retailers and suppliers is selected input factors. The input parameters selected for the retailers are a_i, b_i and γ_i; and m_k, d_k and W_k for suppliers. Considering a 2 retailers- 2 suppliers GP problem, there will be 12 input parameters in this experiment, if 3 levels are considered for each factor, if one were to run a full factorial experiment as previous chapters, then there be $3^{12} = 531{,}441$ number of test cases to run which is almost impossible to do. Thus, a Plackett-Burman (PB) design is used to reduce the number of runs needed for this study. Next, based on the PB design, a set of 49 test cases are created for each of the 4 industries mentioned in previous cases. Table 14 displays the PB design for 12 factors.

Table 14. PB design table for 12 factors

Run Order	a_1	b_1	γ_1	a_2	b_2	γ_2	m_1	d_1	W_1	m_2	d_2	W_2
1	1	-1	-1	-1	-1	1	-1	-1	-1	-1	1	-1
2	1	1	-1	-1	-1	-1	1	-1	-1	-1	1	1
3	1	1	1	-1	-1	-1	-1	1	-1	-1	1	1
4	1	1	1	1	-1	-1	-1	-1	1	-1	1	1
5	1	1	1	1	1	-1	-1	-1	-1	1	1	1
6	-1	1	1	1	1	1	-1	-1	-1	-1	-1	1
7	1	-1	1	1	1	1	1	-1	-1	-1	1	-1
8	1	1	-1	1	1	1	1	1	-1	-1	1	1
9	1	1	1	-1	1	1	1	1	1	-1	1	1
10	1	1	1	1	-1	1	1	1	1	1	1	1
11	-1	1	1	1	1	-1	1	1	1	1	-1	1
12	-1	-1	1	1	1	1	-1	1	1	1	-1	-1
13	1	-1	-1	1	1	1	1	-1	1	1	1	-1
14	-1	1	-1	-1	1	1	1	1	-1	1	-1	1
15	1	-1	1	-1	-1	1	1	1	1	-1	1	-1
16	-1	1	-1	1	-1	-1	1	1	1	1	-1	1
17	1	-1	1	-1	1	-1	-1	1	1	1	1	-1
18	1	1	-1	1	-1	1	-1	-1	1	1	1	1
19	1	1	1	-1	1	-1	1	-1	-1	1	1	1
20	-1	1	1	1	-1	1	-1	1	-1	-1	-1	1
21	-1	-1	1	1	1	-1	1	-1	1	-1	-1	-1
22	1	-1	-1	1	1	1	-1	1	-1	1	1	-1
23	-1	1	-1	-1	1	1	1	-1	1	-1	-1	1
24	-1	-1	1	-1	-1	1	1	1	-1	1	-1	-1
25	1	-1	-1	1	-1	-1	1	1	1	-1	1	-1
26	1	1	-1	-1	1	-1	-1	1	1	1	1	1
27	-1	1	1	-1	-1	1	-1	-1	1	1	-1	1
28	1	-1	1	1	-1	-1	1	-1	-1	1	1	-1
29	1	1	-1	1	1	-1	-1	1	-1	-1	1	1
30	-1	1	1	-1	1	1	-1	-1	1	-1	-1	1
31	-1	-1	1	1	-1	1	1	-1	-1	1	-1	-1
32	-1	-1	-1	1	1	-1	1	1	-1	-1	-1	-1
33	1	-1	-1	-1	1	1	-1	1	1	-1	1	-1
34	-1	1	-1	-1	-1	1	1	-1	1	1	-1	1
35	1	-1	1	-1	-1	-1	1	1	-1	1	1	-1
36	-1	1	-1	1	-1	-1	-1	1	1	-1	-1	1
37	1	-1	1	-1	1	-1	-1	-1	1	1	1	-1
38	1	1	-1	1	-1	1	-1	-1	-1	1	1	1
39	-1	1	1	-1	1	-1	1	-1	-1	-1	-1	1
40	-1	-1	1	1	-1	1	-1	1	-1	-1	-1	-1
41	-1	-1	-1	1	1	-1	1	-1	1	-1	-1	-1
42	-1	-1	-1	-1	1	1	-1	1	-1	1	-1	-1
43	1	-1	-1	-1	-1	1	1	-1	1	-1	1	-1
44	-1	1	-1	-1	-1	-1	1	1	-1	1	-1	1
45	-1	-1	1	-1	-1	-1	-1	1	1	-1	-1	-1
46	-1	-1	-1	1	-1	-1	-1	-1	1	1	-1	-1
47	-1	-1	-1	-1	1	-1	-1	-1	-1	1	-1	-1
48	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
49	0	0	0	0	0	0	0	0	0	0	0	0

Table 15- Table 18 displays the ANOVA table for each industry case, glancing at these tables it can be concluded that the factors that are significant on the response factor is different depending on the industry. In Case 1 and Case 2 the retailer parameters a_i and b_i as well as the suppliers' base price m_k are significant in determining the output factor, in case 2, the price discount factor for supplier 1 was significant though. In Case 3 only the retailer parameters a_i and b_i are important. Case 4 is the only industry case that all the input parameters have a significant impact on the response factor, except the competition factor γ_i. It is interesting that the supplier capacity W_k is significant only industry 4.

Table 15. ANOVA analysis for profit function- GP- 2 retailers- 2 suppliers Case 1

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	13	14805954752	1138919596	97.54	0.000
Linear	12	14792516826	1232709735	105.57	0.000
ai_1	1	4318768656	4318768656	369.87	0.000
bi_1	1	3106366287	3106366287	266.04	0.000
gai_1	1	22133627	22133627	1.90	0.177
ai_2	1	4084029577	4084029577	349.77	0.000
bi_2	1	3153715837	3153715837	270.09	0.000
gai_2	1	49205	49205	0.00	0.949
mk_1	1	12349973	12349973	1.06	0.311
dk_1	1	17009196	17009196	1.46	0.236
Wk_1	1	9635415	9635415	0.83	0.370
mk_2	1	53872613	53872613	4.61	0.039
dk_2	1	3082042	3082042	0.26	0.611
Wk_2	1	11504399	11504399	0.99	0.328
Curvature	1	13437926	13437926	1.15	0.291
Error	35	408672910	11676369		
Total	48	15214627662			

Table 16. ANOVA analysis for profit function- GP- 2 retailers- 2 suppliers Case 2

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	13	3.84464E+14	2.95742E+13	109.73	0.000
Linear	12	3.83689E+14	3.19741E+13	118.64	0.000
ai_1	1	1.34780E+14	1.34780E+14	500.09	0.000
bi_1	1	5.84188E+13	5.84188E+13	216.76	0.000

gai_1	1	4.69116E+11	4.69116E+11	1.74	0.196
ai_2	1	1.22076E+14	1.22076E+14	452.95	0.000
bi_2	1	5.41893E+13	5.41893E+13	201.06	0.000
gai_2	1	74830386784	74830386784	0.28	0.602
mk_1	1	4.52347E+12	4.52347E+12	16.78	0.000
dk_1	1	3.56071E+12	3.56071E+12	13.21	0.001
Wk_1	1	7.11529E+11	7.11529E+11	2.64	0.113
mk_2	1	3.82985E+12	3.82985E+12	14.21	0.001
dk_2	1	4.61038E+11	4.61038E+11	1.71	0.199
Wk_2	1	5.94495E+11	5.94495E+11	2.21	0.146
Curvature	1	7.74790E+11	7.74790E+11	2.87	0.099
Error	35	9.43291E+12	2.69512E+11		
Total	48	3.93897E+14			

Table 17. ANOVA analysis for profit function- GP- 2 retailers- 2 suppliers Case 3

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	13	4.92846E+15	3.79112E+14	90.79	0.000
Linear	12	4.92308E+15	4.10257E+14	98.25	0.000
ai_1	1	1.46439E+15	1.46439E+15	350.70	0.000
bi_1	1	1.02077E+15	1.02077E+15	244.46	0.000
gai_1	1	9.62092E+12	9.62092E+12	2.30	0.138
ai_2	1	1.36769E+15	1.36769E+15	327.54	0.000
bi_2	1	1.03953E+15	1.03953E+15	248.95	0.000
gai_2	1	1.20119E+11	1.20119E+11	0.03	0.866
mk_1	1	6.13926E+11	6.13926E+11	0.15	0.704
dk_1	1	8.35034E+11	8.35034E+11	0.20	0.657
Wk_1	1	4.21018E+12	4.21018E+12	1.01	0.322
mk_2	1	2.46270E+12	2.46270E+12	0.59	0.448
dk_2	1	9.35237E+12	9.35237E+12	2.24	0.143
Wk_2	1	3.48014E+12	3.48014E+12	0.83	0.368
Curvature	1	5.37598E+12	5.37598E+12	1.29	0.264
Error	35	1.46148E+14	4.17567E+12		
Total	48	5.07460E+15			

Table 18. ANOVA analysis for profit function- GP- 2 retailers- 2 suppliers Case 4

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	13	11642675636	895590434	127.77	0.000
Linear	12	11612436260	967703022	138.05	0.000
ai_1	1	1977090542	1977090542	282.05	0.000

bi_1	1	3232435704	3232435704	461.14	0.000
gai_1	1	24286028	24286028	3.46	0.071
ai_2	1	1817473040	1817473040	259.28	0.000
bi_2	1	3511666757	3511666757	500.98	0.000
gai_2	1	285162	285162	0.04	0.841
mk_1	1	287113364	287113364	40.96	0.000
dk_1	1	164991118	164991118	23.54	0.000
Wk_1	1	99629396	99629396	14.21	0.001
mk_2	1	188626151	188626151	26.91	0.000
dk_2	1	148676875	148676875	21.21	0.000
Wk_2	1	160162123	160162123	22.85	0.000
Curvature	1	30239376	30239376	4.31	0.045
Error	35	245337911	7009655		
Total	48	11888013547			

Next, Figure 24- Figure 27 displays the main effect of the input parameters on the response factor in each factor, checking these graphs, one can draw conclusions on how each parameter is affecting the response factor in each industry. Overall, the demand factors a_i and b_i have the biggest impact on the retailers' profit and their effect is opposite of each other, a_i has increasing effect on the profit levels, while b_i has decreasing effect on the profits. Competition factor has a minimal effect on the response factor, and it is mostly a negative effect on the retailers' profit which is expected to happen through a lower retail price for the consumers. Suppliers' price factors m_k and $d_k$also have low impact on the profits except in industry 4, and their effect has a decreasing effect on profits, which means through a higher base price and lower discount rates; same can be said about the supplier capacity W_k, it has higher impact on the profits in industry case 4 compared to other industries and its' effect has an increasing effect on them which means working with larger suppliers is beneficial for the retailers.

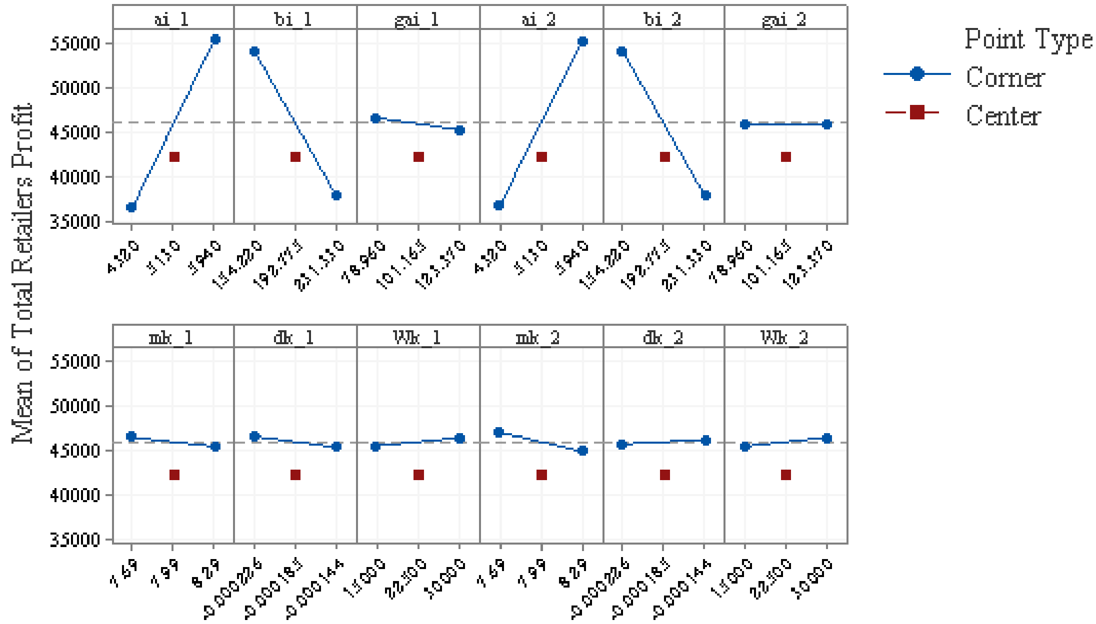

Figure 24. Main effects plot for retailer profit- GP- 2 retailers- 2 suppliers Case 1

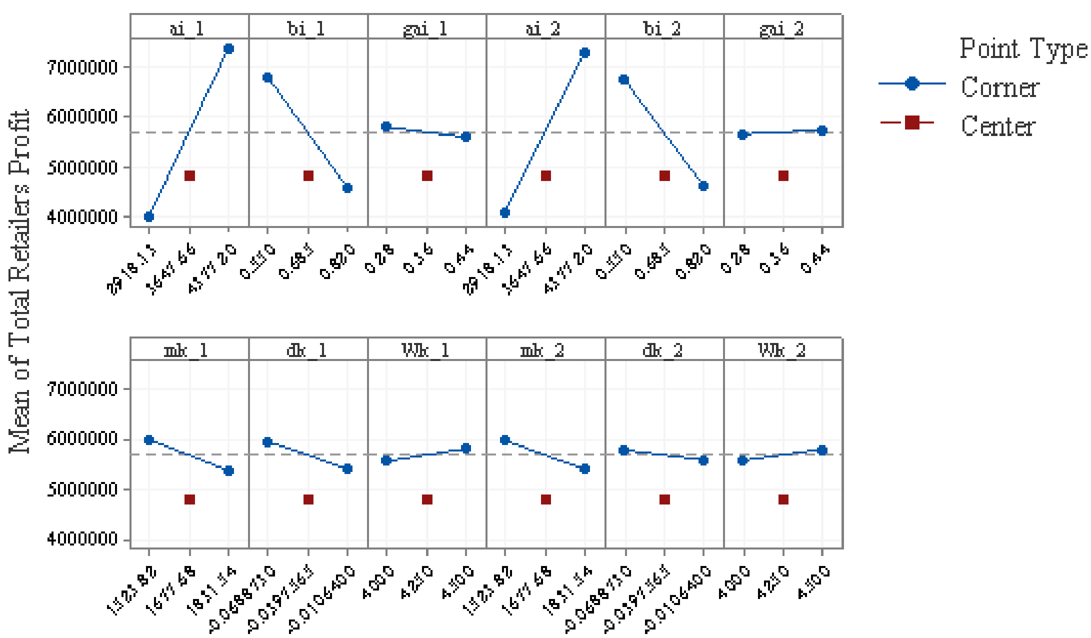

Figure 25. Main effects plot for retailer profit- GP- 2 retailers- 2 suppliers Case 2

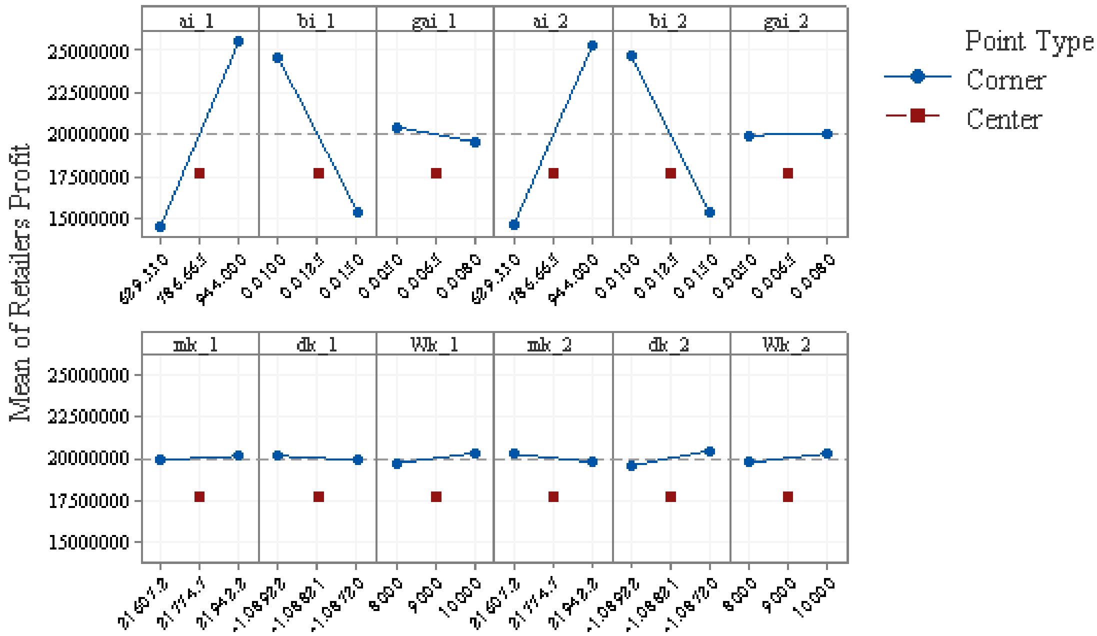

Figure 26. Main effects plot for retailer profit- GP- 2 retailers- 2 suppliers Case 3

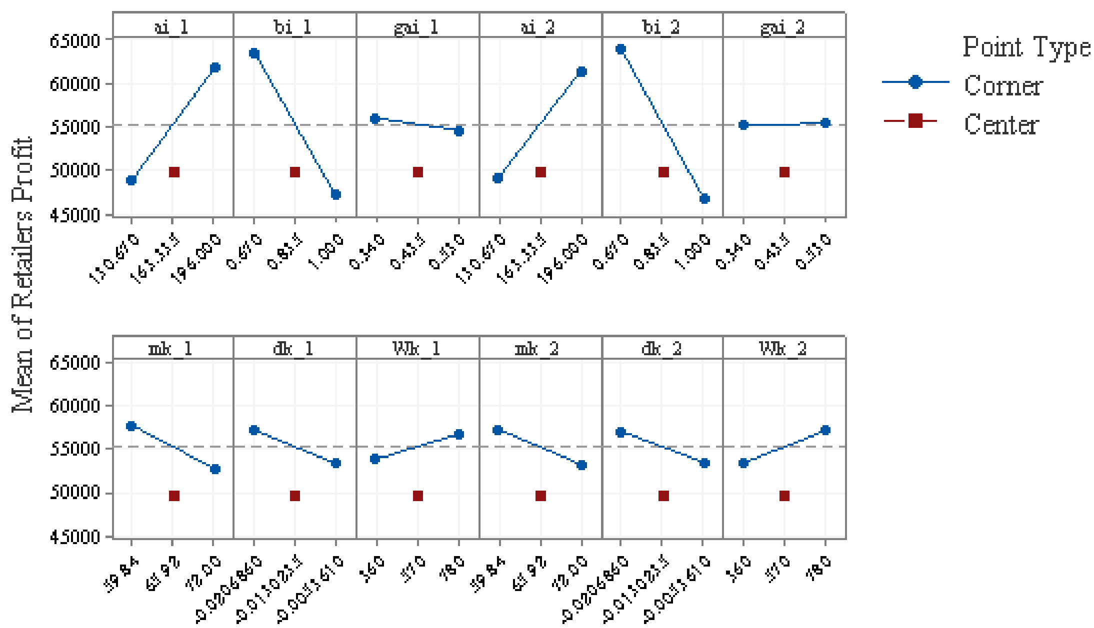

Figure 27. Main effects plot for retailer profit- GP- 2 retailers- 2 suppliers Case 4

4.4 Summary

In this chapter, the general GP model including multiple capacitated suppliers and asymmetric retailers with competition and stochastic, price sensitive demand was modelled and studied. To tackle the problem, it was split to two sub-problems i.e. Retailers' Problem and Suppliers' Problem. The Retailers' Problem addresses the decisions that needs to be taken regarding the retailers i.e. pricing and ordering decisions given a wholesale cost, while the suppliers problem addresses the decision of assigning the retailers' orders to each supplier to minimize the total cost given each supplier's pricing function. Each of these sub-problems were modeled and solved using analytical and numerical approaches. After solving these two problems, they were brought together to solve the main problem by connecting the output of these problems together. The numerical analysis and results for 2-retailers and 2-suppliers problem concludes that:

1. Base demand a_i and demand factor b_i is always a significant factor regardless of the industry. Base demand always has an increasing effect on the retailers' profit i.e. larger demand base always results in higher returns for the retailers, while the demand elasticity parameter always has a decreasing effect on the profits, a more elastic demand base always gets better pricing from the retailers.
2. Based on the experiments in this research, the competition factor has a low effect on the retailers' and is not a significant factor for the profits.
3. On the supply side, the base price m_i is the most significant parameter, The base price is a significant in all industries except in industry 3. Its impact is always a decreasing effect on the retailer profits where it was a significant factor.

4. The discount parameter d_i was only a significant factor in industry 4, in industry 2 it was significant only for one of the suppliers which gives a mixed signal on its general effect on the retailers' profit.
5. The suppliers' capacity parameter W_k is significant only in the industry 4 and it has always an increasing effect on the retailers' profitability, which concludes that if significant, the larger suppliers provide more impact on the retailers' profitability.

In the next parts of the chapter, first, the suppliers' problem is addressed, the suppliers' problem is the optimal assignment of retailers' orders to the suppliers given their pricing function and capacity information. A Lagrangian multiplier method was proposed to address this problem. In the next part, the Retailers' and Suppliers' problems are merged and solved as one problem using the developed methods. Numerical examples were provided to show the effectiveness of the approach and to study their impact on retailers' profits.

5 OPTIMAL GROUP PURCHASING WITH SUPPLY CHAIN CONTRACTS

Contract can be defined as a set of agreements between business sides to transfer a set of payments and its goal is to align the goal of each supply chain member with the supply chain's goal (Cachon 2003). Even though the literature of contract in supply chain is rich, there is not many studies in the context of GP. So far, the models that are addressed here lacked contract, except the quantity discount pricing between the retailers and suppliers. In this chapter the goal is to find the optimal decisions when contracts are offered to the retailers.

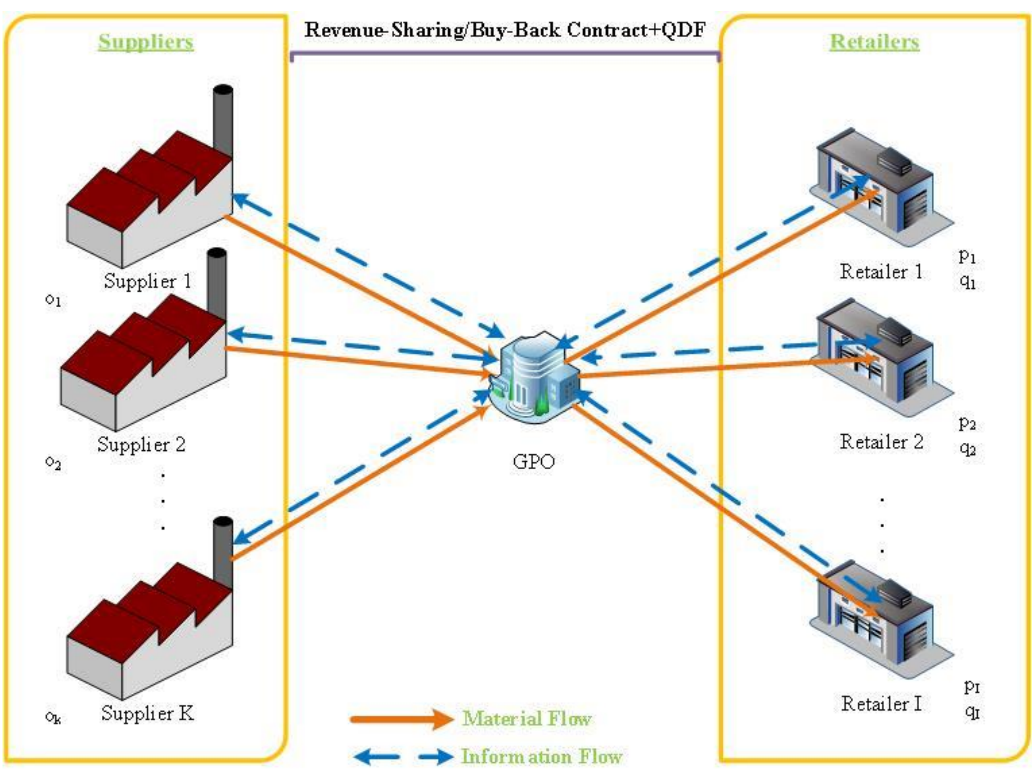

Figure 28. Group purchasing with contracts

5.1 Buyback Contract

In the buyback (BB) contract, like previous cases discussed so far, the retailers pay a price per unit of c as the purchasing cost; but the suppliers pay them back $h \leq c$ for any unsold unit (Cachon 2003). Given that, the retailers' profit function in this case can be rewritten as:

$$E\left[\pi_i\left(z_i, p_i, p_j\right)\right] = \int_A^{z_i} \left(p_i\left[y_i\left(p_i, p_j\right) + u\right] + (h + \mathrm{v_i})[z_i - u]\right) f(u) du + \int_{z_i}^{B} \left(p_i\left[y_i\left(p_i, p_j\right) + z_i\right] - s_i[u - z_i]\right) f(u) du - c\left[y_i\left(p_i, p_j\right) + z_i\right] = \Psi_\mathrm{i}\left(p_i, p_j\right) - L_i(z_i, p_i) \tag{5.1}$$

Where the risk-less profit function $\Psi_i\left(p_i, p_j\right)$ is:

$$\boldsymbol{\Psi_\mathrm{i}\left(\mathrm{p_i}, \mathrm{p_j}\right) = (\mathrm{p_i} - \mathrm{c})\left[\mathrm{y_i}\left(\mathrm{p_i}, \mathrm{p_j}\right) + \mu\right]} \tag{5.2}$$

And the loss due to shortage and surplus, $L_i(z_i, p_i)$ is:

$$\mathbf{L_i(z_i, p_i) = (c - h - v_i)\Lambda(z_i) + (p_i + s_i - c)\Theta(z_i)} \tag{5.3}$$

Assuming the contract terms are given, to find the best order and pricing strategy under this type of contract, the first and second order derivatives with respect to z_i and p_i needs to be studied:

$$\frac{\partial \mathbf{E}[\boldsymbol{\pi}_\mathbf{i}(\mathbf{z}_\mathbf{i},\mathbf{p}_\mathbf{i},\mathbf{p}_\mathbf{j})]}{\partial \mathbf{z}_\mathbf{i}} = -(\mathbf{c}-\mathbf{h}-\mathbf{v}_\mathbf{i}) + (\mathbf{p}_\mathbf{i}+\mathbf{s}_\mathbf{i}-\mathbf{h}-\mathbf{v}_\mathbf{i})[\mathbf{1}-\mathbf{F}(\mathbf{z}_\mathbf{i})] \quad 5.4$$

$$\frac{\partial^2 \mathbf{E}[\boldsymbol{\pi}_\mathbf{i}(\mathbf{z}_\mathbf{i},\mathbf{p}_\mathbf{i},\mathbf{p}_\mathbf{j})]}{\partial \mathbf{z}_\mathbf{i}^2} = -(\mathbf{p}_\mathbf{i}+\mathbf{s}_\mathbf{i}-\mathbf{h}-\mathbf{v}_\mathbf{i})\mathbf{f}(\mathbf{z}_\mathbf{i}) \quad 5.5$$

$$\frac{\partial \mathbf{E}[\boldsymbol{\pi}_\mathbf{i}(\mathbf{z}_\mathbf{i},\mathbf{p}_\mathbf{i},\mathbf{p}_\mathbf{j})]}{\partial \mathbf{p}_\mathbf{i}} = \mathbf{2}(\mathbf{b}_\mathbf{i}+\boldsymbol{\gamma}_\mathbf{i})(\mathbf{p}_\mathbf{i}^\mathbf{0}-\mathbf{p}_\mathbf{i}) - \boldsymbol{\Theta}(\mathbf{z}_\mathbf{i}) \quad 5.6$$

$$\frac{\partial^2 \mathbf{E}[\boldsymbol{\pi}_\mathbf{i}(\mathbf{z}_\mathbf{i},\mathbf{p}_\mathbf{i},\mathbf{p}_\mathbf{j})]}{\partial \mathbf{p}_\mathbf{i}^2} = -\mathbf{2}(\mathbf{b}_\mathbf{i}+\boldsymbol{\gamma}_\mathbf{i}) \quad 5.7$$

Where in 5.6, $p_i^0 = \frac{\mathrm{a_i}+(\mathrm{b_i}+\gamma_i)c+\mu+\gamma_i {}^1/_{n-1}\sum_{j=1, j\neq i}^{n} p_j}{2(\mathrm{b_i}+\gamma_i)}$ is the optimal riskless price which will be the same as the no contract case in 4.1 which makes sense, because BB contract is set to share the risk of shortage or overage due to the stochastic demand and in the riskless scenario the contract would not affect the optimal price. Since p_i^0 is a function of p_i and p_j, Lemma 3 can be used to solve it for known parameters.

Observing 5.5, it can be said that $(p_i + s_i - h - v_i)$ is always positive because p_i and s_i are positive and $h + v_i \leq p_i$. Thus, the term $-(\mathrm{p_i} + s_i - h - \mathrm{v_i})f(z_i)$ is always negative, so the profit function is concave downward in z_i. The same case applies for 5.7, because $b_i \geq 0$ & $\gamma_i \geq 0$. Therefore, $E[\pi_i(z_i, p_i, p_j)]$ is concave downward in both p_i and z_i; thus, the optimization problem can be reduced to a simpler problem. Using Zabel's approach (Zabel 1970), the optimal value of p_i can be found as a function of z_i and substituted in 5.4, then a search over the single variable space will find the optimal z_i. Applying Lemma 4 converts the expected profit function to only a function of z_i and z_j. Lemma 5 applies here as well, since 5.4, the first derivative of the expected profit function with respect to z_i is only a function of z_i, z_i^* can be found without the need to know the values of z_j^*.

If 5.4, the first derivative of the profit function with respect to z_i is called as $R_i(z_i)$. Finding the values of z_i that satisfy the first order condition means finding the roots of $R_i(z_i)$:

$$R_i(z_i) = \frac{\partial E\left[\pi_i\left(\mathrm{z_i}, \mathrm{p_i}(z_i), p_j(z_j)\right)\right]}{\partial \mathrm{z_i}}$$

$$= -(c - h - v_i) \qquad 5.8$$

$$+ \left(\mathrm{p_i^0} + s_i - h - \mathrm{v_i} - \frac{\Theta(z_i)}{2(b_i + \gamma_i)}\right)[1 - F(\mathrm{z_i})]$$

Discovering the values of z_i that satisfy first-order condition is important in finding the optimal profit. Interpreting the shape of the $R_i(z_i)$ based on the problem's parameters is fundamental in finding the optimal z_i; Theorem 6, which is a modified version of Theorem 5 for this problem helps identify the conditions of z_i^*:

Theorem 6: The optimal order and pricing policy in the group buying problem with I competing asymmetric retailers and BB contract is to order $q_i^* = y_i(p_i^*) + z_i^*$ units and sell at the unit price $\mathrm{p_i^*}$, where $\mathrm{p_i^*}$ is determined using Lemma 4 and Lemma 5 and $\mathrm{z_i^*}$ is defined based on the following:

a) If $F(.)$ is a random distribution function, then a complete search over the range $[A, B]$ will determine z_i^*.

b) If $F(.)$ satisfies the condition $2r(z_i)^2 + \frac{dr(z_i)}{dz_i} > 0$ for $A \leq z_i \leq B$ and $r(.) = \frac{f(.)}{1-F(.)}$; then $z_i^*$is the largest z_i in the range $[A, B]$ that satisfies $\frac{dE\left[\pi_i\left(\mathrm{z_i}, \mathrm{p_i}(z_i), \mathrm{p_j}(\mathrm{z_j})\right)\right]}{d\mathrm{z_i}} = 0$

c) If condition b is met and $\mathrm{a_i} - (b_i + \gamma_\mathrm{i})(c - 2\mathrm{s_i}) + \mathrm{A} > 0$, then z_i^* is the unique z in the range $[A, B]$ that satisfies $\frac{dE\left[\pi_i\left(\mathrm{z_i}, \mathrm{p_i}(z_i), \mathrm{p_j}(\mathrm{z_j})\right)\right]}{d\mathrm{z_i}} = 0$.

Proof: the proof is the same as Theorem 5, after replacing 4.15 with 5.8. ■

The stepping-stone for finding optimal parameters for this problem is solving the system of equation to find p_i^0 using Lemma 3; similar challenge that exists for the asymmetric retailers problem in 4.1 persists here; the answer to the symbolic problem grows exponentially with every added retailer. The answer to each problem size can be compacted to some extent, but it cannot be generalized for any I number of retailers.

So far; always the retailers' profit has been considered when doing the analysis in different GP settings. Since a contract should provide incentive for both parties (retailers and suppliers) to be able to bring the parties in the contract; here, its effect on the suppliers will be studied as well. Supplier k's revenue was studied in the previous chapter, in the no contract case with quantity discount pricing it can be displayed as:

$$\Pi_k^n = o_k w_k(o_k) \tag{5.9}$$

In this scenario, the supplier's revenue is only a function of the order quantity it receives from the retailers and is not involved with what happens to the product once they are passed to the retailer.

Contracts define a transfer payment between the supplier and retailer that engages the supplier more to the end market, e.g., in BB contract the supplier shares the cost of over stocking with the retailers by returning them a salvage price h for any unsold unit. To calculate the supplier's revenue in this contract, the transfer payment needs to be considered in profit calculation as well. The expected salvage count can be calculated for each retailer as below:

$$V_i(q_i) = \begin{cases} q_i - D_i(p_i, p_j, \epsilon), & D_i(p_i, p_j, \epsilon) \le q_i, j \neq i \\ 0, & D_i(p_i, p_j, \epsilon) > q_i, j \neq i \end{cases}$$

$$= \begin{cases} q_i - y_i(p_i, p_j) - \epsilon & \epsilon \le q_i - y_i(p_i, p_j), j \neq i \\ 0 & \epsilon > q_i - y_i(p_i, p_j), j \neq i \end{cases} \quad 5.10$$

Since, the suppliers' revenue is evaluated after the retailers' problem is solved, the value of q_i and p_i are known. Thus, it can be assumed that $q_i - y_i(p_i, p_j) = C_i$, where C_i is a constant for each retailer i:

$$V_i(q_i) = \begin{cases} C_i - \epsilon & \epsilon \le C_i, j \neq i \\ 0 & \epsilon > C_i, j \neq i \end{cases} = \int_A^{C_i} (C_i - u) f(u) du = \frac{(C_i - A)^2}{2 * (B - A)} \quad 5.11$$

Assuming the salvage units are compensated by each supplier proportionate to the number of orders they have fulfilled to the retailers; the expected salvage count per unit of product can be calculated as:

$$v = \frac{\sum_i V_i(q_i)}{Q} \quad 5.12$$

Equation 5.12 helps calculate the suppliers' expected revenue function under BB contract by knowing how much the expected transfer payment from each supplier to retailers is:

$$E(\Pi_k) = o_k * [w_k(o_k) - v * h] \quad 5.13$$

The expected revenue of the supplier from each unit sold is the expected transfer payment deducted from sales price, multiply that by the total sales quantity will be the total expected revenue level of supplier. In the next section a set of numerical examples will be provided to study the effect of BB contract on the retailers and suppliers.

5.1.1 Experimental Results

To test the usefulness of the proposed method to solve the GP problem with BB contract and to study the effect of input parameter on the response factors; a set of test cases will be generated and solved in this section. The input parameters selected for the retailers and suppliers are like the ones used in the GP test case as well as the BB contract parameter h. For a 2 retailers- 2 suppliers GP problem, there will be 13 input parameters in this experiment, if 3 levels are considered for each factor, if one were to run a full factorial experiment as previous chapters, then there be $3^{13} = 1{,}594{,}323$ number of test cases to run which is almost impossible to do. A PB design is used to reduce the number of runs needed for this study. Next, based on the PB design, a set of 49 test cases are created for each of the 4 industries mentioned in previous cases.

Table 19- Table 22 displays the ANOVA table for each industry case, glancing at these tables it can be concluded that the factors that are significant on the response factor is different depending on the industry. Only looking at the BB contract factor h, it is not a

significant factor in the experiments that were done in this research in Industries 1-3, only in industry 4 it has a significant impact on the retailers' profit.

Table 19. ANOVA analysis for profit function- GP with BB contract- 2 retailers- 2 suppliers Case 1

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	15077338872	1076952777	93.95	0.000
Linear	13	15063176802	1158705908	101.08	0.000
ai_1	1	4431578403	4431578403	386.59	0.000
bi_1	1	3122402587	3122402587	272.39	0.000
gai_1	1	23853836	23853836	2.08	0.158
ai_2	1	4170209995	4170209995	363.79	0.000
bi_2	1	3207880594	3207880594	279.84	0.000
gai_2	1	276343	276343	0.02	0.878
mk_1	1	13354724	13354724	1.17	0.288
dk_1	1	6413899	6413899	0.56	0.460
Wk_1	1	9589154	9589154	0.84	0.367
mk_2	1	54946783	54946783	4.79	0.036
dk_2	1	11893875	11893875	1.04	0.316
Wk_2	1	10774838	10774838	0.94	0.339
h	1	1771	1771	0.00	0.990
Curvature	1	14162070	14162070	1.24	0.274
Error	34	389747638	11463166		
Total	48	15467086510			

Table 20. ANOVA analysis for profit function- GP with BB contract- 2 retailers- 2 suppliers Case 2

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	3.81622E+14	2.72587E+13	105.27	0.000
Linear	13	3.80830E+14	2.92946E+13	113.13	0.000
ai_1	1	1.33209E+14	1.33209E+14	514.45	0.000
bi_1	1	5.73943E+13	5.73943E+13	221.66	0.000
gai_1	1	5.69062E+11	5.69062E+11	2.20	0.147
ai_2	1	1.20681E+14	1.20681E+14	466.06	0.000
bi_2	1	5.52488E+13	5.52488E+13	213.37	0.000
gai_2	1	1.17411E+11	1.17411E+11	0.45	0.505
mk_1	1	4.25583E+12	4.25583E+12	16.44	0.000
dk_1	1	3.81664E+12	3.81664E+12	14.74	0.001
Wk_1	1	8.32010E+11	8.32010E+11	3.21	0.082
mk_2	1	3.59558E+12	3.59558E+12	13.89	0.001
dk_2	1	5.69669E+11	5.69669E+11	2.20	0.147
Wk_2	1	4.91132E+11	4.91132E+11	1.90	0.177
h	1	50207098563	50207098563	0.19	0.662

Curvature	1	7.92191E+11	7.92191E+11	3.06	0.089
Error	34	8.80380E+12	2.58935E+11		
Total	48	3.90426E+14			

Table 21. ANOVA analysis for profit function- GP with BB contract- 2 retailers- 2 suppliers Case 3

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	4.94484E+15	3.53203E+14	84.09	0.000
Linear	13	4.93946E+15	3.79958E+14	90.46	0.000
ai_1	1	1.46831E+15	1.46831E+15	349.56	0.000
bi_1	1	1.02486E+15	1.02486E+15	243.99	0.000
gai_1	1	9.74283E+12	9.74283E+12	2.32	0.137
ai_2	1	1.37294E+15	1.37294E+15	326.85	0.000
bi_2	1	1.04214E+15	1.04214E+15	248.10	0.000
gai_2	1	1.07981E+11	1.07981E+11	0.03	0.874
mk_1	1	6.28471E+11	6.28471E+11	0.15	0.701
dk_1	1	7.76775E+11	7.76775E+11	0.18	0.670
Wk_1	1	4.21915E+12	4.21915E+12	1.00	0.323
mk_2	1	2.54572E+12	2.54572E+12	0.61	0.442
dk_2	1	9.33842E+12	9.33842E+12	2.22	0.145
Wk_2	1	3.38870E+12	3.38870E+12	0.81	0.375
h	1	4.61950E+11	4.61950E+11	0.11	0.742
Curvature	1	5.38044E+12	5.38044E+12	1.28	0.266
Error	34	1.42816E+14	4.20048E+12		
Total	48	5.08766E+15			

Table 22. ANOVA analysis for profit function- GP with BB contract- 2 retailers- 2 suppliers Case 4

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	11749367468	839240533	117.68	0.000
Linear	13	11718958704	901458362	126.40	0.000
ai_1	1	1988370552	1988370552	278.81	0.000
bi_1	1	3236864096	3236864096	453.87	0.000
gai_1	1	25521697	25521697	3.58	0.067
ai_2	1	1836102039	1836102039	257.46	0.000
bi_2	1	3517433266	3517433266	493.21	0.000
gai_2	1	288604	288604	0.04	0.842
mk_1	1	294470324	294470324	41.29	0.000
dk_1	1	163490152	163490152	22.92	0.000
Wk_1	1	93269595	93269595	13.08	0.001
mk_2	1	192787676	192787676	27.03	0.000
dk_2	1	151908417	151908417	21.30	0.000
Wk_2	1	151814421	151814421	21.29	0.000
h	1	66637865	66637865	9.34	0.004

Curvature	1	30408764	30408764	4.26	0.047
Error	34	242477335	7131686		
Total	48	11991844803			

Next, Figure 29- Figure 32, displays the main effect of the input parameters on the response factor in each industry, inspecting these graphs, similar conclusion can be drawn from them, the effect of the BB contract on the retailers profit is not significant, except in Case 4 where it has an increasing effect on the retailers' profit.

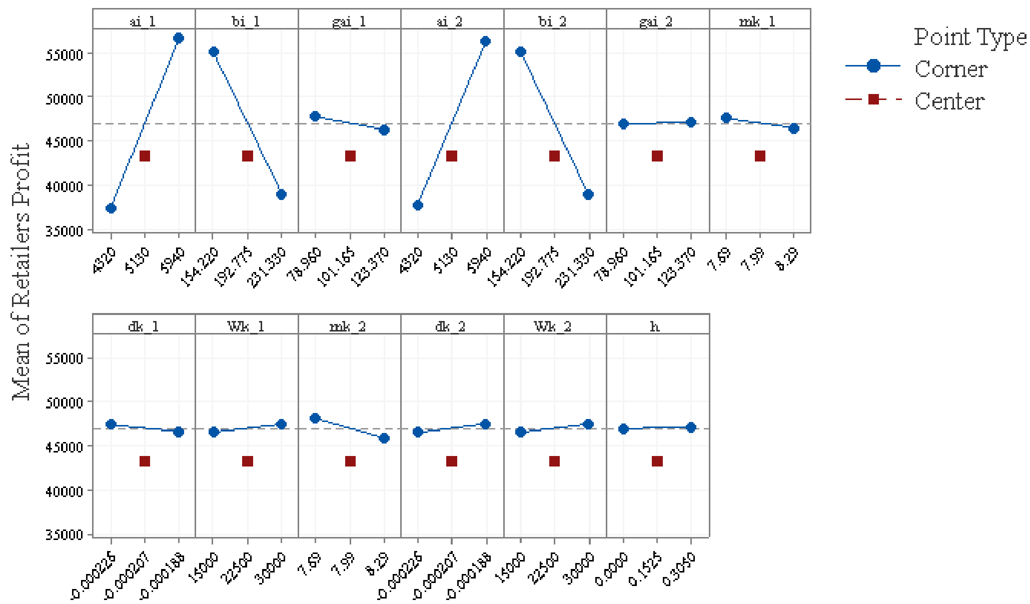

Figure 29. Main effects plot for retailer profit- group purchasing with BB contract-
2 retailers- 2 suppliers Case 1

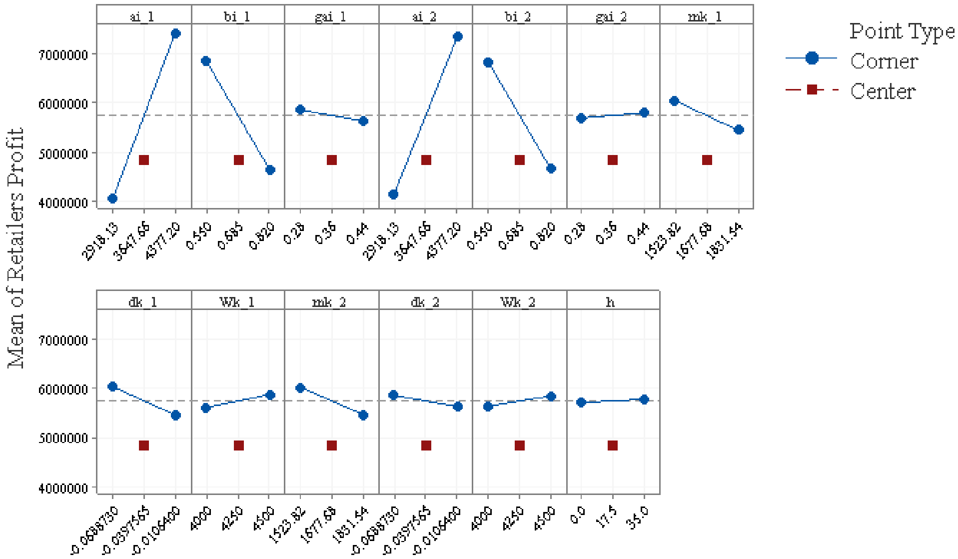

Figure 30. Main effects plot for retailer profit- group purchasing with BB contract- 2 retailers- 2 suppliers Case 2

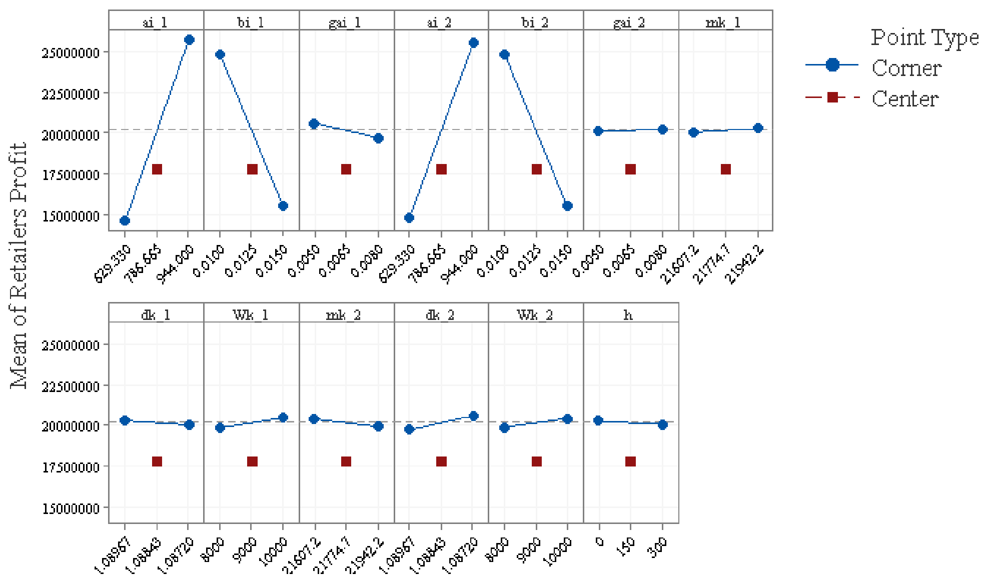

Figure 31. Main effects plot for retailer profit- group purchasing with BB contract- 2 retailers- 2 suppliers Case 3

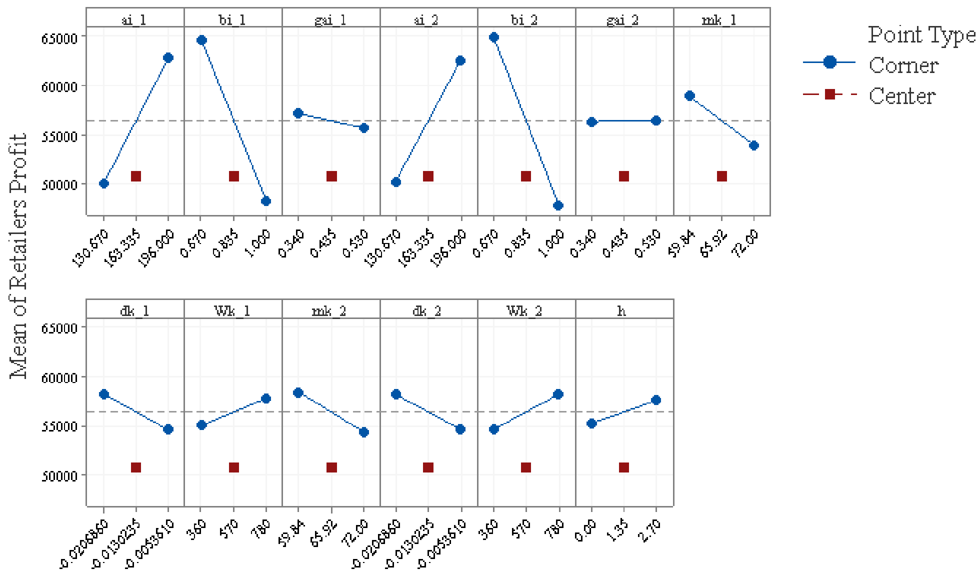

Figure 32. Main effects plot for retailer profit- group purchasing with BB contract- 2 retailers- 2 suppliers Case 4

5.2 Revenue-sharing Contract

In revenue-sharing (RS) contract, the retailer pays back a percentage $(1-\phi)$ of the total revenue back to the supplier at the end of the sale period as well as the purchasing price c (Cachon and Lariviere 2005). In this section, the GP problem will be modeled with RS contract and propose a solution method to find the optimal retailers' decision as well as suppliers' revenue. After solving the problem, the solution will be analyzed to see how the RS contract affects the decision parameters and profits compared to the no-contract case.

The retailer's profit under RS contract can be defined as:

$$\pi_i(z_i, p_i, p_j) = \begin{cases} \phi p_i D_i(p_i, p_j, \epsilon) - cq_i + \phi v_i[q_i - D_i(p_i, p_j, \epsilon)], & D_i(p_i, p_j, \epsilon) \le q_i, j \ne i \\ \phi p_i q_i - cq_i - s_i[D_i(p_i, p_j, \epsilon) - q_i], & D_i(p_i, p_j, \epsilon) > q_i, j \ne i \end{cases} \quad 5.14$$

After substituting $D_i(p_i, p_j, \epsilon)$ with $y_i(p_i, p_j) + \epsilon$ and setting $z_i = q_i - y_i(p_i, p_j)$ the profit function can be displayed as:

$$\pi_i(z_i, p_i, p_j) = \begin{cases} \phi p_i[y_i(p_i, p_j) + \epsilon] - c[y(p_i, p_j) + z_i] + \phi v_i[z_i - \epsilon], \epsilon \leq z_i, j \neq i \\ \phi p_i[y_i(p_i, p_j) + z_i] - c[y(p_i, p_j) + z_i] - s_i[\epsilon - z_i], \epsilon > z_i, j \neq i \end{cases} \quad 5.15$$

Assuming $f(u)$ and $F(u)$ are probability and cumulative density functions and μ is the mean of the random variable ϵ; the expected profit function can be presented as:

$$E[\pi_i(z_i, p_i, p_j)] = \int_A^{z_i} \phi(p_i[y_i(p_i, p_j) + u] + \mathrm{v_i}[z_i - u])f(u)du + \int_{z_i}^{B} (\phi p_i[y_i(p_i, p_j) + z_i] - s_i[u - z_i])f(u)du - c[y_i(p_i, p_j) + z_i] = \Psi_\mathrm{i}(p_i, p_j) - L_i(z_i, p_i) \quad 5.16$$

Where the risk-less profit function $\Psi_\mathrm{i}^\mathrm{r}(p_i, p_j)$ is:

$$\Psi_i(p_i, p_j) = (\phi p_i - c)[y_i(p_i, p_j) + \mu] \quad 5.17$$

And the loss due to shortage and surplus, $L_i(z_i, p_i)$ is:

$$L_i(z_i, p_i) = (c - \phi v_i)\Lambda(z_i) + (\phi p_i + s_i - c)\Theta(z_i) \quad 5.18$$

Assuming the contract terms are given, to find the best order and pricing strategy under RS contract, the first and second order derivatives with respect to z_i and p_i needs to be studied:

$$\frac{\partial E\left[\pi_i\left(z_i, p_i, p_j\right)\right]}{\partial z_i} = -(c - \phi v_i) + [\phi(p_i - v_i) + s_i][1 - F(z_i)] \qquad 5.19$$

$$\frac{\partial^2 E\left[\pi_i\left(z_i, p_i, p_j\right)\right]}{\partial z_i^2} = -[\phi(p_i - v_i) + s_i]f(z_i) \qquad 5.20$$

$$\frac{\partial E\left[\pi_i\left(z_i, p_i, p_j\right)\right]}{\partial p_i} = 2\phi(\mathrm{b_i} + \gamma_i)(p_i^0 - p_i) - \phi\Theta(z_i) \qquad 5.21$$

$$\frac{\partial^2 E\left[\pi_i\left(z_i, p_i, p_j\right)\right]}{\partial p_i^2} = -2\phi(b_i + \gamma_i) \qquad 5.22$$

Where $p_i^0 = \frac{\mathrm{a_i} + (\mathrm{b_i} + \gamma_i)\frac{c}{\phi} + \mu + \gamma_i {}^{1}/_{I-1} \sum_{\substack{j=1 \\ j \neq i}}^{n} p_j}{2(\mathrm{b_i} + \gamma_i)}$ is the optimal riskless price which will be the same as the no contract case if $\phi = 1$; which means that the retailer is keeping all the revenue.

Since p_i^0 is a function of p_i and p_j, Lemma 3 can be used to solve it for known parameters.

Observing 5.20, it ca be shown that $[\phi(p_i - v_i) + s_i]$ is always positive since $\phi \geq 0$ and $v_i \leq p_i$, thus $\phi(p_i - v_i)$ and s_i are positive. As a result, the term $-[\phi(p_i - v_i) + s_i]f(z_i)$ is always negative, so the expected profit function is concave downward in z_i. The same case applies for 5.22, because $\phi \geq 0$, $b_i \geq 0$ & $\gamma_i \geq 0$. Therefore, it can be concluded that $E\left[\pi_i\left(z_i, p_i, p_j\right)\right]$ is concave downward in both p_i and z_i; thus, the optimization problem can be reduced to a simpler problem using Zabel's approach (Zabel 1970) by finding the optimal value of p_i as a function of z_i and substituting it in $\frac{\partial E[\pi_i(\mathrm{z_i}, \mathrm{p_i}, p_j)]}{\partial \mathrm{z_i}}$, then a search over the single variable space will find the optimal z_i. Applying Lemma 4 converts the expected profit function to only a function of z_i and z_j. Lemma 5 applied here as well, since the first derivative of the expected profit function with respect to z_i is only a function of z_i, z_i^* can be found without the need to know the values of z_j^*.

If the first derivative of the profit function with respect to z_i is called $R_i(z_i)$. Finding the values of z_i that satisfy the first order condition means finding the roots of $R_i(z_i)$:

$$R_i(z_i) = \frac{\partial E\left[\pi_i\left(\mathrm{z_i}, \mathrm{p_i}(z_i), p_j(z_j)\right)\right]}{\partial \mathrm{z_i}}$$

$$= -(c - \phi v_i) + \left[\phi(\mathrm{p_i^0} - \mathrm{v_i} - \frac{\Theta(z_i)}{2(b_i + \gamma_i)}) + s_i\right][1 - F(\mathrm{z_i})] \qquad 5.23$$

Finding the values of z_i that satisfy first-order condition is important in finding the optimal profit. Interpreting the shape of the $R_i(z_i)$ based on the problem's parameters is fundamental in finding the optimal z_i; Theorem 7, which is a rewrite of Theorem 5 for this problem helps identify the conditions of z_i^*:

Theorem 7: The optimal order and pricing policy in the group buying problem with n competing asymmetric retailers and RS contract is to order $q_i^* = y_i(p_i^*) + z_i^*$ units and sell at the unit price p_i^*, where p_i^* is determined using Lemma 3 and Lemma 4 and z_i^* is defined based on the following:

a) If $F(.)$ is a random distribution function, then a complete search over the range $[A, B]$ will determine z_i^*.

b) If $F(.)$ satisfies the condition $2r(z_i)^2 + \frac{dr(z_i)}{dz_i} > 0$ for $A \le z_i \le B$ and $r(.) = \frac{f(.)}{1-F(.)}$; then z_i^* is the largest z_i in the range $[A, B]$ that satisfies $\frac{dE\left[\pi_i\left(\mathrm{z_i},\mathrm{p_i}(z_i),\mathrm{p_j}(\mathrm{z_j})\right)\right]}{d\mathrm{z_i}} = 0$

c) If condition b is met and $a_i - (b_i + \gamma_i)(c - 2s_i) + A > 0$, then z_i^* is the unique z in the range $[A, B]$ that satisfies $\frac{dE[\pi_i(z_i, p_i(z_i), p_j(z_j))]}{dz_i} = 0$.

Proof: the proof is the same as Theorem 5, after replacing 4.15 with 5.23. ■

Fundamental step in solving this problem is solving the system of equation to find p_i^0 using Lemma 3; here also, the symbolic solution grows exponentially with the size of the problem, and it is not possible to find the result for general n-retailer problem.

Unlike the BB contract, where the supplier's revenue is dependent on the unsold units; in RS contract the supplier's profit is a function of number of sold units. The transfer payment between the supplier and retailer is a percentage $(1-\phi)$ of the retailers' earned revenue which includes the revenue from salvaged units. To calculate the supplier's revenue in this contract, the transfer payment needs to be calculated. The expected share of the retailers' revenue which is transferred to the suppliers can be calculated for each retailer as a fraction of each retailer's revenue:

$$S_i(q_i) = \begin{cases} (1-\phi)p_i D_i(p_i, p_j, \epsilon) + (1-\phi)v_i[q_i - D_i(p_i, p_j, \epsilon)], & D_i(p_i, p_j, \epsilon) \le q_i, j \ne i \\ (1-\phi)p_i q_i, & D_i(p_i, p_j, \epsilon) > q_i, j \ne i \end{cases} \quad 5.24$$

By substituting $D_i(p_i, p_j, \epsilon)$ with $y_i(p_i, p_j) + \epsilon$ and defining $z_i = q_i - y_i(p_i, p_j)$, supplier's RS can be displayed as:

$$S_i(q_i) = \begin{cases} (1-\phi)p_i[y_i(p_i, p_j) + \epsilon] + (1-\phi)v_i[z_i - \epsilon], & \epsilon \le z_i, j \ne i \\ (1-\phi)p_i[y_i(p_i, p_j) + z_i], & \epsilon > z_i, j \ne i \end{cases} \quad 5.25$$

Assuming $f(u)$ and $F(u)$ are probability and cumulative density functions and μ is the mean of the random variable ϵ; the expected RS function can be presented as:

$$E[S_i(q_i] = \int_A^{z_i} (1-\phi)\left[p_i\left[y_i\left(p_i,p_j\right)+u\right]+v_i[z_i-u]\right]f(u)du$$

$$+\int_{z_i}^{B} (1-\phi)p_i\left[y_i\left(p_i,p_j\right)+z_i\right]f(u)du \quad 5.26$$

$$= (1-\phi)\left[p_i\left[y_i\left(p_i,p_j\right)+\mu\right]+v_i\Lambda(z_i)-p_i\Theta(z_i)\right]$$

The above function is the expected value of RS generated by retailer, since the suppliers' revenue will be calculated after the suppliers' problem is solved, the $S_i(q_i)$ can be calculated by knowing p_i^*, p_j^* and z_i^*. The total RS earned by all retailers is the sum of this parameter for all retailers. Assuming that ϕ is unique, it is expected that each supplier receives a fraction of the total RS proportionate to the number of items fulfilled to the retailers. The expected RS per sold item can be calculated as:

$$s = \frac{\sum_i S_i(q_i)}{Q} \quad 5.27$$

5.27 helps calculate the suppliers' expected profit function under RS contract by knowing how much the expected transfer payment to supplier is for each unit sold:

$$E(\Pi_k) = o_k * [w_k(o_k^s) + s] \quad 5.28$$

The expected revenue of the supplier is the sum of sales price and expected RS per unit, multiplied by the total sales quantity. In the next section numerical examples will be provided to study the effect of RS contract on the retailers and suppliers.

5.2.1 Experimental Results

A set of experiments are created based on Table **4** to test the usefulness of the proposed method to solve the GP problem with RS contract and to study the effect of input parameter on the response factors. The input parameters selected for the retailers and suppliers are like the ones used in the GP test case as well as the RS contract parameter ϕ. 3 levels are selected for ϕ; 1, 0.985, 0.97 where 1 means 100% of the revenue belongs to the retailers. Considering the selected parameters, for a 2 retailers- 2 suppliers GP problem, there will be 13 input parameters in this experiment, considering 3 levels for each factor, there will be 3^{13} combinations. A PB design is used to reduce the number of runs needed for this study. Next, based on the PB design, a set of 49 test cases are created for each of the 4 industries mentioned in previous cases.

Table 23- Table 26 displays the ANOVA table for each industry case, glancing at these tables it looks like the RS contract factor ϕ is significant in all industries, the rest of the factors have similar significance as in GP problem studied in 4.3.

Table 23. ANOVA analysis for profit function- GP with RS contract- 2 retailers- 2 suppliers Case 1

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	13949932450	996423746	86.49	0.000
Linear	13	13937306893	1072100530	93.06	0.000
ai_1	1	3955622662	3955622662	343.37	0.000
bi_1	1	2931358581	2931358581	254.46	0.000
gai_1	1	25791591	25791591	2.24	0.144
ai_2	1	3824824111	3824824111	332.01	0.000
bi_2	1	2879857904	2879857904	249.99	0.000
gai_2	1	164511	164511	0.01	0.906
mk_1	1	8765579	8765579	0.76	0.389
dk_1	1	7412053	7412053	0.64	0.428
Wk_1	1	7536529	7536529	0.65	0.424
mk_2	1	60069456	60069456	5.21	0.029
dk_2	1	2893357	2893357	0.25	0.619
Wk_2	1	7404110	7404110	0.64	0.428
f	1	225606449	225606449	19.58	0.000
Curvature	1	12625557	12625557	1.10	0.303
Error	34	391681532	11520045		
Total	48	14341613982			

Table 24. ANOVA analysis for profit function- GP with RS contract- 2 retailers- 2 suppliers Case 2

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	3.68804E+14	2.63432E+13	88.41	0.000
Linear	13	3.68106E+14	2.83159E+13	95.03	0.000
ai_1	1	1.25682E+14	1.25682E+14	421.80	0.000
bi_1	1	5.46314E+13	5.46314E+13	183.35	0.000
gai_1	1	7.15359E+11	7.15359E+11	2.40	0.131
ai_2	1	1.18937E+14	1.18937E+14	399.16	0.000
bi_2	1	4.80829E+13	4.80829E+13	161.37	0.000
gai_2	1	57137435736	57137435736	0.19	0.664
mk_1	1	4.49596E+12	4.49596E+12	15.09	0.000
dk_1	1	3.44742E+12	3.44742E+12	11.57	0.002
Wk_1	1	6.88491E+11	6.88491E+11	2.31	0.138
mk_2	1	4.85356E+12	4.85356E+12	16.29	0.000
dk_2	1	2.55615E+11	2.55615E+11	0.86	0.361
Wk_2	1	2.26647E+11	2.26647E+11	0.76	0.389
f	1	6.03212E+12	6.03212E+12	20.24	0.000
Curvature	1	6.97915E+11	6.97915E+11	2.34	0.135
Error	34	1.01309E+13	2.97968E+11		
Total	48	3.78935E+14			

Table 25. ANOVA analysis for profit function- GP with RS contract- 2 retailers- 2 suppliers Case 3

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	4.60771E+15	3.29122E+14	78.57	0.000
Linear	13	4.60265E+15	3.54050E+14	84.52	0.000
ai_1	1	1.33273E+15	1.33273E+15	318.16	0.000
bi_1	1	9.52236E+14	9.52236E+14	227.33	0.000
gai_1	1	1.10428E+13	1.10428E+13	2.64	0.114
ai_2	1	1.27618E+15	1.27618E+15	304.66	0.000
bi_2	1	9.35880E+14	9.35880E+14	223.42	0.000
gai_2	1	2.03734E+11	2.03734E+11	0.05	0.827
mk_1	1	1.16126E+12	1.16126E+12	0.28	0.602
dk_1	1	9043386692	9043386692	0.00	0.963
Wk_1	1	3.41044E+12	3.41044E+12	0.81	0.373
mk_2	1	3.51254E+12	3.51254E+12	0.84	0.366
dk_2	1	8.83715E+12	8.83715E+12	2.11	0.156
Wk_2	1	2.01747E+12	2.01747E+12	0.48	0.492
f	1	7.54370E+13	7.54370E+13	18.01	0.000
Curvature	1	5.05629E+12	5.05629E+12	1.21	0.280
Error	34	1.42420E+14	4.18883E+12		
Total	48	4.75013E+15			

Table 26. ANOVA analysis for profit function- GP with RS contract- 2 retailers- 2 suppliers Case 4

Source	**DF**	**Adj SS**	**Adj MS**	**F-Value**	**P-Value**
Model	14	11804507271	843179091	111.50	0.000
Linear	13	11777784052	905983389	119.80	0.000
ai_1	1	1886057335	1886057335	249.40	0.000
bi_1	1	3249131366	3249131366	429.64	0.000
gai_1	1	9845467	9845467	1.30	0.262
ai_2	1	1877821502	1877821502	248.31	0.000
bi_2	1	3196545925	3196545925	422.69	0.000
gai_2	1	7096029	7096029	0.94	0.340
mk_1	1	293292669	293292669	38.78	0.000
dk_1	1	181517933	181517933	24.00	0.000
Wk_1	1	120341930	120341930	15.91	0.000
mk_2	1	244557359	244557359	32.34	0.000
dk_2	1	112127608	112127608	14.83	0.000
Wk_2	1	114968332	114968332	15.20	0.000
f	1	484480598	484480598	64.06	0.000
Curvature	1	26723218	26723218	3.53	0.069
Error	34	257120925	7562380		
Total	48	12061628196			

Next, Figure 33- Figure 36 displays the main effect of the input parameters on the response factor in each industry, inspecting these graphs, similar conclusion can be drawn from them, the effect of the RS contract on the retailers profit is significant in all cases with an increasing effect with increase in ϕ, existence of an RS contract results in lower profit levels for the retailers.

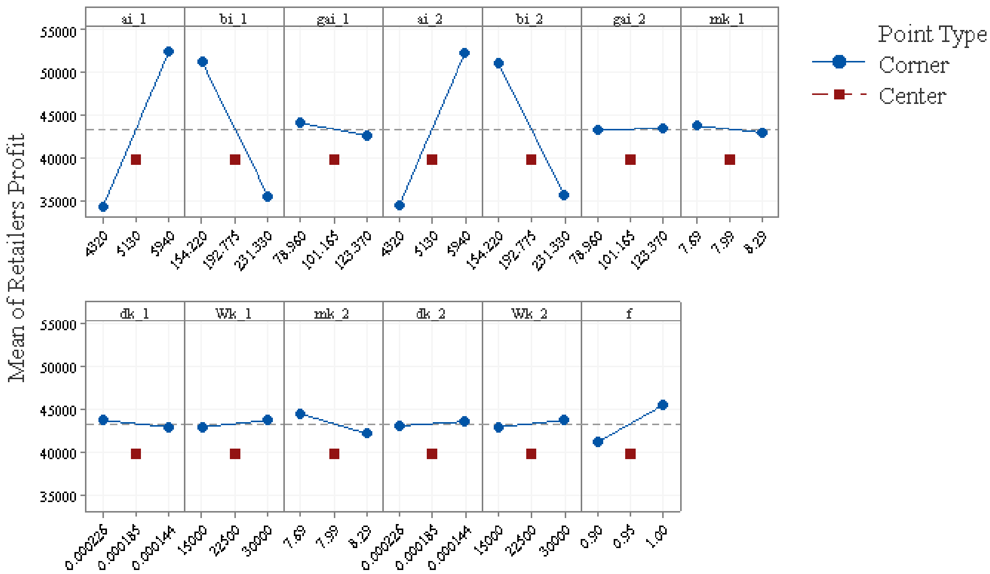

Figure 33. Main effects plot for retailer profit- group purchasing with RS contract- 2 retailers- 2 suppliers Case 1

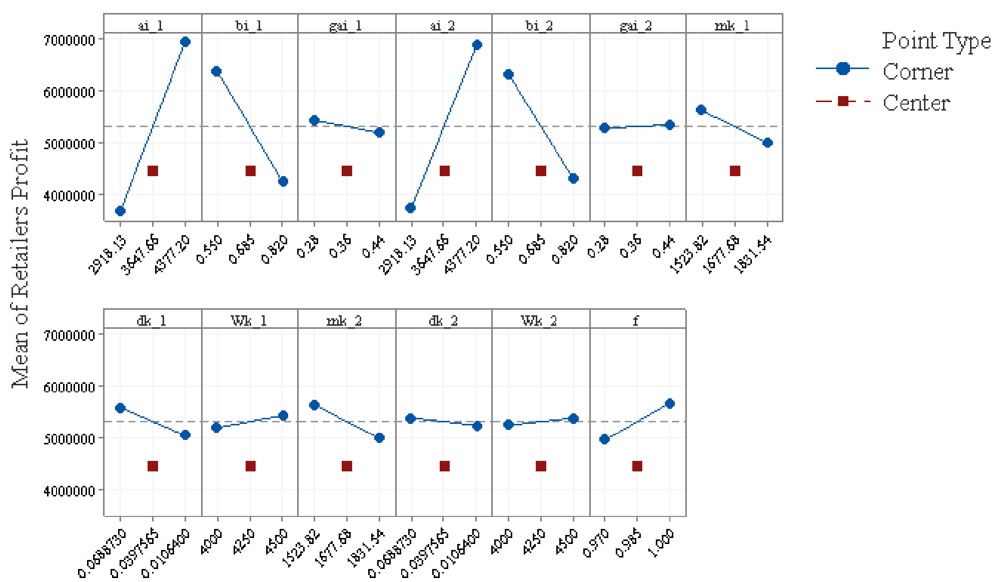

Figure 34. Main effects plot for retailer profit- group purchasing with RS contract- 2 retailers- 2 suppliers Case 2

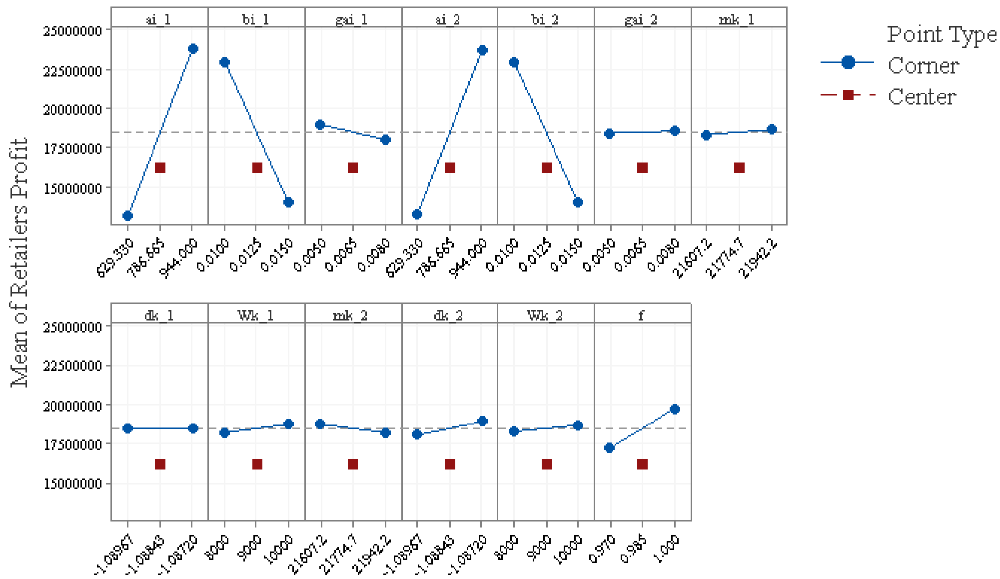

Figure 35. Main effects plot for retailer profit- group purchasing with RS contract- 2 retailers- 2 suppliers Case 3

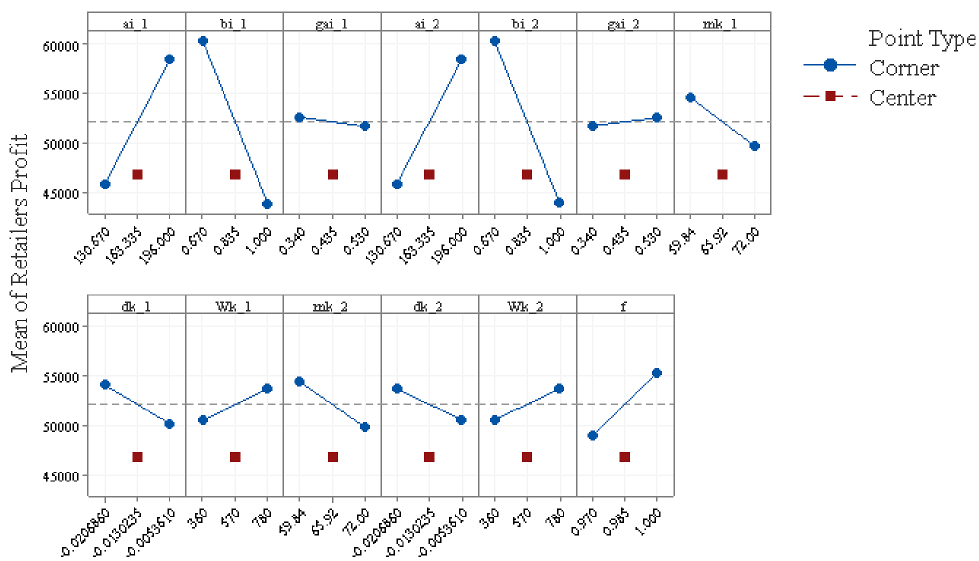

Figure 36. Main effects plot for retailer profit- group purchasing with RS contract- 2 retailers- 2 suppliers Case 4

5.3 Summary

In this chapter two types of contracts i.e., buyback and revenue-sharing contracts are introduced into the GP problem and a solution is provided to these problems. Retailers profit and suppliers order assignment and revenue are modeled and analytically studied to identify the conditions for optimal decisions. Numerical experiments for the industry cases are provided to test the applicability of the provided solutions and to test the effect of contracts on the decision parameters. The numerical experiments with these problems reveal that:

1. Under Buyback contract, the suppliers share the risk of over-stocking with the retailer buy transferring a per excess unit cost to the retailers; the parties could benefit or lose compared to the no contract case, depending on other parameters of the problem:
    a. The significance of the demand and pricing parameters are similar to the no contract case.
    b. The buyback parameter h is significant only in the industry case 4, in other industries, it does not have a significant impact on the retailers' profitability.
    c. If significant, the buyback contract, results in an increased profit for the retailers if all other parameters are fixed.
2. In Revenue-Sharing contract, the suppliers receive part of the retailers' revenue from the sale, thus their profit is dependent on the market demand. Based on the numerical experiments with this contract:
    a. The significance of the demand and pricing parameters are like the no contract case.
    b. The revenue-sharing contract parameter ϕ has a significant impact on the retailers' profit in all industry cases.

c. Where significant, the existence of revenue-sharing contract results in lower profit levels if other parameters are fixed.

6 CONCLUSIONS

This research addresses four research questions by exploring three categories of problems in a GP environment.

Research questions 1 is addressed in Chapter 3 with the newsvendor problem with QDF as well as GP with symmetric retailers. First problem is a modified newsvendor problem facing a price sensitive demand and QDF pricing from supplier, then the problem is extended to a GP problem with symmetric retailers. Two approaches are proposed to find the optimal price and order quantity for the newsvendor, one is a 2-step heuristic based on a fixed cost newsvendor problem that is deployed in a simulation optimization algorithm to solve the problem at hand; the second method is an innovative 1-step method developed specifically for this problem which solves the problem with fewer steps. Using the proposed 1-step method, a full-factorial analysis was done for input parameters where demand parameters a_i and b_i as well as supplier parameters m_k and d_k are used as factors and the retailer profit as response factor. Three levels are selected for each input factor and a full factorial analysis is ran for all the combinations of these factors for on four industries:

1. All the one-way interactions of the input parameters have a significant impact on the retailer profit in all industries.
2. The two-way interactions of the input parameters are significant in all industries, except in industry 3 where the interaction of $b_i * d_k$ and $m_k * d_k$ are not significant. Which means that the impact of parameters b_i and m_k on the response factor is not dependent on the level of parameter d_k.
3. The demand parameters a_i and b_i have the highest impact and their relationship is nonlinear with the response factor:

a. a_i has increasing impact on the response factor, with the increase in a_i, the retailer profit increases.

b. b_i has decreasing impact on the response factor, with the increase in b_i, the retailer profit decreases.

4. The supplier parameters m_k and d_k have lower effect on the retailers' profit compared to the demand parameters, their impact is always a decreasing impact on the retailers' profit:

   a. In case 1, these parameters have a linear relationship with the response factor.

   b. In case 2 and case 4 the relationship is a nonlinear one.

   c. In case 3, even though these parameters are significant based on the ANOVA table, the effect cannot be verified when looking at the main effects plot. The effect looks minimal, and it cannot be said what type of relationship they have.

Next, the GP problem with symmetric retailers is addressed; both methods that were developed for the newsvendor problem were extended to solve this problem with any number of symmetric retailers. Using the proposed extended method, a full-factorial analysis was done for demand parameters a_i and b_i and supplier parameters m_k and d_k as well as the number of retailers I were selected as factors and the retailer profit as response factor. Three levels are selected for each input factor and a full factorial analysis was ran for all the combinations of these factors for on four industries:

1. Much like the newsvendor case, all the one-way interactions of the input parameters have a significant impact on the retailer profit in all industries.

2. Regarding the two-way interactions, it is different based on the industry:
    a. In case 1 the two-way interactions of $b_i * m_k$, $b_i * d_k$ and $m_k * d_k$ are not significant.
    b. In case 2 and 4 the $b_i * m_k$ is not significant.
    c. In case 3, the two-way interactions of $a_i * d_k$, $b_i * d_k$ and $m_k * d_k$ are not significant.
3. The additional parameter I, is significant in all cases and it has an increasing impact on the retailers' profitability:
    a. There are between %14-%22 increase in expected profit levels for each retailer which each additional retailer joining the GP.
    b. In industry 2, the retailers benefit more compared to other industries by joining GP. On average the retailers benefit %1.9 more compared to other industries through GP.
    c. In industry 3, the retailers' gain the least benefit through GP; retailers benefit on average %3.9 less compared to other industries through GP.

In Chapter 4, the research questions 2 and 3 are addressed by modelling the GP with competitive asymmetric retailers and asymmetric suppliers. The GP problem was broken down to two sub-problems to solve: the retailers' problem and the suppliers' problem. Both the problems were analyzed analytically, and solution methods are proposed to solve them. For the retailers' problem the solution method is based on fixing the cost and then solving the symmetric retailers' problem via first solving a system of equations to find the risk-less optimal price p^0 and then one can find the optimal price and order quantity for each retailer. In the supplier problem the provided solution is based on the Lagrangian method which helps identify the order assignment to each supplier considering their capacity and QDF pricing such that the purchasing cost is minimized for the retailers. Once these sub-problems are solved separately, the sub-problems are put together and solved as one problem through a consequential solution until the solution to both problems converge. To experiment with the problem, the input parameters for the retailers are a_i, b_i and γ_i and for the suppliers m_k, d_k and W_k. Considering asymmetric retailers and suppliers, the number of input parameters for sensitivity analysis grows quickly, e.g., for a 2-retailer and 2-supplier problem, there will be 12 input parameters and for the 3-retailer and 3-supplier problem, there will be 18 input parameters. Due to the number of input parameters, running a full factorial analysis is not possible, even for the 2-retailer and 2-supplier problem considering three levels for each parameter. Thus, a Plackett-Burman design is used to experiment with the test cases, which helps significantly lower the number of runs to identify the significant input factors. Running a PB design with center point for all industries, the results from the experiments can be summarized as below:

1. Demand parameters a_i and b_i are significant factors in all industries.
2. The competition factor γ_i is not a significant factor in any of the industries; even though based on the main effect plot it does have an impact on the profit levels.
3. Supplier pricing parameter m_k is significant for all suppliers in industries 2 and 4, this cannot be said for industries 1 and 3.
4. Supplier pricing parameter d_k is significant for all suppliers only in industry 4, which is not the case in other industries.

5. It is interesting that Supplier capacity factor W_k is not significant in industries 1, 2 and 3; but in industry 4 it is significant for all suppliers.
    a. The impact of the supplier capacity on the retailers' profit is an increasing impact i.e., increased supplier capacity results in increased retailers' profit; in other words, doing GP with larger suppliers increases the GP benefit.

The research question 5 is addressed in Chapter 5, where two types of contracts are introduced to the GP problem. The first contract discussed is Buyback contract where in the suppliers share the inventory risk with the retailers by returning a payment of $h \leq c$ for any unsold units to the retailers. The second type of contract is a Revenue-sharing contract where the retailers share their revenue with the suppliers. For both contracts a solution method is provided to find the retailers' profit and suppliers' revenue under these contract types. Next, numerical analysis is done on a GP with 2-retailer and 2-supplier to study the effect of each input factor as well as the contract on the response factor:

1. Taking the retailers' profit as the response factor, in the Buyback contract the numerical results concludes that:
    a. Demand parameters a_i and b_i is significant in all industries.
    b. The competition factor is not a significant factor in any of the industries.
    c. The supplier parameter m_k is significant only in industries 2 and 4, in industry 1 it was significant only for one supplier.
    d. The supplier parameter d_k is significant only in industry 4, in industry 2 it is significant for only one of the suppliers.

e. The supplier capacity parameter W_k is significant only in industry 4.

f. The Buyback contract parameter h is not a significant factor in all the industries, except in industry 4. Checking the main effect plot; in industry 4 the contract has an increasing effect on the retailers' profit, in other industries it does not seem to have a noticeable impact.

2. Taking the retailers' profit as the response factor, in the Revenue-sharing contract the numerical results concludes that:

   a. Demand parameters a_i and b_i is significant in all industries.

   b. The competition factor is not a significant factor in any of the industries.

   c. The supplier parameter m_k is significant only in industries 2 and 4, in industry 1 it was significant only for one supplier.

   d. The supplier parameter d_k is significant only in industry 4, in industry 2 it is significant for only one of the suppliers.

   e. The supplier capacity parameter W_k is significant only in industry 4.

   f. The Revenue-sharing contract parameter ϕ is a significant factor in all the industries. Based on the main effect plot, entering the Revenue-sharing contact results in lower profit levels for the retailers.

This research addresses the GP problem in different settings and explores each problem from an analytical view to find insights for finding optimal decisions for the retailers. To study the applicability of the solution methods, a set of parameters from 4 industries are provided where 3 of the cases are from the literature and 1 is from a case study company. The data from each industry is used to generate test cases and derive insights on how each parameter can impact the retailers.

Even though the author of this research tried to look at the GP problem from different angels, there is still room for further exploration in this field:

1. The models developed in this research consider a non-profit seeking GPO, considering a profit seeking GPO would add to the complexity of the problems and comparing its' impact on the effect of GP on supply chain members could be valuable.
2. This research considers an additive demand function in all cases, other demand functions such as a multiplicative demand function could be studied in the GP this context.
3. The analytical studies provided in this research is done regardless of the distribution of the stochastic factor ϵ, but the numerical examples are ran only for a uniform distribution; experimenting with other distribution functions such as normal distribution can be interesting.
4. This research looks at the problems only from an analytical view, using this method provides a solid insight into the optimal solutions, but lacks flexibility and is time-consuming; deploying other approaches such as linear or non-linear programming could yield results faster and may be more flexible in solving larger problems.
5. Implementing the solutions developed here on more industries could give insight on its impact/usefulness in other industries.
6. In this research the pricing function is the same if whether a purchase is being made through GP or not, considering a case where the suppliers optimize the pricing function would be interesting.

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APPENDIX I: RESTRUCTURING OF THE EXPECTED PROFIT FUNCTION FOR THE NWESVENDOR PROBLEM

The expected profit function defined as below:

$$E\left[\Pi_{\mathrm{i}}\left(z_i, p_i, p_j\right)\right]$$

$$= \int_{A}^{z_i} \left(p_i\left[y_i\left(p_i, p_j\right) + u\right] + \mathrm{v_i}[z_i - u]\right) f(u)du$$

$$+ \int_{z_i}^{B} \left(p_i\left[y_i\left(p_i, p_j\right) + z_i\right] - s_i[u - z_i]\right) f(u)du - c\left[y_i\left(p_i, p_j\right) + z_i\right]$$

Adding and deducting u and μ to $y_i\left(p_i, p_j\right) + z_i$ in the second integral and in the last clause will change the function to:

$$E\left[\Pi_{\mathrm{i}}\left(z_i, p_i, p_j\right)\right]$$

$$= \int_{A}^{z_i} \left(p_i\left[y_i\left(p_i, p_j\right) + u\right] + \mathrm{v_i}[z_i - u]\right) f(u)du$$

$$+ \int_{z_i}^{B} \left(p_i\left[y_i\left(p_i, p_j\right) + z_i + u - u\right] - s_i[u - z_i]\right) f(u)du$$

$$- c\left[y_i\left(p_i, p_j\right) + z_i + \mu - \mu\right]$$

Simplifying the above integral will result in:

$E\left[\Pi_{\mathrm{i}}\left(z_i, p_i, p_j\right)\right] = (p_i - c)\left[y_i\left(p_i, p_j\right) + \mu\right] - (\mathrm{c} - \mathrm{v_i}) \int_{A}^{z_i} (z_i - u) f(u)du - (p_i + s_i - c) \int_{z_i}^{B} (u - z_i) f(u)du$, in which the first part is $\Psi(p_i) = (p_i - c)\left[y_i\left(p_i, p_j\right) + \mu\right]$ and the rest is $L(z_i, p_i)$. ■

APPENDIX II: PROOF OF COROLLARY I: OPTIMAL ORDER AND PRICE IN THE NEWSVENDOR PROBLEM WITH ADDITIVE DEMAND AND UNIFORM DISTRIBUTION

For Corollary 1 to be true, we need to prove that $2r(z)^2 + \frac{dr(z)}{dz} > 0$ for $A \leq z \leq B$, where $r(.) = \frac{f(.)}{[1-F(.)]}$:

If $z \sim U[A,B] \Rightarrow \begin{cases} f(z) = \frac{1}{B-A} \\ F(z) = \frac{z-A}{B-A} \end{cases} \Longrightarrow r(z) = \frac{1}{B-z}, \frac{dr(z)}{dz} = \frac{1}{(B-z)^2} \Longrightarrow$

$2r(z)^2 + \frac{dr(z)}{dz} = \frac{2}{(B-z)^2} + \frac{1}{(B-z)^2} = \frac{3}{(B-z)^2}$, since $B \geq z$ we can say this argument is always positive, except at $B = z$ where it is undefined. Since $R(B) < 0$ and $\frac{d^2R(z)}{dz^2}$ is positive for $[A,B)$, $R(z)$ is still considered monotone or unimodal and thus the (Theorem 1. (b)) from (Petruzzi and Dada 1999) holds. ■

APPENDIX III: OPTIMAL ORDER AND PRICING POLICY IN THE NEWSVENDOR PROBLEM WITH QDF AND $e = 1$

Proof for Theorem 1: by replacing the $p(z)$ and $e = 1$ in the profit function the first derivative is:

$$\frac{\partial \mathrm{E}[\pi(\mathrm{p(z)},\mathrm{z})]}{\partial z} = -(m-v) + \left(p^0 + s - v - \frac{\Theta(z)}{2b}\right)[1 - F(z)]$$

To find the extremum points of $\mathrm{E}[\pi(\mathrm{p(z)},\mathrm{z})]$, we need to find the zeros of the first derivative, assuming $R(z) \equiv \frac{\partial \mathrm{E}[\pi(\mathrm{p(z)},\mathrm{z})]}{\partial z}$:

$$\frac{dR(z)}{dz} = \frac{d}{dz}\left[\frac{\mathrm{dE}[\pi(\mathrm{p(z)},\mathrm{z})]}{dz}\right] = -\frac{f(z)}{2b}\left\{2b\left(p^0 + s - v\right) - \Theta(z) - \frac{1-F(z)}{r(z)}\right\}$$

Where $r(z) = f(z)/[1 - F(z)]$ is hazard rate. The second derivative of $R(z)$ is:

$$\frac{d^2R(z)}{dz^2} = \left[\frac{dR(z)/dz}{f(z)}\right]\frac{df(z)}{dz} - \frac{f(z)}{2b}.\left\{[1-F(z)] + \frac{f(z)}{r(z)} + \frac{[1-F(z)][dr(z)/dz]}{r(z)^2}\right\}$$

$$\Rightarrow \left.\frac{d^2R(z)}{dz^2}\right|_{dR(z)/dz=0} = -\frac{f(z)[1-F(Z)]}{2br(z)^2}\left\{2r(z)^2 + \frac{dr(z)}{dz}\right\}$$

If $F(.)$ satisfies the condition of $2r(z)^2 + \frac{dr(z)}{dz} > 0$, it implies that $R(z) \equiv \frac{\partial \mathrm{E}[\Pi(\mathrm{p(z)},\mathrm{z})]}{\partial z}$ is concave and thus has at most two roots. Additionally, since $R(B) = -(m-v) < 0$ then $R(z)$ has either one root which indicates that there is a change of sign from positive to negative and thus the root is a local maximum for $\mathrm{E}[\Pi(\mathrm{p(z)},\mathrm{z})]$. If $R(z)$ has two roots the larger one is a local maximum and the smaller one is a local minimum for $\mathrm{E}[\Pi(\mathrm{p(z)},\mathrm{z})]$. Also, assuming $R(z)$ is concave, a sufficient condition for $\mathrm{E}[\Pi(\mathrm{p(z)},\mathrm{z})]$ to have one root is that $R(A) > 0$:

$$R(A) = -(m-v) - \left[p^0 + s - v - \frac{\Theta(A)}{2b}\right].[F(A)-1]$$

$$= -(m-v) + \left[\frac{a+bm+\mu}{2b} + s - v - \frac{\mu - A}{2b}\right]$$

We can simplify it by looking at $2bR(A)$:

$$2bR(A) = -2b(m-v) + [a + bm + \mu + 2b(s-v) - (\mu - A)] = a - b(m - 2s) + A \qquad \blacksquare$$

APPENDIX IV: OPTIMAL ORDER AND PRICING POLICY IN THE NEWSVENDOR PROBLEM WITH QDF AND $e = 0$

Proof for Theorem 2: by replacing the $p(z)$ and $e = 0$ in the profit function the first derivative is:

$$\frac{\mathrm{dE[\Pi(p(z),z)]}}{dz} = -(m - v + d) + \left(p^0 + s - v - \frac{\Theta(z)}{2b} + \frac{d}{2}\right)[1 - F(z)]$$

To find the extremum points of $\mathrm{E[\Pi(p(z),z)]}$, we need to find the zeros of the first derivative, assuming $R(z) \equiv \frac{\partial \mathrm{E[\Pi(p(z),z)]}}{\partial z}$:

$$\frac{dR(z)}{dz} = \frac{d}{dz}\left[\frac{\mathrm{dE[\Pi(p(z),z)]}}{dz}\right] = -\frac{f(z)}{2b}\left\{2b\left(p^0 + s - v + \frac{d}{2}\right) - \Theta(z) - \frac{1 - F(z)}{r(z)}\right\}$$

Where $r(z) = f(z)/[1 - F(z)]$ is hazard rate. The second derivative of $R(z)$ is:

$$\frac{d^2R(z)}{dz^2} = \left[\frac{dR(z)/dz}{f(z)}\right]\frac{df(z)}{dz} - \frac{f(z)}{2b}.\left\{[1 - F(z)] + \frac{f(z)}{r(z)} + \frac{[1 - F(z)][dr(z)/dz]}{r(z)^2}\right\}$$

$$\Rightarrow \left.\frac{d^2R(z)}{dz^2}\right|_{dR(z)/dz=0} = -\frac{f(z)[1 - F(Z)]}{2br(z)^2}\left\{2r(z)^2 + \frac{dr(z)}{dz}\right\}$$

If $F(.)$ satisfies the condition of $2r(z)^2 + \frac{dr(z)}{dz} > 0$, it implies that $R(z) \equiv \frac{\partial \mathrm{E[\Pi(p(z),z)]}}{\partial z}$ is concave and thus has at most two roots. Additionally, if $R(B) = -(m - v + d) < 0$ then $R(z)$ has either one root which indicates that there is a change of sign from positive to negative and thus the root is a local maximum for $\mathrm{E[\Pi(p(z),z)]}$. If $R(z)$ has two roots the larger one is a local maximum and the smaller one is a local minimum for $\mathrm{E[\Pi(p(z),z)]}$. Also, assuming $R(z)$ is concave, a sufficient condition for $\mathrm{E[\Pi(p(z),z)]}$ to have one root is that $R(A) > 0$:

$$R(A) = -(m - v + d) - \left[p^0 + s - v - \frac{\Theta(A)}{2b} + \frac{d}{2}\right].[F(A) - 1]$$

$$= -(m - v + d) + \left[\frac{a + bm + \mu}{2b} + s - v - \frac{\mu - A}{2b} + \frac{d}{2}\right]$$

We can simplify it by looking at $2bR(A)$:

$2bR(A) = -2b(m - v + d) + [a + bm + \mu + 2b(s - v) - (\mu - A) + bd] = a - b(m + d - 2s) + A$ ■

APPENDIX V: OPTIMAL ORDER AND PRICING POLICY IN THE NEWSVENDOR PROBLEM WITH QDF AND $e = -1$

Proof for Theorem 3: By replacing the $p(z)$ and $e = -1$ in the profit function the first derivative is:

$$\frac{\mathrm{dE}[\pi(\mathrm{p(z)},\mathrm{z})]}{dz} = -(m-v) + [1-F(z)]\left[\frac{p^0 + d(a+z) - \Theta(z)/2b}{1+db} + s - v\right]$$

$$-2bd\left[\frac{a+z}{b} - \frac{p^0 + d(a+z) - \Theta(z)/2b}{1+db}\right]$$

To find the extremum points of $\mathrm{E}[\pi(\mathrm{p(z)},\mathrm{z})]$, we need to find the zeros of the first derivative, assuming $R(z) \equiv \frac{\partial \mathrm{E}[\pi(\mathrm{p(z)},\mathrm{z})]}{\partial z}$:

$$\frac{dR(z)}{dz} = \frac{d}{dz}\left[\frac{\mathrm{dE}[\pi(\mathrm{p(z)},\mathrm{z})]}{dz}\right]$$

$$= -\frac{f(z)}{2b(1+db)}\left\{2b\left[p^0 + d(a+z) + (1+db)(s-v)\right] - \Theta(z)\right.$$

$$\left. - \frac{2bd + [1-F(z)]}{r(z)}\right\} - \frac{2bd}{2b(1+db)}\{2 - [1-F(z)]\}$$

$$= -\frac{f(z)}{2b(1+db)}\left\{2b\left[p^0 + d(a+z) + (1+db)(s-v) + \frac{2d}{f(z)}\right] - \Theta(z)\right.$$

$$\left. - \frac{4bd + [1-F(z)]}{r(z)}\right\}$$

Where $r(z) = f(z)/[1-F(z)]$ is hazard rate. The second derivative of $R(z)$ is:

$$\frac{d^2R(z)}{dz^2} = \left[\frac{dR(z)/dz}{f(z)}\right]\frac{df(z)}{dz}$$

$$- \frac{f(z)}{2b(1+db)}\left\{2b\left[d - \frac{2d.df(z)/dz}{f(z)^2}\right] + [1 - F(z)] + \frac{f(z)}{r(z)}\right.$$

$$\left. + \frac{[4bd + \ [1 - F(z)]].dr(z)/dz}{r(z)^2}\right\} \Rightarrow \left.\frac{d^2R(z)}{dz^2}\right|_{dR(z)/dz=0}$$

$$= -\frac{f(z)}{2b(1+db)r(z)^2[1-F(z)]}\left\{2bd\left[f(z).r(z) - \frac{2.df(z)/dz}{[1-F(z)]}\right]\right.$$

$$\left. + 2f(z)^2 + [1 - F(z)][4bd + \ [1 - F(z)]].dr(z)/dz\right\}$$

If the following condition is satisfied, it implies that $R(z) \equiv \frac{\partial \mathrm{E}[\pi(\mathrm{p(z),z})]}{\partial z}$ is always concave and thus has at most two roots:

$$2bd\left[f(z).r(z) - \frac{2.df(z)/dz}{[1-F(z)]}\right] + 2f(z)^2 + [1 - F(z)][4bd + \ [1 - F(z)]].dr(z)/dz > 0$$

Additionally, if $R(B) = \ -(m - v) - 2d\left[\frac{a+B-bp^0}{1+bd}\right] < 0$ then $R(z)$ has either one root which indicates that there is a change of sign from positive to negative and thus the root is a local maximum for $\mathrm{E}[\pi(\mathrm{p(z),z})]$. Even if $R(z)$ has two roots the larger one is a local maximum and the smaller one is a local minimum for $\mathrm{E}[\pi(\mathrm{p(z),z})]$. Also, assuming $R(z)$ is concave, a sufficient condition for $\mathrm{E}[\pi(\mathrm{p(z),z})]$ to have one root is that $R(A) > 0$:

$$R(A) = -(m-v) + [1-F(A)]\left[\frac{p^0 + d(a+A) - \Theta(A)/2b}{1+bd} + s - v\right]$$

$$- 2bd\left[\frac{a+A}{b} - \frac{p^0 + d(a+A) - \Theta(A)/2b}{1+bd}\right]$$

$$= -(m-v) + \left[\frac{p^0 + d(a+A) - (\mu - A)/2b}{1+bd} + s - v\right]$$

$$- 2bd\left[\frac{(1+bd)(a+A) - bp^0 - bd(a+A) - (\mu - A)/2}{b(1+bd)}\right]$$

$$= -(m-s) + \left[\frac{bp^0 + bd(a+A) - (\mu - A)/2}{b(1+bd)}\right]$$

$$- 2bd\left[\frac{(a+A) - bp^0 - (\mu - A)/2}{b(1+bd)}\right]$$

$$= -(m-s) + \left\{\frac{[p^0(2bd+1) - d(a+A) + (2bd-1)\,(\mu - A)/2b]}{(1+db)}\right\}$$

Which can be simplified by looking at $(1 + \text{db})R(A)$:

$(1 + \text{bd})R(A) = -(1+bd)(m-s) + p^0(2bd+1) - d(a+A) + (2bd -$

$1)\,(\mu - A)/2b$ ■

APPENDIX VI: OPTIMAL ORDER AND PRICING POLICY IN THE GP PROBLEM WITH SYMMETRIC RETAILERS AND $e = -1$

Proof for Theorem 4: by replacing the $p_i(z_i)$ and $e = -1$ in the profit function the first derivative is:

$$\frac{\mathrm{dE}[\pi(\mathrm{p_i}(\mathrm{z_i}), \mathrm{z_i})]}{dz}$$

$$= -(m_i - v_i) + [1 - F(z_i)]\left[\frac{p_i^0 + Id_i(a_i + z_i) - \Theta(z_i)/2b_i}{1 + b_i Id_i} + s_i - v_i\right]$$

$$- 2b_i Id_i \left[\frac{a_i + z_i}{b_i} - \frac{p_i^0 + d_i I(a_i + z_i) - \Theta(z_i)/2b_i}{1 + b_i Id_i}\right]$$

To find the extremum points of $\mathrm{E}[\pi(\mathrm{p_i}(\mathrm{z_i}), \mathrm{z_i})]$, we need to find the zeros of the first derivative, assuming $R(z_i) \equiv \frac{\partial \mathrm{E}[\pi(\mathrm{p_i}(\mathrm{z_i}), \mathrm{z_i})]}{\partial z_i}$:

$$-\frac{f(z_i)}{2b_i(1 + b_i Id_i)}[2b_i[p^0 + d_i I(a_i + z_i) + (1 + b_i Id_i)(s_i - v_i)] - \Theta(z_i)]$$

$$+ \frac{[1 - F(z_i)]}{2b_i(1 + b_i Id_i)}[2b_i d_i I + 1 - F(z_i))]$$

$$- \frac{2b_i Id_i}{2b_i(1 + b_i Id_i)}\{2 - [1 - F(z_i)]\}$$

$$-\frac{f(z_i)}{2b_i(1 + b_i Id_i)}\left[2b_i[p^0 + Id_i(a_i + z_i) + (1 + b_i Id_i)(s_i - v_i)] - \Theta(z_i)\right.$$

$$\left. - \frac{[1 - F(z_i)]}{f(z_i)}[2b_i Id_i + 1 - F(z_i))]\right] - \frac{2b_i Id_i}{2b_i(1 + b_i Id_i)}\{2 - [1 - F(z_i)]\}$$

$$\frac{dR(z_I)}{dz_i} = \frac{d}{dz_i}\left[\frac{\mathrm{dE}[\Pi(\mathrm{p_i}(\mathrm{z_i}),\mathrm{z_i})]}{dz_i}\right]$$

$$= -\frac{f(z_i)}{2b_i(1+b_iId_i)}\Bigg[2b_i[p^0 + Id_i(a_i+z_i) + (1+b_iId_i)(s_i-v_i)]$$

$$-\Theta(z_i) - \frac{2b_iId_i + [1-F(z_i)]}{r(z_i)}\Bigg] - \frac{2b_iId_i}{2b_i(1+b_iId_i)}\{2-[1-F(z_i)]\}$$

$$= -\frac{f(z_i)}{2b_i(1+b_iId_i)}\Bigg\{2b_i\Bigg[p^0 + Id_i(a_i+z_i) + (1+b_iId_i)(s_i-v_i)$$

$$+\frac{2Id_i}{f(z_i)}\Bigg] - \Theta(z_i) - \frac{4b_iId_i + [1-F(z_i)]}{r(z_i)}\Bigg\}$$

Where $r(z_i) = f(z)/[1-F(z)]$ is hazard rate. The second derivative of $R(z)$ is:

$$\frac{d^2R(z_i)}{dz_i^2} = \left[\frac{dR(z_i)/dz_i}{f(z_i)}\right]\frac{df(z_i)}{dz_i}$$

$$-\frac{f(z_i)}{2b_i(1+b_iId_i)}\Bigg\{2b_i\Bigg[Id_i - \frac{2Id_i.\,df(z_i)/dz_i}{f(z_i)^2}\Bigg] + [1-F(z_i)] + \frac{f(z_i)}{r(z_i)}$$

$$+\frac{\big[4b_iId_i + [1-F(z_i)]\big].\,dr(z_i)/dz_i}{r(z_i)^2}\Bigg\} \Rightarrow \left.\frac{d^2R(z_i)}{dz_i^2}\right|_{dR(z_i)/dz_i=0}$$

$$= -\frac{f(z_i)}{2b_i(1+b_iId_i)r(z_i)^2[1-F(z_i)]}\Bigg\{2b_iId_i\Bigg[f(z_i).\,r(z_i)$$

$$-\frac{2.\,d_if(z_i)/dz_i}{[1-F(z_i)]}\Bigg] + 2f(z_i)^2$$

$$+[1-F(z_i)]\big[4b_iId_i + [1-F(z_i)]\big].\,dr(z_i)/dz_i\Bigg\}$$

If the following condition is satisfied, it implies that $R(z_i) \equiv \frac{\partial \mathrm{E}[\pi(\mathrm{p_i}(\mathrm{z_i}),\mathrm{z_i})]}{\partial z_i}$ is always concave and thus has at most two roots:

$$2b_iId_i\left[f(z_i).r(z_i) - \frac{2.d_if(z_i)/dz_i}{[1-F(z_i)]}\right] + 2f(z_i)^2$$

$$+ [1-F(z_i)]\big[4b_iId_i + [1-F(z_i)]\big].dr(z_i)/dz_i > 0$$

Additionally, if $R(B) = -(m_i - v_i) - 2b_iId_i\left[\frac{a_i+B_i}{b_i} - \frac{p_i^0+d_iI(a_i+B)-0/2b_i}{1+b_iId_i}\right] = -(m_i - v_i) - 2Id_i\left[\frac{a_i+B-b_ip_i^0}{1+b_IId_i}\right] < 0$ then $R(z_i)$ has either one root which indicates that there is a change of sign from positive to negative and thus the root is a local maximum for $E[\pi(p_i(z_i), z_i)]$. Even if $R(z_i)$ has two roots the larger one is a local maximum and the smaller one is a local minimum for $E[\pi(p_i(z_i), z_i)]$. Also, assuming $R(z_i)$ is concave, a sufficient condition for $E[\pi(p_i(z_i), z_i)]$ to have one root is that $R(A) > 0$:

$$(m_i - v_i) + [1-F(z_i)]\left[\frac{p_i^0 + Id_i(a_i+z_i) - \Theta(z_i)/2b_i}{1+b_iId_i} + s_i - v_i\right]$$

$$- 2b_iId_i\left[\frac{a_i+z_i}{b_i} - \frac{p_i^0 + d_iI(a_i+z_i) - \Theta(z_i)/2b_i}{1+b_iId_i}\right]$$

$$R(A) = -(m_i - v_i) + \ [1 - F(A)]\left[\frac{p_i^0 + Id_i(a_i + A) - \Theta(A)/2b_i}{1 + b_i Id_i} + s_i - v_i\right]$$

$$- 2b_i Id_i\left[\frac{a_i + A}{b_i} - \frac{p_i^0 + Id_i(a_i + A) - \Theta(A)/2b_i}{1 + b_i Id_i}\right]$$

$$= -(m_i - v_i) + \left[\frac{p_i^0 + Id_i(a_i + A) - (\mu - A)/2b_i}{1 + b_i Id_i} + s_i - v_i\right]$$

$$- 2b_i Id_i\left[\frac{(1 + b_i Id_i)(a_i + A) - b_i p_i^0 - b_i Id_i(a_i + A) - (\mu - A)/2}{b_i(1 + b_i Id_i)}\right]$$

$$= -(m_i - s_i) + \left[\frac{b_i p_i^{0^0} + b_i Id_i(a_i + A) - (\mu - A)/2}{b_i(1 + b_i Id_i)}\right]$$

$$- 2b_i Id_i\left[\frac{(a_i + A) - b_i p_i^0 - (\mu - A)/2}{b_i(1 + b_i Id_i)}\right]$$

$$= -(m_i - s_i)$$

$$+ \ \left\{\frac{[p_i^0(2b_i Id_i + 1) - Id_i(a_i + A) + (2b_i Id_i - 1)\,(\mu - A)/2b_i]}{(1 + b_i Id_i)}\right\}$$

We can simplify it by looking at $(1 + \mathrm{b_i Id_i})R(A)$:

$(1 + \mathrm{b_i Id_i})R(A) = \ -(1 + b_i Id_i)(m_i - s_i) + p_i^0(2b_i Id_i + 1) - Id_i(a_i + A) + (2b_i Id_i - 1)\,(\mu - A)/2b_i$ ■

APPENDIX VII: OPTIMAL ORDER AND PRICING POLICY IN THE GP PROBLEM WITH ASYMMETRIC RETAILERS

Proof for Theorem 5: Identifying the values of z_i that maximize expected profit is equivalent to finding the roots of $R_i(z_i)$:

$$\frac{dR_i(z_i)}{dz_i} = -\frac{f(z_i)}{2(b_i+\gamma_i)}\left[2(b_i+\gamma_i)(p_i^0+s_i-v_i) - \Theta(z_i) - \frac{\left(1-F(z_i)\right)^2}{f(z_i)}\right]$$

Which can be simplified by substituting $\frac{f(.)}{1-F(.)}$ with hazard rate, $r(.)$ (Barlow and Proschan 1975):

$$\frac{dR_i(z_i)}{dz_i} = -\frac{f(z_i)}{2(b_i+\gamma_i)}\left[2(b_i+\gamma_i)(p_i^0+s_i-v_i) - \Theta(z_i) - \frac{1-F(z_i)}{r(z_i)}\right]$$

To analyze the shape of the function the second derivative of $R_i(z_i)$ needs to be analyzed:

$$\frac{d^2R_i(z_i)}{dz_i^2} = -\frac{df(z_i)}{dz_i}.\frac{1}{2(b_i+\gamma_i)}\left[2(b_i+\gamma_i)(p_i^0+s_i-v_i) - \Theta(z_i) - \frac{\left(1-F(z_i)\right)^2}{f(z_i)}\right] - \frac{f(z_i)}{2(b_i+\gamma_i)}\left[\left(1-F(z_i)\right) + \frac{f(z_i)}{r(z_i)} + \frac{(1-F(z_i))(drz_i/dz_i)}{r(z_i)^2}\right]$$

Which can be re-written as:

$$\frac{d^2R_i(z_i)}{dz_i^2} = \frac{df(z_i)}{dz_i}.\left[\frac{dR_i(z_i)/dz_i}{f(z_i)}\right] - \frac{f(z_i)}{2(b_i+\gamma_i)}\left[\left(1-F(z_i)\right) + \frac{f(z_i)}{r(z_i)} + \frac{[1-F(z_i)](drz_i/dz_i)}{r(z_i)^2}\right]$$

At the extremum points of $R_i(z_i)$:

$$\frac{d^2R_i(z_i)}{dz_i^2} = -\frac{f(z_i)[1-F(z_i)]}{2(b_i+\gamma_i)r(z_i)^2}\left[2r(z_i)^2+\frac{dr(z_i)}{dz_i}\right]$$

The above expression follows that if $2r(z_i)^2+\frac{dr(z_i)}{dz_i}>0$, it implies that $R_i(z_i)$ is concave in all extremum points which follows that it is either monotonic or unimodal with a maximum which means that $R_i(z_i)$ has at most two roots. Considering the range of $z_i \in [A,B]$; we know that $R_i(B) = -(c_i - v_i) < 0$, so if $R_i(z_i)$ has one root then there should be a change of sign and the root corresponds to a local maximum of $E\left[\pi_i\left(z_i, p_i(z_i), p_j(z_j)\right)\right]$, if $R_i(z_i)$ has two roots the larger one is a local maximum and the smaller refers to a local minimum. In either case, $E\left[\pi_i\left(z_i, p_i(z_i), p_j(z_j)\right)\right]$ has only one local maximum which is the closest point to B that satisfies $R_i(z_i) = 0$. The condition for unimodality of $E\left[\pi_i\left(z_i, p_i(z_i), p_j(z_j)\right)\right]$ is $R_i(A) > 0$ or $2b_iR(A) > 0$, where:

$$2(b_i+\gamma_i)R_i(A) \\ = \mathrm{a_i} - (\mathrm{b_i}+\gamma_i)(\mathrm{c_i} - 2\mathrm{s_i}) + \mathrm{A} + p_i^s$$

Where $p_i^s = \frac{\gamma_i}{I-1}\sum_{\substack{p_j=1 \\ p_i \neq p_j}}^{n} p_j$; the value of p_i^s is not known before solving the problem, but we know that it is non-negative; so $2(b_i+\gamma_i)R_i(A)^- = a_i - (b_i+\gamma_i)(c_i - 2s_i) + A$ is a lower bound for $2(b_i+\gamma_i)R_i(A)$ and we can focus on this part to find out the uniqueness of the root z_i.

■

APPENDIX VIII: OPTIMAL RISKLESS PROFIT FOR ASYMMETRIC RETAILERS WITH 4 & 5 RETAILERS

Optimal Riskless profit for 4 & 5 retailer problem. The optimal riskless price p_i^0 is found by solving the system of equation described in 4.12; for 4-retailers' problem p_i^0 can be found as following for retailer 1:

$$p_1^0 =$$

$$-\frac{\begin{array}{c}6\gamma_1\gamma_2 b_3 s_4 + 6\gamma_1\gamma_2 b_4 s_3 + 6\gamma_1\gamma_3 b_2 s_4 + 6\gamma_1\gamma_3 b_4 s_2 + 6\gamma_1\gamma_4 b_2 s_3 \\ +6\gamma_1\gamma_4 b_3 s_2 - 6\gamma_2\gamma_3 b_4 s_1 \\ -6\gamma_2\gamma_4 b_3 s_1 - 6\gamma_3\gamma_4 b_2 s_1 + 36\gamma_1 b_2 b_3 s_4 + 36\gamma_1 b_2 b_4 s_3 + 36\gamma_1 b_3 b_4 s_2 + 216 b_2 b_3 b_4 s_1 \\ +\gamma_1\gamma_2\gamma_3 s_4 + \gamma_1\gamma_2\gamma_4 s_3 + \gamma_1\gamma_3\gamma_4 s_2 - 2\gamma_2\gamma_3\gamma_4 s_1\end{array}}{\begin{array}{c}12\gamma_1\gamma_2 b_3 b_4 + 12\gamma_1\gamma_3 b_2 b_4 + 12\gamma_1\gamma_4 b_2 b_3 + 12\gamma_2\gamma_3 b_1 b_4 + 12\gamma_2\gamma_4 b_1 b_3 + 12\gamma_3\gamma_4 b_1 b_2 \\ -432 b_1 b_2 b_3 b_4 + \gamma_1\gamma_2\gamma_3\gamma_4 + 4\gamma_1\gamma_2\gamma_3 b_4 + 4\gamma_1\gamma_2\gamma_4 b_3 + 4\gamma_1\gamma_3\gamma_4 b_2 + 4\gamma_2\gamma_3\gamma_4 b_1\end{array}}$$

The optimal riskless price p_i^0 for retailer 1 in the 5-retailer problem is as the following:

$$p_1^0 =$$

$$
\frac{\begin{gathered}
8s_4\gamma_2\gamma_1b_3\gamma_5 + 8s_4\gamma_3\gamma_2\gamma_1b_5 + 64s_4\gamma_2\gamma_1b_3b_5 + 64s_4\gamma_5\gamma_1b_3b_2 \\
+64s_4\gamma_3\gamma_1b_2b_5 + 512b_5\gamma_1b_3b_4s_2 \\
-64\gamma_2s_1b_3b_4\gamma_5 + 4096b_2s_1b_3b_4b_5 - 64b_2s_1\gamma_3\gamma_4b_5 \\
-64b_2s_1\gamma_3b_4\gamma_5 - 64b_2s_1b_3\gamma_4\gamma_5 \\
-64\gamma_2s_1\gamma_3b_4b_5 - 16\gamma_2s_1\gamma_3\gamma_4b_5 - 16\gamma_2s_1\gamma_3b_4\gamma_5 - 64\gamma_2s_1b_3\gamma_4b_5 \\
-16\gamma_2s_1b_3\gamma_4\gamma_5 + 8s_5\gamma_4\gamma_2\gamma_1b_3 \\
+64s_5\gamma_3\gamma_1b_2b_4 + 64s_5\gamma_4\gamma_1b_3b_2 + s_5\gamma_4\gamma_3\gamma_2\gamma_1 + 8s_5\gamma_4\gamma_3\gamma_1b_2 \\
+64s_5\gamma_2\gamma_1b_3b_4 + 8s_5\gamma_3\gamma_2\gamma_1b_4 \\
-3\gamma_2s_1\gamma_3\gamma_4\gamma_5 + 8b_2\gamma_3\gamma_1s_4\gamma_5 + \gamma_2\gamma_3\gamma_1s_4\gamma_5 + 8s_3\gamma_4\gamma_1b_2\gamma_5 \\
+64s_3\gamma_2\gamma_1b_4b_5 + 64s_3\gamma_5\gamma_1b_4b_2 \\
+64s_3\gamma_4\gamma_1b_2b_5 - 16b_2s_1\gamma_3\gamma_4\gamma_5 + 512s_5\gamma_1b_3b_4b_2 + 8s_3\gamma_4\gamma_2\gamma_1b_5 \\
+8s_3\gamma_2\gamma_1b_4\gamma_5 + 64\gamma_5\gamma_1b_3b_4s_2 \\
+s_3\gamma_4\gamma_2\gamma_1\gamma_5 + \gamma_4\gamma_3\gamma_1s_2\gamma_5 + 8\gamma_4\gamma_3\gamma_1s_2b_5 + 8\gamma_3\gamma_1s_2b_4\gamma_5 \\
+64\gamma_3\gamma_1s_2b_4b_5 + 512b_5s_4\gamma_1b_3b_2 \\
+8\gamma_4\gamma_1b_3s_2\gamma_5 + 64\gamma_4\gamma_1b_3s_2b_5 + 512b_5s_3\gamma_1b_4b_2
\end{gathered}}{\begin{gathered}
-128\gamma_3b_1\gamma_2b_4b_5 - 32\gamma_3b_1\gamma_2b_4\gamma_5 - 32\gamma_3\gamma_1b_2b_4\gamma_5 - 6\gamma_4\gamma_2\gamma_1b_3\gamma_5 - 32\gamma_4\gamma_2\gamma_1b_3b_5 \\
-128\gamma_4\gamma_3b_1b_2b_5 - 128\gamma_5\gamma_3b_1b_4b_2 - 32\gamma_4\gamma_3b_1\gamma_2b_5 - \\
128\gamma_3\gamma_1b_2b_4b_5 - 128\gamma_4\gamma_1b_3b_2b_5 \\
-128\gamma_5\gamma_1b_3b_4b_2 - 6\gamma_4\gamma_3\gamma_2\gamma_1b_5 - 6\gamma_4\gamma_3\gamma_1b_2\gamma_5 - 32\gamma_5\gamma_4\gamma_3b_1b_2 - 32\gamma_4\gamma_3\gamma_1b_2b_5 \\
-128\gamma_5b_1b_3b_4\gamma_2 - 6\gamma_5\gamma_3\gamma_2\gamma_1b_4 - 32\gamma_4\gamma_1b_3b_2\gamma_5 - 128\gamma_2\gamma_1b_3b_4b_5 + 8192b_1b_2b_3b_4b_5 \\
-32\gamma_3\gamma_2\gamma_1b_4b_5 - \gamma_4\gamma_3\gamma_2\gamma_1\gamma_5 - 6\gamma_4\gamma_3b_1\gamma_2\gamma_5 - 128\gamma_5\gamma_4b_1b_2b_3 - 32\gamma_2\gamma_1b_3b_4\gamma_5 \\
-32\gamma_4b_1b_3\gamma_2\gamma_5 - 128\gamma_4b_2b_3\gamma_2b_5
\end{gathered}}
$$

Comparing p_i^0 between 4 and 5 retailer case, one can notice how much the result grows by only adding one retailer to the problem. ■