MATHEMATICS READINESS OF ENTERING COLLEGE STUDENTS

by

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A dissertation submitted to the faculty of
The University of North Carolina at Charlotte
in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in
Curriculum and Instruction

Charlotte

2018

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ABSTRACT

CATHERINE EDILIA BLAT. Mathematics readiness of entering college students. (Under the direction of DR. DAVID K. PUGALEE)

The purpose of this descriptive research study was to identify key mathematics competencies first-time college students need to succeed in entry-level college mathematics courses. The study was conducted at a large, urban public, research university where between 20-31 percent of its new incoming freshmen were placed in developmental mathematics from 2010 through 2013. In the initial part of the research (Phase 1), a pilot study was conducted utilizing historical student data from mathematics placement tests (MPT). Participants in the pilot were new entering freshmen completing the MPT during student orientation in the summer preceding their entrance in the university. Students' performance on the MPT test questions were used to identify mathematical competencies differentiating students' placements in the various entry-level mathematics courses, hence depicting their level of mathematics readiness. Demographic data and incoming characteristics were also considered. Pilot study data demonstrated deficiencies in questions requiring operations with rational numbers and rational expressions. On average, less than 50 percent of the students placing in Developmental Mathematics, College Algebra, or Precalculus answered those questions correctly. A follow-up study was conducted to confirm the results obtained in Phase 1 through observations and artifacts examination of an entry-level mathematics class. Results from Phase 2 confirmed the results from Phase 1 and identified operations with negative numbers as an important concept affecting student preparedness. This study extends the

mathematics education research by providing specific mathematics competencies affecting students' mathematics preparedness entering a 4-year institution.

DEDICATION

First, I want to thank God for giving me the strength and talent to fulfill my goal to earn a doctorate. He was my stronghold in times of trouble.

This dissertation is dedicated to my family and friends without their support I would have never been able to complete this degree.

To my beloved husband, Enrique, who agreed to put our lives on hold while I worked on this program. Thanks for your continuous love, support, and encouragement throughout this journey. I could not have made it through without the many meals and cups of coffee you fixed for me and for always being there to support me when I had a meltdown.

To my children Irene and Daniel who allowed me to take time away from them to work in achieving this goal. Thanks for your love, your understanding and for your words of encouragement and reassurance throughout the years.

To the staff of the University Center for Academic Excellence who stood by me and pushed me to continue when my energy failed me. In many ways, you have been part of my journey.

Lastly, I want to thank my colleagues and friends at the University of North Carolina at Charlotte. I appreciate your support and encouragement throughout this process.

ACKNOWLEDGEMENTS

I would like to express my sincere appreciation to my committee chair, Dr. David Pugalee for his guidance and encouragement. Dr. Pugalee went beyond of what was expected to help me succeed in completing this dissertation. He took an interest in me and provided me with the guidance and support I needed to complete this process.

I also want to thank the members of my committee, Dr. Michelle Stephan and Dr. Chuang Wang. Thanks for your support, advice and invaluable input that helped me grow in my knowledge and research skills.

I am indebted to my colleague Dr. Kim Buch for agreeing to serve in my committee as the Graduate School representative. Thanks for taking the time to review my dissertation and for providing me with feedback and support. I appreciate your willingness to be an active member of the committee.

I would like to add a special acknowledgement to Dr. Adalira Saénz-Ludlow who allowed me to conduct research in her class. Thanks for your willingness to open your class to me and for sharing with me your extensive experience. I appreciate your support and encouragement throughout the years.

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LIST OF ABBREVIATIONS

ACT	American College Testing
AP	Advanced Placement
BLS	Bureau of Labor Statistics
CBMS	Conference Board of the Mathematical Sciences
CCSS	Common Core State Standards
CD	Commercially Developed curriculum
CFA	Confirmatory Factor Analysis
Dev. Math	Developmental Mathematics
DWLS	Diagonally Weighted Least Square
ECHS	Early College High School
EFA	Exploratory Factor Analysis
GPA	Grade Point Average
HS GPA	High School Grade Point Average
IRB	Institutional Review Board
K-12	Kindergarten through twelfth grade
Math SAT	Mathematics SAT Score
MPT	Mathematics Placement Test
NAEP	National Assessment of Educational Progress
NCES	National Center for Education Statistics
NCTM	National Council of Teachers of Mathematics
NSF	National Science Foundation

QN	Measure for negative number operation
QR	Measure for rational number operations
QS	Expression simplification measure
QV	Equation solving measures
STEM	Science, Technology, Engineering, and Mathematics
TIMSS	Trends in International Mathematics and Science Study
UCSMP	University of Chicago School Mathematics Project
SES	Socio-Economic Status

CHAPTER 1: INTRODUCTION

In recent years, a higher percentage of students are graduating from high school with more mathematics courses than the number required to graduate. The number of students taking Algebra II grew from 40 percent in 1982 to 62 percent in 1998 (Barth, 2002). In addition, more students are taking precalculus or calculus in high school. The number of high school graduates who completed precalculus or calculus tripled from 1982 to 2004 (Dalton, 2007). Nonetheless, the mathematics competency of high school graduates has not improved significantly. According to 2015 *Nation's Report Card* scores in Mathematics and Reading at Grade 12 (National Center for Education Statistics, 2015), only 25 percent of the students in grade 12 performed at the *Proficient* level or above in mathematics and there has not been a significant change in this value since 2005. In fact, fewer 12^{th} graders performed at the *Proficient* level in 2015 than in 2013 and the percentage of 12^{th} graders performing below the *Basic* level was higher in 2015 than in 2013.

Higher education institutions offer remedial education to reduce the gap between mathematically prepared and unprepared students. The percentage of students needing to take remedial mathematics courses in college has been increasing. The National Council of Teachers of Mathematics (NCTM) Research Committee (2011) reports that in the year 2000, 22 percent of first-year students in two-year and four-year institutions were placed in remedial mathematics courses compared to 11 percent of the students who were 12^{th} graders in 1992. Underrepresented minorities, students from urban high schools and low socio-economic homes are disproportionately represented in remedial courses (Adelman, 2004b).

With the increase of students seeking college degrees, the number of students requiring remedial education is also expected to increase (Xu, Hartman, Uribe, & Mencke, 2001). Remedial education is costly. A study conducted by the Alliance for Excellent Education (2011) found that the cost of remediation for public institutions for students enrolled in the 2007-2008 academic year was $3.6 billion. Jimenez, Sargrad, Morales, and Thompson (2016) estimated that remedial courses in 2014 costed students and their families $1.3 billion.

Graduation and persistence are also a concern for students in remedial education. Students taking remedial courses are less likely to graduate (Conley, 2007). Only 27 percent of students enrolled in two or fewer remedial mathematics courses earned a Bachelor's degree after eight years from high school graduation compared to 58 percent of the students not enrolled in postsecondary remedial courses (U.S. Department of Education National Center for Education Statistics, 2004). Dropout rates among community college students registered in remedial classes significantly increases for students who need remediation in three or more courses (Deil-Amen & Rosenbaum, 2009).

Completing college-level mathematics courses influences students' college completion. Budny, LeBold, and Bjedov (1998) found that performance in mathematics courses affected retention and graduation in engineering. Entry-level mathematics courses have become obstacles and dampers for many students. A study conducted by

Adelman (2004a) among students taking college courses between 1992 and 2000 showed that a high proportion of withdrawals, repeats, and failures were concentrated in college mathematics courses. College Algebra, Precalculus, and Calculus were included in the top 20 courses students withdrew, repeated, or failed (Adelman, 2004b). Lack of success in entry-level mathematical courses can result in students changing majors or leaving college.

A study conducted by Lee (2012) using mathematical achievement data from preschool to higher education suggested that fulfilling national and state mathematics proficiency requirements yield different results with regards to higher education degree attainment. Results demonstrated differences between actual and desirable mathematics achievement levels for college readiness at the national level. The required mathematics achievement to complete a college degree varies depending on whether the degree is from a two-year or four-year institution. Students meeting the average state's mathematics proficiency standard were successful at completing an associate degree in a two-year college. To complete a bachelor's degree in a four-year institution, students need to meet or exceed the "high" level in the international test, Trends in International Mathematics and Science Study (TIMSS) or the *Proficient* level for the national test, National Assessment of Educational Progress (NAEP). Lee (2012) suggests that there is a misalignment between the K-12 curriculum performance standards and the college mathematics readiness criteria. It would be desirable to have students be mathematically prepared to meet the national standards which will allow them to pursue or not the college degree of their choosing.

Statement of the Problem

Conley (2008) defines college readiness as "the level of preparation a student needs in order to enroll and succeed, without remediation in a credit-bearing general education [mathematics] course at a postsecondary institution that offers a baccalaureate degree" (p. 4). The American College Testing (ACT) college readiness assessment defines students as Mathematics College Ready if they have a 50 percent probability of earning a grade of B or better and a 75 percent probability of obtaining at least a grade of C in College Algebra (ACT, 2014). *The Condition of College & Career Readiness 2016* report (ACT, 2016) indicated that only 41 percent of the 2016 high school graduates were Mathematics College ready. In *What's Wrong with College Algebra* (2008), Gordon reports that only half of all students successfully complete college algebra courses.

To identify the factors predicting high college academic achievement in the sciences, Benbow & Arjmand (1990) conducted a longitudinal study among gifted students. Results from their research also found that precollege curricula in mathematics and sciences, family background and educational encouragement, attitudes towards mathematics and sciences, and ability were predictors of high achievement in college mathematics and sciences even for high achieving students. An additional significant finding of this study is that there were differences in college performance due to gender and that the educational aspirations of women declined and their attrition increased when they reached college.

The gap existing between high school and college mathematics preparation merits an analysis of the reasons why students are not ready for college mathematics.

Understanding the differences between students placing into college mathematics and students needing remediation will help identify the specific mathematical concepts all students need to master in order to meet the demands of college. A study conducted for the Department of Education Office of Vocational and Adult Education (Golfin, Jordan, Hull, & Ruffin, 2005) found that published literature does not reveal a consistent definition of mathematics standards required for college-level mathematics. According to this study, students in college-level mathematics need to have a foundation in geometry, trigonometry, algebra I, algebra II and basic statistics. Problem solving, critical thinking, and the ability to communicate mathematically were also identified as skills to succeed in college-level mathematics.

Mathematical competence can lead to higher paying jobs. A report by the Pew Research Center, *7 facts about the STEM workforce* (Graf, Fry, & Funk, 2018), indicates that full time college educated workers who have a Science, Technology, Engineering, and Mathematics (STEM) major and work in a STEM field earn $80,011 compared to $60,828 for other majors. STEM majors who work in other fields also earn more than non-STEM majors do, $71,000 vs. $60,000. Currently, there are outstanding career opportunities for people with training in mathematically intensive fields. Data-driven science is changing the processes of innovation and learning in this century. The focus on big data calls for college graduates better prepared for jobs requiring computational and statistical skills (Saxe & Braddy, 2015). The *Bureau of Labor Statistics* report predicts that the number of jobs requiring college degrees will increase by 16.5 percent from 2010 to 2020 (Bureau of Labor Statistics (BLS), 2012). It is also predicted that by 2020, 65 percent of all jobs and 92 percent of the STEM jobs will require post-secondary education and training (Achieve, 2017). In addition to career and employment issues, society benefits from college graduates who are trained in higher mathematics, who can apply their mathematics understanding in their lives and their communities.

The focus of published literature seems to be on the sociological factors affecting college readiness and the impact of remedial education while neglecting to provide a clear delineation of critical mathematical competencies needed to meet the requirements of college-level mathematics (Atuahene & Russell, 2016). Institutions have different expectations of what does it mean to do college-level work (Attewell, Lavin, Domina, & Levey, 2006, p. 887). Among the measures of college readiness are high school courses taken and their level of difficulty as well as test scores in state tests, Advanced Placement (AP) course tests, and admission tests (ACT & SAT) (Conley, 2008). To succeed in college, students need to be able to complete college-level course. Nevertheless, the essential mathematical competencies required to succeed in college-level mathematics are not specified.

Purpose of the Study

This descriptive research study was conducted to identify key mathematics competencies needed by first-time college students to be successful in entry-level college mathematics courses. Furthermore, this study provides a deeper understanding of first-time college students' comprehension of key mathematical competencies. The study was conducted at a large, urban public, research university where between 500 and 900 new freshmen per year were placed in developmental mathematics from 2010 through 2013.

This represents between 20-31 percent of its new incoming freshmen. For the College of Engineering, approximately 30 percent of the new freshmen are not eligible to register for their first mathematics course in the curriculum, Calculus I (Tolley, Blat, McDaniel, Blackmon, & Royster, 2012). Students not meeting the requirements for college-level mathematics are required to complete a developmental mathematics course before they can proceed to their entry-level mathematics course. Enrolling in remedial education lengthens the students' time in college and increases their cost of earning an education.

This explanatory sequential mixed-methods research design includes both quantitative and qualitative data. In the initial quantitative part of the study (Phase 1), a pilot study was conducted utilizing historical student data from mathematics placement tests (MPT) of entering freshmen college students between the fall of 2010 and the fall of 2013. Students' performance on the MPT test questions were used to identify mathematical competencies differentiating students' placements in the various entry-level mathematics courses, hence depicting their level of mathematics readiness. Students placing in developmental mathematics are considered the least prepared with students placing in Calculus I are deemed mathematics college ready. Participants in the pilot were new entering freshmen completing the MPT during student orientation in the summer preceding their entrance in the university. The pilot study data demonstrated deficiencies in questions requiring operations with rational numbers and rational expressions including rationalizing a denominator, simplifying a complex fraction, simplifying a rational function, solving a rational equation, multiplying rational expressions, adding/subtracting rational functions, and ratio and proportion operations. For these questions, on average, less than 50 percent of the students placing in Developmental Mathematics, College Algebra, or Precalculus answered the questions correctly. Other detailed results of the pilot will be discussed later. Based on the pilot data, a follow-up study was conducted. Additional data analysis was conducted on the data collected in Phase 1 and performance in key mathematical competencies by students in an entry-level mathematics course were evaluated to confirm the results obtained in the pilot (Phase 2). Students' responses to homework and test questions containing rational number operations and other key mathematical competencies were examined. The analysis was supplemented with classroom and help sessions' observations to gain additional understanding of students' comprehension of operations with rational numbers and other identified mathematical competencies. Data from the two phases of this study were used to answer the following questions.

Research Questions

1. What mathematics competencies characterize students at different levels of mathematics college readiness?
2. What demographic factors and incoming data (Mathematics SAT score, intended major, high school GPA) characterize students at different levels of mathematics college readiness?
3. What is the level of understanding of key mathematics competencies for incoming students placed in an entry-level mathematics course?

To determine the mathematics course for which students were eligible to register, a mathematics placement test developed in-house is given to new students during student orientation or before registering for classes. Placement data to conduct the pilot were obtained from mathematics placement tests results for fall 2010, 2011, 2012, and 2013. As part of Phase 1 of the study, students' performance on each question of the mathematics placement test was evaluated to determine the mathematics competencies most likely to affect their college-level mathematics placement. In this context, mathematics readiness is determined by the students' placement in the various mathematics entry-level courses. Students placing in developmental mathematics are considered the least prepared and students placing in Calculus I are assumed to be the most prepared.

Significance of the Study

This study examined the mathematics readiness of first-time college students as determined by their understanding of fundamental mathematical concepts. It will fill a void in the existing literature. Most published research focuses on various factors affecting college mathematics readiness and on the results of mathematics remediation but not on the degree of understanding fundamental mathematical concepts. Demographics and other background characteristics of students in remedial courses have been the focus of several research studies. For example, Deil-Amen and Rosenbaum (2009) identified race, parents' college education, and being full time versus part-time as factors affecting persistence of community college students registered in remedial courses. Benbow and Arjmand (1990) found that even among gifted students, there were differences in mathematics achievement in college related to gender and that the educational aspirations of women declined and their attrition increased when they reached college.

Studies have also been conducted on enrollment in mathematics remedial courses and its impact on mathematics achievement and college completion. Adelman and Attewell reported that compared to other subjects, mathematics remediation is the most needed by students (Adelman, 2004b; Attewell et al., 2006). Chen and Simone (2016) indicated that among the 2003–04 beginning postsecondary students who first enrolled in public two-year and four-year institutions, 59.3 percent and 32.6 percent respectively took a remedial course in mathematics compared to 28.1 and 10.8 percent respectively who took an English remedial course.

Various studies examined the effect of mathematics remediation on college success. Registering for remedial courses significantly increases dropout rates (Deil-Amen & Rosenbaum, 2009). Enrollment in remedial courses also lowers four-year institution students' chances of graduation (Adams et al., 2012; Attewell et al., 2006). In four-year public colleges, first-time freshmen registered in remedial mathematics are more likely to dropout from school or to transfer to a two-year college compared to students not in remedial mathematics (Bettinger & Long, 2004; Conley, 2008). Many students enrolled in remedial courses withdraw or do not attend class since in most institutions these are non-credit courses. Only 33 percent of students taking remedial mathematics courses pass the class (Bailey, Jeong, & Cho, 2010).

Bahr (2007, 2008) discussed the impact of the level of mathematics remediation needed to achieve college-level mathematics readiness. Bahr reported that among community college students, those starting at the lowest level of remedial mathematics, basic arithmetic, have a very low probability of earning a passing grade in a college-level mathematics class. However, students who successfully completing remedial courses can pass college-level mathematics have the same rate of graduation/transfer to a four-year institution as students who did need remedial education. Similarly, students in two-year colleges who successfully completed their remedial courses were more likely to graduate than students who never took remediation (Attewell et al., 2006).

As the previous studies show, mathematics preparedness is a complicated matter. College mathematics preparedness is influenced by gender, ethnicity, SES, mathematics courses taken in high school (Adams et al., 2012; Benbow & Arjmand, 1990; Long, Iatarola, & Conger, 2009), and type of institution (Attewell et al., 2006). To rely on community colleges to address the mathematics preparedness gap has proven helpful in some situations but retention and graduation are a concern (Aud et al., 2012; Provasnik & Planty, 2008). A more proactive approach is needed to address this situation. Little can be done to change students' demographics or courses taken before coming to the University. Knowledge on which specific mathematics competencies students are experiencing difficulties will assist in developing focused strategies to enhance student understanding of these key mathematics competencies. Targeted interventions can take place before students enter college or throughout their first semester to ensure that, students can perform in their entry-level mathematics course and in future coursework. Identifying weaknesses in key mathematics competencies and addressing them will assist in helping students to be prepared to enter the college in the appropriate mathematics course required by their area of study.

One implication of this study is to provide valuable information for aligning high school and college mathematics curricula and for developing effective strategies to close the mathematics preparedness gap through adaptive learning strategies, summer programs, tutoring, or instruction technology. By knowing the critical mathematics competencies affecting mathematics placement, students can be advised to take summer courses to enhance their preparation on those essential skills. In addition, results from this study will benefit mathematics instructors developing developmental education courses to make informed decisions on what specific content to include. Finally, K-12 mathematics instructors can be informed of key mathematics competencies students need to understand to place in their designated college entry-level mathematics course. K-12 instructors and administrators can make curriculum changes to ensure students are college ready when they graduate from high school.

This chapter described mathematics preparedness and the implications for students entering college. Lack of adequate mathematics preparedness results in students not being successful at completing entry-level mathematics courses. This may extend students time to graduation or prevent graduation. An alternative path is to take remedial courses to acquire the mathematics preparation to succeed in college-level mathematics. Remediation increases the cost of college for institutions and students, and it may have a negative impact on student progression towards their degree. Limited mathematics preparedness is prevalent among underrepresented minorities and students in lower socio-economic households who already experience academic challenges in their transition to college. A better approach may be to identify key mathematical competencies needed to succeed and provide efficient ways to reduce the mathematics preparedness gap between high school and college mathematics.

The remaining chapters are organized as follows. A literature review is presented in Chapter 2 that further describes the mathematics preparedness of entering students. This chapter will also provide details the challenges students faced in understanding rational number operations and the other identified key mathematical competencies. Chapter 3 will describe the methodology used to conduct this study. Quantitative methods were used to collect and analyze the data obtained in Phase 1 of the project. Qualitative data collection from observations and artifacts was used for the second phase of the study. Chapter 4 includes the results and data analysis. Finally, a discussion of the results, next steps and implication for future research are presented in Chapter 5.

CHAPTER 2: LITERATURE REVIEW

Mathematics Readiness

There are several definitions of mathematical readiness. High school graduation is not accepted as evidence of mathematics preparation because of variations in rigor and course content existing among schools. As indicated in the introduction, Conley (2008) defines college readiness as "the level of preparation a student needs in order to enroll and succeed, without remediation in a credit-bearing general education [mathematics] course at a postsecondary institution that offers a baccalaureate degree" (p. 4). Similarly, The *Closing the Expectations Gap report* (2015) states that "college readiness means that a high school graduate has the knowledge and skills necessary to qualify for and succeed in entry-level, credit-bearing postsecondary course work without the need for remediation" (Achieve, 2015, p. 6). Several measures are used to assess college readiness including transcript analysis (Adelman, 2006), standardized test scores (ACT, 2016), and enrollment in remedial courses.

The National Assessment of Educational Progress (NAEP) provides results on subject matter achievement for grades 4, 8 and 12 including mathematics. It measures students' knowledge and skills in mathematics and students ability to apply mathematical knowledge in problem solving. NAEP evaluates performance in number properties and operations; measurement; geometry; data analysis, statistics, and probability; and algebra. The achievement levels are Basic, Proficient, and Advanced (National Center for Education Statistics, 2015). In 2015, only 25 percent of 12^{th} grade students performed at or above the Proficient level in mathematics (National Center for Education Statistics, 2015).

The release of the report *A Nation at Risk: The Imperative for Educational Reform* released by the National Commission on Excellence in Education (1983b) resulted in five recommendations to improve education in the United States. Among the recommendations were to strengthen the high school graduation by requiring all students seeking a diploma to take four years of English, three years of mathematics, three years of social studies, and half a year of computer science. As a result, the percentage of students who took mathematics courses in high school increased from 1990 to 2009 with except Algebra I. This is likely because currently, many students complete Algebra I before high school (Aud et al., 2012). Early College High Schools (ECHS) is a program aiming at easing the transition from high school to college. The Bill & Melinda Gates Foundation (2009) supports this initiative. ECHSs are frequently opened in college campuses and target students from disadvantaged backgrounds. These schools allow students to enroll in classes that count toward both high school and college credit. Students graduate from high school with college credits and with a better understanding of what college-level courses demand (Le & Frankfort, 2011).

Steps are being taken in K-12 education to make sure all students graduate ready for college, work, and life. The Common Core State Standards (CCSS) initiative was launched in 2009 to ensure that all students are prepared for freshman-level courses, entry-level careers, and workforce training programs. As of August 2015, 42 states, the District of Columbia, four territories, and the Department of Defense Education Activity have adopted the CCSS in English language arts and mathematics (CCSS, 2017a). The CCSS for mathematics proposed shifts from previous standards in three areas.

1. Greater focus on fewer topics – Spend more time and energy in developing on key concepts for each grade. The desired outcome is that students will gain conceptual understanding, procedural skills, and the ability to apply concepts in and out of the classroom.
2. Coherence: Linking topics and thinking across grades – The CCSS standards are designed to connect concepts across grades. Students will build new understanding based on foundations built on previous grades. Topics are not presented in isolation, but connections are made to other mathematical concepts.
3. Rigor: Pursue with equal intensity conceptual understanding, procedural skills and fluency, and applicability. – Rigor means that students will acquire a deep understanding of mathematical concepts to use mathematical knowledge in all three approaches (CCSS, 2017b).

A 2010 report on the CCSS initiative compiled by ACT (2010) found that only 34 percent of the 11th graders tested were performing at the college level in the Number and Quantity category. This category involves arithmetic with polynomials and rational functions and reasoning with equations and inequalities, which is troublesome since this is one of the most fundamental mathematics categories in the Common Core Standards for 11th graders. Only one-third of the students tested met the college and career ready level. While these were preliminary results, the trends indicate that additional efforts are needed to ensure students graduate college and career ready without the need of remediation (ACT, 2010).

National standardized tests like the ACT and the SAT have become primary measures of mathematics readiness. ACT produces annual reports on college and career readiness. The most recent ACT report on *The Condition of College & Career Readiness 2017* indicated that only 41 percent of the 2017 high school graduates were College Mathematics ready (ACT, 2017). According to ACT, students meeting this benchmark have a 50 percent chance of earning a B or better and a 75 percent chance or higher of scoring a C or better in College Algebra. ACT College Readiness Benchmarks are obtained from analysis of data including first-year students' course grades and ACT Mathematics test score data. The total test score is based on 24 items in Pre-Algebra/Elementary Algebra; 18 items in Algebra/Coordinated Geometry; and 18 items in Plane Geometry/Trigonometry. From these data, predictive values of success in College Algebra, defined by course grade attainment, are determined (ACT, 2014).

As described in the *ACT Technical Manual* (ACT, 2014), the following items are included in each subject area:

a. Pre-Algebra. Items in this content area focus on operations using whole numbers, decimals, fractions, and integers; place value; square roots and approximations; the concept of exponents; scientific notation; factors; ratio, proportion, and percent; linear equations in one variable; absolute value and ordering numbers by value; elementary counting techniques and simple probability; data collection, representation, and interpretation; and understanding simple descriptive statistics.

b. Elementary Algebra. Items in this content area focus on properties of exponents and square roots, evaluation of algebraic expressions through substitution, using variables to express functional relationships, understanding algebraic operations, and the solution of quadratic equations by factoring.
c. Intermediate Algebra. Items in this content area focus on understanding of the quadratic formula, rational and radical expressions, absolute value equations and inequalities, sequences and patterns, systems of equations, quadratic inequalities, functions, modeling, matrices, roots of polynomials, and complex numbers.
d. Coordinate Geometry. Items in this content area focus on graphing and on the relations between equations and graphs, including points, lines, polynomials, circles, and other curves; graphing inequalities; slope; parallel and perpendicular lines; distance; midpoints; and conics.
e. Plane Geometry. Items in this content area focus on the properties and relations of plane figures, including angles and relations among perpendicular and parallel lines; properties of circles, triangles, rectangles, parallelograms, and trapezoids; transformations; the concept of proof and proof techniques; volume; and applications of geometry to three dimensions.
f. Trigonometry. Items in this content area focus on understanding trigonometric relations in right triangles; values and properties of trigonometric functions; graphing trigonometric functions; modeling using trigonometric functions; use of trigonometric identities; and solving trigonometric equations (ACT, 2014, p. 10).

Because the ACT is a commercially available test, the company does not disclose detailed results for each these sections but only aggregate mathematics results.

Remedial Education

Despite these K-12 initiatives, the number of students needing to take remedial mathematics or developmental mathematics courses in college continues to increase. The fall 2010 Conference Board of the Mathematical Sciences Survey (2013) indicates that 57 percent of students at two-year colleges and 23 percent at four-year institutions take at least one developmental mathematics course. The proportion of students needing remediation for mathematics is more extensive than for writing and reading. In fall 2000, 22 percent of the entering freshmen required remediation in mathematics, 14 percent in writing, and 11 percent in reading (Corbishley & Truxaw, 2010, p. 4). According to Chen and Simone (2016), among the 2003–04 beginning postsecondary students who first enrolled in public two-year and four-year institutions, 59.3 percent and 32.6 percent respectively took a remedial course in mathematics compared to 28.1 and 10.8 percent respectively who took an English remedial course.

Higher education institutions offer remedial education to reduce the gap between mathematically prepared and unprepared students. Approaches vary depending on the mission and type of school, the type of students served, and on the extent in which remedial education is integrated with the college level curricula and with the academic departments (Perin, 2002). However, as the demand for remedial education increases and the resources decrease, both community colleges and universities are less inclined to provide postsecondary remedial education (Ignash, 1997). Two-year institutions have a higher need for remedial courses (Attewell et al., 2006). Consequently, community colleges are the primary providers of remedial education (Adelman, 2004a; McCabe & Day Jr, 1998; Parsad & Lewis, 2003).

A report from the National Center for Education Statistics (Parsad & Lewis, 2003) reveals that 98 percent of the public two-year institutions provide one or more college-level remedial education courses compared to other type of institutions. Community college students tend to have lower graduation rates than students in four-year institutions. Students who start in four-year schools are more likely to graduate in 6 years than students who transfer from public two-year institutions. According to the report, *The Condition of Education* 2017 issued by the U.S. Department of Education (McFarland et al., 2017), approximately 81 percent of first-time-full-time students who entered four-year institutions in 2014 returned the following year to continue their studies. Instead, at two-year institutions, the retention rate for those who started school in 2014 was 61 percent. In four-year institutions, 59 percent of first time-full-time students who began seeking a bachelor's degree in fall 2009 completed a bachelor's degree at that institution within six years. Comparatively, 29 percent of students beginning at a community college in fall 2012 graduated within 150 percent of the normal time required for the program (McFarland et al., 2017). Among the community college first-time freshmen who intended to transfer to a four-year college, 39 percent had left school by 2006 without completing a degree or certificate program (Provasnik & Planty, 2008). Allocating remedial education solely to community colleges will result in fewer students completing their degrees.

Remedial education expands educational opportunities for entering post-secondary students who lack the appropriate academic skills. Eliminating developmental coursework beyond community colleges will affect at least 35 percent of first-year developmental students who have deficits in mathematics (Parsad & Lewis, 2003). An examination of mathematics education in the U.S. identifies the continuing need for developmental mathematics services at all levels of the postsecondary education continuum. Requiring underprepared students to take remediation at two-year schools will likely reduce the number of university graduates in the country (Duranczyk & Higbee, 2006). While Caucasians constitute the highest number of students in developmental education, African Americans and Latino students are disproportionately represented in remedial courses. As reported in *Remediation Higher Education's Bridge to Nowhere* (Adams et al., 2012), of the students needing remediation in four-year colleges, 39.1 percent are African Americans an 20.6 percent are Latinos compared to 13.6 percent white students.

Adelman (2004b) conducted a study on the significant elements of the post-secondary academic experience and attainment of traditional-age students from 1972-2000. The study indicated that 36 percent of Caucasians and 38 percent of Asians were enrolled in developmental coursework, compared with 62 percent of African Americans and 63 percent of Latino students. The disparity of these proportions reflects the under preparation for college experienced by historically disadvantaged groups.

Students from less-affluent families and for whom English is not their first language are also over-represented in remedial courses (Attewell et al., 2006). The same is true for students from the lowest socio-economic status. As anticipated, students in rural areas and urban high schools are more likely to need remedial education. Adelman (2004b) reported that students from urban high schools were more likely to be taking remedial courses compared to students from suburban and rural high schools. A study conducted by Attewell et al. (2006) determined that 40 percent of students who previously attended a rural high school took remediation courses in college, compared to 38 percent of students from suburban high schools and 52 percent of students from urban high schools. In summary, race, ethnicity, socio-economic status (SES), native language, and location of high school must be considered when discussing developmental education because students from these groups constitute a large percentage of the enrollment in remedial courses. Restricting access to remedial education will primarily reduce the number of low-income and minority students who have the background to succeed in receiving university degrees.

Students who do not attend college or attend and fail because of the lack of adequate preparation have fewer chances to prosper in the modern economy. According to the Pew Research Center report , *The Rising Cost of not Going to College* (2014), college graduates earn $17,500 more annually than employed young adults with only a high school diploma do. Lower earnings by those who fail to graduate from college results in less revenue for local, state, and federal governments in the form of income, property and taxes.

Understanding the Gap

There is a need for additional research into the reasons behind the existing gap between high school and college mathematics. Various studies have been conducted to analyze the factors affecting mathematical preparedness of students entering secondary education. Long et al. (2009) conducted a study to examine the gaps in readiness for college mathematics due to differences in mathematics courses taken by students while in high school. Mathematics readiness, in this study, was defined by the scores obtained by students in the statewide college placement test. Using data from Florida public school students entering Florida postsecondary institutions, Long et al. (2009) concluded that taking mathematics courses beyond the minimum expected to graduate improves college mathematics readiness, with the most significant gains resulting from completing Algebra II. Results from their study also indicate that Latinos, African Americans, and poor students had lower mathematics readiness rates than White and Asian students. According to Long et al. (2009), enrolling blacks, Latinos and needy students in the same high school mathematics courses that whites and non-poor students take could reduce the college gap in mathematics readiness by 28, 35, and 34 percent respectively. Furthermore, this study showed slight gender differences in mathematics readiness with males having a slight advantage. However, the difference in mathematics readiness between males and females could not be explained by completion of advanced mathematics high school courses since women tend to take more advanced courses compared to men.

Post et al. (2010) conducted a similar study where they examined the performance in university-level mathematics as a function of the curriculum used in high school. Results indicated that the high school mathematics curriculum was not a factor in student grades, mathematics courses taking patterns or number of college mathematics courses taken. The curricula tested were a commercially developed (CD) curriculum, the NSF curriculum, and the University of Chicago School Mathematics Project (UCSMP). For students in the lower scoring ranking of the ACT, there was a difference on the initial mathematics course level enrollment for the CD and the UCSMP curricula. Students who had the CD curriculum enrolled in higher-level mathematics courses.

The highest level of mathematics taken a high school has been identified as being a factor predicting college-level mathematics readiness (Long et al., 2009). Students from low socio-economic status (SES) are more likely to attend schools where the highest level of mathematics offered is algebra II (Adelman, 2006). A study conducted by Riegle-Crumb (2006) found that African American and Latino students of both genders generally start high school in lower mathematics courses compared with their white peers. Minority female students are less likely to reach comparable levels of mathematics in comparison with white female students by the end of high school. Lower percentages of African American and Latino females begin high school taking Algebra I making it challenging to achieve a high-level mathematics by the time they finish high school. African American and Latino males are less likely to begin high school in Algebra I. Their performance in Algebra I is also below the performance of their male peers in the course. Thus, minority students are at a disadvantage in attaining high-level mathematics courses in high school.

There is a lack of criteria to define college mathematical readiness. Lee (2012) conducted a logic regression analysis utilizing several national databases and identified desirable mathematics achievement test scores for college readiness. According to Lee, admission into and successful completion of degrees in different types of institutions require different levels of mathematics achievement in K-12 education. Successful completion of four-year degrees demands mathematics performance at or above the "high" level of the Trends in International Math and Science Studies (TIMSS) benchmark or at the "proficient" or higher level of performance in the National Assessment of Educational Progress (NAEP) national test. On the other hand, for successful completion of two-year degrees, students need to perform at the average level of the state's mathematics proficiency test. Lee's study also found that students from disadvantaged minority groups had a lower performance level than other groups. Their scores did not meet the goal for two year-degrees institutions.

Hagedorn, Siadat, Fogel, Nora, and Pascarella (1999) also examined the differences between remedial and non-remedial mathematics students. This study considered the following factors: gender and ethnicity; family income and educational level, encouragement to enroll in college; high school racial composition; high school mathematics level, GPA and study habits; college mathematics level, study habits, and perception of college teaching. Results from their research indicate that remedial students were more likely to be women and members of underrepresented minority groups. In comparison with remedial mathematics students, non-remedial mathematics students were more likely to:

- have parents with higher education degrees
- come from a family with high socio-economic status
- receive encouragement to attend college
- live in neighborhoods and attend high schools composed primarily of non-minority groups
- spend more time studying in high school
- have higher high school GPA
- work collaboratively in college
- rank college-level teaching higher
- get higher scores in mathematics achievement tests.

Similar to other studies, this report highlights the various external factors affecting mathematics college readiness. In addition, it establishes that students enrolling in remedial mathematics classes start their post-secondary education at a considerable disadvantage.

Corbishley and Truxaw (2010) present mathematics preparedness as perceived by college mathematics teaching faculty. They conducted a study to obtain the perception of faculty about mathematics readiness of incoming freshmen and their assessment on which mathematical topics are essential for success in college-level mathematics. Mathematical readiness is defined for this study as “the degree to which a student is predicted to succeed in the college environment in mathematics” (Corbishley & Truxaw, 2010, p. 72). A survey was distributed among faculty from five four-year institutions and three two-year institutions. Faculty perceptions were that the average incoming freshmen were not ready for college mathematics. The mathematical constructs evaluated by this study were subject knowledge, number sense, measurement and data, and reasoning and generalization. Faculty rated the students' skills across all contents as poor or very poor. Among the subjects that were identified by the faculty as being very important and needing improvement were algebraic reasoning, geometry and number sense, including elementary mathematics procedures and ability to use and understand fractions. Corbishley and Truxaw (2010) proposed to address the concerns expressed by this study and recommended that precollege mathematics courses should emphasize the previously mentioned competencies. This study provides a more detailed description of the conceptual factors affecting mathematical college readiness. However, these results are based on faculty perceptions and not on direct measurements of students' mathematical competencies.

Conceptual Foundations for College-Level Mathematics

The most common entry-level mathematics course in four-year institutions is College Algebra. Research indicates that having a thorough understanding of fractions is critical for success in algebra (Driscoll, 1982; Hackenberg, 2013; Kieren, 1980; National Mathematics Advisory Panel, 2008; Wu, 2001). Fraction magnitude understanding has also been shown to be a predictor of mathematics achievement. In a study conducted in Belgium, China, and the U.S., consistent relations between students' fraction understanding and overall mathematics achievement was observed (Torbeyns, Schneider, Xin, & Siegler, 2015).

Inadequate understanding of rational number concepts and difficulties manipulating fractions persist beyond the pre-high school years. According to Behr et al. Behr, Lesh, Post, and Silver (1983), only 1/3 of the 13-year-olds and 2/3 of the 17-year-olds can add fractions with different denominators. Understanding rational numbers is essential for learning algebra, for succeeding at advanced mathematics, and for being competitive in today's workforce. Rational numbers comprehension provides the foundation for learning of algebraic operations and is vital for improving one's ability to handle situations and problems in the real world (Behr et al., 1983; Fuchs et al., 2014; National Mathematics Advisory Panel, 2008).

Rational Number Understanding

A rational number is defined as a number that can be expressed in the form *a/b*, where *a* and *b* are integers, and *b* is not equal to zero. Rational numbers concepts are considered the most complex and most important concepts students have to acquire in middle school (Behr et al., 1983). According to Moss (2005) understanding rational numbers is challenging because students need to develop a multifaceted knowledge network with new concepts, facts and symbols. This new knowledge system is based in multiplicative rather than additive number relations. Several factors contribute to the difficulties students encountered in understanding rational numbers.

Rational numbers representations.

Rational numbers can have different representations. Rational numbers can take the form of fractions, decimal numbers and percentages. Students need to understand not only the symbolism used for each of these forms but also the relationships among them (Sowder, Philipp, Armstrong, & Schappelle, 1998). For example, one-half (1/2) can be represented as 2/4, 0.5, or 0.500. In this example, the 2/4 represents a part-whole relationship while the 0.5 is the quotient decimal representation. Having several representations of a single quantity is confusing to students. To complicate matters, decimals, fractions and percentages are frequently taught as separate topics (Moss, 2005).

Rational numbers concepts.

Rational numbers can be interpreted in a variety of ways referred to as subconstructs. Kieren (1976) and Behr et al. (1983) have identified five ways through which rational numbers subconstruct: part-whole, ratio, quotient, measure, and operator. Kieren (1988) sees the part-whole subconstruct not as a separate construct but as a specific case of the measure subconstruct. He proposed that students must understand each subconstruct independently and jointly to have a general understanding of fractions.

Part-whole subconstruct. The part-whole subconstruct of fractions consists of the situation in which a continuous quantity or a set of discrete objects are partitioned into parts or sets of equal size. In this case, the fraction represents the relation between the number of parts of the partitioned unit and the total numbers of parts in which the unit is partitioned. In the part-whole subconstruct, the numerator of the fraction should be less than the denominator. According to Kieren (1988), this subconstruct is considered fundamental for all future interpretations. Students' difficulties in algebra can be traced back to a lack of understanding of earlier fraction ideas (Behr et al., 1983).

Ratio subconstruct. The ratio subconstruct conveys the idea of a comparison between two quantities of the same type (Lamon, 2012). This situation does not represent the partitioning of one object. The two quantities in a ratio change together, that is, the relationship between the two quantities implies a proportion. It does not change. This is an important concept to understand fraction equivalence and for problem solving in related physical situations.

Operator subconstruct. When fractions are interpreted as operators, rational numbers are seen as functions applied to a number, object, or set (Behr, Harel, Post, & Lesh, 2012). Behr et al. (2012) refer to fractions as operators a stretcher/shrinker and as a duplicator/partition-reducer. The difference between the two is that in the stretcher/shrinker case you have the same number of parts but of a different size while in the duplicator/partition-reducer you have a different number of units of the same size.

Quotient subconstruct. For the quotient subconstruct, any fraction can be thought of as the number resulting from a division operation. Therefore, the fraction x/y refers to the numerical value resulting from dividing x by y, where x and y are whole numbers (Kieren, 2012). In this case, the x represents something that would be partitioned not the number of parts of the whole. In addition, there is no constraint on the size of the fraction. In the quotient subconstruct, the x could be smaller, larger, or the same as y. In addition, the quotient subconstruct by definition is related to linear equation solving and represents a point of connection to the algebra of equations.

Measure subconstruct. The measure subconstruct expresses a fraction as two closely interrelated and interdependent ideas. One is the idea of the quantitative value of a fraction. It represents the size of the fraction, for example, ¼ of an inch. The next idea is associated with a measure assigned to a unit fraction defined as *1/a* that is used repeatedly to determine a distance from a certain point. That is why this subconstruct of the fraction concept has been associated with using the number line or other measuring devices like a ruler (Charalambous & Pitta-Pantazi, 2006).

Rational number understanding is difficult because of the amount of new and complex material students need to acquire. In addition, students' prior knowledge and experience with whole-numbers does not contribute to the learning of rational numbers. In school, students are first introduced to natural numbers. In addition, in their daily lives students encounter natural numbers more frequently than rational numbers. However, "rational number knowing is not just an extension of whole number knowing" (Kieren, 2012, p. 56). Research indicates that students' whole number knowledge acts as an obstacle for developing rational number knowledge (Ni & Zhou, 2005; Vamvakoussi & Vosniadou, 2010).

Whole number bias.

According to Ni and Zhou (2005), "whole number bias refers to a robust tendency to use the single-unit counting scheme to interpret instructional data on fractions" (Ni & Zhou, 2005, p. 28). Consequently, children fail to perceive whole numbers as units that could be dissected. This can lead to misconceptions when manipulating and performing operations with rational numbers. The whole number bias reflects a faulty understanding of the rational number concept. As students learn about fractions, they have difficulty separating the concepts of fractions that seem to be consistent with what they already know about whole numbers. Their tendency is to apply their whole number knowledge to understand fractions (Ni & Zhou, 2005).

Students tend to see a fraction as two separate quantities instead of one (Vamvakoussi, Van Dooren, & Verschaffel, 2012). As a result, when comparing fractions students will assume that the fraction 1/6 is a larger quantity than 1/3 because 6 is bigger than 3. Similarly, when adding fractions, they may add numbers across numerators and denominators.

Natural numbers are discrete. Students have the perception that each number has a sole successor (Vamvakoussi & Vosniadou, 2010). With rational numbers, there are infinite numbers between two “consecutive” numbers. Operations are also different with rational numbers. With natural numbers, addition and multiplication always result in larger numbers while subtraction and division always result in a smaller number (Vamvakoussi et al., 2012). Stafylidou and Vosniadou (2004, p. 505) summarized the differences between natural and rational numbers on Table 1.

Table 1
Differences between natural numbers and fractions

Numerical Value	Natural Number	Fraction
Symbolic Representation	One number (presupposition of discreteness)	Two numbers and a line (presupposition of density)
Ordering	Supported by the natural numbers' sequence (counting on)	Not supported by the natural numbers' sequence
	Existence of a successor or a preceding number	There is no unique successor or a unique preceding number
	No number between two different numbers	Infinity
Relationship to the unit	The unit is the smallest number	No unique smallest number
Operations		
Addition-subtraction	Supported by the natural numbers' sequence	Not supported by the natural numbers' sequence
Multiplication	Multiplication makes the number bigger	Multiplication makes the number either bigger or smaller
Division	Division makes the number smaller	Division makes the number either smaller or bigger

Stafylidou and Vosniadou (2004) conducted a study among students ranging from elementary to high school to investigate the effect of natural number knowledge in the development of the concept of fractions. According to their investigation, through this process, students develop synthetic transitional models causing misconceptions. They identified the following explanatory frameworks:

Fraction as Two Independent Natural Numbers – For students in this explanatory framework, each number corresponds to a symbol. In their representation of fractions, as revealed from their answers concerning the smallest/biggest fraction and the ordering of fractions the numerators and denominators, were treated as if they were separate natural numbers.

Fraction as Part of a Unit – The students who adopt the second explanatory framework believe that fractions always represent quantities smaller than the unit does. This idea is compatible with the way fractions are usually taught initially, as a part of something. This framework seems to represent a transitional phase in the process of understanding fractions.

Fraction as a Relation between Two Numbers – Students adopting this explanatory framework have understood that a fraction can be smaller, equal, or even bigger than the unit can. In addition, they understand that there can be fractions with numerators larger than the denominator. They understand improper fractions.

Stafylidou and Vosniadou (2004) concluded that students, in the development of fraction concepts, will not adopt the concept immediately but will interpret fractions in ways that attempt to reconcile their initial ideas about number with the new information. Moss (2005) proposes that in transitioning from natural numbers (whole numbers) to rational numbers, students encounter a number of challenges because of the shift of numbers expressing a fixed quantity to numbers expressing a relationship to other numbers.

Other factors affect rational numbers' understanding

According to Schneider and Siegler (2010), the whole number bias is only part of the problem in understanding fractions' arithmetic. Understanding of whole numbers is one source of ideas about how to solve fractions arithmetic problems, but other types of numerical knowledge also need to be incorporated. J. L. Booth, Newton, and Twiss-Garrity (2014) identified fraction magnitude knowledge as critical for understanding fraction equivalence and proportionality concepts. They found that students having a better understanding of fraction magnitudes when they begin learning algebra content learned more content than students who have a poor fraction magnitude understanding. Accordingly, they recommend ensuring that students have a solid foundation in fractions before they start learning algebra and utilizing remediation for algebra students who do not have the fraction knowledge needed.

Conceptual understanding of rational numbers has been correlated to mathematical achievement (Siegler, Thompson, & Schneider, 2011). Research also shows that rational number understanding is critical for success in algebra. Results from their study indicated that knowledge of fraction magnitude is related to students' performance in algebra. According to J. L. Booth et al. (2014), fraction magnitude knowledge represents a deeper understanding of fractions. This result suggests that improving students' skills in operating with fractions would lead to improved performance in Algebra.

A possible explanation for the relation between rational number understanding and algebra performance is that rational numbers are included in the conceptual field of multiplicative structures. Conceptual fields are defined as “a set of situations, the mastering of which requires the mastery of several concepts of different nature” (Vergnaud, 1988, p. 141). According to Vergnaud (1988), the concepts of fraction, ratio, rate, rational number, multiplication, division, dimensional analysis, linear and n-linear functions, vector spaces are interconnected and it is difficult to study the acquisition of one of those concepts independently of the others. This suggests that students, who do not have a clear understanding of rational numbers, may have difficulties in solving equations or working with functions when they reach algebra courses.

Vergnaud (1983) proposes that concepts derive from other concepts and they do not develop in isolation. In addition, cognitive boundaries between concepts are not always well defined. Similar to Kieren (1976) and Behr et al. (1983), Vergnaud (1983) identified different meanings of the expression *a/b*. Table 2 summarizes the various meanings.

Table 2
Meanings of Fractions

	Value	Relation	Units/ categories	Example
Part-whole fractions	<1	2 quantities of same nature included in each other	= (scalar)	# boys/ # children
Part-part ratios	<1 or >1	Quantities of same nature not included in each other	= (scalar)	# boys/ # girls
Rates	<1 or >1	Quantities of different nature	≠ (could be functions)	distance/ time

As shown in Table 2, part-whole fractions and part-part ratios are scalars, meaning just regular numbers. However, rates are expressed as a quotient, e.g., 55 miles/hour. Table 2 illustrates the need to study rational numbers as multiplicative structures. Since a given situation does not involve all the properties of a concept, its analysis requires understanding several concepts. Prior to learning fractions, students learn that whole numbers can be associated to quantities by counting. Fractions on the other hand cannot easily be directly associated to quantities. Fractions represent relationships between two quantities. The relationships between the quantities vary depending on whether the quantities are of the same nature, different nature, and if they are included in each other or not. To address all properties of a fraction concept, you must refer to several and various kinds of situations. Understanding these concepts is not easy and takes time. Only when all different meanings are synthesized, the rational number concept can be understood.

Understanding rational numbers is essential for learning algebra, for succeeding at advanced mathematics, and for being competitive in today's workforce. Rational numbers are also necessary for daily activities like following recipes, calculating discounts, car mileage efficiency, making unit conversions, interpreting drawings, and financial statements (Behr et al., 1983; Fuchs et al., 2014; Moss, 2005; National Mathematics Advisory Panel, 2008).

In spite of numerous discussions about mathematics preparedness of students as they leave high school (ACT, 2016; NAEP, 2016; National Center for Education Statistics, 2015; National Commission on Excellence in Education, 1983a), higher education institutions continue to enroll students who are not ready to complete college-level mathematics. Students' exposure to higher-level mathematics courses than the courses required aid in minimizing the effects of transitioning into college-level mathematics (Long et al., 2009). However, not all students have the opportunity to take higher-level mathematics before attending college due to lack of availability, lack of encouragement from family and teachers, or lack of prerequisites from middle school (Hagedorn et al., 1999). The Common Core State Standards (CCSS, 2017a) is part of the efforts being taken in the K-12 systems to ensure mathematics readiness of all students when they graduate. Nevertheless, it is up to higher education institutions to address the disparities in mathematics readiness and make sure that students have the adequate skills to enroll and progress in college-level mathematics successfully. This study will investigate college students' understanding of rational numbers as it affects mathematics college readiness. The results from this study can be used to inform the design of effective interventions.

CHAPTER 3: METHODS

The primary goals of this study are to identify key mathematical concepts first-time incoming college students need to enroll and successfully complete entry-level college mathematics courses and to obtain a deeper understanding of incoming college students' knowledge of key mathematical concepts.

An explanatory, two-phase, sequential mixed method design was used to answer the research questions:

1. What mathematics competencies characterize students at different levels of mathematics college readiness?
2. What demographic factors and incoming data (Mathematics SAT score, intended major, high school GPA) characterize students placing at different levels of mathematics college level readiness?
3. What is the level of understanding of key mathematics competencies of incoming students at an entry-level mathematics course?

In Phase 1, a pilot, historical data from mathematics placement tests were collected and analyzed to determine primary areas of difficulty encountered by incoming students. Data from the pilot (described later) identified rational number operations as a major area of conceptual difficulty for entering freshmen. Hence, in the second component of this study, Phase 2, performance of fall 2017 entering freshmen registered in an entry-level mathematics course was evaluated to confirm the results obtained in the Phase 1 pilot. An error analysis of students' responses to test questions was conducted to explain students' understanding of operations with rational numbers, negative signs, and linear equations.

In this chapter, first, the setting, participants, data collection and initial data analysis for Phase 1 will be described. Second, additional analysis of Phase 1 data will be shown that corroborate the results obtained in the pilot. Finally, Phase 2 participants, data collection and analysis are presented.

Setting

This study took place in a public southeastern urban research university. This University is located in an urban city in North Carolina and currently serves about 29,000 full-time and part-time students. Until fall 2014, an in-house, mathematics placement test was given to all incoming students during student orientation. The total score in the mathematics placement test was used to determine the students' placement in the available entry-level mathematics courses: Developmental Mathematics, College Algebra, Precalculus, and Calculus I.

The Developmental Mathematics course prepares students to succeed in College Algebra. This course includes a review of elementary algebra, exponents and radicals, polynomial and rational functions, equations and inequalities. The College Algebra course covers fundamental algebra concepts. It is the basic mathematics course for students not majoring in mathematics, engineering, or the physical sciences. The Precalculus course is designed for students who plan on taking Calculus I. It includes functions and graphs, linear and quadratic functions, polynomial and rational functions, exponential and logarithmic functions, and trigonometry. Calculus I is designed for students planning to major in mathematics, science, or engineering. Content includes elementary functions, derivatives and their applications, and introduction to definite integrals ("Undergraduate Catalog 2017-2018 ", 2017).

Phase 1 – Pilot

Participants.

The participants were new first-time freshmen entering the University in the fall 2010 – fall 2013. They were selected to ensure homogeneity of the mathematics placement data. While differences in their mathematical preparedness for college were expected, only new freshmen were included to minimize student's dissimilarities including time elapsed between high school graduation and beginning of college, age, and admission criteria. These characteristics could be factors affecting student performance in the placement test. Results from the mathematics placement tests for freshmen entering the University between the fall of 2010 and the fall of 2013 were collected. Fall terms were selected because most new first-time freshmen enter four-year institutions in fall terms. The number of new freshmen completing the placement test was 2,905 in fall 2010; 3,118 in fall 2011; 3,275 in fall 2012; and 3,031 in fall 2013.

Instrument.

The mathematics placement test was developed by the University's Department of Mathematics and Statistics. It consists of 25 multiple-choice questions on basic algebra skills. Students must complete the test in 30 minutes without a calculator. Students were given the mathematics placement test as part of the student orientation activities in the summer before their entrance to the University. The same version of the test was given to all students coming in any given term. The general topics included in the test did not change between entry years but the questions changed. The Mathematics Placement Tests are included in Appendices A-D.

The final mathematics placement test score is based on the number of correct answers. Each correct answer counts as one point towards the total score for a maximum of 25 points. Zero points are assigned to incorrect answers. According to University policy, students earning a score between 0 and 10 are placed in Developmental Mathematics; students earning a score between 11 and 13 are eligible to take College Algebra; students scoring between 14 and 17 can register for Precalculus; and students earning a score of 18 or higher can opt to register for Calculus I.

Performance in the mathematics placement test was used to evaluate students' mathematics preparedness. This assessment was selected to take advantage of the accessibility to the results and the questions' topics. In addition, results could be connected to students' demographic and academic characteristics available in the University's databases. Students' performance on test questions was matched to students' placement into the various entry-level mathematics courses. These results were used to determine which questions had a higher incidence of failure and consequently to identify key mathematical competencies students need to be ready for college mathematics.

The mathematics placement test includes the following topics: arithmetic of rational numbers, order of operations, operations with algebraic expressions, linear equations and inequalities, factoring and algebraic fractions, exponents and radicals, graphing, fractional and quadratic equations, absolute values, systems of linear equations.

Figure 1 shows the process for matching the results of the Mathematics Placement Test to mathematics readiness. Each Mathematics Placement Test question was matched with a topic. The students' performance on each question was obtained and cross tabulated with the mathematics placement levels. Hence, performance on key mathematical concepts were matched to mathematics placement. For this study, mathematics placement was assumed to be a measure of mathematics readiness. Thus, key mathematical concepts affecting mathematics readiness were identified.

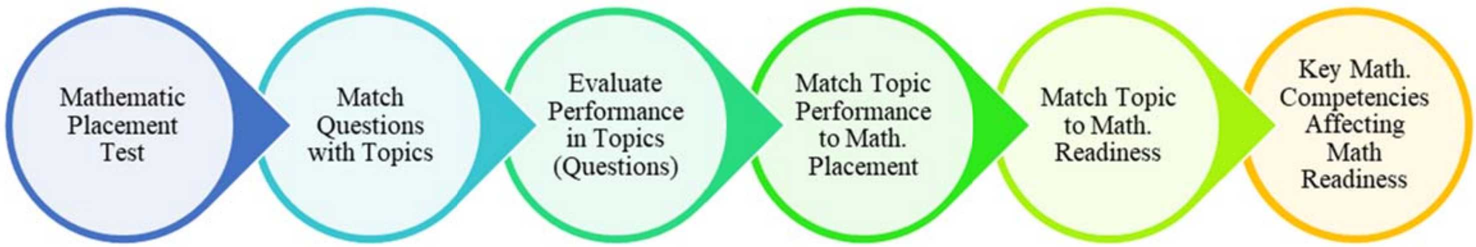


Figure 1. Process for Obtaining Key Mathematical Competencies Affecting Mathematics Readiness

Reliability

Since historical data were used, the reliability of the Mathematics Placement Test was estimated using the single administration method. This method relies on the consistency of the individual's performance from item to item (Thorndike, 1997). Table 3 shows the Unstandardized Cronbach's alpha values for the 25 item Mathematics Placement Tests given from fall 2010 through fall 2013. Unstandardized values were selected because test scores are calculated using raw scores (Falk & Savalei, 2011). As it can be seen, Cronbach's alpha values for all the tests are .8 or higher.

Table 3

Reliability of Mathematics Placement Test

	N (cases)	N (items)	Test Items	Reliability, Cronbach's Alpha Unstandardized
Fall 2010	2,905	25	Q1-Q25	0.8294
Fall 2011	3,118	25	Q1-Q25	0.8205
Fall 2012	3,275	25	Q1-Q25	0.8128
Fall 2013	3.031	25	Q1-Q25	0.8059

Data collection.

Demographic and academic background parameters of new first-time freshmen coming in fall semesters were obtained and analyzed to identify common characteristics among students placing in the various entry-level mathematics courses.

Data collected included:

1. Mathematics placement test results of incoming freshmen in the fall semesters from 2010 through 2013.
2. Demographics and academic characteristics of incoming freshmen in the fall semesters from 2010 through 2013 including gender, ethnicity, mathematics SAT scores, and high school GPA.

Data collection method.

1. Mathematics placement test results were obtained from the University's Department of Mathematics and Statistics for fall 2010 through fall 2013.
2. Demographics and incoming academic data of students were collected from the University' student records database by the researcher.

IRB approval was requested because human subjects were part of this study. A waiver of informed consent was requested for Phase 1 of the study because only historical data were used resulting in minimum harm to the participants.

Data analysis.

1. Students' performance in the University's mathematics placement test was used to identify challenging mathematics competencies using descriptive and inferential statistics. In addition to test results, an answer key was provided by the Mathematics and Statistics Department. MS Excel was used to match correct answers to students' responses and to obtain the test scores for each student. Test results were then used to obtain sample sizes,

means, and standard deviations for each term and mathematics course placement.

2. A four-group independent-samples chi-square test was used to determine if the differences in the students' mathematics placement test score were due to chance.
3. Students' demographic data and incoming academic characteristics were analyzed using descriptive and inferential statistics.
    a. A Chi-square test of independence was conducted to examine the association between gender and frequency of placement in the given mathematics levels. In addition, a one-way ANOVA was conducted to determine if the overall Mathematics Placement Test scores were different for men and women entering the University between the fall semesters of 2010 and 2013.
    b. A Chi-square test of independence was conducted to examine the association between mathematics placement level and ethnicity for all terms. A one-way ANOVA test was run to compare the overall Mathematics Placement Test mean scores for the four entry terms and to determine if there were differences in the test performance among the students from each ethnic group. A Games-Howell post hoc analysis was conducted to determine which ethnic groups had statistically significant different mean Mathematics Placement Test scores.

c. A multiple regression was run to predict Mathematics Placement Test scores from High School GPA, and Mathematics SAT scores.

d. A Chi-square test was conducted to determine if there was an association between mathematics placement and student selection of college for all terms. In addition, Mathematics Placement Test mean scores for the four entry terms for students entering each college were tested to determine if they were statistically significant. A Welch's test was conducted because of unequal sample sizes.

IBM SPSS Statistics 23 software was used to obtain descriptive and inferential statistics parameters.

Results Phase 1 – pilot.

Table 4 shows the Mathematics Placement Test (MPT) mean scores for each mathematics course placement and entry term. Levene's test for Equality of Variances was run before comparing the MPT score means. The significance value was above .05 in Levene's test (.405). Therefore, equal variances can be assumed. Further inspection of the data indicates that the MPT score mean for fall 2013 (13.60) was lower than the means for fall 2010, 2011, and 2012 (15.33, 15.68, and 15.16 respectively), as shown in Table 5. Furthermore, because the MPT scores come from a test, it was expected to have values cluster around the natural limit of 25. As a result, the data distribution was not normally distributed but was slightly skewed. Skewness values for the four entry terms are shown in Table 5. Test score results for the fall 2011 – fall 2012 have negative skew values.

Since the mean score for fall 2013 was lower than for other terms, the skewness was positive as more students had lower MPT scores. The skewness for fall 2010 is positive and very small.

Table 4

Sample size, Math Placement Test Mean and Standard Deviation for New Incoming Freshmen by Course Placement

	Dev Math ($MPT \leq 10$)			College Algebra ($11 \leq MPT \leq 13$)			Precalculus ($14 \leq MPT \leq 17$)			Calculus I ($MPT \geq 18$)		
Term	N	M	SD	n	M	SD	n	M	SD	n	M	SD
Fall 2010	525	8.14	1.75	590	12.04	0.82	791	15.48	1.13	999	20.92	2.23
Fall 2011	524	8.15	1.83	558	12.05	0.82	858	15.46	1.13	1,178	20.91	2.13
Fall 2012	606	8.11	1.82	653	12.01	0.79	918	15.41	1.10	1,098	20.80	2.19
Fall 2013	905	7.89	1.94	696	11.97	0.80	719	15.41	1.10	711	20.65	2.00

Note. MPT = Math Placement Test

Table 5

Sample size, MPT Scores Mean and Standard Deviation for New Incoming Freshmen by Entry Term

Term	n	M	SD	Skewness
Fall 2010	2,905	15.33	5.004	.013
Fall 2011	3,118	15.68	4.997	-.113
Fall 2012	3,275	15.16	4.925	-.006
Fall 2013	3,031	13.60	5.029	.211

Note. MPT = Mathematics Placement Test

A Kruskal-Wallis test was run to compare the MPT score means for the four entry terms and determine if there were differences in the test performance among the students from each cohort. Results from the Kruskal-Wallis test revealed a significant difference in the MPT mean score between the four groups of freshmen coming between fall 2010 and fall 2013 $F(12{,}329)= 103.159$, $p<.001$). Post-hoc Tamhane analysis was conducted to assess if significant differences were present for all the cohorts. As shown in Table 6, there were significant differences among all mathematics placement test mean scores except for the MPT score mean of the students entering the University in fall 2010 and the students entering the University in fall 2012. This can be expected since the sample sizes are large 2905, 3118, 3275, and 3031 for 2010, 2011, 2012, and 2013 respectively. In addition, variations in student mathematics performance among students entering in different years are to be expected. The mathematics placement test score means for students coming into the University between fall 2010 and fall 2012 differed by less than a point. However, the difference in mathematics placement test score with students coming in fall 2013 was between 1.5 and 2.0 points.

Table 6

Differences between MPT Scores Mean for New Incoming Freshmen on a Given Term

Term	Fall 2010	Fall 2011	Fall 2012
Fall 2010			
Fall 2011	-.351*		
Fall 2012	.164	.515*	
Fall 2013	1.722*	2.073*	1.558*

Note. MPT = Mathematics Placement Test
*p < .05

The combined results of all four years of MPT data are shown in Table 7. The overall mathematics placement test score means are very similar for students entering in the fall semesters of the years 2010 – 2012. The overall mean of the MPT for fall 2013 is lower. The standard deviation for all years are nearly identical 5.004, 4.997, 4.925, and 5.029 for fall 2010, 2011, 2012, and 2013 respectively. Therefore, the spread of the scores around the mean for all cohorts is the same.

Table 7

Sample Size, MPT Scores Mean and Standard Deviation for New Incoming Freshmen Entering from Fall 2010 - Fall 2013 by Mathematics Level

Course Placement	n	M	SD
Dev Math (MPT $\leq$10)	2,560	8.1	1.85
College Algebra (11$\leq$ MPT $\leq$13)	2,497	12.0	.81
Precalculus (14$\leq$ MPT $\leq$17)	3,286	15.4	1.11
Calculus I (MPT $\geq$18)	3,986	20.8	2.15

Note. MPT = Mathematics Placement Test Score

To gain a better understanding of the distribution students in each mathematics level, results were cross tabulated using the Mathematics Placement Test (MPT) score ranges defined by the institution were the study was conducted. Students scoring between the MPT were assigned to developmental mathematics. Students scoring between 11 and 13 points in the MPT were assigned to college algebra. Students scoring between 14 and 17 points in the MPT were assigned to precalculus; and students scoring 18 or more points were assigned to calculus I. Table 8 shows the results from the cross tabulation. A chi-square analysis demonstrated that new freshmen coming into the University between fall 2010 and fall 2013 differed significantly in the frequency with which they placed in the various introductory mathematics levels (χ^2 (9, N=12,329) =303.205, p<.001, Contingency Coefficient =.155). The larger percentage of students placing in Developmental Mathematics in fall 2013 was unexpected since the incoming average high school GPA (3.69 vs. ~ 3.50) and average Mathematics SAT score (515 vs.

~ 493) for this cohort were higher than the averages for previous years. See Tables 25 and 26.

Table 8

Number and Percent of New Incoming Freshmen Placing in each Mathematics Level

Term	Dev Math (MPT ≤10)	College Algebra (11≤ MPT ≤13)	Precalculus (14≤ MPT ≤17)	Calculus I (MPT ≥18)	Total
Fall 2010	525 *18%*	590 *20%*	791 *27%*	999 *34%*	2,905
Fall 2011	524 *17%*	558 *18%*	858 *28%*	1,178 *38%*	3,118
Fall 2012	606 *19%*	653 *20%*	918 *28%*	1,098 *34%*	3,275
Fall 2013	905 *30%*	696 *23%*	719 *24%*	711 *23%*	3,031

Note. MPT = Mathematics Placement Test

To address research question 1, mathematics placement test results were cross-tabulated to show the frequency of responses for each entering group of freshmen. The percentage of students selecting the correct answer, for each mathematics classification and term, are shown in Tables 9 – 16. Inspection of Tables 9 – 16 show that students placing in Developmental Mathematics, College Algebra, and Precalculus are more likely to miss questions in a variety of topics with more students missing questions related to fractions and rational number operations and solving equations. The Mathematics Placement Test questions are shown in Appendices A-D. Additional analysis of these data will be conducted in Phase 2.

Table 9

Percent of Fall 2010 Incoming Students Answering Mathematics Placement Test Questions Q1-Q13 Correctly (N= 2,905)

Math Level	Order of operations Q1	Substitution Q2	Simplify complex fraction Q3	Find x and y intercepts Q4	Ratio and proportion Q5	Solve linear equation Q6	Rationalize denominator Q7	Simplify rational function Q8	Evaluate a formula Q9	Substitute and evaluate absolute value Q10	Properties of exponents Q11	Solve rational equation Q12	Multiply rational functions Q13
Dev. Math	11%	69%	22%	25%	49%	56%	17%	16%	23%	64%	53%	7%	14%
College Algebra	24%	86%	38%	33%	74%	78%	33%	32%	47%	80%	68%	11%	20%
Precalc.	39%	92%	61%	43%	85%	88%	52%	54%	66%	89%	76%	21%	29%
Calculus I	65%	96%	88%	66%	97%	95%	81%	88%	92%	96%	92%	58%	61%
Total	40%	88%	59%	45%	80%	83%	51%	54%	63%	85%	76%	29%	35%

Table 10

Percent of Fall 2010 Incoming Students Answering Mathematics Placement Test Questions Q14 – Q25 Correctly (N= 2,905)

Math Level	Properties of exponents Q14	Solve linear inequality Q15	Simplify a radical Q16	Solve a quadratic equation Q17	Solve a system of equations Q18	Properties of exponents Q19	Properties of exponents Q20	Simplify an algebraic expression Q21	Add/Subtract rational functions Q22	Simplify rational function Q23	Simplify an algebraic expression Q24	Graph a linear equation in two variables Q25
Dev. Math	74%	49%	27%	24%	11%	61%	16%	33%	23%	9%	45%	17%
College Algebra	91%	70%	46%	40%	18%	79%	38%	65%	33%	13%	58%	31%
Precalc.	96%	84%	64%	60%	29%	87%	59%	81%	47%	27%	73%	48%
Calculus I	99%	95%	89%	77%	66%	96%	86%	95%	84%	73%	84%	78%
Total	92%	78%	62%	55%	36%	84%	56%	74%	52%	37%	69%	49%

Table 11

Percent of Fall 2011 Incoming Students Answering Mathematics Placement Test Questions Q1-Q13 Correctly (N= 3,118)

Math Level	Order of operations Q1	Substitution Q2	Simplify complex fraction Q3	Find x and y intercepts Q4	Ratio and proportion Q5	Solve linear equation Q6	Rationalize denominator Q7	Simplify rational function Q8	Evaluate a formula Q9	Substitute and evaluate absolute . Q10	Properties of exponents Q11	Solve rational equation Q12	Multiply rational functions Q13
Dev. Math	17%	58%	20%	29%	44%	62%	14%	47%	26%	38%	36%	54%	19%
College Algebra	27%	73%	35%	43%	66%	81%	17%	62%	41%	57%	52%	66%	27%
Precalc.	46%	85%	54%	54%	80%	91%	32%	79%	55%	67%	62%	67%	32%
Calculus I	68%	95%	84%	81%	94%	95%	68%	93%	85%	89%	81%	66%	62%
Total	46%	82%	56%	58%	77%	86%	40%	76%	59%	69%	63%	64%	38%

Table 12

Percent of Fall 2011 Incoming Students Answering Mathematics Placement Test Questions Q14-Q25 Correctly (N= 3,118)

Math Level	Property of exponents Q14	Solve linear inequality Q15	Simplify a radical Q16	Solve a quadratic equation Q17	Solve a system of equations Q18	Property of exponents Q19	Property of exponents Q20	Simplify an algebraic expression Q21	Add/Subtract rational functions Q22	Simplify rational function Q23	Simplify an algebraic expression Q24	Graph a linear equation in two variables Q25
Dev. Math	45%	47%	27%	15%	19%	57%	14%	32%	9%	27%	38%	23%
College Algebra	65%	71%	44%	27%	31%	73%	31%	59%	17%	38%	65%	37%
Precalc.	79%	84%	59%	39%	46%	82%	51%	81%	35%	57%	76%	55%
Calculus I	94%	96%	84%	65%	76%	94%	84%	97%	76%	90%	91%	86%
Total	76%	80%	61%	43%	50%	81%	54%	75%	43%	61%	73%	58%

Table 13

Percent of Fall 2012 Incoming Students Answering Mathematics Placement Test Questions Q1- Q13 Correctly (N=3,275)

Math Level	Order of operations Q1	Simplify an algebraic expression Q2	Simplify complex fraction Q3	Simplify arithmetic expression with absolute . Q4	Properties of exponents Q5	Simplify a radical Q6	Add/Subtract rational functions Q7	Simplify rational function Q8	Evaluate a formula Q9	Solve rational equation Q10	Find x and y intercepts Q11	Properties of exponents Q12	Multiply rational functions Q13
Dev. Math	31%	42%	27%	74%	63%	33%	20%	39%	27%	49%	24%	16%	12%
College Algebra	51%	60%	43%	86%	76%	51%	40%	61%	49%	62%	46%	23%	24%
Precalc.	65%	66%	62%	93%	86%	67%	55%	79%	71%	74%	60%	41%	30%
Calculus I	87%	76%	88%	97%	94%	89%	76%	94%	94%	90%	84%	75%	60%
Total	63%	64%	61%	90%	83%	65%	53%	73%	66%	72%	58%	44%	36%

Table 14

Percent of Fall 2012 Incoming Students Answering Mathematics Placement Test Questions Q14-Q25 Correctly (N=3,275)

Math Level	Properties of exponents Q14	Factor a polynomial Q15	Substitute and evaluate absolute value Q16	Simplify an algebraic expression Q17	Solve a system of equations Q18	Properties of exponents Q19	Rationalize denominator Q20	Solve quadratic equation Q21	Solve linear equation Q22	Simplify rational function Q23	Solve linear inequality Q24	Graph a linear equation in two variables Q25
Dev. Math	40%	15%	52%	18%	6%	42%	13%	18%	70%	12%	43%	25%
College Algebra	56%	29%	72%	36%	13%	58%	26%	27%	87%	23%	63%	39%
Precalc.	72%	40%	87%	54%	25%	73%	41%	44%	93%	35%	75%	53%
Calculus I	90%	64%	97%	78%	57%	87%	75%	75%	98%	61%	90%	74%
Total	69%	41%	81%	52%	30%	69%	44%	46%	89%	42%	72%	52%

Table 15

Percent of Fall 2013 Incoming Students Answering Mathematics Placement Test Questions Q1- Q13 Correctly (N=3,031)

Math Level	Order of operations Q1	Simplify expression Q2	Simplify complex fraction Q3	Solve ratio/proportion equation Q4	Properties of exponents Q5	Simplify a radical Q6	Add/Subtract rational functions Q7	Reduce a rational expression Q8	Evaluate a formula Q9	Solve a rational equation Q10	Find x and y intercepts Q11	Properties of exponents Q12	Multiply rational expressions Q13
Dev. Math	24%	63%	26%	55%	73%	24%	14%	51%	20%	42%	19%	13%	18%
College Algebra	40%	75%	45%	76%	81%	41%	33%	73%	33%	53%	31%	28%	25%
Precalc.	54%	79%	67%	85%	87%	56%	55%	84%	54%	57%	44%	47%	32%
Calculus I	75%	86%	93%	95%	91%	84%	86%	97%	85%	59%	69%	82%	62%
Total	47%	75%	56%	76%	82%	50%	45%	75%	46%	52%	40%	41%	33%

Table 16

Percent of Fall 2013 Incoming Students Answering Mathematics Placement Test Questions Q14-Q25 Correctly (N=3,031)

Math Level	Properties of exponents Q14	Factor a polynomial Q15	Evaluate an absolute value expression Q16	Multiply polynomials Q17	Solve a system of equations Q18	Properties of exponents Q19	Rationalize a denominator Q20	Solve equation by factoring Q21	Solve linear equation in one variable Q22	Reduce a rational expression Q23	Solve a linear inequality Q24	Graph a linear equation in two variables Q25
Dev. Math	15%	25%	49%	32%	27%	45%	9%	22%	50%	17%	29%	29%
College Algebra	35%	36%	68%	61%	44%	61%	22%	32%	71%	43%	42%	47%
Precalc.	56%	46%	81%	78%	61%	71%	42%	46%	83%	60%	53%	65%
Calculus I	87%	73%	94%	92%	85%	87%	75%	71%	93%	90%	69%	84%
Total	46%	44%	72%	64%	53%	65%	35%	42%	73%	50%	47%	54%

The second phase of this study focused on identifying the parameters that characterize students in the various levels of mathematics preparedness, confirming students' challenges with rational number operations, investigating students' understanding of rational numbers, and on developing a model to explain students' college mathematics readiness.

Phase 2

Phase 2 of this study has two purposes. First, to confirm the results obtained in Phase 1 of the project that showed that new freshmen entering college were inadequately prepared to perform rational number operations. Second, to evaluate the level of understanding of the key mathematics competency identified in Phase 1, rational number operations, of incoming students at an entry-level mathematics course. These results were verified through classroom observations and artifacts examination.

Data collected in Phase 1 of the project was subject to further analysis to gain additional understanding about students' mathematics preparedness and to validate preliminary results.

Participants.

Additional participants for the second phase of this project were new freshmen entering in fall 2017 registered for a precalculus course. Freshmen were selected because they had not taken other mathematics courses in college and were not repeating the course. This section of precalculus had students who were not new freshmen. Therefore, not all of the students in the class were part of the study.

Data collection.

Research generated documents were used to collect the new data for Phase 2. The professor teaching the class agreed to provide the researcher access to assignments and tests to be analyzed. An IRB was submitted and students were given informed consent and assent forms. Copies of the informed consent and assent are included in Appendix E and Appendix F. Student work artifacts were used to learn more about students' performance in the key mathematical competencies identified in Phase 1. In addition, class and problem sessions observations were conducted to gather additional information on students understanding of key mathematical competencies.

Instruments.

Selected questions from tests developed by the course instructor were used to analyze student understanding of rational numbers. Questions were selected if they required rational number operations.

Data analysis.

In Phase 1, results from a mathematics placement test completed by entering students were used to identify key mathematical competencies students need to be prepared for college-level mathematics. The tests included 25 questions. Percentages of correct answers were calculated for each question. Results from this pilot revealed that operations with fractions and rational functions were factors affecting mathematics preparedness of new students entering the University. However, only utilizing the percentage of students obtaining the correct answer did not provide sufficient evidence to confirm and identify other constructs. In Phase 2, an Exploratory Factor Analysis (EFA) was conducted to find the number of constructs measured by the Mathematics Placement

Test questions and to help identify those key constructs (Fabrigar & Wegener, 2012). The EFA was conducted with the results from the Mathematics Placement Tests given from fall 2010 – fall 2013. The factors were extracted using a log-likelihood algorithm that utilizes the frequency which with patterns of responses occur (Jöreskog, Olsson, & Wallentin, 2016). LISREL 9.30 was used to conduct the analysis. The EFA was followed by a Confirmatory Factor Analysis (CFA) to estimate and test the model obtained with EFA. Since the answers to the questions on the Mathematics Placement Test are dichotomous, correct (1) or incorrect (0), the factors were extracted using a Diagonally Weighted Least Square (DWLS) fit function (Jöreskog et al., 2016). Raw data were used as the input and the Robust Estimation option was used to account for non-normality of the data (Jöreskog et al., 2016).

Results from artifact examinations were categorized. An error analysis was applied to students' test questions. The nature of the errors commonly made by students was analyzed to obtain information on how the students viewed the problems and the strategies they used to solve them. Consideration of the errors and strategies revealed suggested hypothesis on student learning (Booth, 1981). These data were analyzed to explain students' challenges understanding rational numbers. Annotations were made to responses provided during the class and problem sessions to complement artifact examination results.

Borasi (1987) error analysis guide was used in examining students' wrong results for test questions. The following questions were selected from Borasi's suggested list of questions about "Wrong Results" (Borasi, 1987, p. 6):

Math Content

1. In what sense is the result wrong?
2. Where did the procedure fail? Could it be fixed up and thus lead to different results?
3. What were the assumptions, and are they justified? In what cases?

Nature of Mathematics

1. How can we test whether a correct mathematical procedure was used?
2. How can we decide whether it is appropriate to apply a certain procedure in a given situation?
3. How can we determine the domain of application of a given procedure?

These questions were used to help identify properties of rational numbers that in many cases are taken for granted.

Chapter 3 provided the methodology for data collection and data analysis procedures for this study. Data sources are Mathematics Placement Tests results, student demographic and entering characteristics; and students' written responses to tests in an entry level mathematics course. Mathematics Placement Test Results and entering students characteristics were analyzed utilizing descriptive and inferential statistical analysis. IBM SPSS Statistics 23 software was used for the statistical analysis. An Exploratory Factor Analysis (EFA) was conducted to identify key mathematical competencies affecting mathematics preparedness of entering college students. Subsequently, a Confirmatory Factor Analysis (CFA) analysis was conducted to confirm the results from the EFA. LISREL 9.30 was used to perform the EFA and CFA. An error analysis was used to interpret the results from the entry-level mathematics course test.

Chapter 4 will provide findings from the historical data and factor analysis and the entry-level mathematics course student work analysis. Results from the data analysis of this study will be valuable to identify key competencies students need to be adequately prepared to enter college-level mathematics courses. These findings will provide valuable information that could enhance the K-12 mathematics curriculum and facilitate the development of adaptive learning materials to assist entering college students complete entry-level mathematics courses.

CHAPTER 4: RESULTS

This chapter reports the results of the analysis of the data collected in Phases 1 and 2. Phase 1 data included Mathematics Placement Test historical results and demographics and incoming characteristics for new incoming freshmen entering the university from fall 2010 through fall 2013. Phase 2 data included test questions and classroom observations for new students enrolled in a Precalculus class in fall 2017. These results are presented to answer the following questions:

1. What mathematics competencies characterize students at different levels of mathematics college readiness?
2. What demographic factors and incoming data (Mathematics SAT score, high school GPA, entry college) characterize students placing at different levels of mathematics college readiness?
3. What is the level of understanding of key mathematics competencies of incoming students in an entry-level mathematics course?

Students entering college lack the mathematics readiness required to register for and to complete their entry-level mathematics course. There are key mathematics competencies that affect student preparedness. Data results were analyzed to explain the characteristics of entry-level students' mathematics preparedness.

Research Question 1

What mathematics competencies characterize students at different levels of mathematics college readiness?

Students' responses to the Mathematics Placement Tests were graded according to the provided answer key, and the percentage of students selecting the correct answer were tabulated. Results for the individual terms were shown in Chapter 3 in Tables 9 through 16. Students placing in Developmental Mathematics, College Algebra, and Precalculus, had a higher incidence of missing questions including items related to fractions or rational functions. To summarize these results, questions from Mathematics Placement Tests given from fall 2010 through fall 2013 were grouped by similar topics. A summary of the combined results for all terms is shown in Tables 17 – 19.

Table 17

Number and Percent of Students Answering Order of Operations and Properties of Exponents Questions Correctly

	Order of operations	Properties of exponents	Properties of exponents	Properties of exponents	Properties of exponents
Dev Math (MPT ≤10)	21.5%	32.9%	65.1%	21.0%	43.9%
College Algebra (11≤ MPT ≤13)	36.0%	49.7%	77.4%	40.4%	60.1%
Precalculus (14≤ MPT ≤17)	51.3%	65.2%	85.2%	59.8%	70.6%
Calculus I (MPT ≥18)	73.7%	87.8%	93.9%	86.8%	86.4%
Total	49.3%	62.6%	82.3%	56.5%	68.1%
Total Correct	6,073	7,724	10,144	6,970	8,390
Total Students	12,329	12,329	12,329	12,329	12,329

Results shown in Table 17, show that students in Developmental Mathematics have difficulty solving problems that involve applying the properties of exponents. Except for one of the questions, less than 50% of the students answered the questions correctly. These students are the least mathematically prepared, and they lack understanding of several mathematics concepts. Half or more of the students placing in College Algebra were able to answer properties of exponents correctly in three out of the four questions addressing this topic. Sixty percent or more of the students placing in higher-level mathematics were able to answer these questions correctly.

Table 18 shows the percentage of students answering correctly questions requiring rational number operations. As it can be seen from these results, less than half of the students placing in Developmental Mathematics responded correctly eight of the nine questions in this group. Similarly, for the students placing in College Algebra, less than half of the students answered correctly seven of the nine items including rational number operations. Moreover, less than half of the students placing in Precalculus were able to answer correctly four of the nine questions with rational number operations. These results suggest that students are not adequately prepared to work with mathematical expressions that include rational numbers or expressions.

Table 19 shows the combined percentages of students answering correctly questions requiring solving and simplifying given expressions as well as other questions with other topics. Students placing in Developmental Mathematics had difficulty answering correctly questions requiring solving various types of equations as evidenced by the lower percentages of correct answers. Similarly, students placing in College Algebra faced similar challenges. One possible explanation on why students missed these questions is their difficulties performing operations with negative numbers. All of these questions, required manipulations of negative numbers to find the solution. This postulation was examined by looking at the results of the Precalculus course error analysis discussed later in this chapter.

The results of evaluating the frequency with which new students selected correct answers to the Mathematics Placement Tests from fall 2010 – fall 2013 suggest that a key mathematics competency affecting mathematics preparedness is students understanding of rational number operations. These results also indicate that students placing in Developmental Mathematics lack skills in performing operations with exponents, simplifying algebraic expressions, and solving equations and systems of equations. Students placing in College Algebra faced similar challenges. One possible explanation on why students missed these questions is their difficulties performing operations with negative numbers. All of these questions, required manipulations of negative numbers to find the solution. This proposition was examined by looking at the results of the confirmatory factor analysis discussed later in this chapter.

Table 18

Number and Percent of Students Answering Fractions and Rational Functions Questions Correctly

	Rationalize a denominator	Simplify complex fraction	Simplify rational function	Simplify rational function	Solve a rational equation	Multiply rational functions	Add/ Subtract rational expressions	Evaluate a formula	Solve ratio and proportion equation
Dev Math (MPT ≤10)	12.5%	24.1%	40.0%	16.5%	38.6%	16.2%	16.1%	23.4%	50.3%
College Algebra (11≤ MPT ≤13)	24.4%	40.6%	57.5%	29.3%	48.4%	24.1%	31.3%	42.3%	72.4%
Precalculus (14≤ MPT ≤17)	41.6%	60.8%	73.9%	44.3%	55.4%	30.8%	47.8%	61.7%	82.9%
Calculus I (MPT ≥18)	74.2%	87.9%	92.5%	82.0%	69.1%	60.9%	82.6%	89.2%	95.1%
Total	42.6%	57.9%	69.6%	47.7%	54.9%	36.2%	49.1%	58.7%	77.6%
Total Correct	5,253	7,134	8,576	5,879	6,773	4,459	6,055	7,235	7,027
Total Students	12,329	12,329	12,329	12,329	12,329	12,329	12,329	12,329	9,054

Table 19

Number and Percent of Students Answering Solve & Simplify Questions Correctly

	Simplify a radical	Solve a linear inequality	Solve a linear equation	Solve a quadratic equation	Solve a system of equations	Find x and y intercepts	Graph a linear equation in two variables	Simplify an algebraic expression	Simplify an algebraic expression	Substitute & evaluate absolute value	Substitution
Dev Math (MPT ≤10)	40.2%	58.1%	20.0%	17.1%	23.5%	24.2%	49.2%	28.5%	50.6%	67.1%	43.9%
College Algebra (11≤ MPT ≤13)	60.6%	79.3%	31.5%	26.8%	38.0%	38.9%	65.0%	54.9%	69.4%	81.9%	60.1%
Precalculus (14≤ MPT ≤17)	74.6%	89.1%	46.7%	39.1%	50.6%	54.7%	73.3%	72.8%	81.0%	90.1%	70.6%
Calculus I (MPT ≥18)	88.9%	95.4%	71.8%	69.8%	75.7%	80.1%	84.2%	90.3%	93.6%	95.7%	86.4%
Total	69.2%	82.7%	46.2%	41.9%	50.6%	53.4%	70.1%	65.6%	76.4%	86.4%	68.1%
Total Correct	8,536	10,200	5,695	5,172	6,233	6,585	8,646	8,093	9,424	8,031	8,390
Total Students	12,329	12,329	12,329	12,329	12,329	12,329	12,329	12,329	12,329	9,298	12,329

Research Question 2

What demographic factors and incoming data (Mathematics SAT score, high school GPA, entry college) characterize students placing at different levels of mathematics college readiness?

Demographic and Academic Entering Characteristics

The students' demographic characteristics and academic entering data (Mathematics SAT score, high school GPA, entry college) were obtained from the University database. Descriptive and inferential statistical analyses were conducted to determine the parameters characterizing students at each level of mathematics preparedness.

Gender and Mathematics Preparedness

Table 20 shows the percent of women and men placing in each Mathematics Level. More women placed in Developmental Mathematics compared to men. On the other hand, more men placed in Calculus I compared to women. The gender difference could be the result of more men choosing engineering as a major. The College of Engineering at this institution, on average, enrolls 13 percent of the new freshmen coming in the fall semesters and 58 percent of their incoming class placed in Calculus I. College choice is shown in Table 28. The percent of women placing in Calculus I decreased from 28.0 percent in fall 2010 to 15.4 percent in fall 2013. Both, the percent of men and women placing in Developmental Mathematics increased in fall 2013. In general, more women place in lower level mathematics compared to men. A Chi-square test of independence was conducted to examine the association between gender and frequency of placement in the given mathematics levels. All expected cell frequencies were higher than five. The Chi-square test indicated that there was a statistically significant association between mathematics placement and gender for all terms, $\chi^2(3) = 80.01$, $p < .005$ for fall 2010; $\chi^2(3) = 91.02$, $p < .005$ for fall 2011; $\chi^2(3) = 145.52$, $p < .005$ for fall 2012; and $\chi^2(3) = 166.25$, $p < .005$ for fall 2013.

Mathematics Placement Test mean and standard deviation for males and females placing in each mathematics level were also calculated. Table 21 shows the mean scores for the Mathematics Placement Test within each mathematics level. As it can be seen, the mean score is very similar for men and women. A one-way ANOVA was conducted to determine if the overall Mathematics Placement Test scores were different for new men and women entering the University between the fall semesters of 2010 and 2013. There were no outliers, as assessed by boxplot; data were normally distributed for both men and women; there was homogeneity of variances, as assessed by Levene's test of homogeneity of variances ($p = .977$). The overall Mathematics Placement Test score was lower for women ($M = 13.98$, $SD = 4.966$) than for men ($M = 15.91$, $SD = 4.946$). The difference between the scores was statistically significant, $F(1,12327) = 470.38$, $p < .05$.

Table 20

Number and Percent of New Incoming Freshmen Placing in each Mathematics Level by Gender

Term	Dev Math (MPT ≤10)	College Algebra (11≤ MPT ≤13)	Precalculus (14≤ MPT ≤17)	Calculus I (MPT ≥18)	N	Bar Graph
Female						
Fall 2010	22.8%	22.4%	26.8%	28.0%	1,488	
Fall 2011	21.0%	20.9%	27.3%	30.9%	1,585	
Fall 2012	25.4%	21.9%	27.3%	25.4%	1,588	
Fall 2013	38.0%	25.9%	20.7%	15.4%	1,480	
Male						
Fall 2010	13.1%	18.1%	27.7%	41.1%	1,417	
Fall 2011	12.5%	14.8%	27.8%	44.9%	1,533	
Fall 2012	12.0%	18.1%	28.7%	41.1%	1,687	
Fall 2013	22.1%	20.1%	26.6%	31.1%	1,551	
Total						
Fall 2010	18.1%	20.3%	27.2%	34.4%	2,905	
Fall 2011	16.8%	17.9%	27.5%	37.8%	3,118	
Fall 2012	18.5%	19.9%	28.0%	33.5%	3,275	
Fall 2013	29.9%	23.0%	23.7%	23.5%	3,031	

Note: MPT = Mathematics Placement Test

Table 21

Mathematics Placement Test Mean Score and Standard Deviation by Gender

	Dev Math (MPT ≤10)				College Algebra (11≤ MPT ≤13)				Precalculus (14≤ MPT ≤17)				Calculus I (MPT ≥18)			
	Female		Male		Female		Male		Female		Male		Female		Male	
Term	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*
Fall 2010	8.12	1.704	8.19	1.847	12.01	.829	12.07	.802	15.47	1.134	15.49	1.119	20.74	2.141	21.05	2.287
Fall 2011	8.07	1.808	8.28	1.856	12.08	.827	12.02	.820	15.40	1.117	15.51	1.140	20.80	2.150	20.98	2.108
Fall 2012	8.05	1.923	8.23	1.608	12.04	.794	11.97	.790	15.32	1.070	15.48	1.125	20.51	2.116	20.85	2.198
Fall 2013	7.83	1.980	7.98	1.872	11.99	.817	11.96	.784	15.40	1.115	15.42	1.094	20.51	1.934	20.72	2.027
Total	7.99	1.879	8.14	1.810	12.03	.816	12.00	.798	15.40	1.108	15.48	1.120	20.67	2.110	20.91	2.164

Note. MPT = Mathematics Placement Test

Ethnicity and Mathematics Preparedness

Table 22 shows the number and percent of new incoming freshmen placing in each mathematics level by ethnicity. Except in fall 2013, Caucasians entering freshmen were more likely to place in Calculus I than in other mathematics entry-level courses. In fall 2013, 24 percent of the Caucasians placed in Calculus I compared to 29 percent who placed in Developmental Mathematics. Asian students placed in Calculus I in larger percentages than any other ethnic group: 52 percent in fall 2010, 61 percent in fall 2011, 53 percent in fall 2012 and 38 percent in fall 2013. Latino students are less likely to place in Developmental Mathematics than African American students are (16% vs. 24% in fall 2010; 16% vs. 24% in fall 2011; 22% vs. 25% in fall 2012; and 32% vs. 36% in fall 2013). Latino and Caucasian students placed in Calculus I at similar rates (34% vs. 36%; 35% vs. 39%; 29% vs. 34%; 21% vs. 24%). A Chi-square test indicated an association between mathematics placement and ethnicity for all terms. All expected cell frequencies were greater than five. There was a statistically significant association between ethnicity and mathematics placement, $\chi^2(9) = 47.58$, $p < .005$ for fall 2010; $\chi^2(9) = 80.26$, $p < .005$ for fall 2011; $\chi^2(9) = 71.18$, $p < .005$ for fall 2012; and $\chi^2(9) = 46.79$, $p < .005$ for fall 2013.

Table 23 shows the overall Mathematics Placement Test mean and standard deviation for African American, Asian, Latino and Caucasian students placing in each mathematics level. A one-way ANOVA test was run to compare the overall Mathematics Placement Test mean scores for the four entry terms and to determine if there were differences in the test performance among the students from each ethnic group. Inspection of the data show that there were no outliers and the data was normally distributed for each group, as assessed by boxplot and Shapiro-Wilk test ($p < .05$), respectively. Homogeneity of variances was violated, as assessed by Levene's Test of Homogeneity of Variance (*p* = .006). MPT score was statistically significantly different between different ethnic groups, Welch's F(3, 1790.821) = 87.988, p < .0005. The MPT mean score of African American students was the lowest (*M* = 13.74, *SD* = 4.77) compared to the Latino (*M* = 14.68, *SD* = 4.95), the Caucasian (*M* = 15.03, *SD* = 5.01), and the Asian (*M* = 17.04, *SD* = 4.94) groups. Games-Howell post hoc analysis revealed that the MPT score difference between Africans Americans and Latinos (.94, 95% CI [.42 to 1.46]) was statistically significant (p < .0005) as well as the difference between MPT scores of Caucasians (1.29, 95% CI [.96 to 1.63], p <.0005) and the MPT scores of Asians (3.659, 95% CI [3.07 to 4.25], p <.0005). In addition, there were statistically significant differences in MPT scores between the Latino student group and the Asian group (2.72, 95% CI [2.06 to 3.38], p <.0005) and between the Caucasian group and the Asian student group (2.37, 95% CI [1.84 to 2.89], p <.0005). The Asian students had the highest MPT score average. A summary of these results is shown in Table 24.

Table 22

Number and Percent of New Incoming Freshmen Placing in each Mathematics Level by Ethnicity

Ethnicity	Term	Dev Math (MPT $\leq$10)	College Algebra (11$\leq$ MPT $\leq$13)	Precalculus (14$\leq$ MPT $\leq$17)	Calculus I (MPT $\geq$18)	N	Bar Graph
Afr. Amer.	F10	*24%*	*24%*	*25%*	*27%*	410	
	F11	*24%*	*22%*	*29%*	*26%*	438	
	F12	*25%*	*22%*	*31%*	*22%*	448	
	F13	*36%*	*29%*	*23%*	*13%*	332	
Asian	F10	*8%*	*11%*	*30%*	*52%*	145	
	F11	*6%*	*11%*	*22%*	*61%*	166	
	F12	*8%*	*15%*	*24%*	*53%*	173	
	F13	*19%*	*17%*	*26%*	*38%*	145	
Latino	F10	*16%*	*23%*	*28%*	*34%*	238	
	F11	*16%*	*23%*	*26%*	*35%*	223	
	F12	*22%*	*24%*	*25%*	*29%*	238	
	F13	*32%*	*24%*	*24%*	*21%*	213	
Caucasian	F10	*17%*	*20%*	*27%*	*36%*	1916	
	F11	*16%*	*17%*	*28%*	*39%*	2002	
	F12	*18%*	*19%*	*29%*	*34%*	2111	
	F13	*29%*	*23%*	*24%*	*24%*	2081	
Total	F10	*18%*	*20%*	*27%*	*35%*	2709	
	F11	*17%*	*18%*	*28%*	*38%*	2829	
	F12	*19%*	*20%*	*28%*	*33%*	2970	
	F13	*30%*	*24%*	*24%*	*23%*	2771	

Note: MPT = Mathematics Placement Test

Table 23

Mathematics Placement Test (MPT) Mean Score and Standard Deviation by Ethnicity

	Dev Math (MPT ≤10)			College Algebra (11≤ MPT ≤13)			Precalculus (14≤ MPT ≤17)			Calculus I (MPT ≥18)			Total		
	N	*M*	*SD*	*N*	*M*	*SD*	*N*	*M*	*SD*	*N*	*M*	*SD*	*N*	*M*	*SD*
F10															
African American	97	8.01	1.950	99	12.12	.824	103	15.39	1.157	111	20.38	1.982	410	14.20	4.826
Asian	11	8.09	2.700	16	11.94	.772	43	15.88	1.005	75	21.36	2.480	145	17.69	4.768
Latino	37	7.95	1.649	54	11.98	.812	66	15.55	1.205	81	20.69	2.177	238	15.31	4.844
Caucasian	331	8.24	1.667	382	12.01	.821	523	15.48	1.113	680	20.98	2.227	1,916	15.49	4.990
F11															
African American	104	7.74	1.995	94	11.97	.822	128	15.52	1.072	112	20.35	2.104	438	14.15	4.872
Asian	10	9.00	2.000	19	12.26	.806	36	15.33	1.121	101	21.65	2.104	166	18.45	4.633
Latino	35	8.40	1.499	52	12.33	.760	58	15.66	1.132	78	21.08	2.069	223	15.64	4.849
Caucasian	326	8.21	1.724	343	12.06	.822	561	15.45	1.152	772	20.79	2.095	2,002	15.75	4.907
F12															
African American	112	8.03	1.896	99	12.06	.740	138	15.32	1.107	99	20.37	1.946	448	13.89	4.638
Asian	14	8.57	1.342	26	11.62	.752	41	15.32	1.105	92	21.50	2.115	173	17.50	4.928
Latino	53	7.91	1.863	57	11.96	.844	59	15.19	1.090	69	20.57	2.226	238	14.35	4.972
Caucasian	373	8.17	1.822	409	12.03	.788	607	15.44	1.097	722	20.65	2.179	2,111	15.28	4.850
F13															
African American	118	7.88	1.841	95	11.95	.790	76	15.32	1.146	43	20.77	1.962	332	12.42	4.533
Asian	27	8.70	1.660	25	11.88	.833	38	15.45	.978	55	21.25	2.075	145	15.78	5.097
Latino	68	8.19	1.756	51	11.96	.799	50	15.20	1.143	44	20.73	1.981	213	13.33	4.830
Caucasian	607	7.85	1.957	483	11.98	.806	494	15.43	1.095	497	20.64	2.001	2,081	13.66	5.039

Table 24

Differences between Mathematics Placement Test Score Means for New Incoming Freshmen by Ethnicity for All Terms

	African American	Latino	White
African American			
Hispanic	-.94*		
White	-1.29*	-0.35	
Asian	-3.66*	-2.72*	-2.37*

*Note:**. The mean difference is significant at the .05 level.

High School GPA, Mathematics SAT Score, and Mathematics Preparedness

Tables 25 and 26 show the average high school GPA and Mathematics SAT scores respectively for each entry term and mathematics placement group. A preliminary inspection of these data indicates that students entering the University in fall 2012 and fall 2013 had higher high school GPA, and Mathematics SAT scores. However, in looking at the mathematics placement distribution shown in Table 8, more students placed in lower level mathematics in fall 2012 and fall 2013 compared with students coming in fall 2010 and fall 2011.

A multiple regression was run to predict Mathematics Placement Test scores from High School GPA, and Mathematics SAT scores. Inspection of the data revealed that there were four outliers in fall 2010, four in fall 2011, one in fall 2012 and three in fall 2013. Further review of these students' records indicated that their Mathematics Placement Test scores for the time when they took the test were not an accurate representation of the students' capabilities. In reviewing their records, it could be seen that the students retook the test and obtained higher scores. These students' records were removed from the analysis to avoid incorrect results. The total number of cases were 2,637 for fall 2010, 2,855 in fall 2011, 2,691 in fall 2012, and 2,660 in fall 2013. The analysis was rerun without the students' data, and there were minimal changes to the regression coefficients.

Linearity was assessed by partial regression plots and a plot of studentized residuals against the predicted values. There was independence of residuals, as assessed by visual inspection of plots of the residuals versus independent variables. There was homoscedasticity, as assessed by visual inspection of a plot of studentized residuals versus unstandardized predicted values. There was no evidence of multicollinearity, as assessed by tolerance values greater than 0.1. There were no studentized deleted residuals greater than ±3 standard deviations, no leverage values greater than 0.2, and values for Cook's distance above 1. The assumption of normality was met, as assessed by a Q-Q Plot (Laerd Statistics, 2015). The multiple regression model statistically significantly predicted Mathematics Placement Test scores for fall 2010, $F(2, 2630) = 864.16$, $p < .005$, with adjusted $R^2 = .397$. For the fall 2011 through fall 2013, the following results were obtained $F(2, 2848)=1008.93$, $p< .005$, with adjusted $R^2 = .414$; $F(2, 2687)=932.28$, $p< .005$, with adjusted $R^2 = .407$; and $F(2,2653) = 866.56$, with adjusted $R^2 = .395$, respectively. In all cases, both variables added statistically significantly to the prediction, $p < .005$. High School GPA and Mathematics SAT predict approximately 40% of the variance of the Mathematics Placement Test scores.

Table 25

Sample Size, High School GPA Mean and Standard Deviation for New Freshmen Entering between Fall 2010 – Fall 2013 by Mathematics Level

	Dev Math (MPT ≤10)			College Algebra (11≤ MPT ≤13)			Precalculus (14≤ MPT ≤17)			Calculus I (MPT ≥18)			Total		
	N	Mean	SD	N	Mean	SD	N	Mean	SD	N	Mean	SD	N	Mean	SD
Fall 2010	479	3.48	0.402	553	3.53	0.414	752	3.61	0.42	922	3.83	0.466	2,706	3.65	0.454
Fall 2011	491	3.50	0.408	527	3.54	0.410	811	3.60	0.434	1,100	3.83	0.476	2,929	3.66	0.463
Fall 2012	575	3.55	0.395	625	3.59	0.407	865	3.69	0.428	1,026	3.88	0.461	3,091	3.71	0.449
Fall 2013	868	3.69	0.400	680	3.82	0.403	692	3.84	0.411	692	4.00	0.453	2,925	3.83	0.431

Note. MPT = Math Placement Test Score

Table 26

Sample Size, Math SAT Mean Score and Standard Deviation for New Freshmen Entering between Fall 2010 – Fall 2013 by Mathematics Level

	Dev Math (MPT ≤10)			College Algebra (11≤ MPT ≤13)			Precalculus (14≤ MPT ≤17)			Calculus I (MPT ≥18)			Total		
	N	Mean	SD	N	Mean	SD	N	Mean	SD	N	Mean	SD	N	Mean	SD
Fall 2010	494	494.17	53.741	571	514.54	52.642	764	543.74	54.900	969	594.93	62.964	2,798	546.76	69.101
Fall 2011	499	488.30	48.542	534	513.93	49.990	830	538.13	50.703	1,129	590.73	64.360	2,992	545.35	67.995
Fall 2012	526	496.73	51.091	571	520.47	47.060	799	546.98	55.076	939	597.03	65.399	2,835	548.90	68.083
Fall 2013	827	515.93	49.071	637	538.34	48.875	651	565.73	53.134	626	614.92	60.147	2,741	555.57	64.405

Note. MPT =Math Placement Test Score

Unstandardized and standardized regression coefficients and standard errors can be found in Table 27. Examining the β weights, it can be seem that Mathematics SAT scores had the most substantial impact on mathematics placement (β = .564; β = .578; β = .572; β = .576 for fall semesters 2010, 2011, 2012 and 2013 respectively). This impact is more than three times the effect of high school GPA (β = .168; β = .176; β = .172; β = .156 for fall semesters 2010, 2011, 2012 and 2013 respectively). This result suggests that since high school GPA is a composite score of various subjects, it is not a good predictor of mathematics placement.

Table 27

Summary of Multiple Regression Analysis for MPT from High School GPA and Mathematics SAT Score

	Variable	*B*	SE_B	β
Fall 2010	Intercept	-13.259	.751	
	Math SAT	.04	.001	.564*
	HS GPA	1.812	.170	.168*
Fall 2011	Intercept	14.391	.726	
	Math SAT	.042	.001	.578*
	HS GPA	1.896	.159	.176*
Fall 2012	Intercept	14.375	.749	
	Math SAT	.041	.001	.572*
	HS GPA	1.863	.167	.172*
Fall 2013	Intercept	18.053	.845	
	Math SAT	.045	.001	.576*
	HS GPA	1.798	.178	.156*

Note. *B* = unstandardized regression coefficient; SE_B = Standard error of the coefficient; β = standardized coefficient
*. $p < .05$

College Choice and Mathematics Preparedness

Table 28 shows the percent and number of new entering freshmen placing in the various entry-level mathematics courses by College. The College of Engineering had the largest number of students placing in Calculus I. This is to be expected since most students intending to major in engineering would probably take additional and advanced mathematics courses in high school. As seen in the literature review, taking additional and advanced mathematics courses contributes to students' mathematical readiness (Long et al., 2009). A Chi-square test indicated an association between mathematics placement and student selection of college for all terms. All expected cell frequencies were greater than five. There was a statistically significant association between college selection and Mathematics placement, $\chi^2(21) = 205.53$, $p < .005$ for fall 2010; $\chi^2(21) = 206.87$, $p < .005$ for fall 2011; $\chi^2(21) = 285.82$, $p < .005$ for fall 2012; and $\chi^2(21) = 296.47$, $p < .005$ for fall 2013.

Table 29 shows the mean and standard deviation for students in each college. Inspection of the data show that there were no outliers and the data were normally distributed for each group, as assessed by boxplot and Shapiro-Wilk test ($p < .05$), respectively. Homogeneity of variances was violated, as assessed by Levene's Test of Homogeneity of Variance ($p < .005$). The mean MPT score of the students entering the College of Education was the lowest ($M = 13.93$, $SD = 4.77$) compared to University College ($M = 14.04$, $SD = 4.70$), Health & Human Services ($M = 14.30$, $SD = 4.81$), Liberal Arts & Sciences ($M = 14.33$, $SD = 5.09$), Arts & Architecture ($M = 14.93$, $SD = 5.38$), Business ($M = 15.00$, $SD = 4.79$), Computing & Informatics ($M = 17.18$, $SD =$

4.65), and Engineering (*M* = 18.11, *SD* = 4.58). Mean MPT scores were statistically significantly different for students entering different Colleges, Welch's $F(7, 2775.76) = 159.101$, $p < .005$.

Table 28

Number and Percent of New Incoming Freshmen Placing in each Mathematics Level by College

College	Term	Dev Math (MPT ≤10)	College Algebra (11≤ MPT ≤13)	Precalculus (14≤ MPT ≤17)	Calculus I (MPT ≥18)	N	Bar Graph
Arts & Architecture	F10	*20.1%*	*21.5%*	*20.8%*	*37.5%*	144	
	F11	*19.1%*	*13.9%*	*27.0%*	*40.0%*	115	
	F12	*22.1%*	*16.4%*	*30.3%*	*31.1%*	122	
	F13	*31.2%*	*23.7%*	*25.8%*	*19.4%*	93	
Business	F10	*17.0%*	*17.9%*	*29.4%*	*35.8%*	218	
	F11	*12.5%*	*15.8%*	*30.0%*	*41.6%*	303	
	F12	*13.6%*	*20.8%*	*32.2%*	*33.3%*	360	
	F13	*32.6%*	*23.5%*	*22.7%*	*21.2%*	396	
Computing & Informatics	F10	*6.5%*	*17.6%*	*25.0%*	*50.9%*	108	
	F11	*11.0%*	*7.6%*	*23.7%*	*57.6%*	118	
	F12	*1.5%*	*11.9%*	*29.1%*	*57.5%*	134	
	F13	*12.6%*	*19.9%*	*33.8%*	*33.8%*	151	
Education	F10	*23.7%*	*23.7%*	*25.9%*	*26.7%*	135	
	F11	*19.7%*	*24.4%*	*26.0%*	*29.9%*	127	
	F12	*19.6%*	*21.7%*	*34.1%*	*24.6%*	138	
	F13	*37.6%*	*29.4%*	*21.1%*	*11.9%*	109	
Engineering	F10	*6.6%*	*12.1%*	*21.5%*	*59.8%*	423	
	F11	*3.8%*	*11.2%*	*21.2%*	*63.8%*	392	
	F12	*5.6%*	*9.2%*	*23.6%*	*61.6%*	411	
	F13	*9.0%*	*13.6%*	*27.1%*	*50.2%*	420	
Health & Human Services	F10	*18.8%*	*24.2%*	*27.4%*	*29.6%*	405	
	F11	*21.8%*	*20.3%*	*27.4%*	*30.5%*	325	
	F12	*20.7%*	*17.7%*	*31.5%*	*30.2%*	305	
	F13	*35.6%*	*25.2%*	*20.6%*	*18.7%*	326	
Liberal Arts & Sciences	F10	*21.5%*	*23.3%*	*26.6%*	*28.6%*	790	
	F11	*20.1%*	*18.4%*	*27.7%*	*33.8%*	891	
	F12	*22.6%*	*21.6%*	*26.6%*	*29.2%*	932	
	F13	*35.2%*	*25.3%*	*21.3%*	*18.2%*	751	
University College	F10	*21.4%*	*20.3%*	*32.7%*	*25.6%*	669	
	F11	*19.1%*	*21.1%*	*30.6%*	*29.3%*	834	
	F12	*23.1%*	*25.2%*	*27.3%*	*24.4%*	865	
	F13	*33.9%*	*24.3%*	*24.6%*	*17.2%*	768	
Total	F10	*18.0%*	*20.4%*	*27.2%*	*34.3%*	2,892	
	F11	*16.8%*	*17.8%*	*27.6%*	*37.7%*	3,105	
	F12	*18.4%*	*20.0%*	*28.0%*	*33.6%*	3,267	
	F13	*29.7%*	*23.0%*	*23.8%*	*23.5%*	3,014	

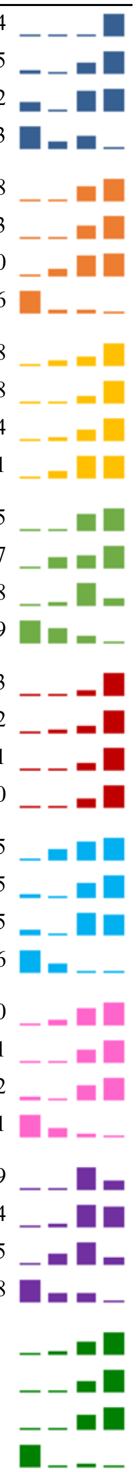


Table 29

Mathematics Placement Test Score and Standard Deviation for New Incoming Freshmen by College for All Terms

College	N	*M*	*SD*
Arts & Architecture	474	14.93	5.375
Business	1,277	15.00	4.794
Computing & Informatics	511	17.18	4.647
Education	509	13.93	4.771
Engineering	1,646	18.11	4.579
Health & Human Services	1,361	14.30	4.808
Liberal Arts & Sciences	3,364	14.33	5.089
University College	3,136	14.04	4.700
Total	12,278	14.95	5.040

Note. MPT = Mathematics Placement Test

The differences in MPT mean scores for students entering the various colleges are shown in Table 30. Games-Howell post hoc analysis revealed that there are statistically significant differences in the mean MPT scores of students entering the colleges of Business, Computing & Informatics, and Engineering and students entering other colleges (Arts & Architecture, Education, Health & Human Services, Liberal Arts & Sciences and University College). In addition, there were statistically significant differences in MPT scores between students entering the College of Arts & Architecture and students entering the College of Education and University College. The College of Engineering students had the highest overall MPT average score, 18.11.

Table 30

Mathematics Placement Test Mean Score Differences by College for All Terms

	Education	University College	Health & Human Services	Liberal Arts & Sciences	Arts & Architecture	Business	Computing & Informatics
Education							
University College	-.11						
Health & Human Services	-.37	-.26					
Liberal Arts & Sciences	-.40	-.29	-.03				
Arts & Architecture	-1.00*	-.89*	-.63	-.60			
Business	-1.07*	-.96*	-.69*	-.67*	-.07		
Computing & Informatics	-3.25*	-3.15*	-2.88*	-2.85*	-2.26*	-2.19*	
Engineering	-4.18*	-4.07*	-3.81*	-3.78*	-3.18*	-3.11*	.92*

Note. MPT = Mathematics Placement Test
*. The mean difference is significant at the .05 level.

Research Question 3

What is the level of understanding of key mathematics competencies of incoming students in an entry-level mathematics course?

Confirmatory Factor Analysis

Phase 2 of this study has two purposes. First, to confirm the results obtained in Phase 1 and evaluate the level of understanding of key mathematics competencies identified in Phase 1. Results from Phase 1 suggest that new freshmen entering college are inadequately prepared to perform rational number operations. An Exploratory Factor Analysis (EFA) was employed to determine the links among the 25 items in the

Mathematics Placement Test and the minimum number of factors that would account for the covariance among these variables. It was assumed that the factors would be correlated because all measures involved mathematical concepts (Byrne, 1998). Therefore, a rotated solution to the EFA was chosen. Joreskög (2016) recommends selecting the reference variables rotated solution because they have the largest factor loadings and makes any factors that are not statistically significant equal to zero. The EFA analysis identified four factors. Measures (questions) were grouped according to the topics identified in Phase 1 of the project to develop a hypothetical model. The four factors were labeled Ratio, Neg(ative), Solve, and Simpl(ify) corresponding to the Mathematics Placement Test topics rational number operations, negative number operations, equation solving, and simplification of expressions. A Confirmatory Factor Analysis (CFA) was conducted to help identify factors affecting mathematics preparedness as measured by the Mathematics Placement Test questions (Fabrigar & Wegener, 2012). The proposed model was used to confirm that the covariance of the measures included in the model would represent the covariance of the population. Questions that measured the previously identified topics were selected as part of the measurement model. Figure 2 shows the hypothetical model to be used for the CFA. As shown in Table 4, more students placed in lower level mathematics in fall 2012 and fall 2013. Consequently, a separate CFA was performed for each entry term. The analysis was conducted utilizing the results from the Mathematics Placement Tests given from fall 2010 – fall 2013. LISREL 9.30 was used to perform the analysis. Loading factors will be shown for the individual models.

In the model, QR represents the measures for rational number operations; QN represents the negative number operation measures; QV represents the equation solving measures; and QS represents the simplifying measures. The question topics for all fall 2010-2013 are the same, but the question numbers may be different for different years. The Mathematics Placement Test questions are included in Appendices A-D.

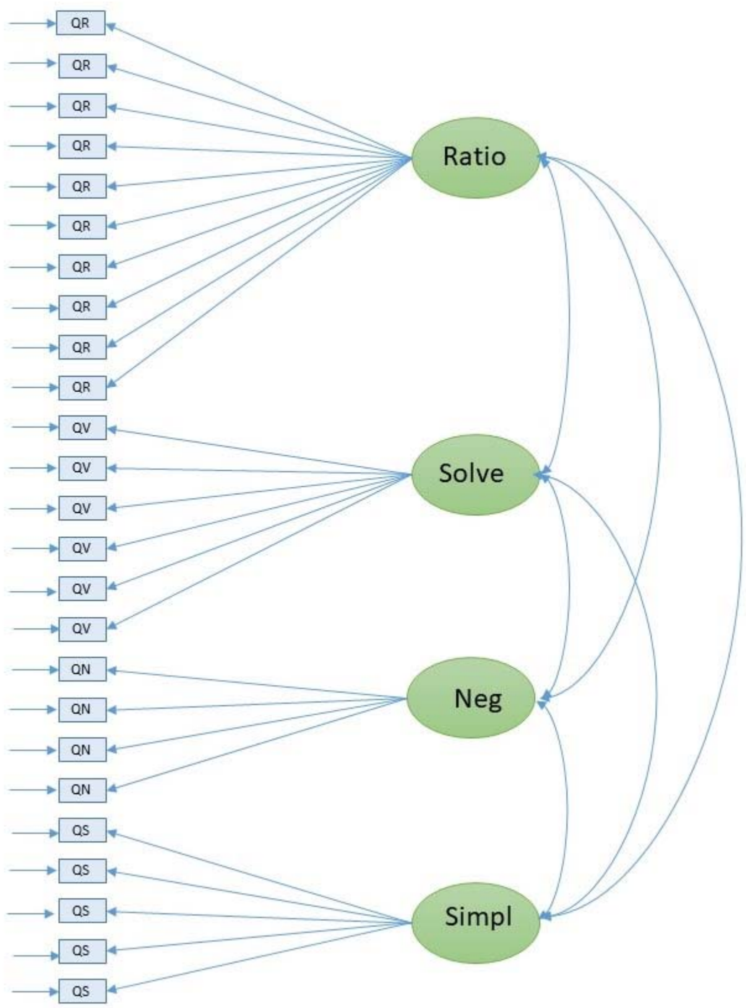


Figure 2. Hypothesized CFA model for Mathematical Key Competencies Measured by the Mathematics Placement Test

Model Identification

The hypothesized model has 25 observed variables (p) and 4 factors (m). This information was used to determine if four factors could be extracted from the data provided. Per Brown (2015), the number of elements in the correlation or covariance matrix (a) and the number of parameters that can be estimated (b) can be obtained using the following equations:

$$a = [p * (p + 1)]/2$$

$$b = (p * m) + \frac{[m * (m + 1)]}{2} + p - m^2$$

With *p,* representing the number of observed variables and *m* the number of factors. For the hypothesized model *a* = 325 and *b* = 119. Since *a* is greater than *b,* the model is over-identified, meaning that the number of observed variables exceeds the number of parameters to be estimated. Consequently, a unique solution could be found.

The following indices were used to assess the goodness of fit of the model, χ^2, χ^2/df ratio (best if less than 2.0), Non-Normed fit index (NNFI, best if 0.90 or greater), normed fit index (NFI, best if 0.9 or greater), and root mean square error of approximation (RMSEA, best if 0.05 or less). Figures 3 – 6 show the path diagram representing the relationship between the latent variable and the independent variables (questions) for each entry term.

Fits to the four-factor models were tested initially using the results from the EFA and the topic identification for the measures for all terms. However, in all four data sets, it was found beneficial to reduce the model to a two-factor model. This option was chosen for two reasons. First, to avoid empirically underidentified models. Empirically under-identification occurs when characteristics of the input matrix cause the analysis not to yield a unique set of parameter estimates (T. A. Brown, 2015; Kline, 2011). One way to avoid this situation is to ensure that at least two measures are associated with every factor. Consequently, when there were less than two measures associated with a factor, that factor was eliminated. Second, some observed variables were not included in the model because of their reliability to measure its underlying construct given by the Squared Multiple Correlation (SMC) or $R_{SMC}{}^{2}$ (Kline, 2011). $R_{SMC}{}^{2}$ is the "proportion of variance in the measure accounted for by the factor" (Ullman, 2013, p. 733). The minimum desired value for the loading of a variable is .32. However, Comrey and Lee suggested that .45 is considered fair and .55 is considered good (as cited in Ullman, 2013). As suggested by Ullman (2013), sometimes is convenient to select a cutoff to facilitate the factor interpretation. For that reason, in this study, loadings of 0.25 or larger were considered.

Fall 2010 EFA and CFA Models

Loading factors resulting from the EFA analysis for fall 2010 are shown in Table 31. These results were used to develop the model to be used as input for the CFA analysis. Measures with loading values above .25 were selected to ensure that each factor was associated with at least two measures. These values are shown in bold font in Table 31.

Table 31

Loading Factors Obtained from the EFA Analysis for the Fall 2010 Four-Factor Model

Question	Negative	Ratio	Solve	Simplify	Unique Variance
QQ6	**0.918**	0	0	0	0.157
QQ15	**0.362**	0.128	**0.280**	-0.115	0.700
QQ5	**0.305**	**0.307**	**0.444**	-0.424	0.623
QQ4	0.125	0.043	**0.458**	-0.252	0.839
QQ18	0.107	0.089	**0.699**	-0.289	0.608
QQ2	0.105	**0.378**	0.138	-0.158	0.814
QQ25	0.079	0.225	**0.452**	-0.139	0.691
QQ9	0.079	**0.436**	**0.522**	-0.396	0.575
QQ21	0.057	**0.342**	0.169	0.232	0.536
QQ10	0.054	**0.463**	0.054	-0.049	0.758
QQ20	0.039	**0.289**	**0.445**	-0.035	0.575
QQ8	0.036	**0.288**	**0.655**	-0.277	0.520
QQ1	0.028	**0.592**	0.169	-0.229	0.664
QQ7	0.021	**0.308**	**0.512**	-0.231	0.646
QQ12	0.019	0.17	**0.759**	-0.326	0.538
QQ3	0.017	**0.351**	**0.577**	-0.333	0.599
QQ11	0.013	**0.295**	**0.335**	-0.257	0.829
QQ24	0	0	0	**0.550**	0.697
QQ14	0	**0.618**	0	0	0.618
QQ23	0	0	**0.764**	0	0.417
QQ17	-0.008	0.242	**0.251**	0.037	0.783
QQ13	-0.031	-0.06	**0.628**	-0.031	0.681
QQ16	-0.044	**0.387**	**0.354**	-0.086	0.660
QQ22	-0.053	0.165	**0.554**	-0.072	0.645
QQ19	-0.082	0.229	0.126	**0.270**	0.734

Before testing the final hypothesized model, a one-factor model was tested to examine unidimensionality. A one-factor model was tested and found not to fit the covariance matrix ($\chi2$ = 668.18, *df* =275, *p*<.00001; RMSEA [90%CI] =.0577, .0611; CFI=.981; GFI=.992). All parameter estimates were statistically significant. An examination of the standardized residuals indicated a misfit of the data, with standardized residuals as high as 6.99 (between QQ15 and QQ6). These results suggest that the data may be multidimensional.

After conducting the CFA analysis with the four-factor model, only measures where the R^2, the proportion of variance in the measure accounted for by the factor, was .25 or larger were retained. Consequently, a two-factor model was obtained. Figure 3 depicts the model for fall 2010. As can be seen from the results of the Robust Diagonally Weighted Least Squares method estimation, the model is a good fit for the data. The overall model fit indices suggested an improvement in the fit ($\chi2$ = 68.50, *df* =43, *p*=.00799; RMSEA [90%CI]=.0388, .0486; CFI=.997; GFI=.998). All parameter estimates were statistically significant, and the standardized loadings ranged from .296 to .507.

The model Chi-square is greater than zero, and it is statistically significant at the .05 level. While this may suggest that the covariance of the model does not fit the covariance of the population, it is most likely due to the large sample size. All standardized residuals are smaller than 3. Table 32 shows all the standardized loading factors, measurement errors, and R^2 (the proportion of variance accounted for in the data that is explained by the latent factor).

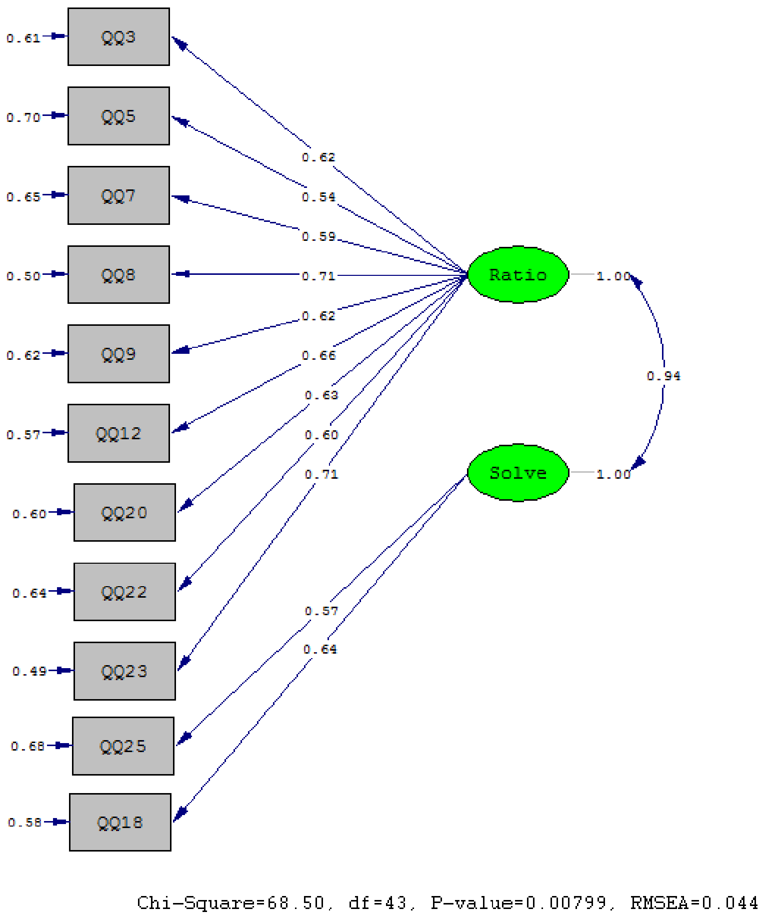


Figure 3. Confirmatory Factor Analysis of Key Mathematical Competencies for Mathematics Preparedness for Students Entering in Fall 2010

The measures associated with the latent variable Ratio included in the CFA Model were:

QQ3	QQ5	QQ7	QQ8	QQ9	QQ12	QQ20
$\frac{3}{3+\frac{1}{4}}$	$\frac{3}{x}=\frac{2}{5}$ $x=?$	$\frac{10}{\sqrt{14}}$	$\frac{9a^2+3a}{3a}$	$L=\frac{3}{8}F+14,$ $L=32,\ F=?$	$If\ \frac{1}{x-3}+7=\frac{x}{x-3}$ $then\ x=?$	$\left(\frac{a^2}{5b}\right)^{-2}$

QQ22	QQ23
$\frac{2}{r}-\frac{9}{s}$	$\frac{10x^2+30}{2x^2+6}$

The measures associated with the latent variable Solve included in the CFA Model were:

Q18	Q25
$x + 4y = 10$ $2x - 8y = 9,$ $x = ?$	Graph $x - y = -3$

Table 32

CFA Standardized Factor Loadings for the Two Factor Model of Mathematical Competencies for Students entering in Fall 2010

Item	Ratio	Solve	*Error*	R^2
QQ3	.624		.611	.389
QQ5	.544		.704	.296
QQ7	.592		.650	.350
QQ8	.711		.495	.505
QQ9	.620		.616	.384
QQ12	.655		.571	.429
QQ20	.633		.600	.400
QQ22	.596		.645	.355
QQ23	.712		.493	.507
QQ25		.566	.680	.320
QQ18		.645	.584	.416

Note. All factor loadings were significant at the .05 level.

The relationships between the two factors were obtained through LISREL estimates using the Robust Diagonally Weighted Least Squares method. The correlation between the factors rational number operations and equation solving was strong (*r=.943*). The results from this analysis suggest that the questions selected are appropriate measures for rational number operations and equation solving.

Fall 2011 CFA Model

Loading factors resulting from the EFA analysis for fall 2011 are shown in Table 33. These results were used to develop the model to be used as input for the CFA analysis. Measures with loading values above .25 were chosen to ensure that each factor was associated with at least two measures. Values are shown in bold font in Table 33.

Before testing the two-factor model, a one-factor model was tested to examine unidimensionality. The one-factor model was tested and found not to fit the covariance matrix ($\chi2 = 796.44$, $df = 275$, $p < .00001$; RMSEA [90%CI]=.0568, .0605; CFI=.976; GFI=.990). All parameter estimates were statistically significant. An examination of the standardized residuals indicated a misfit of the data, with standardized residuals as high as 9.22 (between QQ15 and QQ6). These results suggest that the data may be multidimensional.

Table 33

Loading Factors Obtained from the EFA Analysis for the Fall 2011 Four-Factor Model

Question	Negative	Ratio	Solve	Simplify	Unique Variance
QQ19	0	0	0	**0.539**	0.709
QQ23	0.110	0.163	-0.016	**0.517**	0.539
QQ24	**0.316**	-0.222	0.216	**0.446**	0.646
QQ13	-0.030	0.164	-0.011	**0.401**	0.745
QQ21	**0.333**	0.068	0.173	**0.329**	0.519
QQ20	0.214	**0.324**	-0.06	**0.319**	0.534
QQ11	0.078	0.127	-0.009	**0.274**	0.837
QQ7	0.060	**0.449**	-0.106	**0.263**	0.594
QQ16	0.189	0.200	0.038	0.234	0.717
QQ25	0.159	**0.270**	0.067	0.229	0.655
QQ22	0.073	**0.543**	0.005	0.190	0.474
QQ8	0.220	**0.266**	-0.039	0.159	0.734
QQ18	0.105	**0.321**	0.094	0.155	0.691
QQ14	0.260	0.140	0.131	0.141	0.722
QQ17	-0.005	**0.403**	-0.011	0.139	0.754
QQ15	-0.013	0.090	**0.629**	0.120	0.475
QQ4	0.146	0.262	0.019	0.114	0.795
QQ9	0.060	**0.521**	-0.076	0.083	0.667
QQ5	0.057	**0.270**	**0.31**	0.055	0.672
QQ6	0	0	**0.747**	0	0.442
QQ2	**0.647**	0	0	0	0.581
QQ3	0	**0.639**	0	0	0.592
QQ1	**0.528**	0.138	0.008	-0.037	0.631
QQ10	0.083	**0.466**	0.052	-0.066	0.740
QQ12	0.051	0.045	-0.058	-0.108	0.989

After conducting the CFA analysis with the four-factor model, only measures where the R^2, the proportion of variance in the measure accounted for by the factor, was .25 or larger were retained. Consequently, a two-factor model was obtained. Figure 4 depicts the model for fall 2011. The results of the Robust Diagonally Weighted Least Squares method estimation show that the model is a good fit for the data. The overall model fit indices suggested an improvement in the fit ($\chi2 = 60.442$, $df = 42$, $p = .0324$; RMSEA [90%CI]=.0352, .0447; CFI=.998; GFI=.998). All parameter estimates were statistically significant, and the standardized loadings ranged from .496 to .742. QQ5 and QQ15 were allowed to correlate to obtain a better fit to the model.

The model Chi-square is greater than zero, and it is statistically significant at the .05 level. While this may suggest that the covariance of the model does not fit the covariance of the population, it is most likely due to the large sample size. All standardized residuals are smaller than 3. Table 34 shows all the standardized loading factors, measurement errors, and R^2 (the proportion of variance accounted for in the data that is explained by the latent factor).

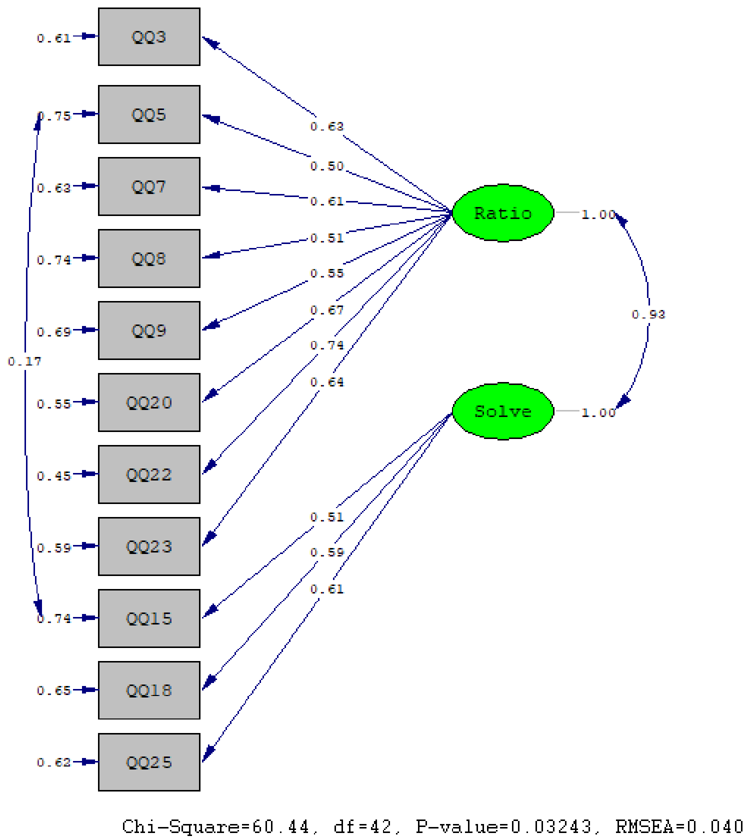


Figure 4. Confirmatory Factor Analysis of Key Mathematical Competencies for Mathematics Preparedness for Students Entering in Fall 2011

The measures associated with the latent variable Ratio included in the CFA Model were:

QQ3	QQ5	QQ7	QQ8	QQ9	QQ20
$\frac{5}{5+\frac{1}{3}}$	$\frac{2}{x}=\frac{3}{7}$ $x=?$	$\frac{6}{\sqrt{15x}}$	$\frac{8x^3+10x}{2x}$	$C=\frac{5}{9}(F-32)$, $C=20, F=?$	$\left(\frac{5a}{b^2}\right)^{-2}$

QQ22	QQ23
$\frac{a}{6b}+\frac{a}{5b}$	$\frac{x^2-25}{x^2-10x+25}$

The measures associated with the latent variable Solve included in the CFA Model were:

QQ15	QQ18	QQ25
$7x - 8 < 2x + 9$ *is equivalent to* ?	$2x + y = 4,$ $x - 2y = 5,$ $x = ?$	Graph $2x - y = 1$

Table 34

CFA Standardized Factor Loadings for the Two Factor Model of Mathematical Competencies for Students entering in Fall 2011

Item	Ratio	Solve	*Error*	R^2
QQ3	.626		.608	.392
QQ5	.496		.754	.246
QQ7	.607		.632	.368
QQ8	.506		.744	.256
QQ9	.554		.693	.307
QQ20	.669		.552	.448
QQ22	.742		.450	.550
QQ23	.638		.593	.407
QQ15		.506	.744	.256
QQ18		.588	.654	.346
QQ25		.614	.623	.377

Note. All factor loadings were significant at the .05 level.

The relationships between the two factors were obtained through LISREL estimates using the Robust Diagonally Weighted Least Squares method. The correlation between the factors rational number operations and operations with negative signs was strong (r=.928). The results from this analysis suggest that the questions selected are appropriate measures for rational number operations and equation solving.

Fall 2012 CFA Model

Loading factors resulting from the EFA analysis for fall 2012 are shown in Table 35. Results from the EFA analysis were used to develop the model to be used as input for the CFA analysis. Measures with loading values above .25 were selected to ensure that each factor was associated with at least two measures. The values are shown in bold font in Table 35.

Before testing the two-factor model, a one-factor model was tested to examine unidimensionality. The one factor model was tested and found not to fit the covariance matrix ($\chi 2$ = 693.38, df =275, p<.00001; RMSEA [90%CI]=.0517, .0553; CFI=.979; GFI=.985). All parameter estimates were statistically significant. An examination of the standardized residuals indicated a misfit of the data, with standardized residuals as high as 7.408 (between QQ16 and QQ4). These results suggest that the data may be multidimensional.

Table 35

Loading Factors Obtained from the EFA Analysis for the Fall 2012 Four-Factor Model

Question	Negative	Ratio	Solve	Simplify	Unique_Var
QQ7	0	**0.678**	0	0	0.541
QQ23	0.001	**0.562**	0	0.223	0.554
QQ12	0.019	**0.474**	0.155	0.064	0.632
QQ8	0.026	**0.430**	0.191	0.033	0.663
QQ13	-0.008	**0.429**	0.097	0.046	0.748
QQ21	-0.076	**0.392**	0.088	**0.296**	0.662
QQ20	-0.036	**0.38**	0.179	0.222	0.639
QQ17	**0.277**	**0.38**	0.111	-0.074	0.666
QQ3	0.159	**0.359**	0.141	0.077	0.671
QQ6	0.158	**0.346**	0.103	0.058	0.727
QQ18	0.079	**0.340**	0.222	0.138	0.632
QQ9	0.187	**0.339**	**0.277**	0.032	0.567
QQ15	0.032	**0.333**	0.072	0.113	0.802
QQ14	0.075	0.24	0.239	0.04	0.774
QQ5	0.07	0.232	0.111	0.069	0.859
QQ16	**0.616**	0.198	-0.098	0.121	0.470
QQ10	0.133	0.169	0.075	0.182	0.824
QQ1	**0.264**	0.158	0.222	0.031	0.724
QQ19	0.056	0.152	0.161	0.175	0.829
QQ2	0.141	0.111	0.086	0.008	0.927
QQ24	-0.119	0.104	0.165	**0.485**	0.682
QQ25	-0.082	0.023	**0.480**	0.136	0.725
QQ22	0	0	0	**0.769**	0.408
QQ11	0	0	**0.675**	0	0.544
QQ4	**0.653**	0	0	0	0.573

After conducting the CFA analysis with the four-factor model, only measures where the R^2, the proportion of variance in the measure accounted for by the factor, was .25 or larger were retained. Consequently, a two-factor model was obtained. Figure 5 depicts the model for fall 2012. The results of the Robust Diagonally Weighted Least Squares method estimation show that the model is a good fit for the data. The overall model fit indices suggested an improvement in the fit ($\chi2$ = 40.921, *df* =25, *p*=.0234; RMSEA [90%CI] =.0402, .0521; CFI = .998; GFI = .998). All parameter estimates were statistically significant, and the standardized loadings ranged from .559 to .828. QQ23 and QQ9 were allowed to correlate to obtain a better fit to the model.

The model Chi-square is greater than zero, and it is statistically significant at the .05 level. While this may suggest that the covariance of the model does not fit the covariance of the population, it is most likely due to the large sample size. All standardized residuals are smaller than 3. Table 36 shows all the standardized loading factors, measurement errors, and R^2 (the proportion of variance accounted for in the data that is explained by the latent factor).

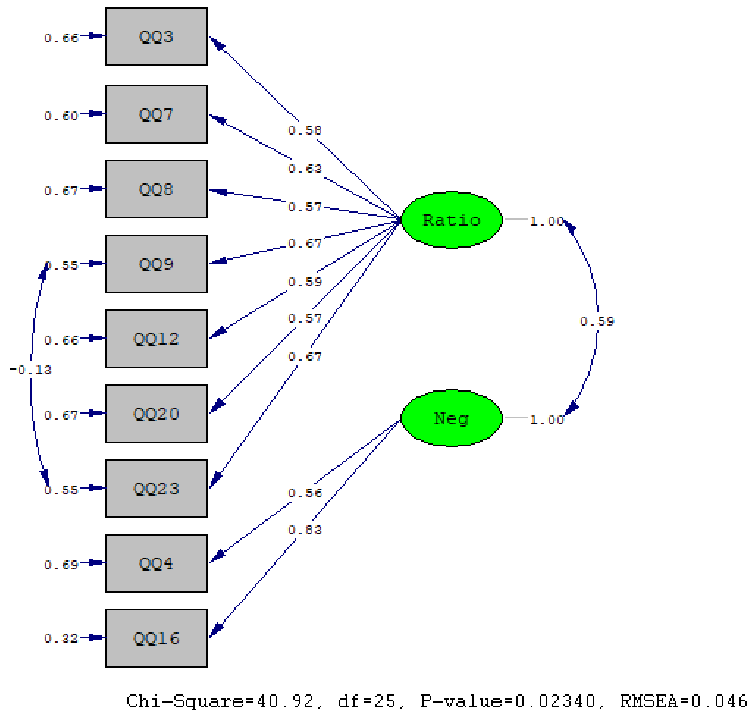


Figure 5. Confirmatory Factor Analysis of Key Mathematical Competencies for Mathematics Preparedness for Students Entering in Fall 2012

The measures associated with the latent variable Ratio included in the CFA Model were:

QQ3	QQ7	QQ9	QQ12	QQ20	QQ23
$2+\frac{1}{3}$	$\frac{5}{x}-\frac{2}{w}$	$L=\frac{5}{8}F+15,$ $L=28\ F\ =?$	$If\ x=\frac{5}{4}$ $then\ x^{-2}$	$\frac{6}{\sqrt{14}}$	$\frac{8x+32}{8x-32}$

The measures associated with the latent variable Neg included in the CFA Model were:

QQ16	QQ4
$a=-3,$ $\lvert a-2\rvert-\lvert -3a\rvert=?$	$6-\lvert -2\rvert=?$

Table 36

CFA Standardized Factor Loadings for the Two Factor Model of Mathematical Competencies for Students entering in Fall 2012

Item	Ratio	Negative Number	*Error*	R^2
QQ3	.581		.663	.337
QQ7	.631		.602	.398
QQ8	.572		.672	.328
QQ9	.667		.554	.446
QQ12	.586		.657	.343
QQ20	.572		.673	.327
QQ23	.670		.552	.448
QQ4		.559	.687	.313
QQ16		.828	.315	.685

Note. All factor loadings were significant at the .05 level.

The relationships between the two factors were obtained through LISREL estimates using the Robust Diagonally Weighted Least Squares method. The correlation between the factors rational number operations and operations with negative number operations was moderate (r=.587). The results from this analysis suggest that the questions selected are appropriate measures for rational number operations and negative number operations.

Fall 2013 CFA Model

Loading factors resulting from the EFA analysis for fall 2013 are shown in Table 37. These results were used to develop the model to be used as input for the CFA analysis. Measures with loading values above .25 were selected to ensure that each factor was associated with at least two measures. Values are shown in bold font in Table 37.

Table 37

Loading Factors Obtained from the EFA Analysis for the Fall 2013 Four-Factor Model

Question	Ratio	Negative	Simplify	Solve	Unique Variance
QQ22	0	0	0	**0.718**	0.485
QQ24	0.07	-0.224	-0.002	**0.632**	0.730
QQ25	**0.305**	-0.069	0.033	**0.346**	0.722
QQ23	**0.338**	0.054	0.118	**0.314**	0.535
QQ18	**0.471**	-0.201	0.042	**0.306**	0.690
QQ21	**0.502**	-0.249	0.040	0.189	0.765
QQ13	**0.336**	-0.133	0.118	0.166	0.815
QQ20	**0.535**	0.032	0.062	0.119	0.557
QQ19	0.011	0.189	0.179	0.117	0.826
QQ15	**0.266**	0.018	0.114	0.077	0.836
QQ11	**0.391**	0.054	0.012	0.048	0.787
QQ3	**0.680**	0	0	0	0.538
QQ4	0	**0.561**	0	0	0.685
QQ14	0	0	**0.919**	0	0.155
QQ12	0.189	0.029	**0.601**	-0.005	0.438
QQ5	0.028	0.144	0.080	-0.013	0.955
QQ17	**0.282**	**0.329**	0.107	-0.035	0.637
QQ10	0.166	-0.028	-0.03	-0.059	0.984
QQ9	**0.664**	0.016	0.009	-0.061	0.575
QQ16	**0.316**	**0.273**	0.018	-0.065	0.746
QQ7	**0.505**	**0.285**	0.062	-0.097	0.502
QQ6	**0.437**	0.187	0.074	-0.107	0.679
QQ2	-0.062	**0.381**	0.031	-0.110	0.915
QQ8	**0.307**	**0.336**	0.066	-0.133	0.702
QQ1	0.184	**0.505**	-0.053	-0.138	0.734

Before testing the two-factor model, a one-factor model was tested to examine unidimensionality. The one factor model was tested and found not to fit the covariance matrix ($\chi2$ = 818.80, *df* =275, *p*<.00001; RMSEA [90%CI]=.0547, .0584; CFI=.971; GFI=.989). All parameter estimates were statistically significant. An examination of the standardized residuals indicated a misfit of the data, with standardized residuals as high as 10.868 (between QQ14 and QQ12). These results suggest that the data may be multidimensional.

After conducting the CFA analysis with the four-factor model, only measures where the R^2, the proportion of variance in the measure accounted for by the factor, was .25 or larger were retained. Consequently, a two-factor model was obtained. Figure 6 depicts the model for fall 2013. The results of the Robust Diagonally Weighted Least Squares method estimation show that the model is a good fit for the data. The overall model fit indices suggested an improvement in the fit ($\chi2$ = 44.184, *df* =25, *p*=.0103; RMSEA [90%CI] =.0395, .0519; CFI=.998; GFI=.999). All parameter estimates were statistically significant, and the standardized loadings ranged from .560 to .849. The covariance errors of QQ23 and QQ9 were allowed to correlate to obtain a better fit to the model.

The model Chi-square is greater than zero, and it is statistically significant at the .05 level. While this may suggest that the covariance of the model does not fit the covariance of the population, it is most likely due to the large sample size. All standardized residuals are smaller than 3. Table 38 shows all the standardized loading factors, measurement errors, and R^2 (the proportion of variance accounted for in the data that is explained by the latent factor).

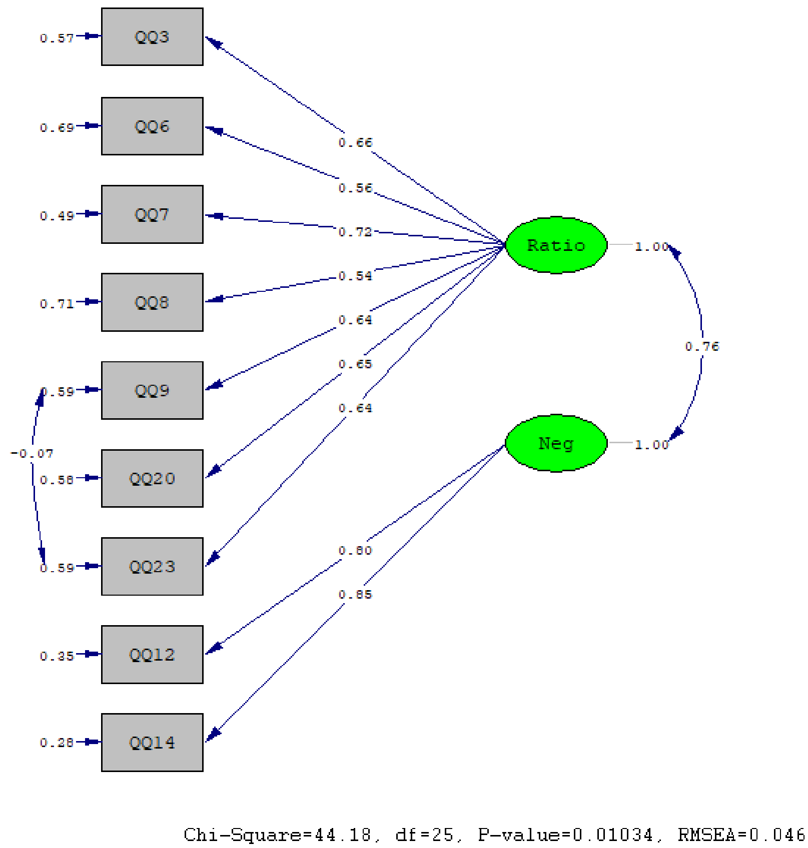


Figure 6. Confirmatory Factor Analysis of Key Mathematical Competencies for Mathematics Preparedness for Students Entering in Fall 2013

The measures associated with the latent variable Ratio included in the CFA Model were:

QQ3	QQ6	QQ7	QQ8	QQ9	QQ20	Q23
$\dfrac{5}{5+\frac{1}{2}}$	$\sqrt{72x^{10}y^2}$	$\dfrac{a}{5b}+\dfrac{a}{2b}$	$\dfrac{6x^5+4x}{2x}$	$C=\frac{5}{9}(F-32)$, $C = 35\ F = ?$	$\dfrac{9}{\sqrt{15x}}$	$\dfrac{x^2-64}{x^2-16x+64}$

The measures associated with the latent variable Neg included in the CFA Model were:

QQ12	QQ14
$If\ x = \frac{2}{3}\ then\ x^{-3}$	$\left(\frac{4a}{b^7}\right)^{-3}$

Table 38

CFA Standardized Factor Loadings for the Two Factor Model of Mathematical Competencies for Students entering in Fall 2013

Item	Ratio	Simplify	*Error*	R^2
QQ3	.658		.567	.433
QQ6	.560		.687	.313
QQ7	.717		.486	.514
QQ8	.540		.709	.291
QQ9	.640		.590	.410
QQ20	.650		.578	.422
QQ23	.640		.590	.410
QQ12		.805	.352	.648
QQ14		.849	.280	.720

Note. All factor loadings were significant at the .05 level.

The relationships between the two factors were obtained through LISREL estimates using the Robust Diagonally Weighted Least Squares method. The correlation between the factors rational number operations and operations with negative number operations was strong (r=.758).

For all models except for the fall 2011 model, the initial analysis started with 10 measures for rational number operations. The fall 2011 test did not have a ratio and proportion question. Some of the indicators were not included in the model if the loading factor was less than 0.5. The value of the loading factor squared corresponds to the proportion of variance accounted for in the data that is explained by the given factor. For a loading factor equal to 0.5, the value of R_{SMC}^2 is 0.25 or 25%. R^2 is also referred to as the Squared Multiple Correlation (SMC) or R^2 (Kline, 2011). Results of the CFA indicated that the two latent variables were well constructed by the selected questions from the Mathematics Placement Test as demonstrated by goodness-of-fit indices, factor loadings, the percent of variance accounted for in the data, R^2, and standardized residuals.

While there were some variations among the models for each academic year, the result of the hypothesized models suggests that the rational number operation questions measured the rational number operations construct. Similarly, the questions included in each model to measure negative number operations and equation solving fit the hypothesized models. Therefore, the Mathematics Placement Test is an appropriate tool in providing information to identify key competencies of entering students.

Entry-Level Mathematics Course Study

Phase 2 of this study aimed to confirm the findings found in the pilot study and to gain additional understanding of students' comprehension of key mathematical competencies. Participants in Phase 2 were new students entering the University in the fall registered in an entry-level mathematics course. An error analysis of students' responses to test questions was conducted.

The entry-level mathematics course selected was a Precalculus class offered in fall 2017. There were 43 students registered in the class with 35 new students. Twenty-four of the students signed informed consents or assents to provide the researcher with access to their homework, tests, and exams as well allowing the researcher to conduct observations during the class and review sessions. The class met on Mondays, Wednesdays, and Fridays for one hour and fifteen minutes. In addition, help sessions led by a graduate student were offered every Tuesday and Thursday. Students had an e-textbook with numerous resources.

A very experienced instructor taught the class. The instructor had 22 years of experience teaching college algebra, precalculus, business calculus, and calculus at the institution where the study took place. In addition, she taught precalculus, calculus I, II, and III, and abstract and linear algebra in a foreign institution for 15 years. The class followed a traditional style with the instructor leading all lecture periods. She provided the students with handouts containing additional class notes and practice problems. The instructor dedicated some lecture periods to work example problems highlighting potential areas of difficulty. Students had many opportunities to practice concepts taught in class and to ask for help.

Students attended class regularly throughout the semester. However, they did not take notes, and they did not ask many questions. The help sessions had low attendance. Only 3-4 students attended the sessions regularly. In the help sessions, they asked questions about the homework. When they did not ask questions, the student leader would ask them to come to the board and solve preselected problems similar to homework questions. While students knew how to apply concepts learned in class, algebra was the major cause of difficulty they had in solving problems. For example, students were asked to simplify the following expression:

$$\frac{9+\frac{1}{x-3}}{9+\frac{1}{x+3}}$$

when students arrived at this expression:

$$\frac{9x-26}{9x+26}$$

they asked if they could just cancel the 9x on the numerator with the 9x in the denominator. This notion was observed again in students' answers to test questions.

Five tests and one final exam were given throughout the semester. All tests and final exam questions were analyzed. Sample tests are included in Appendices G-K. In Phase 1 of the study, operations with rational numbers were identified as a key competency differentiating students' mathematics preparation. Consequently, questions requiring rational number operations were selected for examination. In addition, common errors encountered in tests' responses were studied to evaluate students' level of understanding of identified key competencies. The errors were analyzed, and results were summarized.

Precalculus Test #1 Results

Table 39 shows the percent of students answering the questions selected in Test #1 and the percent of students answering the questions incorrectly.

Table 39

Results for Selected Questions from Precalculus Test #1 (N=25)

Questions	Q1	Q9	Q10	Q11	Q14
Answered	79.0%	92.0%	83.3%	70.8%	95.8%
Incorrect	52.6%	77.3%	60.0%	52.9%	12.5%
Rational Number	5.3%	4.5%	45.0%	5.9%	0.0%
Negative Sign	21.1%	27.3%	N/A	N/A	N/A
Square Binomial	N/A	N/A	N/A	29.4%	N/A
Distributive Property	N/A	4.5%	N/A	5.9%	N/A
Other	26.3%	45.5%	25.0%	11.8%	12.5%

Question Q10 had the most students answering the question incorrectly due to incorrect rational number operations. The question required students to simplify this expression:

$$\frac{8+\frac{1}{x-2}}{8-\frac{1}{x+2}}$$

The most common mistake observed in the students' responses was treating the elements on the numerator and the denominator as individual items that could be separated at will. For example, students would cancel the number 8 in the numerator with the number 8 in the denominator.

$$\frac{8+\frac{\cancel{1}}{\cancel{x}-2}}{8-\frac{1}{\cancel{x}+2}}=\frac{\frac{1}{x-2}}{\frac{1}{x+2}}=\frac{x+2}{x-2}$$

Sometimes, the cancellation would go further canceling the variable *x*,

$$\frac{8+\frac{1}{x-2}}{8-\frac{1}{x+2}}=\frac{-\frac{1}{2}}{\frac{1}{2}}=-1$$

In other cases, students would solve the expression correctly almost to the end and then cancel out terms.

$$\frac{8x^2+x-30}{8x^2-x-30}=\frac{x-30}{-x-30}=\frac{x}{-x}=-1$$

This pattern was not observed if the operations on the numerator and denominator involved addition or subtraction of numbers. For example, 96% of the students answered question Q14 correctly. This question asked students to look up values on a graph and perform a given calculation with the values obtained. The final step resulted in this rational number expression,

$$\frac{-1+3}{5-(-2)}=\frac{2}{7}$$

The remaining 4% answered the question incorrectly because they were not able to complete the steps required to attain this expression.

Precalculus Test #2 Results

In Test #2, two questions addressed the identified competencies question Q13 and question Q20. Results for Test #2 are shown in Table 40.

Table 40

Results for Precalculus Test #2 Selected Questions (N=25)

Questions	Q13	Q20
Answered	80.0%	88.0%
Incorrect	40.0%	45.5%
Rational Number	15.0%	4.5%
Neg Sign	20.0%	36.4%
Other	5.0%	4.5%

Question 13 had the largest percentage of students missing the question due to errors with rational number operations. The question required students to evaluate the function at the indicated value,

$$\text{Find } \frac{f(x+h)-f(x)}{h} \ \ when\ f(x) = 7x + 4$$

The most common error was confusing the division and subtraction operations of rational numbers. All three students solved the problem in this way,

$$\frac{7x + 4 + h - 7x - 4}{h} = \frac{h}{h} = 0$$

Students confused $h/_h$ with *h*-*h*.

Also shown in this answer is the students' failure to apply the distributive property. The expression 7(*x* + h) + 4 was solved as 7*x* + 4 + *h*. Students also incorrectly distributed the negative sign. Four students attempted to solve this question by multiplying only the first term in the binomial by (-1),

$$\frac{7x + 7h + 4 - 7x + 4}{h} = 7 + \frac{8}{h}$$

Question Q21 was included because it also shows students difficulty performing operations involving a negative sign. The last step in this question required students to calculate f^{-1}. Eight out of the ten students who answered the question incorrectly did not seem to understand that raising an expression to a negative number means taking the inverse of the expression.

Precalculus Test#3 Results

Questions Q2, Q3, Q7 and Q11 were selected from Test #3 to illustrate students' challenges operating with negative signs and applying the distributive property. Across these questions, several students failed to distribute a negative sign or a factor through a binomial expression. Students also failed to answer correctly questions where a number was raised to a negative power. Results for Test#3 analysis are shown in Table 41.

Table 41

Results for Precalculus Test #3 Selected Questions (N=24)*

	Q2	Q3	Q7	Q11
Answered	100.0%	91.7%	100.0%	91.7%
Incorrect	25.0%	36.4%	54.2%	18.2%
Neg sign	N/A	9.1%	50.0%	13.6%
Power (-1)	4.2%	4.5%	N/A	N/A
Distr Prop	8.3%	4.5%	N/A	N/A
Other	12.5%	16.7%	4.2%	4.5%

Note. One student did not take the test.

For Q2 and Q3 students were asked to solve the given equations by equating the exponents.

$$\text{Q2: } e^{x-5} = \left(\frac{1}{e^4}\right)^{x+1}$$

$$\text{Q3: } \left(\frac{1}{3}\right)^{5x+4} = 9^{x-5}$$

In both cases, students found it challenging to distribute the exponent of *e*, (4), in Q2 or the negative sign (for both Q2, [-4], and Q3, [-1]) across the entire binomial. Examples of the equations after equating the bases for Q2 and Q3 respectively,

Q2: *4 not distributed across,* $x - 5 = -4x + \langle \quad \rangle 1$ instead of $x - 5 = -4x - 4$

Q3: *(-) not distributed across,* $5x + 4 = 2(x - 5)$ instead of $-5x - 4 = 2(x - 5)$

In questions, Q7 and Q11 students were asked to use the properties of logarithms to expand and compress logarithmic expressions. In both cases, students had difficulties distributing a negative sign through a binomial expression as shown next

Q7: *(-) not distributed across,* $-\log(3(x+5)^2) = -\log 3 + 2(\log(x+5))$ instead of,

$-\log 3 - 2\log(x+5)$ and

Q11: *(-) not distributed across,* $-\ln x - \ln(x^2 - 5) = ln\frac{x^2-5}{x}$ instead of,

$-[\ln x + \ln(x^2 + 5)] = \ln\frac{1}{x(x^2-5)}$.

Precalculus Test #4 Results

For Test #4, question Q1 was selected. Table 42 shows the results for Test #4. Question Q1 was chosen because 52.4 percent of the students responding to this question chose an incorrect answer. Students squared a binomial expression incorrectly as illustrated in the following expression,

$(\cos\alpha - \sin\alpha)^2 = \cos^2\alpha - \sin^2\alpha$, instead of, $= \cos^2\alpha - 2\cos\alpha\sin\alpha + \sin^2\alpha$

Students who answered this question wrong varied in test score from 20 to 93 out of 100, suggesting that this is not a misconception of low performing students. Moreover, this test took place within a month of the end of the semester. By that time, students had seen several examples of squaring binomial expression.

Table 42

Results for Precalculus Test #4 (N = 22)*

	Q1
Answered	88.0%
Incorrect	57.1%
Binomial	52.4%
Other	4.8%

**Note*. Three students did not take the test.

Precalculus Test #5 Results

Only one question was selected from Test #5, Q12. Results for Q12 in Test #5 are shown in Table 43. The selected question required students to simplify a trigonometric expression. It was required to perform rational number operations to answer this question. Four students left the question blank, and only 22 percent of the students answered the question correctly. As it can be seen from Table 43, 22 percent of the students answered Q12 incorrectly due to errors performing rational number operations. The most common error resulted from dividing rational expressions. Students failed to multiply all the components of the numerator by the denominator when they attempted to "flip and multiply" as shown

Q12: Simplify the trigonometric expression

$$\frac{\tan^3 x + \tan x}{\cot x}$$

Sample student solution:

$$\frac{\tan^3 x + \tan x}{\frac{1}{\tan x}} = \tan^3 x + \tan x \times \tan x = \tan^3 x + \tan^2 x.$$

In other cases, students would divide the expression correctly but would not know how to simplify this expression by factoring $\tan^2 x$, out of $\tan^4 x + \tan^2 x$ to obtain the desired final answer

$$= \tan^2 x(1 + \tan^2 x) = \tan^2 x \sec^2 x.$$

The Other category in Table 43 refers to students showing incomplete answers that were hard to categorize.

Table 43

Results for Precalculus Test #5 (N=22)*

	Q12
Answered	88%
Incorrect	78%
Rational Number	22%
Factorization	22%
Other	33%

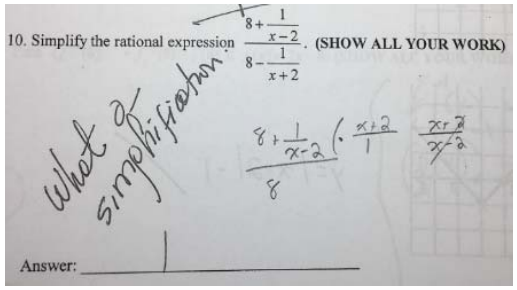
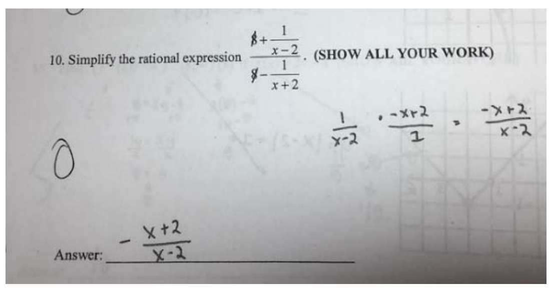
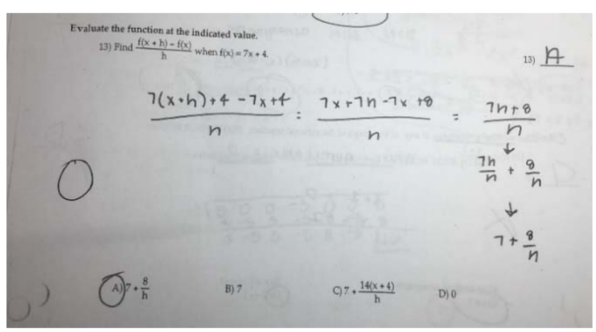
Note. Three students did not take the test.

Sample Student Responses

Table 44 shows examples of student responses indicating common errors students made when performing rational expressions and negative sign operations.

Table 44

Examples of Students' Responses

Error		Example
Flip and multiply		Flipped only a part of the expression 
Confuse division with subtraction		Q10, T1 $\frac{8+\frac{1}{x-2}}{8-\frac{1}{x+2}}=\frac{8}{8}+\frac{\frac{1}{x-2}}{-\frac{1}{x+2}}=0+\frac{-x+2}{x-2}$ 
Distribute (-)		Q13, T2 $\frac{7(x+h)+4-(7x+4)}{h}=\frac{7(x+h)+4-7x+4}{h}$ 

Additional Results

Because the final exam was multiple choice and was graded using Scantron, only a few students included all the steps used to obtain their answer. Consequently, no questions were selected for analysis from the final exam. As a general observation, students who showed their work demonstrated a clear understanding of the concepts addressed by the various questions. Their responses indicated that throughout the semester students moved from procedural understanding towards a more conceptual understanding. One simple example is on proportional reason. Students were asked in one of the tests to convert an angle from radians to degrees. In the test, most students would show the following response when converting $\frac{25\pi}{4}$ degrees,

$\frac{25\pi}{4}$ x $180 = 1125°$, instead of showing

$$\frac{25\pi}{4} \text{ x } \frac{180}{\pi}$$

This answer suggested that students were merely operating in a procedural mode of "multiply by 180" to get degrees instead of looking at the relation that π radians = 180°. On the final exam, when students were asked to convert 144 to radians, students' responses were

$$144° \text{ x } \frac{\pi}{180}.$$

In addition to examining the test questions for errors, questions were scanned for what students understood about rational number operations. Students were able to demonstrate their understanding of rational numbers when solving trigonometry problems. In Test #5, students were asked to find the least positive coterminal angle for

$\frac{25\pi}{4}$. Students demonstrated ability to add and subtract rational numbers as well as their understanding of equivalent fractions, for example,

$$\frac{25\pi}{4} - \frac{8\pi}{4} = \frac{17\pi}{4} - \frac{8\pi}{4} = \frac{9\pi}{4} - \frac{8\pi}{4} = \frac{\pi}{4}$$

Students were asked to use the unit circle to find exact values of trigonometric functions. Student work in tests showed their ability to partition the unit circle as requested. For example, students were asked to find the exact value of $\tan\frac{5\pi}{6}$. Students drew a circle and divided it into sections with $\frac{\pi}{6}$ angles until the $\frac{5\pi}{6}$ angle was identified to answer this question. With that information, students obtained the values of the known sine and cosine functions to get the tangent value. See Figure 7.

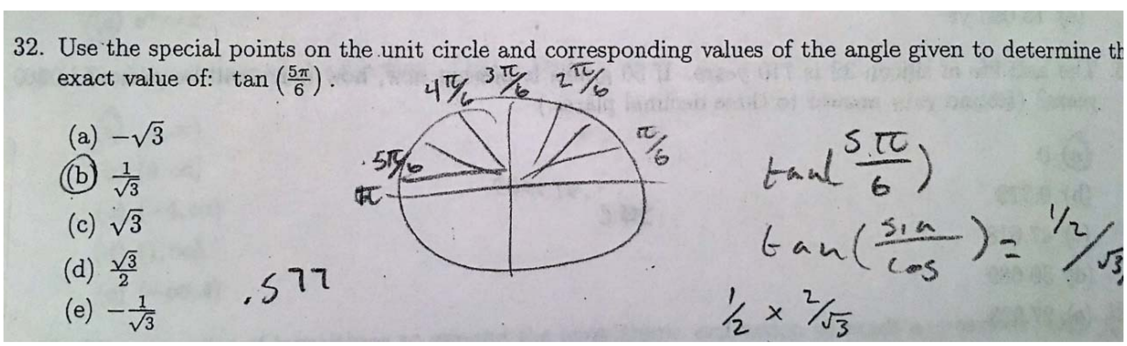


Figure 7. Precalculus Final Exam Question Q32 - Sample Partition

The error analysis conducted on selected questions of the entry-level mathematics course suggests that the results obtained from the pilot were accurate. For the students for which samples of their work were collected in the precalculus course, poor understanding of rational number operations was observed. The most salient observation is the inability of interpreting a rational expression as a relation between two quantities or expressions. Work inspected showed that students perform operations for individual components of the expressions in the numerator and individual components of the expressions in the denominator. This practice is also observed when students perform operations requiring the utilization of the distributive property, as in the case of a negative number multiplying the following expression. Students perform discrete one-on-one operations instead of looking at the relationship expressed by a negative sign in front of an expression.

Students' performance with rational number operations, equation solving, and operations with negative numbers confirm Vergnaud's theory of conceptual fields (Vergnaud, 1983, 1988, 1994). Understanding operations with rational numbers is challenging for students because of the many representations rational numbers may have and the many concepts involved when solving rational expressions and linear equations. Similarly, having a negative sign preceding a number or expression may have various interpretations, e.g., multiplication of an expression by (-1), the subtraction operator, or taking the reciprocal inverse of number. Results from Phase 2 of this study suggest that students lack understanding in interpreting the appropriate operation required by a rational expression or negative sign when solving mathematical problems.

The purpose of this descriptive research study was to identify key mathematics competencies first-time college students need to succeed in entry-level college mathematics courses. The study was conducted at a large, urban public, research university where between 20-31 percent of its new incoming freshmen were placed in developmental mathematics from 2010 through 2013. In the initial part of the research (Phase 1), a pilot study was conducted utilizing historical student data from mathematics placement tests (MPT). Participants in the pilot were new entering freshmen completing the MPT during student orientation in the summer preceding their entrance in the university. Students' performance on the MPT test questions were used to identify mathematical competencies differentiating students' placements in the various entry-level mathematics courses, hence depicting their level of mathematics readiness. Demographic data and incoming characteristics were also considered. Pilot study data demonstrated deficiencies in questions requiring operations with rational numbers and rational expressions. On average, less than 50 percent of the students placing in Developmental Mathematics, College Algebra, or Precalculus answered those questions correctly. A follow-up study was conducted to confirm the results obtained in Phase 1 through observations and artifacts examination of an entry-level mathematics class. Results from Phase 2 confirmed the results from Phase 1 and identified operations with negative numbers as an important concept affecting student preparedness. This study extends the mathematics education research by providing specific mathematics competencies affecting students' mathematics preparedness entering a 4-year institution.

CHAPTER 5: DISCUSSION

This chapter presents a comprehensive summary of the research conducted in this study. These results will help in addressing issues affecting the adequate preparation of students for college-level mathematics. Lack of adequate mathematics preparedness results in students not being able to register for or to complete their entry-level mathematics course resulting in delays in graduation or attrition. The primary goals of this study are to identify key mathematical concepts first-time incoming college students need to be prepared for college-level mathematics and to obtain a deeper understanding of incoming college students' knowledge of key mathematical concepts.

This chapter is organized as follows: first, a summary of the research and results will be presented. Next, conclusions and recommendations for future research and practice are proposed. Finally, the implications of the study will be discussed.

Mathematics preparedness has been the object of many studies and discussions (ACT, 2016; NAEP, 2016; National Center for Education Statistics, 2015; National Commission on Excellence in Education, 1983b). Nevertheless, students coming into higher education institutions lack the mathematics preparedness to enroll or successfully complete college-level mathematics. Among these reports are the *Condition of College & Career Readiness 2016* (ACT, 2016) indicating that only 41 percent of the 2016 high school graduates were Mathematics College ready and *What's Wrong with College Algebra* (Gordon, 2008) reported that only half of all students successfully complete college algebra courses. Published research focuses on sociological and demographic factors affecting college mathematics readiness and on the results of mathematics remediation but not on key fundamental mathematical concepts affecting mathematical readiness. For example,(Deil-Amen & Rosenbaum, 2009) identified race, parents' college education, and being full time versus part-time as factors affecting persistence of community college students registered in remedial courses.

Implementing the Common Core State Standards (CCSS, 2017a) is expected to improve mathematics readiness for all students when they graduate. In the meantime, higher education institutions have to address the disparities in students' mathematics preparedness. Remedial education is one approach utilized by many higher education institutions. However, remedial education is expensive for both students and colleges (Alliance for Excellent Education, 2011; Jimenez et al., 2016). In addition, remedial education is not always effective in providing students with the knowledge needed to succeed and persist in college mathematics (Adams et al., 2012; Attewell et al., 2006; Bahr, 2007, 2008; Bettinger & Long, 2004).

Mathematics preparedness is a complicated matter. Gaining understanding on specific mathematics competencies in which students are experiencing difficulties will assist higher education institutions to develop focused strategies to enhance student success in college mathematics and in future coursework. In addition, K-12 administrators and mathematics curriculum designers can also make use of this information to adjust course content to ensure students are college ready when they graduate from high school.

This study took place in an urban research University with a current enrollment of 29,000 students. The participants were new freshmen entering the University in the fall semester from 2010 to 2013. There were 2,905 students entering in fall 2010, 3,118 in fall

2011, 3,275 in fall 2012, and 3,031 in fall 2013. For Phase 1 of the study, data included demographic and incoming characteristics, and results from the mathematics placement test given to the students during their orientation. Data were analyzed to understand the factors determining mathematics readiness. In the second stage of the study, performance of fall 2017 entering freshmen registered in an entry-level mathematics course was evaluated to confirm the quantitative results obtained from the placement test data.

Students were assigned to a Developmental Mathematics, College Algebra, Precalculus, or Calculus I course according to their mathematics placement test score. From fall 2010 – to fall 2012, on average, 18 percent of the students placed in Developmental Mathematics, 19 percent of the students placed in College Algebra, 28 percent of the students placed in Precalculus and 35 percent placed in Calculus I. In fall 2013, 30 percent of the students placed in Developmental Mathematics, 23 percent placed in College Algebra, 24 percent placed in Precalculus and 23 percent placed in Calculus I.

The purpose of this descriptive research was to assess mathematics preparedness of entering students by answering the following research questions: (1) What mathematics competencies characterize students at different levels of mathematics college readiness? (2) What demographic factors and incoming data (Mathematics SAT score, intended major, high school GPA) characterize students at different levels of mathematics college readiness? (3)What is the level of understanding of key mathematics competencies of incoming students in an entry-level mathematics course? The following sections discuss results for each of the research questions.

Research Question 1

What mathematics competencies characterize students at different levels of mathematics college readiness?

The purpose of this study was to identify key mathematical competencies explaining mathematical readiness utilizing the Mathematics Placement Test questions. Key mathematics concepts were determined by identifying the topics of the questions most missed by students as a function of their mathematics placement. For each term, the percentage of students answering correctly the mathematics placement test questions were obtained.

Mathematics placement test results were cross-tabulated to show the frequency of responses for each entering group of freshmen. Results are shown in Tables 9-16. Data indicate that students placing in Developmental Mathematics, College Algebra and Precalculus, that is, students scoring less than 18 points out of possible 25 in the Mathematics Placement test, were more likely to answer incorrectly questions requiring rational number operations. For all four cohorts, less than 50 percent of the students placing in the lower level mathematics answered those questions correctly. This result suggests that performing rational number operations is a key mathematical competency to be prepared for college-level mathematics. Additional validation related to this result will be further developed with research question 3.

Research Question 2:

What demographic factors and incoming data (Mathematics SAT score, intended major, high school GPA) characterize students at different levels of mathematics college readiness?

Mathematics readiness by gender

Demographic and incoming characteristics were obtained from University databases for students taking the Mathematics Placement Test between fall 2010 and fall 2013. Analysis of the data show that more women than men placing in Developmental Mathematics. Mean scores within each mathematics placement were very similar for men and women (see Table 21). However, the overall Mathematics Placement Test mean score was lower for women (M= 13.98) than for men (M=15.91). The difference in mean scores was statistically significant at $p < .05$.

Research shows that there are many reasons for the difference in mathematics performance between men and women (Halpern et al., 2007). Women get better grades in school in every subject. They also attend college at higher rates than men (Halpern et al., 2007). Yet, men get higher scores than women on standardized tests in math and science from secondary school to graduate school (Benbow & Stanley, 1980, 1983). Curiously, a study conducted by Benbow (1990) found that for high achieving students the differences in mathematics performance between men and women become less noticeable. A similar result was found for this study. The average Mathematics Placement Test score for women pursuing College of Engineering majors, entering to the University between fall 2010 and fall 2013, was slightly higher than the average score for men, 18.5 and 18.1 respectively. The College of Engineering has more rigorous admission requirements than other colleges. Therefore, it could be assumed that in comparison with other students, the students entering the College of Engineers could be described as high achievers.

More men than women pursue Science Technology Engineering and Mathematics (STEM) majors (Hill, Corbett, & St Rose, 2010). For this study, between fall 2010 and fall 2013, only 2.5 percent of the women entering into the University intended to pursue majors in the College of Engineering compared to 24.1 percent of the men. The overall difference in Mathematics Placement Test average scores between men and women may be due to differences between men and women's choice of major.

Mathematics readiness by ethnicity

Mathematics readiness for African Americans, Latinos, Asians, and Caucasian was evaluated. Mathematics Placement Test scores were used as measures of mathematics readiness. Only, those ethnic groups were selected because sample sizes for other ethnic groups, such as Native Americans were comparatively smaller. Between fall 2010 and fall 2012, almost equal percentages of African American students placed in each mathematics level with a slightly larger percentage placing in Calculus I (27 % vs. 24% in other levels). More than half of the Asian Students placed in Calculus I and only 8 percent of the Asian students placed in Developmental Mathematics. Approximately, one-third of the Latino students placed in Calculus I and between 16 and 20 percent placed in Developmental Mathematics on a given year. In looking at the overall Mathematics Placement Test (MPT) score, Asian students have the highest mean score, followed by the Caucasians and Latino, and then by the African Americans. Asian students' MPT scores, on average, were more than 2 points higher in comparison with other ethnic groups. There were statistically significant differences among all means, at the $p<.05$ level, except for the difference between the Caucasians and Latino scores.

According to research, Asians and Caucasians are more likely to take college preparatory classes in high school. On the other hand, African- American and Latino students tend to register for general classes that do not prepare them as thoroughly for college education (Stearns, Potochnick, Moller, & Southworth, 2010). Parents, guidance counselors and teachers of advanced courses may influence course selection in high school. Since African Americans and Latino students are more likely to be the first in their families to go to college, they may not be advised to take advanced courses in high school (Choy, 2001; Horn & Nuñez, 2000; Terenzini, Springer, Yaeger, Pascarella, & Nora, 1996). Since taking more advanced courses has been associated with better mathematics preparedness (Long et al., 2009), there is a possibility that differences in mathematics placement could also be associated with courses taken in high school.

The distribution of Caucasian students in the various mathematics' levels is similar to the distribution of Latinos. This result is in disagreement with some of the published research that indicates that Latino students tend to be less mathematically prepared than Caucasian students are (Adams et al., 2012). Without additional information about the Latinos entering the University between fall 2010 and fall 2013, it is difficult to explain this result. The available data did not include information about parents' college attendance, socio-economic status, or coursework taken in high school. Having that information would have helped in determining if the Latinos coming during those years had similar characteristics to Caucasian students.

Mathematics readiness and high school GPA

High school GPA was tabulated for all years and all Mathematics Placement (see Table 25). The difference in average high school GPA among students placing in Developmental Mathematics and Calculus I was, on average, 0.16 points suggesting that high school GPA is not a determining factor in students' mathematics placement. This is to be expected since GPA is the result of grades in various courses. High school GPA values are the result of grades and achievement in several subjects, not just mathematics. High school GPA alone does provide sufficient information to determine mathematical readiness.

Mathematics readiness and mathematics SAT Score

Students' Mathematics SAT scores of students taking the mathematics placement test were examined for each year and course placement. On average, there was a difference of 150 points between students placing in Developmental Mathematics and Calculus I. This fact was confirmed with the results of multiple regression analysis conducted for the Mathematics Placement Test with high school GPA and Mathematics SAT score. The average adjusted R^2 was approximately 0.4 indicating that Mathematics SAT score and high school GPA explain 40 percent of the variance of the Mathematics Placement. While both variables added statistical significance to the prediction of the Mathematics Placement Test score, the standardized coefficient of the Mathematics SAT score was .57 compared to .17 for the high school GPA. Hence, Mathematics SAT score is a stronger predictor of mathematics readiness than high school GPA.

Mathematics readiness and choice of college

Mathematics Placement Test scores were tabulated by the college to determine if college choice is a factor in mathematics readiness. Students selecting majors in the College of Engineering and the College of Computer and Informatics had the lowest percentage of students placing in Developmental Mathematics. These students in anticipation of being in those majors may have taken additional mathematics courses in high school. Research indicates that taking additional mathematics courses in high school contributes to students being better prepared for college mathematics (Long et al., 2009).

Students enrolled in the College of Arts and Architecture and in the College of Business had more than a third of their students place in Calculus I. They had larger percentages of students placing in Calculus I than the Colleges of Liberal Arts & Sciences and Health and Human Services. Admission into the College of Business and the College of Arts & Architecture at the University where the study was conducted is very competitive. This result suggests that comparatively higher achieving students were applying to these colleges which may have stronger high school preparation than students applying to other colleges.

Mathematics Placement Test mean scores were tabulated for each college. The College of Engineering and the College of Computing and Informatics had the largest means followed by the College of Business and the College of Arts and Architecture. The College with the lowest Mathematics Placement Test mean score was the College of Education. This is a concern for teachers in Elementary Education, for example, teach a variety of subjects to students including mathematics. They need to have a good understanding of mathematics so they can teach it to others.

The difference in Mathematics Placement Test mean scores was statistically significant for the College of Engineering and the College of Computing Informatics. Differences in mean scores between the College of Business and other colleges, except the College of Arts and Architecture, were also statistically significant. These results suggest that students' choice of college may be a factor in students' mathematics readiness. Students applying to technical and competitive majors may be taking advanced courses in mathematics which research suggest that results in better mathematics preparedness (Long et al., 2009).

The demographics and incoming characteristics data results can be made available to University academic advisors and departments to provide students with strategies for success before their registration in college-level mathematics courses. This information can also be disseminated to faculty teaching introductory courses who can direct students to resources that can be used in parallel with course content. Currently, publishers of e-textbooks are including adaptive learning modules and online resources that students can use just-in-time to supplement course materials. Resources are available to all students, but faculty can adapt to the characteristics of their class what gets included as part of their course package.

Targeted interventions can take place before students enter college or throughout their first semester to ensure that, students can perform in their entry-level mathematics course. Identifying weaknesses in key mathematics competencies and addressing them will assist in helping students to be prepared to enter the college in the appropriate mathematics course required by their area of study.

Research Question 3

What is the level of understanding of key mathematics competencies of incoming students in an entry-level mathematics course?

The results from Phase 2 of this study were used to gain additional knowledge about students' understanding the key mathematical competencies that were affecting their mathematics preparation. Results from Phase 1 indicated that new freshmen entering college were inadequately prepared to perform rational number operations. An Exploratory Factor Analysis (EFA) was employed to identify the factors explaining the variations in the students' performance in the Mathematics Placement Test. Initially, using the Mathematics Placement Test topics and the results from the EFA, the model included four factors that were labeled: operations with rational numbers, operations with negative numbers, equation solving, and simplifying algebraic expressions. Results from the Confirmatory Factor Analysis (CFA) analysis pointed to a two-factor measurement model. A CFA was conducted for each cohort entering the University between fall 2010 and fall 2013 to test the hypothesis that mathematics preparedness is a two-factor structure. Each model included a rational number component and a secondary component correlated to operations with rational numbers. LISREL 9.30 was used to perform the analysis.

Results confirmed that all proposed measurement models included rational number operations as a factor. For the cohort entering in fall 2010 and fall 2011, performing operations with linear equations was the factor contributing to the covariance among the measurements. These two factors were highly correlated with *r* values of .943 and .928 for fall 2010 and fall 2011. The high level of correlation can be explained since working with linear equations requires dominion of rational number operations. The slope of a line is a fraction that represents a rate of change. Students also will need to be proficient at using proportional reasoning. Similarly, to solve a system of linear equations students need to have a clear understanding of equivalent fractions and they need to have computational fluency with fraction operations to obtain the solution (G. Brown & Quinn, 2007).

For the cohorts entering the University in fall 2012 and fall 2013, the additional factor included measures associated with negative number operations. For fall 2012, the measures were associated with the distribution of negative sign across an operation including absolute value. In the fall 2013 model, the factor included measures where a mathematical expression was raised to a negative power. Having operations with negative numbers as factors contributing to the measurement of mathematics preparedness is in agreement with the results obtained in Phase 2 of this study which will be discussed next.

Phase 2 of this study aimed to confirm the findings found in the pilot study and to gain additional understanding of students' comprehension of key mathematical competencies. An entry entry-level mathematics class, Precalculus, offered in fall 2017 was selected to conduct Phase 2 of the study. Out of the 43 students registered for the class, 35 met the requirement of being new students. Twenty-four of those students agreed to participate in the research. The class instructor provided the researcher the tests and final exam and allowed the researcher to conduct observations during the class and review sessions. An error analysis of the tests and final exam was conducted. Results confirmed that students struggle with rational number operations. In addition, students work show that students have difficulty performing operations with negative numbers.

When performing rational number operations, students would treat rational expressions as individual elements. This was evident when they canceled single numbers or variables in a numerator expression with single numbers or variables in the denominator expression ignoring other terms and operations. Students' work also showed their difficulty performing rational number operations. They would use multiplication of rational expressions when the problem required division. In addition, students' work shows confusion between division and subtraction of rational number, e.g., $h/h = 0$.

Another common error observed in the Precalculus class was performing operations with negative signs. Students' work showed that students would not distribute a minus sign across an expression. In addition, student work showed that they did not understand what it meant to raise a number to a negative power. For example, students will do $(a)^{-1} = -a$.

There are some commonalities in why students have difficulties performing rational number operations and negative number operations. As it was described in Chapter 2, rational numbers are difficult to understand because they can be conceptualized in a variety of subconstructs: part-whole, ratio, quotient, measure, and operator (Behr et al., 1983; Kieren, 1976). Similarly, according to Gallardo and Rojano (1994), a negative sign can be categorized according to three major functions: unary, binary and symmetric. For the unary function, the minus sign is the symbol included before a number to indicate that: the number is a subtrahend; the number is the opposite of a quantity; a negative number that is the answer to a problem or equation; and a negative natural number.

Gallardo and Rojano (1994) describe the binary function of the minus sign as an operator that can indicate: taking away (e.g., Tommy had 10 crayons, Mary took away 3); completing (e.g., I have 10 inches of ribbon how many more inches I need to have a foot); and taking a difference (e.g., what is the difference between 15 and 7). Thompson and Dreyfus identified an additional binary function the net result of moving along the number line (Thompson & Dreyfus, 1988). The third function defined as the symmetric function. For this function, the minus sign is also an operator that means taking the opposite or inverting the operation (Vlassis, 2004) such as, the role of the minus sign when included in expressions like $3 - [5 - 2(4 - 1)]$.

Negative and rational numbers fit the definition Vergnaud's definition of conceptual fields, "a set of situations, the mastering of which requires a variety of concepts, procedures and symbolic representations tightly connected with one another." (Vergnaud, 1982, p. 36). As described before, both performing operations with rational and negative numbers require the understanding of several concepts that may manifest in variety of situations that have different symbolic representations.

Learning about rational numbers and negative number operations requires a conceptual change. Vosniadou (1994; Vosniadou & Verschaffel, 2004) explains that conceptual change occurs when new information learned comes in conflict with the student's previous knowledge. According to the conceptual change theory, adding new information to an existing conceptual framework or enrichment can be done relatively easy. Revising a concept, on the other hand, it is not as easily accomplished. Conceptual change is difficult because students' conjectures are rooted in and confirmed with their everyday experiences. This is the case for students' experiences with natural numbers and with a negative sign as an operator. In these cases, as seen from this study, misconceptions will result. Vosniadu indicates that "misconceptions are produced when students try to reconcile the inconsistent pieces of information and produce a synthetic model" (Vosniadou, 1994, p. 52). A synthetic model is developed as the student attempts to merge the new knowledge with their previous knowledge. For example, when students face situations where they have to add rational numbers, they add them as natural numbers. Similarly, when students find a negative sign representing the symmetric function, take the opposite of, like in the case of this expression $3 - [5 - 2(4 - 1)]$, their existing knowledge will drive them to perform this operation $3 - 5 - 8 - 2$.

Both Vergnaud (1982) and Vosniadou (2004) agree that conceptual change is a slow process. It requires the reorganization of previous knowledge. Students also need to be made aware of the assumptions or presuppositions they have. This has implications for teaching. Students need to be provided with the appropriate learning environments and meaningful experiences that promote conceptual change. For example, students need to be given opportunities to explain and discuss their understanding so instructors can address them. This can be accomplished in an environment where open discussions can be held without judgment. Instructors can then bring awareness to the students of possible misconceptions. In addition, abundant opportunities to practice and apply the new concepts need to be presented to facilitate the reorganization of knowledge as students become more familiar with the new concept (Vosniadou & Ioannides, 1998). Providing the appropriate time and resources to facilitate conceptual changes for rational number and negative number operations is a necessity to enhance mathematics preparedness of entering college students.

Limitations and Future Work

This study was the first step to assess mathematical preparedness of entering college students. It identified operations with rational numbers and negative numbers as mathematical competencies affecting student preparedness for college-level mathematics. Further research is required to extend the findings of this study. I recommend, developing a new assessment tool that will focus on the two identified competencies and follow the assessment with personal interviews. Many factors affect student performance on a test including physical and mental state at the time of the test, distractions in the class, test format, etc. Following up with interviews may help in interpreting the choice of answer selected by the student.

Phase 2 of the study took place in a single Precalculus class, the third level of entry-level mathematics course. Not all the students are required to take this course as part of their general education requirements. Extending the study to the other entry-level mathematics courses, e.g., Developmental Mathematics, College Algebra, and Calculus I, will allow for comparison of the preparedness of students on each level.

Results from the Exploratory Factor Analysis (EFA) were used to determine the factors accounting for the variance and covariance among a set of measures (Brown, 2015). The obtained model was subsequently tested using Confirmatory Factor Analysis (CFA) for each existing set of Mathematics Placement Test data. The models were not tested for other sets of data. Testing the model for other sets of data would confirm that the hypothesized model represents

This study was conducted with new freshmen entering the University in the fall following their high school graduation. It would be interesting to repeat this study with transfer students. The number of transfer students coming to urban institutions continues to increase. Transfer students performance in entry-level discipline-specific courses lags the performance of students coming into the institution as freshmen. Faculty colleagues suggest mathematics preparedness as one of the reasons for the difference in course performance.

Results of the study also suggest that incoming characteristics may affect students' mathematics performance. To this effect, I suggest investigating if student taking advanced mathematics courses in high school improved their mathematics preparedness. For this study, it was assumed that students pursuing certain majors might have taken advanced courses but the data were not available to confirm that assumption. Similarly, information about parent level of education would help to understand differences in incoming student characteristics.

Conclusions

The purpose of this descriptive study was to examine the effects of a residential learning community and enrollment in an introductory engineering course to engineering students' perceptions of the freshman year experience, academic performance, and persistence. Mathematics Placement Test results were performance of students in an entry-level mathematics course were measured.

Based on the results of these two measurements, operations with rational numbers and with negative numbers were identified as key mathematical competencies affecting the mathematics preparedness of entering college students. In addition, incoming characteristics of new students differentiate their level of mathematics preparedness. Noteworthy were the results for Latinos and women. Based on the results of this study, Latino's mathematics preparedness was similar to Caucasian students. In addition, women entering into the College of Engineering were equally mathematically prepared as men.

This study extends the mathematics education research by providing specific mathematics competencies affecting students' mathematics preparedness entering a 4-year institution. They were identified through four years of entering student data and confirmed in an entering level mathematics course. Findings from this study provide insights for curriculum development used by educators in the K-12 system and for the development of resources needed to enhance students' mathematics performance at the college level.

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APPENDIX A: FALL 2010 MATHEMATICS PLACEMENT TESTS

1. $4[5-4(6-7)] =$

(A) -36 (B) -4 (C) 0 (D) 4 (E) 36

2. If $p=-4$ and $q=-5$, then $p-q=$

(A) -9 (B) -1 (C) 1 (D) 9 (E) 20

3. $\dfrac{3}{3+\frac{1}{4}} =$

(A) 4 (B) 3 (C) $\frac{5}{4}$ (D) $\frac{12}{13}$ (E) $\frac{4}{5}$

4. The graph of the equation $5x-y+20=0$ crosses the $x-axis$ at $x=$

(A) -20 (B) -4 (C) 0 (D) 4 (E) 20

5. If $\frac{3}{x}=\frac{2}{5}$ then $x=$

(A) $\frac{2}{15}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 6 (E) $\frac{15}{2}$

6. If $5x - 4 = 2 - 2x$ then $x =$

(A) $-\frac{6}{7}$ (B) $-\frac{2}{7}$ (C) $\frac{2}{7}$ (D) $\frac{6}{7}$ (E) 2

7. $\frac{10}{\sqrt{14}} =$

(A) $\frac{5\sqrt{14}}{7}$ (B) $\frac{5}{\sqrt{7}}$ (C) $\sqrt{\frac{5}{7}}$ (D) $\frac{\sqrt{7}}{5}$ (E) $\frac{\sqrt{14}}{10}$

8. $\frac{9a^2 + 3a}{3a} =$

(A) $3a + 1$ (B) $9a^2$ (C) $9a^2 + 1$ (D) $4a$ (E) $6a$

9. The length L of a spring is given by $L = \frac{3}{8}F + 14$, where F is the applied force. What force F will produce a length L of 32?

(A) 26 (B) 48 (C) $\frac{242}{3}$ (D) $\frac{298}{3}$ (E) $\frac{368}{3}$

10. If $x = 8$ and $y = -4$, then $|x - y| =$

(A) −12 (B) −4 (C) 4 (D) 12 (E) 32

11. $4^0 5^2 =$

(A) 0 (B) 10 (C) 25 (D) 40 (E) 400

12. If $\frac{1}{x-3} + 7 = \frac{x}{x-3}$ then $x =$

(A) $-\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{10}{3}$ (E) 8

13. $\frac{x^2-9}{2x} \bullet \frac{16}{4x-12} =$

(A) $\frac{3}{2}$ (B) 6 (C) $2(x+3)$ (D) $\frac{2(x-3)}{x}$ (E) $\frac{2(x+3)}{x}$

14. $(-6x)^2 =$

(A) $36x^2$ (B) $6x^2$ (C) $-12x$ (D) $-6x^2$ (E) $-36x^2$

15. The inequality $3x - 2 < 2x + 3$ is equivalent to

(A) $x < -1$ (B) $x < \frac{2}{3}$ (C) $x < \frac{5}{3}$ (D) $x < 5$ (E) $x < 15$

16. $\sqrt{80x^{16}y^4} =$

(A) $4x^8y^2$ (B) $40x^8y^2$ (C) $40x^{16}y^4$ (D) $4x^8y^2\sqrt{5}$ (E) $4x^{14}y^2\sqrt{5}$

17. The solutions of the equation $x^2 + 2x - 24 = 24$ are

(A) -2 *and* 24 (B) -8 *and* 6 (C) 0 *and* -2

(D) -4 *and* 6 (E) 18 *and* 28

18. In the system of equations $\begin{matrix} x + 4y = 10 \\ 2x - 8y = 9 \end{matrix}$, $x =$

(A) 0 (B) $\frac{11}{17}$ (C) $\frac{19}{4}$ (D) $\frac{29}{4}$ (E) 10

19. $(2rs^4)(-7r^2s^3) =$

(A) $-14r^3s^7$ (B) $-14r^2s^{12}$ (C) $-5r^{-1}s$ (D) $-5r^2s^{12}$ (E) $14r^2s^7$

20. $\left(\frac{a^2}{5b}\right)^{-2} =$

(A) $\frac{a^4b^2}{25}$ (B) $\frac{a^4}{5b^2}$ (C) $\frac{a^4}{25b^2}$ (D) $\frac{25b^2}{a^4}$ (E) $25a^4b^2$

21. $7x + 5(x - y) - y =$

(A) $12x$ (B) $4(3x + y)$ (C) $2(6x - y)$

(D) $6(2x - y)$ (E) $2(6x - 5y)$

22. $\frac{2}{r} - \frac{9}{s} =$

(A) $\frac{-7}{r - s}$ (B) $\frac{-7}{r + s}$ (C) $\frac{-7}{rs}$

(D) $\frac{2s - 9r}{rs}$ (E) $\frac{2r - 9s}{rs}$

23. $\dfrac{10x^2 + 30}{2x^2 + 6} =$

(A) 10 (B) 5 (C) $5(x^2 + 1)$ (D) $5x + 5$ (E) $5x^2 + 15$

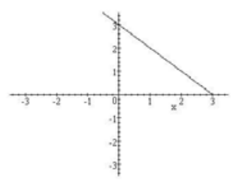
24. $7x - 9(x - 7) + 4(y - 7) =$

(A) $-9x + 4y$ (B) $-9x + 4y + 35$ (C) $-2x + 4y + 35$

(D) $-2x + 4y - 14$ (E) $-2x + 4y - 91$

25. Of the following, which best represents the graph of $x - y = -3$?

(A)



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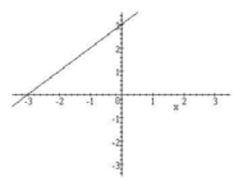
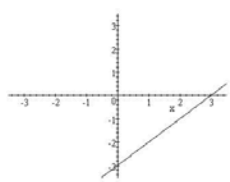
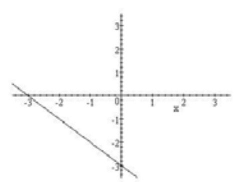


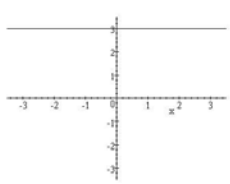
(C)



(D)



(E)



End of Test

APPENDIX B: FALL 2011 MATHEMATICS PLACEMENT TESTS

1. $3[2-4(5-6)] =$

(A) -18 (B) -9 (C) -6 (D) 6 (E) 18

2. If $x = 8$ and $y = -4$, then $x - 4y =$

(A) 24 (B) 8 (C) -8 (D) -24 (E) -128

3. $\dfrac{5}{5+\frac{1}{3}} =$

(A) $\frac{5}{2}$ (B) $\frac{15}{16}$ (C) 3 (D) $\frac{4}{3}$ (E) $\frac{3}{4}$

4. The graph of the equation $x - 5y + 20 = 0$ crosses the $y-axis$ at $y =$

(A) -20 (B) -4 (C) 0 (D) 4 (E) 20

5. If $\frac{2}{x} = \frac{3}{7}$ then $x =$

(A) $\frac{3}{14}$ (B) $\frac{6}{7}$ (C) $\frac{7}{6}$ (D) $\frac{14}{3}$ (E) 11

6. If $8x - 9 = 4 - 2x$ then $x =$

(A) $\frac{13}{6}$ (B) $\frac{13}{10}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$ (E) $-\frac{13}{10}$

7. $\frac{6}{\sqrt{15x}} =$

(A) $\frac{\sqrt{15x}}{6}$ (B) $\frac{2\sqrt{15x}}{5x}$ (C) $\frac{\sqrt{10x}}{5x}$

(D) $\frac{\sqrt{5x}}{2}$ (E) $\frac{2\sqrt{5x}}{5x}$

8. $\frac{8x^3 + 10x}{2x} =$

(A) $9x^2$ (B) $4x^2 + 5$ (C) $4x^2 + 10x$

(D) $8x^2 + 5$ (E) $8x^3 + 8x$

9. If $C = \frac{5}{9}(F - 32)$ and C is 20, then $F =$

(A) 28.9 (B) 52.0 (C) 68.0 (D) 93.6 (E) 212.0

10. If $r = -5$ and $s = 4$, then $|r + s| =$

(A) −9 (B) −1 (C) 1 (D) 9 (E) 20

11. $5^0 3^3 =$

(A) 9 (B) 0 (C) 45 (D) 450 (E) 27

12. If $\frac{3}{x-3} + 7 = \frac{x}{x-3}$ then $x =$

(A) -3 (B) 0 (C) 3 (D) 5 (E) *no* real *solution*

13. $\frac{x^2-16}{2x} \bullet \frac{12}{3x-12} =$

(A) $\frac{2(x+4)}{x}$ (B) $2(x+4)$ (C) $\frac{2(x-4)}{x}$ (D) $\frac{8}{3}$ (E) 8

14. $(-2x)^4 =$

(A) $2x^4$ (B) $-8x^4$ (C) $-2x^4$ (D) $-16x^4$ (E) $16x^4$

15. The inequality $7x - 8 < 2x + 9$ is equivalent to

(A) $x < -\frac{1}{5}$ (B) $x < \frac{8}{7}$ (C) $x < \frac{17}{7}$

(D) $x < \frac{17}{5}$ (E) $x < 119$

16. $\sqrt{18x^6y^{10}} =$

(A) $9x^6y^{10}$ (B) $9x^4y^8$ (C) $9x^3y^5$ (D) $3x^4y^8\sqrt{2}$ (E) $3x^3y^5\sqrt{2}$

17. The solutions of the equation $7x^2 + x + 8 = 16$ are

(A) 1 *and* $\frac{-8}{7}$ (B) −1 *and* $\frac{8}{7}$ (C) 1 *and* −8

(D) −1 *and* $\frac{-8}{7}$ (E) 1 *and* $\frac{8}{7}$

18. In the system of equations $\begin{matrix} 2x + y = 4 \\ x - 2y = 5 \end{matrix}$, $x =$

(A) $-\frac{6}{5}$ (B) 1 (C) $\frac{13}{5}$ (D) 3 (E) 5

19. $(-8x^3y^0)(2x^4y^2) =$

(A) $-16x^{12}$ (B) $-16x^7y^2$ (C) $-6x^7y^2$ (D) $16x^7y^2$ (E) $16x^{12}$

20. $\left(\frac{5a}{b^2}\right)^{-2} =$

(A) $\frac{a^2b^4}{25}$ (B) $\frac{5a^2}{b^4}$ (C) $\frac{25a^2}{b^4}$ (D) $\frac{b^4}{25a^2}$ (E) $25a^2b^4$

21. $3x + 7(x - y) - y =$

(A) $2(5x + 3y)$ (B) $10x$ (C) $2(5x - 4y)$

(D) $2(5x - y)$ (E) $2(5x - 7y)$

22. $\frac{a}{6b} + \frac{a}{5b} =$

(A) $11ab$ (B) $30ab$ (C) $\frac{2a}{11b}$ (D) $\frac{11a}{30b}$ (E) $\frac{11a}{30b^2}$

23. $\dfrac{x^2-25}{x^2-10x+25} =$

(A) 1 (B) $\dfrac{-25}{-10x+25}$ (C) $\dfrac{x+5}{x-5}$ (D) $\dfrac{1}{10x}$ (E) 0

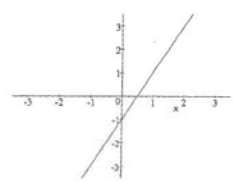
24. $3x-2(x-4)+6(y-5) =$

(A) $x+6y-22$ (B) $x+6y-38$ (C) $x+6y-9$

(D) $-2x+6y-22$ (E) $-2x+6y-1$

25. Of the following, which best represents the graph of $2x-y=1$?

(A)



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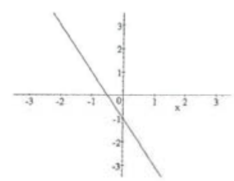
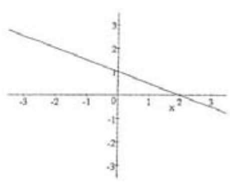
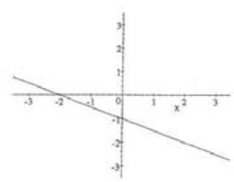


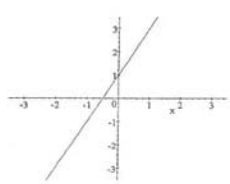
(C)



(D)



(E)



End of Test

APPENDIX C: FALL 2012 MATHEMATICS PLACEMENT TESTS

1. $3 - [5 - 2(4+1)] =$

   (A) 0 (B) 8 (C) −12 (D) 5 (E) 9

2. $3a + 2(a-4) - 5(b-2) =$

   (A) $5a - 5b - 18$ (B) $5a - 5b - 6$ (C) $5a - 5b + 2$
   (D) $2a - 5b + 2$ (E) $5a - 5b - 2$

3. $\dfrac{4}{2 + \frac{1}{3}} =$

   (A) 3 (B) $\frac{7}{3}$ (C) $\frac{12}{7}$ (D) 6 (E) 4

4. $6 - |-2| =$

   (A) 12 (B) 8 (C) −4 (D) 4 (E) −12

5. $(3rs^4)(3r^2s^6) =$

   (A) $9r^3s^{10}$ (B) $6r^2s^{24}$ (C) $6r^3s^{10}$
   (D) $9r^2s^{10}$ (E) $9r^2s^{24}$

6. $\sqrt{32p^{10}q^{30}} =$

(A) $4p^5q^{15}$ (B) $16p^{52}q^{15}$ (C) $16p^8q^{28}$

(D) $4p^8q^{28}\sqrt{2}$ (E) $4p^5q^{15}\sqrt{2}$

7. $\frac{5}{x} - \frac{2}{w} =$

(A) $\frac{3}{x-w}$ (B) $\frac{3}{xw}$ (C) $\frac{3}{x+w}$

(D) $\frac{5w-2x}{xw}$ (E) $\frac{5x-2w}{xw}$

8. $\frac{6r^3+15r}{3r} =$

(A) $7r^3$ (B) $2r^2+15r$ (C) $2r^2+5$

(D) $6r^3+5$ (E) $2r^3+3r$

9. If $L = \frac{5}{8}F + 15$ and L is 28, then $F =$

(A) $\frac{344}{5}$ (B) $\frac{65}{2}$ (C) $\frac{215}{8}$ (D) $\frac{149}{5}$ (E) $\frac{104}{5}$

10. If $\frac{3}{w-4} + 2 = \frac{w}{w-4}$ then $w =$

(A) 6 (B) 11 (C) 1 (D) 13 (E) 5

11. The graph of the equation $3x - 2y + 4 = 0$ crosses the $y - axis$ at $y =$

(A) 2 (B) −2 (C) 0 (D) $\frac{4}{3}$ (E) $\frac{-4}{3}$

12. If $x = \frac{5}{4}$ then $x^{-2} =$

(A) $-\frac{5}{4}$ (B) $-\frac{25}{16}$ (C) $\frac{4}{5}$ (D) $\frac{16}{25}$ (E) $-\frac{16}{25}$

13. $\frac{x^2 - 25}{3x} \bullet \frac{18}{6x - 30} =$

(A) $\frac{5}{6}$ (B) $\frac{x-5}{x}$ (C) $x + 5$ (D) 5 (E) $\frac{x+5}{x}$

14. $\left(\frac{x^3}{3z}\right)^2 =$

(A) $\frac{x^5}{6z^3}$ (B) $\frac{x^6}{9z^2}$ (C) $\frac{x^5}{9z^3}$ (D) $\frac{x^5}{3z^2}$ (E) $\frac{x^6}{6z^2}$

15. Which of the following are factors of $x^4 - 16$?
I. $x + 4$ *II.* $x - 2$ *III.* $x^2 + 4$

(A) *I only* (B) *III only* (C) *I and II only*

(D) *II and III only* (E) *I and III only*

16. If $a = -3$ then $|a - 2| - |-3a| =$

(A) −3 (B) 15 (C) 14 (D) −4 (E) 8

17. $(3x - 4y)^2 =$

(A) $9x^2 + 16y^2$ (B) $9x^2 - 16y^2$ (C) $9x^2 - 24xy + 16y^2$

(D) $6x^2 - 8y^2$ (E) $9x^2 - 24xy - 16y^2$

18. In the system of equations $\begin{matrix} x + 4y = 6 \\ 3x - 5y = 2 \end{matrix}$, $x =$

(A) $\frac{38}{17}$ (B) $\frac{8}{17}$ (C) $\frac{16}{17}$ (D) $\frac{8}{3}$ (E) $-\frac{22}{7}$

19. $2^0 9^2 =$

(A) 1 (B) 18 (C) 81 (D) 162 (E) 0

20. $\frac{6}{\sqrt{14}} =$

(A) $\frac{\sqrt{14}}{6}$ (B) $\frac{3}{\sqrt{7}}$ (C) $\frac{\sqrt{7}}{3}$ (D) $\sqrt{\frac{3}{7}}$ (E) $\frac{3\sqrt{14}}{7}$

21. The solutions of the equation $3x^2 + 14x + 8 = 0$ are

(A) -4 *and* $-\frac{2}{3}$ (B) 2 *and* $\frac{4}{3}$ (C) -2 *and* $-\frac{4}{3}$

(D) -4 *and* $-\frac{3}{2}$ (E) 4 *and* $\frac{2}{3}$

22. The solution to the equation $3x - 2 = x + 4$ is $x =$

(A) 3 (B) $-\frac{8}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) 4

23. $\frac{8x+32}{8x-32} =$

(A) 1 (B) -1 (C) $\frac{x+4}{x-4}$ (D) $\frac{x+32}{x-32}$ (E) 0

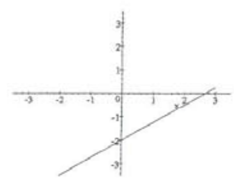
24. The inequality $5x - 2 < 3x + 10$ is equivalent to

(A) $x < \frac{12}{5}$ (B) $x < 6$ (C) $x < -8$

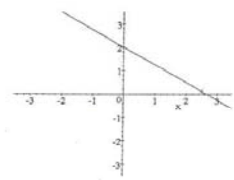
(D) $x < -\frac{4}{3}$ (E) $x > 6$

25. Of the following, which best represents the graph of $3x + 4y = 8$?

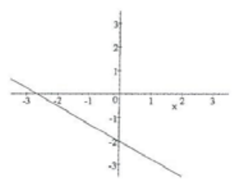
(A)



(B)

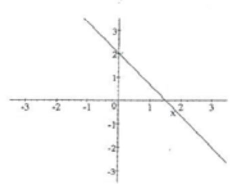


(C)



(D)

(E)



End of Test

APPENDIX D: FALL 2013 MATHEMATICS PLACEMENT TEST

1. $5[14 - 5(14 - 5)] =$

(A) -155 (B) -305 (C) -405

(D) 405 (E) 590

2. $8x - 2(x - 4) + 10(y - 9)$

(A) $6x + 10y - 13$ (B) $-2x + 10y - 82$ (C) $6x + 10y - 98$

(D) $6x + 10y - 82$ (E) $-2x + 10y - 5$

3. $\dfrac{5}{5 + \frac{1}{2}} =$

(A) $\frac{6}{5}$ (B) 2 (C) $\frac{1}{5}$ (D) $\frac{10}{11}$ (E) $\frac{5}{6}$

4. If $\frac{2}{x} = \frac{9}{11}$, then $x =$

(A) $\frac{11}{18}$ (B) $\frac{9}{22}$ (C) $\frac{22}{9}$ (D) 198 (E) $\frac{18}{11}$

5. $(6y^2w^0)(10y^{13}w^8) =$

(A) $60y^{15}w^8$ (B) $60y^{15}$ (C) $16y^{15}w^8$

(D) y^8 (E) $60y^{13}w^8$

6. $\sqrt{72x^{10}y^2} =$

(A) $36x^{10}y^2$ (B) $36x^5y$ (C) $6x^5y\sqrt{2}$

(D) $6\sqrt{2x^5y}$ (E) $36\sqrt{2x^5y}$

7. $\frac{a}{5b} + \frac{a}{2b} =$

(A) $7ab$ (B) $10ab$ (C) $\frac{2a}{7b}$

(D) $\frac{7a}{10b}$ (E) $\frac{7a}{10b^2}$

8. $\frac{6x^5 + 4x}{2x} =$

(A) $5x^4$ (B) $3x^4 + 2$ (C) $3x^4 + 4x$

(D) $6x^4 + 2$ (E) $6x^4 + 2x$

9. If $C = \frac{5}{9}(F - 32)$ and C is 35, then $F =$

(A) 95.0 (B) 37.2 (C) 67.0 (D) 120.6 (E) 347.0

10. If $\frac{10}{x-10} + 3 = \frac{x}{x-10}$ then $x =$

(A) 10 (B) *no* real *solution* (C) –10

(D) 0 (E) 13

11. The graph of the equation $x - 2y + 8 = 0$ crosses the $x - axis$ at $x =$

(A) 4 (B) −4 (C) −8 (D) 2 (E) 8

12. If $x = \frac{2}{3}$ then $x^{-3} =$

(A) $\frac{-27}{8}$ (B) $\frac{3}{2}$ (C) $\frac{27}{8}$ (D) $\frac{-2}{3}$ (E) $\frac{-8}{27}$

13. $\frac{x^2 - 36}{5x} \bullet \frac{35}{10x - 60} =$

(A) $\frac{7x + 42}{10x}$ (B) $\frac{7x - 42}{10x}$ (C) $7x + 42$ (D) 6 (E) $\frac{7}{10}$

14. $\left(\frac{4a}{b^7}\right)^{-3} =$

(A) $64a^3b^{21}$ (B) $\frac{a^3b^{21}}{64}$ (C) $\frac{4a^3}{b^{21}}$ (D) $\frac{-64a^3}{b^{21}}$ (E) $\frac{b^{21}}{64a^3}$

15. Which of the following are factors of $x^4 - 1$?

I. $x + 1$ *II.* $x - 1$ *III.* $x^2 + 1$

(A) *I only* (B) *III only* (C) *I, II, and III*

(D) *I and II only* (E) *II and III only*

16. If $r = 3$ and $s = -2$, then $|r + s| =$

(A) 5 (B) −1 (C) 1 (D) −5 (E) 6

17. $(2x-5y)^2 =$

(A) $4x^2-20xy+25y^2$ (B) $4x^2+25y^2$ (C) $4x^2-10y^2$

(D) $4x^2-25y^2$ (E) $4x^2-20xy-25y^2$

18. In the system of equations $\begin{matrix} 6x+y=1 \\ x-6y=4 \end{matrix}$, $x =$

(A) 27 (B) $\frac{10}{37}$ (C) $\frac{-37}{10}$ (D) 0 (E) 47

19. $5^0 3^3 =$

(A) 450 (B) 0 (C) 45 (D) 9 (E) 27

20. $\frac{9}{\sqrt{15x}} =$

(A) $\frac{\sqrt{15x}}{9}$ (B) $\frac{3\sqrt{15x}}{5x}$ (C) $\frac{\sqrt{3x}}{5x}$ (D) $\sqrt{\frac{3x}{5}}$ (E) $\frac{3\sqrt{5x}}{5x}$

21. The solutions of the equation $6x^2+x+7=14$ are

(A) $1 \textit{ and } \frac{7}{6}$ (B) $1 \textit{ and } -\frac{7}{6}$ (C) $-1 \textit{ and } \frac{7}{6}$

(D) $-1 \textit{ and } -\frac{7}{6}$ (E) $1 \textit{ and } 7$

22. The solution to the equation $5x-12=1-3x$ is $x =$

(A) $\frac{-13}{8}$ (B) $\frac{13}{2}$ (C) $\frac{-11}{2}$ (D) $\frac{11}{8}$ (E) $\frac{13}{8}$

23. $\frac{x^2 - 64}{x^2 - 16x + 64} =$

(A) 1 (B) 0 (C) $\frac{1}{16x}$ (D) $\frac{-8}{-16x+8}$ (E) $\frac{x+8}{x-8}$

24. The inequality $14x - 2 < 5x + 15$ is equivalent to

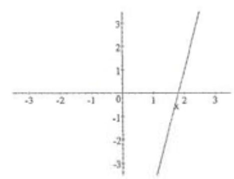
(A) $x < \frac{-13}{19}$ (B) $x < \frac{17}{19}$ (C) $x > \frac{17}{9}$

(D) $x < \frac{17}{9}$ (E) $x > \frac{13}{9}$

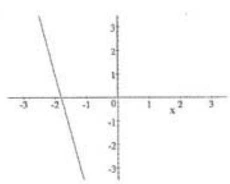
25. Of the following, which best represents the graph of $-5x - y = 9$?

(A)

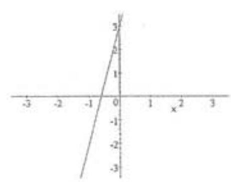
(B)



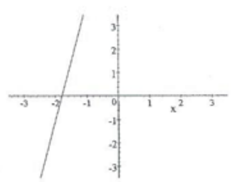
(C)



(D)



(E)



End of Test

APPENDIX E: INFORMED CONSENT

UNC CHARLOTTE

University Center for Academic Excellence
9201 University City Boulevard, Charlotte, NC 28223-0001
t/ 704-687-7841 f/ 704-687-1396 ucae.uncc.edu

Informed Consent for
Mathematics Readiness of Entering College Students

You are invited to participate in a research study. The purpose of this research is to learn more about the mathematics readiness of first time college students as determined by their understanding of key mathematical concepts. This study will provide valuable information for aligning high school and college mathematics curricula and for developing effective ways to close the mathematics preparedness gap through college preparation courses, summer programs, tutoring, or instruction technology.

My name is Cathy Blat and I am the principal investigator for this study. I am a UNC Charlotte PhD student in Curriculum and Instruction in the Math Education Concentration program in the Cato College of Education. I am working under the direction of Dr. David Pugalee also from the Cato College of Education.

You are invited to participate in this study because you are registered in an entry-level mathematics course. The study will last the entire fall 2017 semester. If you volunteer to participate in this study, you will allow me to review the work you complete for assignments and tests for this class to identify factors affecting math preparedness for college. The results of this analysis will not affect your grade in any way. I will also observe the classes and review sessions to gain additional information that will allow me to understand what it means to be Math College ready. You may be invited to participate in follow up meetings to provide clarification on your choice of answers. Whether you choose to participate or not in this research, Dr. Ludlow will not treat you any different.

There are no known risks to participation in this study. However, there may be risks which are currently unforeseeable. There are no direct benefits to you as a study participant. However, results from this study will provide valuable information for better alignment of high school and college mathematics curricula and for developing effective strategies to close the mathematics preparedness gap.

Any identifiable information collected as part of this study will remain confidential to the extent possible and will only be disclosed with your permission or as required by law. As a researcher, I will do everything possible to make sure your data or records are protected and kept confidential. The results from tests and homework as well as notes made from the class and review sessions observations will be kept on the University's password-protected servers. In addition, the results will be coded by a number rather than your name. They will not contain identifying information. When the results of this study are published, participants will be referred to by code numbers, not their names.

Your participation is voluntary. The decision to participate in this study is completely up to you. If you decide to be in the study, you may stop at any time. You will not be treated any differently if you decide not to participate in the study or if you stop once you have started.

UNC Charlotte wants to make sure that you are treated in a fair and respectful manner. Contact the Office of Research Compliance at 704-687-1871 or uncc-irb@uncc.edu if you have questions about how you are treated as a study participant. If you have any questions about the actual project or study, please contact Ms. Catherine Blat (704-687-7841, cmblat@uncc.edu).This form was approved for use on September 7, 2017 for a period of one (1) year.

I have read the information in this consent form. I have had the chance to ask questions about this study, and those questions have been answered to my satisfaction. I am at least 18 years of age, and I agree to participate in this research project. I understand that I will receive a copy of this form after it has been signed by me and the principal investigator of this research study.

____________________________________ ______________________

Participant Name (PRINT) DATE

____

Participant Signature

____________________________________ ______________________

Investigator Signature DATE

APPENDIX F: INFORMED ASSENT

UNC CHARLOTTE

University Center for Academic Excellence
9201 University City Boulevard, Charlotte, NC 28223-0001
t/ 704-687-7841 f/ 704-687-1396 ucae.uncc.edu

Mathematics Readiness of Entering College Students

For Students 15-17 Years Old

My name is Ms. Cathy Blat. I am a PhD student at UNC Charlotte.

You are invited to participate in a research study. Research studies are done to find better ways for helping and understanding people. Your decision to be in this study is voluntary. You do not have to participate in this study if you do not want to. This form will give you information about the risks and benefits of this study so that you can make a better decision about whether you want to take part or not.

PURPOSE OF THE STUDY

The purpose of this research study is to learn more about the mathematics readiness of first time college students as determined by their understanding of key mathematical concepts. This study will provide valuable information for aligning high school and college mathematics curricula and for developing effective ways to close the mathematics preparedness gap through college preparation courses, summer programs, tutoring, or instruction technology. You are invited to participate in this study because you are registered in an entry-level mathematics course.

PROCEDURES

The study will last the entire fall 2017 semester. Dr. Ludlow has given me permission to conduct research in her class. If you volunteer to participate in this study, you will allow me to review the work you complete for assignments and tests for this class to learn more about the factors affecting math preparedness for college. The results of this analysis will not affect your grade in any way. I will also observe the classes and review sessions to gain additional information that will allow me to understand what it means to be Math

College ready. You may be invited to participate in follow up meetings to provide clarification on your choice of answers. Whether you choose to participate or not in this research, Dr. Ludlow will not treat you any different.

There are no known risks to participation in this study. However, there may be risks which are currently unforeseeable. There are no direct benefits to you as a study participant. However, results from this study will provide valuable information for better alignment of high school and college mathematics curricula and for developing effective strategies to close the mathematics preparedness gap.

I will do everything possible to make sure your data or records are protected and kept confidential (not shared with others). The results from tests and homework as well as notes made from the class and review sessions observations will be kept on the University's password-protected servers. In addition, the results will be coded by a number rather than your name.

If you have any questions about this study, please contact me, Cathy Blat (704)-687-7841, cmblat@uncc.edu). You can also call the Office of Research Compliance at 704-687-1871 or email at uncc-irb@uncc.edu. This form was approved for use on September 7, 2017 for a period of one (1) year.

If you want to be in this study, please print and sign your name:

__________________________________ ____________________

Participant Name (PRINT) DATE

____

Participant Signature

__________________________________ ____________________

Investigator Signature DATE

APPENDIX G: PRECALCULUS TEST #1

EXAM # 1 **MATH 1103**

51/75 = 68%

NAME (UPPER CASE LETTER)________________

1. Find the inverse, with respect to composition, of $f(x) = \frac{x-7}{x+5}$ **(SHOW ALL YOUR WORK)**

(a) $f^{-1}(x) = \frac{5x-7}{x-1}$

(b) $f^{-1}(x) = \frac{x-1}{-5x+7}$

(c) $f^{-1}(x) = \frac{5x+7}{x-1}$

(d) $f^{-1}(x) = \frac{5x+7}{1-x}$

$f(x) = \frac{x-7}{x+5}$

$(y+5)x = \frac{y-7}{y+5}(y+5)$

$x(y+5) = y-7$

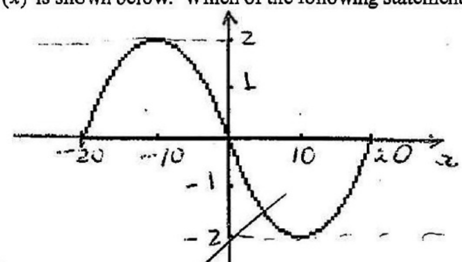
$xy+5x = y-7$

$5x+7 = y-xy$

$5x+7 = y(1-x)$

$\frac{5x+7}{1-x} = y$

2. The graph of $y = f(x)$ is shown below. Which of the following statements is (are) correct.



(-10,2), (10,-2)

I. The **range** of *f* is (-2, 2)

II. *f* is **increasing** on the interval (-20, -10), (10, 20)

III. *f* is decreasing on the interval (-10, 10)

IV. Is *f* **even, odd or neither**? odd

Because it is symmetrical about the origin

V. The **maximum value** of *f* is 2 and **the minimum value of** *f* is -2

3. Given the function $g(x) = 2x - 1$ find $g(\frac{x+7}{2})$. **(SHOW ALL YOUR WORK)**

(a) $-x-6$

(b) $-x+6$

(c) $x-6$

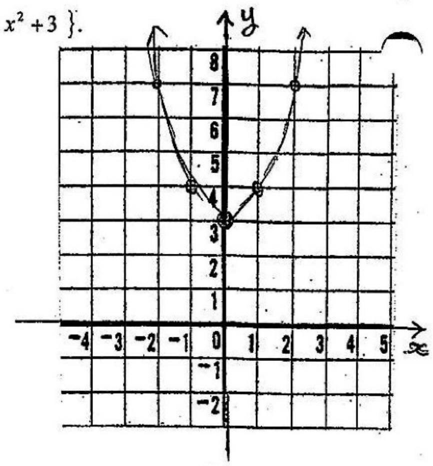
(d) $x+6$

$2\left(\frac{x+7}{2}\right) - 1$

$x+7-1$

$x+6$

4. (A) Use transformations to make the **graph** of $\{(x, y) \mid y = x^2 + 3\}$.



$D = (-\infty, \infty)$
$R = [3, \infty)$

(B) State its **domain** (D) and range (R)

(a) $D = (-\infty, \infty), R = (-\infty, \infty)$

(b) $D = (3, \infty), R = (-\infty, \infty)$

(c) $D = (-\infty, \infty), R = [3, \infty)$

(d) $D = (-\infty, \infty), R = (3, \infty)$

5. Explain why the domain of the function $f(x) = \sqrt{5 - x}$ is one of the following: $(\sqrt{5-x})^2 = (0)^2$

(a) (0,5) . WHY? square root functions can't have a negative under the radical, so x can't be less than 0.

$5 - x = 0$
$+x \quad +x$
$5 = x$

(b) $(-\infty,5]$ WHY?

(c) $[-5,5]$ WHY?

(d) $(-\infty,5)$ WHY?

6. Determine the **slope** and **y-intercept** of the line with equation $8x - 4y - 12 = 0$

$y = 2x - 3$

$8x - 12 = 4y$
$\frac{8x-12}{4} = \frac{4y}{4}$
$2x - 3 = y$

Answer: slope = 2 ; y-intercept = (0, −3)

7. Given $f(x) = x^3 - 5x + 1$, analytically determine whether it is **odd, even, or neither.** **(SHOW ~~ALL YOUR WORK~~)**

odd $-f(x) = f(-x)$ $-y = -x^3 + 5x + 1$ yes

~~$y = x^3 + 5x + 1$~~ not even

$f(-x) = f(x)$ even

Answer: f is odd

8. Find **the equation** of the line through (4, –2) that is **perpendicular** to the line $y = \frac{1}{2}x - 4$.

Write the answer in the slope-intercept form. **(SHOW ALL YOUR WORK)**

$y + 2 = -2(x-4)$

$y+2 = -2x+8$

$-2 \quad -2$

$y = -2x+6$

Answer: $y = -2x+6$

9. Given the function $g(x) = 5x^2 - x + 1$ **find and simplify** $\frac{g(x+h) - g(x)}{h}$.

(SHOW ALL YOUR WORK)

$\frac{5(x+h)^2 - (x+h) + 1 - 5x^2 - x + 1}{h}$

$(x+h)(x+h)$

$x^2 + xh + xh + h^2$

$5(x^2+2xh+h^2) \rightarrow \frac{5x^2 + 10xh + 5h^2 - x - h + 1 - 5x^2 + x - 1}{h}$

$\frac{10xh + 5h}{h}$

$\frac{h(10x+5)}{h}$

$10x+5$ $\quad 5(x+1)$

10. Simplify the rational expression $\frac{8 + \frac{1}{x-2}}{8 - \frac{1}{x+2}}$. **(SHOW ALL YOUR WORK)**

$8 + \frac{1}{x-2} \cdot \frac{x+2}{x+2}$

$8 - \frac{1}{x+2} \cdot \frac{x-2}{x-2}$

$\frac{8(x^2-4)+1}{(x-2)(x+2)}$ Flip and multiply

$\frac{8(x^2-4)-1}{(x-2)(x+2)}$

$\frac{8(x^2-4)+1}{(x-2)(x+2)} \cdot \frac{(x-2)(x+2)}{8(x^2-4)-1} = \frac{1}{-1} = -1$

Answer: -1

0

11. Rationalize the denominator $\frac{5x+1}{\sqrt{2}-1}$. **(SHOW ALL YOUR WORK)**

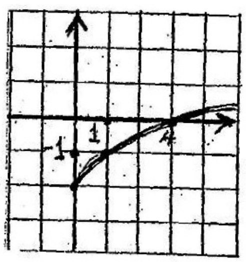
$\frac{(5x+1)^2}{(\sqrt{2}-1)^2}$ $\frac{25x^2+10x+1}{2-1}$ $\frac{25x^2+10x+1}{1}$

0

$(5x+1)^2 \rightarrow (5x+1)(5x+1) \rightarrow 25x^2+5x+5x+1$

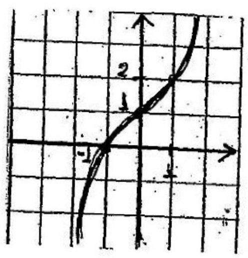
Answer: $25x^2+10x+1$ $\boxed{25x^2+10x+1}$

12. Write the equation of the following graphs.

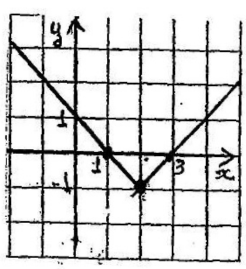


$f(x) = \sqrt{x} - 2$

6



$f(x) = x^3 + 1$



$f(x) = |x-2| - 1$

13. Classify the function $f(x) = \sqrt[3]{\frac{x+7}{x-8}}$ as

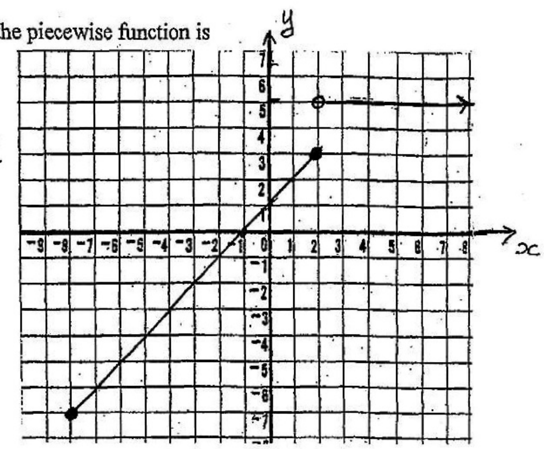
- o Rational function
- o Root function
- o Polynomial function

In addition, **the domain of *f*** is the interval $(-\infty, \infty)$

3

14. The equations that describe the graph of the piecewise function is

$$f(x) = \begin{cases} x+1 & -8 \leq x \leq 2 \\ 5 & x > 2 \end{cases}$$



5

Find $\frac{f(-2)+f(2)}{f(3)-f(-3)}$ is $\frac{2}{7}$

(Below perform your calculation below to justify your answer)

$$\frac{f(-2)+f(2)}{f(3)-f(-3)} = \frac{(-2+1)+(2+1)}{5-(-3+1)} \qquad \frac{2}{7}$$

15. Find $(f^{-1}(8))^{-1} + f^{-1}(8) - f(8)$ if $f(x) = 2x - 8$. **(SHOW ALL YOUR WORK)**

$(f^{-1}(8))^{-1}$

$\frac{8+8}{2} = (8)^{-1} = \frac{1}{8}$

$f^{-1}(8) = \frac{8+8}{2} = 8$

$2(8) - 8 = 8$

inverse: $x = 2y - 8$

$\frac{x+8}{2} = y$

$f(8) = 2(8) - 8 = 8$

$\frac{1}{8} + 8 - 8 = \frac{1}{8}$

5

Answer: 1/8

APPENDIX H: PRECALCULUS TEST #2

94/100

PLEASE SHOW ALL YOUR WORK

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Solve the rational inequality. Express the solution in interval notation. SHOW YOUR WORK.

1) $\frac{3x-1}{x+8} \leq 2$

1) B

$\frac{3x-1}{x+8} - 2 \leq 0$

$\frac{3x-1}{x+8} + \frac{-2(x+8)}{x+8} \leq 0$

$\frac{3x-1-2x-16}{x+8} \leq 0$

$\frac{x-17}{x+8} \leq 0$ $x = -8, 17$

$(-8, 17]$

A) (-8, 3] B) (-8, 17] C) (-8, 17) D) (-8, 3)

Rewrite the quadratic function in standard form by completing the square.

2) $f(x) = x^2 + 6x - 4$

2) D

$x^2 + 6x = 4$ $x^2 + 6x + 9 = 4 + 9$

$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$ $(x+3)^2 = 13$

$(x+3)^2 - 13 = f(x)$

A) $f(x) = (x + 6)^2 - 40$ B) $f(x) = (x + 3)^2 + 13$

C) $f(x) = (x + 6)^2 + 40$ D) $f(x) = (x + 3)^2 - 13$

Find any x-intercepts and any y-intercepts.

3) $f(x) = 2x^2 + 10x + 6$ Give your answers in exact form.

3) B

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\frac{-10 \pm \sqrt{100-48}}{4}$ $\frac{-10 \pm \sqrt{2 \cdot 2 \cdot 13}}{4}$

$\frac{-10 \pm \sqrt{(-10)^2 - 4(2)(6)}}{2(2)}$ $\frac{-10 \pm \sqrt{52}}{4}$ $\left(\frac{-10 \pm 2\sqrt{13}}{4}\right)/2 = \frac{-5 \pm \sqrt{13}}{2}$

A) x-intercepts: $\frac{-10 - \sqrt{13}}{2}$ and $\frac{-10 + \sqrt{13}}{2}$; y-intercept: 0

B) x-intercepts: $\frac{-5 - \sqrt{13}}{2}$ and $\frac{-5 + \sqrt{13}}{2}$; y-intercept: 6

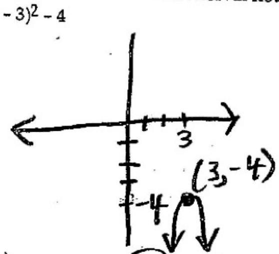
C) x-intercepts: $\frac{-5 - \sqrt{13}}{4}$ and $\frac{-5 + \sqrt{13}}{4}$; y-intercept: 6

D) x-intercepts: $\frac{-5 - \sqrt{37}}{2}$ and $\frac{-5 + \sqrt{37}}{2}$; y-intercept: 0

Find the range of the quadratic function in interval notation. EXPLAIN ON A GRAPH.

4) $f(x) = -7(x - 3)^2 - 4$

4) B



5

A) $[-4, \infty)$ B) $(-\infty, -4]$ C) $(-\infty, 3]$ D) $[-3, \infty)$

Find the remaining zeros of f(x) given that c is a zero. Then rewrite f(x) in completely factored form. SHOW YOUR WORK

5) $f(x) = 3x^4 - 5x^3 + 14x^2 - 20x + 8$; c = 1 is a zero

5) C

1 | 3 -5 14 -20 8
  3 -2 12 -8
3 -2 12 -8 0

$\frac{2}{3}$ | 3 -2 12 -8
  2 0 8
3 0 12 0

5

$3x^3 - 2x^2 + 12x - 8$

$3x^2 + 12$

$\pm \frac{1,8,2,4}{1,3} = \pm(1,8,2,4,\frac{1}{3},\frac{8}{3},\frac{2}{3},\frac{4}{3})$

$\pm \frac{1,12,2,3,6,4}{1,3}$

$\pm(1,12,2,3,6,4,\frac{1}{3},\frac{2}{3},\frac{4}{3})$

A) $4, \frac{2}{3}$; $f(x) = (x - 4)(3x - 2)(x^2 + 1)$

B) $-4, -1, -\frac{2}{3}$; $f(x) = (x - 1)(3x + 2)(x + 1)(x + 4)$

C) $\frac{2}{3}$; $f(x) = (x - 1)(3x - 2)(x^2 + 4)$

D) $-4, -1, \frac{2}{3}$; $f(x) = (x - 1)^2(3x - 2)(x + 4)$

4 | 3 0 12
  12 48
3 12 X

-4 | 3 0 12
  -12 48
3 -12 X

Use the remainder theorem to find the remainder when f(x) is divided by x - c. SHOW YOUR WORK.

6) $f(x) = 5x^6 - 3x^3 + 8$; $x + 1$

6) D

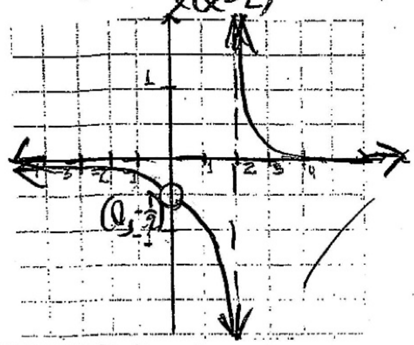
$5(-1)^6 - 3(-1)^3 + 8 = 16$

5

A) 10 B) 6 C) 8 D) 16

For the following rational function, identify the coordinates of all removable discontinuities and sketch the graph. Identify all intercepts and find the equations of all asymptotes. EXPLAIN.

7) $f(x) = \frac{x}{x^2 - 2x}$ $\frac{x}{x(x-2)}$



- $D_f: (-\infty, 0) \cup (0, 2) \cup (2, \infty)$
- Removable discontinuity ? C
- VA : $x = 2$
- HA : $y = 0$
- x-intercepts N/A
- y-intercepts N/A

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Give the equation of the horizontal asymptote, if any, of the function. EXPLAIN.

8) $f(x) = \frac{8x^2 - 4x - 6}{9x^2 - 9x + 7}$ Same degree: $\frac{8}{9}$ 8) C

A) $y = \frac{4}{9}$ B) $y = 0$ C) $y = \frac{8}{9}$ D) none

9) $f(x) = \frac{x + 2}{x^2 - 25}$ Bottom Heavy 9) C

A) $y = 1$ B) $y = -5, y = 5$ C) $y = 0$ D) none

Find the vertical asymptotes, if any, of the graph of the rational function. SHOW YOUR WORK.

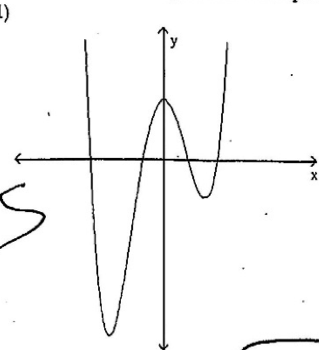
10) $f(x) = \frac{x + 1}{x^2 - 1}$ 10) C

$x^2 - 1$
$(x-1)(x+1)$ $x = 1$

A) no vertical asymptote B) $x = 1, x = -1$
C) $x = 1$ D) $x = -1$

Use the end behavior of the graph of the polynomial function to determine whether the degree is even or odd and determine whether the leading coefficient is positive or negative.

11) 11) B



A) even; negative B) even; positive C) odd; negative D) odd; positive

Use the Factor Theorem to determine whether x - c is a factor of f(x). SHOW YOUR WORK.

12) $f(x) = x^4 + 6x^3 + 8x^2 + 44x - 24;\ x - 6$ 12) B

6	1	6	8	44	-24
		6	72	480	3144
	1	12	80	524	3120

A) Yes B) No

Evaluate the function at the indicated value.

13) Find $\frac{f(x+h) - f(x)}{h}$ when $f(x) = 7x + 4$. 13) A

$$\frac{7(x+h)+4 - 7x+4}{h}$$

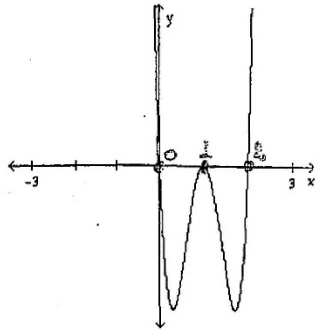
$$7x + 7h + 4 = 7x + 4$$

$$\frac{7h+8}{h} = \frac{7h}{h} + \frac{8}{h} = 7 + \frac{8}{h}$$

A) $7 + \frac{8}{h}$ B) 7 C) $7 + \frac{14(x+4)}{h}$ D) 0

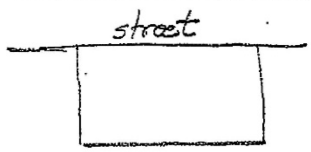
Solve the problem.

14) Which of the following polynomial functions might have the graph shown in the illustration below? 14) C



A) $f(x) = x^2(x - 2)^2(x - 1)^2$ B) $f(x) = x(x - 2)^2(x - 1)$
C) $f(x) = x(x - 2)(x - 1)^2$ D) $f(x) = x^2(x - 2)(x - 1)$

15) A developer wants to enclose a rectangular grassy lot that borders a city street for parking. If the developer has 272 feet of fencing and does not fence the side along the street, what is the largest area that can be enclosed? SHOW YOUR WORK. 15) B



$2x + y = 272$

$xy = A$

$y = 272 - 2x$

$x(272 - 2x) = A$

$-2x^2 + 272x = A$

$\frac{-b}{2a} = \frac{-272}{2(-2)} = \frac{272}{4} = 68 = x$

$y = 272 - 2(68) = 136 = y$

$68(136) = 9{,}248 \text{ ft}^2$

A) 13,872 ft^2 B) 9248 ft^2 C) 4624 ft^2 D) 18,496 ft^2

Solve the polynomial inequality. Express the solution in interval notation.

16) $x^3 + 3x^2 - 40x > 0$ 16) C

$x(x^2+3x-40)$

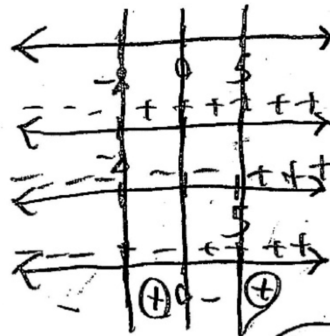
$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{-3 \pm \sqrt{(3)^2-4(1)(-40)}}{2(1)}$

$\frac{-3 \pm \sqrt{9+160}}{2}$

$\frac{-3 \pm \sqrt{169}}{2} = \frac{-3 \pm 13}{2} = \frac{-3+13}{2} = \frac{-3-13}{2}$

$\frac{10}{2}$ (5) $\frac{-16}{2}$ (-8)



$(-8, 0) \cup (5, \infty)$

5

A) $(-8, \infty)$
B) $(-5, 0) \cup (8, \infty)$
C) $(-8, 0) \cup (5, \infty)$
D) $(-\infty, -8) \cup (0, 5)$

Find the domain of the rational function.

17) $f(x) = \frac{2x^2 - 4}{3x^2 + 6x - 45}$ 17) A

$\frac{2(x^2-2)}{3(x+5)(x-3)}$

-135, -9, 15, 6

$(3x^2-9x)+(15x-45)$

$3x(x-3)+15(x-3)$

$(3x+15)(x-3)$

$3(x+5)(x-3)$

$x \neq -5, 3$

5

A) $\{x | x \neq 3, x \neq -5\}$
B) all real numbers
C) $\{x | x \neq -3, x \neq 5\}$
D) $\{x | x \neq 3, x \neq -3, x \neq -5\}$

18) If the parabola $f(x) = x^2 + bx + c$ has it vertex at (1,6), find b and c. A

$(x-1)^2+6$

$x^2-2x+1+6$

x^2-2x+7

5

(A) $b=-2, c=7$ (B) $b=-2, c=5$ (C) $b=2, c=-5$ (D) $b=2, c=-7$

19) What remainder do you get when you divide $3x^{101} - x^{50}$ by $x+1$ A

$3(-1)^{101} - (-1)^{50} = -4$

5

(A) -4 (B) -3 (C) -2 (D) -1

20) $f(x) = \frac{2x+1}{3}$. Calculate $\left(f^{-1}(2)\right)^{-1}$. **Hint: First find $f^{-1}(x)$.** B

$y = \frac{2x+1}{3}$

$x = \frac{2y+1}{3}$

$3x = 2y+1$

$2y+1 = 3x$

$2y = 3x-1$

$y = \frac{3x-1}{2}$

$f^{-1}(x) = \frac{3x-1}{2}$

$\left(\frac{3x-1}{2}\right)^{-1} = \frac{2}{3x-1}$

$\frac{2}{3(2)-1} = \frac{2}{6-1} = \frac{2}{5}$

5

A) 5/2 (B) 2/5 (C) $-5/3$ (D) 3/5

APPENDIX I: PRECALCULUS TEST #3

TEST #3

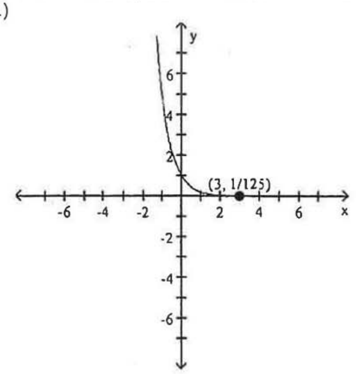
_______________(CAPITAL LETTERS) 83/100

SHOW ALL YOUR WORK

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Determine the correct exponential function of the form $f(x) = b^x$ whose graph is given.

1) 1) D



5

$\frac{1}{125} = b^3$

$\frac{1}{5^3} = b^3 \rightarrow \frac{1}{5}^x$

A) $f(x) = 3^x$ increasing B) $f(x) = 5^x$ increasing C) $f(x) = \left(\frac{1}{3}\right)^x$ D) $f(x) = \left(\frac{1}{5}\right)^x$

Solve the equation.

2) $e^{x-5} = \left(\frac{1}{e^4}\right)^{x+1}$ 2) $X = 1/5$

5

$e^{x-5} = (e^{-4})^{x+1}$

$e^{x-5} = e^{-4x-4}$

$x-5 = -4x-4$
$+5 \quad +4x$

$\frac{5x}{5} = \frac{1}{5}$ $X = 1/5$

3) $\left(\frac{1}{3}\right)^{5x+4} = 9^{x-5}$ 3) $X = \frac{6}{7}$

5

$\frac{1}{3}^{5x+4}$

$3^{-5x-4} = (3^2)^{x-5}$

$-5x-4 = 2x-10$
$+5x \quad +5x$

$-4 = 7x-10$
$+10 \quad +10$

$\frac{6}{7} = \frac{7x}{7}$

SHOW ALL YOUR WORK

Solve the problem.

4) A rumor is spread at an elementary school with 1200 students according to the model $N = 1200(1 - e^{-0.16d})$ where N is the number of students who have heard the rumor and d is the number of days that have elapsed since the rumor began. How many students will have heard the rumor after 5 days?

4) C

$N = 1200(1 - e^{-0.16(5)})$

$1200(.5506)$

660.72

5

A) 1006 students B) 689 students C) 661 students D) 1063 students

Find the domain of the function.

5) $f(x) = \log\left(\frac{x+6}{x-9}\right)$

5) $(9, \infty)$

Set the inequality and solve the inequality

$f(x) = \log\left(\frac{x+6}{x-9}\right)$

0

$x + 6 = 0$ $x - 9 = 0$

$x = -6$ $x = 9$

$\frac{x+6}{x-9} > 0$

F F 0 F T T

-7 -6 0 9 10

$\frac{0}{-15}$ $-\frac{3}{9}$ $(9, \infty)$

SHOW ALL YOUR WORK

Solve the problem.

6) Which has a lower present value: \$40,000 if interest is paid at a rate of 5.71% compounded continuously for 4 years, or \$44,000 if interest is paid at a rate of 3.3% compounded continuously for 57 months? 6) C

Good job!!

$e^{.0571(4)}$

$$\frac{40000}{e^{.0571(4)}} = \frac{Pe^{.0571(4)}}{e^{.0571(4)}} \qquad \frac{44000}{e^{.033(4.75)}} = \frac{Pe^{.033(4.75)}}{e^{.033(4.75)}}$$

5

$P = 31832.23 \qquad P = 37,616.38$

A) Both investments have the same present value.
B) \$44,000 with interest is paid at a rate of 3.3% compounded continuously for 57 months has a lower present value.
C) \$40,000 with interest paid at a rate of 5.71% compounded continuously for 4 years has a lower present value.

Use properties of logarithms to expand the logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

7) $\log \frac{8x^3\sqrt[4]{5-x}}{3(x+5)^2}$ 7) C

$\log 8x^3\sqrt[4]{5-x} - \log 3(x+5)^2$

$\log 8 + 3\log x + \frac{1}{4}\log(5-x) - \log 3 + 2\log(x+5)$

3

A) $\log(8x^3\sqrt[4]{5-x}) - \log(3(x+5)^2)$
B) $\log 8 + 3\log x + \frac{1}{4}\log(5-x) - \log 3 - 2\log(x+5)$
C) $\log 8 + 3\log x + \frac{1}{4}\log(5-x) - \log 3 + 2\log(x+5)$
D) $\log 8 + \log x^3 + \log(5-x)^{1/4} - \log 3 - \log(x+5)^2$

SHOW ALL YOUR WORK

Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

8) $\log x + \log(x^2 - 1) - \log 5 - \log(x - 1)$ 8) C

$\log \frac{x(x^2-1)}{\frac{5}{x-1}} \cdot \frac{x-1}{5} \rightarrow \log \frac{x(x-1)(x+1)}{\frac{x+1}{5}} \rightarrow \boxed{\frac{x(x+1)}{5}}$

5

A) $\log \frac{2x+1}{6-x}$ B) $\log \frac{x(x-1)(x-1)}{5}$

C) $\log \frac{x(x+1)}{5}$ D) $\log \frac{x(x-1)}{5(x-1)}$

Solve the equation.

9) $\log_4 (x + 5) = 3 + \log_4 (x + 2)$ 9) $x = -\frac{1}{2}$

$-\log_4 (x+2)$

$x+2 \cdot \frac{x+5}{x+2} = 3 \cdot x+2$

$x = -\frac{1}{2}$

0 $x+5 = 3x+6$
-6

$x-1 = 3x \Rightarrow \frac{2x}{2} = \frac{-1}{2}$
$-x \quad -x$

10) $9^{7x + 3} = 27$ 10) $x = \frac{-3}{14}$

$(3^2)^{7x+3} = 27$

$3^{14x+6} = 3^3$

5 $14x+6 = 3$
$-6 \quad -6$

$\frac{14x}{14} = \frac{-3}{14}$

SHOW ALL YOUR WORK

Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

11) $\frac{1}{7}[3 \ln (x + 7) - \ln x - \ln(x^2 - 5)]$ 11) C

$\ln \sqrt[7]{\frac{(x+7)^3}{\frac{x}{x^2-5}}} \longrightarrow \ln \sqrt[7]{\frac{(x+7)^3(x^2-5)}{x}}$

0

A) $\ln \sqrt[7]{\frac{(x + 7)^3}{x(x^2 - 5)}}$

B) $\ln \sqrt[7]{\frac{3(x + 7)}{x(x^2 - 5)}}$

C) $\ln \sqrt[7]{\frac{(x + 7)^3(x^2 - 5)}{x}}$

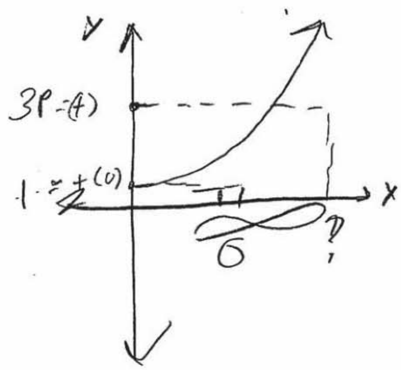
D) $\ln \sqrt[7]{\frac{x(x + 7)^3}{(x^2 - 5)}}$

Solve the problem.

12) How long will it take for an investment **to triple in value** if it earns 12.75% compounded continuously? 12) C

Round your answer to three decimal places.

Write the equation you need, make the graph, and solve the equation. (15 pts)



$A = Pe^{rt}$

$3 = e^{.1275(t)}$

$A = 2e^{.1275(0)}$

$A = 2$

$\ln 3 = .1275(t)$

$\frac{\ln 3}{.1275} = \frac{.1275(t)}{.1275}$

$t = \frac{\ln 3}{.1275} \rightarrow 8.617$

15

A) 5.436 yr

B) 8.999 yr

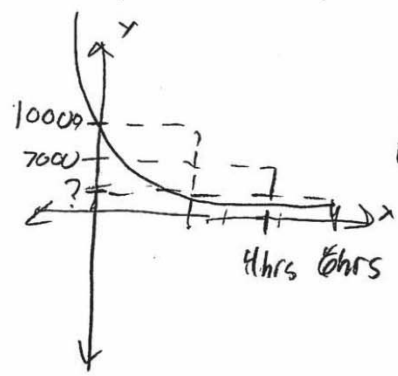
C) 8.617 yr

D) 4.308 yr

SHOW ALL YOUR WORK

13) A bacterial culture has an initial population of 10,000. 13) D
If its population declines to 7000 in 4 hours, what will it be at the end of 6 hours?
Assume that the population decreases according to the exponential model.

Write the equation you need, make the graph, plot your data, and solve the equation(s). (15 pts)



$$\frac{7000}{10000} = \frac{10000e^{r(4)}}{10000}$$

$$\ln\frac{7}{10} = e^{r(4)}$$

$$\ln\frac{7}{10} = \ln e^{r(4)}$$

$$\frac{\ln\frac{7}{10}}{4} = \frac{r(4)}{4}$$

$$r = -.0892$$

15

$$A = 10000e^{(-.0892)(6)}$$

$$A = 5857$$

A) 8156 B) 1500 C) 2929 D) 5857

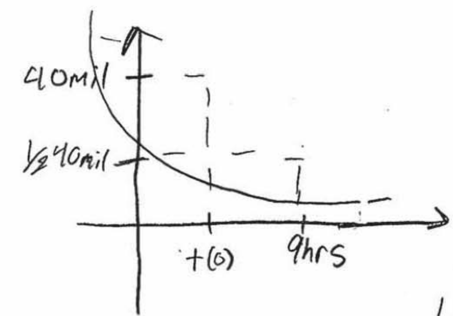
SHOW ALL YOUR WORK

14) The half-life of plutonium-234 is 9 hours. 14) B

If 40 milligrams is present now, how much will be present in 4 days?

(Round your answer to three decimal places.).

Write the equation you need, make the graph, plot your data, and solve the equation(s). (15 pts)



$A = Pe^{rt}$

$\frac{20}{40} = \frac{40e^{r(9)}}{40}$

$\ln \frac{2}{4} = \ln e^{r(9)}$

$\frac{\ln 2/4}{9} = \frac{r(9)}{9}$

$r = -.07702$

$A = 40e^{-.07702(96)}$

$A = .025$

15

A) 1.837 B) 0.025 C) 29.394 D) 19.097

APPENDIX J: PRECALCULUS TEST #4

SHOW ALL YOUR WORK

Exam #4 MATH 1103 / 11/17/2017

NAME

58/90 64/100

1. $(\cos\alpha - \sin\alpha)^2$ equals to which of the following

(a) $\sin^2(x) + \cos^2(x)$

(b) $1 + \cos 2\alpha$

(c) $\cos^2\alpha - \sin^2\alpha$

(d) $1 - \sin 2\alpha$

(e) None of these

$\cos^2\alpha - \sin^2\alpha$

2. Find the general solution of the equation $\sin\alpha = \frac{\sqrt{3}}{2}$

(a) $\alpha = \frac{\pi}{4} + k\pi$.

(b) $\alpha = \frac{\pi}{6} + k\pi$.

(c) $\alpha = \frac{\pi}{3} + k2\pi$ or $\alpha = \frac{2\pi}{3} + k2\pi$.

(d) $\alpha = \frac{\pi}{6} + k\pi$ or $\alpha = \frac{7\pi}{6} + k\pi$.

(e) $\alpha = \frac{\pi}{3} + k\frac{\pi}{2}$ or $\alpha = \frac{2\pi}{3} + k\frac{\pi}{2}$.

5

$\sin\alpha = \frac{\sqrt{3}}{2} = \frac{\pi}{3} + k2\pi$

	k=1	k=2	k=3
$\frac{\pi}{3}$	$\frac{2\pi}{3}$	[illegible]	$\frac{4\pi}{3}$

$\frac{2\pi}{3} + k2\pi$

3. Calculate $\tan\alpha$, if $\sin\alpha = \frac{4}{5}$ and α is in the **second quadrant.**

(Draw an x-y-axes. Draw the appropriate angle and corresponding right-triangle. Solve the problem)

(a) $-3/5$

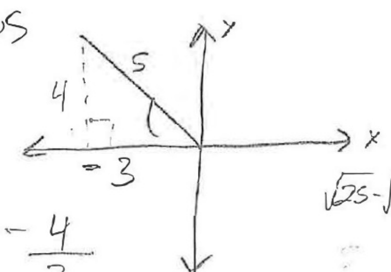
(b) $-4/3$

(c) $3/4$

(d) $4/3$

(e) $-3/4$

$-\cos$



5

$\sqrt{25} - \sqrt{16} = \sqrt{9}$

$-\frac{4}{3}$

3

$\tan = \frac{4}{3}$

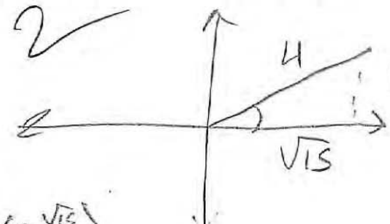
11

SHOW ALL YOUR WORK

4. Calculate $\cos(2\alpha)$, if $\sin(\alpha) = \frac{1}{4}$ and α is in the **first quadrant.**

(Write your formula. Draw an x-y-axes. Draw the appropriate angle and corresponding right-triangle. Solve the problem.)

(a) 7 / 8
(b) 1
(c) $\sqrt{15}/4$
(d) $-\sqrt{15}/4$
(e) 1 / 8



2

$4^2 - 1^2 = c^2$

$16 - 1 = \sqrt{5}$

$\frac{\sqrt{15}}{4}$

$\cos(2\frac{\sqrt{15}}{4})$

$\frac{4}{4} - 2(\frac{1}{4})^2 \rightarrow \frac{4}{4}$

$\frac{4}{4} - 2(\frac{1}{8}) \rightarrow \frac{24}{24} - \frac{2}{8}$

$\frac{4}{4} - \frac{1}{4} \rightarrow \frac{3}{4}$

$\frac{8}{8} - \frac{2}{8} \rightarrow \frac{6}{8}$

5. A) Find **the general solution** of the equation $2\sin(3\theta) + \sqrt{2} = 0$.

General Solution $= \frac{\pi}{4}$

$2\sin(3\theta) + \sqrt{2} = 0$
$-\sqrt{2} \quad -\sqrt{2}$

$\frac{2\sin(3\theta)}{2} = \frac{-\sqrt{2}}{2}$

$\sin(3\theta) = \frac{-\sqrt{2}}{2}$

$\frac{7\pi}{4}$

2

B) Find **the solutions on** the interval $[-\pi, \pi]$. **(Make a table to calculate your solutions)**

$\frac{\pi}{4} + k2\pi$

	k=1	k=2	k=3	k=4
$\frac{\pi}{4}$	$\frac{1\pi}{2}$	$\frac{4\pi}{2}$ 8π	$\frac{7\pi}{4}$	$\frac{11\pi}{4}$

$\frac{\pi}{4}, \frac{\pi}{2}, 2\pi, \frac{7\pi}{4}$

1

5 2

SHOW ALL YOUR WORK

6. Simplify the expression $\cos(x) + ctg(x)\sin(x)$.

$\cos(x) + \frac{\cos x}{\sin x} \cdot \sin x$

$\cos(x) + \cos(x) \to \boxed{2\cos(x)}$

(a) $\cos^2(x)$
(b) $2\sin(x)$
(c) $\sin^2(x)$
(d) $\sin(x)\cos(x)$
(e) $2\cos(x)$

7. Verify the identity $\tan(x)\cos(x) + \csc(x)\sin^2(x) = 2\sin x$.

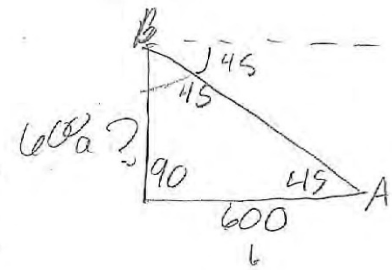
$\frac{\sin x}{\cos x} \cos x + \frac{1}{\sin x} \cdot \sin^2 x \to \sin x$

$\sin x + \sin x \to \boxed{2\sin x}$

8. From a point 600 feet from the base of a tree, the **angle of elevation** to the top of the tree is 45°. Find the height of the tree.

(Draw your figure. Label your figure. Solve the problem.)

(a) 300 *feet*
(b) 600 *feet*
(c) 150 *feet*
(d) 900 *feet*
(e) *none of the above*



$\frac{600}{\sin 45} = \frac{a}{\sin 45}$

$\frac{600 \cdot \sin 45}{}$

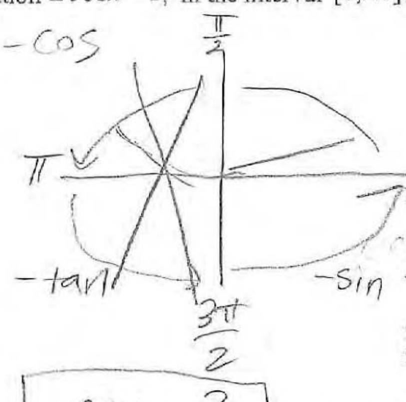
$\frac{424}{\sin 45}$

600ft

15

SHOW ALL YOUR WORK

9. Find the solutions of the equation $2\cos x = 1$, in the interval $[0, 2\pi]$.



$2(-\frac{1}{2}) \to -\frac{2}{2} = -1$

$2(\frac{1}{2}) = 1$

$2\frac{\sqrt{3}}{2} \to \sqrt{3}$

$\frac{2\sqrt{2}}{2} \to \sqrt{2}$

$2(\frac{1}{2}) \to \frac{2}{2} = 1$

$\cos = \frac{2}{2}$

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{6}, \frac{11\pi}{6}$

(c) $\frac{\pi}{3}, \frac{2\pi}{3}$

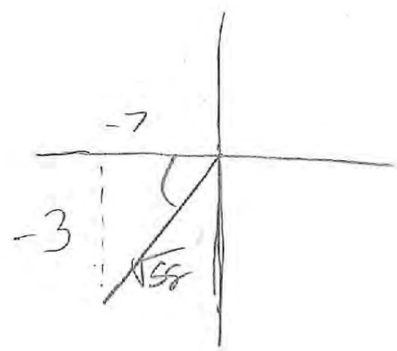
(d) $\frac{\pi}{3}, \frac{5\pi}{3}$

(e) $\frac{\pi}{4}, \frac{7\pi}{4}$

10. Find the exact value of $\sin\ \alpha$ for the angle α in standard position whose terminal side contains the point $(-3, -7)$.

(Draw an x-y-axes.
Draw the appropriate angle and corresponding right-triangle. Solve the problem.)

5



(a) $\sqrt{58}/3$

(b) $3/\sqrt{58}$

(c) $3/7$

(d) $-7/\sqrt{58}$

(e) $-\sqrt{58}/7$

$(-3)^2 + (-7)^2$

$9 + 49$

$\sqrt{58}$

sin $\frac{opp}{hyp}$

7

$-\frac{-3}{\sqrt{58}}$

SHOW ALL YOUR WORK

11. Simplify $\frac{\tan^3(x) + \tan(x)}{\cot(x)}$.

(Show all your work)

$$\text{Cancel}\left(\frac{\left(\frac{\sin(x)}{\cos(x)}\right)^3}{\frac{\cos(x)}{\sin(x)}} + \frac{\frac{\sin(x)}{\cos(x)}}{\frac{\cos(x)}{\sin(x)}} \right) \text{cancel}$$

(a) $\cot^2(x)$

(b) $-\tan^2(x)$

(c) $\tan^2(x)$

(d) $-\tan^2(x)\sec(x)$

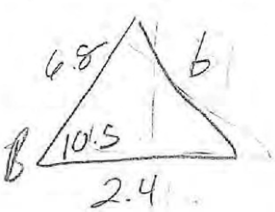
(e) $\tan^2(x)\sec^2(x)$

$$\left(\frac{\sin(x)}{\cos(x)}\right)^2 \to \tan^2(x)$$

2

12. In a given triangle, $a = 6.8$, $c = 2.4$, $\beta = 10.5°$. Find the length of the side b.

(Draw the triangle. Label the triangle. Solve the problem)



Cos Law

$$(b)^2 \to (2.4)^2 + (6.8)^2 - 2(2.4)(6.8)\cos(10.5$$

$$5.76 + 46.24 - 32.64(\cos(10.5$$

$$52 - 32.1$$

$$\sqrt{19.9}$$

$$4.46\ldots$$

5

7

(a) 9.17

(b) 8.25

(c) 5.02

(d) 4.46

(e) None of the above

SHOW ALL YOUR WORK

13. Which of the following equations is **NOT** an identity.

(a) $\tan(x)\cot(x) = 1$

(b) $(\sin(x)+\cos(x))^2 = \sin^2(x)+\cos^2(x)$

(c) $\tan^2(x) = \sec^2(x) - 1$

(d) $\cos^2(x)+\sin^2(x) = 1$

(e) $2\sin(x)\cos(x) = \sin(2x)$.

$\frac{\sin}{\cos} \cdot \frac{\cos}{\sin} \to 1$

5

14. Which of the following expressions equal to $\sin(\alpha - \frac{\pi}{3})$.

$\sin(a - \frac{\pi}{3})$

0

(a) $(\sqrt{3}\sin\alpha - \cos\alpha)/2$

(b) $(\sin\alpha - \sqrt{3}\cos\alpha)/2$

(c) $(\sqrt{3}\sin\alpha + \cos\alpha)/2$

(d) $(\sin\alpha + \sqrt{3}\cos\alpha)/2$

(e) $\sqrt{3}(\sin\alpha + \cos\alpha)/2$

SHOW ALL YOUR WORK

15. Which of the following is equal to $\cos^2(3\alpha)-\sin^2(3\alpha)$.

~~(a)~~ 1
(b) $\cos(3\alpha)$
~~(c)~~ $\sin(6\alpha)$
~~(d)~~ $2\sin(3\alpha)\cos(3\alpha)$
~~(e)~~ $2\cos^2(3\alpha)-1$

O

16. Use the addition subtraction formula to find $\sin(15°)$.

$\sin(45-30)$

$\sin(45)\cos(30)-\sin(30)\cos(45)$

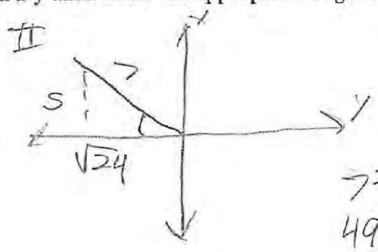
$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4} \rightarrow \frac{\sqrt{6}-\sqrt{2}}{4}$

Answer: $\frac{\sqrt{6}-\sqrt{2}}{4}$ 5.

17. Suppose that the terminal side of the angle α lies in Quadrant II and $\sin(\alpha)=\frac{5}{7}$. Find $\sin(2\alpha)$.

(Draw an x-y-axes. Draw the appropriate angle and corresponding right-triangle. Solve the problem)



$\sin(2a)$

3

$7^2-5^2=\sqrt{24}$

$49-25$

$-\cos$

$\sin(2a)=2\left(\frac{5}{7}\right)\left(\frac{\sqrt{24}}{7}\right)$

$\left(\frac{10}{14}\right)\left(\frac{\sqrt{24}}{7}\right)\rightarrow \boxed{-\frac{10\sqrt{24}}{98}}$

8

APPENDIX K: PRECALCULUS TEST #5

EXAM #5 MATH 1103 / 12/03/2017

NAME [redacted]

1. Among all the angles coterminal with the angle $\frac{25\pi}{4}$, find the *least positive coterminal angle.*

4 8 12 16 20 24

$\frac{24\pi}{4} + \frac{\pi}{4} = \frac{25\pi}{4}$

$6\pi + \frac{\pi}{4} = \frac{25\pi}{4}$

5.0

ANSWER $\pi/4$

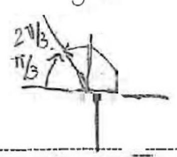
---

2. Find the reference angle of $\frac{28\pi}{6}$.

6 12 18 24

$\frac{24\pi}{6} + \frac{4\pi}{6} = \frac{28\pi}{6}$

$3\pi + \frac{2\pi}{3} = 28\pi$



5.0

ANSWER $\pi/3$

---

3. Find $(\sin^{-1}(\sin\frac{25\pi}{4}))$.

$(\sin^{-1}(\sin \frac{24\pi}{4} + \pi/4) = (\sin^{-1}(\sin 6\pi + \pi/4) = \sin^{-1}(\sin \pi/4) = \pi/4$

5.0

(a) $\frac{\sqrt{2}}{2}$, (b) $-\frac{\sqrt{2}}{2}$, (c) $\frac{\pi}{4}$, (d) $-\frac{\pi}{4}$, (e) $\frac{\pi}{2}$

---

4. Reduce the angle $\frac{25\pi}{4}$ radians to degrees.

$2\pi = 360$

$\frac{24\pi}{4} + \frac{\pi}{4} = 6\pi + \pi/4 = 2\pi + 2\pi + 2\pi + \pi/4 =$

$= 360 + 360 + 360 + 45 = 1125°$

5.0

20

ANSWER 1125°

1

SHOW ALL YOUR WORK TO RECEIVE CREDIT

5. Calculate the *period* and the *principle cycle* of the function $y = 5\tan(3x + \frac{\pi}{4})$.

Period = $\pi / |3| = \pi/3$

$-\pi/2 \le (3x + \frac{\pi}{4}) \le \pi/2$

$-3\pi/4 \le 3x \le \pi/4$

$-\frac{3\pi}{12} \le x \quad \pi/12$

$\pi/3 \div 4 = \pi/12$

5.0

(a) $\frac{\pi}{3}$, $-\frac{\pi}{6} \le x \le \frac{\pi}{6}$; (b) $\frac{\pi}{6}$, $-\frac{\pi}{6} \le x \le \frac{\pi}{6}$; (c) $\frac{\pi}{3}$, $-\frac{3\pi}{12} \le x \le \frac{\pi}{12}$;
(d) $\frac{\pi}{12}$, $-\frac{3\pi}{12} \le x \le \frac{\pi}{12}$; (e) $\frac{\pi}{3}$, $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$

---

6. Calculate $\cos(2\alpha)$, if $\sin(\alpha) = \frac{1}{4}$ and α is in the **first quadrant.**

$\cos^2(a) - \sin^2(a)$

$\cos^2\left(\frac{\sqrt{15}}{4}\right) - \sin^2\left(\frac{1}{4}\right)$

$\frac{15}{16} - \frac{1}{16} = \frac{14}{16} = \frac{7}{8}$

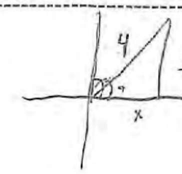
5.0

$4^2 - 1^2 = x^2$

$16 - 1 = x^2$

$15 = x^2$

$\sqrt{15} = x$



(a) 7/8; (b) 1; (c) $\sqrt{15}/4$; (d) $-\sqrt{15}/4$; (e) 1/8

---

7. For what values of x in the interval $(-\pi, \pi)$ does the graph of $f(x) = \csc(2x + \frac{\pi}{2})$ have *vertical asymptotes*? (angles are measured in radians)

(a) $x = -\frac{3\pi}{2}, -\frac{\pi}{6}, \frac{\pi}{6}, \frac{3\pi}{2}$
(b) $x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$
(c) $x = -\frac{\pi}{4}, \frac{\pi}{4}$
(d) $x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$
(e) $x = -\frac{3\pi}{4}, \frac{3\pi}{4}$

5.0

$\sin\left(2x + \frac{\pi}{2}\right) = 0$

$2x + \pi/2 = k\pi$

$2x = -\pi/2 + k\pi$

$x = -\pi/4 + \frac{k\pi}{2}$

15

	0	1	2	-1
$-\pi/4 + \frac{k\pi}{2}$	$-\pi/4$	$\pi/4$	$3\pi/4$	$-5\pi/4$

SHOW ALL YOUR WORK TO RECEIVE CREDIT

8. Verify the identity $\tan(x)\cos(x)+\csc(x)\sin^2(x)=2\sin x$.

$\frac{\sin x}{\cos x}\cdot\cos(x) + \frac{1}{\sin(x)}\cdot\sin(x)\cdot\sin(x) = 2\sin(x)$

$\sin(x) + \sin(x) = 2\sin(x)$

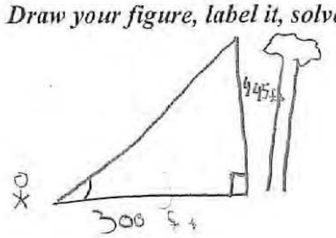
$2\sin(x) = 2\sin(x)$

5 ✓

---

9. You are at 300 feet from the base of a tree. The tree is 445 feet tall. What is the angle of elevation from where you are to the top of the tree? Round your answer to the nearest degree.

Draw your figure, label it, solve the problem.



5 ✓ $\tan^{-1}\frac{445}{300} = 56.01$

(a) 55°; (b) 57°; (c) 58°; (d) 59°; (e) 56°

---

10. Find the solutions of the equation $2\cos x = 1$, in the interval $[0,2\pi]$.

$\cos = \frac{1}{2}$

$\cos x = \frac{1}{2}$ — $\pi/3$, ~~$2\pi/3$~~, ~~$4\pi/3$~~ $-\frac{1}{2}$, $5\pi/3$

5 ✓

15

(a) $\frac{\pi}{3}$; (b) $\frac{\pi}{6}, \frac{11\pi}{6}$; (c) $\frac{\pi}{3}, \frac{2\pi}{3}$; (d) $\frac{\pi}{3}, \frac{5\pi}{3}$; (e) $\frac{\pi}{4}, \frac{7\pi}{4}$

SHOW ALL YOUR WORK TO RECEIVE CREDIT

11. Find the exact value of $\sin\alpha$ when angle α is in standard position and its terminal side contains the point $(-3,-7)$.

Draw an x-y-axes

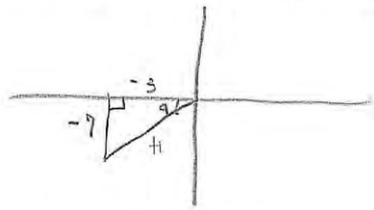
Draw the appropriate angle in standard position and the corresponding right-triangle.

$-7^{(2)} + -3^{(2)} = H^2$

$49 + 9 = H^2$

$58 = H^2$

$\sqrt{58} = H$



$\frac{-7}{H} = \sin = \frac{-7}{\sqrt{58}}$

5 ✓

(a) $\sqrt{58}/3$; (b) $-3/\sqrt{58}$; (c) $3/7$; (d) $-7/\sqrt{58}$; (e) $-\sqrt{58}/7$

---

12. Simplify the trigonometric expression $\frac{\tan^3(x)+\tan(x)}{\cot(x)}$.

$\frac{\tan(x)\cdot\tan(x)\cdot\tan(x) + \tan(x)}{\frac{\cos(x)}{\sin(x)}} = \frac{\frac{\sin}{\cos}\cdot\frac{\sin}{\cos}\cdot\frac{\sin}{\cos} + \frac{\sin}{\cos}}{\frac{\cos}{\sin}} =$

$\tan^4(x) + (\sec^2(x) - 1) = \tan^2 x \cdot \tan^2(x) + (\sec^2(x) - 1) = \tan^4 x + \tan^2 x$

$(\sec^2(x) - 1)\cdot(\sec^2(x) - 1)$

0.0

(a) $\cot^2(x)$; (b) $-\tan^2(x)$; (c) $\tan^2(x)$

(d) $-\tan^2(x)\sec(x)$; (e) $\tan^2(x)\sec^2(x)$

5,0

SHOW ALL YOUR WORK TO RECEIVE CREDIT

13. Use the law of cosines to find the measure of angle A (rounded to the nearest degree) in the figure below.

$25^2 = 32^2 + 37^2 - 2(32)(37)\cos A$

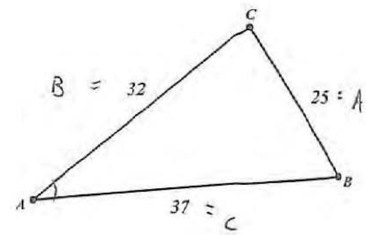
$625 = 1024 + 1369 - 2368 \cos A$

$625 = 2393 - 2368 \cos A$

$-1768 = -2368 \cos A$

$\frac{-1768}{-2368} = \cos A$

$\cos^{-1}\left(\frac{1768}{2368}\right) = A = 41.7$



5.0

(a) $A = 30°$; (b) $A = 37°$; (c) $A = 39°$; (d) $A = 42°$; (e) $A = 44°$

---

14. Which of the following expressions equal to $\sin(\alpha + \frac{\pi}{4})$.

$\sin a \cos \pi/4 + \cos a \sin \pi/4$

$\sin a \frac{\sqrt{2}}{2} + \cos a \sqrt{2}/2$

5.0

$\frac{\sin a \sqrt{2}}{2} + \frac{\cos a \sqrt{2}}{2} = (\sin a \sqrt{2} + \cos a \sqrt{2})/2$

(a) $(\sqrt{2}\sin\alpha - \cos\alpha)/2$; (b) $(\sqrt{2}\sin\alpha - \sqrt{2}\cos\alpha)/2$

(c) $(\sqrt{2}\sin\alpha + \sqrt{2}\cos\alpha)/4$; (d) $(\sin\alpha + \cos\alpha)/2$

(e) $\sqrt{2}(\sin\alpha + \cos\alpha)/2$

10

SHOW ALL YOUR WORK TO RECEIVE CREDIT

14. Write $\cos(ctg^{-1}x)$ as an algebraic expression in terms of x.

(a) $\frac{x}{\sqrt{x^2-1}}$

(b) $\frac{x}{\sqrt{x^2+1}}$

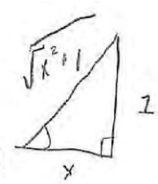
(c) $\frac{x}{\sqrt{1-x^2}}$

(d) $\sqrt{x^2-1}$

(e) $\sqrt{x^2+1}$

$\cot^{-1}(x) = \theta$ $\quad$ $\cot = \frac{\cos}{\sin}$

$\cot\theta = \frac{x}{1}$



$x^2 + 1^2 = b^2$

$x^2 + 1 = b^2$

$\sqrt{x^2+1} = b$

$\cos = \frac{x}{\sqrt{x^2+1}}$

5.0

16. **Without using the calculator** find $\sin(75°)$. Use the addition/ subtraction formula. Give *the exact value.*

$\sin(45° + 30°) = \sin(\pi/4 + \pi/6)$

$\sin\pi/4 \cos\pi/6 + \cos\pi/4 \sin\pi/6$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$

5

ANSWER $\frac{\sqrt{6}+\sqrt{2}}{4}$

17. Suppose that the terminal side of the angle α lies in Quadrant II and $\sin(\alpha) = \frac{5}{7}$.

Find $\sin(2\alpha)$.

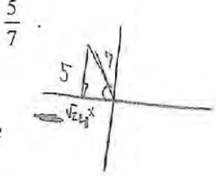
$7^2 - 5^2 = x^2$ $\quad$ $24 = x^2$

$49 - 25 = x^2$ $\quad$ $\sqrt{24} = x$

Draw an x-y-axes.

Draw the appropriate angle in standard position and its corresponding right-triangle

Solve the problem



$\sin(a + a) = \sin a \cos a + \cos a \sin a$

$\frac{5}{7} \cdot \frac{\sqrt{24}}{7} + \frac{\sqrt{24}}{7} \cdot \frac{5}{7}$

$\frac{5\sqrt{24}}{49} + \frac{5\sqrt{24}}{49} = \frac{10\sqrt{24}}{49}$

4.0

ANSWER $\frac{10\sqrt{24}}{49}$

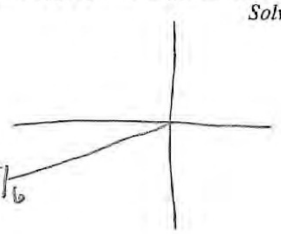
14

SHOW ALL YOUR WORK TO RECEIVE CREDIT

18. **Without using the calculator** find the ***exact value*** of $\sec(\frac{7\pi}{6})$.

Draw an x-y-axes.
Draw the appropriate angle in standard position and its corresponding right-triangle
Solve the problem

$\sec = \frac{1}{\cos}$



$\frac{-\sqrt{3}}{2}, -\frac{1}{2}$ <- $7\pi/6$

$\frac{1}{\frac{-\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$

5

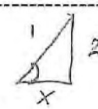
ANSWER $-\frac{2}{\sqrt{3}}$

19. Find $\csc^{-1}(1/2)$. $= \sin^{-1}(2)$ $\csc = \frac{1}{\sin}$

$\csc^{-1}(1/2) = \theta$

$\csc\theta = 1/2$

$\neq -\frac{\pi}{2} \le x \le \pi/2$



5

ANSWER does not exist

20. Find the phase shift, period, and range of the function $f(x) = -5\sin(2x + \frac{2\pi}{3}) + 3$.

Phase Shift $= C/B = \frac{-\frac{2\pi}{3}}{2} = \frac{-2\pi}{6} = -\pi/3$

Period $= 2\pi/B = 2\pi/|2| = \pi$

Range $= (3-5, 5+3)$

$-2, 8$

5

ANSWER
phase shift $= -\pi/3$
period $= \pi$
Range $= [-2, 8]$

15

APPENDIX L: PRECALCULUS FINAL EXAM

MATH 1103 COMMON FINAL EXAM
FALL 2017

Please print the following information:

Instructor:

Section/Time: MWF 9:30 - 10:45
Section 003

The MATH 1103 Final Exam consists of 45 multiple choice questions. They are printed on the front and the back of each page. A special answer sheet is provided so that your answers can be machine graded. You have three hours for the entire test.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet.
- For each question choose the response which *best* fits the question.
- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.
- There is no penalty for guessing. However if you mark more than one answer to a question, that question will be scored as incorrect.
- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.
- *Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.*

At the end of the examination you MUST hand in this booklet, your answer sheet and all scratch paper.

MATH 1103 FINAL EXAM Multiple Choice

1. Find the equation of the line in standard form that is perpendicular to the line $-4x - 9y = -17$; passes through the point $(2, -1)$.

(a) $2x + 9y = -17$
(b) $9x + 4y = 22$
(c) $-4x + 9 = -4$
(d) $9x - 4y = 22$
(e) $9x - 2y = 22$

$-4x - 9y = -17$
$-9y = 4x - 17$
$y = -\frac{4}{9}x + \frac{17}{9}$

$y + 1 = \frac{9}{4}(x - 2)$
$y + 1 = \frac{9}{4}x - \frac{18}{4}$
$y = \frac{9}{4}x - \frac{22}{4}$
$y = 4(9x - 22)$
$4y = 9x - 22$
$-9x + 4y = -22$
$9x - 4y = 22$

2. Find the domain of the function: $g(t) = \sqrt{\frac{5}{t-10}}$.

(a) $(10, \infty)$
(b) $(-\infty, \infty)$
(c) $(-\infty, 10) \cup (10, \infty)$
(d) $[10, \infty)$
(e) $(-\infty, 10]$

3. Evaluate the difference quotient, $\frac{f(x+h)-f(x)}{h}$, for $f(x) = x^2 + 9x - 1$.

(a) $\frac{2x^2+2x+2xh+h^2+h-2}{h}$
(b) $2x + h + 9$
(c) 1
(d) $2x + h - 1$
$\frac{x^2+x+2xh+h^2+h+9}{h}$
(e) $x + 2h - 9$

$f(x+h) = (x+h)^2 + 9(x+h) - 1$
$= x^2 + 2hx + h^2 + 9x + 9h - 1$

$\to \frac{x^2 + 2hx + h^2 + 9x + 9h - 1 - x^2 - 9x + 1}{h} = \frac{2hx + h^2 + 9h}{h}$
$= \frac{h(2x + h + 9)}{h}$

4. Which of the following is true for the function: $f(x) = \frac{x}{x^2-4}$.

(a) $f(x)$ is an even function
(b) $f(x)$ is an odd function
(c) $f(x)$ is a polynomial function
(d) $f(x)$ is not a function
(e) The domain of $f(x)$ is: $(-\infty, \infty)$

$f(-x) = \frac{-x}{-x^2-4} = -\frac{x}{x^2-4}$

5. Given the following piecewise function, find $f(3) \cdot f(10)$.

$$f(x) = \begin{cases} x + 2 & \text{if } -8 \leq x < 5 \\ -9 & \text{if } x = 5 \\ -x + 6 & \text{if } x > 5 \end{cases}$$

(a) 80
(b) −27
(c) 1
(d) −20
(e) 30

$f(3) = 3 + 2 = 5$
$f(10) = -10 + 6 = -4$
$f(3) \cdot f(10) = -20$

17. Solve the polynomial inequality: $x^2 - 8x > -15$

(a) $(5, \infty)$
(b) $(-\infty, 3)$
(c) $(3, 5)$
(d) $(-\infty, 3) \cup (5, \infty)$
(e) $(-\infty, 8) \cup (15, \infty)$

$x^2 - 8x > -15$
$x^2 - 8x + 15 > 0$
$(x-3)(x-5)$
$(-\infty, 3) \cup (5, \infty)$

18. Solve the rational inequality: $\frac{x-17}{x+8} \le 0$.

(a) $(-8, 17)$
(b) $(-8, 3)$
(c) $(-8, 17]$
(d) $(-8, 3]$
(e) $(-\infty, -8) \cup (3, \infty)$

≤ 0
$(-8, 17]$

19. Write the logarithmic equation as an exponential equation: $\ln(x) = 8$.

(a) $8^e = x$
(b) $e^x = 8$
(c) $x^8 = e$
(d) $8x = e$
(e) $e^8 = x$

$e^8 = x$

20. Find the domain of the function: $f(x) = \log(x + 4)$.

(a) $(4, \infty)$
(b) $(0, \infty)$
(c) $(-4, \infty)$
(d) $(1, \infty)$
(e) $(-\infty, 4)$

21. Use properties of logarithms to expand the logarithmic expression as much as possible: $\log_2 \frac{16}{\sqrt{x-1}}$.

(a) $4\log_2 2 - \frac{1}{2}\log_2(x-1)$
(b) $\log_2 16 - \log_2 \sqrt{x-1}$
(c) $4 - \log_2 \sqrt{x-1}$
(d) $4 - \frac{1}{2}\log_2(x-1)$
(e) $\log_2 16 + \frac{1}{2}\log_2(x-1)$

$2^x = 16$
$\log_2 \frac{16}{\sqrt{x-1}}$
$\log_2 16 - \log_2 (x-1)^{1/2}$
$\log_2 16 - \frac{1}{2}\log_2(x-1)$

22. Solve the exponential equation: $5^{x+8} = 4$.

(a) $\{2.06\}$
(b) $\{-1, 00\}$
(c) $\{-7.14\}$
(d) $\{9.16\}$
(e) $\{-5, 23\}$

$5^{x+8} = 4$

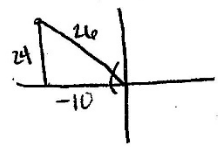
29. Name the quadrant or axes where the terminal side of the angle, 3 radians, lies..

(a) Quadrant 1
(b) Quadrant 2
(c) Quadrant 3
(d) Quadrant 4
(e) x-axis

$3 \cdot \frac{180}{\pi}$

30. The point $(-10, 24)$, lies on the terminal side of angle θ. Find the exact value of the $\sin\theta$.

(a) $-\frac{5}{13}$
(b) $\frac{5}{12}$
(c) $-\frac{12}{13}$
(d) $\frac{5}{13}$
(e) $\frac{12}{13}$



$-10^2 + 24^2 = r^2$

$r = 26$

$\sin\theta = \frac{24}{26} = \frac{12}{13}$

31. Determine the quadrant or axes in which the terminal side of angle θ lies in which: $\cot\theta < 0$ and $\cos\theta > 0$.

(a) Quadrant I
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV
(e) The x-axis

$\cot = \frac{\cos}{\sin}$ -> negative

cos -> positive
sin -> negative

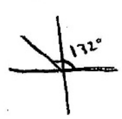
32. Use the special points on the unit circle and corresponding values of the angle given to determine the exact value of: $\tan\left(\frac{5\pi}{6}\right)$.

(a) $-\sqrt{3}$
(b) $\frac{1}{\sqrt{3}}$
(c) $\sqrt{3}$
(d) $\frac{\sqrt{3}}{2}$
(e) $-\frac{1}{\sqrt{3}}$

$\frac{\sin}{\cos}$

33. Determine the reference angle for the angle: $\theta = 132°$.

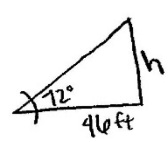
(a) 58°
(b) 48°
(c) 42°
(d) 52°
(e) 38°



$180 - 132° = 48°$

34. From a distance of 46 feet from the base of a flagpole, the angle of elevation to the top of the flagpole is 72°. What is the height of the flagpole rounded to the nearest foot?

(a) 14 feet
(b) 50 feet
(c) 44 feet
(d) 142 feet
(e) 32 feet



$\tan 72° = \frac{h}{46}$

$h = 141.57$ ft

23. Solve the logarithmic equation: $\log_3 x + \log_3 (x - 24) = 4$.

(a) {53}
(b) {27}
(c) {−3, 27}
(d) {14}
(e) No Solution

$\log_3 x + \log_3 (x-24) = 4$
$27-24$
$\log_3 27 + \log_3 3$
$3 + 1 = 4$

24. Find the inverse function of the logarithmic function $f(x) = \log_2 (x + 9) - 7$.

(a) $f^{-1}(x) = \log_9 (x - 7) + 2$
(b) $f^{-1}(x) = \log_2 (x + 7) - 9$
(c) $f^{-1}(x) = 2^{(x+9)} - 7$
(d) $f^{-1}(x) = 2^{(x-7)} + 9$
(e) $f^{-1}(x) = 2^{(x+7)} - 9$

$x = \log_2 (y+9) - 7$
$x+7 = \log_2 (y+9)$
$2^{x+7} = y+9$
$2^{(x+7)} - 9 = y$

25. How long will it take for an investment to double in value if it earns 5.25% compounded continuously? Round your answer to three decimal places.

(a) 20.926 yr
(b) 14.114 yr
(c) 13.203 yr
(d) 6.601 yr
(e) 15.095 yr

$2P = Pe^{rt}$
$2P = Pe^{(0.0525)t}$
$\frac{2P}{P} = e^{(0.0525)t}$
$2 = e^{0.0525t}$
$\ln 2 = 0.0525t$

26. The half-life of silicon-32 is 710 years. If 50 grams is present now, how much will be present in 500 years? (Round your answer to three decimal places.)

(a) 0
(b) 0.379
(c) 47.618
(d) 30.689
(e) 27.923

$A = 50e^{(-710)(500)}$

27. Find the angle of least positive measure that is coterminal with the given angle: $\frac{18\pi}{5}$.

(a) $-\frac{18\pi}{5}$
(b) $\frac{2\pi}{5}$
(c) $\frac{13\pi}{5}$
(d) $\frac{8\pi}{5}$
(e) $\frac{3\pi}{5}$

$\frac{18\pi}{5} - \frac{10\pi}{5} = \frac{8\pi}{5}$

28. Convert 144° into a radian measure.

(a) $\frac{5\pi}{6}$ radians
(b) $\frac{3\pi}{4}$ radians
(c) $\frac{3}{5}\pi$ radians
(d) $\frac{4\pi}{5}$ radians
(e) $\frac{7\pi}{8}$ radians

$144 \cdot \frac{\pi}{180} = \frac{4\pi}{5}$

35. Determine the period and phase shift of the function: $y = 5\sin\left(3x - \frac{\pi}{2}\right)$.

(a) Period: $\frac{3\pi}{2}$ Phase Shift: $\frac{\pi}{6}$
(b) Period: $\frac{2\pi}{3}$ Phase Shift: $\frac{\pi}{2}$
(c) Period: $\frac{2\pi}{3}$ Phase Shift: $\frac{\pi}{6}$
(d) Period: 2π Phase Shift: $\frac{\pi}{2}$
(e) Period: $\frac{\pi}{3}$ Phase Shift: $\frac{\pi}{6}$

$P = \frac{2\pi}{3}$ $\quad \frac{\pi}{2} \cdot \frac{1}{3} = \frac{\pi}{6}$

36. Find the exact angle value in radians of the expression based on the unit circle angles for: $\csc^{-1}\left(-\sqrt{2}\right)$.

(a) $-\frac{\pi}{4}$
(b) $\frac{\pi}{6}$
(c) $\frac{\pi}{4}$
(d) $\frac{\pi}{3}$
(e) $-\frac{\pi}{6}$

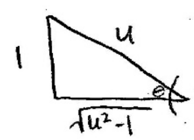
$\csc\theta = -\sqrt{2}$

$\frac{1}{\sin\theta}$

37. Rewrite the following trigonometric expression as an algebraic expression involving the variable u : $\cos\left(\sin^{-1}\frac{1}{u}\right)$.

(a) $\frac{\sqrt{u^2+1}}{u^2+1}$
(b) $\sqrt{u^2-1}$
(c) $\frac{\sqrt{u^2-1}}{u^2}$
(d) $\frac{\sqrt{u^2-1}}{u}$
(e) $\frac{u}{1}$

$\sin^{-1}\frac{1}{u} \rightarrow \sin\theta = \frac{1}{u}$



$1^2 + x^2 = u^2$
$1 + x^2 = u^2$
$x^2 = u^2 - 1$
$x = \sqrt{u^2-1}$

38. Simplify: $1 + \sec^2 x \sin^2 x$.

(a) $\sec x$
(b) $\tan x$
(c) $\cot^2 x$
(d) $\tan^2 x$
(e) $\sec^2 x$

$1 + \sec^2 x \sin^2 x$
$= 1 + \frac{\sin^2 x}{\cos^2 x}$
$= 1 + \tan^2 x$
$= \sec^2 x$

39. Simplify: $\tan\theta\,(\csc\theta - \sin\theta)$

(a) $\cos^2\theta$
(b) $\sin\theta$
(c) $\sin^2\theta$
(d) $\cos\theta$
(e) $\tan\theta$

$\tan\theta(\csc\theta - \sin\theta)$
$= \frac{\sin\theta}{\cos\theta}\left(\frac{1}{\sin\theta} - \sin\theta\right) = \frac{1-\sin^2\theta}{\cos\theta}$
$= \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} = \cos\theta$

40. Which of the following expression is equivalent to $\sin\left(\theta + \frac{3\pi}{2}\right)$.

(a) 1
(b) $\cos\theta$
(c) $\sin\theta$
(d) $-\cos\theta$
(e) $-\sin\theta$

$\sin\left(\theta + \frac{3\pi}{2}\right) = (\sin\theta)\left(\cos\frac{3\pi}{2}\right) + (-1)(\cos\theta)$
$= 0 - \cos\theta$

41. Simplify $\cos(7k)\cos(4k) + \sin(7k)\sin(4k)$ using an appropriate trigonometric identity.

(a) $\cos(11k)$
(b) $\cos(3k)$
(c) $\sin(3k)$
(d) $\sin(11k)$
(e) $\cos(-3k)$

$\cos(7k - 4k)$

$\cos(3k)$

42. If $\tan\theta = \frac{8}{15}$; and the terminal side of θ lies in Quadrant III, then find the exact value: $\sin(2\theta)$

(a) $\frac{161}{289}$
(b) $\frac{240}{289}$
(c) $-\frac{161}{289}$
(d) $-\frac{240}{289}$
(e) $\frac{161}{240}$

$\tan\theta = \frac{8}{15} = \frac{y}{x}$ $y=-8$ $x=-15$ $r=17$

$\sin(2\theta) = \sin^2\theta - \cos^2\theta$

$= \left(\frac{-8}{17}\right)^2 - \left(\frac{-15}{17}\right)^2 = \frac{64}{289} - \frac{225}{289}$

43. Determine a general formula (or formulas) for the solution to the equation: $2\cos\theta - \sqrt{2} = 0$.

(a) $\theta = \frac{3\pi}{4} + 2\pi k$ or $\theta = \frac{5\pi}{4} + 2\pi k$
(b) $\theta = \frac{\pi}{4} + 2\pi k$ or $\theta = \frac{7\pi}{4} + 2\pi k$
(c) $\theta = \frac{\pi}{4} + \pi k$ or $\theta = \frac{7\pi}{4} + \pi k$
(d) $\theta = \frac{3\pi}{4} + \pi k$ or $\theta = \frac{5\pi}{4} + \pi k$
(e) $\theta = \frac{5\pi}{4} + 2\pi k$

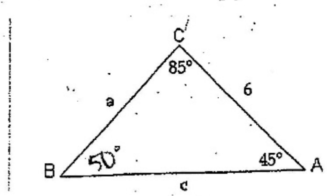
$2\cos\theta - \sqrt{2} = 0$

$2\cos\theta = \sqrt{2}$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

44. Find the values of the missing parts of the triangle using The Law of Sines.



$\frac{\sin(50°)}{6} = \frac{\sin(45°)}{a}$

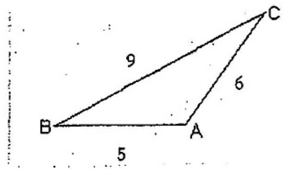
$a = 5.54$

$\frac{\sin(50°)}{6} = \frac{\sin(85)}{c}$

$c = 7.8$

(a) $B = 50°, a = 7.8, c = 5.54$
(b) $B = 50°, a = 5.54, c = 7.8$
(c) $B = 55°, a = 5.54, c = 7.8$
(d) $B = 45°, a = 7.8, c = 5.54$
(e) $B = 50°, a = 7.8, c = 8.7$

45. Find the values of the missing parts of the triangle using The Law of Cosines and The Law of Sines.



(a) $A = 38.9°, B = 109.5°, C = 31.6°$
(b) $A = 38.9°, B = 31.6°, C = 109.5°$
(c) $A = 109.5°, B = 38.9°, C = 31.6°$
(d) $A = 109.5°, B = 31.6°, C = 38.9°$
(e) $A = 109.5°, B = 32.9°, C = 39.8°$

$9^2 = 6^2 + 5^2 - 2(6)(5)\cos A$
$81 = 61 - 60 \cos A$
$20 = -60 \cos A$
$-1/3 = \cos A$
$A = 109.5°$

$6^2 = 9^2 + 5^2 - 2(9)(5) \cos B$
$36 = 106 - 90 \cos B$
$-70 = -90 \cos B$
$\frac{7}{9} = \cos B$
$B = 38.9°$