USING TOOLS TO SUPPORT PRODUCTIVE MATHEMATICAL DISCUSSIONS: A MULTIPLE CASE STUDY

by

Julie Bacak

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Approved by:

____
Dr. Madelyn Colonnese

____
Dr. Drew Polly

____
Dr. Tracy Rock

____
Dr. Michelle Stephan

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ABSTRACT

JULIE BACAK. Using tools to support productive mathematical discussions: A multiple case study. (Under the direction of DR. MADELYN COLONNESE)

Facilitating productive mathematical discussions is considered a core practice of mathematics education. The complexity of this teaching practice presents the need for pedagogical tools to provide structure for preservice teachers (PST) developing their practice, yet little is known about *how* PSTs use these tools. This multiple case study sought to understand what pedagogical tools PSTs use to plan and enact mathematical discussions with elementary students and how they use these tools to support their practice. In particular, this study focused on capturing the experiences of three elementary PSTs as they transitioned from university-based methods course instructions into early clinical teaching experiences in elementary classrooms. These experiences were captured through multiple one-on-one interviews, observations of teaching in clinical classroom settings, and analysis of artifacts of teaching and learning. This study has implications for mathematics teacher education, practice-based teacher education, and the refinement of tools to support teachers' practice facilitating mathematical discussions with students.

Keywords: preservice teacher, productive mathematical discussions, pedagogical tools, math methods instruction, clinical teaching, elementary mathematics, core practices

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My family, both given and chosen, thank you for your unwavering support of my professional journey, even if my choices did not always make sense to you. Dad, since I was little, you always encouraged me to take a critical stance and consider all perspectives. I appreciate the value in that now more than ever. Mom, your selfless love is unmatched. Thank you for always letting me know, “quitting is an option,” knowing full and well I would never give up.

DEDICATION

All my family near and far,
Friends who’re family in my heart,
I owe this to you.

The teachers who inspired me,
The ones who worked beside me,
I owe this to you.

My students who taught me everything,
My students who will teach the future,
I owe this to you.

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LIST OF ABBREVIATIONS

CE	Clinical Educator
PBTE	Practice-Based Teacher Education
PST	Preservice Teacher
TE	Teacher Educator
YLI	Yearlong Internship

CHAPTER ONE: INTRODUCTION

After spending weeks of methods course instruction on why and how teachers should incorporate student-centered discussion into mathematics instruction, a small group of elementary preservice teachers (PSTs) meet to rehearse the lessons they have planned to use with elementary students. Instead of elementary students, PSTs are "teaching" their undergraduate peers in their math methods class. A pair of PSTs present a first-grade level "add to with change unknown" problem for their peers to solve. In planning for the lesson, the PSTs anticipated possible first-grade responses to their task, primarily how students would use direct modeling. When anticipating possible developing ideas first graders might share, they focused mostly on students misreading the problem or not attending to all of the details of the task. Their lesson plan followed a "Launch, Explore, Discuss" model.

After engaging their peers in a launch to provide context for the task, they allot time for their students to explore ways of solving the problem. As they work, the PSTs monitor their peer-students, probing their thinking by referring to a list of go-to questions such as, "Can you solve this problem in a different way?" or, "Explain to me how you came up with your answer." After a few minutes to explore the problem, the PSTs open the floor for volunteers to share their solutions and their mathematical thinking. Nicky, the first peer-student, shares a strategy in which they use direct modeling with counting chips and subtraction to solve. The lead teacher asks, "Do you agree or disagree with Nicky?" All but one peer-student agrees. The teacher turns her attention to Sam, the peer who disagrees, to share their strategy. Sam models the problem with counting chips, recounting several steps in their thinking process, but arrives at an incorrect solution. It is apparent by the faces of both teachers that they do not follow their peer's strategy. The lead teacher turns to the prepared questions in her lesson plan document. "Can *anyone restate Sam's strategy in their own words?" There are no volunteers; the peer-students seem equally perplexed by Sam's strategy. Referring again to her list of questions, the teacher asks, "Do you agree or disagree with Sam's thinking?" All of Sam's peers disagree, yet no one offers a rationale beyond getting different answers. The discussion is at a standstill. Appearing a bit flustered, the lead teacher turns to her teaching partner to consult briefly. They are worried about time. They scroll through their lesson plan one more time but find there is nothing there to help them move forward. They feel stuck.*

I share the story above not because it is unique, but because it represents a common challenge experienced by novice teachers learning to facilitate productive mathematical discussions. According to the eight practices for effective mathematics teaching and learning outlined by the National Council of Teachers of Mathematics (NCTM), teachers should be able to "facilitate meaningful mathematical discourse" to support students in developing shared understanding of mathematical ideas (NCTM, 2014). Classroom discourse provides learners with opportunities to share their ideas, clarify conceptual understanding, construct mathematical arguments, and learn to see mathematical ideas from other perspectives (NCTM, 1991). In discourse-based mathematics instruction, the teacher acts as a facilitator, actively guiding class discussions focused on student thinking (Chapin et al., 2013).

However, facilitating productive mathematical discussions is a complex practice for both professional and prospective teachers, as leading mathematical discussions requires teachers to attend to a range of pedagogical and mathematical knowledge demands simultaneously as they respond to student thinking (Association of Mathematics Teacher Educators [AMTE], 2017; Boerst et al., 2011). For this dissertation, productive mathematical discussions are characterized as discussions that utilize students' ideas in ways that advance the shared understanding of significant mathematics (Lampert, 2001; Stein et al., 2008).

While facilitating productive discussions can also be challenging for experienced teachers, preservice teachers (PST) occupy a unique instructional problem space as they develop their practice. PSTs have less experience with both the mathematical and pedagogical content required to effectively engage learners in mathematical discussions (Ghousseini, 2015). As Ghousseini (2015) explained, PSTs are more likely to rely on instructional maxims than experienced teachers due to their developing pedagogical and content knowledge. One such maxim Ghousseini (2015) observed is PSTs viewing the role of the teacher in the mathematical discussion *always* as an outside manager, giving students control of the conversation. At times, these maxims can provide needed structure for developing teachers. For example, the PST in the opening vignette was able to use "go-to" questions to facilitate connections efficiently and effectively between mathematically valid ideas. However, when these "go-to" questions proved ineffective, the PST lost direction in the discussion; feeling stuck, she was unable to bring meaningful closure to the discussion as planned. In addition, PSTs carry out their instructional practice in existing classroom contexts with established structures for student participation that may or may not align with practices learned from teacher education (Ghousseini, 2015; Munter, 2014). Taking into consideration this unique instructional problem space for PSTs, my study focused directly on PSTs' experiences planning and facilitating mathematical discussions rather than experienced teachers.

This study examined the various pedagogical tools PSTs used to plan and facilitate mathematical discussions with elementary students during their first semester of their yearlong internship (YLI). The purpose of this study was to understand how PSTs used these tools to inform their planning process and their enactment of mathematical discussions with students in their YLI classroom. For this dissertation, pedagogical tools refer to the tools teachers use to "construct and carry out teaching practices," including both broadly applicable conceptual tools and locally applicable practical tools (Grossman et al., 1999, p. 13).

In designing this study, I explored existing research on mathematical discourse in the elementary classroom, theories and practices related to novice teacher learning, and teacher education focused on the practice of facilitating discourse. In order to gain an in-depth understanding of how PSTs use pedagogical tools for discourse in clinical teaching settings, I used a qualitative, multiple case study design to examine PSTs' experiences through interviews, observations, field notes, and artifacts of teaching and learning. Participants in this study were undergraduate, elementary education majors in a large, public university. PSTs had completed two semesters of math methods coursework as they transitioned into their first semester of their YLI in an elementary classroom.

This chapter continues with an overview of the problem surrounding preparing teachers to lead productive mathematical discussions. Following this context, I provide my statement of purpose, research questions, and an overview of the conceptual framework used to guide this study. Next, I outline the significance of my study to mathematics teacher education and how my study benefits elementary PSTs preparing to lead productive mathematical discussions. I include the underlying assumptions of this study and identify limitations. This chapter concludes with a definition of key terms as they are framed in this study and an overview of the subsequent chapters in my dissertation.

Background

Facilitating mathematical discussions is considered a core practice in mathematics education (Janssen et al., 2015). An instructional practice is considered a core practice if it can be used with high frequency, can be enacted with a wide variety of curricular content, and demonstrates the potential to improve student learning outcomes (Grossman & McDonald, 2008). Based on this description, facilitating mathematical discussions is a core practice because participating in discussions helps students construct mathematical knowledge and supports deeper conceptual understanding (Lampert, 2001; NCTM, 1991; Stein et al., 2008). Additionally, facilitating mathematical discussions is an appropriate practice to use across grade levels and mathematical content strands (NCTM, 1991).

Much of the research on preparing teachers to lead mathematical discussions falls within the larger field of practice-based teacher education (PBTE). PBTE focuses on core practices directly related to the work of teaching to develop highly skilled teachers (Forzani, 2014). An assumption of PBTE is that core practices, like facilitating mathematical discussions, are practices that novices can actually begin to master while still maintaining the integrity and complexity of teaching (Grossman, Hammerness, & McDonald, 2009).

Learning experiences within PBTE often follow Grossman, Compton, and colleagues' (2009) framework of pedagogies of practice in professional education. Through cross-professional analysis, they identified three pedagogies of practice needed in professional education: representations, decompositions, and approximations. Representations serve to illustrate elements of professional practice. For teacher education, this includes videos of expert teachers, lesson plans, transcripts, and student work samples. Decompositions of practice break down a larger practice into identifiable parts that can be practiced in a more explicit way. For example, a decomposition of the practice of facilitating mathematical discourse could include a task designed to practice sequencing student work samples to facilitate connections between ideas. Approximations of practice allow novice teachers to engage in experiences resembling real practice that represent some of the complexity of teaching, such as teaching rehearsals. Teaching rehearsals provide PSTs with the opportunity to practice teaching methods by teaching a group of their peers in a simulated classroom environment while receiving coaching feedback from the teacher educator (Lampert et al., 2013). Rehearsals provide a way for PSTs to engage with elements of leading mathematical discussions with more time and opportunities for reflection than would be possible in an actual classroom setting (Anthony et al., 2015; Ball & Cohen, 1999; Lampert et al., 2013)

As previously stated, facilitating productive mathematical discussions is a complex practice for both professional and prospective teachers. Teachers must plan discussion around cognitively demanding tasks that align to mathematical goals (Hiebert et al., 1997; Smith & Stein, 1998, 2018). Centering discussions around student ideas presents a high level of improvisation as teachers elicit and respond to students' thinking in-the-moment without losing sight of the goal of the lesson (Lampert, 2001; Stein et al., 2008). Additionally, supporting discussions that lead to shared understanding of mathematical concepts shifts mathematical authority from the teacher to a shared authority in a classroom discourse community (Hufferd-Ackles et al., 2004; Munter, 2014). Collectively, these characteristics present a view of mathematics instruction that may counter PSTs' previous classroom experiences (Lortie, 1975).

Statement of the Problem

Leading a productive mathematical discussion requires teachers to elicit and respond to student thinking in-the-moment and facilitate meaningful connections to math content and other shared ideas (Stein et al., 2008). This level of in-the-moment thinking is one of the challenges education researchers address in their work with supporting teachers in planning for and enacting mathematical discussions. Without support for intentionally planning for and organizing mathematical discussions, teachers' enactment of discussions lack connection to the mathematical goal and follow a "show and tell" structure of student sharing without a clear rationale for selection and sequence (Sleep, 2012; Smith & Stein, 2018). Much of the research on supporting teacher planning of mathematical discussions is conducted with practicing teachers. While this work can inform practices for preparing PSTs to facilitate discussions, there is a need for more research related to understanding the specific needs of PSTs within the instructional space (Ghousseini, 2015).

Decompositions of practice are commonly used to support PSTs learning to lead mathematical discussions as they allow more time for learning and reflection about specific skills of more complex core practices (Kazemi & Hintz, 2014; Shaughnessy et al., 2019). Yet, Janssen and colleagues (2015) argued that more attention is given to decomposition than *recomposition* of core practices, which can lead to "deskilling" teachers. Sleep (2012) identified a need for frameworks and tools that help break down complex teaching practices into components that can be studied, analyzed, and rehearsed by PSTs. While some tools have been developed to support the decomposition of complex teaching practices, research is lacking in the area of understanding *how* PSTs use the tools created to support their learning of the core practices of teaching (Ghousseini et al., 2015).

Another challenge in preparing PSTs to lead productive mathematical discussions relates to what Kennedy (1999) referred to as the *problem of enactment*, or the disconnect between what PSTs know or say about teaching and what they can actually carry out in practice. If too much focus is placed on the procedures used in facilitating mathematical discussions without enough understanding of the rationale behind the practices, novices experience difficulty translating their learning from teacher education to enactment in authentic classroom settings (Kavanagh et al., 2020; Kennedy, 2016). To address the problem of enactment, teacher educators need to consider ways to help PSTs develop an organized understanding of the skills needed to facilitate productive mathematical discussions in a way that facilitates retrieval and action (National Research Council, 2005). Taking into consideration the problem of enactment, my study captured PSTs' experiences as they transitioned from methods course instruction into their early clinical practice in elementary classrooms.

Research Purpose and Questions

Researchers recommend using tools to provide structure for PSTs learning the skills necessary for leading productive discourse (Ghousseini et al., 2015; Sleep, 2012) and repeated practice with instructional routines to reduce the cognitive demand of ambitious instruction (Ghousseini, 2015; Lampert et al. 2010). Grossman and colleagues (1999) differentiated between conceptual tools and practical tools to support teacher education. Conceptual tools provide frameworks based on general theories of teaching and learning that teachers can use to guide instructional decisions, such as constructivism or scaffolding. Practical tools are the practices and strategies teachers can enact in classrooms to meet the needs of learners, such as using question sequences (Ghousseini et al., 2015) or talk frames (Casa, 2013) to help guide mathematical discussions. While researchers recommend using tools to give structure to PSTs developing practice, research on *how* PSTs use tools designed to support their learning of core practices of teaching is limited (Ghousseini et al., 2015). The purpose of this study was to understand the tools elementary PSTs used to support facilitation of productive mathematical discussions with elementary learners. Beyond simply *what* tools PSTs used, this study sought to understand *how*

PSTs used these pedagogical tools to support their practice. The design of this multiple case study addressed the following research questions:

1. How do elementary preservice teachers apply pedagogical tools to inform their planning process for mathematical discussion?
2. How do elementary preservice teachers use pedagogical tools to facilitate mathematical discussions in clinical teaching experiences?

A qualitative, multiple case study design was used to address these questions. Participants included three elementary PSTs transitioning from math methods course instruction into field experiences during their yearlong internship (YLI). While a variety of tools have been examined in relation to PSTs learning to facilitate mathematical discussions, the voices of PSTs using these tools are often missing in the analysis. Additionally, examination of the use of these tools is often restricted to the methods course setting, which does not take into account the problem of enactment (Kennedy, 1999). This study captured discourse tools used in a clinical teaching context in which PSTs worked with elementary students.

Conceptual Framework

This study draws upon Grossman and colleagues' (1999) application of activity theory to frame how novices use and appropriate tools when learning to teach. Activity theory can be used to understand how PSTs use pedagogical tools to inform and conduct their practice. Activity theory is based on the assumptions that an individual's frameworks for thinking are developed through problem-solving carried out within specific settings (Leont'ev, 1981). When proposing an activity theory framework for teacher learning, Grossman and colleagues (1999) highlighted the roles of activity settings, teacher identity, and pedagogical tools teachers use to construct and carry out their practice. They characterize these tools as either conceptual or practical tools.

Conceptual tools are those that guide decisions about teaching and learning, including frameworks and guidelines that can be applied broadly to instructional practices across strands, such as the Five Practices for facilitating mathematical discussions (Smith & Stein, 2018; Stein et al., 2008). Practical tools are the resources and strategies teachers use to support a specific instructional need, such as lesson planning templates and talk frames to guide discussions (Casa, 2013).

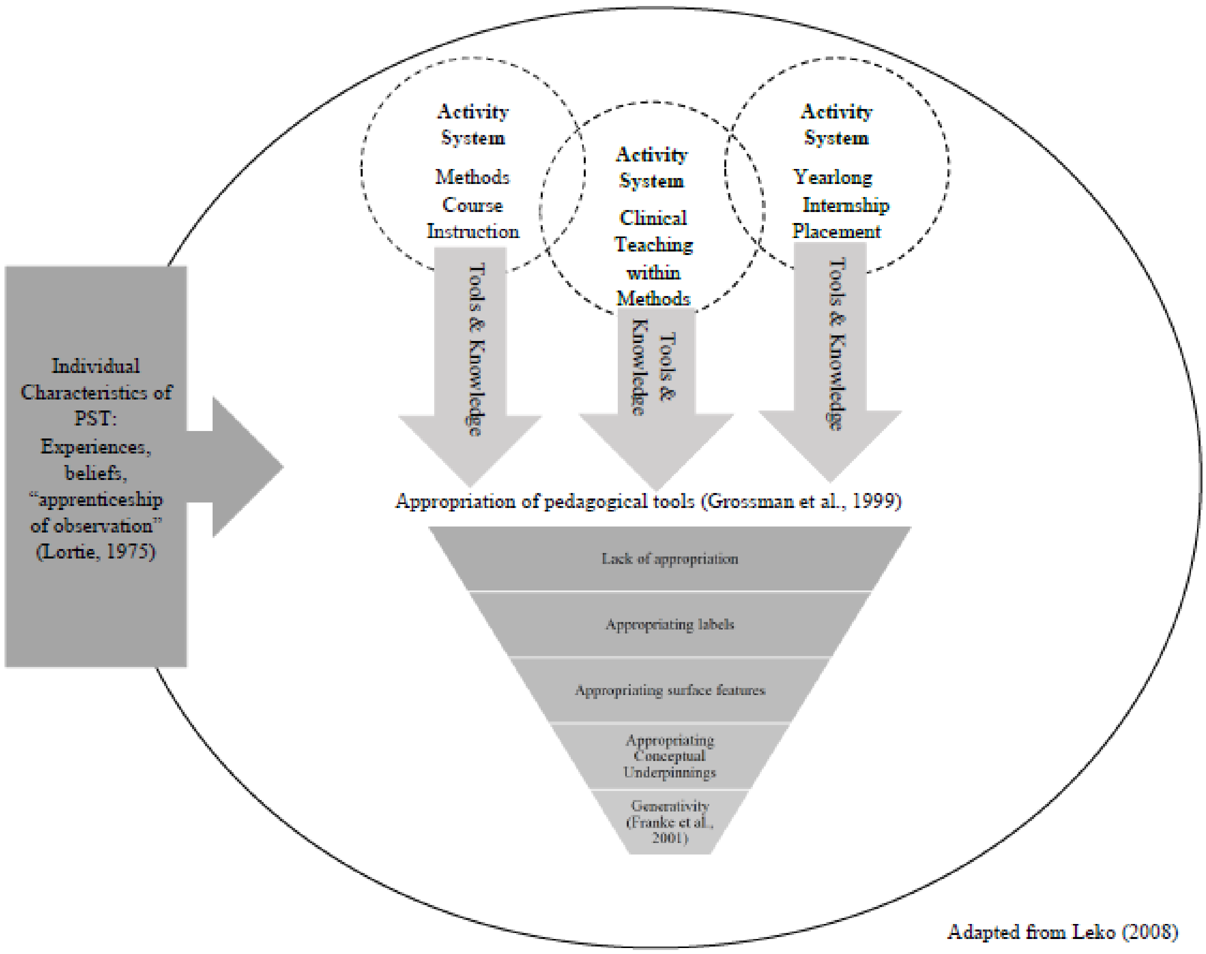
A central concept of activity theory is *appropriation* (Leont'ev, 1981; Wertsch, 1991). When applied to teacher education, *appropriation* refers to the process through which a PST adopts the pedagogical tools from particular social environments (e.g., university methods courses, clinical placements) and through which this process internalizes ways of thinking rooted in specific practices (Grossman et al., 1999), such as using a discussion-based approach to problem solving. According to Wertsch (1991, 1998), the process of appropriation is informed by social factors, as well as goal-directed and tool-mediated actions to scaffold thinking.

Using an activity theory framework, Grossman and colleagues (1999) described five degrees at which novice teachers appropriate pedagogical tools. The lowest level of appropriation is *lack of appropriation*, meaning the novice does not adopt a tool. The next level of appropriation is *labeling* in which the novice is able to name the tool but lacks knowledge of the features of the tool. After *labeling* is *appropriating surface features.* At this level, novices know most of the features of the tool, yet they lack comprehensive understanding of how these features work together conceptually. The next level of appropriation is *appropriating conceptual underpinnings,* which occurs when a novice understands the theoretical basis of the tool. At this level, novices may demonstrate the ability to adapt the tool to new contexts. The final level of appropriation is *achieving mastery*. This level of appropriation is achieved when one possesses the skills to use the tool effectively. This framework can be used to better understand to what degree PSTs use tools for discourse as they transition from a methods course setting into clinical teaching practice.

Figure 1 represents the model of activity theory applied to this study. Through an activity theory lens, how PSTs use and appropriate tools for discourse is influenced by the individual characteristics they bring to teacher learning, including experiences and beliefs shaped by their previous experiences in mathematics classrooms (Lortie, 1975). Tool use and appropriation are influenced by the activity systems in which tools are introduced and put into action. In this study, activity systems impacting tool use include methods course instruction, clinical teaching within methods courses, and clinical placements during the yearlong internship. Central to this conceptual framework is Grossman and colleagues' (1999) levels of appropriation of pedagogical tools. For this study, rather than using the term *mastery* to represent the highest level of tool appropriation, I use Franke and colleagues' (2001) term *generativity. Generativity* refers to an individual's ability "to continue to add to their understanding" (p. 655) and relates to a teacher's ability to use knowledge "as their own to create, adapt, and change" (p. 683). This term promotes a greater sense of the role of a teacher as a lifelong learner than the term *mastery.*

Figure 1

PSTs' Appropriation of Tools for Mathematical Discourse



Significance and Relevance

This study has implications for mathematics teacher education by increasing understanding of *how* PSTs use tools when learning to facilitate productive mathematical discussions and PSTs' perception of how these tools support their developing practice. Though mathematics teacher education has made great strides in articulating core practices to serve as the focus of teacher education, such as eliciting student thinking and facilitating discourse, understanding of how PSTs learn these core practices is still developing (Ghousseini et al., 2015; Kazemi et al., 2016; McDonald et al., 2013). While teacher educators and education researchers have articulated tools and scaffolds to support the decomposition of the complex practice of facilitating classroom discourse, little is known about how these tools are used and understood by PSTs (Ghousseini et al., 2015). This limited understanding is due in part to the lack of representation of PST voices in the existing literature. My dissertation helps to fill this need by focusing on PSTs' experiences through participation in multiple one-on-one interviews throughout the course of the study. In addition to data collection, these interviews provide opportunities for PSTs to reflect on their developing practice with facilitating mathematical discussions.

Few studies related to PSTs learning to facilitate mathematical discussions bridge the methods course and clinical classroom setting. Much of the recent work related to preparing teachers to facilitate productive mathematical discussions is set within the context of methods course instruction (Kazemi & Wæge, 2015; Shaughnessy et al., 2019; Sleep & Boerst, 2012; Tyminski et al., 2014). While these studies help inform the design of methods course instruction, they do little to understand how PSTs use pedagogical tools once they no longer have the structure of the methods course to influence their instructional choices. This study helps to address the need to understand how PSTs use pedagogical tools introduced in methods course instruction by capturing PST experiences as they transition out of methods courses into early clinical teaching practice.

Limitations and Delimitations

I acknowledge that there are some limitations to this study. While case study design allows for rich description within the context of the learning environment, case studies are limited in their generalizability (Guba & Lincoln, 1981; Merriam, 1998). To ensure detailed data collection and analysis, the sample size in my study was intentionally small. Therefore, findings from this study are not generalizable, but they will contribute to the existing literature on PBTE, mathematical discourse, and tools to support PST development. Convenience sampling was used in this study due to accessibility to PSTs enrolled in elementary math methods courses at the participating university. Another limitation is the lack of diversity in the participant sample, as the majority of PSTs enrolled at the university were White females.

Throughout the course of this case study, participants were working in clinical placements at local elementary schools. The lack of control over clinical placement settings presents another limitation. Clinical placements can potentially represent a wide range of diversity in student populations, cooperating teacher experience with facilitating mathematical discussions, and the level of mathematical discourse currently established in the classroom community (Ghousseini, 2015). To help mitigate this limitation, I conducted structured interviews with each clinical educator at the beginning of the study to better understand their classroom context, including how math discussion is typically used. Additionally, I conducted multiple observations of teaching in each setting over the course of the semester which added to my knowledge of the classroom communities each PST was working with for their YLI.

This case study was designed to capture PSTs' use of pedagogical tools to plan and facilitate mathematical discussions in authentic classroom settings. PSTs were not required to use specific tools in their practice. However, since participants were aware of the purpose of the study, mere participation in this study could influence them to use more discussion in their math instruction and use chosen pedagogical tools more deliberately than they would have if they were not part of this study.

One delimitation in this study was the choice to recruit PSTs enrolled in the MAED 3224: Teaching Elementary Mathematics to Elementary Learners, Grades 3-6 methods course. These PSTs enrolled in their first semester of YLI the semester after completing this course. This time between the completion of math methods courses and YLI was selected because PSTs are typically introduced to the practical tools of teaching within the methods course setting (Grossman, Compton et al., 2009). Data collection for this study primarily came from PSTs' participation in the clinical classroom setting, taking into account enactment in authentic classroom settings (Kennedy, 1999). Beginning this case study immediately after PSTs completed their math methods coursework presented a natural transition into classroom field experiences during the first semester of their YLI.

Assumptions

There are several assumptions underlying this study related to PBTE generally and facilitating productive mathematical discussions specifically. These assumptions come from my own interpretation of the research literature, my experience teaching and coaching PSTs and novice teachers, and my perspectives as a classroom mathematics teacher. One assumption underlying this study is that the skills and practices required for teachers to lead productive mathematical discussions are both teachable and learnable. Another assumption is that facilitating productive mathematical discussions is a core practice in mathematics teaching based on the high frequency of the practice, the transferability of skills to a range of curricula, and the benefits to student learning (Grossman & McDonald, 2008). Finally, an underlying assumption of this study is that discourse tools can support PSTs learning to facilitate mathematical discussions by providing a stable structure to reduce the complexity of discourse practice (Ghousseini et al. 2015) while still allowing for adaptation and flexibility in practice.

Definition of Terms

Key terms used throughout this study are briefly defined below. While alternative definitions of these terms are possible, terms are defined below as they are used throughout the study.

- *Appropriation:* According to Grossman and colleagues (1999), appropriation is "the process through which a person adopts pedagogical tools available for use in particular social environments and through which this process internalizes ways of thinking endemic to specific cultural practices" (p. 15). They describe five levels of appropriation ranging from *lack of appropriation* to *mastery*.
- *Classroom discourse community:* In this study, a classroom discourse community refers to a group of learners collaborating to build a shared understanding of mathematical concepts through discussion. In a classroom discourse community, the teacher acts as a facilitator, sharing mathematical authority with students (Hufferd-Ackles et al., 2004; Smith & Stein, 2018).
- *Clinical educator (CE):* In this study, the term clinical educator is used to identify a PST's mentor teacher in their assigned classroom setting. CEs in this study are certified teachers currently teaching grades K-5. While clinical educators are not participants in

this study, the guidance they provide PSTs may impact tool use when PSTs plan and facilitate mathematical discussions.

- *Cognitively demanding tasks:* A cognitively demanding task is a mathematical task that requires students to think abstractly and make connections to mathematical concepts. Smith and Stein (1998) categorize cognitively demanding tasks as those requiring *procedures with connections* or *doing mathematics*.
- *Core practice:* According to Grossman and McDonald (2008), core practices are teaching practices that occur with high frequency, can be enacted across multiple curricula, and are research-based to demonstrate the potential to improve student learning. In addition, core practices are practices that novices can actually begin to master while maintaining the integrity and complexity of teaching. There is general consensus in the field of math teacher education that facilitating discussions is considered a core practice (Janssen et al., 2015).
- *Generativity:* Franke and colleagues (2001) described generativity as an individual's ability "to continue to add to their understanding" (p. 655) and relates to a teacher's ability to knowledge "as their own to create, adapt, and change" (p. 683). In this study, I use this concept of generativity in place of Grossman and colleagues' (1999) use of the *achieving mastery* to frame the highest level of pedagogical tool appropriation.
- *Pedagogical tools*: In this study, I use Grossman and colleagues' (1999) definition of pedagogical tools to include supports used to help teachers "construct and carry out teaching practices" (p. 13). They further define these tools as either *conceptual tools* or *practical tools*:

  - *Conceptual tools*: Conceptual tools are "principles, frameworks, and ideas about teaching [and] learning…that teachers use as heuristics to guide decisions about teaching and learning" (p. 14). Conceptual tools can include theories of learning, such as social learning theory, and broadly defined pedagogical concepts, such as scaffolding, that can be applied across different curricular strands.
  - *Practical tools*: Practical tools include "classroom practices, strategies, and resources…that have local and immediate utility" (p. 14). Examples of practical tools include textbooks and curricular resources, lesson plan models, or three-act tasks (Flynn, 2017; Meyer, 2011).
- *Practice-based teacher education (PBTE):* Practice-based teacher education refers to professional training that focuses on core practices directly related to the work of teaching in an attempt to develop highly skilled teachers (Forzani, 2014). A commonly used framework in PBTE is Grossman, Compton, and colleagues' (2009) pedagogies for professional learning. The framework outlines three elements for professional learning: representations of practice, decompositions of practice, and approximations of practice. Each of these components is described below:
  - *Representations of practice:* Representations of practice are artifacts of teaching practice used to support teacher learning. Examples of representations of practice used in PBTE include video clips of classroom teaching and authentic student work samples.
  - *Decompositions of practice:* Decompositions of practice are used to break down complex teaching practices into smaller parts to support novice learning. For example, when learning to facilitate mathematical discussions, a decomposition of

practice can focus on selecting and sequencing students' mathematical ideas from a set of work samples (Tyminski et al., 2014).

    - *Approximations of practice:* Approximations of practice are opportunities for deliberate practice of the elements of teaching (Janssen et al., 2015). Guided rehearsals are a commonly used approximation of practice in PBTE (Averill et al., 2016; Ghousseini et al., 2015).

- *Preservice teacher (PST)*: I define preservice teachers as undergraduate students who are seeking licensure through an accredited teacher education program. In the literature, PSTs are also referred to as teacher candidates, student teachers, or prospective teachers.
- *Productive mathematical discussions*: In this study, productive mathematical discussions refer to class discussions that use students' thinking about cognitively demanding tasks in a way that advances the shared understanding of significant mathematical concepts (Lampert, 2001; Stein et al., 2008). Discussions can be whole-class or conducted in small groups.
- *Yearlong internship (YLI):* The term yearlong internship (YLI) refers to the final two semesters of teacher education for elementary education majors in the university setting for this study. For PSTs in this study, YLI begins after they complete all methods course instruction. During YLI, PSTs are assigned a clinical educator to work within an elementary classroom setting. In this setting, PSTs gradually take on additional teaching responsibilities as they work to assume full responsibility for the instruction (Kinne et al., 2016).

Chapter Summary

This chapter presented an overview of the purpose and design of the qualitative study. The chapter began with an overview of the rationale for using discourse-based practices in mathematics instruction. While facilitating productive mathematical discussions with students supports a range of positive learning outcomes, it requires teachers to employ a variety of skills and knowledge simultaneously. Education research grounded in PBTE provides strategies and tools for decomposing the complex practice of facilitating discourse for PSTs, yet less is known about *how* PSTs use these tools as they learn to facilitate mathematical discussions. This study sought to understand how PSTs use tools to plan and enact mathematical discussions as they transition from methods course instruction into early clinical experiences teaching in elementary classrooms.

Chapter Two provides a critical review of the literature related to this study. Given the purpose of this study, the following areas are included in the review: the role of discourse in elementary mathematics instruction, theories and practices for novice teacher learning, and teacher education focused on facilitating mathematical discussions. Throughout this review, I highlight research related to the use of tools and supports for teacher learning. In Chapter Three, I describe the context of my research as well as the data collection and analysis methods used to answer the three research questions. Chapter Four presents the findings of this study presented as themes identified both within case and across case. In Chapter Five, I position my findings within the existing literature, discuss the implications for practice from this study, and suggest directions for further research.

CHAPTER TWO: LITERATURE REVIEW

The purpose of this multiple case study is to understand how elementary PSTs use tools to plan and facilitate productive mathematical discussions as they transition from participation in methods course instruction into clinical teaching experiences. To situate my study in the larger field of elementary mathematics teaching and learning, this chapter begins with a review of literature on the role of mathematical discourse in elementary mathematics instruction. This section explores the characteristics of productive mathematical discussions, the changing role of the teacher in a classroom discourse community, and the challenges to incorporating meaningful discussions into mathematics practice. The second section of this review explores various frameworks related to novice learning and development as it applies to teachers, including a practice-based approach to teacher education. In line with the purpose of this study, this section includes a focus on the role of tools in supporting novice teacher learning. The final section of this chapter provides a comprehensive review of the literature related to preparing teachers to facilitate mathematical discussions.

The Role of Discourse in Elementary Mathematics Instruction

In this section, I review literature on the role of discourse in elementary mathematics instruction to better understand how facilitating productive discussions is defined as a core practice in practice-based teacher education (PBTE). I refer to productive mathematical discussions as class discussions that use students' ideas about cognitively demanding tasks to advance the shared understanding of mathematical concepts (Lampert, 2001; Stein et al., 2008). I begin by describing the characteristics of productive mathematical discussions and the potential outcomes for student learning. This literature helps to understand the context of a classroom discourse community, the changing role of the teacher within that community, and the potential challenges teachers face when facilitating discussions. Collectively, this section of the literature review helps understand the complexity of the practice PSTs are preparing for and the rationale for developing this pedagogical practice.

Characteristics of Productive Mathematical Discussion

Productive mathematical discussions provide learners with opportunities to share their thinking, refine their conceptual understanding, develop mathematical arguments, and consider mathematical ideas from other perspectives (NCTM, 1991). Discussion-based instruction can support students in developing mathematical proficiency when students are pushed to reason mathematically, to communicate their ideas, attend to other students' thinking, and to develop conceptual understanding (Franke et al., 2007; Kazemi & Stipek, 2001; Munter, 2014). Taken together, this means productive mathematical discussions are more than just talking about math. Task selection and environment also play a role in supporting purposeful, reasoning-based discourse, as Casa (2014) outlined in the student mathematician discourse framework. Teachers learning to facilitate productive mathematical discussions must consider the aforementioned characteristics when planning and enacting discussions. These characteristics are described in more detail below.

One distinguishing feature of productive mathematical discussions is the emphasis on conceptual thinking. Conceptual approaches to explaining thinking are contextually oriented to the problem and encourage learners to identify relationships between different strategies (Thompson et al., 1994). Mathematical discussions that are conceptually oriented present mathematical tasks that allow students to explore multiple strategies and critically examine errors and contradictions in thinking (Kazemi & Stipek, 2001; Munter, 2014). To facilitate this type of discussion, teachers must have a strong conceptual understanding of the mathematics they teach.

Once teachers set a goal for what they want students to learn, they must select a high-level task that can help students reach that goal (Casa, 2014; Hiebert et al., 1997; Smith & Stein, 1998, 2018). Task selection is critical to ensuring productive mathematical discussion as tasks selected must be able to support conceptual thinking. The quality of the task impacts the quality of the discussion for learning. Hiebert and colleagues (1997) described high-quality math tasks as those which allow for “insights into the structure of mathematics” (p. 23). They described four characteristics of high-quality tasks: (a) the task presents students with something problematic to consider, (b) the task makes the *mathematics* intriguing rather than just the problem context, (c) the task gives students the opportunity to apply skills and knowledge, (d) and the task suits the tools available to solve. Smith and Stein (1998) developed a task analysis guide with general characteristics of lower-level and higher-level demand tasks based on the type of student reasoning required. They categorized lower-level demand tasks as either *memorization* or *procedures without connections*. Higher-level demand tasks are categorized as either *procedures with connections* or *doing mathematics*. Tasks categorized as *procedures with connections* or *doing mathematics* require learners to apply mathematical skills and knowledge to demonstrate mathematical reasoning and problem solving, as described in Hiebert and colleagues’ (1997) characterization of high-quality tasks. These high-quality or higher-level demand tasks provide students with multiple entry points into the problem as well as multiple paths to solving (NCTM, 2014). Tasks of this nature encourage sharing multiple solution strategies within a community of learners. If mathematical discussions are intended to support learners’ ability to reason about mathematical concepts and consider different perspectives, then the tasks selected for discussion must support this kind of reasoning.

Classroom Discourse Communities

One goal of mathematical discussions is to advance shared understanding of mathematical concepts. Research on effective mathematical discourse emphasizes the role of collaboration within a classroom discourse community to foster this shared understanding between learners (Casa, 2014; Hufferd-Ackles, et al., 2004; Lampert, 2001; Munter, 2014; Simon, 1994), which in turn helps nurture a more equitable classroom environment. According to Munter (2014), classroom discourse communities share the following characteristics: (a) math talk is conceptual in nature, (b) teachers' questions facilitate discussion and elicit student thinking, (c) student discussions generate new questions and help students identify areas of confusion, and (d) students hold each other accountable for sharing sufficient mathematical arguments. In this type of community, students share the mathematical authority for determining the validity of ideas (Simon, 1994). This type of learning community must be intentionally crafted.

Creating such a community is no easy task. To develop a nurturing environment that supports students' reasoning-based discourse, teachers must help students explicitly understand their role as mathematical thinkers, speakers, and listeners in a discourse community (Casa, 2014). Hufferd-Ackles and colleagues (2004) applied their findings from a year-long case study in an urban elementary classroom to develop a trajectory to understand the growth and development of a *math-talk learning community*, which they defined as a "community in which the teacher and students use discourse to support the mathematical learning of all participants" (p. 82). Students in this type of community do not depend on the teacher to initiate all student-to-student talk. Applying a socio-constructivist view of learning, Hufferd-Ackles and colleagues (2004) identified four components that capture the growth of a math-talk learning community: (a) questioning, (b) explaining math thinking, (c) source of mathematical ideas, and (d) responsibility for learning. Their trajectory examines movement of these components from that of a traditional, teacher-directed mathematics classroom (Level 0) to a classroom that embraces meaningful, student-centered math talk (Level 3). While this framework was developed based on the growth of an experienced classroom teacher, this trajectory can help understand PSTs' experiences as they challenge their preconceptions about mathematics teaching and learning from their own experiences as K-12 students (Lortie, 1975; NRC, 2005).

Teacher's Role

Within the classroom discourse community, the teacher acts as an active facilitator, eliciting student ideas while steering the discussion towards mathematical goals (Hufferd-Ackles, 2004; Munter, 2014; Sleep, 2012). Facilitating productive mathematical discourse involves attention to planning, eliciting, and responding to student thinking, and facilitating connections between student ideas and the mathematical goals of the task (Smith & Stein, 2018). Teachers must be able to think flexibly about both mathematics content and methods to respond to student thinking (Kavanagh et al., 2020). Facilitating discussions that attend to student thinking and that reach mathematical goals is necessary to foster more equitable, high-quality experiences for students within the classroom discourse community (Hufferd-Ackles et al., 2004; NCTM, 2014). The teacher-as-facilitator role represents a move away from more traditional, teacher-directed practices to instruction centered on student thinking and discovery.

Planning for Discussions

Planning for mathematical discussions requires teachers to identify mathematical goals (Sleep, 2012) and select "worthwhile tasks" that go beyond a strict focus on computational thinking (Smith & Stein, 2018; Thompson et al., 1994). Chapin and colleagues (2013) outlined four steps for planning talk-based lessons: (a) identifying the mathematical goal, (b) anticipating confusion, (c) asking questions, and (d) planning the implementation. This four-step process overlaps with Smith and Stein's (2018) Five Practices, as both attend to focusing on the mathematical goal of the discussion, anticipating student thinking, and considering the sequence of ideas that can support shared understanding of the mathematical goal. Though it is not possible to anticipate all student ideas before the lesson, anticipating student ideas as part of the planning process, including possible developing ideas, helps to mitigate some of the in-the-moment thinking demands of teachers facilitating student-centered discussions.

Eliciting and Responding to Student Thinking

During the discussion, the teacher's role focuses on eliciting and responding to student thinking. Chapin and colleagues' (2013) Talk Moves were developed to provide teachers with strategies for encouraging students to share mathematical ideas, elaborate on their thinking, consider the ideas of others, and facilitate connections between ideas. The Talk Moves include revoicing, repeat/rephrase, agree/disagree and why, add-on, and think time (Chapin et al., 2013). Within a classroom discourse community, the teacher should not be the sole authority to determine the validity of student responses (Simon, 1994), therefore teachers' responses to student thinking should reflect this shared authority.

Early efforts of reform-oriented mathematics teachers to include more discussion in mathematics resulted in instruction that presented student ideas in a disconnected manner without a clear focus on mathematical goals (Stein et al., 2008). Productive mathematical discourse must go beyond just "talk" or a "show and tell" format of sharing ideas without real connections (Smith & Stein, 2018; Stein et al., 2008). By anticipating student ideas when planning for discussion, teachers are better equipped to sequence student ideas in a way that facilitates connections between ideas and to the mathematical goal.

Challenges in Facilitating Mathematical Discourse

The complex nature of effectively facilitating mathematical discourse is a central issue in education research. Facilitating productive mathematical discourse requires teachers to simultaneously attend to conceptual understanding, mathematical goals of the task, and social management while listening and responding to a variety of student ideas (AMTE, 2017; Ghousseini, 2015; Grossman et al., 2019). Leading a student-centered mathematical discussion requires teachers to respond to student thinking in the moment and facilitate meaningful connections to math content and other shared ideas (Lampert, 2001; Stein et al., 2008). This level of improvisation is one of the challenges education researchers address in their work with supporting teachers in planning for and enacting mathematical discussions.

Improvisation presents such a challenge to preparing teachers to facilitate discourse that some research studies remove the role of improvisation completely. Spangler and Hallman-Thrasher (2014) used imaginary task dialogues between students and teachers to prepare for leading discussions in field placements. The task dialogues provide a structure for PSTs to respond to varying levels of correctness in student thinking through writing rather than in-the-moment dialogue. Similarly, Evans and Dawson (2017) used designed student responses to support middle grades teachers in planning for and implementing mathematical discussions with their students. Teachers in this study felt it was much easier to facilitate discussions around these designed responses, however, researchers found that teachers still struggled to facilitate connections between ideas. Additionally, while designed responses do offer opportunities for students to develop shared understanding of math concepts, this structure takes away student agency in discussions.

Productive mathematical discussions are small or whole group discussions that use students' ideas about cognitively demanding tasks to advance the shared understanding of significant mathematical concepts (Lampert, 2001; Stein et al., 2008). This means that not all math talk falls within the category of productive discussion. Stein and colleagues (2008) described how early efforts to build more discussion into reform-based mathematics practice emphasized the social norms of mathematical discussions, helping students feel their contributions were valued. However, less attention was given to how the teacher could facilitate connections between student ideas and steer the discussion towards the mathematical goal. This results in a "show and tell" structure of discussions (Smith & Stein, 2018; Stein et al., 2008). Additionally, teachers may avoid "telling" for fear of disrupting the discovery process, but this fear can result in a lack of guidance towards meaningful mathematical understanding (Lobato et al., 2005; Munter, 2014). Teachers must be careful to maintain a balance between teacher-run and student-run learning communities (Rogoff et al., 1996; Smith & Stein, 2018).

Traditional talk exchanges in mathematics instruction follow an IRE (Initiation, Response, Evaluation) in which the teacher initiates talk by asking a question (often close-ended), the student responds, and the teacher evaluates the correctness of the solution (Mehan, 1979). The IRE model of math talk positions the teacher as the sole authority, which runs counter to the goals of productive mathematical discussions. Helping experienced teachers move away from this IRE format can be challenging. As Herbel-Eisenmann and colleagues (2013) found, experienced secondary mathematics teachers in their study continued to fall back on IRE patterns of math talk even after completing professional development aimed at supporting teachers develop talk moves to support productive discourse. While it could be presumed that PSTs learning to teach have not yet established a habit towards IRE models of talk as they lack professional experience, PSTs likely observed teachers using this format in their own K-12 experiences, thus building preconceptions about what math talk looks like (Lortie, 1975). Additionally, PSTs placed in clinical school settings that use highly structured curriculums may struggle to find ways to incorporate discourse into those existing structures.

Dissertation Contribution to Research on Discourse in Elementary Mathematics

Much of the work on facilitating mathematical discourse and supporting connections in mathematical discussions is done with experienced teachers, not novices. While much can be learned from examining the practices of experienced teachers, supports needed for PSTs developing their practice are different. PSTs often work in established classroom communities that may or may not support math talk. This creates a need to consider how tools to support mathematical discourse are used specifically as supports for PSTs. My study helps address this gap by focusing on the experiences of PSTs as they used tools to plan and enact mathematical discussions in clinical classroom settings.

Teacher Learning and Development

This section of the literature review explores research and theoretical models related to teacher learning and development. This literature provides a framework for understanding how PSTs develop knowledge, skills, and attitudes related to core practices and the factors impacting novice teacher learning. The literature reviewed in this section represents broad theories of novice learning and teacher development, therefore, I begin this review by examining literature that seeks to define the problem space unique to PSTs to help orient these larger theories specifically to PSTs. In this section, I also review literature related to practice-based teacher education (PBTE) as this approach is commonly applied when preparing PSTs to facilitate mathematical discussions. This section concludes with a review of literature specific to the role of conceptual and practical tools in teacher learning and an overview of an activity theory approach to tool appropriation. This literature helps frame my case study as it specifically examined how elementary PSTs use tools for planning and facilitating mathematical discussions.

Defining the Problem Space for PSTs

Much of the research on facilitating productive mathematical discussions is conducted with practicing teachers, often those with many years of experience. While this work informs the field of preparing PSTs to facilitate discourse, PSTs occupy a unique problem space related to facilitating classroom discussions that must be considered when applying lessons learned from research with experienced teachers (Ghousseini, 2015). Cohen and colleagues (2003) presented a model of an interactive view of teaching as an instructional triangle, representing the relationships that concurrently take place between content, the teacher, and students. In promoting the kind of collaboration needed in a classroom discourse community, the teacher supports the relationship between the students and the content by eliciting student thinking, pressing students to explain their reasoning, and representing student thinking to connect to mathematical goals (Lampert, 2001). Establishing these relationships between students and mathematical content relies on maintaining collaborative relationships within the classroom discourse community (Ghousseini, 2015; Lampert 2001). However, PSTs learning to facilitate mathematical discussions do not have the same opportunities for establishing and maintaining these relationships as practicing teachers.

Ghousseini (2015) used the instructional triangle framework (Cohen et al., 2003) as a basis for understanding the instructional space inhabited by PSTs learning within methods classrooms and established classroom communities in clinical settings. Compared to experienced teachers, PSTs experience the relationships within this instructional triangle differently due to their limited knowledge of and experience with the content and pedagogical tools (Ghousseini, 2015), therefore constraining the relationship between teacher and content. Even when PSTs have opportunities to practice instruction within clinical classroom settings, this setting likely already has established norms and relationships between students. When it comes to facilitating mathematical discussions, this presents a barrier to PSTs as leveraging these relationships is critical to the classroom discourse community (Hufferd-Ackles et al., 2004; Ghousseini, 2015; Lampert, 2001). These constraints on the relationships between PSTs, students, and setting contribute to the lack of connection to contextual factors observed in PSTs' practice (Dreyfus, 2004). When analyzing how PSTs use tools to plan and facilitate mathematical discussions, this problem space must be considered.

It is also necessary to consider the role of PSTs' existing conceptions of teaching and learning gathered in their years as K-12 students. Lortie (1975) referred to this as the *apprenticeship of observation*. Unlike other novice professionals, PSTs have years of experience observing professionals in their field of practice. These observations contribute to preconceptions of teaching and learning that may or may not align with the theories and practices shared within teacher education. For PSTs to appropriate reform-based practices, like facilitating student-centered mathematical discourse, they must first address their preconceptions about mathematics teaching and learning (NRC, 2000). Addressing preconceptions is a necessary first step in developing adaptive expertise as a novice teacher (Bransford et al., 2005).

Adaptive Expertise

Achieving adaptive expertise is seen as the "gold standard" for learning (NRC, 2000). Adaptive expertise refers to the ability to recognize when common approaches are not working and adjust routines to respond to new problem situations (Hatano & Inagaki, 1986; Von Esch &

Kavanagh, 2018). Adaptive expert teachers, therefore, are those teachers who recognize when routines are not working and can efficiently respond to adjust their practice (Von Esch & Kavanagh, 2018). In contrast, routine experts are those who have developed efficient instructional routines but lack the ability to respond flexibly when these routines prove ineffective (Hammerness et al., 2005).

Bransford and colleagues from the National Research Council (2005) presented a model for understanding adaptive expertise as it applies to issues of teaching and learning. The model shows a continuum representing the relationship between innovation and efficiency. High efficiency refers to the ability to “rapidly retrieve and accurately apply appropriate knowledge and skills to solve a problem or understand an explanation” (Bransford et al., 2005). Innovation refers to the ability to demonstrate skills and strategies to adapt to new situations. Adaptive experts balance high levels of both innovation and efficiency, allowing them to adapt efficient routines based on context (Bransford et al., 2005; Hatano & Inagaki, 1986). Adaptive experts are often associated with lifelong learners committed to moving beyond existing routines to improve practice (Hammerness et al., 2005).

The ability to adapt instruction based on student thinking is a critical piece of facilitating productive mathematical discussions. Routine experts are those who possess high levels of efficiency but lower levels of innovation. This means routine experts perform particular tasks without having to devote too much attention or resources to achieve them (Hammerness et al., 2005). Without this efficiency, PSTs run the risk of falling into the category of “frustrated novice” (Bransford et al., 2005, p. 49), one who possesses high levels of innovation without the skills to execute these practices with ease. Teacher education programs can help PSTs develop efficiency by providing tools and structures that organize knowledge for teaching in ways that facilitate retrieval and action (NRC, 2000). However, as Dreyfus (2004) explained, novices tend to associate particular actions or procedures they learn in teacher education as maxims for action, which limits their ability to act responsively to student thinking (Jacobs et al., 2010; Kavanagh, Metz et al., 2020). As such, it is important for teacher educators to consider how the strategies and tools they promote contribute to efficient practice while not omitting opportunities for innovation. Teacher educators focusing on core pedagogical practices need to be especially cautious, as Hatano and Inagaki (1986) acknowledged that routine experts can competently conduct a core set of practices efficiently and fluently but lack the flexibility to adapt core approaches when they prove ineffective.

Practice-Based Teacher Education

A practice-based approach to teacher education focuses on core practices directly related to the work of teaching to develop highly skilled teachers (Forzani, 2014). According to Grossman and McDonald (2008), core practices are pedagogical practices that occur with high frequency, can be applied across multiple curricula, and have demonstrated the potential to improve student learning within the educational literature. Facilitating discussions is considered a core practice of mathematics teacher education (AMTE, 2017; Janssen et al., 2015; NCTM, 2014). PBTE attempts to address the perceived disconnect between theoretically rooted teacher education programs and the work of professional teaching. An assumption of PBTE is that core practices can be taught and that novices can actually begin to master these practices while maintaining the integrity and complexity of teaching (Grossman et al., 2019).

A practice-based approach to teacher education is not new. However, earlier attempts at PBTE provided an overly narrow view of skills for teaching that were not well-connected to research. This presented an overly simplified perception of what it takes to be an effective teacher (Zeichner, 2012). Current models of PBTE commonly align with Grossman, Compton, and colleagues' (2009) pedagogies of practice framework. This framework was developed through comparative study of professional education within clinical psychology, teaching, and the clergy. With this framework, they identified three key concepts to be applied within professional education: representations of practice, decompositions of practice, and approximations of practice. Much of the literature related to teachers learning to facilitate mathematical discussions emphasizes decompositions and approximations of practice. This literature will be discussed later in this review.

The Core Practice Consortium (CPC) includes teacher educators across multiple university settings collaborating to identify core practices of education and the pedagogies of teacher education that support PSTs in developing these core practices (Grossman et al., 2019). According to the CPC, core practices support a relational approach to teaching and learning that presents mathematics as more than a set of decontextualized procedures. In addition, the CPC views core practices as "identifiable aspects of teaching that involve the cultivation and orchestration of knowledge, relationship, skill, judgment, and understanding in context-specific enactment" (Grossman et al., 2019, p. 97). Yet the CPC acknowledged that even with a shared vision for PBTE and teacher education pedagogies, diverse perspectives and instructional contexts can influence different outcomes of PBTE. Ball and colleagues (2009) identified a lack of shared resources within teacher education as a challenge to progress in preparing teachers for practice. There is a need for shared materials to help lay the foundation for teaching core practices that can be adapted to fit a variety of instructional contexts and growing knowledge from research.

Critique of PBTE

While PBTE helps mitigate the perceived theory-practice divide in teacher education, critics of PBTE argue that a narrow focus on core practices serves to de-skill teaching (Janssen et al., 2015). Part of this concern relates to the grain-size of decomposed practices. Boerst and colleagues (2011) suggested decomposing ambitious teaching practices, like leading discourse, into a series of nested practices of varying grain size. Decomposing practice into too fine a grain size can result in PSTs developing a narrow set of pedagogical skills. However, using only large grain-sized decompositions can be overwhelming to PSTs and contribute to the perceived disconnect between theory and practice.

The overly simplified view of teaching that critics argue results from PBTE may also connect to novice learners' tendency to associate the procedures they learn in methods courses as maxims or rules for action (Dreyfus, 2004). While it may not be the intention of teacher educators, focusing on core practices can be misinterpreted as a prescriptive approach to instruction, presenting the message that there is a "best" way to conduct a practice regardless of context (Kavanagh et al., 2020). Additionally, there is concern that an attempt at a practice-based approach to teacher education can be misapplied when teacher educators may lack training and experience with PBTE (Ball & Cohen, 1999; Janssen et al., 2015). Understanding these concerns supports critical analysis of this study.

Clinical Teaching in Teacher Education

Another response to the perceived disconnect between university-based teacher education programs and the work of professional teaching includes increased opportunities for clinical teaching experiences for PSTs (Zeichner, 2010). Clinical field placements, often referred to as student teaching, are the culminating experience of teacher education programs in the United States, providing opportunities for PSTs to gradually take over instructional responsibilities with the support of a clinical educator in a classroom setting (Kinne et al., 2016). Novice teachers often consider clinical teaching experiences as the most impactful part of their teacher education program (Clarke et al., 2014; Grossman et al., 2012). Zeichner (2010) advocated for expanded offerings for PSTs to work in clinical education settings as a means to narrow the theory-practice divide. Yet, he cautioned against viewing clinical education settings as little more than a location for PSTs to put into practice knowledge gained from university-based courses. He advocated for a more reciprocal partnership so that university-based teacher educators are also learning from clinical educators to better design instruction around the practical work of teaching.

Clinical teaching experiences provide a rich environment for PST learning and for learning about PSTs' teaching practices. However, there are few studies that explore PSTs' experiences planning for and facilitating mathematical discussions within the clinical classroom setting (Ghousseini, 2015; Sleep, 2012). Sleep (2012) interviewed and observed PSTs working in clinical settings to better understand how teachers steer instruction towards a mathematical goal. However, her selection of the clinical setting was purposeful to better understand the work of teaching and the "bumps and bruises" that are more visible when working with novice teachers, not specifically to better understand PSTs' practice. Ghousseini (2015) used a single case study design to investigate a secondary mathematics PST's experience facilitating connections between students' ideas in mathematical discussions. Observing and interviewing the PSTs' teaching in an authentic classroom setting contributed to the understanding of factors influencing PSTs' practice in clinical settings, including existing classroom structures, PSTs' adherence to rules and maxims for practice, and PSTs' mathematical knowledge.

Tools to Support Novice Learning

Teacher education programs provide PSTs with a variety of tools to support their development of efficient practices. Grossman and others (1999) categorized pedagogical tools as either conceptual tools or practical tools. Conceptual tools are overarching ideas that provide general theories and frameworks related to learning, motivation, and instruction that serve to facilitate teachers' framing of practice (Grossman et al., 1999). Practical tools are often shared within the context of methods courses introduced later in teacher education. Practical tools include the specific practices, strategies, and relationships that teachers can enact in their classrooms (Grossman et al., 1999). Compared to conceptual tools, practical tools are more specific and concrete.

Many of the tools that are highlighted in the literature related to preparing teachers and PSTs to facilitate discourse fall within the practical category. For example, Ghousseini and colleagues (2015) used a question sequence tool with PSTs learning to facilitate discourse. This question sequence provided structure for PSTs enacting lessons in teaching rehearsals, thereby reducing the complexity of responding to student thinking in the moment. Similarly, Herbel-Eisenmann and colleagues (2013) shared a modified version of the commonly used Talk Moves (Chapin et al., 2013) with secondary mathematics teachers working to improve their discourse practices through professional development. In both studies, these practical tools provided needed structure for novices.

While these practical tools provide needed structure and are well-connected to the work of teaching, Ghousseini and others (2015) argue that tools used to support PSTs learning to facilitate mathematical discussions should maintain some element of improvisation so that PSTs can adapt the tool in an unscripted manner. Grossman and colleagues (1999) argued that without understanding the rationale behind these tools, appropriation of the tool is limited. Therefore, tools to support PSTs learning to enact a complex practice, like facilitating discourse, need to be connected to the conceptual underpinnings of the tool and allow room for innovation. Additionally, PSTs must have repeated opportunities to apply these tools in practice (Ghousseini et al., 2015; Lampert et al., 2010).

Appropriation of Pedagogical Tools

While PSTs are introduced to several tools to support instruction within teacher education, the degree to which PSTs appropriate these tools and apply them in future practice varies. Through the lens of activity theory, Grossman and colleagues (1999) described five degrees of appropriation of tools and the factors impacting appropriation at different levels. They describe appropriation as "the process through which a person adopts pedagogical tools available for use in particular social environments and through which this process internalizes ways of thinking endemic to specific cultural practices" (p. 15). These levels of appropriation were used to better understand how PSTs in this case study used tools to plan and facilitate mathematical discourse and how the level of appropriation was impacted by activity setting.

According to Grossman and colleagues (1999), the lowest level of appropriation is *lack of appropriation* whereby the PST does not adopt a tool. A PST may not adopt a pedagogical tool for a variety of reasons, including lack of knowledge, mismatch to the PST's beliefs, or a learning context that does not readily support the use of the tool. The next level of appropriation is *labeling* in which the PST can name the tool but does not know any of the features of the tool. For example, a PST might know that they should include more cognitively demanding tasks within their mathematics instruction, but they do not know the characteristics that distinguish such a task. The third level is *appropriating surface features.* At this level, PSTs know most of the features of the tool, yet they lack a holistic understanding of how these features work together conceptually. The next level of appropriation begins when a PST understands the theoretical basis of the tool, and therefore is at the level of *appropriating conceptual underpinnings.* At this level, PSTs can adapt the tool to new contexts. The final level of appropriation is *achieving mastery*. This level of appropriation is achieved when one possesses the skills to use the tool effectively.

Achieving mastery is unlikely to occur with PSTs engaged in their teacher preparation, as mastery of pedagogical tools is achieved after several years of practice. Additionally, the term mastery presents a view of knowledge and skills as working towards a fixed point rather than one of ongoing growth. Instead of viewing mastery as the highest level of appropriation for this study, I use Franke and colleagues' (2001) definition of *generativity*. According to Franke and colleagues (2001), *generativity* is an individual's ability "to continue to add to their understanding" (p. 655) and relates to a teacher's ability to use knowledge "as their own to create, adapt, and change" (p. 683). Repeated opportunities for practice with tools for discourse across various settings can encourage a level of tool appropriation directed towards generativity.

Previous studies have applied the appropriation of pedagogical tools framework to help understand PSTs' use of tools within the context of literacy education and special education. In a longitudinal study examining how teachers appropriate tools for teaching writing, Grossman and colleagues (2000) followed ten novice teachers from their experiences as PSTs through their first three years of professional teaching. Through this study, Grossman and colleagues (2000) found that the student teaching placement provided a valuable setting for PSTs to experiment with tools acquired from methods instruction and that a reflective stance promoted throughout the teacher education program supported a high degree of tool appropriation. Leko and Brownell (2011) explored special education PSTs' appropriation of tools to teach reading and found that the clinical educator played a significant role in providing opportunities to appropriate knowledge in practice for PSTs. Together, these studies highlight several factors impacting *how* PSTs use tools to support their teaching practice when transitioning from methods course instruction into clinical and professional teaching.

Dissertation Contribution to Research on Teacher Learning and Development

Throughout the course of their teacher education program, PSTs are exposed to a variety of conceptual and practical tools for teaching. However, the degree to which PSTs appropriate these tools and apply them in their future practice is influenced by a variety of factors including preconceptions of teaching and learning, activity setting, depth of knowledge related to the tool, and opportunities for repeated practice. Tools provide needed structure for novices, however, PSTs may inappropriately view these structures as maxims for good teaching and struggle to advance from routine experts into adaptive experts. This dissertation study contributes to research on teacher learning and development by investigating how PSTs use and appropriate tools for discourse during their clinical teaching experiences. Understanding how PSTs use discourse tools at this phase when they are transitioning out of methods course instruction and into classroom practice can contribute to the development of pedagogical tools that support the development of adaptive expertise.

Preparing Teachers to Facilitate Mathematical Discussions

The Association of Mathematics Teacher Educators (AMTE) standards provide guidance for what math teacher education programs should include and what beginning teachers should know and be able to do when entering the classroom. With respect to facilitating mathematical discussions, beginning teachers are not presumed to be proficient in facilitating productive mathematical discussions, but they should have experiences and knowledge of the significance of leading discussions and how it should be done (AMTE, 2017). According to AMTE (2017) standards, well-prepared beginning mathematics teachers should "engage students in whole-class discussions to share, compare, and analyze student strategies and solutions" (p. 16) by providing learners opportunities to reason mathematically about high-level tasks. To prepare PSTs for the complex nature of facilitating discourse, teacher education programs are encouraged to sequence school-based experiences for PSTs to facilitate one-on-one and small group discussions before engaging in whole class discussions. However, most of the research related to preparing PSTs to facilitate mathematical discussions are rooted in the methods course alone and do not include experiences of PSTs working with students in clinical settings.

In this section of the literature review, I provide a critical synthesis of literature related to novices learning to facilitate mathematical discussions. This includes studies within teacher education programs for PSTs as well as studies with practicing teachers who are novices with regard to facilitating productive mathematical discussions. As many of the studies included in this review are grounded in PBTE, studies included in this section are organized as decompositions and approximations of practice as outlined by the pedagogies of practice framework (Grossman, Compton, Igra, et al., 2009). This section concludes with an emphasis on tools used to support teachers developing the core practice of facilitating mathematical discussions.

Decompositions of Practice

Due to the complexity of facilitating productive mathematical discussions, skills are often presented as decompositions of the larger practice (Grossman, Compton et al., 2009). Smith and Stein's (2018) Five Practices are commonly used as a framework for decomposing practice within PBTE. The Five Practices include anticipating, monitoring, selecting, sequencing, and connecting. For example, Tyminski and colleagues (2014) focused on identifying, developing, and discussing the goals for lesson discussion, which falls within what Smith and Stein (2018) described as practice 0. While decompositions of practice may lack authenticity, they afford more time for learning and reflection than could occur within the context of an actual elementary classroom setting (Ball & Cohen, 1999). Research explores different tools that can be used to support PSTs as they work on decomposed skills of the larger discourse practice.

A commonly used or modified tool in decompositions is Chapin and colleagues' (2013) Talk Moves. While not directly designed for PSTs, the Talk Moves provide a guide for strategically eliciting student thinking and encouraging students to consider the thinking of their peers, therefore providing novices with needed structure for a complex practice. Herbel-Eisenmann and colleagues (2013) modified the Talk Moves for use with practicing secondary mathematics teachers to facilitate discourse. They found that the simple act of naming these moves supported teachers in identifying and applying these strategies in their own practice. Similarly, tools and frameworks designed for PSTs can support similar reflective practice.

Tyminski and colleagues (2014) approached the practice of decomposing the practice of facilitating discourse by "frontloading" the planning phase, providing PSTs with student work samples that had already been worked out. By removing the need to anticipate student thinking, PSTs could focus on the practices of selecting and sequencing strategies while planning questions to help steer discussions towards a mathematical goal. Tyminski and colleagues (2014) found that this approach supported PSTs' ability to organize mathematical discussions. Some of the core challenges of preparing teachers to lead productive discussions include the high levels of improvisation and in-the-moment thinking required of the teacher. However, by removing the need to anticipate student thinking and respond in real-time to student ideas, the level of improvisation was not merely reduced, but removed completely from practice.

Shaughnessy and colleagues (2019) described common decompositions used in two different teacher education programs in an effort to support a shared PBTE design. In this study, the practice of discourse was decomposed into four components: planning, launching, orchestrating, and closing discussions. Shaughnessy et al. (2019) found that common decompositions can be used to support PST learning across different teacher education programs. While these common decompositions were found to support a diverse group of teacher educators and their learners, applying these decompositions in a prescriptive way ignores the autonomy of the teacher educator to address the specific needs of their learners in context. Additionally, Kazemi and Hintz (2014) acknowledged that the structure of discourse depends on the mathematical goal. Therefore, decomposing the practice of orchestrating effective discourse does not always adhere to a fixed set of skills.

Decompositions of practice can also become disconnected from authentic practice. Janssen and colleagues (2015) argued that more attention is given to decomposition than *recomposition* of core practices, which can lead to "deskilling" teachers. An underlying assumption of PBTE is that the skills of teaching can be taught and learned. However, Sleep (2012) claimed that the skills for identifying and managing mathematical goals are not currently articulated in a teachable way. Ghousseini and Herbst (2016) advised teacher educators to intentionally combine decompositions of practice with both representations and approximations of teaching related to mathematical discourse to help present a more authentic and meaningful representation of practice and encourage greater transfer to teaching practice.

Approximations of Practice

Approximations of practice for facilitating productive discussions allow PSTs to engage in practices resembling the work of actual teaching, including work with students in clinical placements. When approximations of teaching are used within methods courses, such as teaching rehearsals, PSTs can engage with elements of leading mathematical discussions with more time and opportunities for reflection than are possible with the in-the-moment demands in a classroom setting (Anthony et al., 2015; Ball & Cohen, 1999; Lampert et al., 2013). The AMTE (2017) standards recommend PSTs begin with facilitating discourse with a student one-on-one and in small groups before progressing to leading a whole-class discussion.

Commonly used approximations of teaching practice are teaching rehearsals conducted within methods courses. Though approaches to rehearsals can vary, the general model for rehearsal allows PSTs to enact instruction with their peers acting as students and the teacher educator as an instructional coach (Kazemi et al., 2016; Lampert et al., 2013). While rehearsals are generally considered less rigorous and less authentic than enactment opportunities in actual classrooms, the structure of rehearsals allow for more in-the-moment feedback and opportunities for reflection than a traditional classroom setting permits (Averill et al., 2016; Ball & Cohen, 1999; Wæge & Fauskanger, 2021). Lampert and colleagues (2013) considered teaching rehearsals as clinical practice since they allow novices to engage in the actual work of teaching.

Averill and colleagues (2016) used the context of teaching rehearsals to understand the impact of questioning and coaching feedback on the confidence and capabilities of PSTs facilitating mathematical classroom discussions. They found coaching questions help highlight effective practice, encourage reflection, and promote PSTs' in-the-moment decision making. More recently, Kavanagh, Metz, and colleagues (2020) compared the types of coaching feedback PSTs received during teaching rehearsals and the impact on PSTs' responsiveness to learners.

Feedback to support responsiveness was less focused on the fidelity with which PSTs enacted pedagogical moves and more about the appropriateness of the choice of instructional move within context. Based on their analysis of teaching rehearsals, Ghousseini and others (2015) identified the critical role TEs play in helping PSTs identify challenges and facilitating reflection. While Lampert and colleagues (2013) recognized that coaching interactions during rehearsals may detract from the authenticity of the learning task, novice teachers need this support to prepare for complex tasks in the actual work of teaching.

Kazemi and Wæge (2015) described a less common format for teaching rehearsals in a methods course designed around rehearsals and weekly enactments in elementary classrooms. While the frequency with which PSTs engaged in rehearsals and enactments is less common, it allowed for more immediate application of TE feedback to enactments with children. Kazemi and colleagues (2016) identified time between rehearsal and enactment with children as one of the factors impacting the effectiveness of rehearsals. They found that when PSTs enact lessons with children within hours of rehearsals, they are more likely to focus on management than mathematics when compared to enactments occurring days after rehearsals. Combined, these two studies demonstrate the need to more carefully examine factors influencing the relationship between rehearsals and enactments.

In addition to feedback from TEs, rehearsals provide opportunities for peer feedback from other PSTs. When PSTs have the opportunity to immediately try out content-specific tools and strategies within a learning community of peers, they are more able to effectively enact new practices (Cohen & Hill, 2000). Recently, Wæge and Fauskanger (2021) demonstrated how teacher time outs (TTO) in rehearsals can support collaborative learning of ambitious teaching practices. In their study, teachers spent approximately 60% of rehearsal time on teaching and the other 40% of the time engaged in TTOs. Unlike in other studies of TTOs in rehearsals, the majority of TTOs in this study were initiated by PSTs rather than TEs. These TTOs supported collaborative learning about the principles behind core teaching practices. This study highlights the need to consider how the culture of the classroom community influences the impact of teaching rehearsals on PST practice (Kazemi et al., 2016).

Approximations of practice are not limited to teaching rehearsals. Crespo (2003) used letter writing to help PSTs elicit and respond to student thinking as they learned to pose cognitively demanding tasks. The letter writing format allowed more time for PSTs to reflect on student thinking and pose meaningful problems. In addition, unlike with teaching rehearsals in which peers act as students, this approximation of practice allowed for practice with an authentic audience. Spangler and Hallman-Thrasher (2014) also studied an approximation of practice for PSTs learning to facilitate discourse outside of typical teaching rehearsals. In their study, PSTs completed imaginary task dialogues between teacher and students, with varying degrees of correctness in student thinking represented, before enacting lessons in clinical placements. As with other approximations of practice, these task dialogues afforded PSTs time to reflect and respond. However, the task dialogue structure lacks authenticity and PSTs were not always able to apply the skills they presented in the task dialogue to novel enactments with students in clinical placements. This highlights a need to consider how the level of authenticity within approximations of practice can impact enactment.

Tools to Support Enactment

Various tools for enactment can support PSTs in approximations of practice. Tools and frameworks for enactment provide structure for PSTs learning complex teaching practices, such as facilitating productive mathematical discussions (Ghousseini et al., 2015). Sleep and Boerst

(2012) recommend teacher educators use a combination of hard and soft scaffolds to support and shape PSTs' practice with mathematical discourse. Hard scaffolds are supports designed in advance, such as tools and frameworks used to support planning, while soft scaffolds are in-the-moment supports, such as coaching feedback in rehearsals. In the clinical classroom setting, a soft scaffold could include coaching feedback from the clinical educator.

Teacher questioning is an integral component of leading effective discourse. Ghousseini et al. (2015) recommended the use of question sequences to guide PST questioning. Additionally, they recommend repeated practice with the question sequence tool to build routine and allow opportunities for PSTs to adapt the tool for personal use. Similarly, Kazemi and Hintz (2014) recommended using targeted discussion structures to support PSTs in planning discussions based around specific types of mathematical goals. They designed targeted discussion structures around five different goals: 1) compare and connect, 2) why? Let's justify, 3) What's best and why?, 4) define and clarify, and 5) troubleshoot and revise. Spangler and Hallman-Thrasher (2014) recommended using imaginary task dialogues to support PSTs' development of pedagogical content knowledge related to mathematical discourse, yet they recognized the lack of authenticity in this type of support. Collectively, these recommendations highlight the need for both structure and flexibility within tools to support PST practice.

Dissertation Contribution to Preparing Teachers to Facilitate Mathematical Discourse

A practice-based approach to teacher education does not necessitate more time in the field working in actual classroom settings (Ball & Cohen, 1999). While Lampert and colleagues (2013) considered teaching rehearsals a form of clinical teaching, PBTE conducted within elementary classroom settings provides an authenticity that cannot be replicated with approximations of practice within methods course settings alone. However, few studies of PSTs learning to facilitate mathematical discussions include data collected from clinical settings; most data is collected within methods course assignments and activities. By examining PSTs' use of tools to support the facilitation of productive discourse in the time between methods coursework and professional teaching, my study helps to address a need to consider how tools of enactment are used in clinical settings that do not include the structure provided by methods courses.

A practice-based approach to preparing teachers to facilitate discourse can effectively reduce the complexity of the practice, however common tools for supporting this practice are lacking in the field (Ball et al., 2009). While there is an increasing interest in enactment tools to help novice teachers translate abstract concepts into actionable teaching practice, Ghousseini and colleagues (2015) identified the lack of research related to how novice teachers use these enactment tools to support adaptive practice. My study contributes to this research area by examining *how* PSTs use and appropriate tools for facilitating discourse from methods course instruction into their clinical field experiences. By utilizing a multiple case study design, my study allowed me to deeply examine how different PSTs used tools for discourse across multiple classroom settings.

Chapter Summary

Facilitating productive mathematical discussion is seen as a core practice of mathematics education as it encourages students to think critically and communicate their reasoning as they build their conceptual understanding of key mathematics. Yet learning to plan for and enact this practice is complex for both experienced and novice teachers. Research on facilitating discourse with practicing teachers is informative to teacher education but applying findings from this research to PSTs requires consideration of the unique instructional problem space inhabited by these novice learners. Efforts in teacher education programs to prepare PSTs to facilitate discussions often take a practice-based approach that follows the pedagogies of the practice framework (Grossman, Compton, Igra, et al., 2009) of representations, decompositions, and approximations of practice.

This chapter presents a critical review of the literature on facilitating productive mathematical discussions with a focus on tools for preparing teachers for this practice within PBTE. First, I discussed literature around the role of discourse in elementary mathematics instruction to understand facilitation of mathematical discussions as a core practice and the challenges teachers face when adopting this practice. This background helps to consider the areas in which PSTs may need tools to support effective enactment of this core practice. Then, I explored literature related to teacher learning and development with an emphasis on appropriation of tools. This background helps to frame the challenges of facilitating discourse within the context of novice learning and how pedagogical tools can support PSTs as they develop expertise. Finally, I synthesized research specific to PSTs learning to facilitate mathematical discussions to better understand the strengths and limitations of existing practice. While literature shows a practice-based approach to preparing teachers to facilitate discourse can effectively reduce the complexity of the practice, common tools for supporting this practice are lacking in the field (Ball et al., 2009). In addition, research is needed to understand *how* PSTs use pedagogical tools as they develop their practice, especially in a way that directly includes the voices of PSTs (Ghousseini et al., 2015). My study contributes to this need by utilizing a multiple case study design to examine how PSTs appropriate tools for mathematical discussion across activity settings.

CHAPTER THREE: METHODS

This multiple case study seeks to contribute to mathematics teacher education and the understanding of how preservice teachers (PSTs) use and appropriate tools to facilitate mathematical discussions. In this study, I identified tools PSTs use to plan and facilitate mathematical discussions with elementary students. I also analyzed how PSTs used these tools in their practice and PSTs' perceptions of the support these tools provide. Participants in this case study recently completed their mathematics methods coursework. At the time of this study, participants were completing the first semester of clinical teaching during their yearlong internship (YLI). Positioning this study at this time in participants' teacher education provided insight into how tools for discourse are used during early teaching practice without the structure provided in methods courses.

The purpose of this study was to understand the pedagogical tools PSTs use to facilitate mathematical discussions and how PSTs use these tools as they transition from methods course instruction into clinical teaching in elementary classrooms. The following research questions were addressed in this study:

1. How do elementary preservice teachers apply pedagogical tools to inform their planning process for mathematical discussion?
2. How do elementary preservice teachers use pedagogical tools to facilitate mathematical discussions in clinical teaching experiences?

To answer these research questions, I collected multiple sources of data including interviews, observations, and artifacts of teaching and learning.

This research is significant for several reasons. First, much of the existing literature related to mathematical discourse focuses on the experiences of practicing teachers rather than

PSTs. Since the process of pedagogical tool appropriation refers to the degree to which an individual internalizes available tools (Grossman et al., 1999), gathering insight directly from PSTs is needed to deeply understand this process. Research within teacher education is frequently set solely in the context of methods course instruction with fewer studies connecting to PSTs' experiences enacting practices in clinical settings (Ghousseini, 2015; Sleep, 2012). While case studies set within the methods course setting provide ample details of the instructional setting and PSTs' actions, they often lack the authentic voices of PSTs themselves. Additionally, within methods courses, PSTs might be required to use specific pedagogical tools to complete assignments, which makes it difficult to understand how PSTs would or would not internalize the tool into their practice independently. Including PSTs' voices serves to highlight their experiences using pedagogical tools and their perspectives on which tools were useful in supporting their practice. This multiple case study addressed this gap by exploring PSTs' experiences as they transfer from methods course instruction into early clinical teaching experiences during YLI. In order to capture these experiences, this study included multiple interviews and observations of teaching of each participant.

Additionally, there is a growing interest within practice-based teacher education (PBTE) around developing tools to support PSTs with the complex practice of facilitating productive mathematical discussions, such as question sequences (Ghousseini et al., 2015) or common decompositions of discussion (Shaughnessy et al., 2019). However, little is known about *how* PSTs use these tools (Ghousseini et al., 2015). This study sought to understand how PSTs use tools to support both their planning and enactment of mathematical discussions in elementary classroom settings. Findings from this study related to *how* PSTs use existing tools to facilitate discourse can support the refinement of existing tools or the development of new tools to support

PSTs with the complex practice of facilitating mathematical discussions. Exploring how PSTs use these tools as they transition from methods course instruction into clinical teaching settings can develop an understanding of the transferability of tools across settings, therefore informing both methods course instruction and clinical teaching practice within teacher education.

Chapter Three presents the research design used to complete this multiple case study. The chapter begins with a general overview of the multiple case study design used in this study. Next, I describe the setting and participants, including sampling procedures. Then, I describe the data collection and analytical procedures used. After that, I describe the strategies for quality and my own positionality as the researcher. This chapter concludes with a summary of the study design.

Research Design

For this study, I used a qualitative, multiple case study research design. I chose to use case study for this research since my goal was to develop a deep understanding of *how* PSTs use pedagogical tools as they learn to facilitate productive mathematical discussions. A qualitative case study design is appropriate for in-depth description and analysis within a bounded system (Merriam & Tisdell, 2016). This study describes PSTs' experiences using tools to support planning for and enactment of mathematical discussions. This case study was bounded by both time and setting. As previously described, the practice of facilitating productive mathematical discussions is complex as is the associated process of learning to facilitate such discussions. As Miles and colleagues (2014) described, qualitative data is rich and has the potential to reveal complexity. This potential to reveal complexity is fitting considering the complexity of facilitating mathematical discussions.

Case study allows researchers to explore within the real-world context (Yin, 2018). In this study, the phenomenon under investigation was how PSTs use tools for discourse as they plan for and enact mathematical discussions with elementary learners and their perceptions of the support these tools provide. The context for this study falls within clinical classroom settings as PSTs facilitate mathematical discussions with elementary students during the first semester of their YLI. By exploring the tools PSTs use to facilitate discussion during this phase of their teacher education, I captured a transitional period between methods course instruction and professional teaching. As such, this study has the potential to inform both math methods course instruction and support for PSTs in clinical field placements.

Data collection from multiple sources is characteristic of qualitative case study design (Creswell, 2014; Yin, 2018). For this study, I collected data through observations of teaching, one-on-one interviews, and artifact analysis. Such rich data collection allows for a deeper understanding of *how* PSTs use tools as they transition into teaching in an elementary classroom setting. By focusing data collection on multiple, one-on-one interviews with each participant, PSTs' voices are highlighted to reflect their experiences of how they used tools to support them as they learn to facilitate productive mathematical discussions.

This study employed multiple case study design. Evidence from multiple cases contributes to more robust and powerful findings than those from investigation of a single case (Yin, 2018). Additionally, gathering and analyzing data from multiple cases increases confidence in the findings (Merriam & Tisdell, 2015; Miles et al., 2014). This study included multiple PSTs, representing a range of experiences using tools to plan and facilitate mathematical discussions with elementary students.

Setting and Participants

The primary participants in this study were elementary preservice teachers at a large, public university in the southeastern United States. At the start of this study, participants had just completed their work in math methods courses. Elementary education majors at this university complete a series of two math methods courses, one focused on teaching mathematics grades K-2 and the other focused on grades 3-6. Math methods coursework within this teacher education program includes opportunities to apply methods learned in university courses in elementary classrooms through required clinical hours. All participants in this study completed the grades 3-6 methods course during the spring 2022 semester. The time after completion of math methods courses was selected because PSTs are typically introduced to the practical tools of teaching within the methods course setting (Grossman, Compton, Igra, et al., 2009).

For this study, I worked with three PST participants. The sample size for qualitative case study design is necessarily small in order to develop an in-depth understanding of real-life phenomena (Yin, 2018). This number allowed for in-depth analysis of participants' experiences and the settings impacting the investigation.

Sampling was strategic and purposeful in order to recruit participants that met the participant criteria and permissions needed to access classroom settings (Merriam & Tisdell, 2015; Miles et al., 2013). Additionally, convenience sampling was used as the participants were recruited from a university setting that was accessible to me as an instructor and researcher (Miles et al., 2013). Recruitment occurred at the end of the spring 2022 semester as eligible participants completed the grades 3-6 math methods course. A recruitment script is included in Appendix A. Around this time, participants learned their clinical placements for the fall semester. Once I recruited PST participants meeting the study criteria, I continued the consent process to obtain consent from assigned CEs and permissions from school principals.

To participate in this study, PSTs had to be completing the first semester of their YLI during the fall 2022 semester. Participants for this study were recruited from a cohort of PSTs who teach in their YLI clinical classroom setting 1.5-2 days each week rather than the typical one day. This allowed for more opportunities to observe PSTs engaged in planning and facilitating mathematical discussions throughout the semester. In addition, partnership districts included in this cohort all support discussion for teaching and learning mathematics rather than following a highly scripted curriculum. Based on these criteria, approximately 15 candidates were eligible for participation in this study.

PSTs' inclusion in this study also required permissions from the school setting and the clinical educator (CE) assigned to mentor each PST. School principals provided permission to access the school setting as a research site. CEs provided consent to participate in a structured interview at the beginning of the study related to the nature of mathematical discourse typical in their classroom setting. Their consent also granted access to their classroom setting for observations of PSTs facilitating mathematical discussions with students.

Since observations in this case study occurred within clinical classroom settings, CEs and elementary students were considered as incidental participants. As incidental participants, they are not the subject of the study, however, they were included in observations of PST teaching in the classroom setting and video recordings. As such, a separate consent process was conducted with these participants. Elementary students were also not the subject of the study, but their participation in mathematical discussions was observed and recorded for data collection purposes. In some cases, elementary students' participation in mathematical discussions is included in the data analysis to capture PSTs' response to student ideas. Consent documents were sent home to the parents of each student to collect consent as incidental participants. If a student did not return a signed consent document, video recordings did not capture these students and their ideas were not included in analysis.

The three participants in this study were placed in two local partnership schools. Grace and Jillian are both completing their clinical teaching in fifth-grade classrooms at a K-5, public elementary school in a medium-sized school district outside of a large, urban city. State school report card data indicates that less than 40% of third- through fifth-grade students at this school demonstrated proficiency on recent state math assessments, which is below the state average. Meredith completed her clinical teaching in a third-grade classroom in a K-8 public school in a large-sized school district outside of a large, urban city. State school report card data indicates that about 56% of third- through fifth-grade students at this school demonstrated proficiency on recent state math assessments, which is above the state average. Observations in each classroom setting and interviews with partnership CEs helped to understand the classroom setting in which the participants are completing their clinical work, providing evidence to help define each case. I observed each participant leading elementary students in mathematical discussions at this setting three times throughout the semester. More details about each setting are included in case descriptions in Chapter Four.

Data Collection

Case study design relies on detailed, in-depth data collection from multiple sources (Creswell, 2014; Yin, 2018). Data collection for this study included observations of teaching, interviews, and artifact analysis. The following sections outline the procedures for data collection in this study, including the rationale for each type of data collected.

Interviews with Clinical Educators

To better understand the classroom setting in which each participant is completing their clinical work for YLI, I interviewed each CE at the beginning of the semester using a structured interview protocol (Appendix B). This interview protocol was designed to help capture how math talk is typically used in the classroom, including the frequency and structure of mathematical discussions. These interviews also helped identify some of the pedagogical tools teachers access to support mathematical discussion, including lesson plan models, mathematical tasks, and norms for discussion. Data collected from these interviews helped me define each case by describing the setting in which each PST is completing their clinical teaching. Understanding how mathematical discussion is typically used in this classroom community supported my analysis of how PSTs use tools to plan and facilitate discussions as the clinical setting acts as one of the overlapping activity systems impacting PSTs' appropriation of pedagogical tools.

Observations

For this study, I observed each participant teaching in their clinical classroom setting three times. During each observation, I collected field notes using the Observation of Classroom Teaching protocol (Appendix C). During the initial observation of each participant, I took additional observation field notes to capture details of the classroom setting including the seating arrangement and visual displays. This information supported my understanding of how setting influenced PSTs' use of discourse tools. For example, based on interactions observed and artifacts of discussion posted in the classroom, such as norms for discussion, data collected from this observation can provide evidence as to what extent mathematical discussion is included in regular instruction and what tools are commonly used. Additionally, background information about the classroom environment in which PSTs are teaching helped to define each case in this study.

The discussion-based math lessons I observed were conducted either in small groups or whole class. Observations of teaching were scheduled at approximately three weeks apart, starting in September, to represent a range of times in each participant's clinical teaching. In these observations, I acted as an external observer and did not engage with the teacher or learners in the mathematical discussion. During pre-observation interviews, participants identified discourse tools they would use in the observed lesson. These tools were then included in the observation protocol used for this lesson (Appendix C). This protocol guided me as I collected both descriptive and reflective notes related to how PSTs use the tools they identified. Additionally, I documented descriptive and reflective notes of PSTs use of other tools observed in the lesson that were not identified in the pre-observation interview. Other descriptive notes not related to tool use were gathered to help support analysis of PSTs' reflections on the lesson as captured in post-observation interviews. Video and audio recordings of these classroom observations were also collected and revisited, as needed, during data analysis. These recordings focused on the PST teaching the lesson though images and voices of elementary students were also captured.

Interviews with PSTs

Interview data supported the overall understanding of participants' experiences using tools to plan and enact lessons based around mathematical discussion. Interviews provided an opportunity to gather data in a way that captured PSTs' voices and perspectives firsthand. Research related to preparing teachers to facilitate mathematical discussions that includes PSTs' voices is limited. All interviews were recorded and transcribed using transcription software. Each participant participated in multiple one-on-one interviews throughout this study. Semi-structured interviews were conducted at the beginning and end of this study. Additionally, each participant participated in pre- and post-observation interviews for each mathematical discussion observed in the clinical classroom. Therefore, each PST participated in a total of eight interviews during the course of this study.

Initial Interviews

Each participant was interviewed at the beginning of the study using a semi-structured interview protocol (Appendix D). Initial interviews were conducted before PSTs started work in their clinical classroom during their YLI. Initial interviews helped me learn more about each participant's background, specifically as it relates to their experiences with mathematics teaching and learning. Through an activity theory lens, PSTs' appropriation of pedagogical tools is impacted by their experiences and beliefs (Werstch, 1991), which includes their *apprenticeship of observation* (Lortie, 1975). As such, an understanding of PSTs' previous experiences with mathematics and math talk was needed to support the analysis of PSTs' perceptions and experiences using tools for discourse. Background information about each participant also helped me better define each case when reporting within case findings. Additionally, these initial interviews helped me better understand participants' experiences with planning and facilitating mathematical discussions based on their recent work in mathematics methods courses and how they used discourse tools within their coursework. Each initial interview lasted between 40 and 60 minutes. Participants scheduled initial interviews at a time and location that was convenient to them. For this initial interview, I met with Meredith via Zoom, but interviews with Grace and Jillian were conducted in-person, based on participants' preference.

Pre-Observation and Post-Observation Interviews

Pre-observation interviews provided a space for the PSTs to describe the lesson they would be teaching and share their experiences planning the lesson to be observed. Pre-observation interviews were conducted no more than three days before each lesson observation, though most pre-observations interviews occurred the day before or the day of the observation of teaching. Each pre-observation interview followed the interview protocol provided in Appendix

E. These interviews lasted approximately 20 minutes each. As outlined in the pre-observation interview protocol, participants described the lesson they would teach and shared their experiences planning for the lesson. Additionally, they identified and described the tools they used to plan for discussion and explained how they used these tools to support their planning. Data collected from pre-observation interviews helped me identify what tools PSTs used in their planning and what tools they intended to use in their lesson. Additionally, pre-observation interviews provided an opportunity for PSTs to describe how they used these tools for planning and how they feel these tools support their planning for discussion. This data supports both RQ 1 and 3.

Post-observation interviews allowed PSTs to reflect on the lesson they taught and the mathematical discussion that occurred. These interviews also allowed me to ask questions to clarify my understanding of the data collected in observation field notes. In these interviews, we revisited the tools identified in the pre-observation interview. This allowed PSTs an opportunity to expand on their perceptions of how these tools support their practice now that they have facilitated the discussion with students. Questions and prompts used to guide post-observation interviews are included in Appendix F. Post-observation interviews occurred the day of the observation, typically immediately after the lesson was taught. Due to changes in the daily schedule, Meredith's second post-observation interview was conducted after school, four hours after the observed lesson. Data collected from post-observation interviews primarily supported RQ 2 and 3.

Final Interviews

At the conclusion of the study, each PST participated in a final interview at the end of their first YLI semester. A semi-structured interview protocol was used for this interview

(Appendix G), however, the content of these interviews was adapted to help summarize data collected from observations and previous interviews with each individual participant. The purpose of this final interview was to provide PSTs an opportunity to reflect on the tools they used to support mathematical discourse throughout the semester, including experiences using tools outside of the observed lessons. Final interviews lasted between 45 and 60 minutes. As with initial interviews, participants elected to have their final interviews conducted either in-person or on Zoom. Meredith's final interview was conducted in-person while both Grace and Jillian opted to complete their final interviews via Zoom. Data collected from final interviews supported overall data analysis for all three research questions.

Artifacts

Artifacts of teaching and learning were collected from each participant throughout the study. Participants shared copies of lesson plans used for the observed lessons, including any relevant instructional materials. Additionally, they shared copies of any tools used during the mathematical discussion to capture and organize students' thinking, such as pictures of anchor charts or annotations of students' strategies on the board. Tools were either recorded digitally or via a photograph of handwritten responses. Participants shared these artifacts with me through password-protected Google Drive folders shared between each participant and me. Participants were asked to remove any student identifiers from artifacts before sharing. Data collected from these artifacts was largely used to triangulate data collected from observations and interviews, particularly with regard to understanding *how* PSTs used tools to plan and enact mathematical discussions.

Data Analysis

In case study research, the process of data collection and analysis is recursive, requiring ongoing examination and interpretation of data to unpack emerging findings (Hancock & Algozzine, 2017). Data analysis for this case study occurred in multiple phases alongside the data collection process. Additionally, analysis included both within case and cross case analysis (Stake, 1995; Yin, 2018) during each phase of data collection.

All interviews were transcribed using Otter transcription software and uploaded to NVivo for coding. Field notes were typed and uploaded to NVivo. Any text-based artifacts of teaching and learning shared by participants were organized in NVivo as well. All data was organized within individual case study databases for each participant to preserve the collected data in an accessible way (Yin, 2018). With multiple cases, multiple sources of data, and multiple phases of data collection, the data analysis for this study was necessarily complex. To help bring clarity to this complex process, I begin by describing the informal and formative analysis conducted within cycles of data collection. Then, I describe the formal within case and cross-case analysis conducted to answer the research questions.

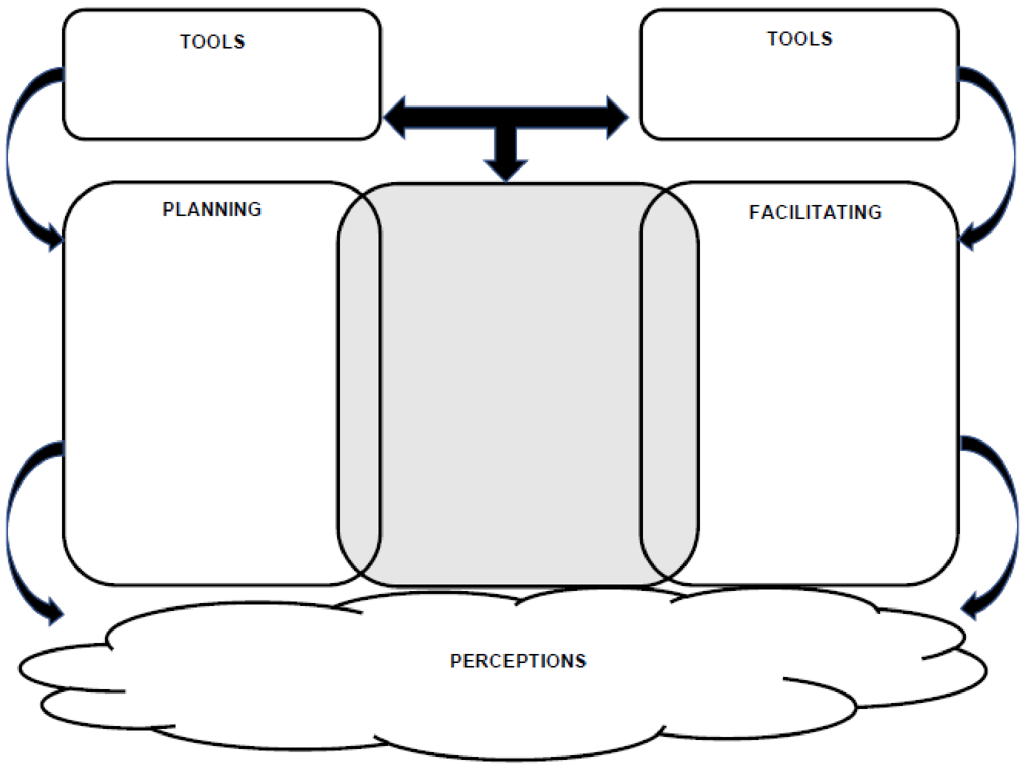
Within Cycle Data Analysis

Data for this study was collected in multiple cycles across five months. In this section, I describe the analysis procedures used to analyze data within cycles to organize data and begin category construction (Merriam, 2002). Data analysis in this phase was both informal and formative as it informed data collection in subsequent cycles. This analysis was primarily conducted within case, though some emerging cross-case themes were noted for further investigation. The overall goal of this phase of analysis was to organize data around the research questions to ensure that adequate data had been collected to answer each question.

While building each case study database, I used Jamboards to organize memos and data I found interesting or relevant to my study (Merriam, 1998). I engaged in this practice after each interview and observation to begin making sense of the data and help guide data collection in future cycles. Each Jamboard included sections for each cycle of data collection: initial, cycle 1, cycle 2, cycle 3, and final. To organize memos, I included a copy of the conceptual framework (Figure 1) for this study in each cycle. For cycles that included observations, I also added sections in the Jamboards to organize visual artifacts, such as pictures of lesson plans or PSTs' handwritten notes used in planning. Additionally, I created a graphic organizer (Figure 2) to help sort the pedagogical tools used within each cycle as tools for planning discussions, facilitating discussions, or both. This graphic organizer also includes a section for me to add notes related to PSTs' current perceptions of how these pedagogical tools support their practice. Using Jamboards to organize the memos made it easier for me to start grouping notes and comments that were related within each data set and make direct connections to the conceptual framework and research questions in this study. Overall, this process helped me develop an initial set of categories and make note of emerging themes within the data that I could reference during formal within case and cross-case data analysis later in the study. Additionally, this informal analysis helped me modify the interview protocol for final interviews (Appendix G) to better address each participant's use of tools for planning and facilitating math discussions throughout the semester.

Figure 2

Within Cycle Pedagogical Tool Use



Formal Data Analysis

I started formal data analysis after all data was collected and analyzed informally, as described above. The first stage of analysis was conducted within case, followed by more comprehensive cross-case analysis (Merriam, 2002; Yin, 2018). Within NVivo, data was coded using an inductive, thematic approach (Charmaz, 2014; Miles et al., 2014; Saldaña, 2016). Data was analyzed for themes related to PSTs' use of pedagogical tools to plan and facilitate mathematical discussions with elementary students in their clinical placements. I began with open coding (Strauss & Corbin, 1990) of interview transcripts, observation field notes, and lesson artifacts to capture the tools PSTs used in planning and enacting discussions and how these tools were used. In this first cycle of coding, data was coded in vivo or with descriptive codes (Saldaña, 2016; Strauss & Corbin, 1990), using the conceptual framework and the research questions as guides. Subsequent coding cycles focused on identifying patterns in the data (Miles et al., 2014) and condensing initial codes into broader, emergent categories related to the research questions.

I started by coding all transcripts, field notes, and artifacts collected from Meredith to develop an initial list of codes. I then coded all transcripts for Grace and Jillian, adding to the initial list of codes, as needed. To address the first and second research questions, I identified the tools PSTs used for planning and enacting discussions. Then, I examined the contexts under which the tools were used and PSTs' descriptions of how they used these tools across both planning and facilitation of discussion throughout the semester. Because there was overlap between how tools were used for planning and facilitation, I combined analysis procedures for the first two research questions. Data analyzed to answer the first research question came from the initial, pre-observation, and final interviews, as well as PSTs' lesson plans. Data used to answer the second research question came primarily from field notes and videos from observations of teaching, post-observation interviews, and final interviews.

Next, I condensed initial codes into related categories, using my research questions and conceptual framework as a guide for grouping codes. I first grouped codes by the tool used to help identify patterns in how specific tools were used by PSTs to plan and enact mathematical discussions. I also grouped codes across tools to identify more general patterns of tool use, using the appropriation of tools framework (Grossman et al., 1999) to support confirmatory coding related to RQs 1 and 2. For example, if a participant named a tool in their planning process but the description of their planning did not yield any evidence of actual use, this might indicate *appropriating labels* of the tool only. I then compared the ways tools were used for planning to the ways tools were used for facilitation of discussions with students. After within case analysis, I identified any themes that occurred across cases in what Stake (1995) referred to as categorical aggregation.

Table 1

Data Collection and Analysis Methods

Research Question	Data Source and Timeline	Data Analysis
How do elementary preservice teachers apply pedagogical tools to inform their planning process for mathematical discussion?	Initial interviews (August 2022)	Open coding
		Pattern coding
	Pre-observation interviews (3 times throughout fall 2022)	Confirmatory coding using the appropriation of pedagogical tools framework (Grossman et al., 1999)
	Final interviews (November/December 2022)	
	Artifacts of teaching and learning used in planning (fall 2022)	Serves to triangulate findings from other sources
How do elementary preservice teachers use pedagogical tools to facilitate mathematical discussions in clinical teaching experiences?	Pre- and post-observation interviews (3 times throughout fall 2022)	Open coding
		Pattern coding
	Final interviews (November/December 2022)	Confirmatory coding using the appropriation of pedagogical tools framework (Grossman et al., 1999)
	Field notes from observations of teaching (3 times throughout fall 2022)	
	Artifacts of teaching and learning from lesson enactment (fall 2022)	Serves to triangulate findings from other sources
	Video recordings of clinical teaching (3 times throughout fall 2022)	

Strategies for Quality

To help increase the trustworthiness of this case study, I triangulated the data by using multiple sources of data to confirm the findings (Denzin, 1978). Multiple sources of data were collected using interviews, observations, and artifact analysis. Data collected from PSTs in interviews was compared to what was observed in practice and what is presented in lesson plans and other artifacts of teaching and learning. Recordings collected from observations of classroom teaching were revisited to support data collected in field notes. Additionally, by engaging with each participant in post-observation interviews, participants had the opportunity to clarify data collected in observations.

Trustworthiness was ensured by conducting ongoing member checking with participants throughout the study (Merriam & Tisdell, 2016). After CE interviews, I shared an interview summary with participants via email. Participants had the opportunity to review the summary and correct any inaccuracies in my initial interpretation. Because my contact with each PST participant was ongoing, member checking was conducted at the start of each new data collection cycle. For example, at the beginning of the Cycle 1 pre-observation interview, I shared my notes from the initial interview with the participant to confirm that my understanding of their responses was accurate. If I needed to clarify anything within the previous transcripts, I was also able to do that during this time with participants.

Due to the qualitative nature of this case study, I, as the researcher, acted as a key instrument of data collection and analysis (Creswell, 2013). As such, throughout the study, I employed strategies to reflect on my own assumptions, biases, and relationship to the study that may have impacted the research. I journaled reflections of my own positionality, or social location within the study, to acknowledge how my own beliefs about mathematical discourse and teacher education were influencing the study (Merriam & Tisdell, 2016). This was most needed when interacting directly with PSTs in interviews to avoid leading questions that reflect my own assumptions. According to Merriam and Tisdell (2016), qualitative researchers need to explain their biases, assumptions, and dispositions regarding the research in order to increase the credibility of their findings. In my positionality statement below, I outline some of my beliefs and experiences related to facilitating mathematical discussions and teacher education.

Positionality Statement

My background in language and literacy education combined with my lifelong love of mathematics influence my choices to regularly incorporate practices commonly used within the humanities (discussion, debate, reflection, and writing) into my mathematics instruction. I have over a decade of experience teaching mathematics to elementary and middle grades learners, and it was through that experience, not my experience in teacher preparation courses, that I learned to facilitate mathematical discussions. I learned to facilitate productive discussions through my own trial and error, by seeking out the knowledge of experts, and by letting my students be my teachers. I believe in the practice of engaging students in mathematical discussions because I have witnessed the lightbulb moments in the reluctant mathematicians who can wield their natural way with words to make mathematical connections they did not believe they could. I have stepped back and watched the collaborative effort of a classroom community as they connected their rough draft thinking (Jansen, 2020) into deep understanding of mathematical concepts. More simply stated, I have seen this kind of teaching help students fall in love with math for the first time.

With as strongly as I feel about the practice of facilitating mathematical discussions, it was important that I constantly reflect on my own perceptions and biases as I conducted this research. Additionally, I had to consider existing relationships with potential participants in this study. As one of the instructors for the MAED 3224, grades 3-6 methods course in the spring semester, about a third of the eligible participants were former students of mine. Additionally, I supported instruction in sections of the MAED 3222, K-2 methods course in the spring 2021 and fall 2021 semesters; other participants were students I worked with in these courses. Using the interview protocols created for this study (Appendices E-F) helped participants differentiate between instructional coaching sessions with me and the pre- and post-observation interviews included in this study. While I did not coach participants in these interviews, often participants engaged in self-reflection while considering the interview questions. As a result, participation in this study could lead to changes in practice and use of pedagogical tools. Throughout the course of this study, I kept a digital researcher journal to reflect on my own beliefs, observations, and initial impressions of the data. Regularly engaging in the reflective journaling process helped me ensure that findings were connected to collected data and not my own beliefs about mathematics teaching and learning.

Chapter Summary

In order to better understand how PSTs use tools to plan and enact mathematical discussions, this qualitative study used a multiple case study design to investigate PSTs' experiences using pedagogical tools to support mathematical discourse as they transition from mathematics methods course instruction into early classroom field experiences during their YLI. Data was collected from three elementary PSTs and their respective CEs. Data included multiple one-on-one interviews, observations of teaching in elementary classrooms, and analysis of artifacts from lesson planning and instruction from participants' YLI. Data analysis was conducted both within case and across cases to answer the research questions.

CHAPTER FOUR: FINDINGS

Chapter Four presents the multiple case studies of the three preservice teachers' (PST) use of pedagogical tools to plan and facilitate mathematical discussions in clinical settings. In this chapter, I answer the research questions by discussing the data collected across the first semester of the PSTs' yearlong internship (YLI), including semi-structured interviews, observations of mathematical discussions, and artifacts of teaching and learning. The purpose of this study was to understand how PSTs use pedagogical tools to plan and facilitate mathematical discussions with elementary students. Findings answer the following research questions:

1. How do elementary preservice teachers apply pedagogical tools to inform their planning process for mathematical discussion?
2. How do elementary preservice teachers use pedagogical tools to facilitate mathematical discussions in clinical teaching experiences?

This chapter presents the findings within each case before examining cross-case patterns between the three cases. For each case, I begin by presenting an overview of the participant's background related to math teaching and learning. This background helped me interpret findings as PSTs' use of pedagogical tools is influenced by their experiences and beliefs (Werstch, 1991). Then, I describe each participant's experiences with mathematical discussion from their methods course and early clinical experiences. I also include a summary of the classroom context in which each PST completed their clinical teaching, focusing on the nature of mathematical discussion commonly practiced by their clinical educator (CE). This information provides insight into the overlapping activity systems in which PSTs appropriate tools and knowledge. The final sections included within each case describe the themes related to tool use in planning and facilitating mathematical discussions in the clinical classroom across the semester. Examples from interviews and observations are shared to support each theme. I conclude this chapter by describing cross-case findings related to all three participants' use of pedagogical tools to plan and facilitate mathematical discussions with their students.

Pedagogical Tools to Support Planning and Facilitation of Mathematical Discussions

While the research questions do not specifically ask *what* tools PSTs used to plan and facilitate mathematical discussions, this information was gathered as part of this study. To help provide an overview of the tools that PSTs in this study used to plan and facilitate math discussions, Table 2 is provided. How PSTs used each of these tools is described in more detail in the within case and cross-case findings.

Table 2

Pedagogical Tools Used for Planning and Facilitating Math Discussions

Pedagogical Tool	Description of the Tool	Connection to Productive Mathematical Discussion
Three-Read Strategy	A protocol used to help students make sense of complex math tasks by reading the task three times with different goals for each read: understand the context, understand the mathematics, and elicit questions about the task (San Francisco Unified School District [SFUSD], 2015)	Use of this tool helps learners focus on understanding language-rich tasks before solving them. When used to launch a task, this strategy can open up complex tasks while maintaining the cognitive demand. This allows more learners to engage in productive exploration of the task to contribute ideas to math discussions.
Launch-Explore-Discuss (LED) lesson model	An indirect lesson framework designed to engage students' thinking about a cognitively demanding task, give students ownership of their mathematical thinking through exploration, and share student ideas through teacher-facilitated discussion (North Carolina Collaborative for Mathematics Learning [NC^{2}ML], 2017).	The LED model is used to plan lessons around cognitively demanding tasks. An effective Launch helps "hook" learners and ensures expectations for exploration are clear. Giving students freedom to explore tasks with limited teacher intervention enables students to own their mathematical thinking. Students can then share their ideas in a student-owned/teacher-directed discussion.
Five Practices	A framework outlining actions teachers can engage in to plan and facilitate meaningful, student-centered math discussions including *anticipating, monitoring, selecting, sequencing,* and *connecting* (Smith & Stein, 2018).	Collectively, engaging in these Five Practices helps moderate the level of improvisation required for teachers to facilitate student-centered discussions that are connected to mathematical goals and avoid a "show and tell" structure.

Talk Moves	A set of actions teachers can use to elicit and respond to student thinking to encourage productive discussion including *revoicing, turn and talk, agree/disagree and why, and wait time (Chapin et al., 2009).*	Talk Moves serve a variety of roles in encouraging productive mathematical discussions including clarifying students' thoughts, deepening students' reasoning, and engaging with others' reasoning.

When specifically analyzing interviews and observations of participants to understand *how* PSTs used the tools described above in their planning process and lesson enactment, three main themes emerged. The three main themes include: (a) nominal use, (b) guiding structure, and (c) connecting to concepts. Within the *connecting to concepts* theme, two subthemes emerged: *crafting questions* and *sharing authority*. A general description of each theme is provided in the narrative below. Data supporting each theme is shared within case and cross-case.

In some cases, PSTs demonstrated only *nominal use* of tools to plan and facilitate discussion. Similarly to how Grossman and colleagues (1999) describe the appropriation of labels or surface features, the nominal use of tools was demonstrated when PSTs named a tool or elements of a tool in their planning or facilitation of mathematical discussions, but their application of the tool did not match the tool they identified. For example, the PST might have identified the model they used for planning their lesson as a Launch-Explore-Discuss format, but the lesson they described was more teacher-directed and did not allow students to explore cognitively demanding tasks. In other cases, PSTs demonstrated nominal use of a tool by identifying a certain tool for facilitation in their written plan or pre-observation interview, but no evidence of actual use of the tool was gathered in observation or post-observation interviews.

The *guiding structure* theme was demonstrated in both PSTs' use of tools for planning and facilitating math discussions. For planning, PSTs relied on elements of a familiar tool to think about the steps they would take to engage students in discussion. For facilitating discussions, PSTs used the tool to carry out these steps in a structured way. The guiding structures theme was demonstrated through both the use of physical structures, such as a visual representation of the planned discussion, or mental models PSTs described to organize the thoughts behind their plan. As theorized by Wertsch (1998), one way tools can mediate a novice's actions is by helping translate an abstract conceptual problem, like learning to facilitate productive math discussions, into a series of concrete operations. The use of pedagogical tools as guiding structures aligns with theory. When PSTs' use of tools demonstrated the *guiding structure* theme, tools were used in a cursory manner. Tools were not adapted to the learning context. At times, PSTs demonstrated use of a tool as a guiding structure when they only used some elements of the tool to support their planning or facilitation of discussions, therefore not demonstrating a comprehensive understanding of the underlying concepts of the tool relative to mathematical discussions.

The final theme was *connecting to concepts*. When a PST used a tool in a way that demonstrated *connecting to concepts*, there was evidence that they could interpret the underlying concepts of the tool to support one of more characteristics of productive math discussions in their planning or enactment. PSTs showed evidence of *connecting to concepts* when they were able to adapt a tool to plan productive math discussions around different mathematical topics and different kinds of tasks. Additionally, the PST were able to adapt the tool to the learning context, such as adjusting plans in the moment to respond to students' thinking. Use of pedagogical tools within the *connecting to concepts* theme also yielded two subthemes: *crafting questions* and *sharing authority with learners.* I elaborate on each of these subthemes in the following paragraphs.

When facilitating productive mathematical discussions with students, the teacher's role is commonly viewed as that of an active facilitator, posing purposeful questions and prompts to elicit student thinking and steer the discussion toward the mathematical goal (Hufferd-Ackles, 2004; Munter, 2014; Sleep, 2012). PSTs commonly used the pedagogical tools described in this study to help them *craft questions* to facilitate discussion with students. Crafted questions were described both in the planning phase and the enactment of discussion as PSTs used tools to help craft questions in the moment to respond to student thinking. Including teacher questions in lesson plans or asking students questions during observations did not always demonstrate the *crafting questions* theme as some questions were focused solely on following step-by-step procedures without connections to underlying mathematical concepts.

The second *connecting to concepts* subtheme is *sharing authority with learners*. PSTs demonstrated sharing authority with students in their use of tools for facilitation of math discussions. This subtheme emerged when PSTs used various tools to encourage peer-to-peer discussion, centered discussion around students' ideas, and included students in determining the mathematical validity of solutions. Table 3 summarizes each of the themes and subthemes described above.

Table 3

Themes in PSTs' Use of Tools for Planning and Facilitating Math Discussions

Theme/ Subtheme	Description	Example
nominal use	Pedagogical tools are named in planning or facilitation of discussion, but the use of the tool does not align with the purpose of the tool. Alternatively, there is no evidence that a tool the participant identified was actually applied in practice.	In Observation Lesson 1, Meredith used the LED lesson model to plan her indirect, problem-solving lesson, but the enacted lesson followed a teacher-directed, guided practice lesson. This demonstrated *nominal use* of the LED model for discussion facilitation.
guiding structures	Some elements of a familiar tool were used to guide decisions in planning or facilitation of math discussions. However, evidence of conceptual understanding of how the elements of the tool worked together to support discussion was not demonstrated and the tool was applied in a cursory way.	In Observation Lessons 2 and 3, Meredith used the LED lesson model as a template for planning the beginning, middle, and end of her lessons, but the identified Launch and Explore did not connect to the goals for discussion.
connecting to concepts	Tool use demonstrated understanding of the underlying concept of the tool as tools were adapted to fit different contexts. Tool use demonstrated connection to one or more characteristics of productive mathematical discussion.	Grace used the LED lesson model to plan both problem-solving and introduction lessons. The plan for the Launch and Explore of the lesson were both connected to the goal for Discussion.
crafting questions	Tool is used to support the PST in crafting purposeful questions to elicit student thinking and connect ideas to the mathematical goal of the lesson.	In all three observation lessons, Jillian used the practice of anticipating student ideas to preplan questions to help students make sense of developing mathematical ideas.
sharing authority	Tool is used to distribute mathematical authority between the students and teacher.	In Observation Lesson 2, Grace used the Talk Move "agree/disagree and why" to invite students to make sense of a peer's developing mathematical idea about volume and surface area.

Case One: Meredith

Individual Characteristics

Throughout all of our interactions, Meredith was very open about her lack of confidence with mathematics and the ongoing challenges she still grapples with as a math teacher. When asked to describe her early experiences with math, Meredith described several factors that contributed to her lack of confidence with math as both a learner and a novice teacher. She recalled asking a lot of questions in math classes, but she was often discouraged when the answers to her questions still did not bring the clarity she needed. "I always asked a lot of questions, but I felt like after I reached a certain amount, I just had to stop. And I was like, 'This teacher just thinks I'm really dumb.'" To mask her discouragement, she learned to pretend she understood the math concepts to avoid pestering teachers with what she perceived were too many questions and to avoid the perception that she was not competent.

While Meredith is an honors student now, she believes her performance in math class in elementary school prevented her from entering the gifted program like her friends when she was young. She recalled teachers making her feel like she and her peers who were not in the gifted program did not need to try more challenging tasks or demonstrate deeper levels of thinking. Part of her motivation to become a teacher stems from her negative experiences in school. She is determined to never make her students feel the way she did when denied access to the gifted program. She believes all students should have access to challenging learning tasks and that teachers should encourage students to share their ideas, even if that means sharing their confusion.

Meredith describes herself as a planner. She keeps organized notes on her iPad, mapping out explicit details of what she needs to get done for both personal and professional tasks.

Planning with this level of detail helps Meredith feel more confident and well-prepared. She admits that she does not do well with spontaneous actions or improvisational thinking, both inside and outside of the teaching context. When completing assignments for courses, for example, Meredith prefers when instructors provide very explicit details and examples of the expectations for the assignment so she can be assured she is on the right track.

Math Methods Courses and Early Clinical Experiences

While Meredith described mostly negative experiences from her K-12 mathematics learning, she "loved every second," of her experience in elementary math methods courses. Unlike the way she learned math, she described the work in her methods courses as hands-on and engaging. She appreciated how the approach to teaching mathematics she learned in her methods courses helped her understand the *why* behind procedures by focusing on conceptual understanding. She also valued the emphasis on problem-solving in context in a way that helped her connect mathematics to a story. Meredith shared how this approach to learning made her forget that she was even doing the mathematics that caused her so much anxiety as a young learner. "It's a story combined with math. Someone like me might not even realize we're doing math, and that's what holds me back so much is knowing I'm doing [a certain kind of math] now."

When specifically reflecting on how her view of math discussions was influenced by her methods course experiences, Meredith emphasized the value of engaging in teaching rehearsals of indirect math lessons. Meredith appreciated that these rehearsals helped her experience what students experience in math discussions, especially since she did not have the opportunity to observe these kinds of indirect lessons in her early clinical experiences in both Kindergarten and fifth-grade classrooms. She also felt rehearsing lessons multiple times throughout her methods courses helped bring meaning to the Launch-Explore-Discuss (LED) model of lesson planning, ultimately making this one of the more memorable practical tools Meredith used for planning and facilitating math discussions. From her methods course teaching experiences, she felt the LED lesson plan format helped her feel organized when planning and implementing lessons in her previous clinical placements.

Another tool Meredith valued from her methods course experience was the three-read strategy for launching a mathematical task. She incorporated this tool into her LED plans as a way to engage students in mathematical discussions around problem-solving tasks. As part of her planning process, she also discussed incorporating elements of Smith and Stein's Five Practices (2018), specifically the value she perceived in *anticipating* student ideas within the planning process. When a student in her fifth-grade clinical placement approached a problem using multiplication rather than division, Meredith felt confident including this idea in discussion because she had planned for it while anticipating talk ideas. "I planned for that one. I had that potential strategy and the reasoning for it. So I used that to kind of put myself in that student's shoes to help them and other students understand that misconception." Overall, Meredith felt that her math methods courses provided her with multiple tools to support planning for and facilitating math discussions with elementary learners.

Yearlong Internship Classroom Context

For her YLI, Meredith is working in a third-grade classroom at a K-8 public elementary school. Her assigned clinical educator (CE) is Ms. Rivenbark. When planning for mathematics lessons and discussions, Ms. Rivenbark and the other teachers at her school have the freedom to choose from a variety of mathematical resources available to teachers in the district and state. There is not a set mathematics curriculum for teachers to follow in this district. Ms. Rivenbark uses the Launch-Explore-Discuss (LED) indirect lesson model to plan daily math lessons. This indirect style of math lessons aligns with the types of lessons Meredith planned, rehearsed, and implemented as part of her methods course experiences. When bringing the whole class together for mathematical discussions, Ms. Rivenbark typically selects the student ideas she wants to highlight in discussion, making sure students are comfortable with sharing out. The ideas she highlights include students sharing their struggles with the task and what they discovered through this challenge. Ms. Rivenbark also uses this part of the lesson to help clear up any misconceptions she noticed in student thinking and provide further explanations, as needed. This intentional selection and sequencing of student talk ideas aligns with elements of the Five Practices (Smith & Stein, 2018) that Meredith has also worked with in her previous math methods work.

Using Tools to Plan and Facilitate Mathematical Discussions

Throughout the observation semester, Meredith used various pedagogical tools to plan and facilitate mathematical discussions with the third-grade students in her clinical placements. To provide context for the observation lessons Meredith planned and taught, I outline the lesson format and the mathematical content focus of each lesson in Table 4. Table 5 outlines the tools Meredith used for both planning and facilitation of math discussions and the dominant theme reflected in her overall use of each tool. In the next section, I elaborate on data collected from interviews and lesson plans to support the themes illustrated in Meredith's planning with each tool. Then, I will share data collected from observations of teaching and debriefing interviews to support the themes that emerged in Meredith's use of each tool for facilitating math discussions with students. I conclude this section with a summary of Meredith's overall tool use.

Table 4

Meredith's Observation Lessons, Fall 2022

Observation Lesson	Lesson Format	Mathematical Content Focus
1	Small group, problem-solving lesson	Multiplication as equal groups
2	Small group, fluency game	Skip counting, multiples of 2 and 10
3		Skip counting, multiples of 4 and 5

Table 5

Meredith's Use of Pedagogical Tools for Planning and Facilitating Math Discussions

Pedagogical Tool	Tool Use for Planning Discussions	Tool Use for Facilitating Discussions
Launch-Explore-Discuss (LED) lesson plan model	guiding structure	nominal use
Three-read strategy	connecting to concepts: crafting questions	connecting to concepts: crafting questions
Five Practices	guiding structure	not demonstrated [a]

[a] Meredith discussed using elements of the Five Practice in pre-observation meetings as a guiding structure for how she planned to facilitate discussion of students' talk ideas, but evidence of use of the Five Practices in observed lessons was not evident.

Tool Use for Planning Math Discussions

The primary pedagogical tools Meredith used for planning math discussions with students were the LED lesson model, the three-read strategy, and elements of the Five Practices. Each of these tools was used to plan at least one of the three math discussions I observed throughout the semester. While Meredith's use of each tool varied across the semester, the themes identified in Table 5 represent the dominant theme represented in pre-observation interviews and lesson planning artifacts.

LED lesson model: Guiding structure. Meredith's use of the LED lesson model for planning math discussions demonstrated the *guiding structure* theme. From both of her math methods courses, Meredith had several opportunities to plan math lessons that included discussion using an indirect lesson plan model, either the LED or the 5E model. Meredith chose to continue using the LED model to support her planning for discussion since she felt this model helped her stay organized and gave her a general structure to follow when planning and teaching her lessons. Meredith used the LED lesson model to plan all three lessons I observed throughout the semester. For Observation Lesson 1, Meredith planned an indirect, problem-solving mini-lesson for a small group of students. The lesson focused on creating visual representations of equal groups to calculate products. For this lesson, Meredith selected three related story problems for students to model and discuss, all related to planning a game night with friends. She selected these specific problems because she liked the "story" element that allowed for discussion of the problem context to avoid students simply "plugging in" quantities to solve without attending to what is going on in the problem. For the Launch of the lesson, Meredith planned to use the three-read strategy to help students make sense of what was going on in each problem before making a plan to solve. For the Explore part of the lesson, Meredith planned to give students access to various manipulatives so they could, "have the freedom to complete the problems using any strategy they want." She planned to discuss students' choice of strategy at the end of the lesson. Some of the questions she had planned included, "What strategy did you use? Let's compare that strategy to another one that was shared." If Observation Lesson 1 provided the only evidence of Meredith's use of the LED lesson model to plan math discussions with students, her tool use could be described as *connecting to concepts* as her application of the tool for planning this lesson demonstrated planning a discussion around students' strategies to approach a cognitively demanding math task. However, as Meredith continued to use the LED lesson model to plan her discussions, she regressed to predominantly *guiding structures*.

For Observation Lessons 2 and 3, Meredith decided to plan her math discussions around fluency games rather than word problems. Ms. Rivenbark suggested Meredith plan lessons to support students' work with skip counting as they continued working on multiplication in class. At quick glance, these lesson plans followed a similar structure to the problem-solving lesson she had planned for Observation Lesson 1, with sections for Launch, Explore, and Discuss. However, these plans were focused on lower cognitive demand tasks, making it difficult for Meredith to use the LED lesson model as more than a guiding structure to help conceptualize the beginning, middle, and end of her lesson plans.

In Observation Lesson 2, the Launch of Meredith's lesson was simply going over the rules of the fluency game, such as what to do if you landed on a space on the game board that did not show the next number in the pattern. The Explore of the lesson was playing the fluency game itself, focused primarily on finding the next number in a counting sequence. Meredith planned two questions for discussion after game play in the Discuss section of her plan: "How do you feel about skip counting after playing this game?" and, "If you did not automatically remember what number came next, what was the strategy you used to find the missing number?" When I asked Meredith about how she planned these questions for discussion, she said she tried to use some question stems she had used previously in other lessons, but she did not feel like any of those questions fit the activity. She also wanted to be sure the discussion captured how students were feeling about the math content, "because if they feel really uncomfortable, then maybe I could change the activity to help fit their comfort level more." She thought that students would benefit from listening to their peers talk about strategies they used to help figure out numbers in the pattern rather than her telling students which strategy to use.

Recognizing that her Observation Lesson 2 plan did not yield productive discussion from students, Meredith made some adjustments to her planning with the LED lesson model for Observation Lesson 3. First, she realized she needed something to “hook” students more during the Launch of her lesson other than just getting them excited about playing a game. She planned to have students use hundreds charts to identify patterns they noticed in the multiples of the factors they would be working with in the skip counting game. While the Explore of the lesson was still a skip counting fluency game, her plan included a note to remind her to “ask questions about what strategy they used *throughout* the lesson,” rather than waiting until the end of the lesson. For the Discuss part of her lesson, she kept the same two questions she used in Observation Lesson 2, but in our pre-observation interview, she also shared that she wanted to connect back to how the patterns students noticed in the hundreds chart during the Launch supported them when playing the game. While Meredith’s use of the LED lesson model to inform her planning in Observation Lesson 3 showed a better understanding of how an effective Launch can support discussion throughout the lesson, her choice to plan the Explore of her second and third lessons around lower-level tasks showed a lack of understanding of how the Launch, Explore, and Discuss work together comprehensively to support mathematical discussion.

Three-read strategy: Crafting questions. Meredith used the three-read strategy to plan the Launch of her problem-solving lesson for Observation Lesson 1. Her use of this tool for planning in this lesson demonstrated *connecting to concepts: crafting questions*. Meredith planned to use the three-read strategy to help students make sense of what was going on in each problem before making a plan to solve. She used the three-read strategy as a tool for facilitating mathematical discussion by opening up the task through teacher questioning so that all students could engage in problem-solving and discussion. Using the three-read strategy as a guide, she scripted out the Launch of her lesson:

Read the scenario and problem 1 without the question prompt.

Teacher: What's going on in the problem? Who or what is involved?

Student: There are friends planning a party. They are going to make sandwiches.

T: Great! Let's read again. How many friends are there? Show me with your counters.

S: There are four friends.

T: Yes. And what other quantity is in the problem?

S: Three sandwiches for each friend.

T: Good. What do you think the question could ask us if we are looking at 3 sandwiches for each of the 4 friends?

S: How many sandwiches do they need in total?

As this excerpt from Meredith's Observation Lesson 1 plan demonstrates, the three-read strategy served as a tool to help Meredith *craft questions* to engage students in one of the problems they would explore and discuss during this lesson. She chose to use this tool because she had used it in her previous clinical teaching when she was in her math methods courses and found it helped her engage students in discussion of math tasks. "It worked really well because students are able to get engaged in the problem context and then feel like they can explore it on their own. Then, we have more to talk about at the end." Meredith also found this tool was beneficial because it gave her a common structure that she could use with a variety of problem-solving tasks.

Five Practices: Guiding structure. Though Meredith did not explicitly identify using the Five Practices as part of her planning process, in our first pre-observation interview she talked in general about engaging in the practice of anticipating student ideas during her planning process and her intentions to facilitate discussion by engaging in the practices of monitoring, selecting, sequencing, and connecting student ideas during the Explore and Discuss parts of Observation Lesson 1. Meredith's use of the Five Practices for planning math discussions demonstrated the *guiding structure* theme. When anticipating student ideas, Meredith used what she had observed in recent class periods to think of the common strategies students might use. "The other day in class, they were doing repeated addition, so a lot of them will probably do that. I know a lot of them might use arrays because we have been practicing that, too." Meredith talked about her plans for the other four practices when talking about how she planned for equitable sharing of ideas in discussion:

> While they're working, I'll look at each student's work and how they worked the problem out. In my head, I can keep track of the different ideas and how to best order and connect them all together. I can check in with students I want to share who might be shyer to offer a conversation starter to them.

Though not explicitly naming the Five Practices as a tool used for planning math discussions, Meredith demonstrated how these practices offered a general structure for her planning. The application of these practices in her written lesson plan was not made visible, such as including a list of anticipated talk ideas sequenced in a way to facilitate discussion. Without more detail about how she planned to specifically use each of these practices, it was unclear if she could demonstrate how these practices worked together conceptually to facilitate productive discussion within her planning process.

Tool Use for Facilitating Math Discussions

When facilitating math discussions with students, Meredith also used the LED lesson model and the three-read strategy. Her use of the three-read strategy demonstrated *crafting questions* in both planning and facilitation. However, her use of the LED lesson plan model for facilitating mathematical discussions shifted from *guiding structure* to *nominal use* only. There was no evidence of Meredith's application of the Five Practices in her facilitation of math discussions with students, even when the practices were applied to her planning.

LED lesson model: Nominal use. Meredith's use of the LED lesson model to plan Observation Lesson 1 demonstrated some connection to the underlying concept of the planning tool. However, the way Meredith used the LED model in her facilitation of math discussions with students demonstrated *nominal use* only. When Meredith planned her first observation lesson, she had a different group of students in mind. On the morning of her lesson, Ms. Rivenbark, her CE, asked Meredith to teach her lesson to the "Green Group" instead, a group of students Meredith described as "striving math learners." Feeling challenged to teach the lesson she had planned to a different group of students than she had intended, during the actual facilitation of the lesson and discussion, Meredith used a more direct, guided practice approach. While she still launched her lesson using the three-read strategy as planned, what Meredith referred to as the Explore of her lesson actually demonstrated direct modeling. First, she modeled how to represent problem one with an array using counting chips. Then, she asked students to create similar arrays to model and solve problem two. For the final problem, she told students they could solve the problem anyway they would like. However, all students continued to use the strategy modeled for them and built arrays with the counting chips. As Meredith looked at the questions she had planned for discussion at the end of the lesson, she realized the questions no longer fit. Some of the questions Meredith asked at the end of her lesson to encourage discussion included, "What operation were we just doing in these problems?" and, "How do you think you can tell when you are looking at a multiplication or a division word problem? Are there certain words they might give you in the problem to help you recognize multiplication or division?" Compared to the discussion she had prepared that would help students draw connections between different strategies they could use to represent multiplication, the in-the-moment questions Meredith asked were mostly close-ended and did not allow for much discussion of student ideas.

During debriefing discussions about her use of the LED model to plan and facilitate math discussions, Meredith admitted that her use of the tool was not always aligned with how she knew the tool was meant to be used. For example, when facilitating the lesson and discussion in Observation Lesson 1, Meredith consciously made the decision to change her plan for the Explore portion of her lesson to include more direct instruction, recognizing that this took away from the purpose of the Explore. "I was trying to have them explore a little bit by letting them use manipulatives, but it wasn't really as much of an exploration as it could have been. It just didn't seem like it was working." Because Meredith observed students in her group struggling to develop their own strategy for solving the problem, she made the decision to work out the problems together. Again, this changed the nature of the mathematical discussion she was able to have with students.

Meredith's use of the LED lesson model to plan Observation Lessons 2 and 3 only demonstrated use of the tool as a *guiding structure*, but her facilitation of these lessons following the structure she created only demonstrated *nominal use* of the LED model. This was partly because Meredith struggled to keep track of the different opportunities for discussion she had planned. For example, though she left a note in her Observation Lesson 3 plan to question students about their strategies throughout gameplay, she forgot to do this during facilitation.

> I think it would have been better if I asked more questions throughout the lesson like I planned. But then it was halfway through, or not even, more like the end of the lesson when I realized that I forgot to do it.

Likewise, while Meredith referred to the hundreds chart during the Launch of her third lesson, she did not use it to help students discuss patterns they noticed between multiples.

> The hundreds chart wasn't really used to support discussion the way we talked about. I kind of just used it randomly. I should have had them use their markers and highlight the patterns so we could talk about it, but it didn't happen.

Given time to reflect on her lesson, Meredith was able to identify ways to make her plans align more with the LED model to facilitate more discussion.

Another reason Meredith's use of the LED model as a tool to facilitate discussions only demonstrated *nominal use* stemmed from the fact that the lessons were planned around lower level tasks. Though one of the two questions Meredith planned for discussion called on students to share the strategies they used when playing the game, students were not challenged to think strategically about the multiples in the sequence. Below, I share the exchange between Meredith and two students, Calvin and Omar, about the strategies they used in the fluency game when they had to skip count by twos.

> Meredith: So I have another question for you. If you didn't automatically know what number came next, what strategy did you guys use?

Calvin: [Pointing to the game board] When I got to 24…no, when I got here I just…No wait! It was when I got here [pointing to 44] I didn't know what was next so I just counted up by ones in my head. Like 45, 46.

M: So you just counted by ones. What about you, Omar?

Omar: When I got stuck, I would just start counting like, 41…no, no wait. I was like 42, 44, or…

Calvin: So basically you just counted up two more like me!

Omar: Yeah, I did the same thing.

When asked to talk about their strategies, students mostly relied on memorization or simply counted up by ones rather than applying a pattern. As we debriefed the lesson, Meredith felt like students struggled to engage in discussion of their strategies because they are not used to verbalizing their thinking. She thought it might help if she had asked more *how* and *why* questions throughout her lesson, but ultimately the task she selected for the lesson did not allow for this type of questioning.

Three-read strategy: Crafting questions. Meredith only demonstrated use of the three-read strategy with her first observation lesson. This is fitting since the Observation Lesson 1 plan was the only plan based around word problems. Similarly to how Meredith used the three-read strategy to craft preplanned questions to support discussion, Meredith used the three-read strategy to craft in-the-moment questions. Meredith used the questions she planned using the three-read strategy and the responses students shared as a point of reference for questioning students when they felt stuck on the task. When discussing her use of the three-read strategy in our first post-observation interview, Meredith shared, "I can ask, 'Remember when you read this? What was going on in the problem again? What did you say we were trying to figure out?'"

In this way, Meredith used the three-read strategy as a tool to *craft questions* to help students make sense of problems in both her planning and facilitation of mathematical discussions.

Summary of Meredith's Tool Use

The first research question in this study seeks to understand how PSTs apply various pedagogical tools to inform their planning for mathematical discussions. Throughout the observation semester, Meredith consistently used the LED lesson model to inform her planning of lessons with math discussion. While her use of the LED model as a planning tool varied throughout the semester, overall, her use of the tool demonstrated use of *guiding structures*. Meredith appreciated that the LED model provided her a familiar structure for planning lessons that focus on mathematical discussion. She could describe each part of the model and how it informed her planning, but her use of the tool did not demonstrate a comprehensive understanding of how the Launch and Explore components work together to ultimately support productive discussion in the lesson. Another tool Meredith used to plan mathematical discussions that demonstrated *guiding structures* was the Five Practices. Meredith used this tool to inform her planning process in a surface level way. The last tool Meredith used to inform her planning for math discussions was the three-read strategy. Her use of this tool was the most authentic to the purpose of the tool as Meredith used this approach of *craft questions* to help students make sense of the tasks in a way that would support more independent exploration, ultimately allowing for a more student-centered discussion of problem-solving strategies.

The second research question seeks to understand how PSTs use pedagogical tools to actually facilitate mathematical discussions with elementary students in their clinical placements. While Meredith's use of the three-read strategy was consistent across planning and enactment, Meredith's use of the LED lesson plan model to facilitate mathematical discussions only demonstrated *nominal use* of the tool. This was demonstrated in a variety of ways. In Observation Lesson 1, though Meredith planned a lesson that would allow for discussion of students' strategies for solving a cognitively demanding task, she enacted the lesson using an "I Do, We Do, You Do," guided practice approach that ultimately lowered the cognitive demand of the task and steered students towards one problem-solving approach. In Observation Lessons 2 and 3, Meredith planned lessons around lower-level tasks, which limited the discussion students could engage in. Additionally, when enacting lessons, Meredith forgot to include several elements of her plan that could have encouraged more discussion, such as connecting procedures to patterns. By not including these elements in her lesson, Meredith was unable to facilitate discussions around students' original ideas.

Case Two: Grace

Individual Characteristics

Like Meredith, Grace also struggled with confidence in her math ability from a young age. She attributes this in part to her participation in a Spanish language immersion program through elementary school. While she valued the opportunity to become fluent in Spanish at such a young age, she struggled to shift from learning math in Spanish to learning math in English once she got to middle school. She was quick to clarify that she was always a proficient math student, but that proficiency did not come as naturally to her as it seemed to come to some of her peers. As a result, she was reluctant to participate in math classes for fear of being wrong.

When asked about her motivation to become a teacher, Grace immediately thought of teachers who inspired her during her K-12 experience. "I specifically think of my third and fifth-grade teachers. They just motivated me to learn and showed me how much of an impact a good teacher can have on a child." She also shared an opportunity she had in high school helping a young student from Ecuador learn to speak English. Witnessing the growth her tutee was able to make in a relatively short amount of time was such a rewarding experience that Grace decided teaching was her calling. Grace also discovered at an early age that teaching others helped her learn. As a kid, she would talk out loud and pretend she was teaching someone the content she was learning. She felt if she could teach someone else, she could understand a concept on a deeper level.

Math Methods Courses and Early Clinical Experiences

While Grace characterized herself as a reluctant math learner from elementary school to high school, she described herself as an active participant in both of her math methods courses in her teacher education program. Like Meredith, her view of teaching mathematics shifted during her methods course experiences. When asked to describe her biggest takeaway about math teaching, she emphasized the value of teaching and learning math in an exploratory way, "allowing students to work through the math concepts, talk about it, use tools, work with their peers, with the teacher as more of the facilitator."

When talking specifically about math discussions, Grace did not feel like she observed productive math discussions in her early clinical experience in fourth grade. She did, however, have the opportunity to lead several small group math discussions with students in her clinical placement as part of her math methods course. A tool Grace used to both plan and facilitate math discussions in her clinical placements was a one-page anticipation guide that her math teacher educator (TE) encouraged PSTs to use, adapted from Smith and Stein's (2018) Five Practices. When planning her discussion, she anticipated as many student strategies as possible, including possible misconceptions, and planned questions to extend students' thinking based on the anticipated strategies. During lesson facilitation, she used the tool as a tracking sheet to monitor the strategies students were actually using, highlighting commonly used strategies, and adding unanticipated strategies, as needed. Grace used the patterns she noticed among the strategies students did and did not use from her anticipation guide to help her refine her plans for subsequent lessons. Like Meredith, Grace felt more comfortable facilitating math discussions with students when she had some sort of visual to guide her instruction. “I made my own little guides. It doesn’t have to be a whole lesson plan, but some sort of chart or bullet points to help me when I’m kind of lost.”

Grace also talked about the ways that she used various Talk Moves (Chapin et al., 2013) to help her facilitate math discussions with students in her early clinical placements. She used the *agree/disagree and why* Talk Move as a way to engage more students in the discussion and also quickly assess students’ understanding of the strategies they were discussing. In one instance, Grace recalls a student sharing a strategy that she did not anticipate in her plan and also was unfamiliar with mathematically. “I didn’t even know how to do it that way. In the moment, I felt stressed.” To help keep the discussion moving forward, Grace tried to revoice the student’s strategy. She encouraged other students in the group to do the same by rephrasing their peer’s strategy in their own words. “Hopefully this doesn’t sound bad, but I was kind of stalling to give myself more time to think about her strategy.” In this way, not only was Grace using the Talk Moves as a tool to elicit deeper student thinking, but she was also using them as a tool to give herself some think time.

Yearlong Internship Classroom Context

For her YLI, Grace is working with Ms. Harkins, a fifth-grade teacher working in the language immersion program at a K-5 public school. As part of the language immersion program, Ms. Harkins works with two classes of fifth-grade students, teaching lessons in English while her teaching partner teaches lessons in Spanish on alternating days. Since Grace is paired with Ms. Harkins as her CE, she also works with both groups of fifth-grade students. Ms. Harkins has freedom to choose from a variety of resources when planning math lessons as there is not a set mathematics curriculum for third through fifth grade in her school district. Mathematics lessons are primarily direct instruction, but Ms. Harkins likes to build in indirect math lessons here and there, depending on the topic and time of year.

Discussion is regularly incorporated into mathematics instruction in this classroom in both small groups and as a whole group. Partner talk occurs daily with students turning and talking with different partners to share their thinking. During indirect math lessons, students can share their ideas with the whole class if they feel comfortable, but this type of discussion occurs less often. If students are working in small groups to explore a task, the group chooses someone to share their ideas with the whole class. Discussion norms in Ms. Harkins classroom are supported by the responsive classroom model. This model encourages mutual respect for ideas, seeing value in mistakes, and helps students understand how to respectfully disagree with other ideas. While not specifically focused on mathematical discussion, the responsive classroom model is reflected in all aspects of the classroom culture.

Using Tools to Plan and Facilitate Mathematical Discussions

Grace used a variety of pedagogical tools to plan and facilitate mathematical discussions with the students in her fifth-grade clinic placement. Table 6 provides an overview of the three observation lessons Grace planned and facilitated during this study. Table 7 outlines the tools Grace used for both planning and facilitation of math discussions and the dominant theme reflected in her overall use of each tool. In the next section, I provide evidence from data collected during interviews and lesson plans to support the themes illustrated in Grace's planning with each tool. Then, I elaborate on data collected from observations of teaching and debriefing interviews to support the themes that emerged in Grace's use of each tool for facilitating math discussions with students in both small group and whole class lessons. I conclude this section with a summary of Grace's overall tool use throughout the semester.

Table 6

Grace's Observation Lessons, Fall 2022

Observation Lesson	Lesson Format	Mathematical Content Focus
1	small group, problem-solving lesson	evaluating numerical expressions, order of operations
2	whole class, concept exploration lesson	volume of rectangular prisms
3	small group, problem-solving lesson	using fractions to model and solve division problems

Table 7

Grace's Use of Pedagogical Tools for Planning and Facilitating Math Discussions

Pedagogical Tool	Tool Use for Planning Discussions	Tool Use for Facilitating Discussions
Launch-Explore-Discuss (LED) lesson plan model	connecting to concepts	guiding structure
Talk Moves	connecting to concepts: crafting questions	connecting to concepts: sharing authority
Five Practices	connecting to concepts: crafting questions	connecting to concepts: sharing authority
Three-read strategy	connecting to concepts: crafting questions	connecting to concepts

Tool Use for Planning Math Discussions

Throughout all three data collection cycles, Grace used the LED lesson plan model, Talk Moves, and the Five Practices to plan mathematical discussions. In Observation Lesson 3, she also used the three-read strategy as a tool for planning discussion. As with Meredith, Grace's use of each tool varied somewhat throughout the semester, but the dominant themes related to Grace's use of each of these tools for planning are outlined in Table 7. In the next sections, I share data collected from pre-observation interviews and lesson planning artifacts to support each theme by tool used for planning.

LED lesson model: Connecting to concepts. Grace's use of the LED lesson model to plan discussions demonstrated *connecting to concepts*. Like Meredith, Grace worked with indirect lesson plan models, like the LED model, in both of her math methods courses. In one interview, Grace explained that this model for planning lessons "feels natural" to her since she used it so frequently throughout her methods courses and earlier clinical experiences. Additionally, Grace's CE, Ms. Harkins, also used the LED model to plan some of her math lessons. Grace used this model to plan both math discussions for students in small groups and for whole-class lessons.

For Observation Lesson 1, Grace planned a small group lesson addressing writing and evaluating numerical expressions using order of operations. During our pre-observation meeting, Grace talked about the challenge she experienced trying to take such a procedural math topic and plan meaningful opportunities for discussion. Ms. Harkins shared a textbook teachers' guide with Grace to help her planning, but Grace found the lesson plans to be too scripted. The lesson also did not allow for any kind of collaboration or math talk beyond asking students to tell the next step in a procedure. Ultimately, Grace still wanted to use the resource Ms. Harkins shared to keep some consistency between the lesson she would teach and the lessons students were used to in this class. "I'm kind of using the textbook as a guide because that's what Ms. Harkins uses." Grace selected a few problems from the textbook that she could base a LED lesson around. "I'm using the same ideas that they had, but I added a lot more questions, extensions, and where I want students to turn and talk to each other." In this way, Grace was able to use the LED lesson model to adapt an existing lesson plan to make the lesson more student-centered and allow more opportunities for students to engage in peer-to-peer discussion.

In Observation Lesson 2, Grace planned a whole-class lesson exploring the volume of rectangular prisms. As the first lesson in this unit, Grace wanted this lesson to include lots of opportunities for hands-on exploration so students could make sense of the meaning of volume and the relationship between volume and area. As with Observation Lesson 1, Grace used the LED lesson model to adapt a volume lesson from the textbook Ms. Harkins shared. Grace found the LED lesson plan model helped her consider how she would engage students in the lesson and access their prior knowledge, provide a hands-on task that students could explore collaboratively, and summarize the big ideas of the lesson through student-centered discussion.

In Observation Lesson 3, Grace used the LED model to plan a small-group, problem-solving lesson. For this lesson, students explored story problems with division. The scenarios presented in the story problems required students to divide a whole number by a whole number resulting in a fraction less than one as the quotient. As with the lesson Grace taught about volume, this concept was new to students. When planning this lesson, Grace considered multiple tasks she could have students explore and discuss, but ultimately selected one focus problem. "I would rather go really in-depth with one problem and discuss the different approaches students used to solve rather than going from problem to problem." She planned her LED lesson around this problem: *Sam, Josie, and Reagan shared 2 boxes of cupcakes. If each friend gets the same amount, how much of a box did each friend get?* Grace purposefully selected a problem context that was familiar to students so they could think of ways to represent the problem even when the math content was new. "With this specific lesson, I think it's important to see what they come up with, in terms of the strategies they use. And to see what they notice as they compare different strategies." Grace used the LED lesson model to plan a discussion around the different ways students approached a problem-solving task, demonstrating that she understood the purpose of the LED model and how task selection connects to productive mathematical discussion.

Talk Moves: Crafting questions. In our initial interview, Grace described the Talk Moves as a tool she used primarily in facilitating mathematical discussions with students. However, in our first pre-observation interview, Grace shared how she also used the Talk Moves as a tool to support her planning. Grace's use of the Talk Moves to inform her planning process demonstrated *connecting to concepts: crafting questions.* As previously mentioned, Grace did not feel like the pre-made lessons her CE shared with her allowed much room for collaboration or discussion. As she planned her first observation lesson, she mapped out various places in the lesson where she could include turn and talks for students to discuss their thinking with a peer before sharing out with the group. In her lesson plan, Grace included a section labeled, "Questions to prompt student thinking." She used various Talk Moves she had learned about in her methods courses and that she observed Ms. Harkins use when teaching to plan these questions. For example, one series of questions Grace planned using the Talk Moves as a guide was, "How can we change the expression so that the value is 40 instead? Do you agree or disagree with ___'s idea? Why?" When asked to share more about how and why she planned these questions for discussion, Grace shared, "These are the questions I've heard Ms. Harkins ask, like, 'Do you agree? Can you add on? What's another way we can think of this?' I'm just really trying to get into the *why* behind their thinking." Each of Grace's subsequent lesson plans included similar applications of the Talk Moves to elicit student thinking through teacher questioning. Grace regularly used the Talk Moves as a tool to adapt existing teaching resources so lessons would include more student-talk. “I guess I started by looking at the resources my CE shared and picked out the parts I wanted to use. Then, I would go back through the plan and think about where I could put in opportunities for students to share their thinking and discuss ideas with peers.” As with the LED lesson model, Grace described using the Talk Moves as a “natural” part of her planning process.

Five Practices: Crafting questions. Unlike Meredith, Grace explicitly identified the Five Practices as a tool she used for planning math discussions. While she did not apply all five of the practices in her planning, she considered how anticipating student ideas helped her plan questions that could engage students in discussion and facilitate connections between students’ talk ideas. In her Observation Lesson 1 plan, Grace outlined anticipated responses students might share, emphasizing possible misconceptions that she would need to address related to the use of parentheses in numerical expressions. Grace felt that anticipating these ideas helped her planning so she could consider how to respond if these ideas emerged in discussion. Grace also felt anticipating student ideas helped reorient her to a math topic she had not worked with in a long time and identify places she might need to consult with her CE for more support during planning.

For Observation Lesson 3, Grace planned a problem-solving lesson around division tasks that yielded fractional quotients. When planning this lesson, she recorded three anticipated student strategies for solving on sticky notes: representing the task with a numerical expression, using a part-whole diagram, and acting out the task with manipulatives. In our pre-observation meeting, Grace spoke a lot about the importance of anticipating student ideas when planning lessons for discussion, but she admitted that this is still difficult for her to do. One reason is because the strategies she sees her students using in class are so different from the types of strategies she was taught to use as an elementary student. “I was trying to think to myself all the different ways to solve the task when I’m not used to doing it like that. I was literally in planning yesterday asking my CE, ‘is this something a kid would do?’”

While Grace struggled to anticipate student ideas efficiently, she does find that this practice helps her preplan questions to ask in the discussion, especially questions she can use to connect strategies students share in discussion.

> I was thinking of the fact that if they all use different strategies, or even if two or three of them use different strategies, how am I going to connect their strategies? Like what types of questions am I going to be asking?

Yet, Grace also recognizes the need to be prepared to ask other questions in-the-moment.

> Truthfully, I don’t know how they’re going to do with this lesson. I could come up with as many questions as I want and possibly none of them would apply. And then it goes back to knowing in the moment, what are you going to ask? What are you going to say to get students to the main point if the things you planned for didn’t happen?

In this way, Grace’s use of the practice of anticipating student ideas as she planned discussions helped her write questions to clarify students’ thinking and consider how she could use questions to facilitate connections between students’ ideas.

Three-read strategy: Crafting questions. The last tool Grace used to inform her process for planning math discussions was the three-read strategy. Grace only applied this tool to plan and facilitate Observation Lesson 3. Like Meredith, Grace’s use of the three-read strategy demonstrated *connecting to concepts: crafting questions*. Grace used the three-read strategy to plan the Launch for her third lesson. She chose this strategy because her CE modeled this method in a recent lesson to help students make sense of the problem context before planning a strategy to solve. Grace also had some experience using this strategy to launch a task from her methods coursework. She used the structure of the three-read strategy to plan the questions she would ask during her Launch to make sure students understood the problem context before exploring the task independently. Grace planned the following questions using this tool: “What is this problem about? What numbers were given in the problem and what do they represent? What are some math questions we can ask using this information?” Since the types of problems students would be working with in her Observation Lesson 3 were new for her students, Grace wanted to make sure students were really able to make sense of the problem context before making a plan to solve. Grace felt the questions she planned using the three-read strategy would support this goal.

Tool Use for Facilitating Math Discussions

Grace used the same pedagogical tools to facilitate math discussions with students that she used to inform her planning. In some cases, the theme demonstrated through tool use for facilitation differed from tool use for planning. Grace’s use of the Talk Moves to inform the planning and facilitation of math discussions both demonstrated the *connecting to concepts* theme, however her use of the Talk Moves to facilitate math discussion demonstrated the *sharing authority with learners* subtheme instead of *crafting questions.* While Grace’s use of the Five Practices to inform her planning demonstrated the *crafting questions* subtheme, her use of the tool for facilitating math discussions demonstrated *sharing authority*. In the next sections, I share the data collected from observations of teaching, post-observation interviews, and the final interview used to demonstrate each theme by tool used for facilitation of discussions.

LED lesson model: Guiding structure. Grace described her use of planning with the LED lesson model as “natural,” since she has had so much experience planning with this tool.

For facilitating lessons with discussion, Grace avoided using the lesson plans she created as scripts. “In terms of the lesson plan I created for myself, I didn’t really use it that much except to look back at what specific questions I wanted to ask.” As a tool for facilitating discussion, Grace used the LED lesson plans more as a visual cue. As such, some elements of the plans she created to support discussion were lost in enactment in her first two observation lessons. While Grace’s use of the LED lesson model to facilitate discussion in Observation Lesson 3 demonstrated *connecting to concepts*, her use of the LED lesson model in Observation Lessons 1 and 2 demonstrated *guiding structures*. As such, I identified *guiding structures* as the main theme demonstrated in Grace’s use of the LED lesson model. Data collected from my observation of these two lessons to support this theme are shared below.

In Observation Lesson 1, while Grace tried to plan a student-centered discussion around evaluating numerical expressions, ultimately Grace found the topic was too focused on procedures for students to discuss different students’ ideas, as planned. The tasks students explored required them to interpret mathematical expressions in words, translate them into numerical expressions, and evaluate them while applying order of operations. Grace was prepared to discuss disagreements students might have about the meaning or the values of the expressions, but this did not occur. Some of the questions Grace had planned no longer fit the discussion. Instead, she asked, “what is something new you learned or something you enjoyed?” The resulting discussion was not focused on the math goal of the lesson, but about students enjoying small group lessons over whole-class math lessons. Grace had a more challenging extension task planned that could have allowed for more discussion, but she did not end up using it. “I had ideas for extensions, but they weren’t completely detailed out in front of me. It was more like, this is hypothetically what I would do." Compared to the discussion Grace had planned, the discussion she facilitated made cursory use of the LED lesson plan model.

Likewise, when students did not make the connections she hoped they would in Observation Lesson 2, Grace struggled to facilitate discussion using the LED lesson she planned. This is due in part to last minute change Grace and her CE made to this lesson the morning of her observation. Rather than having students construct rectangular prisms by cutting our grid paper faces, Grace and Ms. Harkins decided to have students construct prisms using multi-link cubes to explore the concept of volume. With this last minute change in models, students did not see the connection between the area of the base and the volume the way Grace had planned. As a result, the discussion Grace prepared for only somewhat followed her plan:

> It's helpful to have this plan to a certain extent, like to base how the lesson will be structured. But you have to base the discussion off of students' responses. Depending on their responses, the plan I created might not be applicable.

In this case, Grace's use of the LED lesson model to facilitate discussions demonstrates *guiding structure* because she was unable to adapt the plan to facilitate a student-centered discussion that was still connected to the mathematical goal.

Talk Moves: Sharing authority. Grace's use of the Talk Moves to facilitate mathematical discussion demonstrated *connecting to concepts: sharing authority.* While her lesson plans mapped out specific places where she would use Talk Moves to elicit student thinking, when facilitating discussions with students, Grace used additional Talk Moves when she wanted to get more students involved in discussing math ideas.

One example of Grace using the Talk Moves as a tool to share authority with learners was observed in Observation Lesson 1 on order of operations. Grace taught the lesson to a small group of six students. She strategically selected an even group of students to participate in this lesson so that each student would have a peer to discuss ideas with before sharing out with the whole group. But as most of the students in the group finished their exploration of the task, Grace noticed one student, Delia, was still productively working through the task. Grace made the decision to reconfigure her student pairings and create one group of three to allow Delia more think time with the task. “Turn and talk to your partners and share your thinking. Do you agree with your partner? Why or why not? Be ready to share your partner’s idea in your own words.” In this example, Grace used the Talk Moves *agree/disagree and why*, *repeat/rephrase*, and *wait time*. When I asked Grace about this in our post-observation interview, she shared, “I noticed that Delia was taking longer to solve. So whenever I noticed she needed more time, I’d ask the others to turn and talk to a partner so they weren’t just sitting there waiting for her.” She used these Talk Moves to make sure all students were actively engaged in mathematical thinking and ready to contribute ideas to the group discussion. When I asked Grace about her decision to have students share their partner’s thinking instead of their own, she shared, “I don’t think they’re used to talking through their math thinking beyond what’s on their paper. I was trying to get them to explain their thinking better by making sure their partner understood them first.” Grace’s use of these Talk Moves in her Observation Lesson 1 helped to center discussion around students’ mathematical thinking and engaged students in considering others’ ideas.

Grace also used the Talk Moves to *share authority* when facilitating discussion in the whole-class lesson she used to introduce volume in Observation Lesson 2. As before, Grace already mapped out various places in her lesson plan where she would like to include Talk Moves. In some cases, she followed her plan as written. However, she also used the Talk Moves in the moment to respond to student thinking. When one student shared a developing idea about how the volume of a rectangular prism relates to the area of the base of the prism, Grace responded by using *agree/disagree and why*. Grace did not want to simply tell the class this idea was "wrong," so she used this Talk Move to solicit students' help in making sense of this idea. "I prefer going about developing ideas in that kind of way. 'Does someone else want to share what they had in mind that might be similar or different?' versus just being like, 'that's not right.'" In this way, Grace allowed students to *share authority* for determining the mathematical validity of ideas shared in discussion.

Five Practices: Sharing authority. While Grace referenced various elements of the Five Practices in other discussions she planned, it was not until Observation Lesson 3 on division with fractions that she demonstrated use of this tool in discussion facilitation. When I asked Grace about this in our post-observation interview, she felt these practices were easier to apply with this lesson since students could make their thinking about this task visible in a variety of ways. As students explored the division with fractions task described above, they used dry erase markers to represent their thinking on the table, making it easier for her to monitor their thinking. This also helped Grace with selecting and sequencing strategies to discuss:

> With this kind of fraction problem, I knew students would represent their thinking with different models, diagrams, and equations. It was easier for me to select ideas to talk about when there were so many things on the table. Literally! When students really have to apply their own strategy, I can pull from multiple different things and easily ask questions to get them talking to each other and explain their thinking.

As she monitored students' strategies on the table, she compared the strategies students used to the ones she anticipated on the sticky notes and quickly arranged the strategies in a sequence for discussion.

Grace's use of the Five Practices to facilitate discussion in Observation Lesson 3 demonstrated *connecting to concepts: sharing authority.* Her application of the practices of monitoring, selecting, sequencing, and connecting allowed her to facilitate a student-centered discussion with little teacher input. She started the discussion by asking Diego to share his strategy. Though Diego did not represent the problem with an accurate equation, Grace decided he showed a developing idea that "needed to be discussed." As Grace further explained:

> I noticed that Diego had the right setup. He had the two boxes each divided into thirds. And he actually drew out the friends as stick figures, too, and showed they each would get two parts. But he didn't have the right answer. I don't think he was able to put those pieces of his work together to understand that each of those pieces represent one-third of a box. But I liked the way he did it. And his peers could easily follow along with his representation.

As Diego shared how he solved, Grace recreated his strategy on the chart paper for everyone to see. After confirming that she had represented Diego's strategy accurately, Grace asked the students to quietly think about how their strategy was similar to and different from Diego's strategy, adding to their work on the table, if needed. The following excerpt demonstrates how Grace continued to *share authority* with students in the discussion.

> Grace: Jacob, tell us how your strategy was similar or different from Diego's.
>
> Jacob: My strategy is similar because I also drew two boxes like he did and three people.
>
> Grace: So you drew a similar picture to the one Diego drew. What did you do that was a little different? I see some lines in your drawing. What do those represent?
>
> Jacob: It shows sharing the box of cupcakes to each friend.

Grace added the lines from Jacob's drawing to the representation of Diego's strategy on the chart paper.

> Grace: Is this how your lines looked? Ok. And what do each of these pieces in the boxes you shared with each friend represent?
>
> Josie: It represents one-third.
>
> Grace: Do you all agree?

All students showed their agreement that one piece represented one-third of a box using a hand signal. Grace asked Jacob to continue with his strategy. Jacob went on to explain that since each friend would get two pieces of the boxes in his picture and there were a total of six pieces, each friend would get two-sixth of a box of cupcakes. Again, Grace turned to her students to determine the validity of this solution.

> Grace: So you say each friend gets two-sixth a box of cupcakes? Ok, everyone turn and talk to your shoulder partner for a minute. Do you agree or disagree that two-sixths of a box is what is being represented?

Grace circulates as students turn and talk to their neighbor. After about one minute, Grace brings the group back together and asks Soraya to share what she and her partner discussed.

> Soraya: We disagree because the boxes are separate. It's not one big box with six pieces and each person gets two. They are two separate boxes and each person is getting two pieces that are one-third of a box. So we think it's two-thirds, not two-sixths.

Other students demonstrate their agreement with Soraya using their hand signals. It was only then that Grace added her own thinking to relabel Diego's original drawing to show the pieces of each box that each friend would get, helping students better understand where the two-thirds could come from in Diego's representation. By carefully selecting and sequencing which student strategies to share, Grace was able to act as a facilitator while students engaged in a productive mathematical discussion of the task.

Three-read strategy: Connecting to concepts. Grace only used the three-read strategy as a tool to facilitate mathematical discussions in Observation Lesson 3. In class, Ms. Harkins was trying to get students to internalize this process for closely reading a problem and Grace wanted to support that goal in her lesson. As planned, Grace used the three-read strategy to launch the focus task and make sure that students understood the problem context before creating a plan for solving. She even displayed a poster with an outline of the thinking that occurs during each phase of the three-read strategy for students to reference. When describing how she and her students used the three-read strategy to support discussion in her lesson, Grace focused more on the underlying purpose of the three-read strategy as a tool to help all students engage with the problem for discussion rather than strictly following the structure of the routine.

> I know it seems repetitive to them, but even if they're just thinking about these questions as they read, I don't think they actually have to read the problem three times. It's more about thinking through these questions than making sure you follow the exact procedure.

In this way, Grace's use of the three-read strategy as a tool to support facilitating mathematical discussions demonstrates *connecting to concepts*.

Summary of Grace's Tool Use

The first research question in this study seeks to understand how PSTs apply various pedagogical tools to inform their planning for mathematical discussions. The tools Grace used to inform her planning of math discussions were the LED lesson plan model, Talk Moves, the Five Practices, and the three-read strategy. Her use of each of these tools for planning demonstrated the theme *connecting to concepts.* More specifically, her use of the Talk Moves, Five Practices, and three-read strategy all demonstrated the subtheme of *crafting questions* when used as tools for planning. Grace felt that the Talk Moves and the three-reads strategy both provided her a reliable structure to follow when planning questions. Engaging in the practice of anticipating student thinking as one of the Five Practices helped Grace consider how she could use questioning to help students clarify their thinking, respond to developing ideas, and facilitate connections between strategies.

The second research question seeks to understand how PSTs use pedagogical tools to actually facilitate mathematical discussions with elementary students in their clinical placements. All four tools Grace used to inform her planning were also used in her facilitation of mathematical discussions. While her use of the LED lesson model demonstrated *connecting to concepts* in planning, in facilitation, her use of the LED lesson model primarily demonstrated *guiding structures* as she was challenged to facilitate productive discussions when the task exploration she planned did not yield the student responses she planned for. Grace's use of the Talk Moves, Five Practices, and three-read strategy for facilitation of discussion all demonstrated *connecting to concepts*. More specifically, her use of the Talk Moves and Five Practices demonstrated the subtheme of *sharing authority* as she used these tools to help learners take ownership of their mathematical ideas and judge the mathematical validity of strategies and solutions.

Case Three: Jillian

Individual Characteristics

While Meredith and Grace described early math experiences that contributed to a lack of confidence in math as students, Jillian felt very confident as a math learner throughout her K-12 experiences, with a few exceptions. In elementary school, Jillian recalled struggling to grasp the standard algorithm for multiplying multi-digit factors. She preferred to use what she called "the lattice method," but her teacher at the time tried to discourage her from using that strategy. From Jillian's perspective, she felt her teacher did not understand her approach and therefore was never able to help Jillian facilitate meaningful connections to the standard algorithm for multi-digit multiplication. To this day, Jillian struggles to work with the standard algorithm for multiplication.

Despite her struggles with multiplication, Jillian was a confident math student. She would share her ideas in class frequently and she did not hesitate to question her teacher to probe deeper into the *why* of a mathematical procedure. The need for the *why* recurred throughout the experiences Jillian shared. For example, by the time Jillian got to high school, she did not see *why* she had to go to class everyday if she was able to pass the exams without attending. One of her high school math teachers started allowing her to take the unit exams at the start of the unit instead of waiting until the end. If Jillian was able to show she was proficient on the exam, her teacher excused her absences so she could spend more time working in her family's restaurant.

Unlike the other participants in this study, Jillian does not see herself as much of a planner. She has lived with attention deficit hyperactivity disorder (ADHD) since she was a child, something she believes makes it difficult for her to stay organized. However, one asset Jillian perceives results from her ADHD is her comfort with improvisational thinking. "I feel like I am able to blabber out a bunch of questions that are meaningful and relevant off the bat. As long as I have my math worked out in front of me."

Jillian did not feel called to teaching during her K-12 experience. Before switching her major to elementary education, Jillian completed over half the credits she needed to graduate with a degree in computer science. "One day, I just realized that this is not for me. I wasn't failing or anything, and I like coding. But I was getting bored. I'm not the type of person to sit behind a computer all day." When asked why she switched to elementary education, Jillian shared, "I love kids! And I love to talk to people and interact with people." Her clinical work helped affirm that she made the right choice to switch to education. "I couldn't imagine doing anything else. My clinical experience made it set in stone for me. I love doing this!"

Math Methods Courses and Early Clinical Experiences

Jillian credits her early clinical experiences in Kindergarten and fourth-grade classrooms with helping solidify her choice to become a teacher rather than a computer scientist. She loved working with students in the classroom and learning through experience. "Being in that Kindergarten classroom and just seeing how much they grow and change. It was so interesting to watch that progress. I enjoyed every second of it." She also felt called to help students in her clinical placements that were struggling with the expectations of school. "There is one little girl I work with named Caroline. She is constantly getting pulled out of class screaming. I know that might scare some people, but I want to know how to help her and students like her."

When talking about her work in math methods classes, Jillian described the benefit of active learning, ongoing class discussions, and peer mentoring. She particularly enjoyed supporting her peers in the grades 3-6 math methods course as they struggled to make sense of some of the more complex topics, such as fractional reasoning. "There was one girl in my class, Gina. She was always struggling with the math. She was overthinking it. I was able to help her think about the basics, like what would a second or third grader do to solve." Jillian also appreciated the practical scenarios her math methods instructors would provide her. "We would be rehearsing our lessons and our professor would be like, 'One of your students doesn't know how to get started. What do you do?'"

When asked about different pedagogical tools she used to plan and facilitate mathematical discussions in clinical placements as part of her methods coursework, Jillian struggled to name specific tools. Instead, she talked about how helpful it was when lesson planning in her grades 3-6 methods course to have an exemplar lesson plan to use as a guide. She shared the example and a lesson she wrote based on this template with me as we discussed this tool in our initial interview. The example was formatted as a 5E indirect lesson plan. Jillian recalled spending a fair amount of time writing a single lesson plan using this template, but she felt it was time well spent: “I don’t mind doing more because it kind of just helps you be prepared, be over-prepared, for teaching your lesson. And I think that’s a good thing.” She also felt that investing time in writing detailed lesson plans early on in her teaching would benefit her in the long run. “Once you get the hang of it, then you have that reference point to go back to plan similar lessons.” When specifically describing how she felt the 5E lesson plan template supported her in planning and facilitating math discussions with students, Jillian felt the structure of the planning templates she used helped her focus on what the discussion should be about and how she would reach that goal: “It helps me stay on track with where I wanted to be and what I wanted to make a point of discussing with students.”

Though Jillian had difficulty articulating specific pedagogical tools that she used to plan and facilitate discussions from her math methods courses, as we continued talking in our initial interview, it was clear that Jillian had also learned to use the practice of anticipating student talk ideas when planning lessons for discussions as outlined in the Five Practices. Even though Jillian considers herself a strong improvisational thinker, she still sees the benefit in anticipating student ideas when preparing for a math discussion. “It literally just prepared me for whatever was thrown my way.” At times, what was “thrown her way” was a lack of student participation in the discussion, but Jillian found anticipation supported her there, too: “If I got crickets in a discussion, I would be able to pick something up from my plan that I anticipated and ask them what they thought about it.”

Jillian also discussed the perception of the role of discussion in math that she developed from her early math experiences and her experiences in her math methods courses. From her experiences as a math student K-12, Jillian preferred when she had a chance to talk through her ideas in class, but there was not a lot of opportunity for that in her classes. “I’m not one to just listen to a teacher ramble, ramble, ramble. I’m more like a, ‘think about this, do it yourself, let’s come back and talk about it’ kind of person. That’s how my brain works.” Her experience in her math methods courses reinforced Jillian’s appreciation for facilitating mathematical discussions around students’ ideas. “Having different strategies you can use to get to the point and finding the one that works for you is the most important thing, in my opinion.” She also talked about how she sees students benefit from talking through their math ideas, especially students who are struggling with a concept:

> Giving a student who is struggling a bit the chance to talk out loud, it helps them work through it, like hearing themselves talk it out. They might even catch their own mistakes because they have to slow down their brain as they talk through their ideas.

Overall, Jillian shared that she believes students should have many opportunities to engage in mathematical discussion each day.

Yearlong Internship Classroom Context

For her YLI, Jillian is also working in the fifth grade in the same school as Grace. Jillian’s CE is Ms. Becker. Since Ms. Becker is not part of the language immersion program at the school, she only teaches one group of students. Like her colleague Ms. Harkins, Ms. Becker has the freedom to choose instructional resources for planning math lessons without adhering to a set curriculum. Ms. Becker describes her approach to teaching math as a balance between indirect, exploratory lessons and direct teaching. She tries to incorporate math discussion daily through partner talk, small group discussions in math centers, and whole class lessons. Students also engage in written math discourse in their math journals. The math discussions I observed Jillian facilitate throughout the semester were all conducted with small groups during their math centers.

Using Tools to Plan and Facilitate Mathematical Discussions

Table 8 provides an overview of the three observation lessons Jillian planned and facilitated during this study. Compared to Grace and Meredith, Jillian was less explicit about the pedagogical tools she used to plan and facilitate math discussions. When we talked about her planning process in interviews throughout the semester, rather than naming specific tools, Jillian described the practices she engaged in how these practices supported her planning and facilitation of discussions with students. As we discussed these practices, together we identified the tools listed in Table 9 as the pedagogical tools she used to plan and facilitate discussions in the three observation lessons. Table 9 also outlines the dominant theme reflected in Jillian's overall use of each tool. In the next section, I provide evidence from data collected during interviews and lesson plans to support the themes illustrated in Jillian's planning with each tool. Then, I elaborate on data collected from observations and post-observation interviews to support the themes that emerged in Jillian's use of each tool for facilitating math discussions with students. I conclude this section with a summary of Jillian's overall tool use throughout the semester.

Table 8

Jillian's Observation Lessons, Fall 2022

Observation Lesson	Lesson Format	Mathematical Content Focus
1	small group, problem-solving lesson	operations with multi-digit whole numbers
2		connecting to the standard algorithm for multi-digit multiplication
3	small group, guided skills practice	evaluating numerical expressions, order of operations

Table 9

Jillian's Use of Pedagogical Tools for Planning and Facilitating Math Discussions

Pedagogical Tool	**Tool Use for Planning Discussions**	**Tool Use for Facilitating Discussions**
Launch-Explore-Discuss (LED) lesson plan model	nominal use	nominal use
Three-read strategy	not demonstrated [a]	nominal use
Five Practices	guiding structure	not demonstrated [b]
Talk Moves	connecting to concepts: crafting questions	connecting to concepts: sharing authority

[a] Jillian did not describe use of this tool for planning.

[b] Jillian discussed using elements of the Five Practices in pre-observation meetings as a guiding structure for how she planned to facilitate discussion of students' talk ideas, but evidence of use of the Five Practices in observed lessons was not evident

Tool Use for Planning Math Discussions

While both Meredith and Grace continued to write their math lesson plans in a format similar to the format required in their math methods courses, Jillian's plans for discussion were less structured. Jillian was also less explicit about the pedagogical tools she used to inform her planning process, speaking more about the practices she engaged in when planning rather than naming specific tools that she used. From our pre-observation interviews and the planning materials Jillian shared with me, I identified the tools Jillian used to plan discussions as the LED lesson model, the Five Practices, and the Talk Moves. Jillian's use of these tools to plan mathematical discussions was consistent across all three observation lessons. Themes related to Jillian's use of these tools for planning are outlined in Table 9. In the following sections, I share data collected from pre-observation interviews and lesson planning artifacts to support each theme by tool used for planning.

LED lesson model: Nominal use. Jillian's use of the LED lesson model to plan mathematical discussions demonstrated *nominal use*. As previously mentioned, Jillian's written plans were less detailed than the plans Meredith and Grace shared with me. Her lesson plans read more like "notes to self" that would be difficult for another reader to interpret without guidance. Though her plans were not written with separate sections for the Launch, Explore, and Discuss like the indirect lesson plan she shared with me from her methods of coursework, when discussing her planning process during our pre-observation interviews, she referred to the parts of her lesson as launch, explore, and discuss. Similarly to how Meredith used the LED lesson model to plan when facilitating her first lesson, what Jillian referred to as an indirect, LED lesson really demonstrated a more direct approach to having students solve story problem tasks.

For Observation Lesson 1, Jillian planned a small-group lesson for students to discuss different strategies for solving multi-step story problems requiring multiple operations. Ms. Becker, her CE, shared a packet of story problems from a standardized test bank for Jillian to use when planning her lesson. Jillian selected several multi-step story problems to use, as many as she felt time would allow, rather than focusing discussion on one main task with opportunities for related, follow-up tasks as would typically be seen in an LED lesson. She planned to work through the first problem as an example, asking students questions along the way to help guide her procedures. “I want them to help me talk through the problem because I don’t want them doing it wrong. So I’ll ask them, ‘What would we do first here? Who wants to tell me?’” She then planned to have students work through similar problems independently before bringing them back together to discuss the answers to each problem. The discussion Jillian planned focused more on correct answers and students talking through the steps they used to solve rather than their understanding of the problem context. This does not align with the goals of the LED model and therefore demonstrates *nominal use*.

For Observation Lesson 2, Jillian planned another small group lesson with the goal of helping students see connections between the box method for multi-digit multiplication that they were comfortable with and the standard algorithm for multiplication. This time, Jillian selected one multiplication story problem for students to solve using both computation methods. “Ms. Becker and I are really trying to push the standard algorithm since that’s part of the fifth-grade standards, but right now they’re really good using the box method.” She wanted to facilitate a discussion that would allow students to see for themselves the connections between the two methods so they could be comfortable using either approach, something she was not comfortable with at their age.

> I understand that the standard algorithm is part of the fifth-grade standard, but if you’re struggling with that method and you can solve it another way, then I’m not going to force the standard algorithm down your throat. So, if we can make the connections between the methods, then you should be able to choose what works for you.

Planning a discussion that would help students notice these connections could align with the LED lesson model, but again, Jillian attended more to procedures and guiding students to focus on very specific parts of the computations to notice commonalities. “They need you to put the answer in front of them. Like, I’ll probably cover up part of the box and ask, ‘What do you notice about *this* row in the box and *this* number in the computation using the standard algorithm?’” By closing off students’ opportunities to notice connections for themselves, Jillian planned another lesson that did not align with the LED lesson model she referenced.

For Observation Lesson 3, Jillian decided not to use the LED lesson model. She planned a small group lesson focused on evaluating numerical expressions using order of operations. Ms. Becker, gave Jillian a packet with various expressions for students to evaluate applying order of operations. Like Grace in Observation Lesson 1, Jillian struggled to plan a discussion-based lesson for such a procedural topic. One concern she had was that unlike with other lessons she planned around materials her CE provided, these expressions did not have the context of a word problem to open up discussion. During our pre-observation meeting, I probed Jillian to share more about how she planned to facilitate discussion with students about the expressions she selected for the lesson. “I want to give them a few expressions to do on their own. I’ll watch their work. Then, I want them to walk us through how to evaluate the expression without me modeling it.” When I asked her about questions she might ask to elicit student thinking, her questions focused on procedures only. “I’m probably going to ask, ‘where do we start?’ Like, ‘what is the first thing we do and why is that the first thing, according to our procedure.’” While Jillian did not claim to use the LED lesson model to plan this lesson with discussion, I wanted to share evidence from our interviews to support why Jillian did not feel she could use the planning tool with this math topic.

Five Practices: Guiding structure. Jillian's use of the Five Practices to plan mathematical discussions demonstrated *guiding structure*. When planning each of the observations of math discussions, Jillian always engaged in the practice of anticipating student ideas. To help anticipate these ideas, Jillian made sure to work through the math tasks herself as part of her planning process. Jillian found that when she actually took the time to work out the tasks herself instead of relying on the answer key, she was better able to anticipate what students might do.

Though Jillian used the practice of anticipating student ideas to help think of questions she could ask students in discussion, she focused more on procedures students would use to solve and the computational errors they might make to help her think of questions she could ask to help them fix these mistakes. She felt that by anticipating common procedural errors students might make, she could help them notice these errors before they went too far down an incorrect path. "If I see a student beginning to make a mistake I anticipated when I was working through the task, I can immediately be like, 'Oh, I made that mistake, too. Let's talk about it.'" While Jillian engaged in anticipation to help think of questions she could ask students during discussion, since her preplanned questions focused solely on following step-by-step procedures, Jillian's use of this tool only demonstrated *guiding structure* rather than connecting to concepts.

Part of the reason Jillian focused so much on procedures when anticipating students' ideas was to make sure that she would be able to talk to students about the different mathematical procedures that they were familiar with since the procedures she typically used sometimes differed from the ones Ms. Becker encouraged students to use. "I tried to expect a lot of wrong approaches and then questions like, 'What do I do first? 'You multiply the two numbers? So am I going to multiply 100 times 200 first?' And then they'll be like, 'No, no, you need to start in the ones place first!'"

Since Jillian anticipated some developing ideas students might demonstrate when planning Observation Lesson 3, I asked her to share how she could use those ideas to help facilitate group discussion. Again, her prompts focused primarily on procedures: "So, if one of the students shares, 'we're going to do 16 plus 2 first,' I would say, 'well, let's hold on. Let's refer back to the steps in our procedure that we follow.'" But ultimately, Jillian did not want to wait to discuss these developing ideas during the group discussion, preferring to help students notice their errors during their independent practice time: "I don't want them to be practicing wrong. And when they're doing their practice problems, I'm usually watching them and asking them like, 'why did you do that?'" Even though Jillian tried to frame her questioning with *how* and *why*, the responses she anticipated students would give to these questions still focused on following procedures only without connections to underlying math concepts.

Talk Moves: Crafting questions. Jillian's use of the Talk Moves to plan mathematical discussions demonstrated *connecting to concepts: crafting questions*. When describing where she planned to use different Talk Moves to encourage mathematical discussion, Jillian first considered the goal of the discussion she wanted to facilitate with students. This helped her think about where in her plan it would make the most sense to use a specific Talk Move. For example, in Observation Lesson 2, the goal of the discussion Jillian planned was to help students notice connections between the box method and the standard algorithm for multiplication. Jillian considered various Talk Moves she could use to facilitate this noticing without telling students what they should see. "I could easily just point out the connections and be like, 'Have you guys noticed that?' But I am trying to get them to see it before I say it." To facilitate this noticing, Jillian planned different opportunities to use *turn and talks*, *agree/disagree and why*, and *add-on*. With both methods annotated on the board, she planned to cover up certain sections of the computation to help students focus on specific areas to discuss with their partner. "Chat with your partner real quick. What do you notice? Make sure you and your partner agree before we share out." When students shared what they noticed, Jillian planned to use the *agree/disagree and why* Talk Move to invite more students into the discussion. "And if we disagree, we always do it respectfully. 'But what do you think about what your friend just said? Can you add on to it?'" While Grace explicitly mapped out in her lesson plans the questions she would ask using different Talk Moves, Jillian made mental notes of where in the lesson these Talk Moves would help encourage student discussion. Jillian's use of the Talk Moves for planning demonstrated *connecting to concepts: crafting questions* because she planned questions that could deepen students' reasoning and help orient students to others' reasoning through peer-to-peer talk.

Tool Use for Facilitating Discussions

When facilitating math discussions with students, Jillian also used the LED lesson model and the Talk Moves. Though she did not share the three-read strategy as part of her planning for discussions, she did identify this strategy as a tool when facilitating discussions in her first two observation lessons and post-observation interviews. From observations of teaching and post-observation interviews, there was no evidence of Jillian's application of the Five Practices in her facilitation of math discussions with students, even though she applied elements of this tool in planning. Themes related to Jillian's use of these tools for facilitating math discussions with students are outlined in Table 9.

LED lesson model: Nominal use. Jillian's use of the LED lesson model to facilitate mathematical discussions demonstrated *nominal use*. This is not surprising since Jillian's use of the LED lesson model to plan lessons also demonstrated *nominal use* of the tool as her plans focused mostly on discussing mathematical procedures in a step-by-step manner. Jillian used her planning notes as a reference when facilitating her lessons. Though Jillian talked about questions she planned to ask students to elicit discussion, none of these questions were written in her lesson plans or the notes she used when facilitating discussions. “I just have notes on the multiplication parts, really. I really struggle with that and I don’t want to confuse them by showing them a different way than what they are used to.”

The fear of confusing students while they discussed different strategies also impacted other decisions Jillian made as she facilitated this lesson. For example, in Observation Lesson 1, most of the students in the group used standard algorithms to calculate both the subtraction and multiplication portions of the multi-step problem. One student, Adam, reasoned through the multiplication step more efficiently by using partial products. When I asked Jillian why she chose not to discuss Adam’s strategy and only focus on the standard algorithms, she shared, “With this particular group, I definitely wouldn’t have him share because I know the others would just get very confused.” As we talked more about the students in her group, Jillian described them as mostly “striving math learners” who also struggled with reading comprehension. She was concerned that sharing too many different strategies in the amount of time they had would be too overwhelming for this particular group. Rather than trying to make connections between different strategies in the discussion, she felt it was better to stay focused on one approach that was most common among the students. In a LED lesson, the kind of discussion Jillian avoided is exactly the kind of discussion that is encouraged. Just like her use of the LED model to plan discussions demonstrated *nominal use*, her facilitation of those lessons using these plans also demonstrated nominal use of the tool.

Three-read strategy: Nominal use. Jillian's use of the three-read strategy for facilitating math discussion demonstrated *nominal use*. Though not explicitly shared in her plans for discussion, Jillian started both Observation Lesson 1 and 2 by doing multiple reads of the problem, some teacher-led and some student-led. She also referenced use of the three-read strategy in our post-observation interviews. With each read, she would question students to check to see what they thought the problem was about. However, rather than focusing on understanding the context to make sense of the mathematics required to solve, Jillian's questions focused more on keywords in the problems and procedures that could be followed to solve. This excerpt from Observation Lesson 1 provides an example of Jillian's questioning after reading a problem three times with students:

Jillian: What are we going to do first? What does "how much more" imply?

Carmen: Subtraction?

Jillian: What makes you think subtraction? How do we know?

When none of the students answer Jillian's question. She continues:

Jillian: If I had 10 cookies and Carmen had 2, how many more do I have? Would you add them?

Multiple students together: No!

Jillian: No, we'd subtract them. Remember, we've got to think about what it's asking.

When I asked Jillian about why she used multiple reads of each problem with students in this manner, she referenced the three-read strategy that she learned about in her methods courses. She used this strategy because she wanted to help students make sense of the problem and consider a reasonable solution. She observed students in her clinic placement rushing to simply plug in numbers to an operation without really thinking about what the question was really asking, and so she wanted to take the time to read each problem multiple times to make them think about what the question was asking. In the post-observation interview for Lesson 1, she shared:

> Sometimes, context for the answer will be sitting there right in the question, so let's make sure we're not doing a bunch of unnecessary work. So I want to make sure we are pulling out the information we need first and then think about that information in context and what we're really solving for. Because like 1,500 cats? That's not the correct answer, right? So I just used that approach to make sure they know where they're pulling information from, but I want to get them to tell me rather than me handing that information to them.

While Jillian's intentions behind using the three-read strategy with students align with the underlying purpose of the tool, in practice, she did not execute the strategy with the goals for each read in mind. Her questioning also focused more on finding keywords that would indicate a certain procedure rather than solutions that would be reasonable within the given problem context. For this reason, Jillian's use of the three-read strategy to facilitate discussion with students demonstrates *nominal use* only.

Talk Moves: Sharing authority. Jillian's use of the Talk Moves to facilitate discussions with students demonstrated *connecting to concepts: sharing authority*. While the lessons Jillian planned did not give students much authority to solve tasks using a strategy of choice, Jillian frequently used the Talk Moves to orient students to others' thinking and allow students to determine the mathematical validity of ideas shared in discussion. When discussing her use of the Talk Moves during our final interview, Jillian shared, "I don't want to be the one that's talking the whole time, or just like lecture style where students are listening and listening to like 'this is the answer, and this is how you're going to do it.'" She felt the Talk Moves helped her give more opportunities for students to have some control in the discussion.

When facilitating discussions with students, Jillian used various Talk Moves to encourage students to clarify their thinking and help orient students to their peers' talk ideas throughout all three observation lessons. When debriefing Observation Lesson 1, Jillian talked about how she used the revoice Talk Move.

> Sometimes they'll be talking about their idea and not even really understanding what they are saying. So in those moments, I'll be like, 'Okay, so were you kind of saying this? Or was it something different?' And that just helps the student sum up their thought process a little better.

Jillian used this Talk Move to make sure she was understanding her student and that other students in the group were able to follow along with their peer's reasoning.

While Jillian did consider the Talk Moves in her planning for discussions, she really saw them as more of a tool that she could use in the moment when facilitating discussions. "I don't necessarily plan to be like, 'Oh, I'm going to have students turn and talk during this time.' It's more like the looks on their faces tell me I think we would benefit from talking about this without me just rambling on." In Lesson Observation 2, Jillian demonstrated how she used the Talk Moves in this more improvisational way when a disagreement occurred between two students. The task required students to calculate 366 times 57. Carmen solved using the standard algorithm. Raheem solved using the box method. When it came time to discuss the task, Carmen and Raheem disagreed on the correct product. Jillian used this opportunity to share authority with learners, using various Talk Moves to invite the rest of the group to discuss both Carmen and Raheem's thinking and help determine the correct product. After both Carmen and Raheem shared their strategies and justified the product they calculated, the following discussion occurred:

> Jillian: Okay, agree/disagree? After hearing both Carmen and Raheem, which one sounds like the more logical answer given our context? Turn and talk to your shoulder partner first.

After giving students a minute to talk to their shoulder partner, Jillian continued:

> Jillian: Alright. Now that you've had a chance to talk to your neighbor about Carmen and Raheem's ideas, what do you think, Adam?
>
> Adam: I agree with Carmen. I didn't solve it like she did, I solved like Raheem. But I disagree with what he said 60 times 50 equals. It should be 3,000 not 300 because it only takes six 50s to make 300.
>
> Jillian: What do you mean by, 'it only takes six 50s to make 300?'
>
> Adam: Like, two 50s make one hundred. So for 300, you'd have to use two 50s three times, and that's six.
>
> Jillian: Josh, can you explain what Adam just said in your own words?
>
> Josh: I think he's saying that if you add up six groups of 50 you get 300, and I agree with that.

In the excerpt above, Jillian used the *repeat/rephrase* and *agree/disagree and why* Talk Moves to orient students to others' thinking. She also used *revoicing* to help Adam clarify his thinking. In our post-observation meeting, Jillian shared that she was really excited this disagreement came up because, "everybody was really involved in discussing the problem. I wanted to step back and give students time to think about the steps they were following and share their understanding of *what* they were doing and *why*." As this example shows, Jillian used the Talk Moves to *share authority with learners* as she responded to students' thinking during math discussion.

Summary of Jillian's Tool Use

The first research question in this study seeks to understand how PSTs apply various pedagogical tools to inform their planning for mathematical discussions. The tools Jillian used to inform her planning of math discussions were the LED lesson plan model, Talk Moves, and the Five Practices. Her use of the LED lesson model demonstrated the *nominal use* theme because the lessons she planned using this model limited opportunities for exploration and discussions focused on students identifying step-by-step procedures for solving rather than students' own strategies. Likewise, when using the Five Practices to inform her planning, Jillian's focus on procedures in the questions she preplanned only demonstrated the *guiding structure* theme rather than *connecting to concepts.* Jillian's use of the Talk Moves to inform her planning demonstrated *connecting to concepts: crafting questions*. She considered the Talk Moves she could use to question students and help them work through their own misconceptions.

The second research question seeks to understand how PSTs use pedagogical tools to actually facilitate mathematical discussions with elementary students in their clinical placements. When facilitating discussions, Jillian used the LED lesson model, the three-read strategy, and the Talk Moves. As with her use of the LED lesson model for planning, Jillian's use of the LED lesson model as a tool to help facilitate discussions demonstrated *nominal use* only. The discussions she planned with this model emphasized following step-by-step procedures as did her facilitation of these discussions. Jillian's use of the three-read strategy also demonstrated *nominal use*. While she labeled the multiple readings with questioning she used as the three-read strategy, her questions focused on identifying keywords to indicate a specific operation rather than making sense of the problem context as a whole. Finally, Jillian's use of the Talk Moves demonstrated *connecting to concepts: sharing authority*. Jillian used various Talk Moves as a tool to orient students to others' thinking and give students authority to determine the mathematical validity of ideas shared in discussion.

Cross-Case Analysis

In this section, I describe commonalities across PSTs' tool use for planning and facilitating math discussions. While within case findings were organized by tool, I organize these cross-case findings by theme to better understand similarities in the kinds of tool use PSTs demonstrated. First, I describe the cross-case findings related to PSTs' use of pedagogical tools to inform planning for mathematical discussions. Then, I describe the cross-case findings related to PSTs' use of pedagogical tools to facilitate mathematical discussions with elementary students.

Tool Use for Planning Discussions

All three PSTs used the LED lesson model and elements of the Five Practices to inform their planning of mathematical discussions. Grace and Meredith both used the three-read strategy to plan mathematical discussions, and Grace and Jillian both used the Talk Moves to inform their planning. Across cases, PSTs' use of pedagogical tools to plan mathematical discussions demonstrated *guiding structure* and *connecting to content*. In general, the *connecting to concepts* themes were more commonly demonstrated within the use of pedagogical tools for planning discussions compared to tool use for facilitating discussions. Next, I discuss commonalities and differences identified across cases that demonstrated these two themes.

Guiding Structure

The *guiding structure* theme was demonstrated in both Meredith's and Jillian's use of the Five Practices to plan mathematical discussions. All three PSTs focused on the practice of anticipating likely student responses to inform their planning of mathematical discussions. By anticipating likely student responses to the tasks they planned to use to engage students in mathematical discussion, all three PSTs felt better prepared to respond to students' talk ideas. However, according to the Five Practices (Smith & Stein, 2018), the practice of anticipating also includes considering questions to ask students who produce the anticipated responses.

When Meredith engaged in anticipating to inform her planning of Observation Lesson 1, she considered the most common strategies she expected students to use based on what she had observed in recent lessons with Ms. Rivenbark. She did not, however, prepare questions she could ask students to help clarify or extend their thinking. When Jillian used anticipation as part of her planning process, she mostly considered the common errors she expected to see in students' work so she could be prepared to address them. While Jillian did use the practice of anticipating likely student ideas to plan questions she could ask students, her questions focused solely on procedures, such as "what should we do first?"

Another commonality between the way that Meredith and Jillian used the Five Practices for planning discussions is that neither PST formally documented their anticipated strategies in their written lesson plans. For Meredith, this is partly because she finds the practice of engaging in anticipating likely student responses as something she automatically does now when planning. "I kind of just go through it in my head a bit. I'll think, 'it's possible they might use this operation instead,' or whatever. I don't formally do this on paper much anymore like I did for my classes." For Jillian, she does not see as much value in writing out specific questions she might ask students because students might respond differently than she anticipated. "I need to see how they are feeling about the problems in order to ask the questions that I think are beneficial. I can't just brainstorm an amazing question in advance!" Unfortunately, by only considering the likely student responses and not how they as the teacher would respond to these ideas, it was difficult to see how either PST used these anticipated strategies to inform instructional decisions in planning or facilitation of mathematical discussions.

Connecting to Concepts

The *connecting to concepts: crafting questions* theme was demonstrated in both Meredith's and Grace's use of the three-read strategy to plan discussions with students. Both PSTs appreciated that the three-read strategy provided them with a common structure for close reading of a word problem that could easily be adapted to a variety of tasks. When asked why they chose to use this tool to help them plan their discussion, both Grace and Meredith described how they had used this tool to launch tasks for discussion with students in previous clinical placements and found the strategy helped open up the task so all students could develop a plan for solving. As far as how the tool supported her planning process, Grace shared, "It definitely helped me come up with questions to ask them, like what is happening in the story? What do we know?" Similarly, Meredith appreciated that this tool helped her quickly think of the questions she could ask students during the Launch of her lesson: "I know engaging students in the beginning is really important to the discussion you can have later. So I wanted to make sure we were really talking about the task they would solve first." Evidence of *connecting to concepts* was seen in both PSTs' lesson plans and the way they described using this tool in their pre-observation interviews.

The *connecting to concepts: crafting questions* theme was also demonstrated in both Grace's and Jillian's use of the Talk Moves to inform their planning for math discussions. Both

PSTs felt the Talk Moves provided them with "go-to" actions they could use to help students articulate their mathematical ideas and connect to others' ideas. When asked why they used this tool to plan questions to facilitate discussion, both Grace and Jillian described how they had used different Talk Moves in the past and how their current CE used these Talk Moves with students across different content areas. As Jillian described:

> These are just the kinds of questions that I hear Ms. Becker and other teachers using all the time with students. 'Agree or disagree? Turn and talk about this. Can anyone add on to what was just shared?' I feel like the kids are just used to those questions, which helps.

Grace echoed this when she explained the Talk Moves she used to preplan questions in the discussion as the questions she always hears Ms. Harkins using. Multiple opportunities to work with this tool supported the PSTs' ability to effectively incorporate Talk Moves into their planning process.

Tool Use for Facilitating Discussions

All three PSTs used the LED lesson model and the three-read strategy as tools to facilitate mathematical discussions with elementary students. Grace and Jillian both used the Talk Moves when facilitating discussions. Across cases, PSTs' use of pedagogical tools to facilitate mathematical discussions demonstrated *nominal use* and *connecting to content*. Next, I discuss similarities and differences identified across cases that demonstrated these two themes.

Nominal Use

Nominal use of a pedagogical tool was demonstrated in both Meredith's and Jillian's use of the LED lesson model to facilitate discussions with students. Though both Meredith and Jillian framed the lessons they taught in terms of launch, explore, and discuss, the lessons they actually facilitated were more teacher-directed than student-centered. Additionally, both PSTs focused math talk around following procedures to solve a task to make sure all students arrived at the correct solution rather than discussing different ways students approached the tasks through open-ended exploration.

In both cases, Meredith and Jillian were reluctant to give students more freedom to explore tasks because the students they were working with in small groups were “striving math learners.” This was their way of describing students who were performing below grade level standards in mathematics. For example, in Observation Lesson 1, Meredith used the LED lesson model to plan a lesson with math discussion that would demonstrate *connecting to concepts*. However, when she realized the students she would be teaching this lesson to were the “green group,” she changed her plans and modeled the building arrays method for students to follow instead of giving students a chance to explore more independently. Even when discussing the array strategy they modeled together, Meredith relied on calling on one student, Layla, to answer most of her questions. “I didn’t feel like the other students in the group were really getting it the way that Layla was. I didn’t want to put them on the spot to answer a question that they wouldn’t know the answer to.” Similarly, Jillian demonstrated this resistance to opening up discussion to different student strategies when she chose not to include Adam’s partial product strategy in Observation Lesson 1 because she was worried “others would get very confused.”

Grace’s use of the LED lesson model to facilitate discussions with students demonstrated *guiding structure* rather than *nominal use*. However, like the other participants, her use of the LED model for facilitation was less connected to the underlying concepts of the tool than the way she used the tool to inform her planning of mathematical discussions. Grace felt she had the most difficult time applying the LED lesson model to Observation Lesson 1 since the topic of evaluating numerical expressions using order of operations seemed like such a procedural topic.

Likewise, Jillian did not even attempt to use the LED lesson model when planning her lesson and discussion around the same topic in Observation Lesson 3. This shows that PSTs felt certain math topics limited their ability to plan and facilitate student-centered mathematical discussions.

Connecting to Concepts

Grace and Meredith both demonstrated the *connecting to concepts* theme when using the three-read strategy as a tool to facilitate mathematical discussions. Meredith specifically demonstrated the *crafting questions* subtheme. Both felt use of this tool was easier to execute according to their plan because the structure of the tool made it easier for the types of questions they asked to become routine. Additionally, both used the three-read strategy during the Launch of their lesson, making the use of this tool less contingent on students' responses than questions or prompts they had planned to use later in their lessons.

Grace and Meredith both used the three-read strategy as an informal assessment tool when facilitating discussions with students. As Grace shared in our final interview, "In that moment, the three-read strategy kind of lets me check for understanding. Can they make a plan?" She added on to this to elaborate on how having this quick check for understanding supports when facilitating discussion with students: "And it's pretty effective in helping me facilitate discussions just because it allows me to think through what is happening in the lesson. What questions do I need to be asking?" Meredith similarly valued the use of the three-read strategy as a check for understanding that she could use to further guide her facilitation of discussion:

> I feel like that kind of gives a basis for where they are at. So then later on, I am able to say, 'Well, let's look back. Remember when you said the problem was asking this? How can you incorporate that information into your strategy?' That type of thing.

In this way, both Grace and Meredith used the three-read strategy to not only open up tasks for discussion with students, but also to help them gauge students' understanding of the task and respond accordingly.

Grace and Jillian both demonstrated *connecting to concepts: sharing authority* in their use of the Talk Moves as a tool to facilitate mathematical discussions with students. As with using this tool as a tool for planning discussions, both PSTs felt comfortable using this tool because they had many opportunities to practice with this tool in previous teaching experiences and their CEs frequently used Talk Moves to facilitate discussions across different content areas. Both used the Talk Moves as both a tool to preplan opportunities to share authority with students and also to respond to students' in-the-moment thinking. They both recognized the need to pay attention to students' verbal and nonverbal cues to make decisions about when to use Talk Moves during facilitation. As Grace shared in our final interview:

> At this point, I have been working with these students for a couple of months now. I feel like I can understand when they're confused based off their tone of voice or their body language. If I can pick up on those cues, then it helps me think, okay, we need to turn and talk to our partners about this and try to figure it out before I ask anyone to share out with the class.

For Grace, her decision to use certain Talk Moves was about making sure more students felt comfortable sharing their ideas with the whole group.

Jillian also used students' nonverbal cues to help her decide when to use certain Talk Moves to facilitate discussion, as I shared previously when she described basing her decision on the "looks on their faces." For Jillian, using different Talk Moves to share authority with students was more about ensuring the teacher was not dominating the discussion. "I don't like when the teacher holds more of the discussion than students. But sometimes they need help clearly talking through their thought process. And so those little things, like, 'what you're saying is…' really help with that." Both Jillian and Grace felt comfortable using a range of Talk Moves, such as *wait time*, *turn and talk*, *agree/disagree and why*, and *revoicing* to include all students in the discussion.

Chapter Summary

In this chapter, I answered the two research questions by presenting the data collected in this multiple case study from semi-structured interviews, observations of teaching with mathematical discussion, and artifacts collected from lesson planning and lesson facilitation. Three major themes related to PSTs' use of pedagogical tools to plan and facilitate mathematical discussions emerged: *nominal use, guiding structures*, and *connecting to concepts*. In addition, within the connecting to concepts theme, the subthemes of *crafting questions* and *sharing authority* were also demonstrated in PSTs' tool use. Themes were presented within case by pedagogical tools used for planning and facilitating mathematical discussions. Cross-case findings were arranged by theme to highlight similarities and differences between how PSTs demonstrated the different types of use with the same pedagogical tool for planning or facilitating discussions. In Chapter Five, I will further interpret these findings through the conceptual framework used to guide this study. This chapter will also include implications for practice in teacher education and recommendations for further research.

CHAPTER FIVE: DISCUSSION

The purpose of this study was to understand how PSTs use pedagogical tools to inform their planning process and their enactment of mathematical discussions with students in their YLI placements. In this chapter, I present a summary of this multiple case study and conclusions drawn from the data shared in the previous chapter. This chapter also includes implications for practice in mathematics teacher education and clinical teaching related to PSTs use of tools to support their practice with mathematical discussions. This chapter concludes with recommendations for further research and a summary of the study as a whole.

Revisiting the Need for this Study

Facilitating productive mathematical discussions is a complex practice that requires teachers to attend to a range of pedagogical and mathematical knowledge demands simultaneously in response to student thinking (AMTE, 2017; Boerst et al., 2011). This practice is especially challenging for PSTs who have less experience with both pedagogical and mathematical content knowledge (Ghousseini, 2015) and must teach in existing classroom settings that may or may not support the practices they learned earlier in their teacher preparation program (Ghousseini, 2015; Munter, 2014). Within teacher education, there is an increasing interest in enactment tools to help PSTs translate abstract concepts into actionable teaching practice (Ghousseini et al., 2015; Wertsch, 1998), there is a lack of research related to *how* novice teachers use these tools within their practice, particularly in clinical settings. The purpose of this study was to understand how elementary PSTs use various pedagogical tools to plan and facilitate mathematical discussions with elementary students. This study focuses on PSTs' use of tools because, according to Wertsch (1991, 1998), the process of appropriating an ambitious teaching practice is informed by social factors and tool-mediated actions to scaffold novice thinking. This multiple case study was set during the first semester of PSTs' yearlong internship (YLI), which occurred during the semester immediately following the completion of a two-course sequence in mathematics methods for prospective elementary teachers. This multiple case study addressed two research questions:

1. How do elementary preservice teachers apply pedagogical tools to inform their planning process for mathematical discussion?
2. How do elementary preservice teachers use pedagogical tools to facilitate mathematical discussions in clinical teaching experiences?

When planning for mathematical discussions with students, PSTs in this study frequently used the Launch, Explore, Discuss (LED) lesson model (NC^{2}ML, 2017), the three-read strategy (SFUSD, 2015), the Five Practices for Orchestrating Productive Mathematical Discussions (Smith & Stein, 2018), and various Talk Moves (Chapin et al., 2013). PSTs also used these tools to support their facilitation of mathematical discussions with students in their clinical placement. While their use of each of these tools varied throughout the semester, three major themes emerged related to PSTs' tool use: nominal use, guiding structure, and connecting to concepts. In the next section, I further discuss these findings in context of the current literature.

Connecting Themes to Conceptual Framework

According to Grossman and others (1999), "appropriation refers to the process through which a person adopts the pedagogical tools available for use in particular social environments and through this process internalizes ways of thinking endemic to specific cultural practices" (p. 15). While this study focused on tool *use* rather than the process of tool *appropriation*, the themes I identified in PSTs' tool use overlap with the levels of appropriation Grossman and colleagues (1999) outlined in the appropriation of pedagogical tools framework. In the following sections, I connect each of the themes found in this study to the most relevant level of appropriation from this framework. Though the themes from this study share some characteristics with the levels of appropriation, ultimately I chose to define the themes found in this study with different terms as my study focused only on tool use, not the internalize thinking associated with tool appropriation.

The lowest level of appropriation in Grossman and colleagues' (1999) framework is *lack of appropriation*. *Lack of appropriation* means that the novice did not adopt a tool into practice. Since this study examined PSTs' use of tools, the *lack of appropriation* category is not relevant as I argue that even *nominal use* of a tool implies some degree of appropriation.

The first theme I identified related to PSTs' use of pedagogical tools for planning and facilitating math discussions is *nominal use*. *Nominal use* of a tool was demonstrated in two ways: (a) the PST named a tool in their practice but there was no evidence that the tool was used, or (b) the PST identified a tool in their practice but applied the tool in a way that did not align with the purpose of the tool. *Nominal use* aligns most closely with Grossman and colleagues' (1999) description of *appropriating labels.* When a novice appropriates the labels of a tool only, they are able to name a tool, but they do not know the features of the tool. However, PSTs in this study sometimes demonstrated *nominal use* of a tool even when they showed knowledge of the tool beyond its labels, as in Meredith's use of the LED lesson model.

The second theme I identified related to PSTs' use of pedagogical tools to plan and facilitate mathematical discussions was *guiding structures*. When PSTs' tool use demonstrated *guiding structures*, they used some elements of a tool in a cursory way with little attention to how the different elements of the tool worked together to support productive mathematical discussion. This theme most closely aligns with *appropriating surface features* (Grossman et al., 1999). When novices appropriate surface features of a pedagogical tool, they demonstrate that they know most of the features of the tool, but they lack a comprehensive understanding of how those features work together conceptually. In this study, when PSTs used tools as guiding structures, they were drawn to using a familiar structure, even if they were only selecting some features of the tool. This can be seen in both Meredith's and Jillian's selective use of the Five Practices. The *guiding structures* theme identified in this study differs from the *appropriating surface features* category identified by Grossman and colleagues (1999) in that PSTs could demonstrate a comprehensive understanding of the tool in the way they used the tool for planning, but they did not use the tool in a way that showed evidence of this when using the tool for facilitating math discussions, as can be seen in Grace's use of the LED lesson model.

The last theme I identified in this study related to PSTs' use of pedagogical tools to plan and facilitate mathematical discussions was *connecting to concepts*. This theme relates most closely to *appropriating conceptual underpinnings* (Grossman et al., 1999). According to Grossman and colleagues (1999), when novices appropriate the conceptual underpinnings of a pedagogical tool, they are able to adapt the tool to new contexts. In this study, when PSTs demonstrated use of a tool in a way that showed evidence of connecting to concepts, they were able to use the tool in a way that connected with one or more of the characteristics of productive mathematical discussions, such as centering discussion around students' ideas. They were also able to adapt the tool to fit different instructional or mathematical contexts. This theme and its subthemes, crafting questions and sharing authority, were more commonly demonstrated in the way PSTs used tools for planning mathematical discussions. This further supports what Kennedy (1999) refers to as the *problem of enactment,* or the disconnect between what PSTs know or say about teaching and what they can actually demonstrate in practice. In the next section, I further discuss the findings from this study related to each of these themes.

Discussion of Findings

This study examined how PSTs used pedagogical tools to inform their planning for mathematical discussions and also their facilitation of those discussions with elementary students. As outlined in Chapter Four, I found that the three PSTs in this study demonstrated nominal use of pedagogical tools, used tools as guiding structures to plan and facilitate discussions, and were able to connect to the underlying concepts of tools in order to craft questions or share authority with students in the discussion. In this section, I apply the activity theory framework introduced in Chapter One as well as the literature outlined in Chapter Two to help better understand the different ways PSTs in this study used pedagogical tools to inform their practice with mathematical discussions.

Discussion Worthy Mathematics

Productive mathematical discussions use students' ideas about cognitively demanding tasks to advance the shared understanding of significant mathematical concepts (Lampert, 2001; Stein et al., 2008), an ambitious practice for even experienced teachers. The tools PSTs in this study used to plan and facilitate mathematical discussions with students were designed to help mitigate some of the complexity of planning and facilitating discussions with students. However, in multiple lesson observations, PSTs either planned discussions around lower cognitive demand tasks or they reduced the cognitive demand of the task through direct teaching of procedures. Additionally, PSTs described the challenges they faced when trying to apply certain pedagogical tools to planning and facilitating mathematical discussions around math topics that they viewed as primarily procedural in nature.

Procedural Math Topics

PSTs in this study consistently identified word problems or story problems that allowed students to solve using a strategy of choice as mathematics worthy of discussion. However, there were certain math topics that PSTs did not think could lead to deep mathematical discussion because they saw the topic itself as highly procedural. When PSTs viewed math content as highly procedural, their use of pedagogical tools to plan and facilitate mathematical discussions demonstrated *nominal use* or *guiding structure*. In some cases, when PSTs viewed math content in this way, they opted not to use certain pedagogical tools that they previously used to plan and facilitate discussions because they did not see how the tool aligned with the topic.

In Observation Lessons 2 and 3, Meredith's use of the LED lesson model demonstrated *guiding structure* in planning and *nominal use* in facilitation. When Meredith planned mathematical discussions around skip counting fluency games, she felt the lesson model she typically used to plan discussions with students did not fit the mathematical content students would be working with in this lesson. Rather than focusing on strategies for solving word problems, students needed to demonstrate that they could quickly and accurately identify the next multiple in a pattern. Additionally, once Meredith started planning discussions around fluency games rather than word problems, she no longer used the three-read strategy that she found so supportive in her previous clinical experiences planning and facilitating discussions.

During the observation semester, both Grace and Jillian were tasked with planning and facilitating mathematical discussions for fifth grade students to evaluate numerical expressions using order of operations. Seeing this topic as highly procedural, Jillian did not even attempt to plan an LED lesson around this topic. She planned a discussion to help students notice similarities between related numerical expressions, but ultimately students' participation in the discussion involved sharing the next step to follow or the result of a simple computation. Grace did use the LED lesson model to plan and facilitate her discussion of this topic with students. However, when actually facilitating the discussion with students, Grace ended up focusing more on what students enjoyed in the lesson rather than the mathematical goal. When reflecting on what she would have done differently in the discussion to connect more to the math content, Grace shared:

> I didn't really know how to deepen their understanding of order of operations through discussion. There wasn't really room for me to make connections between ideas. Maybe I could have given them a more challenging problem, but it's been a long time since I've worked with this topic.

To facilitate productive mathematical discussions with students, teachers must be able to think flexibly about the math content they are discussing (Kavanagh et al., 2020). When PSTs viewed the math content as focused only on procedures, they struggled to demonstrate this flexibility in their planning and facilitation of math discussions.

Task Selection

When teachers present students with mathematical tasks that allow learners to explore multiple strategies and critically examine errors and contradictions in thinking, mathematical discussions can go beyond a focus on procedures to help develop students' conceptual understanding (Kazemi & Stipek, 2001; Munter, 2014). While all three PSTs referenced the Five Practices as a tool they used to support their practice, none of them explicitly referred to Practice 0: setting goals and selecting tasks (Smith & Stein, 2018). In this study, the ways PSTs used tools to plan and facilitate discussion was impacted by the tasks they selected or the way they chose to present the tasks to students. As described above, Meredith was only able to demonstrate use of the LED lesson model as a *guiding structure* when planning discussions around the fluency games she selected for Observation Lessons 2 and 3. However, her use of the same tool demonstrated *connecting to concepts* when she planned a discussion around the different approaches students could use to solve a multiplication story problem. This indicates that the degree to which PSTs could connect to the conceptual underpinnings of a tool for planning mathematical discussions is impacted by the tasks they select.

In Observation Lessons 1 and 2, Jillian planned discussions around cognitively demanding math tasks, but her use of the LED lesson model and the three-read strategy within these lessons demonstrated *nominal use* of each of these tools. While the tasks Jillian selected could allow learners to explore and discuss multiple strategies, the way she planned and facilitated discussions with students using the LED model and three-read strategy overly emphasized procedures and limited students' contribution of their own ideas to the discussion. Through the activity theory framework used in this study, I interpreted Jillian's focus on procedures when facilitating discussions through her past experiences as a mathematics learner and the established norms in her YLI classroom. In high school, Jillian was able to pass her math courses without attending class by simply passing the exams. From her experience, achievement in mathematics was about getting the right answer. Additionally, PSTs carry out their instructional practice in existing classroom contexts with established structures for student participation that may or may not align with practices learned from teacher education (Ghousseini, 2015; Munter, 2014). Though Jillian did not always agree with it, Ms. Becker wanted Jillian to use her small group lessons to help students become more comfortable with using certain mathematical procedures, such as the standard algorithm for multiplication. An emphasis on the use of an algorithm is likely due to outside pressures to meet growth and proficiency goals on state exams for this grade level. As such, Jillian's *nominal use* of pedagogical tools to plan and facilitate mathematical discussions resulted in part from the conflict she felt between using the practices she learned in her methods courses and complying with her CE's goals for students. While tools can act as mediators to shape actions and beliefs, tools alone cannot change existing beliefs or cause specific actions (Werstch, 1991). Additionally, the activity setting can influence the way teachers use certain pedagogical tools, even if this tool use does not align with the teacher's personal beliefs about teaching and learning mathematics.

Of all the mathematical discussions I observed in this study, the most authentic use of pedagogical tools to plan and facilitate mathematical discussions with students was seen in Grace's Observation Lesson 3. In this lesson, Grace's use of the LED lesson model, the three-read strategy, the Five Practices, and the Talk Moves all demonstrated *connecting to concepts*. Grace attributed this to the types of tasks she was able to select when planning a discussion around division with fractions. In our third post-observation interview, she shared:

> I feel like this topic really allowed me to pick tasks that were open to multiple representations that students could use. And that just makes it easier for me to get them talking about the math and ask them questions to get them thinking about what they did.

In previous lessons, when Grace was not able to plan her discussions around word problem tasks, she struggled to effectively apply certain tools, like the LED lesson model.

Discussion for All

In this study, both Jillian and Meredith described the students they worked with in their small groups as "striving math learners," meaning they were not currently meeting the grade level standards in mathematics. Facilitating discussions that attend to student thinking to reach mathematical goals is one way to foster more equitable, high-quality experiences for students within a classroom discourse community (Hufferd-Ackles et al., 2004; NCTM, 2014). However, when working with these "striving math learners," both Jillian and Meredith focused discussions more on procedures than students' thinking about mathematics concepts. Their focus on procedures with their learners contributed to their *nominal use* of the LED lesson model.

When Meredith used the LED model to plan her first observation lesson and discussion, she did not know which students she would be working with for this lesson. The plan she created for Observation Lesson 1 actually demonstrated *connecting to concepts*. However, when Ms. Rivenbark asked her to work with the "green group," students who were not meeting grade level standards, Meredith felt overwhelmed. She did not plan her lesson with these students in mind. She believed these students would benefit more from following along with her as she modeled the problems rather than letting them explore the tasks on their own, as planned. As she continued to work with this group of students, her use of the LED model to plan and facilitate discussions continued to deviate from the purpose of the tool as she continued to plan discussions around lower cognitive demand tasks for students in the "green group."

When using the three-read strategy to launch tasks for discussion with students, Jillian demonstrated *nominal use* of the tool. Rather than helping students make sense of the context of the problem, she directed the questions she used during the multiple reads to identify keywords that would help students set up the correct computations. Jillian frequently told me, "I don't want them practicing wrong," and would make an effort to correct students' mistakes before they spent too much time applying an incorrect operation or moving to the next step in a procedure with a computational error. At times, she also chose not to discuss different strategies students used to solve for fear of confusing the students she saw struggling the most. When describing what she thought students could benefit from in the discussion, Jillian again focused on procedures without connections. “Jamie, she needs the procedures. Like, she needs to go over basic multiplication. Enzo just needs me to help him set up the problem because he doesn’t always understand the context, but then he’s good.”

While both Meredith and Jillian said they believe that engaging all students in mathematical discussions can help them make deeper connections to mathematics concepts, when working with “striving math learners,” they struggled to foster these deep connections. Rather than giving students opportunities to explore cognitively demanding tasks and facilitate student-centered discussions, they regressed to more conventional teaching methods when working with learners who were not yet meeting grade level standards (Bransford & Stein, 1993; Hogan et al., 2003). Their turn back to conventional methods may have stemmed from the emphasis on standardized tests in these grades and the belief that these methods prepare students to answer test items. This is problematic because research shows that when students have opportunities to explore and discuss rich problem-solving tasks, their achievement in mathematics improves (Hiebert & Wearne, 1993; Hung, 2015). The very students who could benefit the most from discussing cognitively demanding tasks were not given access to this opportunity.

Getting to Know Learners

To help mitigate the theory practice divide in teacher education, some teacher educators are calling for more opportunities for clinical practice within teacher education programs (Zeichner, 2010). At the time of data collection, all PSTs in this study were completing the first semester of a two semester clinical placement, or their yearlong internship (YLI). PSTs will complete their more formal student teaching within the same classroom they are currently assigned, working with the same CE and the same students. From what I observed in this study, one factor influencing how PSTs used different pedagogical tools to plan and facilitate discussions with their students was their knowledge of the students themselves. As described in Chapter 4, both Grace and Jillian used the Talk Moves to share authority with their students in discussions by attending to the verbal and nonverbal cues students shared to indicate the need for peer-to-peer discussion or to clarify an idea a peer shared. In fact, Jillian even identified her knowledge of her students as an informal tool she used when planning discussions. "Just knowing the students that are sitting in front of me and being able to consider what they are really going to benefit from in the discussion. I didn't have that as much when planning lessons before." In this way, using tools in authentic classroom settings helped PSTs plan and facilitate mathematical discussions that were more responsive to student thinking.

Planning vs. Enactment

PSTs' use of tools to plan and facilitate mathematical discussions in this study supports what Kennedy (1999) describes as the *problem of enactment*, or the disconnect between what novice teachers say or know about teaching and what they can actually demonstrate in practice. In this study, this was demonstrated when PSTs' use of a tool to plan a math discussion was more conceptually oriented to the purpose of the tool than their use of the tool to facilitate mathematical discussions. For example, Grace consistently demonstrated *connecting to concepts* in her use of the LED model to plan discussions, but mostly demonstrated use of the tool as a *guiding structure* when facilitating discussions. Grace felt she had more opportunities to use tools to plan discussions than actually facilitate discussions in her teacher preparation program. She shared:

> I feel so comfortable planning. I can write out this glamorous lesson plan with all of these wonderful things. But if I don't have the opportunity to actually teach those lessons, then I won't know what to do. What's important to me is being able to, in the moment, figure out what students need. What are we going to do? Because the lesson plan isn't always going to happen or things are going to go differently. I feel like as a teacher, you need to be flexible and know those moves to make, but I just don't have that experience yet.

A common response to the theory-practice divide in teacher education is to increase the amount of time PSTs have to teach in clinical settings (Zeichner, 2010). As Grace shared, she feels her ability to use tools to plan discussions does not mean as much when she does not also have a chance to facilitate these discussions with actual elementary students.

Similarly, Meredith's use of the LED model demonstrated *guiding structure* in planning, but only *nominal use* in facilitating discussions. Meredith attributed this to her lack of confidence in mathematics. "I know my confidence level with math is much lower. And I don't know if that is the reason. But if something just doesn't go as well as I want, I'm just not fully confident to turn it around." When I asked Meredith about tools or strategies she has used that help her feel more confident during lesson enactment, she shared her practice with scripting lessons for literacy. "Scripts have helped me in the past. I think I just need to make sure to incorporate that every time until I get the hang of it." When I asked her if she worried her script would not fit students' responses, she elaborated:

> I think it's important for me to at least have a little cheat sheet to look at, if needed. Like when I'm reading a book and I know I want to ask a question, I put a sticky note on the page to remind me. So I think I need to find that kind of thing, but for math discussions.

Meredith felt scripting what she wanted to say helped her feel more confident facilitating the discussions she planned, but Ghousseini and others (2015) caution against tools that give too much structure to the discussion as they leave little room for improvisation. Based on their work, PSTs should be able to adapt tools used to facilitate mathematical discussions in an unscripted manner.

Research shows that when novice teachers are easily overwhelmed, they quickly regress back to conventional ways of teaching (Bransford & Stein, 1993; Hogan et al., 2003). To some degree, both Grace and Meredith demonstrated this regression when they planned more ambitious discussions than they were able to facilitate. When Meredith was overwhelmed by working with the "green group," she shifted from the indirect, student-centered lesson she planned to a more teacher-directed lesson in Observation Lesson 1. When Grace felt overwhelmed by working with math content she was less familiar with (Observation Lesson 1) or made last minute changes to the models used in the exploration (Observation Lesson 2), the discussions she facilitated deviated from the mathematical goal. However, Grace's ability to plan and facilitate a productive discussion with students during Observation Lesson 3 and to use a variety of pedagogical tools in a way that demonstrated *connecting to concepts* shows that this regression to conventional teaching methods can rebound quickly when the PST has the opportunity to reflect on their practice with an experienced CE mentor. I elaborate on the role of the CE in the next sections.

Multiple Representations of Tool Use

In order for novices to adequately appropriate and use pedagogical tools, they must understand the rationale behind the tool (Grossman et al., 1999) and have multiple opportunities to apply these tools in practice (Darling-Hammond, 2014; Ghousseini et al., 2015; Lampert et al., 2010). Findings from this study support the need for multiple opportunities for practice with a tool as well as multiple opportunities to see the tool in use by a more experienced mentor. In this study, when PSTs were able to demonstrate tool use that showed *connecting to concepts*, not only did they have multiple opportunities to work with the tool themselves throughout their teacher preparation program, but they frequently observed their CE using the tool to plan or facilitate mathematical discussions with students.

Methods Course and Early Clinical Experiences

Throughout this study, when I asked PSTs about why they chose to use certain pedagogical tools in their planning or facilitation of math discussions with students, the most common response they shared was some variation of, "I used this tool frequently in my methods courses and it really helped me." In this study, when PSTs used tools in ways that demonstrated *connecting to concepts*, they commonly described use of this tool across a range of contexts. For example, when Meredith reflected on her use of the three-read strategy, she described one of her math teacher educators modeling this practice in her first math methods course:

> We'd read the problem, and we'd really break it down, like what was happening. And then actually asking, 'what do you think this problem could be asking?' before giving that information. I just thought that was really cool because it combined a story with the math.

She used this tool to plan and facilitate discussions with students in both of her math methods courses, as well. "I used three-reads when I was leading a discussion with fifth-grade students. They were really able to connect what they read to the math they needed to solve, so that was good for discussion." Even though her Observation Lesson 1 did not go as she planned, Meredith's use of the three-read strategy for both planning and facilitation still demonstrated *connecting to concepts*, which I argue was supported by the multiple opportunities Meredith had to work with this tool in different contexts.

Clinical Educator Model

According to Hill (2012), when CEs facilitate and mentor similar practices to those promoted within the university-based teacher preparation program, PSTs are better able to connect theory to practice. Additionally, Leko and Brownell (2011) found that CEs play a significant role in helping PSTs appropriate tools into practice through their mentorship and modeling. In this study, when CEs used the tools PSTs were familiar with from their methods courses to plan or facilitate mathematical discussions with students, PSTs' use of the tool demonstrated *connecting to concepts*. For example, both Jillian and Grace used the Talk Moves (Chapin et al., 2013) to plan and facilitate discussions with students in ways that demonstrated *connecting to concepts* through their crafting of preplanned and in-the-moment questions and their shared authority with learners. Both PSTs cited using these Talk Moves throughout both of their methods courses and early clinical experiences. They also both credited their CEs for modeling use of these Talk Moves with students during YLI. In addition to using frequent *turn and talks* and using the *agree/disagree and why* talk move, Jillian shared:

> Ms. Becker is always using different Talk Moves to help students be more clear when they are sharing their ideas. Like, when she revoices them, she'll write out literally what they're saying on the board, even if she knows that's not what they mean, to help them be more precise in their explanations.

Because she watched how Ms. Becker used these different Talk Moves with students, Jillian felt more comfortable using them as a tool for planning and facilitating math discussions.

Grace also demonstrated *connecting to concepts* in her use of the LED lesson model for planning mathematical discussions and in her use of the three-read strategy for both planning and facilitation of discussions. Compared to her peers in this study, Grace demonstrated more authentic use of the LED lesson model. While Grace continued to use the LED lesson model because it felt “natural” after using the model throughout her teacher preparation program, she continued to hone her use of the tool through feedback from her CE, Ms. Harkins. As a recent graduate of the same teacher preparation program, Ms. Harkins also used the LED lesson model to plan mathematical discussions with students. Though she admittedly did not use this lesson model to plan all her math lessons, she used it with enough frequency that she felt comfortable adding comments throughout Grace’s plans to help her plan more productive discussions with this tool. Ms. Harkins also used the three-read strategy as a tool to help students make sense of the cognitively demanding math tasks they would discuss. While Grace did have experience using this tool in her methods courses, she opted to use this tool to plan and facilitate mathematical discussions in Observation Lesson 3 primarily because she had observed Ms. Harkins using this method with students and wanted to help students become more accustomed to this routine. Because Ms. Harkins was familiar with these tools from her own practice, she was able to help Grace reflect on her own use of these tools. “My CE does a really good job giving me tips here and there on my plans. And she does a really good job helping me think through decisions in-the-moment.” Grace’s use of these pedagogical tools in a way that connects to the conceptual underpinnings of the tools supports research that finds a high degree of tool appropriation occurs when PSTs have the opportunity to experiment with pedagogical tools acquired in their methods courses in a student teaching setting that promotes a reflective stance (Grossman et al., 2000).

Implications for Practice

Facilitating productive mathematical discussions is an ambitious teaching practice that requires teachers to simultaneously attend to conceptual understanding, mathematical goals of the task, and social management while listening and responding to a variety of student ideas (AMTE, 2017; Ghousseini, 2015; Grossman et al., 2019). According to Thompson and colleagues (2013), novice's appropriation of ambitious teaching practices can be accelerated when they use tools and routines that, "embody pedagogical ideas congruent with ambitious teaching; focus on the relationship between teaching practice and student thinking; can be used directly to plan for, enact, or assess instruction; and are used in collaborative settings across preservice and in-service contexts" (p. 607-609). This study has several implications for teacher education. More specifically, the findings from this study have implications for the way pedagogical tools are introduced and used within mathematics methods courses and how clinical teaching experiences are structured to foster continued use of tools to support PSTs with facilitating productive mathematical discussions. In this section, I will further discuss these implications.

Introduction and Use of Pedagogical Tools in Methods Courses

Supporting Tool Use for Planning Discussions

When examining novice science teachers' use of pedagogical tools to support ambitious teaching practices, Thompson and colleagues (2013) found tool-based routines supported novices' development of ambitious practices when the tool "contained both specified pedagogical and conceptual components, meaning that is suggested a set of specific teaching routines linked to conceptual ideas about an ambitious practice" (p. 605). In this study, multiple PSTs' use of the Talk Moves and the three-read strategy demonstrated *connecting to concepts*.

Both of these tools specify teaching routines that PSTs were easily able to put into practice when planning and facilitating discussions. This indicates that math methods courses introduce PSTs to not only broadly applicable conceptual tools (Grossman et al., 1999), but also practical tools that they can use to easily translate concepts into practice.

Findings from this study indicate that PSTs were more likely to use pedagogical tools to plan and facilitate math discussions in ways that demonstrated *connecting to concepts* when they had multiple opportunities to work with the tool in a variety of contexts. As Meredith described with the three-read strategy, this tool was modeled by both of her math teacher educators. She also had the opportunity to use this tool during peer rehearsals and with elementary students in one of her early clinical placements. Sleep and Boerst (2012) recommend teacher educators (TE) use a combination of hard and soft scaffolds to support PSTs' developing practice with planning and facilitating mathematical discussions. The tools themselves, like the three-read strategy, offer hard scaffolds. Having multiple opportunities to use the tool across different contexts offers different soft scaffolds, such as in-the-moment coaching feedback from the TE and peer feedback during lesson rehearsals. Providing multiple opportunities for PSTs to use the same pedagogical tool throughout their methods courses also helps support appropriating tools in a more flexible way (Grossman et al., 1999; Hammerness et al., 2005).

Research indicates that when students have the opportunity to engage with problem solving tasks and classroom discourse, both self-image and mathematics achievement are positively impacted (Hiebert & Wearne, 1993; Hung, 2015). During initial interviews, all three PSTs in this study described teaching with math discussion as a way to engage all students in developing problem solving and deeper understanding of mathematics concepts. However, in practice, PSTs in this study struggled to apply pedagogical tools in a way that encouraged productive mathematical discourse from all learners. As previously described, both Meredith and Jillian demonstrated *nominal use* of some tools when working with “striving math learners.” Additionally, when asked about support she still needs to effectively plan and facilitate math discussions with students, Grace shared, “I think I would benefit from just having more strategies of what to do in a situation where you're in a class, and there is such a variety of needs.” TEs need to be mindful of the examples of productive mathematical discussions they present in methods courses so PSTs can better understand how to apply tools for math discussions across different settings. When introducing pedagogical tools to help PSTs plan and facilitate math discussions, TEs must also explicitly connect the ways these tools can be used to engage students of all abilities in meaningful discussion. For example, when using the Five Practices, TEs can provide PSTs with student work samples, including developing ideas, to practice selecting and sequencing talk ideas for discussion. PSTs can collaborate with peers and their TE to consider meaningful ways to include developing math ideas in the discussion. Recently, Schmidt and colleagues (2022) identified potential biases teachers can demonstrate when using each of the Five Practices, such as “ignoring unique solution paths to not ‘complicate’ the discussion” (p. 853), something both Grace and Jillian demonstrated in this study. When presenting PSTs with the Five Practices as a tool to plan and facilitate discussions, TEs can concurrently present these biases to encourage PSTs to reflect on their own biases as they plan discussions for different groups of learners. TEs can also support PSTs in developing a more equitable view of mathematical discussions by supporting their ability to select discussion worthy tasks, as described in the next section.

Selecting Discussion Worthy Tasks

Productive mathematical discussions are facilitated around cognitively demanding tasks that align to mathematical goals (Hiebert et al., 1997; Smith & Stein, 1998, 2018). In this study, PSTs' nominal use of pedagogical tools was commonly related to the cognitive demand of the task at the center of the discussion. At times, PSTs planned their discussions around lower level tasks and struggled to apply the tool authentically, as in Meredith's use of the LED lesson model to facilitate discussions. Other times, the PST lowered the cognitive demand of the task by engaging in direct teaching of mathematical procedures before giving students time to explore tasks, as Jillian did in Observation Lessons 1 and 2. Additionally, PSTs struggled to identify discussion worthy tasks for mathematics content they considered highly procedural.

Dreyfus (2004) cautioned that PSTs tend to interpret procedures they learn in methods courses as maxims for teaching action, which can limit their ability to apply these procedures flexibly. When asked to describe the kinds of tasks they felt they could plan productive mathematical discussions around, all three PSTs in this study described story problems that align with one or more of the cognitively guided instruction (CGI) problem types (Carpenter et al., 2014). As such, Grace found it easy to plan and facilitate a LED lesson around a partitive division task. However, when the math topic PSTs planned discussions around did not align with one of the CGI problem types, they found it difficult to make the tools they typically used to plan and facilitate discussions fit the math content.

After discussing the lessons they planned around order of operations, I asked both Grace and Jillian if there were other math topics that they would likewise be challenged to plan discussions around using the LED model. Grace reflected on lessons she and Ms. Harkins recently taught related to graphing on the coordinate plane. "I don't really see how you could plan the Explore of a lesson around that topic. It's not like you're strategizing. You're just finding an answer of what goes where." While Grace did plan a LED lesson around the volume of rectangular prisms, Jillian shared that she did not think this was a topic that could lead to much discussion with students:

> It's not a discussion, in my opinion. I mean, you're finding the volume. There are a couple of different ways you can do it: you could break it apart, you could start with the base. But at the end of the day, you need to find the length, width, and the height to find the volume.

PSTs' responses indicate the need for math teacher educators to help PSTs identify discussion worthy math tasks beyond those that fit the CGI problem types. This would help minimize PSTs development of strict maxims for using tools like the LED lesson model and the Five Practices with only a limited range of math topics and problem types. Math methods courses can help PSTs develop strategies for opening up tasks to make even seemingly procedural math topics more discussion worthy. In addition to the CGI problem types, PSTs can practice using tools to plan and facilitating math discussions using other cognitively demanding structures, such as "Which One Doesn't Belong?" or Open Middle tasks (Kaplinsky, 2019).

Use of Pedagogical Tools in Clinical Practice

In this study, when CEs regularly used the same pedagogical tools to plan and facilitate math discussions that PSTs learned from their math methods courses, PSTs were more likely to demonstrate use of those pedagogical tools in a way that demonstrated *connecting to concepts*. This can be seen in both Grace and Jillian's use of the Talk Moves, as well as in Grace's use of the LED lesson model and the three-read strategy. While mentor teachers are often selected to host PSTs based on their willingness to serve rather than how closely their beliefs and practices align with those of the teacher preparation program (Clarke et al., 2014), PSTs benefit when they are paired with CEs whose practices are more aligned with those used in the teacher preparation program (Hill, 2012). Ideally, teacher preparation programs could strategically select local CEs whose practices with mathematical discussion align with PSTs were prepared for in their methods courses. PST participants were purposefully selected for this study from partnership districts that support the use of discussion for teaching and learning mathematics rather than using highly scripted curricular resources. However, this recruitment criteria did not require that CEs use pedagogical tools for planning and facilitating math discussion that align with those used in PSTs teacher preparation program. As an alumna of the same teacher preparation program, Ms. Harkins was familiar with the tools Grace was using to plan and facilitate math discussions and was therefore able to provide Grace with meaningful mentorship with these tools. When recruiting CEs whose practices are aligned with those of the teacher preparation program is not possible, universities can offer professional development opportunities for potential CEs to help familiarize them with the practices and tools PSTs are familiar with using to teach math.

Clinical experiences must be carefully designed to support PSTs with both enacting ambitious core practices and reflecting on their developing practice. According to Hammerness and others (2005), teachers are better able to enact new practices effectively when they have the opportunity to try out newly learned content-specific strategies and tools, then engage with a learning community of colleagues to continually refine their use of these strategies and tools. For PSTs, their CE mentor can make up part of this reflective learning community, but PSTs learning to use these content specific tools can also benefit from participating in learning communities that include other PSTs and their CE mentors. In this study, Grace and Jillian were assigned to the same grade level in the same school for their YLI. As such, they both participated in a professional learning community (PLC) with Ms. Harkins, Ms. Becker, and the other fifth-grade teachers in the school. As Jillian shared in one of our post-observation interviews, “Ms. Becker obviously shares resources with me, but I also get resources from Grace and her CE, too. Some of it works for me, some of it doesn’t, but I appreciate the support.” Strategically designing clinical experiences so that PSTs are able to continue working in learning communities that include both preservice and inservice teachers can support PSTs reflective practice with tools for mathematical discourse. Additionally, placing groups of PSTs in the same setting would also help facilitate ongoing coaching from a university supervisor that can also support PSTs use of these tools in their practice.

Findings from this study showed that PSTs were more likely to demonstrate *nominal use* of a tool when facilitating mathematical discussions with students identified as performing below grade level standards in mathematics. Teachers’ beliefs about the kinds of math they think their students can and cannot do affects the way they plan instruction (Cavanna et al., 2015). Some studies indicate that knowledge PSTs gain from their CEs and their clinical experience outweighs the knowledge gained from university coursework (Tellez, 2008). As such, it is important that PSTs are placed with CEs who support and model asset-based approaches to teaching mathematics and hold high expectations for all students.

Future Research

This study adds to the research around mathematical discourse in elementary classrooms and teacher education. More specifically, this study adds to the research around preparing teachers to facilitate mathematical discourse. My study contributes to these fields by helping better understand how PSTs use different pedagogical tools to support their planning and facilitation of mathematical discussions with elementary students and to what extent PSTs were able to translate the tools from methods courses into practice. By positioning this study within participants' YLI, I was able to examine PSTs' tool use during a transitional time between formal methods coursework and professional teaching. Future research is needed to enhance understanding of how PSTs and other novice teachers use pedagogical tools to support their work with core practices, such as facilitating class discussions.

In this study, I examined the ways PSTs used a range of pedagogical tools to inform their planning and facilitation of mathematical discussions with elementary students. I intentionally left this study open to various tools in order to better capture the pedagogical tools PSTs would voluntarily continue using even when these tools were not required, as when completing assignments for methods courses. Additionally, many of the tools PSTs used in this study overlapped, such as using the three-read strategy to launch tasks as a part of an LED lesson. As a result, findings from this study are consequently broad so that themes could address the range of tools used. Future studies should more closely examine the use of a specific tool to support planning and facilitating mathematical discussions within the clinical setting. For example, just as Ghousseini and colleagues (2015) explored how a specific question sequence supported PSTs' adaptive practice within the context of lesson rehearsals, future research can explore how enactment tools, like this question sequence, are used by PSTs in clinical settings.

This study was set during the first semester of participants' YLI, which is the semester immediately following completion of required methods courses in their teacher education program. This time period was selected because PSTs are typically introduced to the practical tools of teaching within the methods course setting (Grossman, Compton et al., 2009). Additionally, compared to studies set within methods courses, few studies captured PSTs' planning and facilitation of mathematical discussions within clinical fieldwork. To enhance our understanding of how PSTs use pedagogical tools to support their planning and enactment of mathematical discussions with students during the transition from methods coursework to professional teaching, future studies should span these various settings in an in-depth, longitudinal study from methods coursework through the first several years of professional teaching. A study of this nature could help better understand novice teachers' path towards generativity (Franke et al., 2001) or regression to more conventional teaching methods (Bransford & Stein, 1993; Hogan et al., 2003).

Another reason I chose to position this study during participants' YLI is that novice teachers often cite clinical teaching as the most impactful part of their teacher preparation program (Clarke et al., 2014; Grossman et al., 2012). Though not the focus of this study, my findings suggest different ways that the clinical setting and PSTs' collaboration with CEs impacted PSTs' tool use. To better understand the role clinical settings and CEs play in PSTs' use of pedagogical tools, future studies should include CEs as co-participants in the study. In addition to the sources of data used in this study, data collection could include observation of planning meetings between PSTs and CEs, observations of CEs facilitating mathematical discussions, and analysis of curricular resources available to support mathematical discussion within the clinical setting.

Chapter Summary

In this chapter, I discussed the findings through the lens of the conceptual framework guiding this study as well as existing research on mathematical discourse, preservice teacher development, and clinical education. PSTs' use of pedagogical tools to plan and facilitate mathematical discussions was influenced by how these tools were used in their math methods courses and how these tools were used by their CEs. Additionally, PSTs' tool used was influenced by their beliefs about discussion worthy mathematics, including the types of tasks they can use and who can engage in student-centered discussions. Based on this analysis, I provided several implications for both teacher education and considerations for clinical teaching. I also proposed directions for future research that could help better understand how novice teachers use pedagogical tools to plan and facilitate mathematical discussions from teacher preparation to early professional practice.

Conclusion

This study shows that PSTs' use of pedagogical tools to plan and facilitate mathematical discussions with students can vary as they transition out of methods courses and into classroom teaching. Overall, the PSTs in this study wanted to teach mathematics through discussion and found the tools they used in their methods courses supported their practice in some way. While it is not possible to provide PSTs with pedagogical tools to avoid every challenge of planning and facilitating productive mathematical discussions, tools like the Talk Moves and the Five Practices can help bring structure to ambitious teaching practices (Ghousseini et al., 2015; Sleep, 2012). But these tools can only do so much. Novices need both hard and soft scaffolds to support development of ambitious practices (Sleep & Boerst, 2012). In our final interviews, each PST shared the soft scaffolds they still need to support their practice with mathematical discussions that a tool alone could not provide.

I introduced this study by sharing the experience of a PST rehearsing a math discussion with her peers because I believe it is important to elevate the experiences and voices of the PSTs I include in my research. Accordingly, I want to conclude this study with the words of my participants and what they feel they still need to develop their practice with math discussions. For Meredith, it was practice to build confidence: “I like when my CE pushes me to teach more lessons. And I kind of do self-rehearsals. Just running it through with myself and instilling that confidence. Doing it over and over until I feel good about it." Grace felt reflection was needed to improve her practice: "My CE helps me in the moment, but I don't have a lot of time for reflection on my teaching. To sit down and think through all the things I need to about this one lesson. " And for Jillian, she hopes that social support from mentors and peers will still be there when she finishes her teacher education program: "I need to see how other people approach discussing these strategies with students. And having that peer support to lean on or discuss different ways to help students. I still need that support."

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APPENDIX A: IN-PERSON RECRUITMENT SCRIPT

Elementary Pre-service Teacher Presentation Script: Using Tools to Support Productive Mathematical Discussions

Julie Bacak, a Ph.D. candidate in the Curriculum and Instruction program, is seeking participants for her dissertation study. She is interested in understanding elementary preservice teacher experiences using tools to plan for and enact lessons to facilitate mathematical discussions with elementary learners in clinical placements during the yearlong internship (YLI). You are invited to participate in this qualitative case study. Participants will meet with Ms. Bacak throughout the fall 2022 semester to participate in one-on-one interviews and observations of teaching in your clinical placement. Additionally, participants are asked to share artifacts, such as lesson plans, with the researcher.

Throughout this study, participants will be asked to:

- Engage in a one-on-one interview before the start of YLI teaching to share experiences and knowledge of mathematical discussions. This interview will last between 45 minutes to an hour and will be scheduled at your convenience. This interview will be conducted via Zoom and audio-recorded for transcription purposes.
- Allow the researcher to observe you teaching at least 3 lessons involving mathematical discussion in your clinical placement. These observations will include pre- and post-observation interviews. These interviews will last approximately 20 minutes. Pre- and post-observations can occur in-person or via Zoom. All interviews will be audio recorded. With consent from all participants (clinical educator, elementary students), observations of teaching will be video recorded.
- Engage in a one-on-one interview at the end of the fall 2022 semester to reflect on your experiences using tools to support mathematical discussions. This interview will last between 45 minutes to an hour and will be scheduled at your convenience. This interview will be conducted via Zoom and audio-recorded for transcription purposes.

Your decision regarding participation in this study will have no impact on your performance in this or any other course in your teacher preparation program. The findings from this project will help better understand how PSTs use tools to plan and facilitate discourse and how they feel these tools support their practice. Your insights will help in the completion of dissertation research.

You may benefit from opportunities for reflection throughout this study and an increased focus on incorporating discussion into your mathematics instruction. This study does not involve any risk to you as a participant. Your identity will not be shared in any reporting of the findings.

At this time, I will share a copy of a consent form for you to review. Please make sure you meet the criteria described before giving your consent. To participate in this study, you must:

- Be an elementary education major
- Complete the MAED 3224 course in spring 2022
- Register to begin YLI in fall 2022 in one of the partnership district cohorts

If you agree to participate in this study, please sign and complete the information in the box on the left. Please keep in mind that your participation in this study is completely voluntary and you may withdraw from the study at any time. Additionally, please note that consent from your clinical educator (CE) and school site are also needed. If you consent to participate at this time but the researcher cannot collect consent from your CE, you will no longer be eligible to participate in this study.

If you do not want to participate, please print your name in the box on the right.

If you have any questions or concerns about this request or the study itself, feel free to contact Ms. Bacak at jabacak@uncc.edu or her faculty advisor, Dr. Colonnese at madelyn.colonnese@uncc.edu.

APPENDIX B: CLINICAL EDUCATOR INTERVIEW

Structured interview
Length: 10-15 minutes, audio-recorded
Schedule: Conduct at the beginning of the semester before any observations of teaching; can be in-person on the day of the first observation or via Zoom prior to the first observation date
Purpose: gather background information about the clinical classroom setting to help define case; understand how mathematical discussions are typically used in this classroom community

Participant (Pseudonym):
Date/Time:
School location/Grade level:

- Do you use a set curriculum to plan and teach mathematics?
    - If so, what curriculum do you use?
    - If not, what resources do you typically use to plan and teach mathematics lessons?
- Do you typically teach mathematics with direct instruction or indirect instruction? [ELABORATE]
- Describe how discussion is included in mathematics learning in your classroom.
    - Are discussions typically done in small groups or whole class?
    - How often do students engage in mathematical discussions?
    - Is discussion mostly teacher to student, student to student, or a combination?
    - How do you plan for mathematical discussions?
    - How do you decide which students will share their thinking during a mathematical discussion?
    - Do you have any established norms for class discussion? If so, please share.
- Is there anything else about your classroom setting that you think I should know before I come into begin observations of the preservice teacher leading mathematical discussions with students?
- Do you have any questions for me at this time?

APPENDIX C: OBSERVATION OF CLASSROOM TEACHING

Participant (Pseudonym):
Date/Time:
School location/Grade level:
Number of student participants:
Lesson topic/Task for discussion:

Setting (first observation only):

Descriptive Notes	Reflective Notes
Classroom set-up (i.e., table arrangement) Artifacts related to discourse (ex. Math talk norms, anchor charts, sentence starters):	

Description/Context of observation: (whole group/small group, collaborative/independent work, etc.)

Tools Referenced in Pre-Observation Interview	Tools Observed in Use

Field Notes:

Descriptive Notes	Reflective Notes
Start time: End time:	

Note-taking guide:

- Draw a diagram of the classroom layout and participants to track discourse patterns (who is speaking and to whom? what percentage of the students contribute ideas to the discussion?)
- Timestamp approximately every 5 minutes
- Note the materials/tools PSTs are using
- Capture what the PST is saying/doing during the discussion
- Capture student responses to PST prompts
- With permission, collect photos or copies of artifacts of teaching and learning related to mathematical discussion (lesson plans, graphic organizers, recorded talk ideas)
- Immediately after the observation, review field notes and add anything you recall seeing but did not note in-the-moment; complete reflections in researcher journal

APPENDIX D: INITIAL INTERVIEW PROTOCOL

Semi-structured interview
Length: 45-60 minutes, audio-recorded
Schedule: Conduct between the end of the spring 2022 semester and the beginning of YLI clinical experiences in the fall 2022 semester
Purpose: gather background information about the participant to help define case; understand experiences learning about mathematical discussion in methods coursework; identify tools for discussion shared in methods coursework; understand early experiences planning and facilitating discussions
Steps:

1. Review the expectations for study participation.
2. Review the definitions of terms used in this study: *productive mathematical discussion, practical tools, conceptual tools*
    - *Productive mathematical discussions*: In this study, productive mathematical discussions refer to class discussions that use students' thinking about cognitively demanding tasks in a way that advances the shared understanding of significant mathematical concepts (Lampert, 2001; Stein et al., 2008). Discussions can be whole-class or conducted in small groups.
    - *Practical tools*: Practical tools include "classroom practices, strategies, and resources…that have local and immediate utility" (p. 14). Examples of practical tools include textbooks and curricular resources, talk frames (Casa, 2013), or three-act tasks (Flynn, 2017; Meyer, 2011).
    - *Conceptual tools*: Conceptual tools are "principles, frameworks, and ideas about teaching [and] learning…that teachers use as heuristics to guide decisions about teaching and learning" (p. 14). Conceptual tools can include theories of learning, such as social learning theory, and broadly-defined pedagogical concepts, such as scaffolding, that can be applied across different curricular strands.
3. Set up the recording device.
4. Begin interview.

Participant (pseudonym):
Date/time of Interview
Location of interview:
YLI Placement (if known):
MAED 3222 (instructor and semester):
MAED 3224 (instructor and semester):

Early Experiences with Mathematics Teaching and Learning:

- Tell me a bit about your own experiences learning mathematics.
    - Describe the school(s) you attended.
    - How confident did you feel about your knowledge and abilities in mathematics?
    - Describe the structure of math classes that you can recall.

  - How was discussion incorporated into your mathematics learning experiences?
    - Describe your comfort level contributing to a discussion/answering questions in a math class.
- Describe how and why you chose to become an elementary teacher.

Experiences in Methods Course Instruction:

- Describe your experience in university math methods courses.
  - What are some of your big takeaways about learning to teach elementary mathematics so far?
- What did you learn about facilitating mathematical discussion:
  - Purpose of mathematical discussion/why do we use it?
  - How do teachers plan for productive mathematical discussions with students?
  - How do teachers facilitate productive mathematical discussions with students?
- What tools were you introduced to in your math methods coursework to support you in planning for and facilitating mathematical discussions?
  - Which ones stand out to you? Why?
  - (For each tool shared) What is the purpose of this tool?
  - (For each tool shared) How have you used these tools?
  - (For each tool shared) How has this tool supported your understanding of discussion?
  - (For each tool shared) How has this tool supported your ability to plan for discussion?
  - (For each tool shared) How has this tool supported your ability to facilitate discussion?
  - Any tools shared that you did not find useful? Explain.

Early Clinical Experiences

- Describe an example of mathematical discussion that you observed during your clinical placements from your methods coursework.
  - How did the teacher facilitate discussion?
  - Did you observe the teacher using any tools to facilitate discussion? If so, describe.
- Describe your experiences planning for mathematical discussions in your clinical placements from your methods coursework.
  - What tools, if any, did you use to plan for discussion?
    - How did you use this tool?
    - How did this tool support your planning?
- Describe your experiences leading mathematical discussions in your clinical placements from your methods coursework.
  - What tools, if any, did you use to lead the discussion?
    - How did you use this tool?
    - How did this tool support your facilitation of discussion?

- What questions do you have about mathematical discussions at this time?

APPENDIX E: PRE-OBSERVATION INTERVIEW PROTOCOL

Length: 15-25 minutes, audio-recorded
Conduct within one week prior to observation of teaching
Steps:

1. Review the expectations for study participation.
2. Review the definitions of terms used in this study: *productive mathematical discussion, practical tools, conceptual tools*
3. Set up the recording device.
4. Begin interview.

Participant (pseudonym):
Date/time of Interview
Location of interview:
Date/time of Observation:
Location of observation/grade level:

- Ask the participant to share a copy of their lesson plan and any lesson materials they will be using for the lesson to be observed. Briefly describe the lesson/task you have planned.
  - Topic and standard to be addressed
  - What materials will you use in this lesson to support student discussion?
  - What do expect learners will know and be able to do at the end of this lesson and discussion?
  - How will you check for understanding?
- Describe how you planned for this lesson.
  - How did you consider mathematical discussion in your planning process?
  - What tools did you use to support your planning for discussion?
  - Why did you decide to use these tools?
  - How did you use these tools?
  - How did these tools help you with your lesson planning?
  - What challenges/concerns, if any, did you encounter when planning for discussion?
  - How does your plan allow for equitable discussion of student ideas?
- Are there any tools you plan to use during the discussion or the lesson? If so, describe these tools.
  - What tools do you plan to use? Why?
  - How do you plan to use these tools to facilitate discussion?
- What questions do you have about planning for and facilitating mathematical discussions at this time?

APPENDIX F: POST-OBSERVATION INTERVIEW PROTOCOL

Length: 15-25 minutes, audio-recorded
Conduct the day of the observation
Steps:

1. Review the expectations for study participation.
2. Review the definitions of terms used in this study: *productive mathematical discussion, practical tools, conceptual tools*
3. Set up the recording device.
4. Begin interview.

Participant (pseudonym):
Date/time of Interview
Location of interview:
Date/time of Observation:
Location of observation/grade level:

- Describe how they felt the lesson went, particularly in regard to the discussion students engaged in.
    - Revisit the mathematical goal shared in the pre-observation interview. Do you believe this goal was met? Explain.
    - Describe how you elicited student thinking in the discussion.
    - Describe how you facilitated connections between ideas during the discussion.
    - Describe how you included and honored a range of student ideas in the discussion.
    - Describe a strength of this discussion.
    - Describe a challenge you experienced or a concern you had when facilitating this discussion.
- Review the tools identified in the pre-observation interview.
    - How were these tools used to facilitate discussion?
    - How do you feel these tools supported you in facilitating discussion with your students?
- Review field notes from observation. Ask clarifying questions, as needed, related to notes documented during the observation.
- What changes would you make to the lesson you taught to help facilitate productive mathematical discussion?
- At this time, how do you feel about incorporating discussion into your math instruction?
- What questions do you have about planning for and facilitating mathematical discussions at this time?

APPENDIX G: FINAL INTERVIEW PROTOCOL

Semi-structured interview (questions will be adapted based on previous interviews and observations)
Length: 45-60 minutes, audio-recorded
Schedule: Conduct after all observations of teaching; end of fall 2022 semester (late November/December)
Purpose: summarize and clarify data collected throughout earlier interviews and observations; describe additional tools used not captured in previous interviews and observations; PSTs reflect on their experiences using tools to support mathematical discourse
Steps:

1. Before interview: Review previously collected data to revise questions to fit each participant's experiences and the clarifications needed in the data.
2. Review the expectations for study participation.
3. Review the definitions of terms used in this study: *productive mathematical discussion, practical tools, conceptual tools*
4. Set up the recording device.
5. Begin interview.

Participant (pseudonym):
Date/time of Interview
Location of interview:

Summarize and Clarify (as needed):

- Let's revisit the lesson you taught on (name lesson in question; revisit artifact from observation). Talk to me about:
    - why you chose that tool.
    - how you used that tool.
    - how you feel that tool supported you in incorporating discussion into your mathematics instruction.

Extend:

- Throughout the semester, I have observed you using (list tools) to plan and facilitate mathematical discussions with elementary students. Outside of the lessons I observed, what other tools did you use to plan and facilitate mathematical discussions?
    - Why did you choose to use these tools?
    - How did you use these tools?
    - How did these tools support you?

Reflect:

- Recall throughout the semester you described using the following tools to plan and facilitate discussion (go through tools shared in previous interviews). (For each tool) How did this tool:
    - help you effectively plan for math discussion?
    - facilitate productive mathematical discussion?
    - support you overall in incorporating discussion into your math instruction?
- Which tools do you think you will continue to use in planning for and facilitating mathematical discussions? Explain.

- During our last interview, you described feeling (share information from the interview) about incorporating discussion into your mathematics instruction. Now that your first semester of YLI has come to an end, how do you feel about incorporating discussion into your math instruction?
    - Feelings about planning for discussion?
    - Feelings about facilitating discussion?
- What questions do you still have about incorporating discussion into your math instruction?
- What support do you feel you still need to effectively incorporate discussion into your math instruction?
- What other thoughts would you like to share as we conclude this interview?