IMPROVED VEHICLE SUSPENSION CONTROL ARM MODEL USING EXPERIMENTAL DEFLECTION MEASUREMENTS

by

Gaurav Sandesh Doshi

A thesis submitted to the faculty of
The University of North Carolina at Charlotte
in partial fulfillment of the requirements
for the degree of Master of Science in
Mechanical Engineering

Charlotte

2015

Approved by:	
Dr. Peter Tkacik	
Dr. Mesbah Uddin	
Dr. Pussall Kaonini	

©2015 Gaurav Sandesh Doshi ALL RIGHTS RESERVED

ABSTRACT

GAURAV SANDESH DOSHI. Improved vehicle suspension control arm model using experimental deflections. (Under the direction of DR. PETER TKACIK)

The suspension system plays an important role in automobile performance, safety and noise level reduction. In a suspension system, a control arm is a crucial component. The control arms have different shapes depending upon the model and the purpose of the vehicle. The control arm having a wishbone or 'A' shape carries the maximum load from the shocks helping the wheels to maintain their geometry with the road and thus resulting in a steady ride and good handling. After absorbing the impact that the road has on the wheels the control arm either absorbs it or deflects it to the coils of the suspension depending on the type of suspension. The control arm being a vital component of the vehicle suspension system, it becomes necessary to study the mechanical behavior under varying load conditions.

This thesis describes an improved vehicle suspension control arm finite element model supported by deflections recorded from experiments. It also presents the strength and compliance analysis of a suspension control arm using the load cell output from an Instron 4400- Universal Testing Machine (UTM). Initially stress analysis is done on the control arm model in SolidWorks to get a measure of the stresses and deflections induced in a control arm. Accordingly the test rig is also designed in SolidWorks which helps model the control arm on the UTM. The experimental test rig was manufactured and made ready for testing. The control arm is then fixed in the setup at two inboard ends and the outboard ball joint has an attachment for application of the load through combination of mechanical connectors. The load is applied to the ball joint of the control arm through a tapered hole

in the attachment to the load cell. Displacement results for the load applied are recorded by UTM. Data from UTM is transferred to the external computer through use of General Purpose Instrument Bus (GPIB). The graphs for Force-Elongation and Stress-Strain are plotted and presented from the results. These results are in accordance with the theoretical curves. The strength and the compliance for the control arm is then analyzed from the plotted curves and recorded values. Internal deflections of the control arm (metal deflection not including the deflection of the rubber bushing) are measured with a dial indicator mounted on the metal at one end of the control arm and measured at the other end for loading in both directions i.e. longitudinal and lateral directions. Also the displacements from FEA model of control arm are measured. Careful improvements or mesh refinements to the model were made until the experimental results matched the FEA results to a predefined accuracy in order to get an improved vehicle suspension control arm model. Lastly, the model was done for the lower front control arm of an Acura MDX sport utility vehicle (SUV) for the 2007-2015 model years.

DEDICATION

I would like to dedicate this document to my parents, Jyoti and Sandesh Doshi and my sister Shweta Doshi for their love and support.

ACKNOWLEDGMENTS

I thank my advisor, Dr. Peter Tkacik for his valuable guidance with vision and patience. He made this project a great learning experience for me. Without his support, this project would not have been possible.

I would like to thank Dr. Mesbah Uddin and Dr. Russell Keanini for their help and suggestions, and also for being on the committee for this project.

I would also like to thank Mr. Luke Woroneicki, Mr. Vernon Hutchins and Mr. Kile Stinson for their help in getting me acquainted with the necessary equipment and also for their help in acquiring and validating the results. I would also like to thank Mr. Franklin Green for helping me with my doubts regarding the setup of the Instron machine.

I would like to thank my fellow students, Swapnil Patil, Jugal Popat, Harsh Patel, and Akshar Patel for their help and unique inputs time to time.

I would also like to thank Jui Bhagat and Mayur Dudhani for always being there for me.

Lastly I would like to thank all my relatives, friends, and everyone else for supporting me, and for making my stay at UNCC memorable.

45

45

45

TABLE OF CONTENTS

	LIST OF FIGURES				
CHAPTER 1: INTRODUCTION					
	1.1.	Introduction	1		
	1.2.	Scope of Study	5		
	1.3.	Overview of Project Work	5		
	CHAPTER 2	:: LITERATURE REVIEW	8		
	2.1	Introduction	8		
	2.2	History of Vehicle Suspension System	8		
	2.3	Types of Vehicle Suspension System	10		
	10				
2.3.2 Rear Suspension					
	2.4	Control Arm	15		
	2.5	Solid Modelling	16		
	2.6	Tensile Testing on the Instron	18		
	2.7	Stress-Strain Curves	20		
CHAPTER 3: METHODOLOGY					
	3.1	Introduction to Instron – Universal Testing Machine	24		
	3.2	Design and Manufacturing of a Test Rig	28		
	3.3	Procedure	43		

CHAPTER 4: RESULTS AND CONCLUSIONS

Longitudinal Loading

Results

4.1.1.

4.1

			viii
	4.1.2.	Lateral Loading	49
	4.1.3.	Displacement Results Using Clamping Dial Indicator	51
	4.1.4.	FEA Displacements	51
4.2	Conclu	asions	53
4.3	Future	Scope	55
REFEREN	CES		57

LIST OF FIGURES

FIGURE 1.1: Typical MacPherson strut front suspension	2
FIGURE 1.2: SAE axis convention	2
FIGURE 1.3: Typical Universal Testing Machine (UTM)	4
FIGURE 1.4: Instron machine arrangement for tension and compression	6
FIGURE 2.1: Single pivot front axle	8
FIGURE 2.2: Macpherson strut	12
FIGURE 2.3: Double wishbone suspension	13
FIGURE 2.4: Multi-link rear suspension	14
FIGURE 2.5: Acura MDX lower control arm	15
FIGURE 2.6: 2-dimensional drawing	17
FIGURE 2.7: Extrusion boss and extrusion cut	18
FIGURE 2.8: Fillet	18
FIGURE 2.9: Typical tensile test specimen	19
FIGURE 2.10: Typical force-elongation curve	21
FIGURE 2.11: Typical stress-strain curve	21
FIGURE 2.12: Stress-strain curve	23
FIGURE 3.1: Functional block diagram	25
FIGURE 3.2: Front panel	27
FIGURE 3.3: 3-dimensional drawing of control arm	29
FIGURE 3.4: Lateral loading	30
FIGURE 3.5: Longitudinal loading	31
FIGURE 3.6: Base plate with Instron bolt pattern	32

FIGURE 3.7: End-plate mounting	33
FIGURE 3.8: Bushing mounting (one of two mountings)	34
FIGURE 3.9: Assembly for Fy (lateral) loading	35
FIGURE 3.10: Ball-joint connector 2-dimensional	36
FIGURE 3.11: Ball-joint connector 3-dimensional	36
FIGURE 3.12: UTM connector 1	37
FIGURE 3.13: UTM connector 2	38
FIGURE 3.14: UTM connector	38
FIGURE 3.15: Assembly for Fx (longitudinal) loading	40
FIGURE 3.16: Setup for lateral loading	41
FIGURE 3.17: Setup for longitudinal loading	42
FIGURE 3.18: Displacement measurement with clamping dial indicator	44
FIGURE 4.1: Sample data for longitudinal testing	46
FIGURE 4.2: Load-displacement curve	47
FIGURE 4.3: Stress-strain curve for longitudinal loading	47
FIGURE 4.4: Stress-strain for test rig and rubber bushing (left end of Figure 4-3)	48
FIGURE 4.5: Stiffness curve (right end of Figure 4-3)	48
FIGURE 4.6: Stress-strain curve for lateral loading	49
FIGURE 4.7: Stress-strain for test rig and rubber bushing (left end of Figure 4-6)	50
FIGURE 4.8: Stiffness curve (right end of Figure 4-6)	50
FIGURE 4.9: Displacement for longitudinal loading	51
FIGURE 4.10: Displacement results for lateral loading	51
FIGURE 4.11: Deflections for longitudinal loading	52

	xi
FIGURE 4.12: Displacement for lateral loading	53
FIGURE 4.13: Results using AVE	56

CHAPTER 1: INTRODUCTION

1.1 Introduction

Suspension systems have been widely used from the horse-drawn carriage to the modern vehicles today. Evolution in the automobile industry brought the need for better suspension systems. The suspension system is responsible for supporting the vehicle, driving comfort, handling qualities and safety of the vehicle. This is mainly because the suspension system carries the weight of the vehicle body and helps in diminishing all the forces transferred to the chassis from the impact of road irregularities. It connects the wheel and the assembly of the axle to the main body. Springs, dampers and rubber mountings are used to separate the connections of vehicle suspension components. The suspension system is present at both the sides, front and rear end of the vehicle. Conventional suspension systems come with springs attached to the rigid beam axle which is commonly seen in the front axle of commercial vehicles. This is known as beam axle suspension system. In the independent suspension system each wheel is free to move in vertical direction with little effect on the suspension on the opposite end of the axle. When considering suspension design, disturbances can be classified into two categories, road and load disturbances. Disturbances having characteristics of high magnitude and low frequency (such as on hills) and with small magnitude and high frequency (such as road roughness) would be considered road disturbances. Load disturbances are due to variation in the load such as

loads induced by braking, acceleration and cornering. Therefore a good suspension will be the one resisting all the disturbances or minimizing transfer to the chassis. [1]

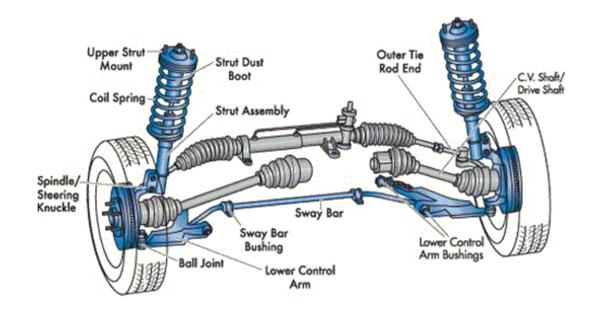


FIGURE 1.1: Typical MacPherson strut front suspension [2]

FIGURE 1.1 is a typical MacPherson strut front suspension. It is also very close to the front suspension of Acura MDX. The analysis in this work was done on an Acura MDX right front lower control arm.

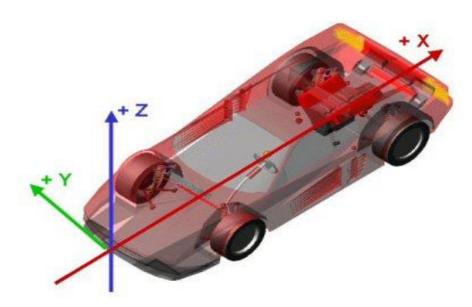


FIGURE 1.2: SAE axis convention

According to Thomas Gillespie, the primary functions of any Suspension system are: [3]

- 1. To provide vertical compliance and allow the wheels to follow the uneven surface without transferring the vibrations from irregularities on the road to the chassis.
- 2. To maintain the camber angle of the wheels and steering axis inclination.
- To react to the control forces produced by the tires such as longitudinal forces, Fx
 (acceleration and braking), lateral forces Fy (cornering), and braking and driving
 torques My.
- 4. To-resist or minimize the roll and pitch of the chassis while travelling on the uneven road surface or while cornering.
- 5. To maintain the contact between tires and the road surface with minimal load variations.

The suspension system being an important assembly of the vehicle, it is necessary to study it in detail. The suspension system includes various suspension components to make the whole system work properly. One of the main components is the Control Arm.

The control arm is a crucial component of the suspension system. Also known as the A-arm due to its shape, it is a mechanical link transferring forces and controlling tire geometry. The Macpherson strut has a single lower arm and coil spring and shock absorber assembly on the top while an alternative design, the Double wishbone has upper and lower control arms with a king pin. The control arm plays the main role in stability of the vehicle that is maintaining proper alignment between the motion of the wheels and the chassis or body frame of the vehicle. It is made from high quality and durable materials and is a necessary component in every vehicle running on the road. Failure of the control arm would

cause unwanted movement of the tire which may lead to road accidents. So it is necessary to make sure the control arm installed is in proper working condition and gets regular inspection. Therefore, it is necessary to do proper analysis of the control arm to decrease the wear and tear on it. [4] [5] [6]

The Instron 4400 series- Universal Testing Machine (UTM) is used in the thesis for load testing. The typical setup of UTM is shown in FIGURE 1.3. This is a material testing instrument but is designed to test variety of systems. It consists of a load frame in which there is space for testing material to be mounted and applies tension or compression load. It also includes a control console which helps in test setup, providing calibration and test operating controls. The details of UTM will be discussed later.



FIGURE 1.3: Typical Universal Testing Machine (UTM)

1.2 Scope of Study

The main focus in this project will be in improving the vehicle suspension control arm FEA model using the deflections recorded from the experiments. It also describes the strength and compliance analysis on Lower Control arm using the Instron Model 4400 Universal Testing Machine. The scope of the study is as follows:

- i. Modelling of control arm in SolidWorks
- ii. Modelling of the Test-rig components in the SolidWorks.
- iii. Manufacturing of the components.
- iv. Analysis of the Control arm on Instron- Universal testing machine.
- v. Refinement of the Control arm FEA model using the experimental deflection measurements.

1.3 Overview of Project Work

The front lower control arm of Acura MDX vehicle suspension system is used for all the modelling and experimental work. For doing strength and compliance analysis on the control arm, the testing was to be done on the UTM. The UTM had no arrangements to hold the control arm or to make tests on control arm. So there was a necessity of designing a fixture/test rig which could hold the control arm. Before designing the test rig, a few factors were to be considered such as where the load was going to be applied on the control arm and which ends would be fixed. Considering the location and application of control arm, the load from the tires is applied through the ball joint. Since the ball-joint can resist only forces and not moments, only X, Y, Z forces need to be considered. Since the control arm pivots around the bushings and allows free motion of the ball joint in the Z direction (vertical), the ball joint only sees forces in the X direction (Longitudinal or braking) and Y

direction (Lateral or cornering). There was a necessity in designing a test rig in such a way that the load was applied on the ball joint in these two directions, Fx (longitudinal) and Fy (lateral) as in respective of the vehicle and keeping other two ends of the control arm fixed.

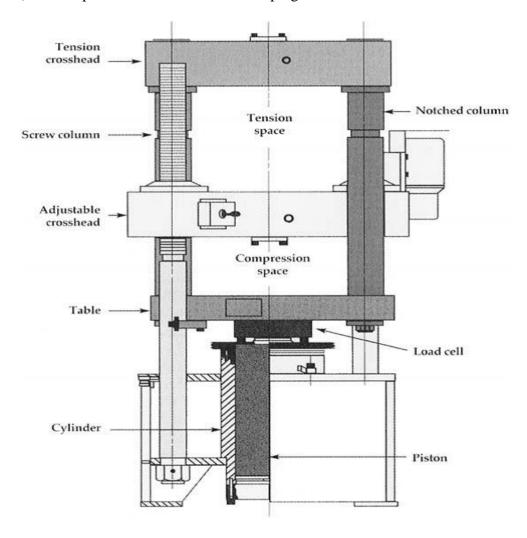


FIGURE 1.4: Instron machine arrangement for tension and compression

SolidWorks was used to design the test rig. Initially the replica of the control arm was modelled in SolidWorks. After this, various designs were made for test rig (UTM attachments) of the control arm. They were modelled in the SolidWorks and were constantly modified to make the project cost efficient and to avoid manufacturing

difficulties. The test rig was designed considering mainly the X and Y direction of the load application on the ball joint of the control arm.

After the designing of test rig, manufacturing was done in the Kulwicky machine shop and all the parts were assembled and welded as designed in the Solidworks.

CHAPTER 2: LITERATURE SURVEY

2.1 Introduction

This chapter begins by explaining the need for development of vehicle suspension systems and different stages of the suspension system from the suspension system used in horse driven carriage. It then explains the different types of suspension systems used in vehicles today. The chapter introduces the control arm and also includes introduction to the solid modelling. Lastly, the tensile testing and Stress - Strain curve are discussed in this chapter.

2.2 History of Vehicle Suspension System

Decades ago, the suspension of the vehicle was considered to be a single solid axle which was pivoted in the center and was attached to the horse in the front.

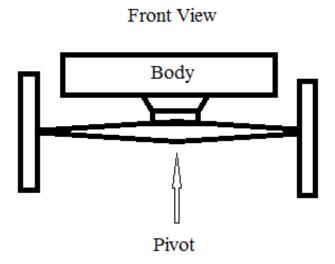


FIGURE 2.1: Single pivot front axle

The horse supplied the power and direction to vehicle. The suspension system consisted of four flexible leaf-springs placed in four corners. At that time instead of worrying about the suspension problems or about the comfort of the ride, the drivers had bigger problems to worry about. The drivers had to think about keeping the vehicle stable over big stones and potholes on the road. Applying a brake was also the problem as any imbalance in the side-side braking would affect the vehicle stability. The single pivoted solid axle was then replaced by the Ackerman axle in which the axle was fixed and the steering took place through two pivot points, one on each end of the axle connected by two kingpins. The evolution of the automobile suspension took place continuously from Ackerman axle to Elliot axle, then to Reverse Elliot axle. [7]

Over the years it was learned that if the amount of unsprung weight was large, in other words the weight which was not supported by the springs (wheels, axle, suspension parts, etc.) was large, the ride smoothness would be lower. Straight axles were too heavy and they contributed a lot to the unsprung weight. Therefore due to harsh ride and poor handling characteristics there was a need for new design, which led to modern automobiles with complex control algorithms. [7]

The suspension systems is mainly made of all the components that makes connection between the vehicle body and the tires. Every suspension system is designed to provide 1) ride-comfort, 2) road-holding and 3) proper handling. The three principles of any suspension system that is required to satisfy proper springing action of motor vehicle are [8]:

Reducing the weight of the components which receive the road shocks to minimum
or reduction of the un-sprung weight.

- 2. Reducing the roll and pitch of the vehicle with the help of suitable design.
- 3. To absorb all the impacts with the help of springing device.

2.3 Types of Vehicle Suspension System

The suspension system is the key component of the vehicle which helps in isolating the automobile body from all the shocks induced from the irregular road surface. It also guides the wheels when the vehicle is in motion.

2.3.1 Front Suspensions

The front suspensions are classified into dependent suspensions and independent suspensions. Dependent suspensions are the one which were used in previous vehicles and very rarely used in modern vehicles because of various disadvantages. The most common type of dependent front suspension is the solid beam axle. Due to the solid beam axle, the mating wheels were dependent on each other and hence the name. The main disadvantage of the solid beam axle is the large un-sprung weight. As the dependent suspension maintains a constant camber and also offers high ground clearance, it is more common on off-road application vehicles and on vehicles that need to carry large loads. [5]

In an independent suspension wheel is supported independently by a coil or spring. It helps in providing better ride comfort and steering. Some of the advantages of independent suspension are [8]:

- 1. Improved steering and stability.
- 2. Increased riding comfort.
- 3. Un-sprung weight is reduced.
- 4. Improved braking.

The common types of front independent suspension are the single control arm suspension also known as 'Macpherson strut' and the 'Double wishbone suspension' also known as 'Double A- arm suspension'. Many of the independent suspensions were tried but discarded later due to many reasons. Macpherson strut and the double wishbone suspension are only two concepts in various forms which are used as front independent suspension.

The suspension with single control arm is better known as 'Macpherson strut' type suspension. Earle S. Macpherson originally designed the Macpherson strut for all the four wheels of the vehicle but nowadays it's mostly used for the front suspension only. The Macpherson strut is a combination of shock absorber and the concentric coil spring integrating into a single-piece design. The single lower control arm controls the lateral and longitudinal location of the wheel. Spindle load and road shocks are directly transferred to the spring keeping the chassis away from the shocks and this results in smoother ride. [9]

Advantages of Macpherson strut:

- 1. Simplicity in Design.
- 2. Low Manufacturing cost.



FIGURE 2.2: Macpherson strut [9]

Disadvantages of Macpherson strut:

- Difficulty in vertical movement of the wheel without altering camber angle change, sideways movement or both.
- 2. Does not give good handling as compared to the double wishbone or multi-link suspension, as it gives less freedom for camber change.

Double wishbone suspension or double A-arm suspension is an independent suspension which consists of two wishbone shaped arms to locate the wheel. Each wishbone arm has a ball-joint at the knuckle or kingpin and is joined at two points on the chassis.



FIGURE 2.3: Double wishbone suspension [9]

This double arm design allows to greater camber correction and other parameters. The lower arm takes the maximum load and the upper arm is usually shorter to increase camber correction.

Advantages of double wishbone suspension:

- 1. Decreased steering vibration.
- 2. Easy to work out the effect of each joint resulting in good kinematics and therefore the wheel motion can be optimized.
- 3. Easy to control the parameters such as camber angle.

Disadvantages of double wishbone suspension:

- 1. Space consuming and complex in design.
- 2. Due to larger number of components, it might take longer for service.

2.3.2 Rear Suspensions

Same as the front suspensions, rear suspension system is also of dependent and independent types. Dependent rear suspension used in the automobile are twist beam, live

and dead axles. Twist beam suspension is inexpensive and compact and mainly used in small cars where package space is limited. Live rear axles are typically used in large pickup trucks or SUV's due to its high unsprung mass. It often consists of longitudinal leaf spring which attaches the axle to the chassis.

The most common types of rear independent suspension are the Chapman strut and the multi-link. The Chapman strut was developed by Colin Chapman famous for his 'Lotus' formula one team and his 'Lotus' sports cars. The Chapman strut is essentially a non-steering rear suspension version of the ubiquitous front MacPherson strut suspension. The multi-link is complicated collection of links which allows camber and toe control with suspension deflections but is an interference geometry and therefore requires rubber bushings to avoid binding up. Due to the greater flexibility, multi-link suspension are the most used rear suspension. [5]



FIGURE 2.4: Multi-link rear suspension [9]

2.4 Control Arm

Most suspension systems mentioned above have a part common in them and that is the control arm. It is the crucial component in the suspension system and is necessary on any vehicle for stability of the vehicle. As mentioned previously the control arm plays an important role by maintaining proper alignment between the motion of wheels and the chassis or body frame of the vehicle.

As the tire goes up and down in the chassis (jounce and rebound) the control arm defines the path of the ball-joint and thus controls the camber. On the front MacPherson strut suspension, the steering is defined by the tie rod (connected to the steering rack) and the vertical load is held by the spring on the strut. A lateral torsion (anti roll) bar may attach to increase axle roll stiffness.

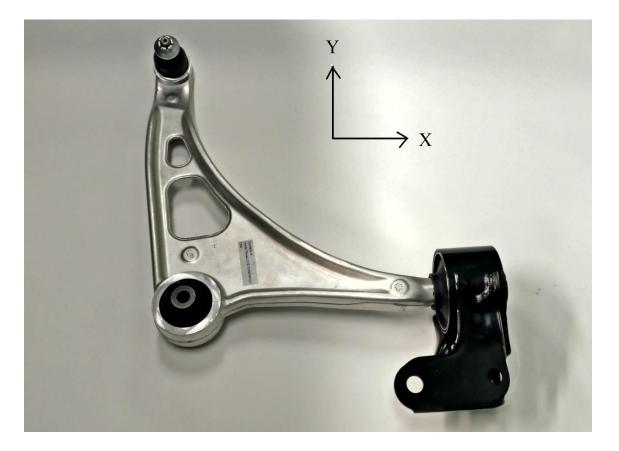


FIGURE 2.5: Acura MDX lower control arm

As shown in FIGURE the control arm has a metal body with ball joint to attach to the strut bottom and bushing and metal attachment to fix on chassis. The control arm bushing consists of inner metal sleeve covered by the rubber bushing and outer metal sleeve. The bushing acts as a hinged dampener, cushioning the suspension and thus providing a quiet ride. [4]. The control arm design is of three types based on how the ball joint is attached to the control arm and are as follows,

- Unitized control arm: The ball joint for this arm is built-in and the whole control arm comes in a single piece.
- 2. Press in: This type of control arm is mostly used in heavy vehicles. The ball joint is removable for this control arm via an interference press fit.
- 3. Bolt in: Allows attachment of the ball joint via bolts.

2.5 Solid Modeling

One of the crucial steps in the design process is the solid modelling. The 2-dimensional drawings takes a lot of time in drawing sketches and also they do not provide proper visualization. Solid modelling allows you to create the 3-dimensional drawing without actually making the prototype and also provides the basis for manufacturing any part and assembly over a wide range of applications. The 3-dimensional modelling software used for this project was SOLIDWORKS 2014. It is the product of Dassault Systems, who also made other software programs like CATIA which is similar to SOLIDWORKS.

Designing a three-dimensional model in SOLIDWORKS includes initial drawing of a 2-dimensional sketch and then using the options such as extrude or extrude-cut in the third direction. This will convert the 2-dimensional drawing into 3-dimensional. There are

various functions like Fillet, Chamfer, Shell, Hole wizard, Mirror, Draft etc. As shown in FIGURE 2.6 the 2-dimensional sketch is then extruded to 3-dimensional sketch and later using extruded cut, the circular cut is made as shown in FIGURE 2.7. The shell function is used to give specified thickness to the 3-dimensional block. Also fillet is used to give a rounded corner of various radius to the sharp edges. The fillet function is used to give rounded corner to the sharp edges as shown in FIGURE 2.8.

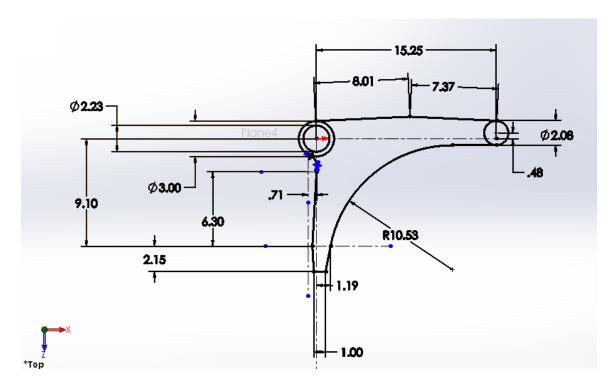


FIGURE 2.6: 2-dimensional drawing

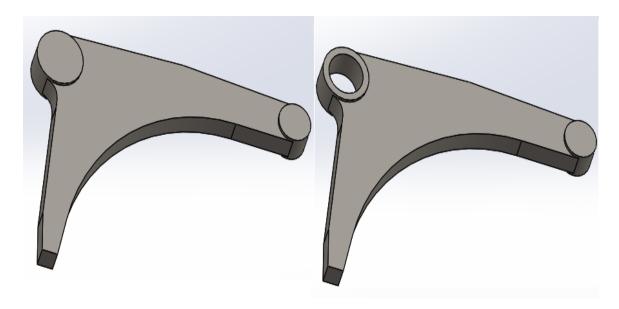


FIGURE 2.7: Extrusion boss and extrusion cut

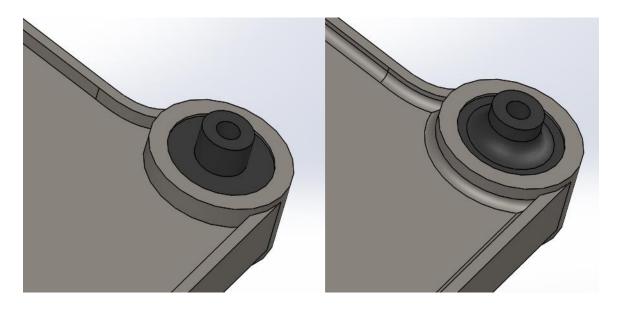


FIGURE 2.8: Fillet

2.6 Tensile Testing on the Instron

Tensile testing (also called as tension testing) is one important material science test.

In this testing the sample may be subjected to controlled tension force until failure occurs.

The results are then used in selecting materials and dimensions for various engineering applications and for quality control. Ultimate tensile strength and maximum elongation can

be measured directly from the tensile test of a special test shape. Once these measurements are done, Young's modulus, Poisson's ratio and yield strength can be determined. For Isotropic materials uniaxial tensile testing is used to obtain the mechanical properties while biaxial tensile testing is used for anisotropic materials such as composite materials and textiles.

The strength of the material is always considered an important factor. The strength considered can be in terms of stress required to cause the necessary plastic deformation or maximum stress the material can withstand without failure. These factors are then used in engineering design to develop safety factors. Apart from the strength, ductility of the material is another important factor which is sometimes considered in the engineering design. It is measure of how much the material deforms before it fails. If the material has low ductility, it offers less resistance to the fracture and is considered brittle.

Usually the tensile specimen is a standardized sample with two shoulders and a gage in between. The shoulders are large enough to be properly gripped. The gage section should have smaller cross-section so that the deformation and failure can occur in this area and gage length should be relatively large as compared to the gage diameter. The typical tensile specimen is shown in the FIGURE below.

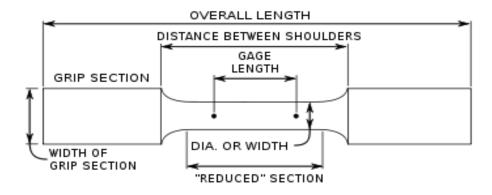


FIGURE 2.9: Typical tensile test specimen

The testing involves of placing the specimen in the universal testing machine and applying controlled tension until the specimen fails. During this process, the elongation is recorded against the applied force. The elongation data is then manipulated so that it does not remain specific to the geometry of the specimen. The elongation data is used in calculation of the engineering strain, ε , using the equation

$$\varepsilon = \frac{\Delta L}{Lo} = \frac{L - Lo}{Lo}$$

Where ΔL is the change in gage length, Lo the initial gage length and L the final gage length. The applied force is used to calculate the engineering stress, σ , using the equation,

$$\sigma = \frac{F}{A}$$

Where F is the tensile force applied and A is the nominal cross-section of the specimen.

The Instron machine does these calculations and plots the graph for stress-strain curve.

2.7 Stress-Strain Curves.

Stress-Strain curves show important mechanical characteristics of the material. A tensile test includes mounting of specimen in the load frame and subjecting it to constant tensile force until failure. The force applied and the elongation of the specimen is recorded. This test gives complete tensile profile of the specimen and helps in plotting of stress-strain curves.

The data recorded from the tensile test gives the force-elongation curve. The force-elongation curve is converted to stress-strain curve which is identical in shape to the force-elongation curve. The advantage of dealing with stress-strain curve is that this curve is independent of the specimen dimensions. An example of both the curves are shown FIGURE 2.10 and FIGURE 2.11.

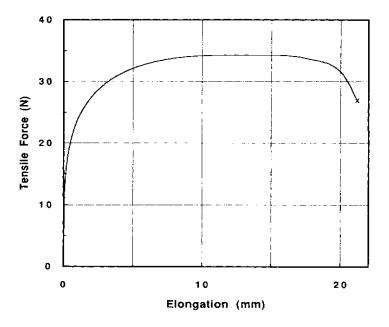


FIGURE 2.10: Typical force-elongation curve [10]

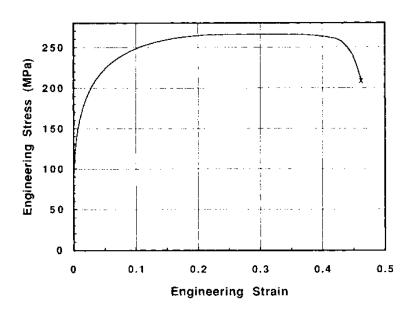


FIGURE 2.11: Typical stress-strain curve [10]

When the solid material is subjected to small stresses, it obeys the relationship defined by Hooke's law. That is the relationship between applied force and elongation the specimen exhibits is linear or the ratio of stress to strain is constant.

$$E = \frac{\sigma}{\varepsilon}$$

'E' slope of the line where stress is proportional to the strain and is called Modulus of Elasticity or Young's Modulus. The Modulus of Elasticity is a measure of stiffness of the material but this applies only when the material is loaded in the linear region of the curve. In this linear region, irrespective of the load applied on the material the material will return to its exact same condition when the load is removed. After this linear region, Hooke's law is not obeyed and there will be some permanent deformation in the material. This point is called the Elastic limit. The material will react plastically beyond this point. The value of the stress applied at this point is called the 'yield strength' (Sy). The maximum load the material sustains during the test to failure is the ultimate tensile strength (Sut). This may or may not be equal to the break point and depends upon the material being used for the testing. All the parameters explained above are shown in the FIGURE below for a typical stress-strain curve.

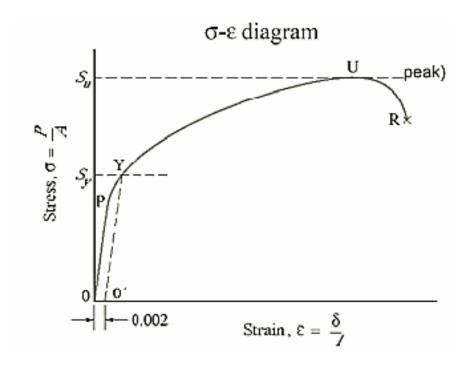


FIGURE 2.12: Stress-strain curve

As shown in FIGURE 2.11, Yield point is represented by 'Y', Ultimate tensile strength is represented by 'U' and the fracture point is shown by 'R'.

Stiffness is defined as the measure of the resistance offered by elastic body to deformation. The inverse of stiffness is compliance.

CHAPTER 3: EXPERIMENTAL SETUP

3.1 Introduction to Instron - Universal Testing Machine

The Instron 4400 R series Universal Testing Machine (UTM) is used in this project. It is a material testing machine and wide range of materials can be tested on this machine. It consists of a load frame in which our control arm will be mounted and tested for tension or compression. It also consists of a control console which helps with the calibration, test operating controls and also the test setup.

Our Instron series 4400 R has a front panel which offers complete interaction with the machine through numeric keypads, buttons, switches and a digital display to show all the real time values of load, extension and strain. The complete machine can be operated from the control panel. When the test is being conducted all the instantaneous values for load, extension and strain are tracked and can be viewed on the front panel. The break and peak values which get stored can be viewed after the test. Several output devices are available for viewing this results. Basically the mechanical system is made up of two major subsystems:

- Crosshead drive and control system which applies compressive or tensile loading to the specimen.
- 2. The load weighing system which measures the loading of a specimen.

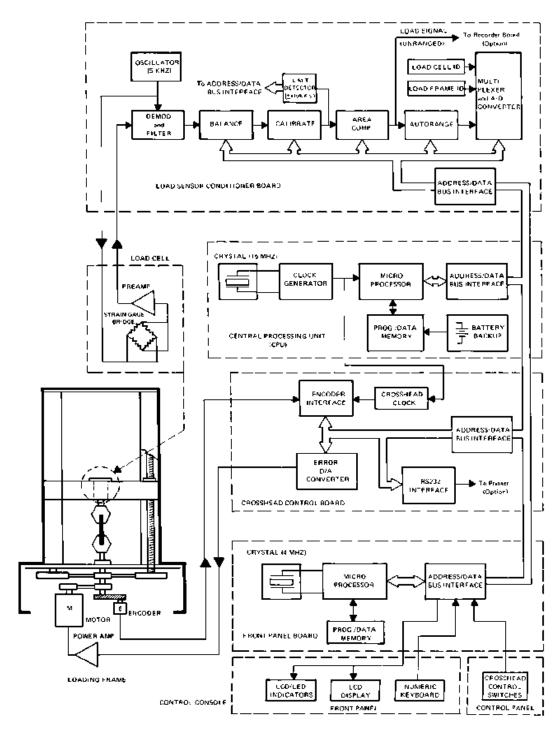


FIGURE 3.1: Functional block diagram [11]

The interfacing of these two systems and overall signal flow within the instrument is shown in the block diagram above. All the operations are controlled by the central processing unit (CPU).

The crosshead control board shown allows to program the speed of the crosshead and also provides digital control of the crosshead position.

The load sensor conditioner board is controlled by CPU and it allows calibration and balance procedures to be performed automatically once initiated at the front panel. It also gives digital load signal output which can be recorded through various types of readout devices.

The main components of the control console front panel are shown in the FIGURE 3.2. It consists of four main sections [11]

1. Main Panel Section

The main panel section consists of test function entry keys, a numeric keypad and status indicators on the left. Units used currently are also displayed in this area. The panel provides the functions such as load cell calibration, strain transducer calibration, and crosshead speed selection, area compensation, testing area definition, printer operation and IEEE bus enable/disable options. The IEEE-488 interface is General Purpose Instrument Bus (GPIB) which allows control of all the procedures through a computer.

2. Display Panel Section

The display panel section includes of all the LCD displays and control keys at the top to display the readings for the load, strain and extension. It helps to keep a track during the test. All the peak and break values are saved which can be viewed anytime during the test or after the test. Indicator displays the current selection.

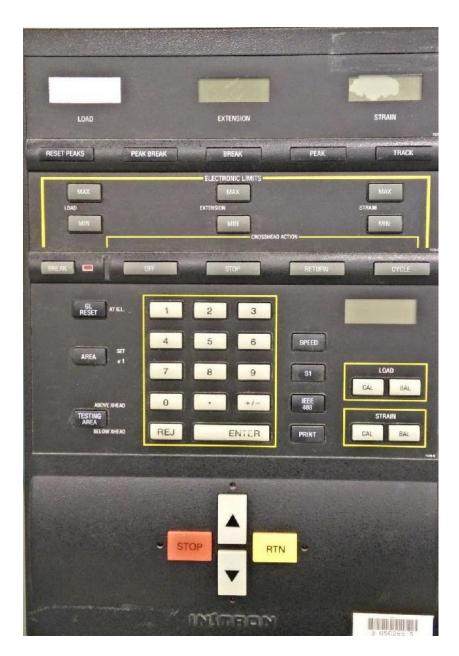


FIGURE 3.2: Front panel

3. Limits Panel section

Some action need to be taken when the system reaches the maximum and minimum values of load, extension or strain as this helps in protecting the specimen from any damage and also to protect the grips and the test fixtures from crosshead over travel and possible collision. The limit panel section allows us to assign what system does

when this maximum or minimum values are reached. Limits should always be set and system should be assigned a suitable action before starting of each test. The limits can be set through the numeric keypad using the LCD display on the main panel section. The actions which can be assigned are STOP, RETURN, CYCLE and OFF. The STOP will stop the crosshead at current position, RETURN will return it to gauge length, CYCLE will change direction at the limit and OFF will not take any action. The status indicator is lit to show the current action assigned and whenever the STOP or RETURN action occurs the related lamp starts flashing.

4. Crosshead control section

The Crosshead control section allows to control the crosshead manually. The STOP, UP, DOWN, RETURN keys present on bottom of the front panel are indicators besides it which is lit when the specific key is in action. The STOP button will completely stop the crosshead, UP and DOWN will move the crosshead up and down at programmed speed and RETURN will return the crosshead at a speed which increases exponentially to the maximum and then decreases exponentially to stop crosshead at gauge length.

3.2 Design and Manufacturing of the Control Arm Test Rig

The primary objective of designing a test rig was to make the control arm available for testing by making suitable attachment to the Instron machine. This was done by fixing the control arm on the base plate and then making the ball joint available for application of the load. The load is applied to the ball joint with an attachment to the load cell. The secondary objective was to make design which will be easy for manufacturing and cost efficient.

The design process started with modelling of control arm in SOLIDWORKS with approximate dimensions (not accurate). This was to get a proper image of the test rig design. The control arm used for testing is Acura MDX right front suspension lower control arm for model years 2007-2014. The modelled control arm was used as base for designing the components of test rig. The 3 dimensional model of control arm is show below.

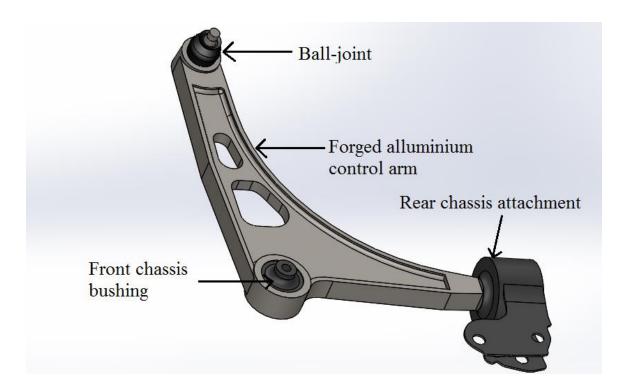


FIGURE 3.3: 3-dimensional drawing of control arm

The actual test rig was designed after completing the design of the control arm. The purpose of the design was to hold the control arm on the Instron and make the ball joint available for loading. An initial design was made but then discarded due to the machining difficulties. Later it was decided to divide the test rig into 4 different parts such as ball joint assembly, base plates, bushing mountings to hold the bushing and the end-plate mounting that can hold the metal plates of the control arm.

Before starting to design anything, the main point to be considered was the direction of the loading. When the control arm fits on the vehicle, the load is applied on the ball joint in Fy (lateral) and Fx (longitudinal) directions in respective of the vehicle. The direction of the loading is represented in the figure below. The force Fy accounts for the lateral loading as shown in FIGURE 3.4 and similarly the force F_x for the longitudinal loading as shown in FIGURE 3.5.

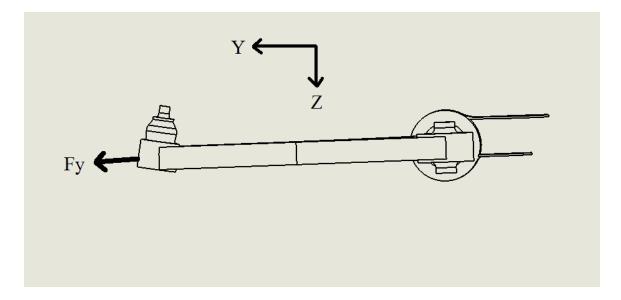


FIGURE 3.4: Lateral loading

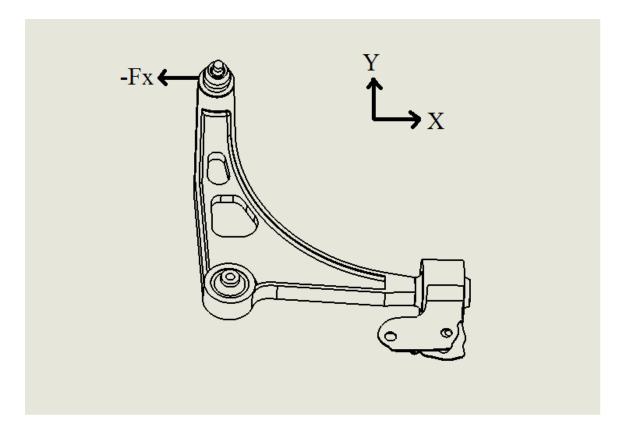


FIGURE 3.5: Longitudinal loading

Following is the design for Fx (longitudinal) and Fy (lateral) loading:

4.1.1 Fy (lateral) Loading:

For designing of base plate, a steel channel or C-beam was used. Channel was chosen over the rectangular and hollow tubing. It gave the required strength keeping the weight of the beam low and also provided welding advantages in further designing. The bolt pattern was matched to the adjustable crosshead on the Instron machine. Material selected was AISI 1018 low carbon steel due to its toughness, strength, cost, excellent weld-ability. The large section of channel was cut using the plasma cutter initially and then on the horizontal band-saw for accuracy. The bolt pattern from Universal testing machine was measured and holes were drilled in accordingly using the drill-press. The 3-dimensional drawing is shown in FIGURE 3.6.

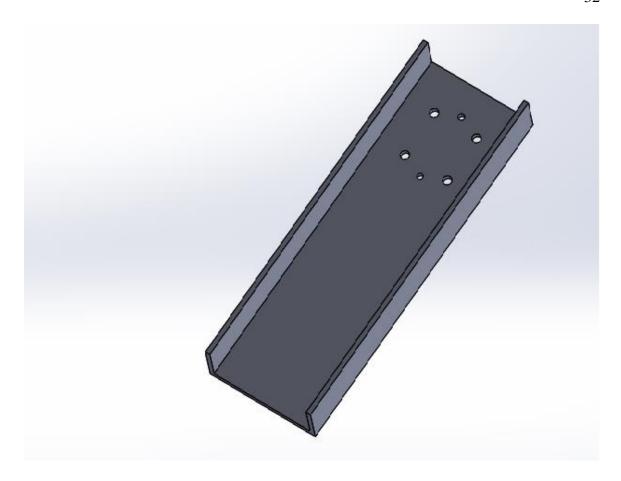


FIGURE 3.6: Base plate with Instron bolt pattern

After this the dimensions between the flanges of the control arm rear bushing was measured. Then end-plate mounting was designed in SolidWorks with exact dimension which will fit in this and hold the control arm. Material selected was again AISI 1018 low carbon steel due to same advantages as before. The end-plate mounting was manufactured on a milling machine. After manufacturing the end-plate mounting, the hole pattern from the control arm plates was transferred to the mounting and drilled accordingly. The 3-dimensional drawing for the designed end-plate mounting is shown in FIGURE 3.7.

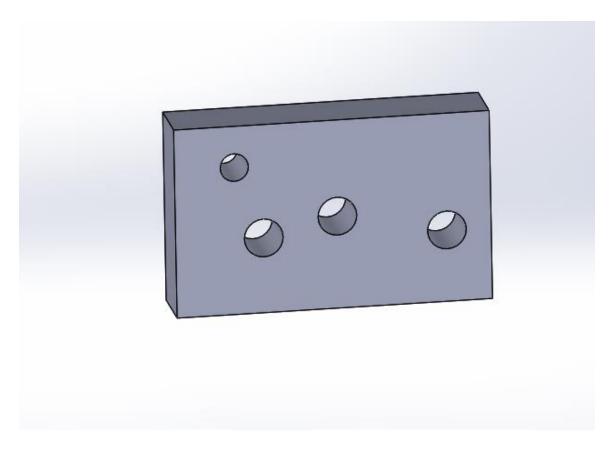


FIGURE 3.7: End-plate mounting

The Instron bolt pattern drilled on the channel, the control arm front bushing and the ball joint were supposed to be exactly one above other. This resulted in a fitment problem with the mounting for the front bushing attachment.

The front bushing mounting was designed as a wide clevis with two identical plates. A long bolt went through one plate, some spacers, the front control arm bushing, more spacers, and finally the other plate. The 3-dimesional drawing of one plate is shown in FIGURE 3.8.

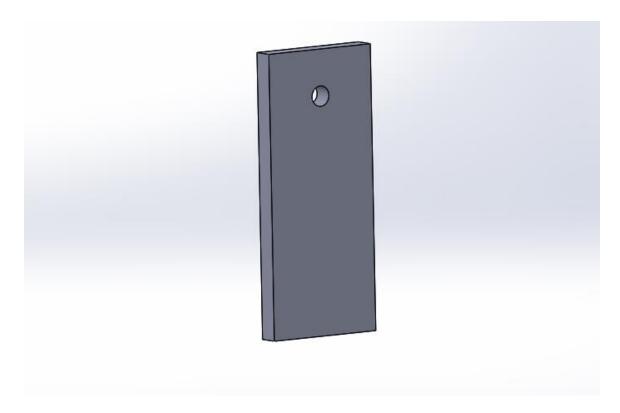


FIGURE 3.8: Bushing mounting (one of two mountings)

The bushing mounting was manufactured on vertical band-saw and drill press. The 3-dimensional drawing is shown above. The position of the bushing mounting to hold the control arm bushing was to be decided taking into consideration the bolt pattern on the c-beam, the bolt head thickness and the weld thickness. After designing and manufacturing above components, the end-plate mounting and the bushing mounting were MIG welded to the channel. The 3-dimensional drawing of the assembly for all the 3 parts mentioned above is shown below.

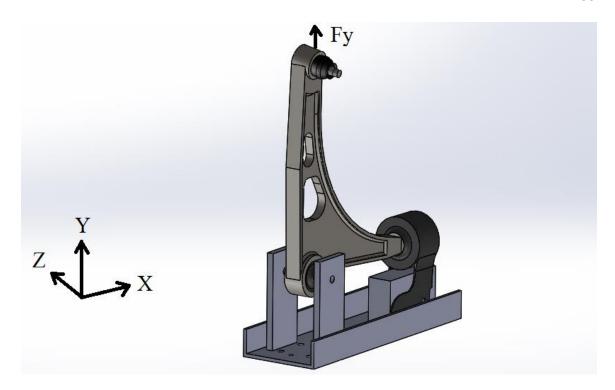


FIGURE 3.9: Assembly for Fy (lateral) loading

The most difficult part in designing the test rig was the design of the ball-joint assembly to hold the ball joint to the control arm. It is subjected to direct loading from the load cell on the Instron machine. The axis of the ball joint was inclined at an angle from horizontal. Also the ball joint had a taper of 1:8. So it was necessary to design something that will hold the ball joint and exert a vertical load on it. A ball joint connector was designed with 1:8 taper in it which will hold the ball joint with the castellated nut. After fitting the ball-joint connector onto the ball-joint, it has to remain inclined to the vertical axis due to inclination of the ball-joint.

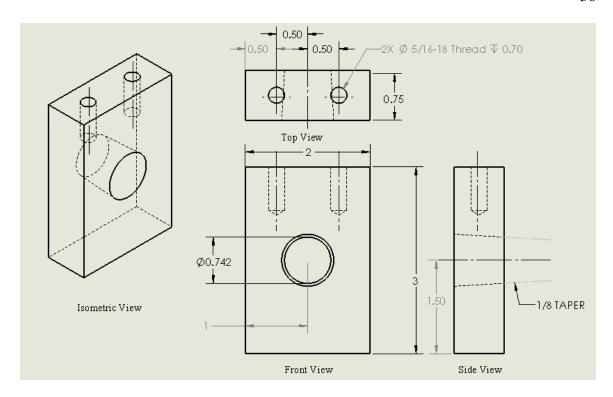


FIGURE 3.10: Ball-joint connector 2-dimensional

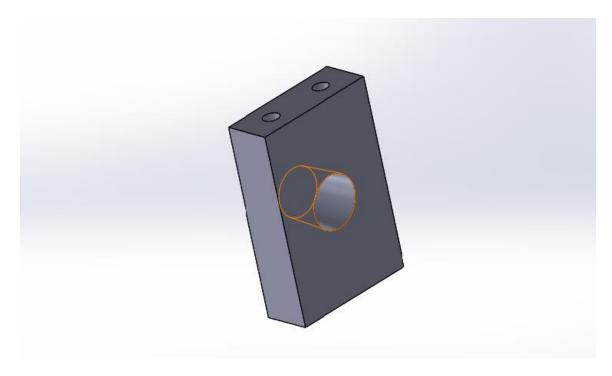


FIGURE 3.11: Ball-joint connector 3-dimensional

The ball-joint connector attaches to the UTM connector (two parts). The UTM connector consists of thick plate (FIGURE 3.12) welded to a circular rod (FIGURE 3.13). Dowel hole will be drilled in the circular rod. The UTM connector will be bolted to the ball-joint connector on one end and will be fixed to the grip attached to the load cell on the Instron.

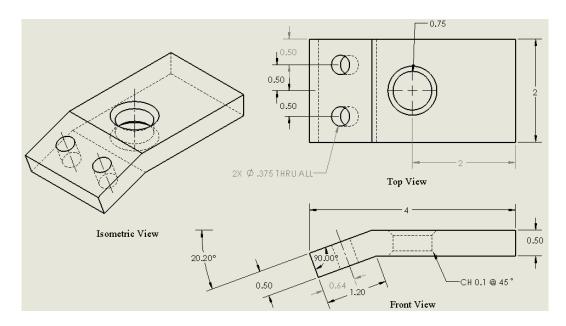


FIGURE 3.12: UTM connector 1

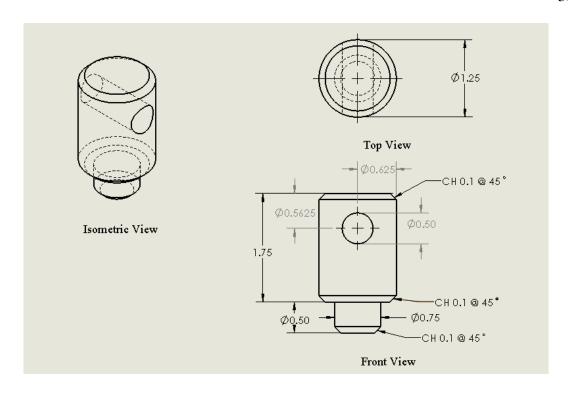


FIGURE 3.13: UTM connector 2

The 3-dimensional drawing for UTM connector is shown below.

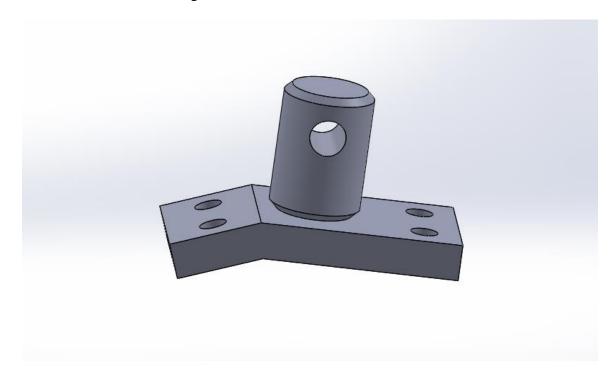


FIGURE 3.14: UTM connector

As seen the UTM connector has arrangements for bolts on both sides. This will be for attaching to the ball joint connector for loading in lateral and longitudinal directions. The dowel pin will fix this UTM connector to the Instron grip which is attached to the load cell. Material used for the UTM connector is 4130 Alloy steel due to its high strength, good weld-ability and easy machinability. It is manufactured using milling machine.

4.1.2 Fx (longitudinal) loading:

The above arrangement was designed and manufactured for Fy (lateral) loading. Minor changes were to be made for Fx (longitudinal) loading. Another base plate was designed and welded to the initial base plate as shown in the 3 dimensional FIGURE below. A pair of diagonal brackets were added to support the two channels for extra strength. Also little changes were made in ball joint assembly for the longitudinal loading.

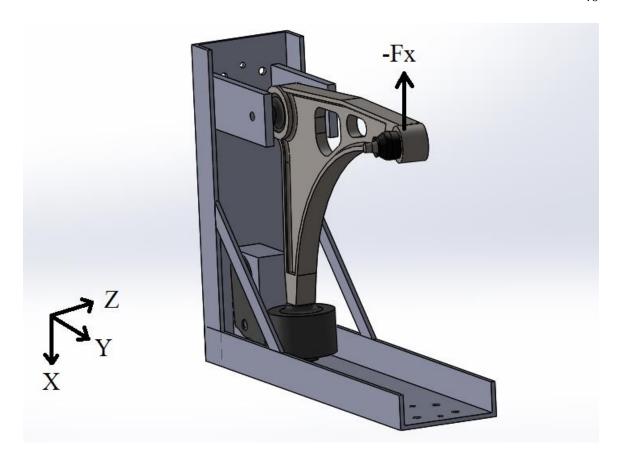


FIGURE 3.15: Assembly for Fx (longitudinal) loading

The parts were manufactured and welded as shown in the FIGURE 3.15 and the assembly setup for longitudinal loading was ready.

4.1.3 The complete assembly:

The setup was ready for longitudinal and lateral testing. The images of the test rig setup is shown below in FIGURE 3.16 and FIGURE 3.17 for lateral loading and longitudinal loading.



FIGURE 3.16: Setup for lateral loading

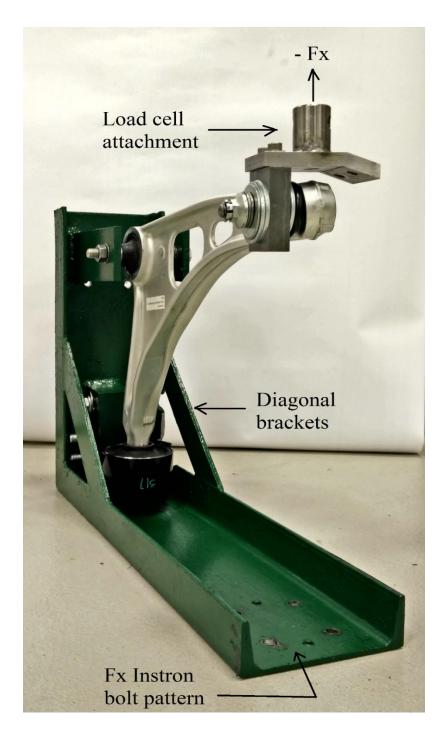


FIGURE 3.17: Setup for longitudinal loading

3.3 Procedure

- 1. The control arm was fixed in the test rig with proper bolts and nuts to hold it firmly on the test rig during the whole test.
- 2. The test rig was bolted to the Instron machine without fixing the ball-joint assembly.
- 3. Instron machine was switched-on on low gear as all the testing was to be done at low speeds like 0.02 inch/min.
- 4. Load calibration was done in order to make sure the load displayed on the front panel is adjusted to zero before starting the test.
- 5. The load cell was fixed on the Instron machine and then the ball joint assembly was fixed on the load cell with the help of the grip.
- 6. G.L reset key is pressed to enter the current spacing between the grips into the memory.
- 7. The specific test method was selected depending upon the lateral testing or longitudinal testing and the file name is given to save the output of the test conducted.
- 8. After hitting the 'Test now' button the test starts and the crosshead starts to move (slowly).
- 9. The elongation is recorded for the force applied and is saved in the memory. Stress and strain values are automatically calculated.
- 10. The force-elongation and stress-strain curves are plotted from the data.
- 11. Displacements were also measured at the ends of the control arm with the help of a clamping dial indicator and recorded every 100 lbf. The setup for the displacement measurement for lateral and longitudinal loading are shown below.

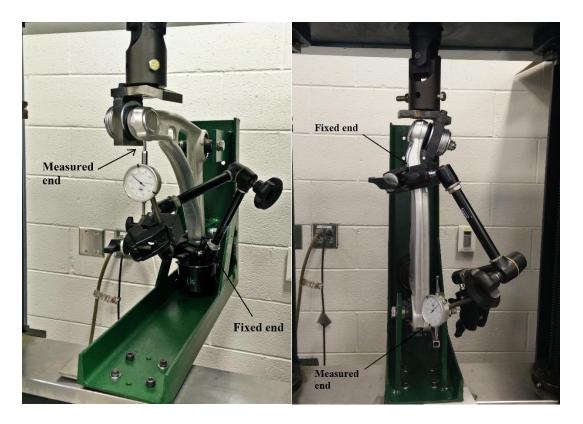


FIGURE 3.18: Displacement measurement with clamping dial indicator

- 12. Displacements form SolidWorks were analyzed and compared with the displacement values from the experiments.
- 13. Chapter 4 discusses the results and conclusions from all above experiments.

CHAPTER 4: RESULTS AND CONCLUSIONS

4.1 Results

The test rig setup and the force and displacement measurements was discussed in chapter 3. For this project the crosshead speed is set to 0.1 inch/min. That is the extension of the control arm will take place at 0.1 inch/min. The Instron series IX software is used to record the elongation for the force applied. The output of the program is saved to Series IX data output file (*.MRD and *.txt). This consists of all the input parameter's set for the test, maximum load and displacement, no. of data points and the values for load and elongation.

All the measurements are done in English units i.e. load is measured in pound-force (lbf) and elongation is measured in inches (in).

4.1.1. Longitudinal Fx Loading

After setting up all the initial parameters the test was started. The load was gradually increased and the displacement was recorded at specific intervals. The load was increased till 3000 lbf and the test was aborted. All the values for load and displacement were obtained and values for stress and strain were calculated. Stress-strain graphs were plotted.

Some values for raw data for longitudinal testing and calculated stress and strain values are shown below in FIGURE 4.1.

Data F	oints	Displacement (in)	Load (lbf)	Stress (psi)	Strain (in/in)
79	91	1.31651	2892.87988	2314.3039	0.057239565
79	92	1.31816	2899.31982	2319.4559	0.057311304
79	93	1.31985	2905.77002	2324.616	0.057384783
79	94	1.3215	2912.20996	2329.768	0.057456522
79	95	1.32319	2919.72998	2335.784	0.05753
79	96	1.32484	2925.09985	2340.0799	0.057601739
79	97	1.32649	2930.45996	2344.368	0.057673478
79	98	1.32818	2936.90991	2349.5279	0.057746957
79	99	1.32983	2943.34985	2354.6799	0.057818696
80	00	1.33152	2949.79004	2359.832	0.057892174
80)1	1.33317	2956.23999	2364.992	0.057963913
80)2	1.33486	2962.67993	2370.1439	0.058037391
80	03	1.33651	2968.05005	2374.44	0.05810913
80)4	1.33816	2974.48999	2379.592	0.05818087
80)5	1.33985	2980.92993	2384.7439	0.058254348
80	06	1.3415	2987.37988	2389.9039	0.058326087
80)7	1.34319	2992.75	2394.2	0.058399565
80	08	1.34484	2999.18994	2399.352	0.058471304
80)9	1.34652	3005.62988	2404.5039	0.058544348

FIGURE 4.1: Sample data for longitudinal testing

FIGURE 4.2 and FIGURE 4.3 below shows the load-displacement curve and stress-strain curve respectively for the data recorded by the Instron machine through the series IX software for longitudinal testing.

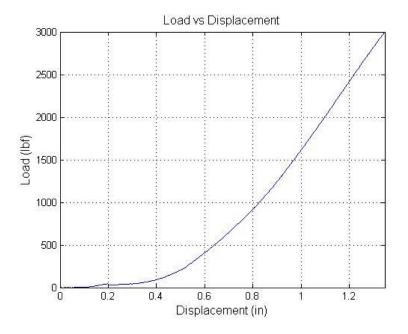


FIGURE 4.2: Load-displacement curve

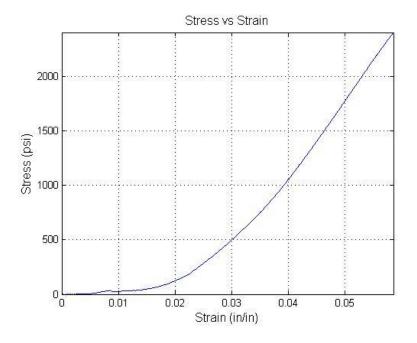


FIGURE 4.3: Stress-strain curve for longitudinal loading

As seen in the FIGURE 4.3, initially there is lot of variation in the stress-strain curve before the curve gets linear and it is shown clearly in FIGURE 4.4. The graph shown in FIGURE 4.4 can again be divided into two parts. First shows the compliance offered due to the test-

rig and second part shows the compliance offered by rubber bushing. The stiffness offered by the control arm is seen in the following curve as shown in FIGURE 4.5.

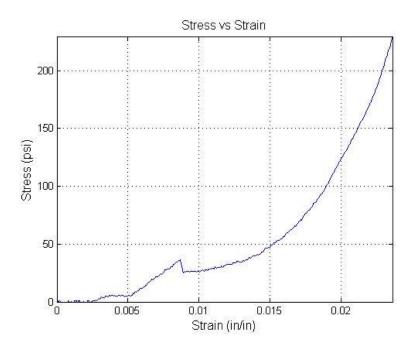


FIGURE 4.4: Stress-strain for test rig and rubber bushing (left end of Figure 4-3)

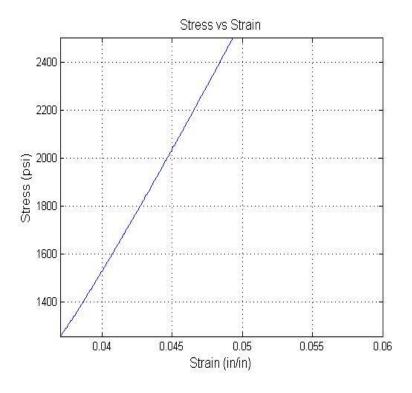


FIGURE 4.5: Stiffness curve (right end of Figure 4-3)

4.1.2. Lateral Fy Loading

Similar test setup was done and test were conducted for Lateral loading. The load was gradually increased up till 1800 lbf and then stopped. The results are given below.

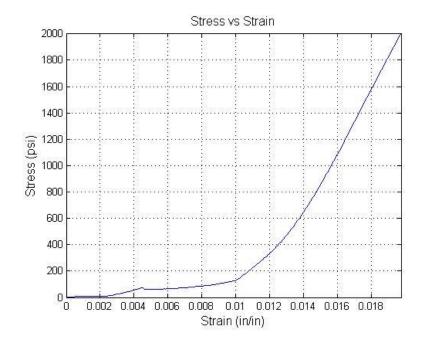


FIGURE 4.6: Stress-strain curve for lateral loading

The stress-strain curve for the lateral testing is shown above in FIGURE 4.6. The compliance offered by the test rig and the rubber bushing is shown in FIGURE 4.7 and the stiffness offered by the control arm is shown in FIGURE 4.8.

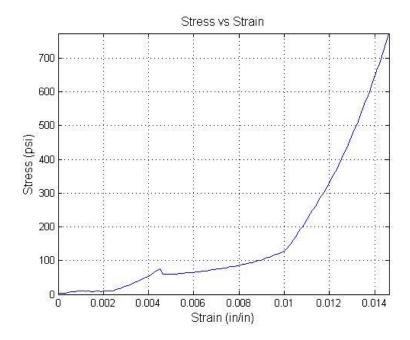


FIGURE 4.7: Stress-strain for test rig and rubber bushing (left end of Figure 4-6)

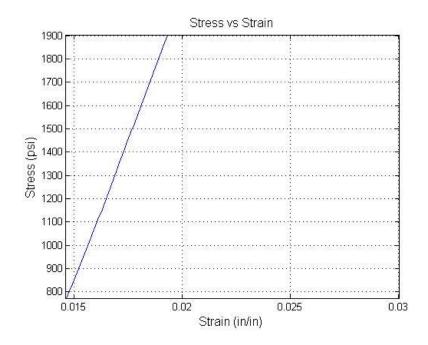


FIGURE 4.8: Stiffness curve (right end of Figure 4-6)

4.1.3. Displacement Results Using Clamping Dial Indicator

The main aim was to find the displacement at the end of control arm (metal displacement and not due to bushing deformation) near the ball-joint holder. The dial indicator was clamped to measure the displacement as shown in the setup previously. Displacement for every 100 lbf of load was manually recorded. Similar tests were done for both longitudinal and lateral testing and the results are shown below.

Load (lbf)	Displacement (in)
100	0.01
500	0.066
1000	0.126
1500	0.175
2000	0.247
2500	0.298
3000	0.359
3500	0.423

FIGURE 4.9: Displacement for longitudinal loading

Load (lbf)	Displacement (in)
100	0
1500	0.001
2400	0.002
3200	0.003
3800	0.004

FIGURE 4.10: Displacement results for lateral loading

4.1.4 FEA Displacements

After the experiments, displacement analysis is done on the FEA control arm model in SolidWorks. There was a difference between the experimental displacements and the FEA model displacements for both lateral and longitudinal testing. Iterations were made to find out the reason for discrepancy. The control arm model was measured more accurately at the critical points. These changes were applied to the FEA control arm model. After

applying the changes, displacement analysis was done again in SolidWorks. The results obtained from SolidWorks are shown below.

In the FEA model, the unloaded dial indicator clamp arm was included to more accurately account for the control arm bending (not just stretching) under load. Without this, the readings were off by tenfold.

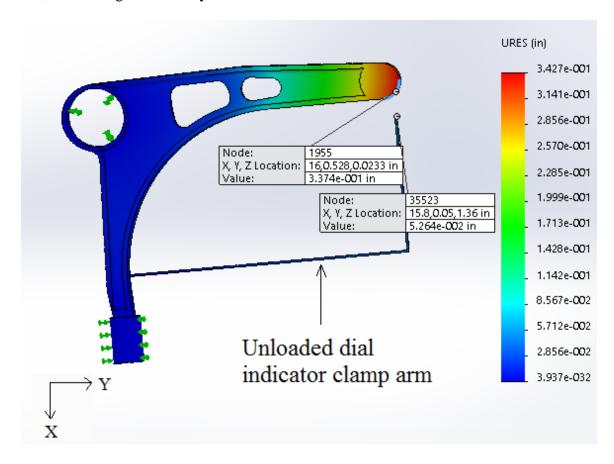


FIGURE 4.11: Deflections for longitudinal loading

As seen in the figure above, the net displacement at the edge where readings were taken through clamping dial indicator for 3500 lbf is 0.39004 inches. The experimental measurements showed displacement about 0.423 inches. As the error is less than 10 %, this gives the better control arm model.

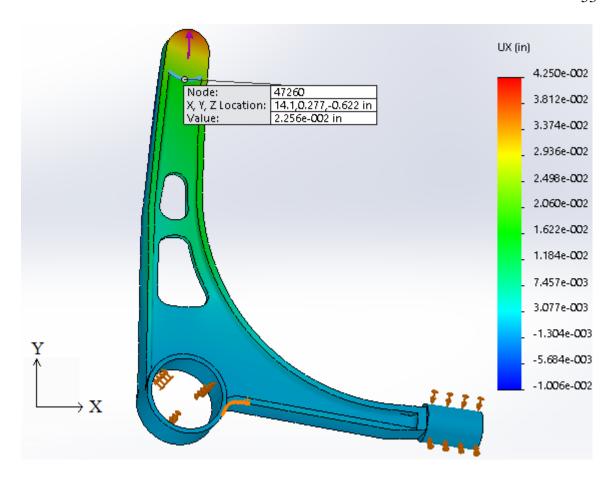


FIGURE 4.12: Displacement for lateral loading

As seen in the figure above, the displacement measured was 0.00225 inches and the displacement measured from the experiments was 0.004 inches at 3800 lbf.

With such less difference in the values for displacements, this is definitely a better FEA control arm model which can be used further for different analysis.

4.2 Conclusions

The main aim of the thesis was to get an improved FEA control arm model and do strength and compliance analysis by measuring displacement for every pound-force applied. The test rig designed proved to be accurate for measuring displacements. Several conclusions can be drawn from initial testing of the setup. It was difficult to get the exact initial parameters such as gauge-length for the testing due to complex shape of the control

arm. This difficulty can be eliminated by using newer versions of Instron which comes with better software which also allows the use of advanced video extensometers.

After the test was done, stress and strain values were calculated for each test and stress-strain curve was plotted. The graphs plotted are in accordance with the graphs obtained from the Honda Research Institute. The initial part of the stress-strain curve shows the compliance offered by the test rig. This can be reduced by designing more accurate test rig with zero compliance to get more correct results. The later part of the stress-strain curve shows compliance offered by the control arm bushing and then the stiffness offered by the control arm. When the stress-strain curves are compared for longitudinal and lateral testing, it is observed that control arm is much stiffer when mounted in lateral direction than the longitudinal direction. Compliance offered by control arm bushing in longitudinal direction is more than in lateral direction. Stiffness is more in lateral direction mainly due to the shape of the control arm. Also the control arm bushing is more active in longitudinal loading due to its location and direction of the loading.

The displacements measured at the ends of 'A' shaped control arm from the experiments were compared with the displacements obtained from the FEA model of the control arm. The control arm model not being accurate to the actual dimensions led to discrepancy in both values. Iterations were performed to find the source of the discrepancy. The control arm was measured more accurately at the critical points. It was found out that dimensions of the spline profile on the control arm beneath the ball-joint were more crucial and made a lot of difference in the results. The dimensions of the FEA model were modified to get the approximately same displacement results with less than 10% error and thus obtaining an improved control arm model based on the experimental deflections. It was

also clear that for future measurements, the FEA model should include an accurate model of the dial indicator fixture.

The test rig designed proved to be perfect for doing all the above non-destructive test. Changes can be made in the test rig by adding mechanical linkages and making a similar one available for testing for all types of 'A' shaped control arms.

4.3 Future Scope

A repeat test of this approach on a different control arm should be done using what was learned in optimizing our FEA model. After modelling, a new experimental test should be performed and the results should be compared to the FEA model. These independent results will clarify robustness of the approach. That is, if they match well, we have demonstrated the efficacy of this modelling protocol.

The Instron 4400 is very old (1970's) and doesn't allow use of the new technology available. Also the software used was outdated. The later versions of this software helps in determining the results much more accurately as they allow using of Advanced Video Extensometers (AVE). They help in determining the accurate stress-strain profile for application of the load. DIC Replay software package comes with AVE which helps in analyzing stress and displacement of a component more easily and correctly. It also helps in visualizing and detecting cracks that are not visible by eye. Therefore use of newer Instron machine with better software can give results free from error. As shown in the FIGURE 4.13 the stress concentration can be directly obtained on the screen.

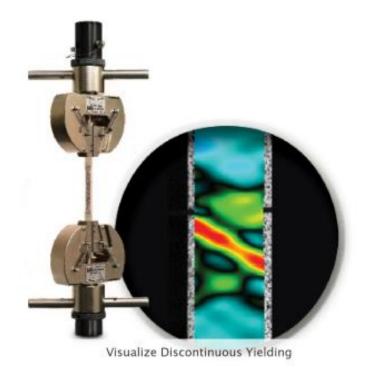


FIGURE 4.13: Results using AVE [12]

The test rig designed was useful in determining the results for Acura MDX front suspension lower control arm. The design for the test rig can be modified, and it can be made more flexible to accommodate any vehicles lower control arm. Few mechanical linkages can be added onto the test rig to help fix any control arm easily for the testing.

REFERENCES

- [1] F.-C. Wang, "Design and Synthesis of Active and Passive Vehicle Suspension," 2001.
- [2] "Southeast Auto Service," 2014. [Online]. Available: http://www.southeastautoservice.com/services/suspension.html.
- [3] T. D. Gillespie, Fundamentals of Vehicle Dynamics, Society of Automotive Engineers, Inc., 1992.
- [4] N. Kumar and V. RK, "Analysis of Front Suspension Lower Control Arm of an Automobile Vehicle," p. 51, 2013.
- [5] H. M. Mohyaldeen, "Analysis of an Automobile Suspension Arm using robust Design Method," 2011.
- [6] D. Knowles, Automotive Suspensions and Steering Systems, Cengage Delmar Learning, 2007.
- [7] L. A. Fornasieri, J. P. Kushnerick, K. A. Freeman, D. F. Morgantini and R. J. Rivele, in *Brakes, Steering and Suspension 1980-87*, 1988, pp. 3-7.
- [8] P. O. Aiyedun and B. O. Bolaji, "Automobile Engineering," [Online]. Available: http://unaab.edu.ng/attachments/470_MCE%20504%20Automobile%20Engineering%20Web%20Note%20-%20Dr%20B%20O%20Bolaji.pdf.
- [9] C. J. Longhurst, "The Suspension Bibles," 2014. [Online]. Available: http://www.carbibles.com/suspension_bible.html.
- [10] J. R. Davis, "Tensile Testing," ASM International, 2004, pp. 1-3.
- [11] "Instron Model 4400 Universal testing machine," 1995. [Online]. Available: http://fab.cba.mit.edu/content/tools/instron/M10-94400-1.pdf.
- [12] "Instron," Instron, [Online]. Available: http://www.instron.us/wa/product/DIC-Replay.aspx?ref=https://www.google.com/.

- [13] "Ball-joint, Control arm, Tie-rod Inspection tips," 2010. [Online]. Available: http://www.knowyourparts.com/technical-articles/ball-joint-control-arm-and-tie-rod-inspection-tips/.
- [14] A. M. Bash, "Design lower arm using optimum approach," 2011.
- [15] D. Silsby and F. Song, "Optimization of front lower arm," 2008.
- [16] N. R. Hema Kumar, Automobile chassis and body engineering, pp. 31-37.
- [17] S. J. Heo, D. O. Kang, J. H. Lee, I. H. Kim and S. H. Darwish, "Shape optimization of lower control arm considering multi-disciplinary constraint condition by using progress meta-model method," *International Journal of Automotive Technology*, vol. 14, no. 3, pp. 499-505, 2013.
- [18] S. Raghavendra, "COMPLIANT MULTI-LINK VEHICLE SUSPENSIONS," 2008.
- [19] D. Roylance, "MITOPENCOURSEWARE," 23 8 2001. [Online]. Available: http://ocw.mit.edu/courses/materials-science-and-engineering/3-11-mechanics-of-materials-fall-1999/modules/ss.pdf.