A DESCRIPTIVE STUDY OF ELEMENTARY MATHEMATICALLY PROMISING STUDENT INTERACTIONS WITH COGNITIVELY DEMANDING MATH TASKS

by

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A dissertation submitted to the faculty of
The University of North Carolina at Charlotte
in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in
Urban Education Curriculum & Instruction

Charlotte

2022

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ABSTRACT

WENDY M LEWIS A Descriptive Study of Elementary Mathematically Promising Student's Interactions with Cognitively Demanding Math Tasks.
(Under the direction of DR. MADELYN COLONNESE & DR. ANDREW POLLY)

National mathematics achievement results show that elementary students in the United States are not increasing in cognitive ability or critical thinking skills (NAEP, 2020). For this increase, mathematically promising students require more opportunities for cognitively demanding mathematics instruction. As a result, this descriptive study focused on the interactions and emergence of mathematical practices in seven third-grade students with a series of five tasks. The seven third-grade students were identified by their teachers as curious and mathematically promising. The tasks used in the two suburban classroom observations of the study were from the Tools 4 NC Teachers framework. Data sources collected included pre-and post-focus group audiotapes, classroom observations via audio and video, field notes, document analysis of student work, and a teacher debrief form. Blumer's theory of social constructivism (1969) and Lesh and colleagues' representational translation model (1987) guided this study. Findings from the students' interactions with the tasks showed they used a variety of interpersonal interactions, interactions between teacher and student, and visual representations. Students used mathematical writing to justify their reasoning in the tasks and reflection to communicate their conceptual mathematical understanding. Students grew in their understanding of the mathematical practices of perseverance through problem-solving, productive struggle, the construction of arguments, and the ability to make connections. These findings indicate the importance of ongoing curriculum development, including differentiated teacher guidance for mathematically promising students. This study's findings will also support mathematics teachers and leaders with a student-centered approach to teaching inquiry-based mathematics.

Keywords: cognitively demanding tasks, inquiry-based instruction, mathematical promising, Mathematical Practices, student interactions

ACKNOWLEDGEMENTS

More than anything else, I am sincerely grateful for the mentorship and guidance of my co-chairs: Dr. Madelyn Colonnese and Dr. Drew Polly. Your encouragement, ongoing support, and mentorship made me feel so connected and gave me the persistence to push forward during every roadblock along the way. From tips on writing organization to small hints about research, I have truly learned from your example. To Dr. Cindy Gilson and Miranda Fitzgerald, your tips on my research design and feedback, as well as your passion for gifted education, have been a valuable resource that has really helped me refine my research design. Dr. Amy Good, you also helped fuel my passion for connecting my ideas to the field as well as giving your perspective to connect my research to what I learned in my strand concentration in Elementary Education.

I also commend Dr. Bettie Ray Butler, Dr. Grant Wiggan, and Dr. Chance Lewis for the urban education courses at the heart of this program. These helped me remember why and gave me the knowledge I needed to connect research to practice. Each faculty member acknowledged above has contributed to my education in a meaningful way as an educator, scholar, student, and human being. Thank you for contributing to my education and for your ongoing support and presence in my life.

I would also like to thank the many colleagues in the program who were passionate and provided their efforts, contributions, and perspectives to help shift my views and push my thinking. I commend Dr. Christine Weiss, who continues collaborating with me to attend professional development and write conference presentations. Your dedication to our pilot study work in the summer of 2021 led to this work. I also wish to acknowledge and thank the executive director of the curriculum and instruction program, Dr. Tina Heafner, for her communication and leadership throughout the program. You are the glue that has molded this venture into a true work of art, and I could not have reached this milestone without you.

Finally, I am deeply thankful to the school administrator at the elementary school where I conducted the study, the two third-grade teachers, and the seven students who were part of my project. Without you, this study would have been impossible. Thank you for your dedication to your students, your collaboration with me, and your desire to grow and challenge your students with this experience.

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CHAPTER I: STUDENT INTERACTIONS AND EXPERIENCES WITH COGNITIVELY DEMANDING TASKS IN ELEMENTARY MATHEMATICS

Introduction

Since the turn of the century, mathematics education research has taken a "social turn" (Lerman, 2000). For more than two decades, the National Council of Teachers of Mathematics has urged teachers to move away from a mathematical environment with more direct instruction in which the teacher acts as the knowledge bearer and students act as knowledge repositories and towards one in which students are part of a sense-making community (NCTM 1989, 2014; Skemp, 1987). Classroom communities also need to be environments grounded in conceptual thinking and reasoning. As a result, the National Teachers Council of Mathematics began setting standards in the late seventies and eighties. However, it continued charging math teachers to teach in an inquiry-based environment with high cognitive demand (NCTM, 1989). In fact, the National Council of Teachers of Mathematics (1989) suggested that students should be doing cognitively demanding mathematics instead of just acquiring basic operations and terminology. Furthermore, cognitively demanding tasks have been defined as tasks that require students to think conceptually and make connections that lead to different opportunities to see mathematics from various perspectives and to understand mathematics content from a deep conceptual level (O'Connell, 2010; Smith & Stein, 1998). This study defines cognitively demanding tasks as a set of problems or a single complex problem that focuses on students' attention to a particular mathematical idea (Smith & Stein, 1998; Stein et al., 1996).

Furthermore, over a decade of research has consistently shown that student learning gains occur in classrooms where the highest level of cognitively demanding mathematical tasks is consistently maintained throughout instruction (Boaler & Staples, 2008; Hiebert & Wearne, 1993; Stein & Lane, 1996; Stigler & Hiebert, 2004). Recently, *The Principles to Actions: Ensuring Mathematical Success for All* (2014) teaching standards called for effective mathematics teaching through the use of cognitively demanding tasks as one way to motivate students to build new mathematical knowledge through problems (NCTM, 2014; Smith & Stein, 1998).

However, despite the strong research in mathematics for cognitively demanding mathematical instruction as a classroom practice, recent studies have only focused more on cognitive outcomes such as achievement gains or content learned (e.g., Smith & Stein, 1998; Stein et al., 1996; Yackel & Cobb, 1996), rather than outcomes of students' perceptions and experiences when interacting with cognitively demanding tasks within elementary classroom mathematics (Hiebert & Wearne,1993; Huinker & Bill, 2017). Besides, moving to a mathematical environment that encourages reasoning and inquiry-based problem-solving in all students entails focusing on research built on a mathematical triad of interaction with students, teachers, and cognitively demanding mathematical tasks (Ball & Cohen, 1993, 2000). A high-quality inquiry-based mathematical environment, in particular, includes multiple ways for students to interact with cognitively demanding tasks, such as visual, physical, symbolic, verbal, and contextual representations (Ball, 1993; Lesh et al., 1987; Nasir & Mckinney de Royston, 2013; Yackel & Cobb, 1996). Therefore, the elementary mathematics teaching field should focus on students' representations of and interactions with mathematics because they can lead to new real-world problem-solving opportunities and experiences for students (Boaler & Brodie, 2004; Dominguez, 2016; Huinker & Bill, 2017). Moreover, student interactions with cognitively demanding tasks help them become doers of mathematics and open windows of opportunity that can enhance their reasoning (Dominguez, 2016; Munter & Haines, 2019; NCTM, 2000, 2014; Schwartz, 2000).

Research on inquiry-based mathematics classroom environments has also focused on the social nature of learning activities in mathematics classrooms (Cobb et al., 1991; Yackel & Cobb, 1996). Some studies, in particular, have focused on implicit norms in classrooms, participation structures, and collaboration, showing that students learn best in collaborative environments full of oral and written discourse (Smith & Stein, 1998; Stein & Lane, 1996; Yackel & Cobb, 1996). Indeed, the National Council of Teachers of Mathematics, the Commission on Standards for School Mathematics (1989, 2000, 2014), and the National Research Council (1989) emphasize the need to teach mathematics to all children in order to help them make connections with their everyday world, engage them in doing mathematics, and help them construct meaning. The *Principles to Action* (2014) recently urged teachers to create learning environments that promote inquiry, reasoning, and problem-solving. In fact, elementary school mathematics programs are still transforming teaching practices and taking action to improve mathematics education (Huinker & Bill, 2017). For example, the National Council of Teachers of Mathematics (2014) recommended that teachers should motivate students through exploring and solving problems by selecting tasks with multiple entry points that have low floors and high ceilings so students can represent mathematics with various representations (Boaler & Brodie, 2004; Boaler, 2008; Flores, 2007; NCTM, 2014). The NCTM also recommended that teachers implement high-cognitive tasks regularly to support and encourage students through various solutions during task implementation (NCTM, 2014). These produsctive mathematical practices move away from teacher-based direct instruction to a more conceptual-based understanding, naturally allowing for multiple ways students can interact and engage in mathematics and opening windows of opportunity (Ball, 1993; Dominguez, 2016; Doyle, 1988; Nasir, 2002).

There are many benefits to teaching cognitively demanding tasks to elementary students, including increased mathematical achievement and content proficiency, which can impact the students' mathematical talent or the classroom's socio-mathematical environment (Boaler & Brodie, 2004; Yackel & Cobb, 1996). Mathematically promising students are talented individuals who think in a way that generates new ideas and deepen the meaning of existing ones (Deal & Wismer, 2010; Johnson et al., 2017; Sheffield, 1999). Specifically, mathematically promising students may transfer ideas and patterns to unusual situations, make connections between unrelated topics, and have a strong desire to question and go beyond what has been introduced (Gavin, 2011; Johnson et al., 2017). According to research, cognitively demanding task enactment may benefit mathematically promising students by increasing conceptual understanding, mathematical reasoning, making connections with mathematics, representations with problem posing, creativity, and problem-solving development, but most importantly, by shifting student interactions and experiences (Gavin, 2011; Johnson et al., 2017; Singer & Voica, 2012; VanTassel-Baska, 2021). While some researchers have suggested that student engagement with cognitively demanding tasks supports mathematical achievement, making mathematics accessible for all students, more research on mathematically promising student interactions with high cognitive demanding tasks is needed (Boaler & Staples, 2004; Huinker & Bill, 2017; NCTM, 2014**).**

Smith and Stein (1998) divided tasks into two broad categories: low cognitive demand tasks, which focus on memorization or procedural algorithmic mathematics, and high cognitive demand tasks, which focus on problem-solving. Focusing on mathematical tasks with low cognitive demand and emphasizing only rules, procedures, memorization, and correct answers often occur in elementary mathematics programs (Ball & Cohen, 1999; Goodlad, 1984; Stodolsky, 1988). However, high cognitive tasks include tasks with procedures and connections, such as doing math, which was the focus of this study.

Although increasing students' overall mathematical proficiency in the United States has been one reason for a shift in focus to highly cognitively demanding instruction in most elementary classrooms, this research focuses on the benefits of cognitive demand for mathematically promising students. By shifting their focus to mathematical practices and adapting tasks to differentiate for mathematical promise, teachers should elicit students' explanations of their mathematical thinking while simultaneously teaching content to all students with highly cognitively demanding tasks (VanTassel-Baska, 2021). In addition, such tasks provide numerous opportunities for all students to master rigorous content while maintaining an equitable growth mindset (Boaler & Staples, 2008; Flores, 2007; Munter & Haines, 2019; NCTM, 2014).

Additionally, ensuring experiences with higher levels of thinking and supporting mathematically promising students' mathematical understanding are practices supported by cognitively demanding tasks that extend and enrich more capable learners. Tasks play a significant role in determining the mathematics that students will see in the classroom (Doyle, 1988). Tasks also determine the concepts that students discover and can assist students in making connections with prior knowledge and exploring and connecting mathematical ideas (Dominguez, 2016; Mutner & Haines, 2019). However, little research has been conducted on mathematically promising student interactions and experiences with cognitively demanding tasks, especially concerning classroom socio-mathematical norms (Dominguez, 2016; Johnson et al., 2017; Yackel & Cobb, 1996).

Over the last two decades, scholars in the field of mathematics education have discovered that different tasks provide diverse learning opportunities for student learning and thinking (Hiebert & Wearne, 1993; Stein et al., 2009). Furthermore, the mathematical tasks in which students participate can shape their learning opportunities and experiences with mathematics as a whole and assist them in improving their mathematical reasoning (Hiebert & Wearne, 1993; Stein & Lane, 1996; Watson & Mason, 2007). According to the *Professional Standards for the Teaching of Mathematics* (NCTM, 2014), classrooms should be environments where students are encouraged to discuss their ideas with one another, where intellectual risk-taking is nurtured through the value of student thinking, and where sufficient time and encouragement are provided for exploration of mathematical ideas. Besides, an environment with socio-mathematical norms is important for cognitively demanding tasks because it encourages students to interact with cognitively demanding tasks (Nasir, 2002; Yackel & Cobb, 1996). Indeed, students' interactions with cognitively demanding tasks help them think and engage in the mathematics classroom (Doyle, 1983). Furthermore, mathematically promising students benefit from opportunities to engage in cognitively demanding tasks that mirror the unfamiliar, challenging, and multifaceted problems in the real world for which we are preparing them (Gavin, 2011; NCTM, 2014). Task enactment in the classroom is also even more important because the opportunities for students to engage actively in reasoning, sense-making, and problem-solving provided by interacting with tasks may lead to a deep understanding of mathematics (Huinker & Bill, 2017; NCTM, 2014). Based on social constructivism, this research analyzes students’ experiences while interacting with cognitively demanding mathematics tasks (Blumer, 1969; Vygotsky, 1987). Also, the emergence of conceptual mathematical thinking and mathematical practices is shown through oral discourse, mathematical writing, and representational modalities observed through classroom socio-mathematical norms (Casa et al., 2016; Gavin, 2016; Lesh et al., 1987; Yackel & Cobb, 1996).

Statement of the Problem

The National Council of Teachers of Mathematics stated in *The Principles to Action* (2014) that all students require cognitively demanding mathematics instruction (NCTM, 2014; Stein et al., 2014). Furthermore, the literature indicates that students benefit from opportunities when they engage in problematic tasks; students require tasks that may take some time to complete and that reflect the unfamiliar, challenging problems in the real world for which we are preparing them (Boaler, 2008; Flores, 2007; Hiebert & Wearne, 1993; Kisa & Stein, 2015). Also, students must engage with tasks that present low floors and high ceilings so they can all access mathematics and productively struggle at different paces, depths, and times in a real-world context (Flores, 2007; Huinker & Bill, 2017).

According to Huinker and Bill's (2017) *Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5,* learning about student perspective in mathematics classrooms is important so students can effectively problem solve, make connections, communicate, and justify their thinking in mathematics classrooms (Huinker & Bill, 2017; NCTM, 2014). While the relationship between students and cognitively demanding mathematics is clearly articulated or agreed upon in the field, less is known about student perspective and how mathematically promising students engage through interactions with mathematical tasks (Stein & Lane, 1996). Moreover, previous studies about mathematical interactions with cognitively demanding tasks have studied the role or preparation of pre-service and in-service teachers, as well as how tasks are implemented, and discovered that teachers tend to use mathematical lessons that foster conceptual understanding in their students (Kisa & Stein, 2015; Stigler & Hiebert 2004). However, even though teachers believe these tasks are best for students, they often have difficulty implementing cognitively demanding tasks without lowering the demands of the task (Stein et al., 1989; Stein & Smith, 1998).

Furthermore, little research has been conducted in the field of elementary mathematics on the role of mathematically promising students when interacting with cognitively demanding tasks (Ainley & Margolinas, 2015; Gavin et al., 2011, 2016; Johnson et al., 2017; Nasir, 2002). No research has been conducted on the organic experiences of mathematically promising students with cognitively demanding tasks. In fact, many studies indicate that students with mathematical talent are not given enough opportunities to participate in high-cognitive demand classroom environments (Gavin, 2011; Sheffield, 1999; VanTassel-Baska, 2021). Besides, scholars have argued that understanding how learners develop a sense of membership in practice and the extent to which youth are identified as "learners" and "doers" of mathematics is critical to understanding learning and engagement in mathematical activity (Ball & Cohen, 1990; Martin et al., 1997; Nasir, 2002).

Purpose of Study

Several types of mathematics tasks influence students' interactions, and students' experiences with tasks may vary (Clarke & Helme, 1998; Johnson et al., 2017). Mathematical tasks need to consider how individuals interact with and problematize them. According to Clarke and Helme (1998), students interpreted tasks differently than the situations described in the task. Students also interact with mathematical tasks based on their symbolic interactionism, oral discourse, metacognitive reflection, and mathematical writing experiences (Casa et al., 2016;

Pugalee, 2004). Previous research has also shown that students' interactions with mathematical tasks improve their creativity, critical thinking, mathematical reasoning, and problem-solving skills, which improve their mathematics achievement and attitude towards mathematics (Nasir, 2002; Hiebert & Wearne, 1993). Furthermore, research has shown that strong metacognition can influence students' mathematical identities, especially those who are mathematically promising (Gavin, 2011; Nasir, 2002; Sheffield, 1999). According to researchers, cognitively demanding tasks necessitate interactions among students and assist students in using their diverse mathematical perspectives (Hiebert & Wearne, 1993; Smith & Stein, 1998). In addition, students analyze their thinking with cognitively demanding tasks such as mathematical writing, which may lead to more conceptual interactions and a more complex mathematical perspective (Munter & Haines, 2019).

When studying student interactions, cognitively demanding tasks can make for complex instruction due to the instructional triangle of teachers' intentions, students' perspectives, and the task's original intentions (Ball & Cohen, 1990). While teachers consider the students' interpretation and perspectives of a task (Ainley & Margolinas, 2015; Ball & Cohen, 2000), the student's interaction with the context of a mathematical task remains an individual, dynamic process that can take various forms (Boaler, 1993b; Boaler & Brodie, 2004). As a result, this study aimed to examine various interactions and the enactment of tasks within the elementary mathematics classroom of mathematically promising students, specifically the theory and methods underlying such research. The next sections examine the theoretical framework and discuss the specific methods used to implement this study.

Research Questions

RQ 1: How did elementary mathematically promising students interact with cognitively demanding mathematical tasks?

RQ 2: How did mathematically promising students use mathematical practices as they completed cognitively demanding tasks?

Overview of The Theoretical Framework

When investigating cognitively demanding tasks, the theory of social constructivism suggests that instructional materials and their meanings serve as the foundation for internal representations (Blumer, 1969). According to Blumer (1969), students' internal and mathematical representations can be shaped as they interact with cognitively demanding tasks in the context of the larger environment. Therefore, the theory seems to focus on students' social interaction opportunities in all instructional situations, including those involving the use of instructional representations (Yackel et al., 1991; Yackel & Cobb, 1996). Besides, individual students' constructive activities are affected by the problems and conflicts that arise during social interactions. Therefore, students' mathematical learning in the classroom should help develop and reflect their individual practices and beliefs. Students may interact verbally, but others may prefer a skills-based approach with direct teaching over a discovery-based approach. To level the playing field, instructional approaches should mirror students' prior experiences and interactions (Dominguez, 2016).

Conclusively, this study employs theoretical traditions to investigate how elementary students' mathematical identities aid in the development of their interactions with cognitively demanding mathematical tasks. This study specifically used social constructivist theory to examine the social interactions with mathematical discourse, mathematical writing, and the socio-mathematical classroom norms that are influenced by students' mathematical experiences when interacting with tasks (Blumer, 1969; Cobb et al., 1988; Smith et al., 2008; Stein et al., 1996; Yackel & Cobb, 1996). Finally, this study explored students' mathematical experiences and perceptions of how elementary inquiry-based mathematics classrooms contribute to the development of mathematically promising students as doers of mathematics (Ball, 1993).

Overview of Context and Method

This study utilized a qualitative thematic analysis to examine the experiences, perspectives, and student interactions with cognitively demanding mathematical tasks. Furthermore, this study collected qualitative data based on a descriptive qualitative study methods approach (Creswell, 2013), a model where the researcher interacted as an observer within the constructivist context of two third-grade classrooms in order to seek and understand the human context. Accurate data were collected from a four-week classroom study with third-grade participants (n = 7) from a classroom in an urban, more metropolitan area in the Southeastern United States. Data were collected using qualitative interviews conducted with two focus groups of seven students. In addition, the researcher collected student work samples from the cognitively demanding tasks, conducted daily classroom observations using field notes, and analyzed data from the selected focus group of three to seven students within the classroom setting who are identified as mathematically promising learners in elementary mathematics. The methodology in this descriptive study was intended to provide greater context for interpreting the relationship between the students' interactions and experiences with cognitively demanding mathematical tasks and their organic experiences within their teacher's classroom (Creswell, 2013).

Specifically, this research analyzed open coding to examine various themes across the qualitative data of observational field notes students. The students' used explanatory, argumentative, and descriptive mathematical writing in this study to communicate their thoughts as they interacted with tasks (Casa et al., 2016). This study defined mathematical writing as forms of prose used to reason about and show their thinking about mathematics in the form of prose, such as symbols, letters, words, phrases, or sentences used via student work with document analysis. The documents were analyzed using preset codes from Principles to Action (NCTM, 2014) and then with the constant comparison method to see how they related to the students' interactions (Kvale & Brinkmann, 2015). The theory in this study was based on the formation of social norms that sustain classroom socio-mathematical reasoning and interactions characterized by explanation, justification, and argumentation (Cobb et al., 1992; Yackel et al., 1991).

Significance Statement

For more than two decades, the National Council of Teachers of Mathematics has been calling for teachers to move away from didactic language (where the teacher acts as the knowledge bearer and students as repositories of this knowledge) towards language that positions students as members of a sense-making community (NCTM, 1989, 2000, 2014). Despite extensive research in mathematics on cognitively demanding mathematical instruction as a classroom practice, recent studies have focused solely on cognitive outcomes in isolation (Smith & Stein, 1998; Stein et al., 1996). In order to develop an understanding of students' experiences and perspectives within the socio-mathematical norms of the classroom and the role teachers, tasks, and materials, as well as their own metacognitive interactions, may play in this process, research focusing on student interactions with cognitively demanding tasks is required. Students' interactions may be metacognitive or reflective, involve oral or written communication with others, or involve manipulatives or other materials. The impact of classroom norms and how these elements work together to influence the mathematical interactions of students when interacting with tasks with multiple representations was the focus of this study (Ball, 1993; Nasir, 2002; Nasir & Mckinney de Royston, 2013; Yackel & Cobb, 1996). Mathematics teachers are tasked with developing and transforming practices that result in novel opportunities that mimic real-world, sociocultural experiences in which students become doers of mathematics (NCTM, 2000; Munter & Haines, 2019; Schwartz, 2000).

Furthermore, the significance of studying student interactions with cognitively demanding tasks is that students change their interpretations and conceptual understanding based on how they perceive each other's mathematical activity within the classroom norms (Bauersfeld, 1980; Cobb et al., 1989; Yackel & Cobb, 1996). This had major implications for the nature of classroom interactions, also known as the Instructional Triangle (Ball & Cohen, 2000; Cohen et al., 2003). However, classroom interactions should center on mathematical reasoning and evidence between teachers, students, and cognitively demanding tasks. For students to develop the ability to formulate problems, explore, conjecture, and reason logically to see if something makes sense, classroom discourse must be founded on mathematical evidence (NCTM, 1991, 2014).

In inquiry-based mathematical classrooms, students with mathematical promise interact and engage with their peers to make sense of mathematics (NCTM, 2014; Stein & Lane, 1996). This study rejects the notion that mathematical meaning can only be discerned through external representations; instead, it argues that mathematical meanings are the product of students' interpretations and perceptions in response to cognitive demands and their socio-mathematical environment (Cohen et al., 2003; Yackel & Cobb, 1996).

This study was carried out in response to several research findings, which signal the need for more attention to cognitively demanding instruction in elementary mathematics classrooms, especially for students with mathematical talent. First, recent data from the National Association of Educational Progress (2012) and NAEP (2020) show that students are still unprepared to solve cognitively demanding mathematical problems, with a national scale score of 244 and 241 for fourth graders, respectively. This data indicated that more work is needed to improve students' achievement in how they problem-solve interactions (NAEP, 2020) (see Figure 1). However, making mathematics tasks more rigorous is not enough to enhance student achievement; teachers need to pay more attention and respond to student interactions, especially with students who are gifted in mathematics (Plucker et al., 2013). This study focused on how socio-mathematical norms and interactions of mathematically promising students may impact their cognitive understanding of mathematics (Yackel et al., 1991; Yackel & Cobb, 1996).

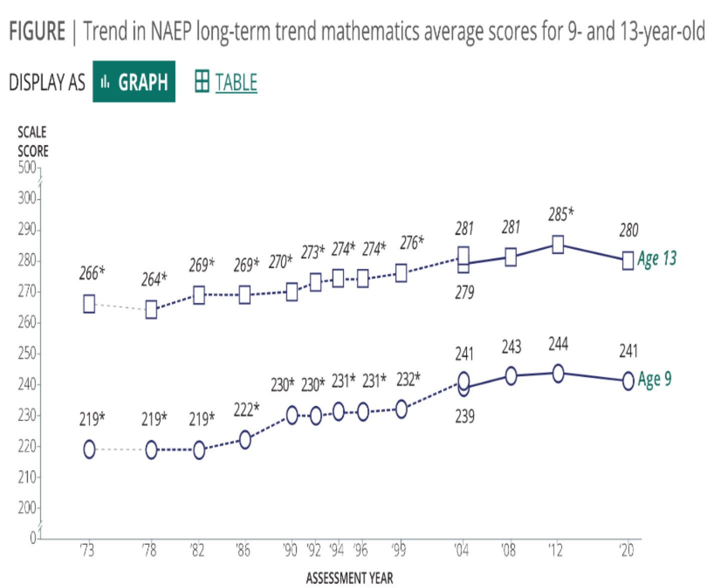

Figure 1

NAEP 2020 Math scores

Recent research in the field of mathematics suggests that teachers should carefully select mathematical tasks so students can have ample opportunities to connect to their identities and open doors of opportunity for students to bring in their prior knowledge and pose their own problems (Dominguez, 2016; Gavin, 2011). When elementary teachers encourage students to use their identities, it can profoundly impact how students interact with mathematics by encouraging them to share their ideas through problem-solving and opening doors for students to explore the “essence” of mathematics (Dominguez, 2016; Mann, 2006).

Most recent studies about students’ mathematical interaction with tasks have focused on pre-service teacher training rather than students, especially mathematically promising elementary students (Gavin, 2011; Johnson et al., 2017; Olawoyin et al., 2021). Although student interactions are an important practice in elementary mathematics classrooms, no studies have specifically investigated how mathematically promising student interactions with cognitively demanding mathematics at the elementary level, particularly investigating the socio-mathematical norms of the classroom and the emergence of the mathematical practices. Some recent studies have investigated the use of pictorial representations used by students to understand cognitively demanding tasks, but no knowledge from students' interactions in elementary grades classrooms have been added to studies (Ainley & Margolinas, 2015; Johnson et al., 2017, Olawoyin et al., 2020; Smith et al., 2008; Smith & Stein, 1998). Furthermore, there is little to no research currently examining how mathematical writing develops in elementary school (Kosko & Zimmerman, 2017). Even though studies over the last 25 years indicated a lack of opportunities for students to engage in writing in mathematics, very few studies had examined the development of mathematical writing, especially among elementary mathematically promising students (Kosko, 2016; Kosko et al., 2009; Pugalee, 2004). However, only recently has the use of mathematizing with mathematical writing been explored (Casa et al., 2022).

This study expanded on recent contributions of cognitively demanding tasks by investigating the intersection of student interactions of oral discourse, writing, and socio-mathematical norms in order to provide a comprehensive picture of the Instructional Triangle of interaction (Ball, 1993; Cohen et al., 2003; Yackel & Cobb, 1996). The study was also founded on empirical evidence about how mathematically promising students' interactions with cognitively demanding tasks impact their mathematical identity and the emergence of mathematical practices while exploring cognitively demanding tasks. The findings revealed that this is the first known study to use a two-classroom study in elementary mathematics focused on mathematically promising students. In addition, instead of focusing on one student subgroup, the data focused on the interactions and experiences of students and looked across the data for thematic interpretations within the organic data collected. Finally, this study also drew on small national and international studies to help fill gaps in the existing literature in elementary mathematics classrooms and the impact of student interactions with cognitively demanding mathematical tasks.

Summary

This dissertation is divided into five chapters, with the current chapter serving as an overview of the study. It provides preliminary evidence for the importance of focusing on student interactions with highly cognitively demanding tasks as well as evidence of thinking within the socio-mathematical norms of mathematics classrooms. Chapter Two provides an overview of the cognitively demanding task literature from 1996, when Stein and Lane's work on cognitively demanding tasks was established, to the present. It includes research evidence relevant to the student's perspective as well as socio-mathematical reasoning. Chapter Two also reviews the literature on mathematically promising student interactions with cognitively demanding tasks within the instructional triangle. Chapter Three outlines the study's descriptive and qualitative methodology. Thematic analysis and open coding are used in this chapter to examine the experiences of the two third-grade small groups of mathematically promising students in this case study. Data were collected and analyzed on how students' mathematical practices emerge as influential while the mathematically promising students explore cognitively demanding mathematical tasks within the context of the socio-mathematical norms of the classroom. Chapter Four provides the study results used to answer the two key research questions. Finally, Chapter Five positions the results of this study within the larger fields of mathematics and gifted education literature through a presentation of key findings, significance, and recommendations for policy, practice, and future study.

Definition of Key Terms

Cognitive demand: Tasks that ask students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that require students to think conceptually and that stimulate students to make connections lead to a distinct set of opportunities for student thinking (Smith & Stein, 1998).

Curriculum materials: As defined by Ball and Cohen (1990), curriculum materials include textbooks, teachers' guides, and other materials such as replacement units and instructional materials kits (p. 14). Since curricular material respects the nature of the mathematical tasks they contain, distinct types of tasks influence students' interactions.

Conceptual Mathematical Thinking: Thinking concepts through a variety of pathways, mathematical thinking is integral to the mathematical content and creation of concepts (Casa et al., 2022).

Discourse: Student mathematical discourse, defined in our study as "the act of [students] articulating mathematical ideas or procedures" (Weaver et al., 2005, p. 3), has been identified as a key element in students' cognitive development (Lampert, 1990; Yackel et al., 1991).

Mathematical discourse: This is defined in our study as "the act of [students] articulating mathematical ideas or procedures" (Weaver et al., 2005, p. 3) and has been identified as a key element in students' cognitive development (Forman, 1996; Lampert & Cobb, 2003; Yackel et al., 1991).

Mathematical task: A mathematical task is a set of problems or a single complex problem that focuses students' attention on a particular mathematical idea (Stein et al., 1996).

Mathematical representation: concrete, visual, numerical, graphical, pictorial, or symbolic components that allow mathematical ideas to be interpreted (Lesh et al., 1987; Tripathi, 2008).

Mathematically promising students: Those with the potential to become leaders and problem solvers of the future with a strong reasoning ability but have not yet had the opportunity to tackle high-level problems (Gavin, 2011; Sheffield et al., 1999).

Mathematical writing: A written form of prose used to reason and communicate mathematical ideas in the form of prose, such as a symbol, figure, label, word, or sentence (Casa & Cohen, 2003; Casa et al., 2016).

Productive Struggle: Significant, durable academic learning is difficult. When students expend effort to grapple with perplexing problems or make sense of challenging ideas, they engage in the process of productive struggle—effortful practice that goes beyond passive reading, listening, or watching—that builds useful, lasting understanding and skill (Gavin, 2011).

Social constructivism: Learners actively construct their knowledge through experiences and interactions with others, using different strategies to rely upon their prior knowledge, situation, and the type of learning materials (Blumer, 1969; Vygotsky, 1987).

Socio-mathematical norms: Classroom norms that are specific to mathematical aspects of student activity (Yackel & Cobb, 1996).

Interactions: "Interaction" refers to no form of discourse but to teachers' and students' connected work, extending through days, weeks, and months. Instruction evolves as tasks develop and lead to others, students' engagement and understanding wax and wane, and the organization changes (Lampert, 2001).

Limitations

This study was guided by social constructivism as it framed student interactions within the mathematics classroom with cognitively demanding tasks. Students' interactions and experiences were studied as they interacted with tasks through various oral, written, symbolic, and representational means within the classroom socio-mathematical norms or classroom norms specific to the students and mathematics within the classroom of study. Student interactions were limited by the classroom norms and instructions of the teacher. Furthermore, during data collection, interactions were restricted to a single group or case.

CHAPTER II: LITERATURE REVIEW

This chapter provides an overview of the literature on cognitively demanding tasks, ranging from the work of Stein and Lane (1996) and Smith and Stein (1998) to current studies in the field of elementary mathematics. It also includes research evidence on socio-mathematical norms beginning in 1988 (Cobb et al., 1988). Chapter Two also focuses on how mathematically promising students' interactions with cognitively demanding tasks relate to social constructivism theory. Some background on teacher movements and how teachers' instructional practices impacted task interaction are provided.

Background

The National Council of the Teachers of Mathematics advocates for the development of mathematics classrooms that foster critical thinking. According to the organization, one of the most important reasons teachers should use strategies that elicit thinking and challenge students is to foster mathematical reasoning and productive mathematical environments (NCTM, 2000). The *Principles to Action Standards* encourage teachers to incorporate discourse into their teaching and learning practices; these standards require teachers to assist students in developing conceptual knowledge, encourage discourse and interaction, and pose meaningful problems (NCTM, 2014).

According to current mathematics education research, effective teaching is centered on the use of cognitively demanding mathematical tasks (NCTM, 2014; Polly & Hannafin, 2011). Teachers are encouraged to provide opportunities for students to interact with and explore cognitively demanding tasks. Smith and Stein (1998) classified tasks into four categories based on their level of cognitive demand (NCTM, 2014; Smith & Stein, 1998; Stein et al., 2007). The different classifications of mathematical tasks are shown in Table 1 (see Appendix E). While each type of task has its place in mathematics classrooms, recommendations emphasize the importance of student interactions with cognitively demanding tasks. This research may also have implications for teachers in terms of noticing and maintaining cognitive demands, as well as developing problem-solving and perseverance practices by exploring and solving tasks with high cognitive demands (NCTM, 2014).

Purpose

This literature review aims to compile current and relevant literature on how elementary mathematics students interact with cognitively demanding tasks based on their perspective and mathematical reasoning, with a focus on students who show mathematical promise. Cognitively demanding tasks have been studied for decades, ever since Smith and Stein (1998) classified math tasks as having low and high cognitive demands (see Table 1). Previous research has focused heavily on a shift in standards (NCTM, 2000), mathematical practices (NCTM, 2014), and teaching standards (CCSSI, 2010a). This literature review also investigated background studies that have shown how students interact within tasks, what representations students use to interact with high cognitively demanding tasks, and how the socio-mathematical norms and social constructivist interactions of the classroom may influence these interactions and thus mathematical thinking and reasoning (Huinker & Bill, 2017; Lesh et al., 1987, Yackel & Cobb, 1996).

Furthermore, the literature review explored the categorization of cognitively demanding tasks, why tasks are relevant for all students, why cognitively demanding tasks should be implemented, and how students interact with mathematical tasks. Also covered in this literature review are how cognitively demanding tasks can help develop mathematical talent and how students can reason, represent, justify, and make connections through task interactions. Further review will show how the socio-mathematical norms of the classroom influence the development of shared meaning in the instructional triangle among the teacher, task, student, and materials (Cohen et al., 2003; Yackel & Cobb, 1996). Finally, further sections of this review explored several ways students interact with tasks through oral discourse, mathematical writing, problem posing, and other mathematical representations (Lesh et al., 1987).

Methods

This literature review supports a social constructivist approach to mathematics. For student interactions within productive mathematical communities to help students open new windows of thinking, mathematics communication, such as questioning and classroom norms, needs to be set up and rooted in Vygotskian sociocultural theory (Dominguez, 2016; Vygotsky, 1987). We must also create a socially constructive environment where all students can enter tasks with prior knowledge and experience (Ball, 1993). Unfortunately, before Smith and Stein's (1998) initial study on cognitively demanding tasks, only a few studies documented ways students interact with cognitively demanding instruction.

Furthermore, recent research on the broad topic of cognitive demand focuses on narrowing cognitive demand categories into low cognitive demand and high cognitive demand. Specifically, this research investigated mathematically promising elementary students' interactions with cognitively demanding tasks by focusing on four broad categories of how students interact with tasks (NCTM, 2014):

1. How students preserve, explore, and reason through cognitively demanding tasks
2. How students take responsibility for making sense of tasks by drawing on and making connections with prior knowledge
3. How students interact with mathematics using tools and representations as needed to support their thinking (Tripathi, 2008)
4. How students approach mathematical solutions and justify their strategies to one

another.

These categories were examined through the lens of the Instructional Triangle of interaction with the student, teacher, and task within the classroom environment (Ball & Cohen, 1990; Cohen et al., 2003; Huinker & Bill, 2017). These categories are drawn from the recent book *Taking Action: Implementing Effective Mathematics Teaching Practices* and were originally noted in the *Principles to Action* teaching standards as critical student actions when promoting problem-solving within the mathematics classroom (Huinker & Bill, 2017; NCTM, 2014).

This literature review begins by focusing on how students interact with cognitively demanding tasks by reviewing the following topics: categorization of tasks, benefits, and reasons for implementing cognitively demanding tasks. This research is heavily focused on how students interact with the tasks. It also shares findings on how students interact by persevering, exploring, reasoning, justifying their thinking, and how they represent and connect with mathematics (NCTM, 2014). Furthermore, how teacher moves influence student interactions with tasks aid in connecting the teacher's role in the instructional triangle in task enactment. The review investigates why cognitively demanding tasks are relevant, as well as their benefits in elementary mathematics for mathematically talented students. Finally, the research shows how classroom norms and teacher moves, such as the five practices for oral discourse, productive struggle, and teacher observation, can influence student interactions with cognitively demanding tasks and influence students' emergence of Mathematical Practices (Ball & Cohen, 2000; Cobb et al., 1991; Martin et al., 2017; Smith & Stein, 2000; Yackel & Cobb, 1996).

Finally, the framework of this literature review synthesizes commonalities among findings and shifts toward future implications for implementing cognitively demanding tasks with all elementary students, especially to promote reasoning and complex problem-solving for students with mathematical promise by providing a series of tasks that have high ceilings and low floors, which will open doors of opportunity for all students (Dominguez, 2016; Flores, 2007; Huinker & Bill, 2017). Cognitively demanding tasks are beneficial for all students because they have multiple entry points and are open-ended, but they offer added benefits to mathematically promising students. Johnson and Sheffield (2013) encouraged the use of standards for mathematical practices for mathematically promising students. The mathematical practices used within this study emphasize creative problem-solving and encourage students to engage in complex, real-world mathematical thinking.

Previous research on social constructivist theory found that students interacted with cognitively demanding tasks through various tasks set in primary classrooms; however, middle school classrooms were not excluded (Blumer, 1969, Flores, 2007; Johnson et al., 2017). This study focuses on classroom studies involving cognitively demanding tasks in K-5 classrooms. Many qualitative case studies of various sizes were included. Several of the larger studies included in this literature review, such as Hufferd-Ackles et al. (2004), examined social interactions (Yackel & Cobb, 1996; Vygotsky, 1987). Learning is a social process; without student interactions, true mathematical thinking cannot occur (Sfard, 2001).

According to Vygotsky (1987), we cannot understand what students are thinking without language. Hence, the importance of oral and written discourse, as well as other forms of language, in helping students internalize and conceptualize mathematics through social experiences. Social experiences and interactions within a mathematics class can take on various representations, such as visual, contextual, verbal, physical, and symbolic, as shown in Figure 2 (Lesh et al., 1987). Specifically, students can interact with tasks in numerous ways, such as through oral discourse, symbolic interaction, mathematical writing, problem-solving, or cognitive meaning.

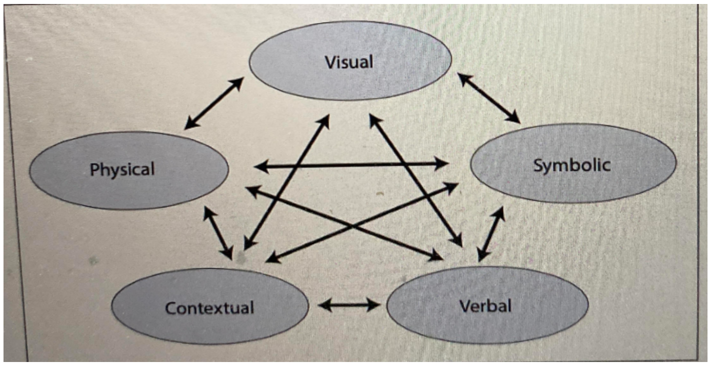

Figure 2
Lesh and colleagues (1987)

"Representations and Translations among Representations in Mathematics Learning and Problem Solving." In Problems of Representation in the Teaching and Learning of Mathematics, edited by Claude Janvier, pp. 33–40. Mahwah, NJ: Lawrence Erlbaum Associates, 1987.

When students interact with tasks, they may memorize information, follow procedures, think deeply, and reason (Hufferd-Ackles et al., 2004). This study found that high achievers had more conceptual explanations during high cognitive tasks using *Mathematics Plus* (1992). In another study, Kisa and Stein (2015) suggested that problem-solving allowed more student interaction. However, Kisa and Stein (2015) also found that the cognitive demand of the task does not guarantee high-level thinking and interactions among students, thus indicating that task interaction is a social experience. As a result, teachers who facilitate cognitively demanding tasks can ignite the socio-mathematical reasoning of students and be catalysts for conversation and student interactions. The key theme of the theoretical framework of this study was that social interactions are required for students to have conceptual and meaningful interactions in a cognitively demanding environment.

Literature Review

Categorization of Cognitively Demanding Tasks

Cognitively demanding tasks are important for student learning in mathematics because mathematical tasks influence how students must learn mathematics in the classroom (NCTM, 2014; Smith & Stein, 1998). There are two types of cognitively demanding mathematical tasks: low cognitive demand tasks and high cognitive demand tasks (Smith & Stein, 1998). Based on Smith and Stein's (1998) original framework, the levels indicate the type of thinking required to solve the tasks (see Table 1). Tasks can be categorized into *lower-level cognitive demand problems* and can be classified as memorization. Routine exercises that involve memorizing formulas, algorithms, or procedures without connection to the underlying concepts or meaning are classified as *procedures without connections.* With *low-cognitive-demand* tasks, there is no connection to the concepts or meaning behind the mathematical procedures used.

High-level cognitive demand tasks also have two classifications, procedures with connections and mathematics (Smith & Stein, 1998). Tasks focusing on the use of broad general procedures for developing a deeper understanding of concepts and ideas can usually be represented in multiple ways and require a degree of cognitive effort to complete successfully. Other *higher-level cognitive demands* are often referred to as doing mathematics (Smith & Stein, 1998). *High-level cognitive demand tasks* require complex thinking or exploration to investigate the nature of mathematical concepts, processes, and relationships (Smith & Stein, 1998).

Benefits and Reasons for Cognitively Demanding Task Implementation

Raising students' overall level of mathematical proficiency in the United States can be seen "as both a matter of national interest and a moral imperative" because teaching in most elementary classrooms emphasizes rules, procedures, memorization, and correct answers (Ball & Cohen, 1999, 2000). Cognitively demanding tasks promote understanding. Over a decade of research has consistently found that the greatest student learning gains occur in classrooms where the high-level cognitive demands of mathematical tasks are consistently maintained throughout the instructional episode (Boaler & Staples, 2008; Hiebert & Wearne, 1993; Stein & Lane, 1996; Stigler & Hiebert, 2004; Tarr et al., 2008). Cognitively demanding instruction can be seen as "doing mathematics," which Stein et al. (2000) defined as investigating complex relationships involving conjectures and metacognition.

Furthermore, cognitively demanding tasks should broaden and strengthen mathematical understanding (Smith & Stein, 1998). Tasks that emphasize computation and memorization and have a low cognitive demand on students often require procedural skills without student understanding. According to research, low-level procedural tasks are not beneficial for students' problem-solving, especially for those who are mathematically talented (Gavin et al., 1996). Instead, focusing on contextual, more cognitively demanding problems assists students in attending to the concepts underlying the problems (Stein & Lane, 1996). Furthermore, many scholars emphasize that *real* problem-solving involves working on unfamiliar, out-of-context, open-ended problems, providing students with real challenges and rich tasks and contexts (Anderson, 2003; Kilpatrick et al., 2001; Schoenfeld, 1992). On the other hand, open-ended tasks have multiple solutions, varied pathways to solve and record the solution, and are non-routine. In contrast, unfamiliar tasks are closed, not regularly encountered, and involve non-routine problems that do not advance students' conceptual thinking forward. Recent research in middle school mathematics has also revealed how students develop meaningful content by solving tasks (Johnson et al., 2017).

Furthermore, cognitively demanding tasks are beneficial in the classroom because tasks that emphasize computation and memorization result in students learning procedural skills without understanding why they work. Moreover, cognitively demanding tasks emphasize the importance of creative problem-solving and investigation activities that present real-life fictional situations to mathematically promising students (Singer et al., 2011). Finally, the need for challenging tasks allows more opportunities to foster potential mathematical talent, as evidenced by research on mathematically gifted and promising students (Diezmann & Watters, 2002; Johnson, 2000; Sheffield, 1999).

Benefits of Cognitively Demanding Tasks for Nurturing Mathematical Promise

Students with mathematical promise who exhibit thinking and problem-solving abilities require a greater depth and breadth of topics as well as open-ended opportunities for solving more complex problems and opportunities (Sheffield, 1994). A substantial body of research supports the conclusion that students with mathematical promise require advanced materials and curricula in order to realize their full potential (VanTassel-Baska, 1995, 2020). Indeed, cognitively demanding tasks are required for students with mathematical promise in order to foster curiosity and develop creativity and scientific thinking skills (Manuel & Freiman, 2017; Singer et al., 2016). Meeting the needs of mathematically promising students entails more than just procedural problem-solving; it also entails cognitive rigor and problem-posing (Leikin, 2009; Mann, 2006; Sheffield, 1999). Furthermore, cognitively demanding tasks allow students to learn new mathematical content while increasing their commitment to the learning tasks, even if they are more difficult (Manuel & Freiman, 2017).

Ways Students Interact with Tasks

According to the *Principles of Action,* students can interact with tasks by persevering in exploring and reasoning through them (NCTM, 2014). Students may also take responsibility for making sense of tasks by drawing on and connecting prior understanding and ideas, as well as using tools and representations to support their thinking and problem-solving as needed. Accepting and expecting that their classmates will use a variety of solution approaches, students will discuss and justify their strategies to one another. In some ways, their caution echoed that of Doyle (1988). He suggested that some students may want more procedure-based approaches that allow them to learn concepts directly rather than doing so on their own through discovery approaches.

Students can interact with tasks in a variety of ways, including cognitively demanding tasks such as persevering, exploring, and reasoning through tasks (NCTM, 2014). Students can also make connections through reasoning and prior knowledge. Furthermore, students can interact with cognitively demanding tasks using tools and representations to support their thinking (Tripathi, 2008). With these interactions, students connect multiple ideas using symbols to communicate an idea or draw a diagram or picture in oral discourse, written discourse, and classroom norms (Cohen & Ball, 1990; Tripathi, 2008). Finally, students can use mathematical thinking to justify their solutions by creating a systematic list, talking about their ideas, writing down their thoughts in writing, using or interpreting graphs, breaking a complex task down into smaller ones, using the calculator, relating a new problem to a previous one, or problem-posing (Johnson et al., 2017; Silver, 1994; Stein et al., 1996).

Why Student Interactions Are Necessary

Cobb and colleagues (1992) believed that communication in the mathematics classroom is a process of mutual orientation rather than simply transmitting information. As a result, while interacting with cognitively demanding mathematical tasks, students should pursue their own and others' mathematical activities. In doing so, a few things will happen. First, mathematical classroom communication and the development of discourse communities in classrooms are stressed in reform documents (NCTM, 2000). Second, discourse research in other scientific disciplines has led to the development of theoretical perspectives and analytical constructs that apply to mathematics education. Third, because scholars have defined mathematics as a discourse, cognitively demanding interactions should naturally include this type of communication (Moschkovich, 2002; Sfard, 2008).

Students Interact by Persevering, Exploring, and Reasoning with Cognitively Demanding Tasks

According to research, mathematical writing is a tool that improves students' ability to reflect, strategize, and communicate, and it is essential for students to engage in mathematics in order to focus their thinking and sharpen their problem-solving skills (Martin et al., 2017). The use of writing as a tool for mathematics learning is well documented in the literature (Casa et al., 2022; Martin et al., 2017; Polly & Hannafin, 2011). Mathematical writing is one way for students to interact with cognitively demanding tasks by reasoning their mathematical thoughts using prose, such as symbols, letters, words, phrases, and sentences (Casa et al., 2016). Students benefit from the opportunity to write in mathematics because it allows them to improve their thinking and convey their ideas clearly, concisely, and conceptually (Martin et al., 2017). The benefits of written reflection have been noted in the research surrounding metacognition, self-evaluation, and self-regulation strategies (Martin et al., 2017). However, few studies have examined how elementary students engage in reflective metacognitive interactions, such as writing in mathematics (Martin et al., 2017). Furthermore, procedural learning continues to dominate mathematics instruction, limiting opportunities for students to explore their mathematical thinking, conceptual learning, and process reflection (Martin et al., 2017; Polly & Hannafin, 2011; Pugalee, 2014; Stein et al., 1996).

Furthermore, Mathematical Practice 1 requires students to make sense of problems and persevere in solving them (NCTM, 2010). Student mathematicians must also enjoy creating their own new problems to solve or problem-posing (Silver, 1994). Through problem-posing and exploration of cognitively demanding tasks, students gain conceptual understanding, mathematical creativity, and perseverance in problem-solving (Lewis & Colonnese, 2021; Smith & Stein, 2008). Teachers should also use discussion-based and reflective pedagogy to support student learning and assist mathematically promising students in analyzing and solving problems by asking questions that connect to previous learning (Gavin, 2011; Smith, 1996).

Students Interact with Tasks by Making Sense and Connections with Mathematics with Prior Knowledge

When first learning about mathematics, teachers should consider their students' prior knowledge and experiences, as well as how these impact their engagement in tasks. One-way students can connect prior knowledge and make connections with cognitively demanding tasks is through problem-posing. Another way students interact with cognitively demanding tasks is by creating original problems or reformulating problems (Matsko & Thomas, 2014; Silver, 1994). Problem-posing allows students to reason at the highest level of cognitive demand and truly demonstrate their understanding of a problem. Furthermore, Silver (1997) proposed that true problem-solving involves problem-posing and that true inquiry-based mathematics instruction assists students in becoming more autonomous and respecting mathematics when confronted with cognitively demanding activities. Literature indicates that teachers can learn more about their students the more they can create spaces for student success in which they can become aware of how students think (Cai et al., 2005; Johnson et al., 2017; Silver, 1994; Watson & Mason, 2007).

Metacognitive thinking, which promotes student awareness and regulation of thinking during task enactment, is another way students make sense and connections with mathematics when interacting with cognitively demanding tasks. According to research, students help manage their own thinking during various learning situations, including mathematics and problem-solving (Hufferd-Ackles et al., 2004; Koszko & Zimmerman, 2017). When mathematics teachers use writing and mathematical exploration opportunities (Martin et al., 2017), the teacher benefits from information provided by students in terms of their learning experiences. For example, students who write about their mathematical reasoning use higher-level thinking skills and develop metacognitive skills, so they should talk about their reasoning and listen to others' explanations (Bell & Bell, 1985; Pugalee, 1997). These thinking skills can help students improve their mathematical reasoning and problem-solving abilities and thus deepen their understanding (Pugalee, 2004).

Students Interact by Approaching Mathematical Solutions, Perceptions, and Justifying their Strategies with Oral Discourse

According to the *Professional Standards for the Teaching of Mathematics* (NCTM, 1999), classrooms should be environments in which students are encouraged to discuss their ideas with one another, where intellectual risk-taking is nurtured through respect and valuing of student thinking, and where adequate time and encouragement are provided for mathematical idea exploration. One-way students can interact with cognitively demanding tasks is through oral discourse. According to the National Council of Teachers of Mathematics (2000, 2014), mathematics communication is essential to learning. Students communicate with a discourse by engaging actively in reasoning, arguing their opinion, and talking with other scholars. As a result, the discourse on mathematical ideas allows students with a mathematical promise to defend and prove their ideas to each other, leading to accurate generalizations (Blanton, 2004; Boaler & Brodie, 2004).

Mathematical discourse is individual utterances made by students; however, it can also be viewed as a whole, connected body of responses between teachers and students (Olawoyin et al., 2020). Facilitating meaningful mathematical discourse requires effective mathematics teaching among students in order to develop a shared understanding of mathematical ideas through the analysis and comparison of student approaches and arguments (Huinker & Bill, 2017). In addition, effective mathematics teaching engages students in discourse to advance the mathematical learning of the entire class. Mathematical discourse includes the deliberate exchange of ideas in the classroom, as well as other forms of verbal, visual, and written communication (Sfard, 2001).

When interacting with tasks, students must do more than just follow the steps. Student interactions involving oral discourse have been identified as a key element in students' cognitive development (Forman, 1996; Lampert & Cobb, 2003; Yackel et al., 1991). Furthermore, discourse is the art of presenting and explaining ideas, reasoning, and representations to one another in groups or pairs. When students participate in mathematical discourse, they can carefully listen to and critique their peers' reasoning as well as provide counter-examples. Students may also experiment with other students' strategies and ask questions to learn how their thinking differs (Huinker & Bill, 2017).

Mathematically promising students were assigned tasks that determined what they learned and how they came to think about, develop, use, and make sense of mathematics. Cognitively demanding tasks engage students at a deeper level by requiring interpretation, flexibility, and the construction of meaning (Stein et al., 1996). Students frequently interact with tasks by describing and justifying their reasoning. This can happen during the engagement of the task or when discussing solutions to the task (Smith & Stein, 1998). In a study with second- and third-grade students, Yackel and Cobb (1996) discovered that allowing students to explain and justify their thinking helped them offer different solutions than those already presented (mathematical difference and sophistication). As students develop their hypotheses and explanations, they may consider how their explanation compares to others (Feldman, 1987).

One-way students with mathematical promise can formulate problems, explore, conjecture, and reason logically to determine whether something makes sense is through oral discourse (NCTM, 1991). Furthermore, oral discourse in the mathematics classroom allows students to share ideas and clarify understandings, build persuasive arguments about why and how things work, develop a language for expressing mathematical ideas, and learn to see things from different perspectives (NCTM 1991, 2000). A primary mechanism for developing conceptual understanding and meaningful mathematics learning is the discourse that focuses on tasks that promote reasoning and problem-solving (Michaels et al., 2008). When students interact with cognitively demanding tasks, the promotion of oral discourse results in an interactive nature of discussions.

Since the implementation of the NCTM standards in 2000, there has been a strong emphasis on problem-solving and thinking elicitation. Several cases in the field have shown that the use of oral discourse is a way students interact with cognitively demanding materials (Ambrose, 2008; Mason, 2000; Turner, 2015). According to Carpenter and colleagues (2003, p. 6), students who learn to articulate and justify their mathematical ideas through their own and others' mathematical explanations, as well as provide a rationale for their answers, develop a deep understanding that is critical to their future success in mathematics and related fields.

Students Interact with Mathematics by Using Tools and Representations to Support Thinking

The creation of a learning environment in which students use multiple representations to collect and communicate mathematical ideas is suggested as a major responsibility for teachers (Lesh et al., 1987; Van de Walle & Lovin, 2005). Mathematics education literature has frequently maintained that student representations should be interpreted socially and physically as mathematical phenomena (Ainley & Magnolias, 2015; Blanton, 2008; Johnson et al., 2017; Webb, 2009). During cognitively demanding tasks, teacher and student actions emphasize the use of connections among mathematical representations to deepen student understanding of concepts and procedures, support mathematical discourse among students, and serve as tools for solving problems (Huinker & Bill, 2017). As students used and made connections among contextual, physical, visual, verbal, and symbolic representations, they grew in their appreciation of mathematics as a unified, coherent discipline (Lesh et al., 1987; Tripathi, 2008). The teacher and student actions depicted in the diagram connected to this research provide a summary of what teachers and students do when teaching and learning mathematics using mathematical representations (see Figure 2).

Students interacted with cognitively demanding tasks by describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations to make sense of and understand mathematics. According to previous and current research, choosing which mathematical representations to use to solve cognitively demanding problems is one way mathematically promising students may interact with tasks (Huinker & Bill, 2017; Lesh et al., 1987; VanTassel-Baska, 2020). Students can also use diagrams to help them understand problems or contextualize mathematical concepts by connecting them to real-world scenarios.

According to Casa and Cohen (2003), mathematical writing involves using prose, which could be letters, words, sentences, or figures, to demonstrate their thinking. Drawing also fosters these reasoning opportunities because that is how children represent mathematics. Although children's written and drawn representations of mathematics differ, both are considered part of mathematics instruction. According to NCTM (2000), the purposes of mathematical writing should include communicating about mathematical ideas in order to analyze and evaluate the thinking of others, building conceptual mathematical knowledge, using mathematical language to express ideas, organizing and consolidating mathematical thinking, communicating clearly by sequencing and elaborating on ideas, and using writing representations such as diagrams, numbers, and symbols to connect mathematical concepts and represent real-world relationships (NCTM, 2000, 2014). Furthermore, mathematically promising students may demonstrate their mathematical understanding in novel ways. Research has shown that mathematical writing of all types is meaningful to learning mathematics and represents student thinking (Pugalee, 1997; Gavin, 2016).

Furthermore, when students interact with cognitively demanding tasks, their understanding of mathematics deepens, and mathematical practices emerge as a result of the mathematical structures and task enactment (Huinker & Bill, 2017; Zimba, 2011). The general classification scheme for representational types shown in Figure 2 (see below) reveals significant connections between contextual, visual, verbal, physical, and symbolic representational types (Lesh et al., 1987; Tripathi, 2008). These various mathematical representations enable students to examine concepts through a variety of lenses, with each lens providing a unique perspective that enriches the picture (concept) and interacts with mathematics (Van de Walle, 2005). Furthermore, students’ ability to move flexibly among representations is related to their success with problem-solving (Huinker, 2013; Stylianou & Silver, 2004). Students use mathematical representations as tools to solve problems and interact with cognitively demanding tasks (Lesh et al.,1987; 2008). Interacting with cognitively demanding tasks enables teachers to *elicit and gather* evidence of student understanding from their representations while monitoring key points during instruction (NCTM, 2000). The interaction of the task, teacher, and students, known as the Instructional Triangle (see Figure 3) (Cohen & Ball, 1999, 2000), determines the nature of the opportunity for students to think and reason in the classroom (Kisa & Stein, 2015). These opportunities allow students to emerge as thinkers and doers of mathematics as well as grow in their application of mathematical practices (CCSI, 2010: NCTM, 2014).

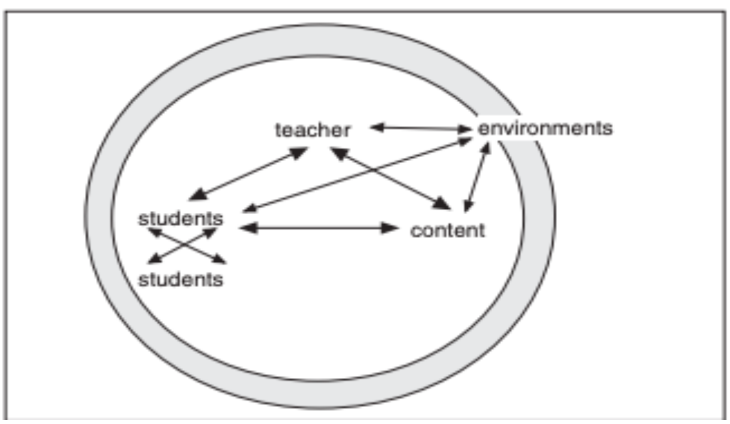

Figure 3
Cohen et al., 2003 Instructional Triangle

Furthermore, when mathematically gifted students process problems, they may generalize and discern mathematical structures, think analogically, and visualize problems and/or relationships. The teachers make the decision to make structures that allow for mathematical practices to emerge available to students. Still, the concrete or representative view of the available mathematics may influence how they perceive and thus solve mathematical tasks. Teachers' known goals and understanding of their students inform their practices as they plan for classroom instruction (Lampert, 1990).

How Classroom Environment Promotes a Shared Meaning and Understanding

Developing reasoning and a deep understanding of mathematics are noted by the *Principles to Action Standards* as characteristics of inquiry-based classrooms (NCTM, 2014). Students can interact in cooperative learning groups and solve cognitively demanding tasks to make sense of mathematical ideas. Students can also benefit from opportunities to work on tasks that are problematic and may take some time to complete. However, not every task provides the same opportunities for students to think and learn (Hiebert et al., 1997; Stein et al., 2009). Such tasks mirror the unfamiliar, difficult, and multifaceted problems for which we prepare them in the real world (NCTM, 2014). Such tasks promote fluency and have a place in the curriculum; however, math application should be a goal of task instruction (NCTM, 2014). Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning, and it is lowest in classrooms where the tasks are routinely procedural (Boaler & Staples, 2008; Hiebert & Wearne,1993; Stein & Lane, 1996).

To create this type of classroom environment, teachers and students collaborate to assist students and help them use mathematics. Bauersfeld (1988) discovered that group interaction is essential in the classroom. When teachers facilitate and attend to student thinking during task interaction, they can construct questions and provide feedback, resulting in increased student interactions (Yackel & Cobb, 1996).

Teachers can shape students' mathematics perceptions and create new windows for growth and thinking, from the learning communities teachers and students co-create in their classrooms to their daily instruction (Boaler & Staples, 2008; Dominguez, 2016). Furthermore, when teachers, like students, actively construct representations of mathematics in their world, the learning situation becomes one in which students are separated from fixed mathematical relationships in a pre-structured environment. Internal representations are often located in students' heads during high-demanding cognitive tasks. In contrast, external representations located in the environment (von Glasersfeld, 1987) are constructed within a shared interpretive framework that constitutes the basis of communication for members of a community (Blanton & Kaput, 2003). The active construction model of learning implies that students build on and modify their current mathematical ways of knowing. Therefore, a detailed understanding of how students interpret situations is crucial to both mathematics instructional development and teaching.

Moreover, focusing on students' interpretations and classroom socio-mathematical practices will not only help students interpret cognitively demanding tasks but will also help students self-reflect and grapple with problems where they may not even know where to begin problem-solving (Huinker & Bill, 2017; Yackel & Cobb, 1996). If students simply dig in and begin experimenting with different strategies to find connections between the problem and other areas of mathematics. When interacting with tasks, elementary students usually work behind closed doors and rarely speak to each other (Dominguez, 2016). As a result, they generate novel ideas to test. Students may also persevere in finding solutions to problems. When students work together, they need more teaching, filling the air with ideas about how to solve problems or what makes sense when interacting with cognitively demanding tasks (Cheng et al., 2011). When students are pushed to articulate their ideas, they produce better sentences that reflect their mathematical thinking, and productive mathematics interactions can occur (Cheng et al., 2011).

In addition, inquiry-based mathematics instruction emphasizes the interaction between teachers and students, as well as how the socio-mathematical norms of the classroom influence the facilitation and learning from cognitively demanding tasks (Cohen et al., 2003; Tools 4

Teachers, 2019). Nurturing these norms entails influencing students' beliefs about their role, the teacher's role, and the nature of mathematical activity in general as classroom norms are guided and recognized (Cobb et al., 1992). For example, Yackel and Cobb (1996) discovered that challenging mathematics activities implemented with high expectations set by teachers contribute to mathematical learning. The socio-mathematical norms are founded on von Glaser Feld’s (1987) constructivism and Cobb et al. (1992) interaction and learning in the mathematics classroom. These norms emphasize the learning-teaching process, which includes the implicit and explicit negotiation of mathematical meanings. During these negotiations, the teacher and students develop the mathematical reality that is assumed to be shared and serves as the foundation for their ongoing communication (Cobb et al., 1992; Yackel & Cobb, 1996).

Student Interactions with Tasks Are Impacted by Teacher Moves

Seventy years ago, mathematician and mathematics educator Pólya (1945) offered advice to math colleagues that still holds to this day: “The teacher should help, but not too much and not too little, so that the students shall have a reasonable share of the work” (p. 1). For students to have productive interactions with cognitively demanding tasks, the teacher’s orchestration of these tasks is key to setting up the norms within the classroom. Since the second aspect of teacher noticing is reasoning about or interpreting classroom interactions, teacher noticing influences how students interact with tasks (Kisa & Stein, 2015; Sherin et al., 2011). As students interact with cognitively demanding tasks, teachers must learn to pay more attention to student interactions and conversation than mathematics itself (Pólya, 1945).

More recently, Sherin et al. (2004, 2011) demonstrated how teacher noticing could assist students in organizing and consolidating their mathematical thinking. During mathematics tasks, teachers should help students analyze and evaluate the mathematical thinking and strategies of others, as well as use mathematical language to express mathematical ideas and justify their thinking (NCTM, 2000, 2014). Furthermore, when teachers observe student thinking and strategies, they encourage them to use a variety of approaches and strategies to make sense of and solve tasks. Posing high-demand tasks on a regular basis encourages mathematically promising students to explore tasks without taking over their thinking and is an important practice in promoting teacher noticing.

Another teacher action that may impact classroom and student interaction during task implementation is how teachers set up mathematics in the classroom, which does not guarantee that students will think and reason in more cognitively complex ways (Kisa & Stein, 2015). According to research, high-cognitive-demand tasks are the most difficult to implement correctly and are frequently transformed into less demanding tasks during instruction (Stein et al., 1996; Stigler & Hiebert, 2004). What is important is that a task allows students to actively engage in reasoning, sense-making, and problem-solving so that they can develop a deep understanding of mathematics (NCTM, 2014). Therefore, teachers should focus on students' thinking and sense-making efforts as they interact with tasks.

Furthermore, teachers can modify tasks in a variety of ways to increase or decrease cognitive demand or interactions (Stein et al., 2000). Teachers should implement cognitively demanding tasks regularly, consistently, and without lowering their demands (Boston & Smith, 2009). However, several studies in elementary mathematics classrooms have found that teachers frequently reduce cognitively demanding tasks by breaking tasks into smaller subtasks, focusing on correct answers and procedures, or adapting the tasks (Doyle, 1988; Romanagno, 1994; Smith & Stein, 1998). Besides, how teachers maintain cognitive demand impacts student interactions with cognitively demanding tasks. Indeed, multiple studies show that teachers frequently assign tasks with low cognitive demands that focus on procedural mathematics even when tasks are written or intended to have high cognitive demand, therefore decreasing student interactions with the tasks (Henningsen & Stein, 1997; Martin et al., 2017; Stein et al., 1996).

Moreover, the teacher's task design does not guarantee that students will demonstrate complex mathematical thinking (Stein et al., 1996). However, when a teacher presses for explanations or elicits thought, they help to maintain a high level of cognitive demand, which may increase students' deeper interactions with tasks (Boaler & Staples, 2008; Hiebert & Wearne, 1993; Stein & Lane 1996; Stigler & Hiebert, 2004; Tarr et al., 2008).

Additionally, Hufferd-Ackles et al. (2004) also described a framework for transitioning to a discourse-centered classroom community. When students interact with cognitively demanding tasks, they examine how teachers and students progress through levels in shifting from a classroom in which teachers play the leading role in pursuing student mathematical thinking to one that is inquiry-based, where teachers assist students in taking on important roles in discussions and representing their mathematical thinking. The framework divides growth into five categories (Hufferd-Ackles et al., 2004). When teachers press students, especially those who are mathematically talented, to respond to tasks, they foster student engagement, justification, and connection. Smith and Stein (1998) researched five practices that facilitate oral discourse with math tasks and outlined them in their *Practices for Orchestrating Mathematical Discussion*:

1. Anticipating student responses to cognitively demanding mathematical tasks
2. Monitoring students' responses to the tasks during the explore phase
3. Selecting students to present their mathematical responses during the discuss-and-summarize phase of the task
4. Purposefully sequencing the student responses that will be displayed

5. Sharing out-of-class strategies and discussing how they solved tasks

Teachers assist students in making mathematical connections between their responses, the responses of other students, and key mathematical ideas. Given the amount of research in mathematics, it has been discovered that instruction designed to promote understanding and problem-solving expertise as it relates to different learning outcomes involves oral discourse (Hiebert & Wearne, 1993). Students' justification of thinking also assists teachers in maintaining cognitive demand at a high-level during mathematics instruction (Smith & Stein, 1998). However, simply providing teachers with challenging mathematics activities is insufficient for their implementation (Leikin, 2011). According to Leiken (2011), teachers must be provided with multiple opportunities to advance their knowledge and develop commitment and beliefs in their own and their students' abilities for high-level mathematical performance. To implement these effective mathematical teaching practices, Sheffield (2009) recommended that teachers pose problems that allow all students, including the most talented, to struggle. These recommendations include expecting coherent explanations and critiques of unique and creative solutions, giving formative and summative assessments that provide opportunities for students to reason, create problems, generalize patterns, solve problems in unique ways, and connect various aspects of mathematics; and acting as a role model who is comfortable with making mistakes and demonstrating the joy of solving difficult problems (Sheffield, 2009).

Characteristics of Mathematically Promising Students

According to Gavin and colleagues (2016), mathematically promising students benefit from advanced mathematical activities focusing on mathematical modeling of real-world problems, such as those associated with cognitively demanding tasks. When learning mathematics, students with mathematical promise may exhibit any or all of the following characteristics: first, mathematically promising students are adaptable to problem-solving because they can easily switch strategies (Gavin et al., 1996; Gavin, 2011). Mathematically promising students tend to see the world through a mathematical lens and rapidly and broadly generalize mathematical relations and operations (VanTassel Baska, 2021). Furthermore, students with mathematical talent have limitations because they can skip steps when solving problems. The use of cognitively demanding tasks also assists students in entering the task where their brain can access whatever mathematics is required to solve it, even if a concept is accelerated. Furthermore, mathematically promising children are often inclined to learn things independently and are tempted to solve problems beyond their current abilities using novel methods, introducing substantial amounts of error and frustration (Freehill, 1961).

Cognitively demanding tasks can also benefit mathematically promising students because one of these students' characteristics is the ability to view the world logically. Mathematical tasks provide logical real-world problems, even though they may be poorly structured (Silver, 1994). Furthermore, formalization is a characteristic of mathematically promising students because they can see the overall structure of a problem and generalize from examples. Asmus (2016) tested Käpnick's items with second graders and found the following characteristics of mathematical talent in early primary school children:

- ability to memorize mathematical issues by drawing on identified structures
- ability to construct and use mathematical structures
- ability to switch between modes of representation
- ability to reverse lines of thought
- ability to capture complex structures and work with them
- ability to construct and use mathematical analogies

- mathematical sensitivity
- mathematical creativity

Furthermore, the ability to rapidly apply mathematical concepts, identify patterns, think abstractly, use flexibility when approaching problem-solving, and transfer mathematical concepts to an unfamiliar situation, as well as use persistence and resilience in solving challenging problems, are characteristics of mathematically promising students (Stepanek, 1999).

Mathematically Promising Students Need Cognitively Demanding Tasks

Teachers should use mathematics instruction that has low entry points and high ceilings, so students will benefit most when they can elicit their original thinking and rely more on their problem-solving efficiency and interactions with others (Jacob & Andrew, 2008; Mason & Watson, 2007; Turner, 2015; Yackel & Cobb, 1996). In addition, teachers can deepen the mathematical understanding by employing a "toolbox" of strategies that benefit mathematically promising students, such as justifying and proving the reasons behind arithmetic operations, solving challenging problems in a variety of ways, and allowing students to pose and solve related problems (Gavin, 2011; Sheffield, 1999).

As teachers facilitate cognitively demanding tasks, they should be careful to include relating the task to what students already know, investigating the problem, evaluating the findings, and discussing solutions, as well as facilitating opportunities for students to pose problems and explore. Furthermore, encouraging multiple solutions, models, methods, and problem-posing has been shown to be effective in developing students' mathematical promise (Gavin et al., 2016; Sheffield, 1999). Teachers can also encourage increased student interaction with tasks by providing multiple entry points through the use of various tools and representations (Smith & Stein, 1998).

According to the National Association of Gifted Standards and other studies in the field, the elementary mathematics curriculum should adapt to the student's needs rather than forcing students to adapt to a curriculum (NAGC, 2019; Tomlinson & Eidson, 2003; VanTassel-Baska, 2020). Tasks should push students to the point of frustration or boredom. Gavin et al. (2016) also conducted a comprehensive study of mathematical creativity, concepts, and problem-solving and found that mathematical creativity is an indicator of mathematical promise. Furthermore, stretching understanding through the creation of new knowledge is especially important for mathematically promising students who are frequently confronted with repetitive tasks, memorized algorithms, or arithmetic skills they have already mastered. As described in Chapter Three, this study was guided by the need for this challenging curriculum and mathematical creativity to be the driving forces behind task interactions among students, teachers, and curriculum.

CHAPTER III: RESEARCH DESIGN AND METHODOLOGY

Overview

This study aimed to examine elementary mathematically promising students' experiences while exploring cognitively demanding mathematical tasks. The study specifically examined the following research questions:

RQ 1: How did elementary mathematically promising students interact with cognitively demanding mathematical tasks?

RQ 2: How did mathematically promising students use mathematical practices as they completed cognitively demanding tasks?

This interpretive study used a descriptive approach developed by Creswell (2013) to examine and interpret experiences and student interactions with cognitively demanding tasks (Creswell, 2013; Miles & Huberman, 2004). This study also used qualitative interpretive methods, where the researcher was an observer within the constructivist context of two 3rd-grade mathematics classrooms to seek and understand the students' experiences. Finally, this study focused on the interactions and use of mathematical practices of the mathematically promising students while they participated in math tasks within the classroom (Ainley & Margolinas, 2015; Cohen et al., 2003; Nasir, 2002; Yackel & Cobb, 1996).

Furthermore, the data or units of inquiry helped the researcher in understanding processes over time and in providing detailed information about the small groups of students whose teachers had identified as mathematically promising or in need of a challenge and ill-structured mathematics instruction in their classroom setting (Kvale & Brinkman, 2015; Ravitch & Carl, 2019). These students were chosen because their teachers felt they demonstrated an opportunity for engagement in mathematics and creativity and had already mastered grade-level content.

Moreover, this descriptive qualitative methodology provided greater context for understanding the relationship between the student's interactions with cognitively demanding mathematical tasks, the student's organic experiences with such tasks, and how they influenced the classroom (Ravitch & Carl, 2019). Within the context of a real classroom, the researcher was the primary instrument in the study to collect data through video and audio observation, as well as student-written artifacts from the tasks. Also, in-depth focus groups, classroom observations, and a review of documents (assessment of students' work on tasks) were all part of the study. In this descriptive study, the verbal, written, and socio-mathematical norms of the participants' interactions, as well as vignettes of students' experiences, were highlighted as forms of discourse. Besides, these experiences were collected from anecdotal field notes and memos based on lesson observations and analysis of student groups, including drawings, problem-solving, and mathematical writing samples (Creswell, 2013; Ravitch & Carl, 2019). The methodology used was intended to provide greater context for understanding the relationship between students' interactions with cognitively demanding mathematical tasks, the organic experiences of the mathematically promising students with such tasks, and how they impacted students' mathematics learning and engagement (Ravitch & Carl, 2019).

In addition, the descriptive qualitative research involved in this study constituted experience and reflective activity situated within the socio-mathematical norms of the classroom and the thick description of the context of two 3rd-grade inquiry-based classrooms (Cobb et al., 1992; Yackel & Cobb, 1996). The use of qualitative interpretative research for this study was chosen to demonstrate the natural experiences of the students and the environment being studied, as well as to describe the nature of the material and data generated from the study (Clandinin & Connelly, 2000; Creswell, 2013). Another reason for choosing descriptive qualitative research for this study was that the findings from this study could influence the process of making sense of mathematics instruction and creating or implementing future pedagogy, especially for mathematically promising students (Barnett, 1998; Shifter, 1996). For example, Cooney (1999, p. 184) advocated for the development of a framework for conceptualizing teachers' ways of knowing, which would contribute to our insight and wisdom in mathematics teacher education. However, the possible use of student interactions and perceptions as a pedagogical tool for enhancing *mathematical* learning has rarely been explored.

Setting and Participants

Setting

The study was conducted in two elementary mathematics classrooms where the teachers were familiar with cognitively demanding tasks and the students were already familiar with various classroom norms, such as using math talk with a partner to discuss and share their ideas and eliciting their thinking through questions. The study was conducted in a public school in North Carolina during a four-week period in the spring of 2022. All participants attended or worked at Riverview Elementary (pseudonym), a suburban community in the southern United States, where the school's mission was to differentiate the social and emotional needs as well as the challenges of each child for them to achieve their full potential regularly. After receiving approval from the school's principal, I was invited to present my study to a group of two teachers whom the principal had suggested as potential participants in the study.

Teacher Participants

Two study participants were teachers A and B, each with over twenty years of teaching experience. Both teachers in the study were selected from a purposeful sample of two teachers with experience teaching cognitively demanding tasks. Both teachers were committed to being part of the study and participated fully in the entire study. Besides, both teachers had previously received professional development in task-based instruction; one had a teaching certificate in gifted education (North Carolina Academic and Intellectually Gifted Certificate [AIG]). As a district math coach, I collaborated with these teachers and students during the enactment of the math activities. Prior to the study, I had already interacted with the teachers through professional learning and team planning and also observed them teach math. In addition, during the study, I served as an observer. At the same time, the students completed the task while the classroom teacher facilitated it, and I kept my subjectivity and bias in check, as explained below (Creswell, 2013; Crossman, 2020).

Recruitment and Student Participants

The study began with the recruitment of twelve 3rd -grade students. Their teachers selected them because they believed these students would benefit from more rigorous mathematics instruction, therefore forming a purposeful sample. Their teachers also identified these students as mathematically promising because they believed that they would benefit from advanced mathematical activities that focused on mathematical modeling of real-world problems, like those of cognitively demanding tasks. Teachers also indicated that some students had “curtailment” because they often skipped steps when problem-solving. In addition, they also felt these students were mathematically promising because they could learn things on their own and were often tempted to solve math questions using novel methods that might be beyond their current grade level (Freehill, 1961; Gavin, 1996).

Furthermore, the seven student participants in this study (see Table 5) were full-time students at one K-3 school in a suburb of a major city in the southeastern United States. The purposefully selected group of seven students is described below in Table 5. Each student was given a pseudonym. Factors such as student race, underrepresentation, and the final AIG identification made by the district where the study was conducted are included in the table to help the reader understand the participants. Brief narratives showing some characteristics of the mathematical promise of each student can also be found below.

Table 5

Mathematical Promising Student Participants

Student Letter	Teacher	Student Race	Student AIG qualification after study	Underrepresented group
A	A	Indian Female	Identified as AIG Both	x
B	B	Asian Male	Identified as AIG Both	x
C	A	Black Female	Not identified	x
D	B	White Male	Identified as AIG Math	
E	B	Hispanic Male	Identified as AIG Math	x
F	A	Hispanic Female	Not identified	x
G	A	White Female	Identified as AIG Reading	x

Note. Confidential Data from Participants in Classrooms A & B (April 2021)

Students' Academically Gifted Identification

Despite the fact that ability is only one factor of mathematical promise, it is often used to determine formal placement into Academically Gifted Local Education Associations or LEAs. Prior to the start of this study, the participants, and all students at Riverview Elementary participated in a universal district screener, the CoGAT 7. During the study, the teachers were unaware of any information on the formal aptitude of these students; therefore, their instructional decisions were not biased. Upon the study's conclusion, five students were officially identified by the following LEA criteria as academically gifted, as shown in Table 5. The main pathway for admission into the local academically gifted program is an ability score of 85% or higher on either the quantitative or qualitative section of this assessment. To be identified as gifted, students were expected to score 40 out of 50 points on a rubric in either or both subject areas. Up to thirty points could be earned for aptitude and twenty points for state achievement scores. Additionally, students from underrepresented groups also had additional opportunities for points from a HOPES teacher survey or environmental considerations if they were from underrepresented populations.

Procedure

As the researcher, I participated in the interpretation of student interactions. This aided me in gathering data on the experiences that influenced student-student interactions and how the students applied mathematical techniques in their mathematics classes (Creswell, 2013). Within the qualitative case study, I positioned myself as an observer, a co-learner, and a co-inquirer. I selected a purposeful sample of two elementary mathematics teachers with experience in facilitating cognitively demanding math tasks in their classrooms. This study included seven student participants from both classrooms; the teachers had identified these students as mathematically talented based on traits such as high ability, creativity, and the ability to skip steps and ask questions beyond what was required for grade level curriculum (Creswell, 2013; Robinson, 2014).

In order to answer all questions related to the study, I observed the interactions of mathematically promising students during a series of five 30- to 45-minute activities in two classrooms (see Appendix F). Accurate data was collected from the four-week descriptive and qualitative study in two classrooms in a suburban school district in the southeastern United States. Furthermore, data were collected using one focus group from each classroom with all the student participants. Through weekly classroom observation of the cognitively demanding tasks, I collected student work samples, then reflected and recorded field notes and memos about the activities and interactions examined within the classroom setting.

Data Collection

Data were collected to answer research questions about how mathematically promising students interact with cognitively demanding tasks. The study was conducted over the course of four weeks in the spring of 2022. Data collection began by first recruiting and obtaining teachers, parents, and student consent for the research. Then, pre-focus groups were conducted via audio recording. Next, I observed a small group of students for one or two days each week while they completed a series of five cognitively demanding tasks. The timeline in Table 3 (see Appendix E) provides the data and time when I observed each task. The teachers chose a series of tasks, including the initial and final tasks from the third-grade fraction cluster (Tools 4 Teachers, 2019). In addition, I observed the tasks selected and documented the sessions in audio and video (see Table 3).

During the weekly classroom observations, I acted as an observer with these questions in mind. Did they interact with other students? Did they use metacognitive strategies and think aloud? Did they immediately draw a symbolic representation of the mathematics they understood, or did they ask for help from the teacher? In the classroom, I observed the “explore phase" of the task and students' engagement as they solved problems throughout the exploration and/or discussion phase. I kept taking field notes and videotaped the entire small group of students as they interacted with the exercises and discussed their responses. As the students were reading, responding, and doing mathematics with the cognitively demanding tasks, I used the audio recording to capture their conversations and interactions (Ball & Cohen, 1993; Miles & Huberman, 2004).

Furthermore, while teachers engaged students in the problem-solving of each cognitively demanding task in the research, they were confused at times on how to guide students through the unfamiliar scenarios described in each task or how to facilitate the task without lowering the cognitive demand. According to research, tasks with high cognitive demands are the most difficult to implement and are often transformed into less demanding tasks during instruction (Stein et al., 1996; Stigler & Hiebert 2004). For this reason, I coached the teachers during a 20-minute pre-task weekly meeting to talk about task selection, facilitation, and teaching strategies between each task. I specifically addressed the task enactment and discourse that the teacher offered for the student to engage in during the task. Student collaboration and ways students may represent or use discourse during the tasks were also discussed. In addition, the study's teachers and I spoke about what they would notice and what questions they might ask during the task enactment. Then, after the facilitation of each task, the teachers filled out a brief Google form (see Appendix C) sharing their reflections on how the students used mathematical practices. In the form, the teachers shared any questions or modifications they needed guidance before facilitating the next task in the study.

Research Question 1: Data Collection.

To answer Research Question One, I videotaped and audiotaped the cognitively demanding tasks and observed student interactions while the student communicated and worked on the task. Videotaping was chosen for this research based on previous findings from Cobb and Jackson (2011), which indicated that various aspects of a high-quality setup of a mathematical task arose from watching video recordings of elementary mathematics teachers' instructions, all of whom were attempting to implement cognitively demanding math tasks. Also, this study's data was based on video and audio recordings, so student interactions were effectively captured. During the observations, I specifically videotaped the smaller table group of mathematically promising students during all parts of the cognitively demanding tasks using school-owned Swivel equipment.

Furthermore, all aspects of the cognitively demanding task were audiotaped, including when the teacher introduced the task to the students, throughout the task's problem-solving engagement, and after the students discussed the task. I also used Otter.ai (Liang & Fu, 2016) audiotaping transcription software and gave each student participant a letter identifier between the small group pairs within the table group. This helped me listen to student interactions and metacognitive thinking and better understand their "windows of thinking" (Dominguez, 2016).

To answer Research Question One, I also conducted two focus groups, a pre-and a post-group, to analyze student responses, emphasizing how the students viewed their interactions with cognitively demanding tasks (see Appendix A and B). The focus groups occurred at the beginning and end of the study after all mathematical tasks had been taught (see Table 3). The students were interviewed in small groups for up to 30 minutes in a separate room within their school building using audio recording. During the interviews, I focused on students' experiences with the cognitively demanding tasks and their reflections about their interactions during each lesson to gather a view of their mathematical experiences and engagement with cognitively demanding tasks (see Appendix A and B).

Research Question 2: Data Collection

I gathered data for Research Question Two by analyzing how mathematically gifted students used mathematical practices as they completed cognitively demanding tasks. During all class observation, I took open field notes and paid close attention as I watched student interactions, examining all evidence of students' mathematical experiences and engagement with the cognitively demanding tasks (Miles & Huberman, 2014). After each observation, I took the field notes and categorized them into the same four broad categories in a table based (see Table 2) on the preset codes of how students interacted with cognitively demanding tasks according to the *Principles of Action* as I did for Research Question One (NCTM, 2014).

To answer Research Question Two, I also used document analysis to collect, analyze, and hand-code students' work samples from cognitively demanding tasks. Then I made a summary memo about each task. According to Ravitch and Carl (2019), fieldwork and data collection should be systematic. As such, I engaged in memo writing throughout the study and focused on how students used mathematical practices to solve each task. First, I began with a memo after the pre-focus group meeting. I also used the memos to record my general impressions after recording each task and reading the teacher's comments in the Google debrief form. Then, I wrote one final memo after the post-focus group. In the memos, I merged vignettes from the document analyzed and what I observed in the classroom with students' interactions within the socio-mathematical norms. These memos were then used for my thematic coding and the pre-and post-focus group transcripts. In the Appendix, I have included some extracts from my notes and analysis so you can understand how they led to the findings of this research question (see Table 9).

Records: Instruments and Data Collection

I used several methods of data collection to align with my research question. Table 4 in the Appendix shows the methods of data collection I used. Table 4 (see Appendix E) describes the data collected type, the data collection origin, the data analysis method, and the research question alignment. The study's main data collection methods were observational field notes and memos, audio- and videotaped classroom observations, pre- and post-focus groups, a student working via document analysis, and a teacher Google debrief form.

Furthermore, the research also used mathematical activities from the North Carolina Open Educational Resource Tools for NC Teachers (https://tools4teachers.com). These resource tasks were chosen using the cognitive demand framework as the basis for task selection in this study (Smith & Stein, 1998). Although these tasks were designed for the core curriculum, I worked with the teachers to train them on the use of mathematical practices and instructional strategies to distinguish them within the context of their classrooms. The lessons have been used by the school district selected within the study as the basis for cluster mathematical planning; as such, the teachers in the study were familiar with the curriculum.

Within the curriculum, I used the tasks within Cluster (Unit) Seven, *Understanding Fractions as Part of a Whole,* which focused on four new content standards at this grade level (see Table 4). I also selected one to two cognitively demanding tasks per week, which each classroom teacher facilitated with a purposefully selected sample group of four female students (Classroom A) and three male students (Classroom B). The students were purposefully selected because their teachers suggested they possessed characteristics of mathematically promising students (Gavin, 2016; Sheffield, 1999). All of these students achieved proficiency or higher on all formative assessments in class, exceeding grade-level expectations. The teachers also noted that students naturally had a lot of questions and out-of-the-box ideas to share during classroom discussions; as such, they felt they had mathematical potential. Finally, teachers wanted the students to be challenged because they wanted to see what they could do with the opportunities given because they seemed bored with basic procedures in math class (Creswell, 2013; McCormick, 2016).

Furthermore, all the tasks classified by standard within the study (see Table 4) were available to teachers to select from. Within the study, both sets of student participants were required to solve the first task, "*Piece of Yarn,"* and the final task, "*Sharing Licorice,"* which included making connections using a length model. Teachers then selected three other tasks from the mathematical standards (NC.3.NF1, NC.3NF.2, NC.3NF.3, & NC.3 NF.4) to best meet the differentiated needs of their learners (see Table 4) for a minimum of five tasks. At the beginning of the study, the teachers within the study and I looked at the sequence of tasks for this cluster. We addressed task selection and assignment before we began the study, as well as at each pre-task meeting. We also discussed any other prior knowledge or information the teachers wanted to share about their students and the rigor they hoped to facilitate during the study.

Throughout the study, I met with the teacher participants for approximately twenty minutes after each task to debrief (see Table 3). I discussed the tools, student grouping, and how the teacher facilitated the task to differentiate each student's needs. I also coached the teachers about any changes they felt were necessary prior to the next task. To mention a few, I discussed any questions or ideas the teacher had about adding rigor, eliciting the thinking of their students, or differentiating the task prior to the next task in the series. Following each teacher pre-task meeting, I allowed the teachers to ask questions and receive coaching support via the Google debrief form before I observed, audio recorded, and videotaped the next task. As noted in

Appendix C, the form helped teachers note the mathematical practices students used to solve the tasks.

After the initial task and teachers' meeting, I decided to rearrange the tasks (see Table 5) due to teacher recommendations. Teacher B, for example, noted that drawing number lines was difficult for this group of students, so she asked me to select tasks where they had to draw and partition number lines. I also observed students' interactions as well as teachers' perceptions of the tasks. I used a Google debrief form for the teachers to reflect upon their instructional practices after each meeting and observation to help guide the study's conceptual framework. Although students' interactions and evidence were the focus of the study, teachers' interactions and practices with students and the task played a role in the interactions with the socio-mathematical norms of the classroom (Ball & Cohen, 2003; Yackel & Cobb, 1996).

Procedure

First, I created an informed consent form and confirmed participation in the study through a secure paper document. Then, I scheduled a meeting with both teacher participants to recruit them for the study, give them the consent forms, and ask them to return the forms that week. Throughout this process, I acknowledged the participants' rights and preserved their identities (Ravitch & Carl, 2019). Then, I developed a pre-and post-focus group protocol with questions before (see Appendix A) and after (see Appendix B) the study that focused on Research Questions One and Two, student interactions with cognitively demanding tasks, and which mathematical techniques these mathematically promising students used while they completed cognitively demanding mathematical tasks.

Before and after I observed the series of five tasks (see Appendix F), I used the focus group protocol to conduct a focus group with a small group of students (see Appendix A and B) (Kvale & Brinkmann, 2015). I held both pre- and post-focus group sessions in a separate room for 30 minutes with all student participants during enrichment time in the morning when other students were completing differentiated work or working in small groups with the teacher so as not to disrupt other classroom teachings. In addition, Table 3 (see Appendix E) shows the timeline of my study across the four-week period and when the focus groups were conducted. I also created memos to reflect on each focus group, particularly to connect my themes for Research Question Two.

Data Analysis

Thematic analysis is a process of analyzing data that focuses more on situated, interpreted, and lived experiences than transcendental experiences (Ravitch & Carl, 2019). After coding the significant statements and creating a code book, I used these codes to form sub-themes and themes in response to the two research questions. I also examined data and answered research questions by interpreting and analyzing the qualitative case study data on students' thoughts and interactions. The data analyzed included videotaping, audiotaping, focus groups, field notes, memos, and document analysis of the students' mathematical tasks. The experiences were viewed as socially situated knowledge constructions, valuing the messiness, discourse, and detailed description of the student's experiences during the tasks (Ravitch & Carl, 2019).

Coding Process

To begin coding, I read and reread the transcripts (Ravitch & Carl, 2019, p. 266) and began developing an iterative code set (see Table 1). Then, to gather all my data sources, I began a secure Google spreadsheet of carefully color-coded descriptive codes and themes. According to Maxwell (2013), these substantive codes are often generated from inductive, open coding "through multiple readings for each kind of coding." As such, I took the analyzed open codes I created from the observations and field notes and created axial or pattern coding for connections between codes (Miles et al., 2014). After that, upon triangulating the data through the constant comparison method and color-coding my code book to help me find the themes and sub-themes, I added notable quotes from each data source and participant to the code book.

Furthermore, I compared codes and created categories that linked codes together (Glaser & Strauss, 1967). Next, I assigned codes to each theme and went through chunks of data to see what codes were related to each in order to generate findings. I also looked for repetition, strong emotions, and language, as well as an agreement between individuals and a disagreement between individuals (Ravitch & Carl, 2019). Once I developed many codes, I began defining them in the spreadsheet. Short sentences and phrases were also created (see Table 7). I tried to make the definitions clear and concise as I reflected on the codes and used them to connect them to my memos and research questions.

According to Glaser and Strauss (1967), the comparative analytical method can be applied to social units of any size to generate a theory. Therefore, after collecting additional data, I used this constant comparison method to return to analyzing and coding the data, and from that analysis, I developed a process to inform the next iteration of data collection. I also continued comparing my codes and color-coding common themes between the observations, focus groups, and document analysis as suggested by research until a strong theoretical understanding of students' interactions and experiences during the series of tasks emerged. Below, I explain the process of data analysis I used to analyze each source separately and then triangulate to generate my themes and findings for each research question.

Research Question 1. To analyze Research Question One, "How did mathematically promising students interact with cognitively demanding math tasks," Miles and Huberman's (1994) inductive coding were applied, and I used a thematic analysis approach. I first analyzed each source of data collection one at a time. Then, I analyzed students' physical, symbolic, visual, written, and oral representations from the cognitively demanding using preset codes to categorize the raw words from the transcripts of the video and audio-taped lessons to categorize various student interactions based on a provisional "start list" of codes from the mathematical practices in *Principles to Action* (see Table 5) (Kvale & Brinkmann, 2015; Miles & Huberman, 1994; 2014, NCTM, 2014; Ravitch & Carl, 2019). The following categories come from the recent work, "*Taking Action: Implementing Effective Mathematics Teaching Practices* and the *Principles to Action*." These practices were originally noted as critical student actions that the National Council of Teachers of Mathematics (2014) suggested that all students show if they are truly interacting with tasks that promote reasoning and problem-solving (Huinker & Bill, 2017; NCTM, 2014, p. 24). Table 6 below shows the preset codes developed from the *Principles to Action* (NCTM, 2014, p. 24) that I used as the preset codes for this study.

Table 6

Pre-Set Mathematical Practice Codes

Practice #	Code	Practice category
1	PER	"Perseverance: (PER) Are students persevering in exploring and reasoning through tasks?" (NCTM, 2014, p. 24)

2	MCK	"Making Sense/Connections: (MCK) Are students taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas?" (NCTM, 2014, p. 24)
3	MR	"Multiple Representations & Tools: (MR) Are students using tools and representations as needed to support their thinking and problem-solving?" (NCTM, 2014)."
4	JS	"Solutions and Strategies & Justifications: (JS) Are students accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another and will analyze the frequency of each type of code to determine which type of interaction occurs most frequently?" (NCTM, 2014, p. 24)

Note: Adapted from NCTM, Principles to Action (2014)

I began the data analysis process with the audio- and video-transcribed observations from Teacher A. Then I moved on to the analysis of Teacher B, followed by the pre-and post-student focus group transcripts. Then, I began highlighting and assigning codes to significant statements and generating themes and subthemes from the classroom observations and focus group transcripts.

Classroom Observation Analysis

Audio and Video Transcripts. To answer Research Question One, I analyzed Teacher A's audio-taped classroom observations, followed by Teacher B's audio recordings. I also carefully listened to each audio recording using Otter.ai software. Next, Otter.ai software (Liang & Fu, 2016) was used to analyze audio transcripts and recordings of the set of five tasks. Each task took an average of twenty-five to fifty minutes to complete, record, and then be transcribed and uploaded into a secure Google Drive.

Next, I began the process of open-coding the audio transcripts, highlighting text sections, and labeling them (Ravitch & Carl, 2019). I first began color-coding common words and phrases that kept occurring, like "*compare*," "*show me*," etc., and then made a list of common raw words in each transcript. Next, I color-coded the transcript to find open codes (see Table 8). Once I found groups of phrases related to the preset words or similar, I took the phrases of the same color and pasted them together in chunks in a secure Google sheet. The analysis of the classroom observation data was similar in that I viewed the video clips multiple times for each of the five cognitively demanding tasks.

I also coded the transcripts, searched for direct statements from the audio recordings, and then watched and compared the videotaped interview with my field notes. As I watched student interactions from the videotapes, I added comments to the audiotape transcripts and wrote comments about each lesson to describe student interactions on the Google coding spreadsheet. In Chapter Four, I will present findings from my analysis in a table and give examples of these open codes.

Field Notes. I created encrypted Google documents with field notes for each classroom observation to analyze them after each task. I also read and studied the field notes while I played the videotaped observations of the classroom tasks. In addition, I specifically looked at the notes I made on the field notes relating to Table 6 (see above) and the preset mathematical practices and interactions. While I read the field notes, I replayed and stopped the video every time the teacher engaged the focus group and looked at the students' verbal and nonverbal cues. I used the same color-coding scheme to classify the field notes that I used to identify the categories (see Table 7). For example, I classified the use of mathematical representations in purple, math knowledge shared by students in yellow, mathematical writing examples in green, and questions the students asked each other, or examples of oral discourse in red. Finally, I used audiotaped observations to look deeper into the data.

Pre- and Post-Focus Groups. According to Kvale & Brinkmann, a focus group contains six to ten general subjects (Kvale & Brinkmann, 2019). When analyzing the data from this study, my focus group interviews were composed of seven students from both classrooms. The pre-and post-focus groups were audiotaped using Otter.ai software (Liang & Fu, 2016), and each took an average of 30 minutes. They also allowed me to learn more about the experiences and backgrounds of each student's learning in mathematics when interacting with the tasks and their conceptual understanding when exploring and solving the cognitively demanding mathematics tasks (see Appendix A & B).

Furthermore, the focus groups were methods of data collection used in this study to encourage a variety of viewpoints from the students and to discuss topics that would create a permissive atmosphere for personal growth with students, task interactions, and the emergence of mathematical practices through the study. I listened to the audio recordings of each focus group two times. First, I looked at the quotes and reactions from students that specifically aligned with both research questions. Then I began highlighting the transcripts from the focus group audio recordings (see Appendix A & B) and made a list of raw keywords generated in the transcripts. From the pre-focus group transcript, the following raw keywords were coded multiple times during the 17-minute audio-recorded transcript: solving, math, problems, question, task, fractions, write, challenge, student, test, draw, strategy, answer, classroom, tools, teacher, pictures, addition, told, and symbols.

In my first reading of the 21-minute post-group transcript, the keywords fraction, tasks, math, number, talk, helped, student, symbols, write, tools, models, solve, multiplication, learn, line, pictures, drawing, split, paper, and partner stood out as important. I then transferred the highlighted raw words and open codes to my spreadsheet along with the original open codes from the observations. Next, I grouped the keywords into codes to generate themes. Table 7 below shows the color-coding scheme I used to analyze and generate my axial codes. Finally, I moved all the highlighted codes into my secure Google sheet, where I began organizing them by theme.

Table 7

Focus Group Open Coding Color Scheme

Color code	Open code
orange	Fractions
Pink	Difficulty
blue	Feelings about math
teal	Comparing
red	Directions from investigator
purple	Representations in math

Note. From original transcript

Research Question Two Analysis

To analyze data from Research Question Two from the focus groups regarding how the mathematically promising students used mathematical practices to complete the cognitively demanding tasks, I used Otter.ai software (Liang & Fu, 2016) to code audio transcripts from the audiotaped focus groups. To further dig into Research Question Two, I used the constant comparative method to compare the codes from the document analysis, student work samples' memos, the focus group transcripts, and the Google debrief form (Glaser & Strauss, 1967). During the data analysis, I focused heavily on coding the students' work from the tasks with document analysis to answer Research Question Two.

With Research Question 2, the codes were inductive, and meaning naturally emerged from the data (Corbin & Strauss, 2015). Furthermore, I first collected the codes from Research Question Two in a separate table on the same Google spreadsheet as Research Question One. Next, I used the same broad categories from Table 6 (see above) and the *Principles to Action to* organize the raw words from the focus group transcripts and memos. I then used interpretive open codes from the focus group transcripts and document analysis of the student work samples to see what patterns emerged from the data (Maxwell, 2013; Ravitch & Carl, 2019). Below, I describe each method of data analysis and how it led to the major findings for Research Question 2.

Document Analysis. For the document analysis of student work from the series of five cognitively demanding tasks (see Appendix F), I hand-coded the work from both classrooms A and B and saw how the student worked on each of the tasks connected to mathematical practices (see Table 6) found with the classroom observations and focus group data. Sociologists typically use document analysis to verify their findings (Angrisano & Mays de Perez, 2000). However, using document analysis allowed for triangulation of the data and confidence in the trustworthiness (credibility) of the findings. In addition, the documents captured the students' strategies for visual and written representation of mathematics, modeling mathematics, as well as justification of their thinking through mathematical writing (see Appendix D).

Document Analysis and Writing. I used document analysis to analyze the student work samples as mathematical writing from each of the five observations. Mathematical writing is one way for students to deal with cognitively demanding tasks using prose, symbols, letters, words, phrases, and sentences to reason their mathematical thought (Casa et al., 2016). Supported by the concepts presented in the research about the mathematical promise, the findings of this research showed the connection with mathematical writing as students showed their reasoning with explanatory, descriptive, and argumentative writing during the tasks (Casa et al., 2016).

In the document analysis, descriptive mathematical writing was used as Student C described what she was thinking aloud as she partitioned the number line. (see Figure 7). Figure

8 shows an illustrative example of mathematical writing as Student E wrote, “Green equals a half because 4/8 equals a half.” Also, Figure 9 shows students’ mathematical writing, which is argumentative and descriptive. Additionally, student E clarified his knowledge by looking at the number line and using writing to explain his thinking.

Document Analysis Codes. From the student work samples from each task, I created a separate table to list the codes from the document analysis of student work for each task (see Table 9). I then hand-coded all of Classroom A’s work samples with A1-A5 and Classroom B’s with B1-B5 in my coding sheet. These codes helped me specify how students used mathematical practices to complete the series of tasks in the study.

To find out the various practices students used to complete tasks, the following abbreviations were used to code and tally occurrences of representations and all interactions students used while solving cognitively demanding tasks during the study. Table 9 (see Appendix D9) reveals the results of the codes collected from Document Analysis, Drawings (D), Comparison of Fractions (C), Labeling (L), Partitioning (P), Explaining (E), Modeling(M), Symbolic reasoning(S), Metacognition (MT), Struggle/Frustration (SF), and Mathematical Operation (MO). Some of these interactions relate directly to mathematical practice, such as M modeling, while others are types of interactions. Table 11 also shows the connection between mathematical practices and document analysis coding.

Google Debrief Form and Memos. Despite the fact that the study focused primarily on student interactions, teachers' activities, such as noticing, questioning, and establishing a socially constructed space where these students with similar abilities (mathematically promising peers) could interact, were noted in the memos. In order to truly capture the use of the mathematically promising students’ skills during task enactment, I used the Google debrief form (see Appendix

C) to help coach the teachers in the study to look deeply into the interactions and how to help build these practices within the conceptual framework of the study. After each task, teachers noted changes and observations of the mathematical practices with the mathematically promising students' interactions during the cognitively demanding tasks (see Appendix F).

Generally, the results from the Google debrief form were used to educate the teachers on how to enhance students' mathematical thinking using the conceptual framework of the instructional triangle (Cohen et al., 2003). I also compared the outcomes between tasks and wrote a summary memo of what practices were emerging with the students. Results were relevant to all themes and coded within the codebook (Ball & Cohen, 2003) (see Appendix D), which revealed that students were challenged by the tasks in the study and were able to make multiple representations and interactions; however, their ability to persevere, reason, and justify their thinking with mathematical tasks increased (see Appendix C).

Finally, as a mathematics coach, I used the results from the Google Debrief form to guide the teachers on how to adjust instruction to bring forth Mathematical practices in students. Appendix C shows the questions I asked teachers after each lesson. Table 11 below shows the four questions I asked and how they related to the practices. I also included the figures in the appendix to help interpret the data below.

Table 11

Google Debrief Form & Mathematical Practices

Debrief Form Question	Mathematical Practice	Teachers' Indication of Practices	Coaching Move
Did the students use multiple representations of their mathematical thinking during the	MP 4-Model with Mathematics MP 7- Look for the Use of Structure	(see Figure 10) -45% somewhat used representations -45% consistently	Teachers reminded students of various representations during the launch of tasks -Teachers engaged

task?		used representations	students during the explore part with questions about their representations (I coached them to use more open questions here)
How do you think the students persevere through the task?	MP 1-Make Sense of Problems & Persevere through Reasoning	(see Figure 11) - Teachers rated students' perseverance from 7-10. - 7(27.3%) - 8(9.1%) - 9(45.5%) - 10(18.2%)	-Teacher A stopped mentioning a time limit after Task 1 -More manipulative choices were mentioned to remind students of multiple representations during task launch -Teachers used similar class experiences to help students make connections
Do you feel students were able to make sound arguments and justifications?	MP 3-Construct Viable Arguments and Critique the Reasoning of Others	(see Figure 12) - Teachers were surprised 10% - Some were better than others (10%) - Many were confused (10%) - For the most part (10%) - Yes, somewhat (10%) - Yes, completely (50%)	-I coached teachers to ask more open questions -*Teacher A* decided to stay with her group, and she did more anticipating their strategies before teaching the task. -*Teacher B* chose to give 5 minutes of silent inquiry time, knowing her students needed more to discuss in the social space, so their arguments stayed focused on mathematics
How do you feel the	MP 1 Make Sense of	(see Figure 13)	Teacher A decided to

students' made connections to the task?	Problems & Persevere through Reasoning MP 3- Construct Viable Arguments and Critique the Reasoning of Others	Teachers rated making connections between a 6-10, 6- 9.1% 7-45.5% 8- 36.4% 10- 9.1%	use real-life objects such as licorice, ribbon, and a weather chart with her Launch once Teacher B shared her Launch strategy in the Pre-task meeting

Focus Groups. Focus groups allowed students to share their experiences and how they completed the tasks in the study using mathematical practices (see Appendix A & B), and I used the focus group transcripts to analyze both researched questions as described above. Also, for Research Question 2, I looked deeply into the mathematical strategies that students used as they completed tasks. Table 10 (see Appendix E) summarizes the students' thoughts shared in focus groups about how they used the mathematical practices and how I coded their thoughtful comments about how they solved the tasks with whatever mathematical practice they used.

Triangulation

For Research Question 1, I compared the triangulated data from the research questions by analyzing the field notes and memos, the coded video and audio recordings, and the focus group data to code the students' interactions. Also, I used multiple data collection measures, which revealed how students interacted with cognitively demanding tasks. This helped me interpret the data more deeply than just using one data collection method in isolation. In addition, the classroom environment, instructional moves by the teacher, or how students' mathematical reasoning and practices were impacted by task interaction.

Furthermore, to ensure a complete and thorough analysis, I triangulated the three methods of data collection for Research Question 2 during analysis, including the focus group transcripts, document analysis of student work, and the teacher Google debrief form. I also classified all the data collected during the analysis of Research Question 2 as belonging to one of the four broad Mathematical Practices defined by NCTM (2014) as the ways students interact with mathematical tasks (Corbin & Strauss, 2015; Huinker & Bill, 2017; Ravitch & Carl, 2019). By examining the study's data, I was able to emerge with mathematical practices (CCSI, 2010a), particularly in the post-focus group interview and their environment, through written and oral forms of communication (Yackel & Cobb, 1996).

Trustworthiness

In this study, trustworthiness was established because of the criterion-based sample; participants were unique to this study and not transferable to another study (Ravitch & Carl, 2019). Even though the specific sample and demographics could be similar in a future study, the data could not be replicated due to the rich and authentic nature of the data collection (Lincoln & Guba, 1985). Furthermore, the study also ensured dependability because the data was member checked by the researcher's advisor. The study also ensured trustworthiness through confirmability because of its interpretive but reflective nature (Ravitch & Carl, 2019). Finally, the constant comparison of the data during data collection and analysis ensured that the data analysis process was complex (Corbin & Strauss, 2015).

The researcher was also self-critical of the research, thus making it valid by writing a subjectivity statement prior to each visit and having a pre-task meeting with each teacher to help ensure reflective thoughts and personal biases were kept out of the recorded research, making it as objective and applicable to the field as possible and ensuring trustworthiness.

Reliability and Credibility

This interpretive study was reliable through multiple methods and triangulated data recording since recording devices and the transcription of digital files were triangulated with handwritten field notes and student work via document analysis (Ravitch & Carl, 2019). To operationalize these terms, long engagement in the field and the triangulation of data sources, methods, and investigators helped to establish credibility. In addition, credibility was also established due to the criteria established for the study's participants.

Limitations

This study was limited because the participants were selected based on purposeful selection. With a purposeful selection of participants, there is always the chance that the participants did not reflect the opinions and views of others in a similar population or chose not to participate in the study. The study can also be considered limited because the participants were chosen from a classroom of students and may be limited by researcher bias. However, since I have experience working as a mathematics teacher, mathematics coach, and gifted specialist in elementary classrooms, my experience gives me a strong opinion about using cognitively demanding instruction.

Subjectivity

Educational researchers like myself should be immersed in and passionate about their theoretical frame, the position of epistemology, and values so that those reading their studies can determine whether the work is applicable in their educational setting. Upon initiating this research, I acknowledged my intellectual and creative stance as a former math teacher, gifted specialist, math coach, educational leader, and researcher in the field. Before collecting data and constructing my participants' stories and identities, I reflected on my own story. I started with my roots, then my family's culture and learning assets, in order to truly look at how my background has contributed to my views of mathematics, culture, and creative learning and thinking. As a researcher, I also reflected upon my personal history buried in my heart and mind so I could truly understand the students' backgrounds and embody a culturally sustaining worldview (Paris & Alim, 2017).

Furthermore, as a white female, this background has influenced me as a researcher. I grew up as a mathematically promising student with mathematical creativity and a very abstract thinker, but I lacked opportunities in a rural school because of my culture. As such, I bring a passion to my work for equity and opportunity for all students. As a young girl, I moved to the city (according to Mamaw because they had a Walmart) at seven. When neighbors saw my banjo-picking, bearded father, they called me a “Redneck.” Upon entering my new school, the teachers knew I came from a “poor” school in the country. So, without any testing or talking to me, they put me in the lowest reading group when I moved to a new school based on bias. It took years for teachers to notice my talents. My family had to fight for me to be tested for the gifted and talented program, and four years later, I was finally placed in advanced classes in middle school. As a growing teen, I was continually exposed to a passion for math by my dad, a carpenter, and my grandma, an accountant.

Moreover, my roots in labor and creative ways of life, such as cooking, planting flowers, and exploring the world, as well as my aesthetic, naturalistic love for the mountains and outdoors, came from my Mamaw. Then, in high school, college, and as an adult, my world was continually surrounded by creativity and scholarship. I became a North Carolina Teaching Fellow graduate, so my roots remained in western NC until I left college. No matter what level of education I have obtained, I know I have a biased lens because I am in the predominant group as an educator. However, as a doctoral student, candidate, and researcher throughout the urban education program, I have learned to balance my hidden biases and experiences and keep this subjectivity in check, so my research is not biased.

To further account for this subjectivity, the researcher included triangulation with data collection methods (Miles & Huberman, 2004). Furthermore, I added reliability measures to the study by having the teacher member check the evaluation rubrics and transcripts. In addition, with a researcher observing how a classroom affected the students' behavior, the participants could deviate from their normal approach to tasks and become self-conscious or behave embarrassedly during observations or focus groups. Thus, I placed myself as the researcher in a slightly distant proximity within the classroom to ensure the camera was videoing students without interfering with their conversations. I also used audio recorders to pick up the intimate interactions between partner groups at the larger table group.

Besides, the classroom observations for the study were made over four weeks to reduce the observation effect. This timeline helped make the video recording devices and the researcher a part of the classroom environment. Since the participants were unaware of the study's actual purpose during data collection, this should have reduced the possibility of behaving or responding in a manner that would "please" the researcher. Thus, the focus groups were not conducted at the initiation of the study or after the observations to allow for a true picture of the experiences of the study and less limitation to the data.

In addition, the regular classroom teachers were involved in the process from the initiation of the study and discussions about the chosen students and the results that occurred throughout the data collection process. The researcher also reviewed the collected data and verified that the behaviors, interactions, and characteristics of the students observed were consistent with how each normally behaves. The researcher and teacher further ensured that the student behaviors were consistent with the characteristics normally displayed in the classroom.

Conclusively, the parameters I used were open for data analysis, as I did not categorize or rate student problem-solving accuracy. Instead, I looked for ways students interacted with the cognitively demanding tasks and emerged in their mathematical practices, such as through Making Sense/Connections with Mathematics (MCK), Multiple Representations (MR), Perseverance when Exploring and Reasoning with Mathematics (PER), and what justifications and solutions strategies (JS) they used during tasks. Did students accept and expect that their classmates used a variety of solution approaches, and did they discuss and justify their strategies to one another? I analyzed the frequency of each type of code to determine which type of interaction occurs most frequently. Lastly, I looked at the data to analyze whether students were using tools and representations as needed to support their thinking and problem-solving when interacting with cognitively demanding tasks to help code the field notes, video recording, focus groups, and student work samples (Huinker & Bill, 2017; Miles & Huberman, 2004; Ravitch & Carl, 2019).

Delimitations

This study was delimited to student outcomes on standardized achievement tests. Therefore, conclusions were not to be extended beyond cognitively demanding mathematical tasks, student perceptions, and student interactions within the socio-mathematical norms of the elementary mathematics classroom. In order to ensure the protection of this study's participants, the researcher carefully followed the guidelines outlined by the Institutional Review Board (IRB). The first consideration involved collecting signed, informed consent statements from all participants. The following safeguards were also outlined in the informed consent statement:

- Participants' real names may have been captured in the video or audio but were not used in the written report for confidentiality purposes. Instead, pseudonyms were assigned to all participants in all verbal and written records, especially when transcribing the audio and video data.
- All materials were locked in a file cabinet to safeguard confidentiality. No videotapes, transcription notes, field notes, or observation notes were used for any purpose other than this study. All related paper materials will be kept in the researcher's locked file cabinet.
- Participation in this study was on a voluntary basis. No children were spoken to or questioned without written consent from legal guardians. Participants had the right to withdraw from this study at any time without penalty.

Summary

During the study, the co-construction of meaning occurred between the researcher and participants. Therefore, the data gathering and analysis process occurred organically throughout the series of tasks. Furthermore, collecting student interactions and perceptions may also help mathematicians and teachers hear content struggles, make sense of students' prior knowledge, and create meanings as they tell or "show" "us" what happened to them. Therefore, it is hoped that the information gained from this descriptive qualitative case study research will help provide descriptions of student interactions, the construction and reconstruction of mathematical perceptions, and evidence of social mathematical environments within elementary mathematics classrooms, particularly for students who need rigorous mathematical thinking. However, although this study was based on the collection of data regarding mathematically promising students' interactions with mathematical tasks, the findings may also be used to suggest that, using highly cognitively demanding mathematics tasks along with the above-listed safeguards, permission was secured from the data collection site to do the study in the school. In addition, a timeline was provided, indicating the projected times for each phase of this study. Socio-mathematical inquiry-based norms as pedagogical tools may have the potential to influence teaching pedagogy.

This study sought to explore student interactions and experiences by employing the qualitative tools of pre- and post-focus groups, classroom audio recording, and videotaped observations, including field notes. Through the voices of students who have experienced success or failure in mathematics, the researcher aimed to examine the perceptions exhibited during a series of cognitively demanding mathematical tasks. This research also presented challenges, as qualitative data can be an in-depth venture to collect, organize, code, and interpret. In addition, the researcher addressed these validity and reliability challenges by planning a triangulation strategy that encourages further accuracy through cross-checking during the interpretation of data. The results shared below provide detailed descriptions of the participants' experiences working with cognitively demanding math tasks through textual descriptions, structural descriptions, and a synthesis of the data.

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CHAPTER IV: FINDINGS

Overview

This qualitative descriptive study aimed to explore the interactions of seven mathematically promising students and investigate how they used mathematical practices to complete a series of cognitively demanding tasks. Using audio- and videotaped classroom observations and field notes, pre-and post-focus groups (see Appendix A & B), a Google debrief form (see Appendix C) for teachers, and document analysis of student work (see Appendix E). Chapter IV also provides the results of the pre-and post-focus group data, which were conducted with the seven student participants in the study. In this chapter, a synthesis of codebook samples and the major themes aligned with each research question from the audio- and videotaped classroom observations, the audiotaped focus groups, document analysis of student work samples, the Google Debrief form, and the field notes from both classrooms will be shared.

Participant Summaries

Participants in Classroom A

Teacher A had taught for twenty-two years and had over a decade of experience teaching math to second and third-graders. She used tasks solely for assessment purposes while teaching second grade. Once she began teaching third-grade several years ago, she began using the Tools for Teachers task as an early finisher and a way to challenge her students, who she felt needed more rigor or higher-level thinking. She pushed herself to adapt her questioning while going through the study with her students. Of the four student participants in Classroom A, all four were female, of various ethnicities, and displayed a variety of characteristics of mathematical promise as described below.

Student A. Student A was an Asian female and was identified as mathematically gifted after the end of the study. She showed her mathematical promise by being very verbal during the study. She was a very abstract thinker and often skipped steps and thought much more deeply about the mathematics of the task. For example, she said in Task 1, "*Piece of Yarn*," "I drew this out and wondered if I'm supposed to use a decimal to divide this fraction." 4/3 would be 1.33 about her knowledge of mathematics. She was very expressive with her feelings about drawing, etc. For example, she said, "Oh my God, this is so hard," expressing her feelings about the *Sharing Licorice* task and partitioning number lines.

Student C. Student C was a noticeably quiet African American female who read and tried to solve problems independently during the study. She displayed her mathematical promise through careful observation and personal study of the problems with her partner. She relied on multiple representations, such as visuals and drawings, to help make sense of the cognitively demanding tasks during the observations.

Student F. Student F was a Hispanic female. She tended to get teachers' feedback on her work in mathematics during the study. She showed her mathematical promise with her mathematical sensitivity and the careful precision of her mathematical writing as she meticulously shaded her area models and used mathematically descriptive writing under them. As noted in video observations, she spent time reading details of the problems carefully and silently and did not talk or interact with the rest of the group during Task 1, "*Piece of Yarn*."

Student G. Student G was a white female student who productively struggled through drawing number lines in Task 5, "*Sharing Licorice*." She used her mathematical creativity to turn her paper over and draw on the back. Although her mathematical writing seemed messy in the artifacts, she showed her mathematical promise by using fraction bar manipulatives to solve tasks. She was confused by Task 2, "*Measuring Rainfall,"* because of the table. However, she was able to understand comparing fractions by comparing her representations with those of her partner. She seemed to enjoy collaborating and using tools to solve problems, and she used mathematical creativity to make her own "rope" on the back of Task 5, "*Sharing Licorice*."

Participants in Classroom B

Teacher B. Teacher B has taught kindergarten through third grade in urban and suburban schools for the past nineteen years. She has a Master of Arts in Curriculum and Instruction as well as a National Board Certification. During her six years as a teacher at Riverview Elementary, she placed a strong emphasis on Social Emotional Learning. Recently, she has been more focused on Culturally Responsive Teaching and SIOP training in order to assist in meeting the diverse populations of multilingual learners. With this training, she has pushed herself to make many real-life connections. While launching the tasks, she felt this was related to her professional development training.

This year, she explored using more cognitively demanding math tasks, such as those used in the study, along with SIOP training. Moreover, her purpose in using task-based instruction was to help her students, particularly her ELL students, grasp how to think through real-world situations. Even though she believed that working with tasks might be difficult for some students, she felt it resulted in effective peer-to-peer math discussion and vocabulary use that scaffolded more understanding.

Below, I describe the three mathematically promising students in Teacher B's classroom. The student participants were three males of different ethnicities. At the start of the study, students B, D, and E were randomly assigned letters while participating in the focus groups.

Student B. Student B was an Asian student who displayed his mathematical promise with an articulate and precise math vocabulary. He expressed his mathematical promise often by questioning others in the group. He also displayed a strong ability to memorize mathematical issues by drawing on identified structures. For example, when comparing fractional assignments, he reminded his peers that if “the numerators are the same that the fraction with the bigger denominator is actually smaller because it has more pieces.” He also showed characteristics of his mathematical promise as he interacted with tasks. During each classroom observation, he did not look up from the paper when given five minutes of silent thought time, thus demonstrating he was inclined to learn new things on his own and attempt novel problems without any frustration (Freehill, 1961; Gavin, 2011).

Student D. Student D was a white student who was energetic and displayed characteristics of being twice exceptional with ADHD. He showed his mathematical promise during the study through his ability to switch between modes of representation to compare fractions. As evident in the video observation of Tasks 3 and 4, he compared fractions using fraction bars. Comparing this physical representation with the other students’ representations helped him reason and make sense of the task. During the study, he often initiated play and enjoyed laughing and manipulating the fraction bars. However, although he struggled with mathematical writing and drawing representations, the social constructivist environment helped him talk about his thinking easily with the group.

Student E. Student E was a charismatic, well-spoken Hispanic male student. He was very vocal and consistent with reading and rereading the tasks for clarity. He showed his mathematical promise with the use of multiple representations and detailed drawings of the models, as well as by displaying perseverance as he worked with the group. Another strength of his was asking the other group members their thoughts and pushing the group to compare their responses to each other. For example, he said, "They are different. The numerators are the same (if you cannot). If the numerators are the same. You can look at the denominators and see which one is larger." He also showed one example of this when Student B created area models to compare 2/6 to 2/4 on the board. Then he asked other group members, "Explain to me how you did that." He constantly questioned other students' information to continue digging deep into the mathematics of each task and interacting with his peers. He was identified as mathematically gifted at the end of the study.

Results

RQ 1: How did elementary mathematically promising students interact with cognitively demanding math tasks?

RQ 2: What mathematical practices did mathematically promising students use when completing cognitively demanding math tasks?

The results provided in this analysis highlight the responses from the five cognitively demanding task observations, pre-and post-focus groups, field notes and memos, a teacher Google debrief form and the document analysis of student work by participants. First, I will present the outcomes of themes produced from the data collection for each research question; then, I will go into each form of data gathering and analysis, as well as how I framed the overarching themes to align with my research questions and the preset codes. I began investigating my research questions, and preset themes (see Table 6) were major mathematical practices considered evident in the inquiring mathematics classroom. Going into the study, I looked for evidence of these within the classrooms (Hiebert & Wearne, 2017; NCTM, 2014). The codes, themes, and connections to the research questions are provided and described here to frame this descriptive case study. I created categories to code the information within the code book (see Table 7). To that end, this study sought to understand the overarching central question of the authentic interactions of elementary mathematically promising students and was specifically guided by the following research questions:

Research Question 1: How Do Elementary Mathematically Talented Students Interact with Cognitively Demanding Tasks?

Research Question One identified five themes about how students interact with cognitively demanding tasks. Furthermore, students in the study demonstrated their mathematical promise via reasoning, perseverance, exploration, and the use of mathematical precision. Their collaboration and interactions showed extraordinarily high levels of inquiry. Since students required little coaching from the teacher, they were able to demonstrate all types of multiple representations indicated in the conceptual framework of Lesh's Multiple Representational Translation Model (Lesh et al., 1987; Tripathi, 2008). Below, I outline the themes that emerged from Research Question One and provide supporting data and examples.

In addition, the themes for Research Question One were all connected to the study's conceptual frameworks. For example, within the instructional triangle (Cohen et al., 2003), the major themes of this study revealed that students interacted with interpersonal collaboration with other students and the teacher to make sense of and persevere through the cognitively demanding tasks. Also, students interacted with tasks by comparing visual representations of fractions to solve tasks. Finally, they proved and explained their thinking and reasoning about cognitively demanding tasks through oral argumentation, mathematical writing, and intrapersonal communication to reason and solve tasks.

Theme 1: Interpersonal Communication: Students Used Interpersonal Interactions to Solve Tasks.

Sub-theme 1*: Interpersonal Interactions Help Students Inquire, Think Deeply, and Solve Cognitively Demanding Tasks*

Student Actions. The main takeaway from Research Question One was how students used interpersonal relationships and thorough investigation to address problems. Throughout the study, students engaged with one another, and the task was to engage in deep thought, ask questions, and challenge the beliefs of other students. For example, in *Measuring Rainfall* Task 2 student E from Classroom B asked, “Are you sure, How many days did it start raining?” Then he said, “Can you see if they are 100 percent sure it is going to rain? Is the forecast, right?” This series of questions showed he was thinking deeply about the content of the task, and the dialog with his partner was crucial in illuminating mathematical reasoning.

Students in Classroom A also naturally engaged within smaller partner groups when solving the tasks. In order to understand the assignment, they would turn to face each other, reread their questions, and enlist the assistance of their partners. For example, students A and F would debate where to place symbols on the number line, as was clear from the audio and video observation in Classroom A. The task also required Student F to construct the fractions represented by the shapes, after which he would turn to Student A and ask, “What do you think? Let us compare our work.” As they worked on Task 1, *Piece of Yarn”* (see Appendix F), and Task 3, “*Comparing Fractions with Number Lines”* (see Appendix F)*,* Students C and G in Classroom A murmured to one another, “Did you draw the same pictures as me? Do you think we should work through it together?”

Also, in Task 4, "*Comparing Fractions*, "students from Classroom B also questioned each other. For example, student B said, ``I think pair 2 is longer. What do you think?" As the study progressed, students, particularly in Classroom B, began to interact and communicate informally. For example, when looking at 11/4, Student B asked the group, "Bro what do you think this means? So, this is 3 wholes?" A few examples during Task 5, "*Sharing Licorice"* (see Appendix F), that students asked each other in classroom B included Student B, who said, "That is supposed to be 11/4" and Student C, who asked, "They are fourths?"

Sub-theme 2*: Students Verbalize Mathematical Knowledge to Others to Solve Cognitively Demanding Tasks.*

Student Actions. Inquiry-based education aims to improve students' conceptual mathematical knowledge as well as their ability to express their conceptual understanding and mathematical knowledge. While two of the participants in Classroom A were very quiet in their interactions until late in the series of tasks, some students in the entire group and small group promptly verbalized their knowledge to each other. For example, in Task 1, Student A and Student F were recorded turning to each other and saying, "Oh, I get it, it is like our fraction wall. We need to label equivalent fractions on the number line." By the end of the study, in Teacher A's classroom, Student F was almost tricked by the number line in Task 5, *Sharing Licorice*. In fact, Student F noticed that the number line in the task had no labels. She asked her group if that was okay. She said, "I know 8/8 makes a whole, so we need to use that knowledge to help us label the number line."

Many verbal ideas were shared in Teacher B's classroom, especially in Task 2, *Measuring Rainfall*. For example, Student E looked at the chart and read the fractions aloud to the group, stating that even though they are in a certain order on the chart, they are not from least to greatest. Therefore, we must put them on a number line and look at their numerators to order them. When examining various fractions in the chart in Task 3, "*Comparing Fractions*, "Student E told the group, "They are different. The numerators are the same." Student B said, "I know there are 8/8 in a whole. When you are comparing fractions, the smaller numerator is the smaller fraction. If the numerators are the same, you can look at the denominators and see which one is bigger." Student D responded, "Yeah, so, I knew that Friday and Thursday were paired up." These assertions of knowledge and interactions of oral discourse between the students and the task opened windows for other students to promptly share their thoughts and for the students to work together to effectively preserve and solve the task (Dominguez, 2016).

Sub-theme 3*: Students Watch and Observe the Work of Other Students to Interact With Tasks.*

Student Actions. During the videotaped observations and field notes, particularly at the beginning of the tasks during the launch, students watched the board and the teacher for directions. Students were often seen staring at the written task on the screen and rereading it silently. Students were also looking at and noticing the task representations, words, and labels, like the words blue, green, and red on the number line for Task 1, *Piece of Yarn*. Once each task was launched, various behaviors of student observation were observed. For example, field notes and videotaped observations showed that Students A and F stared at the pictures of the area models for over thirty seconds at the launch of Task 4, *Comparing Fractions,* before interacting with their partners. In fact, during Task 3, *Comparing Fractions,* Student A said to her partner, Student F, "I want to look at yours." To interact with the task, students naturally wanted to watch and observe the work of others. During Task 5, *Sharing Licorice,* all students in both classrooms were watching the board when the teacher launched the task. Students in Teacher B's classroom were observed staring intently and silently at the number line and fraction 24/8 for over one minute before any active collaboration to explore and reason about the solutions to the task occurred. In addition, the field notes and video observation showed that Student E in Classroom B also interacted with the task by observing the board to see if the teacher had partitioned or labeled the number line to help him solve or make sense of the task.

Sub-theme 4: ***Active Collaboration Was Encouraged and Explored Within the Instructional Triangle of the Math Classroom***

Teacher Actions. Throughout the classroom observations within the study, both teachers continuously encouraged students to interact to solve the series of cognitively demanding tasks. Still, this theme was very apparent in Classroom A. Collaboration to make sense of the tasks was also noted between the students in both classrooms.

Student Actions. Furthermore, students desired interpersonal communication to help them reason and preserve through the tasks to interact with and solve mathematics. This often presented itself in the form of students asking each other questions or observing other students' work within the focus group. For example, in Task 2, *Measuring Rainfall* with the Chart, Student E discreetly asked his peers a series of questions, "How many days has it started raining? So, they can see if they are 100 percent sure if it is going to rain and they got the forecast, right?" His questions not only encouraged collaboration, but he also sought the feedback of his peers to know if he was on the right track. During Task 3 in Classroom B, Student D said to Students B and E, "Look at my number line, I have fourths. What do you guys have?" to collaborate and get at the thoughts of his classmates. In Classroom A, Student G said, "I wanna look at yours?" Student A responded, "Sure. Let us compare our thoughts." When both students were stuck in Task Five, *Sharing Licorice*, they continually conversed and analyzed the number line.

Throughout the study, students interacted with each other and with the tasks in order to solve problems. Student E grew in his reasoning several times during the classroom observation and focus groups. He would ask the group, “Guys, let us think about this again. This does not make sense. Do you think that is right?” The continued active collaboration through oral discourse helped students make sense of the mathematics in the tasks.

Theme 2: Interpersonal Interactions Between the Student and Teacher Helped Students Solve Tasks

Sub-theme 1: ***Teachers Encouraged Listening, Engaging, and Noticing Students in Active Collaboration.***

Teacher Actions. Throughout the study, interactions were encouraged by both teachers within the case study. Teachers encouraged active listening and questioning in order to interact with students and help them think more deeply. Within the first task, Teacher B said to the students, “Listen, Listen!” Teacher A began the first task by saying, “Listen to the story.” Then she launched the problem. Teachers' questions encouraged students to interact at a deeper level. For example, in the *Comparing Fractions* task, Teacher B said, “You see the different ways to represent the same problem, right?” “How does that help you if you know the numerators are the same? That should help you be able to produce another way?” Teacher A asked. Furthermore, Teacher A often said, “You may work with a partner like you've been doing with these tasks” at the initiation of each task.” Teacher A was very specific in her feedback to the students. She suggested, “When they talk to each other, talk to your partner. Work together. If you have an idea, talk about it. See if that is the same idea that your partner has.” Then, after the first two tasks, Teacher A stated in the Google debrief form that she noticed students were sharing their thoughts and ideas more. Although Teacher B did not directly tell the students to collaborate as much, the videotaped observations revealed much collaboration between students. With the reflection in the Google form, both teachers felt that students communicated well with each other about their thoughts and that similar patterns continued throughout the series of tasks.

Sub-theme *2: Teachers Encourage Interpersonal Interactions by Asking and Answering Open Questions Within the Instructional Triangle*

Teacher Actions. Throughout the study, teachers facilitated the tasks by asking open questions and maintaining ongoing conversations within the classroom. Both teachers would frequently engage the small group of students as they interacted to solve each task. For example, Teacher A said to students, with all the tasks, "You are going to talk to each other about what you are thinking in your brain." When comparing fractions on the number line, teachers engaged students by asking, "So when you are looking at the number line, do you think the triangle is half of that number? "Think about the marks you are making." She added.

Teacher Actions. In Classroom B, the teacher engaged all groups of students and encouraged thinking through a series of questions. When engaging students in Task 5, *Sharing Licorice*, Teacher B said, "What is it labeled as? What is given to you? What do you know about 12/4? If it equals 3 wholes? Now, I want you to think about what you have done and talk to each other." The teachers' series of questions engaged students in active collaboration and encouraged them to interact with each other to solve the tasks.

Theme 3: Students Interact with Tasks by Comparing Visual Representations of Fractions

Sub-theme 1*: Students Interacted with the Representations Such as Drawings, Models, and Number lines Provided on the Tasks to Solve the Tasks*

Student Actions. Throughout the study, students in both classrooms interacted with tasks that required them to solve fractions using various visual representations of fractions, such as area models, number lines, and symbolic representations. In most cases, the task instructions encouraged the use of mathematical representations. For example, such a task in the series began with a picture or some kind of mathematical representation (see Appendix F). Task 1, *Piece of Yarn*, Task 3, *Comparing Fractions with a Number Line,* and Task 5, *Sharing Licorice,* all contained a number line length model representation of fractions for students on the task. During Task 4, *Comparing Fractions*, where only fraction comparisons were listed, students were asked to draw models or use symbols to solve their interactions with the tasks using multiple representations. Lastly, Task 2, *Measuring Rainfall,* included a chart with fractions listed in random order indicating inches of rainfall, and no other visual representations of the fractions were present. These models embedded within the tasks were also examples of how students were able to study and make connections throughout the study's series of tasks.

Student Actions. Findings from the audio and video transcripts, as well as field notes, revealed examples of students comparing their models, drawings, and representations with each other in order to collaborate and represent mathematical understanding to solve tasks. Student C said, "I like to draw what you think and then color what your shades are when they are the same pieces. I am thinking that if you have the same denominator, it is helpful when comparing fractions." Also, student G was working with student C on this task and said, "If it is equal, you divide it in the middle." In Task 4, *Comparing Fractions,* students compared fractions, expressing equality in both classrooms. Students D and B both expressed that they look at the numerators to compare fractions if the denominators are the same. Specifically, Student B said, "2/3 is, um, if you have the same denominator, 2 is also greater than one, so ⅔ is greater than one-third if the numerator is bigger."

Sub-theme 2: *Students Drew Their Own Representations Such as Area and Length Models and Pictures to Solve Tasks.*

Area Models. In order to solve tasks, students drew their own pictures and models. When analyzing student work (see Figure 3), all students drew pictures of area models to solve the *Comparing Fractions* tasks. In Task 4, students were given directions for drawing an area model (see Appendix F). One explanation by Student G was, "I lined up the 2/4 and 1/8 and drew models. I stacked the area models on top of each other. Then I realized they were the same length." Then Student B stated that when he drew the area model, the shaded part for 4/6 took up more area than 2/6, implying that it is greater.

Length Models. To solve the cognitively demanding tasks, students drew various length models, such as number lines. Students also labeled number lines and wrote explanations to help them understand the representations in the tasks (see Figures 4-8). For example, Task 4, "*Comparing Fractions"* (see Appendix F), requires students to write an explanation using symbols such as a circle, square, or triangle. One-way students interacted with length models was by partitioning and labeling the given models in the task. For example, Student G circled the fraction on the number line to indicate her solutions. Also, in Task 3, *Comparing Fractions,* students drew partitions on a number line to help them understand the task. In the video observation, Students C and G pointed to the circle, star, and triangle and told each other they had to find fractions between these. For Task 5, "*Sharing Licorice*," Student D stated, "I started dividing up the number line. Once I tried to partition the number line, I got frustrated. So, I looked at the paper again and thought, ``I'm going to draw a new number line."

Sub-theme 3*: Students Used Physical Representations (math manipulatives) to Solve Tasks*

During all the tasks, teachers encouraged students to use physical representations and math manipulatives to solve the tasks. Before starting each task, students brought out the fraction wall diagrams they had made in their classrooms to display the equivalent fractions. Furthermore, the videotaped observations revealed that students used the fraction paper wall of equivalence they had made in class prior to the study to help them compare fractions, as it is a visual representation of the fraction tiles for all tasks. In addition, to complete tasks, students used math manipulatives such as fraction tiles. The fraction tiles were positioned in the center of the floor, as seen in the video observations of Teacher B. Also, Student D, in Classroom B, used the fraction bars extensively to help him solve Task 3, *Comparing Fractions*. He also rebuilt a fraction wall with bars when he got stuck on how to solve the task.

Furthermore, in Classroom A, Student F relied on using fraction bars. Beginning with Task 1, video observations revealed Student F's reliance on the fraction bars to help her set up the task. In order to link contextual models to the task, teachers also provided rulers, pieces of yarn, licorice, and pieces of ribbon. Hence, students interacted with these physical materials to help them interact with the problems. For example, Student A asked if she could use a ruler to partition the number line in Task 1, "*Piece of Yarn*." While in Task 5, Sharing Licorice, student F asked if they could use the yarn or ribbon from Task 1 to help them measure the number line.

Additionally, the teacher promoted the use of visual aids such as anchor charts, fraction tiles, and their fraction wall in both classrooms. During Task 1, "*Piece of Yarn*, "Teacher A told the students, "Take the tool you think you need to use what other tools are using right now. If you need to, use your fraction paper. You took out your own and there are more tiles up there you can borrow." Another example is when Teacher B said, "At the beginning of the engaging portion of Task 2, "*Measuring Rainfall*," remember, you have tools on your desk. You have tools on the carpet here. Whatever you might need to help you. You have tools on the walls. Whatever you might need to help." Both teachers encouraged students to use math manipulatives to solve the tasks.

Sub-theme *4: Students Used Physical Gestures to Respond to Cognitively Demanding Tasks*

During the audio and videotaped observations, students used several physical cues to interact with the cognitively demanding tasks, their peers, and the teacher. Even though students in both classrooms raised their hands to respond to teacher questions, the study's video observations and field notes captured a variety of physical and silent bodily responses. For example, during Tasks 4 and 5, students in Teacher B's classroom frequently pointed at the number lines and even the questions within the task. For example, student E said, "Guys, we know if 8/8 is here in the number line [as he pointed], then it is longer than one whole." In addition, the students even moved their bodies towards each other when discussing the tasks, while student D displayed frequent movement to get materials, play, and laugh. The interactions characterized under this theme included watching and observing the students as the teacher posed the task and other students shared their results.

Theme 4: Students Proved and Explained Thinking and Reasoning about Cognitively Demanding Tasks

A mathematical practice that was visible in the videotaped observations of the tasks was encouraging students to defend their ideas and solutions to each other. Furthermore, the study's findings were in line with *Principles to Action,* which states that in the socio-mathematical context of the classroom, students should justify their solutions and explain how they arrived at them. This theme revealed that during the task's launch, exploration, and discussion phases; students interacted to solve problems by sharing their thinking within the focus group of mathematically talented students. Within the study, teachers asked the students to prove their answers. For example, in the *Piece of Yarn* Task 1 (see Appendix), Teacher B said, “How can you use a number line to prove that your answers are correct?” Students were also encouraged to share their thoughts with their partners. In Task 5, *Sharing Licorice* (see Appendix F), Teacher A said, “You are going to talk about what you are thinking with your partner.” Then, Student E said, “Let me hear someone else's explanation why,” and Student A responded, “She’s correct when she says 1/2.” With this final task, Student F had full-paragraph explanations on her paper.

Theme 5: Students Interacted with Reflective Communication to Reason and Solve Tasks

Reflective communication with oneself was one-way students persevered and made sense of their interactions with tasks. Although there was a constant hum of voices and oral discourse in this social constructivist environment, video observations revealed many independent and quiet thinking moments. Furthermore, the findings from classroom observations and focus groups revealed that students often used reflection and metacognitive thought to interact with tasks by asking themselves questions or writing notes to themselves. Moreover, in Classroom A, students often lean down on the table, studying the task deeply at the start of the exploration phase or when productively struggling near the end of the task. In Classroom A, Student F shared a metacognitive strategy she used in the post-focus group: “I found helpful how like, I did not know what a word meant, like, um, when it was a word problem. I do not know how to say it, but I know what I meant by that. So, I just started to think of what it meant. Um, but I remember one of them like it said on record. I did not know what that meant. I was just thinking in my brain, ‘What could record mean?’” Student A also shared a metacognitive strategy by saying, “I normally say, say in my head, like how am I able to solve this? Like, how am I going to do it?”

An example of this type of reflective thinking is when students say things in their heads and use metacognitive strategies to solve problems. Besides, a few codes revealed that students used metacognition and Mathematical practice four to justify their thinking and persevere with reasoning. For example, during Tasks 3 and 4 of the *Comparing Fractions* tasks, Students A and B were observed staring off into space and deep in thought. While in the pre-focus group, Student A shared that she "normally says the math problem in her head" and asked herself, "How am I able to solve this?" When asked in the pre-focus group how they interacted with tasks, Student A said, "I normally say, say in my head, like how am I able to solve this? Like, how am I going to do it." Student B said that when he reads a problem, he thinks about it in his head for a minute and holds the number in his brain. Then he will hold on to it until he needs it later or tells his partner (Student E).

During Task 5, "*Sharing Licorice,"* Student C from Classroom A thought aloud about how to reflect on and interact with the task. During the video observations and the post-focus group, Student C shared that she asks herself questions and tries to restate the problem in her own words. She said, "The more I think in my head about how I am solving the problem, the better I can solve it."

Summary of Themes for Research Question 1

From Research Question 1, five themes emerged that described how the mathematically promising students in the study interacted with cognitively demanding tasks. Themes one, two, and five in the classroom's instructional triangle were directly connected to interpersonal communication (Cohen et al., 2003). While themes Three and Four showed students interacting with tasks using written, verbal, symbolic, and visual representations that all correspond with the conceptual framework of multiple representations to model mathematics within the socially constructive space of their small group (Lesh et al., 1987). Also, students could interact with tasks due to the space and ability to choose their materials, student groups, and how they solved tasks. The social nature and use of discourse during the task's launch, exploration, and discussion phases also led to many interactions, connected the framework of social constructivism to the themes generated, and gave examples from my codebook of raw data (see Table 7).

Research Question 2: What Mathematical Practices Did Mathematically Promising Students Use to Complete Cognitively Demanding Mathematical Tasks?

Theme 1: Students Solved Tasks by Making Sense of Problems and Persevering Through Them

Sub-theme 1: *Students Reasoned with Perseverance to Complete the Series of Tasks*

Throughout the study, students attempted to solve problems and make sense of them. Results from Table 10 (see Appendix) demonstrate how students discussed their use of sense-making while using Mathematical Practice 1 in the focus groups. Students persevered through Task 5, "*Sharing Licorice,*" by using social interactions and productive failure. Through their social interactions, students could talk and reason about the task and figure out how to partition 3 wholes into 24 pieces. Table 11 provides examples of direct findings and codes from the data that show student perseverance with tasks.

Student Actions. Students used Mathematical Practice 1 throughout the study. Findings from the focus groups showed that the students felt unable to give up on the problems and used their tenacity to complete cognitively demanding tasks. For example, during the pre-focus group, students said, "Geometry was hard, and the test questions on their benchmarks were challenging with pictures." By the end of the post-focus group, students gave more examples of their own feelings of success from their perseverance with tasks. For example, students indicated that partitioning the number lines and explaining math in writing with symbols, length models, and charts in addition to words was challenging; but it helped them understand mathematics. Table 9 (see Appendix) provides examples of how students used productive struggle to solve tasks.

Teacher Reflections. Findings from the Google debrief form also support the idea that perseverance seems to increase throughout the series of tasks in the study. For example, when asked, "How did students persevere and reason with the task?" At the beginning of the study, Teachers A and B rated Task 1, "Piece *of Yarn,"* with a rating of 7. By completing Task 5, "*Sharing Licorice*," both teachers rated their student's perseverance as a ten from both teachers. Teacher A also mentioned that they noticed the students experimenting with different strategies. For example, teacher A observed one strategy of perseverance and sense-making. She says, "When I asked a question, if the students didn't know what to do, they often reread the problem or talked through their solutions together." Teacher A also noted in the Google debrief form that she felt her students did not completely finish Task 1, "Piece of Yarn," so she planned to change her questioning and scaffold more to help the students make more sense of the task in her launch (see Figure 5).

Subtheme 2: Students Used Metacognitive Strategies with Problem Solving

Mathematical Practice 1 focuses on students' ability to understand and persevere through problems (CCSI, 2010a). It also represents students' ability to interact with themselves via metacognition by explaining the meaning of a problem and looking for entry points to the solution of a task. Data from the observations suggested that students used metacognitive strategies to reason and solve problems.

Teacher Actions. Teachers reported that students used Mathematical Practice 1 as they completed tasks. In addition, students used metacognitive strategies when thinking through tasks to understand mathematical problems and persevere through them. Also, findings show that teachers modify their teaching to encourage students to engage in this practice. For example, Teacher B, in particular, encouraged her students to use more independent reflection and thinking time at the launch of the task. After debriefing Task 1, *Piece of Yarn*, Teacher B felt her students were not thinking deeply enough about the task, and she expressed in the Google debrief form that she did not want to give them too much scaffolding. To influence students to make more sense of their thinking, she used five minutes of personal work and reflection time starting with the initiation of Task 2.

To further build sense-making, she also used think-aloud. She practiced some questions for Task 2; “Like If I read this problem, I might be thinking that I know several fractions like 4/8 that are the same as ½ inch of rain,” I also might be thinking that I can represent the fractions in the chart in many ways. So, I might visualize how to see the fractions before I start writing.” After practicing a think-aloud at the beginning of Task 2, *Measuring Rainfall,* Teacher B encouraged her students to reflect and interact deeply with the question before collaboration by saying, “For the first 5 minutes of the task, I want you to work quietly at your own seat, then they move to the group table.”

Teacher Actions. Findings from this research question also showed that teachers used oral discourse and the reading of cognitively demanding tasks to assist students in understanding the problems. This theme was divided into several subthemes. First, the teacher and students read directions together to begin each task, but during the observations, teachers frequently recited statements made by the students about the tasks to encourage reflection. Teachers also read the problem aloud and reread the problem at the beginning of the study or when students were having difficulty. For example, Student F and Teacher A reread problems to solve orally. Teacher A began each task by saying to her class, “Let's read it again to see if that helps.” This reading strategy assisted students in making sense of the tasks before interacting within the small group.

Student Actions. As seen in the video observations, students used reflective thinking to complete tasks. Even Student D maintained silence while reading the problem twice. Throughout the tasks, every student worked more deliberately and with greater focus. Also, during Task 2, Student E frequently raised his hand to signal the teacher that he had read the passage, taken notes, and immediately written down steps to help solve the problem after reading. When the students began talking and interacting about the task, Student E emerged as a leader, sharing his thoughts. “Guys I was thinking, we need to give all the fractions a common denominator so we can see which is equal.” I was trying to make the designs in my head and see what ⅜ would look like; what did you guys think?” This reflection naturally resulted in more engaged teamwork during the task enactment.

In addition, the post-focus group findings from Classroom B showed that metacognitive thinking occurred before partner sharing as part of the Instructional Triangle of interaction (Cohen et al., 2003). During the post-focus group, students B, D, and E from Classroom B said the teacher gave them some quiet time to solve the problem. First, they thought about all the ways they could solve it; then, they wrote them down on paper. Student B also added that taking the time to think through possible solutions for the problem enabled him to connect what he had learned in class and ensure he was not guessing. He, therefore, believed that he improved in problem-solving and teamwork.

Theme 2: Students Used Multiple Representations and Tools When Modeling Their Mathematical Thinking

Throughout the study, visual, symbolic, pictorial, written, and verbal forms of mathematics were all present in the task as multiple representations of mathematics (Lesh et al., 1987). To conclude, the students' mathematical practices used to model mathematics during the study analyzed relationships between all representations of mathematics, including numbers, symbols, words, and models. Furthermore, students routinely modeled mathematics with their interpretations by asking questions like, "Does they make sense?" "What tool should I use to show my thinking?" thus helping themselves improve the model if it has not served its purpose. In addition, the student work analysis, post-focus groups, and memos showed evidence of mathematical modeling in both study classrooms.

Student Actions: Classroom Observations. Data from classroom observations showed that students frequently used picture comparison to solve cognitively demanding tasks, modeled with representations, and used Mathematical Practice 4. In focus groups and observations, it was clear that students used area models, length models, pictures, drawings, and physical manipulatives to complete tasks. For example, in Task 1, the *Piece of Yarn* task, students used only their fraction walls to relate the number line to their prior understanding. Also, during Task 3, in Classroom A, *Measuring Rainfall* (see Appendix D3), students used their fraction walls to make sense of the table (see Figure 6). Additionally, in Task 1, "*Piece of Yarn"* (see Appendix D), Students C and G merely drew models on their papers (see Figure 7). Still, they decided to use fraction bars to create a few of the task's fractions in order to arrange them in numerical order. Unfortunately, in Classroom B, during *Comparing Fractions,* Task 3, Student D could not express his comparisons in words, but he eagerly got up to use the fraction bars to create and rebuild each set of fractions in the task. For example, Student D said, “This is what I did with my number line to compare.”

Focus Groups. In the post-focus groups, students indicated that the manipulatives and fraction tools helped them complete tasks when they were unsure of how to represent the problems. Furthermore, the students in the post-focus groups gave some examples of modeling mathematics using tools. For example, Student F said, “I had to use the fraction bars. I can line up two bars, like two fourths and one half and easily compare them. Using them helped me feel better when comparing fractions.” Student D also added that modeling with the fraction bars helped him split the number line into thirds and then just keep on doing it like (umm), especially on Task 5, *Sharing Licorice*. “It really helped me like them. “So, I just liked the fraction bars.” Student F also shared in the focus group that she did not understand the number line one, so she just drew an area model to help her see what was going on in the number line in order to understand it better. Besides, Student D also said he liked using fraction bars to split up the numbers into thirds.

During the post-focus groups, students said that using different representations to model mathematics helped them enjoy the study. For example, when using the area models, Student C would pile hers on top of one another (see Figure 3). Also, in classroom A, Student A shared, “We like the pictures best.” Student F echoed this indicating, “I think like, I usually do words and like symbols are technically everything that you write down like that write down word, symbols and numbers.” Further analysis of student work via document analysis revealed that students' model drawings, partitioning number lines, and symbolic reasoning were all very frequent and related to codes revealing over 60 interactions within the series of five tasks (see

Table 9). Figure 7 shows many area models drawn throughout the tasks when observing the student's work.

Student Actions. After speaking with the teachers and reviewing the students' work, it was noted in the memo for Task 3, *Comparing Fractions on a Number Line*, that six out of seven students were correctly able to label all the partitions and represent the task with at least three types of representations: area, length, and symbolic representations. As they continued through the series of tasks, the students used various mathematical modeling techniques and presented their representations to their peers and the class.

Teacher Actions. Comparison of models and symbolic reasoning was also encouraged by the teacher. Teacher B explicitly asked, "How many of you drew an area model?" Teacher B also shared her observations about how students used multiple representations to help them solve cognitively demanding tasks. Furthermore, she had specifically observed her students using a fraction paper tool (a fraction wall to show equivalence) that they made to help them compare fractions. They did make an effort to explain why without much prodding. Teacher B encouraged the use of tools by saying, "So when you use a number line, a picture, that may help you visualize. You may use the area models like the ones on the poster over there. You must show me how you get your answer."

Teacher Reflections. When I met with teachers to do the task debrief, they both noted how their students used Mathematical Practice 4 frequently to solve tasks. Findings from the Google debrief form showed that 90% of the time, students constantly used representations in their interactions with visual, symbolic, and physical representations, as observed by their teachers (see Figure 5). For example, teacher B initially noted in the Google Debrief form that in

Task 1, "*Piece of Yarn*," students used the fraction paper tool they made to help them compare fractions as it is a visual representation of the tiles and number lines. By completing task four, "*Comparing Fractions*," her observations had become more in-depth while the students were modeling with mathematics. According to Teacher B, "One student thinks beyond 3rd grade level with the task, three of the students drew out their strategy to answer questions. I noticed two students did not make the connection with number lines and area models without proving that 2/6 and 4/6 could be compared on the same number line and started to draw two number lines (one for each fraction)."

Throughout the study, Teacher A paid close attention to her students' representations and modeling of mathematics. She stated in the Google debrief form that students' used their understanding of equivalence and modeling mathematics on the number line to complete Tasks 1 and 3. She said, "I feel like overall many students had a pretty good understanding that the labels on the number line were able to help them identify the missing numbers." However, a few students were unable to figure out how to label the items with the information given.

Theme 3*:* Student Used Stamina and Ability to Productively Struggle When Solving Cognitively Demanding Math Tasks

Student Actions: Focus Groups. According to evidence from the audio-and videotaped observations as well as the focus groups, students used productive struggle when solving cognitively demanding tasks. Their work with productive struggle was influential throughout the tasks and helped them to become thinkers and doers of mathematics. Furthermore, during the pre-focus group, students had the perception that hard math was regrouping and geometry. For example, student C said, "Like shapes and stuff that's kind of confusing." Also, student B shared his frustration by saying, "Regrouping is just so hard." While student A gave a specific example of being confused on a recent assessment by saying, "It showed this rectangle, and it said the area but there's another rectangle inside of it, but it also showed the area of that, and you have to add the areas of that together. I found that kind of confusing."

Post Focus Groups. After the post-focus groups, six out of the seven students agreed that Task 5, "*Sharing Licorice,"* was the most challenging task they had completed in math. "It stumped me," Student E said. Student D also mentioned that he could not complete one of the tasks, and his paper was blank. Student B said, "because it was like you had to split it up into three parts. It was very tiring for your brain because it made you do like 24 little marks" (see Figure 4). Student A expressed her frustration about *Sharing Licorice* saying, "Partitioning the number line was hard," "it was like, "splitting it up, had me thinking "Oh my God, how many pieces have I done already." Also, Student G stated that she did not really understand the number line task, so she drew an area model to help, while Student F felt that *Measuring Rainfall*, Task 2, was the hardest. She said that there were so many lines on the task that you could not see which one was which. According to Student F, she felt she was not good at comparing fractions, so she struggled with this task. However, she stated that after using the fraction bars and doing another comparing fractions problem with the number line, she felt she improved with comparing fractions.

Furthermore, the productive struggle was crucial in revealing a greater lesson in the post-focus group. After expressing how frustrating the previous task was and how they used productive struggle to solve it, Student F said, "You should not give up on anything. Like you should keep going." She said solving these challenging tasks helped her grow stronger, and now she knows she can solve any math problem and not give up in life when things get hard.

Document Analysis. Overall findings from the document analysis of the tasks showed that the students completed 33/35 tasks as they successfully struggled through each task. The tasks, teacher, and fellow students all assisted the students in solving problems independently. During the study, I noticed that while one student knew how to complete the task, she struggled to explain her thinking and was unable to understand certain probing questions. While students were able to at least start labeling, some were stuck and were getting a little frustrated. However, being able to discuss in small groups appeared to help them start to put together some understanding of mathematics.

Sub-Theme: Students Grew as Productive Strugglers Through Emotional Reactions When Solving Cognitively Demanding Tasks

Student Reflections. During the focus groups, students revealed various emotional interactions when solving cognitively demanding situations. When asked during the pre-focus group, "What was their favorite part of solving challenging math tasks?" students expressed a variety of emotions. Students D and F began the study by complaining that challenging problems took too long. Also, Student E considered challenging problems such as multiplication and division. Student C said, "shapes are confusing for me to remember like the names of "parallelograms and stuff." In addition, Student A expressed her frustration with the state assessments' confusing pictures and diagrams. During the pre-focus group, she described a problem with 20 bars that she thought represented 5 x 4, but it was five times 40, and she found the pictures confusing. After the study, students expressed a range of emotions, including Student E, who said, "It was very tiring for your brain,"

Student Actions. Although students found these math tasks to be frustrating, the productive struggle of this study revealed that they enjoyed solving cognitively demanding tasks.

For example, in the post-focus group, Student D stated that finding equivalent fractions was enjoyable. Student A also stated that math “calms me down” while Students G and C smiled about their experience and echoed, "I really like finding equivalent fractions.” At the end of the study, Student D from Teacher B’s classroom and Student F from Teacher A’s classroom both agreed that they enjoyed the tasks despite how difficult and time-consuming they were.

Teacher Actions. In the Google debrief form, the teachers also echoed this theme of excitement for productive struggle. For example, teacher B noted that although her students were excited to try something new, they were also wary at the beginning of the study because some students had difficulty eliciting deeper understanding. She also observed that some of her students struggled to stay on task at first, but by the end of the study, they were so engrossed that “I had to make them stop working. Other students in the class who I often do not hear from were excited as well.”

Theme 4: Students Used Their Ability to Construct Viable Arguments and Critique the Reasoning of Others to Solve Tasks

Sub Theme 1: Students Emerged as Doers and Thinkers of Mathematics by Justification of Thinking within the Instructional Triangle of Tasks, Teacher, and Amongst Themselves.

Justification of Thinking and Argument were obvious codes for Research Question 2, indicating that students were emerging as doers and thinkers of mathematics with this Mathematical Practice throughout the study of students' learning with the cognitively demanding tasks. However, both teachers in the study agreed that students’ abilities to justify and argue their solutions were inconsistent (see Figure 7).

Teacher Actions. With Task 1, *Piece of Yarn,* and Task 2, *Measuring Rainfall*, teachers indicated that this mathematical practice of justification was inconsistent. They were unsure how to question their students and how to scaffold the task without lowering the cognitive demand. Teacher A said, "students were most inconsistent with their arguments." After Task 3, *Comparing Fractions*, Teacher A mentioned in her debrief form that she noticed students explaining their strategy to help their partner understand. Also, the results of audio and video observation show a few codes for arguments, but at this point in the study, the students were not sharing how they solved the problems or arguing that their solutions were correct; they were just saying, "I think they did well share their evidence and work after the first task of the study." According to Teacher A, "Some students have a tough time showing a deeper level of understanding and have difficulty with tasks that aren't 'right there.'" By completing Task 5, *"Sharing Licorice,"* teachers indicated in the form that students were able to share and mathematicalize their thinking in a much more articulate way.

Sub-theme 2: *Students Used Mathematical Writing to Express Their Justifications and Argue their Conceptual Understanding of the Tasks*.

Evidence from the document analysis, focus groups, and teacher debrief form all showed several sub-themes with how students used Mathematical Practice Four and mathematical writing to complete tasks.

Document Analysis. According to document analysis, students used Mathematical Practice 4, mathematical explanatory, descriptive, and argumentative writing to justify and explain their thoughts about how they solved problems. Tables 9 and 11 show examples of the types of mathematical writing and how students used this mathematical practice to demonstrate an understanding of the tasks. Students used explanations in four out of five tasks, with students in classroom B using ⅓ more explanations than students in classroom A.

Mathematical writing and argumentation were also evident, as labeling was one of the most frequently discovered codes in document analysis (see Table 9). Since mathematical writing can be a symbol or form of prose, students justified their mathematical thinking and representations with mathematical writing by using short phrases and labels such as inequality symbols, words, and numbers. The appendix shows examples of how classroom A students labeled the tasks with words and names to help them solve the problem (see Figure 4).

Focus Groups. The post-focus group transcript also included reflections from students on how they completed tasks using Mathematical Practice 4 and critiqued their reasoning through argumentation. During the post-focus groups, Student B explained how he used the numbers, writing, and words in the problem to support his answer: "Like something that kept on repeating was like supporting your answer by using numbers, writing, or using words." Also, in the post-focus group, Student G shared some interesting thoughts about the purpose of mathematical writing or using prose to show symbols, words, or phrases to demonstrate evidence of her mathematical thinking. She said, "the tasks gave her so much room on the paper to show her thinking. She couldn't complete her work without explaining how she did it or describing what was going on in her brain to others." Her motivations for writing mathematically. "The writers of the tasks give you directions to make sure you know what you are doing. Say I only use numbers and they ask you to use words and you do not know what to use for words. Because your words and numbers must go together. Words cannot just be numbers. The words explain what you are thinking, and they want to make sure you know everything you are doing."

Theme 5: Students Used Mathematical Precision and Ability to Make Connections with Mathematics

Sub-theme 1: ***Tasks Influenced Students to Attend to Precision with Mathematics***.

Student Actions. The task results show that students completed tasks by using Mathematical Practice 6 and paying attention to precision. Table 9 shows how students completed tasks involving document analysis, specifically symbols and partitioning. Furthermore, Table 11 in the appendix shows how students used mathematical precision when solving cognitively demanding tasks.

Document Analysis. **Comparison.** Findings from all five tasks showed that students drew area models and partitioned number lines. Consistent with Lesh's Representational Translational Model (1987), students used comparison symbols greater than or less than, as well as drawing two area models side by side. as shown in Figure 4, other students showed comparison using written words (see Appendix D). Furthermore, Task A3 and Task B3 (see Appendix F) specifically asked students to use symbols to make comparisons with Task 5, *Sharing Licorice,* when they were asked to partition a number line. As students partitioned the number line in this task, creating the number line themselves required precision, and this was something they had never done before.

Observations. Results from document analysis and memos revealed that students became more precise as the study progressed. In Task 1, *Piece of Yarn*, Students A and G were noted to draw some marks on the number line to label fractions. Student A was very meticulous and showed mathematical precision as she divided the number line. When the number line in Task 3, *Comparing Fractions on a Number Line,* did not have all of the equivalent fractions drawn in, Students C and G worked together to iterate units to measure where to split the number line into sections (see Figure 5-6). By completing Task 5, "*Sharing Licorice,"* the students were able to partition the number line into sections (see Figure 7-8). Students A and F even used a ruler to try and measure the number line in order to divide it into exact sections.

Teacher Actions. Labeling was a strategy when students got stuck. According to Teacher A, “Many students were able to at least start labeling, however, some were stuck and were getting a little frustrated. However, when they can discuss in small groups it seems to help them start to put together some understanding.”

Sub-theme 2: *Students Reasoned by Making Connections Between and Among Math Concepts to Solve Tasks*

Students Actions. Throughout the study, students were able to identify important quantities or fractions in each task and make connections between and among the fraction tasks as well as other math topics. For example, in Task 1, “*Piece of Yarn”* (see Appendix D1), and Task 5, *“Sharing Licorice”* (see Appendix D5), students made connections between the number lines that were present in the tasks, flowcharts in the classroom, and the fraction strip walls they had made in class. Student C said, “this reminds me of the task with yarn and ribbon except it has no numbers.” “Lots of these tasks have number lines and are like each other; number lines have lots of fourths and eights.” Student D said. While Student B said, “Yeah, equivalent fractions are like division 8 divided 2 is 4.” When students made connections between math concepts using tools like diagrams, two-way tables, graphs, flowcharts, and formulas, they used Mathematical Practice 1 to persevere in problem-solving.

During the post-focus group, students shared some of the ways they made connections when solving tasks. “Student B shared that the series of tasks encouraged him to use more words when I am doing math like usual. I like to record that 2/4 + 4/8 is equal to one in just a number. I now understand how numbers are related to fractions and adding. I like how I figured out this thorough explanation instead of just memorizing it.”

Sub-theme *3: Tasks and Teachers Connected Real Life to Mathematics and to the Tasks Themselves*

Teachers Actions. Throughout the study, teachers made comments and asked questions to help students make connections between the tasks, other mathematics they had learned, and real-life situations. Below are a few examples of data collected from the Google debrief form and videotaped observations that matched this theme: Teacher A said, “Do you want to measure with something else that is flexible, and it just keeps losing its spot? That could happen, could not it.” With task 3, *Comparing Fractions on a Number Line,* Teacher A continued to expand students’ knowledge and the development of this practice. In her launch of this task, Teacher A said, “Remember, we did it with our rope number line. Teacher A: Okay, well, you must think about first you got to compare them you got to see which one is longer.” Even though Teacher A mentioned she did not feel as confident in her delivery of the first task, she felt she gave away too much or insufficient information, which may have caused some confusion among her students. After the first debrief memo, memos revealed some influence from Teacher A. She intentionally made connections among the tasks and used real-life objects like ribbons, rulers, and licorice so her students could see the similarity between the tasks. She also wanted to ensure she had the right amount of scaffolding and background knowledge for the students. At the end of the study, she felt her students did well because of their background knowledge of fractions learned over the last few weeks. She also mentioned how important she realized it was to give her students the right background.

In addition, memos after Task 1 showed that Teacher B immediately launched the first task by saying, "A lot of times when people buy ribbon or use something, they use ribbon to make something like the bows for a wreath or your hair. Or whatever you might be using wrapping paper." She intentionally launched each task with a real-life connection. In the debriefing moments, she said her work with SIOP and Culturally Responsive Literacy gave her some ideas about how to launch the tasks.

Summary of Themes from Research Question 2

In summary, the data presented for Research Question 2 revealed five major themes about how mathematically promising students used mathematical practices to solve tasks. The data sources provided after these themes show evidence to support the conclusions I drew from the themes. Furthermore, Table 9 shows additional examples of the mathematical practices and how students used them throughout the study tasks (see Table 9). Besides, the following themes were noted:

Theme 1: Students completed tasks by making sense of problems and persevering in solving them.

Theme 2: Students used multiple representations and tools to complete tasks and model mathematics.

Theme 3: Tasks and the socially constructed space facilitated students' ability to productively struggle when solving cognitively demanding math tasks.

Theme 4: Students used their ability to construct viable arguments and critique the reasoning of others to solve tasks.

Theme 5: Students used mathematical precision and the ability to make connections with mathematics.

Summary of Findings

As a result of this study, teacher educators, math coaches, gifted specialists, and administrators may better understand the actual interactions and application of mathematical practices between students and tasks, as well as between students and teachers, when posing cognitively demanding tasks. This research may potentially highlight and lead to the development of a curriculum to help teachers teach, better engage questions, and generate productive failure and problem-posing during cognitively demanding task enactment. This research will also foster conceptual mathematical thinking and collaboration among curriculum coaches, specialists, and teachers who work with mathematically promising students.

CHAPTER V: DISCUSSION, CONCLUSION, & RECOMMENDATIONS

Overview

This chapter begins with a summary of the study's problem, purpose, research questions, and methodology. In this chapter, I will summarize the findings drawn from the data in Chapter IV. Then, based on the findings and their applicability to elementary mathematics education and gifted education, I will discuss the implications and make recommendations for future research. The remaining part of this chapter will include the following: a summary of the study, major findings and connections to literature, major findings and connections to theoretical frameworks, implications, limitations, and recommendations for future research based on the findings of this study.

The Problem

Students who are mathematically promising in the United States are well behind their peers in state and international assessments of mathematics achievement. While this issue can be broadly addressed with all students in the United States, the achievement of students identified as mathematically promising on the most recent National Assessment of Educational Progress (NAEP, 2020) exam also revealed that few students have an advanced understanding of mathematics. For example, on the NAEP Assessment, only 9% of 4^{th} graders scored at the NAEP Advanced level (NCES, 2019).

Furthermore, the underrepresentation of student groups of color in gifted and talented programs may have a negative impact on long-term educational attainment (Triplett & Ford, 2019). In addition to the excellence and achievement gaps shown nationwide by the NAEP (2020) (see Figure 1), reports like these show the persistence of inequity among our mathematically promising students. Besides, the problem exists not only with assessments but also with opportunities for advanced learning. For example, in a 2019 report, *E(race)ing Inequity,* approximately 11% of students in North Carolina Public Schools were considered academically or intellectually gifted (AIG) (Triplett & Ford, 2019). According to Tripplet and Ford (2019), inequity in AIG high school math classes is prevalent in North Carolina because Asian and White students are over-represented compared to their percentage of the total state student population. However, in comparison to their proportion of the total state student population, American Indian, Black, and Hispanic students were under-represented (see Figure 9) (Triplett & Ford, 2019). In addition, race and ethnicity are significant predictors of differential AIG designations, net of all other relevant factors. Therefore, it is recommended that mathematically promising students, especially those from underrepresented races, be included in similar studies (Gavin, 2011).

Recently, there has been more local research and curriculum development for inquiry-based mathematics instruction, which may increase achievement at advanced levels in mathematics, especially in the Southeastern United States. Also, some state programs have developed new curriculum materials for all students using task-based instruction and trained teachers on how to implement cognitively demanding tasks, especially at the elementary level (Gavin & Casa, 2016; Tools 4 Teachers, 2019). In addition, many studies have been conducted over the past few decades indicating the need for cognitively demanding math instruction, starting with Stein and Lane (1996) and Smith and Stein (1998). However, prior studies dug deep into the roots of mathematical promise and students' mathematical identity (Ainley & Margolinas, 2015).

Purpose of the Study

The purpose of this descriptive qualitative study was to explore the interactions and use of mathematical practices of seven mathematically promising third-grade students as they completed a series of cognitively demanding math tasks within the socio-mathematical norms of their classrooms. Using pre- and post-focus groups, classroom audio and videotaped observations, a teacher Google debrief form, field notes, as well as document analysis of student work, this research described the experiences of mathematically promising third graders from two classrooms. Therefore, the findings of this study reveal joy and successful learning outcomes when implementing inquiry-based tasks that focus on student interactions and the use of mathematical practices within a social-cultural space. Specifically, the study described student interaction, students' oral, written, symbolic, physical, and nonverbal representations, as well as emotional reactions to cognitively demanding math tasks. Also, the study described how the students used mathematical practices such as persevering through reasoning, constructing viable arguments, using abstract and quantitative reasoning, as well as precision when solving tasks.

In addition, this study aimed to address gaps in the research, which focuses on how students interact during cognitively demanding math tasks. This study's contributions to mathematically promising students are needed to support the current field of gifted education and the nurturing of mathematical promise. This research would address critical issues in the field of elementary mathematics education as well as potential solutions for addressing the need for inquiry-based instruction for mathematically promising students in mixed-ability classrooms (Gavin, 2011; Johnson et al., 2017; Sheffield, 1999).

Findings In Relation to Research Question One

Research Question One: How did Elementary Mathematically Promising Students Interact with Cognitively Demanding Tasks?

The purpose of Research Question One was to examine the relationship between the student’s interactions with one another and with the series of tasks, as well as how the students interacted with the teacher to complete the tasks. Furthermore, one of the most important emphases in past and current NCTM standards (1989; 2014) was to make problem-solving or problem-posing a central focus of school mathematics. One of the primary goals of mathematics teaching, as echoed in this study, was for students to solve complex problems involving social interactions (Stanic & Kilpatrick, 1988).

With Research Question One, the following themes emerged, which indicated that students worked together in a socially constructed space to interact and solve tasks. Their interactions involved reflection, communication, and the connection of visual representations of mathematics. Interaction with cognitively demanding tasks using representations.

Theme 1: Interpersonal Communication: Students Used Interpersonal Interactions to Solve Tasks

Theme 2: Interpersonal Interactions Between the Student and Teacher that Helped Students Solve Tasks

Theme 3: Students Interact with Tasks by Comparing Visual Representations of Fractions to Solve Cognitively Demanding Tasks

Theme 4: Students Prove and Explain their Thinking and Reasoning about Cognitively Demanding Tasks

Theme 5: Students Interacted with Reflective Communication to Reason and Solve Tasks

Findings in Relation to Research Question 2

Research Question 2: How Did Mathematically Promising Students use Mathematical Practices as They Completed Cognitively Demanding Tasks?

Research Question Two delved deeper into how the students used mathematical practices while interacting with cognitively demanding tasks. Analysis of the focus group transcripts, work samples from the document analysis, and the Google debrief form determined how the students used mathematical practices to complete tasks (see Appendix C). According to the study's findings, students used five major mathematical practices to complete tasks (see Table 10). Also, the findings of Research Question 2 are relevant to current research in mathematics education because teachers used instructional strategies to facilitate students' use of mathematical practices.

Furthermore, current research also aligns with this study's findings regarding the types of questions teachers ask during task enactment. It has been suggested that teachers ask leading questions to enable students to develop reasoning and conceptual mathematical understanding (Carpenter et al., 2015; Kisa & Stein, 2015). To that end, the teachers in this study encouraged students to ask each other questions, which aligns with the Mathematical Practices (CCSI, 2001). In addition, research has shown that teacher questioning has the potential to generate students' responses about their mathematical thinking, problem-solving, and strategies (Hufferd-Ackles et al., 2004; Martin et al., 2017). This study revealed this research to be true, as the development of students' ability to persevere and reason through tasks was facilitated by the teacher's effective use of questions in interpersonal interactions.

In relation to Research Question Two, Mathematical Practice 4 calls for mathematical practice and the use of appropriate tools strategically, which is also evident in this research. For example, students used tools such as rulers to help them with Task 2, *Measuring Rainfall*. Students also used rulers to be precise in partitioning their number lines in Task 5, "*Sharing Licorice.* "Furthermore, focusing on research based on a mathematical triad of interaction, known as the "Instructional Triangle" with students, teachers, and cognitively demanding mathematical tasks, is necessary to make the transition to inquiry-based teaching, which encourages reasoning and problem-solving from all students (Cohen et al., 2003).

Discussion in Response to Conceptual Frameworks

To better understand the interactions of mathematics among students and the actions required of teachers to nurture students' mathematical promise and conceptual understanding of tasks, I used two conceptual frameworks to guide this study: Lesh's (2008), as shown in Figure 2, and Cohen and colleagues' (2003) Instructional Triangle, as shown in Figure 3, in order to better understand the interactions of mathematically promising students with cognitively demanding tasks**.**

Lesh's Model of Mathematical Translations of Representations

According to Lesh's (1987) Model of Multiple Representations, mathematics is the study of the interrelationships between concepts and ideas. In mathematics instruction, multiple representations should include concrete, verbal, numerical, graphical, contextual, pictorial, and symbolic components, as shown in Figure 2. Within the findings of this study, all modalities of interaction were noted when students solved tasks within the audio and videotaped tasks. Some tasks within the study (see Appendix F) involved more physical representations, while others involved more symbolic representations. Therefore, I recommend the implantation of a series of tasks in future studies to allow students time and space to utilize multiple representations of mathematics, such as drawings, manipulatives, mathematical writing, symbols, and discourse.

With the study's series of tasks, students could make multiple connections with mathematical visual and symbolic representations. Within the study, Task 4, *Comparing Fractions,* focused on the comparison symbols of greater than and less than. While Task 1 involves a "*Piece of Yarn,"* Task 3 involves *Comparing Fractions with a Number Line*. The number line representation was also emphasized in Task 5: "*Sharing Licorice."* While Task 2, *Measuring Rainfall,* used a table, the students were able to confidently solve a cognitively demanding task without prior exposure to scaffolded problems.

Throughout the study, Lesh's Model of Mathematical Representations framework, as shown in Figure 2 (Lesh et al., 1987), was linked to Research Question One and NCTM's effective teaching practices of mathematical representations (NCTM, 2014), as well as the standards for mathematical practice. When students modeled mathematics to solve tasks, they identified important quantities in a practical situation and mapped their relationships using tools such as diagrams, two-way tables, graphs, flowcharts, and formulas. Finally, they analyzed those relationships mathematically in order to draw conclusions (CCSM, 2010a).

Inquiry-Based Math Instruction and the Instructional Triangle

Recently, the launch, explore, and discuss (LED) framework has been at the forefront of curriculum and instruction in elementary and secondary mathematics across the country, particularly in North Carolina (NC2ML, 2019). In order to demonstrate the relevance and importance of teaching a connected series of tasks, I chose the Tools for Teachers (2019) tasks for this study from an entire unit. Furthermore, teachers should differentiate and adapt instructional strategies for mathematically promising students using core curriculums like the one used in this study. Teachers should also choose tasks in a series so the prior knowledge of the multiple representations present could help support inquiry-based math instruction, which has been recently cited as an instructional practice that supports students with mathematical promise (Lesh et al., 1987; Van-Tassel Baska, 2021).

Furthermore, the conceptual framework of the Instructional Triangle (Cohen et al., 2003) depicts interactions between teachers, tasks, and students. For example, students' interactions with tasks within this study showed two major types of interaction relating to Research Question 1. When students interacted with the tasks, Theme 1 showed that students interacted with tasks interpersonally. Whereas, Theme 2 showed that students interacted with tasks interpersonally. Also, the Instructional Triangle of Cohen and colleagues (2003) (see Figure 3) depicted a triad of interpersonal communication between teacher and students as well as the task. Equally, this study found that observational thinking, physical gestures, and symbolic thinking, such as social interactions with Theme 1 of Research Question 1, were all present.

Discussion in Relation to Theoretical Framework: Inquiry Into Social Space Nurtured Gifts and Talents

The social constructivist environment and mathematically creative space in the study were created by Teachers A and B (Blumer, 1969). Both teachers gave the students room to spread out and represent their ideas using the tools of their choice, such as markers, manipulatives, pencils, or technology-based manipulatives, at a small group kidney table. Most importantly, the students could interact with one another in this environment due to a few key instructional practices used by their teachers (Yackel & Cobb, 1996). Furthermore, the teachers asked inquiry-based, open questions and encouraged the use of multiple representations within tasks. Due to the fact that they did not assign partners, the teachers in both classrooms allowed these students to work in groups as they chose with other mathematically talented peers while remaining in a separate area of the room with peers of similar abilities. For example, the students in Classroom A naturally formed two partner groups to divide the tasks. While Students A and G were both very vocal and shared almost every thought they had aloud with each other, two of the students interpreted mathematics more through explanatory writing and comparing their solutions.

Furthermore, this space within the socio-mathematical norms enabled the three boys in Classroom B to bring out their mathematically promising characteristics naturally. Interactions also occurred because Teacher B gave students this space. However, although she did not assign group roles to the students, she did use a different strategy, giving them five minutes of silent inquiry after the launch of each task. After that, she allowed the students to work together. Moreover, to meet the needs of her students, she adapted her instruction, as suggested by NAGC (2014) Standard 3, by differentiating the tasks. Besides, Teacher B did not assign group roles; instead, she made it clear to the students that they needed to demonstrate their thinking and explain their reasoning in more than one way. Due to the setup of the classroom social norms, Student B immediately shone as the group facilitator. He carefully read each question while the students naturally turned to face each other. In addition, Student D was a twice-exceptional student, so his ability to spread out and use the fraction bars to demonstrate his thinking and modeling of the task was impressive. Despite his processing issues, his written work did not reflect his oral understanding. He could laugh and enjoy the tasks while sharing his thoughts with the group. Finally, because he was the quiet student who stayed focused and wanted to keep the group on task, Student E naturally emerged as the questioner. The cooperative group also naturally enabled students to learn from each other (VanTassel-Baska, 2021). Equally, the students naturally encouraged each other to fail productively. They persisted in attempting to solve tough tasks, like *Theme 5,* in which students interacted with the behaviors of reflecting on metacognition, making notes to themselves, speaking quietly to themselves, and rereading tasks. All of these involved interaction between students and tasks alone, not with other students.

Discussion In Relation to Literature

Connection to Mathematically Promising Students

The findings of this study show that students displayed several characteristics of mathematically promising students like those in recent studies (Deal & Wisener, 2019; Gavin, 2011; Rotigel & Fello, 2004). Furthermore, the students' awareness and curiosity about numbers, their ability to solve work and think about abstract mathematical patterns and relationships, their ability to transfer mathematical reasoning to new and novel situations, and their ability to mathematical problems using flexible and creative thinking rather than sequential or standard forms of reasoning were a few themes that aligned with current research in gifted education and this study. In addition, research has shown that students with mathematical promise need opportunities to solve complex problems using non-algorithmic thinking, such as the problems under consideration in this study (Diel & Wismer, 2020; Gavin, 2011; Sheffield, 1999).

According to the findings of Gavin's (2011) study of mathematically promising students, students from underrepresented populations and schools, such as Riverview Elementary, need opportunities to explore mathematical tasks and socially interact with highly cognitively demanding tasks. This school and study setting provided opportunities for the students to fulfill their mathematical promise while also connecting this research to the current field of gifted education. More studies using a socially constructed space are recommended in the future.

Findings Show Connection with Gifted Standards and Best Practices and Standards-Based Curriculum: Tools 4 Teachers Tasks

The findings of this study agree with Skemp's (1987) as well as Smith and Stein's (1998) original research that students require mathematical tasks other than procedures and low-level algorithmic thinking. This study also aligned with the NCTM Principles to Action (2014) because the standards require task implementation to be part of standards-based instruction. Furthermore, tasks with a high cognitive demand promote reasoning and students' exploration of mathematical concepts and align with Standard 3 of the National Association of Gifted Standards (2014). According to the NAGC's (2019) standards, educators should apply evidence-based models of curriculum and instruction. First, Tools 4 Teacher's Tasks are research-based (NC2ML, 2019; Smith & Stein, 1998) and should be considered best practices for inquiry-based instruction when teachers adapt their instructional practices to best implement and enact such tasks with mathematically promising students. Furthermore, the study's findings, especially NAGC (2019) Standard 3, *Curriculum & Instruction* (see Table 12 below), show how NAGC *Standard 3: Curriculum & Instruction* aligns with best instructional practices for mathematically promising students.

According to the NAGC standards (2019), teachers must respond to the needs of students with mathematical promise by planning, selecting, adapting, and creating a curriculum that employs a repertoire of instructional strategies to ensure specific student outcomes. In Table 12 below, I show how the standards and the teachers' instructional moves lead to favorable learning outcomes and a socially productive inquiry-based learning environment for mathematically promising students. Also, teaching mathematically promising students should be seen as a mosaic of different instructional strategies selected based on the instructional purpose of the lesson to be taught. Table 12 below shows how NAGC Standard 3 of curriculum planning and differentiation relates to the findings of this study. As you can see, several sub-standards, such as 3.1.3 and adaptation of curriculum, as well as 3.2.2 and the use of connections, were incorporated into the work of this study.

Finally, in this time when public education is often underfunded, and teachers believe they cannot meet the needs of mathematically promising students with their curriculum, the findings of this study provide recommendations for how educators can successfully use the free state-granted work of the NC2ML (2019) with mathematically promising students by adjusting the socio-mathematical norms and placing students in a smaller ability group while facilitating tasks.

Table 12

NAGC Standards Evident with Mathematically Promising Students Task Based-Inquiry

NAGC Standard	Substandard Evident in this Study	Tools 4 Teachers Tasks Are Beneficial for MP Students
3.1 **Curriculum Planning.** Students with gifts and talents demonstrate academic growth commensurate with their abilities each school year.	.3.1.3 “Educators *adapt*, modify, or replace the *core or standard curriculum* to meet the interest, strengths, and needs of students with gifts and talents and those with special needs such as twice exceptional, highly gifted, and English language learners.” 3.1.4. “Educators design differentiated curriculum that incorporates advanced, conceptually challenging, in-depth, and complex content for students with gifts and talents.” 3.1.6- “Educators pace instruction based on the learning rates of students with gifts and talents and compact, deepen, and accelerate curriculum as appropriate.”	Teachers adapted the core curriculum by choosing tasks for mathematically promising students. -Mathematical tasks from Tools 4 teachers in the study were challenging and can be classified as procedures with connections. -Tasks were multiple steps and in-depth -Tools 4 Teachers’ tasks were open-ended, and they were encouraged to use open questions. -The focus group is an ability group for students to work at their own pace. Teacher A did offer a task extension with different numbers for Student A

3.3- **Responsiveness to Diversity.** Students with gifts and talents develop knowledge and skills for living in and contributing to a diverse and global society.	3.3.2. "Educators encourage students to connect to others' experiences, examine their own perspectives and biases, and develop a critical consciousness."	-Themes shows teachers connected tasks to real life, specifically -Task 1 *Piece of Yarn* -Task 2 *Measuring Rainfall* -Task 5 *Sharing Licorice* -Teachers connected the tasks to other classroom lessons and experiences in mathematics.
3.4-**Instructional Strategies**	3.43- Educators use problem-solving models 3.44- Educators use inquiry models to meet the needs of mathematically promising students	-The Tools 4 Teachers Framework uses Launch, Explore, and Discuss where the exploration part of the task allowed for student inquiry. -Teachers used the LED Framework and differentiated for the MP students by asking them open questions and allowing them space to explore tasks in a group with other mathematically promising students.
3.5- **Culturally Relevant Curriculum**	3.5.1-Educators develop and use challenging culturally responsive curriculum 3.5.3- Educators use curriculum for deep explorations of culture and language	Tools 4 Teachers' curriculum used a variety of students' names and materials, such as yarn, licorice, stars, and rain from various cultures. -Students were able to explore vocabulary and language with fractions by working in a single-ability cooperative group during the explore phase -All students benefited from the mixed whole group launch and discussed phase

3.6 **Resources**	Educators pace instruction based on the learning rates of students with gifts and talents and compact, deepen, and accelerate curriculum as appropriate.	-This study allowed students to benefit because a high-quality set of tasks which were formed by a state fun grant of teachers were used -The students in the study were also allowed to use training camp online manipulatives, fraction bars, real-life objects, and their drawings

Teacher Actions and Instructional Strategies Benefit Mathematically Promising Students

Recent work from Van-Tassel Baska (2021) proposes teaching students with mathematical promise like a mosaic when placed in mixed ability groups. Besides, teachers should consider instructional strategies that provide mathematically promising students with a variety of differentiated instruction. Based on the findings of this study, teachers should provide opportunities for mathematically promising students to work in a socially constructed space with peers of similar abilities. Van-Tassel Baska (2021) stated that some materials have been researched and shown to be beneficial for all students. Also, the findings of this research show that cognitively demanding tasks are beneficial to mathematically promising students (Gavin, 2011; Van-Tassel Baska, 2021).

According to the findings of this study, teachers should adapt mixed-ability classrooms to facilitate Tools 4 Teachers tasks using the *Launch, Explore, and Discuss model* (Tools, 2019; NC2ML, 2019). Firstly, Tools 4 Teachers' classroom tasks focus on a non-didactic approach in which the reasoning does not focus on the correct answer. Secondly, Mann (2006) stated that tasks should provide these open-ended, inquiry-based problems using a variety of methods, which align with the cognitively demanding tasks of this study (Smith & Stein, 1998: Tools 4 Teachers, 2019).

Inquiry-Based Instruction is Relevant for Mathematically Promising Students

This study's findings align with the National Association of Gifted Children's (2019) standards and a plethora of research in gifted education calling for high-quality inquiry-based instruction for students with mathematical promise in order to develop conceptual mathematical thinking and reasoning (Gavin, 2011, 2016; Henningsen & Stein, 1997; Lewis & Colonnese, 2021). Also, recent scholarship connects to this study because both emphasize that *real* problem-solving involves working on unfamiliar problems, out of context, and open-ended, as well as offering students' real challenges by providing rich tasks and contexts (Anderson, 2003; Kilpatrick et al., 2001; Schoenfeld, 1992).

Furthermore, current research in gifted education confirms her findings, stating that mathematically promising students often require less teacher scaffolding to solve problems and frequently skip steps (Gavin, 2011; Van-Tassel-Baska, 2021). In fact, students with mathematical promise may be able to interpret and analyze math problems more quickly and accurately than their teachers and require "ill-structured problems" (Deal & Wismer, 2010; Gavin, 2011; Mann, 2006; Rotigel & Fello, 2004; Tomlinson, 1997).

Findings Were Consistent with Other Elementary Studies with Mathematical Promise

The goal of math instruction for mathematically promising students should be to encounter mathematical habits of the mind, including mathematical creativity, collaboration, skepticism, etc. (Van-Tassel et al., 2003). Findings from this study were similar to those of Katherine Gavin and Tutita Casa (2016) in their study of nurturing young student mathematics using curriculum from their M2, Mentoring *Young Mathematicians, and Project M3, Mentoring Mathematical Minds, because* both studies were grounded in mathematics and gifted education. In addition, both studies also foster challenging and motivating students to solve highly cognitively demanding math problems (Gavin & Casa, 2016) to show the correlation between achievement and using a curriculum that would develop the mathematical talent of students. Similarly, just like the students in Gavin and Casa's (2016) study, the students in this study used sophisticated strategies to show their mathematical creativity.

Furthermore, recent research in gifted education has also emphasized that mathematically promising students benefit from student collaboration and affirms that students' achievement drastically improves when they have opportunities to collaborate (Dorzi et al., 2021; VanTassel-Baska, 2021). Also, cooperative learning demonstrates the positive effects of interdependence while highlighting the importance of personal accountability among students. For example, using turns, talks, and group structures to discuss mathematics, such as those used in this study, allowed students to interact, question, talk, and learn how students from different cultural groups desire to communicate as well as open doors and windows into their thinking (Ball, 1993, Dominguez, 2016; Johnson et al., 2017). In addition, this study revealed evidence of student collaboration and interpersonal interactions. Such unstructured conversations have been identified as a key method in research for developing mathematical identities and communities in students (Johnson et al., 2017; Nasir, 2002).

Implications for Mathematics Instruction

Elementary Teachers Should Use Tasks with Multiple Representations to Nurture Mathematical Promise

"Children need opportunities to explore cognitively demanding math tasks to determine how to solve tasks, creative mathematical representation, find solutions, and reason mathematically about their work" (Carpenter et al., 2015). This study's series of tasks allowed for productive struggle and perseverance with cognitively demanding tasks. When students grappled with perplexing problems or made sense of challenging ideas, they engaged in the process of productive struggle, which shifted students from passively watching the teacher control the learning to an active, productive mathematical environment full of conceptual understanding and meaningful application of mathematics (Martin et al., 2017; Polly, 2017).

For mathematics to move forward and promote inquiry-based instruction in elementary mathematics classrooms that will give all students access to highly cognitively demanding math tasks, the implications of having a classification system of representations for mathematics will help teachers to differentiate mathematics and encourage deeper reasoning in mathematically promising students. Another reason for using multiple representations when solving tasks is the student's ability to transfer and connect knowledge from one idea to another. For example, in Task 5, "*Sharing Licorice*," a student might connect this task to measuring rope for a basketball net or ribbon for cheerleading hair bows instead of using pretend licorice. Hence, rather than just posing problems of high cognitive demand, teachers can set up family surveys, talk to students, and use photos or pictorial imagery to launch tasks. Besides, symbolic, physical, written, and oral representations were all recorded in the given study; however, adding and taking more photos of students in their natural environments and playing with cognitively demanding tasks are recommended to help increase students' authentic mathematical creativity and curiosity so they can pose. In addition, students and teachers can share and use outside experiences related to stories, encourage storytelling, and, like discussions in tasks, continue to reflect upon their practice and the context in which cognitively demanding tasks are enacted. Finally, children need opportunities to explore, solve, and create mathematical representations in order to solve tasks and reason mathematically about their work (Carpenter et al., 2015).

Furthermore, this study's findings are consistent with recent work in gifted education, noting that multiple representations benefit mathematically promising students (VanTassel-Baska, 2021). Within this study, the teacher used the fraction bars and paper model diagrams daily in classroom tasks so that students could draw these representations in their work or use a hand-drawn paper model of them to solve problems. In the future of mathematics education, more studies using Lesh's representational model (1987) should be conducted to help determine how teachers can best facilitate the use of multiple representations as they adapt tasks to meet the needs of mathematically promising students. (Lewis & Colonnese, 2021; Olawayin et al., 2021).

Implications for Coaches and Teachers to Adapt Tasks to Facilitate Use of Mathematical Practices

The Standards for Mathematical Practice in the Common Core Mathematics Standards (CCSSI, 2010a) recommended that teachers provide opportunities for students to make sense of mathematics by persevering through problem-solving, modeling mathematics, constructing viable arguments, critiquing others' reasoning, attending to precision while communicating, and reasoning quantitatively while solving and discussing mathematical tasks, to name a few. Equally, the findings in Chapter Four and Table 14 show the data sources for Research Question 2, indicating that students used mathematical practices to solve tasks. In addition, students used the mathematical practice of justification and argumentation since teachers provided time for students to have oral discourse with partners and small groups. The teachers in both classrooms in the study were observed to facilitate instruction and engage the students with precision by asking them to clarify their thinking and explain why. The teachers also promoted best practices for oral discourse and discussion by allowing for task sharing during the discussion.

Findings Revealed Connection-Making Lead to Mathematical Modeling

Modeling Mathematics.

This study's findings revealed that students were able to make connections with other mathematics they had learned as well as real-life objects to model mathematics (MP 4). For example, in Task 1*, "Piece of Yarn,"* students were given a chance to connect mathematical concepts with real-life objects, such as cutting yarn and drawing number lines, and in Task 5, "*Sharing Licorice,"* they shared a piece of licorice and portioned a number line. In fact, three of this study's five tasks (see Appendix F) had a real-life context that students used as a frame of reference for mathematical models. For example, Teacher B used a piece of yarn when introducing Task 1, "*Piece of Yarn*," to her class. She used a culturally relevant approach because she was unsure if all her students were familiar with yarn. In addition, other research findings have shown that contextualizing assignments gives students a chance to engage with tasks that serve as mirrors and windows into their thinking and lives (Dominguez, 2016; Terada, 2022).

Findings from this study also revealed mathematically promising students going beyond their comfort zones and delving deeply into solutions to problems. Also, in order to create equity in mathematics and inquiry-based mathematics classrooms in elementary schools where mathematically promising students are interacting with the Instructional Triangle (Cohen et al., 2003), students should learn mathematics with understanding to actively build new knowledge based on prior experience. Besides, this study was found to have clear connections with other mathematics as well as connections with real-life objects. Throughout the study, teachers encouraged connections with real-life objects as well as other mathematics topics students learned simultaneously as they interacted with the tasks. These connections enabled mathematically promising students to perform calculations backward, approach math in unusual ways, and use reasoning abilities (Gavin & Casa, 2016; Huinker & Bill, 2017).

Teachers Noticing and Questioning Encouraged Perseverance.

Teachers' effective use of questions has the potential to generate students' responses about their mathematical thinking, problem-solving, and strategies (Hufferd-Ackles et al., 2004). The findings of Research Question Two regarding teachers' interpersonal interactions align with much previous and current research about the importance of teachers' use of questioning and task enactment to drive student reasoning (Kisa & Stein, 2015; Lewis & Colonnese, 2021; Martin et al., 2017). Within this study, the students asked each other questions. Teachers were also observed asking open-ended questions to help clarify the task. Both teachers in the study stated that they purposefully avoided giving away too much information with their questions during debriefing sessions in order to maintain the cognitive demands of the task, which has been noted in elementary mathematics research (Carpenter et al., 2015; Johnson et al., 2017; Kisa & Stein, 2015). Furthermore, this study echoed previous research suggesting that when questioning students during tasks, teachers should use a student-centered approach to inquiry and base their questions on the student's explanation and reasoning.

Implications for Problem Posing and Mathematical Creativity (Mathematical Practice 9)

The National Council of Teachers of Mathematics and the National Association of Gifted Children call for teachers to engage students in problem-solving and problem-posing (NCTM, 1991, 2014; NAGC, 2019). One of the characteristics of mathematically promising students displayed in the study that has been noted in past research is their ability to solve given problems well. However, although posing their problems can be challenging for some students, it can take their mathematical promise to the next level. Problem posing is one strategy recommended in current and past research that can improve students' problem-solving skills while deepening their conceptual understanding of mathematics (Lewis & Colonnese, 2021; Silver, 1997). Furthermore, problem posing is recommended to expand mathematically promising students' natural mathematical creativity, or Mathematical Practice Nine, by connecting mathematics to authentic experiences of students. By expanding tasks beyond those in this study to other tasks that are similar, student-created tasks, students will continue to emerge in mathematical creativity and conceptual understanding.

However, although teachers' observations and inquiries were at the forefront of influencing the mathematical interactions with their students, teacher questioning also helped students solve problems. Hence, future studies and teachers should deliberately encourage elementary-aged students to ask challenging questions of their own. Furthermore, it is also recommended that elementary teachers encourage student thinking and creativity by adding problem posing as an extension to the discussion of mathematical tasks in classrooms (English, 1997; Mann, 2006; Lewis & Colonnese, 2021; Olawayin et al., 2021).

Implications for Using Tasks with Mathematically Promising Students for Productive Failure

When planning math instruction, math teachers should use culturally responsive tasks that connect students to their lives and experiences both inside and outside of the classroom. First, mathematically promising students require instruction that allows them to switch between modes of representation and provides flexibility in solving problems, even when the problems are difficult, ill-structured, and not focused on procedures (Van-Tassel-Baska, 2021). For example, the tasks in Tools 4 Teachers exposed students to various mathematical representations, such as number lines, area models, charts, and symbols, to conceptualize fractions.

Furthermore, recent field research and this study's findings show the use of productive failure. Throughout the study, productive failure was created in a socially constructed space where the teachers used the task framework and launch portion to activate the prior knowledge of all students. Teachers also took the time to learn what knowledge students brought to school from their various cultures, as evident in the classroom observations. For example, if they felt that a task, such as *Piece of Yarn* and Task 5, *Sharing Licorice,* or discussing an item that students' cultures were unfamiliar with required a physical representation, they brought it into class to allow all students to have the same assessment of the task and asked them if they knew what yarn and licorice looked like. Besides, the teachers knew that placing math concepts in a real-world context is a great way to help students connect diverse cultural experiences and develop mathematical identities (Johnson et al., 2017; Nasir, 2002).

Moreover, current research supports the study's findings by emphasizing the importance of using standards-based math lessons that connect students' language and backgrounds in order to foster mathematical identity development and open doors of opportunity for students with potential (Dominguez, 2016; Gavin, 2011; Nasir, 2002).

In addition, mathematics teachers are called to "support productive struggle in learning mathematics" (National Council of Teachers of Mathematics [NCTM], 2014, p. 48). According to recent research in mathematics education, the emergence of productive failure is imperative for true problem-solving to emerge and for students to be doers and thinkers of mathematics (Amindon et al., 2020; Polly, 2017; Terada, 2022). Since both teachers felt their students needed more challenge and sought to adapt their instruction for each task to help students experience some small group teaching and discovery-based learning, productive failure was fostered in the mathematically promising students in this study. This instructional adaptation of practices helped students construct their knowledge and mathematical knowledge (Amindon et al., 2020; Polly, 2017; Terada, 2022).

When students productively struggle while learning mathematics, research has shown that they outperform similar-ability students who are not given opportunities for productive struggle (Blanton & Kaput, 2003; Terada, 2022), thus making the argument for the productive failure seen in this study to be replicated in classrooms and additional studies. Productive failure also allows students to explore their positive and negative feelings towards mathematics in past studies (Goldin, 2000a; Hannula, 2015; Terada, 2022). However, recent studies in mathematics education suggest that fostering productive failure with cognitively demanding tasks should be the primary focus of mathematics instruction rather than just content (Casa et al., 2022; Terada, 2022).

Because emotional interactions were at the interplay of the interpersonal interactions and productive failure themes in this study, findings from such studies align with these recommendations in the field. In focus groups, all the students indicated that the tasks were challenging. Also, several students echoed Hannula's (2015) findings, claiming that struggling through a task helped them find joy and pride in their work. Therefore, classroom implications from this study suggest that teachers and coaches should continue to study the connection between productive failure in task interactions and the associated emotions.

Summary of Implications for Tasks

Due to multiple entry points and open-endedness, tasks with high ceilings benefit all students but provide additional benefits to mathematically talented students (Flores, 2007). Furthermore, Johnson and Sheffield (2013) advocated using mathematical practices standards for mathematically promising students. The emphasis on the practice of mathematical creativity and encouraging mathematically talented students to engage in complex, real-world mathematical thinking are the two aspects of the mathematical practices presented in this study. According to this study's findings, elementary mathematics classrooms, teachers, and instructional leaders should modify their instructional strategies so that mathematically talented students can rely more on their ability to solve problems quickly and effectively and on their interactions with others than on learned procedures (Gavin, 2011; Jacob & Andrew, 2008; Mason & Watson, 2007).

Recommendations for Future Research

Based on the results of this study, several additional areas of future research relating to mathematically promising students and their interactions with cognitively demanding math tasks could be explored. Furthermore, the tasks in this study were a connected series of tasks that focused on standards specific to Cluster 7 in third grade (Tools 4 Teachers, 2019). Over the course of four weeks, the context of the study was developed in the classroom. With isolated tasks, the study's findings would not be possible. Besides, findings indicate that future research can extend beyond to investigate the use of the Tools 4 teachers' curriculum with mathematically promising students at other K-2 grade levels. Additionally, more research into how to use the tasks to formatively assess students' conceptual thinking and use of mathematical practices with mathematically promising studies can continue to connect the often disconnected fields of mathematics and gifted education. Finally, teachers, tasks, and student interactions form a triangle, and more research into tasks using this framework will help teachers use effective instructional practices for mathematically gifted students.

Recommendations for Future Studies with Tools 4 Teachers Tasks

Based on the findings of this study, Tools 4 Teachers' formative assessment tasks (2019) align with many characteristics of mathematically promising students. Besides, all of the tasks were cognitively demanding and could be classified as procedures with connections. Also, students were asked to perform calculations, but post-test questions were open-ended and in the form of word problems. When students were asked to compare fractions, they were frequently asked to explain their reasoning and connect their understanding with more than one mode of thinking. Throughout this study, teachers allowed students to share their thoughts during the task enactment.

Firstly, it is recommended that more similar studies be conducted to better understand the social interactions of other grade levels of elementary mathematically gifted students as they interact and solve cognitively demanding tasks. Because this study's content was only focused on Tools 4 Teachers 3rd Grade Cluster 7 Tasks for Fractions, other content areas with fewer mathematical representations and concepts may present different interactions and mathematical practices. In order to understand a broader scope of application and how teachers can use such tasks to best meet the needs of mathematically talented students, more research is needed on Tools 4 Teachers tasks (NC2ML, 2019). Additional research could be conducted at all elementary grade levels, especially grades K-2, with a different cluster or unit to see if the results are similar and transferable. Also, studies should focus on nurturing the potential of mathematically promising students before they are identified as academically gifted. Additionally, various task clusters that promote inquiry and cognitive demand are available within the Tools 4 Teachers (2019) curriculum for all grade levels in elementary mathematics.

Furthermore, just as teachers provided discussion time in the LED framework of this study (NC2ML, 2019), teachers should present students' correct and incomplete solutions or misconceptions and allow students to justify their solutions to help develop their mathematical talent. Table 12 in the Appendix describes the characteristics of the mathematically promising students in this study and how the study's teachers adapted this curriculum to best meet their instructional needs.

Recommendation for Assessments and Adjusting Instruction to Meet the Needs of Students

Standard *3.1.5* of the National Gifted Standards (2019) recommends that educators regularly use pre-assessments, formative assessments, and summative assessments to identify students' strengths and needs, develop differentiated content, and adjust instructional plans based on progress monitoring. In this study, teachers could modify their instruction based on their formative assessment of the tasks without scoring. They were also able to adjust their instruction based on student observation. Based on this study's findings, a second recommendation for future research is to gather more feedback and achievement data from the population of students who participated in this study, such as by correlating the student assessment of the tasks with their summative assessment data from classroom assessments or NC Check-ins.

Furthermore, Lesh et al. (1987) discovered that students in grades 5-8 who were taught using standard-based instruction showed greater achievement on open-ended test questions and problem-solving items than on items assessing procedural knowledge. In this study, the students completed the tasks but were not graded using the provided rubric for *meeting expectations*. However, the study's findings show that students clearly *met expectations* for the tasks. Therefore, future studies should use task rubrics to collect formative assessment data from the students in the study to see the impact of task enactment on student performance longitudinally as students continue interacting with tasks with Tools 4 Teachers (2019).

Recommendations for Research M&P 9 (Mathematical Creativity)

In this study, mathematical creativity and sensitivity have been mentioned as qualities of mathematically promising students. The study's findings also showed that mathematically promising students used a variety of mathematical practices to solve tasks. In addition, this study's social constructivist framework enabled students to construct arguments, persevere through tasks, and productively fail to reason through mathematics. When adapting tasks for mathematically promising students, the Tools 4 Teachers' Tasks (2019) and many curriculums lend themselves to problem posing and adaptations with mathematical creativity. However, these were not directly stated in the tasks.

Furthermore, this study did not investigate mathematical creativity, but it did observe the use of creativity in several cases and the characteristics of some mathematically gifted students. Further research is also needed to investigate the application of mathematical Practice 9 and mathematical creativity when students interact and complete tasks. To connect the research behind this study to the field, teachers should also encourage students' problem-posing and mathematical creativity. According to VanTassel-Baska (2021), the teaching of mathematically gifted students should be seen as a mosaic of different instructional strategies selected based on the instructional purpose of the lesson to be taught. Besides, students require a blend of instructional strategies that promote metacognitive reflection.

Additionally, this study's findings indicate that verbal, physical, written, and symbolic interactions are frequently interwoven into students' representations. Future studies can also incorporate the extensions and problem-posing section into the discussion phase of the task, allowing the mathematical creativity of students to be displayed and researched further. Also, understanding how students use such mathematical practices will help teachers understand how to change their instructional practices.

Limitations of the Study

Qualitative studies are often limited due to the methodology. This study has two potential limitations. First, due to the small sample size (n = 7) chosen for the students in this study based on Creswell's (2013) and Maxwell's (2014) recommendations, this can embed the representation of diversity within the study, such as gender, age, race, or mathematical experience. However, the school diversity of Riverview Elementary and teacher input in the selection of students secured a very diverse sample of students. For example, classroom A was composed of four females of various ethnicities, as described in Chapter Three, while classroom B was composed of three males of diverse ethnicities who were mathematically promising. All these diverse backgrounds may have impacted the study results as characteristics and identifications of mathematically promising students differ (see Table 5).

A second limitation of this study may relate to the transferability of this study to all mathematical content areas and grade levels. This study focused specifically on the mathematically promising students in this third-grade sample. The characteristics of mathematically promising students may differ from another sample of students. Transferability refers to the study data analysis and results not being able to be replicated or transferred to another grade or other contexts (Lincoln & Guba, 1985).

Summary

In summary, this research focused on the interactions and mathematical practices used when seven mathematically promising third-grade students interacted with a series of five cognitively demanding math tasks. Specifically, this research showed how classroom norms and teacher moves, such as the five practices for oral discourse, productive struggle, and teacher observation, impacted student interactions with cognitively demanding tasks and how the students used mathematical practices to complete math tasks (Ball & Cohen, 1999; Cobb et al., 1991; Martin et al., 2017; Smith & Stein, 2000; Yackel & Cobb, 1996). The findings of this study can be used to determine whether students' interactions with cognitively demanding tasks continue to be noticed and used by teachers to push and grow the mathematical creativity and conceptual thinking of mathematically promising students. When teachers use and adapt tasks in an inquiry-based social constructivist environment, students become doers of mathematics and open windows of opportunity that can enhance their reasoning (Dominguez, 2016; Munter & Haines, 2019; NCTM, 2000, 2014; Schwartz, 2000).

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APPENDICES

Appendix A: Student Pre- Focus Group Interview Protocol
Interview Questions (Pertaining to Pre- Focus Group)

Study Title: A Descriptive Study of Elementary Mathematically Promising Students Interactions with Cognitively Demanding Tasks

RQ 1: How did elementary mathematically promising students interact with cognitively demanding math tasks?

RQ 2: What mathematical practices did mathematically promising students' use when completing cognitively demanding math tasks?

What is your student letter? __________

1. Do you like solving challenging math problems? What do you like best about solving them?

2. How do you usually solve difficult math problems best?

3. What tools or strategies help you solve challenging math tasks?

4. Do you talk or discuss your mathematical ideas with other students while solving challenging math tasks? Do you think these interactions help you solve challenging math tasks?

5. a. Do you ask yourself questions or think about your thinking when solving challenging math tasks?

   b. Do you draw pictures, use, or study symbols when solving challenging math tasks?

6. How do you use tools/manipulatives when solving challenging math tasks?

Appendix B: Student Focus Group Post Interview Protocol
Study Title: Elementary Student Interactions with Cognitively Demanding Math Tasks

RQ 1: How do elementary mathematically promising students interact with cognitively demanding mathematical tasks?

RQ 2: What mathematical practices did mathematically promising students' use when completing cognitively demanding math tasks?

What is your student letter? __________

1. What was your favorite part of solving math tasks?

2. Which math task did you find the most challenging? Why?

3. What did you write down or record when doing a math task that was helpful?

4. What questions did you find yourself asking when solving the math tasks?

5. Did you discuss many of your thoughts about the math tasks with your partner or table group?

6. How did these interactions help you solve the math tasks?

7. Did you draw pictures, use, or study symbols when solving math tasks?

8. How did you use tools/manipulatives when solving math tasks?

Appendix C: Google Debrief Form

1. How do you think the students persevered through the task?

2. Do you feel students were able to make sound arguments and justifications?

3. How did you feel about the students' interactions today during the task?

4. What else have you noticed about how the students interact with tasks?

5. What is your perspective on how the task enactment went today?

6. Is there anything else you would like to change about the study to help with the student interactions?

7 How do you feel the students' made connections to the task? (1-poor to 10-total real-life connections)

Appendix D: Figures

Figures 4-9

Document Analysis Student Work Samples

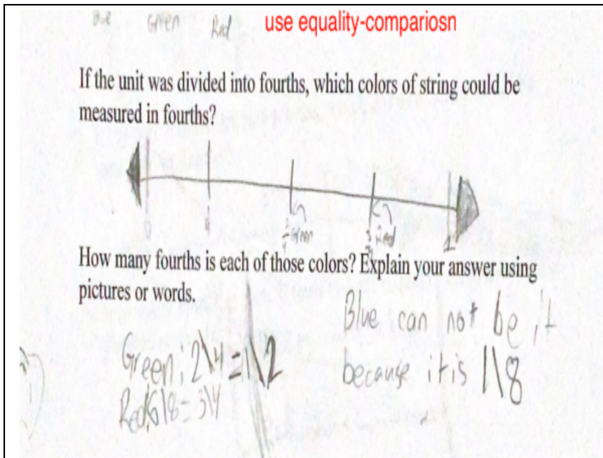


Figure 4
Teacher A, Student C Work Sample
Task 1, Piece of Yarn

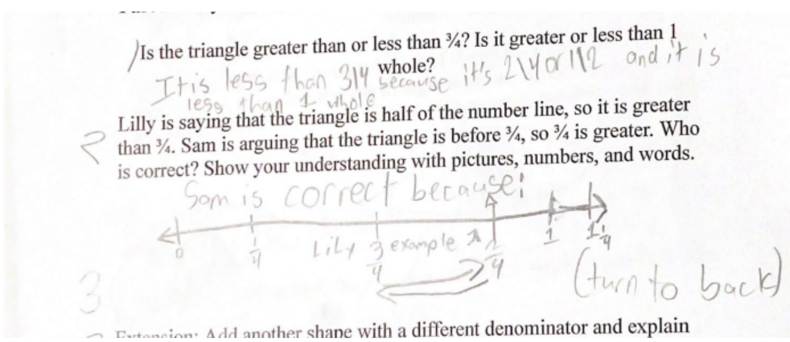


Figure 5
Teacher A, Student C Work Sample
Task 3, Comparing Fractions on a Number Line

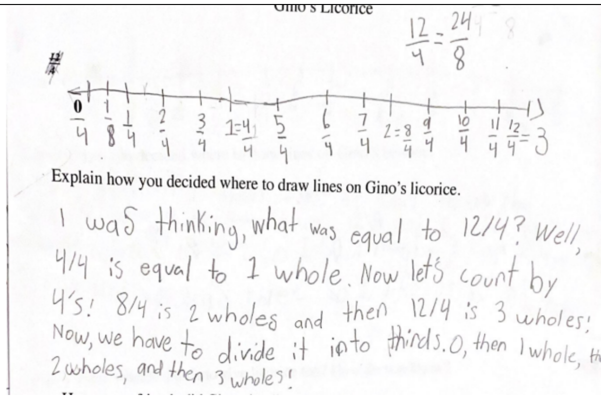


Figure 7, Teacher A, Student C
Task 5, Sharing Licorice

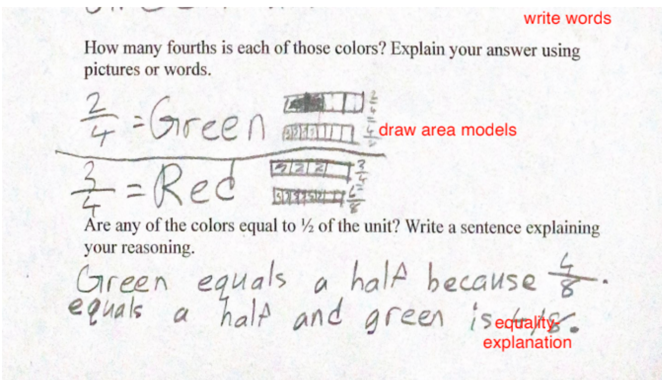


Figure 8
Teacher B, Student E
Note: Task 3, Comparing Fractions on a Number Line

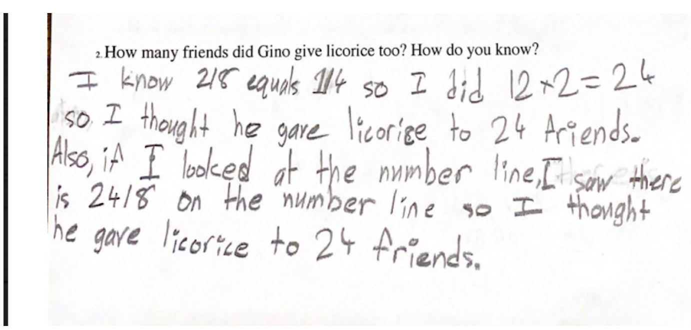


Figure 9

Teacher B5, Student E, *Sharing Licorice*

Figures 10-14

Google Debrief Form Results

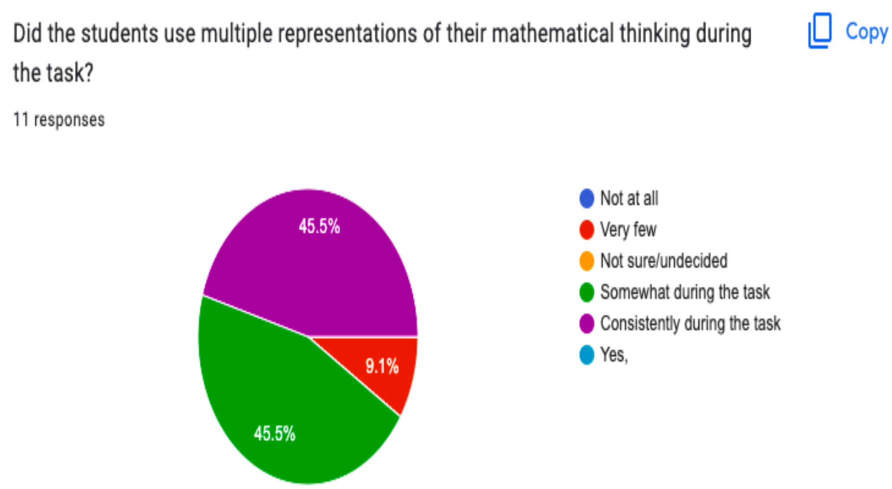


Figure 10

Tasks w/ Multiple Representations Demonstration

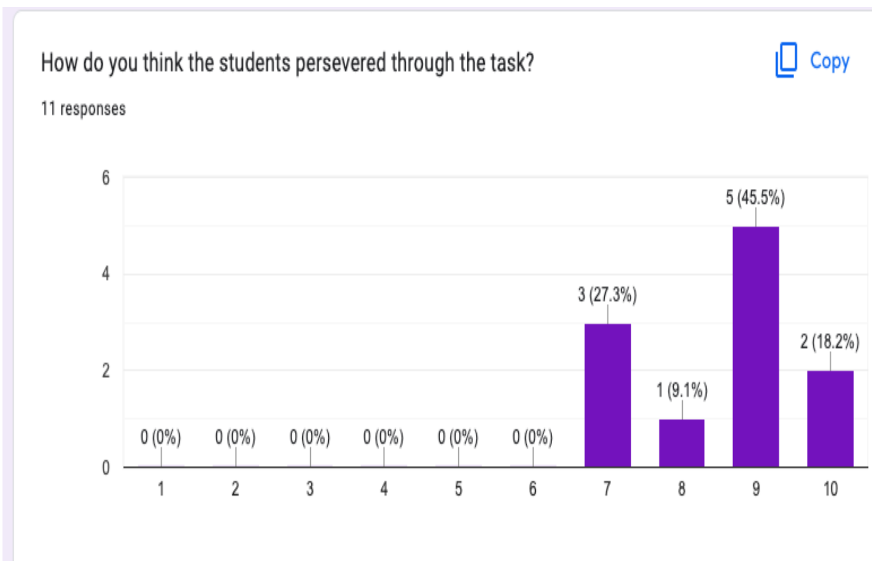


Figure 11

Tasks w/ Perseverance Demonstration

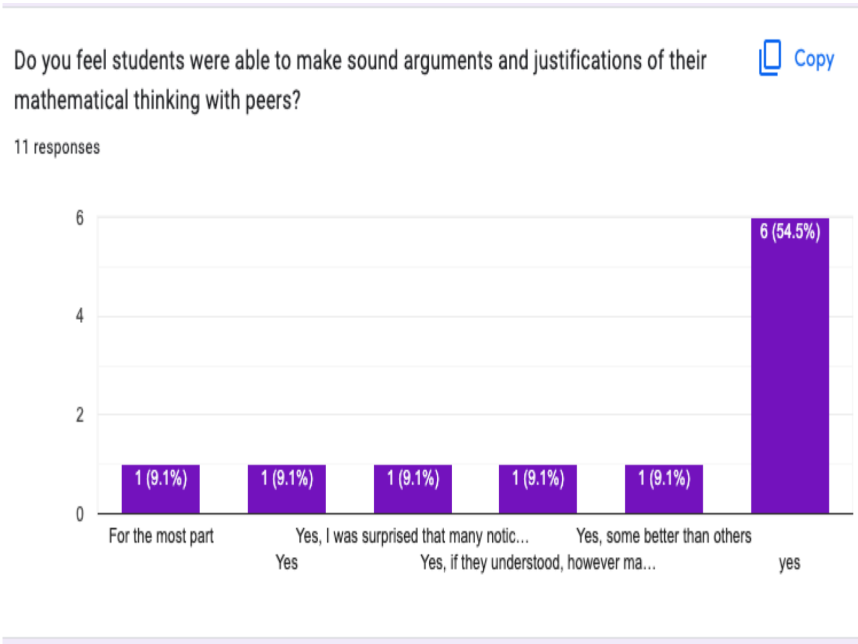


Figure 12

Tasks w/ Argumentation & Justification Demonstration

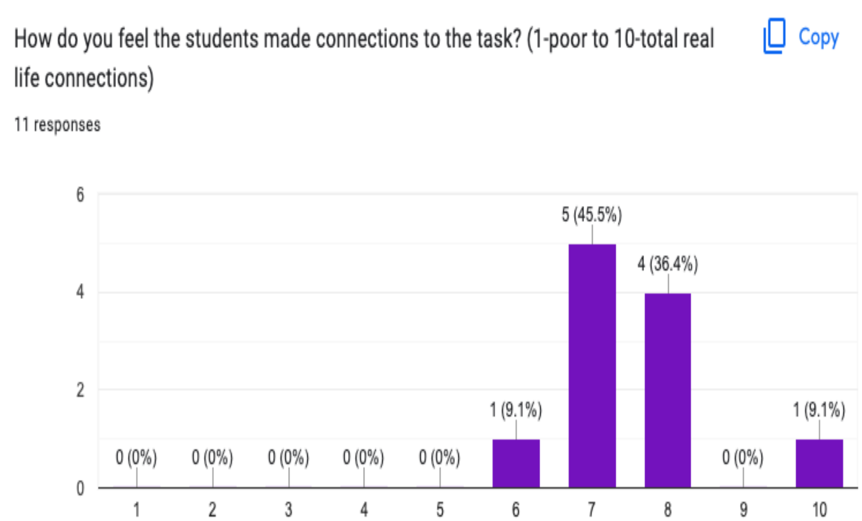


Figure 13

Tasks w/ Making Connections

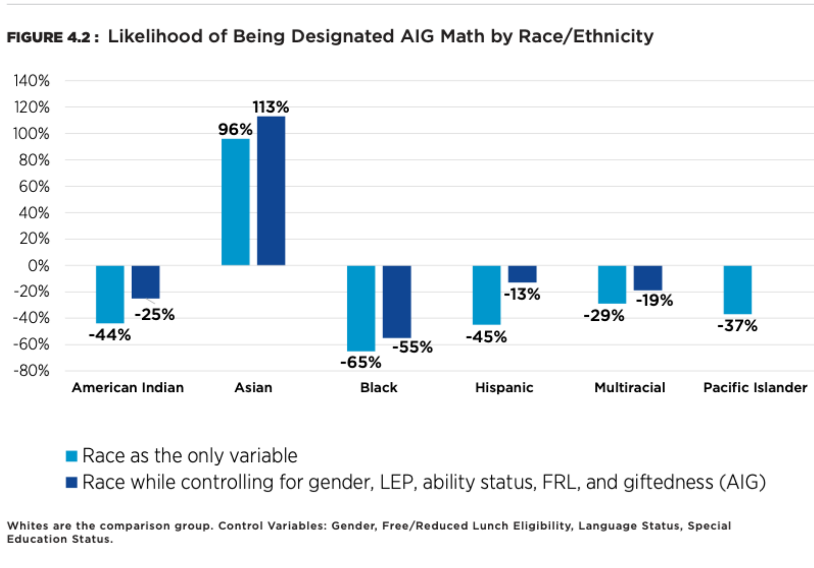


Figure 14

Note: Adapted from Eracing Inequities (Triplett & Ford, 2019)

Appendix E: Data Tables

Table 1

Codebook Definition Table

Codebook Label	Data Source
A1-A5	Audio observations from Teacher A
B1-B5	Audio observations from Teacher B
PRFG	Pre-Focus group interview
POFG	Post Focus group interview
FNA 1	Field notes Teacher A
FNB 1-5	Field notes Teacher B
Video 1-5	Video teacher A
	Video teacher B
Video 1-5	
SATA 1-5	Student artifacts Teacher A
SATB	Student artifacts Teacher B
GDF	Google Debrief Form

Note: Code book abbreviations

Table 2

Student Interactions Mathematical Practices

Math Practice	**Making Sense/ Connections**	**Perseverance**	**Representations & Tools**	**Solution Strategies & Justifications**
Student Interactions Observed With Task Completion	Are students taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas?	Are students persevering in exploring and reasoning through tasks?	Are students using tools and representations as needed to support their thinking and problem-solving?	Are students accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another and will analyze the frequency of each type of code to determine which type of interaction occurs most frequently? (Miles & Huberman, 2004; Ravitch & Carl, 2019)

Note. Adapted from *Principles to Action* (NCTM, 2014).

Table 3

Research Data Timeline

Week # of Study	Data Collection Method/Procedure
1	Teacher & Parent Recruitment & Consent and Student assent form collection Pre-focus group interview with students Task 1 Pre-task meeting w/ teacher Task 1 Teacher Debrief Google form Task 1 observation, and data collection begins
2	**Task 2** Task 2 Pre-task meeting Task 2 observation & data Teacher Debrief Google form -observation, -data collection **Task 3** Task 3 Pre-task meeting Task 3 observation and data Task 3 Teacher Debrief Google form Begin coding audio transcripts
3	**Task 4** Task 4 Pre-task meeting, Task 4 Teacher Debrief Google form observation/data collection, coding continued
4	**Task 5** Task Pre-task meeting Task 5 Teacher Debrief Google form Task 5 observation, data collection, coding continued Post Focus Group Interview Students
5-10	Begin raw word analysis and coding of focus group transcripts Comparison and coding of focus groups Coding of field notes and student work samples Data analysis, Discussion, and Findings write up

Table 4

Research Data Methods Table

Research Question Alignment	Research Method	Data Collection Instrument	Data Analysis Method	Timeline & Outcomes
RQ 1: How did elementary mathematically talented students interact with cognitively demanding mathematical tasks?	Student observations via video recording	Researchers collected observational field notes and videos of classroom observations as the small focus group of students interacted with the mathematical tasks.	Interpretive thematic coding; analyzing by creating open codes. Original field notes were recorded during tasks based on Student Actions from (NCTM 2014, p. 24) *Principles to Action* 1- Exploring & Reasoning 2- Making Sense & Connections 3- Representations & Tools 4- Justification & Approaches	Videotaped & audiotaped each task weekly Transcribed each videotaped task and coded first round the week following each task
RQ 1: How did elementary mathematically talented students interact with cognitively demanding	Pre- & Post-focus group interview	Researchers conducted semi-structured interviews in person at the conclusion of the study	Interpretive thematic coding; analyzing by creating open codes. Themes were created and categorized from	Conducted a pre-focus group with both small groups of classroom students at the initiation of the study

mathematical tasks?			interview protocol transcripts Open field notes were collected and coded, and created into memos	Conducted a post-focus group interview at the conclusion of the study Recorded and transcribed each focus group interview with “Otter.ai” Use open coding to look for themes and compare sociomathematical norms of classrooms
RQ 2: What mathematical practices did mathematically promising students use when completing cognitively demanding mathematical tasks?	Google Debrief Form (teachers) Document Analysis (Student work samples) Pre- & Post-focus groups	The researcher collected samples of student work from task enactment and exploration The tasks were open-coded for document analysis The google debrief form shared teachers' observations of mathematical practices in students	Student work samples were collected and analyzed according to math practices (NCTM, 2014). The audio transcripts of pre- and post-focus groups were compared	Constant comparison method to analyze evidence of learning to other methods of data collection

Table 5

Tools 4 Teachers 3rd Grade Cluster 7 Tasks In Study Table

Standard	**Formative Tasks**
3. NF. 2(Task 1)	*A Piece of Yarn*
3. NF. 3 (Task 2)	*Measuring Rainfall*
3. NF. 4 (Task 3 & 4)	*Comparing fractions* *Comparing fractions with a number line*
Culminating task (Task 5)	*Sharing licorice*

Note: Adapted from NC Tools 4 Teachers (2019)

Table 8

Themes from Research Questions Table

	RQ 1- Student interaction with Tasks	**RQ 2 - Mathematical Practices Used with Tasks**
Intrapersonal Communication	Students work independently to solve cognitively demanding tasks. (PER)	
Interpersonal Communication leads to reasoning & sense-making tasks		Students persevere through tasks (PER)
	Active collaboration is encouraged and explored within the instructional triangle of the math classroom. (PER)	
	Interpersonal interactions help students inquire, think deeply, and solve cognitively demanding tasks. (PER)	
	Reflective Communication: Students interacted with reflective communication to reason and solve tasks. (PER)	
Solving Tasks with Multiple Representations	Students compare visual representations of fractions to solve cognitively demanding tasks (MR).	Students use multiple representations & tools when emerging in their thinking & understanding to model with mathematics (MR)
	Students use physical gestures to respond to cognitively demanding tasks. (MR)	Students solve cognitively demanding tasks with mathematical writing & visual representations (MR)

Communication of Mathematical Knowledge Leads to the Justification of Tasks	Students verbalize mathematical knowledge to solve cognitively demanding tasks. (PER)	Distinct types of oral discourse: multiple readings/revoicing help students solve cognitively demanding tasks
	Students prove and explain their thinking and reasoning about cognitively demanding tasks with written statements (JS)	
Knowledge & Connections with Real Life & Mathematics Leads to Inquiry-Based Thinking & Mathematical Precision	Making connections with prior knowledge and real-life objects helps students solve cognitively demanding tasks (PCK)	Metacognitive thinking & encouragement of inquiry allows for conceptual thinking & supports students' emergence of precision when solving cognitively demanding tasks
Productive struggle, emotional response		Students have emotional reactions when solving cognitively demanding tasks Students productively struggle and when solving cognitively demanding math tasks (JS)
		Tasks influence student stamina and ability to productively struggle and when solving cognitively demanding math tasks (JS)
Teachers Support Task Enactment & Growth of Practices	Interpersonal interactions between the student and	Teacher encourages mathematical sense-making with perseverance and

	teacher helped students solve tasks	precision of cognitively demanding tasks (PER)

*N*ote: Themes adapted from findings of RQ 1 & RQ 2 based on original preset codes

Table 9

Document Analysis Coding Table

Code	Quantity Class-room A	# Of Tasks Code occurred	Quantity Classroom B	# Of Tasks Code Occurred	Total Quantity	Math Practice
Comparison of Fractions	5,6,3, 3, = 17	4	7,1, 7, = 15	3	32	MP 7
Labeling	7, 2, = 9	2	3, 1, 2 = 6	3	15	MP 4
Drawing models	4, 2, 5 = 11	3	3, 3, 2, 3, 2 = 13	5	24	MP 4
Mathematical Operation	5,	1	2, 2	2	9	MP 2
Symbolic reasoning	5, 5, = 10	2	3, 4 = 7	2	17	MP 8
Metacognition	2	1	0	0	2	MP 3
Explanation	5, 7, 3, 5, = 20	4	1, 3, 5, 2 =10	4	30	MP 3
Partitioning Number Line	2, 3, 3, = 8	3	3, 3, 5= 11	3	19	MP 6
Struggle Frustration	2	1	0	0	2	MP 1

Note: *Hand tallied and coded from original student work samples for RQ2*

Table 10

Mathematical Interactions Used by Students

Student	Mathematical Reasoning	Multiple Representations	Perseverance/Productive Struggle
Student A	"I thought the answer has to be a decimal, so it helped me to do it."	"So, like the first one, we had to cut it into eight, but we can't cut thirty-six into eights equally."	"Cause like, it was like, splitting it up, like, oh my God, how many pieces I have already done that! (With expression) "
Student B		Partitioning the number line was hard. But it helped me see the math.	"Because like that you had to split it up into like, three parts and then draw it like seven lines in it. And it was very tiring for your brain. And then it made you do extra work at the end. Partitioning the number line was hard. "
Student C		"The thing that helped me most with drawing was drawing like that division thing. That was the most helpful.	
Student D	"I used symbols a lot. Like less than and equal to. They helped me solve the problem."	"I like the way you can split them into like, what's a little into them?"	
Student E	I learned that we had to multiply it to find the equivalent	"I think like I usually do; words and symbols are	"Doing fractions was my favorite part" I liked how I started

	fraction like 2/3 is equivalent to 4/6. Like if you do two times two is four and three times or is the equivalent for me outside	technically everything that you write down like that, write down words, symbols and numbers."	learning more about equivalent fractions.
Student F	"It's kind of not a tool but like my teacher really helped me make sense of the problems."	" I really liked finding out what is equivalent to the fractions. I like the way you can split them into like, what is a little into them?"	"Like, we all have different talents, and I am not that good at fractions. So, my partner really helped me with that because, like, I was stuck."
Student G	"At first, I did not really understand the number line one. So, I would just like to draw an area model to help me see what was going on in the number line to understand it better.	"I had to use the Fraction Bars. They were not like fraction paper. They are the fraction bars. So, if I can do like two fourths on easy got two fourths, I could compare him to wherever needed."	"The Rainfall task. It was hard because there were so many lines. I could not see which one was which, the sixth one was the fifth one."

Note: From Post Focus Group Transcript

Appendix F: Formative Assessment Tasks

Task 1

A Piece of Yarn

NC.3.NF.2 and NC.3.NF.3 A Piece of Yarn	
Domain	Number and Operations – Fractions
Cluster	Understand fractions as numbers.
Standard(s)	**NC.3.NF.2** Interpret fractions with denominators of 2, 3, 4, 6, and 8 using area and length models. • Using an area model, explain that the numerator of a fraction represents the number of equal parts of the unit fraction. • Using a number line, explain that the numerator of a fraction represents the number of lengths of the unit fraction from 0. **NC.3.NF.3** Represent equivalent fractions with area and length models by: • Composing and decomposing fractions into equivalent fractions using related fractions: halves, fourths, and eighths; thirds and sixths. • Explaining that a fraction with the same numerator and denominator equals one whole. • Expressing whole numbers as fractions and recognize fractions that are equivalent to whole numbers.
Materials	Student activity sheet, paper, pencils, white boards, and dry-erase markers (optional)
Task	**Part 1**: Suni was using the following yard stick to measure pieces of yarn for her art project. This ruler shows how much yarn she cuts for each color. What fraction of a unit does she need of each color? **Part 2**: If the unit were divided into fourths, which colors of string could be measured in fourths? How many fourths is each of those colors? Explain your answer using pictures or words.

	Part 3: Are any of the colors equal to ½ of the unit? Write a sentence explaining your reasoning.

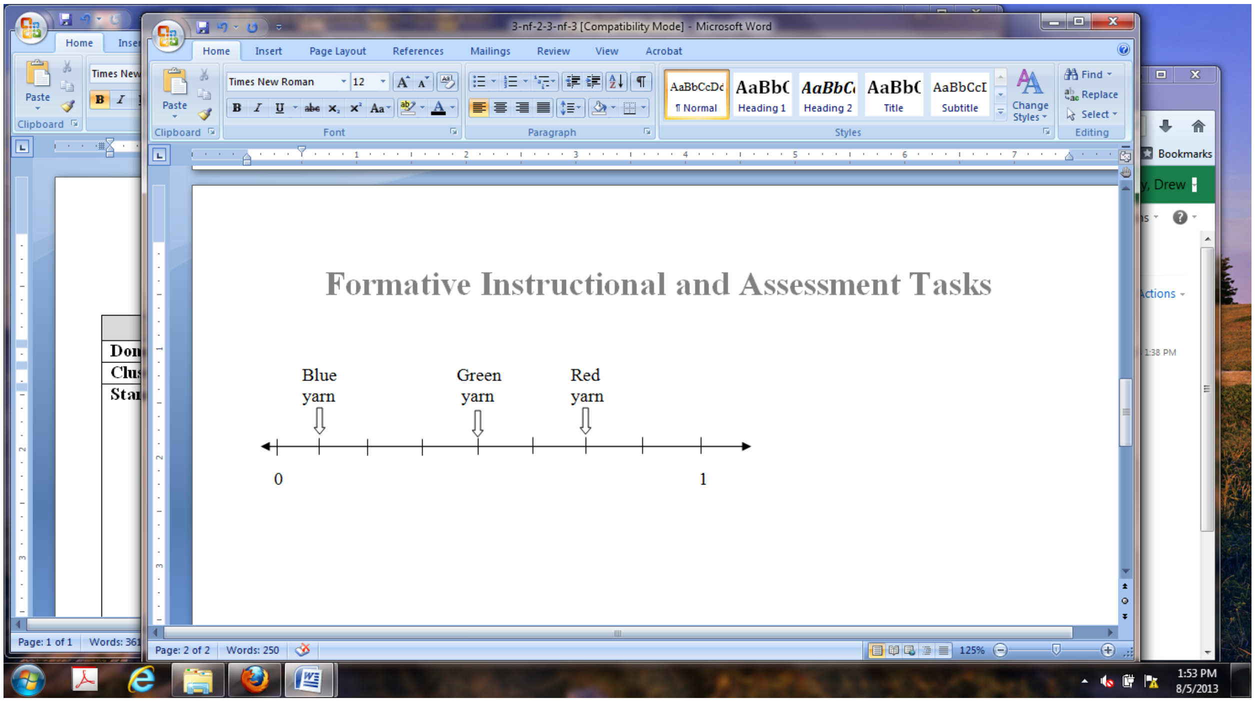
Rubric		
Level I Not Yet	1. **Level II** 2. Progressing	**Level III** Meets Expectation
● Incorrect answer and work	● Finds the correct answer, but there may be inaccuracies or incomplete justification of solution **<u>OR</u>** ● Uses partially correct work but does not have a correct solution	● Accurately solves Part 1: Blue: 1/8, Green: 4/8, Red 6/8. ● Accurately solves Part 2: Green and Red can be measured in fourths. Green: 2/4. Red: 3/4. ● Accurately solves Part 3: Green. ● Write clear and appropriate explanations.

**Level IV would include other equivalent fractions.*

Standards for Mathematical Practice
1. Makes sense and perseveres in solving problems.
2. Reasons abstractly and quantitatively.
3. Constructs viable arguments and critiques the reasoning of others.
4. Models with mathematics.
5. Uses appropriate tools strategically.
6. Attends to precision.
7. Looks for and makes use of structure.
8. Looks for and expresses regularity in repeated reasoning.

A Piece of Yarn

Suni was using the following yard stick to measure pieces of yarn for her art project. This ruler shows how much yarn she cuts for each color.



What fraction of a unit does she need of each color?

If the unit were divided into fourths, which colors of string could be measured in fourths?

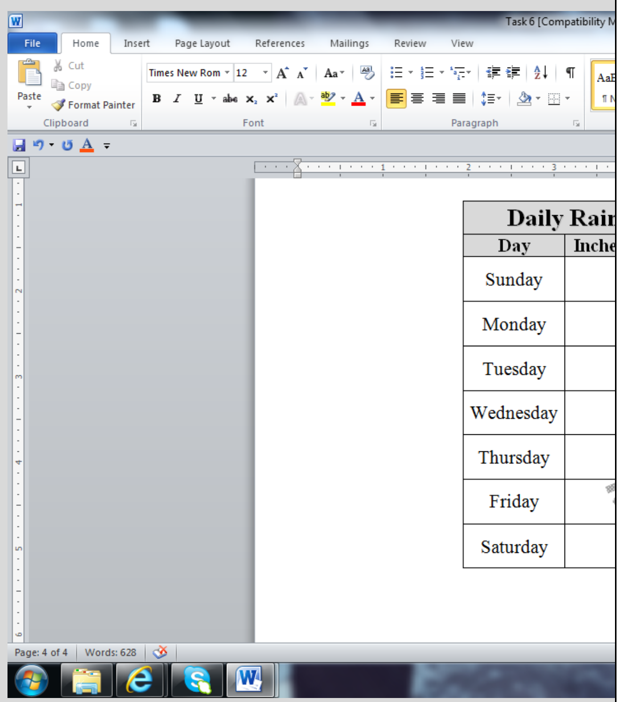
How many fourths is each of those colors? Explain your answer using pictures or words.

Are any of the colors equal to ½ of the unit? Write a sentence explaining your reasoning.

Task 2

Measuring Daily Rainfall

NC.3.NF.3 Measuring Daily Rainfall	
Domain	Number and Operations - Fractions
Cluster	Understand fractions as numbers.
Standard(s)	**NC.3.NF.3** Represent equivalent fractions with area and length models by: • Composing and decomposing fractions into equivalent fractions using related fractions: halves, fourths, and eighths; thirds and sixths. • Explaining that a fraction with the same numerator and denominator equals one whole. • Expressing whole numbers as fractions and recognize fractions that are equivalent to whole numbers.

Materials	Measuring Daily Rainfall handouts, fraction manipulatives, pencils, paper
Task	● Distribute Measuring Daily Rainfall handouts. ● Read: *Since the local weatherman predicted rain for the whole week, Ms. Moore's class decided to measure the amount of daily rainfall. The chart below shows their data. Use this chart to answer each question.*  ● Read each question aloud: ○ *Did more rain fall on Sunday or Tuesday?* ○ *Which day had less rain: Monday or Wednesday?* ○ *Someone erased part of Friday's measurement! If an equal amount of rain fell on Thursday and Friday, what is Friday's measurement? Prove that your answer is correct using objects, drawings, a number line, or words.*

	○ *What is another way to record the amount of rain that fell on Saturday? Use objects, drawings, a number line, or words to explain why you can represent this measurement in more than one way.*

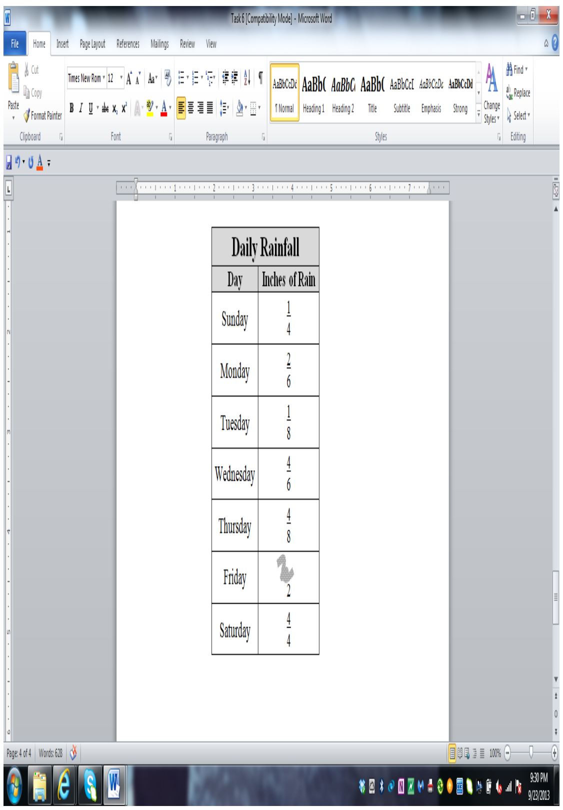
Rubric		
Level I Not Yet	3. **Level II** 4. Progressing	**Level III** Meets Expectation
● Student work is inaccurate, incomplete, or off task.	Students do 1-3 of the following: ● identifies that more rain fell on Sunday ● identifies that less rain fell on Wednesday ● determine that ½ inch of rain fell on Friday and justifies solution ● identifies a fraction or whole number equal to 4/4 and explains that any equivalent fraction can be used to name this amount.	Students do all the following: ● identifies that more rain fell on Sunday ● identifies that less rain fell on Wednesday ● determine that ½ inch of rain fell on Friday and justifies solution ● identifies a fraction or whole number equal to 4/4 and explains that any equivalent fraction can be used to name this amount.

Standards for Mathematical Practice
1. Makes sense and perseveres in solving problems.
2. Reasons abstractly and quantitatively.
3. Constructs viable arguments and critiques the reasoning of others.
4. Models with mathematics.
5. Uses appropriate tools strategically.
6. Attends to precision.

7. Looks for and makes use of structure.	
8. Looks for and expresses regularity in repeated reasoning.	

Since the local weatherman predicted rain for the whole week, Ms. Moore's class decided to measure the amount of daily rainfall. The chart below shows their data.

Use this chart to answer each question.

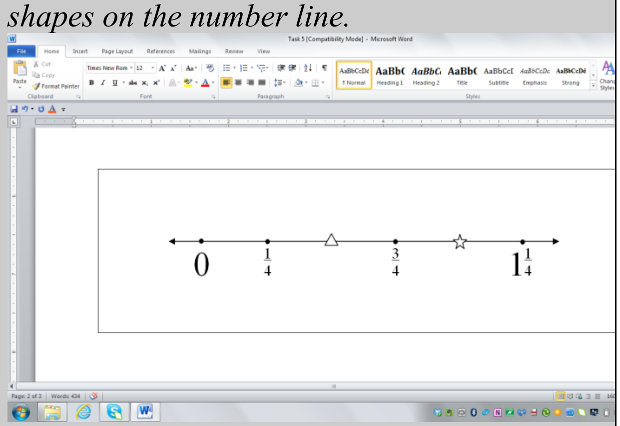


Daily Rainfall	
Day	Inches of Rain
Sunday	$\frac{1}{4}$
Monday	$\frac{2}{6}$
Tuesday	$\frac{1}{8}$
Wednesday	$\frac{4}{6}$
Thursday	$\frac{4}{8}$
Friday	[illegible] / 2
Saturday	$\frac{4}{4}$

5. What is another way to record the amount of rain that fell on Saturday? Use objects, drawings, a number line, or words to explain why you can represent this measurement in more than one way.

Task 3

Comparing Fractions on a Number Line

NC.3.NF.4 Comparing Fractions with a Number Line	
Domain	Numbers and Operations-Fractions
Cluster	Understanding Fractions as Part of a Whole
Standard(s)	**NC.3.NF.4** Compare two fractions with the same numerator or the same denominator by reasoning about their size, using area and length models, and using the >, <, and = symbols. Recognize that comparisons are valid only when the two fractions refer to the same whole with denominators: halves, fourths, and eighths; thirds and sixths.
Materials	Activity sheet, pencil, tools as needed
Task	Hand out the activity sheet to each student. Read each part of the problem to the students before they begin working. *Part 1: Label the fractions represented by the shapes on the number line.*  *Part 2: Lilly and Sam need to use the number line to solve this problem: Is the triangle greater than or less than ¾?* *Lilly is saying that the triangle is half of the number line, so it is greater than ¾. Sam is arguing that the triangle is before ¾, so ¾ is greater. Who is correct? Show your understanding with pictures, numbers, and words."*

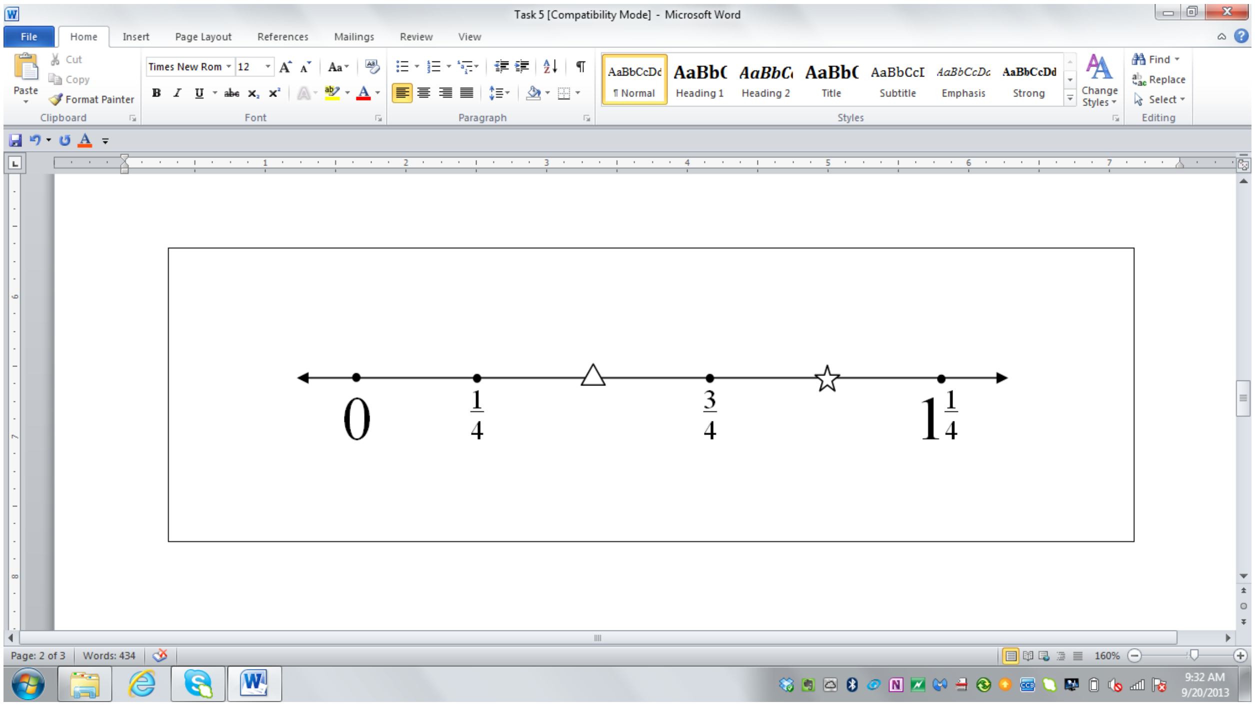
Rubric (include a statement of purpose of rubric--for teacher decision making rather than evaluation)		
Level I	**Level II**	**Level III**

Not Yet	Progressing	Meets Expectation
• Students are unable to identify the correct fraction to the shape on the number line • Student is unable to use reasoning to compare the fractions	• Students can identify the correct fraction to the shape on the number line but CANNOT use reasoning to compare the fractions • Students can compare the fractions but are unable to express reasoning with pictures, numbers, or words.	• Students can identify the fraction to the shape on the number line • Students can use reasoning to compare the fractions • Students can show their understanding with pictures, numbers, or words

Standards for Mathematical Practice
1. Makes sense and perseveres in solving problems.
2. Reasons abstractly and quantitatively.
3. Constructs viable arguments and critiques the reasoning of others.
4. Models with mathematics.
5. Uses appropriate tools strategically.
6. Attends to precision.
7. Looks for and makes use of structure.
8. Looks for and expresses regularity in repeated reasoning.

Task 3: Comparing Fractions with Number Lines

Part 1: Label the fractions represented by the shapes on the number line.



Part 2: Lilly and Sam need to use the number line to solve this problem:

Is the triangle greater than or less than ¾? Is it greater or less than 1 whole?

Lilly is saying that the triangle is half of the number line, so it is greater than ¾. Sam is arguing that the triangle is before ¾, so ¾ is greater. Who is correct? Show your understanding with pictures, numbers, and words.

Extension: Add another shape with a different denominator and explain your reasoning.

Task 4

Comparing Fractions

NC.3.NF.4 **Comparing Fractions**	
Domain	Number and Operations - Fractions
Cluster	Understand fractions as numbers.
Standard(s)	**NC.3.NF.4** Compare two fractions with the same numerator or the same denominator by reasoning about their size, using area and length models, and using the >, <, and = symbols. Recognize that comparisons are valid only when the two fractions refer to the same whole with denominators: halves, fourths, and eighths; thirds and sixths.
Materials	Number line, fraction models, paper, pencils
Task	Part I: Below are measurements of ribbon in feet. For each pair of ribbons, draw a picture to determine which is longer. ● Pair 1: **2/3 2/4** ● Pair 2: **2/6 4/6** Part II: Determine which fraction in each set is larger. Explain your reasoning using only words and numbers (without using models or number lines). ● Pair 3: **1/3 2/3** ● Pair 4: **3/6 3/4**

Rubric		
Level I Not Yet	6. **Level II** 7. Progressing	**Level III** Meets Expectation
● Students do not achieve the correct answer and use inappropriate solution strategy.	● Student determines which fractions are larger but provides limited to no reasoning. **OR** ● Student provides some sound reasoning but is unable to determine which fractions are larger in each set.	● Student accurately determines which fraction in each set is larger: ○ Set 1: 2/3 ○ Set 2: 4/6 ○ Set 3: 2/3 ○ Set 4: 3/4 ● Students use visual models or number lines to accurately explain which

		fractions in Sets 1-2 are larger. ● Student uses sound reasoning to explain how the larger fractions in Sets 3-4 were determined (i.e., When looking at the fractions in Set 4, the student recognizes that there are three pieces in each fraction. Since fourths are larger than sixths, three fourths would be larger than three sixths.)

Standards for Mathematical Practice
1. Makes sense and perseveres in solving problems.
2. Reasons abstractly and quantitatively.
3. Constructs viable arguments and critiques the reasoning of others.
4. Models with mathematics.
5. Uses appropriate tools strategically.
6. Attends to precision.
7. Looks for and makes use of structure.
8. Looks for and expresses regularity in repeated reasoning.

Task 4

Comparing Fractions

Part I: Below are measurements of ribbon in feet. For each pair of ribbons, draw a picture to determine which is longer.

- Pair 1: **2/3** **2/4**
- Pair 2: **2/6** **4/6**

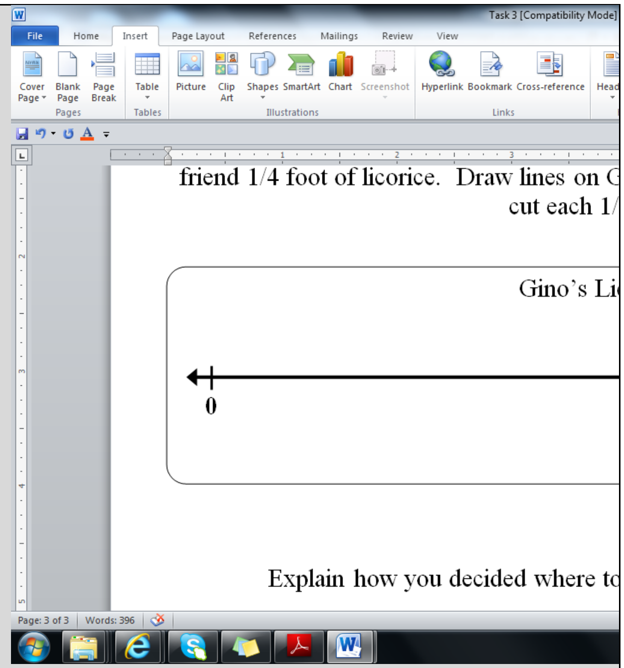
<u>Part II:</u> Determine which fraction in each set is larger. Explain your reasoning using only words and numbers (without using models or number lines).

- Pair 3: **1/3   2/3**
- Pair 4: **3/6   3/4**

Task 5

Sharing Licorice

NC.3.NF.2 Sharing Licorice	
Domain	Number and Operations - Fractions
Cluster	Understand fractions as numbers.
Standard(s)	**NC.3.NF.2** Interpret fractions with denominators of 2, 3, 4, 6, and 8 using area and length models. • Using an area model, explain that the numerator of a fraction represents the number of equal parts of the unit fraction. • Using a number line, explain that the numerator of a fraction represents the number of lengths of the unit fraction from 0.
Materials	Sharing Licorice handouts, paper, pencils, rulers
Task	<u>Part 1:</u> ● Distribute Sharing Licorice handouts. ● Draw students' attention to the image of Gino's licorice.



- Read: *Gino has 8/4 feet of licorice to share with his friends. He decides to give each friend 1/4 foot of licorice. Draw lines on Gino's licorice to show where he should cut each 1/4 foot.*

<u>Part 2:</u>

- Read: *Explain how you decided where to draw lines on Gino's licorice.*

Rubric		
Level I Not Yet	8. **Level II** 9. Progressing	**Level III** Meets Expectation
• Students use inappropriate solution strategies and do not obtain the correct solution.	• The student places some fractions in the correct location, and partially explains why each fraction is placed in its location. ***<u>or</u>*** • Student places all fractions in the correct location but does not have sound reasoning	• Students accurately place all fractions on the number line. • Student correctly explains why each fraction is placed in its correct location.

	to prove his/her solution strategies.	

Standards for Mathematical Practice
1. Makes sense and perseveres in solving problems.
2. Reasons abstractly and quantitatively.
3. Constructs viable arguments and critiques the reasoning of others.
4. Models with mathematics.
5. Uses appropriate tools strategically.
6. Attends to precision.
7. Looks for and makes use of structure.
8. Looks for and expresses regularity in repeated reasoning.

Sharing Licorice

Gino has 8/4 feet of licorice to share with his friends. He decides to give each friend 1/4 foot of licorice. Draw lines on Gino's licorice to show where he should cut each 1/4 foot.

Gino's Licorice

0

Explain how you decided where to draw lines on Gino's licorice.

2. How many friends did Gino give licorice too? How do you know?