THREE ESSAYS ON CAPITAL INSURANCE AND TOO BIG TO FAIL BANKS

by

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ABSTRACT

KATERINA IVANOV. Three essays on capital insurance and too Big to fail banks. (Under the direction of DR. WEIDONG TIAN)

This research study presents an insurance framework of the bank capital by introducing a new type of capital, namely, an insurance capital. A bank pays the insurance capital to an entity which injects a pre-determined payout of capital during the period of systemic crisis. The pre-determined payout relies on the aggregative loss of a bank sector, so this contract between the bank and the entity is a capital insurance contract. In a rational equilibrium setting, the entity charges an appropriate premium while the banks purchase an optimal amount of the insurance.

Chapter I presents a welfare analysis of several capital insurance programs in a rational expectation equilibrium setting. We first characterize explicitly the equilibrium of each capital insurance program. Then, we demonstrate that a capital insurance program based on the aggregate loss is better than the classical insurance when those big financial institutions have similar expected loss exposures. By contrast, the classical insurance is more desirable when the bank's individual risk is consistent with the expected loss in a precise way. Our analysis shows that the capital insurance program is a useful tool to hedge the systemic risk from the regulatory perspective.

As an extension, Chapter II demonstrates that, both the entity and the banks have motivations to participate in this capital insurance program due to their increased expected utilities (welfare) respectively. The total systemic risk ex post within the capital insurance program is reduced and can be even removed eventually after re-

peatedly entering the capital insurance program.

In Chapter III, we develop a rational expectation equilibrium of capital insurance to identify too big to fail banks. We show that (1) too big to fail banks can be identified by loss betas, a new systemic risk measure through this equilibrium analysis, of all banks in the entire financial sector by an explicit algorithm; (2) the too big to fail feature can be largely justified by a high level of loss beta; (3) the capital insurance proposal benefits market participants and reduces the systemic risk; (4) the implicit guarantee subsidy can be estimated within this equilibrium framework; and (5) the capital insurance proposal can be used to resolve the moral hazard issue. The model is further tested empirically to identify too big to fail banks during both pre-crisis and pro-crisis periods. Implementing the proposed methodology, we document that the too big to fail issue has been considerably reduced in the pro-crisis period. As a result, we demonstrate that the capital insurance proposal could be a useful macro-regulation innovation policy tool.

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CHAPTER 1: A WELFARE ANALYSIS OF CAPITAL INSURANCE

1.1 Introduction

This paper presents a welfare analysis of recently proposed capital insurance programs in a rational expectation equilibrium setting. The idea of the capital insurance is motivated to resolve the "too big to fail" issues. As those "too big to fail" banks or companies which are "financial in nature" (thereafter, banks) [1] expect the capital injection from the central bank in time of financial distress, the banks might enact in a risk-taking manner and put the central bank, regulator and all taxpayers in a fragile financial position. In the capital insurance program (see Kashyap, Rajan and Stein (2009)), the bank pays some amount as a premium or reserve to central bank which, in turn, would inject funds to the banks in future financial failure. The capital insurance program is motivated to protect taxpayers and economy as a whole at the presence of big financial predicament. Our purpose is to study whether this capital insurance idea works or not from its welfare perspective.

Capital insurance is very different from current capital regulation implemented in BASEL II and BASEL III. It is also different from the Dodd-Frank Act which posts several prudential standards and new stringent capital requirements to banks

[1] Under the standards set forth in section 113 of the Dodd-Frank Act,a bank holding company or "nonbank financial company" poses a potential systemic risk if "material financial distress at the company, or the nature, scope, size, scale, concentration, interconnectedness, or mix of the activities of the company, could pose a threat to the financial stability of the United States." Therefore, we focus only on these companies with systemic risks (too big to fail).

with systemic risks. According to the capital regulation requirement, the amount of capital reserve or economical capital amount depends on the risk of loss portfolio and the riskiness of the bank itself. The risker the bank, the higher the economical capital; the economical capital is higher for a bank with weak credit situation than for the strong counterpart while assuming the portfolio is identically the same. Therefore, the economical capital idea depends on both the individual bank's riskiness and individual loss portfolio.

By contrast, the capital insurance, in essence, is an insurance contract, and the capital insurance idea casts all banks together from the market level. On the one side, the central bank is an insurer of the contract and receives an insurance premium with the obligation to inject funds to save the bank in financial distress. On the other side, the bank is an insured in this contract agreement. As the central bank represents the taxpayer in this structure, the insurer of the contract is a taxpayer, and the premium represents a special purpose tax in the sense described by Acharya et al (2010). In contrast to the traditional insurance contract, the contract redemption is contingent on the aggregate loss, and the insured event is contingent on the systematic event in the economy.

The rational expected equilibrium of the capital insurance program is explained as follows. The central bank issues insurance contracts to the banks, and the banks purchase these contracts that are placed on the market. The central bank predicts the correct optimal demand from the banks with a given premium structure, so the central bank maximizes the welfare with the premium structure as characterized. Consequently, both the demand (from the banks) and the supply (from the central bank) are determined uniquely in a rational expectation equilibrium.

In this paper, we assume the insurance contract payout has been placed on as proposed by the capital insurance program. Therefore, we do not address the optimal capital insurance design problem. Instead, we consider two capital insurance programs. In the first one, the insurance contract insures the aggregate loss of all banks. In the second one, each bank buys insurance that depends on the aggregate loss of all banks except for the insured bank's own loss portfolio. For comparison purpose, we further consider the situation when each bank purchases the insurance that relies on its own loss portfolio. This is a "classical insurance" by terminology in this paper, and it has the same indemnity as the traditional coinsurance contract. As the premium structure depends on all loss portfolios of the banks, those loss portfolios together affect each bank's coinsurance demand. Therefore, the classical insurance in our setting is different from the traditional coinsurance contract in equilibrium.

We demonstrate that many factors affect the welfare analysis and the chosen capital insurance program. First, the proposed two capital insurance programs are distinguished from each other by the correlation structure. A low correlation environment ensures a low welfare of the contract based on the aggregate loss except for individual bank's loss. Therefore, the aggregate insurance is better than the other one. In fact, when each loss portfolio can be observed completely by all banks and the central bank, and the bank does not manipulate the book loss, the aggregate insurance ensures a higher welfare than another one in general.

Second, both the specific risk and the systematic risk components of the individual loss are important ingredients to compare the classical insurance and the aggregate insurance contracts. These two components play a crucial role in the classical demand analysis of the coinsurance contract (for a mean-variance insured), see Gollier (2011). We demonstrate that the way how each bank's specific risk and systematic risk components behave together in the market has significant effect on the comparison analysis. When a higher individual risk corresponds to a higher expected loss per each volatility unit, we say that the market displays an ordering loss market. Otherwise, the market is a disordering loss market.[2] We show that the classical insurance works better in the ordering loss market, while the aggregate insurance is more beneficial to the central bank in the disordering loss market. Hence, our result is significantly disparate from the optimal sharing rules in a pure exchange market.[3] The optimal insuring rule, in our equilibrium, relies on the aggregate loss portfolio in a more complicated way. Literally, the way how the loss portfolios are connected to each other implies different welfare outcome of the insurance program.

Third, the way how the systematic risk is distributed among each bank is also captious for a comparative welfare analysis of the insurance contracts. If each bank contributes equally or very close to each other in the total systematic risk, we show that the aggregate insurance ensures a higher welfare. Therefore, it is a more desirable insurance program than the classical insurance one. Wagner (2010) shows that the diversification might enhance the systemic risk while it reduces each institution's individual probability of failure, so a full diversification is not always beneficial from

[2]Precisely, when a risk-adjusted covariance of loss portfolio is co-monotonic to the Sharpe ratio of the loss portfolio, we say it is an ordering loss market. If these both sequences are counter-monotonic to each other, we say the market is a disordering loss market. See Propositions 4,6 and 7 below.

[3]By Borch (1962), the optimal sharing rules must be increasing with respect to the aggregate endowment. Our setting is different from Borch's equilibrium setting in the presence of the central bank.

the systemic perspective. According to our result, the aggregate insurance offers a solution in a full diversification situation to reduce the systemic risk.

The remainder of the chapter is organized as follows. Section 1.2 introduces the setting and characterizes the equilibrium. Section 1.3 presents the comparison of three types of capital insurance programs by the welfare analysis developed in the equilibrium. Section 1.4 offers discussion and implications of our theoretical results. Moreover, we explain how to implement the capital insurance program in practice and how to identify the "too big to fail" banks from the regulatory perspective. Section 1.5 briefly describes conclusions of the conducted analysis, and all proofs are stated in the Appendix A. Appendix B presents the equilibrium in a general situation and identifies these "too big to fail" banks by using this capital insurance program.

1.2 The Model

There are N big banks indexed by $i = 1, \cdots, N$ in one-period economic world. Each bank is endowed with a loss portfolio $X_1, \cdots, X_N$, respectively. These loss portfolios are defined on the same state space Ω, and all banks have the same beliefs on the nature of state. This common belief is represented by one probability measure P on the state space. However, these bank's loss portfolios can be significantly different. We assume that each bank is risk-averse, and the preference of risk is interpreted by a utility function $U_i(\cdot)$. The bank's initial wealth is given by W_0^i for each bank $i = 1, \cdots, N$, respectively.

There is a government entity such as Financial Stability Oversight Council (FSOC) in Dodd-Frank Act or a central bank, which sells the insurance contract to each bank.

Each bank is either voluntarily or enforced to purchase the insurance contract by paying particular amount as a premium, and a fund commitment is guaranteed by central bank in a bad business situation in the future. The premium amount can be treated as a special tax purpose rate for each bank as suggested by Acharya et al (2010). The fund commitment offered by the government entity is the indemnity of the insurance. Alternatively, these insurance contracts can be issued by a reinsurance company which is able to diversify the reinsurance risk. For simplicity, we name the insurer as a regulator.

The prototype insurance structure has the indemnity $I_i(X, X_i)$ that depends on the individual book loss X_i and the aggregate loss X. The aggregate loss $X = \sum_{i=1}^{N} X_i$. This insurance contract is called a "capital insurance" as it depends on the aggregate loss being realized in the future. The capital insurance contract is different from the classical contracts in which $I_i(X, X_i)$ is irrelevant to the aggregate loss X and, instead, depends on the individual loss X_i. Following the classical insurance literature (Arrow (1963) and Raviv (1979)), we assume that insurance premium is determined by $(1 + \rho)\mathbb{E}[I(X, X_i)]$, where ρ is a load factor. For simplicity, we assume that the loss factor is the same across the bank industry, but it is possible to consider a bank-specific premium structure in the extended analysis. The loss factor is characterized by the regulator in equilibrium that will be explained shortly.

Given a load factor ρ, each bank chooses the best available insurance contract to maximize the expected utility (see Arrow (1963)):

$$\mathbb{E}[U_i(W^i)] = \mathbb{E}\left[U_i\left(W_0^i - X_i + I_i(X, X_i) - (1 + \rho)\mathbb{E}[I_i(X, X_i)]\right)\right]. \tag{1.1}$$

The regulator is risk-neutral and receives the premium for each contract. The welfare of the regulator is

$$W^r = \sum_i (1+\rho)\mathbb{E}\left[I_i(X, X_i)\right] - \sum_i I_i(X, X_i) - \sum_i c\left(I_i(X, X_i)\right), \tag{1.2}$$

where $c\left(I_i(X, X_i)\right)$ represents the cost for the regulator to issue the contract $I_i(X, X_i)$. The cost can be fixed, a constant percentage of the indemnity, or can depend on a drastic market event. To focus on the analysis of insurance program, we assume that the cost structure is a constant for each bank. The regulator's objective is to determine the best premium structure given the optimal demand for each bank (with any a given load structure ρ) as well as to maximize the expected welfare. Clearly, the insurance $I_i^*(X, X_i)$ in equilibrium depends on both the demand (from all banks) and the supply (from the regulator) and relies on the load factor ρ^* proposed by the regulator. We don't distinguish between the welfare and the expected welfare when there is no confusion in the rest of this paper.

In this paper, we focus on the following three capital insurance programs:

- Aggregate Insurance: $I_i(X, X_i) = \alpha_i X$, where $\alpha_i \geq 0$.
- Classical Insurance: $I_i(X, X_i) = \alpha_i X_i$, where $\alpha_i \geq 0$.
- Aggregate-Cross Insurance: $I_i(X, X_i) = \alpha_i \hat{X}_i$, where $\hat{X}_i = \sum_{j \neq i} X_j$ is the total loss except for the insured bank's loss, and $\alpha_i \geq 0$.

In each case, bank i chooses the best coinsurance parameter a_i. The optimal a is written as $a(\rho)$ to highlight its dependence on the load factor. The first insurance contract depends solely on the aggregate loss X, so it is called "aggregate insurance".

The coinsurance parameter α_i represents the percentage of the aggregate loss that is insured for the bank i. Clearly, this coinsurance parameter depends on how much the individual bank's loss risk contributes to the aggregate loss, as will be seen later. The second insurance contract is a standard one, initiated by Arrow (1963) and is termed as "classical insurance". However, the premium structure in traditional insurance contract is either given exogenously or depends on the specific loss portfolio in equilibrium. Therefore, our classical insurance is different from those traditional insurance contracts in a rational expectation equilibrium. The last insurance contract is motivated differently. Because of the possibility of the bank's manipulation of the loss report on X_i, as discussed in Chiappori and SalaniÃľ (2000) in a similar context, there is a moral hazard issue in case $I_i(X, X_i)$ is related to X_i. To resolve it, Kashyap et al (2008) introduces the aggregate-cross insurance idea in which the bank insures the total risks of all banks except for the bank's itself risk. The aggregate-cross insurance contract is inspired by the idea outlined in Kashyap et al (2008).

In what follows, we impose two assumptions to simplify the discussions.

Assumption I. Each bank is a mean-variance agent with the reciprocal of risk aversion parameter $\gamma_i > 0$. We also assume zero (or constant) cost structure for each contract. [4]

Assumption II. There exists no asymmetric information between each bank and the regulator. The loss portfolio X_i is equivalently identified by the bank and the regulator, and both the bank and the regulator make decision based on the same

[4] We follow the same mean-variance setting as in Mace (1991), in which the aggregate uncertainty insurance is considered, as we focus on the aggregate or systematic risk.

interpretation of the loss portfolio.

We now move to present our equilibrium analysis on each capital insurance program. We also examine how these loss portfolios affect each insurance contract as well as the welfare. Moreover, we examine which insurance contract is desirable from the perspectives of the regulator and the bank.

1.2.1 Aggregate Insurance

We characterize the equilibrium precisely for the aggregate insurance. We start with the bank i's rational decision by assuming that the insurance contract has been placed on the market.

Optimal load factor for bank i can be characterized as follows. Bank i's objective is to find suitable coinsurance parameter α_i to maximize

$$\max_{\alpha_i \geq 0} \mathbb{E}[W^i] - \frac{1}{2\gamma_i} Var(W^i), \tag{1.3}$$

where $W^i = W_0^i - X_i + a_i X - (1+\rho)\mathbb{E}[a_i X]$ is the terminal wealth for the bank i. Given the load factor ρ, the optimal a_i for the bank i is[5]

$$a^{i,a}(\rho) = \frac{Cov(X_i, X) - \rho\mathbb{E}(X)\gamma_i}{Var(X)}, \tag{1.4}$$

if $Cov(X_i, X) - \rho\mathbb{E}(X)\gamma_i \geq 0$; otherwise, $a^{i,a}(\rho) = 0$. The symbol "a" represents the "aggregate insurance". We use $a^{i,a}(\rho)$ to highlight the effect of the load factor ρ for the bank i. Optimal load factor for regulator can be characterized as follows. The regulator predicates the demand from the bank i as $a^{i,a}(\rho)X$ correctly for each bank

[5]It is easy to see that $Var(W^i) = Var(X_i) + a_i^2 Var(X) - 2a_i Cov(X_i, X)$. Then, $a^{i,a}(\rho)$ follows from the first-order condition in (3.1).

$i = 1, \cdots, N$. Therefore, by plugging equation (1.4) into equation (1.5) and assuming that $Cov(X_i, X) \geq \rho \mathbb{E}(X)\gamma_i$, the equilibrium welfare is

$$\mathbb{E}(W^r) = \rho \mathbb{E}(X) - \rho^2 \sum_i \frac{\gamma_i \mathbb{E}(X)^2}{Var(X)}. \tag{1.5}$$

By using the formula (1.5) and its first-order condition, the best load factor is determined by the regulator as

$$\rho^{*,a} = \frac{1}{2\sum_i \gamma_i} \frac{Var(X)}{\mathbb{E}(X)}. \tag{1.6}$$

Consequently, under this premium structure, we obtain the following characterization of the equilibrium.

Proposition 1.1 Assume for each $i = 1, \cdots, N$,

$$\frac{Cov(X_i, X)}{Var(X)} \geq \frac{1}{2} \frac{\gamma_i}{\sum_i \gamma_i}. \tag{1.7}$$

Then the optimal load factor $\rho^{*,a}$ is given by (1.6), the welfare for the aggregate insurance is

$$\mathbb{E}(W^{*,a}) = \frac{1}{4\sum_i \gamma_i} Var(X), \tag{1.8}$$

and the best coinsurance parameter for the bank i in this aggregate insurance contract is

$$a^{i,a} = \frac{Cov(X_i, X)}{Var(X)} - \frac{1}{2} \frac{\gamma_i}{\sum_i \gamma_i}. \tag{1.9}$$

Proof: Under condition (1.7) and the choice of $\rho^{*,a}$ by equation (1.6), we observe that $Cov(X_i, X) \geq \rho \gamma_i E[X]$. Therefore, $a^{i,a}(\rho)$ is given by equation (1.4), and the equilibrium welfare is obtained in (1.5). Then, the equilibrium follows from the non-standard first-order condition. A general solution is presented in Appendix C. $\square$

There are several remarkable points about the aggregate insurance by using Proposition 1.1. First, the welfare estimated by the regulator depends on the variability of the aggregate loss, the systematic risk. The higher the variability, the higher the expected welfare. The smaller the variability, or alternatively, the more stable the aggregate loss is, the smaller the welfare. More interestingly, the welfare does not depend on the expected aggregate loss $\mathbb{E}[X]$. Therefore, only the aggregative risk variability contributes to the welfare. Hence, Proposition 2.1 supports the aggregate insurance idea to reduce the systemic risk.

Second, the optimal coinsurance parameter $a^{i,a}$ for bank i is the difference between the "beta", $\frac{Cov(X_i,X)}{Var(X)}$ [6], and the individual risk aversion parameter γ_i comparing with the total risk aversion among the banks $\sum_i \gamma_i$. The higher the beta, the larger $a^{i,a}$; so the bank i purchases insurance on a larger proportional on the systematic risk when we replace the return by the loss variable in calculating returns. It is intuitively appealing because higher beta implies larger contribution of the bank i to the systematic risk, or the bank i has a higher systemic risk. To hedge the systemic risk, the bank needs to insure a larger amount of the systematic risk. Moreover, the relationship between the bank i's risk aversion and the other bank's risk preferences is also important for the aggregate insurance. Higher $\frac{\gamma_i}{\sum_i \gamma_i}$ implies the less risk aversion of the bank i and, thus, a smaller a_i.

[6] It is the beta in the capital asset pricing model when the loss variable is replaced by the return variable.

Third, note that [7]

$$\sum_i a^{i,a} = \frac{1}{2}, \tag{1.10}$$

the total aggregate insurance indemnity for regulator is $\sum_i I_i(X, X_i) = \frac{1}{2}X$. It states that exactly half of the systematic risk is insured in this program. The number 1/2 comes from the mean-variance setting and does not have any specific meaning. But, a crucial insight at this point is that the aggregate loss is not fully insured in this equilibrium insurance market, which is similar to the classical result for the standard coinsurance contract.

1.2.2 Classical Insurance

For comparative purpose, we next consider the classical insurance, $I_i(X, X_i) = a_i X_i$. By the same idea, we characterize $a^{i,c}(\rho)$, ρ^*, and the welfare sequentially. The equilibrium is summarized as follows.

Optimal load factor for bank i can be characterized as follows.

$$a^{i,c}(\rho) = \max\left\{1 - \frac{\rho\mathbb{E}(X_i)\gamma_i}{Var(X_i)}, 0\right\}, \tag{1.11}$$

where the symbol "c" represents the "classical insurance". Optimal load factor for regulator can be characterized as follows. Given the above optimal load factor $a^{i,c}(\rho)$, and assuming $\frac{\rho\mathbb{E}(X_i)\gamma_i}{Var(X_i)} \leq 1, i = 1, \cdots, N$, the welfare is obtained as follows:

$$\mathbb{E}(W^r) = \rho\mathbb{E}(X) - \rho^2 \sum_i \frac{\gamma_i \mathbb{E}(X_i)^2}{Var(X_i)}. \tag{1.12}$$

[7] Since $X = \sum_i X_i$, $\sum_i Cov(X_i, X) = Var(X)$.

Therefore, the optimal load factor from the regulator's perspective is

$$\rho^{*,c} = \frac{1}{2} \frac{\mathbb{E}(X)}{\sum_i \frac{\gamma_i (\mathbb{E}(X_i))^2}{Var(X_i)}}. \tag{1.13}$$

We have the following result.

Proposition 1.2 Assume that for each $i = 1, \cdots, N$,

$$\frac{\mathbb{E}(X)}{\sum_i \frac{\gamma_i (\mathbb{E}(X_i))^2}{Var(X_i)}} \frac{\gamma_i \mathbb{E}(X_i)}{Var(X_i)} \leq 2. \tag{1.14}$$

Then, the optimal load factor is determined in (1.13). The welfare of the classical insurance is

$$\mathbb{E}(W^{*,c}) = \frac{1}{4} \frac{\mathbb{E}(X)^2}{\sum_i \frac{\gamma_i \mathbb{E}(X_i)^2}{Var(X_i)}}, \tag{1.15}$$

and the best coinsurance parameter for the bank i in this classical insurance contract is

$$a^{i,c} = 1 - \frac{1}{2} \frac{\mathbb{E}(X)}{\sum_i \frac{\gamma_i (\mathbb{E}(X_i))^2}{Var(X_i)}} \frac{\gamma_i \mathbb{E}(X_i)}{Var(X_i)}. \tag{1.16}$$

Proof: Same as the proof of Proposition 1.1. □

According to Proposition 1.2, the welfare estimated by the regulator in the classical insurance depends on both the expectation and the variance of individual loss as well as the expectation of the aggregate loss, whereas the variability of the aggregate loss doesn't contribute to the estimated welfare directly. In fact, the correlation structure of the loss portfolios $(X_1, \cdots, X_n)$ is not involved in the insurance contract at all. Therefore, the welfare depends only on the marginal distribution but not on the joint distribution of loss portfolios. Obviously, this should be seen as a limitation of the classical insurance to address the systemic risk. We will compare the classical insurance with the aggregate insurance in details in the next section.

It is interesting to look at the optimal coinsurance parameter α_i for the bank i in the classical insurance contract. While keeping the risks on other banks fixed, the higher $Var(X_i)$, the higher α_i. A larger insurance is required for a higher individual risk. It is straightforward to verify that for large values of $\mathbb{E}[X_i]$, the optimal coinsurance parameter is increasing with respect to the increase of $\mathbb{E}[X_i]$. As the premium structure depends on all loss portfolios $\{X_1, \cdots, X_n\}$, the risks of other banks affect the classical insurance demand in this setting.[8]

1.2.3 Aggregate-Cross Insurance

At last, we consider the aggregate-cross insurance $I_i(X, X_i) = \alpha_i \hat{X}_i$. By definition, it focuses on the insurance of all banks except the insured bank in the market. Optimal load factor for bank i can be characterized as follows. It is easy to derive $a^{i,ac}(\rho)$ in this situation as

$$a^{i,ac}(\rho) = \max\left\{\frac{Cov(X_i, \hat{X}_i) - \rho\mathbb{E}(\hat{X}_i)\gamma_i}{Var(\hat{X}_i)}, 0\right\}, \tag{1.17}$$

where the symbol "ac" represents the "aggregate-cross insurance".

Optimal load factor for regulator can be characterized as follows. By plugging formula (1.17) into formula (1.5) and assuming that $Cov(X_i, \hat{X}_i) \geq \rho\mathbb{E}(\hat{X}_i)\gamma_i$, we have

[8] It is different from a traditional insurance contract on individual loss exposure. The load factor for a traditional insurance contract is either given exogenously or depends on the specific loss vector in equilibrium. The classical insurance in our setting, however, is characterized in a rational expectation equilibrium with banks and a regulator.

$$\mathbb{E}(W^r) = \rho \sum_i \mathbb{E}(\hat{X}_i) \frac{Cov(X_i, \hat{X}_i) - \rho \mathbb{E}(\hat{X}_i)\gamma_i}{Var(\hat{X}_i)}, \tag{1.18}$$

and

$$\rho^{*,ac} = \frac{1}{2} \frac{\sum_i \mathbb{E}(\hat{X}_i) \frac{Cov(X_i, \hat{X}_i)}{Var(\hat{X}_i)}}{\sum_i \frac{\gamma_i \mathbb{E}(\hat{X}_i)^2}{Var(\hat{X}_i)}}. \tag{1.19}$$

Therefore, we obtain the following proposition which proof is similar to Proposition 1.1 and Proposition 1.2.

Proposition 1.3 Assume for each $i = 1, \cdots, N$,

$$\frac{\sum_i \mathbb{E}(\hat{X}_i) \frac{Cov(X_i, \hat{X}_i)}{Var(\hat{X}_i)}}{\sum_i \frac{\gamma_i \mathbb{E}(\hat{X}_i)^2}{Var(\hat{X}_i)}} \mathbb{E}(\hat{X}_i)\gamma_i \leq 2Cov(X_i, \hat{X}_i). \tag{1.20}$$

Then, the welfare of the aggregate-cross insurance is

$$\mathbb{E}(W^{*,ac}) = \frac{1}{4} \frac{\left(\sum_i \mathbb{E}(\hat{X}_i) \frac{Cov(X_i, \hat{X}_i)}{Var(\hat{X}_i)} \right)^2}{\sum_i \gamma_i \frac{\mathbb{E}(\hat{X}_i)^2}{Var(\hat{X}_i)}}, \tag{1.21}$$

and the best coinsurance parameter for the bank i in this aggregate-cross insurance contract is

$$a^{i,ac} = \frac{Cov(X_i, \hat{X}_i)}{Var(\hat{X}_i)} - \frac{1}{2} \frac{\sum_i \mathbb{E}(\hat{X}_i) \frac{Cov(X_i, \hat{X}_i)}{Var(\hat{X}_i)}}{\sum_i \frac{\gamma_i \mathbb{E}(\hat{X}_i)^2}{Var(\hat{X}_i)}} \frac{\mathbb{E}(\hat{X}_i)\gamma_i}{Var(\hat{X}_i)}. \tag{1.22}$$

By Proposition 1.3, the expected welfare in aggregate-cross insurance contract depends positively on covariance between the individual bank's loss X_i and the aggregate loss except for the insured bank's loss, $\hat{X}_i$, for each bank i. The intuition is simple:

higher correlation coefficient $corr(X_i, \hat{X}_i)$ results in higher expected welfare from the regulator's prospective.

In contrast to the classical insurance, the aggregate-cross insurance depends on the correlation structure of the loss portfolios. We see easily that when X_i and $\hat{X}_i$ are uncorrelated for each i, both the estimated welfare and the optimal coinsurance a for bank i in this aggregate-cross insurance contract equal to zero. In particular, when all banks' loss portfolios are independent, there is no necessity to buy the aggregate-cross insurance.

The next result illustrates the main insights of these three insurance contracts when the loss risk factors are uncorrelated. We say one contract is preferred to another one as long as the former has higher welfare than the later.

Proposition 1.4 Assume the loss portfolios are uncorrelated, i,e., $Cov(X_i, X_j) = 0, \forall i \neq j$. Then, both the aggregate insurance and the classical insurance are preferred to the aggregate-cross insurance. Moreover,

1. If the risk-adjusted variance vector $\left(\frac{Var(X_i)}{\gamma_i}\right)$ and the Sharpe ratio vector $\left(\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}\right)$ are co-monotonic,[9] then the classical insurance is preferred to the aggregate insurance.

2. If the risk-adjusted variance vector $\left(\frac{Var(X_i)}{\gamma_i}\right)$ and the Sharpe ratio vector $\left(\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}\right)$ are counter-monotonic, and there exists one "too big to fail" bank in the sense that $\mathbb{E}[X]^2$ is close to $\sum_i \mathbb{E}[X_i]^2$, then the aggregate insurance is preferred to

[9] Given two vectors $a = (a_1, \cdots, a_n), b = (b_1, \cdots, b_n)$, a and b are counter-monotonic if $(a_i - a_j)(b_i - b_j) \leq 0, \forall i, j$, and this inequality is strictly; a and b are co-monotonic if $(a_i - a_j)(b_i - b_j) \geq 0, \forall i, j$, and one inequality is strictly.

the classical insurance.

Proof: See Appendix A. □

There are several points in Proposition 1.4. First of all, the relationship between the risk-adjusted variance and the Sharpe ratio across the banks plays a crucial role in comparing the classical insurance and the aggregate insurance. As each X_i represents the loss portfolio, we assume positive expected loss in our analysis. Its variance $Var(X_i)$ represents the individual risk of the bank i. Similarly, we use the terminology "Sharpe ratio" to represent the expected loss per each volatility unit. Both the risk-adjusted variance $\frac{Var(X_i)}{\gamma_i}$ and the Sharpe ratio represent two important factors to characterize the loss risk for bank i.

Secondly, when these individual banks' risk-adjusted variance has the same order as the Sharpe ratio, i.e. a higher risk-adjusted variance is consistent with a higher Sharpe ratio, we say that the risk-adjusted variance is co-monotonic to the Sharpe ratio. In this case, the bank sector is in an ordering loss market because a higher expected loss ensures a higher variance. Proposition 1.4 states that a classical insurance is a better contract from the regulator's perspective in the ordering loss market.

Thirdly, in the disordering loss market in which a higher risk-adjusted variance is always linked to a smaller Sharpe ratio, at the presence of few banks with very large expected loss, Proposition 1.4 ensures that aggregate insurance is more beneficial insurance contract. To explain it, say bank 1 is big enough such that $\mathbb{E}[X_1] >> \mathbb{E}[X_2], \cdots, \mathbb{E}[X_n]$[10]. In this case, the bank 1's expected loss is so big that the total expected aggregate loss $\mathbb{E}[X]$ is close to $\mathbb{E}[X_1]$, then $\mathbb{E}[X]^2$ is close enough to $\sum \mathbb{E}[X_i]^2$.

[10] We write $x >> y$ to denote $y/x \to 0$.

Therefore, the aggregate insurance issued to other banks with the small losses together would benefit to the regulator.

We next move to the more interesting situation in which each loss contributes to the systematic risk, so these loss portfolios are correlated.

1.3 Systematic Risk and Comparative Analysis

In this section, we examine closely which insurance contract should be preferred to another one from the perspective of the regulator as well as the bank. For this purpose, we assume the contribution of each bank to the market risk is given exogenously. It is natural to examine the question in a one-factor model. A multi-factor model shares the same insights as a one-factor model.

Suppose $X_i = \eta_i Y + \epsilon_i$, where ϵ_i is a white noise with zero mean and variance σ_i^2. Y represents a market (or systematic) risk factor, and each ϵ_i represents the specific risk of bank i. The aggregate loss $X = \sum_i \eta_i Y + \sum_i \epsilon_i = \eta Y + \epsilon$, where $\eta = \sum_{i=1}^n \eta_i$. Write $\hat{X}_i = \hat{\eta}_i Y + \hat{\epsilon}_i$, where $\epsilon = \sum_{i=1}^n \epsilon_i$, $\hat{\eta}_i = \sum_{j=1, j\neq i}^n \eta_j$, $\hat{\epsilon}_i = \sum_{j=1, j\neq i}^n \epsilon_j$.

We first consider one special case for which specific risks equal to zero. By using equations (1.8), (1.15), (1.21), we have the following result.

Proposition 1.5 If there is no specific risk in the market, then the welfare is equivalent for all three types of insurance contracts. Precisely, if each $\sigma_i = 0$, then

$$\mathbb{E}(W^{*,a}) = \mathbb{E}(W^{*,c}) = \mathbb{E}(W^{*,ac}) = \frac{Var(Y)}{4\sum_i \gamma_i}\eta^2 > 0. \tag{1.23}$$

In general, when the systematic risk factor is highly volatile, that is, $Var(Y)$ is high, then these three contracts offer the same welfare asymptotically. Precisely, when

$Var(Y) \to \infty$,[11]

$$\mathbb{E}(W^{*,a}) \sim \mathbb{E}(W^{*,c}) \sim \mathbb{E}(W^{*,ac}) \sim \frac{Var(Y)}{4\sum_i \gamma_i}\eta^2. \quad (1.24)$$

Proof: See Appendix A. □

Proposition 1.5 states that if $Var(Y)$ is extremely large relative to company's specific risk, then from the regulator's prospective the welfare of all three types of insurance contracts is almost identical and positively depends on both $Var(Y)$ and the aggregate contribution of all banks to the market risk, $\sum_i \eta_i$. Alternatively, when the individual risks are immaterial comparing to the systematic risk, these three contracts in essence provide the same welfare. Therefore, the capital insurance idea does not work particularly well under some circumstances with extremely high systemic risk factor or extremely small specific risks.

Proposition 1.6 If the risk-adjusted individual risk vector $\left(\frac{Var(X_i)}{\gamma_i}\right)$ is co-monotonic to the Sharpe ratio vector $\left(\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}\right)$, then the classical insurance is preferred to the aggregate insurance in the sense that $\mathbb{E}[W^{*,a}] < \mathbb{E}[W^{*,c}]$.

If the risk-adjusted individual risk vector $\left(\frac{Var(X_i)}{\gamma_i}\right)$ is counter-monotonic to the Sharpe ratio vector $\left(\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}\right)$, and the expected aggregate loss $\mathbb{E}[X]$ is large enough, then the aggregate insurance is preferred to the classical insurance in the sense that $\mathbb{E}[W^{*,c}] < \mathbb{E}[W^{*,a}]$.

Proof: See Appendix A. □

Proposition 1.6 has the same insight as Proposition 1.4, but Proposition 1.6 holds in a general correlated market environment. In the ordering loss market such that

[11] By two functions $f \sim g$ we mean that $\lim_{Var(Y)\to\infty} \frac{f}{g} = 1$.

a higher risk-adjusted variance corresponds to a Sharpe ratio, the classical insurance works better. In the disordering loss market, however, the aggregate insurance contract should be preferred to the classical one when the expected total risk $\mathbb{E}[X]$ is a big concern. Indeed, both Proposition 1.4 and Proposition 1.6 demonstrate in different market situations that aggregate insurance is a good design when the individual risk and the Sharpe ratio display a negative relationship for each bank.

To finish this section, we compare the aggregate-cross insurance with the classical insurance.

Proposition 1.7 If the expected losses across the banks are fairly close, the risk-adjusted variance is co-monotonic to the Sharpe ratio, and the risk-adjusted correlated variance $\rho_i^2 \frac{Var(X_i)}{\gamma}$ is co-monotonic to the Sharpe ratio of its dual risk $\frac{\mathbb{E}[\hat{X}_i]}{\sqrt{Var(\hat{X}_i)}}$, where ρ_i is the correlation coefficient between X_i and $\hat{X}_i$ for each $i = 1, \cdots, N$, then $\mathbb{E}[W^{*,ac}] < \mathbb{E}[W^{*,c}]$.

Proof: See Appendix A. □

As shown in Proposition 1.6, the classical insurance is preferred to the aggregative insurance when risk-adjusted variance is co-monotonic to the Sharpe ratio. Therefore, Proposition 1.7 shows us that both the aggregative-type insurances (i.e. aggregate and aggregate-cross insurance contracts) are not supportive under the situations described in Proposition 1.7.

1.4 Discussions

Under what circumstance should the capital insurance programs be implemented and how it should be implemented? In this section, we show several important insights based on our theoretical results.

1.4.1 Disordering Loss Market and Ordering Loss Market

According to Proposition 1.4 and Proposition 1.6, based on our welfare analysis, the aggregate insurance contract should be insured by the regulator in the disordering loss market. When the individual risk of loss $Var(X_i)$ is mismatched with the expected loss per unit, the loss in each bank displays the disordering loss market.

There are two important situations in which the disordering loss market occurs. The first situation is when the contribution to the aggregate loss of each back is fairly close, and each bank has fairly close preference to the risk. In other words, when the aggregate loss is almost equally distributed among the banks, it is a disordering loss market. To see this, we assume $\gamma_i = \gamma$ for all i. Clearly, the risk-adjusted variance $\frac{Var(X_i)}{\gamma_i}$ is counter-monotonic to $\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}$. Therefore, both Proposition 4 and Proposition 6 ensure that aggregate insurance is better than the classical insurance contract.

We describe the second situation in one-factor model. We argue that when the individual risk mainly comes from the specific risk in each bank, this is another example of the disordering loss market. Write $X_i = \eta_i Y + \epsilon_i, i = 1, \cdots, N$. When a higher individual risk $Var(X_i)$ corresponds to a higher $\frac{Var(\epsilon_i)}{Var(X_i)}$, the market can be described as the "disordering loss market". To demonstrate, we assume again $\gamma_i = \gamma$ for all i. Note that $\frac{Var(X_i)}{\mathbb{E}[X_i]^2} = Var(Y) + \left(\frac{\sigma_i}{\eta_i}\right)^2$, and $\frac{Var(\epsilon_i)}{Var(X_i)}$ is increasing with respect to $\frac{\sigma_i}{\eta_i}$. Then, under this assumption, $\frac{Var(X_i)}{\gamma_i}$ is co-monotonic to $\frac{Var(X_i)}{\mathbb{E}[X_i]^2}$; thus counter-monotonic to $\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}}$, this is a disordering loss market. Hence, the aggregate insurance is a better insurance program when the specific risk plays a dominate role inside the individual risk.

Table 1.1 demonstrates the first situation as described. There are 10 big banks in the market, and each bank has the same expected loss as $\eta_i = 0.1$ for all $i = 1, \cdots, 10$. For simplicity, we assume that the variance of the systematic risk factor Y equals to one, and each $\gamma_i = 1$. However, the specific risk in each bank varies from 10% to 40%. Table 1 displays the negative relationship between the risk-adjusted variance and the Sharpe ratio of loss portfolio among these 10 banks. Therefore, Table 1 shows one example of the disordering loss market, and we know that the aggregate insurance is a preferred program by Proposition 6. Moreover, by numerical computations, $\frac{Cov(X_i,X)}{Var(X)} > 0.06 > \frac{1}{2N}$ for each $i = 1, \cdots, N$. Hence, the equilibrium of the aggregate insurance is given explicitly in Proposition 1.1.

The second situation is shown in Table 1.2, in which $\frac{\eta}{\sigma}$ is increasing with respect to η. In this case, these banks have different expected loss, ranging from $0.1\mathbb{E}[Y]$ to $0.55\mathbb{E}[Y]$. As shown, there is a negative relationship between the risk-adjusted variance and the Sharpe ratio of loss portfolio among these 10 banks; hence, Table 1.2 shows another example of the disordering loss market. By numerical computations, $\frac{Cov(X_i,X)}{Var(X)} > 0.08 > \frac{1}{2N}$ for each $i = 1, \cdots, N$. Hence, the equilibrium of the aggregate insurance is given explicitly in Proposition 1.1.

On the other hand, when the individual risk $Var(X_i)$ is opposite to the percentage of the specific risk, $\frac{\sigma_i^2}{Var(X_i)}$, the classical insurance is better. In general, when a higher systemic risk corresponds to a smaller specific risk, the classical insurance is better than the aggregate insurance. Table 1.3 displays an example of the ordering loss market in which the classical insurance program should be preferred to the aggregate insurance.

Through these examples we have shown that the specific risk is critical in comparing those capital insurance programs. If the specific risks can be ignored, these three insurance contracts offer similar welfare. Equivalently, when the systematic risk is extremely large, it does not matter which capital insurance program should be issued, as it is demonstrated by Proposition 1.5.

1.4.2 Low Correlation Market and High Correlation Market

The correlation structure affects the capital insurance program. On the one hand, we have seen by Proposition 1.4 that aggregate-cross insurance is not a good choice in a low-correlated market. A low correlation parameter comes from large specific risks. In other words, if specific risks are sufficiently large enough comparing with the systemic risk component, aggregate-cross insurance does not add welfare. On the other hand, when the specific risks are very small, Proposition 1.5 ensures that aggregate-cross insurance does not add welfare over the aggregate insurance either. Low specific risks correspond to high (or even perfectly correlated) correlation coefficient among the loss portfolios. Therefore, the aggregate-cross insurance does not work better in either a low or a high correlation environment under Assumption I and Assumption II.

Actually, in the absence of asymmetric information, we argue that aggregate-cross insurance does not work better than the aggregate insurance in general. To see this, we assume that η_i is the same for all i, and σ_i is the same for all i. Then, each pair of banks has the same correlation coefficient written as τ. By straightforward calculation, we have

$$\mathbb{E}(W^{*,ac}) = \tau^2\mathbb{E}(W^{*,a}) = \tau^2\mathbb{E}(W^{*,c}). \tag{1.25}$$

Therefore, the lower the correlation coefficient τ, the smaller expected welfare of the aggregate-cross insurance. Overall, $\mathbb{E}(W^{*,ac}) < \mathbb{E}(W^{*,a}) = \mathbb{E}(W^{*,c})$. When all banks contribute to the systematic risk equally, and specific risks are also similar; the aggregate-cross insurance is not as good as two other insurance programs.

1.4.3 Systemic Risk

There are many different interpretations about the systemic risk. Some authors suggest to use the default probability of the whole financial system (see, for instance, Pritsker (2012)). Other authors suggest to use the Shapley values to estimate the systemic risk (see Bluhm et al (2013)).[12] It is beyond the scope of this paper to develop a systemic risk theory as we focus on the effect of the capital insurance. Rather, we indicate that the aggregate insurance is a useful tool to deal with the systemic risk by using two interpretations of the systemic risk.

First, we view systemic risk as the likelihood of the aggregate loss meets a threshold. Precisely, the higher probability $P(X \geq L)$, the higher the systemic risk. In the aggregate insurance, the post-aggregate insurance becomes

$$\sum X_i - \sum \alpha_i X = \frac{1}{2}X. \tag{1.26}$$

Clearly, the ex post aggregate loss is smaller than the ex ante aggregate loss X. Therefore, the aggregate insurance, indeed, reduces the systemic risk.

[12] See Billio et al (2009), Eisenberg and Noe (2001), Choi and Douady (2012).

Second, we consider the systemic risk for each individual bank in a one-factor model. Before purchasing the aggregate insurance, the systematic risk contribution of the bank i is η_i. We assume that γ_i is the same across the banks. Then, the coinsurance percentage for the bank i is

$$\alpha_i \geq \frac{\eta_i \eta Var(Y)}{\eta^2 Var(Y) + \sigma^2} - \frac{1}{2N}. \tag{1.27}$$

Hence, the contribution to the systematic risk of the bank i, after purchasing the aggregate insurance, is

$$\eta_i - \alpha_i \eta \leq \frac{\eta_i \sigma^2}{\eta^2 Var(Y) + \sigma^2} + \frac{1}{2}\frac{\eta}{N}. \tag{1.28}$$

When the number of banks, N, is large enough, or when the variability of the systemic risk, $Var(Y)$, is sufficiently large, we see that $\eta_i - \alpha_i \eta < \eta_i$. Therefore, the systemic risk of each bank i is reduced after purchasing the aggregate insurance.

1.4.4 Identification and Implementation of Too Big to Fail Banks

Suppose the disordering loss market occurs; according to our theory, the aggregate insurance program is a desired regulatory tool to solve the "too big to fail" issue. Nevertheless, there are two fundamental questions to be solved as follows.

1. How to implement the aggregate insurance program? i.e., How to characterize the equilibrium in a general situation?

2. How to distinguish the "too big to fail" banks that are enforced to purchase the aggregate insurance from the other banks? Alternatively, how to identify those "too big to fail" banks?

We illustrate our solutions to these questions by an example, while a general solution is given in Appendix B.

To explain the answers to the questions above, we consider 15 banks, and the loss portfolio of each bank follows a one-factor model. The systematic risk factor is represented by Y with $\mathbb{E}[Y] = Var(Y) = 1$. Each bank has the same expected loss $0.05\mathbb{E}[Y]$, but the specific risk varies differently. In fact, σ_i moves from 40% to 12%. Proposition 1.6 implies that the aggregate insurance is more desirable than the classical insurance. It is also easy to see that $\frac{Cov(X_i,X)}{\gamma_i}$ is decreasing from $i = 1$ to $i = 15$. However, as shown in Table 1.4, condition (1.7) in Proposition 1.1 is not always satisfied. To be precise, for the last 5 banks, $\frac{Cov(X_i,X)}{Var(X)} < \frac{1}{2N}, i = 11, 12, 13, 14, 15$.

Appendix B presents a general solution of the equilibrium without condition 1.7. The equilibrium problem and how to identify the "too big to fail" problem are solved simultaneously. As the risk-adjusted covariance sequence $\frac{Cov(X_i,X)}{\gamma_i}$ is decreasing for $i = 1, \cdots, N$, we know that the sequence $\frac{\sum_{j=1}^{i} Cov(X_j,X)}{2\sum_{j=1}^{i}\gamma_j}$ is decreasing for $i = 1, \cdots, N$ as well. The first step is to find an unique number n such that

$$\frac{Cov(X_i, X)}{\sum_{k=1}^{n} Cov(X_k, X)} \geq \frac{\gamma_i}{2\sum_{k=1}^{n}\gamma_k}, i = 1, \cdots, n; \tag{1.29}$$

and

$$\frac{Cov(X_i, X)}{\sum_{k=1}^{n} Cov(X_k, X)} < \frac{\gamma_i}{2\sum_{k=1}^{n}\gamma_k}, i = n+1, \cdots, N. \tag{1.30}$$

In this example, we find out $n = 13$ (see Table 1.5). Therefore, the first 13 banks, but not the first 10 banks, are "too big to fail" banks that should be required to purchase the aggregate insurance. The last two banks can be ignored in this aggregate insurance program. The second step is to determine the optimal load factor ρ^* in the aggregate insurance program, which is

$$\rho^* = \frac{1}{\mathbb{E}[X]} \frac{\sum_{i=1}^n Cov(X_i, X)}{2\sum_{i=1}^n \gamma_i} = 0.081. \tag{1.31}$$

At last, the optimal co-insurance parameters for the first 13 banks are

$$a^{i,a}(\rho^*) = \frac{Cov(X_i, X) - \rho^* \gamma_i \mathbb{E}[X]}{Var(X)}, i = 1, \cdots, 13. \tag{1.32}$$

The last two banks do not buy the aggregate insurance as $a^{i,a}(\rho^*) = 0, i = 14, 15$. The equilibrium and relevant computation are displayed by Table 1.5. We observe that the optimal co-insurance parameter decreases with respect to $\frac{Cov(X_i,X)}{\gamma_i}$, a measure of the systemic risk of these "too big to fail" banks.

1.5 Conclusion

In this paper, we present a welfare analysis of several capital insurance programs in equilibrium. We show that aggregate insurance ensures a higher welfare if each big bank has similar systematic risk. The classical insurance program, however, has a higher welfare when the individual bank's risk is positively related to the expected loss per each volatility unit. In general, aggregate-cross insurance does not add more welfare if there exists no asymmetric information concern. Overall, we demonstrate that the capital insurance program is a useful regulatory tool to address the "too big to fail" issue.

Table 1.1: Example 1 of a disordering loss market

This table displays a disordering loss market when each bank has the same expected loss in one-factor model. Therefore, the aggregate insurance is a better capital insurance program by Prop 1.6. It can be checked that the condition in Prop 1.1 is satisfied, so the equilibrium of the aggregate insurance is given in Prop 1. We assume $\gamma_i = 1$ for each $i = 1, \cdots, N$. There are $N = 10$ banks.

Bank	η	σ	Risk-adjusted Variance	Sharpe ratio
1	0.1	0.40	0.170	0.243
2	0.1	0.35	0.133	0.275
3	0.1	0.30	0.100	0.316
4	0.1	0.26	0.078	0.359
5	0.1	0.23	0.063	0.399
6	0.1	0.20	0.050	0.447
7	0.1	0.18	0.042	0.486
8	0.1	0.15	0.033	0.555
9	0.1	0.12	0.024	0.640
10	0.1	0.10	0.020	0.707

Table 1.2: Example 2 of a disordering loss market

This table displays a disordering loss market when the percentage of specific risk in the individual risk is increasing with respect to the individual risk. Therefore, the aggregate insurance is a better insurance program than the classical insurance program by Prop1.6. It can be checked that the condition in Prop 1.1 is satisfied, so the equilibrium of the aggregate insurance is given in Prop 1.1. We assume $\gamma_i = 1$ for each $i = 1, \cdots, N$. There are $N = 10$ banks.

Bank	η	σ	Risk-adjusted Variance	Sharpe ratio
1	0.10	0.200	0.050	0.447
2	0.15	0.315	0.122	0.430
3	0.20	0.440	0.234	0.414
4	0.25	0.575	0.393	0.399
5	0.30	0.720	0.608	0.385
6	0.35	0.875	0.888	0.371
7	0.40	1.040	1.242	0.359
8	0.45	1.215	1.679	0.347
9	0.50	1.400	2.210	0.336
10	0.55	1.595	2.847	0.326

Table 1.3: An example of an ordering loss market

This table displays an ordering loss market when the percentage of specific risk in the individual risk is decreasing with respect to the individual risk. Therefore, the classical insurance is a better insurance program than the aggregate insurance program by Prop1.6. We assume $\gamma_i = 1$ for each $i = 1, \cdots, N$. There are $N = 10$ banks.

Bank	η	σ	Risk-adjusted Variance	Sharpe ratio
1	0.10	0.400	0.170	0.243
2	0.15	0.350	0.145	0.394
3	0.20	0.300	0.130	0.555
4	0.25	0.260	0.130	0.693
5	0.30	0.230	0.143	0.794
6	0.35	0.200	0.163	0.868
7	0.40	0.180	0.192	0.912
8	0.45	0.150	0.225	0.949
9	0.50	0.120	0.264	0.972
10	0.55	0.100	0.313	0.984

Table 1.4: Example 3 of a disordering loss market

This table displays a disordering loss market when each bank has the same expected loss in a one-factor model. Therefore, the aggregate insurance is a better program by Prop 1.6. However, the condition in Prop 1.1 is not satisfied as shown for $i = 11, 12, \cdots, 15$. There are $N = 15$ banks, and each $\gamma_i = 1$.

Bank	η	σ	Risk-adjusted Variance	Sharpe ratio	$\frac{Cov(X_i,X)}{Var(X)}$
1	0.05	0.40	0.1625	0.124	0.1170
2	0.05	0.38	0.1469	0.130	0.1070
3	0.05	0.36	0.1321	0.138	0.0990
4	0.05	0.34	0.1181	0.145	0.0907
5	0.05	0.32	0.1049	0.154	0.0829
6	0.05	0.30	0.0925	0.164	0.0755
7	0.05	0.28	0.0809	0.176	0.0686
8	0.05	0.26	0.0701	0.189	0.0622
9	0.05	0.24	0.0601	0.204	0.0563
10	0.05	0.22	0.0509	0.222	0.0509
11	0.05	0.20	0.0425	0.243	0.0459
12	0.05	0.18	0.0349	0.268	0.0414
13	0.05	0.16	0.0281	0.298	0.0374
14	0.05	0.14	0.0221	0.336	0.0338
15	0.05	0.12	0.0169	0.385	0.0307

Table 1.5: Implementation of example 3

This table displays the equilibrium of Example 3. We note that when i starts from 14, $\frac{Cov(X_i,X)}{\gamma_i}$ is strictly greater than $\frac{\sum_{j=1}^{i} Cov(X_j,X)}{2\sum_{j=1}^{i}\gamma_j}$. Then, the last two banks are not "too big to fail". The optimal load factor is $\rho^* = 8.1\%$.

Bank	$\frac{Cov(X_i,X)}{\gamma_i}$	$\frac{Cov(X_i,X)}{Var(X)}$	$\frac{\sum_{j=1}^{i} Cov(X_j,X)}{2\sum_{j=1}^{i}\gamma_j}$	
1	0.1975	0.1170	0.09875	8.10 %
2	0.1819	0.1070	0.09485	7.18 %
3	0.1671	0.0990	0.09108	6.30 %
4	0.1531	0.0907	0.08745	5.47 %
5	0.1399	0.0829	0.08395	4.69 %
6	0.1275	0.0755	0.08058	3.95 %
7	0.1159	0.0686	0.07735	3.27 %
8	0.1051	0.0622	0.07425	2.63 %
9	0.0951	0.0563	0.07128	2.03 %
10	0.0859	0.0509	0.06845	1.49 %
11	0.0775	0.0459	0.06575	0.99 %
12	0.0699	0.0414	0.06318	0.54 %
13	0.0631	0.0374	0.06075	0.14 %
14	0.0571	0.0338	0.05845	0
15	0.0519	0.0307	0.05628	0

CHAPTER 2: THE BANK CAPITAL: AN INSURANCE PESPECTIVE

2.1 Introduction

One of the lessons learned from the financial crisis of 2007-2009 is that regulatory supervision of financial institutions needs a major overhaul. The bail-out of Bear Sterns and AIG, the desperate buyout of Merrill Lynch from Bank of America and Washington Mutual from JP Morgan, the public assistance of Citigroup, Goldman Sachs, Morgan Stanley and Bank of America as well as the freeze of the financial system after Lehaman Brothers' bankruptcy have indicated the increasing demand for significant revision of the financial risk management. In particular, the massive amount of explicit and implicit guarantees and outright infusion of taxpayers' money to cover the financial losses due to the excessive risk - taking behavior of financial institutions have become a serious scrutiny. Many regulatory changes have been implemented in the financial market. For instance, the Dodd-Frank Act has been passed in the U. S. Congress, the Basel Committee has moved to strengthen the bank regulation with BASEL III, and Volcker Rule has been adopted formally by financial regulators to curb Bank-Risk hedging. At the same time, many researches have conferred the "too big to fail" problem. (See Bluhm et al. 2013; Billio et al. 2009; Hansen, 2013; Pritsker, 2012.)

In this chapter, we discuss this "too big to fail" issue from the insurance perspective and focus on the guaranteed fund commitment to the financial institutions in an adverse business situation. As those "too big to fail" banks or companies which are "financial in nature" (thereafter, banks) expect the capital injection from the central bank in time of financial distress, the banks might enact in a risk-taking manner and put the central bank, regulator and all taxpayers in a fragile financial position. Therefore, in the capital insurance program, which was first introduced briefly in Kashyap, Rajan and Stein (2008) and studied extensively in Panttser and Tian (2013), the banks are requested to pay some amount as a premium or reserve to an insurer, say, a central bank, which in turn would inject guaranteed funds to the banks in a future financial failure. As the insurer injects a "guarantee amount of capitals" to strengthen a financial institution in a bad time either explicitly or implicitly, a major insight of the capital insurance program is to ask for an upfront premium from the financial institution for this kind of contingent guarantee.

By its nature, capital insurance is different from current capital regulation implemented in BASEL II and BASEL III. It is also different from the Dodd-Frank Act which posts several new prudential standards and stringent capital requirements for banks with systemic risks. According to the capital regulation requirement, the amount of capital reserve or economic capital amount depends on the risk of loss portfolio and the riskiness of the bank itself. The risker the bank, the higher the economic capital; the economic capital is higher for a bank with weak credit situation than for the strong counterpart while assuming the portfolio is identically the same. Hence, the economic capital idea depends on both the individual bank's riskiness and the individual loss portfolio.

By contrast, the capital insurance, in essence, is an insurance contract, and the capital insurance idea casts all banks in a bank sector together from the market level. On the one side of the capital insurance program, the insurer of the contract receives an insurance premium with the obligation to inject funds to save the bank in financial distress. On the other side, each bank is an insured in this contract agreement. When an insurer is a government entity (as we will argue later, they are reasonable insurer candidates) and, thus, represents the taxpayer in this structure; the insurer of the contract is a taxpayer, and the premium represents a special purpose tax in the sense described by Acharya et al (2010). As a key distinction to the traditional insurance contract, the contract redemption is contingent on the aggregate loss, and the insured event is contingent on the systematic event in the economy. Therefore, the premium amount in the capital insurance is different from conventional bank capitals, but shares several common features with an insurance premium in the insurance market.

In this chapter, the capital insurance premium is viewed as an "insurance capital", and we examine several important economic elements of the insurance capital in a rational expected equilibrium setting. The rational expected equilibrium of the capital insurance program can be explained briefly as follows. The insurer issues insurance contracts to the banks, and the banks purchase these contracts that are placed on the market. The insurer predicts the correct optimal demand from the banks with a given premium structure. Then, the insurer maximizes the welfare with the premium structure as characterized. Consequently, both the demand (from the banks) and the supply (from the insurer) are determined uniquely in a rational expectation equilibrium.

This chapter is organized as follows. In Section 1.2 we review current bank regulations and the motivations of capital insurance program in a brief manner. In Section 1.3 we develop a theoretical framework of capital insurance. We first present a quick review of the classical insurance literature that dated back to Arrow (1961), Borch (1962) and Raviv (1978). Then, we present the rational equilibrium setting for a general capital insurance program. In this chapter, a detailed analysis of capital insurance will be focused on one special type of capital insurance - aggregate (capital) insurance, which is a coinsurance contract written on the aggregate loss.

Panttser and Tian (2013) consider several capital insurance problems and conduct a comprehensive welfare analysis of these capital insurance programs. In particular, Panttser and Tian (2013) demonstrate that aggregate insurance contributes the highest welfare (for the regulator) among these capital insurance problems under some circumstances. So in this chapter, we focus on the aggregate insurance as one illustrative example. Section 1.4 presents the theory of Panttser and Tian (2013) for the aggregate insurance. We show that regulator's expected utility is always positive. We also present an algorithm to identify "too big to fail" banks from the regulator's perspective within this capital insurance framework.

In Section 1.5, we demonstrate that purchasing capital insurance also rewards the banks due to its increased expected utility. Moreover, we show that, after implementing the capital insurance, the entire systemic risk is reduced significantly, and the ex post systemic risk component, " beta" [13], of each bank becomes stable. Furthermore,

[13]By borrowing a terminology of CAPM, a bank's beta is defined as a ratio of the covariance between this bank's loss portfolio with the aggregate loss portfolio to the variance of the aggregate loss portfolio.

after repeatedly entering this capital insurance program, the systemic risk can be removed virtually. Hence, this capital insurance idea reduces the systemic risk and provides motivations for all market participants.

In Section 1.6, we come back to the "too big to fail" issues in more details. We demonstrate that the beta vector is sufficient to identify "too big to fail" in the bank sector. We show that "too big to fail" banks must have large systemic risk component; however, some banks with relatively large systemic risk component are not necessarily "too big to fail" from the capital insurance perspective, since other banks' insurance program might reduce the systemic risk substantially. Conclusions and comments are provided at the end of the chapter. All proofs are stated in the Appendix C.

2.2 Bank Capital Requirement and Motivation

Bank capital requirement is a framework on how banks deal with their capitals. From the regulatory perspective, a bank should hold a sufficient capital buffer to absorb losses on some bad scenarios. If a large (in a systemic sense) bank defaults or a bank-run event occurs, government intervention is plausible to several extents through deposit insurance or a TRAP program.

The traditional risk-based view on capital requirement as proposed in BASEL II depends on several capital ratios which are percentages of a bank's capital to its risk-weighted view. For instance, Tier 1 capital ratio is a percentage of Tier 1 capital to the risk-weighted asset. Leverage ratio is a percentage of the Tier 1 capital to the average total consolidated asset, and an equity ratio is the percentage of the equity to the balance sheet asset. In each capital ratio category, minimal capital ratio has to be satisfied in the traditional capital requirement.

The recent financial crisis has shown the inadequacy of the Basel approach in strengthening the financial system because, in the bad time, it will become hard for the bank to raise capital through equity issuance. When the quantity of "high-quality" capital under Basel II fell, this risk-based capital has been proved unreliable as a measure of risk. While a plausible approach, for instance, in Basel III, is to increase those capital ratios to strengthen the balance sheet, the question how to choose capitals to take consideration of all parties' interest appropriately is still mainly unresolved.

Recently, contingent capital (CC) has gained increasing endorsements among regulators, researchers, financial institutions and investors. Dodd-Frank Act has mandated a study on contingent capital. This new regulation proposal argues that properly structured contingent capital bonds provide incentive for financial institutions to deal with serious financial difficulty before possible government intervention. Basel Committee also considers the role of regulatory capital requirements of contingent capital instruments, which can convert into common equity during financial distress [14]. Among many CC proposals, the trigger events are often realized when the certain bank-specific trigger indicator, such as the market price of common stock or some risk-adjusted capital ratios, falls below a threshold. Systematic trigger indicators are also advocated in some CC proposals. For the study of contingent capital we refer to Coffee (2011), Sundaresan and Wang (2013) and Tian (2013). Other recent sugges-

[14] See, for instance, "Proposal to ensure the loss absorbency of regulatory capital at the point of non-viability", by Basel Committee on Banking Supervision, October 2010. In the Swiss Contingent Capital Proposal, the total capital will be increased to 19 percent: 10 percent in Common Equity Tier 1 and 9 percent in contingent capital.

tions on bank capital requirement can be seen in Admati et al (2011), Hellwig (2009), Zingales and Hart (2010) and many others.

Kashyap, Rajan and Stein (2008) propose an alternative approach, namely, capital insurance. Its basic insight is simple. Since the bank has to increase capital requirement, say, from 8 percent to 10 percent, the bank needs additional $10 billion capital. Instead of raising $10 billion in new equity, the bank is given another option through an insurance mechanism. In this insurance mechanism, each bank acquires an insurance policy that pays off $10 billion upon the occurrence of a systemic "event". The insurer in the insurance policy can be any investor such as pension fund or a sovereign wealth fund, and the insurer receives the insurance premium from the bank. The insurer would put $10 billion into a custodial account, i.e. a "lock box", which would be returned to the issuer if there is no systemic event over the life of the policy.

There are some economic advantages behind this capital insurance idea. First, since $10 billion goes into a custodial account, the bank manager has no full access to these fund, so this idea can resolve some governance issues. In the case of the straight equity issue, however, the $10 billion goes to the bank's balance sheet. Second, $10 billion is a state-contingent amount upon on an event, so it can align resources with investment opportunities on a state-by-state basis. In this way, it resembles the contingent capital in some features. As shown in Tian (2012), contingent capital offers a reasonable investment tool, in particular, during bad business time. Hence, capital insurance might have some appealing features to both the banks and the insurer (the investor). Third and might be the most important point, the event can be designed in a systemic sense, for instance, when the aggregate loss of the whole bank sector crosses a threshold. The last point is a key distinction between the capital insurance and the classical insurance as will be seen shortly.

Before finishing this section, we point out several issues underlying the capital insurance idea which are not addressed in Kashyap et al (2008)'s somewhat illustrative framework. Just name a few, how to characterize the premium of the insurance policy? whether it benefits to the bank upon the capital insurance implementation? if it is the government entity instead of the private-sector investors to insure the policy, whether it has social benefits? We next propose a framework of capital insurance and use it to examine these questions related to capital insurance.

2.3 A Capital Insurance Framework

In this section, we propose a rational equilibrium framework of capital insurance by building on some ideas in standard insurance literature. Within this framework, we are able to answer the following two fundamental questions: how to determine the premium for the capital insurance contract for the insurer and how to characterize the optimal capital insurance for the banks. The answers to these questions lead to a welfare analysis from both the insurer's (in Section 1.4) and the banks's (in Section 1.5) perspectives. For this purpose, we start with a brief discussion of classical insurance.

2.3.1 Classical Insurance

We consider a standard one-period insurance contract, [15] in which the insurer receives upfront premia from the insured at the initial time, and in exchange to these premia, it is obligated to provide coverage at the end of the period. The aggregate amount of loss portfolio for the insured in the future is denoted by X. Its initial wealth W_0 is composed of the collected premia and its own capital. Without background (initial) risk, we often assume that W_0 has no uncertainty while X does, even though it is possible to extend the discussion on background risk in great length. At the end of the period, its final wealth is determined by $\widehat{W} = W_0 - X$ if no insurance is purchased. For our purpose of capital insurance later, the insurer is assumed to be risk-neutral[16].

Specifically, an insured purchases the insurance contract from an insurer by paying an initial premium P. When X is observed and realized, an indemnity $I(X)$ is transferred from the insurer to the insured. Then the insured's final wealth becomes $W = W_0 - P - X + I(X)$. The indemnity $I(X)$ is understood as a function of the loss variable X. In classical insurance literature (see, for example, Arrow (1971) and Raviv (1979)), the coverage $I(X)$ is often assumed to be non-negative and not to exceed the size of the loss. Standard examples of $I(X)$ in the marketplace include coinsurance $I(X) = aX$ for $0 \leq a \leq 1$, deductive $I(X) = \max\{X - K, 0\}$ with a

[15] For discussions on multi-period insurance market, we refer to Janssen and Karamychew (2005) and Venezia and Levy (1983). It has been widely recognized that the welfare improvement is possible only when the indemnity depends on the path of loss variables. Otherwise, there exists no essential distinction between the one-period and the multi-period insurance contract.

[16] One justification of the risk-neutral assumption in the insurance literature is that many insurers hold a well-diversified portfolio.

deductible level K and a cap insurance $I(X) = \min\{X, L\}$ for a cap level $L > 0$.[17] The insured can choose how many insurance contracts need to be purchased. For instance, under some circumstance, the coinsurance coefficient a can be determined while the premium structure is known. Gollier (2001) offers a lucid introduction to insurance market in this regard.

In the insurance contract literature, the authors often impose a premium principle in the form of

$$P = \mathbb{E}[I(X) + C(I(X))], \tag{2.1}$$

where the cost function $C(.)$ is non-negative and satisfies $C'(\cdot) > -1$. Notice that this premium principle underlies the fundamental risk pooling idea in insurance: a key insight to deal with risk in the insurance market. To see it, assume the insurer issues the insurance contract $I(X)$, and there is sufficiently large number of insureds in the insurance market with identical independent distribution (IID) loss $X_1, \cdots, X_n, \cdots$. Then the liability for the insurer becomes $I(X_1)+\cdots+I(X_n)$. The law of large number ensures that

$$\frac{I(X_1) + \cdots + I(X_n)}{n} \to \mathbb{E}\left[I(X)\right]. \tag{2.2}$$

Therefore, the risk for the insurer has been wiped out almost surely as long as there is sufficiently large number of insureds, and these loss variables $\{X_1, \cdots, X_n, \cdots\}$ of the insureds are IID. Figure 2.1 presents the framework of the classical insurance market. Obviously, this independent distribution assumption does not hold anymore

[17] See Froot (2001) for many other insurance contracts in his clinal examination in a reinsurance market and Froot et al (1993) on how those state-contingent contract used in the corporate risk management.

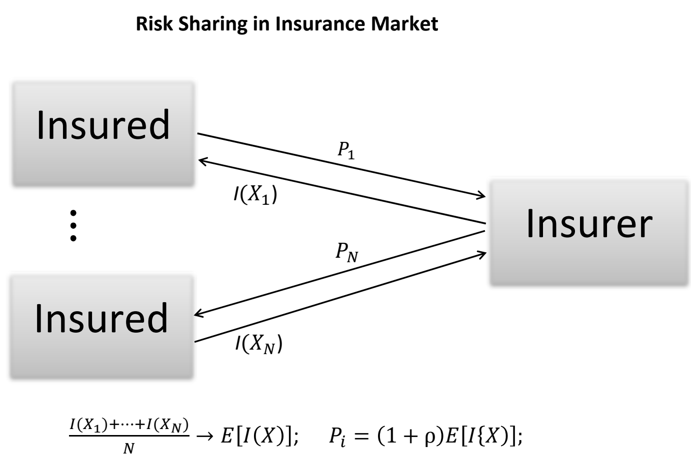

Figure 2.1: How classical insurance market works

for the bank sector in the presence of systemic risk. This is where capital insurance emerges as an innovation to deal with the systemic risk. In the classical insurance literature, there have been a large group of studies on the optimal design of the insurance contract, from the perspective of either an insurer or an insured or in an equilibrium framework. For instance, Arrow (1963) shows the deductible policy is optimal when the premium depends on the expected payoff of the policy only. Raviv (1979) extends Arrow's analysis to the convex cost structure. Huberman, Mayers and Smith (1983) introduce concave cost structure and find that deductible indemnity might not be optimal. Bernard and Tian (2009, 2010) study the optimal insurance design under risk management consideration. The optimal design problem has been also well studied in the presence of moral hazard or adverse selection. See Rothschild and Stiglitz (1976).

2.3.2 A Model of Capital Insurance

In a banks sector, there are N banks indexed by $i = 1, \cdots, N$. Each bank is endowed with a loss portfolio, $X_1, \cdots, X_N$, respectively. These loss portfolios are generated from each bank's business plan so they can be significantly different but correlated by nature. For simplicity, these loss portfolios are defined on the same state space Ω, and all banks have the same beliefs on the nature of state. This common belief is represented by one probability measure P on the state space. We assume that each bank is risk-averse, and its preference to risk is interpreted by a utility function $U_i(\cdot)$. The bank's initial wealth is given by W_0^i for each bank i. We assume no background risk for each bank.

There is an issuer in the capital insurance contract. While Kashyap et al (2008) suggest that investors in a private-sector might be the seller of the capital insurance, we have a broad view at this point. Recalling the role of AIG as the last resort of providing the credit default swap pre-crisis [18] and the fact that the government has injected \$ 182 billions dollars into AIG to save the financial system in jeopardy, we argue that the private-sector market power might be not significant as one thought, in particular, when a systemic risk is on a high level. Therefore, in addition to investors, we suggest this insurer could be a government entity such as Financial Stability Oversight Council (FSOC) in Dodd-Frank Act or a central bank, which sells the insurance contract to each bank. Each bank is either voluntarily or enforced to purchase the insurance contract by paying particular amount as a premium provided

[18] Credit default swap is an insurance contract written on the default event of a company or a portfolio.

the fund commitment is guaranteed by central bank in a bad business situation in future. Since we will demonstrate that banks will be better off from the insurance contract (in Section 1.5 below), the bank's decision is evident as long as the market performs well. If the government entity collects the premium from the banks, the fund commitment offered by the government entity is the indemnity of the insurance, then the premium amount can be treated as a special tax purpose rate for each bank as suggested by Acharya and Pedersen (2010). Besides the investor or a government entity, these insurance contracts can be also issued by a reinsurance company which is able to diversify the reinsurance risk. In each case, the bank is always an insured party and pays a premium to purchase a capital insurance. In the contrast to the traditional bank capital, this premium doesn't sit on the asset side on the bank's balance sheet. In our setting, we do not distinguish the role of seller in details and just name the insurer as a regulator.

For any bank i, the prototype insurance structure has the indemnity, $I_i(X, X_i)$, in which both the individual book loss X_i and the aggregate loss X are involved together. X represents the aggregate loss: $\sum_{i=1}^{N} X_i$. We call this kind of insurance contract a "capital insurance" as long as it depends on the aggregate loss being realized in the future to some extent. Evidently, the capital insurance contract is different from the classical contracts in last section, where $I_i(X, X_i)$ is irrelevant to the aggregate loss X and, instead, depends on the individual loss X_i.[19] Following the classical insurance literature (Arrow, 1963; and Raviv, 1979), we make use of the following

[19]In Panttser and Tian (2013), they study the classical coinsurance contract in the same rational equilibrium setting and compare whether capital insurance is better than the classical insurance.

linear insurance premium for bank i:

$$P_i = (1+\rho)\mathbb{E}[I(X, X_i)], \tag{2.3}$$

where ρ is a load factor. In other words, the cost function $C(\cdot)$ in the premium principle displays a linear structure: $C(t) = (1+\rho)t$. For simplicity, we also assume that the loss factor is the same across the bank industry, but, it is possible to consider a bank-specific premium structure in the extended analysis.

Given a load factor ρ, each bank chooses the best available insurance contract to maximize the expected utility:

$$\mathbb{E}[U_i(\tilde{W}^i)] \equiv \mathbb{E}\left[U_i\left(W_0^i - X_i + I_i(X, X_i) - (1+\rho)\mathbb{E}[I_i(X, X_i)]\right)\right]. \tag{2.4}$$

The regulator is risk-neutral and receives the premium for each contract. The terminal wealth of the regulator is:

$$W^r \equiv \sum_i (1+\rho)\mathbb{E}\left[I_i(X, X_i)\right] - \sum_i I_i(X, X_i) - \sum_i c\left(I_i(X, X_i)\right), \tag{2.5}$$

where $c\left(I_i(X, X_i)\right)$ represents the cost for the regulator to issue the contract $I_i(X, X_i)$. This regulatory cost can be fixed, a constant percentage of the indemnity or depend on a drastic market event. To focus on the analysis of insurance program, we assume that the cost structure is a constant for each bank. The regulator's objective is to determine the best premium structure given the optimal demand for each bank (with the load structure ρ) as well as to maximize the welfare (the expected utility). Clearly, the final insurance contract $I_i^*(X, X_i)$ in equilibrium depends on both demand (from all banks) and supply (from the regulator) and relies on the load factor ρ^* proposed by

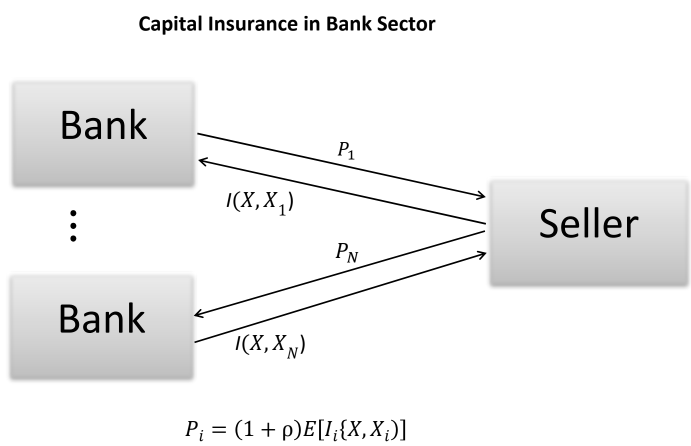

Figure 2.2: How capital insurance market works

the regulator. Figure 2.2 displays the mechanism of the capital insurance market. To explain the insight, we focus on one type of capital insurance contract in this chapter - aggregate insurance: $I_i(X, X_i) = a_i X$, where $a_i \geq 0$. The aggregate insurance is a coinsurance contract written on the aggregate loss. Each bank i chooses the best coinsurance coefficient a_i for itself. The optimal coinsurance coefficient a will be written as $a(\rho)$ to highlight its dependence on the load factor. [20]

2.4 Effects to Insurer

Similar to Panttser and Tian (2013), we first impose two assumptions in subsequent discussions.

Assumption I. Each bank is a mean-variance agent with the reciprocal of risk aversion parameter $\gamma_i > 0$. We also assume zero (or constant) cost structure for each

[20] We refer to Panttser and Tian (2013) for a welfare analysis of several capital insurance programs.

contract.

Assumption I is fairly standard in literature, for instance, Mace (1991) addresses the aggregate uncertainty insurance under the same assumption. Similarly, as our topic is on the systemic risk, it is natural to follow the same assumptions on the insured's risk preference as in Mace (1991). Moreover, the cost structure is immaterial for the regulator in estimating the social welfare, so a zero cost structure is also a reasonable assumption.

Assumption II. There exists no asymmetric information between each bank and the regulator. The loss portfolio X_i is equivalently identified by the bank and the regulator, and both the bank and the regulator make decision based on the same interpretation of the loss portfolio.

Comparing with Assumption I, Assumption II might be subject to confrontation given its limitation. Asymmetric information or adverse selection are important issues in studying insurance economics. In the original plan of Kashyap et al (2008), they do suggest to use all losses except for the individual bank's loss. Say, bank i purchases an insurance whose indemnity depends on $X - X_i = \sum_{j \neq i} X_j$[21]. However, this kind of design is still problematic, as Kashyap et al (2008) further argue that bank i can still manipulate its loss portfolio X_i to affect $X - X_i$ given the correlation structure between X_i and other X_j; the systemic risk component for each bank does play a role in the market. Since there is no easy way to deal with the asymmetric information issue in capital insurance currently, Assumption II serves as a benchmark to a more

[21] This insurance contract is termed as an aggregate-cross insurance contract in Panttser and Tian (2013).

general discussion on capital insurance.

The next proposition precisely presents the equilibrium for the aggregate insurance (Panttser and Tian (2003)).

Proposition 2.1 Assume for each $i = 1, \cdots, N$,

$$\frac{Cov(X_i, X)}{Var(X)} \geq \frac{1}{2}\frac{\gamma_i}{\sum_i \gamma_i}. \tag{2.6}$$

Then the optimal load factor ρ^* is given by

$$\rho^* = \frac{1}{2\sum_i \gamma_i}\frac{Var(X)}{\mathbb{E}(X)}. \tag{2.7}$$

The best coinsurance parameter for bank i in this aggregate insurance contract is

$$a_i(\rho^*) = \frac{Cov(X_i, X)}{Var(X)} - \frac{1}{2}\frac{\gamma_i}{\sum_i \gamma_i}. \tag{2.8}$$

Finally, the welfare for the regulator is

$$\mathbb{E}(W^*) = \frac{1}{4\sum_i \gamma_i}Var(X). \tag{2.9}$$

From Proposition 2.1 (and subsequent discussions), bank i's beta,

$$\beta_i \equiv \frac{Cov(X_i, X)}{Var(X)}, \tag{2.10}$$

plays an essential role in the equilibrium of capital insurance. As long as its beta is greater than $\frac{1}{2}\frac{\gamma_i}{\sum_i \gamma_i}$, the coinsurance coefficient is simply a difference between its beta and $\frac{1}{2}\frac{\gamma_i}{\sum_i \gamma_i}$. This result demonstrates that beta β_i captures its systemic risk component. To hedge the systemic risk for bank i, this capital insurance program allows bank i to purchase an insurance with payout $a_i X$.

According to Proposition 2.1, the issuance of capital insurance benefits to the regulator because of a positive welfare. Moreover, the welfare estimated by the regulator depends on the variability of the aggregate loss. The higher the variability, the higher the welfare. The smaller the variability, or alternatively, the more stable the aggregate loss is, the smaller the welfare. We will define the systemic risk as this variability of the aggregate loss (see Definition 1 and its justifications below). Therefore, the higher the systemic risk inside the bank sector, the better the social benefits. We also note that the welfare does not depend on the expected aggregate loss $\mathbb{E}[X]$, only the systemic risk contributes to the welfare.

If we look at the following decomposition of the systemic risk:

$$Var(X) = \sum_{i=1}^{N} Cov(X_i, X), \tag{2.11}$$

it is evident to see the contribution of each bank to the systemic risk, and the total beta becomes 1:

$$\sum_{i=1}^{N} \beta_i = \sum_{i=1}^{N} \frac{Cov(X_i, X)}{Var(X)} = 1. \tag{2.12}$$

Following from the last formula, we have

$$\sum_{i} a_i(\rho^*) = \frac{1}{2}, \tag{2.13}$$

the total aggregate insurance indemnity for regulator is $\sum_i I_i(X, X_i) = \frac{1}{2}X$. It states that exactly half of the aggregate loss is insured in this program. The number 1/2 comes from the mean-variance setting and might not have any specific meaning. But a crucial insight at this point is that the aggregate loss is not fully insured in this equilibrium insurance market, which is similar to the classical result for a standard coinsurance contract.

Proposition 2.1 motivates the following formal definition regarding to systemic risk and insurance capital.

Definition 2.1 The systemic risk ex ante in the bank sector is the variance of the aggregate loss, $Var(X)$, in the bank sector. The systemic risk component of bank i is its beta β_i. The premium for purchasing the capital insurance for the bank, $(1+\rho^*)a_i(\rho^*)\mathbb{E}[X]$, is an insurance capital of bank i.

We point out that other interpretations about the systemic risk are plausible in literature. Some authors, such as Pritsker (2012), suggest to use the default probability of the whole financial system. Other authors suggest to use the Shapley values to estimate the systemic risk (see Bluhm et al, 2013). See also Billio et al (2009), Rochet (2009), Eisenberg and Noe (2001), Choi and Douady (2012) and Panttser and Tian (2013). In this chapter, we confine ourself one important feature of the systemic risk, that is, the variability of aggregate loss.

2.4.1 Extension and Too Big To Fail

Proposition 2.1 states that the bank needs to purchase capital insurance if all banks have high systemic risks, or equivalently, their betas are large enough. What if some banks have relatively smaller betas in the market? Can we simply remove these banks with smaller betas and apply Proposition 2.1 for other banks to derive the equilibrium? Whether these banks with large betas are exactly "too big to fail" banks which should purchase capital insurance? Whether the expected utility of the regulator is always positive? To answer these questions, we now extend our previous analysis into a general setting.

First, bank i's objective is to find suitable coinsurance coefficient a_i to maximize

$$\max_{a_i \geq 0} \mathbb{E}[\tilde{W}^i] - \frac{1}{2\gamma_i} Var(\tilde{W}^i), \tag{2.14}$$

where $\tilde{W}^i = W_0^i - X_i + a_i X - (1+\rho)\mathbb{E}[a_i X]$ is the ex post terminal wealth for the bank i after purchasing the capital insurance. Similarly, we use $W^i = W_0^i - X_i$ to represent the ex ante wealth of bank i without buying any capital insurance. Given a load factor ρ, by the first-order condition in (2.14), the optimal a_i for the bank i is

$$a_i(\rho) = \max\left\{\frac{Cov(X_i, X) - \rho\mathbb{E}(X)\gamma_i}{Var(X)}, 0\right\}. \tag{2.15}$$

Second, by plugging equation (2.15) into equation (2.5) and then by maximizing the regulator's expected utility, we obtain

$$\rho^* = argmax_{\{\rho \geq 0\}} \rho \sum_{i=1}^{N} \max\left\{\frac{Cov(X_i, X) - \rho\mathbb{E}(X)\gamma_i}{Var(X)}, 0\right\}. \tag{2.16}$$

So the optimal coinsurance coefficient for each bank $i = 1, \cdots, N$ is

$$a_i(\rho^*) = \max\left\{\frac{Cov(X_i, X) - \rho^*\mathbb{E}(X)\gamma_i}{Var(X)}, 0\right\}. \tag{2.17}$$

Definition 2.2 Bank i is "too large to fail" in the sense of capital insurance if bank i has to purchase capital insurance, that is, $a_i(\rho^*) > 0$.

It remains to derive an explicit expression of the load factor in equation (2.16). For this purpose, we re-order the bank index and still use $i = 1, \cdots, N$, such that

$$\frac{Cov(X_1, X)}{\gamma_1} \geq \frac{Cov(X_2, X)}{\gamma_2} \geq \cdots \geq \frac{Cov(X_N, X)}{\gamma_N}. \tag{2.18}$$

It follows that the sequence $\frac{\sum_{j=1}^{i} Cov(X_j, X)}{2\sum_{j=1}^{i}\gamma_j}$ is decreasing for $i = 1, 2, \cdots, N$. Hence, there exists a unique number n, such that

$$\frac{Cov(X_i, X)}{\gamma_i} > \frac{\sum_{k=1}^{i} Cov(X_k, X)}{2\sum_{k=1}^{i}\gamma_k}, i = 1, \cdots, n \tag{2.19}$$

and

$$\frac{Cov(X_i, X)}{\gamma_i} \leq \frac{\sum_{k=1}^{i} Cov(X_k, X)}{2\sum_{k=1}^{i}\gamma_k}, i = n+1, \cdots, N. \tag{2.20}$$

If for each $i = 1, \cdots, N$, $\frac{Cov(X_i, X)}{\gamma_i} > \frac{\sum_{k=1}^{i} Cov(X_k, X)}{2\sum_{k=1}^{i}\gamma_k}$, we set $n \equiv N$.

We know that (see Panttser and Tian, 2013, Appendix B for detail) the optimal load factor is

$$\rho^* = \frac{1}{\mathbb{E}[X]}\frac{\sum_{i=1}^{n} Cov(X_i, X)}{2\sum_{i=1}^{n}\gamma_i}. \tag{2.21}$$

The optimal coinsurance coefficient is

$$a_i(\rho^*) = \frac{Cov(X_i, X) - \rho^*\gamma_i\mathbb{E}[X]}{Var(X)}, i = 1, \cdots, n; \tag{2.22}$$

and $a_i(\rho^*) = 0$, for $i = n+1, \cdots, N$. Therefore, we have the following result to identify "too big to fail" banks.

Proposition 2.2 In a bank sector with N banks with risk aversion parameters γ_i, $i = 1, \cdots, N$, we index these banks, $i = 1, \cdots, N$, such that $Cov(X_i, X)$ is decreasing, or equivalently, its beta sequence β_i is decreasing. Let n be the unique number, such that

$$\frac{Cov(X_i, X)}{\sum_{k=1}^{i} Cov(X_k, X)} > \frac{\gamma_i}{2\sum_{k=1}^{i}\gamma_k}, i = 1, \cdots, n, \tag{2.23}$$

and

$$\frac{Cov(X_i, X)}{\sum_{k=1}^{i} Cov(X_k, X)} \leq \frac{\gamma_i}{2\sum_{k=1}^{i}\gamma_k}, i = n+1, \cdots, N. \tag{2.24}$$

Then the first n banks are "too big to fail" banks which need to buy the capital insurance $a_i(\rho^*)X$ with premium $(1+\rho^*)a_i(\rho^*)\mathbb{E}[X]$, where ρ^* and $a_i(\rho^*)$ are given in equation (2.21) and equation (2.22), respectively. Other banks, $i = n+1, \cdots, N$, do not purchase capital insurance since they are not "too big to fail".

The next result demonstrates that the welfare for the regulator is always non-negative after the issuing the capital insurance. The proof is straightforward and omitted.

Proposition 2.3 In a general aggregate capital insurance setting as above, the welfare of the aggregate insurance for the regulator, $\mathbb{E}[W^*]$, is

$$\mathbb{E}[W^*] = \frac{1}{4\sum_{i=1}^{n}\gamma_i}\frac{(\sum_{i=1}^{n} Cov(X_i, X))^2}{Var(X)}. \tag{2.25}$$

2.5 Impacts on Banks

In the last section, we have seen that capital insurance indeed adds social benefits if the insurer is a government entity, which is obligated to inject capital in a bad business time. It is a good news for the regulators or government. As long as the banks pay upfront premiums, the guarantee to inject funds to save those "too big to fail" banks in a period of financial crisis will not cause taxpayer cry. Panttser and Tian (2013) have studied extensively the welfare of several capital insurance programs. The purpose of this section is to study its effect on the banks as issued.

We first demonstrate that the bank is also better off by purchasing the capital insurance. To see it, recall that $\tilde{W}^i$ and W^i are the terminal wealths of bank i with and without the capital insurance program, respectively. The next proposition ensures that the bank's expected utility increases with a purchase of capital insurance in equilibrium; hence, the banks are motivated to participate in the capital insurance market.

Proposition 2.4 After purchasing the capital insurance, the expected utility of bank i is increased by

$$\frac{1}{2\gamma_i Var(X)}\left\{Cov(X_i, X) - \frac{\gamma_i}{2\gamma}Var(X)\right\}^2,$$

where $\gamma = \sum_{i=1}^{N}\gamma_i$. For simplicity, we assume that the condition (2.6) in Proposition 2.1 holds, and this assumption can be relaxed by using Proposition 2.2.

According to Proposition 2.4, purchasing the capital insurance provides a higher expected utility for the bank. So from a bank's prospective, there is no side effect to enter a capital insurance market. Moreover, the positive benefits to bank i depend on both the systemic risk, $Var(X)$, and its beta, β_i, monotonically and separably. Precisely, the benefit is written as

$$\frac{1}{2\gamma_i}\left(\beta_i - \frac{\gamma_i}{2\gamma}\right)^2 Var(X).$$

Therefore, the higher the systemic risk (aggregate variance), $Var(X)$, the higher the benefit for bank i; the higher the difference between the beta and $\frac{\gamma_i}{2\gamma}$, the higher the benefit for bank i. By combining Proposition 2.1 and Proposition 2.2, $Var(X)$ is an essential ingredient to address the benefits for both the regulator and the banks; and both the regulator and the banks should demand capital insurance when the systemic risk is large.

How about the expected wealth of bank i after purchasing the capital insurance? As insurance contract always provides negative expected value, it is not hard to imagine that $\mathbb{E}[\tilde{W}^i] = \mathbb{E}[W^i] - \rho a_i \mathbb{E}[X] < \mathbb{E}[W^i]$. However, the bank's portfolio becomes less risky after purchasing the capital insurance. It is natural to see how much risk is reduced with a unit of the expected value decreased. The following formula, whose proof is given in Appendix B, describes the change of risk per unit of the expected value changing:

$$\frac{Var(\tilde{W}^i) - Var(W^i)}{\mathbb{E}[\tilde{W}^i] - \mathbb{E}[W^i]} = \gamma_i + 2\gamma\beta_i. \tag{2.26}$$

Again this ratio is determined entirely by beta.

So far, we have demonstrated that beta plays a key role in the capital insurance equilibrium. The next proposition is about the beta ex post and the systemic risk ex post in the capital insurance market.

Proposition 2.5 Let $\tilde{X}_i \equiv X_i - a_i X + (1+\rho) a_i \mathbb{E}[X]$ be the loss portfolio of bank i ex post in the capital insurance program, and X_i be the ex ante loss portfolio. The ex post aggregate loss portfolio is $\tilde{X} = \sum_{i=1}^{N} \tilde{X}_i$. Then the systemic risk is reduced by 75 percent since the ex post aggregate variance becomes

$$Var(\tilde{X}) = \frac{1}{4} Var(X). \tag{2.27}$$

The ex post beta of bank i becomes a constant:

$$\tilde{\beta}_i \equiv \frac{Cov(\tilde{X}_i, \tilde{X})}{Var(\tilde{X})} = \frac{\gamma_i}{\gamma}. \tag{2.28}$$

Proposition 2.5 has an important implication regarding the systemic risk. Let us assume that bank i repeatedly purchases a capital insurance by following the above approach. As its ex post beta is $\frac{\gamma_i}{\gamma}$, after the second time purchasing capital insurance, applying Propositions 1-2, the benefit to the regulator is $\frac{1}{4\gamma}\frac{1}{4}Var(X)$, the benefit to bank i is $\frac{1}{2\gamma_i}\left(\frac{\gamma_i}{2\gamma}\right)^2\frac{1}{4}Var(X)$, and the coinsurance coefficient is still $\frac{\gamma_i}{\gamma}$. After m times, the systemic risk becomes $\left(\frac{1}{4}\right)^m Var(X)$, the benefit to the regulator is $\frac{1}{4\gamma}\left(\frac{1}{4}\right)^m Var(X)$, the benefit to bank i is $\frac{1}{2\gamma_i}\left(\frac{\gamma_i}{2\gamma}\right)^2\left(\frac{1}{4}\right)^m Var(X)$. Hence, we have the following result.

Proposition 2.6 After finitely many times of implementing the capital insurance, the systemic risk can be reduced as much as possible. The capital insurance does not benefit to neither the regulator nor the banks when the systemic risk is reduced entirely.

2.6 Identify Too Big To Fail Banks

In this section, we illustrate one important application of the capital insurance, that is, how to identify "too big to fail" banks.

According to Proposition 2.2, identifying "too big to fail" banks is more complicated than to find banks with large beta. In fact, the set of banks with high betas, $\beta_i \geq \frac{1}{2N}$, is not necessarily the same as the set of "too big to fail" banks characterized in Proposition 2.2. We use several examples to illustrate the method in details.

In the first example, we consider 15 banks in a bank sector, and the loss portfolio of each bank follows a one-factor model. Specifically, $X_i = \alpha_i Y + \epsilon_i, i = 1, \cdots, 15$. The systematic risk factor is represented by Y with $\mathbb{E}[Y] = \$1$ billion and $Var(Y) = 100\%$. Each bank has the same expected loss $0.05\mathbb{E}[Y]$, but the specific risk (i.e., the variance of ϵ_i) varies across the banks. We assume that σ_i moves from 40% to 12%, and each $\gamma_i = 1$. Clearly, $\frac{Cov(X_i,X)}{\gamma_i}$ decreases from $i = 1$ to $i = 15$. Moreover, except for the last bank, all other banks' betas are greater than $\frac{1}{2N}$. For the last bank, $\frac{Cov(X_i,X)}{Var(X)} < \frac{1}{2N} = 3.33\%, i = 15.$[22]

We now identify those "too big to fail" banks by using Proposition 2.2. We find that the number n as defined in Proposition 2.2 is 13. Therefore, the first 13 banks are "too big to fail" banks that should purchase the capital insurance. Even the 14th bank has a large beta, 3.38 %, it does not need to buy a capital insurance because its systemic risk is relatively small; and importantly, given the correlation structure among these 15 banks, the capital insurance of the first 13 largest banks is helpful to resolve the systemic risk issue raised from the 14th banks. We next calculate the insurance capital for these first 13 banks. The optimal load factor ρ^* in the aggregate insurance is

$$\rho^* = \frac{1}{\mathbb{E}[X]} \frac{\sum_{i=1}^{n} Cov(X_i, X)}{2\sum_{i=1}^{n} \gamma_i} = 8.1\%. \tag{2.29}$$

The computation results are given in Table 2.1 (this table slightly extends Table 5 in Panttser and Tian, 2013).

Table 2.1 demonstrates that a bank with a larger beta (the 14th bank in this bank sector) does not have to buy a capital insurance as it is not a "too big to fail" bank; but, a "too big to fail" bank must have a large systemic risk component, ie., a high beta factor. To demonstrate this subtle issue, we consider another example in which the banks with larger betas are exactly the same as "too big to fail" banks.

In the second example, there exists $N = 15$ banks in the bank sector, and the loss

[22] In Panstter and Tian (2013), it is incorrectly stated that the last four banks do not satisfy the condition in Proposition 2.1.

portfolio follows a one factor model $X_i = \eta_i Y + \epsilon_i$, where $\eta_i = 0.04 + 0.002 * (i-1), \sigma_i = 0.45 - 0.02 * (i-1)$, $\mathbb{E}[Y] = 1$ billion and $Var(Y) = 50\%$, where σ_i is the standard deviation of noise ϵ_i for $i = 1, 2, \cdots, 15$. Clearly, the covariance vector $Cov(X_i, X)$ is decreasing with respect to i. As displayed in Table 2.2, the first 12 banks have larger betas, $\beta_i > \frac{1}{2N}, i = 1, \cdots, 12$. We also find out that these 12 banks are exactly "too big to fail" banks according to Proposition 2.2. The optimal load factor in equilibrium is 8.1 %.

In Example 3, we do not assume any factor assumption on the loss portfolio. Instead, we assume that any two different banks' loss portfolios have the same correlation coefficient ρ in the bank sector with N banks. Assume that $Var(X_i) = k^{2i}\sigma^2$ for $0 < k < 1$. Then, $Var(X_i)$ is decreasing. Moreover, we assume that

$$(1-\rho)k^N \geq \rho(k + \cdots + k^N).$$

Thus, $Cov(X_i, X)$ is decreasing for $i = 1, \cdots, N$ under this assumption. Table 2.3 demonstrates that "too big to fail" banks must have large betas, but the inverse statement is not true. In fact, only the last bank has a small beta; however, only the first three banks are "too big to fail".

By example 3, we notice that the "too big to fail" theory in Proposition 2.2 depends virtually on the beta vector of the banks. Proposition 2.2 can be interpreted slightly different in terms of the banks' beta vector, as follows.

Proposition 2.7 Given a bank sector with N banks with beta $\beta_i, i = 1, 2, \cdots, N$. We assume that $\beta_1 \geq \beta_2 \geq \cdots \geq \beta_N$, and each bank has the same risk aversion parameter. Let n be the unique number, such that

$$\frac{\beta_i}{\sum_{k=1}^{i} \beta_k} > \frac{1}{2i}, i = 1, \cdots, n; \frac{\beta_i}{\sum_{k=1}^{i} \beta_k} \leq \frac{1}{2i}, i = n+1, \cdots, N. \tag{2.30}$$

Then the "too big to fail" banks are exactly banks $i = 1, \cdots, n$. Moreover, the optimal coinsurance coefficient a_i for bank $i, i = 1, \cdots, n$, is

$$a_i = \beta_i - \frac{1}{2n} \sum_{k=1}^{n} \beta_k. \tag{2.31}$$

However, these beta vectors are not enough to compute the insurance capital.

Proposition 2.7 ensures that the beta vectors of loss portfolio alone enable us to identify "too big to fail" banks for the regulator and the coinsurance coefficient for the banks. But these beta vectors are not enough to compute the insurance capital charged by the regulator. Since $(1 + \rho^*)\mathbb{E}[X] = \mathbb{E}[X] + \frac{1}{2n} \sum_{i=1}^{n} Cov(X_i, X)$, the regulator also needs information about the expected aggregate loss $\mathbb{E}[X]$ and the covariance $Cov(X_i, X)$ for computing the insurance capital.

2.7 Conclusions

This chapter presents an insurance perspective to bank capital, namely, an insurance capital. This insurance capital is a premium paid by the banks to an insurer of a capital insurance contract, which is written on the aggregate loss in the entire bank sector. By using a simple coinsurance-type insurance contract (aggregate insurance), we demonstrate how the insurer and the banks trade in a rational equilibrium setting; so the banks purchase appropriate capital insurance, and the insurer offers an appropriate premium level. We show that this capital insurance idea is largely promising to resolve the systemic risk management issue, because of the following results: (1) the insurer is better off to issue the capital insurance; (2) banks are better off to increase their expected utilities; (3) systemic risk is reduced significantly ex post; and (4) this capital insurance program enables the regulator to identify which banks are "too big to fail" and, therefore, should purchase the capital insurance contract.

However, our analysis in this chapter should be examined with cautions, more being an introduction instead of a complete theory of the insurance capital. Our numerical computation shows that the insurance capital might be too high comparing with other bank capitals because it insures all loss aggressively. Since each bank has its own hedging programs for the market portfolios, those capital insurance with deductible indemnity on aggregate loss might be more appealing to the market: as long as the aggregative loss hits a threshold, the insurance is triggered to be active and the insurance payout for the bank is a proportion of the deductive. Specially, $I_i(X, X_i) = a_i \max\{X - L, 0\}$ for each bank, where L is a trigger level for the aggregate loss, and a_i is a coinsurance coefficient over the total deductible for bank i. Indeed, Arrow (1973) demonstrates that such a deductible is optimal for the insured when the premium is a linear function of the expected indemnity in the classical insurance setting. Whether Arrow's deductible theorem holds in the capital insurance setting? What is the best possible insurance capital from the regulator's perspective? We hope this chapter serves a basis for future study on the insurance capital and sheds light on the study of the systemic risk.

Table 2.1: "Too big to fail" banks - example 1

This table displays a bank sector with 15 banks and identifies "too big to fail" banks following the presented capital insurance approach. The loss portfolios follow a one-factor model, $X_i = \eta_i Y + \epsilon_i$; $\mathbb{E}[Y] = 1$ billion and $Var(Y) = 5\%$, and the standard deviation of ϵ_i moves from 40% to 12 % equally from $i = 1$ to $i = 15$, while η_i moves from 4% to 6.8% equally. We also assume that each γ_i is the same across the banks and equals to 1. We note that when $i = 13, 14, 15$, $\frac{Cov(X_i,X)}{\gamma_i}$ is strictly greater than $\frac{\sum_{j=1}^{i} Cov(X_j,X)}{2\sum_{j=1}^{i}\gamma_j}$. Then, the last two banks are not "too big to fail". The optimal load factor is $\rho^* = 8.1\%$. The expected total aggregate loss is $\eta\mathbb{E}[Y] = 0.81$ billion.

Bank	$\frac{Cov(X_i,X)}{\gamma_i}$	$\frac{\sum_{j=1}^{i} Cov(X_j,X)}{2\sum_{j=1}^{i}\gamma_j}$	$\frac{Cov(X_i,X)}{Var(X)}$	a_i	$(1+\rho^*)a_i$
1	0.2106	0.1053	0.1226	8.46 %	9.14 %
2	0.1934	0.1010	0.1126	7.46 %	8.06 %
3	0.1770	0.0968	0.1031	6.50 %	7.03 %
4	0.1614	0.0928	0.0940	5.59 %	6.04 %
5	0.1466	0.0889	0.0854	4.73 %	5.11 %
6	0.1326	0.0851	0.0772	3.92 %	4.23 %
7	0.1194	0.0815	0.0695	3.15 %	3.40 %
8	0.1070	0.0780	0.0623	2.43 %	2.62 %
9	0.0954	0.0746	0.0556	1.75 %	1.89 %
10	0.0846	0.0714	0.0493	1.12 %	1.21 %
11	0.0746	0.0683	0.0435	0.54 %	0.59 %
12	0.0655	0.0653	0.0381	0.01 %	0.01 %
13	0.0571	0.0625	0.0332	0	0
14	0.0495	0.0598	0.0288	0	0
15	0.0427	0.0573	0.0248	0	0

Table 2.2: "too big to fail" Banks - example 2

This table displays a bank sector with 15 banks and identifies "too big to fail" banks following the presented capital insurance approach. The loss portfolios follow a one-factor model, $X_i = 0.05Y + \epsilon_i$; $\mathbb{E}[Y] = 1$ billion and $Var(Y) = 100\%$, and the standard deviation of ϵ_i moves from 40% to 12 % equally from $i = 1$ to $i = 15$. We also assume that each γ_i is the same across the banks and equal to 1, for simplicity. We note that when $i = 14, 15$, $\frac{Cov(X_i,X)}{\gamma_i}$ is strictly greater than $\frac{\sum_{j=1}^{i} Cov(X_j,X)}{2\sum_{j=1}^{i}\gamma_j}$. Then, the last two banks are not "too big to fail". The optimal load factor is $\rho^* = 8.1\%$. The expected total aggregate loss is $\eta\mathbb{E}[Y] = 0.75$ billion.

Bank	$\frac{Cov(X_i,X)}{\gamma_i}$	$\frac{Cov(X_i,X)}{Var(X)}$	$\frac{\sum_{j=1}^{i} Cov(X_j,X)}{2\sum_{j=1}^{i}\gamma_j}$	a_i	$(1+\rho^*)a_i$
1	0.1975	0.1170	0.09875	8.10 %	8.75 %
2	0.1819	0.1070	0.09485	7.18 %	7.76 %
3	0.1671	0.0990	0.09108	6.30 %	6.81 %
4	0.1531	0.0907	0.08745	5.47 %	5.91 %
5	0.1399	0.0829	0.08395	4.69 %	5.07 %
6	0.1275	0.0755	0.08058	3.95 %	4.27 %
7	0.1159	0.0686	0.07735	3.27 %	3.53 %
8	0.1051	0.0622	0.07425	2.63 %	2.84 %
9	0.0951	0.0563	0.07128	2.03 %	2.20 %
10	0.0859	0.0509	0.06845	1.49 %	1.61 %
11	0.0775	0.0459	0.06575	0.99 %	1.07 %
12	0.0699	0.0414	0.06318	0.54 %	0.59 %
13	0.0631	0.0374	0.06075	0.14 %	0. 15 %
14	0.0571	0.0338	0.05845	0	0
15	0.0519	0.0307	0.05628	0	0

Table 2.3: "Too big to fail" banks - example 3

This table displays a bank sector with 5 banks and identifies "too big to fail" banks following the presented capital insurance approach. In this example, any two different banks have the same parameter $\rho = 10\%$, and $Var(X_i) = k^{2i}\sigma^2$, where $k = 0.8$. In this example, we see that "too big to fail" banks can be find out using beta vector entirely.

Bank	β_i	$\frac{\beta_i}{\sum_{k=1}^{i}\beta_k}$	Coinsurance coefficient a_i
1	0.4690	1	35.24 %
2	0.3206	0.4060	20.40%
3	0.2215	0.2191	10.49 %
4	0.1548	0.1328	3.82 %
5	0.1095	0.0859	0

CHAPTER 3: IDENTIFY TBTF BANKS AND CAPITAL INSURANCE

3.1 Introduction

We develop a new methodology to identify too big to fail (TBTF) banks[23]from a regulatory perspective. Since the too big to fail issue is virtually linked to the implicit guarantee subsidy[24], this methodology also sheds a light on the assessment of the implicit subsidy. We introduce a new systemic risk measure, loss beta, by conducting an equilibrium analysis of TBTF banks and demonstrate that this loss beta concept captures some essential economic elements of the TBTF issue.

The financial crisis 2007-2009 sparks substantial research interests in measuring the systemic risk recently. Acharya et al (2012), Brownless and Engle (2011) document that time-varying correlation structure play a crucial role in their systemic risk measurements (See also v-lab webpage in New York University); and it is well documented that the time-varying correlation coefficients among big financial institutions are broadly positives. Consequently, several approaches have been proposed to cast the connectivity and correlative features among top banks in studying the systemic risk, including Adrian and Brunnermeier (2010)'s CoVaR approach conditional on

[23]The term "too big to fail" is frequently interchanged with other terms such as "too important to fail" (TITF), "too interconnected to fail" (TITF) or "global systemically important banks" (G-SIBs) with might be slightly different contexts. A bank is deemed to be TBTF in this paper if the bank has implicit government guarantee during a crisis.

[24]The implicit (guarantee) subsidy, or alternatively, capital surcharge, is often estimated by funding costs with and without the guarantee. See, for instance, IMF (2014) and Green/EFA group report (2014). See also O'Hara and Shaw (1990) in the context of deposit insurance; and BCBS (2013) for assessment methodology.

financial institutions being in a state of financial distress; the network approach by Acemoglu et al (2013); the default probability of the whole financial system developed by Shin (2008); the marginal expected shortfall measure approach in Acharya (2009), Brownless and Engle (2011) and Acharya et al (2012), and the CDS premium approach in Zhou, Huang and Zhu (2009). Hansen (2012) documents the challenge to measure the systemic risk, and a comprehensive survey of systemic risk measures is presented by Bisias et al (2012).[25] None of these approaches, however, explores an equilibrium mechanism in which banks and regulator interact with each other in their best interests.

In this paper, we study a rational expectation equilibrium by suggesting that TBTF banks have to pay insurance premium up front to exchange for its implicit guarantee subsidy. Specifically, we view the agreement between the bank and the regulator (or a government entity), which injects the guaranteed capital as an insurance contract and we call it a capital insurance contract. In this framework, each bank predicts the best insured amount whenever the pricing structure of the capital insurance is given by the seller. On the other hand, the seller of the capital insurance fully predicts each bank's optimal insured amount, determines the optimal pricing structure, and simultaneously identifies those banks which are willing to purchase this kind of capital protection, henceforth, too big to fail banks. The idea of capital insurance to study the systemic risk is first briefly proposed by Kashyap et al (2008). It is also resemble to the special tax program proposed in Acharya et al (2010) in which the insurance

[25] Other notably papers include Allen and Gale (2000); Hellwig (2009); Lehar (2005); Battiston et al (2012); Billio et al (2012); and Rochet (2009).

premium is viewed as special tax for too big to fail.[26]

We characterize explicitly the equilibrium of the capital insurance market. By conducting this equilibrium analysis, we demonstrate several positive effects of a capital insurance proposal. Specifically, the social welfare for the regulator is shown to be positive and the total systemic risk is reduced with the implementation of the capital insurance market. Too big to fail banks are also beneficial by purchasing the capital protection in the capital insurance market, and those banks with larger systemic risk components enjoy more expected utility enhancing. Moreover, the capital insurance market can be used by the regulator to reveal banks' true loss portfolios and identify TBTF banks correctly in the presence of moral hazard among banks and the regulator. Overall, we demonstrate that the capital insurance proposal could be a useful macro-regulation policy tool to address the TBTF issue.[27]

In deriving the capital insurance equilibrium, we introduce a new systemic risk measure, loss beta, which is defined as a ratio of the covariance between a bank's loss portfolio with the aggregate loss portfolio in the entire bank sector to the variance of the aggregate loss portfolio. We provide an algorithm to identify TBTF banks by merely using loss betas of all banks. We show that not only banks with large loss betas are TBTF; Conversely, TBTF banks must have large loss betas. Therefore, the too big to fail feature is largely captured by the loss beta measure. We also implement this approach by using several different capital insurance contracts in an empirical

[26]Therefore, the developed equilibrium in this article can be also viewed as an equilibrium of a special tax program.

[27]Classical prudential regulation theory of banks is explained in Dewatripoint and Tirole (1994); Hanson, Kashyap and Stein (2011). See also Aiyar, Calomiris and Wieladek (2014) for a comprehensive discussion on bank capital regulation.

study. We find out that TBTF banks can be consistently identified with this approach over the pre-cris and pro-cris period; and this empirical analysis suggests that the too big to fail concern has been considerably reduced after the financial crisis.

This article merges two important strands of previous research: the financial innovation and the classical insurance literature. By viewing capital insurance as an innovation in a capital market, we explore a similar framework examined in Allen and Gale (1994) to characterize the equilibrium among a group of buyers and a seller in the presence of one financial innovation. Further, we follow Harris and Raviv (1995) to study the optimal payoff structure within a given specification form of the payoff structure of financial innovation. On the other hand, treating the capital insurance as an insurance contract between banks and regulator, we develop the model by drawn on some essential insights in Borch (1962), Arrow (1964) and Ravi (1979). However, it is worth noting that the presented framework itself is different from the classical insurance setting in which the law of large numbers (risk-pooling principle) holds under an independent assumption of the individual risk across a group of insureds. Indeed, the failed risk-pooling principle with correlated underlying risks is a challenge in measuring the systemic risk, and this paper suggests that capital insurance is useful to address the correlated risk management problem.

Given its concentration on loss portfolios, our approach to the systemic risk leads to starkly difference between our systemic risk measure with other systemic measures that based on classical beta, downside beta or tail beta (Bawa and Lindenverg, 1977; Hogan and Warren, 1974; Van Oordt and Zhou, 2014). For instance, Benoit et al (2012) in a recent empirical study shows that from both theoretical and empirical perspective, the marginal expected shortfall measure introduced in Acharya (2009), Brownless and Engle (2011), Acharya et al (2012) is largely explained by the classical betas of banks; and the classical beta of financial institution captures the interconnectedness in the financial sector to some degree but adds little to rank too big to fail banks.

The article proceeds as follows. In Section 3.2 we present a theory of capital insurance. In Section 3.3 we report our empirical analysis and illustrate some implementation issues. Section 3.4 concludes and all proofs are given in Appendix C.

3.2 Theory of Capital Insurance

3.2.1 Model Setup

There are N financial institutions, namely banks, indexed by $i = 1, \cdots, N$, in a financial sector. Each bank is endowed with a loss portfolio, $X_1, \cdots, X_N$, respectively. These loss portfolios are presumed to have systemic risk components and given exogenously. There is a capital insurance market in which each bank decides to purchase or not a capital insurance contract to hedge the systemic risk. The prototype capital insurance contract's payoff structure (or indemnity in insurance terminology) is $I_i(X, X_i)$ for bank i where X represents the aggregate loss, $X = \sum_{i=1}^{N} X_i$, of the financial sector.

We follow standard insurance literature (Arrow, 1963; and Raviv, 1979) to apply a classical linear insurance premium principal. Specifically, the insurance premium P_i for bank i to pay for is, $P_i = (1+\rho)\mathbb{E}[I(X, X_i)]$, where ρ is a load factor that is determined by the seller. It is convenient for now to assume a constant loss factor across the financial sector, and we explain in Section 3.3 how to investigate a bank-specific premium structure in an extended analysis.

In this paper, we focus on the following capital insurance contract, $I_i(X, X_i) = a_i Z$ for each bank i, where a_i is a nonnegative coinsurance coefficient and $Z = I(X)$ is an arbitrarily specification of indemnity that relies on the aggregate loss. Bank i chooses the best coinsurance coefficient a_i, and the optimal coinsurance coefficient is written as $a_i(\rho)$ to highlight its dependence on the load factor ρ.

Each bank i, $i = 1, \cdots, N$, is risk-averse, and its risk preference is represented entirely by the mean and the variance of the wealth with the reciprocal of risk aversion parameter $\gamma_i > 0$.[28] Given a load factor ρ, bank i solves an optimal portfolio problem by choosing the best coinsurance coefficient:

$$\max_{\{a_i \geq 0\}} \left\{ \mathbb{E}[\tilde{W}^i] - \frac{1}{2\gamma} Var(\tilde{W}^i) \right\}, \tag{3.1}$$

where $\tilde{W}^i = W_0^i - X_i + a_i Z - (1+\rho)\mathbb{E}[a_i Z]$ is the ex post terminal wealth for the bank i after purchasing the capital insurance and W_0^i is the initial wealth of bank i. We assume now there is no background risk in this section and we explain how to extend our results into a situation with background risk in Section 3.3. Similarly, $W^i = W_0^i - X_i$ represents the ex ante wealth of bank i before buying capital insurance. Moreover, we assume that each $\gamma_i = \gamma$ for $i = 1, \cdots, N$, so these banks are distinguished from each other due primarily to their different loss portfolios.[29]

By the first order condition in (3.1), the optimal coinsurance coefficient for bank i

[28]Mace (1991) addresses the aggregate uncertainty insurance under the same assumption.

[29]It is easy to extend it into a general situation in which γ_i varies, and the main insights are similar.

is given by

$$a_i(\rho) = \max\left\{\frac{Cov(X_i, Z) - \rho\mathbb{E}(Z)\gamma}{Var(Z)}, 0\right\}. \tag{3.2}$$

The seller of capital insurance contracts can be a private-sector, reinsurance company, a central bank or a government entity such as Financial Stability Oversight Council (FSOC) in Dodd-Frank Act, which is universally named as a regulator. The regulator is assumed to be risk-neutral and receives the insurance premium from each capital insurance contract. Therefore, the terminal wealth of the regulator is

$$W^r = \sum_{i=1}^{N}(1+\rho)\mathbb{E}\left[a_i Z\right] - \sum_{i=1}^{N} a_i Z - \sum_{i=1}^{N} c\left(a_i Z\right), \tag{3.3}$$

where $c\left(a_i Z\right)$ denotes the cost for the regulator to issue the contract $a_i Z$. This regulatory cost $c(\cdot)$ can be a fixed cost, a constant percentage of the indemnity or a general function of the indemnity. Without loss of generality and to focus on the equilibrium analysis of TBTF, we assume that the regulatory cost is a constant for each bank.[30]

Given the optimal demand for each bank (with a load factor ρ) in (3.2), the regulator is presumed to maximize the expected welfare $\mathbb{E}[W^r]$ by determining the best load factor ρ and the optimal insurance premium in (3.2). Specifically, by plugging equation (3.2) into equation (3.3), the regulator's optimal load factor is derived from the following optimization problem:

$$\max_{\{\rho>0\}} \rho \sum_{i=1}^{N} \max\left(\frac{Cov(X_i, Z) - \rho\gamma\mathbb{E}[Z]}{Var(Z)}, 0\right) \tag{3.4}$$

and the optimal coinsurance coefficient for each bank $i = 1, \cdots, N$ is given by $a_i(\rho^*)$,

[30] We refer to Huberman, Mayers and Mayers (1982) for other cost structures in insurance literature.

where ρ^* is the optimal load factor in (3.4). In the end, the capital insurance's payoff for each bank i, $a_i(\rho^*)Z$, relies on both demand (from all banks) and supply (from the regulator) in a rational expectation equilibrium.

In light of the non-concavity feature of its objective function, the regulator's optimization problem (3.4) is non-standard; thus, its solution cannot be easily characterized by virtue of the first order condition. In Appendix C, we elaborately reduce the optimization problem (3.4) to a set of standard optimization problems; and as a consequence, solve the existence of the equilibrium.

Definition 3.1 With a capital insurance $Z = I(X)$, the loss beta of bank i is $\frac{Cov(X_i,Z)}{Var(Z)}$. Bank i is deemed to be TBTF, from the capital insurance $Z = I(X)$ perspective, if its optimal coinsurance coefficient $a(\rho^*)$ is positive. The capital insurance premium, $(1 + \rho^*)a(\rho^*)\mathbb{E}[Z]$, is an insurance capital for bank i.

Clearly, the capital insurance premium offers an assessment of the implicit subsidy from an insurance perspective.

3.2.2 Identifying TBTF Banks

By virtue of equation (3.2), bank i is too big to fail as long as its loss beta, $Cov(X_i, Z)/Var(Z)$, is large enough such that

$$\frac{Cov(X_i, Z)}{Var(Z)} > \rho^* \left(\gamma \frac{\mathbb{E}[Z]}{Var(Z)}\right). \tag{3.5}$$

But the optimal load factor ρ^* in (3.5) is subject to determined endogenously. The optimal load factor is solved by (3.4), and it depends on all loss portfolios information, in particularly, all banks' loss betas. Therefore, one individual bank's loss beta is not sufficient to recognize whether it is too big to fail or not; rather, we have to implement the methodology in the financial sector as a whole to identify all TBTF banks simultaneously. Roughly speaking, a bank is TBTF only when its loss beta is relatively large compared with other banks' loss betas in the same financial sector.

Again, because of its non-standard feature, it is plausible to have multiple optimal solutions in (3.4) and thus multiple equilibria in the capital insurance market. We argue that this plausible multiple equilibria issue is not serious though[31]. Notice that the higher the load factor is, the less banks are identified as TBTF and those identified TBTF banks have to pay higher insurance premiums. In contrast, a smaller load factor ensures a larger number of TBTF banks whereas each TBTF bank pays a smaller insurance premium. Evidently, the regulator is willing to choose the smallest load factor, among many solutions of ρ^*, to enlarge the number of TBTF banks under monitoring even though the expected welfare for the regular is indifferent. Those banks with higher systemic risk components also desire a smaller load factor because of smaller insurance premiums. Only banks with relatively small loss betas have benefited from a higher load factor, because these banks are otherwise characterized as TBTF and forced to pay insurance premiums. For these reasons, it is reasonable to choose the smallest load factor for the regulator in the presence of possible multiple optimal solutions in problem (3.4).

As shown in Appendix C, the following simple algorithm identifies TBTF banks by merely using of loss betas.

[31]However, the multiple equilibrium issue might be very severe in some economic contexts. See, for instance, Diamond and Dybvig (1983), Sundaresan and Wang (2013).

Step 1. Let $\beta_i = \frac{Cov(X_i,Z)}{Var(Z)}$, and reorder that $\beta_1 \geq \cdots \geq \beta_N > 0$. We omit those banks with negative or zero loss betas.

Step 2. Let $\tau_m = \frac{1}{2m}\sum_{i=1}^{m}\beta_i$ for $m = 1, \cdots, N$. Define $\overline{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\}$ for $m = 1, \cdots, N-1$ and $\overline{\tau}_N = \tau_N$.

Step 3. Compute $B_m = h_m(\overline{\tau}_m)$ for each $m = 1, \cdots, N$, where $h_m(\tau) = \sum_{i=1}^{m}(\beta_i\tau - \tau^2)$.

Step 4. Compute m^* as $argmax_{1\leq m\leq N}B_m$, and choose the smallest m^* if there exist multiple solutions of m^*.

Step 5. Bank i is TBTF if and only if $\beta_i > \overline{\tau}_{m^*}$, for $i = 1, \cdots, N$.

The next proposition shows that the bank with the highest loss beta must be a TBTF bank.

Proposition 3.1 Among all banks in a financial sector, the bank with the highest loss beta must be too big to fail.

By Proposition 3.1, there do exist TBTF banks in any financial sector. Therefore, the capital insurance is of necessary from the regulatory perspective.

We provide several examples of identifying TBTF banks with the above algorithm.

Example 1. If each bank contributes equivalently to the systemic risk in the sense that $\frac{Cov(X_i,Z)}{Var(Z)} = c$ for any $i = 1, \cdots, N$ and a positive number c, then each bank is TBTF and the optimal load factor is $\frac{c}{2\gamma}\frac{Var(Z)}{\mathbb{E}[Z]}$. Moreover, the optimal coinsurance coefficient for each bank is its half loss beta.

Example 1 follows easily from Proposition 3.1, in which each bank has the same loss beta; therefore, each bank is too big to fail. The optimal load factor and the corresponding coinsurance coefficient can be calculated easily.

Example 2. Consider a financial sector with two banks, $i = 1, 2$, and assume that $Cov(X_1, Z) \geq Cov(X_2, Z)$. Then each bank is TBTF if $Cov(X_1, Z) = Cov(X_2, Z)$; and only bank 1 is TBTF if, and only if the following condition holds.

$$1 < \frac{Cov(X_1, Z)}{Cov(X_2, Z)} \leq \frac{1}{\sqrt{2} - 1}$$

The first case in Example 2 follows easily from Example 1. Assume that $Cov(X_2, Z) < Cov(X_1, Z)$. Then only the first bank is TBTF, by using the algorithm, if and only if $h_2(\overline{\tau}_2) \geq h_1(\overline{\tau}_1)$. It is easy to verify that, the last inequality holds if and only if $\frac{Cov(X_1,Z)}{Cov(X_2,Z)} \leq \frac{1}{\sqrt{2}-1}$.

The next example is concerned with a financial system with more than 3 banks, in which only one bank is TBTF if this bank's loss beta significantly dominates all other banks' loss betas.

Example 3. Given a loss beta structure such that $\frac{Cov(X_i,Z)}{Var(Z)} = c\tau^{i-1}$ for each $i = 1, \cdots, N$, a positive number c and a positive number $\tau \in (0, 1)$, only the first bank is TBTF when τ is small enough. Moreover, the optimal load factor is $\rho^* = \frac{c}{2\gamma} \frac{Var(Z)}{\mathbb{E}[Z]}$.

Example 3 is interesting in its own right. Even though some banks contribute positively to the systemic risk and banks are heavily correlated, those banks might still not be TBTF banks, given the fact that by insuring the bank with the most significant systemic risk exposure, other banks' systemic risks can be insured to some extent. Example 3 illustrates an essential insight of the capital insurance proposal, which in contrast with the network approach (Acemoglu et al, 2013) to the systemic risk that connectedness amongst the banks play a key role.

3.2.3 Positive Social Values

The following result affirms a positive social value of the capital insurance market.

Proposition 3.2 With an immaterial regulatory cost, the expected welfare of the capital insurance market for the regulator, $\mathbb{E}[W^r]$, is always positive.

Generally speaking, the expected welfare for the regulator depends on many market factors such as all banks' loss betas in a financial sector. Under what circumstance the social value is positively related to loss betas or negatively affected by the loss betas? There is no clear-cut on a comparative analysis given the complexity of the equilibrium. Remarkably, Proposition 3.2 demonstrates a positive effect of the capital insurance market for all possible loss portfolios.

We next study the effect of the capital insurance market to TBTF banks. While TBTF banks are identified by the regulator, an important question arises. Whether these TBTF banks are willing to purchase capital insurance contracts on their interests? What happens if these TBTF banks do not purchase the capital insurance? or even if they are forced to purchase the capital insurance by a regulator, are they intend to manipulate the loss portfolio because the purchase decisions are against their willingness? The next result resolves this potential conflict interest between the regulator and TBTF banks.

Proposition 3.3 The expected utility of a TBTF bank is strictly increased after purchasing the capital insurance. Moreover, the higher the loss beta of a TBTF bank, the higher the improved expected utility of the bank.

Not only are TBTF banks willing to purchase the capital insurance contracts, but also the banks with higher loss betas have more ex post benefits, so those banks are more motivated to participate in this capital insurance market. Both Proposition 3.2 and Proposition 3.3 together ensure Pareto improvement by implementing a capital insurance market.

3.2.4 Aggregate Capital Insurance

In this section, we specialize the capital insurance - aggregate capital insurance - by assuming that the indemnity, Z, is the aggregate loss. With the aggregate capital insurance, we show that TBTF banks must have large loss betas, a somewhat converse statement of Proposition 3.1.

The optimal coinsurance coefficient of the aggregate insurance for a TBTF bank i is

$$a_i(\rho^*) = \frac{Cov(X_i, X)}{Var(X)} - \rho^* \frac{\gamma \mathbb{E}[X]}{Var(X)}, \tag{3.6}$$

in which the second component on the right side of (3.6) is the same for all banks. The first component is (by abuse of notation) its loss beta of the loss portfolio,

$$\beta_i = \frac{Cov(X_i, X)}{Var(X)}. \tag{3.7}$$

We define concretely the systemic risk from both the market level and the individual bank perspective in an aggregate capital insurance market.

Definition 3.2 The systemic risk ex ante in the bank sector is the variance, $Var(X)$, of the aggregate loss in the financial sector. The systemic risk component of bank i is its loss beta, $\frac{Cov(X_i,X)}{Var(X)}$.

Proposition 3.4 The loss beta of a TBTF bank in the aggregate capital insurance market must be greater than or equal to $\frac{1}{2N}$.

In Example 2, each bank has the same loss beta and belongs to TBTF banks, so each loss beta $\beta_i = 1/N$ because the sum of all loss betas is 1. In spite of all possible loss portfolios, Proposition 3.4 shows that all TBTF banks's loss betas must be bounded below by $\frac{1}{2N}$, a fairly tight distribution-freelower bound of loss betas for all TBTF banks.

We turn next to the systemic risk. By using our systemic risk measurements, we demonstrate that the systemic risk is indeed reduced in the entire financial sector by the next result.

Proposition 3.5 In a positive correlated risk environment in the sense that $Cov(X_i, X_j) \geq 0, \forall i, j = 1, \cdots, N$, the total systemic risk in the financial sector is strictly reduced after implementing the aggregate capital insurance.

3.2.5 Moral Hazard

We have so far assumed that the regulator recognizes all banks' true loss portfolios in the capital insurance market. However, the asymmetric information about loss distributions between banks and the regulator could distort the insurance premium, the optimal indemnity, and probably affect entirely the major insights of the capital insurance market. The objective of this subsection is to examine the moral hazard issue between banks and the regulator. We show that the regulator is able to reveal each bank's true loss portfolio in the capital insurance market and to identity TBTF banks correctly; the banks are also aware of regulator's ability to recognize the true loss portfolios. Hence, the true loss portfolios are reported in the presence of the capital insurance market.

Precisely, each bank i's true loss portfolio is denoted by X_i, but this bank's reporting loss portfolio to the regulator is $\hat{X}_i$. We write $\hat{X}_i = X_i + \epsilon_i$, for $i = 1, \cdots, N$ and each ϵ_i has mean 0 and variance σ_i^2. We assume that these noise terms, $\epsilon_1, \cdots, \epsilon_N$, are independent from each other, Moreover, these noise terms are independent from banks' true loss portfolios $\{X_1, \cdots, X_N\}$. For regulator, the aggregate loss is $\hat{X} \equiv \sum_{i=1}^{N} \hat{X}_i$, but it might be not the true aggregate loss of the market due to the asymmetric information on the loss distributions.

We consider two kinds of moral hazard. First, we assume that these banks know the true loss portfolios each other but they collectively report "wrong" loss portfolios to the regulator. This case is called a collective moral hazard (see Farhi and Tirole, 2012, in a similar context). Second, these banks do not know the true loss portfolios each other. In other words, each bank misrepresents its loss portfolios to anyone else to take information advantage in the capital insurance market. This case is termed as a mutual moral hazard. In what follows, we show that the regulator is able to reveal the true loss portfolios and identify TBTF banks with the help of the aggregate capital insurance in these two cases, respectively.

3.2.5.1 Collective Moral Hazard

Since bank i knows all true loss portfolios in this collective moral hazard situation, bank i's optimal coinsurance coefficient, if being positive with a given load factor ρ, is determined by equation (3.6). Moreover, even though the true loss portfolio X_i and the true aggregate loss portfolio X might be unknown to the regulator, the regulator fully observes $a_i(\rho)$ for each $i = 1, \cdots, N$ from the capital insurance market. The next proposition shows that, given the information set $\left\{a_i(\rho), \hat{X}_i; i = 1, \cdots, N\right\}$, the regulator is able to identify σ_i^2 for each bank i.

Proposition 3.6 Given a load factor ρ with $a_i(\rho) > 0, i = 1, \cdots, N$, the variances $\{\sigma_1^2, \cdots, \sigma_N^2\}$ can be derived uniquely by the data set $\left\{a_i(\rho), \hat{X}_i; i = 1, \cdots, N\right\}$.

As the regulator offers the capital insurance contracts with vary load factors, the regulator is able to identify the variances, σ_i^2, $i = 1, \cdots, N$, of the error terms of the loss portfolios. Notice that these banks are not necessarily to be TBTF since the load factor might be not the optimal load factor though. However, knowing σ_i^2, both the "true" covariance $Cov(X_i, X) = Cov(\hat{X}_i, \hat{X}) - \sigma_i^2$ and the "true" variance $Var(X) = Var(\hat{X}) - \sum_{i=1}^N \sigma_i^2$ are known. Therefore, the optimal load factor problem of the regulator, that is, the problem (3.4), is reduced to be

$$\max_{\{\rho>0\}} \rho \sum_{i=1}^{N} \max\left(\frac{Cov(\hat{X}_i, \hat{X}) - \sigma_i^2 - \rho\gamma\mathbb{E}[\hat{X}]}{Var(\hat{X}) - \sum_{i=1}^N \sigma_i^2}, 0\right). \tag{3.8}$$

Problem (3.8) can be solved exactly as in solving problem (3.4). Thus, the regulator is able to identify all TBTF banks correctly in this collective moral hazard situation.

3.2.5.2 Mutual Moral Hazard

In a mutual moral hazard situation, bank i is only aware of its own loss portfolio X_i and "reported" loss portfolios $\hat{X}_j, j \neq i$, of all other banks. Then, from bank i's perspective, the aggregate loss portfolio is $X_i + \sum_{j \neq i} \hat{X}_j$, which is $\hat{X} - \epsilon_i$. Consequently, bank i's terminal wealth in equation (3.1), after purchasing capital insurance, is replaced by $W_0^i - X_i + a_i(\hat{X} - \epsilon_i) - (1+\rho)\mathbb{E}[a_i(\hat{X} - \epsilon_i)]$. As a result, the first order condition yields the optimal coinsurance coefficient for bank i,

$$\overline{a}_i(\rho) = \max\left\{ \frac{Cov(X_i, \hat{X} - \epsilon_i) - \rho\gamma\mathbb{E}[\hat{X} - \epsilon_i]}{Var(\hat{X} - \epsilon_i)}, 0 \right\}. \tag{3.9}$$

Proposition 3.7 In a positive correlated risk environment in the sense that $Cov(X_i, X_j) \geq 0, \forall i, j = 1, \cdots, N$, the regulator is able to identify TBTF banks correctly in a mutual moral hazard situation. Precisely, given a load factor ρ with $\overline{a}_i(\rho) > 0, i = 1, \cdots, N$, the variances $\{\sigma_1^2, \cdots, \sigma_N^2\}$ can be derived uniquely by the data set $\left\{\overline{a}_i(\rho), \hat{X}_i; i = 1, \cdots, N\right\}$.

Since the noises' variances $\{\sigma_i^2; i = 1, \cdots, N\}$ can be solved by the regulator, the regulator knows $Cov(X_i, \hat{X} - \epsilon_i) = Cov(\hat{X}_i, \hat{X}) - \sigma_i^2$ and $Var(\hat{X} - \epsilon_i) = Var(\hat{X}) - \sigma_i^2$. Then, the optimal load factor for the regulator is reduced to be

$$\max_{\{\rho > 0\}} \rho \sum_{i=1}^{N} \overline{a}_i(\rho) \equiv \rho \sum_{i=1}^{N} \max\left(\frac{Cov(\hat{X}_i, \hat{X}) - \sigma_i^2 - \rho\gamma\mathbb{E}[\hat{X}]}{Var(\hat{X}) - \sigma_i^2}, 0 \right). \tag{3.10}$$

Again, Problem (3.10) can be solved similarly by a method explained in Appendix C. Therefore, the regulator can identify all TBTF banks in this mutual moral hazard situation.

We have developed the equilibrium analysis of the capital insurance market and shown the advantages of the proposed capital insurance market in several aspects (Proposition 3.1 to Proposition 3.7). We also justify in theory that the loss betas capture significant component of the systemic risk. We next illustrate how our theoretical results can be implemented empirically.

3.3 Empirical Analysis and Implementation

In this section, we first present an empirical analysis by following the methodology in Section 2. We apply several capital insurance contracts to identify TBTF banks. Then we discuss some implementation issues and make some comments to extend the framework.

3.3.1 Data

In our empirical analysis, we identify TBTF banks over the period from 2004 to 2012 on the year by year basis. There are 14 big financial institutions during the pre-financial crisis period from 2004 to 2008 in our sample. The institutions are in groups of banks, insurance companies, investment firms and government sponsored enterprises. They are: Freddie Mac, Fannie Mac, American International Group, Merrill Lynch, Bank of America, Bear Sterns, Citigroup, Goldman Sachs, JP Morgan, Lehman Brother, Metlife, Morgan Stanley, Wachovia and Wells Fargo. For simplicity, we use the corresponding symbols "3FMCC*1000", "3FNMA", "AIG", "BAC2", "BAC", "BSC.1", "C", "GS", "JPM", "LEHMQ", "MET", "MS", "WB" and "WFC" to represent these 14 big financial institutions, respectively. Only 10 financial institutions out of 14 left in the market after financial crisis, so we report TBTF banks from these ten banks over the pro-crisis period 2009-2012. We obtain information on the bank characteristics such as total assets, total equity and number of shares outstanding from Compustat and stock returns data from CRSP.

Similar to Adrian and Brunnermeier (2010), we compute the asset loss portfolio for each financial institution i, $i = 1, \cdots, N$. For this purpose, we define the following variables:

- L_t^i: the leverage ratio of institution i at time t, the ratio of total asset value over the total equity value;
- M_t^i: the market capitalization of institution i at time t;
- Y_t^i: the profit and loss of institution i at time t, that is, $Y_t^i \equiv L_t^i \cdot M_t^i - L_{t-1}^i \cdot M_{t-1}^i$;
- X_t^i: the loss portfolio of institution i at time t, that is, $X_t^i \equiv \max\{-Y_t^i, 0\}$.
- X_t: the aggregate loss portfolio at time t, $X_t = \sum_i X_t^i$.

Since the number of banks in our sample changes before and after financial crisis, we conduct our analysis for two sub-periods pre-crisis (2004-2008) and pro-crisis (2009-2012) separably.

3.3.2 Identify TBTF Banks Empirically

In the following empirical analysis, we use two types of capital insurance contracts, deductible insurance and cap insurance contracts, respectively. A deductible capital insurance has a payoff structure $Z = \max\{X - L, 0\}$ where L is an exogenously given deductible level. The deductible capital insurance is inspired by the classical deductible insurance contract, which is optimal for the insured with a linear premium principle (Arrow, 1965). On the other hand, a cap contract with a payoff structure $Z = \min\{X, L\}$ is shown to be optimal for insurer under some assumptions in Raviv (1979), where L represents a capped level for the loss. Aggregate capital insurance is a special deductible contract with zero deductible level or a special cap contract with infinitely large cap level. For a robust purpose, we examine three different levels of L including $L = 0.1\mathbb{E}[X]$, $L = 0.2\mathbb{E}[X]$ and $L = 0.5\mathbb{E}[X]$ in both deductible and cap insurance contract, where $\mathbb{E}[X]$ is the expected aggregate loss portfolio across all the banks in our sample. In total, six capital insurance contracts are used in implementing the methodology.

Our identification of TBTF banks are presented in Table 3.1 - Table 3.9 on the year by year basis.

Table 3.1 displays the procedure of identifying TBTF banks in 2004 with these six different capital insurance contracts, in which TBTF banks are reported for both deductible insurance and cap insurance contracts in red and blue colors, respectively. We highlight m^* and $\overline{\tau}_{m^*}$ for each contract. By using three deductible insurance contracts, only "BAC" is identified as TBTF. However, there are additional three TBTF banks, 3FNMA, AIG and MS, if cap insurance contracts are employed. In a certain degree, it is not a surprise that there are more TBTF banks from a cap insurance market than a deductible insurance market because a cap contract itself is optimal from seller's perspective (Raviv, 1979), and we observe similar patterns in Table 3.2- Table 3.9 as well. Moreover, these four banks, BAC, 3FNMA, AIG and MS, are TBTF banks in each cap insurance market, and they have the highest loss betas even in each deductible market. It demonstrates that these four banks indeed have significant systemic risk exposures.

Identifying TBTF banks becomes more interesting and serious in 2005 than in 2004, as reported in Table 3.2. In Table 3. 2, there are five TBTF banks, 3FNMA, AIG, MS, BAC2 and JPM, in each deductible market. Notice that these five banks are also TBTF banks in each cap insurance market, but the cap insurance market reveals more TBTF banks in 2005. When the cap level is given by $L = 0.1\mathbb{E}[X]$, there are 10 TBTF banks in total; and there are seven TBTF banks when the cap level is higher ($L = 0.2\mathbb{E}[X]$ or $L = 0.5\mathbb{E}[X]$). In other words, five new banks are TBTF banks with the first cap contract and two new banks are TBTF by using other cap insurance contracts. As a summary, at least seven banks are deemed to be too big to fail from the regulator's perspective, by implementing the capital insurance market. In these seven banks, 3FMNA, AIG, MS, BAC, BAC2, JPM and 3FMCCC*1000, two banks, BAC and 3FMCC*1000, are not identified as TBTF banks in deductive insurance market but both of them have large loss betas right next to those other five TBTF banks in each deductible insurance market.

Table 3.3 displays TBTF banks in 2006. This table also demonstrates some important differences between the deductible contract and the cap insurance contract. As illustrated in Table 3.3, only one "WFC" is identified as TBTF in each deductible insurance market. On the right side of Table 3.3, however, there are many more TBTF banks; there are 10, 9, and 8 TBTF banks in each cap insurance market with different cap level, respectively. In each cap insurance market, WFC has the highest loss beta so it is TBTF naturally (Proposition 3.1), but there are at least seven other banks which are deemed to be TBTF banks in each cap insurance market. It is interesting to check positions of LEHMQ in Table 3.3. LEHMQ is TBTF in each cap insurance market. More importantly, LEHMQ has very high loss beta so as large systemic risk exposure: it has the third largest loss beta persistently in each cap insurance market and the second highest loss beta persistently in each deductible market. The latter point is worth mentioning because LEHMQ is not identified as TBTF just because another bank's loss beta dominates all other banks' loss betas (as explained in Example 3).

2007 is important in many aspects to understand the financial crisis because some critical issues regarding the mortgage-backed securities and CDO market have been emerged in the market. The identification of TBTF banks, reporting in Table 3.4, is fairly consistent with the substantial systemic risk issue occurred in this year. First of all, comparing with only one TBTF bank in 2006 in each deductible insurance market, there are ten TBTF banks in 2007 when we make use of the same deductible contracts. Second, these ten TBTF banks are fairly the same as TBTF banks from the cap insurances perspective. Over the entire pre-crisis period, 2007 is the only one year in which deductible markets and cap insurance markets identify TBTF most consistently.

Owing to several dramatic market events in 2008, we have to be deliberate with regard to the data analysis. Because of well known events happened on Bear Sterns (BSC1), Lehman Brother (LEHMQ), Merrill Lynch (BAC2) and Wachovia (WB), the loss portfolios of these four banks are under scrutiny. Moreover, because of significant losses across the financial sector in 2008, some cap insurance contracts might not work well in 2008 anymore. For instance, the variance of Z is almost zero when the cap level is set too low in 2008 such as $L = 0.1\mathbb{E}[X]$. Therefore, the top cap insurance market on the right side in Table 3.5 should be read with diligence because of some negative loss betas. Still, we find that those TBTF banks in 2007 are either TBTF banks or have high level loss betas in each capital insurance market in 2008. By combining Table 3.4 and Table 3.5 together, the TBTF issue is so significant that should be alarmed seriously for the regulator.

Over the post-crisis period (2009-2012), only ten banks left in the original financial sector. The TBTF banks in 2009 are identified and reported in Table 3.6. As observed, the TBTF issue is still very serious because there are four banks, "AIG", "WFC", "JPM" and "BAC", are deemed to be TBTF banks in each capital insurance market. This is the second year (the first time is on 2007) when both deductible and cap insurance market identify identical TBTF banks. This list of TBTF banks is clearly intuitively appealing because "AIG" plays a crucial role in its CDS issuance and other three are the largest three commercial banks in U.S.

The TBTF issue has been reduced considerably after 2009 according up to our empirical analysis. As shown in Table 3.7-3.9, only GS is identified as TBTF between 2009-2012. This fact might result from our construction of asset loss portfolio, because the leverage ratio is of essential in this construction and GS has relatively large leverage ratio. Given its substantially large loss beta comparing with all other banks, only the bank, GS, with the highest loss beta is TBTF (as illustrated in Example 3). From the regulatory perspective, it shows some positive signs on the TBTF issues but they should pay a closer attention to GS to reduce its leverage ratio.

Our empirical results can be summarized as follows.

(1). Deductible capital insurance markets with different deductible levels identify TBTF banks consistently in each year.

(2). Cap insurance markets with vary cap levels also identify TBTF banks fairly consistently.

(3). In general, TBTF banks in deductible market are very likely TBTF banks in cap insurance markets, but not vice versa. When a bank is deemed to be TBTF bank in both deductible and cap insurance market, it should have large systemic risk.

(4). The regulator should be alerted when both the deductible and the cap market identify a large number of TBTF banks consistently (say in 2007 and 2009).

(5). When one bank has significantly large loss beta comparing with all other banks, only this bank is TBTF according to our presented methodology. In this case, other banks with large loss betas should be analyzed in diligent as well.

(6). The regulator should conduct the TBTF analysis by using several different capital contracts. The regulator should also be careful to construct loss portfolios to analyze the systemic risk.

(7). The TBTF issues has been considerably reduced lately.

3.3.3 Implementation and Comments

In this section, we explain how the previous discussions can be modified or extended in a more general setting. In particular, we discuss how to address the background risk. We also incorporate richer indemnity structure of the capital insurance as well as the general specification of the load factor into the setting.

3.3.3.1 Background Risk

Essential to our methodology is the loss portfolio of each bank as input to identify TBTF banks. Since the loss portfolio construction is related to its systemic risk exposure, the background risk can not be ignored. For instance, when the mortgage-based securities risk is a big concern as in 2007-2008, we can choose X_i to be the loss portfolio concentrated on mortgage-based risk only. In this way, the initial wealth with other possible risk exposures is not deterministic anymore.

Assume the time period starts from time t and all loss portfolios of banks are realized at the next time period $t+1$. Let $\mathcal{F}_t$ denote the information set at time t which is observed by all banks and regulator. The wealth of bank i at time t is $W_{i,t}$. Due to the background risk, $W_{i,t}$ could be correlated with the loss portfolio $X_{i,t+1}$. Let $X_{t+1} = \sum_{i=1}^{N} X_{i,t+1}$ denote the aggregate loss portfolio in the time period $[t, t+1]$, and the capital insurance contract proposed in this time period is a multiple of $Z_{t+1} \equiv I(X_{t+1})$.

First of all, the bank i's terminal wealth at time $t+1$ is $W_{i,t+1} = W_{i,t} - X_{i,t+1} + a_{i,t}Z_{t+1} - (1+\rho_t)\mathbb{E}_t[a_{i,t}Z_{t+1}]$, where $\mathbb{E}_t[\cdot]$ denotes the conditional expectation operator with respect to the information set $\mathcal{F}_t$ and $a_{i,t}$ is the optimal coinsurance coefficient for bank i. Secondly, let $Cov_t(\cdot)$ denote the conditional covariance with respect to the information set $\mathcal{F}_t$. By standard method in Section 2, the optimal coinsurance parameter at time t for bank i is

$$a_{i,t}(\rho_t) = \max\left\{\frac{Cov_t(X_{i,t} - W_{i,t}, Z_{t+1}) - \rho_t \mathbb{E}_t[Z_{t+1}]\gamma}{Var_t(Z_{t+1})}, 0\right\}. \tag{3.11}$$

By comparing equation (3.2) with equation (3.11), it suffices to replace the loss portfolio in equation (3.2) by the difference between the loss portfolio and the initial wealth at time t. Thirdly, the regulator determines the best load factor, ρ_t, at time t, by solving the conditional-based optimization problem

$$\max_{\{\rho_t > 0\}} \sum_{i=1}^{N} \rho_t \max\{Cov_t(X_{i,t} - W_{i,t}, Z_{t+1}) - \rho_t \mathbb{E}_t[Z_{t+1}]\gamma, 0\}. \tag{3.12}$$

Evidently, the last problem can be solved similarly at time t, given the information set $\mathcal{F}_t$.

3.3.3.2 Payoff Structure

While we develop the theory for a class of capital insurance contract, $I_i(X, X_i) = a_i I(X)$, for some function forms of $I(\cdot)$, the payoff structure can be quite general. $I_i(X, X_i)$ can be designed in a way that both the aggregate loss X and the individual loss portfolio X_i are involved for bank i, or $I_i(X, X_i)$ even depends on the entire set of loss portfolios, $\{X_1, \cdots, X_N\}$. For instance, $I_i(X, X_i) = a_i(X - X_i)$, is a contract proposed in Kashyap et al (2008) and studied in Panttser and Tian (2013). As another example, we can consider a general version of the indemnity:

$$I_i(X, X_i) = a_i I(b_1 X_1 + \cdots + b_N X_N), \tag{3.13}$$

where the parameters $b_1, \cdots, b_N$ capture some firm-specific features of the banks and $I(\cdot)$ is a specific functional form. Bank i chooses the coinsurance coefficient a_i.

It is worth mentioning that the methodology developed in Section 2 is different from the classical insurance literature even for a classical coinsurance contract, $I_i(X, X_i) = a_i X_i$. In classical insurance literature, the insureds' loss portfolios are assumed to be independent from each other, so the law of large number is applied. Panttser and Tian (2013) develops an equilibrium analysis following the same methodology in Section 2 for classical coinsurance contracts at the presence of dependent structure among loss portfolios.

3.3.3.3 Loss Factor

Finally, we consider the load factor in the form of $\rho_i = \rho(\theta, X_i)$ to incorporate the firm-specific information such as size, credit risk, liquidity, and its complexity, where θ is a set of parameters and $\rho_i(\theta, X_i)$ is used to compute the insurance premium for bank i. The equilibrium analysis can be developed similarly. For instance, for the capital insurance contract, $I_i(X, X_i) = a_i Z$, bank's i optimization problem is still the same as in equation (3.1) and the optimal coinsurance coefficient is given by

$$a_i(\theta, \rho(\theta, X_i)) = \max\left\{\frac{Cov(X_i, Z) - \rho(\theta, X_i)\mathbb{E}[Z]\gamma}{Var(Z)}, 0\right\}. \tag{3.14}$$

Therefore, the regulator's optimization problem is

$$\max_{\{\theta, \rho(\theta, X_i) > 0\}} \sum_{i=1}^{N} \rho(\theta, X_i) \max\{Cov(X_i, Z) - \rho(\theta, X_i)\mathbb{E}[Z]\gamma, 0\}. \tag{3.15}$$

The equilibrium is solved similarly to the optimization problem described in equation (3.4).

3.4 Conclusions

This paper suggests a new methodology of studying systemic risk from an insurance perspective. By developing an equilibrium analysis of the capital insurance, we show that this capital insurance idea is promising to examine some systemic risk issues because of the following results. (1) The insurer (say, a regulator) is better off to issue the capital insurance and the systemic risk on the market level is reduced. (2) Banks are better off to increase their expected utilities and their systemic risk components are reduced ex post. (3) This capital insurance program enables the regulator to identify which banks are deemed to be TBTF irrespective of absence of moral hazard or not. (4) The TBTF issues can be mainly captured by a high level of loss beta, a new systemic risk measure introduced in this equilibrium approach.

These reported results have some important policy implications and practical appeals. The regulator can design several optimal capital insurance contracts and identifies TBTF banks. The insurance premium received by the regulator can be viewed as a new type of capital - insurance capital, to protect the insured financial institutions in the face of crisis. Finally, the insurance capital can be also used to assess the implied guarantee subsidy for TBTF banks.

Table 3.1: TBTF banks in 2004

This table displays a bank sector with 14 financial institutions and identifies "too big to fail" banks in year 2004 following the capital insurance approach explained in Appendix C. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows. Deductible insurance is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z = max(X - L, 0))$. Cap Insurance is the cap capital insurance contract, in which $Z = min(X, L)$. We utilize the following three deductible levels, L, defined as the percentage of the expected aggregate loss in the banking sector: $L = 0.1\mathbb{E}[X]$, $L = 0.2\mathbb{E}[X]$ and $L = 0.5\mathbb{E}[X]$. $\beta_m = \frac{Cov(X_i,Z)}{Var(Z)}$. $\tau_m = \frac{1}{2m}\sum_{i=1}^{m}\beta_i$ for $i = 1, \cdots, N$. $\overline{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\}$ for $m = 1, \cdots, N-1$ and $\overline{\tau}_N = \tau_N$. The bank i is too big to fail if and only if $\beta_i > \overline{\tau}_{m^*}$, for each $i = 1, \cdots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
	$L = 0.1\mathbb{E}[X]$		$m^* = 1$		$L = 0.1\mathbb{E}[X]$		$m^* = 4$	
1	BAC	0.7447	0.3723	0.3723	3FNMA	2.4512	1.2256	1.9021
2	3FNMA	0.0746	0.2048	0.0746	BAC	1.9021	1.0883	1.0883
3	AIG	0.0524	0.1453	0.0524	AIG	1.0112	0.8941	0.9090
4	MS	0.0431	0.1144	0.0431	MS	0.9090	0.7842	0.7842
5	JPM	0.0223	0.0937	0.0223	JPM	0.5644	0.6838	0.5644
6	BAC2	0.0156	0.0794	0.0156	BAC2	0.4495	0.6073	0.4495
7	3FMCC*1000	0.0106	0.0688	0.0106	3FMCC*1000	0.2942	0.5415	0.2942
8	WFC	0.0074	0.0607	0.0074	WFC	0.2523	0.4896	0.2523
9	WB	0.0058	0.0543	0.0058	GS	0.2313	0.4481	0.2313
10	MET	0.0048	0.0491	0.0048	LEHMQ	0.1893	0.4127	0.1893
11	LEHMQ	0.0039	0.0448	0.0039	WB	0.1854	0.3836	0.1854
12	C	0.0034	0.0412	0.0034	MET	0.1481	0.3578	0.1481
13	BSC.1	0.0032	0.0382	0.0032	BSC.1	0.1055	0.3344	0.1055
14	GS	-0.0024	0.0353	0.0353	C	0.0883	0.3136	0.0883

Table 3.1 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
	$L = 0.2\mathbb{E}[X]$		$m^* = 1$		$L = 0.2\mathbb{E}[X]$		$m^* = 4$	
1	BAC	0.7665	0.3832	<u>0.3832</u>	3FNMA	1.3133	0.6567	1.0647
2	3FNMA	0.0718	0.2096	0.0718	BAC	1.0647	0.5945	0.5945
3	AIG	0.0518	0.1483	0.0518	AIG	0.5642	0.4904	0.5094
4	MS	0.0425	0.1166	0.0425	MS	0.5094	0.4315	<u>0.4315</u>
5	JPM	0.0218	0.0954	0.0218	JPM	0.3131	0.3765	0.3131
6	BAC2	0.0151	0.0808	0.0151	BAC2	0.2478	0.3344	0.2478
7	3FMCC*1000	0.0104	0.0700	0.0104	3FMCC*1000	0.1577	0.2979	0.1577
8	WFC	0.0071	0.0617	0.0071	WFC	0.1407	0.2694	0.1407
9	WB	0.0056	0.0551	0.0056	WB	0.1025	0.2452	0.1025
10	MET	0.0046	0.0499	0.0046	MET	0.0829	0.2248	0.0829
11	LEHMQ	0.0038	0.0455	0.0038	LEHMQ	0.0769	0.2079	0.0769
12	C	0.0033	0.0418	0.0033	GS	0.0746	0.1937	0.0746
13	BSC.1	0.0032	0.0387	0.0032	BSC.1	0.0502	0.1807	0.0502
14	GS	-0.0027	0.0359	0.0359	C	0.0471	0.1695	0.0471
	$L = 0.5\mathbb{E}[X]$		$m^* = 1$		$L = 0.5\mathbb{E}[X]$		$m^* = 4$	
1	BAC	0.8662	0.4331	<u>0.4331</u>	3FNMA	0.6057	0.3028	0.6025
2	3FNMA	0.0569	0.2308	0.0569	BAC	0.6025	0.3020	0.3020
3	AIG	0.0469	0.1617	0.0469	AIG	0.2946	0.2505	0.2703
4	MS	0.0370	0.1259	0.0370	MS	0.2703	0.2216	<u>0.2216</u>
5	JPM	0.0186	0.1026	0.0186	JPM	0.1517	0.1925	0.1517
6	BAC2	0.0114	0.0864	0.0114	BAC2	0.1326	0.1714	0.1326
7	3FMCC*1000	0.0083	0.0747	0.0083	3FMCC*1000	0.0823	0.1528	0.0823
8	WFC	0.0049	0.0656	0.0049	WFC	0.0726	0.1383	0.0726
9	WB	0.0047	0.0586	0.0047	WB	0.0434	0.1253	0.0434
10	MET	0.0039	0.0529	0.0039	MET	0.0356	0.1146	0.0356
11	BSC.1	0.0031	0.0483	0.0031	LEHMQ	0.0355	0.1058	0.0355

Table 3.1 (continued)

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
12	C	0.0030	0.0444	0.0030	C	0.0206	0.0978	0.0206
13	LEHMQ	0.0028	0.0411	0.0028	BSC.1	0.0167	0.0909	0.0167
14	GS	-0.0030	0.0380	0.0380	GS	0.0071	0.0847	0.0071

Table 3.2: TBTF banks in 2005

This table displays a bank sector with 14 financial institutions and identifies “too big to fail” banks in year 2005 following the capital insurance approach explained in Appendix C. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows. Deductible insurance is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z = max(X - L, 0))$. Cap Insurance is the cap capital insurance contract, in which $Z = min(X, L)$. We utilize the following three deductible levels, L, defined as the percentage of the expected aggregate loss in the banking sector: $L = 0.1\mathbb{E}[X]$, $L = 0.2\mathbb{E}[X]$ and $L = 0.5\mathbb{E}[X]$. $\beta_m = \frac{Cov(X_i,Z)}{Var(Z)}$. $\tau_m = \frac{1}{2m}\sum_{i=1}^{m}\beta_i$ for $i = 1, \cdots, N$. $\overline{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\}$ for $m = 1, \cdots, N-1$ and $\overline{\tau}_N = \tau_N$. The bank i is too big to fail if and only if $\beta_i > \overline{\tau}_{m^*}$, for each $i = 1, \cdots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
	$L = 0.1\mathbb{E}[X]$		$m^* = 1$		$L = 0.1\mathbb{E}[X]$		$m^* = 4$	
1	3FNMA	0.2205	0.1103	0.1745	3FNMA	0.2617	0.1308	0.1513
2	AIG	0.1745	0.0987	0.1680	AIG	0.1513	0.1032	0.1277
3	MS	0.1680	0.0938	0.0943	MS	0.1277	0.0901	0.1179
4	JPM	0.0943	0.0822	0.0941	BAC	0.1179	0.0823	0.0999
5	BAC2	0.0941	0.0751	0.0751	3FMCC*1000	0.0999	0.0758	0.0926
6	3FMCC*1000	0.0655	0.0681	0.0655	JPM	0.0926	0.0709	0.0830
7	BAC	0.0650	0.0630	0.0630	BAC2	0.0830	0.0667	0.0796
8	WFC	0.0483	0.0581	0.0483	GS	0.0796	0.0633	0.0636
9	WB	0.0396	0.0539	0.0396	WB	0.0636	0.0598	0.0598
10	BSC.1	0.0224	0.0496	0.0224	WFC	0.0562	0.0567	0.0562
11	C	0.0199	0.0460	0.0199	LEHMQ	0.0337	0.0530	0.0337
12	MET	0.0163	0.0429	0.0163	MET	0.0285	0.0498	0.0285
13	GS	0.0160	0.0402	0.0160	BSC.1	0.0278	0.0470	0.0278
14	LEHMQ	0.0139	0.0378	0.0378	C	0.0223	0.0445	0.0223

Table 3.2 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
	$L = 0.2\mathbb{E}[X]$		$m^* = 5$		$L = 0.2\mathbb{E}[X]$		$m^* = 7$	
1	3FNMA	0.2181	0.1091	0.1753	3FNMA	0.7121	0.3560	0.4214
2	AIG	0.1753	0.0983	0.1705	AIG	0.4214	0.2834	0.3294
3	MS	0.1705	0.0940	0.0946	MS	0.3294	0.2438	0.3076
4	BAC2	0.0946	0.0823	0.0940	BAC	0.3076	0.2213	0.2753
5	JPM	0.0940	0.0752	0.0752	3FMCC*1000	0.2753	0.2046	0.2622
6	3FMCC*1000	0.0635	0.0680	0.0635	JPM	0.2622	0.1923	0.2289
7	BAC	0.0625	0.0627	0.0625	BAC2	0.2289	0.1812	0.1812
8	WFC	0.0479	0.0579	0.0479	GS	0.1747	0.1695	0.1695
9	WB	0.0383	0.0536	0.0383	WB	0.1631	0.1597	0.1597
10	BSC.1	0.0224	0.0493	0.0224	WFC	0.1529	0.1514	0.1514
11	C	0.0200	0.0458	0.0200	LEHMQ	0.0878	0.1416	0.0878
12	MET	0.0161	0.0426	0.0161	BSC.1	0.0652	0.1325	0.0652
13	GS	0.0141	0.0399	0.0141	MET	0.0632	0.1248	0.0632
14	LEHMQ	0.0129	0.0375	0.0375	C	0.0553	0.1178	0.0553
	$L = 0.5\mathbb{E}[X]$		$m^* = 5$		$L = 0.5\mathbb{E}[X]$		$m^* = 7$	
1	3FNMA	0.2370	0.1185	0.2237	3FNMA	0.3957	0.1978	0.2006
2	AIG	0.2237	0.1152	0.2154	AIG	0.2006	0.1491	0.1908
3	MS	0.2154	0.1127	0.1164	MS	0.1908	0.1312	0.1822
4	BAC2	0.1164	0.0991	0.1099	BAC	0.1822	0.1212	0.1416
5	JPM	0.1099	0.0902	0.0902	JPM	0.1416	0.1111	0.1229
6	3FMCC*1000	0.0695	0.0810	0.0695	3FMCC*1000	0.1229	0.1028	0.1211
7	WFC	0.0539	0.0733	0.0539	BAC2	0.1211	0.0968	0.0968
8	BAC	0.0491	0.0672	0.0491	WFC	0.0809	0.0897	0.0809
9	WB	0.0408	0.0620	0.0408	WB	0.0786	0.0841	0.0786
10	BSC.1	0.0287	0.0572	0.0287	GS	0.0453	0.0780	0.0453
11	C	0.0244	0.0531	0.0244	LEHMQ	0.0276	0.0721	0.0276

Table 3.2 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
12	MET	0.0189	0.0495	0.0189	C	0.0270	0.0673	0.0270
13	LEHMQ	0.0148	0.0462	0.0148	BSC.1	0.0269	0.0631	0.0269
14	GS	0.0140	0.0434	0.0434	MET	0.0266	0.0596	0.0266

Table 3.3: TBTF banks in 2006

This table displays a bank sector with 14 financial institutions and identifies "too big to fail" banks in year 2006 following the capital insurance approach explained in Appendix C. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows. Deductible insurance is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z = max(X - L, 0))$. Cap Insurance is the cap capital insurance contract, in which $Z = min(X, L)$. We utilize the following three deductible levels, L, defined as the percentage of the expected aggregate loss in the banking sector: $L = 0.1\mathbb{E}[X]$, $L = 0.2\mathbb{E}[X]$ and $L = 0.5\mathbb{E}[X]$. $\beta_m = \frac{Cov(X_i,Z)}{Var(Z)}$. $\tau_m = \frac{1}{2m}\sum_{i=1}^{m}\beta_i$ for $i = 1, \cdots, N$. $\overline{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\}$ for $m = 1, \cdots, N-1$ and $\overline{\tau}_N = \tau_N$. The bank i is too big to fail if and only if $\beta_i > \overline{\tau}_{m^*}$, for each $i = 1, \cdots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
	$L = 0.1\mathbb{E}[X]$		$m^* = 1$		$L = 0.1\mathbb{E}[X]$		$m^* = 4$	
1	WFC	0.3996	0.1998	0.1998	WFC	1.3649	0.6824	1.0263
2	LEHMQ	0.1471	0.1367	0.1367	MS	1.0263	0.5978	0.9328
3	MS	0.1061	0.1088	0.1061	LEHMQ	0.9328	0.5540	0.9138
4	JPM	0.0746	0.0909	0.0746	GS	0.9138	0.5297	0.8830
5	BAC2	0.0558	0.0783	0.0558	BAC	0.8830	0.5121	0.7260
6	AIG	0.0541	0.0698	0.0541	JPM	0.7260	0.4872	0.6530
7	BAC	0.0475	0.0632	0.0475	BAC2	0.6530	0.4643	0.5625
8	3FNMA	0.0319	0.0573	0.0319	3FMCC*1000	0.5625	0.4414	0.5510
9	WB	0.0209	0.0521	0.0209	AIG	0.5510	0.4229	0.4881
10	GS	0.0188	0.0478	0.0188	3FNMA	0.4881	0.4051	0.4051
11	3FMCC*1000	0.0134	0.0441	0.0134	WB	0.3837	0.3857	0.3837
12	BSC.1	0.0119	0.0409	0.0119	BSC.1	0.2240	0.3629	0.2240
13	C	0.0091	0.0381	0.0091	MET	0.1472	0.3406	0.1472
14	MET	-0.0017	0.0353	0.0353	C	0.0817	0.3192	0.0817

Table 3.3 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
	$L = 0.2\mathbb{E}[X]$		$m^* = 1$		$L = 0.2\mathbb{E}[X]$		$m^* = 9$	
1	WFC	0.4157	0.2078	<u>0.2078</u>	WFC	0.7695	0.3848	0.5890
2	LEHMQ	0.1515	0.1418	0.1418	MS	0.5890	0.3396	0.5290
3	MS	0.1080	0.1125	0.1080	LEHMQ	0.5290	0.3146	0.5032
4	JPM	0.0760	0.0939	0.0760	BAC	0.5032	0.2988	0.4219
5	BAC2	0.0564	0.0807	0.0564	GS	0.4219	0.2813	0.4144
6	AIG	0.0550	0.0719	0.0550	JPM	0.4144	0.2689	0.3714
7	BAC	0.0468	0.0650	0.0468	BAC2	0.3714	0.2570	0.3094
8	3FNMA	0.0321	0.0588	0.0321	AIG	0.3094	0.2442	0.2442
9	WB	0.0207	0.0535	0.0207	3FNMA	0.2420	0.2305	<u>0.2305</u>
10	GS	0.0174	0.0490	0.0174	WB	0.2190	0.2184	0.2184
11	3FMCC*1000	0.0131	0.0451	0.0131	3FMCC*1000	0.1948	0.2074	0.1948
12	BSC.1	0.0120	0.0419	0.0120	BSC.1	0.1045	0.1945	0.1045
13	C	0.0093	0.0390	0.0093	MET	0.0868	0.1829	0.0868
14	MET	-0.0024	0.0361	0.0361	C	0.0544	0.1718	0.0544
	$L = 0.5\mathbb{E}[X]$		$m^* = 1$		$L = 0.5\mathbb{E}[X]$		$m^* = 8$	
1	WFC	1.7067	0.8534	<u>0.8534</u>	WFC	0.4484	0.2242	0.3161
2	LEHMQ	0.5889	0.5739	0.5739	MS	0.3161	0.1911	0.2815
3	MS	0.3864	0.4470	0.3864	LEHMQ	0.2815	0.1743	0.2273
4	JPM	0.2702	0.3690	0.2702	JPM	0.2273	0.1592	0.2116
5	AIG	0.1949	0.3147	0.1949	BAC	0.2116	0.1485	0.2012
6	BAC2	0.1914	0.2782	0.1914	BAC2	0.2012	0.1405	0.1672
7	BAC	0.1501	0.2492	0.1501	AIG	0.1672	0.1324	0.1598
8	3FNMA	0.1126	0.2251	0.1126	GS	0.1598	0.1258	<u>0.1258</u>
9	WB	0.0586	0.2033	0.0586	WB	0.1148	0.1182	0.1148
10	3FMCC*1000	0.0387	0.1849	0.0387	3FNMA	0.1078	0.1118	0.1078
11	BSC.1	0.0373	0.1698	0.0373	3FMCC*1000	0.0744	0.1050	0.0744

Table 3.3 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
12	GS	0.0355	0.1571	0.0355	BSC.1	0.0547	0.0985	0.0547
13	C	0.0321	0.1463	0.0321	C	0.0303	0.0921	0.0303
14	MET	-0.0141	0.1353	0.1353	MET	0.0190	0.0862	0.0190

Table 3.4: TBTF banks in 2007

This table displays a bank sector with 14 financial institutions and identifies "too big to fail" banks in year 2007 following the capital insurance approach explained in Appendix C. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows. Deductible insurance is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z = max(X - L, 0))$. Cap Insurance is the cap capital insurance contract, in which $Z = min(X, L)$. We utilize the following three deductible levels, L, defined as the percentage of the expected aggregate loss in the banking sector: $L = 0.1\mathbb{E}[X]$, $L = 0.2\mathbb{E}[X]$ and $L = 0.5\mathbb{E}[X]$. $\beta_m = \frac{Cov(X_i,Z)}{Var(Z)}$. $\tau_m = \frac{1}{2m}\sum_{i=1}^{m}\beta_i$ for $i = 1, \cdots, N$. $\overline{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\}$ for $m = 1, \cdots, N-1$ and $\overline{\tau}_N = \tau_N$. The bank i is too big to fail if and only if $\beta_i > \overline{\tau}_{m^*}$, for each $i = 1, \cdots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
	$L = 0.1\mathbb{E}[X]$		$m^* = 1$		$L = 0.1\mathbb{E}[X]$		$m^* = 4$	
1	MS	0.1878	0.0939	0.1323	MS	2.0123	1.0061	1.4468
2	BAC2	0.1323	0.0800	0.1084	GS	1.4468	0.8648	1.1686
3	BAC	0.1084	0.0714	0.0925	3FNMA	1.1686	0.7713	1.1598
4	3FMCC*1000	0.0925	0.0651	0.0834	3FMCC*1000	1.1598	0.7234	1.1376
5	3FNMA	0.0834	0.0604	0.0803	BAC2	1.1376	0.6925	1.1294
6	JPM	0.0803	0.0571	0.0589	BAC	1.1294	0.6712	0.9387
7	AIG	0.0589	0.0531	0.0531	JPM	0.9387	0.6424	0.8657
8	LEHMQ	0.0520	0.0497	0.0505	LEHMQ	0.8657	0.6162	0.7710
9	WB	0.0505	0.0470	0.0504	AIG	0.7710	0.5906	0.5906
10	WFC	0.0504	0.0448	<u>0.0448</u>	WB	0.5895	0.5610	<u>0.5610</u>
11	GS	0.0295	0.0421	0.0295	WFC	0.4807	0.5318	0.4807
12	BSC.1	0.0264	0.0397	0.0264	BSC.1	0.4714	0.5071	0.4714
13	C	0.0227	0.0375	0.0227	MET	0.3233	0.4806	0.3233
14	MET	0.0101	0.0352	0.0352	C	0.2737	0.4560	0.2737

Table 3.4 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
	$L = 0.2\mathbb{E}[X]$		$m^* = 10$		$L = 0.2\mathbb{E}[X]$		$m^* = 9$	
1	MS	0.1898	0.0949	0.1339	MS	1.2654	0.6327	0.8666
2	BAC2	0.1339	0.0809	0.1096	GS	0.8666	0.5330	0.7153
3	BAC	0.1096	0.0722	0.0934	BAC2	0.7153	0.4745	0.7147
4	3FMCC*1000	0.0934	0.0658	0.0841	3FNMA	0.7147	0.4452	0.7102
5	3FNMA	0.0841	0.0611	0.0811	BAC	0.7102	0.4272	0.6703
6	JPM	0.0811	0.0577	0.0594	3FMCC*1000	0.6703	0.4119	0.5903
7	AIG	0.0594	0.0537	0.0537	JPM	0.5903	0.3952	0.4984
8	LEHMQ	0.0524	0.0502	0.0510	LEHMQ	0.4984	0.3770	0.4832
9	WB	0.0510	0.0475	0.0510	AIG	0.4832	0.3619	<u>0.3619</u>
10	WFC	0.0510	0.0453	<u>0.0453</u>	WB	0.3469	0.3431	0.3431
11	GS	0.0289	0.0425	0.0289	WFC	0.3053	0.3258	0.3053
12	BSC.1	0.0266	0.0400	0.0266	BSC.1	0.2688	0.3098	0.2688
13	C	0.0229	0.0378	0.0229	MET	0.1802	0.2929	0.1802
14	MET	0.0101	0.0355	0.0355	C	0.1604	0.2777	0.1604
	$L = 0.5\mathbb{E}[X]$		$m^* = 10$		$L = 0.5\mathbb{E}[X]$		$m^* = 10$	
1	MS	0.1995	0.0998	0.1419	MS	0.6470	0.3235	0.3819
2	BAC2	0.1419	0.0854	0.1161	BAC2	0.3819	0.2572	0.3715
3	BAC	0.1161	0.0763	0.0978	GS	0.3715	0.2334	0.3447
4	3FMCC*1000	0.0978	0.0694	0.0878	3FMCC*1000	0.3447	0.2181	0.3343
5	3FNMA	0.0878	0.0643	0.0846	3FNMA	0.3343	0.2079	0.3312
6	JPM	0.0846	0.0606	0.0617	BAC	0.3312	0.2009	0.3108
7	AIG	0.0617	0.0564	0.0564	JPM	0.3108	0.1944	0.2462
8	LEHMQ	0.0544	0.0527	0.0540	AIG	0.2462	0.1855	0.2259
9	WB	0.0540	0.0499	0.0537	LEHMQ	0.2259	0.1774	<u>0.1774</u>
10	WFC	0.0537	0.0476	<u>0.0476</u>	WFC	0.1645	0.1679	0.1645
11	BSC.1	0.0275	0.0445	0.0275	WB	0.1605	0.1599	0.1599

Table 3.4 (continued)

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
12	GS	0.0262	0.0419	0.0262	BSC.1	0.1198	0.1516	0.1198
13	C	0.0240	0.0396	0.0240	C	0.0816	0.1431	0.0816
14	MET	0.0101	0.0371	0.0371	MET	0.0697	0.1353	0.0697

Table 3.5: TBTF banks in 2008

This table displays a bank sector with 14 financial institutions and identifies "too big to fail" banks in year 2008 following the capital insurance approach explained in Appendix C. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows. Deductible insurance is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z = max(X - L, 0))$. Cap Insurance is the cap capital insurance contract, in which $Z = min(X, L)$. We utilize the following three deductible levels, L, defined as the percentage of the expected aggregate loss in the banking sector: $L = 0.1\mathbb{E}[X]$, $L = 0.2\mathbb{E}[X]$ and $L = 0.5\mathbb{E}[X]$. $\beta_m = \frac{Cov(X_i,Z)}{Var(Z)}$. $\tau_m = \frac{1}{2m}\sum_{i=1}^{m}\beta_i$ for $i = 1, \cdots, N$. $\overline{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\}$ for $m = 1, \cdots, N-1$ and $\overline{\tau}_N = \tau_N$. The bank i is too big to fail if and only if $\beta_i > \overline{\tau}_{m^*}$, for each $i = 1, \cdots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
	$L = 0.1\mathbb{E}[X]$		$m^* = 1$		$L = 0.1\mathbb{E}[X]$		$m^* = 4$	
1	3FNMA	0.2606	0.1303	0.1323	WB	1.4994	0.7497	1.3649
2	BAC	0.1323	0.0982	0.1192	AIG	1.3649	0.7161	0.7161
3	BAC2	0.1192	0.0853	0.1185	MS	0.6151	0.5799	0.6151
4	JPM	0.1185	0.0788	0.0998	BAC	0.6151	0.5118	<u>0.5118</u>
5	MS	0.0998	0.0730	0.0799	BSC.1	0.4421	0.4537	0.4421
6	WFC	0.0799	0.0675	<u>0.0675</u>	MET	0.1922	0.3941	0.1922
7	AIG	0.0624	0.0623	0.0623	JPM	-0.1538	0.3268	-0.1538
8	WB	0.0471	0.0575	0.0471	C	-0.3556	0.2637	-0.3556
9	BSC.1	0.0178	0.0521	0.0178	WFC	-1.1150	0.1725	-1.1150
10	MET	0.0134	0.0475	0.0134	GS	-1.3456	0.0879	-1.3456
11	C	0.0132	0.0438	0.0132	3FNMA	-1.5379	0.0100	-1.5379
12	LEHMQ	0.0105	0.0406	0.0105	LEHMQ	-1.6532	-0.0597	-1.6532
13	GS	0.0036	0.0376	0.0036	3FMCC*1000	-2.1146	-0.1364	-2.1146
14	3FMCC*1000	0.0021	0.0350	0.0350	BAC2	-2.3453	-0.2104	-2.3453

Table 3.5 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
	$L = 0.2\mathbb{E}[X]$		$m^* = 6$		$L = 0.2\mathbb{E}[X]$		$m^* = 8$	
1	3FNMA	0.2607	0.1303	0.1323	3FNMA	11.1738	5.5869	10.5320
2	BAC	0.1323	0.0982	0.1192	JPM	10.5320	5.4265	10.4936
3	BAC2	0.1192	0.0854	0.1185	BAC	10.4936	5.3666	9.2761
4	JPM	0.1185	0.0788	0.0998	MS	9.2761	5.1844	8.2246
5	MS	0.0998	0.0731	0.0799	BAC2	8.2246	4.9700	6.4195
6	WFC	0.0799	0.0675	<u>0.0675</u>	WFC	6.4195	4.6766	6.2804
7	AIG	0.0625	0.0624	0.0624	AIG	6.2804	4.4571	5.1676
8	WB	0.0471	0.0575	0.0471	3FMCC*1000	5.1676	4.2230	<u>4.2230</u>
9	BSC.1	0.0178	0.0521	0.0178	GS	3.3960	3.9424	3.3960
10	MET	0.0134	0.0476	0.0134	MET	2.9175	3.6941	2.9175
11	C	0.0133	0.0438	0.0133	LEHMQ	2.2898	3.4623	2.2898
12	LEHMQ	0.0105	0.0406	0.0105	WB	1.9683	3.2558	1.9683
13	GS	0.0036	0.0376	0.0036	C	1.3849	3.0586	1.3849
14	3FMCC*1000	0.0021	0.0350	0.0350	BSC.1	1.0872	2.8790	1.0872
	$L = 0.5\mathbb{E}[X]$		$m^* = 6$		$L = 0.5\mathbb{E}[X]$		$m^* = 7$	
1	3FNMA	0.2650	0.1325	0.1338	JPM	1.2355	0.6177	1.2006
2	BAC	0.1338	0.0997	0.1208	3FNMA	1.2006	0.6090	1.1744
3	BAC2	0.1208	0.0866	0.1196	BAC	1.1744	0.6018	1.0702
4	JPM	0.1196	0.0799	0.1007	MS	1.0702	0.5851	0.9036
5	MS	0.1007	0.0740	0.0808	BAC2	0.9036	0.5584	0.7364
6	WFC	0.0808	0.0684	<u>0.0684</u>	WFC	0.7364	0.5267	0.7304
7	AIG	0.0629	0.0631	0.0629	AIG	0.7304	0.5036	<u>0.5036</u>
8	WB	0.0479	0.0582	0.0479	3FMCC*1000	0.4562	0.4692	0.4562
9	BSC.1	0.0181	0.0527	0.0181	LEHMQ	0.4057	0.4396	0.4057
10	C	0.0134	0.0481	0.0134	GS	0.3909	0.4152	0.3909
11	MET	0.0134	0.0444	0.0134	MET	0.2378	0.3883	0.2378

Table 3.5 (continued)

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
12	LEHMQ	0.0102	0.0411	0.0102	WB	0.2309	0.3655	0.2309
13	GS	0.0032	0.0381	0.0032	C	0.1368	0.3427	0.1368
14	3FMCC*1000	0.0015	0.0354	0.0354	BSC.1	0.1275	0.3227	0.1275

Table 3.6: TBTF banks in 2009

This table displays a bank sector with 10 financial institutions and identifies "too big to fail" banks in year 2009 following the capital insurance approach explained in Appendix C. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows. Deductible Insurance is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z = max(X - L, 0)$). Cap insurance is the cap capital insurance contract, in which $Z = min(X, L)$. We utilize the following three deductible levels, L, defined as the percentage of the expected aggregate loss in the banking sector: $L = 0.1\mathbb{E}[X]$, $L = 0.2\mathbb{E}[X]$ and $L = 0.5\mathbb{E}[X]$. $\beta_m = \frac{Cov(X_i,Z)}{Var(Z)}$. $\tau_m = \frac{1}{2m}\sum_{i=1}^{m}\beta_i$ for $i = 1, \cdots, N$. $\overline{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\}$ for $m = 1, \cdots, N-1$ and $\overline{\tau}_N = \tau_N$. The bank i is too big to fail if and only if $\beta_i > \overline{\tau}_{m^*}$, for each $i = 1, \cdots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
	$L = 0.1\mathbb{E}[X]$		$m^* = 1$		$L = 0.1\mathbb{E}[X]$		$m^* = 4$	
1	AIG	0.3287	0.1643	0.2133	AIG	1.2916	0.6458	1.0060
2	WFC	0.2133	0.1355	0.1987	WFC	1.0060	0.5744	0.9089
3	JPM	0.1987	0.1235	0.1468	JPM	0.9089	0.5344	0.7893
4	BAC	0.1468	0.1109	<u>0.1109</u>	BAC	0.7893	0.4995	<u>0.4995</u>
5	3FMCC*1000	0.0534	0.0941	0.0534	3FMCC*1000	0.3723	0.4368	0.3723
6	MS	0.0271	0.0807	0.0271	GS	0.2945	0.3886	0.2945
7	GS	0.0083	0.0697	0.0083	MS	0.2046	0.3477	0.2046
8	MET	0.0073	0.0615	0.0073	3FNMA	0.1619	0.3143	0.1619
9	C	0.0060	0.0550	0.0060	MET	0.0938	0.2846	0.0938
10	3FNMA	0.0015	0.0495	0.0495	C	0.0324	0.2578	0.2578
	$L = 0.2\mathbb{E}[X]$		$m^* = 4$		$L = 0.2\mathbb{E}[X]$		$m^* = 4$	
1	AIG	0.3455	0.1728	0.2225	AIG	1.2916	0.6458	1.0060
2	WFC	0.2225	0.1420	0.2076	WFC	1.0060	0.5744	0.9089
3	JPM	0.2076	0.1293	0.1522	JPM	0.9089	0.5344	0.7893
4	BAC	0.1522	0.1160	<u>0.1160</u>	BAC	0.7893	0.4995	<u>0.4995</u>

Table 3.6 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
5	3FMCC*1000	0.0546	0.0983	0.0546	3FMCC*1000	0.3723	0.4368	0.3723
6	MS	0.0276	0.0842	0.0276	GS	0.2945	0.3886	0.2945
7	MET	0.0071	0.0727	0.0071	MS	0.2046	0.3477	0.2046
8	GS	0.0064	0.0640	0.0064	3FNMA	0.1619	0.3143	0.1619
9	C	0.0061	0.0572	0.0061	MET	0.0938	0.2846	0.0938
10	3FNMA	0.0004	0.0515	0.0515	C	0.0324	0.2578	0.2578
	$L = 0.5\mathbb{E}[X]$		$m^* = 4$		$L = 0.5\mathbb{E}[X]$		$m^* = 4$	
1	AIG	0.4261	0.2131	0.2756	AIG	0.6497	0.3248	0.4289
2	WFC	0.2756	0.1754	0.2568	WFC	0.4289	0.2696	0.3989
3	JPM	0.2568	0.1598	0.1809	JPM	0.3989	0.2462	0.3400
4	BAC	0.1809	0.1424	0.1424	BAC	0.3400	0.2272	0.2272
5	3FMCC*1000	0.0678	0.1207	0.0678	3FMCC*1000	0.1156	0.1933	0.1156
6	MS	0.0284	0.1030	0.0284	MS	0.0889	0.1685	0.0889
7	MET	0.0096	0.0889	0.0096	3FNMA	0.0882	0.1507	0.0882
8	C	0.0063	0.0782	0.0063	GS	0.0880	0.1374	0.0880
9	GS	-0.0025	0.0694	-0.0025	C	0.0188	0.1232	0.0188
10	3FNMA	-0.0143	0.0617	0.0617	MET	0.0156	0.1116	0.1116

Table 3.7: TBTF banks in 2010

This table displays a bank sector with 10 financial institutions and identifies "too big to fail" banks in year 2010 following the capital insurance approach explained in Appendix C. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows. Deductible Insurance is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z = max(X - L, 0)$). Cap insurance is the cap capital insurance contract, in which $Z = min(X, L)$. We utilize the following three deductible levels, L, defined as the percentage of the expected aggregate loss in the banking sector: $L = 0.1\mathbb{E}[X]$, $L = 0.2\mathbb{E}[X]$ and $L = 0.5\mathbb{E}[X]$. $\beta_m = \frac{Cov(X_i,Z)}{Var(Z)}$. $\tau_m = \frac{1}{2m}\sum_{i=1}^{m}\beta_i$ for $i = 1, \cdots, N$. $\overline{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\}$ for $m = 1, \cdots, N-1$ and $\overline{\tau}_N = \tau_N$. The bank i is too big to fail if and only if $\beta_i > \overline{\tau}_{m^*}$, for each $i = 1, \cdots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
	$L = 0.1\mathbb{E}[X]$		$m^* = 1$		$L = 0.1\mathbb{E}[X]$		$m^* = 4$	
1	GS	0.4945	0.2472	0.2472	GS	5.9744	2.9872	2.9872
2	BAC	0.1343	0.1572	0.1343	JPM	0.6010	1.6438	0.6010
3	WFC	0.1240	0.1255	0.1240	3FMCC*1000	0.5413	1.1861	0.5413
4	JPM	0.1103	0.1079	0.1079	WFC	0.5280	0.9556	0.5280
5	3FMCC*1000	0.1001	0.0963	0.0963	BAC	0.4642	0.8109	0.4642
6	MS	0.0113	0.0812	0.0113	3FNMA	0.1811	0.6908	0.1811
7	MET	0.0072	0.0701	0.0072	MS	0.1583	0.6034	0.1583
8	AIG	0.0059	0.0617	0.0059	MET	0.1106	0.5349	0.1106
9	C	0.0021	0.0550	0.0021	C	0.1072	0.4814	0.1072
10	3FNMA	-0.0004	0.0495	0.0495	AIG	0.0794	0.4373	0.4373
	$L = 0.2\mathbb{E}[X]$		$m^* = 1$		$L = 0.2\mathbb{E}[X]$		$m^* = 1$	
1	GS	0.4981	0.2490	0.2490	GS	5.9744	2.9872	2.9872
2	BAC	0.1372	0.1588	0.1372	JPM	0.6010	1.6438	0.6010
3	WFC	0.1266	0.1270	0.1266	3FMCC*1000	0.5413	1.1861	0.5413
4	JPM	0.1124	0.1093	0.1093	WFC	0.5280	0.9556	0.5280

Table 3.7 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
5	3FMCC*1000	0.1019	0.0976	0.0976	BAC	0.4642	0.8109	0.4642
6	MS	0.0113	0.0823	0.0113	3FNMA	0.1811	0.6908	0.1811
7	MET	0.0073	0.0711	0.0073	MS	0.1583	0.6034	0.1583
8	AIG	0.0059	0.0625	0.0059	MET	0.1106	0.5349	0.1106
9	C	0.0019	0.0557	0.0019	C	0.1072	0.4814	0.1072
10	3FNMA	-0.0006	0.0501	0.0501	AIG	0.0794	0.4373	0.4373
	$L = 0.5\mathbb{E}[X]$		$m^* = 1$		$L = 0.5\mathbb{E}[X]$		$m^* = 1$	
1	GS	0.5075	0.2538	0.2538	GS	3.0825	1.5412	1.5412
2	BAC	0.1483	0.1640	0.1483	JPM	0.2994	0.8455	0.2994
3	WFC	0.1363	0.1320	0.1320	WFC	0.2637	0.6076	0.2637
4	JPM	0.1201	0.1140	0.1140	BAC	0.2430	0.4861	0.2430
5	3FMCC*1000	0.1100	0.1022	0.1022	3FMCC*1000	0.2217	0.4110	0.2217
6	MS	0.0115	0.0861	0.0115	MS	0.0781	0.3490	0.0781
7	MET	0.0074	0.0744	0.0074	C	0.0480	0.3026	0.0480
8	AIG	0.0060	0.0654	0.0060	MET	0.0465	0.2677	0.0465
9	C	0.0015	0.0583	0.0015	AIG	0.0379	0.2400	0.0379
10	3FNMA	-0.0008	0.0524	0.0524	3FNMA	0.0342	0.2178	0.2178

Table 3.8: TBTF banks in 2011

This table displays a bank sector with 10 financial institutions and identifies "too big to fail" banks in year 2011 following the capital insurance approach explained in Appendix C. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows. Deductible Insurance is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z = max(X - L, 0)$). Cap insurance is the cap capital insurance contract, in which $Z = min(X, L)$. We utilize the following three deductible levels, L, defined as the percentage of the expected aggregate loss in the banking sector: $L = 0.1\mathbb{E}[X]$, $L = 0.2\mathbb{E}[X]$ and $L = 0.5\mathbb{E}[X]$. $\beta_m = \frac{Cov(X_i, Z)}{Var(Z)}$. $\tau_m = \frac{1}{2m}\sum_{i=1}^{m}\beta_i$ for $i = 1, \cdots, N$. $\overline{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\}$ for $m = 1, \cdots, N-1$ and $\overline{\tau}_N = \tau_N$. The bank i is too big to fail if and only if $\beta_i > \overline{\tau}_{m^*}$, for each $i = 1, \cdots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
	$L = 0.1\mathbb{E}[X]$		$m^* = 1$		$L = 0.1\mathbb{E}[X]$		$m^* = 4$	
1	GS	0.8334	0.4167	<u>0.4167</u>	GS	7.0885	3.5443	<u>3.5443</u>
2	C	0.0492	0.2206	0.0492	C	0.6449	1.9334	0.6449
3	JPM	0.0439	0.1544	0.0439	JPM	0.3427	1.3460	0.3427
4	MS	0.0272	0.1192	0.0272	3FMCC*1000	0.3012	1.0472	0.3012
5	MET	0.0218	0.0976	0.0218	MS	0.2196	0.8597	0.2196
6	WFC	0.0147	0.0825	0.0147	MET	0.1466	0.7286	0.1466
7	AIG	0.0101	0.0715	0.0101	WFC	0.1104	0.6324	0.1104
8	BAC	0.0018	0.0626	0.0018	AIG	0.0949	0.5593	0.0949
9	3FNMA	-0.0003	0.0557	-0.0003	BAC	0.0085	0.4976	0.0085
10	3FMCC*1000	-0.0103	0.0496	0.0496	3FNMA	-0.0037	0.4477	0.4477
	$L = 0.2\mathbb{E}[X]$		$m^* = 1$		$L = 0.2\mathbb{E}[X]$		$m^* = 1$	
1	GS	0.8492	0.4246	<u>0.4246</u>	GS	7.0885	3.5443	<u>3.5443</u>
2	C	0.0497	0.2247	0.0497	C	0.6449	1.9334	0.6449
3	JPM	0.0449	0.1573	0.0449	JPM	0.3427	1.3460	0.3427
4	MS	0.0278	0.1214	0.0278	3FMCC*1000	0.3012	1.0472	0.3012

Table 3.8 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
5	MET	0.0223	0.0994	0.0223	MS	0.2196	0.8597	0.2196
6	WFC	0.0150	0.0841	0.0150	MET	0.1466	0.7286	0.1466
7	AIG	0.0103	0.0728	0.0103	WFC	0.1104	0.6324	0.1104
8	BAC	0.0018	0.0638	0.0018	AIG	0.0949	0.5593	0.0949
9	3FNMA	-0.0003	0.0567	-0.0003	BAC	0.0085	0.4976	0.0085
10	3FMCC*1000	-0.0113	0.0505	0.0505	3FNMA	-0.0037	0.4477	0.4477
	$L = 0.5\mathbb{E}[X]$		$m^* = 1$		$L = 0.5\mathbb{E}[X]$		$m^* = 1$	
1	GS	0.9053	0.4527	$\underline{0.4527}$	GS	3.2685	1.6343	$\underline{1.6343}$
2	C	0.0509	0.2391	0.0509	C	0.2890	0.8894	0.2890
3	JPM	0.0481	0.1674	0.0481	JPM	0.1575	0.6192	0.1575
4	MS	0.0298	0.1293	0.0298	MS	0.1003	0.4769	0.1003
5	MET	0.0242	0.1058	0.0242	MET	0.0659	0.3881	0.0659
6	WFC	0.0162	0.0895	0.0162	WFC	0.0492	0.3275	0.0492
7	AIG	0.0110	0.0775	0.0110	3FMCC*1000	0.0422	0.2838	0.0422
8	BAC	0.0020	0.0680	0.0020	AIG	0.0411	0.2509	0.0411
9	3FNMA	-0.0003	0.0604	-0.0003	BAC	0.0042	0.2232	0.0042
10	3FMCC*1000	-0.0131	0.0537	0.0537	3FNMA	-0.0009	0.2008	0.2008

Table 3.9: TBTF banks in 2012

This table displays a bank sector with 10 financial institutions and identifies "too big to fail" banks in year 2012 following the capital insurance approach explained in Appendix C. The analysis is performed for two types of capital insurance contracts and for three different deductible levels. Two types of contracts are as follows. Deductible Insurance is the deductible capital insurance contract, in which the indemnity of the contract is given by $Z = max(X - L, 0))$. Cap insurance is the cap capital insurance contract, in which $Z = min(X, L)$. We utilize the following three deductible levels, L, defined as the percentage of the expected aggregate loss in the banking sector: $L = 0.1\mathbb{E}[X]$, $L = 0.2\mathbb{E}[X]$ and $L = 0.5\mathbb{E}[X]$. $\beta_m = \frac{Cov(X_i,Z)}{Var(Z)}$. $\tau_m = \frac{1}{2m}\sum_{i=1}^{m}\beta_i$ for $i = 1, \cdots, N$. $\overline{\tau}_m = \min\{\beta_m, \max(\beta_{m+1}, \tau_m)\}$ for $m = 1, \cdots, N-1$ and $\overline{\tau}_N = \tau_N$. The bank i is too big to fail if and only if $\beta_i > \overline{\tau}_{m^*}$, for each $i = 1, \cdots, N$. Thus, the banks marked in red (blue) are too big to fail if the indemnity of the contract follows deductible insurance (cap insurance).

i	Bank Name	β_m	τ_m	$\overline{\tau}_m$	Bank Name	β_m	τ_m	$\overline{\tau}_m$
	Deductible				**Cap**			
	$L = 0.1\mathbb{E}[X]$		$m^* = 1$		$L = 0.1\mathbb{E}[X]$		$m^* = 4$	
1	GS	0.8872	0.4436	0.4436	GS	4.4281	2.2141	2.2141
2	JPM	0.0423	0.2324	0.0423	JPM	0.3238	1.1880	0.3238
3	MS	0.0206	0.1584	0.0206	MS	0.1848	0.8228	0.1848
4	MET	0.0177	0.1210	0.0177	MET	0.1500	0.6358	0.1500
5	C	0.0145	0.0982	0.0145	3FNMA	0.1366	0.5223	0.1366
6	3FNMA	0.0143	0.0831	0.0143	3FMCC*1000	0.1331	0.4464	0.1331
7	WFC	0.0125	0.0721	0.0125	C	0.1086	0.3904	0.1086
8	AIG	0.0084	0.0636	0.0084	WFC	0.1016	0.3479	0.1016
9	BAC	0.0000	0.0565	0.0000	AIG	0.0593	0.3125	0.0593
10	3FMCC*1000	-0.0027	0.0507	0.0507	BAC	0.0008	0.2813	0.2813
	$L = 0.2\mathbb{E}[X]$		$m^* = 1$		$L = 0.2\mathbb{E}[X]$		$m^* = 1$	
1	GS	0.9311	0.4655	0.4655	GS	4.4281	2.2141	2.2141
2	JPM	0.0434	0.2436	0.0434	JPM	0.3238	1.1880	0.3238
3	MS	0.0209	0.1659	0.0209	MS	0.1848	0.8228	0.1848
4	MET	0.0180	0.1267	0.0180	MET	0.1500	0.6358	0.1500

Table 3.9 (continued)

i	Bank Name **Deductible**	β_m	τ_m	$\overline{\tau}_m$	Bank Name **Cap**	β_m	τ_m	$\overline{\tau}_m$
5	C	0.0148	0.1028	0.0148	3FNMA	0.1366	0.5223	0.1366
6	3FNMA	0.0147	0.0869	0.0147	3FMCC*1000	0.1331	0.4464	0.1331
7	WFC	0.0128	0.0754	0.0128	C	0.1086	0.3904	0.1086
8	AIG	0.0087	0.0665	0.0087	WFC	0.1016	0.3479	0.1016
9	BAC	0.0000	0.0591	0.0000	AIG	0.0593	0.3125	0.0593
10	3FMCC*1000	-0.0043	0.0530	0.0530	BAC	0.0008	0.2813	0.2813
	$L = 0.5\mathbb{E}[X]$		$m^* = 1$		$L = 0.5\mathbb{E}[X]$		$m^* = 1$	
1	GS	0.7848	0.3924	0.3924	GS	12.9880	6.4940	6.4940
2	JPM	0.0347	0.2049	0.0347	JPM	0.8337	3.4554	0.8337
3	MS	0.0160	0.1393	0.0160	MS	0.4698	2.3819	0.4698
4	MET	0.0145	0.1063	0.0145	3FNMA	0.3652	1.8321	0.3652
5	C	0.0119	0.0862	0.0119	MET	0.3558	1.5013	0.3558
6	3FNMA	0.0108	0.0727	0.0108	C	0.2821	1.2746	0.2821
7	WFC	0.0103	0.0631	0.0103	WFC	0.2446	1.1099	0.2446
8	AIG	0.0069	0.0556	0.0069	3FMCC*1000	0.2118	0.9844	0.2118
9	BAC	0.0000	0.0494	0.0000	AIG	0.1659	0.8843	0.1659
10	3FMCC*1000	-0.0056	0.0442	0.0442	BAC	0.0011	0.7959	0.7959

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APPENDIX A: CHAPTER 1. PROOFS.

The proofs rely on the following simple lemma.

Lemma 1 *Given positive numbers b_i, c_i, κ_i for each $i = 1, \cdots, n$,*

1. *If the vector $\kappa = (\kappa_i)$ is co-monotonic to the vector $\frac{b}{c} = (\frac{b_i}{c_i})$, then*

$$\frac{\sum_{i=1}^n b_i\kappa_i}{\sum_{i=1}^n b_i} > \frac{\sum_{i=1}^n c_i\kappa_i}{\sum_{i=1}^n c_i}.$$

2. *If the vector $\kappa = (\kappa_i)$ is counter-monotonic to the vector $\frac{b}{c} = (\frac{b_i}{c_i})$, then*

$$\frac{\sum_{i=1}^n b_i\kappa_i}{\sum_{i=1}^n b_i} < \frac{\sum_{i=1}^n c_i\kappa_i}{\sum_{i=1}^n c_i}.$$

Proof: $\sum b_i\kappa_i \sum c_i - \sum b_i \sum c_i\kappa_i = \sum_{i,j} b_i\kappa_i c_j - \sum_{i,j} b_j c_i \kappa_i = \sum_{i,j}(b_i c_j - b_j c_i)\kappa_i = \sum_{i,j,i<j}(b_i c_j - b_j c_i)(\kappa_i - \kappa_j) = \sum_{i,j,i<j} c_i c_j \left(\frac{b_i}{c_i} - \frac{b_j}{c_j}\right)(\kappa_i - \kappa_j)$. □

Given a vector $a = (a_1, \cdots, a_n)$, we use $VAR(a) = \sum a_i^2 - (\sum a_i)^2$ to represent the variability of the vector a. A small $VAR(a)$ means that those components in a are close to each other. Similarly, we write $\mathbb{E}[a] = \sum a_i$. It is easy to see that $VAR(a) = \frac{1}{2}\sum(a_i - a_j)^2$.

Lemma 2 *Given two sequences of positive numbers $a_i, b_i, i = 1, 2, \cdots, n$,*

- *If those numbers $a_1, \cdots, a_n$ are close enough in the sense that $VAR(a) \le \mathbb{E}[a]^2 VAR(b)/\mathbb{E}[b]^2$, then $\frac{\sum a_i^2}{\sum b_i^2} \le \frac{(\sum a_i)^2}{(\sum b_i)^2}$.*
- *If those numbers $b_1, \cdots, b_n$ are close enough in the sense that $VAR(b) \le \mathbb{E}[b]^2 VAR(a)/\mathbb{E}[a]^2$, then $\frac{\sum a_i^2}{\sum b_i^2} \ge \frac{(\sum a_i)^2}{(\sum b_i)^2}$.*

Proof: By straightforward calculation, we obtain

$$\begin{aligned} \sum a_i^2 (\sum b_i)^2 - (\sum a_i)^2 \sum b_i^2 &= \frac{1}{2}\left\{\sum_{i,j,k}(a_i - a_j)^2 b_k^2 - \sum_{i,j,k} a_i^2 (b_j - b_k)^2\right\} \\ &= \sum b_i^2 VAR(a) - \sum a_i^2 VAR(b). \end{aligned} \qquad \text{(A-1)}$$

When the numbers a_i are close enough, the first term in (A-1) is dominated by the second term. This is the first case. It is the classical Cauchy-Schwartz inequality when $a_1 = \cdots = a_n$. In the second case, the second term is close to zero. $\square$

Proof of Proposition 1.4. Under the uncorrelated assumption, $\mathbb{E}[W^{*,a}] = \frac{\sum Var(X_i)}{4\sum \gamma_i}$. As $\mathbb{E}[X_i] \geq 0$ for each i, we have $\mathbb{E}[W^{*,c}] \geq \frac{\sum \mathbb{E}[X_i]^2}{4\sum_i \gamma_i \mathbb{E}[X_i]^2/Var(X_i)}$. For each $i \neq j$, if $\frac{Var(X_i)}{\gamma_i} < \frac{Var(X_j)}{\gamma_j}$, then by the co-monotonic assumption, $\frac{\mathbb{E}[X_i]}{\sqrt{Var(X_i)}} \leq \frac{\mathbb{E}[X_j]}{\sqrt{Var(X_j)}}$. So, $\frac{\mathbb{E}[X_i]^2}{Var(X_i)} \leq \frac{\mathbb{E}[X_j]^2}{Var(X_j)}$. Therefore,

$$\frac{Var(X_i)}{\mathbb{E}[X_i]^2} \geq \frac{Var(X_j)}{\mathbb{E}[X_j]^2}. \tag{A-2}$$

It means that vectors $\left(\frac{Var(X_i)}{\gamma_i}\right)$ and $\left(\frac{Var(X_i)}{\mathbb{E}[X_i]^2}\right)$ are counter-monotonic. Then by Lemma 1, we obtain (using $b_i = \gamma_i, c_i = \gamma_i \mathbb{E}[X_i]^2/Var(X_i)$ and $\kappa_i = Var(X_i)/\gamma_i$):

$$\frac{\sum \mathbb{E}[X_i]^2}{\sum_i \gamma_i \mathbb{E}[X_i]^2/Var(X_i)} > \frac{\sum Var(X_i)}{\sum \gamma_i}. \tag{A-3}$$

We have proven the first part. As for the second part, assume that the risk-adjusted variance is counter-monotonic to the Sharpe ratio vector. Then by the same idea, we have that

$$\frac{\sum \mathbb{E}[X_i]^2}{\sum_i \gamma_i \mathbb{E}[X_i]^2/Var(X_i)} < \frac{\sum Var(X_i)}{\sum \gamma_i} = \mathbb{E}[W^{*,a}]. \tag{A-4}$$

Therefore, when $\mathbb{E}[X]^2$ is close to $\sum \mathbb{E}[X_i]^2$, we obtain that $\mathbb{E}[W^{*,c}] \leq \mathbb{E}[W^{*,a}]$. The proof is complete. $\square$

Proof of Proposition 1.5. The welfare of each insurance contract in the one-factor model is computed as follows.

$$\mathbb{E}(W^{*,a}) = \frac{1}{4}\frac{\eta^2 Var(Y) + \sigma^2}{\sum_i \gamma_i}. \tag{A-5}$$

$$\mathbb{E}(W^{*,c}) = \frac{1}{4}\frac{\eta^2}{\sum_i \gamma_i \frac{\eta_i^2}{\eta_i^2 Var(Y) + \sigma_i^2}}, \tag{A-6}$$

and

$$\mathbb{E}(W^{*,ac}) = \frac{1}{4}\frac{\left(\sum_i \hat{\eta}_i E(Y)\frac{\alpha_i\hat{\eta}_i Var(Y)}{\hat{\eta}_i^2 Var(Y)+\hat{\sigma}_i^2}\right)^2}{\sum_i \gamma_i \frac{\hat{\eta}_i^2 E(Y)^2}{\hat{\eta}_i^2 Var(Y)+\hat{\sigma}_i^2}} = \frac{1}{4}\frac{\left(\sum_i \frac{\alpha_i\hat{\eta}_i^2 Var(Y)}{\hat{\eta}_i^2 Var(Y)+\hat{\sigma}_i^2}\right)^2}{\sum_i \gamma_i \frac{\hat{\eta}_i^2}{\hat{\eta}_i^2 Var(Y)+\hat{\sigma}_i^2}}. \quad \text{(A-7)}$$

Clearly, when the total $\sigma^2 = 0$, the welfare is identical for all three types of contracts. The second part follows from the same idea. □

Proof of Proposition 1.6.

First, note that $\eta^2 \geq \sum \eta_i^2$ and the function $f(x) \equiv \frac{x^2 Var(Y)+\sigma^2}{x^2}$ is decreasing with respect to x. Then,

$$\frac{\eta^2 Var(Y)+\sigma^2}{\eta^2} \leq \frac{\sum(\eta_i^2 Var(Y)+\sigma_i^2)}{\sum \eta_i^2}. \quad \text{(A-8)}$$

To prove $\mathbb{E}[W^{*,a}] < \mathbb{E}[W^{*,c}]$ under the co-monotonic condition, it suffices to show that

$$\frac{\sum(\eta_i^2 Var(Y)+\sigma_i^2)}{\sum \gamma_i} < \frac{\sum \eta_i^2}{\sum_i \gamma_i \frac{\eta_i^2}{\eta_i^2 Var(Y)+\sigma_i^2}}. \quad \text{(A-9)}$$

In fact, by using the co-monotonic relationship between the risk-adjusted variance and the Sharpe ratio, the risk-adjusted variance is counter-monotonic to the vector $\left(\frac{Var(X_i)}{\mathbb{E}[X_i]^2}\mathbb{E}[Y]^2\right)$. Note that $\mathbb{E}[X_i] = \eta_i \mathbb{E}[Y]$ and $Var(X_i) = \eta_i^2 Var(Y) + \sigma_i^2$. Then, the last inequality (A-9) follows from Lemma 1 for $b_i = \gamma_i, c_i = \gamma_i \frac{\eta_i^2}{\eta_i^2 Var(Y)+\sigma_i^2}$, and $\kappa_i = Var(X_i)/\gamma_i$.

If the risk-adjusted variance is counter-monotonic to the Sharpe ratio across the banks, then by the same proof, we obtain:

$$\frac{\sum(\eta_i^2 Var(Y)+\sigma_i^2)}{\sum \gamma_i} > \frac{\sum \eta_i^2}{\sum_i \gamma_i \frac{\eta_i^2}{\eta_i^2 Var(Y)+\sigma_i^2}}. \quad \text{(A-10)}$$

For a large positive number x, $f'(x) = -\frac{2\sigma^2}{x^3}$ is close to zero, so the curve $y = f(x)$ is almost flat. Then, for a large $\mathbb{E}[X]$, the numbers $\frac{\eta^2 Var(Y)+\sigma^2}{\eta^2}$ and $\frac{\sum(\eta_i^2 Var(Y)+\sigma_i^2)}{\sum \eta_i^2}$ are so close enough that

$$\frac{\eta^2 Var(Y)+\sigma^2}{\eta^2} \sim \frac{\sum(\eta_i^2 Var(Y)+\sigma_i^2)}{\sum \eta_i^2} > \frac{\sum \gamma_i}{\sum_i \gamma_i \frac{\eta_i^2}{\eta_i^2 Var(Y)+\sigma_i^2}}.$$

Equivalently, $\mathbb{E}[W^{*,a}] > \mathbb{E}[W^{*,c}]$. □

Proof of Proposition 1.7. As the risk-adjusted variance is co-monotonic to the Sharpe ratio across each bank, Lemma 1 yields that

$$\frac{1}{4}\frac{\sum \mathbb{E}[X_i]^2}{\sum \gamma_i \mathbb{E}[X_i]^2/Var(X_i)} > \frac{1}{4}\frac{\sum Var(X_i)}{\sum_i \gamma_i}. \tag{A-11}$$

By using Cauchy-Schwartz inequality, $Var(X_i)Var(\hat{X}_i) \geq Cov(X_i,\hat{X}_i)^2$ for each i. We obtain

$$\frac{1}{4}\frac{\sum \mathbb{E}[X_i]^2}{\sum \gamma_i \mathbb{E}[X_i]^2/Var(X_i)} > \frac{1}{4}\frac{\sum Cov(X_i,\hat{X}_i)^2/Var(\hat{X}_i)}{\sum_i \gamma_i}. \tag{A-12}$$

Note that $\frac{Cov(X_i,\hat{X}_i)^2}{Var(\hat{X}_i)\gamma_i} = \rho_i^2 \frac{Var(X_i)}{\gamma}$ where ρ_i is the correlation coefficient between X_i and $\hat{X}_i$. If the Sharpe ratio of the "dual" risk $\frac{\mathbb{E}[\hat{X}_i]}{\sqrt{Var(\hat{X}_i)}}$ is counter-monotonic to the risk-adjusted correlated variance $\rho_i^2 \frac{Var(X_i)}{\gamma}$, then $\rho_i^2 \frac{Var(X_i)}{\gamma}$ is co-monotonic to $\frac{Var(\hat{X}_i)}{\mathbb{E}[\hat{X}_i]^2}$. Again by Lemma 1 (for $b_i = \gamma_i, c_i = \gamma_i \frac{\mathbb{E}[\hat{X}_i]^2}{Var(\hat{X}_i)}$ and $\kappa_i = \rho_i^2 \frac{Var(X_i)}{\gamma}$), we have

$$\frac{\sum Cov(X_i,\hat{X}_i)^2/Var(\hat{X}_i)}{\sum_i \gamma_i} > \frac{\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)^2}{Var(\hat{X}_i)^2}}{\sum \gamma_i \frac{\mathbb{E}[\hat{X}_i]^2}{Var(\hat{X}_i)}}. \tag{A-13}$$

By combing (A-12) with (A-13) together, we obtain

$$\frac{\sum \mathbb{E}[X_i]^2}{\sum \gamma_i \frac{\mathbb{E}[X_i]^2}{Var(X_i)}} > \frac{\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)^2}{Var(\hat{X}_i)^2}}{\sum \gamma_i \frac{\mathbb{E}[\hat{X}_i]^2}{Var(\hat{X}_i)}}. \tag{A-14}$$

Equivalently,

$$\frac{\sum \mathbb{E}[X_i]^2}{\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)^2}{Var(\hat{X}_i)^2}} > \frac{\sum \gamma_i \frac{\mathbb{E}[X_i]^2}{Var(X_i)}}{\sum \gamma_i \frac{\mathbb{E}[\hat{X}_i]^2}{Var(\hat{X}_i)}}. \tag{A-15}$$

When $\mathbb{E}[X_i]$ is distributed equally, or the expected losses are fairly close enough, Lemma 2 ensures that

$$\frac{(\sum \mathbb{E}[X_i])^2}{\left(\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)}{Var(\hat{X}_i)}\right)^2} > \frac{\sum \mathbb{E}[X_i]^2}{\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)^2}{Var(\hat{X}_i)^2}}. \tag{A-16}$$

Finally, by using (A-15) and (A-16) we obtain

$$\frac{(\sum \mathbb{E}[X_i])^2}{\left(\sum \mathbb{E}[\hat{X}_i]^2 \frac{Cov(X_i,\hat{X}_i)}{Var(\hat{X}_i)}\right)^2} > \frac{\sum \gamma_i \frac{\mathbb{E}[X_i]^2}{Var(X_i)}}{\sum \gamma_i \frac{\mathbb{E}[\hat{X}_i]^2}{Var(\hat{X}_i)}}. \tag{A-17}$$

By using Proposition 1.2and Proposition 1.3, we obtain that $\mathbb{E}[W^{*,c}] > \mathbb{E}[W^{*,ac}]$. □

Proof of Proposition 2.3: $\mathbb{E}[U(\tilde{W}^i)] - \mathbb{E}[U(W^i)]$ can be represented by

$$A \equiv -a_i\rho\mathbb{E}[X] - \frac{1}{2\gamma_i}\left\{a_i^2 Var(X) - 2a_i Cov(X_i, X)\right\}.$$

By straightforward computation and the expression of a_i in (2.8), we have

$$a_i^2 Var(X) - 2a_i Cov(X_i, X) = -\frac{1}{Var(X)}\left\{Cov(X_i, X)^2 - \rho^2\gamma_i^2\mathbb{E}[X]^2\right\}. \quad \text{(C-1)}$$

Then by using the expression of $a_i(\rho^*)$ again in $a_i\rho\mathbb{E}[X]$, we have

$$A = \frac{1}{2\gamma_i Var(X)}\left\{Cov(X_i, X) - \gamma_i\rho\mathbb{E}[X]\right\}^2, \quad \text{(C-2)}$$

which implies the formula in Proposition 3.3 by using the formula of the load factor ρ^*. □

Proof of Equation (2.26): The difference between $Var(\tilde{W}^i)$ and $Var(W^i)$, $Var(\tilde{W}^i) - Var(W^i)$, is given by

$$-\frac{1}{Var(X)}\left\{Cov(X_i, X)^2 - \rho^2\mathbb{E}[X]^2\gamma_i^2\right\}.$$

Then we apply the formula of $a_i(\rho^*)$ and ρ^* to derive the formula (2.26). □

Proof of Proposition 2.5: Note that $\sum_i a_i(\rho^*) = \frac{1}{2}$. Then $\tilde{X} = \frac{1}{2}X + \frac{1+\rho}{2}\mathbb{E}[X]$, and the aggregate ex post variance, $Var(\tilde{X})$, is $\frac{1}{4}Var(X)$. Next, we have

$$\begin{aligned} Cov(\tilde{X}_i, \tilde{X}) &= Cov(X_i - a_i(\rho^*)X, \frac{1}{2}X) = \frac{1}{2}Cov(X_i, X) - \frac{1}{2}a_i(\rho^*)Var(X) \\ &= \frac{1}{4}\frac{\gamma_i}{\gamma}Var(X), \end{aligned}$$

where the formula of $a_i(\rho^*)$ in (2.8) is employed in the last equation. □

APPENDIX C: CHAPTER 3. PROOFS

Solution of the Optimization Problem (3.4). We present a solution of the optimization problem (3.4) and the equilibrium in a general situation with different risk aversion parameters γ_i. We re-order the bank sector such that

$$\frac{Cov(X_1, Z)}{\gamma_1 Var(Z)} \geq \frac{Cov(X_2, Z)}{\gamma_2 Var(Z)} \geq \cdots \geq \frac{Cov(X_N, Z)}{\gamma_N Var(Z)}.$$

Moreover, we assume that $Cov(X_i, Z) > 0$ for each bank $i = 1, \cdots, N$, because those banks with negative covariance $Cov(X_i, Z)$ have no contribution to (3.4); thus, those banks with negative or zero covariance $Cov(Z_i, Z)$ should be removed from this setting.

Write $f(\rho) = \sum_{i=1}^{N} \max\{Cov(X_i, Z)\rho - \rho^2\gamma_i\mathbb{E}[Z], 0\}$, and $g_m(\rho) = \sum_{i=1}^{m}\{Cov(X_i, Z)\rho - \rho^2\gamma_i\mathbb{E}[Z]\}$ for each $m = 1, \cdots, N$. Let $A_m = \max_{\rho\in\mathbb{I}_m} g_m(\rho)$, where

$$\mathbb{I}_m = \begin{cases} \left[\frac{Cov(X_{m+1},Z)}{\gamma_{m+1}\mathbb{E}[Z]}, \frac{Cov(X_m,Z)}{\gamma_m\mathbb{E}[Z]}\right], m = 1, \cdots, N-1, \\ \\ \left[0, \frac{Cov(X_N,Z)}{\gamma_N\mathbb{E}[Z]}\right], m = N. \end{cases}$$

We first demonstrate that, noting that $f(0) = 0$,

$$\max_{\rho\geq 0} f(\rho) = \max_{1\leq m\leq N} A_m. \tag{D-1}$$

Therefore, the optimization problem (3.4) is reduced to a sequence of solving A_m, which in turn are solved by a set of standard optimization problem of $g_m(\rho)$.

On one hand, let ρ^* be the one such that $\max_{\rho\geq 0} f(\rho) = f(\rho^*)$. If $Cov(X_i, Z)\rho^* \geq (\rho^*)^2\gamma_i\mathbb{E}[Z]$ for all $i = 1, \cdots, N$, we set $m = N$ and then $\rho^* \in I_N$. Otherwise, there exists a unique number $m = 1, \cdots, N-1$ such that

$$f(\rho^*) = \sum_{i=1}^{m} \left(Cov(X_i, Z)\rho^* - (\rho^*)^2\gamma_i\mathbb{E}[Z]\right),$$

and m is characterized by the following system of inequalities:

$$\begin{cases} Cov(X_i, Z)\rho^* - (\rho^*)^2\gamma_i\mathbb{E}[Z] > 0, \text{ for } i = 1, \cdots, m \\ Cov(X_i, Z)\rho^* - (\rho^*)^2\gamma_i\mathbb{E}[Z] \leq 0, \text{ for } i = m+1, \cdots, N. \end{cases} \tag{D-2}$$

That is, $\rho^* \in \mathbb{I}_m$. Hence, $f(\rho^*) = g_m(\rho^*) \leq A_m \leq \max_{1\leq m\leq N} A_m$. On the other hand, for any $m = 1, \cdots, N$, it is evidently that

$$g_m(\rho) \leq \sum_{i=1}^{m} \max\left(Cov(X_i, Z)\rho - \rho^2\gamma_i\mathbb{E}[Z], 0\right) \leq f(\rho)$$

for any $\rho \geq 0$. Hence, $\max_{1\leq m\leq N} A_m \leq \max_{\rho\geq 0} f(\rho)$. We have thus proved equation (D-1). □

By virtue of (D-1), the equilibrium of the capital insurance market can be solved by three steps as follows.

First. Compute A_m and $\overline{\rho}_m \equiv argmax_{\rho\in\mathbb{I}_m} g_m(\rho)$ for each $m = 1, \cdots, N$.

Let $\rho_m = \frac{1}{2\mathbb{E}[Z]}\frac{\sum_{i=1}^m Cov(X_i,Z)}{\sum_{i=1}^m \gamma_i}$. Then, we can verify that, for $m = 1, \cdots, N-1$,

$$\overline{\rho}_m = \min\left(\frac{Cov(X_m, Z)}{\gamma_m\mathbb{E}[Z]}, \max\left(\frac{Cov(X_{m+1}, Z)}{\gamma_{m+1}\mathbb{E}[Z]}, \rho_m\right)\right) \tag{D-3}$$

and

$$\overline{\rho}_N = \min\left(\frac{Cov(X_N, Z)}{\gamma_N\mathbb{E}[Z]}, \rho_N\right). \tag{D-4}$$

Second. Compute $\max_{1\leq m\leq N} A_m$ and $m^* = argmax_{1\leq m\leq N} A_m$.

It is possible to have multiple m^* and thus multiple equilibrium, because of the non-concavity feature of the objective function $f(\rho)$ for the regulator. As explained in Section 2, it is natural to choose the smallest one among $\{m^*\}$ if there are more than one optimal solutions.

Third. The optimal load factor $\rho^* = \overline{\rho}_{m^*}$.

The bank i is TBTF if and only if $\rho^* < \frac{Cov(X_i,Z)}{\gamma_i E[Z]}$. For these too big to fail banks, the premium or the insurance capital is $(1 + \rho^*)a_i(\rho^*)\mathbb{E}[Z]$.

Algorithm to identifying TBTF banks in terms of loss beta only:

Assume that $\gamma_i = \gamma$ for each $i = 1, \cdots, N$. Then, $A_m = \mathbb{E}[Z]\gamma c^2 \max_{\tau \in J_m} h_m(\tau)$, where $c = \frac{Var(Z)}{\gamma \mathbb{E}[Z]}$, $J_m = [\beta_{m+1}, \beta_m]$ for $m = 1, \cdots, N-1$ and $J_N = [0, \beta_N]$. The algorithm to identify TBTF banks follows easily from the above characterization of the equilibrium in a general situation.

Proof of Proposition 3.1:

Since $g_1(\rho) = Cov(X_1, Z)\rho - \rho^2\gamma\mathbb{E}[Z]$, $g_1\left(\frac{Cov(X_1,Z)}{\gamma\mathbb{E}[Z]}\right) = 0$. Therefore, the optimal load factor ρ^* must be strictly smaller than $\frac{Cov(X_1,Z)}{\gamma\mathbb{E}[Z]} = \max\left\{\frac{Cov(X_i,Z)}{\gamma\mathbb{E}[Z]}, i = 1, \cdots, N\right\}$. By definition 1, those banks with the highest loss beta are too big to fail. □

Proof of Proposition 3.2:

By exploring equation (D-1), it suffices to show that $\max_m A_m > 0$. Actually, when $\frac{Cov(X_1,Z)}{\mathbb{E}[Z]} > \frac{Cov(X_2,Z)}{\mathbb{E}[Z]}$, we must have $A_1 > 0$ since $g_1\left(\frac{Cov(X_1,Z)}{\gamma\mathbb{E}[Z]}\right) = 0$. Assuming $\frac{Cov(X_1,Z)}{\mathbb{E}[Z]} = \frac{Cov(X_2,Z)}{\mathbb{E}[Z]}$, then $A_2 > 0$ unless $\frac{Cov(X_3,Z)}{\mathbb{E}[Z]} = \frac{Cov(X_2,Z)}{\mathbb{E}[Z]}$. Continuing the process we know that one of $A_m, m \in \{1, \cdots, N-1\}$, must be positive unless each $\frac{Cov(X_i,Z)}{\mathbb{E}[Z]}$ is the same positive number. In the last situation, it is easy to verify that $A_N > 0$. Therefore, $\max_{\rho>0} f(\rho) = \max_{\rho\geq 0} f(\rho) > 0$. □

Proof of Proposition 3.3:

Note that $\mathbb{E}[U(\tilde{W}^i)] - \mathbb{E}[U(W^i)]$ is $-a_i\rho\mathbb{E}[Z] - \frac{1}{2\gamma_i}\{a_i^2 Var(Z) - 2a_i Cov(X_i, Z)\}$. For TBTF bank i, $a_i(\rho) = \frac{Cov(X_i,Z) - \rho^*\gamma\mathbb{E}[Z]}{Var(Z)} > 0$. By straightforward computation, we have

$$\begin{aligned}\mathbb{E}[U(\tilde{W}^i)] - \mathbb{E}[U(W^i)] &= \frac{1}{2\gamma Var(Z)}\left(Cov(X_i, Z) - \rho^*\gamma\mathbb{E}[Z]\right)^2 \\ &= \frac{Var(Z)}{2\gamma}\left(\frac{Cov(X_i, Z)}{Var(Z)} - \rho^*\gamma\frac{\mathbb{E}(Z)}{Var(Z)}\right)^2 > 0.\end{aligned}$$

Moreover, assuming $a_i(\rho) > 0$, the higher the loss beta, the higher the expected utility enhance, $\mathbb{E}[U(\tilde{W}^i)] - \mathbb{E}[U(W^i)]$. □

The proof of Proposition 3.4 relies on a simple combinational-type result as follows.

Lemma 3 *Given N positive numbers such that $b_1 \geq b_2 \geq \cdots \geq b_N$ and $\sum_{i=1}^N b_i = 1$.*

If there exists an integer i such that

$$\frac{b_i}{\sum_{k=1}^{i} b_i} > \frac{1}{2i}, \tag{D-5}$$

then $b_i > \frac{1}{2N}$. Moreover, if "$>$" is replaced by $\geq$ in (D-5), then $b_i \geq \frac{1}{2N}$.

Proof: We prove the first part of this lemma while the proof for the second part is the same.

We first consider the case when N is divided by i, that is, $N = mi$ for a positive integer m. Notice that $\sum_{k=1}^{N} b_k = 1$. Since b_k is decreasing for $k = 1, \cdots, N$, we have

$$1 = \sum_{k=1}^{N} b_k \leq m \sum_{k=1}^{i} b_k. \tag{D-6}$$

Then

$$\sum_{k=1}^{i} b_k \geq \frac{1}{m}. \tag{D-7}$$

Hence, by virtue of (D-5),

$$b_i > \frac{1}{2i} \sum_{k=1}^{i} b_k \geq \frac{1}{2i}\frac{1}{m} \geq \frac{1}{2N}. \tag{D-8}$$

The lemma is proved if N can be divided by such an i.

If N can't be divided by i, write $N = mi + t$ for some $0 < t < i$ and $m \geq 1$. We use the decreasing property of b_k again, we obtain

$$\begin{aligned} 1 &= \sum_{k=1}^{N} b_k \\ &= (b_1 + \cdots + b_i) + \cdots + \left(b_{(m-1)i+1} + \cdots + b_{mi}\right) \\ &\quad + (b_{mi+1} + \cdots + b_{mi+t}) \\ &\leq m\,(b_1 + \cdots + b_i) + t b_i. \end{aligned}$$

Therefore,

$$\sum_{k=1}^{i} b_k \geq \frac{1 - t b_i}{m}, \tag{D-9}$$

then by using (D-5), we obtain

$$b_i > \frac{1}{2i}\frac{1-tb_i}{m}, \tag{D-10}$$

which yields (since $N = mi + t$)

$$b_i > \frac{1}{2mi+t} > \frac{1}{2N}. \tag{D-11}$$

This lemma is proved. □

Proof of Proposition 3.4:

By using the solution of Problem (3.4), there are two possibilities for the optimal load factor ρ^*.

Case 1. $\rho^* = \rho_m$ for some m and $\rho_m \leq \frac{Cov(X_m,X)}{\gamma\mathbb{E}[X]}$.

In this case, $\rho_m = \frac{Var(X)}{\gamma\mathbb{E}[X]}\frac{\sum_{i=1}^m \beta_i}{2m}$ and $\frac{Cov(X_m,X)}{\gamma\mathbb{E}[X]} = \frac{Var(X)}{\gamma\mathbb{E}[X]}\beta_m$. Therefore, $\beta_m \geq \frac{\sum_{i=1}^m \beta_i}{2m}$. By using Lemma 1, we have $\beta_m \geq \frac{1}{2N}$.

Case 2. $\rho^* = \frac{Cov(X_m,X)}{\gamma\mathbb{E}[X]}$ for some $m \geq 2$.

In this case, by using the solution of the equilibrium, we have $\frac{Cov(X_m,X)}{\gamma\mathbb{E}[X]} \geq \rho_{m-1}$. Then we have

$$\beta_m \geq \frac{\beta_1 + \cdots + \beta_{m-1}}{2(m-1)}$$

which implies that

$$\beta_m > \frac{\beta_1 + \cdots + \beta_{m-1}}{2m-1}.$$

The last inequality in turn is equivalent to

$$\beta_m > \frac{\beta_1 + \cdots + \beta_m}{2m}.$$

By using Lemma 1 again, $\beta_m > \frac{1}{2N}$. □

Proof of Proposition 3.5:

Notice that after implementing the capital insurance, the loss portfolio is $\tilde{X}_i =$

$-X_i + a_i X - (1+\rho^*)a_i\mathbb{E}[X]$ where $a_i = a_i(\rho^*)$ is the optimal coinsurance coefficient. Thus, the aggregate loss portfolio becomes $\tilde{X} = -X + \sum_{i=1}^N a_i X - (1+\rho^*)\sum_{i=1}^N a_i \mathbb{E}[X]$, and the systemic risk $Var(\tilde{X}) = (1-a)^2 Var(X)$, where $a = \sum_{i=1}^N a_i$. To prove that the total systemic risk is reduced, that is, $Var(\tilde{X}) < Var(X)$, it suffices to show that $0 < a < 1$. First, $a > 0$ because of existence of too big to fail by Proposition 3.1. Second, by using the definition of a_i and the fact that $\rho^* > 0$ in (D-1), we have

$$
\begin{aligned}
a &= \sum_{i=1}^m \left(\frac{Cov(X_i, X) - \rho^*\gamma\mathbb{E}[X]}{Var(X)} \right) \\
&= \sum_{i=1}^m \beta_i - \rho^*\gamma m \frac{\mathbb{E}[X]}{Var(X)} \\
&< \sum_{i=1}^m \beta_i
\end{aligned}
$$

where those banks $i = 1, \cdots, m$ are too big to fail banks. The positive correlated assumption yields that $\sum_{i=1}^m \beta_i \leq \sum_{i=1}^N \beta_i = 1$. □

The proof of Proposition 3.6 depends on the following Sherman-Morrison formula in linear algebra.

Lemma 4 *Suppose A is an invertible $s \times s$ matrix and u, v are $s \times 1$ vectors. Suppose further that $1 + v^T A^{-1} u \neq 0$. Then the matrix $A + uv^T$ is invertible and*

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^T A^{-1}}{1 + v^T A^{-1} u}. \tag{D-12}$$

Proof of Proposition 3.6:

For each $i = 1, \cdots, N$, we have

$$Cov(X_i, X) - \rho\mathbb{E}[X] = a_i(\rho)Var(X). \tag{D-13}$$

Let

$$\hat{a}_i(\rho) = \frac{Cov(\hat{X}_i, \hat{X}) - \rho\mathbb{E}[\hat{X}]}{Var(\hat{X})}. \tag{D-14}$$

By assumption, it is easy to see $Cov(\hat{X}_i, \hat{X}) = Cov(X_i, X) + \sigma_i^2$ and $\mathbb{E}[X] = \mathbb{E}[\hat{X}]$. Replacing $Cov(X_i, X)$ by $Cov(\hat{X}_i, \hat{X}) - \sigma_i^2$ in equation (D-13) and using equation

(D-14), we obtain

$$\begin{aligned} a_i(\rho)Var(X) &= Cov(X_i, X) - \rho\mathbb{E}[X] \\ &= Cov(\hat{X}_i, \hat{X}) - \rho\mathbb{E}[\hat{X}] - \sigma_i^2 \\ &= \hat{a}_i(\rho)Var(\hat{X}) - \sigma_i^2. \end{aligned}$$

Again, by assumption, $Var(\hat{X}) = Var(X) + \sum_{i=1}^{N} \sigma_i^2$. Then, for $i = 1, \cdots, N$ and let $\sigma^2 = \sum_{i=1}^{N} \sigma_i^2$, we have

$$a_i(\rho)(Var(\hat{X}) - \sigma^2) = \hat{a}_i(\rho)Var(\hat{X}) - \sigma_i^2. \tag{D-15}$$

Equivalently,

$$\sigma_i^2 - a_i(\rho)\sigma^2 = (\hat{a}_i(\rho) - a_i(\rho))\, Var(\hat{X}). \tag{D-16}$$

The coefficient matrix of the variance vector, $(\sigma_1^2, \cdots, \sigma_N^2)^T$, in the last equation is

$$\begin{bmatrix} 1 - a_1(\rho) & -a_1(\rho) & \cdots & -a_1(\rho) \\ -a_2(\rho) & 1 - a_2(\rho) & \cdots & -a_2(\rho) \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ -a_N(\rho) & -a_N(\rho) & \cdots & 1 - a_N(\rho) \end{bmatrix}$$

which is written as $I + uv^T$, where I is an identity matrix, $u = (-a_1(\rho), \cdots, -a_N(\rho))^T$ and $v = (1, 1, \cdots, 1)^T$. Furthermore,

$$\sum_{i=1}^{N} a_i(\rho) = 1 - \rho N \frac{\mathbb{E}[X]}{Var(X)} < 1,$$

we have $1 + v^T I^{-1} u = 1 - \sum_{i=1}^{N} a_i(\rho) > 0$. Then the Sherman-Morrison formula (Lemma 2) ensures that the coefficient matrix $I + uv^T$ is invertible. Therefore, the noises' variance vector, $(\sigma_1^2, \cdots, \sigma_N^2)^T$, is uniquely determined by the set $\left\{a_i(\rho), \hat{X}_i; i = 1, \cdots, N\right\}$. The proof is completed. □

Proof of Proposition 3.7:

By assumption, $Cov(\hat{X}_i, \hat{X}) = Cov(X_i + \epsilon_i, X + \sum_{i=1}^{N} \epsilon_i) = Cov(X_i, X) + \sigma_i^2$, and $Cov(X_i, \hat{X} - \epsilon_i) = Cov(X_i, X + \sum_{j \neq i} \epsilon_j) = Cov(X_i, X)$. Then

$$Cov(X_i, \hat{X} - \epsilon_i) = Cov(\hat{X}_i, \hat{X}) - \sigma_i^2. \tag{D-17}$$

Moreover, $Var(\hat{X} - \epsilon_i) = Var(X) + \sum_{j \neq i} \sigma_j^2 = Var(\hat{X}) - \sigma_i^2$. Then, by the definition of $\overline{a}_i(\rho)$, we obtain

$$Cov(\hat{X}_i, \hat{X}) - \sigma_i^2 - \rho\gamma\mathbb{E}[\hat{X} - \epsilon_i] = \overline{a}_i(\rho)Var(\hat{X} - \epsilon_i). \tag{D-18}$$

Therefore,

$$\hat{a}_i(\rho)Var(\hat{X}) - \sigma_i^2 = \overline{a}_i(\rho)\{Var(\hat{X}) - \sigma_i^2\}, \tag{D-19}$$

in which we make use of equation (D-14). Hence, we have

$$\sigma_i^2 - \overline{a}_i(\rho)\sigma_i^2 = \{\hat{a}_i(\rho) - \overline{a}_i(\rho)\}Var(\hat{X}). \tag{D-20}$$

To determine σ_i^2 uniquely, it thus suffices to show that $\overline{a}_i(\rho) < 1$ under assumption on correlated risk environment. In fact, by definition of $\overline{a}_i(\rho)$ and $\mathbb{E}[X] > 0$, we have $\overline{a}_i(\rho)Var(\hat{X} - \epsilon_i) < Cov(X_i, \hat{X} - \epsilon_i)$. Notice that $Cov(X_i, X_j) \geq 0$ in a correlated risk environment, then $Cov(X_i, X) \leq Var(X)$ for each $i = 1, \cdots, N$. Therefore, $Cov(X_i, \hat{X} - \epsilon_i) = Cov(X_i, X) - \sigma_i^2 \leq Var(X) - \sigma_i^2 = Var(\hat{X} - \epsilon_i)$. Therefore, we have proved that $0 < \overline{a}_i(\rho) < 1$. □

Details of Example 3:

We claim that when τ is small enough such that

$$\tau^{m+1} \leq \frac{1}{1 + 2(1-\tau)(m+1)}, m = 0, 1, \cdots, N-1 \tag{D-21}$$

and

$$\tau^m \leq \frac{\sqrt{m+1} - \sqrt{m}}{\sqrt{m+1} - \tau\sqrt{m}}, m = 1, \cdots, N-1, \tag{D-22}$$

then only the first bank is too big to fail. In fact, by formula (D-21), $\tau^{m+1} \leq$

$\frac{1+\tau+\cdots+\tau^m}{2(m+1)}$. Hence, $\rho_m = argmax_{\rho\in\mathbb{I}_m} g_m(\rho)$. Moreover, $g_m(\rho_m) = \frac{(1+\tau+\cdots+\tau^{m-1})^2}{4mc}$ for a constant c which independent of m. The condition (D-22) ensures that $g_m(\rho_m)$ is increasing with respect to m. Therefore, by (D-1), $\max_{\rho\geq 0} f(\rho) = g_1(\rho_1)$, and the optimal load factor is $\rho^* = \rho_1 = \frac{a}{2\mathbb{E}[Z]}$. $\square$