TEACHER PRACTICES, BELIEFS, AND CONCEPTUAL UNDERSTANDING OF MATHEMATICS: A PHENOMENOLOGICAL CASE STUDY OF TEACHERS INSTRUCTING MATHEMATICALLY GIFTED AND PROMISING STUDENTS

by

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ABSTRACT

CHRISTINE H. WEISS. Teacher Practices, Beliefs, and Conceptual Understanding of Mathematics: A Phenomenological Case Study of Teachers Instructing Mathematically Gifted and Promising Students. (Under the direction of DR. DREW POLLY)

Students in the United States are not achieving in mathematics as indicated on the NAEP (2019) exams and other measurements of student achievement (NCES, 2019; OECD, 2019; O'Dwyer, et al., 2015). Mathematically gifted and promising students are especially impacted by this phenomenon, though it is not exactly known what factors contribute to successful teachers of these students. This phenomenological case study focused on the beliefs, instructional practices, and conceptual understanding of mathematics of five teachers in a public charter school for gifted students. Data sources collected included semi-structured interviews, classroom observations, and questionnaires based on Swan's (2006) practices and beliefs research with effective mathematics teachers. To better understand these phenomena, two conceptions of giftedness served as the lenses for this study: Renzulli's Three-Ring Model (1978) and Gagné's Differentiated Model of Giftedness and Talent (1985). Using an interpretive phenomenological analysis several themes emerged in response to each research question. Findings for instructional practices indicated that teachers used both student-centered and teacher-centered practices and consistently utilized differentiated groupings. Additionally, teacher participants believe that gifted students possess both positive traits and challenges, and specifically for math, believe that sense-making is key, and math is a subject students should enjoy. Teachers also noted that school and home environments impact access to an equitable mathematics education for children. Teachers' conceptual understanding of mathematics is guided by their ongoing practice, the curriculum, and math experiences prior to teaching. These findings indicate the importance of ongoing training and professional development in mathematics and gifted education, as well as the recruitment and retention of teachers who possess a strong conceptual understanding of mathematics, a passion for the subject, and a student-centered approach to teaching.

Keywords: mathematically gifted and promising, instructional practices, beliefs, equitable access, teachers' conceptual understanding of mathematics

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I Am From
By Christine H. Weiss

I am from the sweet taste of potential,
    from messy pencil pouches and equations to untangle.
I am from high expectations (but with soft places to land),
    from understanding that when others (even you) think you can't be wrong or lost, or
    heaven forbid, last, I am hoping---insisting, you will be.
I am from the mistakes that become light bulbs, which become doorways, which become
    understanding, and then purpose and passion. I'm from knowing that you are
    more than just the answer in your head or the problem on the page, or the ideas
    yet to come.
I am from a yearning desire to teach you that math is so many things, and not just
    getting the right answer. It's taxes, and llamas, and money systems, and
    probability, and Contig. It's working together, it's playing together.
    But you already know that. This is your world, your playground.
I am from a classroom of absent-minded professors, philanthropists, and perfectionists,
    from big idea people who won't be bullied for their brilliance, who have found their tribe,
    their people,
    who are safe.
I am from curious, collaborative, talkative, and quick,
    from wondering whether it might just be you who solves the world's problems one day,
    who finds a cure,
    for something.
I am from the heart who sees that little girl in my class who says she doesn't like math,
    but I know she loves art. So, I let her draw multiplication monsters and now she
    loves math (but still loves art more).
I am from the soul that worries about kids who come from homes with only a handful of
    books or have parents who can't or won't or don't help them with math
    homework. I wonder, "where is the equity for students in the same school who
    start in such different places?" Who finish in such different places?"
I am from teachers everywhere who see the challenges before us, unlike never before,
    who somehow still believe that there's a child in our classroom who might just
    solve the world's problems, who might find a cure to something, maybe…. maybe
    even this problem that is staring right at us, hidden, but not hidden.

If only we will open our eyes, join hands, and see it, solve it...together.

DEDICATION

I dedicate this dissertation to my incredibly patient and loving husband, Jeff, who never lacked for an encouraging word or act and provided grace, love, and perspective (not to mention, the late-night writing snacks) every step of the way. Your unfailing support and love mean the world to me, and I could not imagine taking on a project of this magnitude without you by my side. To my daughters, Victoria, Hannah, and Sadie, thank you for believing not just in me but in the importance of this research for children everywhere, and for the countless times you asked, "How's it going?". Every time you said how proud you were of my work and of me it helped soften the immense guilt I felt for missing important family time and events.

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I would be remiss if I did not dedicate this work to my professors and advisors at this university, especially Dr. Drew Polly, Dr. Amy Good, and Dr. Tina Heafner, all of whom reminded me that my research mattered. Just like Glinda told Dorothy in T*he Wizard of Oz,* you reminded me I had the power all along to make my way home, I just needed to discover this for myself. Thank you for allowing me to forge my own path but making sure I never ventured too far off course.

And finally, to Kyle, my mathematically gifted and promising fourth-grade student in 1994, who was the first student to make me wonder what it would take to teach him and every student like him, extraordinarily well. It was that wonder, so long ago, that has served as my inner compass and the ultimate research question of all.

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CHAPTER I: INTRODUCTION

Background and Context

A lack of student achievement in mathematics is of critical concern in the United States. Research shows that U.S. students perform poorly in mathematics when compared to each other and to students in other countries. In a report titled *Adding It Up* (National Research Council, 2001), students in the K-8 grades were proficient in rote computation but did not do well on international assessments that require more advanced problem-solving and understanding. Three large scale assessments, two at the international level and one at the national level, are described here and include the most recent results for the mathematics portion of these tests. These results provide concerning evidence of the performance of U.S. students in mathematics.

Teacher preparation and conceptual understanding of mathematics are often cited as a critical factor in student mathematics achievement (Ma, 1999/2010). Some studies indicate that elementary teachers do not have the skills needed to teach mathematics based on the poor preparation received at many undergraduate institutions (NCRTE, 1991). Teacher beliefs about mathematics guide their instructional practices and behaviors towards students (Ernest, 1989; Swan, 2006) and provide valuable information to better understand student achievement.

Mathematics Achievement Data

One way to look at student achievement is through testing data on student performance. National and international assessments are presented and show how U.S. students compare to students in other countries and among themselves.

PISA Assessments

PISA, or the Programme for International Student Assessment, is a worldwide study by the Organisation for Economic Co-operation and Development (OECD) intended to evaluate educational systems by measuring 15-year-old school pupils' scholastic performance on mathematics, science, and reading (Organization for Economic Cooperation and Development [OECD], 2019). PISA defines mathematical literacy as a student's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. The PISA scores describe student performance in each subject area in terms of the levels of proficiency, from the lowest level (Level 1) to the highest (Level 6) (See Appendix A for descriptions of these levels).

The 2018 PISA tests showed that 73% of students in the United States attained Level 2 or higher in mathematics (OECD average: 76%), compared to 93% in Singapore. At a minimum, these students can interpret and recognize, without direct instructions, how a (simple) situation can be represented mathematically. More significantly as it relates to students who are advanced or mathematically gifted, only 8% of U.S. students scored at Level 5 or higher in mathematics (OECD average: 11%) compared to Singapore which had 37%, China (44%), and Korea (21%). A Level 5 or higher indicates that these students can model complex situations mathematically, and can select, compare, and evaluate appropriate problem-solving strategies for dealing with them (OECD, 2019).

TIMMS Assessments

Like the PISA exams, the Trends in International Mathematics and Science Study (TIMSS) is an international comparative study that measures trends in mathematics and science achievement in Grade 4 and Grade 8 every four years. TIMSS tests aim to align broadly with mathematics and science curricula in the participating education systems. This should therefore reflect students' school-based learning. The United States has participated in every administration of TIMSS since its inception in 1995, and the study provides valuable information on how U.S. students compare to students around the world (NCES, 2019).

For the 2019 results, U.S. fourth graders ranked 15th among the 64 participating education systems in average TIMSS mathematics scores. The U.S. average score (535) was not significantly different from the average scores of students in seven education systems. Average scores ranged from 297 in the lowest performing education system (the Philippines) to 625 in the highest performing education system (Singapore). However, the United States had relatively large score gaps between the top- and bottom-performing students in both TIMSS subjects and grades. In eighth grade mathematics, only one of the 45 other education systems (Turkey) had a larger score gap between the top-performing (90th percentile) and bottom-performing (10th percentile) students than the United States. Like the PISA assessments, the top scoring countries included Korea and China, in addition to Singapore which ranked first (NCES, 2020). (See Appendix B for benchmark descriptions and Appendix C for 2019 fourth grade math results and rankings.)

NAEP Exams

A review of the most recent (2018) NAEP (National Assessment of Educational Progress) test of fourth graders' math skills provides a similar narrative for younger students as the PISA data does with older students. The NAEP mathematics assessment measures students' knowledge and skills in mathematics and their ability to solve problems in mathematical and real-world contexts. It is a congressionally mandated project administered by the National Center for Education Statistics (NCES) within the U.S. Department of Education and is the largest continuing and nationally representative assessment of what our nation's students know and can do in select subjects. The average scale scores represent what students know and can do, while the achievement-level results indicate the degree to which student performance meets expectations of what they *should* know and be able to do. Table 1 shows that little change has occurred over the past four testing cycles with roughly 60% of all students scoring below proficient in mathematics. Even more concerning is that fewer than 10% of students show advanced understanding in mathematics and twice that many (or close to 20%) are below basic in their mathematics ability (NCES, 2019).

Table 1

NAEP Average Scale Scores and Percentages at Each Achievement Level for Grade 4 Math

Year	Jurisdiction	All Students	Average Scale Score	% Below Basic	% At Basic	% At Proficient	% At Advanced
2019	National	All students	241	19	40	32	9
2017	National	All students	240	20	39	32	8
2015	National	All students	240	18	42	33	7
2013	National	All students	242	17	41	34	8

Note. Some apparent differences between estimates may not be statistically significant.

Note. From the U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), 2013, 2015, 2017, and 2019 Mathematics Assessments.

As noted, 40% of U.S. 4th graders showed Basic understanding, which is defined as "partial mastery" of the knowledge and skills needed to be proficient in mathematics. These results provide evidence that many of our nation's students are not equipped to be successful in mathematics at the school level which negatively impacts their school experience in later years and beyond. Table 2 shows the NAEP achievement level definitions (See Appendix D for a detailed description of the NAEP levels).

Table 2

NAEP Achievement Level Definitions

Basic	Partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at the grade level assessed.
Proficient	Solid academic performance for the grade level assessed. Students reaching this level have demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.
Advanced	Superior performance for the grade level assessed.

Note. Adapted from the NAEP Glossary of Terms. www.nces.ed.gov (2020).

Teacher Preparation and Conceptual Understanding of Mathematics

These achievement results indicate that U.S. students are not measuring up to their international peers nor achieving on the national level, but the numbers do not tell us why. Various studies have been conducted on the reasons U.S. students struggle in mathematics and researchers consistently find that how well teachers know and understand (or don’t understand) mathematics, particularly mathematical concepts, impacts student achievement (Ball et al., 2005; Ginsburg et al., 2005; Ma, 1999/2010; Southern Regional Education Board [SREB], 2019). Browning et al. (2014) found preservice teachers often struggled with understanding the concepts of decimals and place value, algebraic equations, and representing fractions. One common theme in the study was that these future teachers tend to know how to use algorithms, but often are unable to explain how and why those algorithms work. Other reports expressed similar concern for teacher preparation in mathematics. Just 3% of elementary teachers who responded to a 2018 national survey of science and math education have a mathematics degree, compared to 45% in middle school and 79% of high school teachers of math (SREB, 2019). According to the SREB report, “If a teacher doesn’t understand and can’t explain why a mathematical process works, it is unlikely she can fully teach that process to students or troubleshoot a student’s difficulties in understanding it or mistakes in applying it” (SREB, 2019, p. 9).

Teaching mathematics for conceptual understanding is a primary goal of teacher education (National Council of Teachers of Mathematics [NCTM], 2000). However, this may be challenging as teachers need a specific kind of knowledge to teach conceptual understanding well, including deep conceptual understanding of the mathematics (Ball et al., 2005). The NCTM’s *Principles and Standards for School Mathematics* (2000) aimed to provide clarity on the importance of “learning mathematics with understanding” (p. 20) while also calling for a balance between conceptual and procedural learning: “Students who use facts or procedures without understanding often are not sure when or how to use what they know, and such learning is often quite fragile” (p. 20).

Further, content knowledge in mathematics and conceptual understanding should not be assumed, and therefore one recommendation is to thoroughly study teachers’ mathematical knowledge (Rubenstein et al., 2015). From Ma’s (1999) work, we learn that the lack of mathematical content knowledge and understanding for U.S. teachers is concerning. She suggested that this cycle can only be broken by developing more effective mathematics courses in undergraduate programs.

Mathematically Promising Students

Though the literature explains that teacher conceptual knowledge and understanding is essential for all student achievement, this knowledge is critical for students who are identified as mathematically gifted or promising. Conceptions and definitions of mathematics and giftedness have changed over time. Some terms that have been used to describe what it means to be mathematically gifted include precocious, advanced, highly able, and others (Sheffield, 2016). The term "mathematically promising" was suggested by the NCTM rather than "gifted" to broaden the definition, include a greater range of students, and serve to develop outstanding mathematical ability (Sheffield et al., 1999). The task force was intentional about using the word "promising" to emphasize the goal of including students who have previously been excluded from this group. Students who lack self-confidence or who have been overlooked historically as a group that performs well in math are missing important opportunities to nurture their mathematical promise (Gavin, 2005).

The task force also stated that traditional methods of identifying students such as standardized test scores limit the pool of students. Sheffield et al. (1999) recommended a variety of measures for identification to ensure the broadest number of both females and males from diverse socioeconomic and cultural backgrounds. Krutetski (1976) outlined one of the most widely cited definitions of mathematical giftedness which he described as "a unique aggregate of mathematical abilities that opens up the possibilities of success performance in mathematical activity" (p. 77). Please note for the purposes of this research, and because there is no one standard definition, mathematically promising and mathematically gifted will be used interchangeably and combined as mathematically gifted and promising students (MGPS). It is also important to ensure that students with mathematical promise are included based on the

NCTM's task force recommendation as noted above. See definitions of key terms for more information.

Challenging Mathematics for All

The current study provides insight into the types of instructional practices teachers use with mathematically gifted students, their beliefs about mathematics and gifted students, and their conceptual understanding of mathematics. The TIMMS data presented a significant gap between top and bottom-performing students in the United States and may be explained by certain classroom teaching practices. "Teaching to the test" is a phrase used to explain the approach taken in response to pressure from high stakes testing and has become a formalized instructional practice, particularly for African American students, according to Davis and Martin (2008). This is characterized by using remediation and skills-based instruction over conceptual, higher-order thinking and rich curriculum. "While mastery of low-level content is necessary, it often becomes the ceiling of the mathematics students learn" (Davis & Martin, 2008, p. 11). The authors argued that teachers must "provide these students with a challenging and stimulating mathematics education that assist these students in improving their individual and collective group conditions" (p. 30). Therefore, in addition to an overreliance on using algorithms, an emphasis on low-level mathematics, and lack of opportunity can have damaging results and ultimately exclude whole groups of students from mathematical success.

Similarly, Gavin (2005) noted that many students, particularly students of poverty, often have not had the experiences and opportunities to demonstrate their talent and have instead been taught mathematics as procedures and rules to memorize without the opportunity to solve interesting or challenging problems. Sheffield (2016) cited the myth that only wealthy, White, and Asian males can become strong mathematics students and suggested that we increase the pool of mathematical promise by creating more positive portrayals in schools and media and by providing more engaging opportunities for underrepresented groups such as girls, students of color, and those who live in poverty.

As noted previously, only 8% of U.S. students scored at Level 5 or higher in mathematics using the PISA tests (OECD average: 11%), compared to Singapore which had 37%, China (44%), and Korea (21%). A Level 5 or higher indicates an ability to model complex situations mathematically, and select, compare, and evaluate appropriate problem-solving strategies for dealing with them (OECD, 2019). This is the level at which mathematically gifted students have the potential to operate, but in the United States this is not being realized even though it has long been recognized as a critical issue in U.S. mathematics. In 1980, the National Council of Teachers of Mathematics (NCTM) declared what is now an often-repeated statement: "the student most neglected, in terms of realizing full potential, is the gifted student of mathematics. Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world" (NCTM, 1980, p. 18).

Other researchers have indicated the dire consequences of not properly educating our mathematically gifted young people in terms of impact on the students themselves and our nation (Renzulli, 2005; Sheffield, 2016; Stepanek, 1999). In the United States, the National Science Board report, *Preparing the Next Generation of STEM Innovators* indicated, "The long-term prosperity of our nation will increasingly rely on talented and motivated individuals who will comprise the vanguard of scientific and technological innovation; and every student in America deserves the opportunity to achieve his or her full potential" (National Science Board, 2010, p. v, as cited in Sheffield, 2016). It is clear from these statements that the potential contributions of mathematically gifted students are valued and needed; research needs to address this issue and provide potential solutions.

Identification of Mathematically Gifted and Promising Students

A vital need exists for teachers to recognize and understand mathematically gifted students and to engage with them in a way that positively impacts their mathematical growth or promise. Mathematically gifted students learn differently than their same-age peers and require a differentiated approach to succeed in a typical, mixed-ability elementary classroom. Compared to their classmates, these students learn at a faster pace, perform at higher levels, and can easily become bored, frustrated, or underachieving if not challenged appropriately (McAllister & Plourde, 2008). Krutetski (1976) identified specific qualities of mind that distinguish students who may excel in math versus those who may not. Parish (2014) defined mathematically gifted students as those who possess unusually high natural aptitudes for grasping math concepts.

Teachers of Mathematically Gifted and Promising Students

Though Krutetskii (1976) and others (Leiken, 2011; Sheffield, 1994, 1999, 2016; Singer et al., 2016) provided guidance on the identification of mathematically gifted students, much less is known about the teachers of these students. In interviews with outstanding teachers in Russian schools for advanced mathematics, Karp (2010) found that the teachers interviewed held the belief that a future teacher of mathematically gifted students must have a strong foundational education in mathematics. Mingus and Grassi (1999) researched the type of environment that nurtures the growth of mathematically gifted students, however, the research on how teachers of the mathematically gifted should be prepared and formed over time is scarce (Karp, 2010; Hill & Ball, 2004). Despite having standards from the National Association of Gifted Children-Council on Exceptional Children (NAGC-CEC) (2014) on teacher preparation, these standards, though well-intentioned, do not specifically address the mathematical preparation needed to fully teach and support gifted students as the standards are geared towards gifted education professionals and not classroom teachers, nor do they outline the teacher belief systems that best support MGPS. A need for this research exists to ensure that students with mathematical promise are taught by teachers who are prepared with the conceptual understanding of mathematics and mathematical knowledge for teaching needed to develop student potential fully. Research on the influence of teacher beliefs about mathematics and beliefs about mathematically gifted students on the experience of mathematically gifted students is also limited but is essential to further understand what constitutes a successful teacher of mathematically gifted students.

Conceptual Frameworks

To better understand teacher beliefs about gifted students and mathematics, teacher instructional practices, and the conceptual understanding of mathematics required of teachers to successfully nurture students with mathematical promise, two concepts of giftedness will serve as the lenses for this study, Renzulli's Three-Ring Model (1978) and Gagné's Differentiated Model of Giftedness and Talent (1985). In Renzulli's model, gifted behavior occurs via three basic clusters of human traits, namely, above-average general and/or specific abilities, high levels of task commitment (motivation), and high levels of creativity. The three-ring model is considered an operational one in that it is based on research studies that define giftedness, provides guidance for schools and educators to identify students, and can help determine the type of programming needed that will best meet their needs and help recognize and consider these students as learners with special needs. Determining the type of guidance teachers need and the type of programming that best serves gifted students in general, or in the specific domain of mathematics, makes this an important framework for this study.

Gagné's model of giftedness provided a critique of Renzulli's framework and served to describe a clear distinction between giftedness and talent for which he believed had not been done and that, in fact, the two terms were incorrectly used as synonyms. This work became the Differentiated Model of Giftedness and Talent (DMGT) which was derived from his initial conception of giftedness and added a quantitative benchmark (Gagné, 1999). This framework is important for this study because it emphasizes the importance of environmental factors, such as the teacher and the classroom experience, in developing talent in gifted students. Gagné's main critique of Renzulli's model of giftedness was that if motivation is required in conjunction with the other two clusters it has the potential to ignore students with gifted behaviors. Gagne believed that motivation serves as a major catalyst for the actualization of giftedness to talent. Both frameworks will be described in greater detail in the literature review section of this study.

Subjectivity Statement

It is important for me to acknowledge my own subjectivity to ensure that my research is valid, particularly before data is collected, rather than retrospectively (Peshkin, 1988). In assessing my own identity and the possibility of unintended bias towards the data and the participants from whom data will be collected, I must declare the roles which shape my position as a researcher: 1) I am a White, cisgender female who was identified as gifted during my elementary years in the public schools of Florida, 2) I am a current elementary school principal in an academically rigorous independent school setting, 3) I was a gifted education teacher and specialist, and 4) I was a math coach for elementary teachers. I anticipate that my classroom teacher experience will create a sense of comfort and empathy around shared experiences while observing and interviewing teachers. Admittedly, I realize that my role as a principal may have caused some teachers to feel nervous or overly self-conscious about their teaching abilities.

However, my current position serves as a commonality in my interactions with the school principals or other administrators at the school where I conducted my research.

In practicing the important stance of reflexivity, I acknowledge that just like teachers in other schools, some of the teachers in my own school do not have strong conceptual mathematical knowledge and understanding or an understanding of mathematically gifted students and how best to serve them. As a classroom teacher, I grew in my appreciation for advanced and novel mathematical concepts while trying to meet the needs of certain students. Eventually I started an after-school math club in collaboration with a university mathematics professor for students with a passion, interest, or ability in solving challenging mathematical problems.

Statement of the Problem

Students in the United States are not achieving in mathematics as indicated on the NAEP (2019) exams and other measurements of student achievement (OECD, 2019; O'Dwyer et al., 2015; NCES, 2019). Mathematically gifted students are especially impacted by this phenomenon, though it is not exactly known what factors contribute to successful teachers of mathematically gifted and promising students. Limited in the literature is the influence of teacher beliefs about mathematics and gifted students on the teacher's classroom instructional practices, as well as the teacher's conceptual mathematical understanding as it relates to mathematically gifted students. This study contributes to the literature by identifying the conceptual mathematical knowledge and understanding needed, and the types of beliefs and behaviors, especially instructional practices, exhibited by successful teachers of mathematically gifted students.

This research study is important because of the potential implications for all elementary students, particularly those with mathematical promise, and their success in mathematics. The literature from the past 50 years indicates that students with mathematical promise lack opportunities to nurture their interests and gifts (Sheffield, 1999, 2016; Stepanek, 1999). For students of color, the issues of access and equity are even more pronounced (Davis & Martin, 2008; Tate, 2005). More than two decades ago, the National Council of Teachers of Mathematics (NCTM) commissioned a Task Force whose members were charged with broadening the traditional idea of mathematically gifted to be more inclusive of all students who show promise as a function of their ability, motivation, belief, and experiences or opportunities. The implications of teachers being unprepared to support the needs of mathematically gifted students are likely to have detrimental effects on students being challenged and growing their potential as mathematicians (Sheffield et al., 1995).

These factors highlight the challenges all students in the United States face, and specifically those students identified with having mathematical promise, in meeting their individual potential. While studies have been conducted on teacher preparation in mathematics, less is known about the combined preparation needed for the successful engagement of mathematically gifted and promising students, and the beliefs and instructional practices of teachers who instruct these students.

Purpose of the Study

The purpose of this qualitative phenomenological case study was to explore the experiences of teachers who work with mathematically gifted students through their beliefs, instructional practices, and conceptual understanding of mathematics. Phenomenology is a type of qualitative research used to understand and describe an event from the point of view of the participant (Mertens, 2015) and will be used as a framework to guide this study. To identify how an individual makes sense of an issue or phenomenon, it is important to look at experience to create a deep understanding of the phenomena (Creswell, 2007). Using semi-structured interviews, questionnaires, and observations, I described the phenomenon of teachers' instructing elementary and middle school students who have been identified as mathematically gifted in a K-8 public charter school for gifted students. I specifically chose this setting to study teachers who work exclusively with mathematically gifted students and therefore may possess a unique view of this experience. As a result of this study, teacher educators, gifted specialists, and administrators may better understand the experiences of the teachers, potentially highlighting the teacher preparation, professional development, and recruitment of teachers who work with mathematically gifted students. The question "What are the lived experiences of teachers instructing mathematically gifted students?" is the focus of this study. Specifically, this study will describe teacher beliefs about mathematics and gifted students, teacher instructional practices, and teachers' descriptions of their conceptual understanding of mathematics as it relates to mathematically gifted and promising students. This study addresses the lack of student achievement in mathematics in general and sought to understand the influence these phenomena specifically have on mathematically gifted and promising students.

Research Questions

Based on the literature, there is a need for research that examines the instructional practices teachers use with mathematically gifted and promising students and how teachers' beliefs about gifted students, beliefs about mathematics, and their conceptual understanding of mathematics influence their use of those practices. The following research questions are directly tied to the purpose of this study in response to the problem statement presented. To that end, this study sought to understand the overarching central question or phenomenon of the lived experiences of the teachers of mathematically gifted students, and was specifically guided by the following research questions:

1. What are the teachers’ instructional practices with mathematically gifted and promising students?
2. How are the teachers’ beliefs towards mathematics and beliefs towards gifted students reflected in their instructional practices with mathematically gifted and promising students?
3. How do teachers describe their conceptual understanding of mathematics as it relates to the teaching of mathematically gifted and promising students?

Significance of the Study

This research study is important because of the potential implications for mathematically gifted and promising students and their success in mathematics and future contributions to the fields of mathematics and science. The implications of teachers being unprepared or ill-equipped to support the needs of mathematically gifted students are likely to have detrimental effects on students being challenged and growing their potential as mathematicians (Sheffield et al., 1995). Further, a better understanding of the types of beliefs teachers should hold to be successful with mathematically gifted students may be helpful to better meet the needs of those students and to develop their mathematical promise.

Conducting this research provided greater understanding on the current state of elementary teachers in terms of conceptual mathematical knowledge and understanding, instructional practices, and beliefs towards the subject and students, and how these factors influence the experiences of elementary students, particularly those with mathematical promise.

This understanding could benefit school principals and other school leaders in recruiting and developing educators for these students, as well as teacher educators in designing courses in higher education.

Definitions of Key Terms

The following key terms and concepts will be defined in this study as follows:

Beliefs: A teacher's system of beliefs, values, conceptions, and ideology that teachers may hold about mathematics (Ernest, 1989). "Beliefs underpin personal thought and behavior. They underlie dispositions to engage in certain practices and not others" (Schoenfeld, 1996, as cited by Swan, 2006, p. 59).

Conceptual mathematical knowledge: Knowledge that involves understanding concepts and recognizing their applications in various situations (Ben-Hur, 2006; Ma, 1999/2010).

Conceptual understanding of mathematics: Knowledge that involves understanding concepts and recognizing their applications in various situations; can be referenced to both teachers and students.

Disposition: The values, commitments, and professional ethics that influence behaviors toward students, families, colleagues, and communities and affect student learning, motivation, and development as well as the educator's own professional growth. Dispositions are guided by beliefs and attitudes related to values such as caring, fairness, honesty, responsibility, and social justice (National Council for Accreditation of Teacher Education, p. 53).

Exceptional mathematical promise: includes those who are talented or express higher levels of interest in mathematics as well as students who are identified as gifted in mathematics through a battery of standardized assessments (NCTM Position Statement, 2016).

Mathematical knowledge for teaching (MKT): Professional knowledge of mathematics different from that demanded by other math intensive occupations such as engineering or accounting; involves examination of student work, analyzing student errors, and using models and representations (Ball et al., 2005).

Mathematical promise: A function of ability, motivation, belief, and experience or opportunity (Sheffield et al., 1995).

Mathematically promising students: A term used to describe individuals who have the ability, motivation, and belief to perform at an above average level in mathematics.

Mathematical giftedness: A unique aggregate of mathematical abilities that opens the possibilities of successful performance in mathematical activity (Krutetskii, 1976, p. 77); the ability to generalize easily, extend, create, and invent new methods of solving mathematical problems; to strive "for the cleanest, simplest, shortest and thus most 'elegant' path to the goal" (Krutetskii, 1976, p. 187). Additionally, this is described by Krutetski (Schindler & Rott, 2016) as the ability for logical thought in quantitative and spatial relationships and to think in mathematical symbols; mathematical memory and a mathematical cast of mind (striving to make the phenomena of the environment mathematical, noticing spatial and quantitative relationships everywhere.

Procedural Knowledge: "Procedural knowledge is made up of two distinct parts. One part is composed of the formal language, or symbol representation system, of mathematics. The other part consists of the algorithms, or rules, for completing mathematical tasks" (Hiebert & Lefevre, 1986, p.6).

Organization of the Study

This dissertation utilized a qualitative case study approach to gather data from classroom observations, questionnaires, and semi-structured interviews. These data sources were used to examine and respond to the three research questions and better understand the lived experiences of teachers instructing mathematically gifted and promising students. The remainder of the study is organized in the following manner. Chapter II presents a review of the related literature and evolving data regarding mathematics education, gifted education, teacher preparation, and mathematics equity and access. Chapter III delineates the research design and methodology of the study. The setting, description of the participants, instruments used to gather the data, and the procedures followed are described. An analysis of the data and discussion of the findings are presented in Chapter IV. The summary, conclusions, and recommendations of the study are outlined in Chapter V. Finally, the study concludes with the references and appendices.

Summary

Teachers who lack conceptual understanding of mathematics and the beliefs that support effective instruction of mathematically gifted and promising students will limit those students' opportunity and potential to be challenged and to grow as mathematicians (Sheffield et al., 1995). Additional studies are warranted to determine how well teachers are prepared to teach mathematics effectively, especially to those who are mathematically advanced or gifted. Further, there is a need to provide insight into what instructional practices teachers use with mathematically gifted and promising students and the beliefs of those teachers towards mathematically gifted and promising students. This chapter provided an overview of the problem regarding the overall lack of success for U.S. students in mathematics and the study's purpose, which was to better understand the lived experiences of teachers who instruct mathematically gifted students. The conceptual frameworks used as lenses for this research are referenced in Chapter I and presented in greater depth in Chapter II. A review of the literature is provided in Chapter II followed by a description of the methods used in the study in Chapter III, the research findings in Chapter IV, and the discussion and conclusion in Chapter V.

CHAPTER II: LITERATURE REVIEW

Introduction

This study focused on the lived experiences of teachers working with mathematically gifted learners through their instructional practices, conceptual understanding of mathematics, and beliefs regarding mathematics and gifted children to better understand the implications for student achievement in mathematics and future contribution to the field. The existing literature provides context for the identification and needs of gifted learners in general, the mathematically gifted more specifically, and teacher beliefs about gifted students. Also provided is a review of elementary teacher preparation for mathematics instruction, their conceptual understanding and knowledge of mathematics, and the beliefs teachers hold regarding mathematics teaching and learning. The present study adds to the current scholarly contributions to the field by further describing the instructional practices in math classrooms, student access to challenging mathematics, and the overall lack of achievement in mathematics for U.S. students, especially for mathematically gifted and promising students.

This chapter will first provide an overview of gifted education, including identification of students, national and state standards, differentiation of instructional practices, and teacher beliefs about gifted students. The next main section will focus on mathematics teaching and learning, including teacher preparation, standards, measurement of math teaching, and teacher conceptual understanding and knowledge of mathematics. Included in this section will be teacher beliefs about mathematics and issues of equity and access to challenging mathematics. To synthesize the first two sections, a review of the literature pertaining to the identification and traits of mathematically gifted students, their experiences, and the teachers of these students will be provided. Finally, two conceptual frameworks that offer a lens through which this study will be positioned are provided.

Gifted Education Overview

Brief Historical Review of Giftedness

The insidious nature of the history of gifted education cannot be overlooked, particularly when the critical need exists to address a longstanding practice of excluding whole groups of individuals based on a dominant culture mindset. Galton's scientific efforts to study hereditary genius were based on what he described as eminence and his belief that society could be improved by children who inherited these traits from their parents (Parish, 2014). Galton was a British anthropologist and a pioneer of the eugenics movement who published his research in 1869. In the early part of the 20th century an American psychologist named Lewis Terman researched what he considered mentally "superior" children and was largely responsible for creating the popular myth that these children are superior in every way—physically, emotionally, and intellectually. During this period, Binet developed a scale to measure intelligence which was later modified and is now known as the Stanford-Binet test, a tool that is often used to measure giftedness.

Leta Hollingworth also studied gifted children in the United States in the early 1900s and was considered the first counselor of gifted children, studying their social and emotional characteristics in depth (Silverman, 1993). In what is described as the first comprehensive textbook on giftedness, *Gifted Children: Their Nature and Nurture* (1926), Hollingworth used an intelligent quotient or IQ to describe children who were considered highly gifted with IQs above 140. Sometime later these intelligence tests were challenged as being culturally biased and lacking validity (Whitmore, 1980). In fact, much has been written about the use of intelligence or cognitive ability tests and the impact on diverse children who are underrepresented in gifted arenas (Frasier et al., 1995; Ford, 2004; Reynolds, 1998; Valencia & Suzuki, 2001). The field of gifted education struggles to be an equitable and inclusive space for these children in the United States.

Finally, in the 1980s, three prominent theorists outlined their own definitions of giftedness. Renzulli's (1978) three-ring conception of giftedness is described as the interaction of above-average ability, task commitment, and creativity. Sternberg (1985) developed a triarchic theory of intelligence that added a category of contextual giftedness which went beyond the IQ measurement derived from standardized tests, and Gagné posited his model in contrast to Renzuilli's, stating gifted traits do not necessarily translate into talent. In other words, some children may be gifted but not talented. Those gifts may not be revealed due to circumstances such as poverty, learning disabilities, cultural barriers, teacher expectations, and classroom or school opportunities (Ford, 2004). More recently, gifted education scholars (Campbell, 2012; Collins et al., 2019) have focused specifically on the underrepresentation of certain groups of students, such as Black girls, who are often overlooked because teachers base their recommendations on Black girls' behaviors. Novak et al. (2020) acknowledged the long history in this country of underrepresentation of culturally, linguistically, and economically diverse (CLED) gifted students. Though this problem has received substantial attention, a gap in equity and access to gifted programs persists, primarily due to practices related to teacher referrals, assessments, and support structures. According to the authors these issues have their origin in one common underlying issue and that is the teacher's lack of cultural knowledge and competency related to gifted youth. Professional development grounded in equity and social justice topics is critical to address and solve the issue of disproportionality in gifted programs.

Professional learning in culture and cultural differences must be continuous and substantive (Novak et al., 2020).

Identification of Gifted Students

Identifying gifted students in the United States has a long history, but the process was first defined by the federal government in the 1972 Marland Report in Public Law 91-230, Section 806 (Marland, 1972): "Gifted and talented children are those identified by professionally qualified persons, who by virtue of outstanding abilities are capable of high performance." The National Association of Gifted Children (NAGC, 2021) noted that roughly 6-10% of students are gifted and could use additional support in the classroom. However, misunderstandings abound regarding the characteristics of giftedness and no single definition of gifted exists across states or local school districts (McIntyre, 2016). Many students go unrecognized and therefore "un-taught" as their behaviors may be overlooked or misrepresented by teachers who are unaware of what mathematical giftedness looks like in elementary age children**.**

It is no wonder that teachers are unprepared to properly identify gifted students from all backgrounds based on the lax and inconsistent requirements during pre-service education and in-service professional development. The NAGC's *State of the States* report (2015) found that most general education teachers were unlikely to be required to receive any training or professional development in gifted and talented education. Only one state (Nevada) required a separate course in gifted education at the pre-service level. Twelve states reported that all pre-service teacher candidates were required to receive coursework by teacher preparation programs or by Local Education Agencies (LEAs). Thirty-nine states reported requirements for general education teachers to receive professional development on gifted students *after* initial certification, with only five states requiring this training through policy, but without any set number of hours.

Twenty-three states leave it up to LEAs due to state policy (5), or absence of state policy (18), while another 11 make it voluntary (NAGC, 2015). This issue is even more complicated when incorporating the issues of bias based on race towards students of color.

In its position statement on giftedness and best practices, the NAGC (2019) noted that gifted students come from all racial, ethnic, cultural, and socio-economic populations. Schools may contribute to an underrepresentation of these groups through identification practices that do not cast a wide net to include all students. Some gifted students may be twice exceptional, possessing a disability in one or more domains (dyslexia or Autism, for example). NAGC recommends a strengths-based, talent-focused approach for identification and programming models.

The state of North Carolina, where this study occurs, mandates that public schools identify and serve academically or intellectually gifted (AIG) students in grades K-12. The North Carolina Department of Public Instruction (1996) defined AIG students using this definition:

> Academically or Intellectually Gifted (AIG) students perform or show the potential to perform at substantially high levels of accomplishment when compared with others of their age, experiences, or environment Academically or Intellectually Gifted students exhibit high-performance capability in intellectual areas, specific academic fields, or in both the intellectual areas and specific academic fields. Academically or Intellectually Gifted students require differentiated educational services beyond those ordinarily provided by the regular educational program. Outstanding abilities are present in students from all cultural groups, across all economic strata, and in all areas of human endeavor. (Article 9B (N.C.G.S. § 115C-150.5))

State (North Carolina) and National Gifted Standards

Following the adoption of the state's definition of giftedness, North Carolina developed program standards to serve as a framework for local agencies (North Carolina Academically or Intellectually Gifted Program Standards, 2009/2021). These standards reflect student identification, programming, differentiated curriculum and instruction, professional development, partnerships, and accountability. Of significance is the state's goal to promote both equity and excellence, henceforth the connection of these standards to North Carolina's Critical Actions to Realize Equity and Excellence in Gifted Education (NC Department of Public Instruction, 2021). A call to reframe your lens, use equitable identification practices, provide a range of services within the program, foster talent development, collect and use meaningful data, and provide focused professional learning opportunities make up the six action steps in this plan.

On the national front, standards for teacher preparation as outlined by the National Association of Gifted Children-Council on Exceptional Children (NAGC-CEC) (2013) focus on expectations for beginning gifted education professionals. Emphasized are learner development and differences, learning environments, curricular content knowledge, assessment, instructional planning and strategies, professional learning and ethical practices, and collaboration. Though important to gifted education specialists, these practices are not directed to classroom teachers who serve on the front line in providing experiences for gifted students. NAGC responded by outlining these standards (pulled from the larger set of standards) for all teachers of gifted students (NAGC, 2014):

All teachers should be able to:

1. recognize the learning differences, developmental milestones, and cognitive/affective characteristics of gifted and talented students, including those

from diverse cultural and linguistic backgrounds, and identify their related academic and social-emotional needs;

2. design appropriate learning and performance modifications for individuals with gifts and talents that enhance creativity, acceleration, depth and complexity in academic subject matter and specialized domains; and
3. select, adapt, and use a repertoire of evidence-based instructional strategies to advance the learning of gifted and talented students.

Closely aligned to the NAGC teacher preparation standards are the national programming standards for gifted students (NAGC, 2019). Like the state of North Carolina's programming standards, these include learning and development, assessment, curriculum and instruction, learning environments, programming, and professional learning.

Differentiation of Instructional Practices

Tomlinson (1995) defined differentiation as the use of a variety of instructional approaches to modify content, process, and products in response to the learning readiness of students. Rotigel and Fello (2004) stressed the importance of assessment to properly identify students who need more than what the grade level experience can provide. Once students are identified, however, the effort required of teachers to differentiate lessons, plan for small groups, and access additional resources is steep. Unfortunately, few published studies examine the effects of differentiation on students or teachers, and fewer label it as differentiation (Rubenstein et al., 2015). Teachers may not be equipped to differentiate content for gifted students due to a lack of time or pedagogical content knowledge and the perception that gifted students would succeed anyway regardless of what teachers do. Teachers may even resort to fun activities due to the "variety of reasons (e.g., time, desire, confidence, ability, and high stakes testing) why differentiation in the regular classroom is challenging for teachers to implement" (Rubenstein et al., 2015, p. 143). Using Tomlinson's Differentiation of Instruction Model and other models for gifted students, Rubenstein et al. (2015) provided third-grade teachers with mathematics curricula that used pre-assessments to place students in flexible groups and then used tiered activities commensurate with the needs of the group. They found that many teachers could differentiate if they have the support and materials.

Singer et al. (2016) described Tomlinson's (1999) view on differentiation and pre-service teacher preparation to meet the needs of gifted and other academically diverse students. Tomlinson found that the teachers affirmed the existence and importance of recognizing student differences and needs, but they utilized vague criteria for identification of needs, expressed incomplete understanding of differentiating instruction, and had few strategies for enacting differentiation. Workshops on differentiation strategies may provide the training needed; yet, when the training was translated into practice, several issues arose due to a lack of collaboration between the coach, the teacher, and the university supervisor.

In her work with teachers of mathematically gifted students, Stepanek (1999) discussed a teacher who differentiated in her non-ability grouped classroom by using pretests and observations to provide appropriately challenging questions, a focus on concepts, and opportunities to go deeper. "Differentiating instruction is difficult. It is not something I feel that I have mastered, because it requires constantly reflecting on what works with my students and what doesn't" (Stepanek, 1999, p. 18).

Teacher Beliefs About Gifted Students

According to Troxclair (2013), attitudes of teachers influence the teaching-learning process and include feelings and cognitive beliefs about someone or something and the behaviors that result from those feelings and beliefs. Furthermore, cognitive beliefs are influenced by information or knowledge that individuals embrace or have been exposed to, such as in a teacher education program. In Troxclair's (2013) study of beginning teachers' perspectives on high ability students she noted, "Teacher education programs provide prospective teachers with knowledge and information that become part of their cognitive belief system" (p. 54). Findings from her study showed negativity towards gifted students based on attitudes reflected in societal myths about ability grouping, acceleration, and elitism. Recommendations were made for providing preservice teachers accurate, research-based information to develop accurate cognitive beliefs.

Research-based practices for serving gifted students identified in the earlier version of the NAGC (2010) *Pre-K-Grade 12 Gifted Programming Standards* include ability grouping, assessment, differentiation, acceleration, and collaboration. However, Johnsen and Kaul (2019) found limited implementation of these practices is related to teacher beliefs which are difficult to change or modify especially if the teacher's professional beliefs and the teaching strategy are incongruent. Beginning teachers possess preconceived beliefs based on the way they were taught and their own experiences in the field. Additionally, beliefs about their own abilities to use some practices or their beliefs about students' abilities can impact how teachers implement certain lessons. Educators must embrace certain beliefs to successfully support GT students (Johnsen & Kaul, 2019).

Matheis et al. (2018) cited research indicating that teachers' beliefs include theories of teaching and learning which are considered subjective, as well as theories about the characteristics of their students. Teachers may hold incorrect beliefs about gifted students' characteristics which can negatively affect their expectations towards gifted students and how they behave towards them. Subjective theories, according to Matheis et al. (2018) are based on stereotypes and assumptions held by individuals. Their study showed teachers held positive ratings of gifted students' intellectual abilities, but inaccurate and negative beliefs about those same students' non-cognitive characteristics, the latter of which lowered their motivation to teach them. A recommendation for teacher education programs to include an emphasis on gifted education was provided.

Mathematics Teaching and Learning

To become mathematically literate and therefore successful in mathematics and problem-solving, students must develop strong foundational understandings in their early elementary years (Southern Regional Education Board [SREB], 2019). In fact, number sense develops well before children enter school and those who engage in math games and number activities with their families are likely to have better math competence than those who do not (Jordan et al., 2009). Many children's math skills are not fully nurtured when they do start school at the elementary level. If students fail to master these early concepts they will struggle later as math concepts build on each other. In a report that outlined how schools could help students develop skills for success as citizens, many of the competencies related to math were also considered 21st-century learning skills. These skills include reasoning, problem-solving, and critical thinking (SREB, 2019). However, as the three data sets shared in this paper have revealed (PISA, TIMMS, NAEP), U.S. students struggle with these higher order thinking skills in national and international assessments. Further exploration in this review of the literature shows that teachers tend to focus more on procedural knowledge over conceptual knowledge when teaching mathematics at the elementary level.

Quality of Mathematics Teaching

One obstacle to the goal of creating mathematically literate and successful students is the quality of mathematics teaching, which is dependent on teacher conceptual understanding of the content (Ball et al., 2005). In Liping Ma's groundbreaking study (1999), she compared the mathematical knowledge of U.S. and Chinese elementary teachers and found U.S. teachers' mathematical content knowledge was weak in comparison to the Chinese teachers. Ma was struck by the general misunderstandings of the U.S. teachers. For example, in her study, 83% of the U.S. teachers vs. 14% of the Chinese teachers displayed only procedural knowledge (and no conceptual knowledge) of subtraction with regrouping.

Data gathered from a qualitative study on the use of pre-differentiated and enriched mathematics curricula (Rubenstein et al., 2015) showed that teachers expressed concern about not understanding the math or making errors, with one teacher noting that she was "comfortable with the third-grade math, but fourth-grade math may be beyond her ability" (Rubenstein et al., 2015, p. 157). When observing classroom teachers teaching math, the researchers found that the teachers made conceptual errors while teaching the study's curriculum. This curriculum was geared towards high-ability students and Rubenstein and her team noted that to successfully differentiate, teachers "must have a conceptual understanding of third-grade mathematics, understand how it builds on previous years, and know how it will prepare students for future classes" (Rubenstein et al., 1995, p. 157).

Classroom Instructional Practices

In a study based on the 1999 TIMMS data, researchers analyzed math classroom observation videos in the United States alongside classrooms in six high-performing countries (O'Dwyer et al., 2015). The researchers noted that teachers in the United States spent more classroom instructional time having students complete low-level and repetitive exercises instead of applying their skills and extending them to new and different problems, ultimately presenting fewer challenging lessons than the other six countries. The video observations also revealed that teachers in the higher-performing countries use strategies that develop conceptual knowledge over procedural skills. In the United States, more time is spent on the review of previously taught material and students experience more interruptions to lessons. U.S. teachers introduce less advanced content and cover an array of unrelated skills and concepts rather than focusing deeply on a single coherent topic (Hiebert et al., 2005). Similarly, Prawat (1989) connected the poor mathematical performance of U.S. students to teachers spending more time on procedural knowledge and fact learning at the expense of conceptual understanding.

Learning in the company of teachers who value the work of students, treat students like sense-makers, and adapt their teaching based on their learners utilize instructional practices in ambitious teaching (Lampert et al., 2013). The authors of this study focused on novice teachers and their use of instructional practices that included managing student engagement and interest, viewing students as competent, eliciting and responding to student contributions to a discussion, capturing student thinking on the board, and facilitating student discourse with each other. These practices are intended to maximize access to meaningful mathematics (Lampert et al., 2013).

Conceptual Understanding and Procedural Skills

Conceptual knowledge and conceptual understanding are terms that are often used interchangeably. Hiebert and Lefevre (1986) defined conceptual understanding as “Knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete pieces of information” (pp. 3-4). The authors also cited the relationship between conceptual understanding and procedural skills as a central issue in mathematics. They define procedural knowledge as two parts: one that consists of the formal language or symbol representation systems, and the other part which includes the algorithms, or the rules, for completing math tasks. Relationships between conceptual and procedural knowledge have a long history in the study of mathematics and are important to study (Hiebert, 1986). If more was understood about how individuals acquire these two types of knowledge and their connection, it would likely reveal some of the issues in math performance. (Hiebert & Lefevre, 1986).

Similarly, Ben-Hur (2006) described *conceptual knowledge* in mathematics as the understanding of concepts and the ability to recognize their applications in a variety of situations. *Procedural knowledge*, on the other hand, involves the use of mathematical skills to solve problems, with the aid of pencil and paper, calculator, and so on. Throughout history, mathematicians have invented procedures based on mathematical concepts. The National Council of Teachers of Mathematics (NCTM) standards require that students know how to do the procedures, but with an understanding of the concepts connected to them.

Ma (1999/2010) studied how elementary teachers in the United States see mathematics from the standpoint of procedural knowledge instead of conceptual knowledge, whereas their Chinese counterparts consider these two aspects connected and interwoven. She noted, “Teachers who had a conceptual understanding of the topic and intended to promote students’ conceptual learning did not ignore procedural knowledge at all. In fact, from their perspective, a conceptual understanding is never separate from the corresponding procedures where understanding lives” (Ma, 2010, p.115). According to Browning et al. (2014), preservice teachers tend to struggle with understanding and representing fractions, understanding decimals and place value concepts, and solving algebraic problems flexibly. The researchers concluded that teachers may have the algorithm to help them solve a math problem, but they are often unable to explain why the algorithm works. If a teacher does not understand and cannot explain why a process works, it will be difficult to fully teach that process or troubleshoot a student's difficulties or mistakes when students try to apply it themselves.

Teacher Conceptual Understanding of Mathematics

Conceptual understanding applies to teachers as well as students. Based on the research cited previously, conceptual understanding of mathematics will be generally defined here as knowledge that involves understanding concepts and recognizing their applications in various situations. Liping Ma (1999), in her study on U.S. and Chinese elementary teachers, compared conceptual understanding to procedural understanding in what she described as knowledge packages. She found that those teachers who possessed only procedural understanding in subtraction with regrouping, for example, could not explain the concept. In fact, one teacher (from the United States) analogized the algorithm as similar to a mother borrowing sugar from a neighbor! In Ma's description of a teacher with conceptual understanding of the same math topic, the teacher described the concept of decomposing a higher value unit and understood the procedures, concepts, and basic principles of the concept itself. For U.S. teachers, only 17% displayed conceptual knowledge of regrouping in subtraction, compared to 86% of the Chinese teachers (Ma, 1999).

Profound Understanding of Fundamental Mathematics

Ma was surprised at the stark differences between the U.S. and Chinese teachers and was particularly struck by the general misunderstandings of the U.S. teachers regarding math concepts. At the core of her research, she found that Chinese teachers are much more likely to have what she called "profound understanding of fundamental mathematics" (2010, p. xx). They do not necessarily know more than U.S. teachers, but what they do know, they know more deeply and flexibly. Like the first scenario regarding subtraction with regrouping, she describes a second scenario, this one involving multi-digit number multiplication. Here, she found that 61% of the U.S. teachers and 8% of the Chinese teachers could not provide conceptual explanations for the procedure of multiplication. Recall the PISA data which showed the math achievement of Chinese students to be much greater when compared to U.S. students.

Instructional Practices for Conceptual Understanding

Students need regular opportunities to reflect on their learning with a knowledgeable teacher and one another to develop conceptual understanding (Ben-Hur, 2006). Hiebert and Grouws (2007) suggested two key instructional practices to promote conceptual understanding in students. The first one is allowing students to struggle with problems and the second practice is for teachers to be explicit when discussing conceptual relationships. Utilizing these approaches provide students with the support needed to apply their skills and knowledge to new contexts and problems.

Mathematical Knowledge for Teaching

Numerous studies have investigated teacher knowledge of mathematics and how it influences teaching effectiveness (Begle, 1972; Fennema & Franke, 1992; Hill et al, 2004; Ma, 1999/2010). In their study of preservice teachers, Bartell et al. (2013) investigated the role of content knowledge and the teachers' ability to recognize students' mathematical understanding and found that content knowledge alone is not enough to support the preservice teachers' analysis of the way children think mathematically. Notably, Schulman (1986) studied teacher knowledge extensively and developed a framework that included three categories of content knowledge. The first, subject matter content knowledge, includes what teachers need to know in a particular domain. This differs from pedagogical content knowledge which Schulman described as the way a teacher represents the content for others to comprehend, including an understanding of the preconceptions and misconceptions learners bring to the table. The third category is curricular knowledge, as in the textbooks and materials of a subject that teachers must know well. In Ball et al. (2001), the authors lament the field's insufficient understanding of the mathematical knowledge (what Schulman would see as the subject matter content knowledge) it takes to teach well. They recognize, however, that even a specialized package of math knowledge does not ensure the teacher will be equipped with the flexibility needed to manage a complex practice.

Ball and her colleagues began to connect mathematical knowledge to teaching practice and reference the pedagogical knowledge for teaching derived from Schulman's work (1986). This knowledge, which they called a "mathematical knowledge for teaching" (Ball et al., 2005), is a kind of professional knowledge of mathematics that differs from the kind of math needed in physics, accounting, or engineering. Categorizing this kind of teaching, Ball and her colleagues developed the concept of Mathematical Knowledge for Teaching (MKT), which they later refined as a framework to define and categorize four primary domains: common content knowledge, specialized content knowledge, knowledge of content and students, and knowledge of content and teaching (Ball et al., 2007). Thames et al. (2008) noted that mathematical knowledge for teaching (MKT) elaborates Schulman's (1986) pedagogical content knowledge rather than replaces it. To help students be successful mathematicians, teachers must be able to use error analysis, representations, and other common tasks of teaching that involve both mathematical reasoning and pedagogical thinking (Ball et al., 2005).

To test their hypothesis on teachers' mathematical knowledge for teaching (MKT) and the impact on student learning, Ball and her team (2005) created a domain map that described the topics and knowledge to be measured, namely number and operations, and patterns and algebra, and designed over 250 multiple choice items designed to measure teachers common and specialized knowledge for teaching. The goal of the study was to determine if high performance on these assessments was related to effective instruction. In an analysis of more than 700 elementary teachers, the researchers found that teachers' performance on the assessments significantly predicted the size of student gain scores on a standardized test, even though they controlled for variables of teacher experience, credentials, and student SES.

One additional finding from this same study was that higher-knowledge teachers tend to teach non-minority students, leaving diverse students with less-knowledgeable teachers. The researchers argued for the critical need to invest in the quality of mathematics content knowledge for teachers in diverse schools and point to the impact of their study's summer professional development institutes which resulted in greater performance on the teacher assessments (Ball, 2005).

Criticisms of Measuring Teacher Knowledge

It should not be assumed that all teachers have the kind of content knowledge needed to teach math well. One recommendation is to conduct more research that includes measures of teachers' mathematical knowledge (Rubenstein et al., 2015). How to effectively measure a teacher's content or subject matter knowledge and mathematical knowledge for teaching has not been resolved and, in some cases, is disputed and controversial. Testing teachers, conducting large scale studies, or using students' standardized test scores as measures draw sharp criticism from those who say these efforts "deprofessionalize" teachers (Ball et al., 2005). Some believe that a teacher's professional knowledge should not be challenged and that all teachers deserve authority by the nature of their profession. However, the researchers argued that these criticisms run counter to the essence of the critical agenda that faces the professional teaching community. They believe that "Isolating aspects of knowing mathematics different from that which anyone who has graduated from sixth grade would know and demonstrating convincingly that this knowledge matters for students' learning, is to claim *skill* in teaching, not to *deskill* it" (Ball et al., 2005, p. 45).

Teacher Preparation in Mathematics

Ball et al. (2005) also acknowledged that most U.S. adults struggle in math because they are the graduates of the very system that needs improvement. One solution is to require teachers to study more mathematics or stipulate a subject matter major. Another solution suggested that teachers should be recruited from more selective colleges. Critics wonder if teachers need advanced courses like calculus or only what they will teach to elementary students. Many of these solutions are unproven and even controversial (Ball et al., 2005), and more research that compares teacher mathematical preparation and student achievement is needed.

In 2001, the National Research Council published a study on pre-K-8 math teaching and learning, titled *Adding It Up*. The report outlined the changes in instruction from the 1950s, which saw a greater emphasis placed on conceptual understanding but then swung back to an emphasis on computational fluency in the 1970s. These and other back-and-forth changes have been detrimental to math learning according to the group. The committee proposed five intertwined strands of proficiency for students that included both conceptual and procedural abilities, adaptive reasoning, a productive disposition, and the ability to represent and solve math problems (National Research Council, 2001). Although math standards are stronger and higher today, they cannot be implemented without well-trained teachers. As the committee noted,

The mathematical education they received, both as K-12 students and in teacher

> preparation, has not provided them with appropriate or sufficient opportunities to learn mathematics. As a result of that education, teachers may know the facts and procedures that they teach but often have a relatively weak understanding of the conceptual basis for that knowledge. Many have difficulty clarifying mathematical ideas or solving problems that involve more than routine calculations. (National Research Council, 2001, p. 372)

Further evidence points to the critical issue that elementary teachers do not have the skills needed to teach mathematics based on the poor preparation received at many undergraduate institutions. The National Center for Research on Teacher Education (NCRTE, 1991) study showed that most U.S. teacher preparation programs in universities focus on how to teach math rather than on the math itself. More recent evidence highlighted the specifics on the content offered in these programs. For example, in 2016, the National Council on Teacher Quality (NCTQ) reviewed 860 such programs and concluded that only 13 percent covered critical math topics, including numbers and operations, algebra, geometry, and data and probability (Durrance, 2019). Similarly, but at the graduate level, a 2018 review of graduate elementary preparation programs found that just 1% of 201 programs covered these same topics (Durrance, 2019). This lack of teacher preparation across the United States may be indicative of why one in four teacher candidates failed the math portion of a common elementary licensing exam the first time they took it, according to data in NCTQ's 2019 report *A Fair Chance* (Putman & Walsh, 2019).

Ma (1999) saw three distinct periods during which teachers' subject matter knowledge of mathematics could be developed. In China, this begins when teachers are still students themselves and where they attain mathematical competence. It then grows in teacher education programs, and finally, the third period occurs during their teaching careers when they obtain a "profound understanding of fundamental mathematics" (PUFM). This is not the case with U.S.

teachers who are not likely to acquire competence in K-12 school settings and are unlikely to acquire it later (Ma, 1999).

Standards for Mathematics Teacher Preparation Programs

The mission of the Association of Mathematics Teacher Educators (AMTE) is to promote the improvement of mathematics teacher education in pre-K-12 (AMTE, 2017). The organization has outlined a vision for standards and indicators for teacher candidates and teacher preparation programs. Those standards are based on five key assumptions: (1) Ensuring the success of every learner demands a deep, integrated focus on equity in every program that prepares teachers of mathematics; (2) Teaching mathematics effectively requires career-long learning about teaching mathematics; (3) Learning to teach mathematics requires a central focus on mathematics; (4) Multiple stakeholders should be responsible for and invested in preparing teachers of mathematics; and (5) Those involved in mathematics teacher preparation must be committed to improving their effectiveness in preparing future mathematics teachers.

One of the four AMTE standards for well-prepared beginning teachers of mathematics indicates that candidates must possess robust knowledge of mathematical and statistical concepts that frame the foundation of what they will encounter in the classroom. Pedagogical knowledge for teaching mathematics, understandings of student knowledge and skills, and the teacher’s role as advocates for all students make up the remaining standards.

A consistent theme resonates through the recommendations of educational researchers, policy makers, and professional organizations. Focused attention and collaboration among all stakeholders—policy makers, teacher education programs, and pre-K-12 school systems—must be in place to improve the quality of mathematics teaching. This should begin with a universal understanding of the content knowledge and pedagogical practices elementary teachers need and then measuring this to ensure that teachers are prepared before stepping into the classroom.

Mathematics for All

Another consistent theme related to preparing teachers of mathematics is ensuring that ALL students experience success in math. However, it is clear from the literature that all students do not have the same mathematical experiences based on factors of race, ethnicity, and gender. Yet, some scholars refute the current reform movements as they focus on a universal approach to teaching and learning mathematics. Gutierrez (2018) called for a central focus on Black, Indigenous, and Latinx students in ways that build upon their strengths rather than helping them "do well by Whitestream standards" (p. 2). She provided a strong counter to the idea of training teachers to look for misconceptions as this may be an effort to dehumanize a student's home language or home country's algorithm. Gutierrez sought to rehumanize teacher practices and policies in mathematics and preferred rehumanizing as an ongoing process over the well-known term equity. Rehumanizing practices include an acknowledgement of culture/histories, and feelings of joy and ownership.

Davis and Martin (2008) offered two important insights regarding African American students' experience in math classrooms. First, many African American students experience low-quality mathematics instruction designed purely to pass a standardized math test rather than develop critical thinking and problem-solving skills (Davis, 2008; Lattimore, 2001, 2003, 2005a; Tate, 1993). Remedial courses and strategies become the experience of these students to comply with federal, state, and local goals. This comes in the form of worksheets, practice, and other low-level mathematics work that emphasize repetition, drill and memorization, and test-taking skills (Davis, 2008; Ladson-Billings, 1997; Lattimore, 2001, 2003, 2005a).

Consequently, if African American students receive low-quality instruction in mathematics during their elementary and middle school years, their access to advanced courses becomes severely limited as these standardized tests then become a “gatekeeper” in providing African American students access to higher-level mathematics, gifted and honors programs, and future aspirations (Berry, 2005; Davis, 2008; Lattimore, 2001, 2003, 2005a; Moody, 2003, 2004; Oakes, 1990; Sheppard, 2006).

Teacher Characteristics and Beliefs About Mathematics

Teacher characteristics that can be described by their dispositions, beliefs, and attitudes are well documented in the literature. The importance of a sense of joy and excitement about mathematics cannot be underestimated. Members from a task force sponsored by the National Council of Teachers of Mathematics (NCTM) noted that teachers should have access to professional development to address issues and questions such as these:

> Do teachers themselves have the mathematical power to make connections and the mathematical sophistication to see the big picture? Do states require a strong mathematical background for all teachers of mathematically promising students? Are teachers modeling the joy of learning mathematics and are they learners of mathematics themselves? (Sheffield et al., 1995, p. 6)

Elementary teachers often identify more as readers and writers rather than mathematicians, making it easier for students to reflect that identity in themselves. To eliminate the inadequate cycle of math learning in one’s own school experience requires that teachers also see themselves as learners of mathematics. Improvement in student mathematics achievement is dependent on the ability of teachers to be competent and confident in this subject (Sheffield, 1995). In their study on the traits and characteristics that define a good teacher of mathematics, Baier et al. (2019) found that in terms of profession-specific teacher variables, teachers who reported more enthusiasm were more likely to have higher ratings in all dimensions of instructional quality except for cognitive activation even when other predictors were considered. This finding is in line with previous research and points to significance of teacher enthusiasm for overall high instructional quality. Sheffield (1994) posited that teachers who understand mathematically promising students and who engage in their own deep mathematical thinking are needed to serve the mathematically promising.

Three Philosophies of Mathematical Beliefs

Other research has been conducted regarding the beliefs teachers hold about mathematics and from a variety of categories. These include the role of the teacher and student in the classroom, perspectives on how students learn mathematics, and teacher beliefs about mathematics' structure and nature (Campbell et al., 2014). Paul Ernest (1989) outlined a model that emphasized the importance of teachers' beliefs concerning the nature of mathematics and the process of teaching and learning mathematics. Ernest described beliefs as a teacher's system of beliefs, conceptions, values, and ideology and outlined three philosophies that teachers may hold about mathematics. The first is a **problem-solving** view, where mathematics is seen as a field that is continually expanding; the second is a **Platonist** view which sees mathematics as a unified body of knowledge, one that is discovered; and the third is referred to as the **instrumentalist** viewpoint that sees mathematics as a useful, but unrelated collections of facts, rules, and skills. Ernest indicated that a teacher's beliefs about mathematics can have practical applications in the classroom. For example, a problem-solving mindset may mean that the teacher is open to multiple ways of solving a problem, whereas a Platonist or instrumentalist view might result in the teacher modeling that there is just one “right” way to solve a problem (Ernest, 1989).

Swan’s Beliefs Questionnaire

Malcolm Swan (2006) designed a mathematical beliefs questionnaire based on the work of Ernest (1991) who outlined three components to a teacher’s belief system: the teacher’s conception of the nature of mathematics as a subject for study, the nature of mathematics teaching, and the process of learning mathematics. Swan’s instrument was also influenced by Askew et al.’s (1997) report on effective mathematics teachers. Askew characterized the orientations of teachers to the three components as **transmission, discovery,** or **connectionist**. A transmission view sees mathematics as a series of rules that must be formally taught to students and the practiced until mastery is obtained. The discovery orientation sees math as a human creation and encourages students to learn through exploration with the teacher serving as facilitator. A network of ideas that teachers and students must construct together makes up the connectionist view of mathematics where the teacher has a proactive role in challenging students. Swan noted that beliefs cannot be observed, but it is possible to look for consistencies in how teachers describe their reasons for engaging in certain practices (Swan, 2006).

Mathematically Gifted and Promising Overview

So far, this review has centered on the literature of gifted education and mathematics education and will now shift to a synthesis of these two fields to focus on mathematically gifted and promising students and the teachers who instruct them.

Characteristics of Mathematically Gifted and Promising Students (MGPS)

Russian psychologist Vadim Krutetskii (1976), in his groundbreaking empirical study on children, identified qualities of mind that distinguish a student who can excel at mathematics.

Krutetskii's work led to a set of mathematical abilities to be significant for developing a theory of mathematical giftedness which he grouped into categories of obtaining, processing, and retaining mathematical information (Schindler & Rott, 2017). These abilities were closely interrelated and influenced one another and include the ability to think in mathematical symbols, mathematical memory, and a mathematical cast of mind, among others. A mathematical cast of mind means "striving to make the phenomena of the environment mathematical, constantly urging to pay attention to the mathematical aspect of phenomena" (Krutetskii, 1976). In other words, students with this disposition see the world through the lens of mathematics in a wide variety of situations even if those situations do not seem mathematical on the surface. Krutetskii believed that the way to identify mathematically gifted students was to observe them as they solved problems (Gavin, 2005).

Parish (2014) stated that mathematical giftedness should not be confused with terms often used to describe good math students, such as bright, highly able, or high achievers as some gifted children are not necessarily high achievers, quick workers, or those who finish their work early. Ensuring that mathematically gifted students' needs are recognized and met is an effort in student equity. Parish (2014) defined mathematically gifted children, "as those who possess unusually high natural (or instinctual) aptitudes for understanding mathematical concepts, and who therefore differ substantively to their peers in the way they view, understand, and learn mathematics" (p. 515).

Singer et al. (2016) outlined characteristics of mathematical ability that can be seen in younger children before they begin their formal schooling. Self-initiating games with numbers and patterns, a fascination with numbers, and the ability to concentrate for extended periods of time on certain tasks are some of the traits. Once students arrive in classrooms their teachers can then compare them to one another wherein they see some students show a keen interest, ability, and excitement in mathematical problem solving, the ability to think in abstract and logical terms, and a desire to work on special projects related to math (Bicknell, 2008).

Experiences of Mathematically Gifted and Promising Students

In a review of empirical research from the 1970s and 1980s on programs specifically designed for mathematically gifted students, Sowell (1993) found that acceleration, ability grouping, and some technology-based instruction provided mathematically gifted students opportunities to succeed, while most enrichment curricula (four out of five reviewed) showed inconclusive evidence of value and efficacy for such students. This was primarily based on the lack of a clear definition of enrichment programs and a lack of clarity on what qualified students to be labeled as gifted. One study, however, showed significant gains in achievement in an enrichment program for 4th-6th graders when compared to a regular program in math. Similarly, teachers were unclear in what is meant by mathematically gifted and tended to identify students with the best grades or fastest computation (McAllister & Plourde, 2008). When not properly identified and asked to complete grade-level mathematics, students may lose interest in learning, exhibit poor behavior, and eventually become underachievers.

More recently, and specific to mathematics, the National Council of Teachers of Mathematics (NCTM) articulated a position statement (NCTM, 2016) on providing opportunities for students with “exceptional mathematical promise,” defined as “those who express higher levels of interest in mathematics and those who are identified through standardized assessments” (NCTM, 2016, p.1). These students demonstrate focused interest, are eager to try difficult problems, and solve problems in creative, different ways and teachers have a responsibility to identify and nurture them. Considered as those who have the potential to become the leaders and problem-solvers of the future, mathematically promising was meant to go beyond the concept of mathematically gifted who were traditionally defined as the top three to five percent of students based on a standardized mathematics test. This outdated idea of what it means to have mathematical promise means that only a small portion of the population has access to challenging, high-interest mathematics**.** The NCTM Task Force defined mathematical promise as a function of these variables, all of which are fluid and can be developed: ability, motivation, belief, and experience (Sheffield, 2003).

According to Applebaum et al. (2008), mathematically promising students need multiple opportunities to foster creativity, curiosity, and mathematical understanding. Mathematical tasks need to be challenging, with procedures not readily available. In their study they found that mathematically promising students are often learning in heterogeneous classrooms with teachers who lack knowledge of mathematics and of gifted learners. Teachers may also view these students as behavior problems when students choose their own way to solve problems or perform their tasks quickly and become bored. The researchers cite the need for more rigorous research on teacher preparation for mathematically promising students.

Mun and Hertzog (2016) studied a STEM enrichment program for gifted students who experienced mathematically challenging tasks not typically taught in school. The researchers discovered four main strategies teachers used in the program that influenced student interest and enjoyment in mathematics: fostering a collaborative and supportive environment, developing the identity of a mathematician, using open-ended questions for conceptual exploration, and discovery through play.

Teachers of Mathematically Gifted and Promising Students

One key to providing learning environments that nurture powerful enrichment experiences is the preparation and professional development of teachers, especially in mathematics. From the NCTM Task Force (Sheffield et al., 1995), the members noted that teachers should have access to professional development to address issues and questions such as these: “Do teachers themselves have the mathematical power to make connections and the mathematical sophistication to see the big picture? Do states require a strong mathematical background for all teachers of mathematically promising students? Are teachers modeling the joy of learning mathematics and are they learners of mathematics themselves?” (Sheffield et al., 1995, p. 6). Mingus and Grassl (1999) found that teachers must be well versed in math content and willing to take risks to support the mathematically gifted. A community that values mathematics within school districts that support academic programs also influences student success.

Karp (2010) outlined what an effective math teacher of gifted students should possess: knowledge and love of mathematics, skill in promoting students’ learning, and a view of teaching as a creative art. Further, they must have a “deep-rooted familiarity with the difficult problems of school mathematics,” and mentoring by more experienced teachers. “A teacher who does not have this kind of creative potential cannot teach such kids” (Karp, 2010, p. 277).

However, the kind of training teachers receive to work with any gifted students varies by state and few states require that all classroom teachers receive training to address the educational needs of advanced learners (NAGC, 2020). According to the gifted education teacher preparation standards (NAGC-CEC, 2014), all teachers should be able to recognize the characteristics of gifted and talented students and design appropriate learning and performance modifications for them. Leikin (2011) stressed the importance of teacher preparation to effectively teach mathematically gifted students through a series of questions: “Should the teachers of gifted be gifted? Should the teachers be creative to develop students' creativity? How can teachers' creativity be characterized both from the mathematical and from the pedagogical points of view? What are the desirable qualities of teachers' knowledge, beliefs, and personality that make them creative and gifted teachers?” (Leiken, 2011, p. 182).

To further develop the implementation of enrichment experiences for mathematically gifted students, a ninth standard for mathematical practice was recommended in a joint statement by NAGC, NCTM, and NCSM (National Council of Supervisors of Mathematics): “Solve problems in novel ways and pose new mathematical questions of interest to investigate. The characteristics of the new proposed standard would suggest that students are encouraged and supported in taking risks, embracing challenges, solving problems in a variety of ways, posing new mathematical questions of interest to investigate, and being passionate about mathematical investigations” (Johnsen & Sheffield, 2012, pp. 15–16).

According to Sheffield (1994), several traits characterize good teachers of gifted mathematics students and could be described as traits for all good math teachers. These include an enthusiasm for mathematics, the ability to convey the beauty of mathematics, confidence in their own math abilities, a willingness to admit a lack of knowledge and then a desire to model seeking answers, a strong mathematical background, and ongoing professional development work. They should also possess “a flexibility and a willingness to be co-investigators with the students. Students will frequently ask questions that lead the class in directions not foreseen by the teacher. Teachers should be ready and willing to follow the lead of the students as they investigate unplanned areas” (Sheffield, 1994, p. xviii).

Conceptual Frameworks

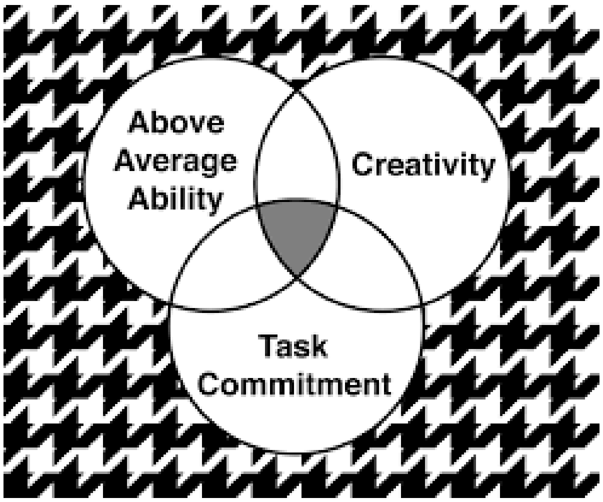
To better understand the conceptual understanding of mathematics and the beliefs and practices required of teachers to successfully nurture students with mathematical promise, two models of giftedness will serve as the lenses for this study: Renzulli's Three-Ring Model (1978) and Gagné's Differentiated Model of Giftedness and Talent (1985). These models were chosen as they help frame the research purpose and questions in this study, namely, to better understand the experiences of teachers of the mathematically gifted and promising. Renzulli's model describes the importance of utilizing more than one trait to identify programming for students to nurture their potential and avoid missing students whose test scores may miss a prescribed cut-off. This approach allows for more students to be recognized and their promise to be developed. Gagné's conception of talent development identifies six domains of giftedness (beyond intellectual) and the catalysts (environment and support) that help develop those gifts into talents. The impact of classroom teachers who are in a prime position to serve as catalysts for the development of talent in young and promising students cannot be underestimated.

Renzulli's Three-Ring Model of Giftedness

In Renzulli's framework, shown in Figure 1, gifted behavior occurs via three basic clusters of human traits, namely, above-average general and/or specific abilities, high levels of task commitment (motivation), and high levels of creativity. As noted in the Schoolwide Enrichment Model (a program for developing giftedness), gifted behaviors can be found in certain people (not all people), at certain times (not all the time), and under certain circumstances (not all circumstances) (Renzulli, 1978). According to the Three-Ring model (Renzulli, 2016), no one piece of criteria should be used to identify intelligence or giftedness. Those who are recognized based on specific accomplishments or creative endeavors possess a well-defined set of traits, all of which connect and relate to one another.

Figure 1

Renzulli's Three-Ring Model of Giftedness (2016)



Each of these traits has a specific definition or characteristics of behavior. In the first cluster, above-average ability includes general and specific ability. General ability is defined as the capacity to process information and engage in abstract thinking, while specific ability is the capacity to acquire knowledge or skill to perform in a specialized area such as mathematics, for example. Renzulli cautioned against the use of a single test score to identify giftedness as it may leave out creative and productive individuals. Task commitment is considered more than just motivation and represents energy that is brought to bear on a specific area of performance and with a specific problem or task one has encountered. Creativity makes up the third cluster of traits in Renzulli's model though it may be difficult to measure this accurately using traditional tests of creativity. It is worth noting that creative, genius, and gifted are words that are often used synonymously (Renzulli, 1978). Renzulli's full definition of giftedness based on the three-ring tasks framework posits that:

> Giftedness consists of an interaction among three basic clusters of human traits — these clusters being above-average general abilities, high levels of task commitment, and high levels of creativity. Gifted and talented children are those possessing or capable of developing this composite set of traits and applying them to any potentially valuable area of human performance. Children who manifest or are capable of developing an interaction among the three clusters require a wide variety of educational opportunities and services that are not ordinarily provided through regular instructional programs. (Renzulli, 1978, p. 9)

The Three-Ring model is considered an operational one in that it is based on research studies that define giftedness, provides guidance for schools and educators to identify students, and can help determine the type of programming and teachers needed that will best meet the needs and help recognize and consider these students as learners with special needs. According to Renzulli (2016), two types of giftedness exist, and both are important and often interact: schoolhouse giftedness and creative productive giftedness. Schoolhouse giftedness is test-taking and lesson-learning giftedness, and generally represents students with higher IQ scores who are also successful in school. But Renzulli cautioned that if schools eliminate the opportunities for students who miss the cut-off for some gifted programs, it is akin to a basketball coach who eliminates kids who are below a specific height. In other words, test scores are not the only predictor for success in schools any more than basketball height is the only predictor for success (Renzulli, 2016).

Creative productive giftedness is represented by three phenomena of creativity as described by Csikszentmihalyi (1996). The first phenomenon relates to those individuals who are considered brilliant by their unique and stimulating thoughts, while the second is based on those who experience the world in novel ways, a personal creativity that impacts just them. The third phenomenon is the type of creativity that impacts others and is caused by those who have changed the world in some significant way. It is this third phenomena of creativity that Renzulli suggested should be developed the most in schools, and most people who earn recognition for their contributions tend to have all three of the interlocking traits described in the model. Considering the teachers who are most equipped to develop this type of creativity, particularly mathematical creativity, is of significant importance. Renzulli provides a clear reminder, “The task of providing better services to our most promising young people cannot wait until theorists and researchers come up with an unassailable ultimate truth, because such truths probably do not exist. We must draw our circles larger so that we don’t overlook any young person who has the potential for high levels of creative productivity” (Renzulli, 2016, pp. 85-86).

Gagné’s Differentiated Model of Giftedness and Talent

Gagné’s conception of giftedness provided a critique of Renzulli’s framework of giftedness and served to describe a clear distinction between giftedness and talent for which he believed had not been done and that, in fact, the two terms were incorrectly used as synonyms. He first defined these terms as follows: “Giftedness corresponds to competence which is distinctly above average in one or more domains of ability. Talent refers to performance which is distinctly above average in one or more fields of human performance” (Gagné, 1985, p. 108). This initial work became the Differentiated Model of Giftedness and Talent (DMGT), which derived from his initial research but added a quantitative benchmark (Gagné, 1999). This framework is important for this study because it emphasizes that one can be gifted, but not talented (as in the case of an underachiever), but not the other way around. In fact, he suggested that one replace the “and” with “or.” This concept of giftedness also espouses the idea of specific ability domains, to which mathematics may be applied, and a way to measure the success of a gift or aptitude when or if it becomes an actual achievement. Further, Gagné believed that individuals (teachers, peers) and provisions (curriculum, pacing, pedagogy) served as catalysts towards producing talents from gifted learners.

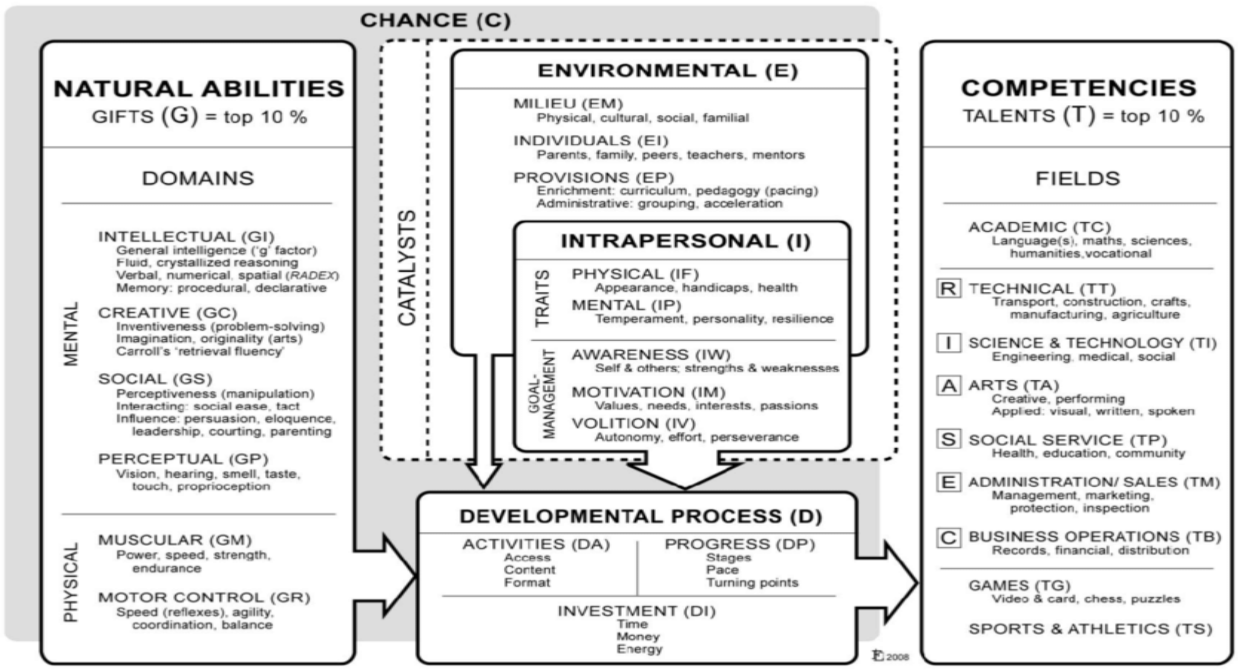
To better understand the knowledge and beliefs needed by teachers to help mathematically promising students become successful and use their gifts, it is helpful to distinguish between these two definitions. According to Gagné's definition and model,

> Giftedness designates the possession and use of untrained and spontaneously expressed superior natural abilities (called aptitudes or gifts), in at least one ability domain, to a degree that places an individual at least among the top 10% of his or her age peers. Talent designates the superior mastery of systematically developed abilities (or skills) and knowledge in at least one field of human activity to a degree that places an individual within at least the upper 10% of age peers who are or have been active in that field or fields. (Gagné, 1999, p. 230)

Five aptitude domains, intellectual, creative, socio-affective, sensorimotor, and "others" (e.g., extrasensory perception), can be observed in every task children are confronted with during the course of their schooling. Of further significance to the present study is Gagné's belief that motivation and environment can serve as a bridge or catalyst between a gift and a talent, where exceptional gifts can become exceptional talents. His main critique of Renzulli's model is that if motivation is required in conjunction with the other two clusters, it has the potential to ignore students with gifted behaviors. Specifically, regarding underachieving children, individuals can possess exceptional abilities without those abilities translating into any kind of academic talent. Figure 2 outlines Gagné's (2011) Differentiated Model of Giftedness and Talent.

Figure 2

Gagné’s *Differentiated Model of Giftedness and Talent*



Summary

The review of the scholarly literature provided an overview of the past and present research that has been conducted regarding gifted education, mathematics, and mathematically gifted and promising students and the teachers who teach them. Emphasis was placed on studies that described the conceptual mathematical understanding and mathematical knowledge for teaching that is needed by teachers to be successful and the characteristics and beliefs that support that success. Studies outlining efforts to provide challenging mathematics to all students in a way that meets their unique needs were also highlighted. To successfully complete this study and embark on the research proposed, it is essential to possess the depth and breadth of knowledge in the field as it pertains to the experiences of teachers of mathematically gifted students. Furthermore, a review of the literature is important to determine the factors that have contributed to the problem of student achievement in mathematics and the specific impact of advanced learners. Through this study it is hoped that the data collected will contribute to and strengthen the existing literature on the conceptual understanding, practices, and beliefs of teachers instructing mathematically gifted and promising students.

CHAPTER III: METHODOLOGY

Overview

The purpose of this qualitative phenomenological case study was to understand the lived experiences of elementary teachers who teach mathematically gifted and promising students and how the teachers' conceptual understanding of mathematics and beliefs about math and gifted students are reflected in their instructional practices. I specifically chose to study teachers who work exclusively with mathematically gifted students and who may possess a unique view of this experience. As a result of this study, teacher educators, gifted specialists, and administrators may better understand the experiences of the teachers, potentially highlighting teacher preparation, professional development, and recruitment of teachers who work with mathematically gifted students. This chapter describes the research design and process of carrying out the study. This includes the procedures and methods of collecting and analyzing data. Detailed information is provided regarding the research instruments used in the data collection process and the data analysis conducted for this study.

Research Design

For this study, a phenomenological case study approach was used to uncover the essence of the experiences of teachers working with mathematically gifted students by using rich, detailed descriptions of the teachers’ experiences as described by the teachers themselves. As the human research instrument, I wrote the questionnaires and interview protocol and conducted the interviews in a one-to-one setting, recognizing that as the instrument I bring my own set of values, biases, and assumptions to the process (Mertens, 2015; Pezalla et al., 2012). I gathered multiple sources of data from the participants and utilized trustworthy methods to analyze the data to determine the essence of the teachers’ experience. Phenomenology is a type of qualitative research which has its roots in philosophy and has been studied in various forms (Moustakas, 1994). Researchers using this design are interested in the analytical and descriptive experiences by individuals in their everyday world, which is referred to as the “lifeworld” in phenomenology (Creswell, 2013). In phenomenological studies, the researcher’s focus is to collect and analyze data in a way that accurately describes the phenomenon or participants’ experiences while avoiding any preconceived assumptions (Groenewald, 2004).

The goal of phenomenological studies is to explain the meaning of perception and phenomenon, and to respond to basic questions such as "What is the experience of the individual regarding the phenomenon?" and "What are the settings and conditions affecting the experience regarding this phenomenon?” (Creswell, 2007, pp. 61-62). In using this approach to answer these questions, researchers are interested in describing what the participants have in common with the phenomenon and to condense those experiences into essential meanings (Creswell, 2007). The purpose of this study was to better understand the lived experiences of teachers instructing mathematically gifted and promising students, thus making a phenomenological approach the most appropriate methodology (Creswell, 2013; Moustakas, 1994). The three research questions for this study were guided by the basic principles of phenomenology which should, according to Moustakas (1994) “give a direction and focus on meaning, and in themes that sustain an inquiry, awakening further interest and concern, and account for our passionate involvement with whatever is being experienced” (p. 94).

Case Study

This study employed case study methodology to examine the experiences of teachers instructing mathematically gifted and promising students. Case studies are an investigative approach that describe specific and complex phenomena to discover new and richer understandings of the phenomenon of interest. A case may be based on any number of units of analysis including an individual or group of individuals, a classroom, a school, or even an event (Mertens, 2015). In this study, the case was bounded by the group of teachers at one school site who were recruited and chose to participate in the study. The most important aspect of case study, according to Merriam (1998) is determining that the case is a bounded unit, a thing, a single entity, a unit around which there are boundaries. Merriam describes it this way, “I can fence in what I am going to study” (p. 27).

In this study, the focus was the lived experiences of teachers working with mathematically gifted and promising students. This phenomenological approach was used to answer the overarching central question: What are the experiences of teachers working with mathematically gifted and promising students? In phenomenology, the researcher is interested in the lived experiences of the people who are connected to the issue or problem being studied (Kvale, 1996). This study was conducted using questionnaires, semi-structured interviews, and classroom observations. The goal of a phenomenological research design is to understand and describe an event from the point of view of the participants in the study and to emphasize the individual’s subjective experience (Mertens, 2015).

Research Questions

Based on the literature review in Chapter II, there is a need for research studies that examine the instructional practices teachers use with mathematically gifted and promising students and how teachers’ conceptual understanding of mathematics and their beliefs influence their use of those practices. The following research questions are directly tied to the purpose of this study in response to the problem statement presented. To that end, this study sought to understand the overarching central question or phenomenon of the lived experiences of the teachers of mathematically gifted and promising students. This study was specifically guided by the following research questions:

1. What are the teachers' instructional practices with mathematically gifted and promising students?
2. How are the teachers' beliefs towards mathematics and beliefs towards gifted students reflected in their instructional practices with mathematically gifted and promising students?
3. How do teachers describe their conceptual understanding of mathematics as it relates to the teaching of mathematically gifted and promising students?

Setting and Participants

Site of Research

Since the research required in-person classroom observations and on-site interviews, the location was bounded by a group of teachers in one school located in a major metropolitan city in a southeastern U.S. state. The research site was a K-8 charter school for highly gifted students referred to in this report using the pseudonym *Sycamore School.* This setting was chosen because the students have already been identified as gifted as part of the entrance criteria and because the teachers there work exclusively with gifted students. The school was also chosen because of its reputation for having a strong mathematics program and because of the way it differentiates for students within the classroom and through ability grouping.

Overview of School Site

Established in 2000, Sycamore School is a public, non-profit charter school serving the unique social, emotional, and intellectual needs of one of the most underserved children in school settings—the highly gifted. Currently the school enrolls 385 students in grades K-8. For the academic year of 2020-2021 a total of 374 students made up the student body. Based on gender, there are 194 male students (51.9%) and 180 female students. For race/ethnicity, 126 students are White (33.7%), 8 students are Black (2.1%), 10 students are Hispanic (2%) 12 students are two or more races (3%), and 218 students are Asian (58.3%). Five percent of the students are classified as economically disadvantaged. In both math and English language arts the overall school score was 95% and the school received an A for academic achievement by the state.

These children represent only 2% of the population, and come from all socioeconomic groups, backgrounds, and cultures. When highly gifted children's special needs are not met, they have a greater risk of social and emotional problems compared to their age mates. As a result, many will underachieve and/or drop out of high school or college. Highly gifted children from disadvantaged backgrounds are doubly at risk of academic failure if their needs are not met. The school's mission states that it exists to provide a differentiated, challenging, and equitable learning environment that supports the distinctive intellectual, social, and emotional needs of highly gifted children from diverse backgrounds and enables them to form meaningful relationships with their intellectual peers.

One goal of Sycamore School's 2020-2025 strategic plan is to enhance the marketing efforts to reach students who may benefit from the program. This includes a marketing plan to increase awareness in the region about the school, especially during the months leading up to the application period. The school is also interested in attracting underserved populations and has indicated another strategic goal that will ensure measurable progress is made in attracting and enrolling underserved populations, while celebrating the diversity that already exists within the school. Further, Sycamore School wishes to intentionally market such school offerings as transportation, meal service, and before and after school care to attract families from diverse backgrounds and institute processes that attract and enroll a diverse student body. This year the school has implemented a pilot program for bus transportation in the hopes of attracting students in underrepresented communities and plan to expand this program to meet the needs of more students.

Admissions Criteria

Prospective students are identified by a comprehensive process that includes cognitive testing. Selection is by lottery if the number of qualified applicants exceeds open spaces. As a public, non-profit charter school, the school does not discriminate based on race, color, disability, gender, gender expression, or national origin. The goal of Sycamore School is to serve highly gifted students, including those who might not be identified if decisions were made strictly based on test scores. Identification is not based on a single measure and therefore admission to the school utilizes multiple indicators of giftedness. Personal nomination forms and teacher questionnaires elicit relevant behavioral information about the potential applicant and are used by the admissions team to make admissions decisions about applicants whose scores on an acceptable cognitive abilities test lies between two and three standard deviations above the mean (130-145). Applicants whose cognitive abilities test score lies at least three standard deviations above the mean (145+) on an acceptable cognitive abilities test are automatically deemed to meet the admissions criteria of Sycamore School.

Mathematics at Sycamore School

In addition to the core subjects of math, language arts, science, and social studies, the students participate in SEL (social emotional) lessons daily, as well as enrichment courses (music, art, P.E.), and afterschool activities and sports. For mathematics specifically the school uses the *Dimensions Math® PK-5* (Singapore Math Inc., 2022) curriculum for grades K-4. Students in K-2 classes remain in their homeroom for math where differentiation occurs through flexible ability grouping and curricular compacting. Beginning in third grade, students are placed into two or possibly three ability groups for math instruction based on test data, teacher recommendations, and information processing speed. In middle school, the pathway broadens to provide three paths for students. A student's pathway will determine the number of high school math courses taken at the school. In grades 5 through 8 students progress through middle school and some high school curriculum. The Sycamore School uses a variety of instructional resources for these courses. In middle school, instruction is based on the standards issued by the National Council of Teachers of Mathematics (NCTM).

The Singapore Math model or approach that is the hallmark of the *Dimensions Math* curriculum relies on bar model drawing to solve word problems, uses mental math to teach students to solve problems in their head, focuses on a concrete-pictorial-abstract progression, and revolves around key number sense strategies taught in stages over time. Mathematics is taught not as a series of rote rules to be memorized, but from the framework of understanding the "why" behind the mathematics. The program teaches for mastery, and it is expected that students move through this in different stages.

Recruitment of Participants

A purposeful sampling method was utilized to recruit and secure participants for the study. Individuals were chosen according to the core constructs of the research questions. Purposeful sampling allows researchers to intentionally select certain individuals and settings based on a certain experience or knowledge of a specific phenomenon (Ravitch & Carl, 2021). It is widely used for identification and selection of individuals or groups who are particularly knowledgeable or experienced with a specific phenomenon of interest (Patton, 2002). The following sequence of events describe the recruitment of the participants in the study:

- During the summer of 2021, I contacted the director of a local K-8 charter school for highly gifted students in one large, urban school district in the southeastern United States via email based on a prior connection to the school and the recommendation of a dissertation committee members who was familiar with the school and had conducted professional development on site. The email highlighted a description of the study, including purpose, method, significance, timeframe, and other relevant information (Mertens, 2015).
- Once the director expressed interest and a desire for more information, I suggested a follow-up Zoom conference meeting. During the meeting I introduced myself and shared my research proposal for consideration. The director agreed that the study had value and would be welcome at the school. She requested I complete and submit a written research proposal form which is regularly used by the school for outside research studies. This proposal was submitted and then approved approximately one month later. Part of the approval process required that the director share the proposal form with the Board of Trustees of the charter school.
- A follow-up phone call was completed to plan for the first visit. Teachers who were currently teaching mathematics to second grade students and up were requested to be considered for the study. A date was established for me to make my initial visit to the school where I met and presented my proposed study to the teachers, and I received a tour of the school.

- I presented my study to eight teachers whom the director had invited as possible participants. At the conclusion of my presentation, three teachers gave me signed consent forms. After the presentation I emailed the remaining teachers to ask if they would like to participate and two additional teachers said yes and submitted their consent forms. I then emailed all five participants to schedule interviews and observations. No additional teachers agreed to be participants in the study.

Teachers in different grade levels, comprising both elementary and middle school, were chosen to ensure that a variety of experiences would be recorded. These requirements were based on the research questions as presented in the study which focus on teachers' perceived descriptions of their conceptual mathematical understanding, their instructional practices, and the teachers' beliefs about mathematics and gifted children. The goal for the number of participants in the study was 4-6 teachers. Creswell (2013) recommended a sample size of five to 25 individuals for phenomenological studies. Five participants agreed to participate in the study and signed consent forms. One individual teaches second grade, two teach third grade, one teaches fifth grade, and the final participant teaches eighth grade. The eighth-grade class has a fourth-grade student placed in the group based on the student's ability and need for grade level acceleration.

Data Collection Process

Once I confirmed the participation of each of the five participants, I scheduled interviews, which were either in person or via Zoom. Using a spreadsheet, I created pseudonyms for each participant. I conducted three interviews in person at the school site and in the teacher's classroom during a planning period or after school. Two interviews were conducted via Zoom. Before I began each interview, I reiterated the purpose of the study and their role as a participant, reminding them they could withdraw from the study at any time. I worked hard to establish rapport with each of the participants and to mitigate any presumed or present power dynamics in my role as a researcher or even as a school principal.

Using a hand-held Sony recorder for the in-person meetings and Zoom audio recording for those that were virtual, I began the interview using an interview protocol based on a semi-structured approach and designed to inform the research questions. I had completed a pilot study at my school over the summer which allowed me to gather critical feedback about the questions from the teacher participant in the pilot study. The interview protocol was also reviewed by two peers and a university professor in a qualitative research class in which I was enrolled the same semester as my data collection. Those three individuals provided feedback and suggestions for improvement. I used this feedback to create the final interview protocol which I used with the five participants of this study (See Appendix E for the Teacher Participant Interview Questions).

With each interview I began with open-ended questions about the participant's background, years of teaching, strengths, a source of accomplishment, and their best math lesson ever. I stayed closely to the questions but asked follow-up and clarification questions as necessary and appropriate. The categories of my interview questions included teacher experiences with mathematically promising students, followed by teacher beliefs about mathematics and beliefs about mathematically gifted students, their own conceptual understanding of mathematics, and finally, issues of equity related to student access to challenging mathematics. I was intentional in my effort to establish rapport with all of the participants.

Before the interview I asked teachers to complete the online Classroom Instructional Practices questionnaire via a Google Form (See Appendix F) and after the interview I gave them a paper copy of the Teacher Beliefs questionnaire (See Appendix G). The classroom observations were the final part of data collection. I asked the teachers to answer five pre-observation questions, either in person or in writing (See Appendix H), and then following the observation I conducted a debrief using the post-observation questions (See Appendix I) and collected the responses in my notebook. Following the holiday break, I presented each participant with a gift card and a thank you note to express my appreciation for their time and willingness to be part of the study. See Table 3 for demographic data on each participant.

Table 3

Research Participants

Participant Pseudonym	Grade Level	Education (Gifted)	Years Teaching	Years at Sycamore School
Dave	7-8	Some gifted ed classes	35+	8
Linda	5	Some gifted ed workshops	11	1
Barry	3	AIG Certificate[a]	19	8
Katie	2	AIG Certificate	15	15
Natalie	2	AIG Certificate	13	2

Note. [a]An Academically or Intellectually Gifted (AIG) certificate requires a 12-credit hour sequence of courses and provides a consistent, cohesive structure for teachers seeking to add on the North Carolina Standard Professional 1 (SP1) AIG Professional Educator's license.

Data Collection

This qualitative study used several instruments to collect data in response to the three research questions. Teacher participants were asked to complete two questionnaires based on Swan's (2006) work with teachers of mathematics in England. The teacher beliefs questionnaire was used to survey the participants' beliefs towards mathematics, teaching, and learning. The classroom instructional practices questionnaire asked teachers to self-report on their use of instructional practices in the mathematics classroom. This questionnaire was administered via a Google Form, while the teacher beliefs questionnaire was completed on a paper form. Questionnaires are self-reported instruments to gather the thoughts, feelings, attitudes, beliefs, values, perceptions, personality, and behavioral intentions of the individual completing it (Johnson & Christensen, 2020).

In addition to the questionnaires, I collected data through semi-structured interviews with each teacher, and one classroom observation for each teacher. The classroom observations included pre and post interview questions; and I used an observation instrument to collect data in addition to field notes. More information about each data collection instrument is provided following Table 4 which shows the alignment of the research questions, focus, and data collection sources.

Table 4

Research Design

Research Question	Research Question Focus	Data Source
RQ 1: What are the teachers' instructional practices with mathematically gifted and promising students?	Instructional practices	Classroom observations (video and field notes) Teacher interviews Questionnaires
RQ 2: How are the teachers' beliefs towards mathematics and beliefs towards gifted students reflected in their instructional practices with mathematically gifted and promising students?	Beliefs & Instructional Practices	Teacher interviews Questionnaires Connecting questionnaire data to data from RQ 1

RQ 3: How do teachers describe their conceptual understanding of mathematics as it relates to the teaching of mathematically gifted and promising students?	Conceptual understanding of mathematics	Teacher interviews

Teacher Beliefs and Instructional Practices Questionnaires

For these data collection instruments, I used Swan's (2006) questionnaires of Teacher Beliefs and Classroom Instructional Practices (Appendices F and G respectively). These were validated through a national development project promoting student understanding of algebra. Swan and his team used the questionnaires to evaluate the effects of an intervention of professional development and other resources used with teachers. Swan noted that most research on beliefs uses qualitative methods and that to develop teaching practices it is essential to study beliefs as "changes in them may be the clearest measure of a teacher's professional growth" (p. 59). He further cited the constant constraints teachers face daily which can often lead to inconsistencies between what a teacher believes and what they do in classroom practices.

Swan developed the Beliefs survey from Paul Ernest's (1991) research that suggests a teacher's belief system has three components: the teachers' conception of the nature of mathematics as a subject for study (M), the nature of mathematics teaching (T), and the process of learning mathematics (P). Derived from research by Askew et al. (1997), a teacher's orientation to each of these categories can be characterized as transmission (T), discovery (D), or connectionist (C). A teacher with an orientation towards **transmission** sees math as a series of rules that must be directly taught to students and practiced until fluent. A **discovery** orientation is one where teachers believe that students learn through exploration and the teacher takes on the role of facilitator. The **connectionist** orientation sees math as a network of ideas that students and teachers construct together through collaboration. Participants were asked to give each statement in the three sections (Mathematics, Teaching, Learning) a percentage, so that the sum of the three percentages in each section is 100 (See Appendix G).

The Instructional Practices questionnaire consisted of 25 classroom practices containing 13 behaviors categorized as teacher-centered and 12 behaviors categorized as student-centered. Teacher-centered describes practices one might expect to originate from a transmission-oriented belief system. In this setting the teacher is the primary director of the work, teaching students in a whole class model with an emphasis on fluency over understanding. There is less creativity and a focus on transmitting definitions and practicing methods. Student-centered practices come from a constructivist position where teachers teach students as individuals, allow them to create their own methods, are flexible about what is covered, and see mathematics as a subject open for discussion.

This questionnaire asked participants to respond using a Likert scale of 1 to 5. The response indicators were as follows: 1- almost never, 2- occasionally, 3- half the time, 4- most of the time, and 5- almost always. Likert scales are the familiar five-point bipolar response used in many questionnaires today. The scales typically range from low to high, asking respondents to indicate how much they believe to be true or not or agree or disagree (Allen & Seaman, 2007).

Semi-structured Interviews

I conducted the semi-structured interviews with each teacher participant either in person (three interviews) or via Zoom (two interviews) and audio-recorded the interviews to aid in the data analysis. Each interview lasted approximately one hour and 15 minutes which included time to establish rapport before launching into more formal questions. This type of approach to interviews allowed for consistency across all participants regarding the questions asked, as well as an opportunity for natural conversation and a deeper dive into the participants' lived experiences. According to Brinkman and Kvale (2015), phenomenology has been important in clarifying the mode of understanding in a qualitative research interview to focus the interview on the experienced meanings of the participants' life world. The semi-structured interview attempts to understand themes of the lived everyday world based on the perspectives of the interviewees. It is like an "everyday conversation, but as a professional interview it has a purpose and involves a specific approach and technique" (p. 31).

Classroom Observation

The last stage of the data collection process consisted of classroom observations of each participant teaching math which I videotaped using an iPad. For a qualitative research study, the use of observation as part of the data collection allows the researcher to note a phenomenon within the field setting (Angrosino, 2007). My role as a researcher in this setting was that of a complete observer (Mertens, 2015), not participating in the lesson nor discussion. Prior to the observation, I asked teachers to share a little bit about the lesson I would see, including the objectives, the instructional practices they planned to use, and where they thought students might struggle.

I chose the Mathematics Classroom Observation Protocol for Practices ($MCOP^2$) from the research of Gleason et al. (2015) to use as a guide for the observation. The development of the $MCOP^2$ (see Appendix J) was validated via feedback from 164 external experts. Results indicated that it measures how well teacher actions align with practices recommended by national organizations (Gleason et al., 2015). The protocol uses a two-factor structure of teacher facilitation and student engagement. I chose this instrument because of its emphasis on teacher practices around conceptual understanding and student engagement, and its alignment with the Standards for Mathematical Practices (SMPs). Further, the instrument was relatively simple to manage based on my experience of observing hundreds of math lessons as a former math coach, academic dean, and currently as an elementary school principal.

It should be noted that the $MCOP^2$ form is designed to measure the activities occurring in a mathematics classroom during a single lesson, but its best use is that of a formative instrument. It is recommended that the researcher or observer use it with different class settings over time. While all items in the protocol are desired or ideal qualities of a mathematics classroom, it is not expected that that they will all be observed during one lesson. For the purposes of this research study, however, my goal was to use the protocol as a guide as I observed the teacher's actions and practices during my observation.

As I observed the teacher and students, I kept notes in a journal and on a printed copy of the $MCOP^2$ instrument. After the observation I met briefly with the teacher as a follow-up using the post observation questions as a guide. These questions included how they thought the lesson went overall, did all students have access to the lesson, what instructional practices they used, and if they achieved the learning objectives for the lesson. I hand-wrote the teachers' responses on a printed copy of the post-observation interview document.

Data Analysis

Qualitative data analysis is intentional scrutiny of the data that takes place throughout the research process (Ravitch & Carl, 2021). Data was analyzed throughout the data collection process to determine the phenomenological experiences of the participants in relation to the research purpose and questions in this study. Data analyzed included the questionnaires, individual interviews, and classroom observations videos and observation instrument. This information was carefully compiled and organized into Google documents and spreadsheets and maintained in a secure, password-protected, and locked computer. The interviews were audio recorded and the observations were videotaped. Using an online transcription program, I uploaded the recordings to create transcripts which were then read through, edited to match the words spoken, and uploaded to a word document. Transcripts are a representation of data but may also be a form of interpretation and should be as verbatim as possible to maintain fidelity (Ravitch & Carl, 2021). Additionally, the use of field notes during the observations allowed for analysis to occur during and after the data collection process. Although an application was used to transcribe the interviews, I engaged in multiple readings of the transcripts to fully immerse myself in the data I collected. This included listening to the recordings multiple times and watching the classroom videos, all before assigning codes to the data analyzed. Additionally, I printed copies of each interview transcript and made notes by hand and then typed samples of descriptive, in vivo, and process codes into a codebook instead of copying and pasting the data. This allowed me to further immerse myself in the words and experiences of my participants. To analyze the video recordings of the classroom observations I initially took notes of the practices in a journal and typed paraphrased quotes on a Google Doc before merging those into a codebook.

Interpretive Phenomenological Analysis (IPA)

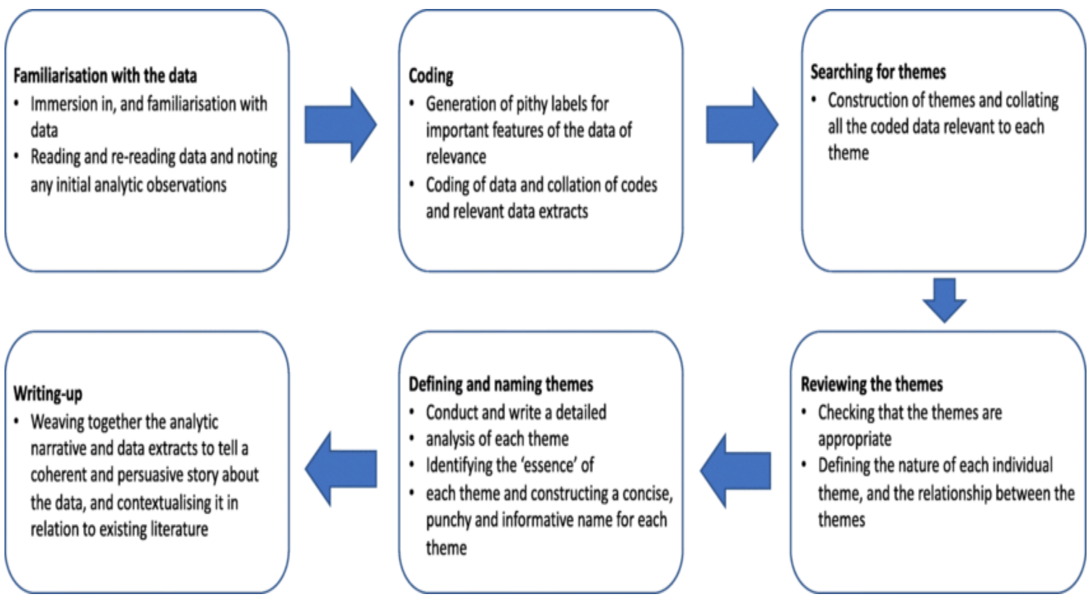
In analyzing my data in this phenomenological study, I followed an interpretive phenomenological analysis (IPA) approach. IPA is a process of analyzing data that focuses more on situated, interpreted, and lived experiences than on transcendental experiences. (Johnson & Christenson, 2020). Its main concepts stem from several philosophers, including Husserl, Heidegger, and Merleau-Ponty (Smith et al., 2009). Using this as a framework, the researcher analyzes the data, moving back and forth between the shared experiences of the participants and their own interpretation of what those experiences mean. The researcher’s goal is to understand their participants' understanding of their experiences (Peoples, 2021). IPA research combines an empathic, insider's perspective, with a questioning, interpretive stance (Smith et al., 2009). In effect, the analysis moves away from what the participant is saying and becomes more interpretive on the part of the researcher. IPA researchers aim to be empathic, while also questioning or puzzling over what their participants are saying. This can also be described as attempting to understand, as in trying to see what it is like for someone, but also trying to analyze, illuminate, and make sense of something.

The two foundational philosophies of phenomenology are Husserl's transcendental phenomenology and Heidegger's hermeneutic phenomenology. Interpretive Phenomenological Analysis aligns with Heidegger's hermeneutic circle wherein the data is analyzed by breaking down the information into parts, synthesizing, and then looking at the whole again. The parts make sense of the whole, and the whole makes sense of the parts until an understanding of the phenomenon emerges (Peoples, 2021; Smith et al., 2009). This circle approach sees engagement with the data as more circular than linear, more of a spiraling process of assigning meaning as new data is introduced and the meaning is revised. Heidegger also believed that the researcher's biases are part of the research process and cannot be set aside, but instead should be implicitly stated (Peoples, 2021).

There is no single method for working with data when using an IPA process. The essence of IPA is contained in its analytic focus (Smith et al., 2009) which centers on the participants' efforts to make sense of their experiences. Interpretive Phenomenological Analysis follows strategies that are much like the familiar step-by-step process of Thematic Analysis (See Figure 3 for a description of the steps of Thematic Analysis) (Braun & Clarke, 2006). The steps for IPA can begin with a close, line by line analysis of the data, followed by the identification of patterns (themes) in individuals and across cases, the development of a full narrative, and finally a reflection from the analyst on the processes utilized (Smith et al., 2009). Though thematic analysis tends to follow a linear design, the actual analysis of this data utilized a more circular or spiral direction as each type of data was analyzed separately participant by participant. Once new data was pulled in, it required a step of circling back to the codes and synthesizing each type of data in relation to the other and re-evaluating and even re-naming the themes.

Figure 3

Thematic Analysis Steps



I spent considerable time familiarizing myself with the transcript data from the interviews, observations, and questionnaire responses. In IPA, Smith et al. (2009) recommended initial noting on the transcripts with no rules or requirements, but ultimately the analyst is looking to describe what is important to the participants and the meaning of those things for the participant.

Following the reading and re-reading of data and the note-taking process, I began to assign open codes to the statements and created a codebook with terms for each individual participant. Open coding allows the data to be highlighted and labeled for regularly occurring phrases and terms connected to the research purpose and questions (Ravitch & Carl, 2021). Open coding also allows the researcher to “analyze the data for significant statements, meaning units, textual and structural description, and description of the essence” (Creswell, 2013, p. 105).

The next stage of analysis involved searching for themes that connected to the codes I had generated. I kept my research questions front and center at this stage and looked for themes that would provide answers to these questions. Reviewing the themes may include numeration, (i.e., counting the frequency a theme is reported). While this is not the only indicator of importance it can help determine patterns within and across data (Smith et al., 2009). In an IPA process, this stage includes mapping the themes and then figuring out how they fit together in clusters or related themes for each participant (Smith et al., 2009). I completed this process for each participant until all the participants’ data had been analyzed. Clustering the data into themes allowed me to get closer to the essence of the phenomenon of my research.

Looking for patterns across individual participants led to the next stage of naming, defining, and then renaming the sub-themes and the themes. The sub-themes were reviewed and scrutinized to determine if any changes needed to be made before re-naming and defining them, and then doing the same with the themes, which again is a more circular than linear process. I connected the sub-themes and themes to each of the three research questions of teacher instructional practices, teacher beliefs of mathematics and gifted education, and the conceptual understanding of mathematics in relationship to the experience of mathematically promising students. Data saturation was determined once there were no longer any new themes, and the research questions were sufficiently addressed. The final stage of this data analysis involved the construction of a narrative, or story that described the phenomenon of the lived experiences of teachers instructing mathematically gifted and promising students.

Trustworthiness

The trustworthiness of a study should meet the requirements of four main criteria: credibility, transferability, dependability, and confirmability (Lincoln & Guba, 1985). Credibility refers to the faithfulness of the description of the phenomena; transferability suggests the results are applicable in other contexts; dependability refers to the degree at which other researchers could follow the steps and decisions made by the researcher; and confirmability refers to logical inferences made based on the results of the findings (Webb & Welsch, 2019).

Credibility

Multiple data collection methods in the form of interviews, observations, and questionnaires helped to triangulate the findings for credibility. The pre- and post-observation questionnaires provided the participants the opportunity to state their intentions before the class and what they believed occurred during the class. The use of the participants' words and quotations in this report provides an insider's view of the participant's experience (Webb & Welsh, 2019).

A journal was kept during the process of data collection and analysis to monitor for individual biases and change in understanding over time, further ensuring that the research is credible (Mertens, 2015). Presenting my own bias in a journal and in field notes was critical to achieve credibility. I aimed to be fully open and transparent in discovering new ideas and themes related to the phenomenon being explored. During the data collection and analysis period, I met regularly with a peer (the colleague with whom I conducted the pilot study) to gather feedback about the methods and results to create accountability and honesty.

Transferability

Rich description (Mertens, 2015; Ravitch & Carl, 2021) aided in the transferability of the research process by describing sufficient details of the participants and selection process, the setting in which the research was conducted, and the context so that readers may be able to connect the findings to their situations. The use of technology or other tools, and specific quotes from the participants were provided as part of this thick description.

Dependability and Confirmability

Dependability was achieved by providing a thorough description of the data collection methods and the connection and relevance to the research questions. It was further achieved with an audit trail where a reflexive journal was kept, as well as all field notes, transcripts, and raw data. According to Nowell et al. (2017), an audit trail provides readers with an understanding of the decisions made during the study and the rationale for those decisions. Confirmability was achieved by maintaining a reflexive journal and the appropriate handling of the data.

Data saturation occurred when all data had been collected and when the process of analysis provided answers to the research questions. Writing drafts and creating visual displays of the data was part of an ongoing process throughout the study to better prepare for the presentation of the analysis and findings.

Risks, Benefits, and Ethical Considerations

One risk for the teacher participants could have been a sense of discomfort with being interviewed and observed by an outsider, holding a fear that their performance and character were being evaluated and may come under scrutiny or even shared with others who hold power within the school. Prior to the interview I worked to present myself as a friendly and curious peer through communication via email. And, during the interview itself I frequently connected with the teachers by discussing my own classroom experiences with gifted students and my interest in mathematics. I intentionally made every effort to not discuss my role as a school principal or even as a researcher to further mitigate any perceived or real feelings of power differentials.

A benefit of participating could include a greater understanding of the underlying beliefs that form “the why” of a teacher’s instructional practice and craft, especially in their work with mathematically gifted students. This understanding may lead to further self-reflection and even a desire to improve or refine their work with gifted students. Additionally, engaging as a co-researcher and hearing about the experiences of others who are doing the same work may provide a sense of comfort (a benefit) in hearing about shared experiences from colleagues. Teachers may increase their collaboration, desire for professional growth and training, and a stronger sense of purpose in working with mathematically gifted and promising children.

To maintain ethical standards according to Moustakas (1994), this study established clear agreements with the research participants, recognized the necessity of confidentiality and informed consent, and developed procedures for ensuring full disclosure of the nature, purpose, and requirements of the study. Consent forms for all participants and all aspects of data collection, as well as pseudonyms for the participants and schools, were used consistently and great care was taken to ensure confidentiality in every circumstance. As was stated on the consent forms participants could withdraw at any time from the study. Interview transcripts, Google Form results from questionnaires, demographic data, classroom video, reflexive journals kept by the researcher, and all other data was kept on a secure, password-protected computer with access granted only to the researcher. Data will be destroyed after five years following the study. Every effort was made to ensure that the participants felt their contributions were valued and secured properly.

Summary

This chapter provided an overview of the research design, procedures, and methods of data analysis. The purpose of this study was to describe and synthesize the lived experiences of teachers working in a charter school for gifted students and instructing mathematically gifted and promising students. Specifically, the goal of the study was to better understand the teachers' conceptual understanding of mathematics and their beliefs about mathematics and beliefs about gifted learners and how these factors were reflected in their instructional practices. Data analysis involved a collection of data including questionnaires, semi-structured interviews, and classroom observations. This chapter outlined the use of an Interpretive Phenomenological Analysis (IPA) design of the study, a research timeline, description of the setting and participants, the data collection tools and their descriptions, and the procedures for collecting and analyzing the data. Descriptions were provided on the trustworthiness of the study and the ethical considerations for collecting and using the data.

CHAPTER IV: FINDINGS

Overview

The purpose of this qualitative phenomenological case study was to explore the experiences of teachers who work with mathematically gifted and promising students through their beliefs, instructional practices, and conceptual understanding of mathematics. Using semi-structured interviews, questionnaires, and observations, I described the phenomenon of teachers instructing elementary and middle school students who have been identified as mathematically gifted in a K-8 charter school for gifted students. I specifically chose this setting to study teachers who work exclusively with mathematically gifted students and therefore may possess a unique view of this experience. As a result of this study, teacher educators, gifted specialists, and administrators may better understand the experiences of the teachers, potentially highlighting the teacher preparation, professional development, and recruitment of teachers who work with mathematically gifted and promising students.

Chapter IV provides detailed descriptions of the five teacher participants including their background, experiences, and their current work at Sycamore School. Following this are the results of the two questionnaires completed by participants, one on teacher beliefs and another one on classroom instructional practices. Using an Interpretive Phenomenological Analysis (IPA) approach, I first analyzed each source of data collection one participant at a time, assigning codes to significant statements and generating sub-themes and themes from the interview transcripts and the classroom observations. I then used all of the data sources, including the responses to the questionnaires, to respond to the three research questions and generate themes. The codes, themes, and connection to the research questions are provided and described here.

The central question “What are the lived experiences of teachers instructing mathematically gifted and promising students?” framed this phenomenological case study. To that end, this study sought to understand the overarching central question or phenomenon of the lived experiences of the teachers of mathematically gifted students, and was specifically guided by the following research questions:

1. What are the teachers’ instructional practices with mathematically gifted and promising students?
2. How are the teachers’ beliefs towards mathematics and beliefs towards gifted students reflected in their instructional practices with mathematically gifted and promising students?
3. How do teachers describe their conceptual understanding of mathematics as it relates to the teaching of mathematically gifted and promising students?

The results shared below provide detailed descriptions of the participants’ experiences in working with mathematically gifted and promising students through textual descriptions, structural descriptions, and a synthesis of the data. A summary of the themes found during data analysis concludes the chapter.

Setting and Participants

The five participants in this study are full-time teachers at one K-8 charter school where the school’s mission is to serve academically gifted students in an urban city located in the southeastern United States. Once I received approval from the Director of the school (Sycamore School is the pseudonym), I was invited to present my study to a group of eight teachers whom the Director had invited to consider being part of the study. Five teachers committed to being part of the study and each of them participated fully in the entire study. Teaching experience ranged from 9 years to over 35 years. All have received professional development in gifted education, and two have their teaching certificate in gifted education (North Carolina Academic and Intellectually Gifted Certificate or AIG). Once the participants confirmed, I assigned each a pseudonym which helped to ensure confidentiality and helped me to organize the data more effectively.

Participant Summaries

Natalie

Natalie teaches third grade mathematics at Sycamore School and is in her second year at the school and in this grade level. This is her 13th year as a classroom teacher; she taught in a large urban public school district for 11 years prior to moving to her current school. During those 11 years, she stayed in one elementary school teaching both third and fifth grade classrooms for seven years. Natalie indicated that many of her students at that school were identified as gifted and received services by the school system. It was during this time she became interested in working with gifted students and moved to a magnet school for gifted students in the same school district and then taught third grade there for four years. While at the magnet school she learned about the gifted charter school and applied for a position to teach.

In her first year at Sycamore School, the first year in her career she did not teach math, Natalie taught English Language Arts and Science. This year Natalie is one of three third grade teachers and is teaching all core subjects, but teachers group the students into three different math classes based on ability and the teachers rotate who teaches each group. Students do not necessarily remain in their math groups as a rule and teachers may move them into any of the groups throughout the year based on student readiness. Natalie has a bachelor's and a master's degree in elementary education and an AIG education certificate from the state. She noted that this was her first year using *Dimensions Math® PK-5* (Singapore Math Inc., 2022), the school's math curriculum.

Dave

I interviewed Dave via Zoom and asked him to share his professional background and teaching journey overall. Dave has been teaching for over 35 years and has been at the Sycamore School for about half of that time, close to 15 years. He has taught at small private Christian schools in middle and high school. Dave also worked at a large K-12 private Christian school where he taught junior high and high school classes with a focus on Algebra 1, Algebra 2, and Geometry in the high school. For a few years he served as the dean of students and then became the junior high principal and taught a few classes as well which he described as "very, very time-consuming." Much of his time was spent on student discipline, but Dave also taught an SAT math prep class for one quarter. His role as principal lasted about 10 years during which time, he was unable to teach any classes.

Due to health issues, Dave took a break from education and did not believe he would ever return. However, he saw an advertisement for a math teacher at Sycamore School and returned to the classroom as a part-time middle school teacher in eighth grade for the first year and moved into a full-time role shortly after. This is Dave's eighth year at Sycamore School. He has a bachelor's degree and has taken gifted education classes through an online program but has not yet received a gifted certificate. He is currently teaching Math 1, Math 2 to seventh- and eighth-grade students, and Math 3. A fourth-grade student has been accelerated to his Math 2 seventh-grade class this year and this was a primary reason I was interested in inviting Dave to be a part of this study. Dave is interested in coursework on learning styles and considers his participation in several math competitions throughout the year (up to 10) as a major source of professional development for him. Dave considers his involvement in math competitions to be a form of professional development as he can connect with other teachers and experience a variety of problem-solving opportunities with his team and other teams.

Linda

I interviewed Linda over Zoom and broke the interview time into two separate sessions back-to-back. This is Linda's first year at Sycamore School and her 11th year as a teacher. Most of her experience has been teaching middle school mathematics, apart from last year when she taught a virtual first grade class during the first full year of the pandemic. It was also her first year living in this community after relocating from another state. Linda completed her K-12 schooling and then attended a community college for one year. Soon after she moved to another town to attend a college that was associated with her faith denomination and there, she earned a bachelor's degree in education with a focus on special education. Linda's education program provided teaching experiences in a different school setting each semester, rather than making students wait until their senior year before stepping into a classroom.

Although she initially wanted to be with younger children, her professors encouraged her to complete a student teaching assignment in high school (ninth grade). It was there that she began to appreciate the teaching and learning environment of older students. She took a state certification test in mathematics because she felt confident in that area and because the state required a double certification (another focus area in addition to special education). With teaching certificates in math and special education, Linda recognized that both are areas of high need in public schools. Linda shared that her special education training has helped her in the gifted education classroom because there is a need for her to differentiate for the range of students in her classroom. She stated, "Every kid needs some sort of differentiation or accommodation." Linda taught middle school math at another school prior to joining Sycamore because she wanted a regular teaching position and not in special education. Currently, she teaches four sections of fifth and sixth grade math students whose course work focuses on everything from pre-Algebra to Math 1.

Katie

A second-grade teacher at Sycamore School, Katie first came to the school in 2005 as a parent and then accepted a kindergarten assistant position the next year. Both of her daughters graduated from Sycamore School. After four years of teaching kindergarten, Katie earned her teaching certificate and moved into a lead teacher role in the second grade where she has been ever since (15 years). With a background in public health education (she has a bachelor's degree and master's degree in the field), Katie worked in public health and spent time in Chad, Africa, with the Peace Corps. She earned her teaching certificate and has her Academically Gifted Certification.

The interview took place in Katie's classroom, which is based on a fictional country grounded in student-driven rules, jobs, and discipline. They have recently created their own monetary system. The group studies civilizations and systems as part of their yearlong curriculum and students run the class meetings where a secretary records the minutes and students propose solutions to any problems, they place on the agenda. For mathematics, the students are in different coursework with the *Dimensions Math® PK-5* (Singapore Math Inc., 2022) based on readiness, but Katie teaches all of the students. She has a full-time assistant who supports the work of the classroom with small, differentiated groups. Students are typically working in one of three leveled areas, such as the skills and concepts of addition and subtraction, addition and subtraction with regrouping, or starting the foundation of multiplication. Katie uses the same content, but she modifies the pacing based on the needs of the students.

Barry

Barry is a third-grade teacher at Sycamore where she has been teaching for eight years (all in third grade), but prior to joining the faculty at Sycamore she taught in a public school system for four years. This is her 19th year in education, and she has taught in grades four, five, and six. Barry earned a bachelor's degree in elementary education and taught in the public schools there before staying home to raise her children. In addition to attending a week-long conference on the school's math curriculum, Barry has her AIG certificate from a local university. Currently, Barry teaches the "highest" math group of students who are working on a fifth-grade level in the *Dimensions Math® PK-5* (Singapore Math Inc., 2022) curriculum.

Results

The results provided here highlight the responses from the two questionnaires completed by the participants, which included their beliefs surrounding mathematics, teaching, and learning, as well as their self-reported frequency of use of classroom instructional practices. The semi-structured interviews and classroom observations are also outlined here, followed by an overview of the results framed by each of the research questions.

Classroom Instructional Practices Questionnaire

In response to Research Question One: What are the teachers' instructional practices with mathematically gifted and promising students, the data collection began with a questionnaire on the use of 25 classroom instructional practices that asked participants to self-report the frequency of use of each practice with a 5-point Likert scale of almost never, occasionally, half the time, most of the time, and almost always. This questionnaire was based on the work of Swan (2006) who was interested in establishing a connection between teacher beliefs about teaching and learning and the instructional practices used. The questionnaire was sent to each participant via email as a Google Form and took 20-25 minutes to complete. I used the questionnaire to establish a general understanding of the types of instructional practices the teachers use with their students in the mathematics classroom.

The 25 classroom practices contain 13 behaviors categorized as teacher-centered and 12 behaviors as student-centered. Teacher-centered describes practices one might expect to originate from a **transmission-oriented** belief system. In this setting, the teacher is the primary director of the work, teaching students in a whole class model with an emphasis on fluency over understanding. There is less creativity and a focus on transmitting definitions and practicing methods. Student-centered practices come from a **constructivist** position where teachers teach students as individuals, allow them to create their own methods, are flexible about what is covered, and see mathematics as a subject open for discussion. Getting through every topic on a syllabus is not necessary and time may be used for exploration. The results of this classroom practices questionnaire are provided in Table 5.

Table 5

Results from the Classroom Instructional Practices Questionnaire

Student or Teacher- Centered	Classroom Practices	M
S	Students learn through discussing their ideas	4.2
S	I encourage students to make and discuss mistakes	4.2
S	Students compare different methods for doing questions	3.8
S	I am surprised by the ideas that come up in a lesson	3.4
S	I encourage students to work more slowly	3.2

S	I draw links between topics and move back and forth between topics	3.2
S	Students work collaboratively in pairs or in small groups	3.2
S	I jump between topics as the need arises	3.2
S	I teach each student differently according to individual needs	3.0
S	Students invent their own methods	2.2
S	I find out which parts students already understand and don't teach those parts	2.0
S	Students choose which questions they tackle	1.8
T	I know exactly what math topics the lesson will contain	4.6
T	Students learn through doing exercises	4.2
T	I tell students which questions to tackle	4.0
T	Students start with easy questions and work up to harder questions	3.4
T	Students work on their own, consulting a neighbor from time to time	3.0
T	I tend to teach each topic separately	3.0
T	I teach the whole class at once	2.8
T	I try to cover everything in a topic	2.8
T	I tend to follow the textbook or worksheets closely	2.8
T	Students use only methods I teach them	2.6
T	I teach each topic from the beginning, assuming they know nothing	2.4
T	I avoid mistakes by explaining things carefully first	2.4
T	I only go through one method for doing each question	1.6

Note. Teachers were asked to rate each behavior or practice on a scale: 5 *almost always*, 4 *most of the time*, 3 *half the time*, 2 *occasionally*, 1 *almost never*. Mean frequencies are shown.

The first 12 practices listed in the table are student-centered and the mean frequencies reported by the teachers ranged from a low of 1.8 to a high of 4.2 and an average of 2.85. The teacher-centered practices had a larger range with mean frequencies from 1.6 to 4.6, and an average of 3.02, which was higher when compared to the student-centered practices. Following Swan's (2006) method for analyzing the questionnaire feedback, the practices statements were then used to construct a practices scale score. An individual score for each participant was obtained by reverse coding student-centered statements and summing the ratings. A score between 25 and 125 could then be used to analyze the reported practices of each teacher with higher scores reflecting teacher-centered practices. Table 6 shows the score for each of the participants in this study.

Table 6

Individual Participants' Classroom Instructional Practices Score

Participant	Score
Dave	76
Linda	74
Barry	78
Katie	64
Natalie	72

Note. A score between 25 and 125 was obtained by reverse coding student-centered statements and summing all 25 of the ratings reported by each participant.

Teacher Beliefs Questionnaire

Research Question Two for this study asked, "How are the teachers' beliefs towards mathematics and beliefs towards gifted students reflected in their instructional practices with mathematically gifted and promising students?" One data source for this research question included the teacher beliefs questionnaire. I provided each teacher with a hard copy of the teacher beliefs questionnaire which is based on Swan's (2006) research on teacher belief systems. Participants were asked to give each statement in the three sections (Mathematics, Teaching, Learning) a percentage, so that the sum of the three percentages in each section totaled to 100. A teacher's orientation to each of these categories can be characterized as transmission (T), discovery (D), or connectionist (C). A teacher with an orientation towards **transmission** sees math as a series of rules that must be directly taught to students and practiced until fluent. A **discovery** orientation is one where teachers believe that students learn through exploration and the teacher takes on the role of facilitator. The **connectionist** orientation sees math as a network of ideas that students and teachers construct together through collaboration. Table 7 provides the results from this questionnaire.

Table 7

Results From the Beliefs Questionnaire

Component/ Characteristic	Statement	Mean weighting (%)	SD
	Mathematics is:		
MT	A given body of knowledge and standard procedures, a set of universal truths and rules which need to be conveyed to students	28	9.79
MD	A creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods	33	21.26
MC	An interconnected body of ideas which the teacher and the student create together through discussion	39	15.16
	Learning is:		
LT	An individual activity based on watching, listening, and imitating until fluency is attained	27.66	11.03
LD	An individual activity based on practical exploration and reflection	31.66	4.94
LC	An interpersonal activity in which students are challenged and arrive at understanding through discussion	40.66	10.41

	Teaching is:		
TT	Structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to “grasp” what is taught	35	16.27
TD	Assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences	41	21.54
TC	A non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on	24	10
Transmission:	Mean weighting of MT, LT, TT	30.22	3.38
Discovery:	Mean weighting of MD, LD, TD	35.22	4.12
Connectionist:	Mean weighting of MC, LC, TC	34.55	7.49

Note. The instruction to teachers was “Give each statement a percentage, so that the sum of the three percentages in each section is 100”. Key to letters: The first letter represents Mathematics (M), Learning (L), or Teaching (T). The second letter refers to Transmission (T), Discovery (D), or Connectionist (C) beliefs.

According to the seminal work of Schoenfeld (1992), beliefs underpin personal thought and behavior. They underlie dispositions to engage in certain practices and not others. The results of the teacher beliefs showed the teachers as a group gave more weight to the **connectionist** statement about mathematics (a mean weight of 39% for connectionist versus 29% for transmission and 33% for discovery). A connectionist belief in mathematics is defined as an interconnected body of ideas which the teacher and the student create together through discussion.

Similarly, for learning, the responses again were weighted heavier in the **connectionist** category (40.66% for connectionist versus 27.66% for transmission and 31.66% for discovery statements). In this category of learning, a connectionist belief system is defined as an interpersonal activity in which students are challenged and arrive at understanding through discussion. This pattern shifted, however, in the last category of teaching where beliefs about teaching were weighted more heavily on a **discovery** role (41% for discovery versus 35% for transmission and 24% for a connectionist approach). The discovery approach to teaching is defined here as assessing when a student is ready to learn, providing a stimulating environment to facilitate exploration, and avoiding misunderstandings by the careful sequencing of experiences. It should be noted that the connectionist approach had the lowest weighting for the category of teaching and the lowest weighting overall (24%) of all nine of the statement weightings for this questionnaire even though it had the highest weighting in the first two categories of mathematics and learning.

In looking at the individual responses to the questionnaire, the beliefs orientation of three of the five teachers revealed a discovery approach, while two teachers showed a connectionist orientation. Based on Swan's (2006) development and interpretation of the instrument, teachers with a connectionist or discovery orientation showed an increase in the frequency of student-centered practices, with connectionist teachers presenting as the most student-centered of all three orientations. This is due to the emphasis teachers place on providing opportunities for students to make connections to prior learning and to discuss ideas and misconceptions. Table 8 shows the responses by participants for the three categories based on teacher beliefs about mathematics, teaching, and learning.

Table 8

Teacher Beliefs Responses by Participant

Orientation	Dave	Linda	Barry	Katie	Natalie
Transmission	90	80	113.33	70	100
Discovery	50	**125**	**133.33**	100	**120**
Connectionist	**160**	95	53.33	**130**	80
Total	300	300	300	300	300

Note. Based on Swan's (2006) criteria a teacher with a **transmission** orientation sees math as a series of rules that must be directly taught to students and practiced until fluent. Teachers who have a **discovery** orientation believe that math is a human creation, students learn through exploration, and the teacher takes on the role of facilitator. Teachers with a **connectionist** orientation see math as a network of ideas that students and teachers construct together through collaboration wherein the teacher is proactive in challenging students.

Semi-structured Interviews

The semi-structured interviews provided data in response to all three of the research questions in this study. Each of the semi-structured interviews took an average of 60 minutes and allowed me to learn more about the experiences and backgrounds of each teacher, their instructional practices, their beliefs about gifted students and mathematics, and the influence that their own conceptual understanding has on the experiences of students. The interviews were recorded and then later transcribed.

Classroom Observations

The third data source involved classroom observations, which took place on site at a mutually agreed upon time. The observations provided data in response to the research questions regarding the instructional practices teachers use with their students, as well as the teachers' conceptual understanding of mathematics. Prior to the observation, I asked each teacher to complete a pre-observation questionnaire, either in person or in writing, to provide an overview of the lesson's objectives, the instructional practices planned for the lesson, the knowledge or understanding students should have prior to the lesson, and any anticipated areas of struggle. I spent one class period, which was approximately one hour, in each of the five participants' classrooms as they taught math to a group of students. This included a second grade, two third grades, a fifth grade, and a seventh-grade classroom. Each observation was videotaped using an iPad set up on a tripod in the back of the room. I also took field notes and used the Mathematics Classroom Observation Protocol for Practices ($MCOP^2$) observation tool (Gleason et al., 2015) as a guide.

Process of Coding the Data

As part of the thematic analysis process, I analyzed the statements made by the teachers during the interviews and categorized them into specific codes based on each of the three research questions. As I looked for sub-themes, I counted the number of code statements that led to the identification of a sub-theme. I then grouped the sub-themes into themes. Table 9 provides an example of codebook data I used for this process (from research question three).

Table 9

Codebook Data Sample

Codes (statements from participants)	Sub-theme	Number of Code Statements	Theme
I know where kids will struggle with a concept	Understanding where students will struggle with a concept	2	Learning and practice are ongoing
I like to show misconceptions when teaching a new concept	Understanding where students will struggle with a concept		
Conceptual understanding is connecting the abstract to something concrete	Connecting the abstract to concrete	3	
Multiplying fractions is easy, drawing it in pictures is a lightbulb moment	Connecting the abstract to concrete		
Breaking it down helped me understand concepts	Connecting the abstract to concrete		
I look at test and standards and content to understand the concepts I am teaching	Ongoing learning and practice with the concepts	3	
I have to brush up on my skills all the time; constantly learning	Ongoing learning and practice with the concepts		
I always do a math puzzle or test before I give it to students	Ongoing learning and practice with the concepts		
Conceptual understanding is confident use of vocabulary	Students must be able to show how they got an answer	3	
Conceptual understanding for students is when they can explain to me how they got the answer	Students must be able to show how they got an answer		
Conceptual understanding for teachers-very important for them to understand the content and concepts I'm teaching and to show students HOW to get the answer	Students must be able to show how they got an answer		
Bar models helped me to make sense of the math	Bar models and the Singapore Math curriculum support teacher conceptual understanding	3	Curriculum supports understanding

Singapore Math has made me a better teacher	Bar models and the Singapore Math curriculum support teacher conceptual understanding		
Singapore Math helps me explain to the kids what it means to get equal parts	Bar models and the Singapore Math curriculum support teacher conceptual understanding		
Singapore math is new for me	New curriculum	3	
Love teaching word problems	New curriculum		
Love teaching word problems in this curriculum	New curriculum		
Math as a kid was very basic, no diving deep	School experiences	5	Math experiences impact understanding
Didn't understand math till college	School experiences		
In college they broke it down for us	School experiences		
Had good math teachers	School experiences		
Took AP Stats and Calculus;worked hard for B	School experiences		
Breaking it down helped me understand concepts	Math ability	5	
Never understood math growing up	Math ability		
I was bad at math.	Math ability		
I was always good at math	Math Ability		
Good at math as a kid	Math ability		
Favorite subject as a child	Enjoyment of math growing up	3	
Did not enjoy math growing up, did not come easily	Enjoyment of math growing up		
Love the light-bulb moment in math	Enjoyment of math growing up		

Research Question One

Research Question One asked: What are the teachers' instructional practices with mathematically gifted and promising students? The purpose of this question was to determine the enacted pedagogies or instructional moves that teachers regularly engage in with their students. The analysis of interview, questionnaire, and observation data revealed the following four themes: (a) students use a variety of means to represent concepts, (b) students communicate their ideas to others, (c) differentiation is the norm, and (d) teacher talk encourages student thinking. Table 10 provides data on the sub-themes, the number of code statements that led to the sub-theme, and the themes for this research question.

Table 10

Sub-themes and Themes from Research Question One

Sub-themes	Number of Code Statements	Theme
Use of manipulatives	4	Students use a variety of means to represent concepts
Big concepts	3	
Bar models	3	
Note Taking skills	3	
Drawing models	2	
Student choice	1	
Students work in groups	6	Students communicate their ideas to others
Real-world problems	2	
Modeling mathematics	2	
School structure	3	Differentiation is the norm
Classroom structure	2	
Questions	6	Teacher talk encourages student thinking
Content delivery	4	
Modeling mistakes	2	

Projects	2
Assessments	1

Students Use a Variety of Means to Represent Concepts

When asked to describe the instructional practices they used that were successful with mathematically promising and gifted students, all participants shared enacted practices that allowed for students to represent concepts in a variety of ways. This theme name was derived from one of the indicators on the $MCOP^2$ (Gleason et al., 2015), and can be defined as the use of models, drawings, graphs, concrete materials, manipulatives, and other tools that are used by the teacher to help focus student thinking on a concept. Students should interact with and develop these representations and not simply observe the teacher using them. Representations can be created by the students or the teacher.

Natalie described a multidisciplinary approach that is hands-on and allows students to choose the best method for solving a problem. She shared, "For me, finding a balance knowing that these kids learn in such different ways, means having a variety of instructional practices. Teaching them several different methods and letting them choose what strategy works best for them is helpful." She teaches her third-grade students to sketch out a problem, show it in a bar model, and work towards the algorithm, rather than starting with the algorithm. Natalie described a student who was "very good at math, but it was not her favorite subject," but she provided the opportunity for the student to show her understanding of groups by creating multiplication monsters (two eyes, each eye has five eyebrows, and so on) as part of an art project. During my classroom observation of Natalie, she directed the students to form partnerships to play *Contig*, a game of strategy using the four basic operations. In the game they must prove to each other that their dice combination equation is correct and then write this in their math journal.

Katie expressed the importance of connecting math with real-world situations and described how her second graders decided to create their own money system for their classroom while learning about money and exchange rates in different countries. She also keeps an estimation jar of various items which changes monthly, and students observe and write down their estimations and then count out the items in groups of ten.

As a middle school teacher, Dave provides guided notes to his students and as they are learning new concepts the students complete the notes. His students use *Desmos*, a software application that provides advanced math functions (like a graphing calculator) for projects that help users visually demonstrate math concepts. One of those projects is the birthday polynomial where their birthday represents coefficients for a polynomial. This work is displayed on posters in Dave's classroom and features student reflections.

Linda, like Dave, teaches her fifth-grade students note-taking skills where she models her thinking in her own notebook projected on the screen while students respond to questions in their notebooks in a guided process. I observed Linda teaching her fifth-grade students an introduction to two-step equations by asking them to complete guided notes that she projected on her screen.

Barry described her students' frequent use of math manipulatives for place value concepts. These place value manipulatives are part of the math curriculum used in grades K-4 at the school. Barry said, "I love teaching word problems. I love getting the kids to break them down and use bar models because I think this is the most eye-opening thing."

During Katie's observation, I saw her teach subtraction with regrouping to a small group of second graders using concrete materials and manipulatives. She modeled a place value chart on a large whiteboard with round magnetic discs and students used their own individual chart with dry erase markers to solve the problem of 462 – 27. Students had their own place value discs for hundreds, tens, and ones that Katie modeled the use of on her chart. As they worked through the problem, the students used these materials to show their understanding of the concept of subtraction with regrouping.

Students Communicate Their Ideas to Others

Teachers are expected to create mathematical communities that include dialogue around the mathematical content based on recommendations by the National Council of Teachers of Mathematics and the Standards for Mathematical Practices (Gleason, et al., 2015). This theme was also derived from one of the practices used in the MCOP^2 observation form and is defined as the way in which students are involved in the communication of their ideas to others (peer-to-peer). This item highlights the importance of all students being active participants in the classroom through peer-to-peer dialogue in pairs, groups, or the whole class. All five teachers talked about the practice of putting students in groups or pairs to work together to understand concepts taught in each of the classrooms. Barry typically posts a word problem on the smart board with a bar model (a visual that is a hallmark of the *Dimensions Math® PK–5* curriculum) but covers up the solution while students work the problem on their own and then discuss with one another. She also shared a pre-pandemic practice that she had used frequently of having students sit together in small groups with a big box of manipulatives they could share and just work on solving problems together.

“Very talkative with a strong need to collaborate,” is how Linda describes the students who excel most in math. “They just really like talking, collaborating, it’s more with each other, rather than, like, with me. The way that they think, they can connect with each other’s minds a little bit better.” In fact, Linda noted that she normally would not use this practice of allowing students to talk this much with each other, but she has adapted her practice because she believes that her students can sometimes learn better from each other than they can learn from her. Linda noted,

> If I just let them have that collaboration or kind of be chatty with one another about a topic or things they discover or notice, I don't have to intervene. I can literally explain something to them and they're like, "I don't know what you're saying." And one of them will be like, "Well, here's what I think she is trying to say."

In other words, it appears that Linda's students actually "translate" to one another what she is saying for better understanding. In Dave's classroom, he described students who are extroverted and talkative and who easily lead discussions. One project Dave described as successful is the video project students complete in groups in his Math 2 class. To understand and represent trigonometry identities, students cut out different colored manipulatives and blend them together. When asked why this practice was successful, Dave said, "I think it's two things. One would be it's new, it's brand new to them, they've never seen it. And number two, I think because they get to work in groups to produce a video. They enjoy doing that." While observing Dave teaching a lesson on probability, I noted a high level of discussion in table groups as they were asked to come up with two different rules for replacing or not replacing different colored marbles in a bag. Students made conjectures to each other, asked questions, and then explained and even argued their thinking to one another. Dave stopped by each table and provided feedback and asked questions, but the students themselves were leading this discussion on their own.

Differentiation is the Norm

The group work also serves as a differentiation practice, but more broadly, students are regularly and flexibly grouped based on readiness for the content. This theme can be defined as the regular and consistent practice of providing different content, small groups based on readiness, or acceleration to an upper grade. This practice was observed in all five teachers' classrooms and in their discussion of their practices. Linda teaches four cohorts of fifth grade but has four different preps for each class based on her students' demonstrated ability and readiness. Dave has a fourth grader who has been placed in his eighth-grade math class based on her ability, which was a primary reason I recruited Dave as a participant. As a third-grade teacher, Barry has the highest math group for the grade level and even that group is divided into two smaller groups; one group is working in the fourth grade *Dimensions Math® Pre-K-5* text, while the other group is doing fifth-grade math work. Barry shared, "One of the boys in our class, he's new to the school this year, he will probably jump up to seventh or eighth grade math next year. He's incredible, he's just that strong." Barry has a teacher assistant who works in the hallway with one group of students who are completing independent work while Barry teaches the other group, and then they switch. The practice of moving students ahead within the class and within the school is a normal practice. The school administration supports this model of acceleration and entrusts teachers with making decisions about what their students need in the classroom. One participant, Barry, remarked on how the teachers are part of the policymaking at Sycamore, and also shared, "This is the only place I've ever been treated like a professional, the only place where what I say matters. My director will say how do you want to arrange the kids? What do you want to do? You're treated like you actually know what you're doing.

Katie described the lesson I would be observing in her second-grade classroom where she had formed three flexible groups of students based on their understanding of the concept of subtraction with regrouping. Students move in and out of these cohorts throughout the year. For this lesson, one group would be working closely with the assistant on bar models. "They're still in the baby stages of bar models. And they're really struggling." Two groups would be working with her using the place value discs and mats, with one group moving more quickly through the content. “We’re extremely lucky that I have 20 students and an assistant. So, I can do three groups of math. My assistant is an engineer by training; we have very different skill sets and we work well together.” Katie described students who tested into fourth grade math but once she is sure they have a strong understanding of the bar models they can move up to third and fourth grade math work in the *Dimensions Math*® Pre-K-5 curriculum (though they stay in her classroom but use different texts). She discussed a student from a previous year who went to the middle school math classes to learn about Pi “because he understood the concept.” She described the importance of having access to older students and grade levels for her younger ones and noted, “Having that flexibility and having access to older kids with older experiences is really nice.”

Teacher Talk Encourages Student Thinking.

Teachers can have a significant impact on the type and level of student thinking simply by the types of questions they ask students. This theme can be defined as the type of language teachers use with students that focuses on high levels of mathematical thinking (analysis, synthesis, evaluation) which can promote reasoning and critique among students (Gleason et al., 2015). Linda specifically mentioned her approach to using the higher levels of Bloom’s taxonomy in her classroom, while Dave noted his practice of asking open-ended questions. I witnessed this in Dave’s Math 2 classroom as he asked open-ended questions about a probability exercise, often starting with “What if...?” One question that Natalie asked was, “How can I decompose 91 into 2 numbers that are easily divisible by 7?” Dave taught a probability lesson in his Math 2 seventh-grade class and asked students to come up with two rules, one with replacing a different colored ball to a bag and one with not replacing the ball to a bag. Dave also discussed the rubric that the school uses to provide feedback for student work which is another way for teachers to encourage student thinking in their classrooms, stating:

> Several years ago, we got together as a math department and decided we would use a rubric to rate three areas: understanding and knowledge which is the lowest level. The second one is problem-solving, and the third one is communication which I would say is the hardest one to do for gifted students.

Linda indicated that she is intentional about modeling her thinking and writing with her fifth-grade students. She also expressed a practice of regularly providing clarifying questions, encouraging students to ask questions at any time, especially if they are not sure about an assignment or struggling with a new concept. Modeling mistakes was an example of an instructional practice that Dave shared which can support student thinking:

> Mistakes are good, they help your brain grow. I try to emphasize this to my kids and it's hard in seventh grade, but then by the time they get to eighth grade they're fine with it. I might ask you to put a problem on the board. And I'm not gonna tell you if it's right or not, just put it up. If it's wrong, it's fine, it's okay. Because other people will learn from your mistake.

In response to the first research question, "What are the teachers' instructional practices with mathematically gifted and promising students?" several themes emerged from the data collected. These four themes included: students use a variety of means to represent concepts, students communicate their ideas to others, differentiation is the norm, and teacher talk encourages student thinking.

Research Question Two

The second research question for this study was, “How are the teachers’ beliefs towards mathematics and beliefs towards gifted students reflected in their instructional practices with mathematically gifted and promising students?” One of the goals for this study was to better understand how teacher beliefs can impact their work with students. The surveys provided data on the beliefs of teachers about mathematics, and then the frequency with which they used certain practices. The teachers also talked about their beliefs towards mathematics and gifted students as part of the interview. I did not use a survey on their beliefs towards gifted students; this data came specifically from the interview questions. After coding individual interviews and then looking for common sub-themes and themes across the data, three themes emerged regarding beliefs about math, while four themes were found for beliefs about gifted students for a total of seven themes in response to this research question. The themes regarding beliefs about mathematics were: (a) sense-making is important in mathematics, (b) math should be something students enjoy, and (c) math is the most important subject. For the beliefs towards gifted students, were the themes: (a) gifted kids want to get to the right answer quickly, (b) positive characteristics of gifted kids, (c) challenges and concerns of gifted students, (d) what gifted kids need from their teachers. The themes that emerged concerning beliefs about equity were a combination of beliefs about math and gifted education. These two themes included: (a) equity is giving every student what they need, and (b) school and home environments impact access to an equitable mathematics education. Table 11 describes the sub-themes, the number of code statements that led to the sub-themes, and the themes for this research question.

Table 11

Sub-themes and Themes from Research Question Two

Sub-themes	Number of Code Statements	Themes
Understanding the math	7	Sense-making is important in mathematics
Foundational skills	2	
Creative and efficient	1	
Math enjoyment	4	Math should be something students enjoy
Students learn at different rates	2	
Math is important	4	Math is the most important subject
Math has a hierarchy	1	
Need math for jobs	1	
Perfectionism	5	Gifted kids want to get the right answer quickly
Product over process	4	
Quick thinkers	3	
Care about social issues	5	Positive characteristics of gifted kids
Curious and smart	4	
Love of learning	4	
Need to collaborate	3	
Creative thinkers	2	
Socially vulnerable	2	
Learn at different rates	9	Challenges and concerns for gifted kids
Lack of executive functioning skills	6	

Self-esteem	5	
Compare and compete with others	3	
Opportunity to learn	1	Equity is giving every student what they need
All kids are capable of learning	1	
Equity would create happier students	1	
Giving kids what they need to be successful	7	
Time, class size, assistants impact equitable access	4	School and home environments impact access to an equitable mathematics education
Teacher education, physical resources, parent support, instructional practices	3	
Lack of family support impacts student success and opportunity	4	
Teacher encouragement	4	What gifted kids need from their teachers
Special school	3	
Special content	3	
Social emotional lessons	2	

Sense-making is Important in Mathematics

This theme can be defined as the significance of placing a greater emphasis on understanding the why and the how behind the procedures and skills related to math concepts. When sharing an accomplishment of which she was most proud, Katie relayed that, "It's really just that starting moment where math makes sense, even for kids who don't like math or feel they're not very good at it. Once that math makes sense, they can really fly." Barry emphasized the importance of understanding the mathematical concepts based on the belief that "these kids can memorize anything." She described a Singapore Math Night at the school as "transformative" when a middle school student who was attending as a sibling made the connection between the x's and y's he was learning in Algebra to the bar models she was teaching to parents. When Barry was teaching multiplication of fractions she described the student's understanding, by stating, "They don't have any idea what it is actually that they are doing. But when they're able to draw it in pictures, it's like they're having lightbulb moments."

When asked about an accomplishment that makes her proud, Katie explained the importance of sense-making in this way,

> Even my high-fliers who come in knowing the answers to everything, making them slow down and really understand what they're doing, then it's like they're exponentially growing after that. Then we can get into more project-based things. Once we have those foundational skills then we can keep going from there.

In a similar reflection, Natalie described how the curriculum ensures that students have a solid foundation of place value, but before she taught the algorithm, she taught them the box method for multiplication (also known as an open array model). Upon learning this and applying it to several problems, she described a student who was finally able to grasp the concept: "He was like 'Wow, this just makes a lot of sense.' Hearing things click, that's when it's the best because they're really getting the math behind it." Linda mentioned several times how she valued the importance of her students understanding math applications, stating, "I think when we can make it applicable to life, I think they're able to connect with it as well." Linda provided examples of lessons that weren't necessarily in the text but were essential for students to understand and make sense of, including sales tax, discounts, and paycheck deductions. Similarly, Katie believes that sense-making is important so that "you're not being ripped off, and you understand what people are saying." She also believes that students need to understand statistics, especially with "what's going on in the world right now."

Barry and Katie emphasized foundational skills in their third grade and second grade math classrooms respectively. Some of Katie's second graders have tested into fourth-grade math but she insists that they have a strong foundation of numbers regardless. Barry highlighted the use of word problems to ensure that her students could make sense of the mathematics behind the problem itself. She cited the practice of breaking down the problem and then using the bar models to understand the problem more deeply.

Math Should Be Something Students Enjoy

Katie shared her belief that enjoying math is "almost more important than just being able to understand math." She believes that "the more you enjoy something the more willing you are to challenge yourself." And that enjoyment leads to a sense of feeling successful, "and everyone likes to feel successful."

Dave felt that it is important for students to enjoy math but noted that not all children do: "I hope they enjoy it, but it's not my number one goal. It's not always going to be enjoyable, sometimes it's hard work." He did indicate, however, that if you can get kids to enjoy math, they'll do better in math. Dave shared, "I'd say the majority of our students in our school probably enjoy math. There's a few who don't, who struggle with it, they hate seeing a math problem. But my goal is to get them not to feel that way." Students learn at different rates and those who struggle don't always enjoy math. Barry acknowledged that, "not every kid's gonna be great in math," while Linda agreed that "tons of kids struggle in math." Katie shared her hope for her students in this way: "My hope here in our little world is they get to enjoy math."

Math is the Most Important Subject

All but one of the participants described math as the most important subject, noting that it is an important part of everyday life, people use it every day, and that students really need it to find jobs later in life. Barry, however, felt that math was not more important than English, but shared that at Sycamore, math was higher in the hierarchy because of the importance that parents placed on it. She felt that it had always been that way. She also implied that because she was the math teacher of the grade, she was seen as more important and more esteemed by the parents. It was interesting to hear Barry's stance on the way the state or public-school systems prioritized reading over math. She remarked on the inequity of not passing students if they fail the reading end-of-grade tests, but it was different if they failed the math end-of-grade tests:

> If you don't pass the math EOG you get to go on. You don't pass the reading EOG, they hold you back. There isn't the same priority for math in big school systems, or any school system. The state doesn't prioritize it.

The participants clearly believed in the importance of mathematics as a subject that students need to understand now in school, but also to help them be successful in the future. This concludes the description of the themes related to beliefs about mathematics. The next four themes reflect the teachers' beliefs towards gifted students.

Gifted Kids Want to Get the Right Answer Quickly

The focus of the second research question was on the beliefs of teachers regarding both mathematics and gifted students, though when teachers spoke of the students, they referred to them as mathematically gifted. While these topics can be combined, it was important to understand the beliefs teachers held about gifted students in general. When asked to describe the characteristics of gifted students or what teachers believed about them, several themes developed around the description of students as quick thinkers and a preference for product over process.

This can be described as the definition for this theme: students are more interested in rapidly arriving at the correct answer, while potentially placing less emphasis on the process of understanding the problem. Barry said, “Gifted kids want instant gratification; they’re not worried about the process. “Lots of them are Kumon kids, they have all the facts, but they don’t necessarily know what they’re doing with the facts.” Dave mentioned that his middle school students often only want to give the answer and not share how they got it. Katie depicted her students as “quick” and being reluctant to check their work.

Traits of perfectionism came up in several of the interviews with teachers as they described students who “aren’t willing to try new things” or they are “more concerned with being right” (Barry). Linda believes that gifted kids “don’t like to be wrong or not know something and that they don’t know what to do when they don’t get something right.” She was not sure if this was a “gifted mind thing or more of a culture thing.” She described the students in her class who are of Asian descent who tell her, “No, I have to get all of them right. If I don’t get all of them right, I have to do them all again at home.”

Positive Characteristics of Gifted Kids

It was clear in the way that the teachers spoke and the responses they shared just how much they appreciate and care for their students. Though these are their perceptions about their students it does not necessarily represent all students, as participants were not asked to quantify their responses to the question. Rather, the participants were asked to describe their students in general. Several sub-themes emerged as teachers described how inquisitive, creative, enthusiastic, curious, and smart many of their students are. “One of the best things about this job is you have kids who really want to learn, it’s awesome. They’re extremely motivated and they want so much to learn more,” according to Natalie. Dave described them as “extroverted, they talk a lot, just like a regular kid," and "they are smarter than me and can sometimes explain to the class better than I can." Linda turned her attention to the way that for her students "education is very valued in their community and home," and "they always want to know the why, so I have to come prepared with the why." Barry and Linda both described the students' need to collaborate, while Natalie shared,

> These are our big idea people. This is the person who is going to be like a philanthropist and come up with a cure for who knows what, in 50 years. I was a high achiever as a student, but I did not have all these ideas that were going to solve all our world problems.

Dave talked about his students' desire to raise awareness and change the world, stating, "Gifted students feel strongly about social issues. I tell them, you will be leaders in our nation soon." Like Natalie's belief about gifted students' contributions to the world, Dave believes that gifted children have the solutions to problems that could help our society. And Linda believes that gifted kids are important to our society, stating, "I think when we have different opinions and different thoughts and ways to process things in life it makes our world unique." Many of the characteristics the participants described, however, could be considered myths or even stereotypes of gifted students and therefore it is important to not automatically assign these traits to all gifted students.

Challenges and Concerns for Gifted Kids

While there were several characteristics described as positive for gifted students, the teachers also shared their own perceived challenges these students face and the concerns they have for their students in facing these challenges as observed by the teachers. Natalie and Linda each described a lack of executive functioning skills in many of their students. Natalie said, "they are like absent-minded professors." And Linda explained that "they have so much going on in their brains that they're forgetful and sometimes seem to lack common sense, but please know I don't mean any of this in negative ways." She provided further detail about the students' lack of organization with their notebooks, binders, and backpacks and how they can "walk out of the classroom and then realize they forgot all of their stuff on their desk." Linda also indicated that they do not always get idioms or understand sarcasm, so she tries to explain this better to them so they will understand. Natalie described two different kinds of students:

> You see two spectrums of kids, one that is like the perfectionist, they want to get everything down. And then you see the absent-minded professors who are doing a million different things in their head and not one of them is the lesson you are teaching, because something is so important that they're focused on, so you see a wide variety of personalities, you see a wide range of emotions. I have a lot of that in my class this year.

Many participants reported concern for their students' self-esteem. Dave noted that many students he has taught lack confidence and feel that no one believes in them. Some are introverted and keep to themselves and seem more sensitive overall. Barry shared that "their attitude can affect their confidence and ability." Oftentimes, the participants noted, gifted students compare themselves with other students, expressing a worry that they are not good enough or that they will not get into the next class. Natalie saw this as a positive characteristic in her third graders, stating, "Mathematically gifted kids want more, they're motivated, engaged, and competitive," but they can also be "perfectionists, impatient, and very hard on themselves."

Across all participants, it was noted that gifted students learn at different rates and that some may even be "reluctant learners" (Barry) and may "struggle silently even though they want to do well and are studious" (Linda). Barry described her students who struggled with word problems, and acknowledged that “everyone’s got different gifts, and some have more than others.”

What Gifted Kids Need from their Teachers

One of the interview questions asked of all teachers was to describe what they believed gifted students need most from their teachers. The responses shared here are based on the individual experiences and interactions of each of these participants with their students. Gifted kids need “an abundance of reassurance” from their teachers, according to Linda as she described her students’ need for her to clarify that how they have solved a problem is right. Linda also expressed the importance for teachers to provide “a ton of movement” for their gifted students. Natalie felt it was important for gifted students to have opportunities for inquiry-based learning and STEM activities. At the same, she also discussed the importance of struggle as something gifted students need to experience,

> I think part of being a gifted math teacher is really, purposely choosing some challenges at the beginning so that kids learn that this should be hard, and it is okay. Because you want them to learn how to struggle through something. You don't want it to be easy because they're going to be great at math no matter what, but we need to teach them the skills to work through something when something is really hard and knowing that it's okay and almost better when you don't get something right away because it's a learning experience.

Dave described the importance of teachers being a cheerleader, a lifelong friend, and someone who will show the students love. He said, “You have to take care of them as if they were your own children.” Several of the teachers discussed the SEL (social emotional learning) lessons that take place weekly at Sycamore School and how important this was for gifted students to experience. Dave emphasized this, “I hope you can convey to teachers that gifted kids need to know that people love them and care for them, and they have confidence in them. Because I don't think teachers realize that these kids lack self-confidence.”

In terms of a special school, such as the one at Sycamore School, Dave believes that gifted students need a special school in which to be successful socially and academically. He said they also need structure and to learn the content based on the standards. On the contrary, Linda believes that gifted students do not necessarily need a special school, but that it benefits other students to keep them in the general population, but only if all schools were able to provide the accommodations and differentiation they need. She also shared that gifted students can still learn from their same-age peers who aren’t labeled gifted and that, “having all of those gifted brains in one classroom can sometimes be seen as a challenge because they all have different pieces of giftedness, and then you're trying to combat all of them.”

Equity is Giving Every Student What They Need

When participants were asked to define equity and to then expound on their beliefs regarding whether every child has access to an equitable mathematics education many responded similarly in that equity is about giving every student what they need. Barry noted that all kids are capable of learning and that if given the right environment can succeed, while Natalie shared that a special school (like Sycamore School) provides an equitable opportunity in academics for those students with high abilities. When specifically prompted about race, ethnicity, or gender, one teacher (Barry) remarked that gender does not impact a student’s success or enjoyment in mathematics and that equitable access was less about race and ethnicity and more about socioeconomic status. Dave feels that race, ethnicity, and gender only impact student enjoyment and achievement if the teacher or student allows for that to be the case, “I don't care what ethnicity or what gender you are, I think you can be successful in math." Dave also stated that the school should ensure that transportation, Internet access, digital devices, and lunch be provided to any student who needs it and that this should not get in the way of a student's access to the school. He also mentioned the importance of not changing the admissions criteria as that would not be an example of equity.

School and Home Environments Impact Access to an Equitable Mathematics Education

Several teachers emphasized that teacher training and professional development impacts the kind of experience students receive in mathematics. Barry said, "The bottom line with anything in education is teachers are going to do it if they're trained. The biggest thing is professional development and making that a priority." Katie indicated that teacher education and training can impact an equitable mathematics education. Other factors cited by the participants from the school level were time (amount of time to teach math), resources, class size, and having an assistant. Linda defined equity as differentiation and student-centered learning and expressed the importance of teachers creating small, differentiated groups to help all students be successful.

The home environment was also prominently featured as having an impact on success for students. Dave stated, "Lack of parental involvement is the number one cause for students to not be successful in mathematics." Natalie noted similarly that some students don't have the support at home in math. Linda discussed the importance of the educator's mindset about a child's culture or background,

> If you have an educator who's close-minded, that believes that students of this ethnicity, or this gender are slower learners, well, you've put a limitation on that child, and no child should ever have a limitation put on them. I think it's when we start putting limitations and labels on certain groups of students it can hinder their learning. Every student has a

brain, and it is up to the adults in their lives, their educators and role models to cultivate that brain.

As a group the teachers acknowledged that certain policies or practices at the school level, such as teacher training, class size, physical and human resources (assistants) all impact the mathematical experiences students receive, and that this can and does vary across school settings. The participants also indicated that the home environment plays a role in a child's access to an equitable mathematics education and experience.

Research Question Three

This research question asked, "How do teachers describe their conceptual understanding of mathematics as it relates to the teaching of mathematically gifted and promising students?" The purpose of this question was to better understand the teachers' own mathematical experience, before teaching and as a teacher now, and how that connects to mathematically gifted and promising students. Three themes emerged in response to this question: (a) learning and practice are ongoing, (b) curriculum supports understanding, and (c) math experiences impact understanding. This question was asked during the interviews and some of the participants referred to the students' conceptual understanding, but this was really about the teacher's own understanding of the content they were teaching daily. Table 12 provides the sub-themes, the number of code statements that led to the sub-themes, and the themes in response to the teachers' description of their conceptual understanding of mathematics.

Table 12

Sub-themes and Themes from. Research Question Three

Sub-themes	Number of Code Statements	Themes

Connecting the abstract to the concrete	3	Learning and practice are ongoing
Ongoing learning and practice with the concepts	3	
Students must be able to show how they got an answer	3	
Understanding where students will struggle	2	
Bar models and Singapore Math support understanding	4	Curriculum supports understanding
New curriculum	3	
Math ability	6	Math experiences impact understanding
School experiences	4	
Enjoyment of math	3	

Learning and Practice Are Ongoing

Noting the changes in mathematical standards and content, Linda stressed that her own conceptual understanding of mathematics requires her to stay abreast of any changes. When students share with her something they learned from a tutor, she researches this and often finds that the way to teach the content has changed. She also prepares for new content by looking ahead to see what she needs to review and be ready to teach:

> I always look ahead to what I'm teaching, and I brush up on my skills all the time. Sometimes I have to sit down and watch videos so I can think of ways to help them understand the lessons a little bit easier. I'm always constantly learning.

When describing her understanding of the mathematics she is teaching, Barry indicated that she always completes the test, or the math puzzle herself before giving it to students.

Likewise, Dave shared how important it is for him to understand the content and concepts he is teaching and to show students *how* to get the answer, stating, “I feel like I have a good conceptual idea for what I teach. If it’s something I haven’t taught in a while, I brush up on it.”

The importance of knowing where kids will struggle and anticipating misconceptions so that she can show them when teaching a new concept is how Natalie described conceptual understanding in relation to teaching students. She also acknowledged that experience over time matters and because she has taught for many years it strengthens her understanding of concepts and helps her provide better instruction to her students.

Curriculum Supports Understanding

Participants frequently referenced the curriculum used at Sycamore School, which is the *Dimensions Math® PK-5* (Singapore Math Inc., 2022) curriculum, though they typically referred to it as Singapore math. Barry noted that the curriculum had made her a better teacher. More specifically, she said,

> The bar models have helped me make sense of the math and help me explain to the kids what it means to get equal parts. I spent an entire day from 8 to 4 pm doing nothing but bar models (in reference to curriculum training). I loved it. Once I got it, I was like, wow, this is just all making sense to me. Whereas you know, as a child, it never did.

Barry also noted that the curriculum starts off easy and then problems begin to represent more difficult concepts. She felt that she would have been more successful in math if she had learned the way she is teaching it now. “If I had learned the way they teach in Singapore math I might have been an engineer.”

When asked to describe her confidence in understanding the why behind certain skills, Natalie shared, “It betters my instruction the better I understand the background of the concept.”

She mentioned when she had once gone from teaching third-grade math to fifth-grade math and there was a learning curve in knowing where the students were going to struggle, which she described as similar to teaching the curriculum at her school since it is new for her. “There’s a big learning curve with me now with Singapore math being my first time teaching this curriculum.” She said that knowing where students are going to struggle is important, but a new curriculum can make that difficult. Katie spoke about having more confidence with using math vocabulary because of the curriculum.

Math Experiences Impact Understanding

This theme primarily described the mathematics experiences the participants had growing up, but also referenced math teaching experience. Natalie felt that her experience teaching, and her own education had equipped her with feeling prepared to teach mathematical concepts to gifted students. She shared that it was her favorite subject as a child and that she always did well in math at school. Natalie also noted that math was more basic when she was in school and that she really didn’t fully understand math until college. While Linda shared, “I was good at math, it came easy to me. I’m dyslexic so I don’t spell well, but I was always good at math. I didn’t have to do spelling to be a math teacher.” Katie indicated that she did not enjoy math when she was growing up, but that she enjoys it now as a teacher. “I had to work really hard at it, I mean I went all the way up to Calculus, but it was one of those things that didn’t come easily.” Barry discussed how “bad” she was at math growing up but felt that if she had learned math the way her students are learning it, she would be an engineer. She feels like math makes sense to her now, but as a child it never did.

Overall, three of the participants (Dave, Linda, and Natalie) indicated that math is a subject they have always enjoyed and have felt good at or successful with. And two participants

(Katie and Barry) expressed how difficult math was for them in school and that they didn’t really enjoy it growing up. Natalie indicated she did not have to work hard at math until Calculus and AP Stats in high school. She also shared that her math experiences in elementary school were different than what is expected of students at her school. “I feel like we weren't, they weren't diving deep, then. Like it was here's the algorithm, you practice it, and that's it, you got it. Whereas now, obviously, we know that's not the right direction to go. But I felt successful as a math student. And I liked the variety that math offered. Like, I liked having geometry. I liked having algebra. And I would say for the most part, I had pretty good math teachers.”

Natalie also shared that she enjoys teaching reading and that she likes to do math outside of school. “I love teaching all subjects, but math and reading are what I'm most passionate about. I like math puns; I like solving math equations.” She noted that she never had an interest in going into a field like finance or accounting, but she did want to be a teacher. “My first consideration when I went off to college was that I'd be like a middle school math teacher. And then I realized that I like elementary school aged kids way better. So yeah, I'm happy with what I do. It's awesome.”

Summary

Chapter IV began with a discussion of the setting and participants for this study including the participant demographics. Next, I presented a profile of each of the five participants, followed by a description of the data collection and data analysis methods. The findings presented included survey data from the teacher beliefs and classroom instructional practices questionnaires. Lastly, I described the codes and themes created in response to each of the three research questions and then synthesized those themes across all participants.

In Chapter V, I will discuss an interpretation of the findings of this study in relation to the literature and conceptual frameworks used, followed by implications and recommendations. The chapter also includes a description of the limitations of the study, recommendations for further research that are grounded in the strengths and limitations of the current study, and a conclusion.

CHAPTER V: DISCUSSION, CONCLUSIONS, AND RECOMMENDATIONS

Overview

The chapter begins with a summary of the problem, purpose, research questions, and methodology of the study. I will then present a summary of the study and important conclusions drawn from the data presented in Chapter IV which includes a discussion of the implications for action and recommendations for future research. The chapter is organized as follows: summary of the study, major findings and their connections to the literature and the conceptual frameworks, implications and recommendations to the field, limitations, and recommendations for future research. Additionally, the findings for each research question will be connected to current literature. Responses to each research question and a discussion of the data in connection with the literature will then be presented. Further, I propose key recommendations to support the work of teachers and schools and the improved achievement of mathematically gifted and promising students. The implications for this study overlap the fields of gifted education and mathematics education with a specific focus on ensuring that teachers have the skills and knowledge related to supporting the academic and social emotional needs of their students. Finally, I will share the limitations and delimitations of this study along with recommendations for future research.

The Problem

Students in the United States are well behind their peers in international assessments of mathematics achievement. While this issue affects all students in the United States the implications for students who have been identified as mathematically gifted and promising are significant. Only 8% of U.S. students scored at Level 5 or higher in mathematics using the PISA tests (OECD average: 11%), compared to Singapore which had 37%, China (44%), and Korea (21%). A Level 5 or higher indicates an ability to model complex situations mathematically, and select, compare, and evaluate appropriate problem-solving strategies for dealing with them (OECD, 2019). Similarly, U.S. student achievement on the most recent NAEP (National Assessment of Educational Progress) exams also indicate that few students show advanced understanding in mathematics (less than 10% of 4th graders) and twice that many (or close to 20%) are below basic in their mathematics ability (NCES, 2019).

Not properly educating our mathematically gifted young people has dire consequences on the students themselves and our nations (Renzulli, 2005; Sheffield, 2016; Stepanek, 1999). In the United States, the National Science Board report, *Preparing the Next Generation of STEM Innovators* indicated, "The long-term prosperity of our nation will increasingly rely on talented and motivated individuals who will comprise the vanguard of scientific and technological innovation; and every student in America deserves the opportunity to achieve his or her full potential" (National Science Board, 2010, p. v, as cited in Sheffield, 2016).

Teacher preparation and conceptual understanding of mathematics are often cited as a critical factor in student mathematics achievement (Ma, 1999/2010). Some studies indicate that elementary teachers do not have the skills needed to teach mathematics based on the poor preparation received at many undergraduate institutions (NCRTE, 1991). Teacher beliefs about mathematics guide their instructional practices and behaviors towards students (Ernest, 1989; Swan, 2006) and provide valuable information to better understand student achievement.

Purpose of the Study

The purpose of this qualitative phenomenological case study was to explore the experiences of teachers who work with mathematically gifted students through their beliefs, instructional practices, and conceptual understanding of mathematics. Using semi-structured interviews, questionnaires, and observations I described the phenomenon of five teachers instructing mathematically gifted students in a K-8 public charter school for gifted students. I chose this setting to study teachers who work exclusively with mathematically gifted students and who therefore may possess a unique view of this experience Specifically, this study described teacher beliefs about mathematics and gifted students, teacher instructional practices, and teachers' descriptions of their conceptual understanding of mathematics as it relates to mathematically gifted and promising students. This study aimed to address the lack of student achievement in mathematics in general and to understand the influence these phenomena have on mathematically gifted and promising students. The contributions of mathematically gifted students are valued and needed, and this research sought to address this issue and provide potential solutions.

Research Questions and Methodology

The following research questions are directly tied to the purpose of this study in response to the problem statement presented. To that end, this study sought to understand the overarching central question or phenomenon of the lived experiences of the teachers of mathematically gifted students, and was specifically guided by these research questions:

1. What are the teachers' instructional practices with mathematically gifted and promising students?
2. How are the teachers' beliefs towards mathematics and beliefs towards gifted students reflected in their instructional practices with mathematically gifted and promising students?
3. How do teachers describe their conceptual understanding of mathematics as it relates to the teaching of mathematically gifted and promising students?

Using an Interpretive Phenomenological Analysis (IPA) approach, I began with the interview data I collected for each of the five teachers. I listened to the recordings and then read and re-read the interview transcripts looking for significant statements for each participant and then across the participants. IPA is a process of analyzing data that focuses more on situated, interpreted, and lived experiences than on transcendental experiences. (Johnson & Christenson, 2020). After coding the significant statements and creating a code book for each interview I used these codes to form sub-themes and then finally themes in response to the three research questions. The analysis of the classroom observations data was similar in that I viewed the video clips multiple times for each teacher, wrote descriptions for teacher moves, and added significant quotes from each participant to the codebook. This data was also used to support the themes generated.

In analyzing the questionnaires, I looked at the beliefs statements first and calculated the *mean* weighting for each of the nine statements on teacher beliefs for mathematics, learning, and teaching to determine if each teacher's beliefs were more aligned with a transmission (T), discovery (D), or connectionist (C) view. To analyze the questionnaire on classroom practices I calculated the mean frequencies as reported by the teachers for each of the 25 practices to determine if the group used more teacher-centered or student-centered practices. I then constructed a practices scale score (from 25 – 125) for each teacher by reverse coding and then summing the student-centered statements for each individual participant. Higher scores indicate more teacher-centered practices.

Summary of Findings

Several themes emerged for each research question after data analysis was complete. This summary provides an overview of the findings connected to each of the three research questions.

Research Question One: Instructional Practices

The first research question focused on instructional practices and the findings showed the different ways teachers guided the students to represent concepts, which included bar modeling, note taking skills, and using manipulatives. Partner and group work were a common practice especially in the interest of discussion and collaboration between and among students. Teachers regularly modeled mistakes and used questions to provoke thinking. The school institutes a common practice of flexible grouping of students at the grade level, within classrooms, and within the school based on readiness. This differentiation practice was the norm at this school, allowing teachers the flexibility and autonomy to use more than one grade level of curriculum within a classroom or a grade level or to recommend acceleration to another grade level when needed. Quantitative survey data from the classroom instructional practices questionnaire revealed that the participants utilized a combination of student-centered and teacher-centered practices in their classrooms. This data was combined with the teachers' responses to the interview questions about the types of instructional practices they enact in their own classrooms. Classroom observation data provided additional insight and information into the types of practices used by the teachers.

Research Question Two: Beliefs About Mathematics and Gifted Students

An analysis of the second question on teacher beliefs highlighted the importance of sense-making in mathematics, student enjoyment of math, and a belief that math is the most important subject for students to study now and in preparation for their future. It was noted that students learn at different rates, but foundational skills and understanding are paramount for success. For their beliefs about gifted students the teachers reported that gifted students value the product over the process; they want to get an answer quickly. Teachers listed a multitude of positive traits that their students exhibit, such as curiosity, a love of learning, a desire for challenge, enthusiasm, and creative thinking.

Additionally, each teacher participant shared their beliefs about the common struggles of their students and concerns for the challenges their students face. Namely, the teachers recognized the social-emotional needs and pressures that their students experience daily and were vocal about supporting these students' ability to navigate challenges with executive functioning, parental pressure, self-esteem, and perfectionist tendencies. Overall, the teachers were passionate about mathematics and working with gifted students. Four of the participants felt that mathematically gifted students need a special school to succeed, not just for the academics but for the social aspect of being accepted for who they are. The survey data collected from the beliefs questionnaire revealed that two of the participants held a mostly connectionist belief orientation about mathematics, teaching, and learning, while three of the participants followed a discovery orientation. This quantitative survey data provided an understanding of what the teachers believed about mathematics while the responses solicited in the teacher interview offered additional and deeper insight into the beliefs as described by the teachers, Further discussion about these orientations will be provided in this chapter.

Beliefs About Equity

Teacher beliefs about equity in relation to their beliefs about mathematics and gifted students suggested two themes. For the first theme, teachers believed that equity is giving everyone what they need; that all children, regardless of their race, ethnicity, or gender can succeed if given the right environment and opportunity. In response to the question, "What would it look like if all students had access to an equitable mathematics education? Katie responded, "A whole bunch of happier math students." And Linda's response to this question highlighted that small group instruction with a focus on comprehending the curriculum was the key to equitable access, “We have to make sure that we're doing our part to take the time and pull small groups or do mini lessons for the students, understanding that not all students need the same lesson.”

The second theme related to equity was: School and home environments impact access to an equitable mathematics education. School resources cited by the teachers included teacher training and professional development in mathematics, small class sizes, having an assistant, math manipulatives, time, and the ability to provide differentiated instruction. The participants also believe that parental support is key for students to be successful in mathematics. Students who don’t have a home environment that supports their mathematical needs or who have parents who are not involved in their child’s education limits a child’s access to an equitable mathematics education.

Research Question Three: Teachers’ Conceptual Understanding of Mathematics

For the third research question which probed the teacher’s description of their conceptual understanding of mathematics several themes emerged. These included an emphasis on the math curriculum (*Dimensions Math®Pre-K-5*) to help the teachers understand math concepts, in particular the bar model approach that is part of Singapore math. All participants shared their own math experiences growing up, with some noting that they were not good in math, and others indicating this as a strength when they were in school. All teachers acknowledged that their understanding of mathematics did not match that of their students, i.e., they believed that the students had a deeper conceptual understanding of mathematics than they did. The two teachers working with older students (fifth grade and seventh grade) stressed the importance of ongoing learning and self-practice to stay current with topics they were less familiar with or hadn't taught in a while and needed to review before teaching to students.

Discussion in Response to the Conceptual Frameworks

To better understand the conceptual understanding of mathematics and the beliefs and practices required of teachers to successfully nurture students with mathematical promise, two conceptions of giftedness served as the lenses for this study: Renzulli's Three-Ring Model (1978) and Gagné's Differentiated Model of Giftedness and Talent (1985). These models were chosen as they helped frame the research purpose and questions in this study, namely, to better understand the experiences of teachers of the mathematically gifted and promising.

Renzulli's Three-Ring Model of Giftedness

Renzulli's model describes the importance of utilizing more than one trait to identify students for programming that will nurture their potential. This approach aims to avoid missing those students whose test scores may be lower than a prescribed cut-off. This design allows for more students to be recognized and their promise to be developed. In the present study teachers responded to questions related to equity and to whether they believed that all students had access to an equitable mathematics education, and what, if any, factors might prevent students from having full access. Two of the themes that emerged in response to Research Question Two about teacher beliefs were 1) Equity is giving everyone what they need and 2) School and home environments impact access to an equitable mathematics education. Renzulli believes that it is critical for schools to identify gifted behaviors in order to provide the resources and programming students need to develop and grow their potential. School factors include time, resources, teacher training, and small, differentiated groups. Said one participant, "I tell students what one person needs you might not need and what you need they might not need. It's not ever going to be equal but meeting you where you are" (Linda). The teachers believed that teacher education is a factor that helps ensure that all students get what they need, which was a definition the teachers used to describe equity.

The three clusters that make up Renzulli's Three-Ring Model include above-average ability, task commitment, and creativity. The participants in this study shared their beliefs that gifted students possess all these attributes. Renzulli (1978) stated that children capable of developing all three clusters require a wide variety of educational opportunities, and that these are not typically provided in regular educational settings. One such opportunity is a special school designed specifically to meet the needs of gifted individuals, such as Sycamore School, the school site for this research. The school uses multiple criteria as part of the admissions process. Renzulli (2016) cautions against using test scores as the only predictor for success in schools. "We must draw our circles larger so that we don't overlook any young person who has the potential for high levels of creative productivity (Renzulli, 2016, pp. 85-86).

Renzulli indicates three phenomena of creativity as described by Csikszentmihalyi (1996), where the third phenomenon is the type of creativity that impacts others and is caused by those who have changed the world in some significant way. The participant, Dave, described his older students as very concerned about social issues. "They want to raise awareness over any of the social issues that are in our society."

Gagné's Differentiated Model of Giftedness and Talent

Gagné's concept of talent development identifies six domains of giftedness (beyond intellectual) and the catalysts (environment and support) that help develop those gifts into talents. The impact of classroom teachers who are in a prime position to serve as catalysts for the development of talent in young and promising students cannot be underestimated. This research study was aimed solely at the teacher--their beliefs, their practices, and their own conceptual understanding of mathematics. Gagné critiqued Renzulli's framework of giftedness because he believed that gifted and talented were not synonyms. He is less interested in identification of students and more focused on what it takes to develop young people's gifts (above-average competence) into talent (above average performance). His model also designates specific ability domains, such as mathematics. One of the teacher participants talked about her role as the math teacher for her grade level and that parents value her more highly because she teaches math. She described math as being at the very top of the hierarchy in the school.

Of critical importance is Gagné's belief that teachers and peers serve as catalysts for producing talents from gifted learners. The teachers at Sycamore School were intent upon helping each child fulfill their potential, but to also see more than just their academic gifts. Dave expressed a desire to help gifted students gain confidence in math, to understand the concepts, and apply them to the real world or to higher math. His intentions reflect Gagné's concept of developing gifts into talents. Linda shared her belief that gifted students need a special school like Sycamore to excel and reach their potential. Several of the participants remarked on the influence of peers, seeing partner and group work as opportunities to discuss math with a like-minded peer.

Finally, Gagné also recognized the impact that curriculum, pacing, and pedagogy can have on developing the potential in students, which he grouped under the Provisions (EP) category of environmental influences. Three of the participants frequently mentioned the role that the school's curriculum played in their own conceptual understanding of mathematics. They also saw its power in strengthening the students' understanding and to see the "why" behind the procedures of math. Provisions also encompassed grouping and acceleration. A theme that emerged from the study was: "differentiation is the norm" at this school. Students are accelerated, when necessary (i.e., the fourth-grade student who is in a seventh-grade math class), and students are ability grouped within each class or the grade level to ensure that students' learning needs are being met.

Discussion in Relation to the Literature

Research Question One

Research Question One asked: What are the teachers' instructional practices with mathematically gifted and promising students? The purpose of this question was to determine the enacted pedagogies or instructional moves that teachers regularly engage in with their students. The analysis of interview, questionnaire, and observation data revealed the following four themes: (a) students use a variety of means to represent concepts; (b) students communicate their ideas to others; (c) differentiation is the norm; and (d) teacher talk encourages student thinking.

The findings of this study align well with the NAGC's (2014) standards for all teachers of gifted students who should be able to "select, adapt, and use a repertoire of evidence-based instructional strategies to advance the learning of gifted and talented students." Tomlinson (1995) defined differentiation as the use of a variety of instructional approaches to modify content, process, and products in response to student readiness. The second and third grade teachers in this study consistently employed a variety of practices, including the use of different math texts, while the seventh-grade teacher (Dave) provided different prompts to small groups as they searched for a probability rule. Consistent with the video study of the 1999 TIMMS data (O'Dwyer et al., 2015) where teachers in high-performing countries spent more time teaching and reviewing concepts than on procedural skills, the teachers at Sycamore School also utilized practices that emphasize understanding of math topics over procedural skills. This included subtraction with regrouping using place value discs, the student creation of rules that apply to different probability situations, similarities and differences between one-step equations and two-step equations, and the use of mental math strategies to determine expressions for a word problem to break down or decompose numbers. In fact, not one of the classrooms observed in this study featured teachers focusing on procedures, though only one observation was conducted in each teacher's classroom. Again, the participants' practices reflected the teachers in the higher-performing countries and not the U.S. teachers who spend considerable time completing low level, repetitive exercises. The emphasis on deep understanding of a single topic was evident across the case study of the school, though researchers (Hiebert et al., 2005; Prawat, 1989) did not find this in U.S. classrooms and have connected such practices to poor math performance. The findings related to instructional practices validate the research of Lampert et al. (2013) who concluded that facilitating student discourse with each other and with the teacher through open-ended questions maximized access to meaningful mathematics.

The findings from the teacher-reported frequencies of 25 instructional practices based on Swan's (2006) research differed dramatically from Swan's findings in that the teacher-centered practices dominated in his results. As reported in Chapter IV the results of the survey for the present study showed that teachers' mean frequencies ranged from a low of 1.8 to a high of 4.2 and an average of **2.85 for student-centered practices** (on a 5-point scale). The teacher-centered practices ranged in mean frequencies from a low of 1.6 and a high of 4.6 with an average of **3.02 for teacher-centered practices**, so somewhat higher when compared to the student-centered practices. This finding shows that the teachers as a group engaged similarly in teacher-centered and student-centered practices, with neither category really dominating. The statements about practices were then used to construct a practices scale score. An individual score for each participant was obtained by the process of reverse coding the student-centered statements and summing the ratings. A score between 25 and 125 could thus be used to analyze the reported practices of each teacher with higher scores reflecting teacher-centered practices. Not surprisingly, the practices score for the participants fell right in the middle (roughly halfway between 25 and 125) with scores ranging from 64 to 78. Further research on the types of practices best suited for mathematically gifted and promising students is recommended to better understand the implications this approach may have on this population of students.

Research Question Two

This research question probed for the teachers' beliefs towards mathematics and gifted students. Some of the findings here were consistent with the work of Ball and her colleagues (2005, 2007) in studies related to "mathematical knowledge for teaching." All participants indicated the importance of knowing when and where students might make errors in their thinking and their propensity for working quickly to the detriment at times of their understanding. The teachers reported and exhibited a strength in possessing curricular knowledge which is consistent with one of Shulman's (1986) three categories of content knowledge. "Sense-making is important" was a key theme for the teachers' beliefs about math wherein they each expressed the importance of students understanding the mathematics, understanding the "why" of math, and emphasizing conceptual understanding over procedural skills. These beliefs are reflected in the findings of Ball, 2005; Hiebert and Grouws; 2007; and Ma, 1999/2010.

The love of mathematics, the belief that math is something students should enjoy, and that math is the most important subject of all was evidenced in the interviews and the classroom observations of all participants. This is well-reflected in the research from Sheffield et al. (1995), Baier et al. (2019), and Karp (2010) all of whom posited that an effective teacher of mathematically gifted students should possess both mathematical knowledge and a love of math. Sheffield et al. (1995) asked, “Are teachers modeling the joy of learning mathematics and are they learners of mathematics themselves?” (p. 6). The assertion by one of the teacher participants in this study (Linda) about mathematics sums this up well,

> Everyone needs it, it’s super important. It’s what I teach every single day and I have to have a passion for it. I think when you have a passion and the true love and desire for what you’re teaching the students see that and they’re able to learn and grow more; they can hear my passion and excitement.

The results from Swan’s (2006) beliefs questionnaire indicated that the math beliefs orientation of three of the five teachers (Linda, Barry, Natalie) revealed a **discovery** approach, while two teachers (Dave and Katie) showed a **connectionist** orientation. Swan’s own findings found teachers exhibiting much more of a transmission orientation for their beliefs about mathematics, teaching, and learning, which is seen as more teacher-directed with a focus on procedures. According to Swan (2006) teachers who fell into the category of connectionist were the most student-centered of all the categories and this describes two of this study’s participants. This position can be distinguished from the others by the emphasis that teachers place on students discussing ideas and mistakes, and the use made of prior learning. In comparing the beliefs orientation with the reported frequency of practices both Dave and Katie answered *almost always* in response to the statement: I encouraged students to make and discuss mistakes. They answered *occasionally* to the question: I teach each topic from the beginning assuming they know nothing. In other words, Dave and Katie’s beliefs about mathematics, teaching, and learning aligned with their reported practices. When I compared this data to their interviews, I found that Dave talked about the value of mistakes, “I model that mistakes are good, other people will learn from your mistakes." The connectionist orientation also views mathematics as a network of ideas that the student and teacher construct together through collaborative discussion. In this instance the teacher has a proactive role in challenging students.

According to Askew et al. (1997) the discovery orientation views mathematics as a human creation and encourages students to learn through their own individual exploration and reflection, trying out different strategies. Efficiency and effectiveness are not important as long as the student gets the answer. Learning is seen as more important than teaching and teachers serve mostly as facilitators. The pace is determined by the students and students must be deemed "ready" before they can learn certain mathematical concepts. Linda, Barry, and Natalie displayed this orientation based on their responses to the beliefs questionnaire. This aligned only with this statement about practices: "I know exactly what math topics the lesson will contain" where they each responded with *almost always.* However, their reported practices in other areas did not seem to match a discovery orientation. While this study did not focus on the students' experience nor their achievement, Askew's (1997) research suggested that students with connectionist teachers made higher average gains in numeracy than other classes of students in the study.

The beliefs expressed by the participants about gifted students which encompassed mathematically gifted and promising students were consistent with the findings of Sheffield et al. (1995), Krutetski (1976), Gavin (2005), and Parish (2014), who identified these students as having a mathematical memory, learn more rapidly than their peers, and who possess unusually high natural aptitudes for understanding math concepts. Contrary to Troxclair's (2013) work where teachers expressed negative attitudes about gifted students, the participants in this study saw gifted students in positive ways based on their characteristics and expressed concern and support for their challenges. The research indicates that mathematically promising students need teachers who are enthusiastic (Baier, 2019), engage in their own deep mathematical thinking (Sheffield, 1994), and a knowledge and love of mathematics (Karp, 2010). The findings of the present study are consistent with this research.

Of significant concern was the participants' responses regarding equitable access to gifted programs or advanced mathematics based on race, ethnicity, and gender. As evidenced in their responses some of the teachers did not express an awareness or understanding about access to high-quality instruction in the younger years, particularly for African American students. The implications for students who are not welcome in these classroom or school spaces are critical. Are these affirming spaces for students of color if the teachers do not recognize the systemic challenges they face daily? The teachers' beliefs regarding equity and access to a quality mathematics' experience did not align with the literature reviewed for this study. The research is clear that low-quality instruction leads to limited access for higher-level mathematics and gifted and honors programs (Berry, 2005; Davis, 2008; Lattimore, 2001, 2003, 2005a; Moody, 2003, 2004; Oakes, 1990; Sheppard, 2006). Standardized tests serve as a gatekeeper for these students and can let them in or out of special programs designed for those with high mathematical ability or promise. Likewise, teachers can serve as gatekeepers, whether through a lack of identification of underrepresented students or a deeper lack of knowledge and training in culturally responsive learning (Novak et al., 2020).

Three of the participants noted that any combination of teacher education, family support, small class sizes, and small teacher to student ratios can all impact equitable access to high-quality mathematics. "All kids if given the right environment can succeed; all kids are capable of learning," remarked Barry. Another participant shared this regarding the reason(s) for a student's lack of success in mathematics, "it could be related to motivation, background knowledge, support at home, support at school. It could be learning disabilities that haven't been diagnosed." While it would certainly be easier to imagine that the right environment is all that it takes for student success, the reality is that marginalized students' development in the STEM (science, technology, engineering, and mathematics) field is "centered on race and gender and is impacted by (un) intentional and (in) formal interactions with other variables" (Collins, et al., 2019, p. 55). An opportunity exists for teachers at the Sycamore School to learn more about the issues of equity and access in education for certain groups of students and how this might impact a student's ability to be enrolled in this kind of special education setting, or if enrolled, how they might need different or additional resources to be successful. Those resources should be centered on professional learning in cultural competency and knowledge.

As noted earlier, Sycamore School's admissions process is not solely based on one test score but includes teacher questionnaires and personal nomination recommendations in addition to a cognitive test. Sycamore School's mission is to serve those students identified as highly gifted, a subset of the general gifted population. Test scores that lie at least three standard deviations above the mean (145) automatically meet the admissions criteria. Those students with scores between two and three standard deviations (130-145) are deemed qualified at the sole discretion of the admissions team. Applicants with scores less than two standard deviations (129 or less) are deemed not qualified for admissions. Enrollment priorities are offered to qualified applicants in this order: children of Sycamore School's staff, siblings of currently enrolled students, legacy siblings, economically disadvantaged students, and returning students (North Carolina Department of Public Instruction/N.C.G.S. 115C-218.45, 2021).

Research Question Three

The influence of Ma's (1999/2010) work comparing U.S. and Chinese mathematics teachers became a key source of inspiration as I initially sought to replicate her tasks with teachers in this study. Instead, the data gathered for this question focused on asking teachers to describe their conceptual mathematical understanding which resulted in themes connected to the role that curriculum plays in a teacher's understanding of math concepts, their own math experiences in school, and the importance of ongoing learning and practice. Ball et al. (2005) concluded that U.S. teachers struggle in their own understanding of mathematics because they are the graduates of the very system that needs improvement. Several participants shared negative experiences with math growing up; they were bad at math (Barry), did not feel like they were taught to think deeply (Natalie), or that it did not make sense (Barry). Professional development in the school's curriculum was cited by two teachers as helping them to have a deeper conceptual understanding, while Dave connected his ongoing growth in mathematics to the competitions he attends with students.

Like the Chinese teachers in Ma's groundbreaking study (1999/2010) the participants in this study indicated that conceptual and procedural knowledge are interwoven. The participants also expressed a greater emphasis on understanding why the algorithm works and an awareness of where students might struggle with a process, which is consistent with the findings of Browning and colleagues (2014). However, it was difficult for some of the participants to define conceptual understanding for themselves or for teachers in general. One teacher described it as knowing what the standards are and staying on top of any changes in the standards, and another noted that it was about understanding place value, division, and equal parts. One teacher analogized conceptual understanding to being able to use vocabulary confidently and to have an abstract image for something concrete, while another participant expressed the importance of understanding the background of a concept and knowing where the struggle or misconceptions for students will occur. A greater awareness of what constitutes conceptual understanding for teachers and not just students is an important area of growth and opportunity for the study's participants and all teachers.

Implications and Recommendations

Recognizing the value that mathematically gifted and promising students bring to our nation and society is of critical importance. According to the National Science Board's (2010) report regarding the identification and development of our nation's human capital, "the U.S. education system too frequently fails to identify and develop our most talented and motivated students who will become the next generation of innovators" (p.5). Nurturing these gifts could result in the creation of new jobs, maintaining our standing as a science and technology global leader, and improving the quality of life for all citizens (National Science Board, 2010).

Teacher Education and Professional Learning

The participants in this study highlighted teacher education and professional development as important for improving the experiences of mathematically gifted and promising students which is also well-documented in the literature on mathematics education (AMTE, 2017; Ball et al., 2005; Ma, 1999/2010; NCRTE, 1991), and in research related to professional development for teachers who instruct mathematically gifted students (Leiken, 2011; NAGC, 2020; Rubenstein et al., 2015; Singer et al., 2016).

Teacher Education in Equity and Access for Underrepresented Students

Teacher growth and development emerged as one of the sub-themes related to beliefs about equity in mathematics, though this was mainly regarded as having more training in differentiation to address the academic needs of students in an equitable manner. Based on the participants inability to recognize the lack of access and equity experienced by students of color it is essential that this is a key focus for teacher growth and development. The teachers' beliefs seemed to align with the false mindset that all students have the same opportunities and access. Data from the current study, however, indicated that teachers were missing the reality that implicit bias exists systemically in all education arenas. The work of Novak and colleagues (2020) identifies seven guiding principles for equity-driven professional learning which are based on NAGC programming standards, social-justice work, and best practices in professional learning. The standards "illuminate a crucial path toward reducing and eliminating the disproportionality that has marred gifted programs for far too long" (p. 181). The authors cite Gorski's (2019) equity principles which include acknowledging that racism exists and then determining how it operates in a school; reviewing processes and policies to determine how they marginalize some students and work to fix them; and addressing the issues of systemic bias, eliminating the context that creates the injustice, rather than trying to fix the kids to meet the needs of the system. While it may be possible to be a highly effective teacher without specific degrees or certificates it is critical that teachers have the appropriate training to ensure that the latest practices and strategies are used consistently and well, particularly in the arena of culturally responsive teaching for students who are historically marginalized and ignored.

Teacher Development and Education in Mathematics

Recall that the participants struggled to identify what conceptual understanding of mathematics entails for teachers themselves, separately from students. Three of the five teacher participants have received a state gifted certificate which is required by the school (two are working on a pathway towards this certification). However, it is well worth noting that this AIG certificate is content agnostic. There does not exist a special certificate or professional development protocol that combines best practices for the teaching of the specific domain of mathematics specifically geared towards mathematically promising students. Furthermore, none of the participants has a degree in mathematics. Ball and her colleagues (2005) recommended that teachers study more mathematics or major in mathematics at the university level. The NCTQ (2016) in its review of U.S. teacher preparation programs concluded that only 13 percent covered critical math topics, which may explain why 1 in 4 teacher candidates failed the math portion of a common elementary licensing exam (Putman & Walsh, 2019).

The participants all indicated a desire for more teacher development in their work with students, and their learning interests including learning styles, building mathematical minds, more work with the curriculum, and incorporating technology with mathematics. According to the school's website and the director, the school provides all teachers with ongoing workshops in gifted education, but less emphasis has been placed on teacher education in mathematics. Based on the literature which cites a need for teachers of mathematics (especially elementary teachers) to have stronger mathematical education and development (Ball et al., 2005; Ma, 1999/2010; AMTE, 2017), it is therefore recommended that teachers engage in more robust and strategic professional development in mathematics. One approach could be the use of Lesson Study as a powerful professional development model. Lesson study has its roots in Japan where teachers research and reflect on their own mathematics lessons and that of their colleagues (Doig & Groves, 2011; Fernandez & Yoshida, 2007). According to the Association of Mathematics Teacher Educators (AMTE, 2017), teaching mathematics effectively requires career-long learning about teaching mathematics. As indicated in recent reviews of teacher education programs most of these programs do not provide teachers with the mathematical knowledge needed to teach well (Durrance, 2019, Ma, 1999/2010). Further, the NCTM recommends professional development for the teachers of students with exceptional mathematical promise to recognize, challenge, and nurture these students (NCTM, 2016), and the NAGC acknowledged their own support of this position (NAGC, 2017).

Recruitment and Development of Teachers

According to the research as cited in this study there is a gap that exists in determining how teachers of the mathematically gifted should be prepared and how they are formed over time (Karp, 2010). One such definition was proposed by Thornton and Peel (1999) who said the effective math teacher of gifted students "has a knowledge and love of mathematics, is skilled in promoting students' learning, and views teaching as a creative art" (p. 304). Based on the findings of this study the participants all shared a common passion and excitement for teaching mathematically gifted and promising students. Additionally, they expressed a strong desire for their students to not only love mathematics, but to understand topics beyond memorization. The teachers also showed great compassion and care for the emotional well-being of their students, noting the need for encouragement and support for their students' social emotional needs. As future teachers are recruited to the school these will be important traits to look for and develop in candidates and teachers as many of these characteristics align with the findings of researchers on the types of teachers that mathematically gifted students need. Karp (2010) insists that teachers of these students must have a "deep-rooted familiarity with the difficult problems of school mathematics," and mentoring by more experienced teachers (Karp, 2010, p. 277). Leiken (2011) questioned whether teachers of mathematically gifted students should be gifted themselves, or whether they should be creative to develop students' creativity. Questions around teacher creativity were not explored in this study but based on the recommendations outlined in the research on teachers who instruct mathematically gifted and promising students (Karp, 2010;

Leiken, 2011; Renzulli, 1975), evidence of this trait should be considered when recruiting new teachers and developing current ones in a school designed to serve gifted students.

Parent Education

Parent and home support were deemed important in the results of this study for students' mathematical success at the highest levels. Teachers noted the strong levels of support by the school's parents and the high value placed on education and achievement in general. However, the participants also cited a perceived misalignment between the goals and expectations from parents about the way that mathematics should be taught at the school. The school does provide math nights and other parent education opportunities, and parent education around this topic is included in the school's strategic plan. The onus is on the school to continue to support and help parents understand the goals of the math program. Further, teachers are cautioned against placing blame on parents for lack of student success and ensure that the child has every opportunity at school to be successful. Since feedback from parents was not included in this study it could be anticipated that their voices might provide additional insight as to why there is a perceived lack of understanding and agreement about the way mathematics should be taught at the school.

An ongoing evaluation and plan for how to support parent understanding about best practices in teaching mathematics for understanding and sense-making is recommended. One of Sycamore's current strategic initiatives is related to parent education and support with an action item to develop a plan for parent education to ensure there is a clear understanding around the academic program and instructional practices of the school. Furthermore, the school plans to survey the parent body every two years to gauge satisfaction and gather feedback.

An improved partnership between school and home around the common expectations for learning could have implications that reduce the stress on children and better support teachers in their implementation of the program. As noted, one of the themes revealed from the analysis of teacher beliefs around equity and access to mathematics was the support students receive at home. As confirmed by Wadham et al. (2020) the child is influenced by both school and home, and it is recommended that both groups work together as partners to find and achieve shared goals for student learning of mathematics. The authors of the study also found that schools with culturally or linguistically diverse families may experience additional difficulties or challenges when engaging with parents. Further, a study conducted by Warren and Young (2002) highlighted the importance of productive partnerships between school and home and indicated that the school must determine the knowledge parents possess or do not possess and then, "incorporate these strengths and weaknesses into two-way conversations when working together to support children's (and adults') literacy and numeracy development" (p. 226).

Limitations of the Study

Qualitative studies often have limitations related to the methodological design chosen for the research. This study had at least three potential limitations which are named here. Though the size of the group was within the range recommended by Creswell (2013) for phenomenological studies, a smaller sample can also impede the diversity of representation with respect to gender, age, race, or experience. In fact, in this study all the participants were White, and all but one was female, so a lack of diverse backgrounds in race and gender may impact the results of the study.

A second limitation of this study was related to the amount and type of data collected and the short window of data collection. Only one interview and one observation were conducted for each participant. The responses may have been richer and more diverse if multiple interviews and observations were collected from each participant over a longer period (data was collected over three months). To address this limitation, I gathered additional data by asking teachers to provide responses to a pre-and post-observation questionnaire which provided additional information to analyze. The data collection was limited only to teachers which may have impacted the findings. Some of the research cited in this study involved gathering feedback from students and measuring this against the teacher data. For this study I chose not to involve student data, due to constraints of time and ability to analyze the information. Similarly, due to time concerns I did not choose to interview administrators of the school, though this feedback could have provided more information about the types of teachers the school recruits or considers effective, as well as efforts around professional development.

A third limitation of this study was the small sample size for the purposes of the questionnaires. Though these were validated instruments (Swan, 2006) the mean frequencies and other statistical results may have been more valid on a larger sample of participants such as the group from Swan's (2006) work which was 64 teachers. However, despite this limitation the orientation types of the five participants which were determined via the beliefs survey aligned closely with the practices survey and the classroom observations.

Recommendations for Future Research

Based on the results of this study, I identify several areas of future research to explore regarding the teachers of mathematically gifted and promising students. First, it is recommended that a similar study be conducted to understand the experiences of teachers working with these students in other settings such as public schools that are not specifically designed to serve gifted students and independent schools. A second recommendation is to modify the current study by gathering feedback, including achievement data, regarding the students of these teachers. Similar feedback should be collected from academic leaders (coaches, facilitators) and administrators in these schools. These other sources of data collection would be in addition to the teacher feedback through interviews, surveys, and observations, thus making it a larger and more comprehensive study overall. Differences in types of school settings and data collection sources could lead to a clearer and more thorough understanding of the phenomenon studied here, while reflecting more diverse perspectives and experiences of the participants. The findings from this type of research could lead policymakers, teacher educators, and school administrators to a better understanding of the type of teacher, specifically regarding their beliefs, practices, and conceptual understanding, who can most effectively serve the mathematically gifted student.

Though not specifically grounded in the research questions of this study, the issue of equity and access to an equitable mathematics education that nurtures the promise and potential of students from diverse populations was explored through the interview questions utilized in the protocol. Results indicated that the participants recognized that school environment factors impact access to an equitable math education, but they did not seem to fully comprehend the impact that low-level instruction in a child's early years of education, particularly for African American students, can have on a child's opportunity for higher-level mathematics and their achievement overall (Davis & Martin, 2008). The authors acknowledge that "While mastery of low-level content is necessary, it often becomes the ceiling of the mathematics students learn," (p. 11). Further, it is critical that teachers must "provide these students with a challenging and stimulating mathematics education that assists these students in improving their individual and collective group conditions," (p. 30).

Furthermore, Ball et al. (2005) found that higher-knowledge teachers tend to teach non-minority students, which leaves diverse students with less-knowledgeable teachers, which again impacts and may limit their current and future education success and opportunities. The critical issue in education today of access to an equitable mathematics education warrants a fuller investigation and multiple studies to determine the types of solutions that will provide all students with equitable access to high quality and challenging mathematics to nurture their potential and promise. The results of this research may also help explain why some teachers do not cite the school environment as a potential root cause of the disparities that many students experience due to their race, gender, or ethnicity. More importantly, research findings may also effect change in the way that teachers are placed into schools and school districts and the type of resources and training that all teachers receive in any school setting.

To determine whether teachers' conceptual mathematical understanding impacts the success of all students and particularly gifted ones more research is needed to determine how best to measure teachers' mathematical understanding and knowledge for teaching (Baier et al., 2019; Ball et al., 2005; Karp, 2010; Ma, 1999/2010; Sheffield, 1995; Thornton & Peel, 1999). The use of interview tasks such as the ones designed and implemented by Ma (1999/2010) could be used in teacher education programs as part of this research. In her study of U.S. and Chinese teachers she asked teachers to describe how they would respond to typical math topics with students. Similarly, the work of Ball and her colleagues (2005), could be studied more closely with preservice and in-service teachers to determine if a correlation exists between teacher knowledge and student achievement. The researchers used a domain map with topics and questions designed to measure the mathematics that teachers need to know well. The findings of such a study on how best to measure teacher conceptual mathematical understanding could improve how well teachers are prepared upon completion of a teacher education program and increase their readiness to provide for the needs of all students, including those with mathematical promise.

Finally, based on the outcomes of this study's use and analysis of the mathematical beliefs and practices questionnaires (Swan, 2006), further research into the ideal types of teachers who can best serve mathematically gifted and promising students is recommended. Notably, an instrument that measures the intersection of beliefs about mathematics and beliefs about gifted students could be designed to study teacher beliefs and their connection to instructional practices more fully. In Swan's (2006) study the research was completed with current teachers, though it is plausible to consider the value of using beliefs instruments with pre-service teachers and then measuring changes at key points of the teacher education program. Swan (2006) wrote that beliefs can become comfortable and may be resistant to change. If the mathematics research community wishes to improve classroom practices, it is important to study the beliefs of teachers as they may indicate best the amount of professional growth teachers experience. Relatedly, further research on the types of practices best suited for mathematically gifted and promising students is recommended to better understand the implications that student-centered and teacher-centered practices may have on this population.

Summary

Based on the critical lack of student achievement in mathematics for all U.S. students, and especially our most talented and motivated ones, this qualitative case study sought to understand the phenomenon of lived experiences of teachers instructing mathematically gifted and promising students in a K-8 public charter school for gifted children. Two conceptual frameworks were employed as the lenses through which to better understand the instructional practices, beliefs, and conceptual understanding of teachers. These were Renzulli's Three-Ring Model of Giftedness (1978) and Gagné's Differentiated Model of Giftedness and Talent 1985).

Conducting this research provided greater understanding on the current state of elementary teachers instructing mathematically gifted and promising students. Specifically, this study contributed to a fuller understanding of teachers' conceptual mathematical understanding, instructional practices, and beliefs towards mathematics and towards gifted students, and how these factors influence the experiences of elementary students, particularly those who are mathematically gifted and promising. The results of this study may benefit school principals and other school leaders in recruiting and developing educators for these students. It may also benefit designers of professional development for current teachers as well as teacher educators in designing courses and programs of study in teacher education programs.

Major findings indicated that the group of teachers was passionate about mathematics and working with gifted students; the school's curriculum and the previous math experiences of teachers played strong roles in the teachers' conceptual understanding of mathematics; and instructional practices allowed for students to demonstrate understanding in multiple ways and emphasized students' communication of their thinking with peers. Additionally, the findings from this study revealed that the teachers collectively identified many of the same positive traits about their students, such as curiosity, love of learning, and high intelligence. They also expressed concerns about the challenges their students faced. Namely, the teachers recognized the social-emotional needs and pressures faced by the students and were vocal about supporting these students' ability to navigate challenges with executive functioning, parental pressure, and perfectionist tendencies. Recommendations were made in this study for parent education that helps parents and caregivers understand the curriculum and instruction of mathematics, including its purpose and benefits to children, and the importance of productive partnerships between school and home.

In response to questions concerning equity and the equitable access to challenging mathematics, more research should be conducted to better understand this issue and what impacts, if any, this may have on the students at the school where this study was conducted. The question of whether students from diverse backgrounds have full access to the opportunities presented at special schools such as this one is a critical one and must be explored further so that solutions can be proposed and potentially adopted.

Based on the findings this study revealed a need for more consistent and strategic teacher education, especially in mathematics and in preparation for working with mathematically gifted and promising students. Further efforts should be made to understand the type of teacher who can most effectively succeed in teaching these students, particularly in terms of their beliefs and instructional practices. Characteristics and skills that lend themselves to increased effectiveness in teaching can be identified and used for more efficient and strategic recruitment of new teachers and the further development of current teachers.

It was over 40 years ago that the National Council for Teachers of Mathematics warned that "the student most neglected, in terms of realizing full potential, is the gifted student of mathematics. Outstanding mathematical ability is a precious societal resource, sorely needed to maintain leadership in a technological world" (NCTM, 1980, p. 18). We are even further removed from the time in our history when the United States set out to recruit and train the "best and brightest" in math and science as a direct response to the Soviet Union's launch of *Sputnik* into space in 1957. And yet it was more recently that our nation's leaders issued this dire warning:

> One of the pillars of American economic prosperity—our scientific and technological superiority—is beginning to atrophy even as other nations are developing their own

> human capital. If we wait for a dramatic event—a 21st-century version of *Sputnik*—it will be too late. There may be no attack, no moment of epiphany, no catastrophe that will suddenly demonstrate the threat. Rather, there will be a slow withering, a gradual decline, a widening gap between a complacent America and countries with the drive, commitment, and vision to take our place (National Science Board, 2010).

As the participants of this study shared, these students are our future leaders, philanthropists, and big-idea people. They are the ones who will come up with the solutions the world needs. And what they need now, and needed over 40 years ago, are teachers and schools that can provide them with the experiences they deserve to fulfill their potential and their promise, not only for themselves, but for the generations of students we missed before and those we have yet to teach.

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Appendix A

Description of PISA Mathematics Literacy Proficiency Levels: 2018 Proficiency Level, Lower Cut Score, and Task Description

Level 6 (669): At Level 6, students can conceptualize, generalize, and utilize information based on their investigations and modeling of complex problem situations, and can use their knowledge in relatively non-standard contexts. They can link different information sources and representations together and flexibly translate amongst them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply this insight and understanding, along with a mastery of symbolic and formal mathematical operations and relationships, to develop new approaches and strategies for attacking novel situations. Students at this level can reflect on their actions and can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments, and the appropriateness of these to the original situation.

Level 5 (607): At Level 5, students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare, and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriate linked representations, symbolic and formal characterizations, and insight pertaining to these situations. Students at this level have begun to develop the ability to reflect on their work and to communicate conclusions and interpretations in written form.

Level 4 (545): At Level 4, students can work effectively with explicit models for complex, concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic representations, linking them directly to aspects of real-world situations. Students at this level can utilize their limited range of skills and can reason with some insight, in straightforward contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments, and actions.

Level 3 (482): At Level 3, students can execute clearly described procedures, including those that require sequential decisions. Their interpretations are sufficiently sound to be a base for building a simple model or for selecting and applying simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They typically show some ability to handle percentages, fractions, and decimal numbers, and to work with proportional relationships. Their solutions reflect that they have engaged in basic interpretation and reasoning.

Level 2 (420): At Level 2, students can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions to solve problems involving whole numbers. They can make literal interpretations of results.

Level 1 (358): At Level 1, students can answer questions involving familiar contexts where all relevant information is present, and the questions are clearly defined. They can identify information and carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are almost always obvious and follow immediately from the given stimuli.

Note. To reach a particular proficiency level, a student must correctly answer most items at that level. Students were classified into mathematics literacy levels according to their scores. Cut scores in the exhibit are rounded; exact cut scores are provided in table A-2 in the Methodology and Technical Notes. Scores are reported on a scale from 0 to 1,000.

SOURCE: Organization for Economic Cooperation and Development (OECD), Program for International Student Assessment (PISA), 2018

Appendix B

Descriptions of the Trends in International Math and Science (TIMSS) 2019 International Benchmarks: Grade 4 Mathematics

Advanced (625): Students can apply their understanding and knowledge in a variety of relatively complex situations and explain their reasoning. Students can solve a variety of multistep word problems involving whole numbers and show an understanding of fractions and decimals. They can apply knowledge of two- and three-dimensional shapes in a variety of situations. Students can interpret and represent data to solve multistep problems.

High (550): Students apply conceptual understanding to solve problems. They can apply conceptual understanding of whole numbers to solve two-step word problems. They show understanding of the number line, multiples, factors, and rounding numbers, and operations with fractions and decimals. Students can solve simple measurement problems. They demonstrate understanding of geometric properties of shapes and angles. Students can interpret and use data in tables and a variety of graphs to solve problems.

Intermediate (475): Students can apply basic mathematical knowledge in simple situations. They can compute with three- and four-digit whole numbers in a variety of situations. They have some understanding of decimals and fractions. Students can identify and draw shapes with simple properties. They can read, label, and interpret information in graphs and tables.

Low (400): Students have some basic mathematical knowledge. They can add, subtract, multiply, and divide one- and two-digit whole numbers. They can solve simple word problems. They have some knowledge of simple fractions and common geometric shapes. Students can read and complete simple bar graphs and tables.

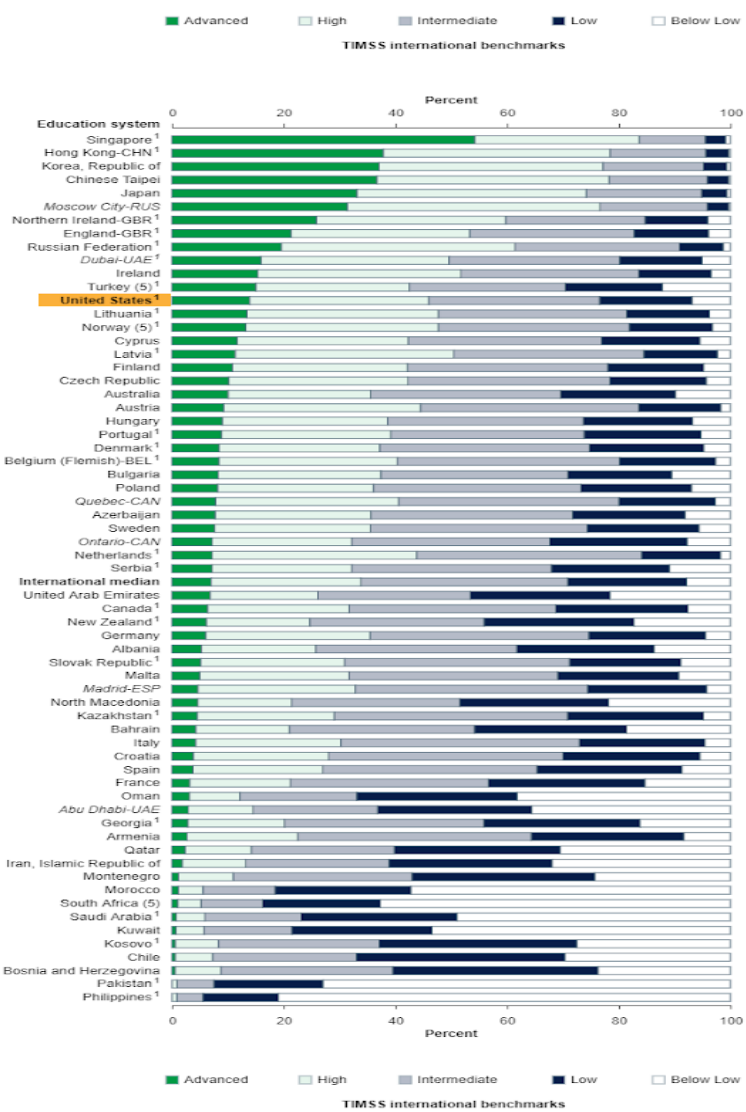
Mullis, I. V. S., Martin, M. O., Foy, P., Kelly, D., & Fishbein, B. (2020). *TIMSS 2019 International Results in Mathematics and Science.* Retrieved from Boston College, TIMSS & PIRLS International Study Center website: http://timssandpirls.bc.edu/timss2019/international-result

Appendix C

Percentages of Fourth-grade Students Reaching the TIMMS Benchmarks in Mathematics by Educational System 2019

Figure M3a. Percentages of 4th-grade students reaching the TIMSS international benchmarks in mathematics, by education system: 2019

Hover over the table labels to view the definitions for each benchmark.



Education system	Advanced (625)	High (550)	Intermediate (475)	Low (400)
Singapore[1]	54*	84*	96*	99*
Hong Kong-CHN[1]	38*	78*	96*	100*
Korea, Republic of	37*	77*	95*	99*
Chinese Taipei	37*	78*	96*	100*
Japan	33*	74*	95*	99*
Moscow City-RUS	31*	77*	96*	100*
Northern Ireland-GBR[1]	26*	60*	85*	96*
England-GBR[1]	21*	53*	83*	96*
Russian Federation[1]	20*	61*	91*	99*
Dubai-UAE[1]	16	50*	80*	95*
Ireland	15	52*	84*	97*
Turkey (5)[1]	15	43	70*	88*
United States[1]	**14**	**46**	**77**	**93**
Lithuania[1]	13	48	81*	96*
Norway (5)[1]	13	48	82*	97*
Cyprus	12	42	77	95
Latvia[1]	11*	50*	85*	98*
Finland	11*	42*	78	95*
Czech Republic	10*	42	78	96*
Australia	10*	36*	70*	90*
Austria	9*	45	84*	98*
Hungary	9*	39*	74	93
Portugal[1]	9*	39*	74	95
Denmark[1]	8*	37*	75	95*
Belgium (Flemish)-BEL[1]	8*	40*	80*	97*
Bulgaria	8*	37*	71*	90*
Poland	8*	36*	73	93
Quebec-CAN	8*	41*	80*	97*
Azerbaijan	8*	36*	72*	92
Sweden	8*	36*	74	94
Ontario-CAN	7*	32*	68*	92
Netherlands[1]	7*	44	84*	98*
Serbia[1]	7*	32*	68*	89*
International median	7*	34*	71*	92
United Arab Emirates	7*	26*	53*	78*
Canada[1]	6*	32*	69*	92
New Zealand[1]	6*	25*	56*	83*
Germany	6*	36*	75	96*
Albania	5*	26*	62*	86*
Slovak Republic[1]	5*	31*	71*	91
Malta	5*	32*	69*	91*
Madrid-ESP	5*	33*	74	96*
North Macedonia	5*	21*	52*	78*
Kazakhstan[1]	5*	29*	71*	95*
Bahrain	4*	21*	54*	81*
Italy	4*	30*	73*	95*
Croatia	4*	28*	70*	95
Spain	4*	27*	65*	91
France	3*	21*	57*	85*
Oman	3*	12*	33*	62*
Abu Dhabi-UAE	3*	15*	37*	64*
Georgia[1]	3*	20*	56*	84*
Armenia	3*	23*	64*	92
Qatar	2*	14*	40*	70*
Iran, Islamic Republic of	2*	13*	39*	68*
Montenegro	1*	11*	43*	76*
Morocco	1 !!*	6*	18*	43*
South Africa (5)	1*	5*	16*	37*
Saudi Arabia[1]	1*	6*	23*	51*
Kuwait	1 ! *	6*	21*	47*
Kosovo[1]	1*	8*	37*	73*
Chile	1*	7*	33*	70*
Bosnia and Herzegovina	1*	9*	40*	76*
Pakistan[1]	# !!*	1 ! *	8*	27*
Philippines[1]	# !!*	1*	6*	19*

\# Rounds to zero.

! Interpret data with caution. Estimate is unstable due to high coefficient of variation (>30 percent and ≤50 percent).

!! Interpret data with caution. Estimate is unstable because the standard error represents more than 50 percent of the estimate.

* $p < .05$. Significantly different from the U.S. percentage at the same benchmark level at the .05 level of statistical significance.

[1] Education system had coverage, sampling, or reliability issues, including issues with the national defined population coverage, issues with satisfying sampling guidelines, and/or concerns about estimation because the percentage of students whose performance is too low to estimate exceeds certain thresholds. Details can be found here.

NOTE: Education systems are ordered by the percentage of students reaching the *Advanced* international benchmark in mathematics. Education systems that are not countries are designated by the appended three-letter international abbreviation for their country. Benchmarking participants are indicated with italics and pink shading. For education systems with a "(5)" after their name, 5 indicates the years of formal schooling; these education systems chose to administer TIMSS at a different grade than other education systems (4 years of formal schooling). The international median represents all participating TIMSS education systems, except the benchmarking participants. Although rounded numbers are displayed, data shown are based on unrounded estimates.

SOURCE: International Association for the Evaluation of Educational Achievement (IEA), Trends in International Mathematics and Science Study (TIMSS), 2019.

Appendix D

NAEP Mathematics Achievement Levels Grade 4 (NCEE, 2019)

Basic. Fourth-grade students performing at the *NAEP Basic* level should show some evidence of understanding the mathematical concepts and procedures in the five NAEP content areas. Fourth graders performing at the *NAEP Basic* level should be able to estimate and use basic facts to perform simple computations with whole numbers, show some understanding of fractions and decimals, and solve some simple real-world problems in all NAEP content areas. Students at this level should be able to use—though not always accurately—four-function calculators, rulers, and geometric shapes. Their written responses will often be minimal and presented without supporting information.

Proficient. Fourth-grade students performing at the *NAEP Proficient* level should consistently apply integrated procedural knowledge and conceptual understanding to problem solving in the five NAEP content areas. Fourth graders performing at the *NAEP Proficient* level should be able to use whole numbers to estimate, compute, and determine whether results are reasonable. They should have a conceptual understanding of fractions and decimals; be able to solve real-world problems in all NAEP content areas; and use four-function calculators, rulers, and geometric shapes appropriately. Students performing at the *NAEP Proficient* level should employ problem-solving strategies such as identifying and using appropriate information. Their written solutions should be organized and presented both with supporting information and explanations of how they were achieved.

Advanced. Fourth-grade students performing at the *NAEP Advanced* level should apply integrated procedural knowledge and conceptual understanding to complex and nonroutine real-world problem solving in the five NAEP content areas. Fourth graders performing at the *NAEP Advanced* level should be able to solve complex and nonroutine real-world problems in all NAEP content areas. They should display mastery in the use of four-function calculators, rulers, and geometric shapes. The students are expected to draw logical conclusions and justify answers and solution processes by explaining why, as well as how, they were achieved. They should go beyond the obvious in their interpretations and be able to communicate their thoughts clearly and concisely.

Appendix E

Teacher Participant Interview Questions

Establishing Rapport (Introduction)

1. Tell me a little bit about yourself.
2. How might you describe your strengths as a teacher?
3. Describe your best mathematics lesson ever.
4. Tell me about a specific achievement or accomplishment you are proud of as a teacher.

Teacher Experiences with Mathematically Promising* Students/Instructional Practices

1. Describe the behaviors and characteristics of students you have taught who have been identified as mathematically gifted and promising.
2. Describe a lesson or math experience that you felt was very successful with mathematically promising students.
3. What were the instructional practices you used that contributed to that success?
4. Tell me about any formal training you have received that supports your work with mathematically promising students.
5. Please share any ways in which you would like to grow or learn more about working with mathematically promising students.

Teacher Beliefs About Mathematics

1. As an individual how important would you say mathematics is in general? Explain.
2. As a teacher, how important would you say mathematics is in your role? Explain.
3. Would you describe mathematics as something you enjoy as an individual?
4. What about as a teacher, do you enjoy teaching mathematics? Say a little more about why this is so.
5. How important is it for students to enjoy and appreciate math? Explain your response.

Teacher Beliefs About Gifted Students

1. Describe the characteristics of gifted students in your classroom or in general.
2. What do you think gifted students need most in terms of their educational experience?
3. What do you think gifted students need from their teachers more specifically?
4. Do you think that gifted students are a valuable resource to our society? Say more about this.
5. Do you think that gifted students need a special school or classroom to be successful academically and emotionally? Explain.

Teacher Beliefs Regarding Equity in Mathematics:

1. What does equity mean to you? (Possible probes/follow-ups):
    1. What would it look like if all students had access to an equitable math education?
    2. What factors might contribute to an equitable math education?

3. Do you believe that all students have equitable access to be successful in mathematics? Explain.
4. What might contribute to a student's lack of success in mathematics? Explain.

Teacher Descriptions of Conceptual Understanding of Mathematics

1. As a teacher, talk about how important you believe mathematical knowledge and your own conceptual understanding of mathematics is in your current teaching role.
2. How would you describe your own mathematical knowledge and conceptual understanding of mathematics?
3. Do you feel prepared mathematically to meet the needs of mathematically gifted and promising students?

Teacher Beliefs About Mathematically Promising* Students

1. Do you believe that some students are able to understand math concepts at different rates? (More quickly or less quickly, for example) Explain.
2. What kinds of math experience(s) do you believe this type of student should have in your classroom or any classroom?
3. Do you believe that race, ethnicity, or gender can impact a student's achievement and enjoyment in mathematics? Explain.

Other Questions/Closing

1. Do you have anything important you would like to contribute before we close today?

Note. *Mathematically promising was defined by the NCTM (National Council of Teachers of Mathematics) in 1994 as those students who have the potential to become leaders and problem-solvers of the future. More specifically, mathematical promise is the function of ability, motivation, belief, and experience/opportunity.

Appendix F

Teacher Questionnaire: Classroom Instructional Practices (Swan, 2006)

Rate on a scale of 1-5
1 almost never, 2 occasionally, 3 half the time, 4 most of the time, 5 almost always

1. Students learn through doing exercises
2. Students work on their own, consulting a neighbor from time to time
3. Students use only methods I teach them
4. Students start with easy questions and work up to harder questions
5. Students choose which questions they tackle
6. I encourage students to work more slowly
7. Students compare different methods for doing questions
8. I teach each topic from the beginning, assuming they know nothing
9. I teach the whole class at once
10. I try to cover everything in a topic
11. I draw links between topics and move back and forth between topics
12. I am surprised by the ideas that come up in a lesson
13. I avoid students making mistakes by explaining things carefully first
14. I tend to follow the textbook or worksheets closely
15. Students learn through discussing their ideas
16. Students work collaboratively in pairs or in small groups
17. Students invent their own methods
18. I tell students which questions to tackle
19. I only go through one method for doing each question
20. I found out which parts students already understand and don't teach those parts
21. I teach each student differently according to individual needs
22. I tend to teach each topic separately
23. I know exactly what math topics the lesson will contain
24. I encourage students to make and discuss mistakes
25. I jump between topics as the need arises

Note. This questionnaire of 25 classroom practices contains 13 behaviors categorized as teacher-centered and 12 behaviors as student-centered. Teacher-centered describes practices one might expect to originate from a transmission-oriented belief system. In this setting the teacher is the primary director of the work, teaching students in a whole class model with an emphasis on fluency over understanding. There is less creativity and a focus on transmitting definitions and practicing methods. Student-centered practices come from a constructivist position where teachers teach students as individuals, allow them to create their own methods, are flexible about what is covered, and see mathematics as a subject open for discussion.

Appendix G

Teacher Questionnaire: Beliefs (Swan, 2006)

Give each statement a percentage, so that the sum of the three percentages in each section is 100.

Mathematics is:
- A given body of knowledge and standard procedures, a set of universal truths and rules which need to be conveyed to students (MT)
- A creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods (MD)
- An interconnected body of ideas which the teacher and the student create together through discussion (MC)

Learning is:
- An individual activity based on watching, listening, and imitating until fluency is attained (LT)
- An individual activity based on practical exploration and reflection (LD)
- An interpersonal activity in which students are challenged and arrive at understanding through discussion (LC)

Teaching is:
- Structuring a linear curriculum for the students; giving verbal explanations and checking that these have been understood through practice questions; correcting misunderstandings when students fail to "grasp" what is taught (TT)
- Assessing when a student is ready to learn; providing a stimulating environment to facilitate exploration; avoiding misunderstandings by the careful sequencing of experiences (TD)
- A non-linear dialogue between teacher and students in which meanings and connections are explored verbally. Misunderstandings are made explicit and worked on (TC)

Key to letters: The first letter represents Mathematics (M), Learning (L), or Teaching (T). The second letter refers to Transmission (T), Discovery (D), or Connectionist (C) beliefs.

Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education*, *75*(1), 58–127. https://doi.org/10.7227/RIE.75.

Appendix H

Teacher Participant Interview Questions: Pre-Observation Interview

Name: ____ Date: ________________

Description of today's lesson (will be used before all observations)

1. What is the mathematical learning objective(s) for today's lesson?
2. Describe the instructional practices, materials, and plan for teaching that you will utilize for today's lesson?
3. What kind of math knowledge or understanding should the students already have for this lesson?
4. Where do you anticipate students will struggle with the content of this lesson?
5. Is there anything else you would like to share with me before the classroom observation today?

Appendix I

Teacher Participant Interview Questions: Post-Observation Interview

1. How do you think your lesson went today in general?
2. Do you believe that students achieved your goals for today in terms of understanding the learning objective(s) for today's lesson?
3. Do you believe that all students, regardless of their gender or race, had full access to the lesson today? Why do you believe so?
4. Which instructional practices (moves, questions, groupings, etc.) were the most effective with your students today? Why do you think so?
5. What evidence of math knowledge and understanding did students display today?
6. Describe your reactions to where the students struggled or did not struggle? Were you surprised or affirmed? Why?
7. Is there anything you wish you had done differently? Why?
8. Describe tomorrow's lesson and what and how you plan to follow up from today's lesson.
9. Is there anything else you'd like to share or discuss?

Appendix J

Mathematics Classroom Observation Protocol for Practices ($MCOP^2$)
(Gleason et al., 2015)

1. Students engaged in exploration/investigation/problem solving. SE*

3 Students regularly engaged in exploration, investigation, or problem solving. Over the course of the lesson, the majority of the students engaged in exploration/investigation/problem solving.
2 Students sometimes engaged in exploration, investigation, or problem solving. Several students engaged in problem solving, but not the majority of the class.
1 Students seldom engaged in exploration, investigation, or problem solving. This tended to be limited to one or a few students engaged in problem solving, while other students watched but did not actively participate.
0 Students did not engage in exploration, investigation, or problem solving. There were either no instances of investigation or problem solving, or the instances were carried out by the teacher without active participation by any students.

2. Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent concepts. SE*

3 The students manipulated or generated two or more representations to represent the same concept, and the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were explicitly discussed by the teacher or students, as appropriate.
2 The students manipulated or generated two or more representations to represent the same concept, but the connections across the various representations, relationships of the representations to the underlying concept, and applicability or the efficiency of the representations were not explicitly discussed by the teacher or students.
1 The students manipulated or generated one representation of a concept.
0 There were either no representations included in the lesson, or representations were included but were exclusively manipulated and used by the teacher. If the students only watched the teacher manipulate the representation and did not interact with a representation themselves, it should be scored a 0.

3. Students were engaged in mathematical activities. SE*

3 Most of the students spend two-thirds or more of the lesson engaged in mathematical activity at the appropriate level for the class. It does not matter if it is one prolonged activity or several shorter activities. (Note that listening and taking notes does not qualify as a mathematical activity unless the students are filling in the notes and interacting with the lesson mathematically.)
2 Most of the students spend more than one-quarter but less than two-thirds of the lesson engaged in appropriate-level mathematical activity. It does not matter if it is one prolonged activity or several shorter activities.

1 Most of the students spend less than one-quarter of the lesson engaged in appropriate-level mathematical activity. There is at least one instance of students' mathematical engagement.

0 Most of the students are not engaged in appropriate-level mathematical activity. This could be because they are never asked to engage in any activity and spend the lesson listening to the teacher and/or copying notes, or it could be because the activity they are engaged in is not mathematical—such as a coloring activity.

4. Students critically assessed mathematical strategies. SE* TF**

3 More than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small-group work, or a student making a comment during direct instruction or individually to the teacher.
2 At least two but less than half of the students critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small-group work, or a student making a comment during direct instruction or individually to the teacher.
1 An individual student critically assessed mathematical strategies. This could have happened in a variety of scenarios, including in the context of partner work, small group work, or a student making a comment during direct instruction or individually to the teacher. The critical assessment was limited to one student.
0 Students did not critically assess mathematical strategies. This could happen for one of three reasons: (1) No strategies were used during the lesson. (2) Strategies were used but were not discussed critically. For example, the strategy may have been discussed in terms of how it was used on the specific problem, but its use was not discussed more generally. (3) Strategies were discussed critically by the teacher, but this amounted to the teacher telling the students about the strategy(ies), and students did not actively participate.

5. Students persevered in problem solving. SE*

3 Students exhibited a strong amount of perseverance in problem solving. The majority of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), the majority of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
2 Students exhibited some perseverance in problem solving. Half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem.
1 Students exhibited minimal perseverance in problem solving. At least one student but less than half of students looked for entry points and solution paths, monitored and evaluated progress, and changed course if necessary. When confronted with an obstacle (such as how to begin or what to do next), at least one student but less than half of students continued to use resources (physical tools as well as mental reasoning) to continue to work on the problem. There must be a roadblock to score above a 0.

0   Students did not persevere in problem solving. This could be because there was no student problem solving in the lesson, or because when presented with a problem-solving situation. no students persevered. That is, all students either could not figure out how to get started on a problem, or when they confronted an obstacle in their strategy, they stopped working.

6. The lesson involved fundamental concepts of the subject to promote relational/conceptual understanding. TF**

3 The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, and the teacher/lesson uses these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.
2 The lesson includes fundamental concepts or critical areas of the course, as described by the appropriate standards, but the teacher/lesson misses several opportunities to use these concepts to build relational/conceptual understanding of the students with a focus on the "why" behind any procedures included.
1 The lesson mentions some fundamental concepts of mathematics but does not use these concepts to develop the relational/conceptual understanding of the students. For example, in a lesson on the slope of the line, the teacher mentions that it is related to ratios, but does not help the students to understand how it is related and how that can help them to better understand the concept of slope.
0 The lesson consists of several mathematical problems with no guidance to make connections with any of the fundamental mathematical concepts. This usually occurs with a teacher focusing on the procedure of solving certain types of problems without the students understanding the "why" behind the procedures.

7. The lesson promoted modeling with mathematics. TF**

3 Modeling (using a mathematical model to describe a real-world situation) is an integral component of the lesson, with students engaged in the modeling cycle (as described in the Common Core State Standards).
2 Modeling is a major component, but the modeling has been turned into a procedure (i.e., a group of word problems that all follow the same form, and the teacher has guided the students to find the key pieces of information and how to plug them into a procedure); or modeling is not a major component, but the students engage in a modeling activity that fits within the corresponding standard of mathematical practice.
1 The teacher describes some type of mathematical model to describe real-world situations, but the students do not engage in activities related to using mathematical models.
0 The lesson does not include any modeling with mathematics.

8. The lesson provided opportunities to examine mathematical structure. (symbolic notation, patterns, generalizations, conjectures, etc.) TF**

3 The students have a sufficient amount of time and opportunity to look for and make use of mathematical structure or patterns.
2 Students are given some time to examine mathematical structure but are not allowed adequate time or are given too much scaffolding so that they cannot fully understand the generalization.

1 Students are shown generalizations involving mathematical structure but have little opportunity to discover these generalizations themselves or adequate time to understand the generalization.
0 Students are given no opportunities to explore or understand the mathematical structure of a situation.

9. The lesson included tasks that have multiple paths to a solution or multiple solutions. TF**

3 A lesson which includes several tasks throughout, or a single task that takes up a large portion of the lesson, with multiple solutions and/or multiple paths to a solution and which increases the cognitive level of the task for different students.
2 Multiple solutions and/or multiple paths to a solution are a significant part of the lesson, but are not the primary focus, or are not explicitly encouraged; or more than one task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.
1 Multiple solutions and/or multiple paths minimally occur, and are not explicitly encouraged; or a single task has multiple solutions and/or multiple paths to a solution that are explicitly encouraged.
0 A lesson that focuses on a single procedure to solve certain types of problems and/or strongly discourages students from trying different techniques.

10. The lesson promoted precision of mathematical language. TF**

3 The teacher "attends to precision" with regard to communication during the lesson. The students also "attend to precision" in communication, or the teacher guides students to modify or adapt non precise communication to improve precision.
2 The teacher "attends to precision" in all communication during the lesson, but the students are not always required also to do so.
1 The teacher makes a few incorrect statements or is sloppy about mathematical language, but generally uses correct mathematical terms.
0 The teacher makes repeated incorrect statements or incorrect names for mathematical objects instead of their accepted mathematical names.

11. The teacher's talk encouraged student thinking. TF**

3 The teacher's talk focused on high levels of mathematical thinking. The teacher may ask lower-level questions within the lesson, but this is not the focus of the practice. There are three possibilities for high levels of thinking: analysis, synthesis, and evaluation. Analysis: examines/interprets the pattern, order, or relationship of the mathematics; parts of the form of thinking. Synthesis: requires original, creative thinking. Evaluation: makes a judgment of good or bad, right or wrong, according to the standards he or she values.
2 The teacher's talk focused on mid-levels of mathematical thinking. Interpretation: discovers relationships among facts, generalizations, definitions, values, and skills. Application: requires identification and selection and use of appropriate generalizations and skills
1 Teacher talk consists of "lower-order" knowledge-based questions and responses focusing on recall of facts. Memory: recalls or memorizes information. Translation: changes information into a different symbolic form or situation.

0 Any questions/responses of the teacher related to mathematical ideas were rhetorical, in that there was no expectation of a response from the students.

12. There was a high proportion of students talking related to mathematics. SE*

3 More than three-quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.
2 More than half but less than three-quarters of the students were talking related to the mathematics of the lesson at some point during the lesson.
1 Less than half of the students were talking related to the mathematics of the lesson.
0 No students talked related to the mathematics of the lesson.

13. There was a climate of respect for what others had to say. SE* TF**

3 Many students are sharing, questioning, and commenting during the lesson, including their struggles. Students are also listening (active), clarifying, and recognizing the ideas of others.
2 The environment is such that some students are sharing, questioning, and commenting during the lesson, including their struggles. Most students listen.
1 Only a few share as called on by the teacher. The climate supports those who understand or who behave appropriately. Or some students are sharing, questioning, or commenting during the lesson, but most students are actively listening to the communication.
0 No students shared ideas.

14. In general, the teacher provided wait time. SE*

3 The teacher frequently provided ample amount of "think time" for depth and complexity of a task or question posed by the teacher or a student.
2 The teacher sometimes provided ample amount of "think time" for depth and complexity of a task or question posed by the teacher or a student.
1 The teacher rarely provided ample amount of "think time" for depth and complexity of a task or question posed by the teacher or a student.
0 The teacher never provided ample amount of "think time" for depth and complexity of a task or question posed by the teacher or a student.

15. Students were involved in the communication of their ideas to others (peer to peer). SE*

3 Considerable time (more than half) was spent in peer-to-peer dialog (pairs, groups, whole class) related to the communication of ideas, strategies, and solution.
2 Some class time (less than half, but more than just a few minutes) was devoted to peer-to-peer (pairs, groups, whole class) conversations related to the mathematics.
1 The lesson was primarily teacher-directed, and few opportunities were available for peer-to-peer (pairs, groups, whole class) conversations. A few instances developed where this occurred during the lesson but only lasted less than 5 minutes.
0 No peer-to-peer (pairs, groups, whole class) conversations occurred during the lesson.

16. The teacher uses student questions/comments to enhance conceptual mathematical understanding. SE* TF**

3 The teacher frequently uses student questions/comments to coach students, facilitate conceptual understanding, and boost the conversation. The teacher sequences the student responses that will be displayed in an intentional order, and/or connects different students' responses to key mathematical ideas.
2 The teacher sometimes uses student questions/comments to enhance conceptual understanding.
1 The teacher rarely uses student questions/comments to enhance conceptual mathematical understanding. The focus is more on procedural knowledge of the task versus conceptual knowledge of the content.
0 The teacher never uses student questions/comments to enhance conceptual mathematical understanding.

***SE Student Engagement is measured**
****TF Teacher Facilitation is measured**

Gleason, J., Livers, S.D., & Zelkowski, J. (2015). *Mathematics classroom observation protocol for practices: Descriptor's manual*. Retrieved from http://jgleason.people.ua.edu/mcop2.html