

COORDINATED OPTIMAL CONTROL OF SMART BUILDINGS AND PV  
SYSTEMS IN ACTIVE DISTRIBUTION NETWORKS

by

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## ABSTRACT

NAVEEN KUMAR KODANDA PANI. Coordinated optimal control of smart buildings and PV systems in active distribution networks. (Under the direction of DR. LINQUAN BAI)

Driven by economic and environmental policies, there is an increase in distributed energy resources (DER) penetration, such as wind farms and rooftop solar panels in the electrical grid system. With advancement in smart technology such as smart meters and smart inverters, the DERs are transforming the traditional consumers into prosumers (Producers and Consumers) that can actively contribute to the power grid operation by providing grid services. This work will investigate the grid management and coordination strategies for distributed photovoltaics (PVs) and smart buildings in distribution power networks.

This research includes two parts. The first part is to establish a centralized optimization framework to effectively coordinate the operations of distributed PVs and Heating, ventilation, and air conditioning (HVAC) unit in smart buildings in the distribution network to minimize the total network losses. The proposed control strategy has been compared with a basic thermostat control logic to demonstrate its effectiveness.

The second part of this research proposes a distributed optimization approach to coordinating distributed PVs and building aggregators. It is usually not feasible for a system operator to directly model and control individual HVAC unit in each building from the grid operation's perspective. Also, the privacy concerns of the customers and other parties in the network need to be considered. In this regard, a decentralized optimization framework is proposed in this work for distributed PVs and building aggregators. The optimization problem is divided as grid operations main problem and aggregators sub-problem. A modified Benders decomposition algorithm is proposed to solve the model. The objective of the main problem is to minimize the total

network losses while maintaining the nodal voltage in the distribution network. The sub-problem represents the operations of buildings. The objective of sub-problem is to minimize the active power consumption by optimally operating the HVAC units over a time period. The Lagrangian dual extracted from sub-problems is used to update the main problem to converge to an optimal solution using the modified Benders decomposition algorithm which is based on the classical Benders decomposition technique.

All the models have been implemented in MATLAB with YALMIP tool box and solved using commercial solvers such as Gurobi or CPLEX to obtain the optimal solutions. Case studies and comparisons have been conducted to verify the effectiveness of the proposed models.

## DEDICATION

To my parents, advisor and teachers for being a great source of support throughout this journey.

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## LIST OF ABBREVIATIONS AND SYMBOLS

## Abbreviations

ADMM	alternating direction method of multipliers
BD	Benders Decomposition.
DR	Demand Response.
DSO	Distribution System Operator.
EV	Electric Vehicles
FERC	Federal Energy Regulatory Commission
GBD	Generalized Benders Decomposition.
GHG	Greenhouse Gases.
HVAC	Heating, Ventilating, and Air-Conditioning.
ISO	Independent System Operator.
LMP	Location Marginal Price.
LP	Linear Programming.
MILP	Mixed Integer Linear Programming.
MIP	Mixed Integer Programming.
MISOCP	Mixed Integer Second Order Cone Programming.
MPC	Model Predictive Control.
OPF	Optimal Power Flow.
PV	Photovoltaics.

RC Resistance-Capacitance.

TCL Thermostatically Controlled Loads.

VER Variable Energy Resources

### Symbols

$\alpha_{n,t}$  On-off status of the HVAC unit in building  $n$  at time  $t$

$\epsilon$  A very small number

$\lambda_{P,m}$  Dual variable representing active power for iteration  $m$

$\mu_{P,m}$  Dual variable representing active power for iteration  $m$

$\sigma^*$  Benders cut variable for main problem

$\tau_{max,n}$  Maximum set temperature limit for building  $n$

$\tau_{min,n}$  Minimum set temperature limit for building  $n$

$A, B, E$  Discrete time building thermal Constants

$a, b, e$  Continuous time building thermal Constants

$Ap_{m,t}$  Slack variables used to relax the boundary constraints for iteration  $m$  at time  $t$

$C$  Thermal capacity

$C_p$  Active power cost per unit

$C_{tmp}$  Location marginal price of active power

$C_{pv}$  Unit power generation revenue of PV

$D$  Set of distribution lines

$DT_n$	Minimum off time for HVAC unit in building $n$
$G_{out,t}$	Heat gain from solar irradiance at time $t$
$H_{n,t}$	Number of hours the HVAC unit in building $n$ has been on or off at the end of time $t$
$i, j, k$	Node index
$I_{max,ij}^2$	Square of maximum current between nodes $i - j$
$K$	Time period
$LB_m$	Calculated lower bound for iteration $m$
$m$	Iteration count
$N$	Set of nodes
$n$	Building index
$N_f$	Set of substation nodes
$NB_j$	Number of buildings at node $j$
$P_{ij,t}^f$	Active power flow between nodes $i - j$ at time $t$
$P_{j,t}^L$	Active power (fixed load) at node $j$ at time $t$
$P_{j,t}^N$	Aggregated active power at node $j$ at time $t$
$P_{j,n}^{HVAC}$	Active power consumption of HVAC at node $j$ for building $n$
$P_{j,t}^{PVmax}$	Maximum active power output of PV at node $j$ at time $t$
$P_{j,t}^{PV}$	Active power output of PV at node $j$ at time $t$
$P_{j,t}^S$	Active power from substation node at node $j$ at time $t$

$P_{j,t}^{TL}$	Total active power at node $j$ at time $t$ for grid operation
$Q_{ij,t}^f$	Reactive power flow between nodes $i - j$ at time $t$
$Q_{j,t}^L$	Reactive power (fixed load) at node $j$ at time $t$
$Q_{j,t}^N$	Aggregated reactive power at node $j$ at time $t$
$Q_{j,t}^{PV}$	Reactive power output of PV at node $j$ at time $t$
$Q_{j,t}^S$	Reactive power from substation node at node $j$ at time $t$
$Q_{j,t}^{TL}$	Total reactive power at node $j$ at time $t$ for grid operation
$R$	Thermal Resistance of building
$r_{ij}$	Resistance in line between nodes $i - j$
$S\alpha_{j,t}$	Summation of HVAC on-off status for node $j$ and time $t$
$S_j^{PV}$	Inverter capacity of PV at node $j$
$t$	Time index
$T_t$	Building indoor Temperature at time $t$
$T_{out,t}$	Building outdoor air temperature at time $t$
$u_{j,t}$	Voltage square of node $j$ at time $t$
$UB_m$	Calculated upper bound for iteration $m$
$UT_n$	Minimum on time for HVAC unit in building $n$
$V_{max,j}^2$	Square of maximum voltage at node $j$
$V_{min,j}^2$	Square of minimum voltage at node $j$
$w_{ij,t}$	Square of Current Between nodes $i - j$ at time $t$

$x_{ij}$  Reactance of between nodes  $i - j$

## CHAPTER 1: INTRODUCTION

In today's modern world, electricity has become a necessity, like food and shelter and is growing in importance. In our day-to-day life, the uses of electricity include lighting, cooking, washing, heating and cooling, transportation, communication, machinery in industries, to name a few.

Electricity production follows the first law of thermodynamics (law of conservation of energy) which states, "Energy can be transformed from one form to another, but can be neither created nor destroyed". Hence electricity is a secondary energy source as its production depends on the conversion of a primary energy from sources such as coal, petroleum, natural gas, solar energy, wind energy, nuclear energy, or biomass. Electricity, like other energy resources, cannot be stored efficiently except in small amounts using devices such as lithium-ion batteries or converting it to another form of energy to be later used, such as pumped-storage hydroelectricity or thermal energy.

Hence to meet the demand of consumers such as residential and industrial, the suppliers should generate the required amount of electricity simultaneously in real-time and use the electric grid to transmit it to its consumers.

### 1.1 Modern electric grid with distributed energy resources

An electric power grid is an interconnected network of generating stations, electric substations, high voltage, and distribution power lines used for delivering electrical power from producers to consumers. One significant benefit of electricity is its long-distance transmission capability. Hence the electrical power is transmitted from producers to consumers located at long distances using power lines.

Based on the size, electrical grids are classified ranging from a single building

network to national grids and transnational grids. Yet the basic structure of buses and flow lines remains the same in the electrical networks. A bus is a graphical node representation in which electrical quantities such as voltage, current, and power flows are evaluated whereas flow lines are lines connecting these buses or nodes that are responsible for transferring the power flow.

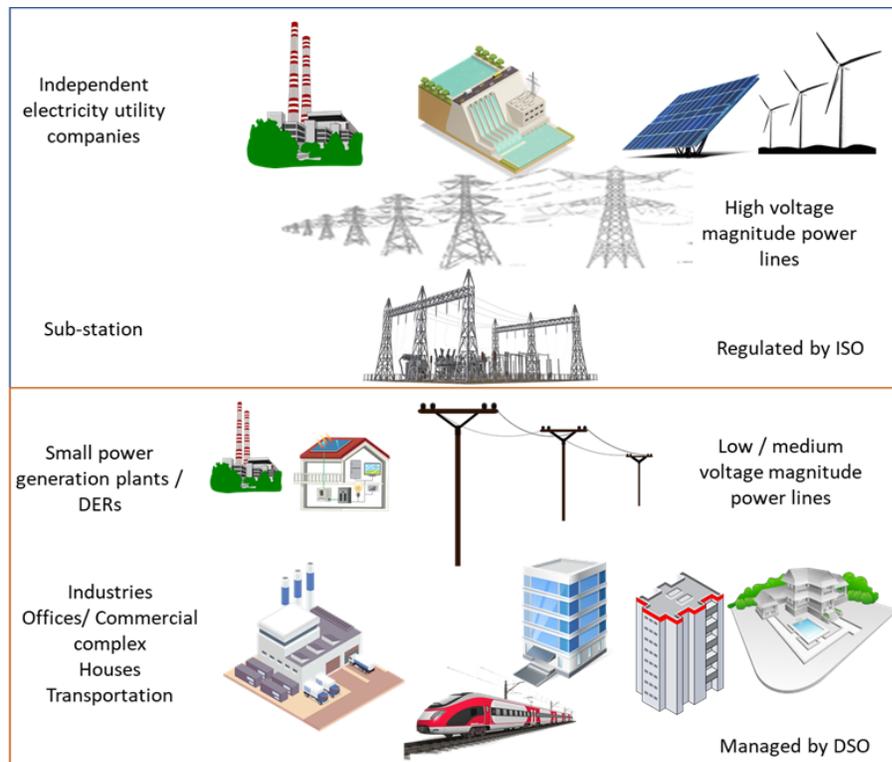


Figure 1.1: Power transmission and distribution structure

The electrical network can be categorized as a transmission network or distribution network depending on the function and voltage as shown in fig.1.1. The transmission networks are responsible for collecting the electric power from producers and transferring it to electric substations at very high voltages to minimize the energy loss due to the resistance and capacitance in the flow lines. The electric substation then reduces the voltage magnitude (step down) and transfers the low voltage electric power to distribution networks whose responsibility is to deliver the electricity to its customers. The distribution networks may include power generation sources

such as small distributed power generators or energy storage devices to provide an uninterrupted power supply.

Due to economic and environmental policies, there is an increase in distributed energy resources (DERs) penetration, such as wind farms and rooftop solar panels in the electrical grid system. These DERs produce a volatile situation which has caused changes in electricity forecasts. To mitigate such issues, the Federal Energy Regulatory Commission (FERC) introduced an Independent System Operator (ISO) a neutral party responsible for the management and control of the electric transmission grid in a state or a region. The roles of ISO include:

- Grid operation: Coordinate and direct the flow of electricity over the region's high voltage transmission systems to maintain safety and reliability.
- Market administration: Design, run, and oversee the markets where wholesale electricity is brought and sold.
- Power system planning: Study, analyze, and plan to meet future electricity needs.

Similarly, the Distribution System Operators (DSO) are responsible for the reliable operation of power distribution network at low and medium voltages. The roles of DSO include:

- Planning, maintenance, and network management.
- Manage power supply outages.
- Energy billing.
- Connection and disconnection and peak load management of DERs

To avoid firms from accessing the market power (monopoly), the ISO, which is a neutral organization based on day-ahead forecasts, calls for price auctions. The firm's

marginal cost of production (System energy price) includes variable costs due to fuel and the other variable operating and maintenance costs [1]. Based on the demand, system energy prices, transmission congestion costs, and cost of marginal losses, the Locational Marginal Price (LMP) is decided. The transmission congestion costs vary based on peak demand. Congestion occurs when parts of the grid operate near their limits and prevent the low priced energy from freely flowing to a specific region in the grid [2].

The LMP represents the cost to buy and sell electric power to different locations within the electricity market. The LMP is classified as day ahead and real-time LMPs. Day ahead LMP represents the day ahead market that lets market participants to buy and sell electricity a day before to avoid volatility and real-time LMPs represent prices in real-time markets that let participants to buy and sell power during the day of operation [3]. Figure 1.2 shows the variation in average day ahead and real time LMPs [4].

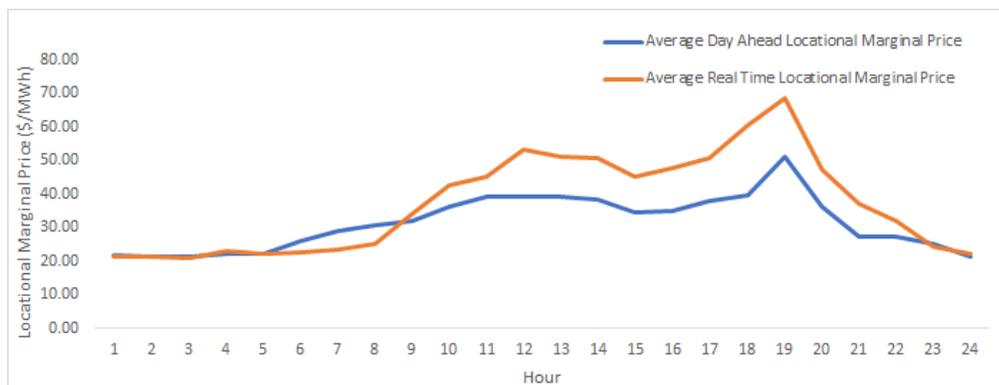


Figure 1.2: Day ahead VS real-time LMP prices[4]

## 1.2 Impact of building energy consumption on environment

The rise in temperatures and global climate change has resulted in instances of a rapid change in biodiversity within complex ecosystems. The industries play a larger role in affecting sustainability on a global scale. The transportation and electricity sectors account for more than 50% of total greenhouse gasses (GHG) emissions in

the US. In 2018, with 26.9% of total GHG emissions, the energy industry stands second, with fossil fuels contributing approximately 63% of total energy production. With an increase in urbanization, electric vehicles (EVs), and electrification of heating equipment, power consumption has shown rapid growth. The increase in capacity requirements promotes energy production from fossil fuels and natural gas. The commercial and residential buildings account for 30-40% of total electricity consumption. The air conditioning systems consume 48%, and lighting systems consume 24% of the total power consumption in commercial buildings [5].

The building sector is a complex system that can improve energy efficiency and sustainability by changing the patterns in which the lighting, heating, cooling, and smart appliances operate [6]. This complex system hence needs an integrated approach towards energy efficiency since various factors affect building energy consumption and carbon footprint. This brings attention to looking at factors that play a key role in building energy consumption simultaneously via a holistic energy model [7]. The commercial building lighting and air conditioning system requirements depend on the occupancy, devices used, and the materials used in the construction of the building.

The appropriate building design has the potential to save 30% of power when compared to conventional buildings that offer the same level of comfort [8]. For commercial building owners to incorporate smart technologies in building construction, analysis of the impact of these technologies on energy savings, and carbon emission reduction can provide a better insight [9]. Renewable energy resources can further help in reducing GHG emissions and optimally control the energy consumption in conventional buildings [10].

### 1.3 Optimal Power flow

The increasing trend in urbanization, EVs, and policies towards GHG emissions has led to the integration of Variable Energy Resources (VER) such as large scale solar and wind farms and DERs such as rooftop solar panels and energy storage into the

power grid with smart technologies such as smart meters, smart appliances, sensors, and control units to form smart grids with bidirectional flow of communication and electric power between suppliers and consumers [11].

The introduction of smart meters has provided its customers with real-time data to capture the patterns of electric energy usage. By implementing necessary control strategies based on the communication between producers and consumers, the total electric energy consumption can be reduced. DERs such as rooftop PVs has introduced a new type of players into the electricity market called prosumers (producers and consumers).

The two most important control parameters that define the electric demand are voltage and line loss. To meet the total power demand at every node in the bus, the necessary voltage magnitude has to be maintained. To achieve this there are multiple ways. One such way is to control the reactive power by finding an optimal way to maintain the voltage and deliver the required electric demand and this is called Optimal Power Flow (OPF).

#### 1.4 Motivation and background

Demand Response (DR) is an incentive approach that changes the electricity usage by end-users from their regular consumption patterns to increase the power grid operation efficiency [12]. Currently, both industries and educational institutes have focused on the OPF of electric power in the power distribution networks, and this thesis aims to formulate a model in the distribution network with the end consumers to minimize the total operational cost for transmitting the electricity. Since the patterns in both power production (VER's) and consumption are variable, modeling them to form an OPF model is a challenging task and energy storage requires high initial investment costs. Hence the better option is to consume the variable power from VERs on the spot as much as possible. Predictive control of Thermostatically Controlled Loads (TCL) in buildings can be used to compensate for fluctuations

in VERs power generation [13]. Hence this research uses TCL's such as HVAC to store the variation in solar energy harnessed using PVs. The operation of HVACs depends on the status of HVAC (On/Off) represented as a binary variable (1-0). The formulation of the OPF model with this binary variable makes the optimization problem a Mixed Integer Non-Linear Programming (MINLP) model. Hence the use of commercial solvers such as Gurobi or CPLEX to solve the MINLP problem in this thesis.

## 1.5 Organization of thesis

This thesis is organized as follows. Chapter 2 provides a brief review related to the concepts used in this thesis. In Chapter 3, a centralised model for optimal control of building loads in a distribution network is introduced. Using the formulations from Chapter 3 a distributed optimization model has been presented in Chapter 4 with the modified Benders decomposition algorithm, which is followed by chapter 5, a case study for both the optimization models discussed in Chapter 3 and Chapter 4. Chapter 6 outlines the conclusion, where the thesis is summarized, and potential future research directions are outlined.

## CHAPTER 2: LITERATURE REVIEW

In this chapter, a review of the previous research in the optimal power flow models of distribution networks, building thermal storage, and decomposition techniques are discussed.

### 2.1 Centralized optimization

The policies towards GHG emissions and concerns related to global warming by individuals have resulted in a drastic increase in the deployment of DERs, especially solar PVs. Due to various factors affecting solar irradiation, such as cloud cover, shade, location, and angle of inclination, the uncontrolled power generation variability has imposed significant challenges to grid stability. An increase in solar penetration causes voltage magnitude variations in the distribution network resulting in degradation of power transformer life due to frequent changes in tap positions [14], [15].

The OPF problem is widely used in fields such as energy management, economic dispatch, congestion management, demand response etc. The OPF models are non-convex, non linear mathematical programs which are NP-hard and are constrained by Kirchhoff's laws. In order to obtain global solution, OPF models are modified and solved as convex optimization problems. The relaxation of non convex constraints or formulation of SOCP, dc power flow approximation are some of the methods used to represent the OPF models as convex optimization problems [16].

The concept of energy storage in the form of thermal energy is demonstrated in various pieces of literature. TCLs, such as HVACs, refrigerators, and electric water heaters, can be used to store the electric power from PVs [13]. Using this concept, [17] has demonstrated that power from PVs can be used to maintain the HVAC using a

quadratic optimization method to optimally dispatch the HVAC power consumption without violating the set temperatures of a room. The average load profile based on outdoor temperature forecast can be used to control the HVAC units for multi-time period load balancing [18].

With advancement in inverter based resources, the PV inverters have been used in distribution networks which can contribute to the line loss minimization and faster voltage regulation by optimally adjusting the active and reactive power outputs. The smart inverters can have the apparent power capacity of 110% of its maximum active power output, leaving 46% of its capacity for reactive power even at full real power output [19]. The Volt-VAr control of IBRs can mitigate large voltage fluctuations due to high penetrations of PV generation and the resulting reverse power flow. [19], [20].

Several DR algorithms have been developed using the centralized optimization framework to solve OPF models but since in the real world, one centralized authority cannot control all the operations as there are third party companies operating at different levels with different goals. The main advantage of centralized algorithms is that they produce the best optimization result possible but need more computational power in addition to stable network communications [21]. Also, in the centralized optimization models the centralized authority has access to all the information related to the companies involved in the DR response, which can violate the privacy policies of these companies.

## 2.2 Decentralized optimization and Decomposition methods

The concept of decentralization (multi level optimization problem) structure has been discussed in many literature to overcome the drawbacks of the centralized models. Lagrangian dual decomposition Alternating Direction Method of Multipliers (ADMM) has been widely used to solve the bi-level (main and sub-problem) structure [22], [23] but since this algorithm has low convergence and as it cannot handle discrete

variables in sub-problems that are used to model binary constraints [24], ADMM cannot be used in mixed-integer nonlinear program (MINLP) sub-problem models. To handle integer sub-problems in two-stage stochastic programming, other algorithms such as Ordinal Optimization (OO) theory can be used, which is a kind of statistical optimization method that includes two primary principles: Order comparison instead of value comparison and goal softening to find the optimal results [25]. OO provides a good enough solution with high probability instead of best solution to reduce the computation time [26]. Because of the goal softening, the accuracy and the local searching ability is inferior and poor. The Column generation (CG) is another algorithm that is used to generate results for sub-problems that contain integer or binary variables. Reference [27] uses CG algorithm in a location transportation problem but one of the main disadvantages is that it may be difficult to determine whether or not a problem can be formulated so that column generation will be beneficial.

Dantzig-Wolfe decomposition (DWD) is a classical algorithm for solving a large-scale linear programs. DWD represents a set of constraints as a set of extreme points and extreme arrays. DWD is used only on the problems that have Non-Integral properties [28]. DWD have been successfully used on DR algorithms [21] but as described the decomposed problems must be continuous linear problems.

Solving MINLP problems is easier using Branch and Bound (BB) algorithm and has been very successful in solving the DR models with other algorithms such as Benders Decomposition [26]. The idea behind BB is to solve continuous relaxations of the original problem and to divide the feasible region, eliminating the fractional solutions of the relaxed problem. Doing this creates a tree of problems from which the integer optimum is found. The hierarchical Benders decomposition with BB algorithm has been used in [29] where the sub main problem uses BB algorithm to solve investment decision and sub-problem is relaxed and solved as specialized Linear Programming (LP) model.

The Heuristic methods are developed to increase the convergence speed and or improve the reliability to find good sub-optimal solutions. The concept of rounding the solution of a continuous nonlinear program subject to linear constraints has been proposed in [30]. Here two algorithms have been proposed one which finds point to be rounded for initial feasible solution and the other which searches for an improved solution within the neighbourhood of a given point. The drawback of heuristic methods is that these are not applicable to other types of problems other than to which these are made.

The Geoffrion's Generalized Benders decomposition (GBD) has been used in DR evaluation, [31] which uses nested Benders decomposition technique to solve central station and distributed power generation, storage, and demand management assets on a linearized electric power transmission network which is a mixed-integer stochastic programming model. The model uses the bi-level structure by using GBD twice, first to communicate between a stochastic linear production costing model for operating central system generation and a nonlinear program for planning central system generation and transmission. Second, between nonlinear program for planning central system generation and transmission, and a mixed-integer program for evaluation of local area distributed resources. Applying GBD twice results in more computation time and computer memory requirements.

To deal with the uncertainty of PV output over a time (multi-time) [32] uses benders decomposition in optimal power flow model which include battery energy storage system and HVAC units. But this model considers all integer and binary variables in main problem. Relaxing the MINLP main problems have been extensively used such as [33], [34] to make the convergence faster.

### 2.3 Benders decomposition

Benders decomposition (BD) is a mathematical programming technique that provides a solution to very large LP problems that are represented as a bi-level structure

problem (main and sub-problems). This algorithm was developed by Jacques F. Benders in 1962. BD tackles the problems that arise with complicating variables (Variables that come in both main and sub-problems) [35].

Consider the following MILP problem

$$f = \text{Minimize } cx + dy \quad (2.1)$$

subjected to:

$$Ax + By \leq b_1 \quad (2.2)$$

$$Cx \leq b_2 \quad (2.3)$$

$$Dy \leq b_3 \quad (2.4)$$

$$y \geq 0 \quad (2.5)$$

$$x \in X \quad (2.6)$$

Where equation 2.2 represent the complicating constraint with  $A, B, C, D, b_1, b_2$  and  $b_3$  representing constants and  $x$  and  $y$  are variables.

The above formulation is decomposed into main and sub-problems as shown below  
main Problem:

$$f^{MP} = \text{Minimize } cx \quad (2.7)$$

subjected to:

$$Cx \leq b_2 \quad (2.8)$$

$$x \in X \quad (2.9)$$

sub-problem:

$$f^{SP} = \text{Minimize } dy \quad (2.10)$$

subjected to:

$$By \leq b_1 - Ax^* : \lambda \quad (2.11)$$

$$Dy \leq b_3 \quad (2.12)$$

$$y \geq 0 \quad (2.13)$$

Where  $x^*$  represents the solution of main problem and  $\lambda$  represents the dual for the complicating constraint.

In case if the sub-problem results in infeasible solution, the complicating constraint is relaxed as shown in the following formulation

Relaxed sub-problem:

$$f^{SP} = \text{Minimize } dy + ez \quad (2.14)$$

subjected to:

$$By + gz \leq b_1 - Ax^* : \lambda \quad (2.15)$$

$$Dy \leq b_3 \quad (2.16)$$

$$y \geq 0 \quad (2.17)$$

Where,  $e$  and  $g$  are constants.

Using the dual variable  $\lambda$  the main problem is updated to provide link between main and sub-problem and is represented as

Relaxed main problem

$$f^{RMP} = \text{Minimize } cx + \sigma \quad (2.18)$$

subjected to:

$$Cx \leq b_2 \quad (2.19)$$

$$\sigma \geq 0 \quad (2.20)$$

$$\sigma \geq f^{SP} + \lambda * (x - x^*) \quad (2.21)$$

$$x \in X \quad (2.22)$$

Here the lower bound (LB) is given by  $f^{RMP}$  and upper bound (UB) is given by  $c * x^* + f^{SP}$

Stopping criteria:

- If  $LB > UB$ , the solution is infeasible.
- If  $LB = UB$ , the solution is optimal.

The BD method is applicable only to LP problems, So Geoffrion in 1972 proposed a Generalized Benders Decomposition (GBD) that was applicable to certain NLP and MINLP problems. The algorithm is as shown in figure 2.1 below.

This method used the concept of optimal cuts (for feasible sub-problem) and feasible cuts (for infeasible sub-problem).

Relaxed main problem

$$f^{RMP} = \text{Minimize } cx + \sigma \quad (2.23)$$

subjected to:

$$Cx \leq b_2 \quad (2.24)$$

$$\sigma \geq 0 \quad (2.25)$$

$$\sigma \geq f^{SP} + \mu * (x - x^*) \quad (2.26)$$

$$0 \geq f^{SP} + \lambda * (x - x^*) \quad (2.27)$$

$$x \in X \quad (2.28)$$

sub-problem:

$$f^{SP} = \text{Minimize } dy \quad (2.29)$$

subjected to:

$$By \leq b_1 - Ax^* : \mu \quad (2.30)$$

$$Dy \leq b_3 \quad (2.31)$$

$$y \geq 0 \quad (2.32)$$

Where  $\mu$  represents the dual for the complicating constraint.

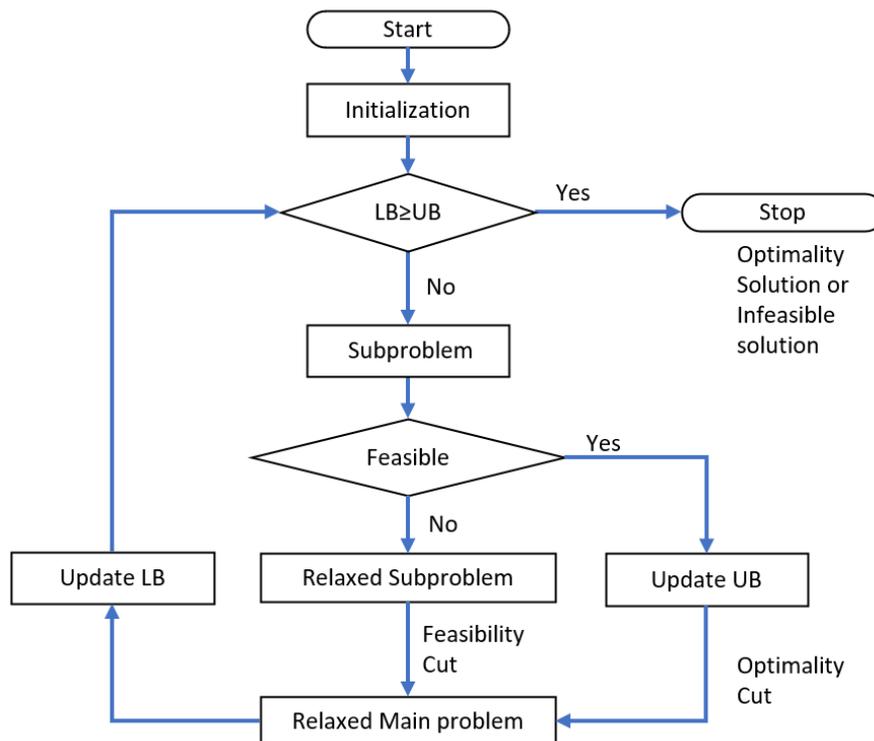


Figure 2.1: Generalized Benders Decomposition Algorithm

In case, if the sub-problem results in an infeasible solution, the complicating constraint is relaxed as shown below.

Relaxed sub-problem:

$$f^{SP} = \text{Minimize } dy + ez \quad (2.33)$$

subjected to:

$$By + gz \leq b_1 - Ax^* : \lambda \quad (2.34)$$

$$Dy \leq b_3 \tag{2.35}$$

$$y \geq 0 \tag{2.36}$$

Where  $\lambda$  represents the dual for the complicating constraint.

## CHAPTER 3: COORDINATED OPTIMAL CONTROL OF PV INVERTERS AND HVAC LOADS IN ACTIVE DISTRIBUTION NETWORK

This section presents the optimization model for optimally controlling the PV inverters and HVAC units to minimize the total network loss in the distribution network for centralized framework. Figure 3.1 shows the centralized framework structure where the entire network control is handled by the grid controller.

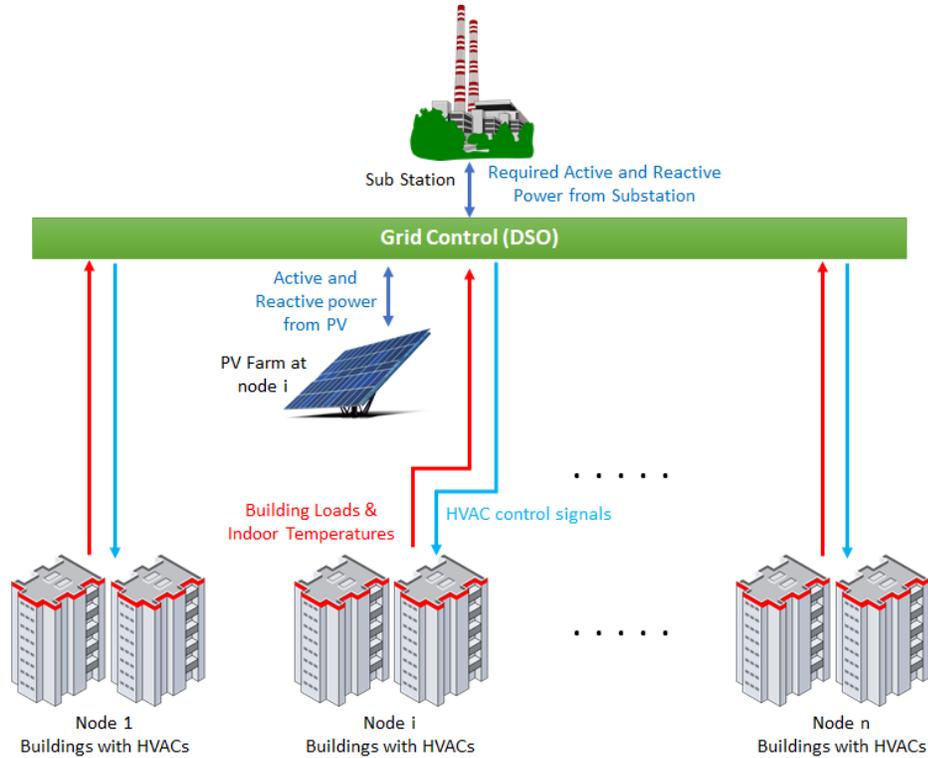


Figure 3.1: Centralized distribution structure

### 3.1 Objective function

The objective of the problem is to minimize the total network loss in the distribution network while maintaining the nodal voltage in the set range. The objective can be represented as shown in equation 3.1.

$$\min \sum_{t=1}^k \sum_{(i,j) \in N} r_{ij} w_{ij,t} \quad (3.1)$$

## 3.2 Constraints

### 3.2.1 PV inverter control

Smart inverters can flexibly control the PV active and reactive power and the total active and reactive power generation cannot exceed the inverter capacity at any point of the given time and can be represented as equation 3.2. Since this equation forms a quadratic function, it is transformed into the second-order cone (SOC) form as shown in equation 3.3

$$(P_{j,t}^{PV})^2 + (Q_{j,t}^{PV})^2 \leq (S_j^{PV})^2 \quad \forall j \in N, \forall t \quad (3.2)$$

$$\|P_{j,t}^{PV} \ Q_{j,t}^{PV}\|_2 \leq S_j^{PV} \quad \forall j \in N, \forall t \quad (3.3)$$

The total active power utilized by the model cannot exceed the maximum active power output of a PV.

$$0 \leq P_{j,t}^{PV} \leq P_{j,t}^{PVmax} \quad \forall j \in N, \forall t \quad (3.4)$$

### 3.2.2 Branch Flow Model

To provide the relation between different nodes, a branch flow model is considered for the power distribution. The constraints of branch flow model are as follows:

- Node balancing equations: For each node in the distribution network, the total power flow into the node should be equal to the total power flowing out of the node. For node  $j$ , the node balance equations are as shown in equations 3.5 and 3.6 where  $i$  represents the parent nodes and  $k$  represents the children nodes.

$$\sum_{k \in N_f} P_{jk,t}^f - \sum_{i \in N_f} (P_{ij,t}^f - r_{jk} w_{jk,t}) = P_{j,t}^N - P_{j,t}^{PV} \quad \forall i, j, k \in N \setminus N_f, \forall t \quad (3.5)$$

$$\sum_{k \in N_f} Q_{jk,t}^f - \sum_{i \in N_f} (Q_{ij,t}^f - r_{jk} w_{jk,t}) = Q_{j,t}^N - Q_{j,t}^{PV} \quad \forall i, j, k \in N \setminus N_f, \forall t \quad (3.6)$$

Similarly, for the substation, the active and reactive power equation are as follows:

$$P_{i,t}^s - \sum_{i \in N} P_{ij,t}^f = 0 \quad \forall i \in N, \forall t \quad (3.7)$$

$$Q_{i,t}^s - \sum_{i \in N} Q_{ij,t}^f = 0 \quad \forall i \in N, \forall t \quad (3.8)$$

- Coordinated optimization active power constraint: In equation 3.5, the active power at node  $j$  is split into base load and HVAC load and is represented as

$$P_{j,t}^N = P_{j,t}^L + \sum_{n \in NB_j} \alpha_{n,j,t} * P_{n,j}^{HVAC} \quad \forall n \in NB_j, \forall j \in N, \forall t \quad (3.9)$$

- Voltage drop equation: When the electric power flows along the distribution line from node  $i$  to the node  $j$ , the voltage drop will be observed. The voltage drop equation is given by

$$u_{j,t} - (u_{i,t} - 2(r_{ij} P_{ij,t}^f + x_{ij} Q_{ij,t}^f) + (r_{ij}^2 + x_{ij}^2) w_{ij,t}) \quad \forall ij \in D, \forall i \in N, \forall t \quad (3.10)$$

- Voltage magnitude limits: The incorporation of PVs can cause variations in voltage, thus it is necessary that the voltage is maintained within limits. The constraint is given by

$$V_{min,j}^2 \leq u_{j,t} \leq V_{max,j}^2 \quad \forall j \in N, \forall t \quad (3.11)$$

- Current constraints: For any line connecting two nodes, the current flowing through it cannot exceed the maximum current capacity, as constrained by

$$0 \leq w_{ij,t} \leq I_{max,ij}^2 \quad \forall ij \in D, \forall t \quad (3.12)$$

- Load flow: The load flow calculations are necessary to determine the steady-state operating characteristics of the power system for a given load, substation real power and voltage conditions. The relationship between active power, reactive power, current and voltage can be expressed as

$$(P_{ij,t}^f)^2 + (Q_{ij,t}^f)^2 = w_{ij,t} * u_{i,t} \quad \forall ij \in D, \forall t \quad (3.13)$$

The above equation 3.13 is relaxed and transformed into a second order cone form as

$$\|2P_{ij,t}^f \ 2Q_{ij,t}^f \ (w_{ij,t} - u_{i,t})\|_2 \leq w_{ij,t} + u_{i,t} \quad \forall ij \in D, \forall t \quad (3.14)$$

### 3.2.3 Building thermal model

- Thermal dynamics model: The typical one-dimensional resistance-capacitance (RC) model is widely used in the literature [17]. Consider the following continuous linear time invariant (LTI) system which is based on the dynamics of the room temperature and outside air temperature.

$$T_{t+1} = \frac{T_{out,t}}{RC} - \frac{T_t}{RC} + \frac{G_{out,t}}{C} + \frac{\alpha_t * (-P_{HVAC})}{C} \quad (3.15)$$

In equation 3.15,  $-P_{HVAC}$  indicates that the HVAC is in cooling mode in the summer. The LTI equation 3.15 is converted to state-space as

$$X_{t+1} = aX_t + bU_t + eV \quad (3.16)$$

Where  $a = \frac{-1}{RC}$ ,  $b = \frac{-P_{HVAC}}{C}$ ,  $e = [\frac{1}{RC} \frac{1}{C}]$ ,  $U = \alpha$  and  $X = T$ .

Since equation 3.16 is a continuous-time model, it is converted to discrete-time model for a given time with zero-order hold and reformulated as equation 3.17.

$$\tau_{t+1} = A\tau_{n,t} + B\alpha_{n,t} + Ed_t \quad \forall n \in NB_j, \forall t \quad (3.17)$$

Since the model is designed to maintain the indoor temperature within set limits, this is represented by equation 3.18.

$$\tau_{min,n} \leq \tau_{n,t} \leq \tau_{max,n} \quad \forall n \in NB_j, \forall t \quad (3.18)$$

The mode of HVAC at time  $t$  is given by  $\alpha_t$  which is a binary variable that controls the HVAC unit to maintain the indoor temperature given by 3.19.

$$\alpha_{n,t} \in [0, 1] \quad \forall n \in NB_j, \forall t \quad (3.19)$$

- Minimum on and off time constraint: To maintain the long life of an HVAC unit, it is necessary to avoid frequent switching operations. Therefore, to enforce the minimum on and off time constraints, equations 3.20 and 3.21 have been used which utilize the difference in the previous range and current status to determine the status for the next time interval.

On time constraints

$$(H_{n,t-1} - UT_n)(\alpha_{n,t-1} - \alpha_{n,t}) \geq 0 \quad \forall n \in NB_j, \forall t \quad (3.20)$$

Off time constraints

$$(H_{n,t-1} - DT_n)(\alpha_{n,t} - \alpha_{n,t-1}) \leq 0 \quad \forall n \in NB_j, \forall t \quad (3.21)$$

To understand the above constraints, it is assumed that at time  $(t - 1)$  the HVAC is on, then  $\alpha_{t-1,n} = 1$ . If  $H_{t-1,n}$  is less than  $UT_n$  which means until time  $t - 1$ , the HVAC unit has not reached the minimum on time, then  $H_{t-1,n} - UT_n$  will result in a negative coefficient. To ensure the left hand side in equation 3.20 to be non-negative,  $\alpha_{n,t-1} - \alpha_{n,t} \leq 0$ . Thus, to satisfy the constraint, the HVAC status for time  $t$  has to be 1 (on).

CHAPTER 4: DECENTRALIZED COORDINATED OPTIMAL CONTROL OF HVAC LOAD AGGREGATORS IN ACTIVE DISTRIBUTION NETWORK

This section presents the optimization model for optimally controlling the PV inverters and HVAC units in the active distribution network for decentralized framework. Figure 4.1 shows a decentralized framework structure where there are multiple entities working on different goals. The aggregator works on minimizing the active power consumption in buildings at each node whereas the grid control work on minimizing the network losses.

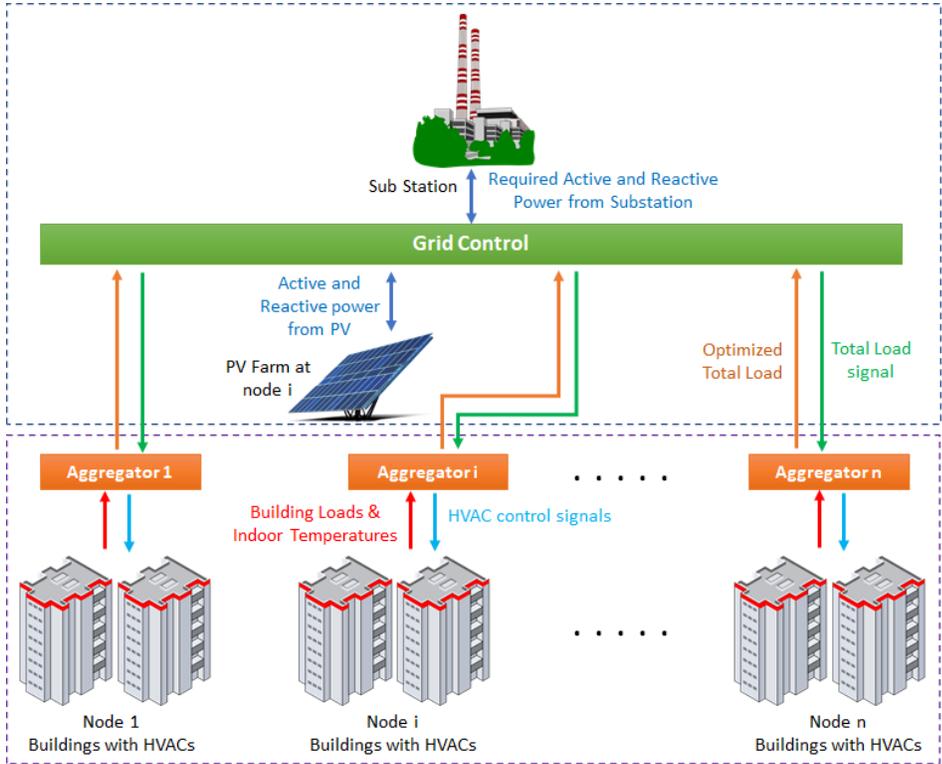


Figure 4.1: Decentralized distribution structure

## 4.1 New centralized model

With multiple entities, in the distribution network, the objective is formulated to (1) minimize the cost of line losses in power distribution network for the grid operator while maintaining the voltage magnitude within range, and (2) minimize the power consumption at each node for buildings, and is as shown in equation 4.1.

$$\text{minimize} \left( C_{lmp} * \sum_{t=1}^k \sum_{(i,j) \in N} r_{ij} w_{ij,t} \right) + \left( C_p * \sum_{t=1}^k \sum_{j \in N} P_{j,t}^N \right) \quad (4.1)$$

Subjected to

$$\|P_{j,t}^{PV} \ Q_{j,t}^{PV}\|_2 \leq S_j^{PV} \quad \forall j \in N, \forall t \quad (4.2)$$

$$0 \leq P_{j,t}^{PV} \leq P_{j,t}^{PVmax} \quad \forall j \in N, \forall t \quad (4.3)$$

$$\sum_{k \in N_f} P_{jk,t}^f - \sum_{i \in N_f} (P_{ij,t}^f - r_{jk} w_{jk,t}) = P_{j,t}^N - P_{j,t}^{PV} - P_{i,t}^s \quad \forall i, j, k \in N \setminus N_f, \forall t \quad (4.4)$$

$$P_{j,t}^N = P_{j,t}^L + \sum_{n \in NB_j} \alpha_{n,j,t} * P_{n,j}^{HVAC} \quad \forall n \in NB_j, \forall j \in N, \forall t \quad (4.5)$$

$$\sum_{k \in N_f} Q_{jk,t}^f - \sum_{i \in N_f} (Q_{ij,t}^f - r_{jk} w_{jk,t}) = Q_{j,t}^N - Q_{j,t}^{PV} - Q_{i,t}^s \quad \forall i, j, k \in N \setminus N_f, \forall t \quad (4.6)$$

$$Q_{j,t}^N = Q_{j,t}^L \quad \forall j \in N, \forall t \quad (4.7)$$

$$u_{j,t} - (u_{i,t} - 2(r_{ij} P_{ij,t}^f + x_{ij} Q_{ij,t}^f) + (r_{ij}^2 + x_{ij}^2) w_{ij,t}) \quad \forall ij \in D, \forall i \in N, \forall t \quad (4.8)$$

$$V_{min,j}^2 \leq u_{j,t} \leq V_{max,j}^2 \quad \forall j \in N, \forall t \quad (4.9)$$

$$0 \leq w_{ij,t} \leq I_{max,ij}^2 \quad \forall ij \in D, \forall t \quad (4.10)$$

$$\|2P_{ij,t}^f \ 2Q_{ij,t}^f \ (w_{ij,t} - u_{i,t})\|_2 \leq w_{ij,t} + u_{i,t} \quad \forall ij \in D, \forall t \quad (4.11)$$

$$\tau_{t+1} = A\tau_{n,t} + B\alpha_{n,t} + Ed_t \quad \forall n \in NB_j, \forall t \quad (4.12)$$

$$\tau_{min,n} \leq \tau_{n,t} \leq \tau_{max,n} \quad \forall n \in NB_j, \forall t \quad (4.13)$$

$$\alpha_{n,t} \in [0, 1] \quad \forall n \in NB_j, \forall t \quad (4.14)$$

## 4.2 Decentralized main problem

To mitigate the drawbacks of centralized network, the decentralized frame work is used. Here, the model is divided as grid operation (main problem) and aggregator (sub-problem). Since, the aggregators are in large numbers, these are considered as sub-problems.

The objective of the grid operation is to minimize the total network loss in the distribution network while maintaining nodal voltage in the secure range. Here the nodal operations are not given importance hence it requires only the aggregated nodal load for modeling. The main problem is formulated as follows

$$f^{MP} = minimize \left( C_{lmp} * \sum_{t=1}^k \sum_{(i,j) \in N} r_{ij} w_{ij,t} \right) \quad (4.15)$$

subjected to

$$\|P_{j,t}^{PV} \ Q_{j,t}^{PV}\|_2 \leq S_j^{PV} \quad \forall j \in N, \forall t \quad (4.16)$$

$$0 \leq P_{j,t}^{PV} \leq P_{j,t}^{PVmax} \quad \forall j \in N, \forall t \quad (4.17)$$

$$\sum_{k \in N_f} P_{jk,t}^f - \sum_{i \in N_f} (P_{ij,t}^f - r_{jk} w_{jk,t}) = P_{j,t}^{TL} - P_{j,t}^{PV} - P_{i,t}^s \quad \forall i, j, k \in N \setminus N_f, \forall t \quad (4.18)$$

$$\sum_{k \in N_f} Q_{jk,t}^f - \sum_{i \in N_f} (Q_{ij,t}^f - r_{jk} w_{jk,t}) = Q_{j,t}^{TL} - Q_{j,t}^{PV} - Q_{i,t}^s \quad \forall i, j, k \in N \setminus N_f, \forall t \quad (4.19)$$

$$u_{j,t} - (u_{i,t} - 2(r_{ij} P_{ij,t}^f + x_{ij} Q_{ij,t}^f) + (r_{ij}^2 + x_{ij}^2) w_{ij,t}) \quad \forall ij \in D, \forall i \in N, \forall t \quad (4.20)$$

$$V_{min,j}^2 \leq u_{j,t} \leq V_{max,j}^2 \quad \forall j \in N, \forall t \quad (4.21)$$

$$0 \leq w_{ij,t} \leq I_{max,ij}^2 \quad \forall ij \in D, \forall t \quad (4.22)$$

$$\|2P_{ij,t}^f \ 2Q_{ij,t}^f \ (w_{ij,t} - u_{i,t})\|_2 \leq w_{ij,t} + u_{i,t} \quad \forall ij \in D, \forall t \quad (4.23)$$

For simplicity, the above formulation is denoted as

$$f^{MP} \quad (4.24)$$

subjected to

$$d(x) \leq b \quad (4.25)$$

Where equation 4.25 represents constraints 4.16 to 4.23

### 4.3 Decentralized sub-problem

Load aggregators act as intermediates to represent corresponding end-users. The purpose of every aggregator in the distribution network is to operate the HVAC units in an optimal way so as to minimize the total power consumed at each node over a time period. The formulation for MILP sub-problem for node  $j$  is given by

$$f_j^{SP} = \min C_p * \left( \sum_{t=1}^k P_{j,t}^N \right) \quad (4.26)$$

Subjected to:

$$P_{j,t}^N = P_{j,t}^L + \sum_{n \in NB_j} \alpha_{n,j,t} * P_{n,j}^{HVAC} \quad \forall n \in NB_j, \forall j \in N, \forall t \quad (4.27)$$

$$Q_{j,t}^N = Q_{j,t}^L \quad \forall j \in N, \forall t \quad (4.28)$$

$$\tau_{t+1} = A\tau_{n,t} + B\alpha_{n,t} + Ed_t \quad \forall n \in NB_j, \forall t \quad (4.29)$$

$$\tau_{min,n} \leq \tau_{n,t} \leq \tau_{max,n} \quad \forall n \in NB_j, \forall t \quad (4.30)$$

$$\alpha_{n,j,t} \in [0, 1] \quad \forall n \in NB_j, \forall t \quad (4.31)$$

For simplicity, the above formulation for node  $j$  is denoted as

$$f_j^{SP} \tag{4.32}$$

subjected to

$$e_j(x) \leq b_j \tag{4.33}$$

$$\alpha_{n,j,t} \in [0, 1] \tag{4.34}$$

$$P_j^N = P_j^{TL} \quad \forall j \in N, \forall t \tag{4.35}$$

Where equation 4.33 represents constraints 4.27 to 4.30 and equation 4.35 acts as boundary variable (Control) that provides relation between the main and sub-problem.

#### 4.4 Modified benders decomposition method

The BD algorithm uses a number of iterations to converge to an optimal solution. In each iteration a benders cut is generated. These cuts help in converging to an optimal solution.

The aggregated nodal load for modeling can be obtained after  $m$  iterations of the sub-problem. The solution from main problem can result in three possible situations:

- The sub-problem is infeasible.
- The sub-problem is feasible but not optimal.
- The sub-problem is feasible and optimal.

In case of infeasibility or non-optimality, the sub-problem result can be assumed to be optimal and benders cuts can be added to ensure that the same result never reoccurs.

#### 4.4.1 Relaxation of sub-problem

Since the sub-problem contains binary variables (on-off status of HVAC units) as shown in equation 4.34 the algorithm is modified to solve the sub-problem in two stages. First, the sub-problem is solved as a MINLP problem with boundary constraints to obtain the summation of HVAC on-off statuses for node  $j$  at time  $t$  ( $S\alpha_{j,t}$ ) which is then fixed in the relaxed linear sub-problem to obtain the Lagrangian dual for the benders cuts used in the main problem.

The relaxed MINLP sub-problem for node  $j$  is formulated as shown below:

$$f_j^{RSP} = f_j^{SP} + \sum_{t=1}^k Ap_t \quad (4.36)$$

subjected to

$$e_j(x) \leq b_j \quad (4.37)$$

$$\alpha_{n,j,t} \in [0, 1] \quad (4.38)$$

$$P_j^N + Ap_t = P_j^{TL} \quad \forall j \in N, \forall t \quad (4.39)$$

For the MINLP sub-problem the linear sub-problems is given by

$$f_j^{LSP} = f_j^{SP} \quad (4.40)$$

subjected to

$$e_j(x) \leq b_j \quad (4.41)$$

$$0 \leq \alpha_{n,j,t} \leq 1 \quad (4.42)$$

$$\sum_{n=1}^{NB_j} \alpha_{n,j,t} = S\alpha_{j,t} \quad (4.43)$$

$$P_j^N = P_j^{TL} : \mu_{j,t,m} \quad \forall j \in N, \forall t \quad (4.44)$$

For the relaxed MINLP sub-problem the linear sub-problem is given by

$$f_j^{RLSP} = f_j^{RSP} \quad (4.45)$$

subjected to

$$e_j(x) \leq b_j \quad (4.46)$$

$$0 \leq \alpha_{n,j,t} \leq 1 \quad (4.47)$$

$$\sum_{n=1}^{NB_j} \alpha_{n,j,t} = S\alpha_{j,t} \quad (4.48)$$

$$P_j^N + Ap_t = P_j^{TL}: \lambda_{j,t,m} \quad \forall j \in N, \forall t \quad (4.49)$$

#### 4.4.2 Modified algorithm

The Grid-aggregator problem can be decomposed using the idea of GBD algorithm where main problem and sub-problem optimize independently and exchange the optimal active load (boundary variable) with each other. The flow chart for modified algorithm is as shown in figure 4.2. The solving procedure is as follows.

- Step 0 (initialization): Set the iteration count  $m = 1$ , lower bound ( $LB$ ) =  $-\infty$ , upper bound ( $UB$ ) =  $\infty$ , the number of optimal cutting planes  $p_m = 0$  and the number of feasible cutting plane  $q_m = 0$ .
- Step 1: (Solve sub-problems): For all  $j$  nodes, solve MINLP sub-problem individually in parallel. if any of the MINLP sub-problem results in an infeasible solution, solve the corresponding relaxed MINLP sub-problem to get the summation of on-off status of HVAC at node  $j$  for time  $t$ .
- Step 2 (Solve linear sub-problems): Using the result from step 1, solve the corresponding linear sub-problems to obtain the dual variables and objective function value ( $S_{j,m}$ ).

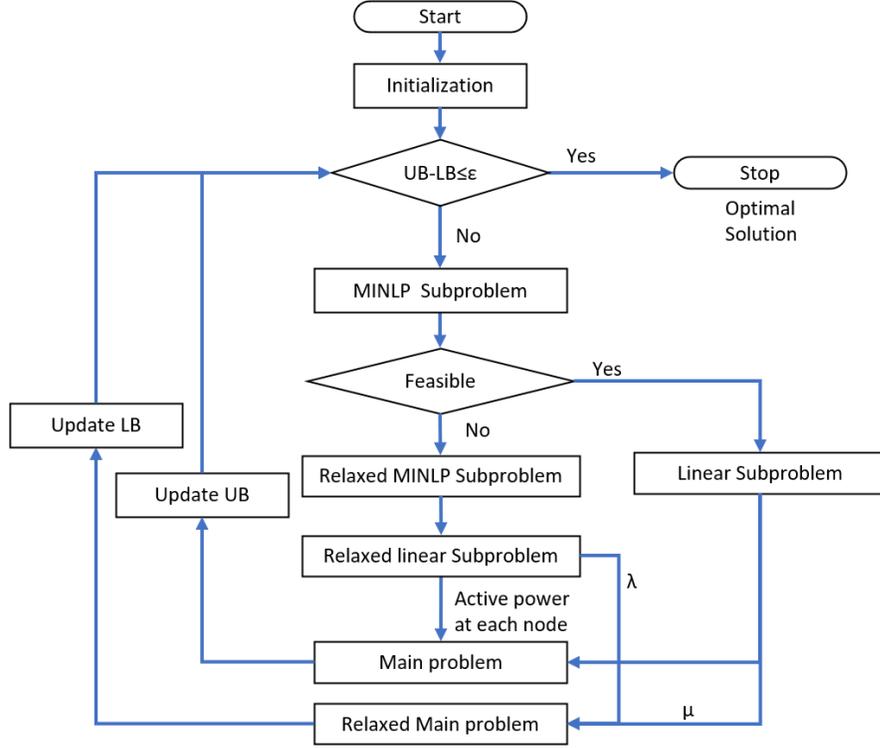


Figure 4.2: Modified Benders Decomposition algorithm flowchart

To reduce the communication between main and sub-problems, the cuts can be represented as follows

For feasible MINLP sub-problem, the cutting plane is given by

$$\inf\{f_j^{SP} + \mu_{j,p}P_{j,p}^N\} = S_{j,p} + \mu_{j,p}(P_j^N - P_j^{TL}) \quad (4.50)$$

Equation 4.50 can be reformulated as

$$L_{j,p}^* = S_{j,p} + \mu_{j,p}P_j^N \quad (4.51)$$

so that the cutting plane of sub-problem  $j$  can be represented as  $L_{j,p}^* - \mu_{j,p}P_j^{TL}$ .

Similarly, for the infeasible MINLP sub-problem, the cutting plane is given by

$$\inf\{f_j^{SP} + \lambda_{j,q}P_j^N\} = S_{j,q} + \lambda_{j,q}(P_j^N - P_j^{TL}) \quad (4.52)$$

Equation 4.52 can be reformulated as

$$L_*^{j,q} = S_{j,q} + \lambda_{j,q}P_j^N \quad (4.53)$$

so that the cutting plane of sub-problem  $j$  can be represented as  $L_*^{j,q} - \lambda_{j,q}P_j^{TL}$ .

- Step 3 (Main problem with cuts): After obtaining cuts from step 2, the main problem can be solved using the relaxed main problem represented as

$$f^{RMP} = f^{MP} + \sum_{j=1}^N \sigma_j \quad (4.54)$$

subjected to

$$d(x) \leq b \quad (4.55)$$

$$\sigma_j \geq 0 \quad (4.56)$$

$$\sigma_j \geq L_{j,m}^* - \mu_{j,m}P_j^{TL} \quad \forall j, \forall m = 1, \dots, p_j \quad (4.57)$$

$$\sigma_j \geq L_*^{j,m} - \lambda_{j,m}P_j^{TL} \quad \forall j, \forall m = 1, \dots, q_j \quad (4.58)$$

- Step 4 (Convergence check): The upper bound ( $UB_k$ ) is the value of main problem calculated by solving the main problem ( $f^{MP}$ ) with  $P^{TL} = P^N$  and the lower bound ( $LB_k$ ) is calculated using the relaxed main problem ( $f^{RMP}$ ).
- Step 5 (Stopping criteria): The algorithm is terminated when

$$gap(K) = UB_k - LB_k \leq \epsilon \quad (4.59)$$

## CHAPTER 5: CASE STUDY

The formulations from chapter 3 and 4 were used in a case study to demonstrate the effectiveness of the proposed models. A modified IEEE-33 node radial distribution network with PVs, Loads and HVAC units were considered. The modified network is as shown in figure 5.1.

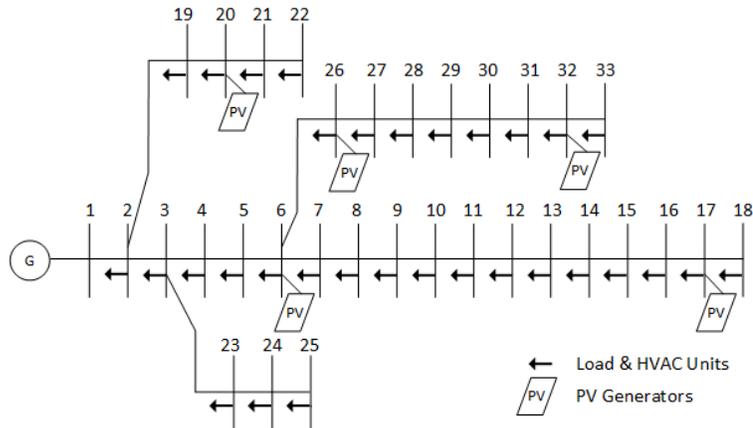


Figure 5.1: Modified IEEE-33 bus radial distribution system

### 5.1 Assumptions

- The capacity of all PV inverters is set to 0.4 MW.
- The cooling power of each HVAC unit is assumed to be 10.512 kW.
- The indoor temperature is set to  $23 \pm 0.5$  °C.
- The minimum on and off of all HVAC units are assumed to be 20 minutes.
- The indoor variations that affect the indoor heat gain such as occupant and devices inside the building is assumed to be constant.

- The initial temperature inside the building are assumed to be within the specified range ( $23 \pm 0.5$  °C).
- The voltage limits are assumed to be 0.95 to 1.05 per unit (p.u)
- The wall thermal constants are assumed to be constant irrespective of the material and window size (Use RC model).
- For centralized model the cost of power is assumed to be uniform.

## 5.2 Data

The active, reactive loads and maximum solar power profiles between 11:00 AM and 2:00 PM with 10-min time interval are selected for this study. The outside temperature, heat gain and thermal dynamics parameters can be found in [17]. The number of buildings at each node of the distribution network is as shown in Table 5.1. The detailed data of the IEEE 33-node system can be found in [36] and [37].

Table 5.1: Number of HVAC Units At Each Node.

Node	Qty	Node	Qty	Node	Qty	Node	Qty
2	40	10	46	18	46	26	42
3	46	11	42	19	42	27	40
4	42	12	40	20	40	28	40
5	40	13	40	21	40	29	42
6	40	14	42	22	46	30	40
7	46	15	40	23	42	31	46
8	42	16	46	24	40	32	40
9	40	17	40	25	46	33	46

### 5.3 Results of coordinated optimal control of PV inverters and HVAC loads in active distribution network

To demonstrate the effectiveness of the proposed model, a comparison is made between the proposed model and two other models including

Model 1: Basic control strategy of a residential thermostat for inside temperature control. The control follows the rules for cooling cycle as follows:

- If inside temperature  $T \geq 23.5C$ , then switch HVAC on
- If HVAC is on and  $T \leq 22.5C$ , then switch off the HVAC.

Model 2: The thermal constraints defined in Chapter 3 is used but without the minimum on and off time constraints.

All three models have been modeled using YALMIP [38] and solved using commercial solver such as Gurobi [39].

The simulation results from all three optimization models are compared in Table 5.2. From this table it can be observed that the model 2 shows less line loss and active power requirement from the substation when compared to the other models. The frequent switching operations of HVAC units in model 2 can cause additional maintenance cost and may not be preferable to the building owners since it does not consider on/off time constraints resulting in the most frequent switching operations. The line loss in the proposed model is significantly lower than Model 1 with active participation of HVAC unit in the grid management. Compared with Model 2, the proposed model can reduce the number of switching operations of HVACs by the incorporation of minimum on/off constraints meanwhile the total line loss is slightly higher. Due to the effective utilization of the PV generation for HVAC loads, model 2 and proposed models perform better than model 1 hence a reduction in active power consumption.

Figure 5.2 shows the voltage variations from simulations for all 3 models. From

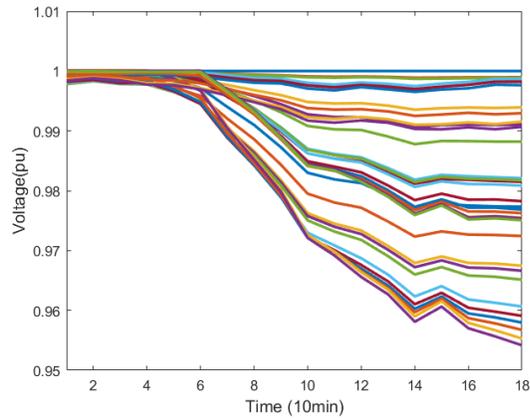
this it can be observed that the voltage variations for all three models are within the assumed specification (0.95 to 1.05 p.u) indicating that the model is effective.

Table 5.2: Comparison of Centralized Model Simulation Results for 3 Hours.

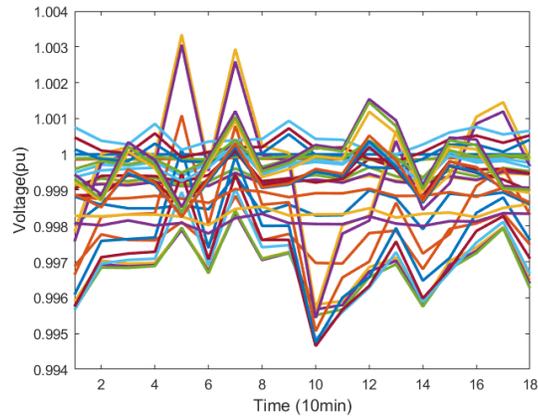
Model	Model 1	Model 2	Proposed
Total Line Loss ( <i>kWh</i> )	383.25	17.38	27.51
Total Active Power from Substation ( <i>MWh</i> )	20.87	3.47	6.19
Total Reactive Power from Substation ( <i>MVarh</i> )	1.82	0.51	0.32
Total number HVAC units switches	520	1141	774
Simulation Time ( <i>Sec</i> )	39.40	4745	473

The voltage variation here depends on the operation of HVAC units. When a large number of HVAC units turn on, the voltage magnitude drops and this can be observed in figure 5.2.(a). Further to understand the effect of voltage variation, node 24 simulation results for temperature and HVAC mode are shown in figures 5.3 and 5.4 respectively. From figure 5.3.(a), it is clear that the mode of HVAC changes only when the temperature reaches the extreme limits which occur mostly at the end of the time period and the same effect can be observed in the figure 5.2. Figure 5.3.(b) shows frequent changes at an interval of 10 minutes indicating the active power is being utilized optimally when compared to other models. Similarly, figure 5.3.(c) shows the changes at an interval of 20 minutes indicating the effect of minimum on/off time constraints on HVAC units. The same effects can be observed on the mode of HVAC as shown in figures in 5.4. Since the operation of HVAC units depend on the available PV power, the HVACs change the mode in contrast to model 1 in model 2 and the proposed models. Hence, the variation of voltage can be observed

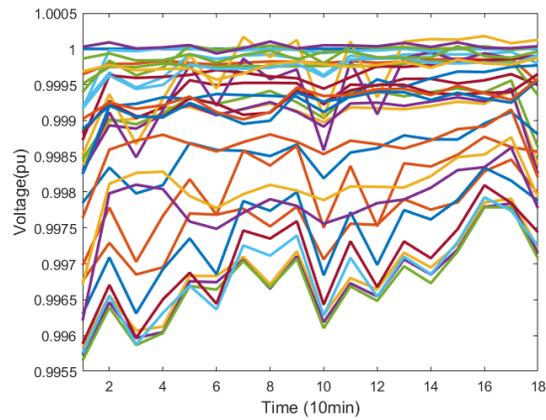
through the time horizon in model 2 and the proposed models.



(a) Model 1 Voltage variation for all 33 nodes

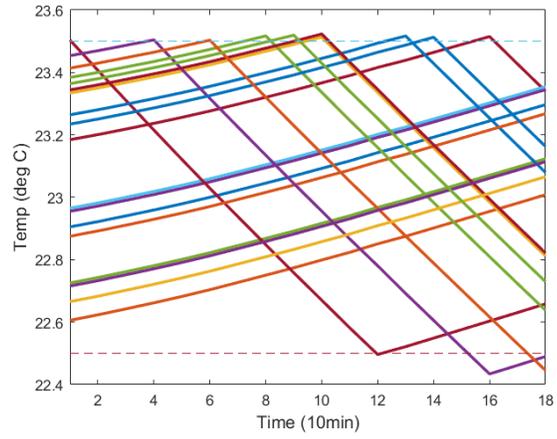


(b) Model 2 Voltage variation for all 33 nodes

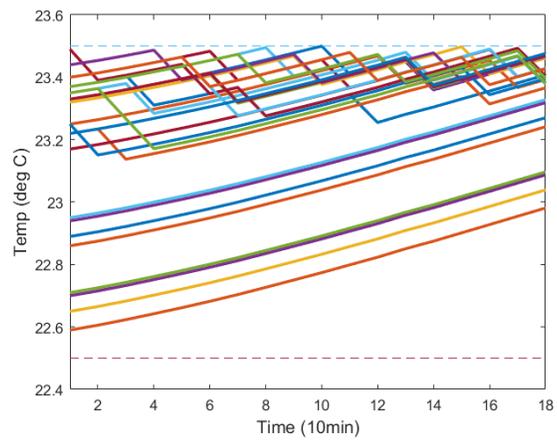


(c) Proposed Model Voltage variation for all 33 nodes

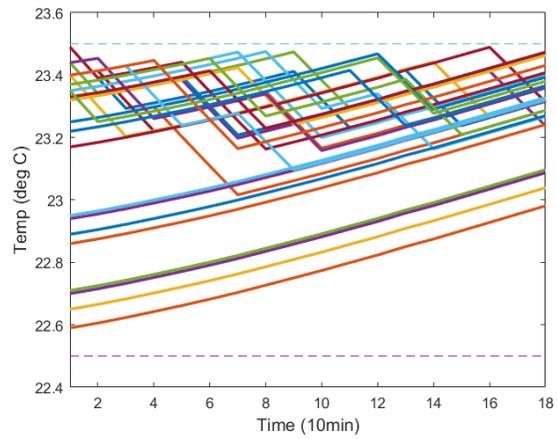
Figure 5.2: Voltage variation for all 33 nodes in centralized frame work.



(a) Model 1 Temperature variation for node 24

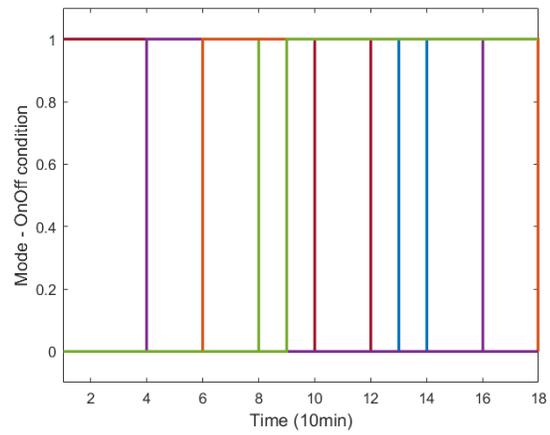


(b) Model 2 Temperature variation for node 24

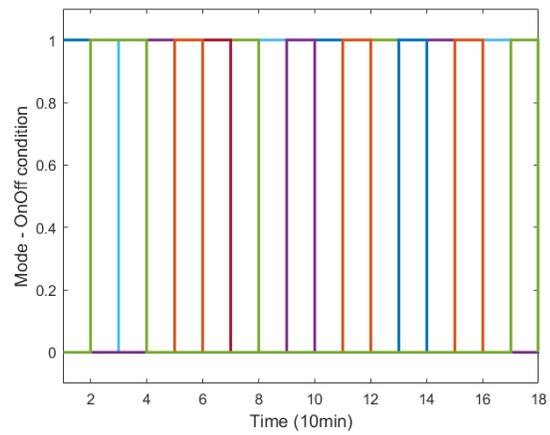


(c) Proposed Model Temperature variation for node 24

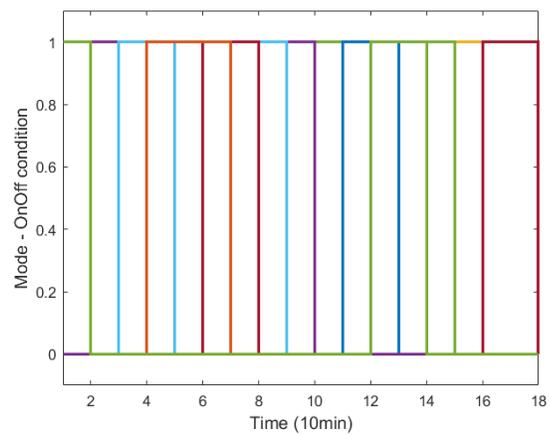
Figure 5.3: Temperature variation for node 24 in centralized frame work.



(a) Model 1 HVAC Status for all buildings at node 24



(b) Model 2 HVAC Status for all buildings at node 24



(c) Proposed Model HVAC Status for all buildings

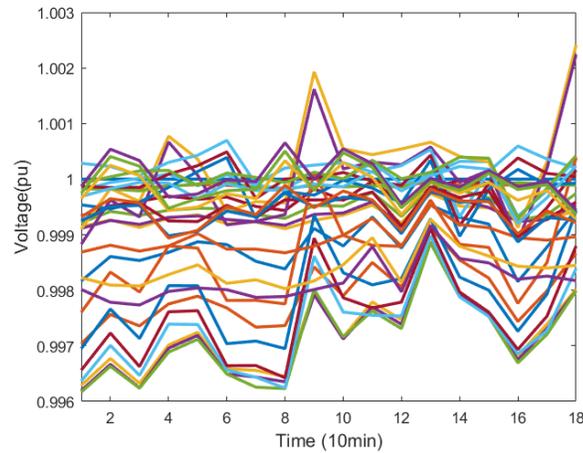
Figure 5.4: HVAC Status for all buildings at node 24 in centralized frame work.

#### 5.4 Results of decentralized coordinated optimal control of HVAC load aggregators in active distribution network

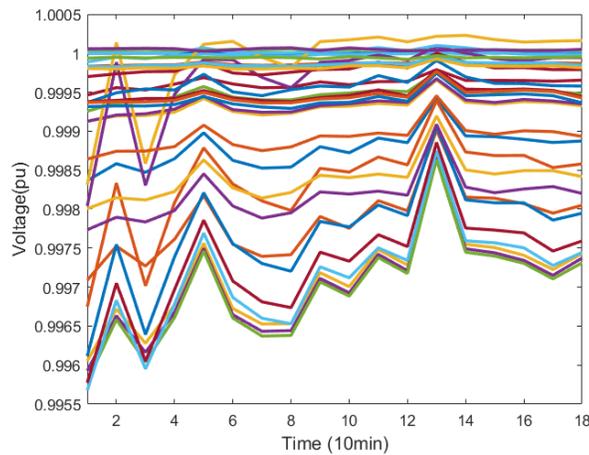
To demonstrate the effectiveness of the decentralized model, comparison with the centralized model is made. By modeling the objective and constraints for both the models using YALMIP [38] and solving them using commercial optimizers such as Gurobi [39] or CPLEX [40], the optimal solutions are obtained and the simulation results are as shown in Table 5.3. Here a comparison of the network losses, total amount of active and reactive power consumption, total number of times HVAC units are switched on, objective function values, and simulation time is shown.

Table 5.3: Comparison of Decentralized Model Simulation Results for 3 Hours.

Model	New centralized model	Decentralized model
Total Line Loss ( $kWh$ )	19.18	21.69
Total Active power consumption ( $MWh$ )	23.3	23.3
Total Active power from PV ( $MWh$ )	19.62	19.33
Total Active power from substation ( $MWh$ )	3.68	3.97
Total Reactive power from substation ( $MVarh$ )	0.29	0.33
Total number HVAC units switches	1097	1129
Simulation Time ( $min$ )	360	13.30
Objective Value (\$)	2609.4	2609.2



(a) Centralized model Voltage variation for all 33 nodes



(b) Decentralized model Voltage variation for all 33 nodes

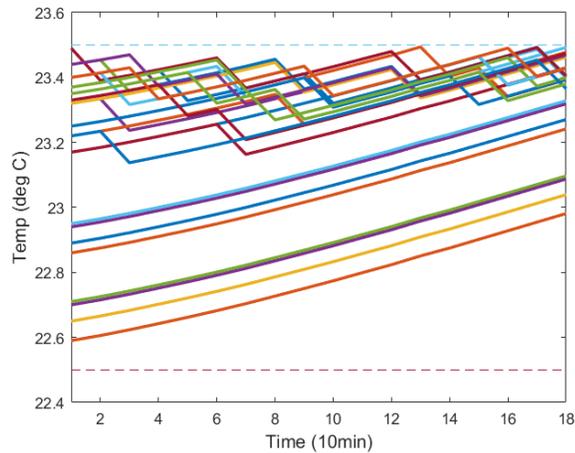
Figure 5.5: Comparison of Voltage variation for all 33 nodes.

The objective function value of the proposed decentralized optimization model is very close to the centralized optimization model. The minor variation in total line loss, total active power and reactive power is due to the weak duality that exist between the relaxed sub-problems and relaxed main problem. It can also be observed that though the power source vary (sub stationion and PV), the total power consumption remains almost same. One benefit of using the proposed decentralized model is the simulation time. Since, the sub-problems run in parallel, the best solutions for each sub-problem can be calculated faster as it contains less binary variables than the

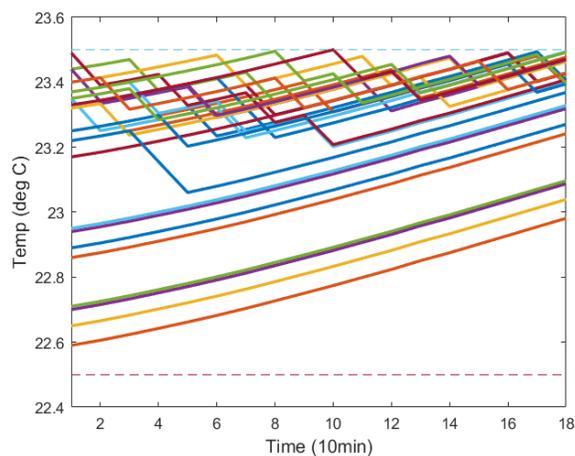
centralized model. As the solver uses BB algorithm to solve the MINLP problem, the simulation time varies depending on the number of binary variables and constraints available.

Figure 5.5 shows the voltage variation for all 33 nodes in centralized and decentralized models. On closer observation slight variations can be observed even though the voltage magnitude is maintained within the specified limits.

The slight variation is due to the fact that the HVACs operate differently in both the models. For better understanding figures 5.6 and 5.7 for node 24 are shown below.



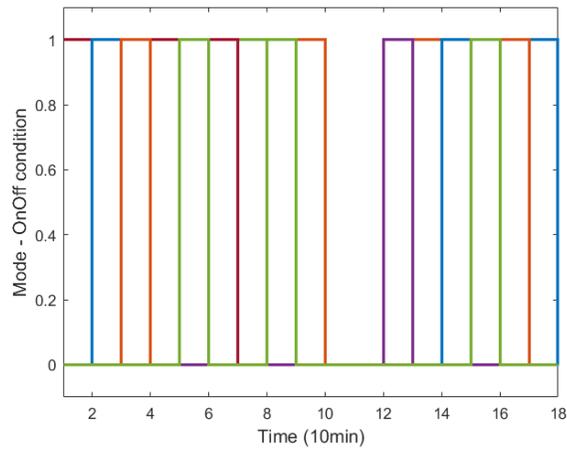
(a) Centralized model temperature variation for node 24



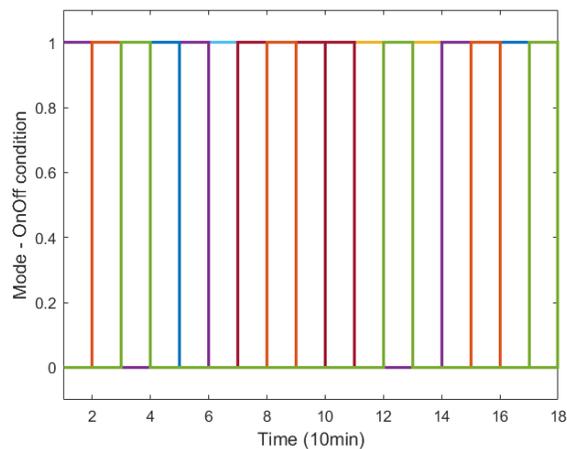
(b) Decentralized model temperature variation for node 24

Figure 5.6: Comparison of temperature variation for node 24.

The mode of HVACs change depending on the availability of PVs active power to each node and the LMP cost that has been used in the main problem to minimize the line loss. From figure 5.6 it can be observed that both the models follow a similar approach of utilizing the active power to run the HVACs optimally to minimize the total power requirement for each node without violating the voltage magnitude and indoor set temperature limits.



(a) Centralized model HVAC mode operation for node 24



(b) Decentralized model HVAC mode operation for node 24

Figure 5.7: Comparison of HVAC mode operation for node 24.

## CHAPTER 6: CONCLUSIONS

This thesis conducted the following research.

- Centralized optimization model for coordinated optimal control of PV inverters and HVAC loads in active distribution network with and without HVAC minimum on/off time constraints.
- Decentralized coordinated optimal control of HVAC load aggregators in active distribution network without HVAC minimum on/off time constraints.

In the first part, a centralized optimization framework to effectively coordinate the operations of distributed PVs and Heating, ventilation, and air conditioning (HVAC) unit in smart buildings in the distribution network to minimize the total network losses is established. Using this model, a new centralized model was formulated and its decentralized model with benders decomposition was solved in the second part.

From both the cases, the effectiveness of the proposed models are demonstrated through a comparative case study.

From the results of centralized optimization model for coordinated optimal control of PV inverters and HVAC loads in active distribution network, the proposed model is effective in reducing the total network loss while maintaining the nodal voltage and regulating the HVAC units to maintain temperature within a specified comfort range in practical applications (using the minimum on/off time constraints).

To mitigate the privacy issues that arise in centralized model, the introduced load aggregators act as intermediate service provider between the grid control and buildings at each node. In the decentralized frame work, the proposed model with decision-making of multiple parties, provides a close enough results to the centralized model

and is effective in minimizing the objective value.

These multi-period MISOCP models proposed in both the cases can be run in a rolling horizon to update the control decisions on PV inverters and HVAC units to optimize the grid operation more effectively.

## 6.1 Future Work

In real situations, the building owners have different mind sets. A study in UK has shown that 40% of programmable thermostat owners did not use programming features and 33% had programming features overridden even though they had the programmable thermostats installed to minimize the power usage based on the set programs [41].

This thesis work did not include the rooftop PVs and electric battery storage devices that are currently in trend to minimize the GHG emissions and/or to minimize the peak load demands using DR program. Also, an assumption was made to eliminate the building occupancy which is a major contributor for inside heat gain in buildings which affects the cooling inside the buildings.

So, depending on these factors, the following cases can be developed.

- Use binary variable to indicate smart thermostat and DR program participation.
- Change the building thermal constraint to include internal heat gain and effects of occupancy.
- Use a data driven and machine learning based approach for the coordinated control of PV and smart buildings.
- Investigate how to enable PVs and buildings to provide grid services such as reserve and frequency regulation.

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