STOCHASTIC AND FUZZY FLEXIBLE AGGREGATE PRODUCTION PLANNING TO MANAGE PLAN STABILITY

by

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ABSTRACT

SETAREH TORABZADEH. Stochastic and fuzzy flexible aggregate production planning to manage plan stability. (Under the direction of Dr. E. C. OZELKAN)

Aggregate production planning generally deals with configuration of an aggregate plan in advance of 6 to 18 periods (e.g. months) to give the organizations an idea about the amount of invested money, utilized capacity, required inventory and any other procurement activities need to be done before the actual times arrives. Inherent uncertainties faced by the planners (caused by unreliable estimates of demand, cost or production processes) could make the production planning a challenging task. That is, the production planners not only have to deal with the available parameters' uncertainties (Demand, cost, etc.), but also, new information which become available with the pass of time, sometimes requires several re-planning activities for the future periods. Stochastic and Fuzzy planning are among the popular techniques to deal with the uncertainties in optimization models. While the stochastic/fuzzy programming techniques provide a more realistic representation of future estimations, the production plans need to be also revised from one planning period to another as time rolls and new information become available (a.k.a. rolling horizon planning). However, frequent re-planning activities and changes in the production plans could result in a state of plan instability causing plan related "nervousness" in manufacturing firms, which could undermine manager's confidence in the system, depriving it of the support needed for successful operations. It could also result in disruptions in the production and delivery systems, which could result in inaccurate personnel scheduling, machine loading, and unnecessary supplier orders (Pujawan and control 2004).

Frozen horizon along with other solution approaches attempt to provide insights on how to mitigate nervousness, however, most of the existing approaches do not consider the flexibility aspect in production plans. Flexible Requirements Profile (FRP) and bi-objective optimization are alternative stabilizing approaches which are the focus of this research. In FRP, flexible bounds are enforced on production plans to maintain the desired degree of flexibility. Instead of 0% flexibility in the case of a frozen period or 100% flexibility in the case of plan to order, FRP model considers different flexibility levels. For the bi-objective optimization approach, the production planning problem can also be formulated with two objectives, where one trades-off between the traditional cost objective and the plan stability objective.

The aim of this research is to address several flexible production planning related open research questions. While deterministic FRP-APP and Bi-Objective APP models have been developed (Demirel 2014) and compared to a traditional deterministic APP model,

1) the deterministic FRP-APP and Bi-objective APP models have not been compared with APP techniques such as Stochastic APP and Fuzzy APP models that are meant to handle uncertainties, and

2) there has not been an attempt to develop Stochastic and Fuzzy FRP-APP and Stochastic and Fuzzy Bi-Objective APP models to deal with planning system uncertainties and

3) also, while FRP-APP was tested with two industry-based case studies and Bi-Objective APP was tested on one industry-based case study, more validation is needed

under different industrial scenarios to conclude about the performance of the FRP-based models.

Therefore, our main research objectives here are:

1) to compare FRP-APP with Stochastic and Fuzzy APP in terms of both plan cost and stability,

2) to develop and compare new "hybrid" Stochastic and Fuzzy FRP-APP models to combine the strengths of stochastic and fuzzy models, which represent input uncertainties more realistically, and FRP models that have better control over plan variability,

3) to develop and compare new Stochastic and Fuzzy Bi-objective APP models as alternate techniques to trade off the traditional cost objective with the stability objective formally following a multi-objective decision making framework, and

4) to conduct extensive testing of the proposed FRP-based and Bi-objective models under various industry scenarios.

Since there are multiple ways to approach stochastic and fuzzy production planning models, for both Stochastic and Fuzzy FRP-APP as well as the Bi-objective Stochastic and Fuzzy APP models that are developed here, we used four of the well-known techniques (two on the stochastic and two on the fuzzy programming) to analyze the effect of specific stochastic and fuzzy approaches on the model performance. More specifically, for the stochastic models, we utilized the Chance-Constraint (CC) and Robust-Stochastic (RS) approaches and for the fuzzy models, we utilized Fuzzy Max-Min (MM) and Fuzzy Ranking (R) approaches. Hence, we will propose in this dissertation eight new APP models, namely: Stochastic CC-FRP-APP, Stochastic RS-FRP-APP, Stochastic CC-BO-

APP, Stochastic RS-BO-APP, Fuzzy MM-FRP-APP, Fuzzy R-FRP-APP, Fuzzy MM-BO-APP, and Fuzzy R-BO-APP. For each of these models, the effect of industry cost structure, demand structure, flexible limits, and the modeling approaches are analyzed using a comprehensive design of experiments analysis to identify influential factors on plan cost and stability.

The results indicate that, for most of the industries tested, the Fuzzy FRP-APP models improve on stability while yielding close cost performance as compared to the Fuzzy APP models. Fuzzy FRP-APP and (non-fuzzy) FRP-APP models show similar performances especially when the Fuzzy R-FRP-APP formulation is used. It is found that lower levels of flexibility limits control stability better as expected, but in general depending on the industry setting and the demand scenario, the cost and stability of the FRP-based models need to be carefully analyzed to choose an ideal flex-limit for practical applications.

The results for the stochastic case show that when Stochastic APP is compared with the FRP-APP, a scenario-based modeling could adversely affect its stability performance. While maintaining the same cost preference as compared to the FRP-APP, the CC-APP, shows more control on the stability of the plans compared to the FRP-APP. The incorporation of the stochastic uncertainty into the FRP-APP formulation, however, can retrieve its better stability performance with improved cost performance as compared to its Stochastic APP counterpart. As a result, Stochastic FRP-APP can be considered as a reliable planning approach to take care of input uncertainty and stability issues at the same time. The stability improvements are more visible using Stochastic RS-FRP-APP with more strict flex-limits. Similar to the fuzzy models, for the stochastic case, a careful selection of flex-limits can further highlight the comparative stability improvement of the Stochastic FRP-APP models as compared to the Stochastic APP planning formulations.

The Bi-objective Stochastic/Fuzzy APP model results indicate that defining the stability as a second objective in the Fuzzy/Stochastic APP formulation could result in more stable plans. In addition, the general observation from different Industry Cases indicates the cost and stability tradeoff performance of the Fuzzy Bi-objective APP are more promising as compared to the Fuzzy FRP-APP, while under stochastic formulations, the Stochastic Bi-objective APP and the Stochastic FRP-APP show more competitive cost and stability performances. This emphasizes the importance of a more careful selection of the specific stochastic/fuzzy technique (i.e. CC vs. RS / MM vs. R) and the weights for the cost and stability objectives in the Bi-objective APP formulation.

The overall results indicate that the proposed Stochastic and Fuzzy FRP-APP and the Stochastic and Fuzzy Bi-objective APP techniques show good potential in terms of stability and cost performance. They also provide more control to a planner to manage plan stability concerns, while representing input data uncertainties more realistically at the same time. For a given industry, these planning techniques require sensitivity analysis with respect to the flexibility limits and objective weight selection, depending on the technique deployed.

DEDICATION

To my Father, Mother and Brothers who have defined the true meaning of Unconditional Love and Support to me throughout my life.

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Chapter 1: INTRODUCTION

1.1 Introduction

A supply chain consists of different players, including suppliers, manufacturers, distribution centers, and the retailers (customers). Since each of these players follow their own objectives, depending on the focal player, the optimization of the supply chain planning problems may have to deal with various (often conflicting) objectives. As an example, while manufacturers would like to have a steady production rate and manage what they produce in an efficient way, the retailers would want to focus on the market demand changes to be responsive to the market as much as possible. Hence due to uncertainties in end customers demand, the retailers may not be eager to purchase large quantities, as they would prefer to place purchase orders as needed following the market changes. As the demand is the main uncertainty factor in today's business worlds (which could be affected by multiple factors such as the uncertainty of the cost, seasonality of the sales, forecasting errors, new market knowledge, making-up damaged shipments), developing aggregate production plans could become a challenging task and the manufacturers could be seriously affected due to frequent production plan updates which could lead to a state of confusion, unresponsiveness, or higher inventory levels and costs. This phenomenon is called "nervousness", or lack of planning stability, and increases the frequency of re-planning activities, which induces further uncertainty into the production plan (Kimms 1998). Manufacturers directly responding to the customers are only as flexible as their supply system allows, and as a result, planning stability becomes more important due to the impact of the plan changes on the supply chain (Meixell 2005). As a result, having an efficient approach for handling the nervousness would not only help the manufacturers to deal with the demand uncertainties better, but also it would help them (manufacturers) to maintain a more stable collaboration with their suppliers, which could consequently help minimizing the adverse effects of uncertainties throughout the supply chain.

One approach to deal with “Nervousness” or “Plan Stability issue” is freezing the master production schedule (MPS) in which no production changes are allowed for a number of periods. Such an approach can result in inventory pileups or even allow shortages and reflect negatively on the manufacturer’s responsiveness and future customer service image (Zhao and Lee 1993). In addition, freezing schedules is undesirable to many manufacturers as it holds back meaningful information about demand patterns that should be made known to the supply network (Meixell 2005). This strategy could be improved by establishing some limits to the amount of changes to the MPS.

In this research, we investigate another nervousness mitigation technique, named “Flexible Requirements Profile (FRP)” by enforcing flexible bounds to the production plans to make sure the production levels would be between dynamically calculated lower and upper bounds during each planning period. In other words, as time rolls to the next planning period, the bounds are dynamically updated due to more visibility to the demand and based on the previous plan. The utilization of this technique in form of an optimization model was first proposed in Demirel (2014) for an aggregate production planning problem. The results show the FRP-based models may result in much more stable production plans with comparable cost values. Demirel (2014) also developed the Bi-objective APP formulation to further analyze the effect of FRP-based Bi-objective formulation on the improvement of the stability. Their overall results show that the proposed Bi-objective APP model is an effective alternate to analyze the trade-off between cost and stability objectives simultaneously and reduce the nervousness in an organization.

While the deterministic FRP-APP and Bi-Objective APP enable planners to protect their developed plans from instability due to new information and uncertainties during re-planning, one alternative approach would be using uncertainty modeling approaches to develop APP plans. This would enable a planner to consider for example different demand uncertainty scenarios more realistically ahead of time. Furthermore, it would be interesting for a planner to develop techniques that can model uncertainty more precisely and handle stability concerns at the same time. Consequently, a more realistic representation of the system uncertainties in the planning model may also affect stability and cost of the production plans.

Two of the well-known uncertainty programming techniques include the Stochastic and Fuzzy programming. Stochastic programming enables planners to incorporate randomness of some input parameters into the planning problem where availability of historical data can help determine the random parameters' distribution functions. The stochastic programming formulations could typically be presented in the form of distribution-based problems, such as Chance-Constraint (CC) programming (Borodin, Dolgui et al. 2016, Moshtagh and Taleizadeh 2017) or the Scenario-based formulations, such as: Stage-based (Santoso, Ahmed et al. 2005, Kazemi Zanjani, Nourelfath et al. 2010) or the Robust-Stochastic (RS) (Pan and Nagi 2010, Mirzapour Al-e-Hashem, Baboli et al. 2013) models. As we will further discuss in the literature review and respective modeling chapters, we will utilize the Stochastic CC and RS techniques to incorporate randomness into the APP models.

Fuzzy programming considers ambiguity of the input data and uses linear functions (to obtain linear programs) to define membership function for the fuzzy objective function and/or the fuzzy constraints and later use related methods to linearize and solve the related mathematical programming problem. Some of the popular fuzzy programming techniques include: Fuzzy Max-

Min (MM) (Baykasoğlu and Göçken 2006, Liang 2008), Ranking (R) (Jiménez, Arenas et al. 2007, Baykasoglu and Gocken 2010), Possibilistic Linear Programming (PLP) (Wang and Liang 2005), or a combination of these techniques (Torabi and Hassini 2008, Torabi, Ebadian et al. 2010). Again, we will utilize the Fuzzy MM and R techniques to incorporate randomness into the APP models as will be discussed in detail subsequent chapters.

Due to the different approaches in the "stability optimized" flexible APP modeling (FRP and Bi-Objective) planning and the uncertainty modeling (Stochastic and Fuzzy), it is worth developing "hybrid" new techniques such as Stochastic/Fuzzy FRP-APP and Stochastic/Fuzzy Bi-Objective APP models. Accordingly, in this research, we would like to further investigate the following main research questions:

1. How does (deterministic) FRP-based planning models compare to Stochastic-APP and Fuzzy-APP in terms of stability and cost?
2. Can the incorporation of FRP into uncertainty modeling techniques (such as stochastic and fuzzy programming) help the production planning performance in terms of cost and stability?
3. How does the cost and stability performance of the Stochastic/Fuzzy APP formulation changes if in addition to the cost minimization, stability is also officially considered as a second objective in its formulation?
4. How do the FRP-based and Bi-objective APP models performance change for different industries?

1.2 Summary of Expected Research Contributions

The research contributions can be summarized as follows:

Contribution 1: Comparison of the FRP-APP model with the Stochastic APP and Fuzzy APP models in terms of plan cost and stability.

Justification: Although the FRP-APP model has been already developed by Demirel (2014) to deal with the effect of the demand variation on plan stability in a rolling horizon framework, its performance has not been compared to the APP models that are meant to deal with demand uncertainty.

Contribution 2: Developing new Stochastic FRP-APP models and comparing its performance with other models in terms of cost and stability.

Justification: While Stochastic APP models exist, there is no Stochastic APP or other stochastic planning models that incorporate FRP. The new Stochastic FRP-APP models will not only provide a more realistic representation of demand uncertainty, it will also provide the planner with a mechanism to consider plan stability and plan cost at the same time.

Contribution 3: Developing new Fuzzy FRP-APP models and comparing its performance with other models in terms of cost and stability.

Justification: The justification is similar to the justification of Contribution 2 above. Again, while Fuzzy APP models exist, there is no Fuzzy APP or other fuzzy planning models that incorporate FRP. The new Fuzzy FRP-APP models will not only provide a more realistic representation of demand uncertainty, but it will also provide the planner with a mechanism to consider plan stability and plan cost at the same time.

Contribution 4: Developing a Fuzzy Bi-Objective APP model.

Justification: An alternate to FRP is consideration of stability as a second objective in the APP model in addition to the traditional cost objective. This would result in a bi-objective optimization problem where the planner would trade-off cost and stability. While Bi-objective

APP model was developed by Demirel (2014), a Fuzzy Bi-Objective APP model does not exist in the literature.

Contribution 5: Developing a Stochastic Bi-Objective APP model.

Justification: The justification is similar to the one for Contribution 4. A Stochastic Bi-Objective APP model does not exist in the literature.

Contribution 6: Validation of FRP-based and Bi-objective APP models' performance across various industry scenarios.

Justification: While FRP APP and Bi-Objective APP models were tested on one to two Industry-based datasets in Demirel (2014), more validation is needed. In this study, we aim to extend the testing to five industry scenarios and perform extensive numerical experiments for additional twelve hypothetical scenarios.

As in this research we will be dealing with multiple APP models, we would like to present a summarized overview of them in Table 1.1 below for better visibility and also use the related abbreviations in the rest of this dissertation. As seen in this table, APP Models 1-7 do exist in the literature, but the eight models listed as Models 8-15 are new and are the contributions of the research presented in this dissertation.

Table 1.1: Aggregate production planning models summary, abbreviations, and contribution

Model Number	Uncertainty Modeling					Stability Control			Model Name	New Models proposed in this dissertation?
	None	Stochastic Programming		Fuzzy Programming		None	FRP	Bi-Objective		
		Chance Constraint (CC)	Robust Stochastic (RS)	Max-Min (MM)	Ranking (R)					
Model 1	x					x			APP	
Model 2		x				x			CC-APP	
Model 3			x			x			RS-APP	
Model 4				x		x			MM-APP	
Model 5					x	x			R-APP	
Model 6	x						x		FRP-APP	
Model 7	x							x	BO-APP	
Model 8		x					x		CC-FRP-APP	x
Model 9		x						x	CC-BO-APP	x
Model 10			x				x		RS-FRP-APP	x
Model 11			x					x	RS-BO-APP	x
Model 12				x			x		MM-FRP-APP	x
Model 13				x				x	MM-BO-APP	x
Model 14					x		x		R-FRP-APP	x
Model 15					x			x	R-BO-APP	x

1.3 Dissertation Outline

The remainder of this research is organized as follows: Chapter 2 provides a comprehensive review of the relevant literature, including: a review of APP problems, rolling horizon procedures, and stability in production planning, followed by bi-objective, fuzzy and stochastic optimization programming research and applications in APP problems. In Chapter 3, FRP-APP in described.

Chapter 4 includes the development and analysis of new Fuzzy FRP-APP models, namely Fuzzy Max-Min (MM-FRP-APP), and Fuzzy Ranking (R-FRP-APP) APP models. Chapter 5 presents two new Stochastic FRP-APP models, namely Stochastic Chance-Constraint (CC-FRP-APP) and Robust Stochastic (RS-FRP-APP) APP models, and corresponding analysis and results. Chapter 6 introduces the new Bi-objective Stochastic and Fuzzy APP models, and the corresponding result. Finally, in Chapter 7, the main conclusions along with the future research directions and suggestions are presented.

Chapter 2: LITERATURE REVIEW

2.1 Introduction

In this chapter, we review the related literature in the aggregate production planning area from different perspectives. We first start with the mathematical programming of aggregate production planning and then continue with the rolling horizon approach which is the main strategy in this research. Next, we review some of the mitigation strategies for dealing with the issue of nervousness in the production plans. After reviewing the literature related to the main idea behind flexible aggregate production planning, we also review the related literature on multi-objective production planning followed by the literature on the fuzzy and stochastic aggregate production planning for which we will propose FRP and bi-objective versions in this research.

2.2 Mathematical Programming in Aggregate Production Planning Problems

There exists a vast literature on the aggregate production planning, in which, meeting customer demand while minimizing the overall cost over a finite horizon by adjusting production, inventory and workforce levels is the main concern. Mathematical programming formulations have been proposed for a wide range of production-related problems since 1950s, addressing problems of long-term aggregate production planning, medium-term allocation of capacity to different products, lot sizing, and detailed short-term production scheduling (Missbauer and Uzsoy 2011). Furthermore, the production planning under uncertainty, mainly due to the demand, selling prices, capacity and cost uncertainties, is quite popular and numerous examples for different uncertainty types could be found in the literature (Thompson and Davis 1990, Ning, Liu et al. 2013). The review of the classical models for production planning under uncertainty could be found in Mula, Poler et al. (2006). In fact, the need for uncertainty consideration into the aggregate production planning problems results from the fact that these problems aim to allocate resources to the future periods according to the current information about future circumstances. The first step for incorporating uncertainty into the planning problems is to determine an appropriate approach to deal with the uncertain parameters (Mirzapour Al-E-Hashem, Malekly et al. 2011). We will investigate more recent literature in the following sections of our literature review.

2.3 Rolling Horizon Models

One of the notable subjects in the area of planning literature is rolling horizon models, where the planning is done iteratively and each plan consists of periods for which there exist real information about the model parameters (current period) while there exist estimation for the future values (future periods). Baker (1977) is one of the first studies to investigate the effectiveness of rolling horizon models in the context of production planning and their results suggest that rolling schedules are quite efficient but the demand pattern and the length of the planning horizon play an important role in the efficiency of the production plans. Another investigation can be found in McClain and Thomas (1977). The results indicate the length of planning horizon has significant impact on cost performance and the fact that the longer planning horizons could be considered less acceptable from managerial perspective. The application of rolling horizon modeling for a production lot-sizing planning problem with stochastic demand can be found in Bookbinder and H'ng (1986). Their optimization model forces a chance-constraint on the probability of stock out in any period. The analysis of results imply that the seasonality parameters of the demand could have significant effect on the quality of the results from cost to stock out values (decreasing trend has the best while increasing trend results in the least favorable results).

Application of rolling horizon-based modelling in production planning is also extended to the multi-level planning in an assembly environment (Simpson 1999), scheduling integration (Li and Ierapetritou 2010), sourcing (Yıldırım, Tan et al. 2005), and routing problems (Bostel, Dejax et al. 2008). A review, implications and future research reviews of the rolling horizon planning in supply chains could be found in Sahin, Narayanan et al. (2013). In addition, a complete review of different horizon-based optimization and operation improvement along with their characteristics including: horizon type, model type, source of horizon, and method are presented in Chand, Hsu et al. (2002). Rolling horizon planning and plan updates as a result of future uncertainties updates could generate a considerable amount of short-run and medium-term adjustment efforts as well as loss in planning confidence, which urges the need for incorporating strategies to mitigate nervousness (Inderfurth 1994).

2.4 Mitigating Nervousness (Instability)

Various approaches have been introduced to minimize the instability (nervousness) under rolling horizon plans. Some possible stability improvement methods specially for the build-to-order systems include: having higher levels of component commonality in the products design structure, excess capacity consideration and also keeping setup costs low which could help dealing with frequent changes in the estimated demand with less negative effect on the effective operation of production systems (Meixell 2005). Production systems which could produce products in small batches as a result of lower setup costs, and also those having fewer machines and operators could more easily deal with changes and the nervousness does not seem to be a major issue for them as compared to production systems with quite large batches (Pujawan and control 2004). Some other well-known approaches include:

Safety stocks: this method considers safety stocks to decrease the amount of instability in the production level as a result of demand violations (Sridharan and LaForge 1989). One major issue with this approach is that the total inventory costs are more likely to rise due to the costs of carrying the buffers (Blackburn, Kropp et al. 1986). Determination of the adequate amount of safety stock could also be a challenge.

Forecast beyond the planning horizon: this approach uses a forecast of demand beyond the planning horizon to protect against an order being placed near the end of the planning horizon. As shown in Carlson, Beckman et al. (1982), the effect of this approach is mixed, with better results in multi-stage production planning settings as there are greater benefits for avoiding changes (Blackburn, Kropp et al. 1986).

Change cost procedure: initially presented by Carlson, Jucker et al. (1979) and later discussed by Blackburn, Kropp et al. (1986), this approach involves modification of the specified setup cost for a period considering sum of the change cost and the old setup costs. As a result, in addition to the regular planning cost, the cost of nervousness which is the change of setup costs is also added and the planning problem aims to balance all costs to determine the least cost production schedule. This approach seems to be practical to be interpreted in form of a multi-objective optimization problem, with varying nervousness cost definitions, such as plan change values, setup change values and even nervousness related workforce change costs.

Freezing the schedule over a time window: this approach is also utilized in multiple studies. As one of the critical factors in utilizing this method, the frozen interval which is the number of scheduled periods for which the schedules are implemented according to the original plan could have a strong effect on the planning costs, where higher frozen interval may increase stock-outs for finished products as a result of lower responsiveness to demand changes (Zhao and Lee 1993). The effectiveness of this approach is also very dependent on the right selection of forecasting methods and related errors in the forecasting process. In addition, controlling the re-planning periodicity (which represents the fraction of the periods at which the re-planning is done to the total frozen horizon periods) to happen after the frozen horizon finish period is proposed as a method for controlling the nervousness. Sridharan, Berry et al. (1987) reasoned that freezing the Master Production Schedule (MPS) can improve the nervousness through limiting the number of schedule changes, but it could also result in increase in production and inventory costs. The results demonstrate that costs are found to be dramatically increasing when the frozen horizon is being applied over more than 50% of the planning horizon. A similar conclusion was derived where frozen schedules were not found cost effective in industries with more optioned products like the automotive industry (Meixell 2005). In the analysis done by Blackburn, Kropp et al. (1986) for a multi-stage lot-sizing MRP system, it is shown despite the potentials of this approach in eliminating the nervous behavior, it also could have potential in reducing benefits specially due to higher chance for higher total setup and holding costs since it does not use the best ordering policy in different stages of the production process.

Flexible fences: The idea of using planning flexible fences is conceptually discussed in a few studies. For example, in Graves (2011) it is discussed that the planners could use time fences to establish varying limits on the amount of changes permitted to their frozen schedule plan and later, these time fences and frozen schedules act as a constraint on the re-planning. Earlier similar conceptual practice is presented in Costanza (1996). While these studies provide basic computational examples, none of these references conducted a research approach to investigate when flexible fences can be useful. In addition, they did not present any optimization-based procedure for applying this concept on production planning problems. Demirel (2014) implemented the flexible fences concept in aggregate production planning optimization models calling it “Flexible Requirement Profile (FRP)”. The results showed that more stable production plans with comparable cost values can be achieved for cases of single and multi-objective deterministic models.

In addition, as an early optimization-based research to improve stability of a rolling horizon planning problem, a dynamic production planning with rolling schedules is presented by Kimms (1998) where over two consecutive planning iterations, if the calculated instability measure (changes in production levels) for overlapping periods is more than what is feasible to the planner, a set of constraints are added to the problem which limit the amount of instability value to a specific tolerance value. More recently, utilization of the stability measure (differences between production levels over consecutive planning cycles) as an objective component, in the MRP and master production schedule development optimization (while having control over both instability measure and its drastic reduction adverse impact on planning cost using multi-objective optimization framework) is discussed in the literature (Herrera and Thomas 2009, Herrera, Belmokhtar-Berraf et al. 2016).

2.5 Multi-Objective Aggregate Production Planning

An extension to the typical cost minimization single objective APP optimization models, is the consideration of multiple (often) conflicting objectives, which result in a multi-objective APP. There are various approaches to deal with multiple objectives:

Goal programming is one of the popular multi-objective programming techniques, where each objective is given a goal or a target value to be achieved. Profit and workforce levels are considered as two goals in the goal programming-based production planning problem addressed in Chen and Tsai (2001). In addition, production cost, carrying and backorder costs, and change in labor levels are defined as goals in Wang and Liang (2004). Production and distribution costs, number of rejected items, and total delivery time are the goals in the production and distribution planning problem in Liang (2007). Other examples of different goals combinations include: cost of logistics and value of purchasing (Torabi and Hassini 2008); production cost, carrying and backordering cost, and change in labor level (Sadeghi, Hajiagha et al. 2013); and production level, storage cost, transportation cost, and distribution cost (da Silva and Marins 2014).

The compromise programming is another technique which aims at finding a compromise solution between the final solution and the utopia point for each objective while the optimum compromise objective function could also be presented in form of a weighted metric (Chang, Eh et al. 1999, Wu and Chang 2004). In this approach, the weights are varied to identify Pareto optimal solutions. The compromise programming formulation in Entezaminia, Heydari et al. (2016), aims at minimizing the total losses of the supply chain as well maximizing total score of product in terms of environmental criteria. Another example can be found in Mirzapour Al-E-Hashem, Malekly et al. (2011) with the objectives of: minimizing the total cost of the supply chain (in form of a robust-stochastic formulation), and maximizing customer satisfaction level.

Finally, the epsilon(ε)-constraint method keeps only one measure in the objective function and all remaining measures (objectives) are represented as inequality constraints to be satisfied (Mavrotas 2009). Again, in this technique, Pareto optimal solutions are identified by varying the epsilon levels. As an example, the multi-objective robust aggregate production planning problem in Al-e, Aryanezhad et al. (2012) uses this technique to simultaneously minimize the weighted sum of the expected and a multiple of the variability of total cost of supply chain, maximizing the customer service level, and also maximizing the weighted average of the workers productivity levels. As another example, Felfel, Ayadi et al. (2016) used the epsilon-constraint method for dealing with the cost and the lost demand level objectives in a multi-site supply chain stochastic planning problem under uncertain demand.

2.6 Fuzzy Programming Models in Production and Supply Chain Planning Problems

In this section, we review the fuzzy programming literature, which was first introduced by Zadeh (1965), and its extensions and applications in the production and supply chain planning problems by classifying our literature under the most popular fuzzy techniques as follows:

2.6.1 Fuzzy Max-Min and weighted-sum Programming Approach

Fuzzy sets and fuzzy logic first introduced by Zadeh (1965), attracted many researchers as it brought a novel yet flexible approach for uncertainty definition. Bellman and Zadeh (1970) extended the fuzzy logic to a mathematical programming context in which the fuzzy uncertainty could exist in the Objective function (OF) coefficients and the constraints of an optimization model. In other words, under consideration of the combined effect of the fuzzy OF (referred to as goals) and the fuzzy constraints, the feasible region could be represented by the intersection of the all membership functions, while the optimum solution would be represented as the maximum membership value in the feasible region (which here is referred as the Max-Min method).

An example of a fuzzy goal programming-based aggregate production planning problem can be found in Wang and Fang (2001). They used the Max-Min method to simultaneously optimize the fuzzy profit and to reduce overhead costs while minimizing the change in workforce levels, having fuzzy inventory and production capacity constraints. The fuzzy goal programming aggregate production planning formulation in Wang and Liang (2004), (Wang and Liang 2005) uses Max-Min optimization technique as well for optimizing the fuzzy goals of: total production cost, carrying and backorder costs, and rate of change in labor levels without any fuzzy constraints. In these studies, the membership functions of the fuzzy goals are formed using initial solutions of each objective using conventional linear programming model. The same approach to form fuzzy goal membership function is used in Liang (2007), Liang (2008) where a fuzzy production transportation planning is modelled. They apply goal programming to a multi-echelon supply chain to optimize the fuzzy goals like: total transportation costs, total number of rejected items, and total delivery time. For a single objective aggregate production planning problem with trapezoidal fuzzy demand and resource levels, Dai, Fan et al. (2003) assumed the fuzzy constraints and a fuzzy goal, and then used the varying values of the fuzzy numbers and based on Decision Maker's (DM) idea, the membership function for the objective (the goal) is formulated to be used in the Max-Min optimization problem. The fuzzy goal programming aggregate production planning problem in Baykasoğlu and Göçken (2006) is modelled as a Max-Min optimization problem where all the goals (profit maximization, workforce change minimization, inventory investment minimization and also backorder cost minimization) are represented in the form of fuzzy constraints with triangular membership functions. The fuzzy Material Requirement Planning (MRP) model in Mula, Poler et al. (2007) with fuzzy resource processing time and resource capacity, as well as fuzzy demand and costs, has both fuzzy total cost objective function and fuzzy constraints with predefined triangular membership functions. Again, the Max-Min technique is applied as a defuzzification method. As another example of the Max-Min method, one can list Tavakkoli-Moghaddam, Rabbani et al. (2007) who again modelled the APP model in a make-to-stock environment to minimize fuzzy cost subject to fuzzy demand and resource usage rate using triangular membership functions. da Silva and Marins (2014) considered a fuzzy goal programming aggregate production planning where a total of 9 fuzzy goals with triangular membership functions (upper and lower bounds are determined by DM) were optimized subject to crisp constraints.

In addition to the Max-Min method, Bellman and Zadeh (1970) presented the idea of Max weighted-sum of membership functions maximization, where the DM has priority (weights) to some of the objectives or goals over the rest. An example can be found in Sadeghi, Hajiagha et al. (2013) who used three goals (total production cost, carrying and backorder cost and rate of change in labor levels) where the membership functions are defined by DM, and the model aims to maximize sum of all membership functions while the aspiration level of the first two goals is desired to be higher than the third membership goal value.

In some other cases, some prioritization constraints are added to the fuzzy model which not only reflect the relation between different membership function values, but also define the minimum aspiration level of each membership function. Application of this technique in the context of APP using goal programming could be found in Belmokaddem, Mekidiche et al. (2009) where three goals of production cost, carrying cost, and labor level change are considered as fuzzy goals. In this study, the objective function was set as maximizing the summation of the membership functions with both singular minimum aspiration levels and pairwise prioritization constraints for the goals. The same approach is also used in Jamalnia and Soukhakian (2009) with one additional goal of customer satisfaction level. Another example can be found in Chen and Tsai (2001) who presented two models. While both models maximize the sum of all membership functions, for the first model, each membership function has a minimum aspiration level constraint, while for the second model, the constraints impose the importance of some goals over some others.

As it can be inferred in the Min-max and the weighted-sum methods, in these fuzzy programming techniques, it is aimed to maximize the aspiration level of either the minimum membership function or the most important ones. These objectives could also be represented in a reverse way in the form of a minimization objective function, that is, the objective function could be represented as a minimization of sum of deviations from the nominal aspiration level values.

The example of such modeling approach could be found in Leung, Wu et al. (2003) where a multi-site aggregate planning problem is formulated as a goal programming problem with fuzzy goals (profit, hiring and layoff level, and availability and utilization of import quota) . In addition, another form would be representing the minimization objective in terms of a weighted-sum minimization, where the weights are determined by DMs as in Mekidiche, Belmokaddem et al. (2013) for a fuzzy APP goal programming model with three goals of production cost, carrying cost and changes in labor level.

2.6.2 Fuzzy Weighted Average Programming Approach

This fuzzy programming technique could be considered as the most straightforward technique for defuzzifying the triangular membership functions for uncertain optimization model parameters. The main idea behind this technique is to use the three estimates for each fuzzy number components (pessimistic, most likely and optimistic) and then use related weights for each estimate to transform the fuzzy number into a crisp equivalent using a weighted sum formula.

In the production and supply chain planning literature, this method is generally used for transforming fuzzy constraints into a crisp one (especially for the right hand side triangular fuzzy numbers) and then join the crisp constraints to the rest of the model defuzzified using other previously introduced techniques, such as: Max-Min and/or weighted-sum programming approaches (Liang 2008, Torabi 1 and Hassini 2009, Azadegan, Porobic et al. 2011) and also the PLP technique which would be introduced in next section.

2.6.3 Fuzzy Possibilistic Linear Programming (PLP) Approach

Naming other fuzzy techniques, another popular fuzzy programming technique for the fuzzy objective functions where the triangular membership function is defined with three prominent points of $(TC^p, 0)$, $(TC^m, 1)$ and $(TC^o, 0)$, is called Possibilistic Linear Programming (PLP). In this approach, which was first introduced by Lai and Hwang (1992), for a minimization problem the solution would be obtained by pushing these critical points toward left and because the vertical coordinates of these critical points are fixed to either 0 or 1, the only item that could be changed is the horizontal coordinates and each objective function would turn into three objectives (goals) to be simultaneously optimized. The utilization of this PLP fuzzy technique and its joint application with Max-Min and/or weighted-sum techniques and weighted average method in production planning for addressing problems with both fuzzy objectives and fuzzy constraints is quite popular in production and supply chain planning. The assemble-to-order production planning in Hsu and Wang (2001) with fuzzy costs is one of the examples in which the PLP technique turns the single objective cost minimization model into a multi-objective model while the model is later optimized using Max-Min technique. The fuzzy aggregate production planning problem in Wang and Liang (2005) with fuzzy cost, resource capacity, resource availability and demand, which has fuzziness in both the objective and the constraints utilizes PLP technique for defuzzification of objective while the constraints are turned into crisp ones using weighted average method. The crisp multi-objective problem is then solved using Max-Min technique. The same defuzzification approach is used in the fuzzy manufacturing distribution supply chain planning problem in Liang and Cheng (2009) and also the aggregate production planning problem of Paksoy, Pehlivan et al. (2010) both with fuzzy goal(s) and capacities. Another example of utilizing the same techniques for a manufacturing distribution supply chain planning under fuzzy demand and cost could be found in Liang (2011). The fuzzy supply chain planning problem in Liang, Cheng et al. (2011) also follows the same approach but as an extension to this technique, the solutions of the Max-Min technique are updated using the Max weighted-sum method. The supply chain fuzzy planning problem in Torabi and Hassini (2008) with fuzzy demand, capacity utilization rates, production capacity, costs, defective rate, and service levels is a comprehensive example with two fuzzy objectives of cost minimization and value of purchase maximization and multiple fuzzy constraints. This research uses both weighted average and PLP methods to turn the fuzzy model into a crisp one and then uses the Max weighted-sum method to solve the final crisp model.

2.6.4 Fuzzy Ranking Programming Approach

Fuzzy ranking method is another method in the fuzzy programming literature which allows the decision makers to work with the concept of feasibility degree and to adjust the solution and make a balance between the feasibility degree of fuzzy constraints and the satisfaction degree of the fuzzy goal(s). One of the useful reference articles explaining the basics of fuzzy sets, fuzzy numbers ranking, and the application of fuzzy rankings in the optimization models with both fuzzy objective(s) and fuzzy constraints is the research done by Jiménez, Arenas et al. (2007). This ranking method is then used in Baykasoglu and Gocken (2010) for a fuzzy multi-objective aggregate production planning problem with fuzzy costs, profit, production capacity and utilization rates, where different versions of the fuzzy ranking method is tested and then the crisp models are solved utilizing a Tabu Search algorithm. In addition, the fuzzy aggregate production planning problem in Tang, Fung et al. (2003) is turned into a crisp model through transforming the fuzzy inventory and capacity constraints into crisp ones using DM satisfaction degree and fuzzy ranking methods. Another example of fuzzy ranking methods utilization can be found in Azadegan, Porobic et al. (2011) where the joint application of weighted average method and fuzzy rankings is applied to a fuzzy manufacturing problem with fuzzy production capacity and production time.

2.6.5 Other Fuzzy Programming Approaches

The methods we have reviewed so far are the most popular and highly utilized techniques in the literature of fuzzy production and supply chain planning. However, there exist some other fuzzy techniques although less popular in the literature.

Credibility theory which is represented using performance criteria could be defined both for the constraints (Ex. credibility service level to impose minimum stock out, chance of balancing the labor level in two successive periods, chance that the hours of labor used by all products not to exceed the maximum available labor level, chance that the hours of machine usage by all products not to exceed the maximum machine capacity, chance that all the warehouse spaces used not to exceed the maximum warehouse space available) and the objective function (Ex. maximizing the credibility that the total fuzzy cost be less than a preselected threshold) could be considered as another fuzzy technique in the literature of aggregate production planning (Ning, Tang et al. 2006, Lan, Liu et al. 2009). As the credibility constraints and the objective function are defined for fuzzy parameters, the solution method is different from other similar constraints and depends on approximation schemes while using heuristic methods (Lan, Liu et al. 2010). In addition, the credibility constraints could be turned into crisp equivalents using different credibility levels and possibility distribution of fuzzy parameters, while the credibility-based objective could be defined using a piecewise credibility function.

Other techniques include: definition of piecewise possibilistic membership functions and substitution of fuzzy parameters with different values in the model constraints (Phruksaphanrat, Ohsato et al. 2011), development of fuzzy Genetic Algorithm (GA) where the fitness of solutions is defined using the degree of constraints satisfaction (Aliev, Fazlollahi et al. 2007), transformation of a fuzzy model into a crisp one using DM satisfaction degree parameters (Tang, Wang et al.

2000), and also transformation of each fuzzy parameter into a membership function formula using related parameters in form of a single value (Tang, Wang et al. 2000).

As a summary of the reviewed fuzzy production and supply chain planning literature, different fuzziness considerations, number of objectives and also the fuzzy programming techniques utilized, Table 2.1 includes the related information of the reviewed articles.

Table 2.1: Literature on application of fuzzy programming in production planning

Paper	Fuzzy parameters	Fuzzy technique	Fuzzy Obj?	Fuzzy constraints?	Number of Objs
Bellman and Zadeh (1970)	General	1. Max-Min 2.Weighted Sum.Memberships	x	x	
Tang, Wang et al. (2000)	Demand Production capacity	Transforming the fuzzy model into aquadratic model using DM satisfaction degrees	x	x	1
Wang and Fang (2001)	Goals	Max-Min	x		3
Chen and Tsai (2001)	Goals	Sum.Memberships	x		5
Hsu and Wang (2001)	Costs	1. Possibilistic Linear Programming (PLP) 2. Max-Min	x		1
Dai, Fan et al. (2003)	Demand Resource level	Max-Min	x	x	1
Tang, Fung et al. (2003)	Demand Production capacity	Transforming the fuzzy inventory and capacity constraints into crisp ones using DM satisfaction degrees and fuzzy ranking methods	x	x	1
Leung, Wu et al. (2003)	Goals	Min deviation of goals from nominal values	x		3
Wang and Liang (2004)	Goals	Max-Min	x	x	3
Wang and Liang (2005)	Demand Product price Subcontract cost Workforce level Production capacity	Max-Min	x	x	2

Paper	Fuzzy parameters	Fuzzy technique	Fuzzy Obj?	Fuzzy constraints?	Number of Objs
Wang and Liang (2005)	Cost Resource capacity Resource availability Demand	1. Weighted average method 2. Possibilistic Linear Programming (PLP) 3. Max-Min	x	x	1
Baykasoğlu and Göçken (2006)	Profit goal backorder goal workforce change goal Inventory investment goal	Max-Min		x	4
Vasant (2006)	Resource level	Substitution of fuzzy parameters with an equation with scaled parameters		x	1
Ning, Tang et al. (2006)	Demand Costs Resource level	Credibility theory	x	x	1
Tavakkoli-Moghaddam, Rabbani et al. (2007)	Demand Usage rate	Max-Min	x	x	1
Liang (2007)	Goals	Max-Min	x		3
Aliev, Fazlollahi et al. (2007)	Costs Profits Demand Transportation level Production capacity	Fuzzy GA	x	x	1
Jiménez, Arenas et al. (2007)	General	Fuzzy ranking	x	x	1
Mula, Poler et al. (2007)	Demand Goal	Max-Min		x	1
Liang (2008)	Goals Demand Resource capacity	1. Weighted average method 2. Max-Min	x	x	2
Liang (2008)	Goals	Max-Min	x		3
Torabi and Hassini (2008)	Demand Capacity utilization rate Production capacity Costs Service level Defective rates	1. Weighted average method 2. PLP 3. Max-Min 4. Sum.memberships	x	x	2

Paper	Fuzzy parameters	Fuzzy technique	Fuzzy Obj?	Fuzzy constraints?	Number of Objs
Jamalnia and Soukhakian (2009)	Goals	Sum.Memberships	x		3
Belmokaddem, Mekidiche et al. (2009)	Goals	Sum.Memberships	x		3
Liang and Cheng (2009)	Goals Resource capacity	1. Weighted average method 2. Possibilistic Linear Programming (PLP) 3. Max-Min	x	x	2
Lan, Liu et al. (2009)	Production cost Inventory cost Demand	Credibility theory	x	x	1
Torabi 1 and Hassini (2009)	Demand Production capacity -Minimum acceptable capacity utilization rate	1. Fuzzy constraints: weighted average method 2. Fuzzy OF: Weighted Sum.Memberships	x	x	4
Lan, Liu et al. (2010)	Production cost Inventory cost Demand	Credibility theory	x	x	1
Paksoy, Pehlivan et al. (2010)	Costs Capacities	1. Weighted average method 2. Possibilistic Linear Programming (PLP) 3. Max-Min	x	x	1
Torabi, Ebadian et al. (2010)	Demand Costs Production capacity Production time	1. Fuzzy OF: Possibilistic Linear Programming (PLP) 2.Fuzzy OF: weighted sum.memberships 3. Fuzzy constraints: weighted average method, fuzzy ranking, and transformation of fuzzy constraints using minimal acceptance level of satisfaction	x	x	1
Baykasoglu and Gocken (2010)	Costs Profits Production capacity -Minimum acceptable capacity utilization rate	Fuzzy ranking	x	x	4
Mula, Peidro et al. (2010)	Demand	weighted average method		x	1
Phruksaphanrat, Ohsato et al. (2011)	Demand	Substitution of fuzzy demand in the constraints using different possibilistic formulations		x	1

Paper	Fuzzy parameters	Fuzzy technique	Fuzzy Obj?	Fuzzy constraints?	Number of Objs
Liang, Cheng et al. (2011)	Cost Resource capacity	1. Weighted average method 2. Possibilistic Linear Programming (PLP) 3. Max-Min 4. Weighted Sum.Memberships	x	x	1
Liang (2011)	Costs Demand	1. Weighted average method 2. Possibilistic Linear Programming (PLP) 3. Max-Min	x	x	1
Azadegan, Porobic et al. (2011)	Production capacity Production time	1.Weighted average method 2. Ranking method		x	1
Yaghin, Torabi et al. (2012)	Costs Inventory space Resource level Subcontracting volume available	1.Fuzzy Constraints : Weighted method using DM degree of optimism 2. Fuzzy OF: Sum.Memberships	x	x	3
Sadeghi, Hajiagha et al. (2013)	All model parameters	Sum.Memberships	x		3
Mortezaei, Zulkifli et al. (2013)	Resource level Costs Technical coefficients	1. weighted average method 2. Max-Min	x	x	2
Mekidiche, Belmokaddem et al. (2013)	Goals	Weighted Sum.violation fraction levels	x		4
da Silva and Marins (2014)	Goals	Max-Min	x		9
Gholamian, Mahdavi et al. (2015)	Costs Sales price Demand Failure rate	1. Fuzzy Constraints: Fuzzy ranking 2. Fuzzy OF: Possibilistic Linear Programming (PLP) 4. Fuzzy OF: Weighted Sum.Memberships	x	x	4

We will later utilize two of the popular fuzzy techniques, which are: Max-Min (As the base technique for other methods such as PLP and Weighted-Sum methods) and also the Fuzzy ranking method which gives the flexibility to the DM to analyze different ranking degrees and their related solution and also as a technique which has not been widely used in the aggregate production and supply chain planning literature while we believe it has good practical potentials from decision making perspective specially in case of having inequality fuzzy constraints. In addition, to the best of our knowledge, no Fuzzy FRP-APP model has been developed before in the literature of aggregate production planning and as a result, the developed models will be novel ones as they take into account both fuzziness and stability considerations in the production planning optimization process.

2.7 Stochastic Programming Models in Production and Supply Chain Planning Problems

This section includes the most popular stochastic programming techniques utilized in the literature of aggregate production and supply chain planning to later utilize in this research. In stochastic programming, two main different methodologies could be utilized for uncertainty representation: scenario-based approaches and distribution-based approaches. In the former approach, a set of discrete scenarios represent how the future uncertainties are forecasted. Each scenario is associated with a probability value, which is the DM's expectation for the occurrence of that specific scenario. The main advantage of this method is the fact that there are no limitations on the number of uncertain parameters, while the main challenge would be anticipating all possible consequences. The latter (like chance-constraint programming) on the other hand, is utilized when only a continuous range of potential future outcomes can be anticipated. The advantage of this method is that by assigning a probability distribution function to the continuous range of possible consequences, the need for forecasting exact scenarios and their probabilities is eliminated. On the other side, the complexity of applying distribution function limits the number of considered uncertain parameters (Mirzapour Al-E-Hashem, Malekly et al. 2011).

2.7.1 Chance-Constraint Programming

The general idea behind chance-constraint programming is meeting one or some specific constraints with at least a probability value because of having uncertainties mainly in their right-hand side. In the field of production and supply chain planning for majority of stochastic cases, the demand is the main source of uncertainty and as a result, the inventory constraint is represented in form of a chance constraint. First application of chance-constraint programming in aggregate production planning can be found in Filho (1999) where the demand and cost parameters are uncertain model parameters. The inventory balance constraint is represented by a chance constraint while the uncertainty in the objective function is represented through the expected value of the inventory cost. The model is applied on a two-product case study where the different customer service level coefficients are considered for sensitivity analysis. The results show a higher responsiveness to customer service level results in higher production cost values. The stochastic multi-period and stochastic supply chain production planning and sourcing problem addressed in Yıldırım, Tan et al. (2005) deals with the randomness in demand and related probabilistic service level constraints. In addition, the cost function is represented in form of an expected inventory holding and production cost function in the planning horizon. As the demand in the inventory constraint makes it an uncertain chance constraint, a customer satisfaction approach is utilized in which the inventory chance constraint is turned into a chance constraint which aims to achieve to a positive net inventory (no shortage).

In addition to the inventory chance-constraints resulting from uncertain demand, the uncertainty in machine breakdowns could also be the source for a chance-constraint production planning as in Nourelfath (2011). The mentioned research assumes the production rate and the customer service level could be random variables resulting from machine breakdowns, and by introducing different service level measures, presents different constraints for the relation between the chance-constraints related to each service level measure. Another example can be found in Borodin, Dolgui et al. (2016) where a component replenishment planning problem for a single-level assembly system with random lead times is modeled in form of a chance-constraint programming problem where the chance-constraint is on the shortage delay for all components. The green supply chain network design problem in Shaw, Irfan et al. (2016) is modeled in form of a chance-constraint problem under uncertainties of supplier, plant and warehouses capacities and the customer demand. All chance-constraints are turned into deterministic equivalents using the cumulative distribution function and standard deviation of each probabilistic parameter under different chance constraint aspiration levels.

2.7.2 Scenario-based Programming

While the chance-constraint programming method is mainly considered as a distribution-based stochastic programming technique where distributional parameters and functions like: mean, standard deviation, and cumulative distributions are used to transform the stochastic model into a crisp equivalent, there are other stochastic techniques which are based on a scenario-based representation of the uncertain parameters into the optimization formulation. One major group of scenario-based formulations are called: stage-based programming formulations, while there exist other scenario-based formulations like: joint robust-stochastic formulation.

2.7.2.1 Stage-based Programming Approaches

The stage-based stochastic formulations are among the popular techniques in the stochastic programming literature which basically divide the decision-making process about the current and future decision variables with respect to the time when more visibility about the uncertainties become available. In the two-stage programming formulation, a set of variables are called: first-stage variables (or the set-up variables that are typically 0-1), can be determined “here and now” before uncertainties are determined or even estimated, while the second stage variables or recourse variables (operational scenario-based variables) are the decision variables which will be determined after estimations about future uncertainties becomes available. Examples of first stage variables in the supply chain planning and design optimization problems include: production decision of a specific product, ordering from suppliers decision, establishment of a production plant, procuring machines; second stage variables could be: volume of produced products, stock volume of products, volume of transported products. There are different studies used the two-stage scenario-based formulation to deal with stochastic uncertainties in the supply chain planning problems: (Alonso-Ayuso, Escudero et al. 2003, Santoso, Ahmed et al. 2005, Mirzapour Al-e-Hashem, Baboli et al. 2013, Shapiro, Dentcheva et al. 2014, Osmani and Zhang 2017). In addition to the two-stage scenario-based formulations, multi-stage programming assumes random processes for model uncertainties and it would not be reasonable to plan for the entire planning horizon, instead, one has to make decisions at successive stages depending on the information available at the current stage. As a result, in a multi-stage decision problem, it is crucial to specify which decision variables depend on which part of the past information (Shapiro, Dentcheva et al. 2014). This characteristic makes the multi-stage decision making a dynamic decision-making process. Examples of multi-stage and also the dynamic programming formulations in the stochastic supply chain and production planning problems can be found in studies like: Escudero, Kamesam et al. (1993), Escudero and Kamesam (1995), Kazemi Zanjani, Nourelfath et al. (2010), Wu, Huang et al. (2015), Fleming, Sethi et al. (1987), Li, Liu et al. (2009).

2.7.2.2 Robust-Stochastic Programming Approaches

It is worth noting that the stage-based stochastic programming approaches mainly focus on optimizing the expected performance of the model over a range of possible scenarios of the random parameters. As a result, the model would optimally perform for the mean sense, and possibly poorly at realization of scenarios like worst case (Mirzapour Al-E-Hashem, Malekly et al. 2011).

One alternative approach would be the scenario-based robust-stochastic formulation which aims to find a solution which is "close" to the optimum and "almost" feasible in response to the changing input data. In other words, this robust-stochastic optimization model is a special type of stochastic nonlinear model which could handle both the problem cost and its variability over various scenarios. Mulvey, Vanderbei et al. (1995) initially proposed the concept of stochastic and later, Leung* and Wu (2004) presented the robust optimization model for the stochastic aggregate production planning problem. In another study Leung, Lai et al. (2007), developed a robust-stochastic model for a multi-site aggregate production planning problem under uncertainty. The application of robust-stochastic programming is continued in other studies as well, where the robust optimization is mainly dealing with the expected total cost and the cost variability due to the demand uncertainty and also the expected penalty for any model related infeasibility related to some scenarios (Pan and Nagi 2010, Zanjani, Ait-Kadi et al. 2010, Mirzapour Al-E-Hashem, Malekly et al. 2011).

As a summary of the reviewed stochastic production and supply chain planning literature, different stochastic parameters, number of objectives and the stochastic programming techniques utilized, Table 2.2 includes the related information of the reviewed articles.

Table 2.2: Literature on application of stochastic programming in production planning

Paper	Stochastic parameters	Stochastic technique	Stochastic Obj?	Stochastic constraints?	Number of Objs
Rakes, Franz et al. (1984)	Constraints right and left hand side coefficients	Chance-constraint		x	1
Fleming, Sethi et al. (1987)	Demand	Dynamic Programming		x	1
Escudero, Kamesam et al. (1993)	Demand Cost	Multi-stage programming	x	x	1
Escudero and Kamesam (1995)	Demand Capacity constraints Dual sourcing	Multi-stage programming	x	x	1

Paper	Stochastic parameters	Stochastic technique	Stochastic Obj?	Stochastic constraints?	Number of Objs
Mulvey, Vanderbei et al. (1995)	General	Robust-Stochastic	x	x	1
Filho (1999)	Demand Inventory Cost	Chance-constraint	x	x	1
Alonso-Ayuso, Escudero et al. (2003)	Deman Price	Two-stage programming	x	x	1
Leung* and Wu (2004)	Demand Inventory cost Labor costs	Robust-Stochastic	x	x	1
Yıldırım, Tan et al. (2005)	Demand	Chance-constraint	x	x	1
Santoso, Ahmed et al. (2005)	Demand Cost	Two-stage programming	x	x	1
Leung, Lai et al. (2007)	Demand Initial inventories Inventory costs Overtime cost Production cost	Robust-Stochastic	x	x	1
Fleten and Kristoffersen (2008)	Prices Reservoir inflows	Multi-stage programming	x	x	1
Li, Liu et al. (2009)	Demand Return amount	Dynamic Programming	x	x	1
Pan and Nagi (2010)	Demand	Robust-Stochastic	x	x	1
Zanjani, Ait-Kadi et al. (2010)	Production capacity	Robust-Stochastic	x	x	1
Kazemi Zanjani, Nourelfath et al. (2010)	Demand Quality of raw material	Multi-stage programming	x	x	1
Mirzapour Al-E-Hashem, Malekly et al. (2011)	Demand Sale price Labor costs Inventory cost Shortage cost Raw material cost Transportation cost Production cost	Robust-Stochastic	x	x	2
Nourelfath (2011)	Machine breakdown	Chance-constraint	x	x	1

Paper	Stochastic parameters	Stochastic technique	Stochastic Obj?	Stochastic constraints?	Number of Objs
Bilsel and Ravindran (2011)	Demand Suppliers' capacities Costs	Chance-constraint	x	x	3
Awudu and Zhang (2013)	Demand Price	Two-stage programming	x	x	1
Mirzapour Al-e-Hashem, Baboli et al. (2013)	Demand	Two-stage programming	x	x	1
Bakhrankova, Midthun et al. (2014)	Raw material quantities finished goods market prices	Expected OF	x	x	1
Wu, Huang et al. (2015)	Cost Waste generation rate	Multi-stage programming Chance-constraint	x	x	1
Borodin, Dolgui et al. (2016)	component procurement lead times	Chance-constraint		x	1
Shaw, Irfan et al. (2016)	Demand Suppliers' capacities Plants' capacities Warehouses' capacities	Chance-constraint		x	1
Moshtagh and Taleizadeh (2017)	Return rate of products Quality of returned material Buyback cost Remanufacturing cost Salvage value	Chance-constraint	x	x	1
Osmani and Zhang (2017)	Bioethanol demand Bioenergy sale price Switchgrass yield	Two-stage programming	x	x	3

We will later use two stochastic programming techniques to develop the stochastic version of the FRP-APP model to see how the incorporation of the stochastic uncertainty would affect its performance. These two techniques are: Chance-Constraint and Robust-Stochastic programming. To the best of our knowledge, no chance-constraint or scenario-based FRP-APP model has been developed before in the literature of aggregate production planning and as a result, the developed models will be novel as they take into account both stochastic uncertainty and stability considerations into the production planning process. Furthermore, as the uncertainty consideration and the mathematical approach in these two techniques are different, we believe these two formulations can make our analysis and conclusions more varied.

2.8 Conclusions

In this chapter, the related literature to our research is reviewed from different aspects. The reviewed literature provides valuable insights about the available techniques and current practices in the APP area. Here are the main conclusions based on the literature review:

- While the FRP method has been (relatively) recently introduced to address APP instability issues, the literature on this technique's performance and its potential benefits is relatively scarce. Furthermore, comparison of this planning approach to the Stochastic/Fuzzy APP models (which are meant to handle planning uncertainties) do not exist. Hence to address these research gaps, 1. We will compare the FRP-APP model to the Stochastic/Fuzzy models and 2. We will conduct more testing of these models under various Industrial scenarios.
- To the best of our knowledge, existing Stochastic/Fuzzy Single/Multi-Objective rolling horizon APP models do not incorporate any stability improvement methods. While in the Stochastic/Fuzzy APP literature, the emphasis has been primarily on handling the randomness/fuzziness in the planning problem, the rolling horizon planning literature has emphasis on the need for stability improvement of the rolling plans. The connection between the Stochastic/Fuzzy APP and rolling horizon planning in terms of stability management seems to be missing. Hence, to close this missing link, we would like to introduce new Stochastic/Fuzzy FRP/Bi-Objective APP models to take advantage of

the strengths of both Stochastic/Fuzzy mathematical programming and FRP/Bi-Objective planning to create more stable and cost-efficient plans. For this purpose, we will propose eight new Stochastic/Fuzzy Bi-Objective/FRP APP models in this research to address these research gaps. More specifically we will propose four new Stochastic APP models and four new Fuzzy APP models as follows: 1. Stochastic CC-FRP-APP, 2. Stochastic RS-FRP-APP, 3. Stochastic CC-BO-APP, 4. Stochastic RS-BO-APP, 5. Fuzzy MM-FRP-APP, 6. Fuzzy R-FRP-APP, 7. Fuzzy MM-BO-APP, and 8. Fuzzy R-BO-APP.

Chapter 3: AGGREGATE PLANNING WITH AND WITHOUT FLEXIBILITY REQUIREMENTS PROFILE

In this chapter we first present the deterministic APP formulation (Missbauer and Uzsoy 2011, Demirel 2014), which is a basis for development of all production planning models developed in this research. Then we will introduce the FRP counterpart of the deterministic APP model referred here as the FRP-APP.

3.1 APP Mixed Integer Linear Formulation

The APP optimization model aims to minimize the total planning cost over a planning horizon. The cost components are: production cost, workforce regular time cost, workforce overtime cost, workforce hiring cost, workforce layoff cost, inventory holding cost and backorder cost. The major constraints are production capacity, workforce level change constraint, and the typical inventory constraint. In this research, we assume a single product planning problem while the presented methods could also be extended to the multi-product case without loss of generality.

The basic model is similar to the APP model utilized by Demirel, Özelkan et al. (2018). Let's start with the introduction of model parameters and decision variable:

Indices:

i: index for planning horizon, $i = 0, \dots, N$

Parameters:

N: Total number of periods in the planning horizon

c^W: labor cost of a worker per hour

c^O: overtime labor cost of a worker per hour

c^H: hiring cost of a worker

c^L: layoff cost of a worker

c^p: unit production cost

h: unit inventory holding cost

b: unit backorder cost

th: total number of working hours per period

m^R: maximum number of units produced per worker per hour

m^O: fraction of total regular worker hours in each period available for overtime

$d_{t,i}$: i-step ahead demand estimated in planning iteration t

I: Initial inventory

W: Initial workforce

Variables:

P_i: i-step ahead planned production level planned

O_i: i-step ahead planned overtime production worker hours

I_i: i-step ahead planned inventory level

B_i: i-step ahead planned backorder level

W_i: i-step ahead planned workforce level

H_i: i-step ahead planned hiring level

L_i: i-step ahead planned layoff level

Note: For all i related parameters and variables, i=0 represents the current and actual values, while i >0 are corresponding to the future periods.

The corresponding MILP formulation is as follows:

Minimize $\sum_{i=0}^{N}(c^w.th.W_i + c^o.O_i + c^H.H_i + c^L.L_i + c^p.P_i + h.I_i + b.B_i)$ (3.1)

Subject to

Initial Inventory: $P_0 = d_0 + I_0 - B_0 - I$ (3.2)

Inventory: $P_i = d_i + I_i - B_i - I_{i-1} + B_{i-1}$ $\forall i = 1, \dots, N$ (3.3)

End Inventory: $I_N \geq I$ (3.4)

Initial Workforce: $W_0 = H_0 - L_0 + W$ (3.5)

Workforce: $W_i = W_{i-1} + H_i - L_i$ $\forall i = 1, \dots, N$ (3.6)

Production Capacity: $P_i \leq m^R.th.W_i + m^R.O_i$ $\forall i = 0, \dots, N$ (3.7)

Overtime Capacity: $O_i \leq th.W_i.m^O$ $\forall i = 0, \dots, N$ (3.8)

$W_i, O_i, H_i, L_i, P_i, I_i, B_i \geq 0$ $\forall i = 0, \dots, N$ (3.9)

W_i, H_i, L_i: integers $\forall i = 0, \dots, N$ (3.10)

As indicated earlier, the objective function aims to minimize the planning cost including workforce, production, inventory and shortage costs over the next N periods. Constraints (3.2) & (3.3) are the inventory balance constraints stating that the total production in each period equals the realized demand for that period plus the net inventory at the end of that period minus the net in hand inventory at the beginning. In addition, Constraint (3.4) makes sure the end inventory is at least equal to the starting inventory. Constraints (3.5) & (3.6) are the workforce balance constraints which make sure the workforce level in each period equals the available workforce at the beginning of that period plus the net changes in the workforce level decided for the same period. Constraints (3.7) & (3.8) are the capacity constraints. Non-negativity of all variables is represented in Constraint (3.9), while Constraint (3.10) enforces the integer values for the workforce related variables.

3.2 Rolling Horizon Planning

In rolling horizon planning, the planning is made iteratively in each period. During each iteration, the initial conditions for inventories and backlogs as well as demand forecasts are updated based on the current actual values, and production plans are generated for future periods. Hence, rolling horizon planning dynamically reflects new information to provide updated plans.

Below, we will first define the notation and then present the rolling horizon-based APP model. For this purpose, we slightly modify the parameters and variables previously defined in to reflect the rolling horizon iterations using index t as follows:

Indices:

i: index for planning horizon, $i = 0, \dots, N$

t: index for rolling horizon, $t = 1, \dots, T$

Parameters:

N: Total number of periods in the planning horizon

c^{w}: labor cost of a worker per hour

c^{o}: overtime labor cost of a worker per hour

c^{H}: hiring cost of a worker

c^{L}: layoff cost of a worker

c^{p}: unit production cost

h: unit inventory holding cost

b: unit backorder cost

th: total number of working hours per period

m^{R}: maximum number of units produced per worker per hour

m^{O}: fraction of total regular worker hours in each period available for overtime

$d_{t,i}$: i-step ahead demand estimated in planning iteration t

I: Initial inventory at the start of planning iteration 1

Variables:

$P_{t,i}$: i-step ahead production level planned in planning iteration t

$O_{t,i}$: i-step ahead overtime worker hours planned in planning iteration t

$I_{t,i}$: i-step ahead inventory level planned in planning iteration t

$B_{t,i}$: i-step ahead backorder level planned in planning iteration t

$W_{t,i}$: i-step ahead workforce level planned in planning iteration t

$H_{t,i}$: i-step ahead hiring level planned in planning iteration t

$L_{t,i}$: i-step ahead layoff level planned in planning iteration t

For each rolling horizon period t, the corresponding APP model MILP formulation is as follows:

(APP)

Minimize $\sum_{i=0}^{N}(c^{w}.th.W_{t,i} + c^{o}.O_{t,i} + c^{H}.H_{t,i} + c^{L}.L_{t,i} + c^{p}.P_{t,i} + h.I_{t,i} + b.B_{t,i})$ (3.11)

Subject to

Initial Inventory: $P_{t,0} = d_{t,0} + I_{t,0} - B_{t,0} - I_{t-1,0} + B_{t-1,0}$ (3.12)

Inventory: $P_{t,i} = d_{t,i} + I_{t,i} - B_{t,i} - I_{t,i-1} + B_{t,i-1}$ $\forall i = 1, \dots, N$ (3.13)

End Inventory: $I_{t,N} \geq I$ (3.14)

Initial Workforce: $W_{t,0} = W_{t-1,0} + H_{t,0} - L_{t,0}$ (3.16)

Workforce: $W_{t,i} = W_{t,i-1} + H_{t,i} - L_{t,i}$ $\forall i = 1, \dots, N$ (3.17)

Production Capacity: $P_{t,i} \leq m^{R}.th.W_{t,i} + m^{R}.O_{t,i}$ $\forall i = 0, \dots, N$ (3.18)

Overtime Capacity: $O_{t,i} \leq th.W_{t,i}.m^{O}$ $\forall i = 0, \dots, N$ (3.19)

$W_{t,i}, O_{t,i}, H_{t,i}, L_{t,i}, P_{t,i}, I_{t,i}, B_{t,i} \geq 0 \qquad \forall i = 0, \dots, N \qquad (3.19)$

$W_{t,i}, H_{t,i}, L_{t,i}$: integers $\qquad \forall i = 0, \dots, N \qquad (3.20)$

The APP model above has the same structure as the model in (3-1)-(3-10) but it has a more dynamic nature: Constraint (3.12) is the initial inventory balance constraint for the current period, in which the production plan corresponds to the actual demand plus the net inventory of the current period minus the net actual inventory resulting from the previous planning horizon. Constraint (3-13) follows the same logic for the determination of the production values for future periods except the fact that the demand values are forecasted for the future periods, and the input inventory is resulting from the optimal inventory levels from the previous planning periods. In addition, Constraints (3.16) and (3.17) are the workforce balance constraints, where the initial workforce for the current period in constraint (3.16) is coming from the actual workforce estimated in the previous planning iteration. In addition, the initial workforce in constraint (3.17) is resulting from the optimal workforce level in the previous planning period.

The issue arising here is the previously mentioned "nervousness" i.e. how to make sure the developed plans are stable/reliable from one rolling horizon planning period to another. This issue will be addressed by introducing the FRP concept into the APP model as discussed next.

3.3 FRP-APP: Flexibility Requirements Profile-based Aggregate Production Planning with Rolling Horizon

The FRP planning imposes flexible fences to the optimization model to maintain the production plans within certain levels (upper and lower bounds). The calculation of bounds is done with the utilization of the previously estimated production levels and flexibility bounds coefficients called "flex-limits". Let $\pm F_i, i = 0,1, \dots N$ represent "flex-limits" over the planning horizon N such that: $F_0 \leq F_1 \leq F_2 \leq \cdots \leq F_N$, which implies less flexibility to the plan changes for near future periods as they are getting closer to the current time. As it will be shown in Propositions 1 and 2 subsequently, the FRP limits will ensure that the deviation in the dynamic planning process stays within the specified ranges while the plan rolls, and the amount of flexibility that is permitted will be higher in distant periods due to the higher degrees of uncertainty. Figure 3.1 illustrates the three incremental levels for the flex-limits values (1%, 3%, 5%). The 5% case results in higher flexibility with less smoothing effect on production levels, while the 1% flex-limits result in less variability in production levels due to tighter bounds.

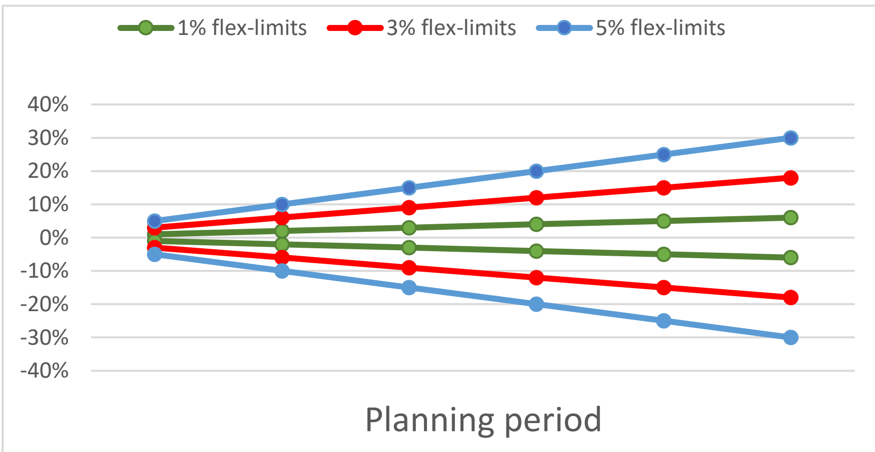

Figure 3.1: Flexibility of different flex-limits bounds on variation of production plans

In order to define and incorporate these FRP related bounds, we will introduce the following notation:

$LB_{t,i}$: i-step ahead lower bound on planned production calculated for planning iteration t

$UB_{t,i}$: i-step ahead upper bound on planned production calculated for planning iteration t

These bounds are updated at the end of each planning iteration $(t-1)$ using the optimal production levels for that period, as well as flexibility limits coefficients, and previous bounds as follows (Demirel 2014):

$$Lower\ Bounds: LB_{t,i} = max(LB_{t-1,i+1}, P_{t-1,i+1}(1 - F_i)) \qquad \forall i = 0, \dots, N-1 \quad (3.21)$$

$$Upper\ Bounds: UB_{t,i} = \min\ (UB_{t-1,i+1}, P_{t-1,i+1}(1 + F_i)) \qquad \forall i = 0, \dots, N-1 \quad (3.22)$$

In the above formulations we set $LB_{t,N} = -\infty$ and $UB_{t,N} = +\infty$. Once these bounds are updated, they are included in the next rolling horizon iteration planning problem as constraints on the production levels as follows:

$$LB_{t,i} \leq P_{t,i} \leq UB_{t,i} \qquad \forall i = 0, \dots, N \qquad (3.23)$$

Hence the FRP-APP Model can be defined as follows:

(FRP-APP) = (APP) + Constraints (3.21)-(3.23)

Considering formulas (3.21) and (3.22), we can show that for two consecutive iterations, the gap between the upper and lower bounds for a specific period gets tighter. Figure 3.2 illustrates how the bounds get tighter for a specific period as time rolls to the next planning iteration where each specific period gets closer to the current period. The arrows in Figure 3.2 show that a future period in rolling iteration t-1 becomes closer in rolling iteration t. For example, a plan for 4 periods ahead or for period 4 during rolling horizon planning iteration 1, becomes 3 periods ahead or moves to period 3 during the second rolling planning iteration since current time rolls to the next period. This is formally stated and proven in Proposition 1 below.

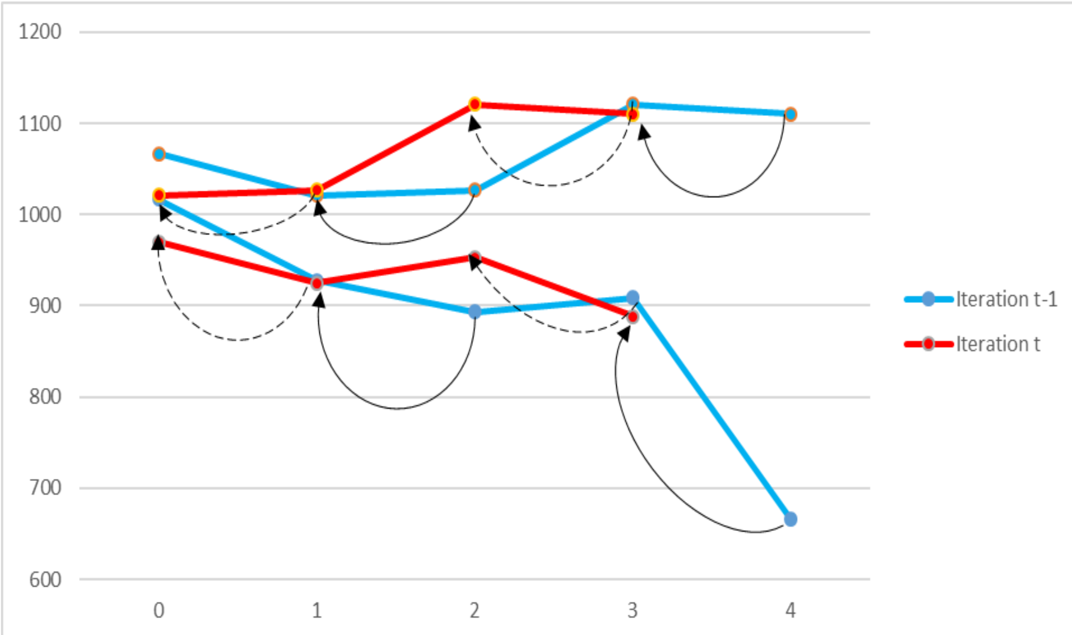

Figure 3.2: Demonstration of flexibility bounds for specific periods during consecutive planning iterations

Proposition 1. The flexible bounds for each specific period get tighter or stay the same as time rolls to the next planning horizon (Demirel, Özelkan et al. 2018).

Proof. Taking into account formula (3.32) we have:

$$LB_{t+1,i} \geq LB_{t,i+1} \qquad \forall i = 0, \dots, N-1 \qquad (3.24)$$

Which is equal to

$$-LB_{t+1,i} \leq -LB_{t,i+1} \qquad \forall i = 0, \dots, N-1 \qquad (3.25)$$

As in formula (3.33) we also have:

$$UB_{t+1,i} \leq UB_{t,i+1} \qquad \forall i = 0, \dots, N-1 \qquad (3.26)$$

Adding the two inequalities (3.35 & 3.36) results in:

$$UB_{t+1,i} - LB_{t+1,i} \leq UB_{t,i+1} - LB_{t,i+1} \qquad \forall i = 0, \dots, N-1 \qquad (3.27)$$

Which results in tighter or equal bounds for two consecutive planning horizons for each period □.

Proposition 2. The production lower bound is always less than or equal the upper bound.

Proof. Let's assume the initial upper & lower bounds (for the first iteration of the plan) are set equal to -∞ & ∞ (meaning the initial plan is developed using the regular APP optimization formulation). As a result, the calculated bounds at the end of iteration 1 to be fed to the second iteration are calculated as the following formula:

As $(1 - F_i) < (1 + F_i)$, the lower bound is greater than the lower bound.

$$LB_{2,i} = P_{1,i+1}(1 - F_i) \qquad \forall i = 0, \dots, N\text{-}1 \qquad (3.28)$$

$$UB_{2,i} = P_{1,i+1}(1 + F_i) \qquad \forall i = 0, \dots, N\text{-}1 \qquad (3.29)$$

Now we have to move on to the next iteration and make sure iteration 3 calculated lower bounds are less than their related upper bounds.

$$LB_{3,i} = \max\left(LB_{2,i+1}, P_{2,i+1} * (1 - F_i)\right) \qquad \forall i = 0, \dots, N - 1 \qquad (3.30)$$

$$UB_{3,i} = \min\left(UB_{2,i+1}, P_{2,i+1} * (1 + F_i)\right) \qquad \forall i = 0, \dots, N - 1 \qquad (3.31)$$

We have:

$$LB_{2,i+1} \leq P_{2,i+1} \leq UB_{2,i+1} \qquad \forall i = 0, \dots, N - 1 \qquad (3.32)$$

As a result:

$$P_{2,i+1} * (1 - F_i) \leq UB_{2,i+1} \qquad \forall i = 0, \dots, N - 1 \qquad (3.33)$$

$$P_{2,i+1} * (1 + F_i) \geq LB_{2,i+1} \qquad \forall i = 0, \dots, N - 1 \qquad (3.34)$$

In addition:

$$P_{2,i+1} * (1 - F_i) \leq P_{2,i+1} * (1 + F_i) \qquad \forall i = 0, \dots, N - 1 \qquad (3.35)$$

So we can make sure:

$$\max\left(LB_{2,i+1}, P_{2,i+1} * (1 - F_i)\right) \leq UB_{2,i+1} \qquad \forall i = 0, \dots, N - 1 \qquad (3.36)$$

And

$$\max\left(LB_{2,i+1}, P_{2,i+1} * (1 - F_i)\right) \leq P_{2,i+1} * (1 + F_i) \qquad \forall i = 0, \dots, N-1 \qquad (3.37)$$

Which is equivalent to:

$$\max\left(LB_{2,i+1}, P_{2,i+1} * (1 - F_i)\right) \leq \min\left(UB_{2,i+1}, P_{2,i+1} * (1 + F_i)\right) \qquad \forall i = 0, \dots, N-1 \qquad (3.38)$$

And

$$LB_{3,i} \leq UB_{3,i} \qquad \forall i = 0, \dots, N-1 \qquad (3.39)$$

This means our proposition is true for the third planning horizon (iteration). We now continue our proof by assuming the proof holds for other planning horizons up to $t = m$, if we can generalize the conclusion to planning horizons $t = m + 1$ we can conclude the proposition holds for all planning horizon. For planning horizon $t = m + 1$ we have:

$$LB_{\mathrm{m}+1,i} = \max\left(LB_{\mathrm{m},i+1}, P_{\mathrm{m},i+1} * (1 - F_i)\right) \qquad \forall i = 0, \dots, N-1 \qquad (3.40)$$

$$UB_{\mathrm{m}+1,i} = \min\left(UB_{\mathrm{m},i+1}, P_{\mathrm{m},i+1} * (1 + F_i)\right) \qquad \forall i = 0, \dots, N-1 \qquad (3.41)$$

Having in mind (3.38), we have:

$$P_{\mathrm{m},i+1} * (1 - F_i) \leq UB_{\mathrm{m},i+1} \qquad \forall i = 0, \dots, N-1 \qquad (3.42)$$

$$P_{\mathrm{m},i+1} * (1 + F_i) \geq LB_{\mathrm{m},i+1} \qquad \forall i = 0, \dots, N-1 \qquad (3.43)$$

In addition:

$$P_{\mathrm{m},i+1} * (1 - F_i) \leq P_{\mathrm{m},i+1} * (1 + F_i) \qquad \forall i = 0, \dots, N-1 \qquad (3.44)$$

$$\max\left(LB_{\mathrm{m},i+1}, P_{\mathrm{m},i+1} * (1 - F_i)\right) \leq UB_{\mathrm{m},i+1} \qquad \forall i = 0, \dots, N-1 \qquad (3.45)$$

And

$$\max\left(LB_{\mathrm{m},i+1}, P_{\mathrm{m},i+1} * (1 - F_i)\right) \leq P_{\mathrm{m},i+1} * (1 + F_i) \qquad \forall i = 0, \dots, N-1 \qquad (3.46)$$

Which results in:

$$\max\left(LB_{\mathrm{m},i+1}, P_{\mathrm{m},i+1} * (1 - F_i)\right) \leq \min\left(UB_{\mathrm{m},i+1}, P_{\mathrm{m},i+1} * (1 + F_i)\right) \qquad \forall i = 0, \dots, N-1 \qquad (3.47)$$

And

$$LB_{m+1,i} \leq UB_{m+1,i} \qquad \forall i = 0, \dots, N-1 \qquad (3.48)$$

The proof is complete and true for each planning horizon□.

3.4 Overall Planning Procedure using FRP-APP

Before concluding this chapter, we would like to summarize the main procedural steps for running the FRP-APP model on a rolling horizon basis.

Step 1: Generate (forecast) demand for $i = 0, \dots, N$

Step 2: Initialize workforce levels, inventories and FRP bounds

Step 3: Solve FRP-APP Model for planning iteration t and develop the optimal plan for the planning horizon for $i = 0, \dots, N$

Step 4: Update flexibility bounds and if needed, demand forecasts, and then roll into the next planning iteration $(t + 1)$

Step 5: Repeat steps 3 and 4 for each t, $t = 1, \dots, T$

Please note that, in essence the above procedure would be the same for APP Model under rolling horizon except for the FRP related steps (Steps 2 and 4).

3.5 Conclusions

In this chapter, we presented the main formulation for the APP and FRP-APP models along with the definition of the flexibility limits and bounds. In addition, both APP and FRP-APP models are modified to apply a rolling horizon planning framework. These formulations will be the basis for the stochastic and fuzzy models that will be developed in subsequent chapters. In addition, the overall rolling horizon procedure presented in this chapter will be later used to run our numerical analysis in Chapters 4, 5, and 6.

Chapter 4: FUZZY AGGREGATE PLANNING WITH FLEXIBLE REQUIREMENTS PROFILE

4.1 Introduction

In this chapter, after reviewing the basics of fuzzy logic and fuzzy programming, we will first present the Fuzzy APP model without the incorporation of FRP, namely the MM-APP and R-APP models. Then, we will develop the two new FRP counterparts, namely the MM-FRP-APP and R- FRP-APP. We use the Max-Min technique due to its popularity and widespread use in the related fuzzy programming literature, and also use the Fuzzy Ranking method, which provides an interactive decision-making tool to the planner. The FRP-APP and the proposed new Fuzzy FRP-APP models will then be compared to existing Fuzzy APP models based on cost and stability using five industry-based case studies under various demand scenarios and flex-limits. Additional industry scenarios will also be presented and analyzed using design of experiments techniques.

4.2 Introduction to Fuzzy Programming

Fuzzy logic provides an alternative approach to represent the uncertainties, especially when enough historical data about uncertainties are not available and when subjective and approximation reasoning are involved to describe the uncertainties and make decisions (Zadeh 1988).

A fuzzy number has a type of imprecision associated with fuzzy sets, which has classes in which, the exists no sharp transition from membership to non-membership (Bellman and Zadeh 1970). As a result, each fuzzy set would be defined using a membership function. As illustrated in Figure 4.1, there exist different ways for defining membership functions (linking each number to its membership value). Two of the commonly used functions are: triangular membership functions (Represented by A is Figure 4.1), and trapezoidal membership functions (Represented by C is

Figure 4.1). There can be other type of functions as well such as: *z*-shape (Represented by B is Figure 4.1), *s*-shape (Represented by D is Figure 1.1), sigmoid (Represented by E is Figure 4.1), and Gaussian (Represented by F is Figure 4.1) (Rajabi, Bohloli et al. 2010).

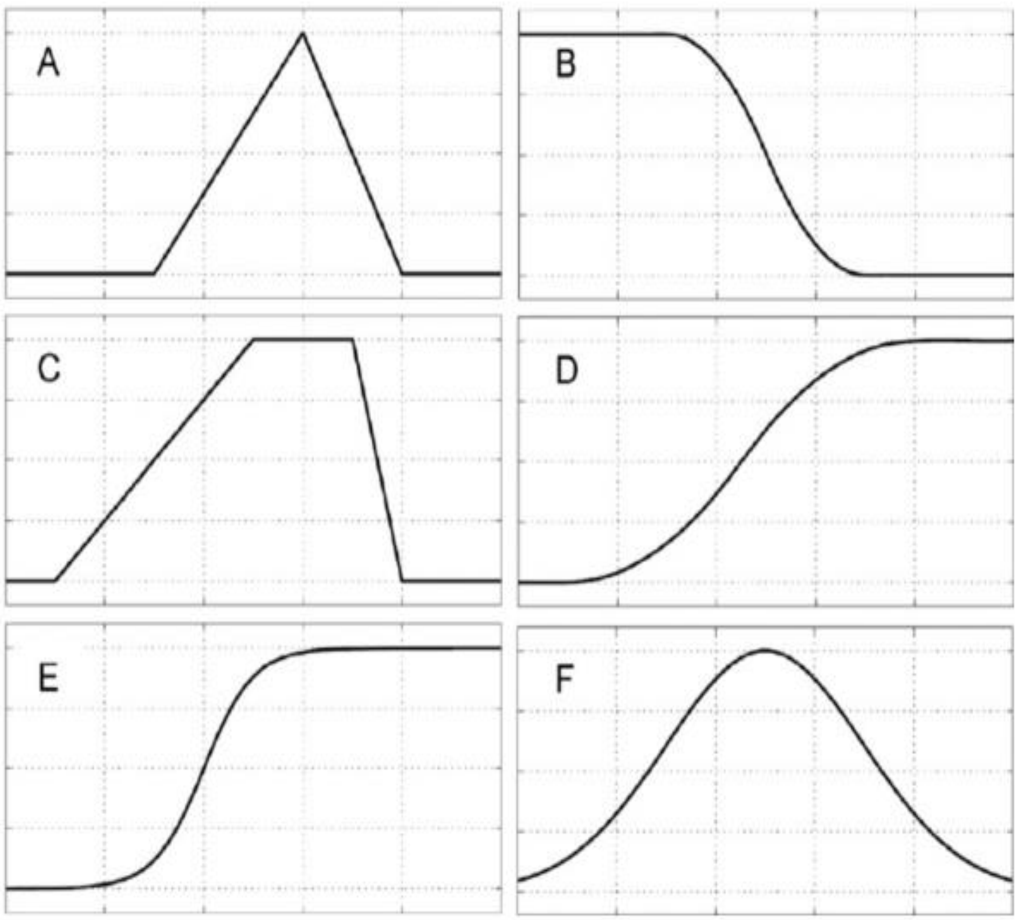

Figure 4.1 Different fuzzy membership function shapes

Due to the popularity and more widespread use (Paksoy, Pehlivan et al. 2010, Azadegan, Porobic et al. 2011, Mortezaei, Zulkifli et al. 2013), we introduce the triangular and trapezoidal membership functions formulations as below:

<u>Triangular membership function:</u>

Let $x \in X$ denote a fuzzy number and μ_X the corresponding membership function, then μ_X depends on three positive scalar parameters a, b, c as follows:

$$\mu(X) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a \le x \le b \\ \frac{c-x}{c-b} & b \le x \le c \\ 0 & c \le x \end{cases} \tag{4.1}$$

<u>Trapezoidal membership function:</u>

In this case μ_X depends on four positive scalar parameters a, b, c, d as follows:

$$\mu(X) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{d-x}{d-c} & c \le x \le d \\ 0 & d \le x \end{cases} \tag{4.2}$$

As a result of fuzzy values in an optimization model, the model is referred as a fuzzy model. We present a fuzzy mathematical formulation to clarify how membership functions for fuzzy objective function and fuzzy constraints in a mathematical optimization formulation can be defined (Tavakkoli-Moghaddam, Rabbani et al. 2007).

$$Z \cong MinZ(x) \tag{4.3}$$

Subject to

$$(Ax)_i \gtrapprox b_i \qquad \forall\, i \tag{4.4}$$

$$(Ax)_j \cong b_j \qquad \forall\, j \tag{4.5}$$

$$x \ge 0 \tag{4.6}$$

In the above formulation, both the objective function coefficients (and as a result, the whole objective value) and the right hand side of the constraints are fuzzy numbers which would turn the objective function and the constraints to the fuzzy formulations with membership functions shown below:

a) Fuzzy objective function membership function (Tavakkoli-Moghaddam, Rabbani et al. 2007):

$$\mu_Z(X) = \begin{cases} 1 & Z(x) \leq Z_l \\ \frac{Z_u - Z(x)}{Z_u - Z_l} & Z_l < Z(x) \leq Z_u \\ 0 & Z(x) \geq Z_u \end{cases} \quad (4.7)$$

In the above formulation, Z_l and Z_u are the upper and lower bounds for the objective function value.

b) Fuzzy constraints membership function (Tavakkoli-Moghaddam, Rabbani et al. 2007):

The first membership function in (4.8) is related to the fuzzy constraint (4.4) and the membership function in (4.9) is related to the fuzzy constraint (4.5). It should be noted that $\Delta_{b_i}$and $\Delta_{b_i}{}'$are the tolerance values for the constraints fuzzy right hand side values. If the tolerance values are equal, the related membership function is symmetric while non-equal values make the membership function asymmetric.

$$\mu_i(X) = \begin{cases} 1 & (AX)_i \geq b_i \\ 1 + \frac{(AX)_i - b_i}{\Delta_{b_j}} & b_i - \Delta_{b_i} < (AX)_i \leq b_i \\ 0 & (AX)_i \leq b_i - \Delta_{b_i} \end{cases} \quad (4.8)$$

$$\mu_j(X) = \begin{cases} 0 & (AX)_j \le b_j - \Delta_{b_j}{}' \\ 1 - \frac{b_j - (AX)_j}{\Delta_{b_j}{}'} & b_i - \Delta_{b_j}{}' < (AX)_j \le b_j \\ 1 - \frac{(AX)_j - b_j}{\Delta_{b_j}} & b_j < (AX)_j \le b_j + \Delta_{b_j} \\ 0 & (AX)_j \ge b_j + \Delta_{b_j} \end{cases} \tag{4.9}$$

Figure 4.2 illustrates the corresponding membership functions of the fuzzy objective function and constraints (Tavakkoli-Moghaddam, Rabbani et al. 2007).

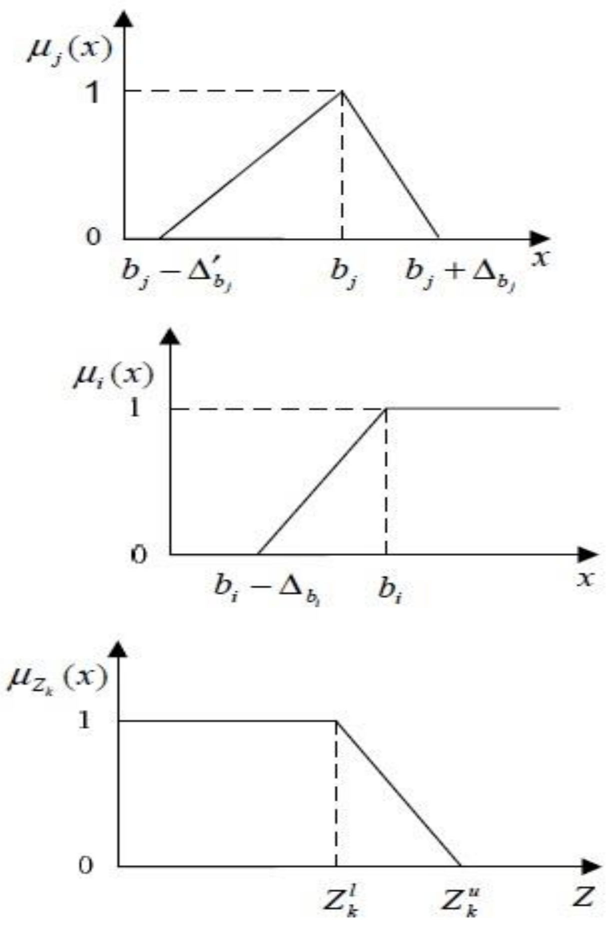

Figure 4.2: Membership functions of the fuzzy objective and fuzzy constraints

In the next sections, we present the Fuzzy Max-Min and the Fuzzy Ranking modeling approaches. For each modeling approach, we present the related Fuzzy APP and also the new related Fuzzy FRP-APP.

4.3 Fuzzy Max-Min Programming

The Max-min technique could be considered as a very flexible fuzzy technique for both single and multi-objective (generally referred to as goal programming) optimization models with or without fuzzy constraints. The reason is that this technique just takes into account the membership functions for all fuzzy parts of the model whether its related to the fuzzy objective or the fuzzy constraint(s). As a result, as long as we are able to come up with fuzzy membership functions for the fuzzy parts of the model, this technique will look for a solution with the emphasis to improve the lowest achievement degree of all fuzzy membership functions.

Figure 4.3 represents how the decision (feasible) region for the optimum solution finding is determined by having maximization-based objective function (goal) and an inequality-based constraints' membership functions. The decision region is the area below the bold curved lines in the figure (Bellman and Zadeh 1970).

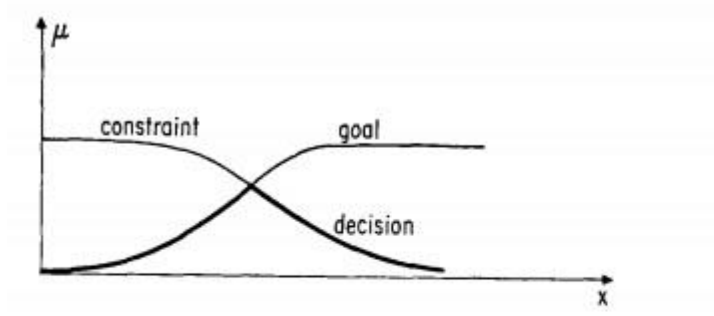

Figure 4.3: Feasible region definition

The below optimization model defined in (4.10)-(4.12) shows a typical fuzzy model where the main sources of fuzziness are related to the objective function (the goal) and the right hand side of the constraints. As a result, both the objective function and the constraints need to be defined by related membership functions.

$$Minimize\ Z \cong Cx \tag{4.10}$$

Subject to

$$Ax \cong \tilde{b} \tag{4.11}$$

$$x \geq 0 \tag{4.12}$$

As we aim to minimize the objective function, the ideal value would be the minimum value; however, if the solution results in any objective value greater than the minimum value, the related membership function would not be equal to 1. In some cases, if the problem is formulated in the form of a goal programming model, the upper and lower values of this membership function would be determined subjectively by the decision maker (Chen and Tsai 2001). However, setting the fuzzy parameters equal to their upper and lower bounds and solving the related optimization model could also result in estimated upper and lower bound values for the objective. Some other techniques include, solving the exact minimization and maximization problems of the same model and form the related membership function (Kumar, Vrat et al. 2006), or just solve the exact minimization problem and consider tolerance values for the objective function to form the membership function. While there are various formulations for membership functions, the most common one is the triangular membership function, which is used here. The formulation of the triangular membership function for a minimization objective function is as follows:

$$\mu_z(x) = \begin{cases} 1 & if\ z(x) \leq z^{min} \\ \frac{[z^{max} - z(x)]}{[z^{max} - z^{min}]} & if\ z^{min} \leq z(x) \leq z^{max} \\ 0 & if\ z(x) \geq z^{max} \end{cases} \tag{4.13}$$

The next step would be the formulation of the membership function for the fuzzy constraints. Let's assume that the left-hand-side of constraint (4.2) for each i is equal to $g(x)_i$ and b_i is its related right-hand-side. The following formulation would be used to form the equality constraint membership function where u_i and v_i are the upper and lower violation levels for the fuzzy right hand side of each fuzzy constraint b_i (Kumar, Vrat et al. 2006):

$$\mu_{ci}(x) = \left\{ \begin{array}{lr} 0 & g(x)_i \leq b_i - v_i \\ \frac{g(x)_i - (b_i - v_i)}{v_i} & b_i - v_i < g(x)_i \leq b_i \\ 1 & g(x)_i = b_i \\ \frac{(b_i + u_i) - g(x)_i}{u_i} & b_i \leq g(x)_i \leq b_i + u_i \\ 0 & b_i + u_i \geq g(x)_i \end{array} \right\} \quad (4.14)$$

In the above formulation, i is an index for the constraint while b_i is the most likely value of the constraints right hand side and v_i is the maximum violation level of the fuzzy right hand side which could be defined using the data related to the historical values of b_i.

Considering all fuzzy constraints and the fuzzy objective function, the solution membership function ($\mu_S(x)$) could be defined as the intersection of all fuzzy membership functions. In the fuzzy logic, the intersection equals to the minimum of two fuzzy values (Zadeh 1965).

$$\mu_S(x) = \mu_z(x) \cap \mu_c(x) = \min\left[\mu_z(x);\ \mu_c(x)\right] \quad (4.15)$$

$$\mu_c(x) = \bigcap_i \mu_{ci}(x) \quad (4.16)$$

The objective becomes to maximize the solution membership function, in order to get the highest degree of membership value, and hence, considering λ, as the variable representing the solution membership function, its optimum value will not be greater than either the objective or the constraints membership functions; we have:

$\lambda \leq [z^{max} - z(x)]/[z^{max} - z^{min}] \rightarrow \lambda[z^{max} - z^{min}] \leq [z^{max} - z(x)]$ (4.17)

$\lambda \leq \left[1 - \frac{\{g(x)_i - b_i\}}{v_i}\right] \rightarrow \lambda.v_i + g(x)_i \leq b_i + v_i$ (4.18)

$\lambda \leq \left[1 - \frac{\{-g(x)_i + b_i\}}{u_i}\right] \rightarrow \lambda.u_i - g(x)_i \leq -b_i + u_i$ (4.19)

As a result, the following formulations can be equivalently used to generate the solution with the highest membership value:

$Maximize\ \lambda$ (4.20)

Subject to

$\lambda[z^{max} - z^{min}] \leq [z^{max} - z(x)]$ (4.21)

$\lambda.v_i + g(x)_i \leq b_i + v_i$ (4.22)

$\lambda.u_i - g(x)_i \leq -b_i + u_i$ (4.23)

$x \geq 0$ (4.24)

4.4 Fuzzy Max-Min APP (MM-APP)

Due to the need for making estimations for future periods' demand in the APP formulation, except the current period demand that we assume to be known, the demand for all other periods ($i \geq 1$) is assumed to be a fuzzy number. In addition, the total planning cost of the APP model in each rolling horizon planning iteration could have desired upper and lower goals (based on the variations in the planning costs expected resulting from the APP and the FRP-APP models) and could be defined by a membership function. As a result, the initial fuzzy formulation of the APP model would be as follows:

Minimize $Z \cong \sum_{i=0}^{N}(c^w.th.W_{t,i} + c^o.O_{t,i} + c^H.H_{t,i} + c^L.L_{t,i} + c^p.P_{t,i} + h.I_{t,i} + b.B_{t,i})$(4.25)

Subject to

$P_{t,0} = d_{t,0} + I_{t,0} - B_{t,0} - I_{t-1,0} + B_{t-1,0}$ (4.26)

$P_{t,i} - I_{t,i} + B_{t,i} + I_{t,i-1} - B_{t,i-1} \cong \tilde{d}_{t,i}$ $\forall i = 1, \dots, N$ (4.27)

$I_{t,N} \geq I$ (4.28)

$W_{t,0} = W_{t-1,0} + H_{t,0} - L_{t,0}$ (4.29)

$W_{t,i} = W_{t,i-1} + H_{t,i} - L_{t,i}$ $\forall i = 1, \dots, N$ (4.30)

$P_{t,i} \leq m^R. th. W_i + m^R. O_i$ $\forall i = 0, \dots, N$ (4.31)

$O_i \leq th. W_{t,i}. m^O$ $\forall i = 0, \dots, N$ (3.32)

$W_{t,i}, O_{t,i}, H_{t,i}, L_{t,i}, P_{t,i}, I_{t,i}, B_{t,i} \geq 0$ $\forall i = 0, \dots, N$ (4.33)

$W_{t,i}, H_{t,i}, L_{t,i}$: integers $\forall i = 0, \dots, N$ (4.34)

Using the Max-Min technique, the resulting MM-APP optimization model is as follows:

(MM-APP)

$Maximize\ \lambda$ (4.35)

Subject to

$\lambda[z^{max} - z^{min}] \leq [z^{max} - z(x)]$ (4.36)

$z(x) = \sum_{i=0}^{N}(c^w. th. W_{t,i} + c^o. O_{t,i} + c^H. H_{t,i} + c^L. L_{t,i} + c^p. P_{t,i} + h. I_{t,i} + b. B_{t,i})$ (4.37)

$\lambda. v_{ti} - P_{ti} + (I_{ti} - B_{ti} - I_{ti-1} + B_{ti-1}) \leq -d_{ti} + v_{ti}$ $\forall i = 1, \dots, N$ (4.38)

$\lambda. u_{ti} + P_{ti} - (I_{ti} - B_{ti} - I_{ti-1} + B_{ti-1}) \leq d_{ti} + u_{ti}$ $\forall i = 1, \dots, N$ (4.39)

Constraints (4.26), (4.28)-(4.34)

$0 \leq \lambda \leq 1$ (4.40)

In this formulation, each inventory constraint which contains the fuzzy demand, as well as the fuzzy goal objective will be used in determining the optimum λ value.

4.5 Fuzzy Max-Min FRP-APP (MM-FRP-APP)

Adding flexible bounds Constraint sets (3.21)-(3.23), we will have the MM-FRP-APP as follows:

(MM-FRP-APP) = (MM-APP) + Constraints (3.21)-(3.23)

In this formulation, the FRP-APP takes into account both the fuzzy demand and upper and lower goal values for the planning costs when developing plans. It also makes sure the FRP bounds are controlling the plan stability for any plan update in the rolling horizon plan development process.

4.6 Fuzzy Ranking Programming

The fuzzy ranking method is generally based on the fuzzy operations of $\tilde{\leqq}$ and $\tilde{\geqq}$ where at least one side includes fuzzy values. It could also be generalized to fuzzy objective functions where the objective value in the solution finding process needs to be compared and ranked considering all other feasible solutions objectives values. There exist multiple fuzzy ranking methods, and in the selection of the fuzzy ranking methods, the shape of the fuzzy numbers, and the ease of computation of the ranking method can be the major factors to be considered (Baykasoglu and Gocken 2010). However, in general, the basic idea behind all methods is to substitute each fuzzy number with a function of its most likely and the bound (violations) values using some coefficients. We here present 3 main fuzzy ranking methods presented in the literature (Tang, Fung et al. 2003, Jiménez, Arenas et al. 2007, Baykasoglu and Gocken 2010).

4.6.1 The signed distance method

For the triangular fuzzy numbers of $\tilde{A}$= $(\underline{a}, a, \overline{a})$ and α-cut (0≤ α≤1), the related signed distance could be defined as follows:

$$d(\tilde{A},0) = \frac{1}{2}\int_0^1 \left[\underline{a} + (a - \underline{a})\alpha + \bar{a} - (\bar{a} - a)\alpha\right] d\alpha = \frac{1}{4}(2a + \underline{a} + \bar{a}) \qquad (4.41)$$

While the signed distance of a trapezoidal fuzzy number $\tilde{A}$= $(a_l, \underline{a}, \bar{a}, a_u)$ and α-cut (0≤ α≤1) is as below:

$$d(\tilde{A},0) = \frac{1}{2}\int_0^1 \left[a_l + (\underline{a} - a_l)\alpha + a_u - (a_u - \bar{a})\alpha\right] d\alpha = \frac{1}{4}(\underline{a} + a_l + a_u + \bar{a}) \qquad (4.42)$$

If $\tilde{A}$ and $\tilde{B}$ are both triangular or trapezoidal fuzzy numbers, the ranking of these two numbers is defined as: $\tilde{A} \leq \tilde{B} \leftrightarrow d(\tilde{A},0) \leq d(\tilde{B},0)$.

4.6.2 Ranking of fuzzy numbers with integral value

This technique was introduced by Liou and Wang (1992) to rank fuzzy numbers with integral values. This method is relatively simple from computational perspective, especially in case of triangular and trapezoidal fuzzy numbers while it could be used for ranking more than two fuzzy numbers simultaneously (Liou and Chen 2006). As each triangular fuzzy number (ex. $\tilde{A}$= $(\underline{a}, a, \bar{a})$) could be defined by two membership functions parts, the left and right hand side of the membership function could be defined as follows (Baykasoglu and Gocken 2010):

$$\mu_{\tilde{A}}^{L}(x) = \begin{cases} \frac{x-\underline{a}}{a-\underline{a}} & \underline{a} \leq x \leq a, \underline{a} \neq a \\ 1 & \underline{a} = a \end{cases} \qquad (4.43)$$

$$\mu_{\tilde{A}}^{R}(x) = \begin{cases} \frac{x-\bar{a}}{a-\bar{a}} & a \leq x \leq \bar{a}, a \neq \bar{a} \\ 1 & \bar{a} = a \end{cases} \qquad (4.44)$$

Then $u_{\tilde{A}}^{L}: [\underline{a}, a] \to [0,1]$ and $u_{\tilde{A}}^{R}: [a, \bar{a}] \to [0,1]$. Since $\mu_{\tilde{A}}^{L}(x)$ and $\mu_{\tilde{A}}^{R}(x)$ are continuous and strictly increasing, the inverse function of $\mu_{\tilde{A}}^{L}(x)$ and $\mu_{\tilde{A}}^{R}(x)$ exist, denoted by $g_{\tilde{A}}^{L}$ and $g_{\tilde{A}}^{R}$, and $g_{\tilde{A}}^{L}: [0,1] \to [\underline{a}, a]$ and $g_{\tilde{A}}^{R}: [0,1] \to [a, \bar{a}]$, respectively. Both $g_{\tilde{A}}^{L}$ and $g_{\tilde{A}}^{R}$ are as follows:

$$g_{\tilde{A}}^{L}(u) = \begin{Bmatrix} \underline{a} + (a - \underline{a})u, \underline{a} \neq a, u \in [0,1] \\ \underline{a} \qquad \underline{a} = a \end{Bmatrix} \tag{4.45}$$

$$g_{\tilde{A}}^{R}(u) = \begin{Bmatrix} \bar{a} + (a - \bar{a})u, a \neq \bar{a}, u \in [0,1] \\ \bar{a} \qquad a = \bar{a} \end{Bmatrix} \tag{4.46}$$

As a result, the integral value for the triangular number $\tilde{A}$ could be calculated as follows:

$$I(\tilde{A}) = (1-\alpha)\int_0^1 g_{\tilde{A}}^{L}(u)du + \alpha\int_0^1 g_{\tilde{A}}^{R}(u)du = \frac{1-\alpha}{2}\underline{a} + \frac{1}{2}a + \frac{\alpha}{2}\bar{a} \tag{4.47}$$

Where $0 \leq \alpha \leq 1$.

The index of optimism α is the representation of the degree of optimism, where higher value of α indicates higher degrees of optimism. A fuzzy number with larger integral value, is a larger fuzzy number.

4.6.3 Ranking of fuzzy numbers through the comparison of their expected intervals

This method is based on the definition of the expected interval and the expected value of a fuzzy number as proposed in Heilpern (1992) and later used for fuzzy modeling optimization by Jiménez, Arenas et al. (2007). Let $\tilde{A} = (a_l, \underline{a}, \bar{a}, a_u)$ denote a trapezoidal fuzzy number defined as follows:

$$\mu_{\tilde{A}}(x) = \begin{Bmatrix} 0 & \forall x \in (-\infty, a_l] \\ f_A(x) & \forall x \in [a_l, \underline{a}] \\ 1 & \forall x \in [\underline{a}, \bar{a}] \\ g_A(x) & \forall x \in [\bar{a}, a_u] \\ 0 & \forall x \in (a_u, \infty) \end{Bmatrix} \tag{4.48}$$

Please note that a triangular fuzzy number is a specific case of trapezoidal fuzzy number where $\underline{a}=\bar{a}$. In order to warrant the existence and inegrability of the inverse functions $f_A^{-1}(x)$ and

$g_A^{-1}(x)$, it is assumed that $f_A(x)$ is continuous and increasing while $g_A(x)$ is continuous and decreasing. The expected interval of this fuzzy number could be defined as follows:

$$EI(\tilde{A}) = [E_1^{\tilde{A}}, E_2^{\tilde{A}}] = [\int_0^1 f_A^{-1}(\alpha)d\alpha, \int_0^1 g_A^{-1}(\alpha)d\alpha] \quad (4.49)$$

Where $\alpha = f_A(x) = g_A(x)$. For a trapezoidal (or triangular) fuzzy number, the expected interval could be re-written as follows:

$$EI(\tilde{A}) = [\frac{1}{2}(a_l + \underline{a}), \frac{1}{2}(\bar{a} + a_u)] \quad (4.50)$$

In addition, for the difference of two fuzzy numbers $\tilde{A}$, $\tilde{B}$, the expected interval is:

$$EI(\tilde{A} - \tilde{B}) = [E_1^{\tilde{A}} - E_2^{\tilde{B}}, E_2^{\tilde{A}} - E_1^{\tilde{B}}] = EI(\tilde{A}) - EI(\tilde{B}) \quad (4.51)$$

The degree in which $\tilde{A}$ is greater than $\tilde{B}$ could be defined as the following (Jiménez, Arenas et al. 2007, Baykasoglu and Gocken 2010):

$$\mu_M(\tilde{A}, \tilde{B}) = \begin{Bmatrix} 0 & if\ E_2^{\tilde{A}} - E_1^{\tilde{B}} < 0 \\ \frac{E_2^{\tilde{A}} - E_1^{\tilde{B}}}{E_2^{\tilde{A}} - E_1^{\tilde{B}} - (E_1^{\tilde{A}} - E_2^{\tilde{B}})} & if\ 0 \in [E_1^{\tilde{A}} - E_2^{\tilde{B}}, E_2^{\tilde{A}} - E_1^{\tilde{B}}] \\ 1 & if\ E_1^{\tilde{A}} - E_2^{\tilde{B}} > 0 \end{Bmatrix} \quad (4.52)$$

If $\mu_M(\tilde{A}, \tilde{B}) = 0.5$ we can say $\tilde{A}$ and $\tilde{B}$ are equal. When $\mu_M(\tilde{A}, \tilde{B}) \geq \alpha$ we will say that $\tilde{A}$ is bigger than or equal to $\tilde{B}$ at least with a degree α, which could be represented by $\tilde{A} \geq_\alpha \tilde{B}$.

In addition, the expected value of the fuzzy number $\tilde{A}$ could be defined according to the below formula (Jiménez, Arenas et al. 2007):

$$EV(\tilde{A}) = \frac{E_1^{\tilde{A}} + E_2^{\tilde{A}}}{2} \quad (4.53)$$

For fuzzy numbers $\tilde{A}$, $\tilde{B}$ and for non-negative values of λ, and γ, the following relationships hold (Jiménez, Arenas et al. 2007):

$$EI\left(\lambda\tilde{A}+\gamma\tilde{B}\right)=\lambda\mathrm{EI}\left(\tilde{A}\right)+\gamma\mathrm{EI}\left(\tilde{B}\right) \tag{4.54}$$

$$EV\left(\lambda\tilde{A}+\gamma\tilde{B}\right)=\lambda\mathrm{EV}\left(\tilde{A}\right)+\gamma\mathrm{EV}\left(\tilde{B}\right) \tag{4.55}$$

As there exist an equality fuzzy constraint in our formulations, the following equality should be true for this constraint:

$$\frac{E_2^{a_i x}-E_1^{b_i}}{E_2^{a_i x}-E_1^{b_i}-(E_1^{a_i x}-E_2^{b_i})}=0.5, i=1,\ldots,m \tag{4.56}$$

Based on (4.56) we have:

$$[0.5E_2^{a_i}+0.5E_1^{a_i}]\,\mathrm{x}=0.5E_2^{b_i}+0.5E_1^{b_i}, i=1,\ldots,m \tag{4.57}$$

Since in our formulations the left-hand-side of the fuzzy inventory constraint is not a fuzzy number, the transformed version of this constraint is:

$$P_{t,i}-I_{t,i}+B_{t,i}+I_{t,i-1}-B_{t,i-1}=0.5E_2^{d_{t,i}}+0.5E_1^{d_{t,i}} \qquad \forall i=1,\ldots,N \tag{4.58}$$

In order to optimize the objective function with fuzzy coefficients, let's consider the objective function in objective (4.10) without considering any uncertainty in constraints. According to Jiménez, Arenas et al. (2007), a vector x^0 is an acceptable optimal solution of this model, if we can verify that:

$$\mu_m(\tilde{c}^t x, \tilde{c}^t x^0)\geq\frac{1}{2}\ \ \forall x \tag{4.59}$$

Thus we have:

$$\tilde{c}^t x \geq_{\frac{1}{2}} \tilde{c}^t x^0 \ \ \forall x \tag{4.60}$$

Therefore, x^0 is a better choice at least to a degree of $\frac{1}{2}$ as opposed to other feasible vectors. The above expression could be then re-written as follows:

$$\frac{E_2^{c^t x} - E_1^{c^t x^0}}{E_2^{c^t x} - E_1^{c^t x} + E_2^{c^t x^0} - E_1^{c^t x^0}} \geq \frac{1}{2} \tag{4.641}$$

Or alternatively

$$\frac{E_2^{c^t x} + E_1^{c^t x}}{2} \geq \frac{E_2^{c^t x^0} + E_1^{c^t x^0}}{2} \tag{4.62}$$

A vector $x^0(\alpha) \in R^n$ is a α-acceptable optimal solution of Model (4.10)-(4.12) if it is an optimal solution to the following problem:

$$\text{Minimize } EV(\tilde{c})x \tag{4.63}$$

Subject to

$$[0.5E_2^{a_i} + 0.5E_1^{a_i}]\ \mathrm{x} = 0.5E_2^{b_i} + 0.5E_1^{b_i}, i = 1, \dots, m, \mathrm{x} \geq 0 \tag{4.64}$$

4.7 Fuzzy Ranking APP (R-APP)

In this research, we are not considering fuzzy values for the cost coefficients of the APP and FRP-APP models, as a result, the R-APP model is as follows:

(R-APP)

Minimize $Z = \sum_{i=0}^{N}(c^w.th.W_{t,i} + c^o.O_{t,i} + c^H.H_{t,i} + c^L.L_{t,i} + c^p.P_{t,i} + h.I_{t,i} + b.B_{t,i})$(4.65)

Subject to

Constraints (4.26), (4.28) -(4.34), (4.58)

4.8 Fuzzy Ranking FRP-APP (R-FRP-APP)

Adding the FRP related constraint to the R-APP formulation, the R-FRP-APP formulation is as follows:

(R-FRP-APP) = (R-APP) + Constraints (3.21)-(3.23)

The R-RFP-APP is also using the ranking method to deal with the fuzzy inventory constraint resulting from fuzzy demand estimations of the future periods. In addition, it controls the stability of the plans through enforcing restrictions on production level changes over various plan updates in a rolling horizon planning approach.

4.9 Computational Study

In this section, a comprehensive analysis on the performance of the FRP-APP, Fuzzy FRP-APP, and Fuzzy APP models will be presented. We initially start with the data structure and the two performance measures for comparing different models, and then present the main results, sensitivity analysis, and also the experimental analysis of models to identify influential factors on models' performance. It should be noted that the data structure and the performance measures will be used in all analysis related to Chapters 4-6.

4.9.1 Data Structure

We use five sets of data corresponding to Textile (Leung, Wu et al. 2003, Demirel 2014), Automotive parts (Sillekens, Koberstein et al. 2011, Demirel 2014), Machinery & Transmission parts (Wang and Liang 2005), Wood & Paper production (Mirzapour Al-E-Hashem, Malekly et al. 2011), and Air Conditioning Units components (Techawiboonwong and Yenradee 2002) as shown in Table 4.1. It should be noted that the values marked by double underscores are missing values in each case and calculated using the mean values of the same parameters in other cases.

These data sets follow a different structure in terms of production, inventory and labor related costs and times, hence they provide a good opportunity for a sensitivity analysis of the proposed planning models' performance.

Table 4.1: Cost and capacity, 5 Case studies

Parameter	***Textile***	***Automotive Parts***	***Machinery & Transmission Components***	***Wood & Paper***	***Air Conditioning Units***
Production cost ($/unit)	6.41	1.8	20	9.03	9
Inventory cost per unit per week ($/unit)	1.92	0.18	0.3	5	6
Backorder cost per unit per week ($/unit)	3.85	3.6	40	2	12.36
Labor cost ($/*person*-hour)	0.80	11.16	10	18	0.6
Overtime labor cost ($/*person*-hour)	1.28	12.28	30	27	1.2
Hiring cost ($/person)	12.82	3571	10	40	36
Layoff cost ($/*person*)	15.38	14286	3	70	29.46
Number of units produced (*unit*/person-hour)	0.57	16.67	20	9.48	0.7

The initial inventory is assumed to be 100 units and the initial workforce is set based on the realized demand in the first period of each planning iteration. Each employee regularly works 8 hours per day, 5 days per week.

We use the demand generation formulation introduced by Demirel (2014) where the demand is assumed to follow a seasonal behavior according to the following formulation:

$$D_t = (a + bt)S_t + e_t \tag{4.66}$$

Where D_t is the demand value at time period t, a is the baseline parameter, b is the trend component, S_t is the seasonal factor at time period t, and e_t is the random error component with a normal distribution $N(0, \sigma^2)$. The variation in the values of demand generation components could result in various demand scenarios. Table 4.2 represents the magnitude of four demand generation components (Demirel 2014). As each of 4 main components in (4.68) has 2 levels, 16 demand scenarios would be generated.

Table 4.2: Demand generation components values

Component	***Low***	***High***
Baseline	1000 units	3000 units
Trend	20	100
Seasonality	1∓0.1	1∓0.3
Magnitude of error	Std=50	Std=100

We will be using the same demand scenarios for all Case studies as the 16 demand scenarios have variation combinations for demand values and it has the potential to give an overview of the effect of different cost structures on the FRP-APP and APP models performance. Table 4.3 presents these scenarios and levels of parameters used for generation of each scenario.

Table 4.3: Demand scenarios

Scenario	***Baseline***	***Trend***	***Seasonality***	***Magnitude of error***
1	Low	Low	Low	Low
2	Low	Low	Low	High
3	Low	Low	High	Low
4	Low	Low	High	High
5	Low	High	Low	Low
6	Low	High	Low	High
7	Low	High	High	Low
8	Low	High	High	High
9	High	Low	Low	Low
10	High	Low	Low	High
11	High	Low	High	Low
12	High	Low	High	High
13	High	High	Low	Low
14	High	High	Low	High
15	High	High	High	Low
16	High	High	High	High

For the crisp models and taking into account the generated demand as the historical data, the demand values of the future periods are forecasted using the Multiplicative Holt-Winter (Triple Exponential Smoothing) forecasting method. This method of forecasting takes into consideration all components of the demand generation formulation where each component has a specific parametric-based formulation. Baseline, Trend, and Seasonality, and the forecasted demand formulation are as follows:

$$\textit{Base}: L_t=\alpha\left(\frac{D_t}{S_{t-s}}\right)+(1-\alpha)(L_{t-1}-T_{t-1}) \tag{4.67}$$

$$\textit{Trend}: T_t=\beta(L_t-L_{t-1})+(1-\beta)T_{t-1} \tag{4.68}$$

$$\textit{Seasonality}: S_t=\Upsilon\left(\frac{D_t}{L_t}\right)+(1-\Upsilon)S_{t-s} \tag{4.69}$$

$$\textit{Forecast}: F_{t+m}=(L_t+mT_t)S_{t+m-s} \tag{4.70}$$

In the above formulation, s is the seasonality length, and m denotes the number of future periods for which the forecasting is done. In addition, $\alpha, \beta, \Upsilon\, \epsilon[0,1]$ are the smoothing parameters. Each demand scenario is forecasted using the above formulation, and is used in crisp models as the forecasted demands. For each demand scenario, we aim to do the planning for the current period and $N = 5$ periods ahead in each planning iteration ($i = 0,1, \dots ,5$) and repeat the planning for $T = 50$ rolling horizon iterations ($t = 1, \dots ,50$). As a result, the forecasted demand for each period and its actual demand generated using the demand generation formulation is initially used to calculate the Mean Square Error (MSE) of the forecasts. After we complete all the forecasts, we do the MSE minimization to come up with the optimum α, β, and Υ, and also the more reliable forecast values to be later used in our test problems.

The fuzzy upper and lower bounds in the membership functions related to the demand values are calculated using the forecasted demand values with pre-specified violation levels. Here, the violation levels are set as: upper violation 15%, lower violation 12%. In addition, for the Max-Min method, the upper and lower bounds for the fuzzy objective function are taken here as the highest and lowest values of the APP and FRP-APP model objective function values, respectively. The membership functions are all formulated in forms of triangular functions which without loss of generality could be reformulated using trapezoidal functions, depending on the data availability for building related fuzzy membership components. In general, triangular and trapezoidal membership functions are preferred in fuzzy programming since they require less data for the construction of the related membership function and provide robust results. It should also be noted that, all fuzzy transformation methods explained earlier could be generalized to other fuzzy memberships as well. However, nonlinear membership functions would result in nonlinear constraints hence nonlinear optimization problems, which can be a justification to utilize triangular or trapezoidal membership functions instead.

Finally, the flex-limits for each period (F_i) are based on the following sets for sensitivity analysis purposes. We believe due to the high range of differences these two flex-limit sets have, it gives variability to the FRP-APP models to better analyze how the cost and stability of this planning approach changes with respect to high and low flex-limit tolerances.

Table 4.4: Flex-limits coefficients

Case	***i=0***	***i=1***	***i=2***	***i=3***	***i=4***	***i=5***
1% Flex Limits	1%	2%	3%	4%	5%	6%
5% Flex Limits	5%	10%	15%	20%	25%	30%

4.9.2 Performance Measures

The first performance measure is the total current cost for $i = 0$ (actual cost) over all planning iterations. Please note that this cost performance measure is related to but different from the cost objective function defined in the optimization models as it just considers the summation of actual costs over all planning iterations as shown in the formula below:

$$Actual\ Cost = \sum_{t=1}^{T} C^{w}.th.W_{t,0} + C^{o}.O_{t,0} + C^{H}.H_{t,0} + C^{L}.L_{t,0} + C^{p}.P_{t,0} + h.I_{t,0} + b.B_{t,0} \qquad (4.71)$$

Another measure is the stability of the production plans. The stability could be defined in different forms like: cost, changes in number of setups, order quantity change, changes in production quantities, or a mixture of these criteria simultaneously (Kadipasaoglu and Sridharan 1997, Pujawan and control 2004). Some other studies such as Herrera and Thomas (2009) proposed use the maximum changes in the production levels in two planning iterations in the rolling horizon as a measure of instability. We use the stability definition presented by Demirel (2014) which measures the total amount of changes in the estimated production level values over consecutive plan updates and then average it over all rolling horizon iterations (starting from iteration 2) and planning horizon periods N.

$$Stability = \frac{\left(\sum_{t=2}^{T}\sum_{i=1}^{N}\left|P_{t-1,i} - P_{t,i-1}\right|^{m}\right)^{1/m}}{(T-1).N} \qquad (4.72)$$

Where m is the compensation parameter (1$\leq m \leq \infty$), $m = 1$ implies full compensation which means the sum of absolute deviations from the production plans. In our research, we consider $m = 1$. In formulation (4.75), as each planning iteration is done, the absolute value of changes in production levels for identical periods are calculated, added and then averaged over all periods and iterations. Please also note that the lower values of this stability measure mean more stable production plans.

4.9.3 Computational Results & Analysis

All our numerical experiments were conducted using the Gurobi Python Interphase a.k.a. Gurobipy (Pedroso 2011) optimization solver on a personal computer with Intel Corei7 6300U CPU (2:4 GHz) with 8.00 GB of RAM.

Figures 4.4-4.23 represent the related results of comparing deterministic and Fuzzy FRP-APP with Fuzzy APP models for different Industry Cases and various demand scenarios using the two fuzzy techniques. The results of all FRP-APP models are presented using the previously introduced 2 flex-limits sets for sensitivity purposes (For further details about the sensitivity of the FRP-APP and sample Fuzzy FRP-APP cost and stability performance with respect to varying flex-limit sets, please refer to Figures 1-20 in Appendix). In addition, the average cost gap and the average instability ratio of FRP-APP, Fuzzy FRP-APP, and Fuzzy APP models are presented in Tables 4.5, 4.6. These gaps are calculated as formulas below (these formulations will be used in Chapters 5, and 6 as well):

$$\text{Cost gap\%} = \frac{Average\ Cost - Best\ average\ cost}{Best\ average\ cost} * 100 \tag{4.73}$$

$$\text{Instability ratio} = \frac{Stability\ measure}{Best\ stability\ measure} \tag{4.74}$$

Looking at the different industry cost and stability results over various demand scenarios indicate that in general, the FRP-APP models show to be very promising planning models to produce the most stable production plans and competitive cost values, with reliable results even if formulated as a fuzzy model. In addition, the general observation in most of the Industry Cases indicate the change of fuzzy technique could have impact on the stability of the Fuzzy APP model (Ranking technique shows more control over the stability of the plans). When fuzziness is also considered in the FRP-APP formulation, the impact of fuzzy technique change on the stability performance of the FRP-APP becomes more smooth moving from Flex-limits 5% to 1%. Another interesting observation is that incorporation of Fuzziness in the FRP-APP formulation specially when formulated with the 1% flex-limit set does not noticeably impact the relative cost and more specifically the stabilizing performance of the Fuzzy FRP-APP as compared to the FRP-APP model. There are some further notes related to each industry that we would like to further discuss them:

Textile Industry (Figures 4.4-4.7) indicate that FRP-APP models with both 1% and 5% flex-limit sets result in very close cost values as compared to the Fuzzy-APP model. Looking at the stability results, the 1% FRP-APP with and without fuzziness seem to result in the most stable production plans. Table 4.6 shows that Fuzzy FRP-APP with 1% flex-limit is able to noticeably improve the stability measure up to 5 times better no matter which fuzzy technique is utilized. We need to note that while the stability improves significantly, depending on which fuzzy technique is used, there is a trade-off with the cost, which increases slightly by 9% (for R-FRP-APP)-13% (for MM-FRP-APP).

The Automotive parts (4.8-4.11) results indicate the 1% FRP-APP models result in very close cost values compared to other models while being able to produce the most stable plans. On the other hand, Figures (4.9) and (4.11) show that stability results deteriorate for the Automotive Parts Industry Case as we move from 1% to 5% flex limits results. Taking into account both the cost and stability, the Automotive Parts Industry Case advocates the utilization of the 1% Fuzzy FRP-APP models as it not only takes into account input parameters' uncertainties, but it makes sure the stability is well addressed while the costs are maintained at relatively low levels. However, the 5% Fuzzy FRP-APP models could lose their favorability as compared to the Fuzzy APP models in terms of stability performance and cannot always guarantee more stable production plans.

For the Wood and Paper Industry (Figures 4.16-4.19), unlike the 1% case, the 5% fuzzy FRP-APP is not constantly and noticeably more stable as compared to the non-FRP models. For this case, the MM-FRP-APP model, especially for the 1% flex-limit set tends to result in more favorable cost and stability results.

The Machinery and Transmission Parts industry Case results (Figures 4.12-4.15) indicate that the 5% flex-limit set in either MM-FRP-APP or the R-FRP-APP methods are more promising for the cost and relative stability performance as compared to the 1% flex-limit set. The cost of 1% Fuzzy FRP-APP has the potential of getting high specially in MM-FRP-APP (up to about 30% higher cost) but a less drastic change (with only 6% higher cost) is observed in the R-FRP-APP.

Finally, for the Air conditioning unit industry Case (Figures 4.20-4.23), none of the FRP-based models stand out in terms of cost and stability. The 1% FRP-APP models seem to improve the stability by 300% but also result in more than 20% cost, which requires a decision-maker to trade-off between stability versus cost. Furthermore, the 5% FRP-APP model does not seem to be attractive since it has no advantage in either cost or stability as compared to the APP model.

Overall, Tables 4.5 and 4.6 indicate the Max-Min technique is slightly better in terms of stability and the Ranking technique is slightly better in terms of cost. The justification of using the Max-Min fuzzy technique however, would be the possibility of using the decision maker’s judgements and/or historical data to form the goal membership function for the objective without paying attention to each cost component, which could make it easier for the decision makers and planners to come up with the fuzzy functions components. The Max-Min fuzzy results show it does not necessarily result in significantly higher cost values for the most stable case (MM-FRP-APP 1%) except for the Machinery and Transmission Parts industry Case. On the other side, the R-FRP-APP shows the potential for handling fuzziness and stability in the planning, resulting in close performance to the crisp FRP-APP.

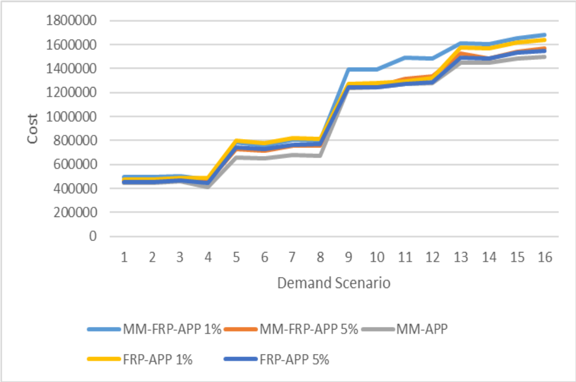

Figure 4.4: Total current cost comparison, FRP-APP, MM-FRP-APP, MM-APP, Textile Industry

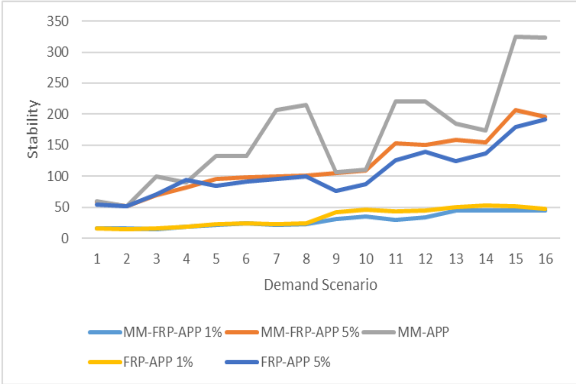

Figure 4.5: Stability comparison, FRP-APP, MM-FRP-APP, MM-APP, Textile Industry

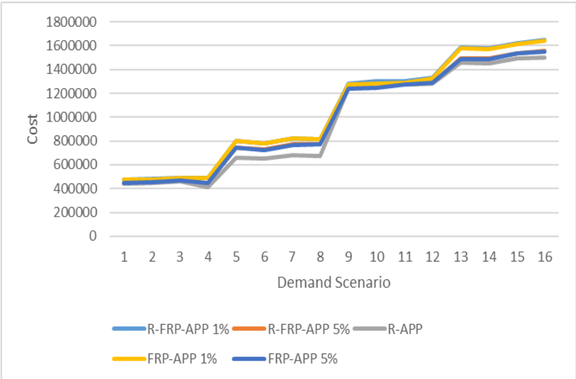

Figure 4.6: Total current cost comparison, FRP-APP, R-FRP-APP, R-APP, Textile Industry

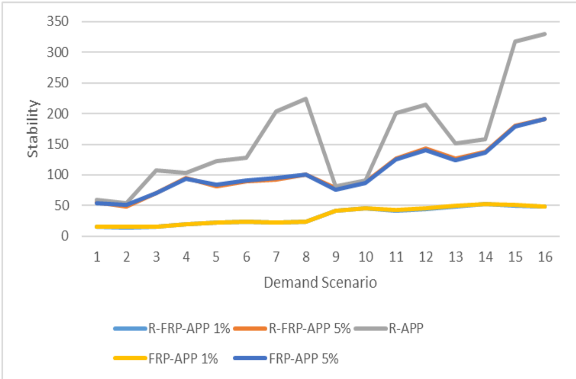

Figure 4.7: Stability comparison, FRP-APP, R-FRP-APP, R-APP, Textile Industry

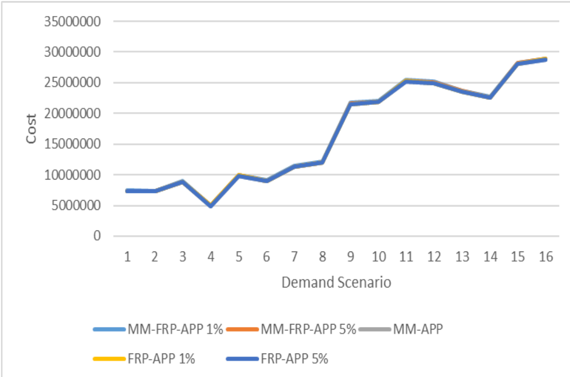

Figure 4.8: Total current cost comparison, FRP-APP, MM-FRP-APP, MM-APP, Automotive Parts Industry

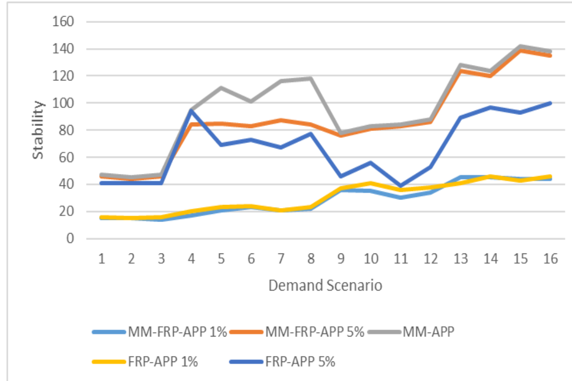

Figure 4.9: Stability comparison, FRP-APP, MM-FRP-APP, MM-APP, Automotive Parts Industry

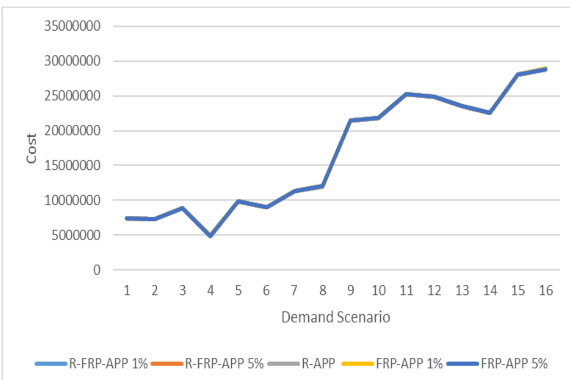

Figure 4.10: Total current cost comparison, FRP-APP, R-FRP-APP, R-APP, Automotive Parts Industry

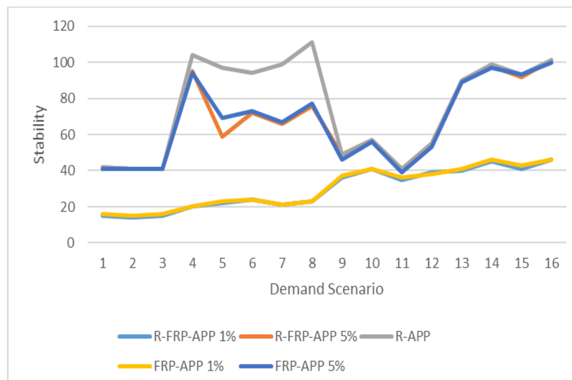

Figure 4.11: Stability comparison, FRP-APP, R-FRP-APP, R-APP, Automotive Parts Industry

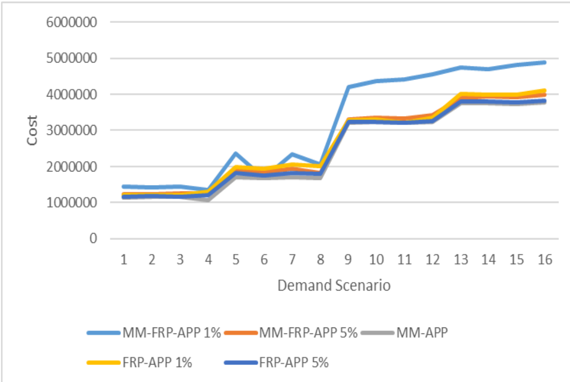

Figure 4.12: Total current cost comparison, FRP-APP, MM-FRP-APP, MM-APP, Machinery and Transmission Industry

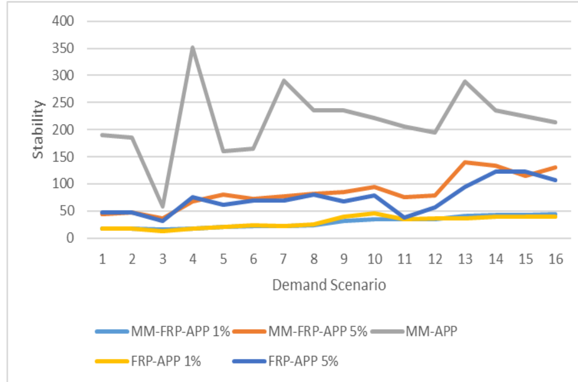

Figure 4.13: Stability comparison, FRP-APP, MM-FRP-APP, MM-APP, Machinery and Transmission Industry

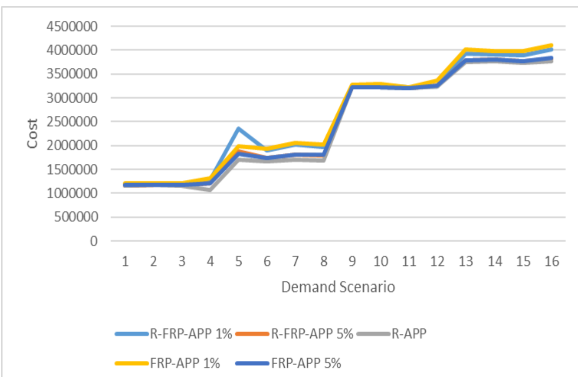

Figure 4.14: Total current cost comparison, FRP-APP, R-FRP-APP, R-APP, Machinery and Transmission Industry

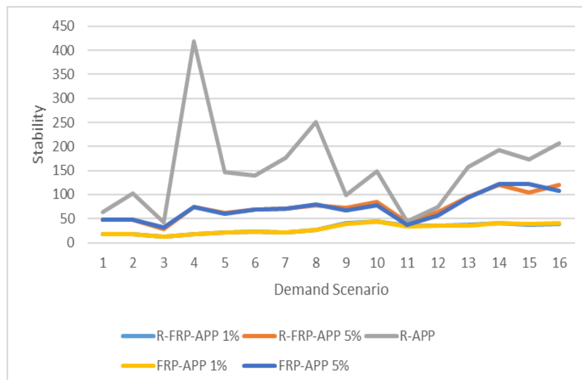

Figure 4.15: Stability comparison, FRP-APP, R-FRP-APP, R-APP, Machinery and Transmission Industry

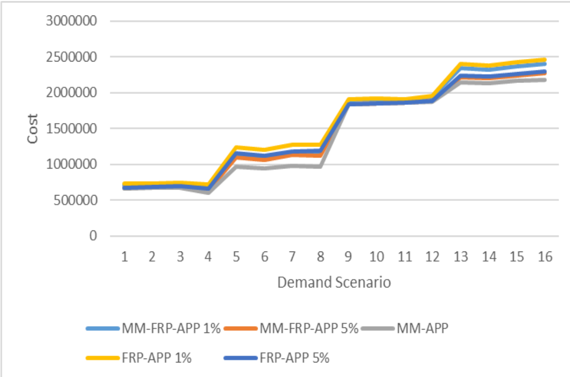

Figure 4.16: Total current cost comparison, FRP-APP, MM-FRP-APP, MM-APP, Wood and Paper Industry

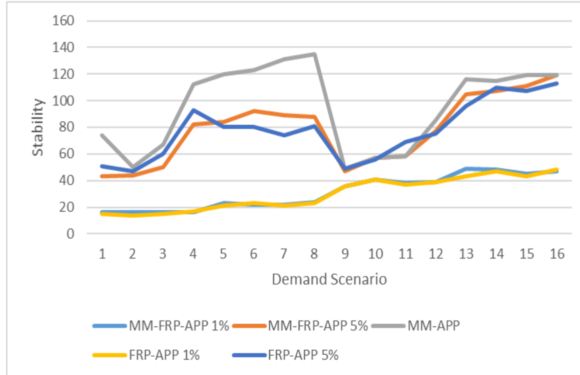

Figure 4.17 Stability comparison, FRP-APP, MM-FRP-APP, MM-APP, Wood and Paper Industry

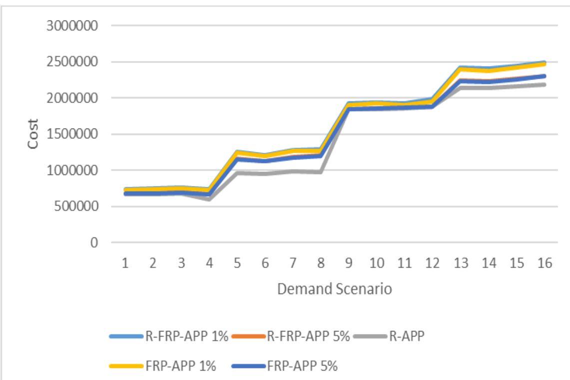

Figure 4.18: Total current cost comparison, FRP-APP, R-FRP-APP, R-APP, Wood and Paper Industry

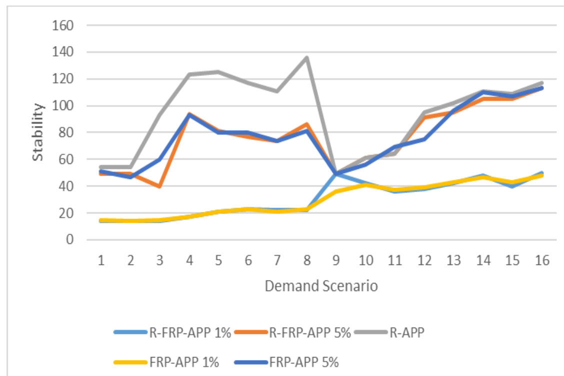

Figure 4.19: Stability comparison, FRP-APP, R-FRP-APP, R-APP, Wood and Paper Industry

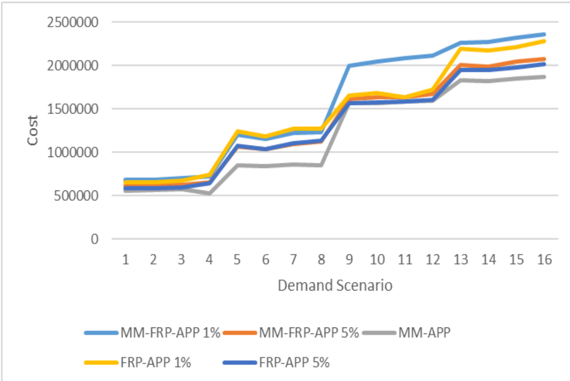

Figure 4.20: Total current cost comparison, FRP-APP, MM-FRP-APP, MM-APP, Air Conditioning Units Industry

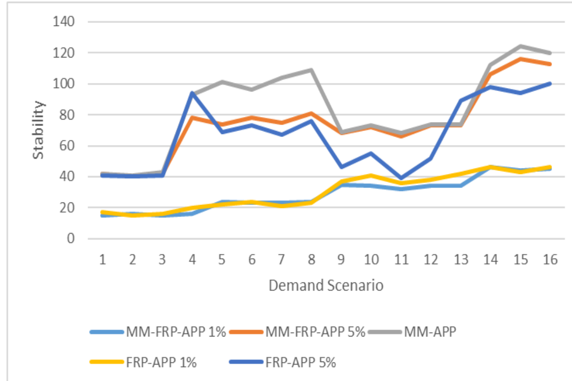

Figure 4.21: Stability comparison, FRP-APP, MM-FRP-APP, MM-APP, Air Conditioning Units Industry

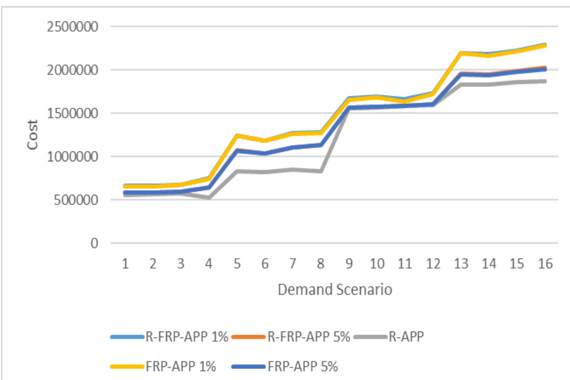

Figure 4.22: Total current cost comparison, FRP-APP, R-FRP-APP, R-APP, Air Conditioning Units Industry

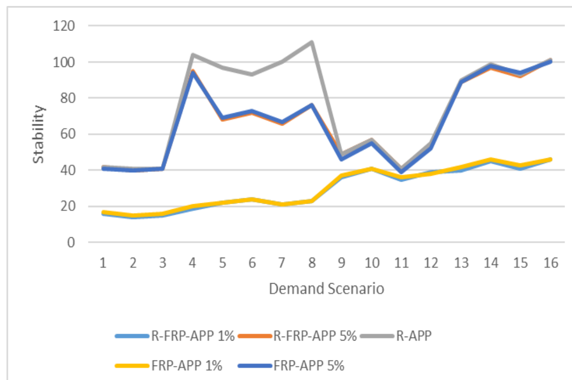

Figure 4.23: Stability comparison, FRP-APP, R-FRP-APP, R-APP, Air Conditioning Units Industry

Table 4.5: Average cost gap percentage of different models in different industries

	Textile	Automotive	Machinery & Transmission	Wood & Paper	Air Conditioning
MM-FRP-APP 1%	13.6	0.9	29.5	8.8	30.2
MM-FRP-APP 5%	4.8	0.4	6.2	4.2	11.7
MM-APP	0.0	0.0	0.0	0.0	0.3
R-FRP-APP 1%	9.4	0.1	6.7	13.5	21.6

R-FRP-APP 1%	4.1	0.0	2.1	6.2	9.3
R-APP	0.3	0.0	0.1	0.0	0.0
FRP-APP.1%	8.8	0.1	7.6	12.3	20.8
FRP-APP.5%	3.7	0.0	2.1	5.8	8.9

*0.0 means the lowest average cost, and a cost gap percentage closer to 0.0 shows a better cost performance.

Table 4.6: Comparison of average instability ratio for different models in different industries

	Textile	Automotive	Machinery & Transmission	Wood & Paper	Air Conditioning
MM-FRP-APP 1%	1.0	1.0	1.0	1.0	1.0
MM-FRP-APP 5%	4.1	3.0	3.0	2.6	2.6
MM-APP	5.8	3.4	7.5	3.2	2.9
R-FRP-APP 1%	1.1	1.0	1.0	1.0	1.0
R-FRP-APP 1%	3.7	2.3	2.6	2.6	2.3
R-APP	5.5	2.6	5.3	3.1	2.6
FRP-APP.1%	1.2	1.1	1.0	1.0	1.1
FRP-APP.5%	3.7	2.3	2.5	2.6	2.3

*1.0 means the best stability control, and a ratio closer to 1.0 shows a better stability performance.

The results presented thus far, show that the APP model, and to some extend the newly proposed Fuzzy FRP-APP models show sensitivity to the demand scenarios, the fuzzy modeling techniques, and also Industry case change. In order to see how each model is statistically affected by each of these factors, and which factors are playing a stronger role, we follow on experimental design approach. Table 4.7 includes the main experimental factors and their related levels. Each demand generation formulation related parameter has 2 levels, while the effect of flex-limits is analyzed considering 4 levels, more specifically, 1%, 3%, 5% and No Flex limits. Please note that the no Flex Limits case corresponds to the APP model without flex limit consideration. The effect of different industries is also tested by considering a 5 level factor, where levels 1,2,3,4,5 are related to the Textile, Automotive Parts, Machinery and Transmission Parts, Wood and Paper, and Air Conditioning Units Industry Cases respectively. The data is analyzed using the General Linear

Model (GLM) with 2*2*2*2*4*2*5=640 scenarios. Figures 4.44 and 4.45 present the main effects of the independent variables for each response variable. The cost seems to show higher sensitivity to changes in demand structure (base, trend and seasonality) and industry, and less sensitivity to the demand error, flex-limits and fuzzy technique. The industry seems to be sharply affecting the cost in the Automotive Parts Industry, which has the highest values for hiring and layoff and also a relatively high workforce cost compared to the other Industry Cases. On the other hand, for stability, all factors seem to be significant, but flex-limits seem to have the highest effect as expected.

Tables 4.8 and 4.9 include the ANOVA (Analysis of Variance) results for plan cost and stability, respectively. Looking at the results, the cost and stability performance measures are affected by the demand structure and industry parameters, while stability is highly affected by fuzzy technique and flex-limits as well. Cost ANOVA results for the second and third level interactions indicate that in general, except for the flex-limits and the fuzzy technique, all other factor interactions seem to be significant for cost. On the other hand, stability ANOVA results for the second and third level interactions indicate that in general, the flex-limits, the fuzzy technique and the demand structure seem to be significant for stability.

Table 4.7: Experimental design components and designs to be tested

Factors	***Number of Levels***	***Levels***
Demand-base	2	Low(1000),High(3000)
Demand-trend	2	Low(20),High(100)
Demand-seasonality	2	Low(+/-0.1),High((+/-0.3)
Demand-magnitude of error	2	Low(std=50),High(std=200)
Flex-limits	4	1%,3%,5%,No-flex-limits
Fuzz-technique	2	Max-Min,Ranking
Industry	5	1,2,3,4,5

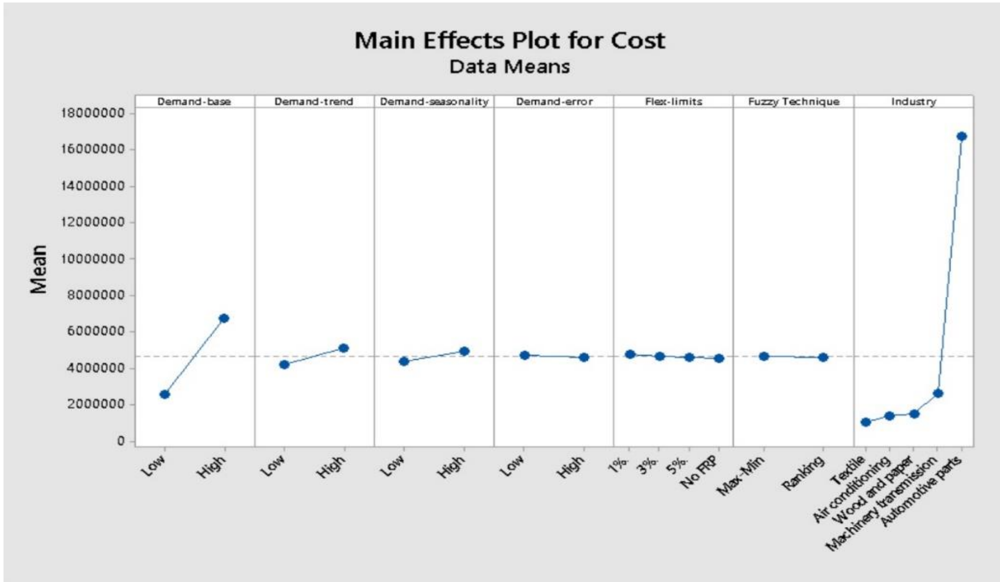

Figure 4.24: Main effects plot for cost, Fuzzy models

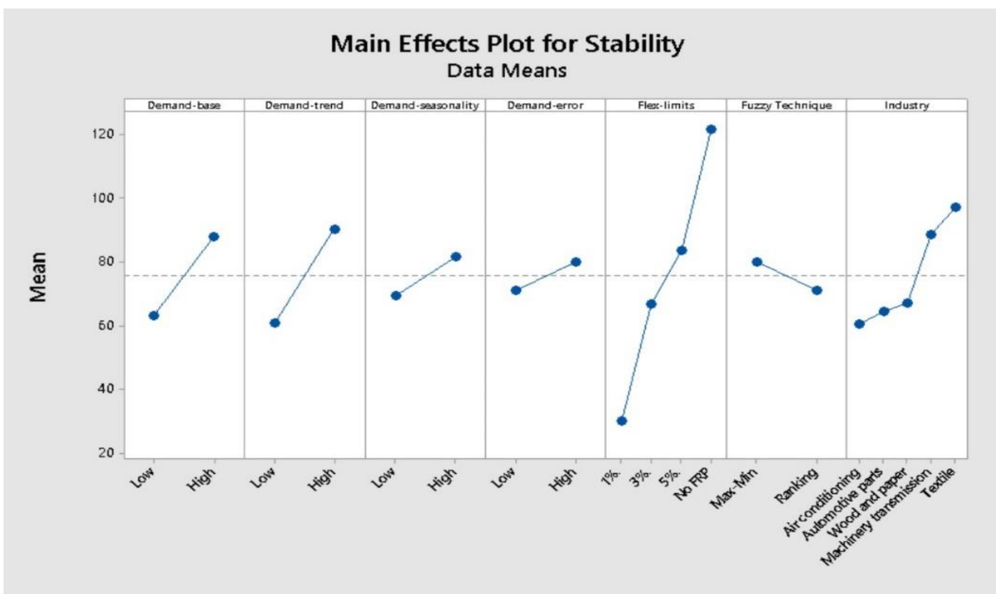

Figure 4.25: Main effects plot for stability, Fuzzy models

Table 4.8: Selected ANOVA results for plan cost, Fuzzy FRP-APP, Fuzzy APP

Source	***DF***	***Adj SS***	***Adj MS***	***F-Value***	***P-Value***
Demand-base	1	2.80E+15	2.80E+15	28477.43	0
Demand-trend	1	1.35E+14	1.35E+14	1372.99	0
Demand-seasonality	1	4.88E+13	4.88E+13	496.04	0
Demand-error	1	2.01E+12	2.01E+12	20.41	0
Flex-limits	3	4.09E+12	1.36E+12	13.86	0
Fuzzy Technique	1	6.55E+11	6.55E+11	6.66	0.01
Industry	4	2.36E+16	5.90E+15	59928.66	0
Demand-base*Demand-trend	1	2.75E+12	2.75E+12	28	0
Demand-base*Demand-seasonality	1	2.09E+13	2.09E+13	212.45	0
Demand-base*Demand-error	1	2.26E+12	2.26E+12	22.93	0
Demand-base*Fuzzy Technique	1	4.05E+11	4.05E+11	4.12	0.043
Demand-base*Industry	4	5.39E+15	1.35E+15	13695.21	0
Demand-trend*Demand-seasonality	1	9.33E+12	9.33E+12	94.81	0
Demand-trend*Demand-error	1	9.19E+11	9.19E+11	9.34	0.002
Demand-trend*Industry	4	1.57E+14	3.91E+13	397.87	0
Demand-seasonality*Industry	4	1.76E+14	4.39E+13	446.08	0
Demand-error*Industry	4	7.81E+12	1.95E+12	19.85	0
Fuzzy Technique*Industry	4	1.13E+12	2.83E+11	2.88	0.023
Demand-base*Demand-trend*Demand-error	1	1.46E+12	1.46E+12	14.81	0
Demand-base*Demand-trend*Industry	4	6.73E+12	1.68E+12	17.11	0
Demand-base*Demand-seasonality*Demand-error	1	1.23E+12	1.23E+12	12.5	0
Demand-base*Demand-seasonality*Industry	4	7.77E+13	1.94E+13	197.56	0
Demand-base*Demand-error*Industry	4	5.63E+12	1.41E+12	14.3	0
Demand-trend*Demand-seasonality*Demand-error	1	7.02E+12	7.02E+12	71.35	0
Demand-trend*Demand-seasonality*Industry	4	3.45E+13	8.63E+12	87.77	0
Demand-trend*Demand-error*Industry	4	5.24E+12	1.31E+12	13.33	0
Error	430	4.23E+13	98369839137		
Total	639	3.25E+16			

Table 4.9: Selected ANOVA results for plan stability, Fuzzy FRP-APP, Fuzzy APP

Source	***DF***	***Adj SS***	***Adj MS***	***F-Value***	***P-Value***
Demand-base	1	99376	99376	328.13	0
Demand-trend	1	139152	139152	459.46	0
Demand-seasonality	1	24988	24988	82.51	0
Demand-error	1	12558	12558	41.47	0
Flex-limits	3	696392	232131	766.47	0
Fuzzy Technique	1	13423	13423	44.32	0
Industry	4	134439	33610	110.98	0

Demand-base*Demand-trend	1	5623	5623	18.57	0
Demand-base*Flex-limits	3	8758	2919	9.64	0
Demand-base*Fuzzy Technique	1	4682	4682	15.46	0
Demand-base*Industry	4	34803	8701	28.73	0
Demand-trend*Demand-error	1	3023	3023	9.98	0.002
Demand-trend*Flex-limits	3	46995	15665	51.72	0
Demand-trend*Industry	4	5056	1264	4.17	0.003
Demand-seasonality*Demand-error	1	4879	4879	16.11	0
Demand-seasonality*Flex-limits	3	28561	9520	31.44	0
Demand-seasonality*Industry	4	19092	4773	15.76	0
Demand-error*Flex-limits	3	4588	1529	5.05	0.002
Demand-error*Fuzzy Technique	1	2209	2209	7.29	0.007
Demand-error*Industry	4	3120	780	2.58	0.037
Flex-limits*Fuzzy Technique	3	9211	3070	10.14	0
Flex-limits*Industry	12	205175	17098	56.46	0
Fuzzy Technique*Industry	4	8448	2112	6.97	0
Demand-base*Demand-trend*Demand-seasonality	1	1693	1693	5.59	0.018
Demand-base*Demand-trend*Demand-error	1	1491	1491	4.92	0.027
Demand-base*Demand-trend*Flex-limits	3	3014	1005	3.32	0.02
Demand-base*Demand-seasonality*Demand-error	1	4457	4457	14.72	0
Demand-base*Demand-seasonality*Industry	4	14749	3687	12.18	0
Demand-base*Demand-error*Flex-limits	3	2397	799	2.64	0.049
Demand-base*Demand-error*Industry	4	3057	764	2.52	0.04
Demand-base*Flex-limits*Fuzzy Technique	3	3379	1126	3.72	0.012
Demand-base*Flex-limits*Industry	12	22608	1884	6.22	0
Demand-trend*Demand-seasonality*Demand-error	1	3455	3455	11.41	0.001
Demand-trend*Demand-error*Flex-limits	3	3690	1230	4.06	0.007
Demand-trend*Demand-error*Industry	4	3507	877	2.9	0.022
Demand-trend*Flex-limits*Industry	12	9958	830	2.74	0.001
Demand-seasonality*Demand-error*Flex-limits	3	4525	1508	4.98	0.002
Demand-seasonality*Flex-limits*Industry	12	31057	2588	8.55	0
Demand-error*Flex-limits*Industry	12	7999	667	2.2	0.011
Flex-limits*Fuzzy Technique*Industry	12	15527	1294	4.27	0
Error	430	130228	303		
Total	639	1800458			

While the five Industry Cases presented earlier provide a fairly good idea about the applicability of the proposed Fuzzy FRP-APP models, more analysis is needed to conclude for other industries. Also, the statistical analysis presented earlier did indicate that industry can be a significant factor on the cost and stability performance, but it did not provide the means to conclude on which industry related cost and capacity factors are more influential on the results. For these reasons, we will present an additional sensitivity analysis through experimental design to further investigate the behavior and performance of the Fuzzy models under other possible industrial variations. This sensitivity analysis will also enable us to take a closer look at each industry related parameter to identify the statistically significant ones in terms of both planning cost and stability.

The experimental design model chosen for this sensitivity analysis is Plackett-Burmann design. This design is selected as opposed to other designs (such as full or partial factorial analysis) since it can handle many factors with fewer number of runs. The selected Plackett-Burman design for the five industry related factors is shown in Table 4.10. As one can see there are 12 different scenarios, which correspond to 12 different possible industries. We would like to note that even with these reduced 12 industry scenarios the total number of computations are quite high since for each scenario, we run 32 scenarios as explained earlier, which reflect the demand plus fuzzy technique variations. Hence for this additional analysis 384 additional optimization simulations were conducted. In order to generate the low and high values for the 12 different scenarios, we utilized the minimum and maximum possible factor values for the 5 Industry Cases analyzed earlier. We would like to remark that the selected industry factors for this analysis shown in Table 4.10, slightly differs from the main industry factors that were listed as in Table 4.1. Two modifications were carried out in Table 4.9 to take care of the dependencies between some of these factors. More specifically, since the magnitude of the hiring and layoff costs in a specific industry are related, we set both of these factors at their high or low values at the same time. As a result, we consider hiring and layoff almost like as a “single factor” from an experimental scenario generation perspective. Another set of variables with dependencies are the regular and overtime workforce cost. For different industry types, the overtime over regular workforce cost ratio is changing from 1.1 to 3, which makes the overtime cost always higher than the regular workforce cost. Using the regular and overtime costs as two factors, could result in designs with very strange ratios that might not be realistic (<1). In order to make it more realistic, we take the regular workforce cost and the ratio of the regular and overtime costs as another factor. This way, we control the amount of overtime cost, taking into account the regular workforce cost. With these modifications, the design consists of 7 factors.

Table 4.10: Plackett-Burman design

Design	C^W	C^O/C^W	C^H (C^L)	C^P	***b***	***h***	m^R
1	Low	High	High	Low	High	Low	Low
2	High	High	Low	High	High	Low	High
3	High	High	High	Low	High	High	Low
4	High	High	Low	High	Low	Low	Low
5	High	Low	High	High	Low	High	Low
6	Low	High	High	High	Low	High	High
7	Low	Low	High	High	High	Low	High
8	Low	Low	Low	Low	Low	Low	Low
9	High	Low	High	Low	Low	Low	High
10	High	Low	Low	Low	High	High	High
11	Low	Low	Low	High	High	High	Low
12	Low	High	Low	Low	Low	High	High

Figure 4.26 includes the average cost performance of the Fuzzy FRP-APP and Fuzzy APP models using either fuzzy technique over 12 Industry Cases, and Figure 4.27 includes the relative stability results. While the overall results show relatively close cost values for all models, there are 3 Industry scenarios (3,5,9) in which the overall cost of all models are higher that other Cases. From Table 4.10, we see that these Cases correspond to the industry scenarios with highest workforce and hiring/layoff costs.

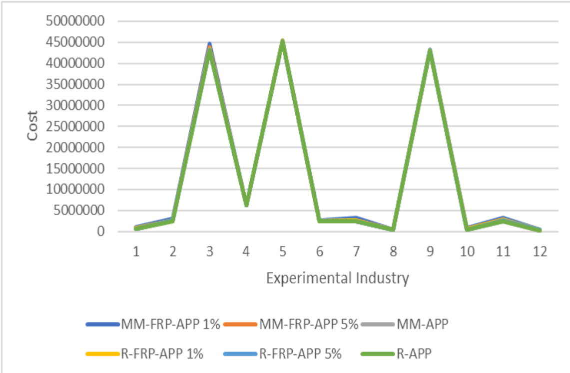

Figure 4.26: Total current cost comparison of Fuzzy FRP-APP and Fuzzy APP model, 12 experimental industries, Averaged over 16 demand scenarios

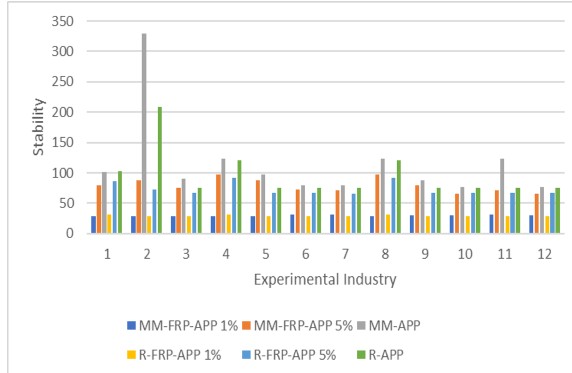

Figure 4.27: Stability comparison of Fuzzy FRP-APP and Fuzzy APP model, 12 experimental industries, Averaged over 16 demand scenarios

The ANOVA results shown in Table 4.11 concur that the cost analysis of the models using each of the fuzzy techniques are unanimously affected by the workforce cost and the workforce hiring and layoff costs. This could explain the sharp change in the cost values in Figure 4.26 for designs 3,5,9 and also the sharp line slope in Figure 4.24. Other influential factors on the stability of the models, especially the Fuzzy APP and Fuzzy FRP-APP 5%, include the inventory cost and the production capacity, which refers to the fact that the inventory considerations could affect the production amount changes over re-planning and hence the stability of the production plans.

Table 4.11: Plackett-Burman design results for influential factors effect on cost and stability, Fuzzy models, averaged demand results

		C^W	C^O/C^W	C^H/C^L	C^P	b	h	m^R
Max-Min	APP Cost	√		√				√
	APP Stability						√	
	FRP-APP Cost 1%	√		√				√
	FRP-APP Stability 1%							
	FRP-APP Cost 5%	√		√				√
	FRP-APP Stability 5%						√	√
Ranking	APP Cost	√		√				√
	APP Stability						√	
	FRP-APP Cost 1%	√		√				√
	FRP-APP Stability 1%							
	FRP-APP Cost 5%	√		√			√	
	FRP-APP Stability 5%						√	√

4.10 Conclusions

In this chapter, we analyzed the performance of the FRP-APP and the APP models with fuzziness in future demand estimations. We used two fuzzy programming techniques called: Fuzzy Max-Min (MM) and Fuzzy Ranking (R) to transform the APP into a fuzzy model and also to developed two new Fuzzy FRP-APP formulations (Fuzzy MM-FRP-APP and Fuzzy R-FRP-APP) to further analyze the performance of the FRP-APP taking into account both input uncertainty and stability concerns at the same time. The analysis of results are done for five Industry-based Cases and twelve hypothetical Industry scenarios (using Design of Experiments techniques) with different cost and capacity characteristics using 16 demand scenarios and 5 levels for flex-limits in the FRP-APP model.

In order to compare the FRP-APP and APP planning models, the FRP-APP is initially assumed to have a deterministic structure which uses the forecasted demand estimations for the planning purposes while the updated estimations are used in different re-planning iterations and the stability is controlled through flex-limits considerations in the model. The APP model on the other hand, uses the fuzziness programming techniques as a mechanism for capturing uncertainties in the planning procedure. Another set of analysis is done using newly developed Fuzzy FRP-APP and also the Fuzzy APP models. This way, the FRP-APP not only takes into consideration the uncertainty of the input parameters in the planning phase, but it also uses the flex-bounds as a stability control mechanism over various plan updates. The Fuzzy APP on the other hand, puts its effort just on handling initial input parameter uncertainties without any concern about plan stability over future plan updates.

The results indicate the FRP-APP and the Fuzzy FRP-APP show almost the same pattern in their cost and stability as compared to the Fuzzy APP models, especially if formulated using the Fuzzy Ranking method. As a result, the Fuzzy FRP-APP formulations results in a robust model as it provides a flexible planning framework capable of taking into account the input data uncertainties when developing production plans, but also having control over plan stability when re-planning is necessary.

The 1% flex limits promise a reliably better stability with occasional higher cost as compared to other flex limits scenarios and specially the APP model. While other flex-limits cases may result in lower cost values, there is a trade-off with the stability performance. Therefore, the planner may need to decide on the best flex-limit based on the desired trade-off for their company.

According to our experimental analysis on the fuzzy technique, demand scenario components, and also cost and capacity structures, the influential factors on the Fuzzy FRP-APP and Fuzzy APP models performances are identified which could be useful to explain why and how each model performance is changing if influential parameters are changed when tested on other Cases with different structures. In general, demand base, trend and seasonality are the most influential factors on cost and stability of different models. In addition, the stability is also affected by the consideration and change of flex-limits magnitude and also the selection of fuzzy technique. The effect of fuzzy technique is more visible in the Fuzzy APP and in the Fuzzy FRP-APP models this effect gets less visible moving from 5% to 1% flex-limits sets.

Taking into consideration the costs and production capacity of different test Cases, the models cost performance are mainly affected by workforce related costs and the capacity of the production systems while the stability shows more vulnerability to the inventory and also the production capacity.

Chapter 5: STOCHASTIC AGGREGATE PLANNING WITH FLEXIBLE REQUIREMENTS PROFILE

5.1 Introduction

In this chapter, we continue our analysis on the FRP-APP and APP models' performance where uncertainties in the optimization models are incorporated using stochastic programming. In other words, the main idea in this chapter is the same as Chapter 4, but all demand uncertainties are modeled as stochastic in the APP and later, on the FRP-APP formulations. We use the robust-stochastic (RS) and the chance-constraint (CC) formulations for developing the stochastic models. The computational and experimental analysis of the models will be in accordance with Chapter 4.

5.2 Introduction to Stochastic Programming

Stochastic programming optimization models deal with the cases where some or all input data of the model are uncertain parameters taking the advantage of the fact that probability distributions related to the uncertain data are assumed to be known or can be estimated (Kazemi Zanjani, Nourelfath et al. 2010). The goal is to find some policy that is feasible for (almost) all possible data instances and optimizes the objective function of the model. Due to the diversity in the way each stochastic parameter is defined and as a result of overwhelming number of scenarios for each uncertainty, solving stochastic optimization problems could result in very huge sizes to deal with. Consequently, the optimization problem could become infeasible to solve due to its complexity even with powerful solvers and computers. However, there are methods to make the problem smaller and easier to solve.

Chance-constraint programming is one of the popular stochastic programming techniques where one or more uncertain constraints are represented in a way to meet a pre-specified fulfillment degree (Wu, Huang et al. 2015). Based on the decision maker's desire, the chance of meeting the uncertain constraints could make those constraints more or less restricting factors in the optimization model. Chance-constraint programming gives flexibility to the decision maker for making the model harder or softer in terms of the restriction imposed to the model for holding the uncertain constraints.

Scenario-based stochastic techniques are another set of stochastic programming where a set of discrete scenarios represent how the future uncertainties are forecasted. Each scenario is associated with a probability value, which could be the decision maker's expectation for the occurrence of that specific scenario. One major group of scenario-based formulations are called: stage-based programming formulations, while there exist other scenario-based formulations like: joint robust-stochastic formulation.

Next, we will present the chance-constraint programming and the robust-stochastic formulations to later transform the APP and the FRP-APP into their stochastic counterparts for the analysis purposes.

5.3 Stochastic Chance-Constraint (CC) Programming

In a chance-constraint approach some of the constraints could be formulated such that they are fulfilled at selected minimum probability levels ($\alpha \leq 1$). A chance constraint with $\alpha=1$ is equivalent to a deterministic constraint. As discussed earlier, chance-constraints in aggregate production and supply chain planning are generally incorporated to represent demand or capacity uncertainties (Bilsel and Ravindran 2011). In addition, the uncertainty could be the case for the cost parameters in the objective function as well. Depending on where the uncertainty exists in the model, the related formulation for transforming the stochastic model into the deterministic equivalent could also change. Here we represent the general formulations techniques under various categories of uncertainties as discussed in Nazemi and Tahmasbi (2013).

Let's assume the following general chance-constraint optimization formulation:

$$Minimize\ F(x) = \sum_{j=1}^{n} c_j\, x_j \tag{5.1}$$

Subject to

$$P\left[\sum_{j=1}^{n} a_{ij}x_{ij} \leq b_i\right] \geq p_i \qquad \forall i=1,\ldots,m \tag{5.2}$$

$$x_j \geq 0 \qquad \forall j=1,\ldots,n \tag{5.3}$$

Where c_j, a_{ij} and b_i could be random variables with normal distributions and p_i are specified probabilities.

<u>Category1</u>: a_{ij} uncertainty only

We assume the stochastic a_{ij} has the mean $\bar{a}_{ij}$ and $Var(a_{ij}) = {\sigma_{a_{ij}}}^2$. In addition, by knowing the multi-variate distribution of $a_{ij}, i = 1,2,\ldots,m; j = 1,2,\ldots,n$ along with the covariance $Cov(a_{ij}, a_{kl})$ between the random variables a_{ij} , we can define d_i as:

$$d_i = \sum_{j=1}^{n} a_{ij}x_j \qquad \forall i=1,\ldots,m \tag{5.4}$$

If $a_{i1}, a_{i2}, \ldots, a_{in}$ are normally distributed, and $x_1, x_2, \ldots, x_n$ are constants (not yet known), d_i will also be normally distributed with a mean value of

$$\bar{d}_i = \sum_{j=1}^{n} \bar{a}_{ij}x_j \qquad \forall i=1,\ldots,m \tag{5.5}$$

And a variance of

$$Var(d_i) = {\sigma_{d_i}}^2 = x^T V_i x \qquad \forall i=1,\ldots,m \qquad (5.6)$$

Where V_i is the ith covariance matrix defined as

$$V_i = \begin{bmatrix} Var(a_{i1}) & Cov(a_{i1,}a_{i2}) & \ldots & Cov(a_{i1,}a_{in}) \\ Car(a_{i2,}a_{i1}) & Var(a_{i2}) & \ldots & Cov(a_{i2,}a_{in}) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ Cav(a_{in,}a_{i1}) & Cov(a_{in,}a_{i2}) & \ldots & Var(a_{in}) \end{bmatrix}$$

As a result, the chance-constraint can be expressed as:

$$P[d_i \leq b_i] \geq p_i \qquad \forall i=1,\ldots,m \qquad (5.7)$$

i.e.

$$P[\frac{d_i - \bar{d}_i}{\sqrt{Var(d_i)}} \leq \frac{b_i - \bar{d}_i}{\sqrt{Var(d_i)}}] \geq p_i \qquad \forall i=1,\ldots,m \qquad (5.8)$$

Where $\frac{d_i - \bar{d}_i}{\sqrt{Var(d_i)}}$ is a standard normal variable with a mean of zero and a variance of one. As a result, the probability of realizing d_i smaller than or equal to b_i can be written as

$$P[d_i \leq b_i] = \varphi(\frac{b_i - \bar{d}_i}{\sqrt{Var(d_i)}}) \qquad \forall i=1,\ldots,m \qquad (5.9)$$

Where $\varphi(x)$ is the cumulative distribution function of the standard normal distribution evaluated at x. If e_i denotes the value of the standard normal variable at which $\varphi(e_i) = p_i \geq 0.5 \ \ (e_i \geq 0)$, then we have:

$$\left(\frac{b_i - \bar{d}_i}{\sqrt{Var(d_i)}}\right) \geq e_i \geq 0 \qquad \forall i=1,\ldots,m \qquad (5.10)$$

Or

$$\bar{d}_i + e_i\sqrt{Var(d_i)} - b_i \leq 0 \qquad \forall i=1,\ldots,m \qquad (5.11)$$

Which is equivalent to

$$\sum_{j=1}^{n} \bar{a}_{ij}x_j + e_i\sqrt{x^T V_i x} - b_i \leq 0 \qquad \forall i=1,\ldots,m \qquad (5.12)$$

If the random variables a_{ij} are independent, the covariance terms will be zero and the related matrix would reduce to the following:

$$V_i = \begin{bmatrix} Var(a_{i1}) & 0 & \ldots & 0 \\ 0 & Var(a_{i2}) & \ldots & 0 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & \ldots & Var(a_{in}) \end{bmatrix} \qquad (5.13)$$

As a result, we will have:

$$\sum_{j=1}^{n} \bar{a}_{ij}x_j + e_i\sqrt{\sum_{j=1}^{n}[Var(a_{ij}){x_j}^2]} - b_i \leq 0 \qquad \forall i=1,\ldots,m \qquad (5.14)$$

<u>Category2</u>: *b_i uncertainty only*

We assume the stochastic b_i has the mean $\bar{b}_i$ and $Var(b_i) = {\sigma_{b_i}}^2$. The chance-constraint in the model could be transformed in the following way:

$$P\left[\sum_{j=1}^{n} a_{ij}x_{ij} \leq b_i\right] = P\left[\frac{\sum_{j=1}^{n} a_{ij}x_j - \bar{b}_i}{\sqrt{Var(b_i)}} \leq \frac{b_i - \bar{b}_i}{\sqrt{Var(b_i)}}\right] = P\left[\frac{b_i - \bar{b}_i}{\sqrt{Var(b_i)}} \geq \frac{\sum_{j=1}^{n} a_{ij}x_j - \bar{b}_i}{\sqrt{Var(b_i)}}\right] \geq p_i \forall i=1,\ldots,m \qquad (5.15)$$

Under normality assumption, $\frac{b_i - \bar{b}_i}{\sqrt{Var(b_i)}}$is a standard normal variable with zero mean and unit variance. We can further continue the above formulation as:

$$P\left[\frac{b_i - \bar{b}_i}{\sqrt{Var(b_i)}} \leq \frac{\sum_{j=1}^{n} a_{ij}x_j - \bar{b}_i}{\sqrt{Var(b_i)}}\right] \leq 1 - p_i \qquad \forall i=1,\ldots,m \qquad (5.16)$$

If E_i represents the value of the standard normal variance at which $\varphi(E_i) = 1 - p_i < 0.5 \quad (E_i < 0)$,

As a result

$$\frac{\sum_{j=1}^{n} a_{ij}x_j - \bar{b}_i}{\sqrt{Var(b_i)}} \le E_i \qquad \forall i=1,\dots,m \qquad (5.17)$$

Or

$$\sum_{j=1}^{n} a_{ij}x_j - \bar{b}_i - E_i\sqrt{Var(b_i)} \le 0 \qquad \forall i=1,\dots,m \qquad (5.18)$$

<u>Category3</u>: *c_j uncertainty only*

Assuming all c_js to be normally distributed random variables, the objective function $F(x)$ will also be a normal random variable. As a result, the mean and variance of F are given by:

$$\bar{F} = \sum_{j=1}^{n} \bar{c}_j\, x_j \qquad (5.19)$$

And

$$Var(F) = x^T V x \qquad (5.20)$$

Where $\bar{c}_j$ is the mean value of c_j and the matrix V is the covariance matrix of c_j defined as:

$$V = \begin{bmatrix} Var(c_1) & Cov(c_{1,}c_2) & \dots & Cov(c_{1,}c_n) \\ Car(c_{2,}c_1) & Var(c_2) & \dots & Cov(c_{2,}c_n) \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ Cav(c_{n,}c_1) & Cov(c_{n,}c_2) & \dots & Var(c_n) \end{bmatrix} \qquad (5.21)$$

Where $Var(c_j)$ and $Cov(c_{i,}c_j)$ denoting the variance of c_j and covariance between c_i and c_j respectively.

A new deterministic objective function for minimization can be formulated as:

$$F(x) = k_1\bar{F}+k_2\sqrt{Var(F)} \quad (5.22)$$

Where k_1 and k_2 are nonnegative constants which indicate the relative importance of $\bar{F}$ and standard deviation of F for minimization ($k_1 + k_1 = 1$). If the random variables c_j are independent, the objective function changes to:

$$F(x) = k_1 \sum_{j=1}^{n} \bar{c}_j\, x_j+k_2\sqrt{\sum_{j=1}^{n} Var(c_j)x_j^2} \quad (5.23)$$

<u>*Category4*</u>: a_{ij}, b_i, c_j *uncertainty*

For this uncertainty situation, the objective function would be treated the same as <u>*Category3*</u>. In addition, as there are uncertainties both in the right and left hand side of the chance-constraints, the whole constraint could be transformed to the following form by introducing new variables h_i:

$$h_i = \sum_{j=1}^{n} a_{ij}x_j - b_i = \sum_{k=1}^{n} q_{ik}y_k \qquad \forall i=1,\ldots,m \quad (5.24)$$

And the chance constraint would be:

$$P[h_i \leq 0] \geq p_i \qquad \forall i=1,\ldots,m \quad (5.25)$$

Since h_i is a function of other uncertain parameters (Assumed to have the same type of distribution functions), the mean and standard deviation of this variable is also a function of known parameters' distribution functions. By defining the mean and variance of the variable h_i the same approach as <u>*Category3*</u> could be applied. More details about this uncertainty category is presented in Nazemi and Tahmasbi (2013).

5.4 Stochastic Chance-Constraint APP (CC-APP)

The main uncertainties in our formulation is related to demand, presented in the form of an equality. Since the probability of a stochastic value being equal to a specific value is zero, we need to transform this constraint into an inequality formulation so that the main idea presented in *Category2* could be utilized. We use the transformation technique introduced by Yıldırım, Tan et al. (2005) and Aouam and Uzsoy (2012) for dealing with stochastic inventory constraints in a production planning problem. The main idea behind this technique is focusing on satisfying the demand in each period (improving the customer satisfaction degree). Let's consider the following inventory balance constraint for a multi-period aggregate production planning problem (Aouam and Uzsoy 2012):

$$I_t = I_{t-1} + P_t - D_t \qquad \forall t=1,\ldots,\text{T} \qquad (5.26)$$

Assuming a random demand, the finished inventory at the end of period t, I_t is also a random variable. Taking the expectation and with repetitive substitution in Constraint (5.26), we will have:

$$E[I_t] = E[I_{t-1}] + P_t - E[D_t] = I_0 + \sum_{j=1}^{t} P_i - \sum_{j=1}^{t} E[D_j] = I_0 + \sum_{j=1}^{t} P_j - \sum_{j=1}^{t} \mu_j \quad \forall t=1,\ldots,\text{T} \qquad (5.27)$$

Using Equation (5.27), the unit inventory holding cost could be multiplied by the right hand side of this equation to estimate the expected inventory cost in each period. Also, considering the service level as a chance-constraint, we have the following formulation:

$$P(I_t \geq 0) \geq \alpha \rightarrow P\left(I_0 + \sum_{j=1}^{t} P_j - \sum_{j=1}^{t} D_j \geq 0\right) \geq \alpha \qquad \forall t=1,\ldots,\text{T} \qquad (5.28)$$

In order to transform Constraint (5.28) into a linear form, we use the cumulative demand distribution function as follows:

$$P\left(\sum_{j=1}^{t} D_j \leq I_0 + \sum_{j=1}^{t} P_j\right) \geq \alpha \rightarrow F_{\sum_{j=1}^{t} D_j}\left(I_0 + \sum_{j=1}^{t} P_j\right) \geq \alpha \qquad \forall t=1,\ldots,\text{T} \qquad (5.29)$$

As a result, the final chance-constraint is equivalent to:

$$I_0 + \sum_{j=1}^{t} P_j \geq F^{-1}_{\sum_{j=1}^{t} D_j}(\alpha) \qquad \forall t=1,\ldots,\text{T} \qquad (5.30)$$

In order to replace the above formulation with a corresponding chance-constraint for each period, the distribution of the aggregated demand up to that specific period ($\sum_{i=1}^{t} D_i$) needs to be identified to be able to compute the inverse α-level cumulative distribution using the aggregate demand. Assuming a Normal distribution for different periods' demand distributions and independence between demand of consecutive periods, the aggregated demand will also have a normal distribution $\sum_{i=1}^{t} D_i \sim N(\sum_{j=1}^{t} \mu_j, \sum_{j=1}^{t} \delta_j^2)$.

Utilizing the above transformation, the CC-APP model is as follows:

(CC-APP)

Minimize $\sum_{i=0}^{N}(c^w.th.W_{t,i} + c^o.O_{t,i} + c^H.H_{t,i} + c^L.L_{t,i} + c^p.P_{t,i} + h.(I_{t,0} + \sum_{j=0}^{i} P_{t,j} - \sum_{j=0}^{i} \mu_{t,j}))$ (5.31)

Subject to

$$P_{t,0} = d_{t,0} + I_{t,0} - I_{t-1,0} \qquad (5.32)$$

$$I_{t,0} + \sum_{j=1}^{i} P_{t,i} \geq F^{-1}_{\sum_{j=1}^{i} D_{t,i}}(\alpha) \qquad \forall i=1,\ldots,\text{N} \qquad (5.33)$$

$$I_{t,0} + \sum_{j=1}^{N} P_{t,j} \geq F^{-1}_{\sum_{j=1}^{N} D_{t,j}}(\alpha) + I \qquad (5.34)$$

$$W_{t,0} = W_{t-1,0} + H_{t,0} - L_{t,0} \qquad (5.35)$$

$$W_{t,i} = W_{t,i-1} + H_{t,i} - L_{t,i} \qquad \forall i = 1, \ldots, N \qquad (5.36)$$

$$P_{t,i} \leq m^R.th.W_i + m^R.O_i \qquad \forall i = 0, \ldots, N \qquad (5.37)$$

$$O_i \leq th.W_{t,i}.m^O \qquad \forall i = 0, \ldots, N \qquad (5.38)$$

$$W_{t,i}, O_{t,i}, H_{t,i}, L_{t,i}, P_{t,i}, I_{t,i} \geq 0 \qquad \forall i = 0, \ldots, N \qquad (5.39)$$

$$W_{t,i}, H_{t,i}, L_{t,i}: \text{integers} \qquad \forall i = 0, \ldots, N \qquad (5.40)$$

5.5 Stochastic Chance-Constraint FRP-APP (CC-FRP-APP)

Adding flexible bounds Constraint sets (3.21)-(3.23), we will have the CC-FRP-APP as follows:

(CC-FRP-APP) = (CC-APP) + Constraints (3.21)-(3.23)

The CC-FRP-APP addresses 3 issues at the same time: 1. Incorporation of stochastic demand in its formulation, 2. Improving customer service levels through enforcing chance-constraints on probabilities of meeting customer demand in each period, and 3. Stability improvement of the planning approach over various rolling horizon planning iterations.

5.6 Robust-Stochastic (RS) Programming

The robust-stochastic formulation is a scenario-based formulation, which addresses both solution and model robustness as two components of the objective function. The solution robustness makes sure the model remains close to optimality for any uncertain scenario realization, while the model robustness part makes sure the model remains “almost feasible” for any scenario realization as further explained below.

Two sets of variables can be defined: design variables and control variables. The design variables are fixed while the control variables are responsible for any realization of uncertain parameters. Let’s assume a finite set of scenarios to model the uncertain parameters and coefficients while with each scenario s, there is an associated probability $P_s(\sum_s P_s = 1)$ and a subset $(\delta_s, B_s, C_s, e_s)$., where B_s and C_s are scenario-based coefficients, e_s is the right-hand-side scenario-based parameter, and δ_s is a scenario-based variable accounting for feasibility adjustment of the constraint over various scenarios (if the model is feasible under all scenarios, δ_s is equal to 0 for all s). Also, a control variable y may be subject to adjustment when one scenario is realized and can be denoted by y_s. The general scenario-based formulation is provided below:

$$Min\ \sigma(x, y_1, y_2, \dots, y_s) + \omega\rho(\delta_1, \delta_2, \dots, \delta_s) \tag{5.41}$$

Subject to

$$Ax = b \tag{5.42}$$

$$B_s x + C_s y_s + \delta_s = e_s \qquad \forall\ s\epsilon S \tag{5.43}$$

$$x \geq 0,\ \delta_s, y_s \geq 0 \qquad \forall\ s\epsilon S \tag{5.44}$$

The first term takes care of a solution's robustness, which mainly consists of the summation of expected objective value and weighted variance as formulated below:

$$\sigma(x, y_1, y_2, \dots, y_s) = \sum_{s\epsilon S} P_s(C^T x + d^T{}_s y_s) + \Lambda \sum_{s\epsilon S} P_s(C^T x + d^T{}_s y_s - \sum_{s'\epsilon S} P_{s'}(C^T x + d^T{}_{s'} y_{s'}))^2 \tag{5.45}$$

Λ is the importance weight for objective variability over various stochastic scenarios and a higher Λ makes the solution less sensitive to the changes in data under various scenarios (Mulvey, Vanderbei et al. 1995). Please note that the quadratic formulation in (5.45) makes it difficult to solve. Yu and Li (2000) proposed a transformation to change equation as follows using the absolute value approach for representing variability.

$$\sigma(x, y_1, y_2, \dots, y_s) = \sum_{s\epsilon S} P_s(C^T x + d^T{}_s y_s) + \Lambda \sum_{s\epsilon S} P_s |C^T x + d^T{}_s y_s - \sum_{s'\epsilon S} P_{s'}(C^T x + d^T{}_{s'} y_{s'})| \tag{5.46}$$

While Equation (5.46) is still a nonlinear function, it can be linearized as follows (Yu and Li 2000, Leung, Lai et al. 2007):

$$Z = Min\ \sum_{s\epsilon S} P_s(C^T x + d^T{}_s y_s) + \lambda \sum_{s\epsilon S} P_s\left[C^T x + d^T{}_s y_s - \sum_{s'\epsilon S} P_{s'}\left(C^T x + d^T{}_{s'} y_{s'}\right) + 2\theta_s\right] \quad (5.47)$$

Subject to

$$C^T x + d^T{}_s y_s - \sum_{s\epsilon S} P_s(C^T x + d^T{}_s y_s) + \theta_s \geq 0 \qquad \forall\ s\epsilon S \quad (5.48)$$

$$\theta_s \geq 0 \qquad \forall\ s\epsilon S \quad (5.49)$$

With this modification, the quadratic formula in (5.45) is initially changed to the mean absolute deviation in (5.46) and finally to the linear formulation (5.47), where the variability of the solution under various scenarios is minimized.

The second term in the objective function ($\rho(\delta_1, \delta_2, \ldots, \delta_s)$) is corresponding to the infeasibility function to penalize the violation of control constraints (Constraint set (5.45)) under some scenarios. Using weight ω the trade-off between solution robustness and model robustness can be modeled. For the model robustness part, the infeasibility variable (δ_s) could be related to any specific feasibility limitation consideration or logical infeasibility in the control Constraint (5.43) for some scenarios. For example, in Leung, Lai et al. (2007) the inventory balance constraint is assumed as a control constraint and shortage level is introduced as the infeasibility variable as no shortage is allowed and the shortage cannot carry over to next periods demand. Another example is presented for the aggregate production planning problem in Mirzapour Al-E-Hashem, Malekly et al. (2011) where the control constraint determines the amount of products transferred to customers' zones and the amount of shortage in each period. This constraint is formulated in such a way that for each period, the customer demand is either fully fulfilled or there exist some shortage as product storage at the customers' zones is impossible, and any positive storage value indicates model infeasibility and gets penalized in the objective function.

5.7 Robust-Stochastic APP (RS-APP)

In our APP formulation, the demand uncertainty over the planning horizon can be defined using scenarios and as a result, the inventory balance constraint becomes the control constraint. The design variables could be defined as the production levels and the workforce variables. Similarly, the scenario-based variables, which will be controlling the scenario-based control constraint, can be taken as the inventory and shortage in each period. Please note that having both inventory and allowing shortages in each period would always result in a feasible solution under different demand scenarios. We now continue with the RS-APP formulation following the Robust-Stochastic formulation.

Let Z_s denote the scenario-based cost objective as follows:

$$Z_s = \sum_{i=0}^{N}(c^w.th.W_{t,i} + c^o.O_{t,i} + c^H.H_{t,i} + c^L.L_{t,i} + c^p.P_{t,i} + h.I_{t,i,s} + b.B_{t,i,s}) \quad (5.50)$$

Then the robust-stochastic APP formulation can be written as follows:

(RS-APP)

$$\text{Minimize } Z = \sum_{s \epsilon S} P_s(Z_s) + \lambda \sum_{s \epsilon S} P_s[Z_s - \sum_{s' \epsilon S} P_{s'}(Z_{s'}) + 2\theta_s] \quad (5.51)$$

Subject to

$$Z_s - \sum_{s \epsilon S} P_s(Z_s) + \theta_s \geq 0 \qquad \forall\, s \epsilon S \qquad (5.52)$$

$$P_{t,0} = d_{t,0,s} + I_{t,0,s} - B_{t,0,s} - I_{t-1,0,s} + B_{t-1,0,s} \qquad \forall\, s \epsilon S \qquad (5.53)$$

$$P_{t,i} = d_{t,i,s} + I_{t,i,s} - B_{t,i,s} - I_{t,i-1,s} + B_{t,i-1,s} \qquad \forall i = 1, \ldots, N, \forall\, s \epsilon S \qquad (5.54)$$

$$I_{t,N,s} \geq I \qquad \forall\, s \epsilon S \qquad (5.55)$$

Constraints (5.35) - (5.38), (5.50)

$$W_{t,i}, O_{t,i}, H_{t,i}, L_{t,i}, P_{t,i}, I_{t,i,s},, B_{t,i,s}, \theta_s \geq 0 \qquad \forall i = 0, \ldots, N, \forall\, s \epsilon S \qquad (5.56)$$

$W_{t,i}, H_{t,i}, L_{t,i}$: integers $\forall i = 0, \dots, N$ (5.57)

It should be noted that Constraint (5.53) is also formulated in a scenario-based format for consistency purposes with the demand, inventory and backorder notations, but this constraint has a deterministic nature as the current period demand has a known single value.

5.8 Robust-Stochastic FRP-APP (RS-FRP-APP)

Adding flexible bounds Constraint sets (3.21)-(3.23), we will have the RS-FRP-APP as follows:

(RS-FRP-APP) = (RS-APP) + Constraints (3.21)-(3.23)

The RS-FRP-APP also considers various stochastic scenarios for the planning problem's future periods' demand estimations and aims at minimizing expected cost and the variations among different planning cost scenarios. In addition, due to the high levels of uncertainty in future demand estimations, in case the future scenarios get updated and a new plan is developed, the FRP bounds instabilities resulting from plan updates.

5.9 Computational Study

To conduct our computational analysis, apart from the forecasted demand to be used in deterministic models, for the periods with scenario-based stochastic demand, we need to generate these scenarios as well. In each rolling horizon iteration (t), the current period ($i = 0$) demand is a known parameter while the future periods' demand values are defined using scenarios (estimated demands). After the current period's plan is implemented, the next planning iteration ($t + 1$), again fixes the current period demand and could use updated demand scenarios for the future periods.

Using demand generation formulation, each period's demand is generated 40 times and later used to estimate the distribution, mean and standard deviation of each period demand for stochastic models. For the case of Robust-Stochastic models, as the number of scenarios to fully represent all possible scenarios for a stochastic parameter could be very large, determining an adequate number of scenarios in a scenario-based formulation could have a noticeable impact on the quality of the stochastic problem solution. One approach would be doing a sensitivity analysis on random numbers of scenarios tested to see how the quality of the solution changes. Another popular technique, typically used when a two-stage or recourse-based stochastic formulation is used, is applying the Sample Average Approximation algorithm, which through sampling replications, gives an estimation of the stochastic solution through finding the solution with the minimum optimality gap (Verweij, Ahmed et al. 2003). Another approach utilized by Mirzapour Al-e-Hashem, Baboli et al. (2013) is using the statistical confidence intervals. The main idea behind this method is to specify the minimum number of scenarios by the preferred level of accuracy of the solution. According to this method, the Monte Carlo sampling variance estimator of the results for a stochastic programming problem, which is independent of the probability distribution of the uncertain parameters can be defined as:

$$S(n) = \sqrt{\frac{\sum_{s=1}^{n}(E(Z)-Z_s)^2}{n-1}} \tag{5.58}$$

Where n is the number of scenarios, $E(Z)$ is the mean of all scenarios objective function values, and Z_s is the total cost of scenario s. Then the confidence interval of $1-\alpha$ is as:

$$[E(Z) - \emptyset_{\frac{\alpha}{2}}\frac{S(n)}{\sqrt{n}}, E(Z) + \emptyset_{\frac{\alpha}{2}}\frac{S(n)}{\sqrt{n}}] \tag{5.59}$$

Where $\emptyset_{\frac{\alpha}{2}}$ is a quantity for which the Equation (5.63) for a standard normal random variable $\varphi \approx N(\mu = 0, \sigma = 1)$ will be satisfied (for example: according to the standard normal distribution table, for $\alpha = 0.05, \emptyset_{\frac{\alpha}{2}} = 1.96$):

$$\Pr\left(\varphi \leq \emptyset_{\frac{\alpha}{2}}\right) = 1 - \frac{\alpha}{2} \tag{5.60}$$

As a result, if the sampling estimator $S(n)$ and the maximum tolerable error (er) for confidence level $1 - \frac{\alpha}{2}$ are given, the minimum required number of scenarios can be determined by:

$$n^{`} \geq [\frac{\emptyset_{\alpha/2} S(n)}{er}]^2 \tag{5.61}$$

Therefore, to determine the minimum number of scenarios $n^{`}$, we can first solve the stochastic programming model with a small number of scenarios n to estimate the required parameters in the above formula. The idea here is to control the variability of the objectives over various scenarios to minimize the sensitivity of the objective values to different scenarios. As our scenario-based formulation also aims at minimizing the variance of objectives over different stochastic scenarios, we expect the minimum required number of scenarios not to be very high. For this aim, we initially run the RS-APP model (as the basic scenario-based model) with $n = 10$ and asses the error to see if we need to run the models with the same or larger number of scenarios. Once the minimum number of scenarios for Stochastic APP is identified, we make sure the minimum number of scenarios we use to run any scenario-based model is greater than the minimum required sample size. We repeat the same for all 5 Cases, and 16 demand scenarios and use the maximum of all identified minimum sample sizes to further continue running scenario-based models.

5.9.1 Computational Results & Analysis

Our RS-APP results with $n = 10$ indicated for most of 5 Cases over 16 demand scenarios, the scenario-based model with $\Lambda = 1$, is able to well capture objective function variability over various scenarios and the error value is less than the desired value. There are a few Cases, however, for which the minimum needs to be not lower than $n^{`} = 20$. This implies that the model can absorb variability and get a good nominated mean objective value for the stochastic problem. For consistency purposes, however, we run all scenario-based models with 50 scenarios. In addition, the α value for all Chance-Constraint models is set equal to 0.75.

The comparative results of FRP-APP, and Stochastic APP compared models with the Stochastic APP model using either Chance-Constraint or the Robust-Stochastic formulations for different industries over various demand scenarios are presented in Figures 5.1-5.20. In addition, we have inserted the graphs demonstrating the sensitivity of the FRP-APP and sample Stochastic FRP-APP cost and stability performance to varying flex-limit sets in Figures 21-40 in Appendix.

On a general observation about the Stochastic APP models compared to the FRP-APP, with a slightly better cost performance, the CC-APP has a noticeable control over the input uncertainty and the stability of the plans as compared to the FRP-APP for almost all flexible bounds scenarios. This trend is more visible for the Automotive (Figure 5.6), Wood and Paper (Figure 5.14), and Air conditioning (Figure 5.18) industries. For the Textile (Figure 5.2) and Machinery and Transmission (Figure 5.10) Cases however, still the FRP-APP with 1% flexible-limits, shows reliable results in terms of stability.

When the problem setting and the available data, urges the utilization of the scenario-based formulation for the planning problems, without a significant adverse effect on the cost gap, the

FRP-APP outperforms the RS.APP in stability for all demand scenarios for different Industry Cases specially if 1% flex-limits are used. These inferences can be observed in the stability figures and also the stability ratio of different models as presented in Table 5.2.

If the FRP-APP is formulated as a stochastic model addressing both input uncertainty and stability concerns at the same time, it can outperform the Stochastic APP in stability regardless of which stochastic technique is used. Another interesting observation is that, Stochastic FRP-APP models show the potential to not only have better stability performance, but also to shrink the cost gap with the Stochastic APP, as compared to the deterministic FRP-APP (Table 5.1). This observation holds for all Industry Cases: Textile (Figures 5.1-5.4), Automotive (Figures 5.5-5.8), Machinery and Transmission (Figures 5.9-5.12), Wood and Paper (Figures 5.13-5.16), Air Conditioning Units (Figures 5.17-5.20). As a result, both stochastic techniques show promising results for the FRP-APP performance as an attractive modeling alternative, where the stochastic model uncertainty is captured besides controlling plan stability and cost.

Another general note on all stochastic models cost and stability specially the Stochastic APP is that the RS formulation tends to result in higher cost and more instable plans. This could be expected specially for the stability since the RS models deal with different scenarios of demand and being able to optimize the problem with the same stability performance as the Chance-Constraint models is more difficult. However, the RS-FRP-APP can still maintain a better planning performance. In addition, more restrictive flex-limit sets further highlight the noticeable stability improvement of the RS-FRP-APP as compared to the RS-APP model without a noticeable negative impact on the cost measure.

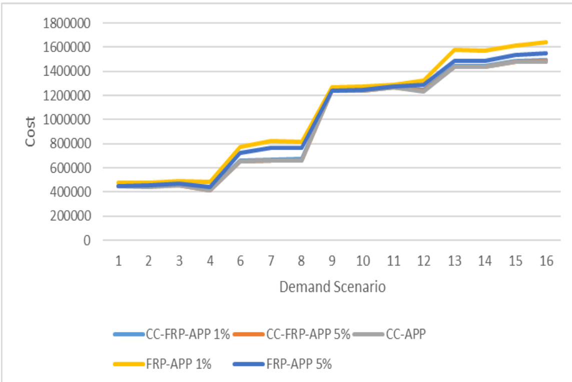

Figure 5.1: Total current cost comparison, FRP-APP, CC-FRP-APP, CC-APP, Textile Industry

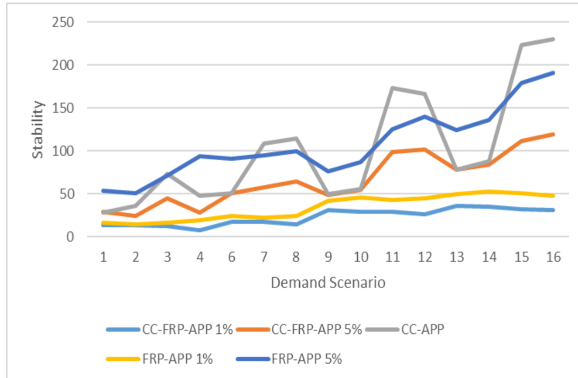

Figure 5.2: Stability comparison, FRP-APP, CC-FRP-APP, CC-APP, Textile Industry

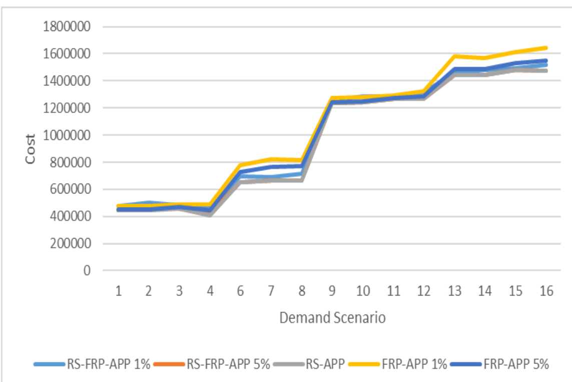

Figure 5.3: Total current cost comparison, FRP-APP, RS-FRP-APP, RS-APP, Textile Industry

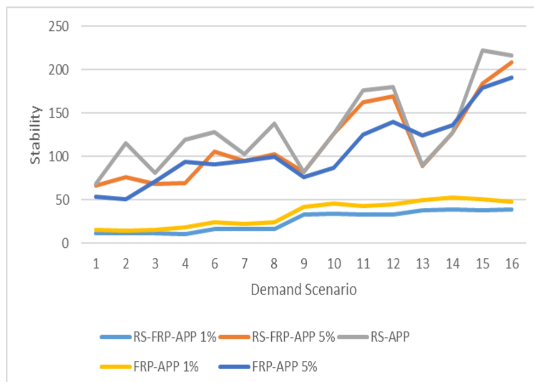

Figure 5.4: Stability comparison, FRP-APP, RS-FRP-APP, RS-APP, Textile Industry

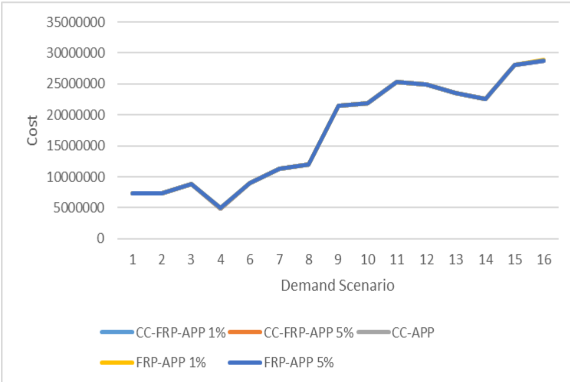

Figure 5.5: Total current cost comparison, FRP-APP, CC-FRP-APP, CC-APP, Automotive Parts Industry

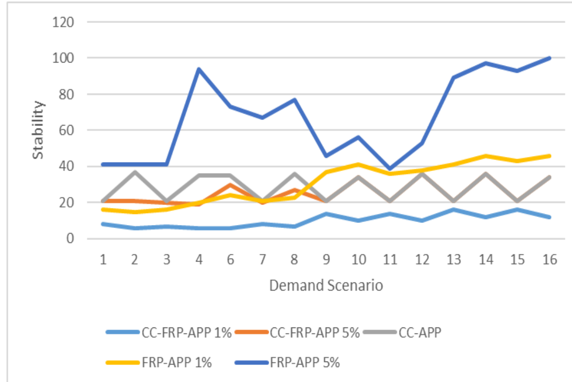

Figure 5.6: Stability comparison, FRP-APP, CC-FRP-APP, CC-APP, Automotive Parts Industry

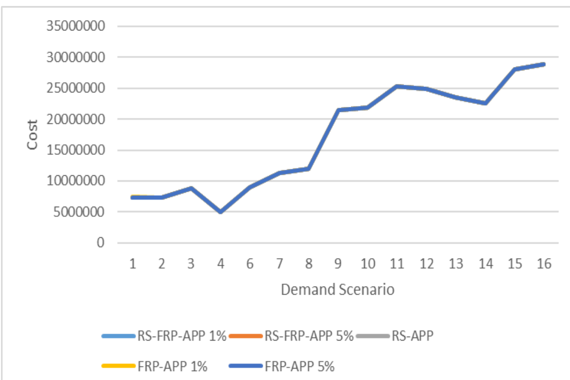

Figure 5.7: Total current cost comparison, FRP-APP, RS-FRP-APP, RS-APP, Automotive Parts Industry

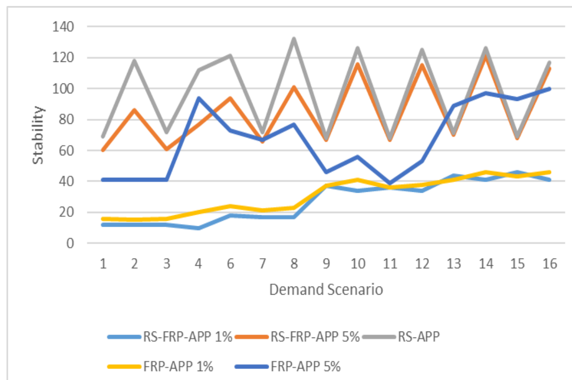

Figure 5.8: Stability comparison, FRP-APP, RS-FRP-APP, RS-APP, Automotive Parts Industry

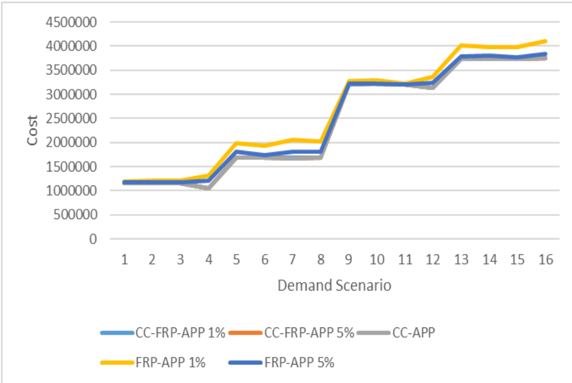

Figure 5.9: Total current cost comparison, FRP-APP, CC-FRP-APP, CC-APP, Machinery and Transmission Industry

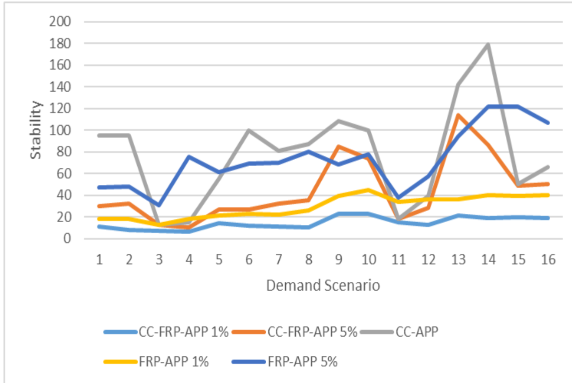

Figure 5.10: Stability comparison, FRP-APP, CC-FRP-APP, CC-APP, Machinery and Transmission Industry

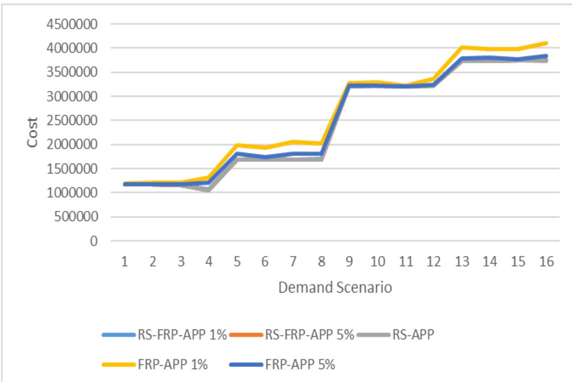

Figure 5.11: Total current cost comparison, FRP-APP, RS-FRP-APP, RS-APP, Machinery and Transmission Industry

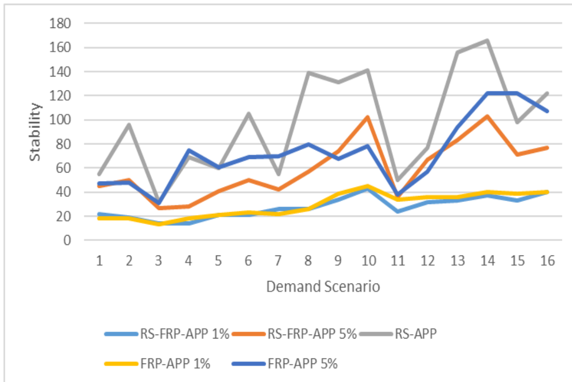

Figure 5.12: Stability comparison, FRP-APP, RS-FRP-APP, RS-APP, Machinery and Transmission Industry

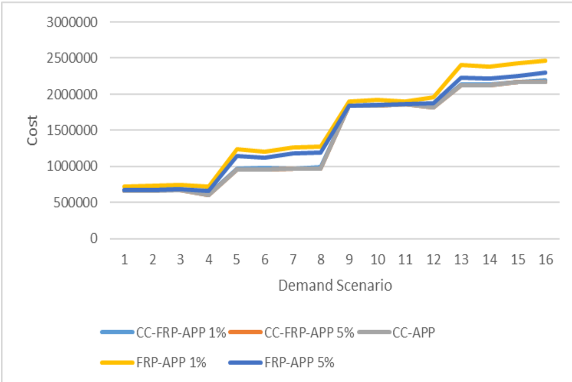

Figure 5.13: Total current cost comparison, FRP-APP, CC-FRP-APP, CC-APP, Wood and Paper Industry

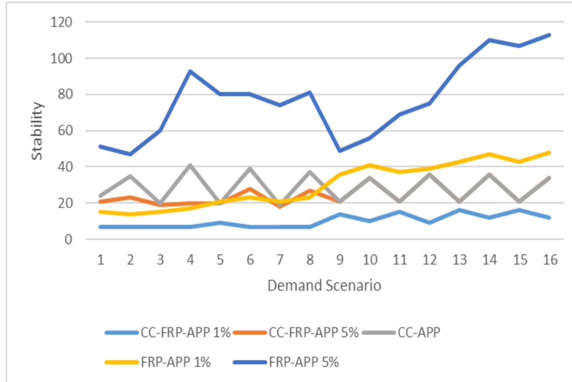

Figure 5.14: Stability comparison, FRP-APP, CC-FRP-APP, CC-APP, Wood and Paper Industry

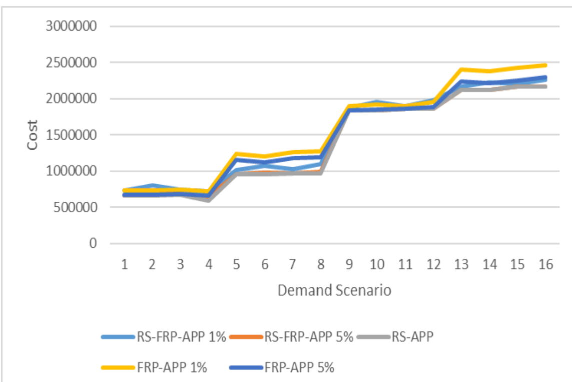

Figure 5.15: Total current cost comparison, FRP-APP, RS-FRP-APP, RS-APP, Wood and Paper Industry

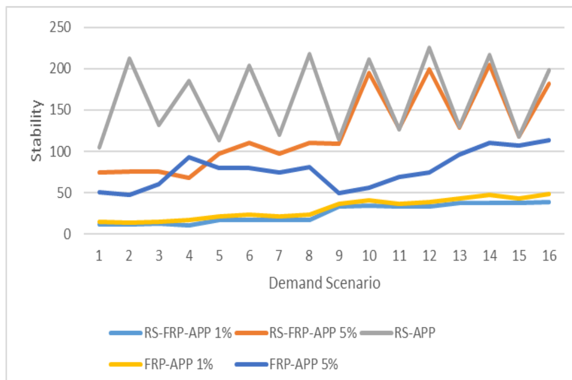

Figure 5.16: Stability comparison, FRP-APP, RS-FRP-APP, RS-APP, Wood and Paper Industry

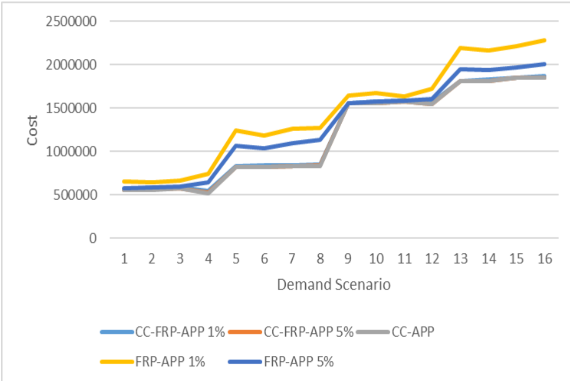

Figure 5.17: Total current cost comparison, FRP-APP, CC-FRP-APP, CC-APP, Air Conditioning Units Industry

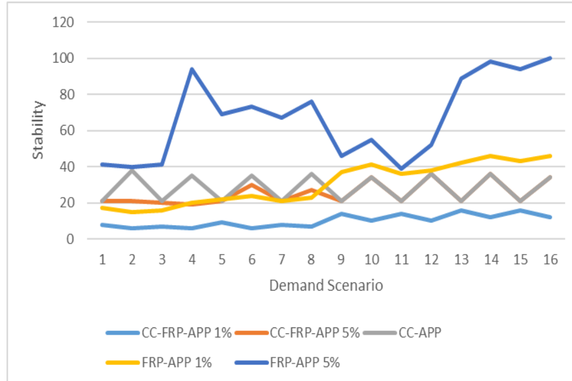

Figure 5.18: Stability comparison, FRP-APP, CC-FRP-APP, CC-APP, Air Conditioning Units Industry

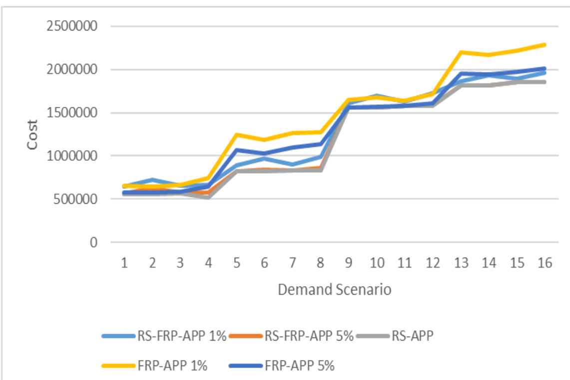

Figure 5.19: Total current cost comparison, FRP-APP, RS-FRP-APP, RS-APP, Air Conditioning Units Industry

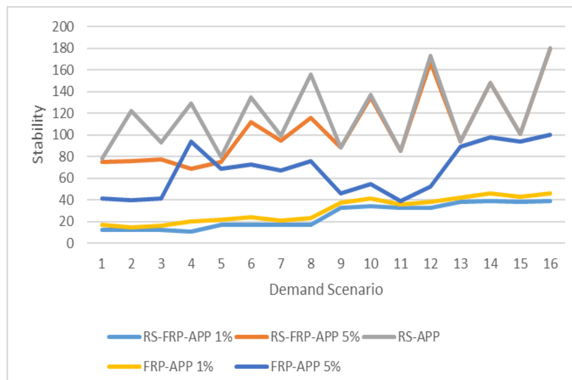

Figure 5.20: Stability comparison, FRP-APP, RS-FRP-APP, RS-APP, Air Conditioning Units Industry

Table 5.1: Average cost gap percentage of different models in different industries

	Textile	Automotive	Machinery & Transmission	Wood & Paper	Air Conditioning
CC-FRP-APP 1%	0.770	0.003	0.057	0.834	1.209
CC-FRP-APP 5%	0.192	0.000	0.000	0.138	0.175
CC-APP	0.000	0.000	0.008	0.000	0.000
RS-FRP-APP 1%	3.862	0.023	0.573	6.131	8.692
RS-FRP-APP 5%	0.463	0.005	0.394	0.888	1.158
RS-APP	0.191	0.003	0.268	0.069	0.206
FRP-APP 1%	9.926	0.159	8.144	12.700	21.618
FRP-APP 5%	4.763	0.061	2.634	6.165	9.639

*0.0 means the lowest average cost, and a cost gap percentage closer to 0.0 shows a better cost performance.

Table 5.2: Comparison of average instability ratio for different models in different industries

	Textile	Automotive	Machinery & Transmission	Wood & Paper	Air Conditioning
CC-FRP-APP 1%	1.00	1.00	1.00	1.00	1.00
CC-FRP-APP 5%	2.82	2.50	3.06	2.47	2.51
CC-APP	4.26	2.80	5.36	2.83	2.81
RS-FRP-APP 1%	1.10	2.66	1.89	2.48	2.50
RS-FRP-APP 5%	4.90	8.37	4.11	12.18	10.51
RS-APP	5.56	9.53	6.69	16.23	11.79
FRP-APP 1%	1.46	3.02	2.02	2.98	3.02
FRP-APP 5%	4.61	6.68	5.03	7.66	6.67

*1.0 means the best stability control, and a ratio closer to 1.0 shows a better stability performance.

As the comparative results, specially the stability performance of the stochastic models developed seem to be sensitive to the demand scenario, stochastic technique change and also occasional changes to different test Cases, we conduct the same experimental analysis as in Chapter 4 on the cost and stability of the stochastic models to different factors : demand generation components, stochastic technique and also cost and capacity parameters of various Case studies to identify influencing factors. Table 5.3 includes the main factors and their level values for running the GLM.

Figures 5.41, 5.42 present the main effect plots for each factor for cost and stability, respectively and the ANOVA results are presented in Tables 5.4, 5.5. The results for cost performance shown in Table 5.4 and Figure 5.41 indicate that, demand structure (base, trend, seasonality) and the industry Case have significant effect. However, flex-limits and stochastic technique change do not seem to have a significant impact. The stability however (as shown in Table 5.5 and Figure 5.42), is highly affected by the flex-limit and the choice of the stochastic technique followed by the Industry Case, and some of the demand related components (base, trend and magnitude of error).

Table 5.3: Experimental design components and designs to be tested, Stochastic models

Factors	***Number of Levels***	***Levels***
Demand-base	2	Low(1000),High(3000)
Demand-trend	2	Low(20),High(100)
Demand-seasonality	2	Low(+/-0.1),High((+/-0.3)
Demand-magnitude of error	2	Low(std=50),High(std=200)
Flex-limits	4	1%,3%,5%,No-flex-limits
Stochastic-technique	2	Chance-Constraint, Scenario-based
Industry	5	1,2,3,4,5

Table 5.4: Selected ANOVA results for plan cost, Stochastic FRP-APP, Stochastic APP

Factor	DF	Adj SS	Adj MS	F-Value	P-Value
Demand-base	1	2.7569E+15	2.7569E+15	28800.91	0
Demand-trend	1	1.12475E+14	1.12475E+14	1175.01	0
Demand-seasonality	1	4.55272E+13	4.55272E+13	475.62	0
Demand-error	1	1.92432E+12	1.92432E+12	20.1	0
Industry	4	2.37667E+16	5.94167E+15	62071.66	0
Demand-base*Demand-trend	1	1.88817E+12	1.88817E+12	19.73	0
Demand-base*Demand-seasonality	1	2.13469E+13	2.13469E+13	223.01	0
Demand-base*Demand-error	1	1.62071E+12	1.62071E+12	16.93	0
Demand-base*Industry	4	5.3844E+15	1.3461E+15	14062.47	0

Demand-trend*Demand-seasonality	1	9.92287E+12	9.92287E+12	103.66	0
Demand-trend*Demand-error	1	1.52266E+12	1.52266E+12	15.91	0
Demand-trend*Flex-limits	3	379873733	126624578	0	1
Demand-trend*Industry	4	1.67263E+14	4.18157E+13	436.84	0
Demand-seasonality*Industry	4	1.79031E+14	4.47578E+13	467.58	0
Demand-error*Industry	4	7.88213E+12	1.97053E+12	20.59	0
Demand-base*Demand-trend*Demand-error	1	1.79566E+12	1.79566E+12	18.76	0
Demand-base*Demand-trend*Industry	4	7.12334E+12	1.78084E+12	18.6	0
Demand-base*Demand-seasonality*Demand-error	1	1.32392E+12	1.32392E+12	13.83	0
Demand-base*Demand-seasonality*Industry	4	7.82942E+13	1.95735E+13	204.48	0
Demand-base*Demand-error*Industry	4	6.12008E+12	1.53002E+12	15.98	0
Demand-trend*Demand-seasonality*Demand-error	1	7.044E+12	7.044E+12	73.59	0
Demand-trend*Demand-seasonality*Industry	4	3.46147E+13	8.65368E+12	90.4	0
Demand-trend*Demand-error*Industry	4	4.7627E+12	1.19068E+12	12.44	0
Error	430	4.11608E+13	95722760258		
Total	639	3.2642E+16			

Table 5.5: Selected ANOVA results for plan stability, Stochastic FRP-APP, Stochastic APP

Factor	DF	Adj SS	Adj MS	F-Value	P-Value
Demand-base	1	95111	95111	840.91	0
Demand-trend	1	15406	15406	136.21	0
Demand-error	1	32690	32690	289.02	0
Flex-limits	3	379875	126625	1119.54	0
Stochastic-technique	1	331695	331695	2932.63	0
Industry	4	59017	14754	130.45	0
Demand-base*Demand-error	1	3195	3195	28.25	0
Demand-base*Flex-limits	3	9361	3120	27.59	0
Demand-base*Stochastic-technique	1	11340	11340	100.26	0
Demand-base*Industry	4	18857	4714	41.68	0
Demand-trend*Demand-seasonality	1	1600	1600	14.15	0
Demand-trend*Demand-error	1	585	585	5.17	0.023
Demand-trend*Flex-limits	3	2180	727	6.43	0
Demand-trend*Stochastic-technique	1	731	731	6.46	0.011
Demand-trend*Industry	4	4546	1136	10.05	0
Demand-seasonality*Flex-limits	3	1892	631	5.58	0.001
Demand-seasonality*Industry	4	37207	9302	82.24	0
Demand-error*Flex-limits	3	24234	8078	71.42	0
Demand-error*Stochastic-technique	1	14194	14194	125.49	0
Demand-error*Industry	4	3223	806	7.12	0

Flex-limits*Stochastic-technique	3	72362	24121	213.26	0
Flex-limits*Industry	12	30824	2569	22.71	0
Stochastic-technique*Industry	4	79108	19777	174.86	0
Demand-base*Demand-trend*Demand-seasonality	1	533	533	4.71	0.031
Demand-base*Demand-trend*Demand-error	1	819	819	7.24	0.007
Demand-base*Demand-seasonality*Industry	4	14053	3513	31.06	0
Demand-base*Demand-error*Flex-limits	3	3617	1206	10.66	0
Demand-base*Demand-error*Stochastic-technique	1	833	833	7.36	0.007
Demand-base*Demand-error*Industry	4	1540	385	3.4	0.009
Demand-base*Flex-limits*Stochastic-technique	3	3220	1073	9.49	0
Demand-base*Flex-limits*Industry	12	13302	1108	9.8	0
Demand-base*Stochastic-technique*Industry	4	6168	1542	13.63	0
Demand-trend*Demand-seasonality*Flex-limits	3	843	281	2.48	0.06
Demand-trend*Demand-seasonality*Industry	4	2653	663	5.86	0
Demand-trend*Flex-limits*Industry	12	4159	347	3.06	0
Demand-trend*Stochastic-technique*Industry	4	1647	412	3.64	0.006
Demand-seasonality*Demand-error*Industry	4	1008	252	2.23	0.065
Demand-seasonality*Flex-limits*Stochastic-technique	3	179	60	0.53	0.664
Demand-seasonality*Flex-limits*Industry	12	32014	2668	23.59	0
Demand-seasonality*Stochastic-technique*Industry	4	1411	353	3.12	0.015
Demand-error*Flex-limits*Stochastic-technique	3	8258	2753	24.34	0
Demand-error*Flex-limits*Industry	12	3818	318	2.81	0.001
Demand-error*Stochastic-technique*Industry	4	1090	272	2.41	0.049
Flex-limits*Stochastic-technique*Industry	12	37157	3096	27.38	0
Error	430	48635	113		
Total	639	1420249			

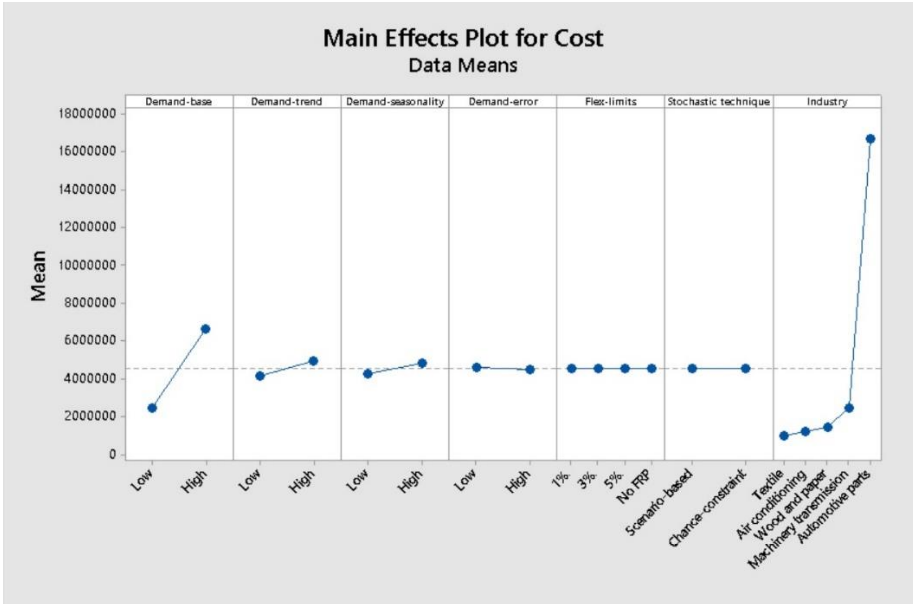

Figure 5.21: Main effects plot for cost, Stochastic models

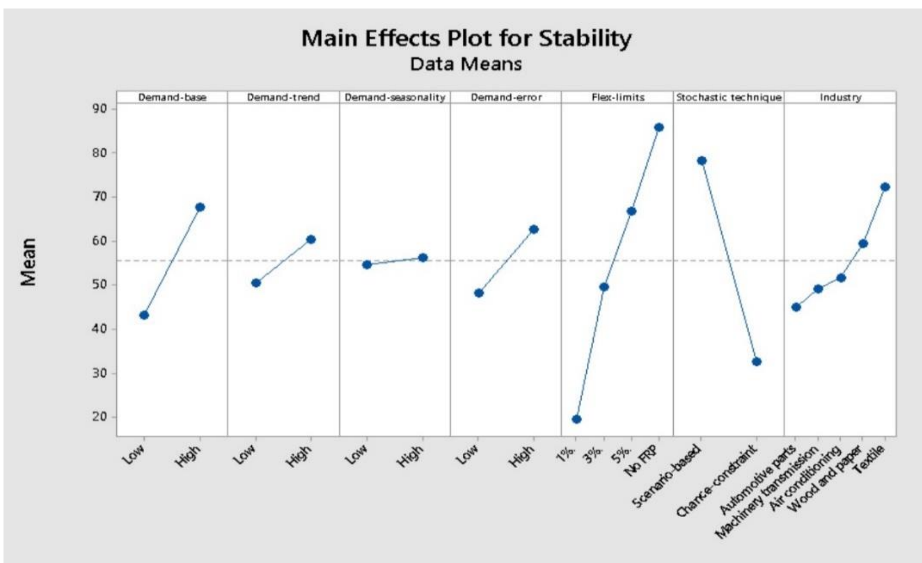

Figure 5.22: Main effects plot for stability, Stochastic models

We further continue our analysis with the Plackette-Burman design as in Chapter 4 using the same design in Table 4.10. Figure 5.23 includes the average cost performance of the Stochastic FRP-APP and Stochastic APP models using either stochastic technique over 12 Industry Cases, and Figure 5.24 includes the relative stability results. While the overall results show relatively close cost values for all models, there are 3 Industry scenarios (3,5,9) in which the overall cost of all models are higher that other Cases. According to Table 4.10, we see that these Cases correspond to the Industry scenarios with highest workforce and hiring/layoff costs.

The ANOVA results shown in Table 5.6 concur that the cost analysis of the models using each of the stochastic techniques are unanimously affected by the workforce cost and the workforce hiring and layoff costs. This could explain the sharp change in the cost values in Figure 5.23 for designs 3,5,9 and also the sharp line slope in Figure 5.21. While the stability of the scenario-based models is mainly affected by inventory and shortage costs, the chance constraint FRP-APP models stability is mainly affected by inventory cost and the chance-constraint APP stability is vulnerable to the changes in all cost components.

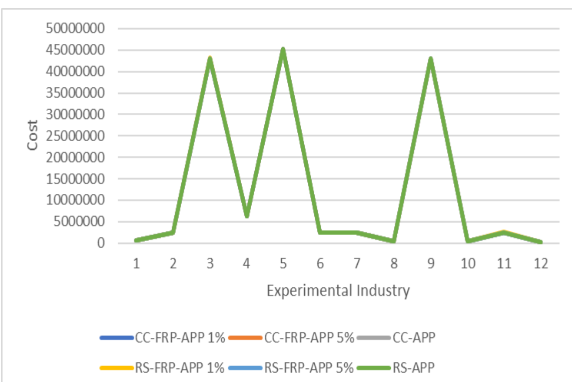

Figure 5.23: Total current cost comparison of Stochastic FRP-APP and Stochastic APP model, 12 experimental industries, Averaged over 16 demand scenarios

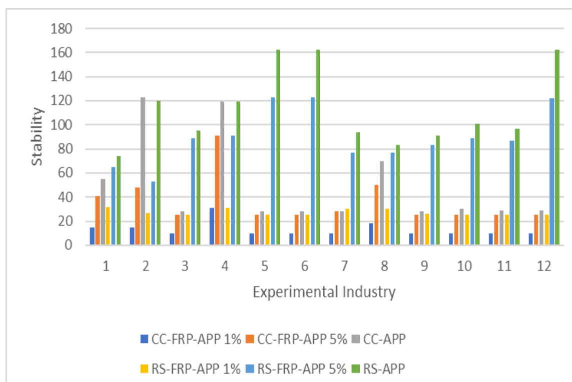

Figure 5.24: Stability comparison of Stochastic FRP-APP and Stochastic APP model, 12 experimental industries, Averaged over 16 demand scenarios

Table 5.6: Plackett-Burman design results for influential factors effect on cost and stability, Stochastic models, averaged demand results

		C^w	C^o/C^w	C^H/C^L	C^P	b	h	m^R
Robust-stochastic	APP Cost	√		√				
	APP Stability					√	√	
	FRP-APP Cost 1%	√		√				
	FRP-APP Stability 1%						√	
	FRP-APP Cost 5%	√		√				
	FRP-APP Stability 5%					√	√	
Chance-constraint	APP Cost	√		√				
	APP Stability	√	√	√	√		√	
	FRP-APP Cost 1%	√		√				
	FRP-APP Stability 1%						√	
	FRP-APP Cost 5%	√		√				
	FRP-APP Stability 5%						√	

5.10 Conclusions

In this chapter, the analysis of the performance of APP and FRP-APP models is continued under stochastic uncertainty. After presentation of the two stochastic techniques, which are: Robust-Stochastic and Chance-Constraint programming, the FRP-APP is compared to the Stochastic APP as two different modeling approaches for dealing with future uncertainties. In addition, the Stochastic uncertainty is officially considered into the FRP-APP formulation and the resulting models are compared with the Stochastic APP model where the APP is just taking care of input uncertainty while the Stochastic FRP-APP has an additional concern of plan stability.

The Stochastic APP stability in general shows high vulnerability to the stochastic programming technique and as a result, when compared with the FRP-APP, a scenario-based modeling could adversely affect its stability performance. While maintaining the same cost preference as compared to the FRP-APP, the CC-APP on the other hand, shows more control on the stability of the plans compared to the FRP-APP.

Adding stochastic uncertainty into the planning formulation of the FRP-APP to incorporate input uncertainty and use FRP bounds to control the stability of the developed plans retrieves its stability preference and reduces the cost gap when compared with the Stochastic APP using either Chance-Constraint or the Robust-Stochastic formulation for different test Cases. Although the Stochastic FRP-APP and Stochastic APP stability results are affected by selection of stochastic technique, the Stochastic FRP-APP can maintain its improved stability performance as compared to the Stochastic APP in all Industry Cases specially when more restrictive flex-limit sets are used. As a result, taking into account both the cost and stability results, the Stochastic FRP-APP can be considered as a reliable candidate planning model when the nature of the problem or the available historical makes it possible to incorporate a stochastic planning formulation.

There are demand scenarios, and test Cases with some variations to the above mentioned results, and as a result, experimental analysis is conducted to identify the most affecting factors on either cost or stability of the Stochastic FRP-APP and Stochastic APP for future use in other Cases with different demand scenarios or cost structures. Our results indicate the demand scenario and industry changes are the most influential factors on stochastic models' cost change, while in addition to these two factors, the stability of the stochastic models is mainly affected by selection of stochastic technique, and also the consideration and changes to the flexible-limits magnitude as well.

Chapter 6: BI-OBJECTIVE STOCHASTIC AND FUZZY APP MODELS

6.1 Introduction

As we have seen in Chapters 3-5, FRP-APP formulation deals with the stability of the production plans using flexibility bounds by limiting the production level change over sequential planning iterations. Our analysis results have shown that the FRP-APP has good potentials to incorporate both input uncertainty and the stability concerns at the same time and still maintain a good cost and stability performance as its Fuzzy/Stochastic APP counterpart. An alternate approach would be formulating a bi-objective optimization where cost and stability are optimized simultaneously. In this chapter, we will develop Bi-objective APP counterparts of the Fuzzy Max-Min (MM), Fuzzy Ranking (R), Robust-Stochastic (RS), and Stochastic Chance-Constraint (CC) APP models. As discussed in the literature review, there are multiple approaches for the Multi-objective, hence Bi-objective decision problems. Here we utilized the compromise technique, mainly because it is one of those techniques that helps maintain the problem linearity to be formulated as a mixed-integer linear program. Below we initially present an introduction to the Multi-objective optimization, followed by the Bi-objective APP formulation, and then continue with the fuzzy and stochastic formulations in subsequent sections.

6.2 Multi-objective Optimization

To deal with the problem of existing more than one objective in an optimization problem, multiple techniques are available. One of the most common and popular classical methods is the Ɛ-constraint method. In this method, one objective is kept as the main objective in the model while the rest of the objectives are kept within user-specified values (Ɛ). The following formulation presents the transformed version of a multi-objective problem with multiple minimization and conflicting objectives (Deb 2014):

Minimize $f_{\mu}(x)$ (6.1)

Subject to

$f_m(x) \leq \mathcal{E}_m$ $\forall m = 1,2,\dots,M$ and $m \neq \mu$ (6.2)

$g_j(x) \geq 0$ $\forall j = 1,2,\dots,J$ (6.3)

$x_i \geq 0$ $\forall i = 1,2,\dots,N$ (6.4)

In the FRP-APP formulation, we use the previously optimized production levels to come up with bounds between which the new production levels could get updated. As a result, the resulting bounds can be considered as constraining Ɛ for the stability objective value while keeping the cost as the main objective in the problem formulation, and hence the FRP-APP formulation uses an indirect format of the Ɛ-constraint to balance both the cost and stability objectives.

Another classical and popular multi-objective method is the compromise programming formulation. This method uses importance weights for different objectives and typically uses the normalized objective values as a "compromise" formulation to sum the weighted normalized objective values. The weights for each objective are the relative importance of each objective as compared to other objectives. In order to normalize each objective, the "Utopia" and "Nadir" points for each objective need to be defined. These two points are the best and worst possible values for each objective and are not target values determined by decision makers as in the goal programming models. After the objectives are normalized, a compromise objective function can be formed by summing the weighted normalized objectives and the problem is then converted to a single-objective optimization problem. The compromise formulation aims at finding the solutions that are closest to the utopia point of each objective. The following formulation presents the compromise formulation of the Model (6.1), (6.4) using weights and Utopia and Nadir points for each objective.

Maximize $F(x) = \sum_{m=1}^{M} w_m \frac{f_m(x) - N_m}{U_m - N_m}$ (6.5)

Subject to

Constraints (6-3), (6-4)

In the above formulation, N_m is the worst outcome for each objective and U_m is the best possible outcome or utopia point for objective m. For a minimization problem, the best and worst outcomes are the minimum and the maximum possible values and the compromise objective is changed to a maximization form to find solutions where each objective is closest to its utopia value.

Although both Ɛ-constraint and compromise programming techniques result in linear programming formulations, in this research we decided to utilize compromise programming. One justification is compromise programing can be seen more practical for planners/decision makers. It requires identification of the preferences (objective weights) for the planner. Given the weights for the planner, the pareto-optimal solution can be identified in a single iteration. On the other hand, Ɛ -constraint technique identifies pareto optimal solutions by varying the epsilon value hence requires multiple iterations.

6.3 Bi-objective APP (BO-APP)

The main formulation for the BO-APP with cost and stability objectives is as follows:

(BO-APP)

Minimize Objective1: $w * \left(\sum_{i=0}^{N-1} \left| P_{t,i} - P_{t-1,i+1} \right| \right)$ (6.6)

Minimize Objective2: $\sum_{i=0}^{N} (c^w . th. W_{t,i} + c^o . O_{t,i} + c^H . H_{t,i} + c^L . L_{t,i} + c^p . P_{t,i} + h. I_{t,i} + b. B_{t,i})$ (6.7)

Subject to

Initial Inventory: $P_{t,0} = d_{t,0} + I_{t,0} - B_{t,0} - I_{t-1,0} + B_{t-1,0}$ (6.8)

Inventory: $P_{t,i} = d_{t,i} + I_{t,i} - B_{t,i} - I_{t,i-1} + B_{t,i-1}$ $\forall i = 1, \dots, N$ (6.9)

End Inventory: $I_{t,N} \geq I$ (6.10)

Initial Workforce: $W_{t,0} = W_{t-1,0} + H_{t,0} - L_{t,0}$ (6.11)

Workforce: $W_{t,i} = W_{t,i-1} + H_{t,i} - L_{t,i}$ $\forall i = 1, \dots, N$ (6.12)

Production Capacity: $P_{t,i} \leq m^R.th.W_i + m^R.O_i$ $\forall i = 0, \dots, N$ (6.13)

Overtime Capacity: $O_i \leq th.W_{t,i}.m^O$ $\forall i = 0, \dots, N$ (6.14)

$W_{t,i}, O_{t,i}, H_{t,i}, L_{t,i}, P_{t,i}, I_{t,i}, B_{t,i} \geq 0$ $\forall i = 0, \dots, N$ (6.15)

$W_{t,i}, H_{t,i}, L_{t,i}$: integers $\forall i = 0, \dots, N$ (6.16)

The first term in the objective function is controlling the total change of the production levels over consecutive planning iterations, while the second conflicting term is the cost objective. It should be noted that in planning iteration t, $P_{t,i}$ is the decision variable while $P_{t-1,i+1}$ is an input parameter previously determined in planning iteration $t-1$.

As the stability objective is formulated in form of summation of absolute values, we need to use a transformation technique to make sure it follows the linear programming requirements. As a result, we do the following transformation in this objective by defining two decision variables ($e1_i \geq 0, e2_i \geq 0$) and also two Constraints (6.18) and (6.19). The transformed formulation for the stability objective function is as follows:

Minimize $(\sum_{i=0}^{N-1} e1_i + e2_i)$ (6.17)

The newly defined constraint are:

$P_{t,i} - P_{t-1,i+1} = e1_i - e2_i$ $\forall i = 0, \dots, N-1$ (6.18)

$e1_i, e2_i \geq 0$ $\forall i = 0, \dots, N-1$ (6.19)

By applying the techniques mentioned earlier, the transformed linearized and standardized BO-APP model is as follows:

$$\text{Maximize } \; w * \frac{\left(\sum_{i=0}^{N-1} e1_i + e2_i\right) - Stab_{max}}{Stab_{min} - Stab_{max}} +$$

$$(1-w) * \frac{\sum_{i=0}^{N} (c^{w}.th.W_{t,i} + c^{o}.O_{t,i} + c^{H}.H_{t,i} + c^{L}.L_{t,i} + c^{p}.P_{t,i} + h.I_{t,i} + b.B_{t,i}) - C_{max}}{C_{min} - C_{max}} \qquad (6.20)$$

Subject to

Constraints (6-8)-(6-16), (6-18), (6-19)

In the above objective function, best (worst) values for stability objective could be obtained through solving the single objective stability (cost) optimization model. For the cost objective, APP model results in the best cost while the worst cost would result from a single objective stability minimization model. In addition, changing w would affect the relative importance of each objective in the optimization process and produces a Pareto-Frontier of non-dominating solutions. Hence, we use the compromise programming formulation (6.20) as the basis for all Stochastic/Fuzzy Bi-objective APP models in the next sub-sections. However, as we will further explain in the sub-sequent sections, depending on the uncertainty modeling technique, we may need to modify the Bi-objective formulation accordingly.

6.4 Fuzzy Max-Min Bi-objective APP (MM-BO-APP)

Recalling from the MM-APP formulation in Chapter 4, λ represents the intersection (minimum) value for all membership functions (fuzzy objective and fuzzy inventory constraints) and the Max-Min formulations aims at finding the maximum intersection value of the membership functions. When transforming MM-APP into a Bi-objective version, there are some notes we need to make: 1. The resulting formulation needs to be formulated in such a way to give the decision maker the option to change the weight of the two main objectives which are: cost and stability, 2.

The intersection of fuzzy membership functions still needs to be taken into account for the maximization as the main idea behind the Max-Min technique. Recalling from the literature review, the Max-Min technique could have an extended version in form of a Max-weighted sum formulation as proposed by Bellman and Zadeh (1970). In this formulation, the relative importance of different membership functions in the model are determined based on the decision maker's preference. We will use the idea of this method to develop the objective of the MM-BO-APP problem. The first part of the objective is a variable (λ_c) related to the intersection of all fuzzy constraints' membership functions (fuzzy inventory constraint for different periods ($i = 1, \dots, N$) in the planning problem). We add a second part to the objective which is the compromise formulation of the cost and stability objectives. This way, the intersection of fuzzy constraints as well as the weight consideration in the compromise programing formulation are both maintained. We would like the Bi-objective formulation to treat both the fuzzy constraints and the compromise cost and stability objectives with equal importance (so we used 0.5 as an equal weight for both parts), while in the compromise formulation, the relative importance of either cost or the stability objectives is defined as in the BO-APP formulation.

(MM-BO-APP)

$$Maximize\ 0.5 * \lambda_c + 0.5 * (w * \frac{(\sum_{i=0}^{N-1} e1_i + e2_i) - Stab_{max}}{Stab_{min} - Stab_{max}} +$$

$$(1-w) * \frac{\sum_{i=0}^{N} (c^w.th.W_{t,i} + c^o.O_{t,i} + c^H.H_{t,i} + c^L.L_{t,i} + c^p.P_{t,i} + h.I_{t,i} + b.B_{t,i}) - C_{max}}{C_{min} - C_{max}}) \quad (6.21)$$

Subject to

$$\lambda_c.v_{ti} - P_{ti} + (I_{ti} - B_{ti} - I_{ti-1} + B_{ti-1}) \leq -d_{ti} + v_{ti} \qquad \forall i = 1, \dots, N \quad (6.22)$$

$$\lambda_c.u_{ti} + P_{ti} - (I_{ti} - B_{ti} - I_{ti-1} + B_{ti-1}) \leq d_{ti} + u_{ti} \qquad \forall i = 1, \dots, N \quad (6.23)$$

Constraints (6.8), (6.10) - (6.16), (6.18), (6.19)

$0 \leq \lambda_c \leq 1$ (6.24)

Another note we would like to make here is that, as the C_{min} in the compromise formulation is defined as the Utopia for the objective, and the cost objective in the MM-APP formulation is assumed to have predetermined upper and lower goal values as z^{max}, z^{min}, the Utopia (C_{min}) for the cost objective is determined by solving the following Max-weighted sum optimization problem.

$$Maximize\ 0.5 * \lambda_c + 0.5 * \left(\frac{\sum_{i=0}^{N}(c^W.th.W_{t,i}+c^O.O_{t,i}+c^H.H_{t,i}+c^L.L_{t,i}+c^P.P_{t,i}+h.I_{t,i}+b.B_{t,i})-z^{max}}{z^{min}-z^{max}}\right) \quad (6.25)$$

Subject to

Constraints (6.8), (6.10) - (6.16), (6.22)-(6.24)

6.5 Fuzzy Ranking Bi-objective APP (R-BO-APP)

The R-BO-APP has the same sets of constraints as the R-APP model, where the inventory constraint related to any future period in the planning problem is formulated using the expected interval values of the fuzzy demand in each period as in Constraint (6.27). However, the Bi-objective formulation requires the compromise formulation in the objective function (6.26) and adding Constraints (6.18) and (6.19).

(R-BO-APP)

$$Maximize\ w * \frac{\left(\sum_{i=0}^{N-1} e1_i + e2_i\right) - Stab_{max}}{Stab_{min} - Stab_{max}} + (1-w) * \frac{\sum_{i=0}^{N}(c^W.th.W_{t,i}+c^O.O_{t,i}+c^H.H_{t,i}+c^L.L_{t,i}+c^P.P_{t,i}+h.I_{t,i}+b.B_{t,i})-C_{max}}{C_{min}-C_{max}} \quad (6.26)$$

Subject to

$$P_{t,i} - I_{t,i} + B_{t,} + I_{t,i-1} - B_{t,i-1} = 0.5E_2^{d_{t,i}} + 0.5E_1^{d_{t,i}} \qquad \forall i = 1, \dots, N \quad (6.27)$$

Constraints (6.8), (6.10) - (6.16), (6.18), (6.19)

6.6 Stochastic Chance-Constraint Bi-objective APP (CC-BO-APP)

The CC-BO-APP model follows the same formulation as the CC-APP for the cost objective while adding the stability objective as another objective to be minimized. The chance-constraint inventory related constraints ((6.29) and (6.30)) as well as the rest of the constraints are still the same as the CC-APP. Again, the stability objective related constraints are added here as well.

(CC-BO-APP)

Maximize $w * \frac{(\sum_{i=0}^{N-1} e1_i + e2_i) - Stab_{max}}{Stab_{min} - Stab_{max}} + (1-w) *$

$$\frac{\sum_{i=0}^{N}(c^W.th.W_{t,i}+c^o.O_{t,i}+c^H.H_{t,i}+c^L.L_{t,i}+c^p.P_{t,i}+h.(I_{t,0}+\sum_{j=0}^{i} P_{t,j}-\sum_{j=0}^{i}\mu_{t,j})) - C_{max}}{C_{min}-C_{max}} \quad (6.28)$$

Subject to

$$I_{t,0} + \sum_{j=1}^{i} P_{t,i} \geq F^{-1}_{\sum_{j=1}^{i} D_{t,i}}(\alpha) \qquad \forall i=1,\ldots,\text{N} \quad (6.29)$$

$$I_{t,0} + \sum_{j=1}^{N} P_{t,j} \geq F^{-1}_{\sum_{j=1}^{N} D_{t,j}}(\alpha) + I \quad (6.30)$$

Constraints (6.8), (6.11) - (6.14), (6.16), (6.18), (6.19)

$$W_{t,i}, O_{t,i}, H_{t,i}, L_{t,i}, P_{t,i}, I_{t,i} \geq 0 \qquad \forall i = 0, \ldots, N \quad (6.31)$$

6.7 Robust-Stochastic Bi-objective APP (RS-BO-APP)

The RS-BO-APP equivalent to RS-APP is as the formulations below (Mirzapour Al-E-Hashem, Malekly et al. 2011). Since the main objective function in RS-APP is summation of expected cost and the cost variance over different scenarios, the compromise formulation is using the maximum (Z_{max}) and minimum objective values (Z_{min}) as the Nadir and Utopia parameters in the Bi-objective formulation. The minimum value is resulting from the RS-APP and the maximum value results from RS formulation with the stability minimization objective.

(RS-BO-APP)

$$Maximize\ w * \frac{(\sum_{i=0}^{N-1} e1_i + e2_i) - Stab_{max}}{Stab_{min} - Stab_{max}} + (1-w) *$$

$$\frac{\sum_{s\epsilon S} P_s(Z_s) + \lambda \sum_{s\epsilon S} P_s[Z_s - \sum_{s'\epsilon S} P_{s'}(Z_{s'}) + 2\theta_s] - Z_{max}}{Z_{min} - Z_{max}} \quad (6.32)$$

Subject to

$$Z_s = \sum_{i=0}^{N} (c^w . th. W_{t,i} + c^o . O_{t,i} + c^H . H_{t,i} + c^L . L_{t,i} + c^p . P_{t,i} + h. I_{t,i,s} + b. B_{t,i,s}) \quad (6.33)$$

$$Z_s - \sum_{s\epsilon S} P_s(Z_s) + \theta_s \geq 0 \qquad \forall\ s\epsilon S \qquad (6.34)$$

$$P_{t,0} = d_{t,0,s} + I_{t,0,s} - B_{t,0,s} - I_{t-1,0,s} + B_{t-1,0,s} \qquad \forall\ s\epsilon S \qquad (6.35)$$

$$P_{t,i} = d_{t,i,s} + I_{t,i,s} - B_{t,i,s} - I_{t,i-1,s} + B_{t,i-1,s} \qquad \forall i = 1, \dots, N, \forall\ s\epsilon S \qquad (6.36)$$

$$I_{t,N,s} \geq I \qquad \forall\ s\epsilon S \qquad (6.37)$$

Constraints (6.11)-(6.14) , (6.16), (6.18), (6.19)

$$W_{t,i}, O_{t,i}, H_{t,i}, L_{t,i}, P_{t,i}, I_{t,i,s}, , B_{t,i,s}, \theta_s \geq 0 \qquad \forall i = 0, \dots, N, \forall\ s\epsilon S \qquad (6.38)$$

6.8 Computational Results & Analysis

We use the same data sets for the Bi-objective fuzzy and stochastic models as previously explained in Chapters 4 and 5. In addition, for sensitivity purposes, we test 5 main scenarios for w, which are: 0, 0.1, 0.5, 0.9, 1. Please note that $w = 0$ is equivalent to the APP and $w = 1$ results in a pure stability minimization problem. We initially run the single objective cost and stability minimization related to each model to come up with the minimum and maximum cost (and objective in the RS-APP) values for each iteration and consider these as estimates of lower and upper bounds for the cost related objective standardization in the bi-objective formulation. In addition, the resulting stability from the cost and stability minimization models is used to estimate $Stab_{max}$, $Stab_{min}$ for normalizing the stability objective.

In the following sections, we will present and discuss the results corresponding to the Fuzzy/Stochastic Bi-objective APP model.

6.8.1 Fuzzy Bi-objective APP Results

The box-plots in Figures 6.1-6.10 shows the distribution of cost and stability values of average FRP-APP, Fuzzy APP, Fuzzy FRP-APP, and the Fuzzy BO-APP models using the two fuzzy technique (Max-Min and Ranking) and different weights for the stability and cost objectives. Figures 6.11-6.20 show the pareto frontier of the cost versus stability for the Fuzzy Bi-objective APP using different importance weights (0, 0.1, 0.5, 0.9, 1) for the stability (cost) objectives and also the Fuzzy FRP-APP with different flex-limits (1%, 3%, 5%). To have a more concise look, in these graphs, the results of each model are averaged over the 16 demand scenarios. In addition, Tables 6.1 and 6.2 also provide insights about the average cost gap and average stability performance of Fuzzy BO-APP, Fuzzy FRP-APP, and Fuzzy APP for different industries for each fuzzy technique.

The box-plots in Figures 6.1-6.10 show although the different planning approaches have very competitive cost values, consideration of the stability as a second objective can noticeably stabilize the resulting plans from the respective APP model specially when using MM-BO-APP formulation. In addition, incorporation of stability as a second objective can result in less variation in the stability of the APP model over various demand scenarios. These improvements are even more noticeable if higher weights are given to the stability objective. The cost performance differences in Table 6.1 as well as the box-plots indicate for the Wood and Paper and Air

Conditioning Units Cases, the Max-Min technique shows potentials for noticeable cost increase for the MM-BO-APP while if formulated using the Fuzzy Ranking method, the combinatorial cost and improved stability measures are much more promising. For other cases however, using either fuzzy technique can result in promising improved stability with comparable cost values as compared to the Fuzzy APP.

In addition, comparing the Fuzzy FRP-APP and the Fuzzy BO-APP planning approaches (Figures 6.11-6.20) indicates the noticeable improved stability of the APP with consideration of the stability objective has the potential to even outperform the Fuzzy FRP-APP in terms of cost and stability in the majority of the industries tested (Textile, Automotive Parts, Machinery and Transmission, and the Air Conditioning Units industries). The Wood and Paper Industry results indicate when a Max-Min formulation is used, the MM-FRP-APP with 1% and 3% flex-limits yield more reliable planning choices in terms of both cost and stability specially when compared with the MM-BO-APP with $w = 0.2$ or higher.

As a result, if formulated using the Max-Min technique, the MM-BO-APP can result in more noticeable stable plans as compared to the MM-FRP-APP, with occasional possibilities to have higher cost, but the Ranking formulation results in more competitive stability for the R-BO-APP and R-FRP-APP with a consistent better cost performance for the R-BO-APP.

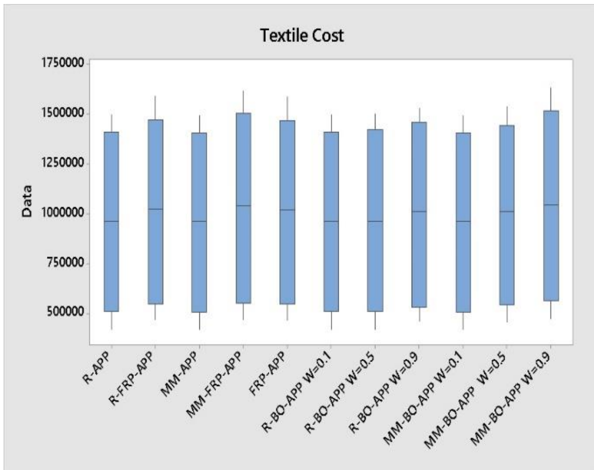

Figure 6.1: Textile cost variation: Fuzzy APP, Fuzzy FRP-APP, Fuzzy Bi-objective APP, FRP-APP

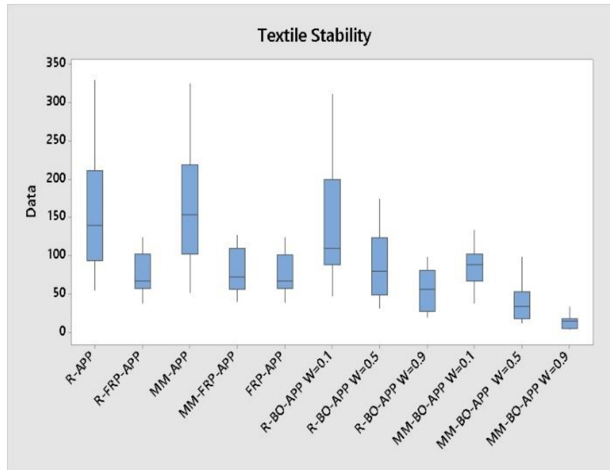

Figure 6.2: Textile stability variation: Fuzzy APP, Fuzzy FRP-APP, Fuzzy Bi-objective APP, FRP-APP

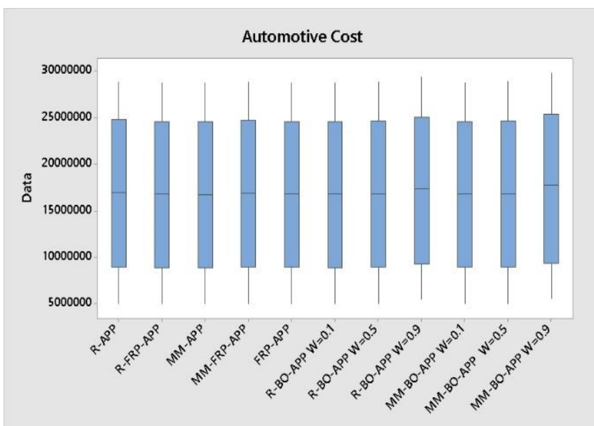

Figure 6.3: Automotive parts cost variation: Fuzzy APP, Fuzzy FRP-APP, Fuzzy Bi-objective APP, FRP-APP

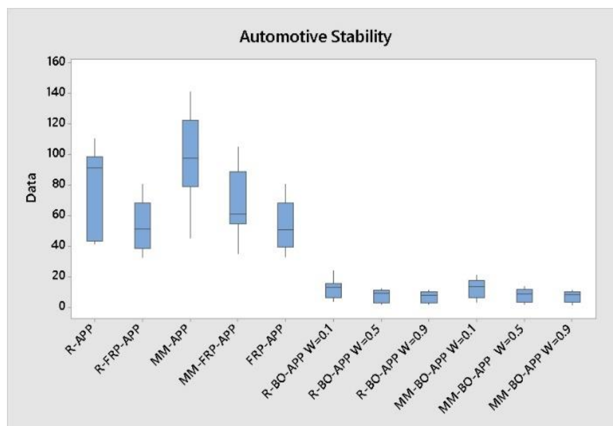

Figure 6.4: Automotive parts stability variation: Fuzzy APP, Fuzzy FRP-APP, Fuzzy Bi-objective APP, FRP-APP

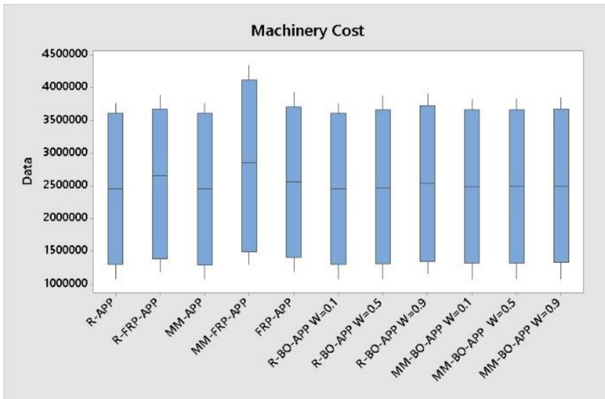

Figure 6.5: Machinery and Transmission cost variation: Fuzzy APP, Fuzzy FRP-APP, Fuzzy Bi-objective APP, FRP-APP

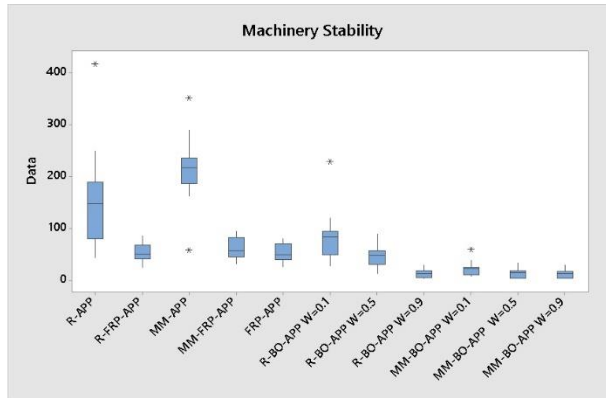

Figure 6.6: Machinery and Transmission stability variation: Fuzzy APP, Fuzzy FRP-APP, Fuzzy Bi-objective APP, FRP-APP

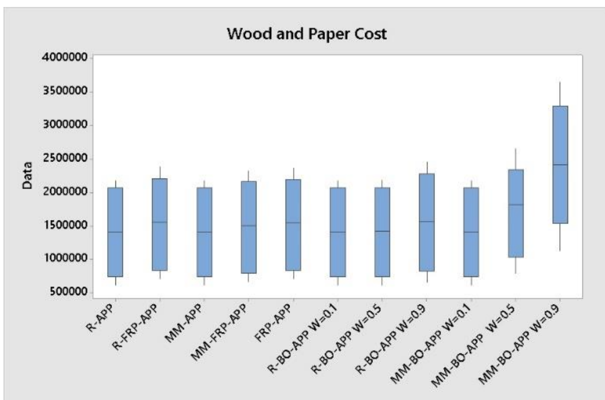

Figure 6.7: Wood and Paper cost variation: Fuzzy APP, Fuzzy FRP-APP, Fuzzy Bi-objective APP, FRP-APP

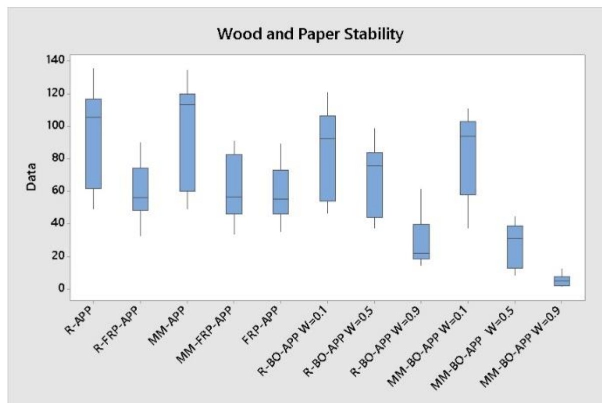

Figure 6.8: Wood and Paper stability variation: Fuzzy APP, Fuzzy FRP-APP, Fuzzy Bi-objective APP, FRP-APP

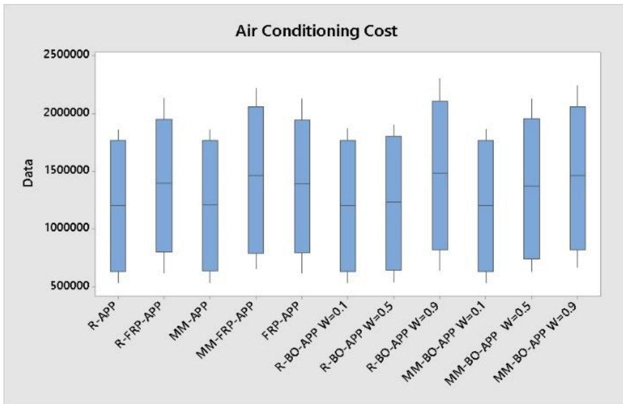

Figure 6.9: Air Conditioning Units cost variation: Fuzzy APP, Fuzzy FRP-APP, Fuzzy Bi-objective APP, FRP-APP

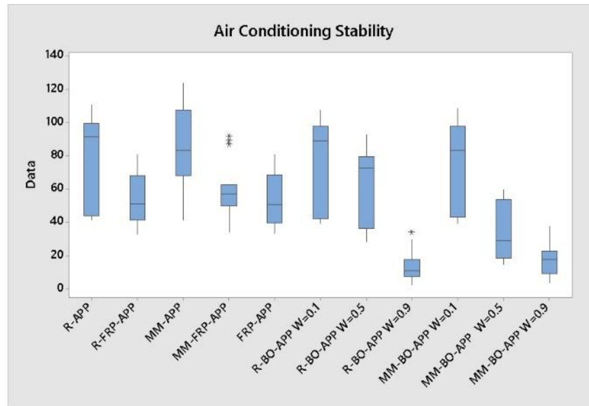

Figure 6.10: Air conditioning stability cost variation: Fuzzy APP, Fuzzy FRP-APP, Fuzzy Bi-objective APP, FRP-APP

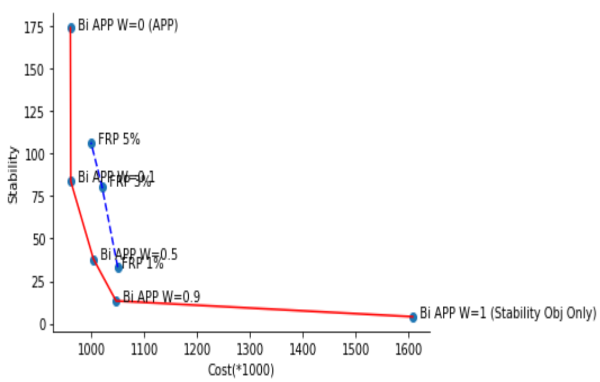

Figure 6.11: Pareto Frontier of Fuzzy MM-BO-APP, Fuzzy MM-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Textile

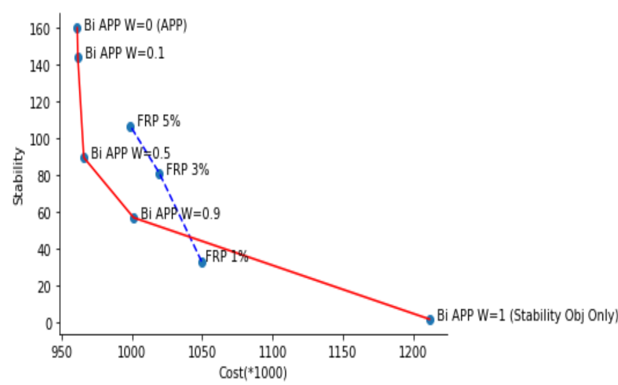

Figure 6.12: Pareto Frontier of Fuzzy R-BO-APP, Fuzzy R-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Textile

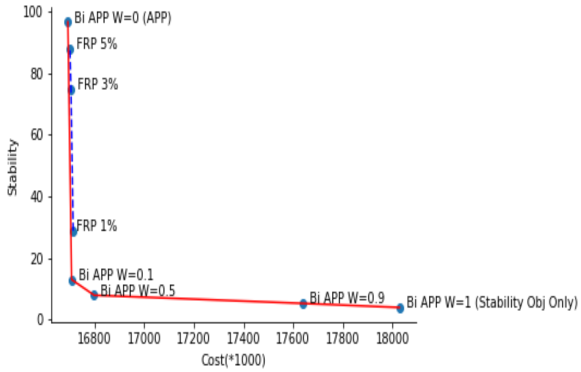

Figure 6.13: Pareto Frontier of Fuzzy MM-BO-APP, Fuzzy MM-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Automotive Parts

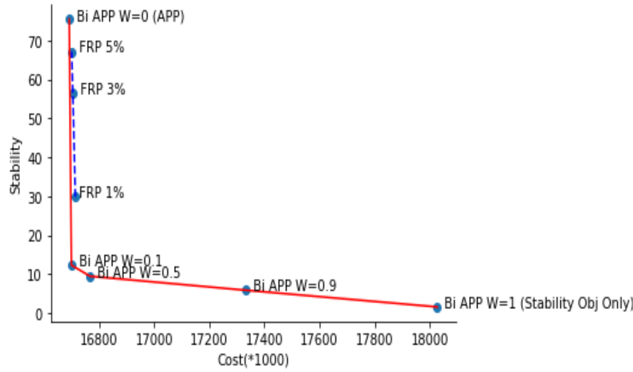

Figure 6.14: Pareto Frontier of Fuzzy R-BO-APP, Fuzzy R-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Automotive Parts

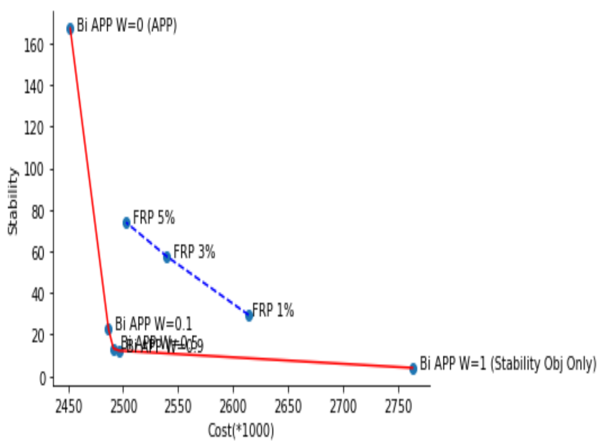

Figure 6.15: Pareto Frontier of Fuzzy MM-BO-APP, Fuzzy MM-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Machinery and Transmission

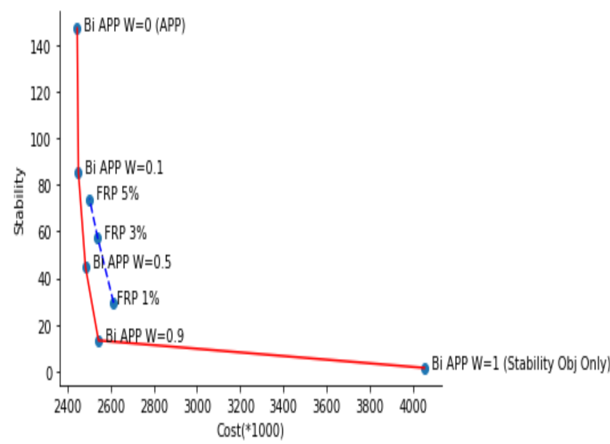

Figure 6.16: Pareto Frontier of Fuzzy R-BO-APP, Fuzzy R-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Machinery and Transmission

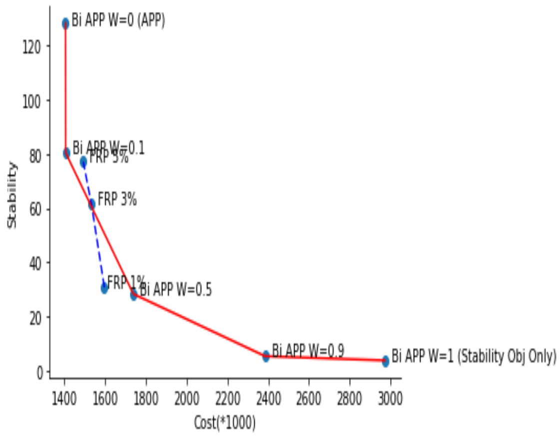

Figure 6.17: Pareto Frontier of Fuzzy MM-BO-APP, Fuzzy MM-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Wood and Paper

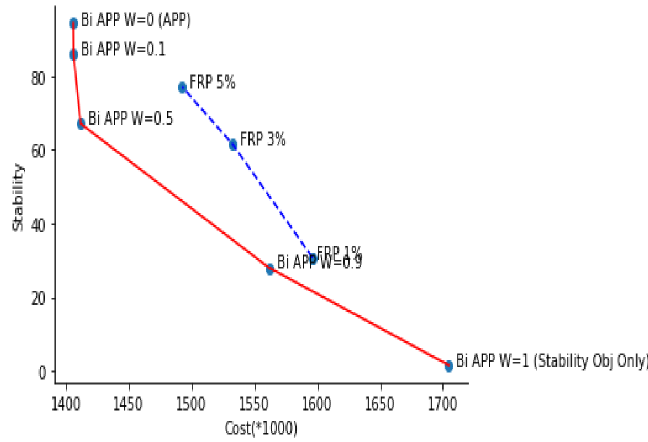

Figure 6.18: Pareto Frontier of Fuzzy R-BO-APP, Fuzzy R-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Wood and Paper

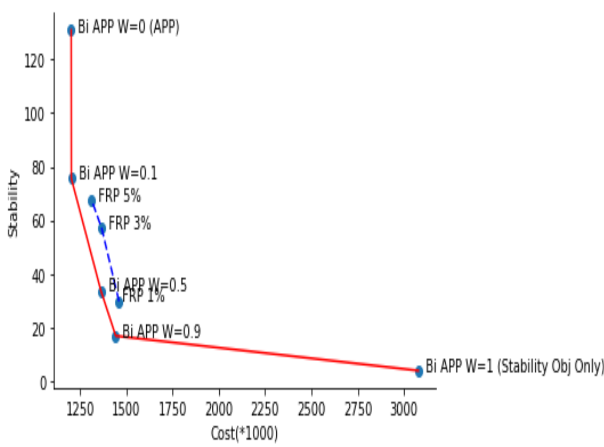

Figure 6.19: Pareto Frontier of Fuzzy MM-BO-APP, Fuzzy MM-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Air Conditioning

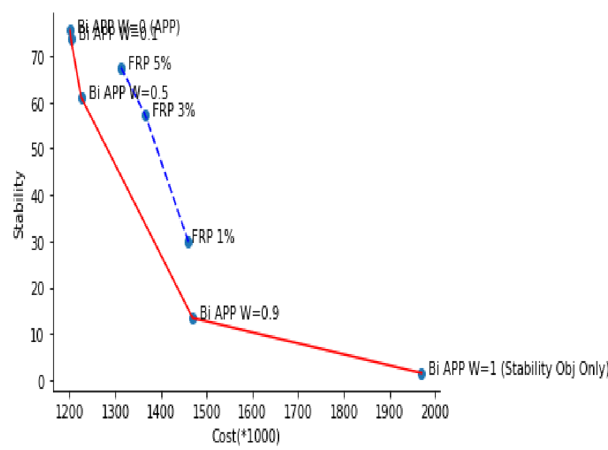

Figure 6.20: Pareto Frontier of Fuzzy R-BO-APP, Fuzzy R-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Air Conditioning

Table 6.1: Average cost gap percentage of different fuzzy models in different industries

	Textile	Automotive	Machinery & Transmission	Wood & Paper	Air Conditioning
MM-FRP-APP	8.9	0.7	15.2	6.2	20.1
MM-BO-APP	4.6	0.6	1.7	23.7	13.4
MM-APP	0.0	0.0	0.0	0.0	0.3
R-FRP-APP	6.5	0.1	4.0	9.4	14.6
R-BO-APP	0.6	0.4	1.4	0.4	2.0
R-APP	0.3	0.9	0.1	0.0	0.0

*0.0 means the lowest average cost, and a cost gap percentage closer to 0.0 shows a better cost performance.

Table 6.2: Comparison of average instability ratio for different fuzzy models in different industries

	Textile	Automotive	Machinery & Transmission	Wood & Paper	Air Conditioning
MM-FRP-APP	2.1	8.9	4.5	2.1	1.5
MM-BO-APP	1.0	1.1	1.0	1.0	1.0
MM-APP	4.4	13.0	16.2	3.4	2.5
R-FRP-APP	2.0	7.1	4.1	2.1	1.6
R-BO-APP	2.4	1.0	3.4	2.4	1.8
R-APP	4.2	10.2	11.4	3.4	2.3

*1.0 means the best stability control, and a ratio closer to 1.0 shows a better stability performance.

6.8.2 Stochastic Bi-objective APP Results

In this section, we present similar analysis and charts as in the fuzzy case for the comparison of the stochastic models. The distribution of cost and stability performance of the stochastic models over 16 demand scenarios using each stochastic technique (Chance-Constraint and Robust-Stochastic) are presented in box-plots in Figures 6.21-6.30, while the Pareto charts of cost and stability of the Stochastic BO-APP and the stochastic FRP-APP (averaged over 16 demand scenarios) for different Industry Cases are presented as in Figures 6.31-6.40. In addition, Tables 6.3, 6.4 give an overall idea of the average cost and stability performance of the Stochastic BO-APP, Stochastic FRP-APP, and Stochastic APP approaches.

The general observation from the box-plots in Figures 6.21-6.30 and also analyzing Tables 6.3, 6.4 indicates that using each stochastic modeling technique, with no noticeable adverse effect on the cost of plan, the Stochastic BO-APP is able to improve the stability of its Stochastic APP counterpart. The results are more noticeable if $w = 0.5$ or higher.

The comparative results of Stochastic FRP-APP and Stochastic BO-APP approaches for the Automotive Industry (Figures 6.23, 6.24, 6.33, 6.34) advocates the utilization of BO-APP as a more cost and stability beneficial planning approach. Other Industry cases show very competitive cost and stability performance for the CC-FRP-APP and CC-BO-APP approaches, and more noticeable stability improvement for the CC-BO-APP if $w = 0.9$ or higher. The RS-FRP-APP and RS-BO-APP comparative results indicate the RS-BO-APP can result in more cost beneficial plans with competitive average stability performance for the two planning approaches, and potentials for further stable plans if RS-BO-APP uses $w = 0.5$ or higher.

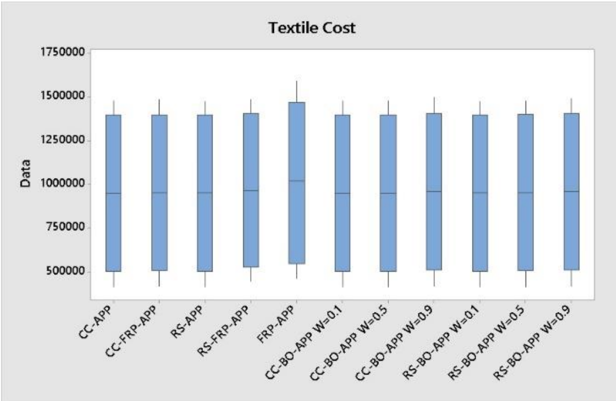

Figure 6.21: Textile cost variation: Stochastic APP, Stochastic FRP-APP, Stochastic Bi-objective APP, FRP-APP

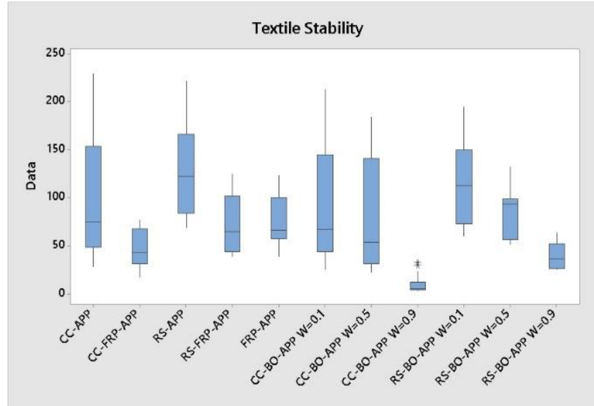

Figure 6.22: Textile stability variation: Stochastic APP, Stochastic FRP-APP, Stochastic Bi-objective APP, FRP-APP

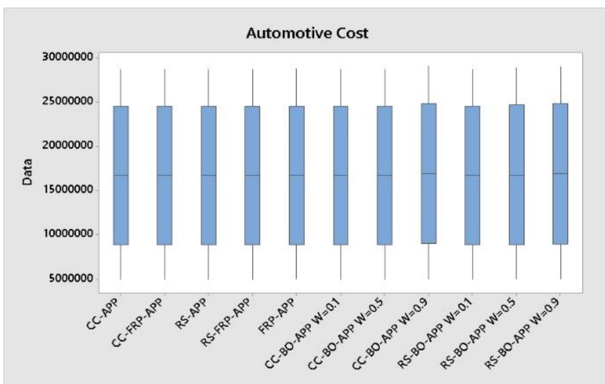

Figure 6.23: Automotive Parts cost variation: Stochastic APP, Stochastic FRP-APP, Stochastic Bi-objective APP, FRP-APP

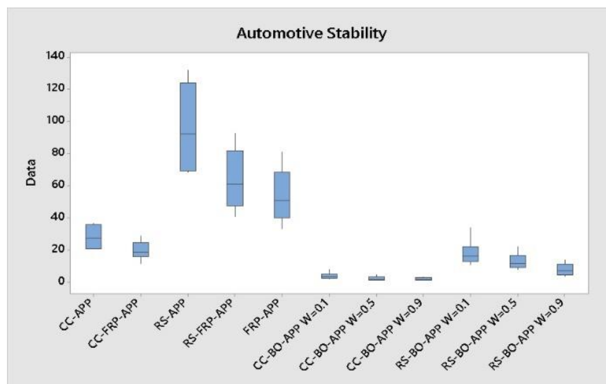

Figure 6.24: Automotive Parts stability variation: Stochastic APP, Stochastic FRP-APP, Stochastic Bi-objective APP, FRP-APP

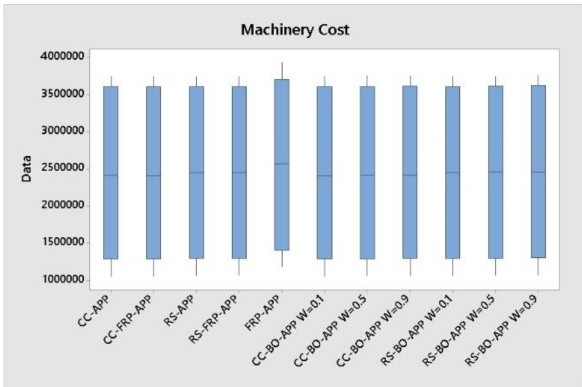

Figure 6.25: Machinery and Transmission cost variation: Stochastic APP, Stochastic FRP-APP, Stochastic Bi-objective APP, FRP-APP

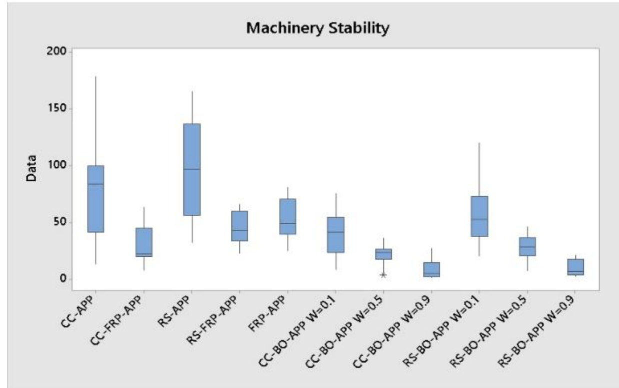

Figure 6.26: Machinery and Transmission stability variation: Stochastic APP, Stochastic FRP-APP, Stochastic Bi-objective APP, FRP-APP

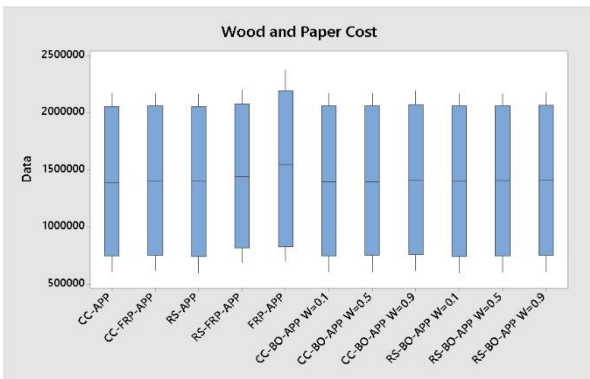

Figure 6.27: Wood and Paper cost variation: Stochastic APP, Stochastic FRP-APP, Stochastic Bi-objective APP, FRP-APP

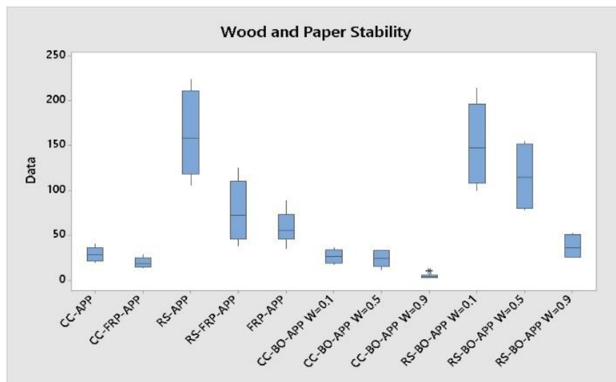

Figure 6.28: Wood and Paper stability variation: Stochastic APP, Stochastic FRP-APP, Stochastic Bi-objective APP, FRP-APP

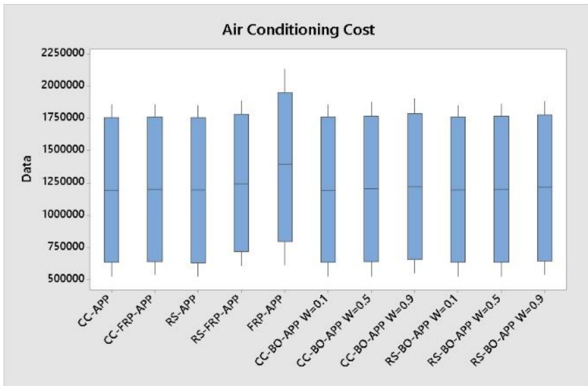

Figure 6.29: Air Conditioning Units cost variation: Stochastic APP, Stochastic FRP-APP, Stochastic Bi-objective APP, FRP-APP

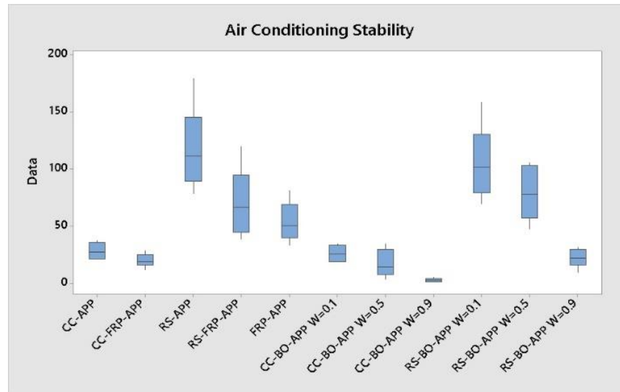

Figure 6.30: Air Conditioning Units stability variation: Stochastic APP, Stochastic FRP-APP, Stochastic Bi-objective APP, FRP-APP

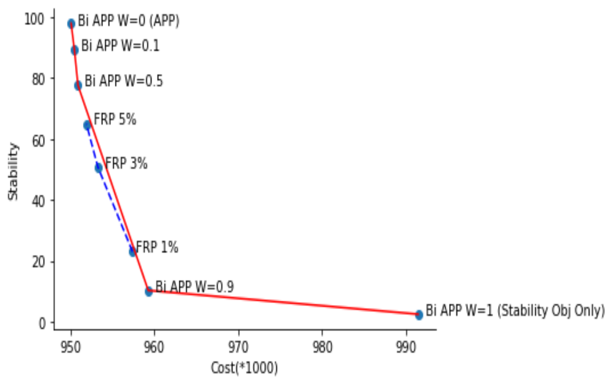

Figure 6.31: Pareto Frontier of CC-BO-APP, CC-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Textile

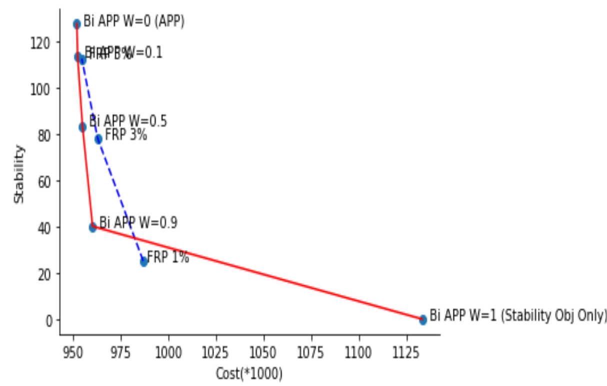

Figure 6.32: Pareto Frontier of RS-BO-APP, RS-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Textile

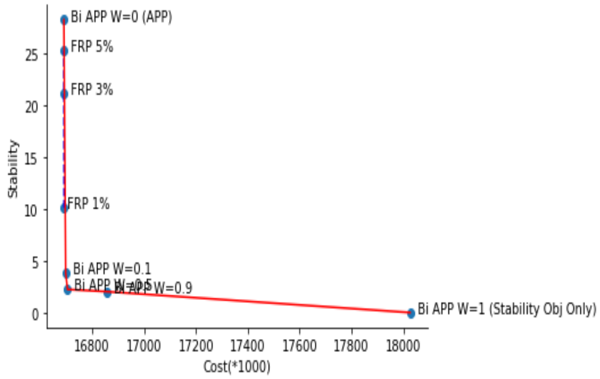

Figure 6.33: Pareto Frontier of CC-BO-APP, CC-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Automotive Parts

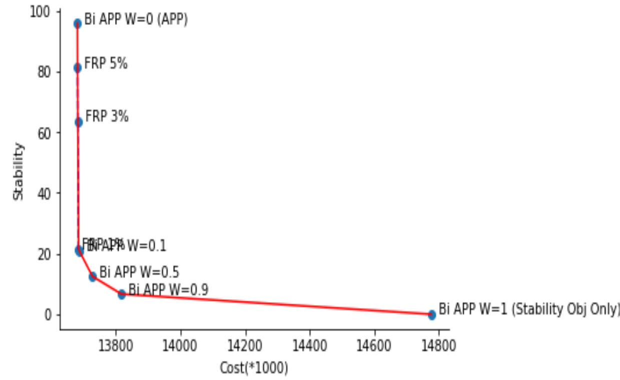

Figure 6.34: Pareto Frontier of RS-BO-APP, RS-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Automotive Parts

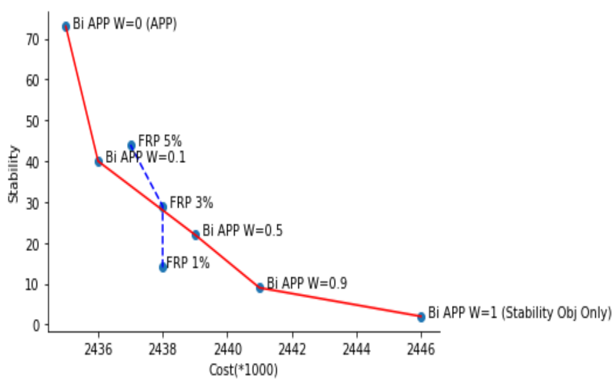

Figure 6.35: Pareto Frontier of CC-BO-APP, CC-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Machinery and Transmission

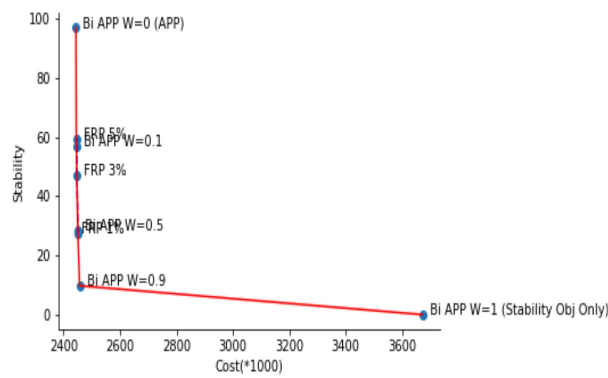

Figure 6.36: Pareto Frontier of RS-BO-APP, RS-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Machinery and Transmission

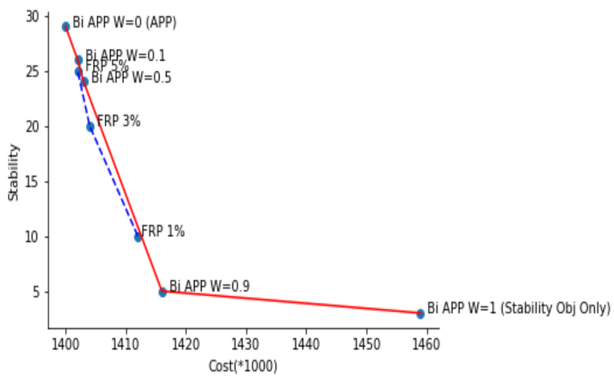

Figure 6.37: Pareto Frontier of CC-BO-APP, CC-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Wood and Paper

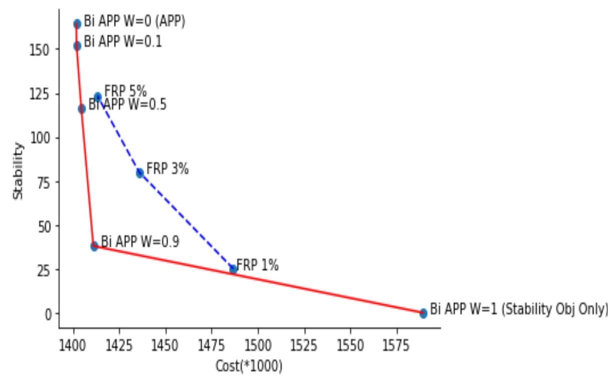

Figure 6.38: Pareto Frontier of RS-BO-APP, RS-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Wood and Paper

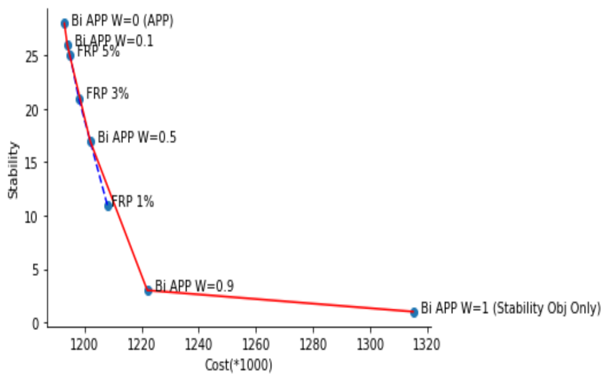

Figure 6.39: Pareto Frontier of CC-BO-APP, CC-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Air conditioning

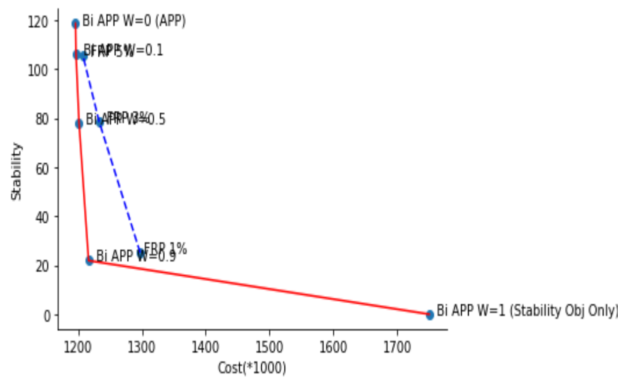

Figure 6.40: Pareto Frontier of RS-BO-APP, RS-FRP-APP 1%, 3%, 5% (Average of 16 demand scenarios), Air conditioning

Table 6.3: Average cost gap percentage of different stochastic models in different industries

	Textile	Automotive	Machinery & Transmission	Wood & Paper	Air Conditioning
CC-FRP-APP	0.4	0.0	0.0	0.4	0.6
CC-BO-APP	0.1	0.1	0.1	0.2	0.7
CC-APP	0.0	0.0	0.0	0.0	0.0
RS-FRP-APP	1.8	0.0	0.4	3.0	4.1
RS-BO-APP	0.5	0.3	0.5	0.3	0.7
RS-APP	0.2	0.0	0.3	0.1	0.2

*0.0 means the lowest average cost, and a cost gap percentage closer to 0.0 shows a better cost performance.

Table 6.4: Comparison of average instability ratio for different stochastic models in different industries

	Textile	Automotive	Machinery & Transmission	Wood & Paper	Air Conditioning
CC-FRP-APP	1.0	8.7	1.4	1.0	1.1
CC-BO-APP	1.6	1.0	1.0	1.2	1.0
CC-APP	2.1	12.6	3.6	1.5	1.6
RS-FRP-APP	1.5	28.0	2.1	4.0	4.1
RS-BO-APP	1.8	5.7	1.3	6.1	4.5
RS-APP	2.7	43.0	4.4	8.6	6.9

*1.0 means the best stability control, and a ratio closer to 1.0 shows a better stability performance.

6.9 Conclusions

The fuzzy results indicate the Bi-objective APP has the potential to noticeably improve the stability of the APP model. In addition, it can further alleviate another issue with the stability of the Fuzzy APP, which is controlling the stability variation over various demand scenarios specially if higher weight is given to the stability objective. In general, the MM-BO-APP shows better stability control over the developed plans, but since it also could result in higher cost values in some Industry Cases , with comparable stability improvement performance, the R-BO-APP results in a better combination of cost and stability behavior as compared to the Fuzzy APP models.

Furthermore, the comparison of the Fuzzy BO-APP and the Fuzzy FRP-APP models shows for most of the test Cases (Wood and Paper, and Air Conditioning Units), the former model can outperform the cost and stability of the latter. The stability preference of the Fuzzy BO-APP becomes more visible if formulated as MM-BO-APP, with occasional chances to result in higher costs. The ranking formulation, however, can result in more competitive stability performance for the R-BO-APP and R-FRP-APP models. The Automotive Industry results, however, indicate very competitive cost values for both Fuzzy BO-APP and Fuzzy FRP-APP with more noticeable stable plans resulting from Fuzzy BO-APP approach as compared to the Fuzzy FRP-APP using either fuzzy technique.

The stochastic results for each of the Chance-Constraint and the Robust-Stochastic methods, indicate the Stochastic BO-APP has potential to improve the stability of its Stochastic APP counterpart, with more promising results for the Robust-Stochastic models comparisons when $w \geq 0.5$. In addition, the comparison of the CC-FRP-APP and CC-BO-APP models indicate very competitive cost and stability performance with possibility of better stability performance if CC-BO-APP uses $w \geq 0.9$. The RS-BO-APP cost and stability results indicate this planning method could result in better cost performance as compared to RS-FRP-APP and also show better control over plan stability if $w \geq 0.5$. The Automotive Industry results, again, indicate very competitive cost values with more noticeable stable plans resulting from Stochastic BO-APP approach as compared to the Stochastic FRP-APP using either stochastic technique.

As a concluding mark, we would like to mention both the Fuzzy/Stochastic FRP-APP and the Fuzzy/Stochastic BO-APP models are promising planning approaches with varying parameters to balance the cost and stability of the developed plans as compared to the regular Fuzzy/Stochastic APP. The FRP follows an incremental flexibility level consideration policy and gives the decision maker or the planner the ability to adjust and decide about a specific level of flexibility for each period in the planning horizon. The BO-APP on the other hand, treats stability as a second objective and the importance weight it gives to the stability objective is not a period-dependent adjustable parameter. Based on the planner preference for adjusting the stability of the planning problems, the fuzzy/stochastic technique utilized, and the industry the models are going to be tested on, the FRP-APP and the BO-APP could be considered as viable candidates, however, a careful selection of either flex-limit sets or the stability objective weight becomes crucial.

Chapter 7: SUMMARY, CONCLUSIONS AND FUTURE DIRECTIONS

7.1 Summary

In this research, several stochastic and fuzzy techniques have been proposed to create flexible aggregate production plans (APPs) to deal with planning variability related "nervousness" under uncertainty by trading off plan cost versus plan stability. More specifically 8 new APP models have been proposed here as follows: Stochastic CC-FRP-APP, Stochastic RS-FRP-APP, Stochastic CC-BO-APP, Stochastic RS-BO-APP, Fuzzy MM-FRP-APP, Fuzzy R-FRP-APP, Fuzzy MM-BO-APP, and Fuzzy R-BO-APP. A comprehensive sensitivity analysis was conducted utilizing experimental design techniques to test the proposed models with respect to different flexibility levels, industry types, demand patterns, and uncertainty modeling techniques.

7.2 Conclusions

The main conclusions of this research can be summarized as follows:

- *Fuzzy FRP-APP Results and Conclusions:*
    - For the majority of the industries tested (Textile, Automotive Parts, Machinery and Transmission Parts, and Wood and Paper), the Fuzzy FRP-APP shows as a promising planning approach to have a noticeable better stable performance with comparable cost values as compared to the Fuzzy APP. This however requires a careful analysis on the performance of Fuzzy FRP-APP with different flex-limit options and the fuzzy technique used.
    - Fuzzy FRP-APP and FRP-APP models show similar performances specially when the R-FRP-APP formulation is used.

- MM-FRP-APP can result in slightly better stability while R-FRP-APP could further improve the cost performance of Fuzzy FRP-APP model. The cost-stability results indicate R-FRP-APP to be a more promising planning approach.
- Depending on the Industry Case and demand structure, careful selection of flex-limits for the FRP-APP can make it a viable planning approach as compared to the Fuzzy APP. The 1% flex limits promise a reliably better stability with occasional higher cost as compared to other flex limits scenarios, while other flex-limits cases may result in lower cost values and less control over plan stability.
- Fuzzy models' cost performance is mainly affected by the demand structure and the industry parameters (especially the workforce related costs)
- Fuzzy models' stability performance is mainly affected by the demand structure, industry parameters (especially inventory cost and also the production capacity), fuzzy technique selection (this is more tangible for the Fuzzy APP), and the flex-limits.

- *Stochastic FRP-APP Results and Conclusions:*
  - Robust- Stochastic technique in general seem to have less control over plan stability as compared to the Chance-Constraint technique. This is because the scenario-based formulation includes more volatility with respect to demand as compared to the distribution-based formulation in Chance-Constraint, and as a result, the stability is more difficult to maintain.
  - CC-APP is a preferred planning approach for developing stable plans with better cost values as compared with the FRP-APP and RS-APP models.

- When FRP-APP is formulated as a Stochastic planning problem, using either RS-FRP-APP or the CC-FRP-APP, it yields noticeably better stability performance, and also close cost values as compared to the Stochastic APP counterparts.
- Stochastic models' cost is mainly affected by demand structure and industry change (specially changes in workforce related costs and production capacity).
- Stochastic models' stability performance is mainly affected by stochastic technique selection, and the flex-limits, followed by the behavior of demand structure and industry parameters (specially inventory cost).

• *Bi-objective Fuzzy Results and Conclusions:*
    - Based on the results, it seems the Fuzzy BO-APP can noticeably improve the stability of the Fuzzy APP. It also seems to result in less variable stability values over different demand scenarios. However, due to the potentials for increased cost values in the Bi-objective APP model as compared to the APP model (as indicated in MM-BO-APP results for the Wood and Paper, and Air Conditioning Units Industry cases), depending on the industry structure, selection of the fuzzy technique could become more important.
    - The Fuzzy BO-APP also yields better cost-stability performance as compared to the Fuzzy FRP-APP, with more noticeable stability improvement with occasional higher cost for the MM-BO-APP and more consistent cost improvement with competitive stability for R-BO-APP.

- *Bi-objective Stochastic Results and Conclusions:*
  - Irrespective of the stochastic technique used (i.e. either chance-constraint or the robust stochastic formulation), the Stochastic BO-APP with comparable cost performance, can result in more stable plans specially with stability importance weight of ≥0.5.
  - The Stochastic CC-FRP-APP and the Stochastic Bi-objective CC-APP models have very competitive results in terms of both cost and stability. On the other hand, the RS-BO-APP shows more promising cost and stability performance results specially with stability importance weight of ≥0.5.
  - In general, the Bi-objective Stochastic/Fuzzy APP models with stability importance weight of ≥0.5 seem to perform well in terms of stability without sacrificing the cost objective.
- *General Results and Conclusions:*
  - FRP-APP models' cost and stability performance are dependent on the flex-limits. FRP-APP models with 1%-3% flex-limits seem to yield more promising stability results. 1% flex-limit results in the most stable plans with a potential to increase the cost of plans.
  - Stochastic/Fuzzy FRP-APP seems to give more control to the planners and decision makers to adjust their level of flexibility levels over the planning horizon compared to the Stochastic/Fuzzy Bi-objective models since the importance weight is fixed for the stability objective.
  - Speaking of the optimal model parameters, both Stochastic/Fuzzy -based methods require sensitivity analysis. FRP-APP requires sensitivity with respect to flex-limits while the Bi-objective APP requires sensitivity with respect to w.

- From a computational complexity perspective, the Gurobipy solver was able to handle all developed models (irrespective of the Industry Case) and reach to an optimal solution for all 50 planning iterations within a few minutes (typically less than 5 minutes). The scenario-based models seem to be the most time-consuming models as compared to other models due to the model size growth based on different possible scenarios. In conclusion, we believe increasing the number of scenarios, size of the planning horizon, number of planning iterations, and having multiple interdependent products in the planning problem could further increase the computational time and effort. However, we still believe the complexity of these problems could be handled by existing solvers within reasonable time.

7.3 Future Directions

Our findings could lead to several directions for further research and analysis in the future:

- *Varying Flex limits:* According to our analysis, the flex limits are among the most influential factors on the relative stability performance of the planning models. In this research, we have incorporated flex-limits with incremental increase from one period to another making the control tighter for current/near periods and looser for future periods. While this approach seems to work, other distributions or selections of flex-limits can be tested for example perhaps adjusting the flex-limits dynamically from period to period based on either planners' preferences and/or demand uncertainties.
- *Time-varying objective function weights:* We have considered constant weight for stability and cost in the bi-objective optimization; however, this assumption can be relaxed by varying the weights over the planning horizon. This approach can enable a planner to have

tighter control of stability in the current and near periods by selecting higher weights for the stability objective. Similarly, for future periods the weights can be reduced.

- *Effect of Forecasting Methods:* In this research the forecasted demands are generated using the decomposition technique for 16 demand scenarios. An investigation on different forecasting methods and other demand patterns can be interesting to further validate the FRP-APP and well as the Bi-objective APP models proposed here.
- *Multi-product planning:* Also, as this research considers the single product case, future research could also include multi-products cases where there could exist shared resources, setups as well as demand correlations among some or all products.
- *Other Stochastic and Fuzzy Techniques:* While we have utilized the most popular stochastic and fuzzy techniques to build the FRP and bi-objective models, other techniques could be utilized such as dynamic, multi-stage planning, and the fuzzy possibilistic linear programming.
- *Other Industry Scenarios:* The results indicate that stability of the models may show vulnerability to mainly the inventory cost and also the production capacity, as both could affect levels of production and as a result, the effort the models put on meeting customers' demand. While we tested five industry-based cases and twelve hypothetical industrial scenarios under 16 demand scenarios and various flex-limits, continuing the sensitivity analysis and applications of the proposed methods on other industry test cases could be interesting and could further validate the models and related conclusions.
- *Other areas of planning applications:* Finally, in addition to the production planning, the FRP-based planning approach as well as the Bi-objective planning models presented here, could be extended to other planning problems such as: lot-sizing and scheduling,

distribution and transportation planning, and resource planning problems in service industries . We also think that the FRP-based planning could be applied to address other stability related concerns, such as: workforce level changes and resource utilization levels especially when workforce satisfaction is important or there exist expensive bottleneck resources in the planning problems.

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APPENDIX A: FUZZY SUPPLEMENTARY GRAPHS ON FRP-APP SENSITIVITY

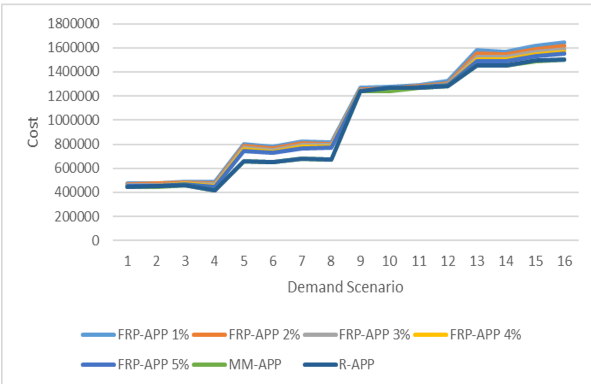

Figure 1: Total current cost comparison, FRP-APP, R-APP, MM-APP, Textile Industry

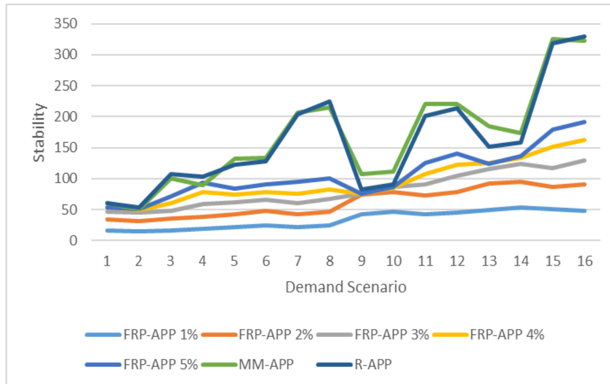

Figure 2: Stability comparison, FRP-APP, R-APP, MM-APP, Textile Industry

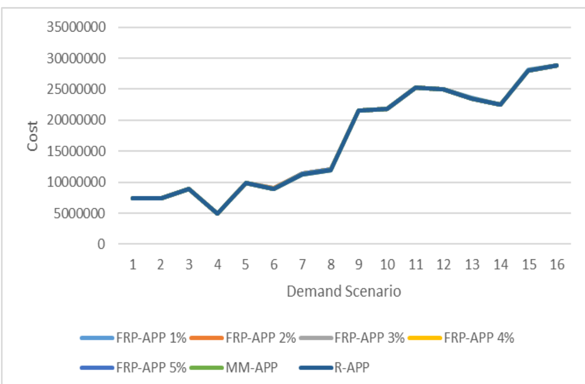

Figure 3: Total current cost comparison, FRP-APP, R-APP, MM-APP, Automotive Parts Industry

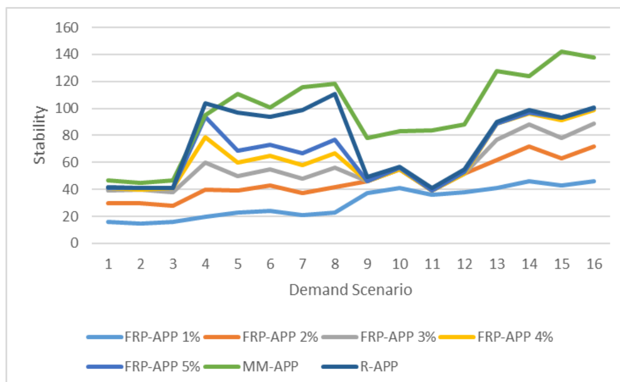

Figure 4: Stability comparison, FRP-APP, R-APP, MM-APP, Automotive Parts Industry

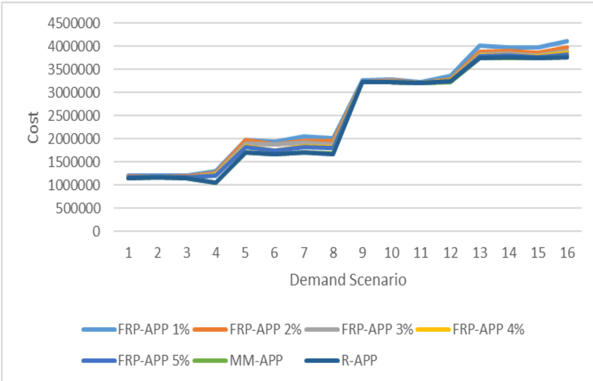

Figure 5: Total current cost comparison, FRP-APP, R-APP, MM-APP, Machinery and Transmission Industry

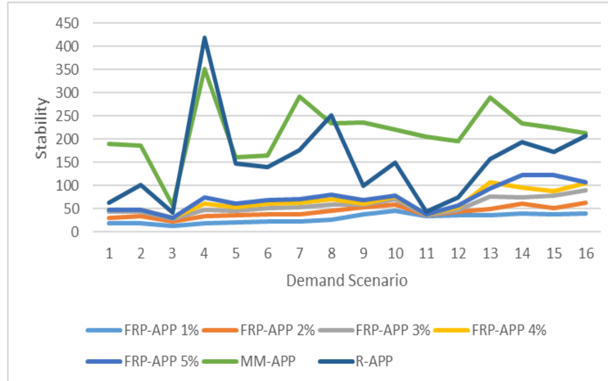

Figure 6: Stability comparison, FRP-APP, R-APP, MM-APP,Machinery and Transmission Industry

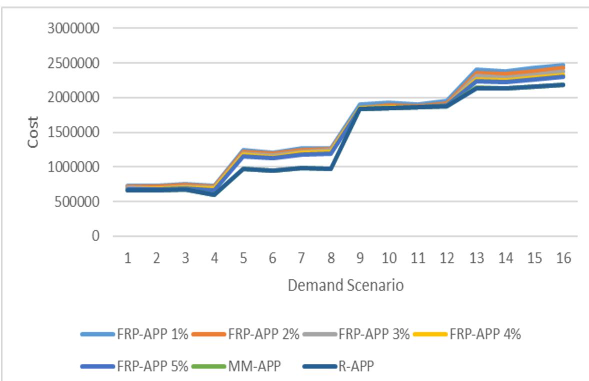

Figure 7: Total current cost comparison, FRP-APP, R-APP, MM-APP, Wood and Paper Industry

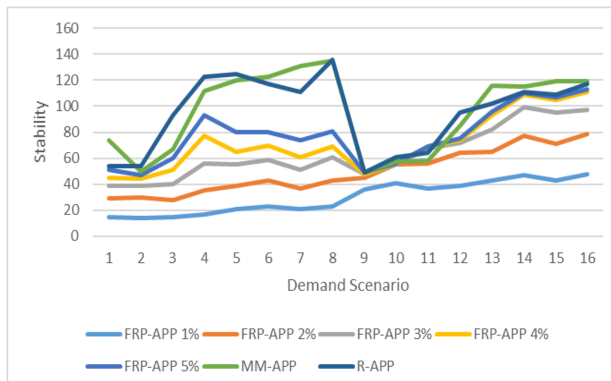

Figure 8: Stability comparison, FRP-APP, R-APP, MM-APP, Wood and Paper Industry

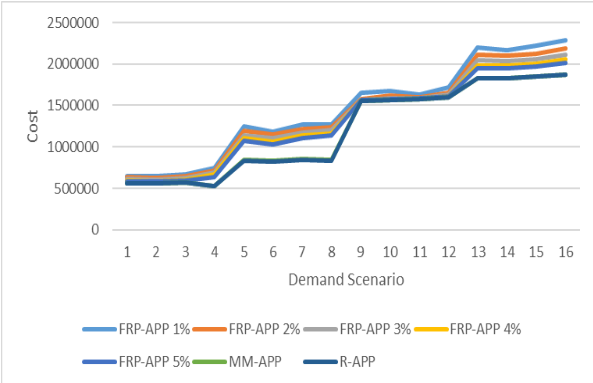

Figure 9: Total current cost comparison, FRP-APP, R-APP, MM-APP, Air Conditioning Unite Industry

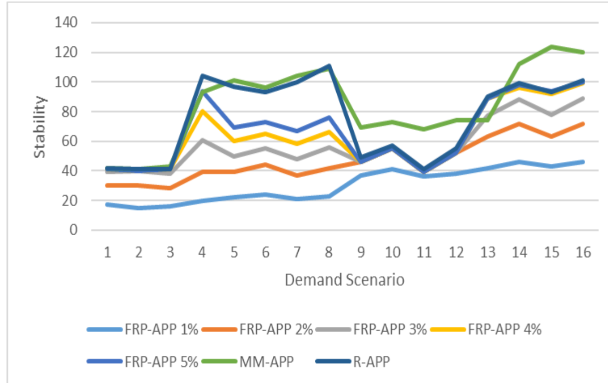

Figure 10: Stability comparison, FRP-APP, R-APP, MM-APP, Air Conditioning Unite Industry

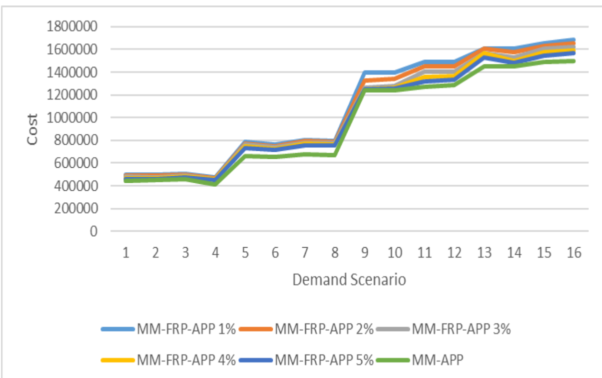

Figure 11: Total current cost comparison, MM-FRP-APP, MM-APP, Textile Industry

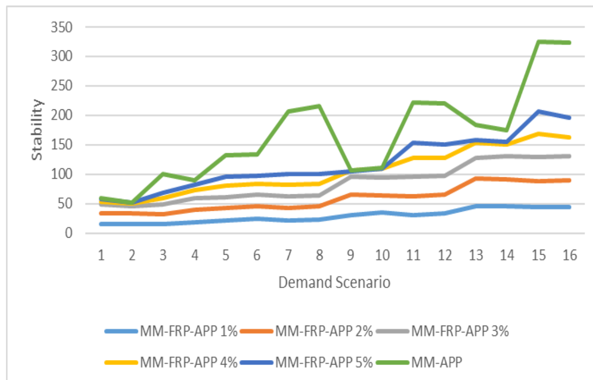

Figure 12: Stability comparison, MM-FRP-APP, MM-APP,, Textile Industry

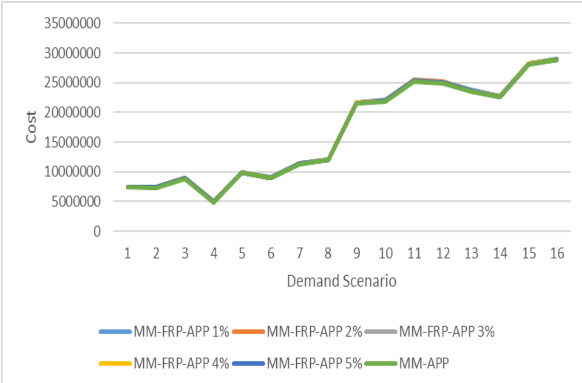

Figure 13: Total current cost comparison, MM-FRP-APP, MM-APP, Automotive Parts Industry

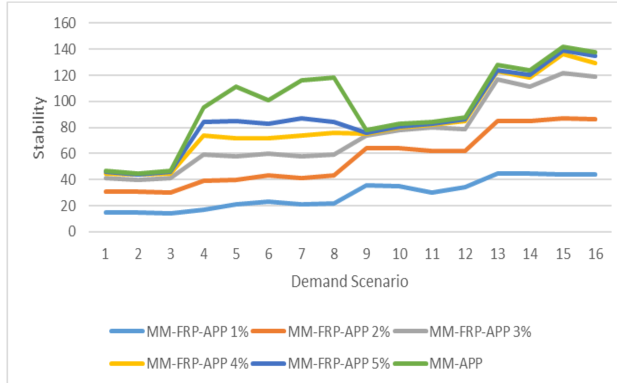

Figure 14: Stability comparison, MM-FRP-APP, MM-APP, Automotive Parts Industry

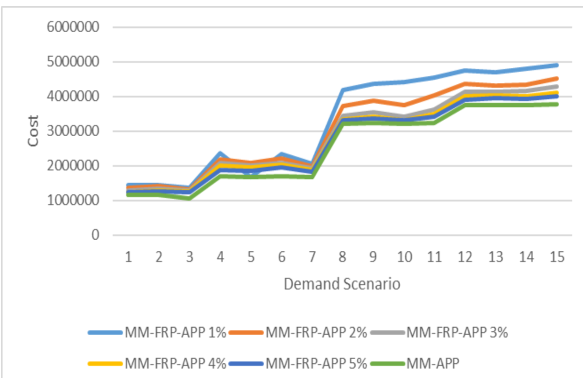

Figure 15: Total current cost comparison, MM-FRP-APP, MM-APP, Machinery and Transmission Industry

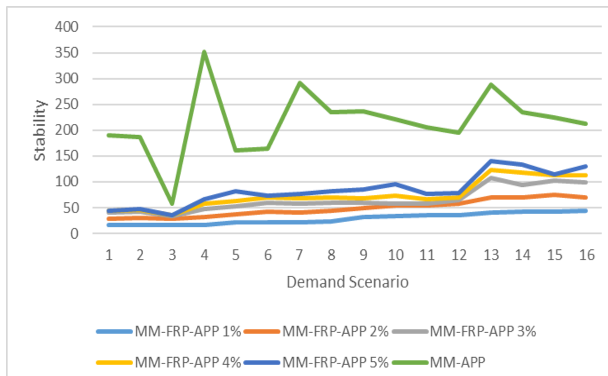

Figure 16: Stability comparison, MM-FRP-APP, MM-APP, Machinery and Transmission Industry

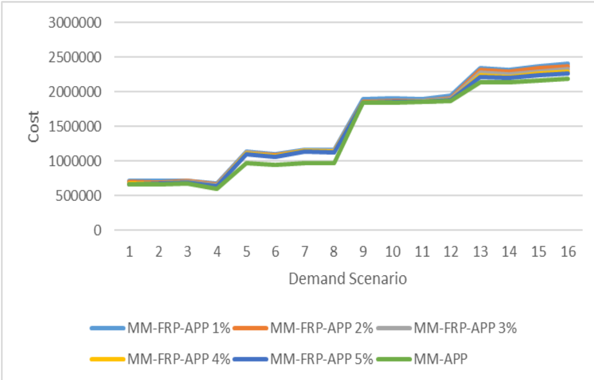

Figure 17: Total current cost comparison, MM-FRP-APP, MM-APP,Wood and Paper Industry

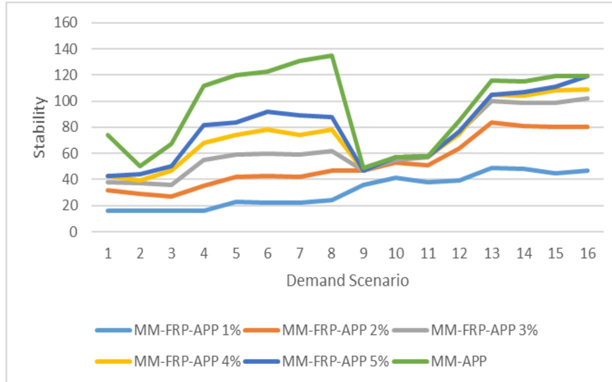

Figure 18: Stability comparison, MM-FRP-APP, MM-APP,Wood and Paper Industry

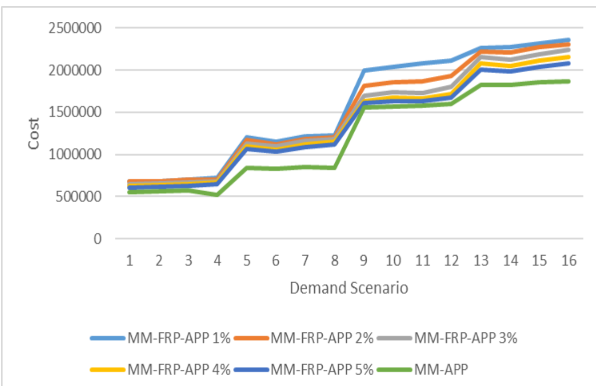

Figure 19: Total current cost comparison, MM-FRP-APP, MM-APP, Air Conditioning Unite Industry

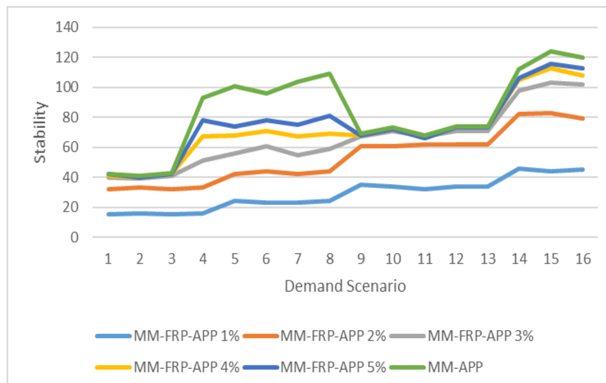

Figure 20: Stability comparison, MM-FRP-APP, MM-APP, Air Conditioning Unite Industry

APPENDIX B: STOCHASTIC SUPPLEMENTARY GRAPHS ON FRP-APP SENSITIVITY

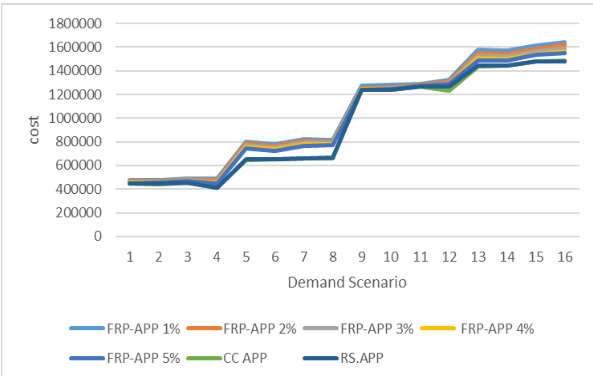

Figure 21: Total current cost comparison, FRP-APP, RS-APP, CC-APP, Textile Industry

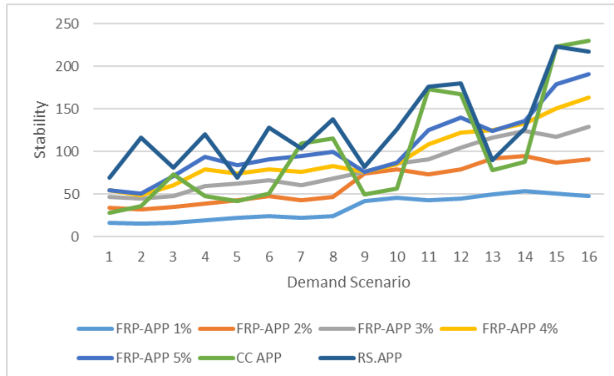

Figure 22: Stability comparison, FRP-APP, RS-APP, CC-APP, Textile Industry

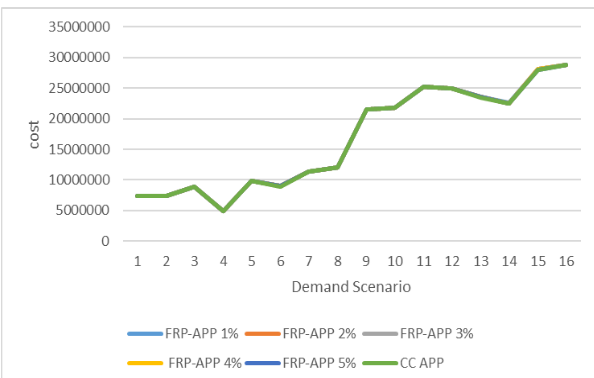

Figure 23: Total current cost comparison FRP-APP, RS-APP, CC-APP, Automotive Parts Industry

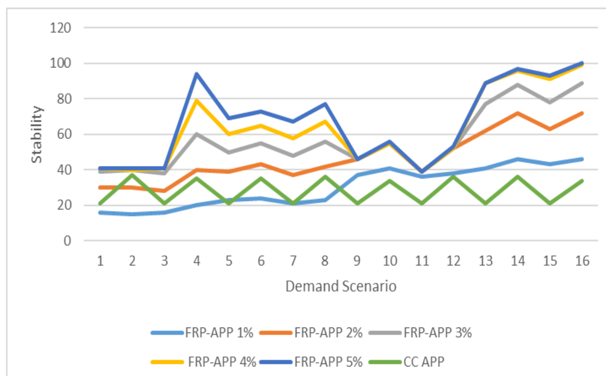

Figure 24: Stability comparison, FRP-APP, RS-APP, CC-APP, Automotive Parts Industry

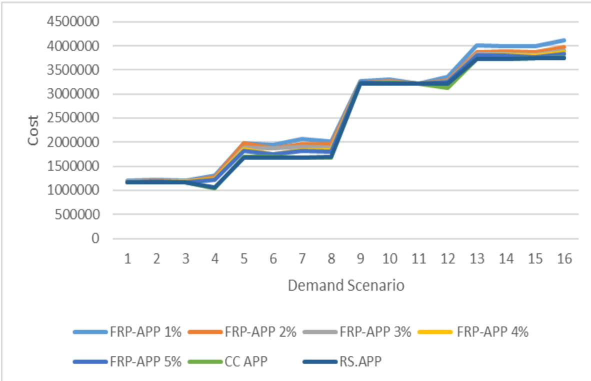

Figure 25: Total current cost comparison, FRP-APP, RS-APP, CC-APP, Machinery and Transmission Industry

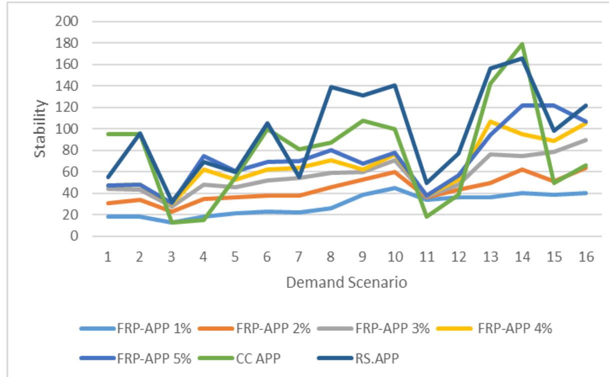

Figure 26: Stability comparison, FRP-APP, RS-APP, CC-APP, Machinery and Transmission Industry

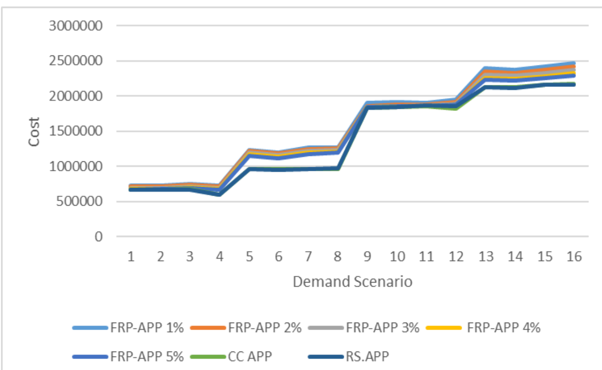

Figure 27: Total current cost comparison, FRP-APP, RS-APP, CC-APP, Wood and Paper Industry

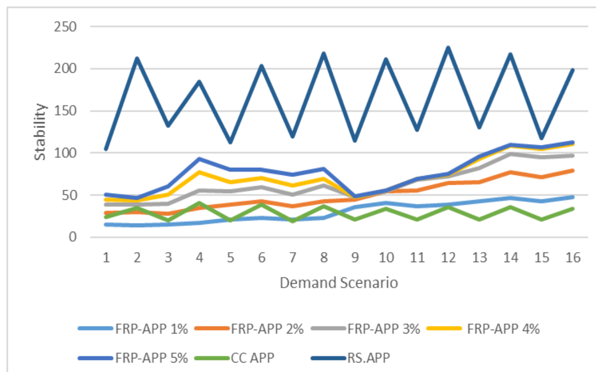

Figure 28: Stability comparison, FRP-APP, RS-APP, CC-APP, Wood and Paper Industry

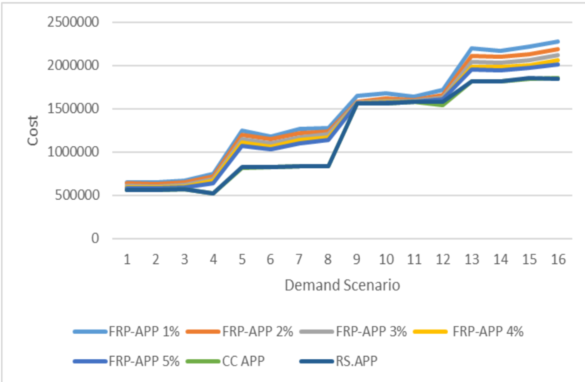

Figure 29: Total current cost comparison, FRP-APP, RS-APP, CC-APP, Air Conditioning Unite Industry

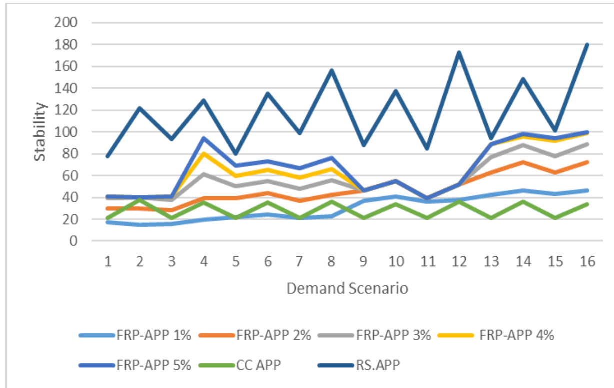

Figure 30: Stability comparison, FRP-APP, RS-APP, CC-APP, Air Conditioning Unite Industry

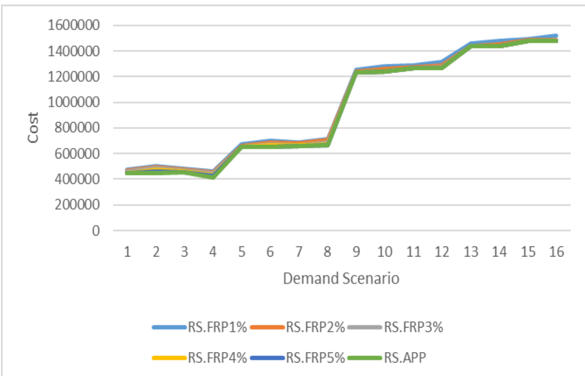

Figure 31: Total current cost comparison, RS-FRP-APP, RS-APP, Textile Industry

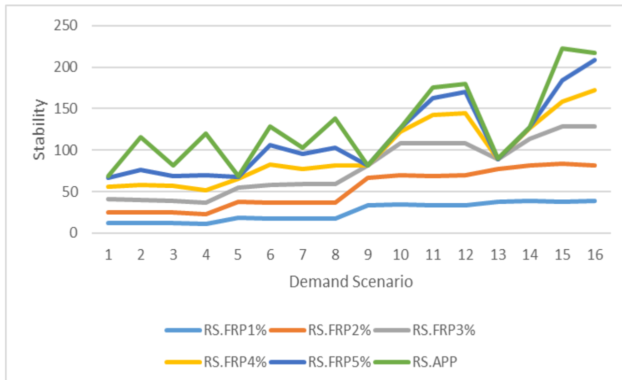

Figure 32: Stability comparison, RS-FRP-APP, RS-APP, Textile Industry

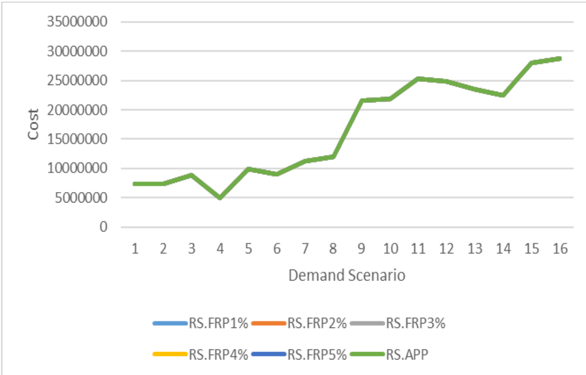

Figure 33: Total current cost comparison RS-FRP-APP, RS-APP, Automotive Parts Industry

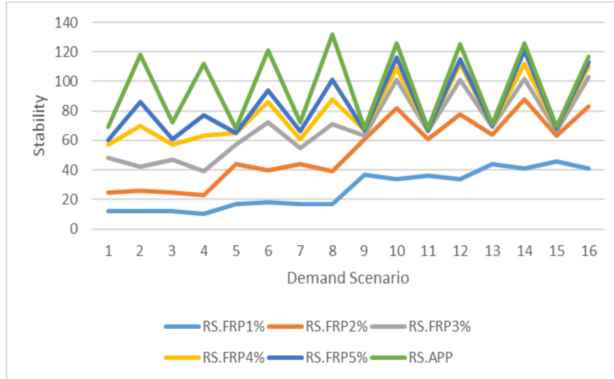

Figure 34: Stability comparison, RS-FRP-APP, RS-APP, Automotive Parts Industry

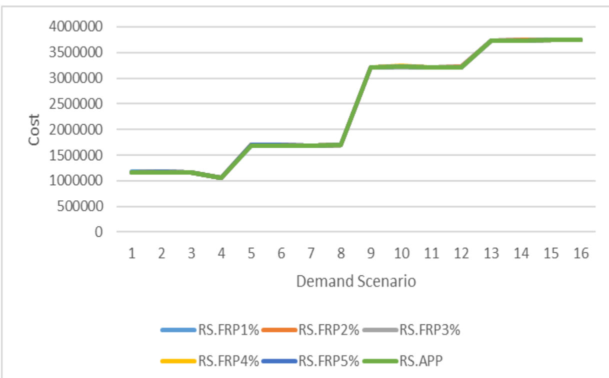

Figure 35: Total current cost comparison, RS-FRP-APP, RS-APP, Machinery and Transmission Industry

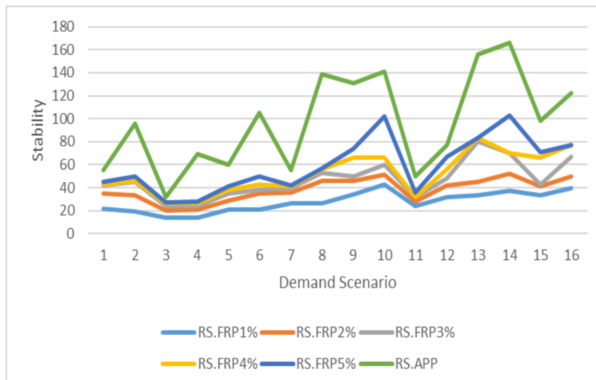

Figure 36: Stability comparison, RS-FRP-APP, RS-APP, Machinery and Transmission Industry

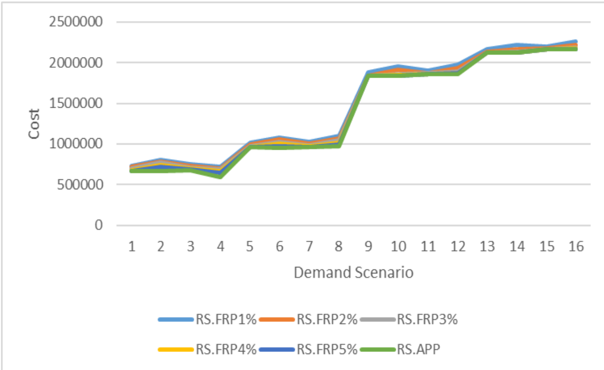

Figure 37: Total current cost comparison, RS-FRP-APP, RS-APP, Wood and Paper Industry

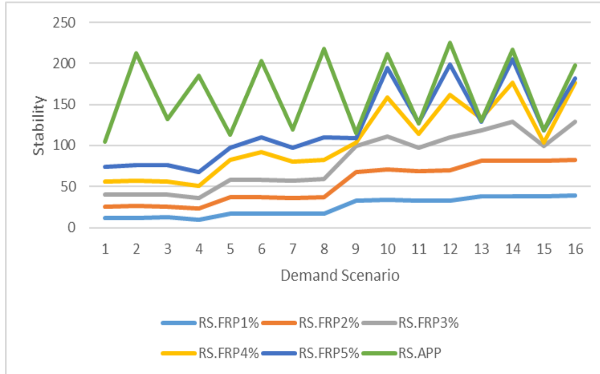

Figure 38: Stability comparison, RS-FRP-APP, RS-APP, Wood and Paper Industry

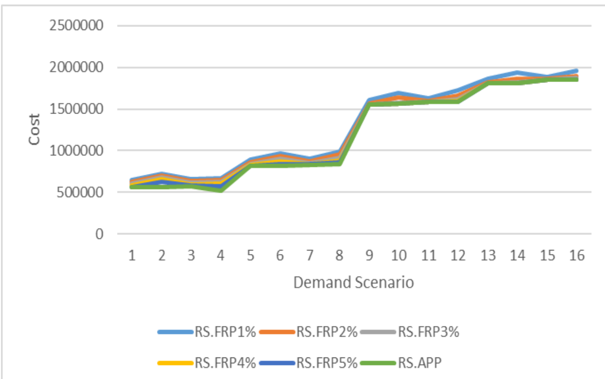

Figure 39: Total current cost comparison, RS-FRP-APP, RS-APP, Air Conditioning Unite Industry

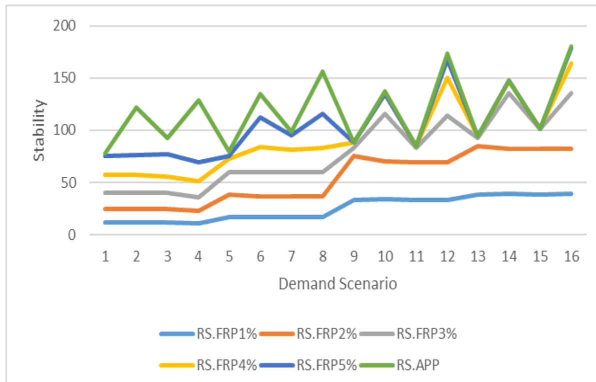

Figure 40: Stability comparison, RS-FRP-APP, RS-APP, Air Conditioning Unite Industry