DURABILITY, INDIRECT BANKRUPTCY COSTS, AND CAPITAL STRUCTURE

by

Dhara G. Shah

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Approved by:

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Dr. David C. Mauer

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Dr. Yilei Zhang

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Dr. Steven P. Clark

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Dr. Artie Zillante

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ABSTRACT

DHARA G. SHAH Durability, Indirect Bankruptcy Costs, and Capital Structure
(Under the direction of Dr. David C. Mauer)

I contribute to the literature by formally modeling the indirect bankruptcy costs for a firm that sells durable goods. Consumers concerned about warranties or future product services penalize the distressed firm by putting off purchases or leaving the market. Subsequent lower demand of durable goods generates lower cashflows for the firm, and these lost profits are larger if the firm produces more durable goods. In a parsimonious framework, I show that the loss in profitability (i.e., indirect bankruptcy costs) significantly lowers the firm's demand for leverage. I also acknowledge that there is an innate consumer demand for durability (i.e., useful life of a product) and show that the firm supplies lower durability than the market demands. Other than the cost of producing durable goods, indirect bankruptcy costs limit the firm's choice of durability. Finally, I also show that if the firm has a valuable innovation option, it exercises the option sooner to offset bankruptcy costs of durability.

Keywords: Durability, Durable Goods, Indirect Bankruptcy Costs, Capital Structure.
JEL Classification Numbers: D11, D20, D21, G32, G33, L15, L68.

Dedications

To my parents G. L. Shah and Ajita G. Shah, for their faith in me.

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Lastly, all remaining errors are my responsibility.

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1. Introduction

Titman (1984) and series of other papers investigate the agency (contracting) problems outside the firm (i.e., the firm's implicit contracts with consumers, with competitors, and suppliers) and their implication for a firm's financing decisions.[1] Titman (1984) emphasizes how consumers induce indirect bankruptcy costs for the firm that produces durable goods. Consequently, the firm's ability to take on more debt is restricted due to rise in such costs. I formally model indirect bankruptcy costs for a firm that produces durable goods with a certain durability (useful life of a product/durable-good) in two different frameworks.[2] I show that indeed, such bankruptcy costs reduce the firm's demand for leverage. Further, in absence of optimal debt contracting, the firm's leverage limits its choice of durability and output. By endogenizing the choice of durability and choice of leverage, I show that under certain conditions optimal durability is decreasing in the tax-rate and demand uncertainty and increasing in demand growth rate. Lastly, I examine these results when the firm has an innovation option and I find that the firm chooses to exercise the option sooner to mitigate durability-induced indirect bankruptcy costs.

A variety of articles examine possible causal linkages between a firm's performance and firms' product quality choices (see for e.g., Rose (1990), Luo and Bhattacharya (2006), and Adelino et al., (2015)). However, there is scant empirical or theoretical research on durability –

[1] For example, see Benoit (1984), Brander and Lewis (1986), and Maksimovic (1988)

[2] Where other papers focus on contracting costs of other stakeholders (consumers in our case), my trade-off model takes these stakeholders' contribution to profitability into account against its costs. I model consumers' demand for durability - which contributes towards profitability. In comparison, the cost is the decline in consumer demand for durable goods when the firm nears bankruptcy. I model these indirect bankruptcy costs as being larger for longer product durability.

the life of a product and firm's profitability.[3] I model the consumer demand for durability (to the best of my knowledge, I am the first to do so) and I show that the firm's profitability is positively correlated with it.[4] My model also recognizes that consumers impose an additional cost on the firm by leaving the market during financial distress. Specifically, since consumers will not be able to service goods or replace parts of goods that they purchased; they will suffer a loss when the firm defaults. Consumers are thus less willing to buy durable goods when the firm defaults, which gives a rise to indirect bankruptcy costs.[5] Thus, my model is a tradeoff model of costs and benefits of product durability.

In this paper I focus on indirect bankruptcy costs that arise due to a firm's product durability, and assume all other bankruptcy costs (e.g., such as filing fees, trustee expenses, legal and accounting fees, and other costs of reorganization) are zero.[6] I show that these indirect bankruptcy costs are a significant determinant of a firm's capital structure. The firm responds to the demand for durability and produces more durable goods, which also extends the firm's commitment to consumers to service its products. When the firm defaults, the stockholders transfer the ownership to debtholders. This transfer of ownership may bring new management, different suppliers, and a change of key personnel. As such, consumers will be concerned over future service difficulties of the durable good. Therefore, as the firm nears default, it faces falling demand and loss in profit, which will be primarily faced by debtholders. The firm's creditors

[3] Whereas product quality has no unanimous definition, product durability (life of a product) is specific and easily measured.

[4] I assume that in absence of debt contracting, the firm's choice of durability is a function of consumer demand for durability, the cost of producing a more durable product, and the structure of the market (as measured by price elasticity of demand). The firm's choice of capital structure further influences it choice of durability.

[5] For example, see Titman (1984), Opler and Titman (1994), and Hortaçsu et al. (2013)

[6] Here on, I will use indirect bankruptcy costs and bankruptcy costs interchangeably to indicate bankruptcy costs arising due to product durability.

expect these ex-post costs and increase the cost of debt, thereby decreasing the firm's demand for leverage.

The firm that sells durable goods has an implicit contract with consumers to service its product until its useful life (durability) is over. Indirect bankruptcy costs arise when the firm defaults and fails to fulfill such agreements.[7] Thus, the firm's financing decision will affect such commitment to consumers, i.e., the greater the durability, the more extended the commitment to service its products, and hence more chances of unfulfilled commitments in default. Thus, these costs progressively make debt costlier in durability and pushes the firm to produce goods with lower durability to offset some of these costs. Lower durability will not only reduce the length of the commitment to consumers, but will also offset the fall in demand for durable goods in default. Thus, firm's sale and profitability improve, and indirect bankruptcy costs decrease.[8]

Altman (1984) defines indirect bankruptcy costs as the lost profits that a firm can be expected to suffer due to significant bankruptcy potential. I extend my basic framework to include such a case. I show that consumers rationally expect firm's imminent bankruptcy and stay away from the market. The firm faces a shift down in demand and sells fewer goods as it approaches default. Total market value of equity goes down and the firm suffers from severe loss in profitability. The indirect bankruptcy costs arising in this case are larger and more impactful on the firm's financing decisions than my basic framework, where the firm faces indirect costs only in default.

[7] In such cases, indirect costs occur when the firm has declared bankruptcy and is attempting to operate and manage a return to financial health. Indirect bankruptcy costs are not limited to firms only in default. Firms which have high probabilities of bankruptcy, whether they eventually fail or not, still can incur these costs.

[8] I do not explicitly model the commitment or warranty services, but model the cost that arises due to these implicit contractual agreements between the consumer and firm.

I also extend my modeling framework to examine indirect bankruptcy costs of a durable goods firm on its invention behavior. Though much of the academic literature in finance has focused on financial markets that play a role in driving technological innovation and commercializing ideas, I focus innovation at a firm level.[9] The firm in my model has an option to innovate, which will alter its product market, i.e., it will reduce the cost of producing durable goods. By exercising this option, the firm can offer more durable and less expensive goods which increase consumer demand. More consumers will participate in the market and the firm will bring more cashflow in. I find that when the firm faces increasing indirect bankruptcy costs due to durability, it exercises this option sooner to offset effects of indirect bankruptcy costs.

To the best of my knowledge, this is the first paper to formalize indirect bankruptcy costs in two different frameworks and show how the firm's capital structure influences product market decisions. Further, I also show the feedback effect of such decisions on the firm's capital structure. This paper contributes to the finance literature that views durability as primarily a pricing factor, which means that a firm producing durable goods has higher systematic risk. Higher systematic risk is associated with lower financial leverage, and so a firm that produces durable goods has lower leverage (Gomes et al. (2009)). Ait-Sahalia et al. (2004) measure risk aversion from consumption data and show that the risk aversion implied by the consumption of durable goods is more than that of non-durable goods, which suggests that the consumption of durable goods is more responsive to macro-economic shocks than the consumption of non-durable goods.[10]

[9] For example, Brown et al., (2009), Comin and Nanda (2014), and Hsu et.el., (2014) look at the role of capital markets on innovation and research and development. Our focus is very specific to firm level, so I do not look at or address questions pertaining to innovation and capital markets or theory of financing innovation or information asymmetry in innovation expenditures.

[10] Ait-Sahalia et al. (2004) address the equity risk premium puzzle by showing that risk aversion measured from consumption data is different for non-durable goods and durable goods. They show that the risk aversion implied by the consumption of durable goods is more than that of non-durable goods, which suggests that the consumption of durable goods is more responsive to macro-economic shocks than the consumption of non-durable goods.

Schwert and Strebulaev (2014) show that higher systematic risk reduces a firm's leverage, which suggests that firms producing durable goods have higher systematic risk, and so have lower leverage. The first paper to establish a link between goods durability and leverage is Lee et al. (2019). They show that durable goods have higher demand elasticity, which exposes durable goods producers to cost shocks, which encourages firms producing durable goods to build financial slack (e.g., reduce leverage to create unused debt capacity, build cash, and have lower dividend payouts).

Titman (1984) predicts lower debt ratios for firms whose liquidation imposes significant costs on its workers, customers, and suppliers. My frameworks concur this finding, and in addition also acknowledges benefits that these customers bring in. I model and show that the firm's cashflow increases in consumer demand for durability. I find that an increase in product durability increases consumer demand for durable goods, which increases the firm's profitability and creates the debt capacity. Thus, in this paper, I show that the firm's capital structure is a result of its trade-off of profitability, resulting from consumer demand for durability, and indirect bankruptcy costs of durability.

While my paper investigates the effect of the firm's capital structure on the choice of product durability (and hence its output), there are numerous economics literature that examine how the firm's financing decision affects its output strategies. For example, Benoit (1984) stresses effects of firm's financial constraint on its entry into the market. Importantly, Brander and Lewis (1986) show how equity's limited liability affects implicit contracting with outside competitors. They show that as firms take on more debt, they will have an incentive to pursue output strategies that raise returns in good states and lower returns in bad states. The basic point is that shareholders will ignore reductions in returns in bankrupt states, since bondholders become the residual claimants. Maksimovic (1988) analyzes the effect of a firm's capital structure on its product market strategy and shows that competition limits the amount of debt the firm can take on. My paper shows that agency costs (i.e., indirect bankruptcy costs) with outside stakeholders limits its choice of capital structure and forces the firm to innovate sooner. I also show that the firm’s financing decision exacerbates these costs and as a result constraints its choice in product market.

Other related literature includes Swan (1970, 1971, 1980), Coase (1972), Bulow (1982, 1986), Rust (1986), and Waldman (1996) who examine on the firm’s choice of durability in presence of second-hand markets of durable goods. They model strategic interactions between the firm and consumers. While I do not attempt to model the dynamics between durable-goods consumers and the firm, I model the consumer demand for durability in a setting which captures the market structure.

Lastly, the links I establish between durability and financing decisions contribute to a stream of literature examining the relation between aggregate dynamics and financing decisions (see Hackbarth et al.,(2006); Bhamra et al, (2010a) and (2010b); Chen (2010); Chen et al., (2016); Chen and Manso, (2017); and Westermann, (2018)). These researches show that higher firm sensitivity to the risk of changing macroeconomic conditions (e.g., decrease in consumer demand as the economy transitions into recession) increase credit spreads and default risk, which encourages more conservative financial policies. Independent from but complementary to this macro risks channel, I jointly determine product durability and the firm’s capital structure. Subsequently, our results show (under certain conditions) the firm supplies greater durability when faced with higher discount rates or demand growth. However, the firm reduces durability as tax rates and demand uncertainty increases.

The remainder of the paper is organized as follows. Section 2 presents the models. Section 3 presents the models for a firm with an innovation option. Section 4 presents numerical results and analysis. Finally, section 5 concludes.

2. Model

I begin by considering consumer demand for durability in Section 2.1, and then analyze the interaction of a profit maximizing firm and consumers demand for durable goods in Section 2.2. Section 2.3 studies the firm's choice of durability under the polar market structure cases of monopoly and perfect competition. Lastly, Sections 2.4 and 2.5 model firm value with debt and equity financing.

2.1. The Consumer

The Bureau of Economic Analysis (BEA) defines durable goods as tangible products that can be stored or inventoried and that have an average service life of three years or greater. Accordingly, I define durability (T) as the usable service life of a product in years. Examining the literature to see how the firm's choice of product durability is influenced by consumer behavior, I quantify the interaction between the producer and the market.

Conn et al. (1972) in their survey of primary markets for durable goods (e.g., TVs, Radios, Vacuum cleaners, etc.) and their disposal show that before the product becomes obsolete, 41% of consumers stopped using the product because they could not decide what else to do with it, 21% of consumers threw away the product because it was too costly to repair, and the remainder sold, donated, or traded the product. For products where performance is an essential function, e.g., refrigerators and cars, the cost and inconvenience incurred from repeat failures encourage consumers to dispose of the product prior to the end of its useful life. In another survey, Tippett and Ruffin (1975) show that for appliances acquired new, the greatest variation from the average retention period was among households that had moved in the 18 months prior to the survey. On average, moving households shortened the service life (i.e., threw away or sold with the house) of their appliances by 60 percent, compared with other households. Estimated service life of appliances owned by households that did not move in the 18 months prior to the survey was more than three times as long as for households that did move. Further, they found that retention was shorter for households in which the head was under age 50 than for all other households (about 20 percent less than for all households).

In comparison, the marketing and management literature study how fashion and aesthetics affect consumer choices ranging from automobiles and housing to clothing and music. This literature finds that the upper class of the market distinguishes itself from the masses by adopting a fresh style every few years. This is often referred to as trickle-down theory. Using changing fashion as an example, new style invades consumer choices, and consumers come to the market to either purchase an additional item or replace an existing one. Products are disposed because of technological or fashion obsolescence.

In summary, after the purchase of a product, the consumer continuously evaluates the costs of operating and maintaining durable goods, efficiency of current performance, and the cost of disposal in comparison to the benefits of a new product with enhanced features and improved efficiency and performance. When the costs of a used durable good outweigh the benefits, the consumer disposes the product regardless of remaining useful life. The upshot is that increasing durability is desirable, but beyond a point it will have little impact on the age of the product disposed.[11]

Based on the preceding discussion, I posit that consumer preference for durable goods and consumer disutility from storing and maintaining a product are both increasing functions of product life, T. I assume that a second-hand market for used durable goods does not exist, i.e., the

[11] See for example Debell and Dardis (1979) and Sproles (1981)

consumer cannot replace a durable good with a used durable good from a second-hand market.[12] Thus, consumer demand for durability (dd) per unit of durable good is increasing at a decreasing rate in T. For convenience, I model these properties with the function:

$$dd = e^{\delta T - \gamma T^2}, \qquad \delta, \gamma > 0, \tag{1}$$

where δ is the durability preference parameter, and γ is the durability disutility parameter.[13] It is straightforward to show that dd is increasing at a decreasing rate within the range $T \in \left(\frac{\delta}{2\gamma} - \frac{1}{\sqrt{2\gamma}}, \frac{\delta}{2\gamma}\right)$, and is decreasing at a decreasing rate within the range $\left(\frac{\delta}{2\gamma}, \frac{\delta}{2\gamma} + \frac{1}{\sqrt{2\gamma}}\right)$, i.e., the sign reversal for the rate of change of dd occurs at $\delta/2\gamma$, which is the point where consumer demand for durability is maximized. Thus, I see from (1) that optimal consumer demand for durability is given by

$$T_{dd}^* = \frac{\delta}{2\gamma}. \tag{2}$$

It is straightforward to show that (2) maximizes consumer demand for durability.[14] T_{dd}^* is increasing in the durability preference parameter (δ) and decreasing in the durability disutility parameter (γ).

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[12] In the seminal paper Coase (1972) showed that secondary market of durable goods limits the extent to which a firm can extract rent from consumers. Specifically, as durable goods in the secondary market serve as a cheaper (almost perfect) substitute to new durable goods, it increases total number of goods available in the market for sale, thereby decreasing the price a firm can charge. In the limit, the firm will lose the complete market power and be able to charge only marginal cost (of producing durable good). While I assume there exist no secondary market of durable goods and hence no strategic interactions between consumers and the firm, I believe that the price elasticity enables me to capture an aspect of available substitutes in the market and the competition that the firm faces.

[13] dd quantifies the consumer preference for the durable good. An increase in dd thus represents the increased consumer willingness to buy the durable good. Perhaps a better functional form of demand for durability is $dd = \delta T - \gamma T^2$, which gives zero preference for a good with zero durability. However, the choice in (1) allows for closed form analytical solutions to the objective function (discussed below). Numerical analysis suggests that other than the corner solution at $T = 0$, the exponential functional form in (1) captures expected behavior of demand for durable goods and gives nonnegative demand for durable goods.

[14] Taking the first and second derivatives of (1) gives, $\frac{\partial(dd)}{\partial T} = (\delta - 2\gamma T)dd = 0, \left(i.e., \ T_{dd}^* = \frac{\delta}{2\gamma}\right)$ and $\frac{\partial^2(dd)}{\partial T^2} = (\delta - 2\gamma T)^2 dd - 2\gamma dd < 0$ at $T = T_{dd}^* = \frac{\delta}{2\gamma}$.

I assume that the durable-goods producer faces quantity demand schedule, q, that is a function of consumer demand for durability, dd, output price, p, and a stochastic demand X. The demand schedule is given by

$$q = \left(e^{\delta T - \gamma T^2}\right) X p^{-\varepsilon}, \quad \varepsilon > 1, \tag{3}$$

where ε is the price elasticity of demand.[15] The consumer's demand for durability enters equation (3) multiplicatively, indicating that an increase in dd increases consumers willingness to buy more durable goods. Further, an increase in T (for $T < T_{dd}^*$) results in an outward shift of the demand curve, i.e., the quantity demanded increases at every price. Equation (3) assumes the firm can influence the output price by choice of quantity, i.e., the inverse demand curve, $p = f(q)$, is downward sloping. Thus, optimal q is where $MR = MC$ instead of where $p = MC$. I assume that demand evolves according to geometric Brownian motion:

$$\frac{dX}{X} = \alpha dt + \sigma dZ, \tag{4}$$

where α is the drift rate, σ is the volatility rate, and dz is the increment of a standard wiener process.

2.2.The (Unlevered) Firm's Profit Maximization Problem

I assume the total cost of manufacturing the durable good is given by

$$TC = q e^{\kappa T}, \qquad \kappa > 0, \tag{5}$$

where κ is the rate at which cost increases in durability, T. Therefore, the marginal cost is

$$MC = e^{\kappa T}, \tag{6}$$

[15] The demand function has constant elasticity of demand. I rule out $\varepsilon \leq 1$, because in this case marginal revenue would be negative.

which embeds the assumption that the marginal cost of producing the durable good is increasing in durability. For non-durable goods, i.e., when $T = 0$, marginal cost is one (numéraire), and market demand for goods is solely determined by price and the stochastic demand (see equation (3)).

Assuming the firm faces a constant tax rate, τ, then from (3) and (6), the firm's after-tax profit is

$$\pi = (1-\tau)(p(q)q - qMC) = (1-\tau)\left(q^{\frac{\varepsilon-1}{\varepsilon}} X^{\frac{1}{\varepsilon}} e^{\frac{(\delta T - \gamma T^2)}{\varepsilon}} - qe^{\kappa T}\right) \tag{7}$$

where $p(q)$ is the inverse demand function (i.e., price as a function of quantity) derived from (3).[16] Note that π in (7) embeds the price adjustment as the firm chooses the optimal quantity to produce.

Maximizing π in (7) with respect to quantity (q) and durability (T) gives the first-order conditions:

$$\frac{\partial \pi}{\partial q} = (1-\tau)\left(\frac{\varepsilon-1}{\varepsilon} q^{\frac{-1}{\varepsilon}} X^{\frac{1}{\varepsilon}} \left(e^{\delta T - \gamma T^2}\right)^{\frac{1}{\varepsilon}} - e^{\kappa T}\right) = 0, \tag{8}$$

and

$$\frac{\partial \pi}{\partial T} = (1-\tau)\left(\frac{\delta - 2\gamma T}{\varepsilon} q^{\frac{\varepsilon-1}{\varepsilon}} X^{\frac{1}{\varepsilon}} \left(e^{\delta T - \gamma T^2}\right)^{\frac{1}{\varepsilon}} - kqe^{\kappa T}\right) = 0. \tag{9}$$

Solving (8) for optimal quantity gives

$$q^* = \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon} X\, e^{(\delta - \varepsilon\kappa)T - \gamma T^2}. \tag{10}$$

Substituting q^* into $p(q)$ gives

$$p(q^*) = e^{\kappa T}\left(\frac{\varepsilon}{\varepsilon - 1}\right), \tag{11}$$

[16] The after-tax profit in (7) assumes full loss offset provisions, i.e. the firm receives a rebate for losses proportional to τ.

which, given $1 < \varepsilon < \infty$, gives the classic result that a monopolist sets quantity so that price is above marginal cost.[17] The formulation embeds perfect competition as the limiting case as ε tends to infinity, i.e., price is equal to marginal cost and markup is zero. Further, notice that the output price is an increasing function of durability, and the difference between output price and marginal cost, i.e., the profit margin $p(q^*) - MC = e^{\kappa T}\left(\frac{1}{\varepsilon-1}\right)$, is an increasing function of durability and the cost parameter, κ.[18] Output, q^*, is the profit maximizing quantity for a given T, which encompasses the market demand for goods and the cost of producing these goods. From (10) and (11), as $T \to \infty$, $p(q^*) \to \infty$ and $q^* \to 0$, indicating it is not optimal for the firm to produce an infinitely durable good.

Substituting (10) into (9) and solving for the profit maximizing choice of durability gives

$$T^* = \frac{\delta - (\varepsilon - 1)\kappa}{2\gamma}, \qquad \delta > (\varepsilon - 1)\kappa, \tag{12}$$

where for $T^* > 0$ I assume that $\delta > (\varepsilon - 1)\kappa$.[19] . Optimal durability is affected by four factors: δ (consumer durability preference), γ (consumer disutility from continuing to hold a durable good), κ (parameter for rate of change of cost with respect to durability), and price elasticity of demand, ε. As seen in equation (12), optimal durability increases in δ, and decreases in $\gamma, \kappa,$ and

[17] Note that for given T, the output price at the optimal quantity is not a function of X, δ or γ. An increase in X or δ or a decrease in γ increases market demand, which encourages the firm to increase output. However, a firm's ability to influence price is unaltered. Specifically, if the firm increases price following an increase in demand (through X, δ or γ), the quantity demanded will decrease and the firm will deviate from producing the optimal amount of output in (10), which will reduce profit. In summary, the firm will be able to sell more quantities without decreasing price.

[18] The output price is a decreasing function of elasticity of demand, $\partial P(q^*)/\partial \varepsilon < 0$, which means that the profit margin, $p(q^*) - MC$, is also a decreasing function of demand elasticity.

[19] The numerator of equation (12) shows T^* increases in consumer preference, δ and a firm's ability to influence price, κ, and decreases in consumer price sensitivity, $\varepsilon\kappa$. Therefore, for a firm with market power, I assume $\delta > (\varepsilon - 1)\kappa$, and a tighter bounder for a firm without market power, $\delta > \varepsilon\kappa$, i.e, consumer prefers durable goods than non-durable goods.

ε. An increase in ε decreases a firm's ability to influence the output price, and the firm's output falls.

Substituting T_{dd}^* in (2) into (12) gives

$$T^* = T_{dd}^* - \frac{(\varepsilon - 1)\kappa}{2\gamma} = T_{dd}^* - \frac{\varepsilon\kappa}{2\gamma} + \frac{\kappa}{2\gamma} \tag{13}$$

Since $\varepsilon > 1$ and $\kappa, \gamma > 0$, I see in (13) that the firm produces a durable product that has lower durability than the durability preferred by consumers. The firm's optimal durability increases in demand for durability and its ability to influence the price $(\kappa/2\gamma)$, and since durable goods are costlier to produce, the firm's optimal durability decreases in price sensitivity $(\varepsilon\kappa/2\gamma)$. If $\kappa = 0$, i.e. the cost of producing durable and nondurable goods are same, then the firm's optimal durability satisfies $T^* = T_{dd}^*$. Alternatively, if κ is large (i.e., $\kappa \geq \delta/(\varepsilon - 1)$), then the firm is better off producing non-durable goods. If $\gamma \to \infty$, i.e. consumers prefer only nondurable goods, then the firm's optimal durability $T^* = T_{dd}^* = 0$.

Coase (1972) showed that a durable-goods monopolist will face competition from its used products. The upshot is that a durable-goods monopolist cannot charge a monopoly price.[20] Bullow (1986) confirms that in order to avoid this time-inconsistency problem the monopolist should build less durable goods. This reduces the time between the replacement sales and restores the firm's ability to charge monopoly prices.

My result complements literature by showing that the firm reduces durability as competition intensifies. When competition increases, consumers will have more product choices and for a given price, quantity demanded will decrease. Thus, the firm will be forced to decrease

[20] Subsequent work has shown that if the firm commits to restrict the supply of the good or lease the good (i.e., internalize the price impact) then the firm will be able to charge a monopoly price.

price and output. In order to counter these effects of competition, the firm reduces durability, which has two opposing effects on the firm's output. First, a decrease in durability decreases the cost of production which increases the firm's quantity choice and decreases the price, which in turn increases the quantity demanded. Second, when the firm reduces durability, consumer demand decreases. The overall effect of reducing durability on output is positive. I summarize this finding in a following proposition.

Proposition 1: A monopolist produces a more durable good than an otherwise identical firm under perfect competition.

***Proof**:* I can characterize perfect competition by marginal cost pricing:

$$p - \frac{\partial TC}{\partial q} = 0, \tag{14}$$

Since, the firm's profit is also a function of durability, I use equation (9) for first order condition of profit maximization, which gives

$$\frac{\partial \pi}{\partial T} = (1-\tau)\left(\frac{\partial(p(q^*)q)}{\partial T} - \frac{\partial TC}{\partial T}\right) = 0 \tag{15}$$

Condition (14) implies zero economic profits when firm increases quantity by one unit and condition (15) follows directly from individual firms' profit maximizing behavior. Substituting the value of p from equation (14) into equation (15), I get optimal durability under perfect competition, which is given by,

$$T_{pc}^* = \frac{\delta - \varepsilon\kappa}{2\gamma} < T^* < T_{dd}^*. \tag{16}$$

Note that, when $\kappa = 0$ or $\lim \gamma \to \infty$, then $T^* = T_{dd}^* = T_{pc}^*$. Specifically, when marginal cost of producing durable good is no different than the marginal cost of non-durable goods, then the firm will always produce goods that satisfies the market demand of durability, independent of a market structure. Similarly, when consumers disutility in durability is very large, i.e.,$T_{dd}^{*} = 0$, then the firm is better off producing non-durable goods. In following sections, I evaluate the firm value when it is unlevered and when it is financed with debt and equity and examine the influence of product durability on the firm's capital structure.

2.3.Unlevered Firm Value

Substituting q^* in (10) into π in (7) gives the firm's profit function as

$$\pi(q^*; T; X) = (1-\tau)\Omega X, \tag{17}$$

where $\Omega = \left(\frac{1}{\varepsilon}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon-1} e^{(\delta-(\varepsilon-1)\kappa)T-\gamma T^2}$. I do not substitute the profit maximizing durability at this point, I work with (17) because it will be convenient to write firm value as a function of T rather than T^*.

Assuming an infinite horizon, I find the value of the firm, $V(X; q^*, T)$, whose profit is given by (13). Assuming risk-neutrality and using standard arguments, firm value must satisfy the equation:[21]

$$\frac{1}{2}\sigma^2 X^2 V_{XX} + \alpha X V_X - rV + (1-\tau)\Omega X = 0. \tag{18}$$

The homogeneous part of equation (18) is a linear combination of two power solutions corresponding to the roots of the quadratic equation $\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - r = 0$. Thus, I have

$$V^h(X; q^*, T) = AX^{\beta_1} + BX^{\beta_2}, \tag{19}$$

where the constants A and B remain to be determined and where

[21] For technical reasons (i.e., no bubbles), I assume $r > \alpha$. We can easily make risk-adjustments by equating the total return on the firm to the risk-adjusted required rate of return.

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{(2r)}{\sigma^2}} \qquad > 1, \tag{20}$$

and

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left[\frac{\alpha}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{(2r)}{\sigma^2}} \qquad < 0.^{22} \tag{21}$$

I also need a particular solution to equation (18). A simple substitution shows that $(1-\tau)\Omega X/(r-\alpha)$ satisfies (18). Therefore, the general solution of (18) is given by

$$V(X;q^*,T) = AX^{\beta_1} + BX^{\beta_2} + \frac{(1-\tau)\Omega X}{r-\alpha}. \tag{22}$$

The firm value in (22) must satisfy the following boundary conditions:

$$V(0;q^*,T) = 0 \tag{23a}$$

and

$$\lim_{X\to\infty}\left(\frac{V(X;q^*,T)}{X}\right) = 0 \tag{23b}$$

Condition (23a) requires that the firm is worthless when the demand X is absorbed at zero, so B must be zero; and condition (23b) is a no bubble condition, which is satisfied when A is zero. Substituting equation (22) into (23a) and (23b) gives

$$V(X;q^*,T) = \frac{(1-\tau)\Omega X}{r-\alpha}. \tag{24}$$

Thus, firm value is the discounted expected value of a growing stream of after-tax profits. Note that Ω depends on $T, \delta, \gamma, \kappa$ and ε. For notational simplicity, in subsequent analysis I suppress the dependence of $V(X;q^*,T)$ on q^* and T and simply write $V(X)$. It is trivial to show that the value

[22] See Dixit and Pindyck (1994) p. 143 for more details about roots of a characteristic equation.

maximizing choice of durability for the unlevered firm is the same as given by equation (12), i.e., the T^* that maximizes (unlevered) profit also maximizes unlevered firm value.

2.4. Model 1: Equity and Debt Financing with Indirect Bankruptcy Costs.

The model 1 below incorporate indirect costs of bankruptcy as borne by the firm that produces durable goods. These indirect costs refer to the loss to consumers and/or stakeholders if the firm were to liquidate. Since, the consumer will not be able to service the good or replace parts of the good that she purchased, she will suffer a loss when the firm defaults. Many stakeholders of the firm (e.g., suppliers) will also incur losses due to excess unsold inventories. In liquidation, the firm's ownership is transferred to bondholders. The transfer of ownership sometimes accompanies closing a profitable line of business, change of suppliers, change of management etc., and as a result, consumers become uncertain about after-sale services and product maintenance. Accordingly, consumers' willingness to buy the durable good declines under different management in bankruptcy. Thus, in the first model, I posit that when the firm transfers ownership to bondholders in bankruptcy, it faces a negative demand shock. I further assume that this shock increases with durability (i.e., the useful life of the product). The longer the life of the product, the higher the expected maintenance cost to consumers, which makes them more uncertain about the company's future and less willing to buy the durable good. Thus, the firm indirectly bears the cost of bankruptcy.[23]

I begin with standard valuation of the firm's outstanding securities. Suppose the firm is partially financed with debt. I assume the firm's debt is permanent with no stated maturity and

[23] See Titman (1984) and Hortacsu et al. (2013) for detailed discussions of these types of bankruptcy costs.

pays a continuous coupon C per unit time. The after-tax cashflow to equity is therefore $(1-\tau)(\Omega X - C)dt$. The general solution for the value of levered equity is

$$E(X) = B_1 X^{\beta_1} + B_2 X^{\beta_2} + (1-\tau)\left[\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right], \qquad X > X_D, \tag{25}$$

where $\Omega \equiv \left(\frac{1}{\varepsilon}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon-1} e^{(\delta-(\varepsilon-1)\kappa)T-\gamma T^2}$, B_1 and B_2 are constants to be determined, $\beta_1 > 1$ and $\beta_2 < 0$ are roots of the characteristic equation, $\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r$, and default boundary, X_D, is endogenously determined by equity holders. The general solution in (21) must satisfy the following boundary conditions:

$$lim_{X\to\infty} E(X) = (1-\tau)\left[\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right], \tag{26a}$$

$$E(X_D) = 0, \tag{26b}$$

and

$$\left.\frac{\partial E(X)}{\partial X}\right|_{X=X_D} = 0. \tag{26c}$$

Condition (26a) states that default becomes irrelevant as X becomes large and it is a standard no-bubble condition. This condition is satisfied when $B_1 = 0$. Condition (26b) assumes that equity has limited liability upon default, and condition (26c) is a standard smooth pasting condition at the default threshold, i.e., the default threshold is optimally determined to maximize the market value of equity. Substituting equation (25) into boundary conditions (26a) - (26c) gives

$$E(X) = (1-\tau)\left[\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right] - (1-\tau)\left[\frac{\Omega X_D}{r-\alpha} - \frac{C}{r}\right]\left(\frac{X}{X_D}\right)^{\beta_2}, \tag{27}$$

where

$$X_D = \left(\frac{\beta_2}{\beta_2 - 1}\right)\left(\frac{C}{r}\right)\left(\frac{r-\alpha}{\Omega}\right). \tag{28}$$

The debtholders receive a continuous coupon payment Cdt in the absence of bankruptcy. The general solution for debt value is

$$D(X) = \frac{C}{r} + B_3 X^{\beta_1} + B_4 X^{\beta_2}, \qquad X > X_D \tag{29}$$

where the constants B_3 and B_4 remain to be determined.

The general solution to (29) must satisfy the following boundary conditions:

$$lim_{X\to\infty} D(X) = \frac{C}{r} \tag{30a}$$

and

$$D(X_D) = V(\phi(T)X_D) \tag{30b}$$

Condition (30a) states that when there is no chance of default, debt is a risk-free perpetuity with coupon payment C. This condition is satisfied when $B_3 = 0$. Condition (30b) states that in bankruptcy (i.e., when $X = X_D$), debt holders receive the altered value of the unlevered firm. Specifically, when equity transfers ownership to debtholders, the firm faces a negative demand shock, $\phi(T), 0 < \phi(T) \leq 1$, with $\phi'(T) < 0$. The unlevered firm value, $V(\phi(T)X_D)$, is evaluated at $X = X_D$ and faces the negative demand shock.[24]

There are two simplifying assumptions in condition (30b). First, other bankruptcy costs, such as filing fees, trustee expenses, legal and accounting fees, and other costs of reorganization are assumed to be zero. Second, consumers don't anticipate the risk of bankruptcy and potential loss of warranties, services, or difficulties in maintaining the durable good until the actual default at $X = X_D$. Consumers penalizes the firm only at the point of default.

Now, I examine the debt structure of the firm by looking at what happens when it defaults. Bankruptcy disrupts business and damages the firm's ability to service durable goods after sale

[24] The value of unlevered firm is as given in (24), $V(X) = (1-\tau)\frac{\Omega X}{r-\alpha}$ and $V(\phi(T)X_D) = (1-\tau)\frac{\Omega\phi(T)X_D}{r-\alpha}$, where $\phi(T)$ represents the negative shock. As consumers rationally anticipate the firm's bankruptcy risk, their willingness to buy durable goods decreases. The longer is the life of the good, the higher the uncertainty associated with maintaining the durable good. Specifically, as durability of the good increases, the shock to demand is larger. Therefore, I posit $\phi'(T) < 0$, i.e., consumer demand upon default,X_D, decreases more with durability, T.

(e.g., parts suppliers are less confident supplying parts to a damaged company, automobile dealers switch makes of cars because they are uncertain about the future health of the business, employees in the service and maintenance departments jump to other jobs because they are concerned about the firm's ability to continue operations, etc.). Thus, even though the company may continue to operate in the hands of bondholders after bankruptcy, it is worth less because of these indirect costs. Note that these costs are associated with the production of a durable good (i.e., non-durable goods do not have similar costs).[25] Substituting (29) into (30a) and (30b) gives risky debt value as

$$D(X) = \frac{C}{r} + \left(V(\phi(T)X_D) - \frac{C}{r}\right)\left(\frac{X}{X_D}\right)^{\beta_2}. \tag{31}$$

The total value of the levered firm, $V^L(X)$, is the sum of the equity and debt values in equations (27) and (31):

$$V^L(X) = (1-\tau)\frac{\Omega X}{r-\alpha} + \frac{\tau C}{r}\left(1 - \left(\frac{X}{X_D}\right)^{\beta_2}\right) - V\left(\left(1-\phi(T)\right)X_D\right)\left(\frac{X}{X_D}\right)^{\beta_2}. \tag{32}$$

I see that the value of the levered firm is equal to the value of unlevered assets plus the present value of expected tax shields minus the present value of expected bankruptcy costs. The last term on the right-hand side of equation (32) represents indirect costs of bankruptcy, i.e., the loss of value when ownership transfers to debtholders in bankruptcy. This loss reflects the decrease in demand for the durable good when the firm declares bankruptcy.

Since, debt holders bear the indirect costs of bankruptcy, they will charge a higher cost of debt. The firm ultimately bears this higher cost of debt and will accordingly borrow less.

[25] I can easily incorporate the direct proportional and/or fixed bankruptcy costs. Since, I want to emphasis the impact of indirect bankruptcy costs, I assume direct costs are zero. This will not change the qualitative results.

Importantly, since bankruptcy costs increase in durability the firm can moderate bankruptcy costs by producing less durable products. I show these results analytically in the following section.

2.4.1. Analytical Results:

The firm maximizes value by optimally choosing durability, T, and coupon on debt, C. This joint optimization is derived from taking first order conditions of equation (32).

The firm-value-maximizing first-order conditions with respect to the coupon, C and durability, T, are

$$\frac{\partial V^L}{\partial C} = \frac{\tau}{r} - \frac{\tau}{r}\left(\frac{X}{X_D}\right)^{\beta_2}(1-\beta_2) + \frac{\beta_2}{r}(1-\tau)\big(1-\phi(T)\big)\left(\frac{X}{X_D}\right)^{\beta_2} = 0. \tag{33}$$

and

$$\frac{\partial V^L}{\partial T} = M_1 V - \frac{\tau C}{r}\beta_2 M_1\left(\frac{X}{X_D}\right)^{\beta_2} + M_1 V\left(\big(1-\phi(T)\big)X_D\right)\left(\frac{X}{X_D}\right)^{\beta_2}(1-\beta_2) + V(\phi'(T)X_D)\left(\frac{X}{X_D}\right)^{\beta_2}. \tag{34}$$

where $M_1 = \delta - (\varepsilon - 1)\kappa - 2\gamma T$.[26] Unlike equation (33), equation (34) is not analytically tractable. Therefore, in discussions below I solve equation (33) for optimal coupon, C^*, substitute it into equation (34), and derive comparative statics with respect to, T_L^*, which is the value maximizing durability for the levered firm.

First, I look at the firm's optimal coupon, C^*. For any level of durability, C^* is given by the C that solves (33):

$$C^* = rX\left(\frac{\Omega}{r-\alpha}\right)\left(\frac{\beta_2 - 1}{\beta_2}\right)\left[(1-\beta_2) - \left(\beta_2\frac{(1-\tau)}{\tau}\big(1-\phi(T)\big)\right)\right]^{1/\beta_2}. \tag{35}$$

[26] $M_1 = M_1(T)$ is the profit maximizing, first-order condition for an unlevered firm, where the firm's profitability is increasing for $M_1(T < T^*) > 0$ and maximized when $M_1(T = T^*) = 0$.

The second-order condition for a maximum is satisfied for $C > 0$ and is given as[27]

$$\frac{\partial^2 V^L}{\partial C^2} = \frac{\beta_2}{C}\frac{\tau}{r}\left(\frac{X}{X_D}\right)^{\beta_2}(1-\beta_2) - \frac{\beta_2^2}{rC}(1-\tau)\big(1-\phi(T)\big)\left(\frac{X}{X_D}\right)^{\beta_2} < 0. \quad (36)$$

As it is evident in equation (31), C^* is a function of durability. Since, I cannot analytically solve for optimal durability, T_L^*, I now analyze the feedback effect of durability on the firm's choice of coupon.[28]

I use the functional form $\phi(T) = 1 - \Delta T,\ \Delta > 0$ and since, $0 < \phi(T) < 1,\ \Delta < \frac{1}{T}$ to examine how C^* varies with product durability, T. From the admissible set of functions for demand shock, the affine function, $1 - \Delta T$, allows for analytical solutions for the firm's optimal policies. In later numerical analysis, I model the demand shock with a quadratic function. While the functional form of $\phi(T)$ determines the level and speed of the shock, the qualitative results do not change.

Since, debtholders pass the indirect costs of durability to the firm by raising the cost of debt, ex-ante I expect the firm's choice of coupon is decreasing in durability. Taking the derivative of C^* with respect to T gives

$$\frac{\partial C^*}{\partial T} = C^*(\delta - (\varepsilon - 1)\kappa - 2\gamma T) + \frac{C^*(1-\tau)/\tau\ (\partial\phi/\partial T)}{(1-\beta_2) - \left(\beta_2 \frac{(1-\tau)}{\tau}\big(1-\phi(T)\big)\right)}. \quad (37)$$

[27] For any coupon $C > 0$, the first term is negative since $\beta_2 < 0$, and the second term is also negative for $\phi(T) < 1$.
[28] Notice in equation (35) when $T = 0$, there is a finite positive optimal C^*. This is because there are no bankruptcy costs at $T = 0$ (I have assumed bankruptcy costs are proportional to the level of durability, therefore, when $T = 0$ there are no bankruptcy costs) and there is tax benefit associated with positive profit (i.e., $\Omega > 0$) of non-durable good. Even though there are no bankruptcy costs at $T = 0$, that shields are lost in bankruptcy so C^* is positive yet finite.

Equation (37) can be written succinctly as $\frac{\partial C^*}{\partial T} = C^*(M_1 + M_2)$, where $M_1 = (\delta - (\varepsilon - 1)\kappa - 2\gamma T) > 0$ for $T \leq \frac{\delta-(\varepsilon-1)\kappa}{2\gamma}$ and $M_2 = \frac{(1-\tau)/\tau\,(\partial\phi/\partial T)}{(1-\beta_2)-\left(\beta_2\frac{(1-\tau)}{\tau}(1-\phi(T))\right)} \leq 0$. Durability has two opposite effects on the optimal coupon. First, an increase in durability increases the firm's profit, thereby increasing the firm's optimal coupon, C^*, through the term M_1. Second, an increase in durability increases the firm's cost of bankruptcy, which decrease the firm's optimal coupon, C^*, through the term M_2.[29] I solve for the level of T at which $\partial C^*/\partial T$ changes from positive to negative by setting (37) equal to zero for demand shock $\phi(T) = 1 - \Delta T$. Noting that (37) is then quadratic equation in T, and solving for the positive root I have

$$T^+_{C^*} = -\frac{-\left(a_1 a_2 \beta_2 + 2\gamma(1-\beta_2)\right) + \sqrt{\left(a_1 a_2 \beta_2 - 2\gamma(1-\beta_2)\right)^2 + 8a_2^2\beta_2\gamma}}{4a_2\beta_2\gamma} \qquad (38)$$

where $a_1 = (\delta - (\varepsilon - 1)\kappa) > 0$, and $a_2 = \frac{\Delta(1-\tau)}{\tau} > 0$. The square root term in equation (38) is positive since $a_1(1-\beta_2) > a_2$, ensures two real roots. It can be shown that $\partial C^*/\partial T$ is increasing for durability $0 < T < T^+_{C^*}$ and decreasing for $T > T^+_{C^*}$.

Thus, durability has two opposing effects on the optimal coupon, C^*. An increase in durability increases the firm's profitability. Thus, holding everything else constant, higher durability increases the firm's debt capacity. In contrast, an increase in durability increases the firm's indirect bankruptcy cost, which increases the cost of debt and decreases debt capacity. Therefore, C^* initially increases in T (profitability effect) but eventually decreases for higher values of T (bankruptcy effect). This result shows us that the firm's capital structure is net effect

[29] The denominator of M_2 is positive since, $\beta_2 < 0$ and $0 < \phi(T) < 1$, and the numerator is negative since $\phi'(T) < 0$.

of a trade-off of profitability and bankruptcy. This is different than what Titman (1984) showed. He showed that firm's choice of debt decreases as it imposes more indirect bankruptcy costs on consumers. His theory presents unidirectional effects of product durability on firm's capital structure. Whereas, this paper accounts for both benefits and costs associated with durability and show that the firm's capital structure is the net result of a trade-off between the two. Next, I show how the firm's choice of coupon feeds into its choice of durability for a general value of shock, $\phi(T)$.

Substituting C^* from equation (35) into equation (34), the first order condition for a firm-value maximizing T becomes:

$$\frac{\partial V^L}{\partial T}\Big|_{C=C^*} = M_1 V + (M_1 + M_2)\left[\frac{\tau C^*}{r}\left(1 - \left(\frac{X}{X_D}\right)^{\beta_2}\right) - V\left(\left(1-\phi(T)\right)X_D\right)\right]$$
$$+ \beta_2 M_2 \left[\frac{\tau C^*}{r} - V\left(\left(1-\phi(T)\right)X_D\right)\right]\left(\frac{X}{X_D}\right)^{\beta_2} \tag{39}$$
$$- V\left(\left(1-\phi(T)\right)X_D\right)\left(\frac{X}{X_D}\right)^{\beta_2} = 0.$$

I can infer from equation (35) that durability influences firm value through four channels. The first term reflects profitability (i.e., durability increases profits, all else equal). The second term shows the marginal effect of durability on, C^*. The third term reflects how durability influences the likelihood and consequence of default. Finally, the last term shows the effect of durability on expected bankruptcy costs.

If I expand C^* in equation (39) to rewrite as $C^* = rX\left(\frac{\Omega}{r-\alpha}\right)\left(\frac{\beta_2 - 1}{\beta_2}\right)M_3^{\frac{1}{\beta_2}}$, then $X_D(C^*) = XM_3^{\frac{1}{\beta_2}}$ and $\left(\frac{X}{X_D(C^*)}\right)^{\beta_2} = \frac{1}{M_3}$. After some simplification equation (39) can be re-written as

$$\frac{\partial V^L}{\partial T}|_{C=C^*} = \frac{1}{\left(\frac{X}{X_D(C^*)}\right)}\left(M_1\left(\left(\frac{X}{X_D(C^*)}\right)+\left(\frac{\tau}{1-\tau}\right)\right)+\phi'(T)\left(\frac{X}{X_D(C^*)}\right)^{\beta_2}\right)$$

$$= M_1\left(1+M_3^{\frac{1}{\beta_2}}\left(\frac{\tau}{1-\tau}\right)\right)+\phi'(T)M_3^{\frac{1}{\beta_2}-1} = 0 \tag{40}$$

Since, I cannot solve (40) explicitly for T_L^*, I compute comparative statics using the implicit function theorem.

2.4.2. Comparative Statics

Equation (40) allows us to write the implicit function theorem as shown below.[30]

$$F\big(T_L^*(C^*)\big) = \frac{\partial V^L}{\partial T} = 0 \tag{41}$$

I have the following results.

Result 1: Optimal durability for the levered firm, T_L^*, is an increasing function of discount rate, r, when the specified set of conditions (see below) are satisfied.

Proof: By the implicit function theorem, I have

$$\frac{\partial T_L^*}{\partial r} = -\frac{\frac{\partial F}{\partial r}}{\frac{\partial F}{\partial T_L^*}} \tag{42}$$

where,

[30] For any variable y, the impliction function $F(T_L^*(y), y)$ allows us to use the chain rule to obtain $\frac{\partial F}{\partial y}[T_L^*(y), y]\frac{dy}{dy} + \frac{\partial F}{\partial T_L^*}[T_L^*(y), y]\frac{dT_L^*}{dy} = 0$, solving for $\frac{dT_L^*}{dr} = -\frac{\frac{\partial F}{\partial y}[T_L^*(y),y]}{\frac{\partial F}{\partial T_L^*}[T_L^*(y),y]}$.

$$\frac{\partial F}{\partial T_L^*} = -2\gamma\left(1 + M_3^{\frac{1}{\beta_2}}\left(\frac{\tau}{1-\tau}\right)\right) + \phi'(T) M_1 M_3^{\frac{1}{\beta_2}-1}$$
$$+ \left(\phi'(T)\right)^2 (1-\beta_2)\left(\frac{1-\tau}{\tau}\right) M_3^{\frac{1}{\beta_2}-2} + \phi''(T) M_3^{\frac{1}{\beta_2}-1}, \tag{43}$$

and

$$\frac{\partial F}{\partial r} = -\frac{\partial \beta_2}{\partial r} M_3^{\frac{1}{\beta_2}} \frac{1}{\beta_2^2}\Big[M_1\left(\frac{\tau}{1-\tau}\right)\{(\log(spd^*) - spd^* + 1)\}$$
$$+ \phi'(T) spd^* \{(\log(spd^*) - (1-\beta_2) spd^* + 1 - \beta_2)\}\Big]. \tag{44}$$

where $\frac{\partial \beta_2}{\partial r} = -\frac{\frac{\partial}{\partial r}\left(\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - r\right)}{\frac{\partial}{\partial \beta}\left(\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - r\right)|_{\beta_2}} = \frac{1}{\sigma^2\beta_2 + \left(\alpha - \frac{1}{2}\sigma^2\right)} < 0$ for $\beta_2 < 0$ and $\sigma^2(0.5 - \beta_2) > \alpha$.

It can be shown that $\frac{\partial F}{\partial T_L^*} < 0$ and sufficient conditions for the equation (39) to be positive are $M_1 = \delta - (\varepsilon - 1)\kappa - 2\gamma T_L^* > 0$, i.e.,$T_L^* < T^*$, and

$$M_1\left(\frac{\tau}{1-\tau}\right)\{\log(spd^*) - spd^* + 1\}$$
$$+ \phi'(T) spd^* \{\log(spd^*) - (1-\beta_2) spd^* + 1 - \beta_2\} > 0. \tag{45}$$ [31]

where state price of default at optimal coupon is, $spd^* = \left(\frac{X}{X_D(C^*)}\right)^{\beta_2} = \frac{1}{M_3}, \phi'(T) < 0$, and $\log(spd^*) - spd^* + 1 < \log(spd^*) - (1-\beta_2) spd^* + 1 - \beta_2$. The first term is positive and denotes marginal benefits associated with durability and the second term is negative and shows marginal costs associated with durability. Therefore, $\frac{\partial T_L^*}{\partial r} > 0$, when marginal benefits of

[31] Equation (43) has first and second term negative since $\phi'(T) < 0$. The third term is positive but relatively smaller for $1 > M_3^{\frac{1}{\beta_2}-2} > 0$ and can be ignored. The last term will always be negative for a concave function choice (for e.g., $\phi(T) = a - bT^2$) or zero for affine function (for e.g., $\phi(T) = 1 - \Delta T$). Thus, $\frac{\partial F}{\partial T_L^*} < 0$. The first and the third term on the right-hand side equation (44) are positive, therefore the equation (45) is the sufficient condition for $\frac{\partial F}{\partial r} > 0$.

durability associated with discount rate outweighs marginal costs of durability associated with discount rate.

Intuitively, as r increases the present value of expected profit decreases and the probability of default increases. However, higher r discounts bankruptcy costs even more, as it is farther in the future. Therefore, lower bankruptcy costs encourage the firm to produce goods with higher durability. In comparison, lower expected bankruptcy costs also encourage the firm to take on more debt, which in turn increases expected bankruptcy costs and forces the firm to reduce durability.[32] Thus, the firm produces more durable good only when overall benefits of durability outweigh total expected bankruptcy costs.

Result 2: Optimal durability for the levered firm, T_L^*, is an increasing function of the growth in the demand, α, when the specified set of conditions (see below) are satisfied.

Proof: By the implicit function theorem, I have

$$\frac{\partial T_L^*}{\partial \alpha} = -\frac{\frac{\partial F}{\partial \alpha}}{\frac{\partial F}{\partial T_L^*}}. \tag{46}$$

Since $\frac{\partial F}{\partial T_L^*} < 0$, the sign of $\frac{\partial T_L^*}{\partial \alpha}$ depends on the sign of $\frac{\partial F}{\partial \alpha}$

$$\begin{aligned}\frac{\partial F}{\partial \alpha} = -\frac{\partial \beta_2}{\partial \alpha} M_3^{\frac{1}{\beta_2}} \frac{1}{\beta_2^2} \Big[M_1 \left(\frac{\tau}{1-\tau}\right) \{(\log(spd^*) - spd^* + 1)\} \\ + \phi'(T) spd^* \{(\log(spd^*) - (1-\beta_2) spd^* + 1 - \beta_2)\} \Big]\end{aligned} \tag{47}$$

[32] Discount rate, r, increases durability, which increases expected bankruptcy costs offsetting partially the effect of discount rate. In comparison, larger durability also increases profitability and hence reduces probability of default. These effects will feed into firm's financing decision, and when net bankruptcy costs reduce in r, the firm will take on more debt.

where, $\frac{\partial \beta_2}{\partial \alpha} = -\frac{\frac{\partial}{\partial r}\left(\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - r\right)}{\frac{\partial}{\partial \beta}\left(\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - r\right)|_{\beta_2}} = \frac{-\beta_2}{\sigma^2\beta_2 + \left(\alpha - \frac{1}{2}\sigma^2\right)} < 0$ for $\beta_2 < 0$ and $\sigma^2(0.5 - \beta_2) > \alpha$.

Sufficient conditions for equation (47) to be positive are $M_1 = \delta - (\varepsilon - 1)\kappa - 2\gamma T_L^* > 0$, i.e., $T_L^* < T^*$ and

$$M_1\left(\frac{\tau}{1-\tau}\right)\{\log(spd^*) - spd^* + 1\} + \phi'(T)spd^*\{\log(spd^*) - (1-\beta_2)spd^* + 1 - \beta_2\} > 0. \tag{48}$$

Therefore, $\frac{\partial T_L^*}{\partial \alpha} > 0$ under conditions mentioned above.

An increase in growth rate of demand shifts future demand up, allowing the firm to sell more goods (at prevailing price) and earn more revenue, which increases firm's expected profit. Accordingly, higher expected profitability decreases probability of default and hence expected bankruptcy costs, which in turn allows the firm to increase durability. As such at higher durability the demand curve will shift up and will allow the firm to charge higher price and increase the profitability.[33] The secondary effect through firm's financing decision reduces product durability.[34] A growth rate increase and hence expected profitability allows the firm to take on more debt, which increases expected bankruptcy cost and pushes the firm to reduce the durability to offset higher bankruptcy costs. Therefore, as long as the primary effect dominates the secondary effect of leverage, optimal durability is increasing in the growth rate.

[33] T^* and T_L^* are larger than T_{PC}^*, i.e., the quantity maximizing durability. Therefore, any increase in durability in the region (T_{PC}^*, T^*), will decrease the quantity. The firm in my setting has the market power, and as a result I see that the firm by increasing durability will produce fewer goods, exert more rent from the consumer and make larger profits.

[34] An increase in durability due to increase in growth rate increases profitability, debt capacity, and also expected bankruptcy costs. These effects will feed into the firm's financing decision. Increase in expected bankruptcy costs will be offset by higher profitability due to higher growth rate and larger durability, which will increase the total debt capacity.

Result 3: Optimal durability for the levered firm, T_L^*, is a decreasing function of uncertainty in demand as long as the specified set of conditions (see below) are satisfied.

Proof: By the implicit function theorem, I have

$$\frac{\partial T_L^*}{\partial \sigma} = -\frac{\frac{\partial F}{\partial \sigma}}{\frac{\partial F}{\partial T_L^*}}. \tag{49}$$

Since $\frac{\partial F}{\partial T_L^*} < 0$, the sign of $\frac{\partial T_L^*}{\partial \sigma}$ depends on the sign of $\frac{\partial F}{\partial \sigma}$:

$$\begin{aligned}\frac{\partial F}{\partial \sigma} = -\frac{\partial \beta_2}{\partial \sigma} M_3^{\frac{1}{\beta_2}} \frac{1}{\beta_2^2} \Big[M_1 \left(\frac{\tau}{1-\tau}\right) \{(\log(spd^*) - spd^* + 1)\} \\ + \phi'(T) spd^* \{(\log(spd^*) - (1-\beta_2) spd^* + 1 - \beta_2)\} \Big]\end{aligned} \tag{50}$$

where, $\frac{\partial \beta_2}{\partial \sigma} = -\frac{\frac{\partial}{\partial r}\left(\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - r\right)}{\frac{\partial}{\partial \beta}\left(\frac{1}{2}\sigma^2\beta^2 + \left(\alpha - \frac{1}{2}\sigma^2\right)\beta - r\right)|_{\beta_2}} = -\frac{\sigma(\beta_2^2 - \beta_2)}{\sigma^2\beta_2 + \left(\alpha - \frac{1}{2}\sigma^2\right)} > 0$ for $\beta_2 < 0$ and $\sigma^2(0.5 - \beta_2) > \alpha$. Sufficient conditions for equation (50) to be negative are $M_1 = \delta - (\varepsilon - 1)\kappa - 2\gamma T_L^* > 0$, i.e., $T_L^* < T^*$ and

$$\begin{aligned}M_1 \left(\frac{\tau}{1-\tau}\right) \{\log(spd^*) - spd^* + 1\} \\ + \phi'(T) spd^* \{\log(spd^*) - (1-\beta_2) spd^* + 1 - \beta_2\} > 0.\end{aligned} \tag{51}$$

Therefore, $\frac{\partial T_L^*}{\partial \sigma} < 0$ under conditions mentioned above.

As demand volatility increases equity delays the decision to default, X_D, which increases the chances of firm's survival and decreases expected bankruptcy costs. As a result, the firm demand's demand for debt and its choice of durability increase. In comparison, an increase in uncertainty also increase the firm's likelihood of default, which increases expected bankruptcy costs, and discourages the firm to take on more leverage and produce less durable good. The firm's choice of leverage and durability will depend on which of these effects dominates.

Result 4: Optimal durability for the levered firm, $T^*_{V^L}$, is a decreasing function of tax rate, τ, if $\frac{\Delta T^*_L(\Delta - M_1)}{1-\beta_2-\frac{\beta_2(1-\tau)}{\tau}\Delta T^*_L} > \left(\frac{\tau}{1-\tau}\right)^2$.

Proof: By the implicit function theorem, I have

$$\frac{\partial T^*_L}{\partial \tau} = -\frac{\frac{\partial F}{\partial \tau}}{\frac{\partial F}{\partial T^*_L}}. \tag{52}$$

Since $\frac{\partial F}{\partial T^*_L} < 0$, the sign of $\frac{\partial T^*_L}{\partial \tau}$ depends on the sign of $\frac{\partial F}{\partial \tau}$:

$$\frac{\partial F}{\partial \tau} = \left[\frac{\big(\delta - (\varepsilon - 1)\kappa - 2\gamma T^*_L - \phi'(T)\big)(1-\beta_2)}{\tau^2}\left(\frac{1-\phi(T^*_L)}{1-\beta_2-\frac{\beta_2(1-\tau)}{\tau}\big(1-\phi(T^*_L)\big)}\right) + \frac{1-\beta_2}{(1-\tau)^2}\right] M_3^{\frac{1}{\beta_2}-1} \tag{53}$$

which is negative when the following condition is satisfied

$$\frac{\big(1-\phi(T^*_L)\big)T^*_L(\phi'(T) - M_1)}{1-\beta_2-\frac{\beta_2(1-\tau)}{\tau}\big(1-\phi(T^*_L)\big)} > \left(\frac{\tau}{1-\tau}\right)^2. \tag{54}$$

Therefore, $\frac{\partial T^*_L}{\partial \tau} < 0$ under conditions mentioned above.

An increase in the tax rate decreases after tax cashflows, which increases the probability of default and expected bankruptcy costs. Thus, the optimal durability is decreasing in the tax rate. The secondary leverage effect of tax-rate can be explained as follows. An increase in the tax rate encourages the firm to take on more debt, which further increases bankruptcy costs.[35] This reinforces the negative effect of the tax rate on optimal durability.

2.5. Model 2: Equity and Debt Financing with Indirect Bankruptcy Costs

In this section, I present a more general setting with indirect bankruptcy costs. In addition to facing cost in bankruptcy, the firm loses a fraction of its customers outside of bankruptcy as the likelihood of bankruptcy reaches a critical level.

I assume the consumer monitors demand and updates her probability of bankruptcy as X gets closer to X_D. Specifically, let demand shift downward at $X_P > X_D$. I model the demand shock at X_P as $\psi(T)X_P$, where $0 < \psi(T) < 1$ with $\psi'(T) < 0$.[36] I specify that X_P is where the present value of expected dividends to equity are zero.[37] At X_P, the probability the firm will continue operations is dwindling, so consumers rationally anticipate that equity holders will put the firm to debt holders soon. Thus, the endogenous value of X_P is given by

$$(1-\tau)\left[\frac{\Omega X_P}{r-\alpha} - \frac{C}{r}\right] = 0 \text{ or } X_P = \frac{C}{r}\frac{(r-\alpha)}{\Omega}. \tag{55}$$

Note that X_P is decreasing in durability when $T < \frac{\delta-(\varepsilon-1)\kappa}{2\gamma}$. As such, if the firm choses to reduce durability, then everything else held constant, it will face the negative demand shock sooner. This is because of reduction in profitability that is associated with durability. As the firm produces less durable products, though it reduces bankruptcy costs, it also reduces total demand of the firm's

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[35] Further, lower product durability decreases profitability and firm's thus firm's choice of leverage. Firm will increase the leverage when the demand for leverage due to higher tax-rate will outweigh the reduced profitability due to lower durability.

[36] This is equivalent to saying that the particular solution of equity is zero.

[37] In this model, the firm receives two negative shocks at X_P and X_D respectively, Appendix B, Figure 2 shows the behavior of X_t before and after as the firm faces these negative shocks..

products. As a result, the firm's overall profitability decreases and hampers the equity's ability to service the debt, i.e., the particular solution for equity reaches to zero sooner.

The values of equity and debt have the same general solutions as in equations (25) and (29) but have different boundary conditions. The general solutions of equity and debt in the region $X \in (X_P, \infty)$ are given by

$$E(X) = B_{P1}X^{\beta_1} + B_{P2}X^{\beta_2} + (1-\tau)\left[\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right], \tag{56}$$

$$D(X) = \frac{C}{r} + B_{P3}X^{\beta_1} + B_{P4}X^{\beta_2}, \tag{57}$$

where $E(X)$ and $D(X)$ are equity and debt values for $X > X_P$. Equity and debt values for the region $X \in (X_D, X_P)$ are given by

$$E_P(X) = B_1(\psi(T)X)^{\beta_1} + B_2(\psi(T)X)^{\beta_2} + (1-\tau)\left[\frac{\Omega\psi(T)X}{r-\alpha} - \frac{C}{r}\right], \tag{58}$$

and

$$D_P(X) = \frac{C}{r} + B_3(\psi(T)X)^{\beta_1} + B_4(\psi(T)X)^{\beta_2}. \tag{59}$$

The general solutions of equity and debt in equations (51) through (54) must satisfy the following boundary conditions.

$$lim_{X\to\infty}E(X) = (1-\tau)\left[\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right], \tag{60a}$$

$$lim_{X\to\infty}D(X) = \frac{C}{r}, \tag{60b}$$

$$E(X_P) = E_P(\psi(T)X_P), \tag{60c}$$

$$D(X_P) = D_P(\psi(T)X_P), \tag{60d}$$

$$\left.\frac{\partial E}{\partial X}\right|_{X=X_P} = \left.\frac{\partial E}{\partial X}\right|_{\psi(T)X=\psi(T)X_P}, \tag{60e}$$

$$\left.\frac{\partial D}{\partial X}\right|_{X=X_P} = \left.\frac{\partial D}{\partial X}\right|_{\psi(T)X=\psi(T)X_P}, \tag{60f}$$

$$E_P(\psi(T)X_D) = 0, \tag{60g}$$

$$\left.\frac{\partial E}{\partial X}\right|_{\psi(T)X=\psi(T)X_D} = 0, \tag{60h}$$

$$D_P(\psi(T)X_D) = V(\phi(T)\psi(T)X_D). \tag{60i}$$

Conditions (60a) and (60b) are standard no bubble conditions. Conditions (60g) and (60h) are value matching and smooth-pasting conditions for equity at the default boundary. Condition (60i) states that debt holders receive the unlevered firm and faces a further negative demand shock in bankruptcy. $V(\phi(T)\psi(T)X_D)$ is the unlevered firm value in bankruptcy (i.e., when $X = X_D$) when faced with a negative demand shock, $0 < \phi(T) \leq 1$ with $\phi'(T) < 0$. Conditions (60c) and (60d) are value matching conditions at X_P. Since, equity and debt holders rationality continuity anticipate what happens when X diffuses to X_P, conditions (60e) and (60f) are market rationality conditions that must hold given full information. Conditions (60a)-(60i) can be solved numerically for constants, $B_{P1}, B_{P2}, B_{P3}, B_{P4}, B_1, B_2, B_3$ and B_4, and the default threshold, X_D. Subsequently, I can jointly optimize levered firm value – the sum of equity and debt values – to compute the optimal coupon, C^*, and durability, T_L^*.

2.6.Comparison of Model 1 and Model 2

In both models, the firm receives a negative demand shock in bankruptcy. In Model 2, the firm additionally receives another demand shock when the net present value of equity is zero, i.e., at $X_P > X_D$. The implication is that the firm faces lower demand and start losing profit once X

hits X_P from above. As a result, the market value of equity will begin to decline, and the probability of default will begin to go up rapidly. Ex-ante, the firm faces larger bankruptcy costs, lower market value of equity, and higher probability of default relative to Model 1. Consequently, the firm's choice of leverage and the firm's choice of durability will be different than in Model 1. The firm can counter larger bankruptcy costs and probability of default by decreasing leverage and durability. However, decreasing leverage will reduce debt tax-shields and decreasing durability will decrease firm's profitability and hence debt capacity. Therefore, the choice of leverage and the choice of durability will depend on these trade-offs. Thus, durability carries both positive and negative consequences and it is unclear which effect dominates. I turn to numerical analysis to examine trade-offs.

2.7.Model 3: Equity and Debt Financing with Indirect Bankruptcy Costs

A third model alternative is to incorporate indirect costs of bankruptcy by changing the parameters of the process for demand, X. I examine this model in detail in Appendix A. I provide a brief discussion of this model formulation.

Assume the consumer rationally anticipates lower resale value of the for a high risk of bankruptcy. As consumers' uncertainty increases, demand growth decreases. Further, demand uncertainty likely increases as growth decreases. Hortaçsu et al. (2013) find that as the firm nears bankruptcy, the prices of their durable goods decrease significantly. While some consumers rush to buy cheaper goods others stay away, and the result is higher volatility of consumer demand. Model 3, discussed in Appendix A, attempts to capture these dynamics (i.e., a decrease in the drift of consumer demand and an increase in the variance of consumer demand as the firm approaches bankruptcy.

3. Option to Innovate

There are theoretical models that investigate the impact of investments that decrees marginal costs under different market structure. Dasgupta and Stiglitz (1980) use a Cournot competition model to analyze the impact of cost-reducing research and development.[38]I assume that the firm has an option to innovate by incurring a fixed cost I. The benefit of this innovation is that the marginal costs decrease κ to $\kappa\theta$, where $\theta \in (0,1)$. When the firm innovates to produce goods at a cheaper cost, it alters the product market by producing more durable goods at a cheaper rate, which allows it to sell a larger number of goods and generate higher profits.

I first examine the case of an unlevered firm and denote its innovation exercise policy trigger as X_{UI}. I then mode to the case of a levered firm, denoting its exercise policy as X_I.

3.1. Unlevered Firm Value

The value of the firm after exercising the innovation option is described by the dynamics:

$$\frac{1}{2}\sigma^2 X^2 V_{\theta,XX} + \alpha X V_{\theta,X} - rV_\theta + (1-\tau)\Omega_\theta X = 0, \tag{61}$$

where $\Omega_\theta X$ is the optimized profit before tax after the firm has exercised the innovation option, with $\Omega_\theta = \left(\frac{1}{\varepsilon}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right)^{\varepsilon-1} e^{(\delta-(\varepsilon-1)\theta\kappa)T-\gamma T^2}$. The general solution of equation (61) is given by

$$V_\theta(X) = \frac{(1-\tau)\Omega_\theta X}{r-\alpha} + A_1 X^{\beta_1} + A_2 X^{\beta_2}, \tag{62}$$

where A_1 and A_2 are constants to be determined and the β_1 and β_2 are the characteristic roots of the equation $\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0$.

The general solution in (62) must satisfy the following boundary conditions:

$$V_\theta(0) = 0 \tag{63a}$$

[38] Neumann et al. (2001) and Hegji (2001) examine models where the firm can make a fixed investment expenditure to reduce marginal costs.

and

$$lim_{X\to\infty}\left(\frac{V_\theta(X)}{X}\right)=0 \tag{63b}$$

Condition (63a) arises from the observation that if X is absorbed at zero, then $V_\theta(0)=0$, which can only be satisfied when $A_2=0$. Condition (63b) is the no bubble condition and is satisfied only if $A_1=0$. Substituting equation (62) into boundary conditions (63a) and (63b) gives firm value after the innovation option is exercised as

$$V_\theta(X)=\frac{(1-\tau)\Omega_\theta X}{r-\alpha}. \tag{64}$$

Consider next the value of the firm before the option is exercised. Firm value must satisfy equation (61), when $\theta=1$. Thus, the general solution is given by

$$V(X)=A_3X^{\beta_1}+A_4X^{\beta_2}+\frac{(1-\tau)\Omega X}{r-\alpha},\qquad X<X_{UI} \tag{65}$$

where constants A_3 and A_4 are to be determined, ΩX is the profit before tax, and X_{UI} is the level of the demand at which the firm exercises the innovation option by paying the fixed cost, I. Firm value, $V(X)$, must satisfy the following boundary conditions:

$$V(0)=0, \tag{66a}$$

$$V(X_{UI})=V_\theta(X_{UI})-I, \tag{66b}$$

and

$$\left.\frac{\partial V(X)}{\partial X}\right|_{X=X_{UI}}=\left.\frac{\partial V_\theta(X)}{\partial X}\right|_{X=X_{UI}}. \tag{66c}$$

Condition (66a) requires the value of the firm to be zero when X goes to zero, which is satisfied when $A_4=0$. Condition (66b) is the value matching condition which says that the net payoff from exercising the innovation option is $V_\theta(X_{UI})-I$. The last condition (66c) is a standard smooth-pasting condition, i.e., $V(X)$ must be continuous and smooth at the innovation exercise boundary, $X = X_{UI}$. Substituting (65) into (66a) - (66c), I find that

$$V(X) = \frac{(1-\tau)\Omega X}{r-\alpha} + \left((1-\tau)\frac{(\Omega_\theta - \Omega)}{r-\alpha}X_{UI} - I\right)\left(\frac{X}{X_{UI}}\right)^{\beta_1}, \tag{67}$$

where

$$X_{UI} = I\left(\frac{\beta_1}{\beta_1 - 1}\right)\left(\frac{r-\alpha}{(1-\tau)(\Omega_\theta - \Omega)}\right). \tag{68}$$

The exercise policy, X_{UI}, is increasing in cost, I, and decreasing in the difference between after-tax profits before and after exercising the innovation option.

3.2.Equity and Debt Financing with Innovation Option

3.2.1. Model 1, Equity and Debt Financing with Innovation Option

Consider first the levered firm's equity and debt values after the innovation option has been exercised. The after-tax cashflow to equity is $(1-\tau)(\Omega_\theta X - C)dt$ and the general solution for the value of levered equity is

$$E_\theta(X) = A_5 X^{\beta_1} + A_6 X^{\beta_2} + (1-\tau)\left[\frac{\Omega_\theta X}{r-\alpha} - \frac{C}{r}\right], \qquad X > X_{D\theta}, \tag{69}$$

where constants A_5 and A_6 are to be determined and $X_{D\theta}$ is the endogenously determined post investment default boundary. The general solution (69) must satisfy the following boundary conditions:

$$lim_{X\to\infty} E_\theta(X) = (1-\tau)\left[\frac{\Omega_\theta X}{r-\alpha} - \frac{C}{r}\right], \tag{70a}$$

$$E_\theta(X_{D\theta}) = 0, \tag{70b}$$

and

$$\left.\frac{\partial E_\theta(X)}{\partial X}\right|_{X=X_{D\theta}} = 0. \tag{70c}$$

Condition (70a) states that default becomes irrelevant as X becomes large. This condition is satisfied when $A_5 = 0$. Condition (70b) assumes that equity has limited liability upon default and condition (70c) is a standard smooth pasting condition at the default threshold, i.e., $X_{D\theta}$ is optimally determined to maximize the market value of equity. Substituting equation (69) into boundary conditions (70a) - (70c) gives

$$E_\theta(X) = (1-\tau)\left[\frac{\Omega_\theta X}{r-\alpha} - \frac{C}{r}\right] - (1-\tau)\left[\frac{\Omega_\theta X_{D\theta}}{r-\alpha} - \frac{C}{r}\right]\left(\frac{X}{X_{D\theta}}\right)^{\beta_2}, \tag{71}$$

where

$$X_{D\theta} = \left(\frac{\beta_2}{\beta_2 - 1}\right)\left(\frac{C}{r}\right)\left(\frac{r-\alpha}{\Omega_\theta}\right). \tag{72}$$

The debtholders receive a continuous coupon payment Cdt in the absence of bankruptcy. The general solution for debt value is

$$D_\theta(X) = \frac{C}{r} + A_7 X^{\beta_1} + A_8 X^{\beta_2}, \qquad X > X_{D\theta} \tag{73}$$

where the constants A_7 and A_8 remain to be determined.

The general solution in (73) must satisfy the boundary conditions:

$$lim_{X\to\infty} D_\theta(X) = \frac{C}{r} \tag{74a}$$

and

$$D_\theta(X_{D\theta}) = V_\theta(\phi(T)X_{D\theta}) \tag{74b}$$

The condition (74a) states that debt is a risk-free perpetuity when there is no chance of default. This condition is satisfied when $A_7 = 0$. Condition (74b) states that in bankruptcy the firm faces a negative demand shock, $\phi(T)$, $0 < \phi(T) < 1$, and the equity holders transfer this diluted firm value to debtholders. In (74b), $V_\theta(\phi(T)X_{D\theta})$ is the unlevered firm value given in equation (59), evaluated at $\phi(T)X_{D\theta}$. Substituting equation (73) into boundary conditions (74a) and (74b) gives risky debt value as

$$D_\theta(X) = \frac{C}{r} + \left(V_\theta((\phi(T)X_{D\theta}) - \frac{C}{r}\right)\left(\frac{X}{X_{D\theta}}\right)^{\beta_2}. \tag{75}$$

The total value of the levered firm after the innovation option has been exercised is the sum of the equity and debt values given in equations (71) and (75) respectively, and is given by

$$\begin{aligned} V_\theta^L(X) = (1-\tau)\frac{\Omega_\theta X}{r-\alpha} + + \frac{\tau C}{r}\left(1 - \left(\frac{X}{X_{D\theta}}\right)^{\beta_2}\right) \\ - \left[(1-\tau)\frac{\Omega_\theta X_{D\theta}\big(1-\phi(T)\big)}{r-\alpha}\right]\left(\frac{X}{X_{D\theta}}\right)^{\beta_2}. \end{aligned} \tag{76}$$

Equation (76) is the value of the levered firm after the innovation option has been exercised, and it is equal to the value of unlevered assets plus the present value of expected debt tax-shields and minus the present value of expected indirect bankruptcy costs.

I now turn to the problem of deriving the firm and levered equity value before the innovation option is exercised.

The general solutions for equity and levered firm values are given by:

$$E(X) = (1-\tau)\left(\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right) + A_9 X^{\beta_1} + A_{10} X^{\beta_2}, \qquad X_D < X < X_I, \tag{77}$$

and

$$V^L(X) = (1-\tau)\left(\frac{\Omega X}{r-\alpha}\right) + \frac{\tau C}{r} + A_{11} X^{\beta_1} + A_{12} X^{\beta_2}, \qquad X_D < X < X_I, \tag{78}$$

where A_9, A_{10}, A_{11}, and A_{12} are constants to be determined, X_D is the endogenously determined demand at which equity will default on debt before the innovation option has been exercised, and X_I is the demand level at which the firm exercises the innovation option. Note that $X_{D\theta} < X_D$, since exercising the innovation option will increase the expected value of profit to the firm, equityholders will subsequently delay the decision to default and continue to earn profit before putting the firm to bondholders.

Equations (77) and (78) must satisfy the following boundary conditions:

$$E(X_D) = 0, \tag{79a}$$

$$\left.\frac{\partial E}{\partial X}\right|_{X=X_D} = 0, \tag{79b}$$

$$V^L(X_D) = V(\phi(T)X_D), \tag{79c}$$

$$E(X_I) = E_\theta(X_I) - I, \tag{79d}$$

$$V^L(X_I) = V_\theta^L(X_I) - I, \tag{79e}$$

and

$$\left.\frac{\partial V^L(X)}{\partial X}\right|_{X=X_I} = \left.\frac{\partial V_\theta^L(X)}{\partial X}\right|_{X=X_I}. \tag{79f}$$

Conditions (79a) and (79b) recognize respectively that equity has limited liability at X_D and X_D is chosen to maximize equity value. Condition (79c) specifies that at X_D the levered firm value equals unlevered firm value (i.e., the value specified in equation (24)) when faced with a negative demand shock in bankruptcy. Equation (79d) is a continuity requirement for the value of equity at X_I, where the right-hand side value of equity is that specified in equation (69). Equations (79e) and (79f) are value matching and smooth pasting conditions for the levered firm value at the innovation option exercise strategy X_I, where the right-hand side levered firm value is specified in equation (76). Substituting equation (72) and (73) into boundary conditions (79a)-(79f) allows for the determination of the constants A_9, A_{10}, A_{11} and A_{12}, the default boundary X_D, and the innovation option exercise boundary, X_I.

3.2.2. Debt and Equity Financing with Innovation Option, Model 2

Next consider the innovation option exercise policy under Model 2, where consumer anticipates the negative consequences of the firm going bankrupt before the firm actually goes bankrupt. First, I consider the equity and the debt after the innovation option has been exercised.

The consumer's perception of expected bankruptcy shifts demand for the durable goods downward at $X_{P\theta} > X_{D\theta}$, where $X_{P\theta} = \frac{C}{r}\frac{(r-\alpha)}{\Omega_\theta}$.

The general solution for the values of equity and debt are given in (25) and (29) and for $X \in (X_P, \infty)$ are given by

$$E_\theta(X) = A_{13}X^{\beta_1} + A_{14}X^{\beta_2} + (1-\tau)\left[\frac{\Omega_\theta X}{r-\alpha} - \frac{C}{r}\right], \tag{80}$$

$$D_\theta(X) = \frac{C}{r} + A_{15}X^{\beta_1} + A_{16}X^{\beta_2}, \tag{81}$$

Equity and debt values for the region $X \in (X_D, X_P)$ are given by

$$E_{P\theta}(X) = A_{17}(\psi(T)X)^{\beta_1} + A_{18}(\psi(T)X)^{\beta_2} + (1-\tau)\left[\frac{\Omega_\theta \psi(T)X}{r-\alpha} - \frac{C}{r}\right], \tag{82}$$

and

$$D_{P\theta}(X) = \frac{C}{r} + A_{19}(\psi(T)X)^{\beta_1} + A_{20}(\psi(T)X)^{\beta_2}. \tag{83}$$

The general solutions of equity and debt in equations (80) through (83) must satisfy the following boundary conditions.

$$lim_{X\to\infty}E_\theta(X) = (1-\tau)\left[\frac{\Omega_\theta X}{r-\alpha} - \frac{C}{r}\right], \tag{84a}$$

$$lim_{X\to\infty}D_\theta(X) = \frac{C}{r}, \tag{84b}$$

$$E_\theta(X_{P\theta}) = E_{P\theta}(\psi(T)X_{P\theta}), \tag{84c}$$

$$D_\theta(X_{P\theta}) = D_{P\theta}(\psi(T)X_{P\theta}), \tag{84d}$$

$$\left.\frac{\partial E_\theta}{\partial X}\right|_{X=X_{P\theta}} = \left.\frac{\partial E_{P\theta}}{\partial X}\right|_{\psi(T)X=\psi(T)X_{P\theta}}, \tag{84e}$$

$$\left.\frac{\partial D_\theta}{\partial X}\right|_{X=X_{P\theta}} = \left.\frac{\partial D_{P\theta}}{\partial X}\right|_{\psi(T)X=\psi(T)X_{P\theta}}, \tag{84f}$$

$$E_{P\theta}(\psi(T)X_{D\theta}) = 0, \tag{84g}$$

$$\left.\frac{\partial E_{P\theta}}{\partial X}\right|_{\psi(T)X=\psi(T)X_D} = 0, \tag{84h}$$

$$D_{P\theta}(\psi(T)X_{D\theta}) = V_\theta(\phi(T)\psi(T)X_{D\theta}). \tag{84i}$$

Conditions (84a) and (84b) are standard no bubble conditions. Conditions (84g) and (84h) are value matching and smooth-pasting conditions for equity at the default boundary. Condition (84i) states that debt holders receive the unlevered firm and face a further negative demand shock in bankruptcy. $V_\theta(\phi(T)\psi(T)X_{D\theta})$ is the unlevered firm value in bankruptcy when faced with a negative demand shock, $0 < \psi(T) \leq 1$ with $\psi'(T) < 0$. Conditions (84c) and (84d) are value matching conditions at $X_{P\theta}$. Since, equity and debt holders rationally anticipate what happens when X diffuses to $X_{P\theta}$, conditions (84e) and (84f) are market rationality conditions that must hold given full information. Conditions (84a)-(84i) can be solved numerically for the constants, $A_{13}, A_{14}, A_{15}, A_{16}, A_{17}, A_{18}$ A_{19}, and A_{20} and the default threshold, $X_{D\theta}$.

Next, I construct equity and debt values before-exercise of the innovation option. The general solutions for equity and levered firm value for the region $X \in (X_D, X_P)$ are given by:

$$E_P(X) = (1-\tau)\left(\frac{\Omega\psi(T)X}{r-\alpha} - \frac{C}{r}\right) + A_{21}(\psi(T)X)^{\beta_1} + A_{22}(\psi(T)X)^{\beta_2}, \tag{85}$$

and

$$V_P^L(X) = (1-\tau)\left(\frac{\Omega\psi(T)X}{r-\alpha}\right) + \frac{\tau C}{r} + A_{23}(\psi(T)X)^{\beta_1} + A_{24}(\psi(T)X)^{\beta_2}. \tag{86}$$

Assuming $X_I > X_P > X_D$, the general solutions for equity and levered firm value for the region $X \in (X_P, X_I)$ are given by:

$$E(X) = (1-\tau)\left(\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right) + A_{25}X^{\beta_1} + A_{26}X^{\beta_2}, \qquad X_P < X < X_I, \tag{87}$$

and

$$V^L(X) = (1-\tau)\left(\frac{\Omega X}{r-\alpha}\right) + \frac{\tau C}{r} + A_{27}X^{\beta_1} + A_{28}X^{\beta_2}, \qquad X_P < X < X_I. \tag{88}$$

Equations (85)-(88) must satisfy the following boundary conditions:

$$E_P(\psi(T)X_D) = 0\,, \tag{89a}$$

$$\left.\frac{\partial E_P}{\partial X}\right|_{\psi(T)X=\psi(T)X_D} = 0\,, \tag{89b}$$

$$\left.\frac{\partial V_P^L}{\partial X}\right|_{\psi(T)X=\psi(T)X_D} = V(\psi(T)\phi(T)X_D)\,, \tag{89c}$$

$$E_P(\psi(T)X_P) = E(X_P)\,, \tag{89d}$$

$$V_P^L(\psi(T)X_P) = V(X_P)\,, \tag{89e}$$

$$\left.\frac{\partial E_P}{\partial X}\right|_{\psi(T)X=\psi(T)X_P} = \left.\frac{\partial E}{\partial X}\right|_{X=X_P}, \tag{89f}$$

$$\left.\frac{\partial V_P^L}{\partial X}\right|_{\psi(T)X=\psi(T)X_P} = \left.\frac{\partial V^L}{\partial X}\right|_{X=X_P}, \tag{89g}$$

$$E(X_I) = E_\theta(X_I) - I\,, \tag{89h}$$

$$V^L(X_I) = V_\theta^L(X_I) - I\,, \tag{89i}$$

$$\left.\frac{\partial V^L}{\partial X}\right|_{X=X_I} = \left.\frac{\partial V_\theta^L}{\partial X}\right|_{X=X_I}. \tag{89j}$$

Conditions (89a) and (89b) are value matching and smooth-pasting conditions for equity at the default boundary. Condition (89c) states that the firm faces a negative demand shock in bankruptcy and receives unlevered value $V(\phi(T)\psi(T)X_{D\theta})$. Conditions (89d) and (89e) are value matching conditions at X_P. Since equity and debt holders rationally anticipate what happens when X diffuses to X_P, conditions (89f) and (89g) are market rationality conditions that must hold given full information. Equation (89h) is a value matching requirement for the value of equity at X_I, where the right-hand side value of equity is that specified in equation (71). Equations (89i) and (89f) are value matching and smooth pasting conditions for the levered firm value at the innovation option exercise demand threshold X_I, where the right-hand side levered firm value is the sum of the equity, specified in equation (71), and the debt, specified in equation (75). Conditions (89a)-(89i) can be solved numerically for constants, $A_{21}, A_{22}, A_{23}, A_{24}, A_{25}, A_{26}$ A_{27}, and A_{28} and the innovation option exercise demand threshold, X_I.

4. Numerical Analysis

The base case parameter values are summarized in Table 1. The initial level of the demand shock (X) is 1. The annualized volatility of the demand shock (σ) is 25%. The proportional tax rate (τ) is 0.35, the annualized risk-free rate (r) is 7%, and the demand shock has zero drift (α).[39] For convenience, I assume consumers' demand preference for durability (δ) is 1.4, consumers' disutility of durability (γ) is 0.05, cost parameter of producing the durable good (κ) is 0.3, and the price elasticity of demand (ε) is 1.5. With these parameters, optimal durability for an unlevered firm (T^*) is 12.5 years.[40] I further assume that the economies of scale parameter (θ) is 0.6 and the fixed costs of investment (I) is 6795.[41]

Table 2 gives a snapshot of comparative statics for all the models. Tables 3 and 4 present results for Model 1 and Model 2. Table 5 present results for Model 3 (I also append results for Model 3 in Appendix B when coupon and durability are fixed). Tables 6 to 9 present comparative statics for Models 1, 2, and 3. Tables 10 and 11 present results for Models 1 and 2 when the firm

[39] See Ju, et al. (2005) and Altman (1991) for recovery rate discussions. The tax rate is similar to Leland and Toft (1996). See Mauer and Ott (2000) and Hackbarth (2008) for the choice of drift rate of diffusion process. While the former paper models the commodity price with convenience yield of 7%, the latter models an earnings process with a "convenience yield" of 7%. Dixit (1991) models market demand with zero drift rate. Our choice of zero drift assume a zero expected demand growth with the dynamics defined by random shocks.

[40] The Bureau of Economic Analysis (BEA) defines durable goods as tangible products that can be stored or inventoried and that have an average service life of three years or greater. According to 2018 consumerreports.org, the average age of all cars is more than 11 years (they use data from the US department of transportation). Miao (2005) assumes price elasticity of demand is 0.75 in a competitive industry and I assume a higher price elasticity of demand for a durable-goods monopolist. Banks et al. (1997), Lewbel (1999), Gowrisankaran and Rysman (2012), Rapson (2014), and Clara (2018) documents larger price elasticity of for durable goods compared to that of non-durable goods.

[41] For simplicity, I assume the fixed cost the firm pays to innovate is similar to the cost a firm may pay for investment in an expansion option. Hackbarth and Mauer (2012) assume an investment cost about 0.5 times the expected present value of the firm's profit before tax when it has no expansion option. In Mauer and Ott (200), the investment cost amounts to approximately 2.5 times the present value of the firm's profit before tax with no option. Therefore, I assume $I = 0.5\left(\frac{\Omega X}{r-\alpha}\right)|_{T=12.5} \approx 6795$. The choice of $\theta = 0.8$ reduces total marginal cost by approximately 50% $(0.5MC = MC_\theta = \exp(\theta\kappa T), 9.5 \leq T \leq 12.5)$, so any choice of $\theta < 0.8$ will create strong incentives for the firm to innovate. I chose θ to be 0.6.

has an option to innovate. Lastly, Table 12 presents comparative statics for models 1 and 2 when the firm has an option to innovate.

Table 2 displays comparative statics summaries of numerical analysis for the firm with and without an innovation option. Column 1 shows parameters and models, columns 2 and 3 show the behavior of the optimal coupon C^* and durability T_L^* for the firm without the innovation option, and columns 4,5, and 6 show the behavior of the optimal coupon (C^*), durability (T_L^*), and demand exercise threshold (X_I) for the firm with an innovation option.

An increase in the discount rate has two opposing effects. First, it reduces expected profits and option value, which increases the chances of default and encourages the firm to reduce leverage and durability and delay the exercise of the option. Second, it reduces future expected bankruptcy costs, which encourages the firm to take on more leverage and produce more durable goods. Thus, the firm's choice of each of these variables depends on which effect dominates and interactions among them.[42] Table 2 summarizes varied dominance of each effect on all models. I discuss later in detail which effect dominates in models 1,2, and 3 for a firm with and without an innovation option.

In comparison, an increase in demand growth increases the firm's profitability, which encourage the firm to take on more leverage, produce more durable goods, and exercise the innovation option sooner. This effect is consistent across models.

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[42] Specifically, the firm's choice of leverage and durability interact and impact on each other and X_I. If firm chooses larger leverage, it increases bankruptcy costs, which discourages the firm to produce more durable goods and encourages the firm to exercise the option sooner (to offset bankruptcy costs). In comparison, if firm produces more durable goods, it has two opposing effects. Increase in durability increases profitability, which increases the firm's demand for debt and allows the firm to exercise the option. Whereas, an increase in durability also increases bankruptcy costs, which decreases the firm's demand for debt and increases the firm's incentive to exercise the option sooner. The firm's choice of demand threshold decreases in each of the cases, either to offset bankruptcy costs or because the option becomes more valuable at higher durability.

An increased in uncertainty (i.e., volatility (σ)) increases option value and decreases probability of default, the firm demands more debt and produces a more durable good. However, increased uncertainty increases the likelihood of default, which increases expected bankruptcy costs. Thus, the firm's demand for leverage decreases and produces less durable goods. Thus, the firm's choice of each of these variables depends on which effect dominates and interactions among these choices. Lastly, note that X_I increases as σ increase. This is the standard result from the real option literature that an increase in uncertainty increases the value of waiting to invest.[43] I can see in table 2 how these effects impact the firm's policies in a different way in all three models.

Lastly, an increase in the tax rate lowers after-tax profits, increases the probability of default and expected bankruptcy costs. However, it also increases the value of debt tax-shields. As a result, the firm's demand for leverage increases, its choice of durability decreases, and its incentive to exercise the innovation option decreases. These effects are consistent across all models.

Table 3 shows Model 1 and Model 2 results in panels A and B respectively for exogenous leverage and durability. Where the first model only emphasizes indirect costs realized at time of default, the second model also takes into consideration the lost profits that a firm is expected to suffer due to the anticipation of bankruptcy at $X_P > X_D$. Table 3, panels A and B show these results. As consumers rationally expects at X_P that firm's default is in near future, they stay away from the market, which lowers demand for durable goods and forces the firm to reduce output. I can look at the market value of equity in panel B, which initially increases and then decreases. This indicates lost profits between X_P and X_D due to indirect bankruptcy costs of durability, which

[43] See for e.g., McDonald and Siegel (1986)

is absent in panel A results of Model 1.[44] Thus, the firm in Model 2 has hampered sales and profitability and also lower market value of equity. Note that in the first model, since the firm faces larger shock only in bankruptcy, the market value of equity is unaltered. The cost of debt (CRS) shows the additional costs imposed on debt holders in Model 2, which is approximately 86 basis points greater than in Model 1.[45] I show similar results for Model 3 in Appendix B.

Next, Table 4 shows results for Model 1 and Model 2 when I first optimize over coupon while holding durability constant, then optimize durability while holding coupon constant, and finally I optimize over both coupon and durability.[46] Table 4, Panels A and B show that an increase in durability increases unlevered value due to higher profitability when $T \leq T^* = 12.5$ years, where T^* is firm's choice of durability under zero leverage. However, it also increases loss of demand and sales in bankruptcy, i.e., increases bankruptcy costs. As a result, as shown in panel A the firm's demand for leverage decreases in durability. In comparison, in Model 2 due to significant bankruptcy potential, the firm faces a decrease in demand and profit prior to default (at X_P), thereby raising bankruptcy costs even more. As shown in panel B, the firm's optimal leverage is significantly less than Model 1 (approximately 40% less).

Table 4, panels C and D show models 1 and 2 results for optimal durability, when firm's leverage is exogenously specified. The impact on firm's choice of durability relative to the impact

[44] The firm's profitability increases in durability until $T = T^* = 12.5$ years, which is the profit maximizing durability for the unlevered firm. Therefore, in both panels, the profitability will decrease after $T = 12.5$ years. However, in panel B, the decrease in market value of equity before $T = 12.5$ years is attributable to the indirect bankruptcy costs of durability.

[45] Model 3 is not directly comparable in bankruptcy and so I exclude Model 3 from direct comparison.

[46] Once, I endogenize over either of the variable or both, the direct comparison by imposing a shock will be moot. Therefore, I simply display results and draw comparison with the base results in table 3.

on leverage in panels A and B is small, this is partly due to the demand for durability.[47] An increase in leverage increases the risk of bankruptcy and hence expected bankruptcy costs. This pushes the firm to produces goods with lower durability to offset some of these costs. Lower durability reduces the length of the commitment to consumers. So, the fall in demand in bankruptcy is lower, which reduces indirect bankruptcy costs. In Model 2, the firm's choice of durability is approximately 1.5% less than that in Model 1.

Finally, panel E shows results when the firm optimally chooses leverage and durability. In both models, the firm produces a less durable good than an otherwise identical unlevered firm. The firm in Model 2 faces larger indirect costs of bankruptcy, which has a significant impact on its choice of leverage, which is 42% less than that in Model 1. The firm's choice of optimal durability in Model 2 is a bit higher (almost similar) to that in Model 1. It reflects lower debt and hence distant bankruptcy compared to Model 1, allowing the firm to supply durability closer to market demand.

Table 5 shows Model 3 results, where in addition to receiving a demand shock $\phi(T)$ in bankruptcy at $X = X_D$, (1) the firm's demand growth rate decreases at X_P (panel A), or (2) the firm's demand volatility rate increases at X_P (panel B), and (3) the firm's demand growth rate decreases and volatility rate increases at X_P (panel C).[48] When the firm faces lower growth in demand for durable goods at X_P, it reduces expected profit and increases probability of default, thereby reduces firm's choice of leverage and durability. The firm's choice of leverage (coupon)

[47] Specifically, though not linear, marginal increase in firm's profitability per unit of durability is larger than marginal increase in firm's net benefit of debt per unit of leverage. Thus, I see relatively larger drop in coupon to offset the bankruptcy cost relative to the drop in durability.

[48] The firm's demand process is altered at X_P forever. Later, when X hits X_D from above, the firm receives a demand shock $\phi(T)$, similar to that in models 1 and 2, which brings down the level of demand to $\phi(T)X_D$. See Appendix A for the formulation.

in this case is approximately 16% lower than that in Model 1. Lower choice of leverage offsets some of the indirect bankruptcy costs due to durability, thereby allowing the firm to increase durability and hence the profit. The value maximizing durability $T = T_L^*$ is12.346 years.[49]

Panel B shows when the firm faces increased uncertainty at X_P, it asymmetrically increases the firm's chances of survival. Increased demand uncertainty increases the firm's cashflow volatility, which works similar to equity's gamble for resurrection.[50] The external shock to uncertainty at X_P increases the firm's chances of survival and increases the distance between X_P and X_D. This in turn increases the likelihood of debt payment. Thus, an increase in the firm's chances of survival reduces expected bankruptcy costs, increases the firm value, allows the firm to take on more debt and produce more durable goods relative to Model 1. The value maximizing durability $T = T_L^*$ is12.331 years.

Finally, Panel C shows results when the firm faces both a decrease in growth and an increase in volatility at X_P. I see the joint effect of these parameter changes in firm value, leverage, and the net benefit of debt.

Table 6 shows comparative statics for discount rate, r, for models 1 (panel A), 2 (panel B), and 3 (panels C, and D). An increase in the discount rate has two opposing effects on a firm's financing decision and its choice of durability. First, at higher discount rates expected profitability declines, which encourages the firm to reduce financial leverage. Accordingly, the firm's expected bankruptcy costs decrease and the firm increases the product durability. Second, a higher discount rate also reduces expected costs of future bankruptcy, which allows the firm to increase

[49] In the contrasting evidence, Kleiman and Ophir (1966) showed that the firm chooses lower durability in a response to a rise in interest rate. However, their argument was from the perspective of expected utility, i.e., expected utility of durable goods decreases when interest rates increase.
[50] Gamble of resurrection: Equity takes on greater risk right before the default, which shifts the excess risk to debt holders but increase equity's convex payoff

leverage and durability. However, leverage and product durability have countervailing effects on each other, as they both increase expected bankruptcy costs and are jointly determined. Therefore, how much debt and/or durability will increase when the discount rate increases depends on which parameter has marginally lower bankruptcy costs. In Model 1, the first effect dominates and as a result, an increase in the discount rate from 3% to 5% (7%) decreases the financial leverage by 5.4% (6.85%) and increases durability by 0.15% (0.23%).

In Model 2, where the firm receives an additional demand shock out of bankruptcy, the second effect dominates. Since Model 2 has relatively larger expected bankruptcy costs than Model 1, any decrease in such costs are valued more in Model 2. Therefore, lower expected bankruptcy costs encourage the firm to increase leverage by 8.31% when the discount rate increases from 3% to 5%. Simultaneously, the firm reduce product durability by 0.041% to offset the bankruptcy costs associated with an increase in the firm's debt. Notice that these offsetting effects are non-linear, which I can observe when discount rates go up to 5% or 7%. In this case, the firm's leverage increases further by 6.31% and firm's durability now increase by 0.024%. The decrease in expected bankruptcy costs at higher discount rates must be large enough to allow the firm to increase leverage and product durability simultaneously.

In Model 3, the out of bankruptcy demand-shock operates through the demand growth rate or the demand volatility rate. Panel C shows result for Model 3 when the demand growth rate decreases at $X = X_P > X_D$. Higher discount rates reduce expected profit and total expected bankruptcy costs. The firm's demand for debt will increase if the latter effect dominates. For the indicated parameter values, the firm's leverage increases by 17.54% (26.45%), when discount rate increases from 3% to 5% (7%). However, higher leverage increases bankruptcy costs, which together with lower expected profit offsets the firm's choice of durability by 0.21% (0.27%).

Table 6, panel D shows results for Model 3 when demand volatility increases at $X = X_p > X_D$. Firm's expected profitability decreases in discount rate, together with an increase in demand volatility rate at X_P, it increases the likelihood of default and expected bankruptcy costs. Consequently, the firm's cost of debt will go up, thereby forcing it to reduce the leverage. As shown in the table, the firm reduces leverage by 7.18% (10.02%) when the discount rate increase from 3% to 5% (7%). Lower expected bankruptcy costs due to the decrease in leverage allows the firm to increase the durability, i.e., durability increases by a small amount as the discount rate increases.

Table 7 shows comparative statics for demand growth rate, α, for models 1 (panel A), Model 2 (panel B), and Model 3 (panels C and D). An increase in the demand growth rate increases expected demand in the future, which allows the firm to sell more goods. Accordingly, the firm's expected revenue and profitability increases, which increases the demand for leverage. Higher expected profitability also decreases the probability of default and expected bankruptcy costs, which encourages the firm to produce a more durable good. Thus, an increase in the demand growth rate increases the firm's financial leverage and durability. As shown in panel A for Model 1, as the growth rate increases from 0% to 2% (4%), the firm value increases by 40.04% (133.47%), the optimal coupon increases by 39.60% (134.25%), and product durability increases by 0.12% (0.24%).

As shown in panel B for Model 2, where the firm receives an additional demand shock outside of bankruptcy at X_P, total expected bankruptcy costs are larger and time to default is sooner in comparison to Model 1. Therefore, any increase in profitability should have a larger impact on the firm's prospects compared to Model 1. I can observe this impact in the firm's leverage choice. As the growth rate increases from 0% to 2% (4%), the optimal coupon increases by 67.15% (223.57%), which shows a larger impact on firm's financing choice. Increased profitability decreases probability of default and expected bankruptcy costs, which allows the firm to increase product durability by 0.065% (0.285%).

Panel C shows result for Model 3 when the demand growth rate decreases at $X = X_P > X_D$., which reduces overall expected profitability for the firm. Thus, in this setting any improvement in growth rate has relatively larger impact on firm's prospect than that in Model 1. I can observe this impact in the firm's leverage and durability choices. As growth rate increases from 0% to 2% (4%), the financial leverage increases by 43.64% (145.97%) and product durability increases by 0.130% (0.332%). Notice that there is a larger impact on firm's financial leverage in Model 2 compared to Model 3. Although Model 2 and Model 3 are not directly comparable, it is clear that the drop in current level of demand has a larger effect on firm policies than shift in the growth or volatility rates of demand.

Panel D shows results for Model 3 when demand volatility increases at $X = X_p > X_D$. Firm's profitability increases in growth rate and its chances of survival increase in increased uncertainty at X_P. Together, it allows the firm to borrow more. As growth rate increases from 0% to 2% (4%) the financial leverage increases by 39.95% (134%) which are approximately equal to that in Model 1. In comparison, the product durability increases by 0.097% (0.203%), which is less than that in Model 1. Relative smaller choice of durability reflects overall larger expected bankruptcy costs in Model 3, which arises at X_P.

Table 8 shows results for comparative statics for demand volatility Model 1 (panel A), Model 2 (panel B) and Model 3 (panels C and D). An increase in σ increases the likelihood of default, which increases expected bankruptcy costs, increases the cost of debt, and forces the firm reduce leverage. Firm's choice of durability also decrease as expected bankruptcy costs increase.

If the lower choice of leverage offset some of the bankruptcy costs, it will allow the firm to produce more durable goods. When uncertainty increases from 15% to 25%, the financial leverage decreases by 18.16% in Model 2, and decrease by 1.88% in Model 1. Similarly, in Model 3, panel A, an increase in volatility rate from 15% to 25% decreases the financial leverage by 6.73% and decreases durability by 0.226%.

In comparison, the firm and the option are valued more in uncertainty. As shown in panel D, an increase in overall uncertainty from 15% to 25%, together with an increase in uncertainty at $X = X_P > X_D$ increase the financial leverage by 2.53%. Higher leverage in turn increases bankruptcy costs, and as a result, the firm's choice of durability decreases by 0.251% .

Finally, table 9 shows comparative statics for tax rate, τ, for Model 1 (panel A), Model 2 (panel B), and Model 3 (panels C and D). Tax rate has a straightforward effect on firm's profitability and financing choice. A higher tax rate lowers after-tax cashflows and encourages debt growth. Lower expected profits and larger debt payments increase the probability of default and expected bankruptcy costs, thereby lowering product durability. Observe in Model 1, an increase in tax rate from 15% to 25% (35%) decreases after tax profit and unlevered firm value by 11.84% (23.67%). Firm's financial leverage increases by 41.35% (71.69%) and product durability increases by 0.58% (1.03%). I see similar qualitative impact of tax rate increase in Model 2 and Model 3.

Table 10, panels A and B report results when the firm has an option to innovate. Both models assume a fixed coupon payment. The innovation option reduces the cost of producing the durable good, allowing the firm to produce at higher durability. When the unlevered firm exercises the innovation option, reduced marginal costs increase its optimal durability from $T =$

12.5 years to $T = 12.77$ years.[51] The firm's innovation option exercise threshold decreases in durability. In absence of financial leverage, the durability increases demand and hence firm's profitability, which enables the firm to pay the fixed cost to exercise innovation option.

In a levered firm indirect bankruptcy costs increase as durability increases. The firm's incentive to exercise the option is higher relative to the unlevered firm due to increased value from debt benefits. However, there are now two opposing effects on firm's innovation option exercise boundary, X_I. As durability increases the firm's profitability increases, allowing it to invest sooner, i.e., X_I decreases. In comparison, an increase in durability also increases indirect bankruptcy costs, which increases the chance of default.[52] The firm will exercise the option even sooner to counter the increase in probability of default. I can tease out the effect of bankruptcy costs on X_I, induced by product durability. When I examine the ratio of X_I/X_{UI}, together with NBD (net benefit of debt), I see slim evidence of the influence of bankruptcy costs on the option exercise boundary. The levered firm boundary, X_I, decreases at faster rate in comparison to X_{UI}, indicating increasing bankruptcy costs in durability. I can confirm this evidence when I look at the innovation option exercise boundary for Model 2, which is lower than that in Model 1 for the same level of durability and leverage. The firm in Model 2 faces larger bankruptcy costs and as a result, it chooses to exercise the option sooner to offset the higher probability of default.

I see in table 10 that the firm exercises the option sooner to offset higher probability of default.[53] Next, I examine the firm's policies when it optimizes first over durability and then over

[51] After innovation, the decrease in marginal cost will increase the unlevered firm's capacity to produce durable good at $T = 13.1$ years. However, the optimal durability that maximizes the firm's profitability is $T = 12.77$ years.
[52] The firm's profitability decreases at $T > 12.5$ years, which increase the probability of default. Further, in model 2, as durability increases, the market value of equity begins to decrease in large indirect bankruptcy costs, which encourages equity to put the firm to debt holders sooner, i.e., the probability of default will increase.
[53] Numerically I can measure that the X_I gets smaller faster then X_{UI}.

the coupon. Table 11, panels A and B show results when the coupon varies and the firm optimizes over durability for Model 1 and Model 2 respectively. As I increase the coupon the probability of bankruptcy increases, driving the choice of durability lower. The firm in Model 2 has a larger drop in durability as it faces larger bankruptcy costs relative to Model 1. The firm's profitability and unlevered value, V decreases in the firm's choice of lower durability, and the firm waits longer before exercising the innovation option. Further, by lowering durability the firm also lowers bankruptcy costs and decreases the chances of default, thereby increasing the firm's incentive to wait before exercising the innovation option. The distance between X_{UI} and X_I increases as the levered firm's net benefits of debt increases.

Table 11, panels C and D provides results for models 1 and 2 respectively, when the firm optimizes over coupon, C^*, as I vary durability, T. The unlevered firm value, V, is maximized when $T = 13.1$ years. Thus, the firm's profitability will increase for $T \leq 13.1$ years. As a result, I see that firm is able to exercise the innovation option sooner. In comparison, the levered firm will also face larger bankruptcy costs in durability, T. For models 1 and 2, I see that these bankruptcy costs outweigh benefits of durability and decreases the firm's demand for leverage. Larger profitability due to durability and lower coupon due to higher bankruptcy costs increase the market value of equity and reduce the leverage ratio. The levered firm also choses to exercise the option sooner, but I can see the incentives are getting even stronger (i.e., X_I decrease faster) as I vary durability, T. I can see this effect by observing that the distance between X_I and X_{UI} is getting larger.

Lastly, panel E show the results for Model 1 and Model 2 when the firm simultaneously chooses the profit maximizing level of coupon, C^*, and durability, T_L^*. The firm in Model 2 chooses lower durability and leverage, has larger market value of equity, and delays the exercise of the innovation option. When the firm exercises the innovation option it reduces the cost of producing the durable good. By exercising the option, the firm can expand output, increase durability and drive up demand. This will increase the firm's cash flows. Thus, I can say the indirect bankruptcy costs arising due to durability increases the firm's incentive to incur the fixed cost to exercise the innovation option sooner.

Table 12 shows comparative statics for models 1 and 2 for discount rate, r, demand growth rate, α, demand volatility rate, σ, and tax rate τ. Panels A and B show an increase in demand growth rate, α, increases firm and option value, which encourages the firm to invest sooner. Further, as α increases the probability of default decreases, allowing the firm to take on more debt and increase durability. Accordingly, larger leverage and durability increase the probability of default and the firm offsets it by exercising the higher valued innovation option sooner. As discussed earlier, the firm in Model 2 has larger bankruptcy costs than in Model 1, which is why I see smaller X_I in Model 2 relative to Model 1.

Panels C and D display the influence of the discount rate on the firm's optimal policies. An increase in the discount rate decreases the expected profitability of the firm and the option value, thereby discouraging the firm to exercise the innovation option sooner. As r increases the cost of debt and the probability of default will increase, which will force the firm to reduce leverage and durability.

Panels E and F show the firm's optimal policies when the volatility of demand increases. An increase in σ decreases the probability of default and increases the value of the option. Accordingly, the equity waits longer to resolve uncertainty and exercises the innovation option at a higher level of demand. Larger option value and lower probability of default reduce expected bankruptcy costs, allowing the firm to increase durability.

Panels G and H display the firm's policies when tax rate increases. An increase in τ decreases firm value and the innovation option is worth less. Although an increase in the tax rate increases the firm's tax benefits, it also increases the associated bankruptcy costs by allowing the firm to take on more debt. Consequently, the firm waits longer to exercise the innovation option, bankruptcy costs increase, and the firm is forced to reduce durability.

5. Conclusions

I model indirect bankruptcy costs in two different settings. These costs are imposed by the firm's consumers if the firm files for bankruptcy. I find that these indirect costs are significant and impact the firm's capital structure significantly. These costs and the firm's capital structure in turn influences the firm's choice of product durability.

I also model the consumer demand for durability and show that the firm's choice of product durability and the output increase in the consumer demand. An increase in durability allows the firm to extract more rent and bring more cashflow in. Consequently, the firm's profitability increases in product durability. In comparison, indirect bankruptcy costs limit the firm to supply less durability than what consumers demands.

Thus, this paper acknowledges both benefits and costs associated with product durability and show the firm's trade off. The firm's capital structure and product market thus interact and impact each other in these frameworks. Finally, the choice of product durability (firm's product market strategy) and firm's capital structure are jointly determined. These policy choices are subject to interest rates, tax rates in the economy and heavily influenced by the market demand and its uncertainties.

It is the demand for durability in the market that encourages the firm to pursue more aggressive behavior in terms of its choice of durability. I anticipate that the equity's choice of durability will be different than the firm's choice. Though, I do not enumerate the exacerbated agency costs between equity and debt due to these indirect costs, I expect limited liability of equity will produce larger durability and aggressive output, which will maximize its value, but will make bondholders worse off. This is the avenue I can explore in future research.

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Appendix A

Model 3: An Alternative Specification to Indirect Bankruptcy Costs

(*A model where financial distress influences the growth rate and/or volatility of consumer demand.*)

In this section I explore one more model variation of altered demand for durable goods, due to the bankruptcy cost of durability. This model is similar to Model 2, where the firm faces two shocks. However, in this model I incorporate one of the shocks in demand parameters rather than level. Specifically, the firm, outside the bankruptcy, faces the first shock in demand parameters, i.e., decrease the growth rate of demand and/or increase the volatility rate of demand. In comparison, the firm upon default, faces the level shock in demand, which it is similar to Model 1 and bankruptcy shock of Model 2. I explain the economics below.

Assume the consumer rationally anticipates lower resale value of the for a high risk of bankruptcy. As consumers' uncertainty increases, demand growth decreases. Further, demand uncertainty likely increases as growth decreases. Hortaçsu et al. (2013) find that as the firm nears bankruptcy, the prices of their durable goods decrease significantly. While some consumers rush to buy cheaper goods others stay away, and the result is higher volatility of consumer demand. Model 3, discussed in Appendix A, attempts to capture these dynamics (i.e., a decrease in the drift of consumer demand and an increase in the variance of consumer demand as the firm approaches bankruptcy.

The firm faces its first shock outside bankruptcy, at $X = X_P,$ which is the demand level where equity's particular solution is zero, i.e., $X_P = \frac{C}{r}\frac{(r-\alpha)}{\Omega}$. Specifically, X_P is the demand level where the firm's expected cash in-flow is equal to its expected cash out-flow. Further, at $X_P > X_D$, the demand growth rate α becomes $\alpha' = \alpha - \Gamma_\alpha(T)$ and demand volatility rate σ becomes

$\sigma' = \sigma\Gamma_\sigma(T)$ where $\Gamma_\sigma(T) > 1$ and $\Gamma_\alpha(T) > 0$ with $\Gamma'_\sigma(T) > 0$ and $\Gamma'_\alpha(T) > 0$.[54] Therefore, the general solutions of equity and debt in the region $X > X_P$ are given by

$$E(X) = B_{P5}X^{\beta_1} + B_{P6}X^{\beta_2} + (1-\tau)\left[\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right], \tag{A1}$$

$$D(X) = \frac{C}{r} + B_{P7}X^{\beta_1} + B_{P8}X^{\beta_2}, \tag{A2}$$

where $E(X)$ and $D(X)$ are equity and debt value for $X > X_P$. β_1 and β_2 solves characteristic equation $\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0$. Equity and debt values for the region $X \in (X_D, X_P)$ are given by

$$E_P(X) = B_5X^{\gamma_1} + B_6X^{\gamma_2} + (1-\tau)\left[\frac{\Omega X}{r-\left(\alpha - \Gamma_\alpha(T)\right)} - \frac{C}{r}\right], \tag{A3}$$

and

$$D_P(X) = \frac{C}{r} + B_7X^{\gamma_1} + B_8X^{\gamma_2}, \tag{A4}$$

where γ_1 and γ_2 solve $\frac{1}{2}(\Gamma_\sigma(T)\sigma)^2\gamma(\gamma-1) + \left(\alpha - \Gamma_\alpha(T)\right)\gamma - r = 0$, with $\Gamma_\sigma > 1$ and $0 < \Gamma_\alpha < 1$.

The general solutions of equity and debt in equations (A1) through (A4) satisfy following boundary conditions.

$$lim_{X\to\infty}E(X) = (1-\tau)\left[\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right], \tag{A5a}$$

$$lim_{X\to\infty}D(X) = \frac{C}{r}, \tag{A5b}$$

[54] If the firm produces more durable good, consumer uncertainty will increase with durability, and growth rate will decrease i.e.$\Gamma'_\alpha(T) > 0$ and volatility will increase $\Gamma'_\sigma(T) > 0$. The consumer becomes more concern about bankruptcy and the loss of service/maintenance, the more the product durability. Gomes et al. (2009) show that durable goods firm have more volatile cashflow and higher systematic risk than non-durable goods. I conjecture that the risk of bankruptcy amplifies the demand volatility as an increasing function of durability.

$$E(X_P) = E_P(X_P), \tag{A5c}$$

$$D(X_P) = D_P(X_P), \tag{A5d}$$

$$E'(X_P) = E_P{}'(X_P), \tag{A5e}$$

$$D'(X_P) = D_P'(X_P), \tag{A5f}$$

$$E_P(X_D) = 0, \tag{A5g}$$

$$E_P{}'(X_D) = 0, \tag{A5h}$$

$$D_P(X_D) = V(\phi(T)X_D). \tag{A5i}$$

Conditions (A5a) and (A5b) are standard no bubble conditions. Conditions (A5g) and (A5h) are value matching and smooth-pasting conditions for equity at the default boundary. Condition (A5i) states that in bankruptcy debt holders receive the unlevered value of the firm net of bankruptcy costs. $V(\phi(T)X_D) = \frac{(1-\tau)\phi(T)X_D}{r-(\alpha-\Gamma_\alpha(T))}$ is the unlevered firm value in bankruptcy when faced with a negative demand shock, $0 < \phi(T) \leq 1,$ with $\phi'(T) < 0$, evaluated at X_D. Conditions (A5c) and (A5d) are value matching conditions at X_P. Conditions (A5e) and (A5f) are market rationality conditions that must hold given full information about what will happen at $X = X_P$. Conditions (A5a)-(A5i) can be solved numerically for constants, $B_{P5}, B_{P6}, B_{P7}, B_{P8}, B_5, B_6, B_7$ and B_8, and the default threshold, X_D.

Now I look at the investment policy under Model 3. In this case the firm receives the first shock when consumers' uncertainty alters demand growth rate and volatility rate. In the following section I examine, the influence of altered demand parameters outside bankruptcy and demand shock in bankruptcy, on the firm's investment policy.

First, I consider the equity and the debt after the growth option has been exercised. I will continue to use shock parameters and value parameters defined earlier. The consumer's perception of expected bankruptcy shifts demand for the durable goods downward at $X_{P\theta} > X_{D\theta}$, where $X_{P\theta} = \frac{C}{r}\frac{(r-\alpha)}{\Omega_\theta}$ and Ω_θ is the before tax profit , after exercising the growth option. Since, $\Omega_\theta > \Omega$, $X_{P\theta} < X_P$, i.e., as the firm's prospects improves consumer's perceived uncertainty decreases, allowing them stay in the market longer.

The values of equity and debt follow the same general solutions as in equations (21) and (25) but have different boundary conditions. The general solutions of equity and debt after the growth option has been exercised in the region $X \in (X_{P\theta}, \infty)$ are given by

$$E_\theta(X) = A_{29}X^{\beta_1} + A_{30}X^{\beta_2} + (1-\tau)\left[\frac{\Omega_\theta X}{r-\alpha} - \frac{C}{r}\right], \tag{A6}$$

$$D_\theta(X) = \frac{C}{r} + A_{31}X^{\beta_1} + A_{32}X^{\beta_2}, \tag{A7}$$

where $E(X)$ and $D(X)$ are equity and debt values for $X > X_{P\theta}$. Equity and debt values for the region $X \in (X_D, X_{P\theta})$ are given by

$$E_{P\theta}(X) = A_{33}X^{\gamma_1} + A_{34}X^{\gamma_2} + (1-\tau)\left[\frac{\Omega_\theta X}{r-\alpha} - \frac{C}{r}\right], \tag{A8}$$

and

$$D_{P\theta}(X) = \frac{C}{r} + A_{35}X^{\gamma_1} + A_{36}X^{\gamma_2}. \tag{A9}$$

where γ_1 and γ_2 solve $\frac{1}{2}(\Gamma_\sigma(T)\sigma)^2\gamma(\gamma-1) + (\alpha - \Gamma_\alpha(T))\gamma - r = 0$, with $\Gamma_\sigma > 1$ and $0 < \Gamma_\alpha < 1$.

The general solutions of equity and debt in equations (A6) through (A9) must satisfy the following boundary conditions.

$$lim_{X\to\infty} E_\theta(X) = (1-\tau)\left[\frac{\Omega_\theta X}{r-\alpha} - \frac{C}{r}\right], \tag{A10a}$$

$$lim_{X\to\infty} D_\theta(X) = \frac{C}{r}, \tag{A10b}$$

$$E_\theta(X_{P\theta}) = E_{P\theta}(X_{P\theta}), \tag{A10c}$$

$$D_\theta(X_{P\theta}) = D_{P\theta}(X_{P\theta}), \tag{A10d}$$

$$\left.\frac{\partial E_\theta}{\partial X}\right|_{X=X_{P\theta}} = \left.\frac{\partial E_{P\theta}}{\partial X}\right|_{X=X_{P\theta}}, \tag{A10e}$$

$$\left.\frac{\partial D_\theta}{\partial X}\right|_{X=X_{P\theta}} = \left.\frac{\partial D_{P\theta}}{\partial X}\right|_{X=X_{P\theta}}, \tag{A10f}$$

$$E_{P\theta}(X_{D\theta}) = 0, \tag{A10g}$$

$$\left.\frac{\partial E_{P\theta}}{\partial X}\right|_{X=X_D} = 0, \tag{A10h}$$

$$D_{P\theta}(X_{D\theta}) = V_\theta(\phi(T)X_{D\theta}). \tag{A10i}$$

Conditions (A10a) and (A10b) are standard no bubble conditions. Conditions (A10g) and (A10h) are value matching and smooth-pasting conditions for equity at the default boundary. Condition (A10i) states that debt holders receive the unlevered firm and faces a further negative demand shock in bankruptcy. $V(\phi(T)X_D) = \frac{(1-\tau)\phi(T)X_D}{r-(\alpha-\Gamma_\alpha(T))}$ is the unlevered firm value in bankruptcy when faced with a negative demand shock, $0 < \phi(T) \leq 1$, with $\phi'(T) < 0$, evaluated at X_D. Conditions (A10c) and (A10d) are value matching conditions at $X_{P\theta}$. Since, equity and debt holders rationally anticipate what happens when X diffuses to $X_{P\theta}$, conditions (A10e) and (A10f) are market rationality conditions that must hold given full information. Conditions (A10a)-(A10i) can be solved numerically for constants, $A_{29}, A_{30}, A_{31}, A_{32}, A_{33}, A_{34}$ A_{35}, and A_{36}, and the default threshold, $X_{D\theta}$.

Boundary conditions (A10a) to (A10i) gives back general solutions of equity and debt specified in (A6) and (A7), the sum of which yields the value of the levered firm post-exercise of the growth option. Next, I look at the equity and debt value before-exercise of the growth option.

The general solution for equity and levered firm value for the region $X \in (X_D, X_P)$are given by:

$$E_P(X) = (1-\tau)\left(\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right) + A_{37}X^{\gamma_1} + A_{38}X^{\gamma_2}, \tag{A11}$$

and

$$V_P^L(X) = (1-\tau)\left(\frac{\Omega X}{r-\alpha}\right) + \frac{\tau C}{r} + A_{39}X^{\gamma_1} + A_{40}X^{\gamma_2}. \tag{A12}$$

I assume the order $X_I > X_P > X_D$. Therefore, the general solution for equity and levered firm value for the region $X \in (X_P, X_I)$ are given by:

$$E(X) = (1-\tau)\left(\frac{\Omega X}{r-\alpha} - \frac{C}{r}\right) + A_{41}X^{\beta_1} + A_{42}X^{\beta_2}, \qquad X_P < X < X_I, \tag{A13}$$

and

$$V^L(X) = (1-\tau)\left(\frac{\Omega X}{r-\alpha}\right) + \frac{\tau C}{r} + A_{43}X^{\beta_1} + A_{44}X^{\beta_2}, \qquad X_P < X < X_I. \tag{A14}$$

Equations (A11)-(A14) must satisfy following boundary conditions:

$$E_P(\psi(T)X_D) = 0\,, \tag{A15a}$$

$$E_P(X_P) = E(X_P)\,, \tag{A15b}$$

$$V_P^L(X_P) = V(X_P)\,, \tag{A15c}$$

$$E(X_I) = E_\theta(X_I) - I\,, \tag{A15d}$$

$$V^L(X_I) = V_\theta^L(X_I) - I\,, \tag{A15e}$$

$$\left.\frac{\partial E_P}{\partial X}\right|_{X=X_P} = \left.\frac{\partial E}{\partial X}\right|_{X=X_P}, \tag{A15f}$$

$$\left.\frac{\partial V_P^L}{\partial X}\right|_{X=X_P} = \left.\frac{\partial V^L}{\partial X}\right|_{X=X_P} , \tag{A15g}$$

$$\left.\frac{\partial V^L}{\partial X}\right|_{X=X_I} = V_\theta^{L\prime}(X_I) , \tag{A15h}$$

$$\left.\frac{\partial E_P}{\partial X}\right|_{X=X_D} = 0 , \tag{A15i}$$

$$\left.\frac{\partial V_P^L}{\partial X}\right|_{X=X_D} = V(\phi(T)X_D) , \tag{A15j}$$

Conditions (A15a) is a value matching condition for equity at the default boundary. Equation (A15i) is a smooth pasting condition of equity at default boundary, X_D. Condition (A15j) states that the firm faces a negative demand shock in bankruptcy and receives altered unlevered value. $V(\phi(T)X_{D\theta})$ is the unlevered firm value in bankruptcy (i.e., when $X = X_D$) specified in equation (59). Conditions (A15b) and (A15c) are value matching conditions at X_P. Since, equity and debt holders rationally anticipate what happens when X diffuses to X_P, conditions (A15f) and (A15g) are market rationality conditions that must hold given full information. Equations (A15d) and (A15e) are value matching conditions and (A15i) is smooth pasting condition at boundary X_I, where the right-hand side equity value is specified in equation (A6) and levered firm value is specified as the sum of the equity and the debt in equations (A6) and (A7). Conditions (A15a)-(A15i) can be solved numerically for constants, $A_{37}, A_{38}, A_{39}, A_{40}, A_{41}, A_{42}, A_{43}$, and A_{44} the investment boundary, X_I.

Appendix B

Model 3: Firm's Policies as We Vary Durability, T, for Exogenously Given Coupon, C

The market demand for durability, $T_{dd}^{*} = 14$ years, the unlevered firm's optimal durability T^{*}=12.5 years.For the base case parameters given in Table 1, Appendix B, panels A shows results for firm's policies under exogenously chosen coupon, C. The firm receives two demand shocks, at X_P and at X_D respectively. The firm receives shocks at X_P, which decreases growth rate α by $\Gamma_{\alpha}(T) = 0.001T$ (Panel A), or increases the volatility rate only $\Gamma_{\sigma}(T) = 1 + 0.05T$ (Panel B) or both, i.e., decreases the growth rate and increases the volatility rate both (Panel C). Additionally, the firm faces shock in bankruptcy $\phi(T) = 1.8534 - 0.0095\text{T}^2$. All panels show unlevered value, V, levered value, V^L, leverage ratio, Lev, net benefit of debt, NBD, credit spread, CRS. X_D is default boundary, $X_P = \frac{C^{*}(r-\alpha)}{r\Omega}$ is the boundary where consumers rationally perceive the firm's default is imminent.

Panel A: Model 3- The firm's demand growth is decreased at X_P and it receives bankruptcy shock at X_D

T	V	X_D	X_P	E	V^L	D	Lev	CRS	NBD
12.250	8805.05	0.2548	0.47	5039.35	10123.80	5084.44	0.50	185.05	1318.75
12.350	8822.68	0.2544	0.47	5055.86	10131.53	5075.68	0.50	186.58	1308.85
12.450	8831.51	0.2542	0.47	5064.00	10129.41	5065.41	0.50	188.38	1297.90
12.500	8832.61	0.2542	0.47	5064.93	10124.63	5059.70	0.50	189.38	1292.02
12.550	8831.51	0.2543	0.47	5063.77	10117.37	5053.60	0.50	190.45	1285.86

Panel B: Model 3- The firm's demand volatility rate is decreased at X_P and it receives bankruptcy shock at X_D

T	V	X_D	X_P	E	V^L	D	Lev	CRS	NBD
12.250	8805.05	0.18	0.47	5184.44	10376.67	5192.23	0.50	166.68	1571.62
12.350	8822.68	0.18	0.47	5201.59	10388.41	5186.82	0.50	167.58	1565.73
12.450	8831.51	0.18	0.47	5210.53	10390.61	5180.07	0.50	168.71	1559.10
12.500	8832.61	0.18	0.47	5211.92	10388.11	5176.19	0.50	169.37	1555.50
12.550	8831.51	0.18	0.47	5211.25	10383.21	5171.96	0.50	170.08	1551.71

Panel C: Model 3- The firm's demand growth rate is decreased, and volatility rate is increased at X_P and it receives bankruptcy shock at X_D

T	V	X_D	X_P	E	V^L	D	Lev	CRS	NBD
12.250	8805.05	0.19	0.47	5164.48	10281.02	5116.54	0.50	179.50	1475.97
12.350	8822.68	0.18	0.47	5181.58	10293.24	5111.66	0.50	180.34	1470.56
12.450	8831.51	0.18	0.47	5190.45	10295.83	5105.38	0.50	181.42	1464.32
12.500	8832.61	0.18	0.47	5191.80	10293.50	5101.70	0.50	182.06	1460.89
12.550	8831.51	0.18	0.47	5191.08	10288.74	5097.67	0.50	182.76	1457.24

Visual Representation of Model 1 and Model 2

Figure 1

This figure illustrates Model 1 and shows the simulated path of a demand shock, X_t, with parameters ($\alpha = 0\%$, $\sigma = 25\%$, $X_0 = 1$). The dash line shows the default boundary, X_D. When X_t hits barrier X_D from above, it jumps down to $0.5X_{t^+}$. The demand gets absorbed at zero.

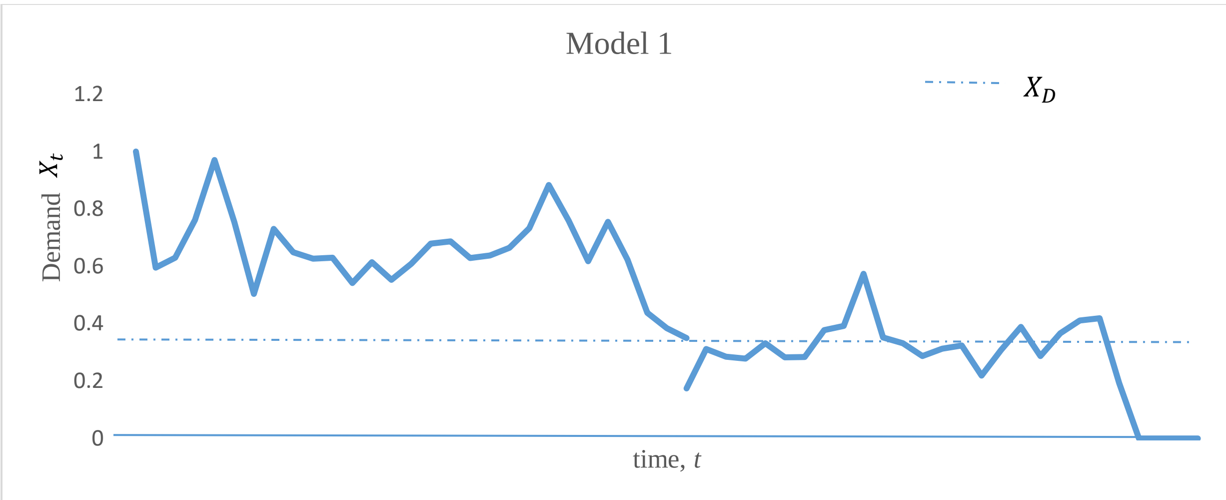

Figure 2

This figure illustrates the Model 2 and shows the simulated path of a demand, X_t, with parameters ($\alpha = 0\%$, $\sigma = 25\%$, $X_0 = 1$). There are two barriers X_P and X_D, $X_D < X_P$. When X_t hits barrier X_P from above, it jumps down to $0.5X_{t^+}$. When X_t hits the second barrier X_D from above, it jumps down further to $0.5X_{t^+}$. Since, the demand has already received the shock at X_P, another downward adjustment at X_D amplifies the previous shock. $X_t$gets absorbed at 0.

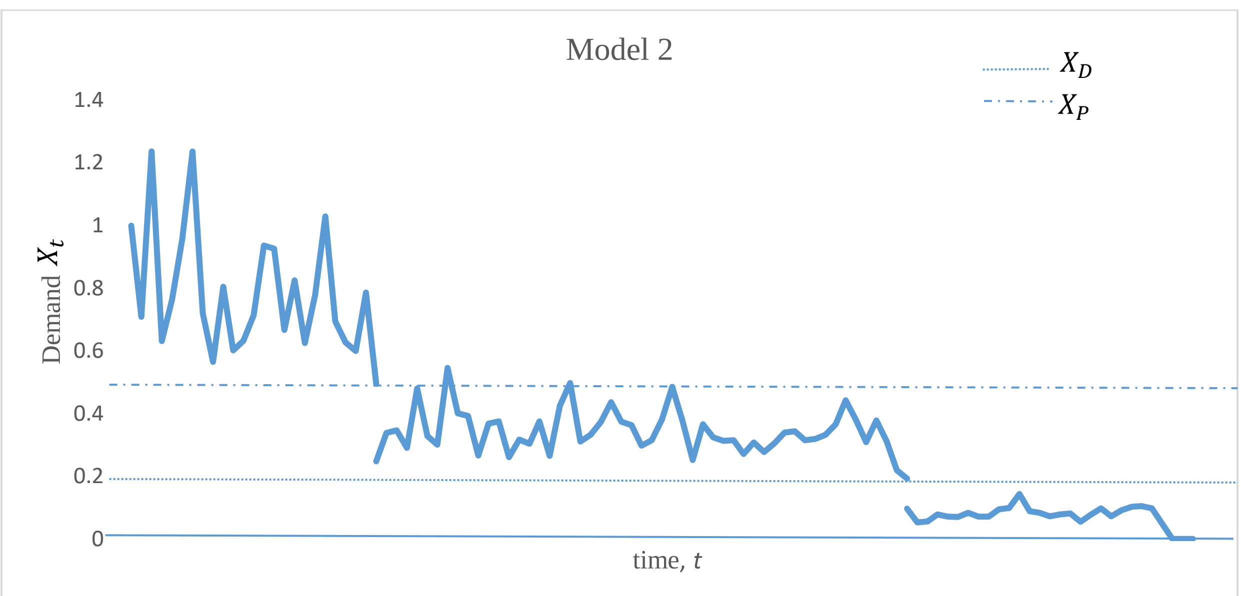

Table 1 Base Case Parameter Values and Table Description

Panel A. Base Case Parameter Values for Numerical Simulations

Parameter	*Value*
Durable goods preference parameter	δ = 1.4
Durable goods disutility parameter	γ = 0.05
Cost parameter†	κ = 0.3
Price elasticity of demand	ε = 1.5
Riskless interest rate	r = 7% annually
Demand volatility rate	σ = 25% annually
Demand growth rate	α = 0% annually
Corporate tax rate	τ = 0.35
Growth option economies of scale	θ = 0.6
Investment cost	I = 6795
Initial level of demand shock	X = 1

Panel B. Layout of Numerical Results by Table

Table 2	Comparative statics snapshot for all three models.
Table 3	Model 1 (section 2.4) and Model 2 (section 2.5) comparison when durability and coupon are given exogenously.
Table 4	Firm's policies in Model 1 (section 2.4) and Model (section 2.5) when (1) Durability is given exogenously and Coupon is given endogenously. (2) Coupon is given exogenously and Durability is given endogenously. (3) Durability and Coupon are both endogenously determined
Table 5	Model 3 (Appendix A) – Firm's policies when coupon is endogenously determined. We also append results for Model 3 in Appendix B when both coupon and durability are exogenously determined.

Table 1 (*contd.*)

Panel B. Layout of Numerical Results by Table (*contd.*)	
Tables 6 to 9	Comparative statics for Model 1 (section 2.4), Model 2 (section 2.5), and Model 3 (Appendix A) when Durability and Coupon are both endogenously determined. (1) When discount rate, r, varies (2) When demand growth rate, α, varies (3) When demand volatility rate, σ, varies (4) When tax rate, τ, varies
Table 10	Model 1 (section 3.2.1) and Model 2 (section 3.2.2) comparison when durability and coupon are given exogenously, and firm has an option to innovate
Table 11	Firm's policies in Model 1 (section 3.2.1) and Model (section 3.2.2) when the firm has an option to innovate and when (1) Coupon is given exogenously and Durability is given endogenously. (2) Durability is given exogenously and Coupon is given endogenously. (3) Durability and Coupon are both endogenously determined
Table 12	Comparative statics for Model 1 (section 3.2.1) and Model 2 (section 3.2.2) when the firm has an option to innovate when both coupon and durability are exogenously determined.

† The cost parameter, κ, determines the marginal cost of producing the durable good.

Table 2
Numerical Comparative Statics for Optimal Policies of a Levered Firm

This table presents how the levered firm's optimal policies (with and without an innovation option) differ across models 1, 2, and 3. Since, Model 3 incorporates shifts in the demand parameters (growth rate α and volatiltiey rate σ) the analysis is grouped by which parameter(s) is (are) shifted. The table shows the direction of the change in optimal coupon, C^*, optimal durability, T_L^* and innovation option exercise threshold, X_I, as I increase the discount rate, r, demand growth parameter, α, demand volatility parameter, σ, and tax-rate, τ, respectively.

Parameter	**Firm without Innovation Option**		**Firm with Innovation Option**		
As r increases	C^*	T_L^*	C^*	T_L^*	X_I
Model 1	Decreases	Increases	Decreases	Decreases	Increases
Model 2	Increases	Convex:	Convex	Decreases	Increases
Model 3 (α)	Increases	Decreases			
Model 3 (σ)	Decreases	Increases		Not computed	
Model 3 (α, σ)	Increases	Decreases			
As α increases	C^*	T_L^*	C^*	T_L^*	X_I
Model 1	Increases	Increases	Increases	Increases	Decreases
Model 2	Increases	Increases	Increases	Increases	Decreases
Model 3 (α)	Increases	Increases			
Model 3 (σ)	Increases	Increases		Not computed	
Model 3 (α, σ)	Increases	Increases			
As σ increases	C^*	T_L^*	C^*	T_L^*	X_I
Model 1	Convex	Decreases	Increases	Increases	Increases
Model 2	Decreases	Decreases	Decreases	Increases	Increases
Model 3 (α)	Decreases	Decreases			
Model 3 (σ)	Increases	Decreases		Not computed	
Model 3 (α, σ)	Convex	Decreases			
As τ increases	C^*	T_L^*	C^*	T_L^*	X_I
Model 1	Increases	Decreases	Increases	Decreases	Increases
Model 2	Increases	Decreases	Increases	Decreases	Increases
Model 3 (α)	Increases	Decreases			
Model 3 (σ)	Increases	Decreases		Not computed	
Model 3 (α, σ)	Increases	Decreases			

Table 3

Model 1 and Model 2 Comparison when Durability and Coupon are Exogeneous

The market demand for durability, $T_{dd}^{*} = 14$ years, the unlevered firm's optimal durability T^{*}=12.5 years. For the base case parameters given in Table 1, Table 3 presents models 1 and 2 results and their comparison for exogenously chosen durability T. Panel A shows Model 1 results when demand shock in bankruptcy is $\phi(T) = 1.8534 - 0.0095\mathrm{T}^2$. Panel B shows Model 2 results when it receives demand shocks at X_P, $\psi(T) = 1.8534 - 0.0095\mathrm{T}^2$ and in bankruptcy $\phi(T) = 1.8534 - 0.0095\mathrm{T}^2$. All panels show levered firm's coupon, C, value, V^L, leverage ratio, Lev, Net Benefit of Debt, NBD, and credit spread, CRS. X_D is the default boundary, and $X_P = \frac{C^*(r-\alpha)}{r\Omega}$ is the boundary where consumers rationally perceive firm's default is imminent.

T	V	X_D	X_P	E	V^L	Lev	C	CRS	NBD
Panel A: Firm's policies in Model 1 when coupon, C, and durability, T, are exogenously given									
12.25	8805.05	0.25	NA	5070.28	10284.74	0.51	450.00	162.98	1479.69
12.35	8822.68	0.25	NA	5086.95	10292.87	0.51	450.00	164.40	1470.19
12.45	8831.51	0.25	NA	5095.30	10291.30	0.50	450.00	166.05	1459.80
12.50	8832.61	0.25	NA	5096.35	10286.86	0.50	450.00	166.97	1454.25
12.55	8831.51	0.25	NA	5095.30	10279.98	0.50	450.00	167.94	1448.47
Panel B: Firm's policies in Model 2 when coupon, C, and durability, T, are exogenously given									
12.25	8805.05	0.31	0.47	4928.85	9673.10	0.49	450.00	248.52	868.05
12.35	8822.68	0.31	0.47	4941.55	9662.14	0.49	450.00	253.27	839.46
12.45	8831.51	0.32	0.47	4945.84	9641.45	0.49	450.00	258.34	809.94
12.50	8832.61	0.32	0.47	4944.82	9627.43	0.49	450.00	261.00	794.82
12.55	8831.51	0.32	0.47	4941.69	9610.97	0.49	450.00	263.75	779.46

Table 4

Firm Policies as a Function of Coupon, C, and Durability, T, for Model 1 and Model 2

The market demand for durability, $T_{dd}^{*} = 14$ years, the unlevered firm's optimal durability T^{*}=12.5 years. For the base case parameters given in Table 1, Table 4, panels A and B report model 1 and 2 results for the firm's optimal coupon under exogenously specified durability, T. Panels C and D show model 1 and 2 results for firm's optimal durability under exogenously specified coupon, C. Finally, panel E shows model 1 and 2 results for firm's optimal coupon and optimal durability. In Model 1 the firm receives demand shock in bankruptcy $\phi(T) = 1.8534 - 0.0095\mathrm{T}^2$. In Model 2, the firm receives demand shocks at X_P, $\psi(T) = 1.8534 - 0.0095\mathrm{T}^2$ and in bankruptcy $\phi(T) = 1.8534 - 0.0095\mathrm{T}^2$. All panels show unlevered firm value, V, levered firm value, V^L, leverage ratio, Lev, net benefit of debt, NBD, credit spread, CRS. X_D is the default boundary, and $X_P = \frac{C^*(r-\alpha)}{r\Omega}$ is the boundary where consumers rationally perceive firm's default is imminent.

T	V	X_D	X_P	E	V^L	D	lev	C^*	CRS	NBD
Panel A. Firm financing choice in Model 1 when durability T is exogeneous										
12.25	8805.05	0.34	NA	3929.89	10405.83	6475.94	0.62	617.15	252.99	1600.78
12.35	8822.68	0.33	NA	3990.52	10405.36	6414.84	0.62	610.18	251.20	1582.68
12.45	8831.51	0.33	NA	4046.63	10394.87	6348.23	0.61	602.73	249.44	1563.36
12.50	8832.61	0.33	NA	4072.94	10385.86	6312.92	0.61	598.83	248.58	1553.25
12.55	8831.51	0.32	NA	4098.06	10374.36	6276.30	0.60	594.82	247.73	1542.86
Panel B. Firm's financing choice in Model 2 when durability T is exogeneous										
12.25	8805.05	0.25	0.38	5661.71	9735.87	4074.16	0.42	358.86	180.82	930.82
12.35	8822.68	0.24	0.37	5731.74	9735.98	4004.24	0.41	352.11	179.34	913.30
12.45	8831.51	0.24	0.36	5793.18	9727.43	3934.25	0.40	345.41	177.96	895.92
12.50	8832.61	0.24	0.36	5820.63	9719.89	3899.26	0.40	342.08	177.29	887.28
12.55	8831.51	0.24	0.36	5845.93	9710.19	3864.25	0.40	338.76	176.65	878.68

T_L^*	V	X_D	X_P	E	V^L	D	Lev	C	CRS	NBD
Panel C. Firm choice of durability in Model 1 when coupon C is exogeneous										
12.428	8830.35	0.19	NA	5842.80	10112.98	4270.18	0.42	350.00	119.64	1282.63
12.407	8828.81	0.22	NA	5460.97	10213.73	4752.76	0.47	400.00	141.62	1384.92
12.384	8826.66	0.25	NA	5090.72	10293.43	5202.71	0.51	450.00	164.93	1466.77
12.359	8823.79	0.27	NA	4732.09	10352.05	5619.96	0.54	500.00	189.69	1528.26
12.332	8820.12	0.30	NA	4385.15	10389.58	6004.42	0.58	550.00	215.99	1569.46
Panel D. Firm choice of durability in Model 2 when coupon C is exogeneous										
12.358	8816.175	0.24	0.37	5743.84	9736.80	3992.96	0.41	350.00	176.54	920.63
12.307	8805.051	0.28	0.42	5327.49	9723.12	4395.63	0.45	400.00	210.00	918.07
12.250	8789.174	0.31	0.48	4916.35	9675.05	4758.70	0.49	450.00	245.64	885.88
12.186	8768.065	0.34	0.53	4510.56	9593.50	5082.95	0.53	500.00	283.68	825.44
12.117	8762.581	0.38	0.58	4126.58	9477.78	5351.20	0.56	550.00	327.81	715.20

Panel E. Firm financing choice and durability choice under Model 1 and Model 2										
	T_L^*	V	X_D	X_P	E	V^L	Lev	C^*	CRS	NBD
Mode1	12.295	8814.14	0.34	NA	3957.94	10406.86	0.62	614.05	252.17	1592.73
Mode2	12.301	8815.14	0.25	0.37	5698.51	9737.01	0.41	355.41	180.05	921.87

Table 5

The Firm's Policies under Demand Parameter Shifts (Model 3)

The market demand for durability, $T_{dd}^{*} = 14$ years, the unlevered firm's optimal durability T^{*}=12.5 years. For the base case parameters given in Table 1, Table 5 represents Model 3 results for exogenously chosen durability, T. The firm in Model 3 faces demand shock at X_P and X_D. The demand shock in bankruptcy at X_D is $\phi(T) = 1.8534 - 0.0095\mathrm{T}^2$ (all panels). The demand shock at X_P which decreases the growth in demand, α by $\Gamma_\alpha(T) = 0.001T$ (Panel A), or increases the volatility rate $\Gamma_\sigma(T) = 1 + 0.05T$ (Panel B) or both (Panel C). All panels show the levered firm's choice of coupon, C^*, value, V^L, leverage ratio, Lev, net benefit of debt, NBD, credit spread, CRS. X_D is the default boundary, and $X_P = \frac{C^*(r-\alpha)}{r\Omega}$ is the boundary where consumers rationally perceive firm's default is imminent.

T	V	X_D	X_P	E	V^L	D	lev	$C*$	CRS	NBD
Panel A. The firm's demand growth rate decreases to $\alpha - \Gamma_\alpha(T)$ when $X = X_P$										
12.25	8805.05	0.29	0.55	4551.35	10147.47	5596.12	0.55	517.55	224.84	1342.42
12.30	8814.96	0.29	0.54	4577.92	10150.98	5573.06	0.55	515.08	224.23	1336.01
12.346	**8822.10**	**0.29**	**0.54**	**4601.30**	**10152.00**	**5550.70**	**0.55**	**512.71**	**223.69**	**1329.90**
12.35	8822.68	0.29	0.54	4603.26	10152.01	5548.75	0.55	512.50	223.63	1329.33
12.40	8828.20	0.29	0.54	4627.30	10150.56	5523.26	0.54	509.82	223.04	1322.37
12.45	8831.51	0.29	0.53	4650.11	10146.65	5496.54	0.54	507.03	222.45	1315.14
12.50	8832.61	0.28	0.53	4671.61	10140.26	5468.65	0.54	504.14	221.87	1307.65
12.55	8831.51	0.28	0.53	4691.80	10131.40	5439.60	0.54	501.15	221.30	1299.90
Panel B. The firm's demand uncertainty (volatility) rate increases to $\sigma\Gamma_\sigma(T)$ when $X = X_P$										
12.25	8805.05	0.28	0.73	3731.52	10606.10	6874.60	0.65	694.37	310.05	1801.07
12.30	8814.96	0.28	0.73	3754.65	10609.03	6854.38	0.65	691.67	309.09	1794.07
12.331	**8820.01**	**0.27**	**0.73**	**3768.45**	**10609.54**	**6841.09**	**0.64**	**689.93**	**308.51**	**1789.54**
12.35	8822.68	0.27	0.72	3776.74	10609.36	6832.62	0.64	688.83	308.15	1786.69
12.40	8828.20	0.27	0.72	3797.89	10607.11	6809.23	0.64	685.83	307.21	1778.92
12.45	8831.51	0.27	0.72	3817.98	10602.28	6784.30	0.64	682.69	306.28	1770.77
12.50	8832.61	0.27	0.71	3837.00	10594.87	6757.86	0.64	679.41	305.36	1762.26
12.55	8831.51	0.27	0.71	3855.02	10584.88	6729.86	0.64	675.98	304.45	1753.38
Panel C. The firm's demand growth rate decreases to $\alpha - \Gamma_\alpha(T)$ and demand uncertainty rate increases to $\sigma\Gamma_\sigma(T)$ when $X = X_P$										
12.25	8805.05	0.25	0.65	4127.46	10398.69	6271.23	0.60	614.40	279.71	1593.64
12.30	8814.96	0.25	0.65	4147.84	10403.65	6255.81	0.60	612.49	279.07	1588.68
12.35	8822.68	0.25	0.64	4167.15	10406.05	6238.91	0.60	610.44	278.44	1583.37
12.372	**8825.38**	**0.25**	**0.64**	**4175.26**	**10406.31**	**6231.04**	**0.60**	**609.50**	**278.17**	**1580.93**
12.40	8828.20	0.25	0.64	4185.31	10405.91	6220.60	0.60	608.26	277.82	1577.72
12.45	8831.51	0.25	0.64	4202.34	10403.22	6200.88	0.60	605.95	277.20	1571.71
12.50	8832.61	0.25	0.63	4218.27	10397.97	6179.70	0.59	603.50	276.58	1565.36
12.55	8831.51	0.24	0.63	4233.06	10390.18	6157.13	0.59	600.92	275.97	1558.68

Table 6

Comparative Statics for Discount Rate, r

The market demand for durability, $T_{dd}^{*} = 14$ years, the unlevered firm's optimal durability T^{*}=12.5 years. For the base case parameters given in Table 1, Table 6 presents comparative statics for discount rate, r, under Model 1 (Panel A), Model 2 (Panel B), and Model 3 (Panels (C, and D). All panels show the optimal coupon, C^{*}, optimal durability, T_L^{*}, levered firm value, V^L, leverage ratio, Lev, net benefit of debt, NBD, credit spread, CRS. X_D is default boundary, $X_P = \frac{C^{*}(r-\alpha)}{r\Omega}$ is the boundary when consumers rationally perceive the firm's default is imminent. The firm faces negative demand shock at X_D in Model 1 and faces two demand shocks, first at X_P and second at X_D, in Models 2 and 3. In all three models, the demand shock at X_D is $\phi(T) = 1.8534 - 0.0095\mathrm{T}^2$. The firm in Model 2 additionally faces a demand shock at X_P which decrease demand by $\psi(T) = 1.8534 - 0.0095\mathrm{T}^2$. The firm in Model 3 faces a demand shock at X_P which decreases demand growth, α, by $\Gamma_{\alpha} = 0.001T$ (Panel C) or increases demand volatility σ, by $\Gamma_{\sigma}(T) = 1 + 0.05T$ (Panel D).

r	T_L^{*}	C^{*}	V	X_D	X_P	E	V^L	D	Lev	NBD	CRS
Panel A. Model 1											
3%	12.262	659.19	20551.3	0.26	NA	10252.5	23435.3	13182.8	0.56	2883.96	200.04
5%	12.281	623.61	12336.1	0.30	NA	5793.63	14354.6	8561	0.6	2018.53	228.43
7%	**12.295**	**614.05**	**8814.14**	**0.34**	**NA**	**3957.94**	**10406.9**	**6448.92**	**0.62**	**1592.73**	**252.17**
Panel B. Model 2											
3%	12.303	308.67	20569.5	0.15	0.33	14806.7	21919.9	7113.16	0.32	1350.42	133.94
5%	12.298	334.33	12340.5	0.21	0.35	8347.77	13422.6	5074.86	0.38	1082.18	158.8
7%	**12.301**	**355.41**	**8815.14**	**0.25**	**0.37**	**5698.51**	**9737.01**	**4038.5**	**0.41**	**921.87**	**180.05**
Panel C. Model 3 with change in demand growth at $X = X_P$											
3%	12.38	405.48	20594.6	0.17	0.43	13447.3	22368.6	8921.3	0.4	1773.99	154.51
5%	12.354	476.62	12352.5	0.24	0.5	7026.1	13895.2	6869.13	0.49	1542.75	193.86
7%	**12.346**	**512.71**	**8822.1**	**0.29**	**0.54**	**4601.3**	**10152**	**5550.7**	**0.55**	**1329.9**	**223.69**
Panel D. Model 3 with change in demand volatility at $X = X_P$											
3%	12.321	766.74	20576.4	0.19	0.81	10228.9	23930.9	13702	0.57	3354.49	259.58
5%	12.325	711.66	12346.7	0.24	0.75	5595.72	14650.3	9054.54	0.62	2303.53	285.97
7%	**12.331**	**689.93**	**8820.01**	**0.27**	**0.73**	**3768.45**	**10609.5**	**6841.09**	**0.64**	**1789.54**	**308.51**

Table 7

Comparative Statics for Demand Growth Rate, α

The market demand for durability, $T_{dd}^* = 14$ years, the unlevered firm's optimal durability T^*=12.5 years. For the base case parameters given in Table 1, Table 7 presents comparative statics for demand growth, α, under Model 1 (Panel A), Model 2 (Panel B), and Model 3 (Panels (C, and D). All panels show the optimal coupon, C^*, optimal durability, T_L^*, levered firm value, V^L, leverage ratio, Lev, net benefit of debt, NBD, credit spread, CRS. X_D is default boundary, $X_P = \frac{C^*(r-\alpha)}{r\Omega}$ is the boundary when consumers rationally perceive the firm's default is imminent. The firm faces negative demand shock at X_D in Model 1 and faces two demand shocks, first at X_P and second at X_D, in Models 2 and 3. In all three models, the demand shock at X_D is $\phi(T) = 1.8534 - 0.0095\mathrm{T}^2$. The firm in Model 2 additionally faces a demand shock at X_P which decrease demand by $\psi(T) = 1.8534 - 0.0095\mathrm{T}^2$. The firm in Model 3 faces a demand shock at X_P which decreases demand growth, α, by $\Gamma_\alpha = 0.001T$ (Panel C) or increases demand volatility σ, by $\Gamma_\sigma(T) = 1 + 0.05T$ (Panel D).

α	T_L^*	C^*	V	X_D	X_P	E	V^L	Lev	NBD	CRS
Panel A. Model 1										
0%	**12.295**	**614.05**	**8814.14**	**0.34**	NA	**3957.94**	**10406.86**	**0.62**	**1592.73**	**252.17**
2%	12.310	857.23	12343.27	0.37	NA	5289.85	14787.87	0.64	2444.60	202.54
4%	12.325	1438.38	20577.99	0.40	NA	8358.95	25048.98	0.67	4470.99	161.82
Panel B. Model 2										
0%	**12.301**	**355.41**	**8815.14**	**0.25**	**0.37**	**5698.51**	**9737.01**	**0.41**	**921.87**	**180.05**
2%	12.309	594.08	12343.12	0.30	0.45	7121.76	14037.27	0.49	1694.15	159.05
4%	12.336	1150.01	20581.73	0.35	0.52	10443.10	24156.38	0.57	3574.65	138.61
Panel C. Model 3 with change in demand growth at $X = X_P$										
0%	**12.346**	**512.71**	**8822.10**	**0.29**	**0.54**	**4601.30**	**10152.00**	**0.55**	**1329.90**	**223.69**
2%	12.362	736.48	12353.89	0.33	0.55	6109.07	14454.13	0.58	2100.24	182.53
4%	12.387	1261.11	20596.27	0.36	0.57	9639.91	24516.24	0.61	3919.97	147.73
Panel D. Model 3 with change in demand volatility at $X = X_P$										
0%	**12.331**	**689.93**	**8820.01**	**0.27**	**0.726**	**3768.45**	**10609.54**	**0.64**	**1789.54**	**308.51**
2%	12.343	965.56	12350.42	0.30	0.726	4920.15	15103.95	0.67	2753.52	248.13
4%	12.356	1614.41	20588.07	0.34	0.728	7614.33	25606.21	0.70	5018.15	197.30

Table 8

Comparative Statics for Demand Volatility Rate, σ

The market demand for durability, $T_{dd}^{*} = 14$ years, the unlevered firm's optimal durability T^{*}=12.5 years. For the base case parameters given in Table 1, Table 8 presents comparative statics for demand volatility, σ, under Model 1 (Panel A), Model 2 (Panel B), and Model 3 (Panels (C, and D). All panels show the optimal coupon, C^{*}, optimal durability, T_{L}^{*}, levered firm value, V^{L}, leverage ratio, Lev, net benefit of debt, NBD, credit spread, CRS. X_D is default boundary, $X_P = \frac{C^{*}(r-\alpha)}{r\Omega}$ is the boundary when consumers rationally perceive the firm's default is imminent. The firm faces negative demand shock at X_D in Model 1 and faces two demand shocks, first at X_P and second at X_D, in Models 2 and 3. In all three models, the demand shock at X_D is $\phi(T) = 1.8534 - 0.0095\mathrm{T}^{2}$. The firm in Model 2 additionally faces a demand shock at X_P which decrease demand by $\psi(T) = 1.8534 - 0.0095\mathrm{T}^{2}$. The firm in Model 3 faces a demand shock at X_P which decreases demand growth, α, by $\Gamma_{\alpha} = 0.001T$ (Panel C) or increases demand volatility σ, by $\Gamma_{\sigma}(T) = 1 + 0.05T$ (Panel D).

σ	T_L^{*}	C^{*}	V	X_D	X_P	E	V^L	Lev	NBD	CRS
Panel A: Model 1										
15%	12.342	625.82	8821.57	0.44	NA	3370.70	10922.75	0.69	2101.18	128.68
25%	**12.295**	**614.05**	**8814.14**	**0.34**	**NA**	**3957.94**	**10406.86**	**0.62**	**1592.73**	**252.17**
35%	12.268	643.61	8808.96	0.27	NA	4309.44	10111.73	0.57	1302.77	409.24
Panel B. Model 2										
15%	12.335	434.27	8820.60	0.38	0.46	4850.64	10278.63	0.53	1458.03	100.06
25%	**12.301**	**355.41**	**8815.14**	**0.25**	**0.37**	**5698.51**	**9737.01**	**0.41**	**921.87**	**180.05**
35%	12.3	316.46	8814.96	0.17	0.33	6217.79	9455.52	0.34	640.55	277.41
15%	12.374	549.73	8825.60	0.40	0.58	3936.556	10671.30	0.63	1845.70	111.93
Panel C. Model 3 with change in demand growth at $X = X_P$										
25%	**12.346**	**512.71**	**8822.10**	**0.29**	**0.54**	**4601.30**	**10152.00**	**0.55**	**1329.90**	**223.69**
35%	12.333	512.52	8820.30	0.23	0.54	5016.251	9857.72	0.49	1037.42	332.59
Panel D. Model 3 with change in demand volatility at $X = X_P$										
15%	12.362	672.88	8824.20	0.39	0.708	3147.229	11083.38	0.72	2259.17	147.87
25%	**12.331**	**689.93**	**8820.01**	**0.27**	**0.726**	**3768.45**	**10609.54**	**0.64**	**1789.54**	**308.51**
35%	12.322	744.54	8818.63	0.21	0.784	4245.604	10325.69	0.59	1507.06	524.55

Table 9

Comparative Statics for Tax Rate, τ

The market demand for durability, $T_{dd}^* = 14$ years, the unlevered firm's optimal durability T^*=12.5 years. For the base case parameters given in Table 1, Table 9 presents comparative statics for tax rate, τ, under Model 1 (Panel A), Model 2 (Panel B), and Model 3 (Panels (C, and D). All panels show the optimal coupon, C^*, optimal durability, T_L^*, levered firm value, V^L, leverage ratio, Lev, net benefit of debt, NBD, credit spread, CRS. X_D is default boundary, $X_P = \frac{C^*(r-\alpha)}{r\Omega}$ is the boundary when consumers rationally perceive the firm's default is imminent. The firm faces negative demand shock at $X_D$in Model 1 and faces two demand shocks, first at X_P and second at X_D, in Models 2 and 3. In all three models, the demand shock at X_D is $\phi(T) = 1.8534 - 0.0095\text{T}^2$. The firm in Model 2 additionally faces a demand shock at X_P which decrease demand by $\psi(T) = 1.8534 - 0.0095\text{T}^2$. The firm in Model 3 faces a demand shock at X_P which decreases demand growth, α, by $\Gamma_\alpha = 0.001T$ (Panel C) or increases demand volatility σ, by $\Gamma_\sigma(T) = 1 + 0.05T$ (Panel D).

τ	T_L^*	C^*	V	X_D	X_P	E	V^L	Lev	NBD	CRS
Panel A: Model 1										
15%	12.424	357.66	11547	0.20	NA	7562.967	11944.54	0.37	397.59	116.28
25%	12.352	505.56	10180.3	0.28	NA	5414.387	11116.96	0.51	936.66	186.55
35%	**12.295**	**614.05**	**8814.14**	**0.34**	NA	**3957.94**	**10406.86**	**0.62**	**1592.73**	**252.17**
Panel B: Model 2										
15%	12.459	170.93	11549.00	0.1196	0.18	9525.00	11739.00	0.19	190.00	72.04
25%	12.393	266.81	10185.64	0.19	0.28	7442.02	10679.97	0.30	494.33	124.01
35%	**12.301**	**355.41**	**8815.14**	**0.25**	**0.37**	**5698.51**	**9737.01**	**0.41**	**921.87**	**180.05**
Panel C: Model 3 with change in demand growth at $X = X_P$										
15%	12.451	280.52	11548.95	0.16	0.23	8343.968	11860.78	0.30	311.83	148.73
25%	12.397	409.9	10186.07	0.23	0.37	6185.262	10945.5	0.43	759.43	179.39
35%	**12.346**	**512.71**	**8822.10**	**0.29**	**0.54**	**4601.30**	**10152.00**	**0.55**	**1329.90**	**223.69**
Panel D: Model 3 with change in demand volatility at $X = X_P$										
15%	12.424	438.36	11547.00	0.17	0.35	6914.50	12034.00	0.43	487.00	156.26
25%	12.368	588.83	10182.60	0.23	0.54	4998.36	11273.53	0.56	1090.93	238.35
35%	**12.331**	**689.93**	**8820.01**	**0.27**	**0.726**	**3768.45**	**10609.54**	**0.64**	**1789.54**	**308.51**

Table 10

Option to Innovate -Model 1 and 2 Comparison when Durability, T, and Coupon, C are Exogenous

The market demand for durability, $T_{dd}^{*} = 14$ years, the unlevered firm's optimal durability with an option to innovate T=13.1 years. For the base case parameters given in Table 1, Table 3 reports models 1 and 2 results with an option to innovate and their comparison for exogenously chosen durability,T. Panel A shows Model 1 results when it receives shock in bankruptcy $\phi(T) = 1.8534 - 0.0095\mathrm{T}^2$. Panel B shows Model 2 results when it receives shocks at X_P, $\psi(T) = 1.8534 - 0.0095\mathrm{T}^2$ and in bankruptcy $\phi(T) = 1.8534 - 0.0095\mathrm{T}^2$. The column Total shock in bankruptcy computes the numerical value of shock faced during bankruptcy. All panels show levered firm's coupon, $C = 450$, value, V^L, leverage ratio, Lev, net benefit of debt, NBD, credit spread, CRS. X_D is default boundary, $X_P = \frac{C^*(r-\alpha)}{r\Omega}$ is the boundary where consumers rationally perceive the firm's default is imminent, X_{UI} and X_I are boundaries when unlevered firm and levered firm will exercise innovation option. Lastly, the ratio X_I/X_{UI} measures the distance between these two boundaries.

T	V	X_{UI}	X_D	X_P	X_I	E	V^L	Lev	CRS	NBD	$\frac{X_I}{X_{UI}}$
Panel A: Firm's policies in model 1 when coupon, C, and durability, T, are exogenously given											
12.40	10058.98	3.46	0.24	NA	3.43	6304.51	11568.62	0.46	154.85	1509.65	0.991
12.50	10094.21	3.42	0.23	NA	3.39	6339.00	11594.99	0.45	156.17	1500.78	0.991
12.70	10131.80	3.35	0.23	NA	3.32	6376.47	11612.30	0.45	159.46	1480.50	0.990
12.90	10124.68	3.30	0.24	NA	3.26	6371.09	11581.27	0.45	163.69	1456.59	0.989
13.10	10072.35	3.26	0.24	NA	3.22	6322.39	11501.07	0.45	168.95	1428.72	0.989
13.20	10029.26	3.24	0.24	NA	3.20	6281.84	11442.45	0.45	171.99	1413.19	0.988
Panel B: Firm's policies in Model 2 when coupon, C, and durability, T, are exogenously given											
12.40	10058.98	3.46	0.30	0.47	3.42	6143.17	11121.42	0.45	203.93	1062.45	0.987
12.50	10094.21	3.42	0.30	0.47	3.37	6172.68	11127.25	0.45	208.25	1033.03	0.986
12.70	10131.80	3.35	0.31	0.47	3.30	6199.70	11100.74	0.44	218.17	968.94	0.985
12.90	10124.68	3.30	0.31	0.48	3.24	6183.08	11021.77	0.44	230.00	897.09	0.983
13.10	10072.35	3.26	0.32	0.48	3.20	6122.20	10888.87	0.44	244.06	816.52	0.982
13.20	10029.26	3.24	0.32	0.48	3.18	6075.15	10801.86	0.44	252.04	772.61	0.981

Table 11

Option to Innovate - Firm's Policies as a Function of Firm's Coupon, C, and Durability, T

The market demand for durability, $T_{dd}^{*}=14$ years, the unlevered firm's optimal durability with an option to innovate, $T=13.1$ years. For the base case parameters given in Table 1, Table 4, panels A and B represent models 1 and 2 results for firm's optimal durability under exogenously chosen coupon, C. Panels C and D show models 1 and 2 results for firm's optimal coupon under exogenously chosen durability, T. Finally, panel E shows models 1 and 2 results for firm's optimal coupon and optimal durability. In Model 1 the firm receives shock in bankruptcy $\phi(T)=1.8534-0.0095\mathrm{T}^2$. In Model 2, the firm receives shocks at X_P, $\psi(T)=1.8534-0.0095\mathrm{T}^2$ and in bankruptcy $\phi(T)=1.8534-0.0095\mathrm{T}^2$. All panels show unlevered value, V, levered value, V^L, leverage ratio, Lev, net benefit of debt, NBD, credit spread, CRS. X_D is default boundary, $X_P=\frac{C^*(r-\alpha)}{r\Omega}$ is the boundary where consumers rationally perceive the firm's default is imminent, X_{UI} and X_I are boundaries when unlevered firm and levered firm will exercise innovation option. Lastly, the ratio X_I/X_{UI} measures the distance between these two boundaries.

Panel A. Firm's policies for Model 1 when coupon, C, is given exogenously and durability T_L^* is optimally chosen.

T_L^*	C	X_{UI}	X_D	X_P	X_I	E	V^L	Lev	CRS	NBD	X_I/X_{UI}
12.723	300.00	3.34	0.16	0.32	3.33	7532.00	11294.97	0.33	97.24	1161.70	0.996
12.708	350.00	3.35	0.18	0.37	3.33	7135.43	11420.10	0.38	116.87	1287.72	0.994
12.691	400.00	3.35	0.21	0.42	3.33	6749.39	11525.96	0.41	137.42	1394.89	0.992
12.672	450.00	3.36	0.23	0.47	3.33	6373.79	11612.76	0.45	158.95	1483.55	0.990
12.653	500.00	3.36	0.26	0.53	3.32	6008.74	11680.73	0.49	181.52	1553.78	0.988

Panel B. Firm's policies for Model 2 when coupon, C, is given exogenously and durability T_L^* is optimally chosen

T_L^*	C	X_{UI}	X_D	X_P	X_I	E	V^L	Lev	CRS	NBD	X_I/X_{UI}
12.640	300.00	3.37	0.21	0.32	3.35	7451.63	11076.36	0.33	127.65	951.18	0.994
12.597	350.00	3.38	0.24	0.37	3.35	7021.56	11122.98	0.37	153.36	1005.01	0.991
12.548	400.00	3.40	0.27	0.42	3.36	6594.87	11139.64	0.41	180.13	1032.38	0.989
12.496	450.00	3.42	0.30	0.47	3.38	6171.70	11127.25	0.45	208.07	1034.24	0.986
12.442	500.00	3.44	0.33	0.53	3.39	5752.15	11086.75	0.48	237.28	1011.66	0.984

Panel C. Firm's policies for Model 1 when durability, T, is given exogenously and coupon C^* is optimally chosen

T	C^*	X_{UI}	X_D	X_P	X_I	E	V^L	Lev	CRS	NBD	X_I/X_{UI}
12.400	674.00	3.46	0.34	0.71	3.39	4766.56	11753.65	0.59	264.64	1694.67	0.980
12.500	666.85	3.42	0.34	0.70	3.35	4845.48	11770.01	0.59	263.03	1675.80	0.979
12.700	650.85	3.35	0.33	0.69	3.28	4985.40	11765.70	0.58	259.91	1633.89	0.979
12.900	632.69	3.30	0.33	0.67	3.23	5099.78	11711.45	0.56	256.93	1586.77	0.979
13.100	612.54	3.26	0.32	0.66	3.19	5186.88	11607.20	0.55	254.06	1534.85	0.979
13.200	601.78	3.24	0.31	0.65	3.17	5219.76	11536.50	0.55	252.67	1507.24	0.979

Panel D. Firm's policies for Model 2 when durability, T, is given exogenously and coupon C^* is											
T	C^*	X_{UI}	X_D	X_P	X_I	E	V^L	Lev	CRS	NBD	X_I
12.40	411.1	3.46	0.1	0.43	3.42	6461.4	11127.7	0.4	181.0	1068.7	0.989
12.50	407.8	3.42	0.1	0.43	3.38	6519.3	11138.5	0.4	182.8	1044.3	0.989
12.70	388.2	3.35	0.1	0.41	3.31	6711.3	11126.2	0.4	179.4	994.43	0.989
12.90	368.4	3.30	0.1	0.39	3.26	6864.0	11068.6	0.3	176.3	943.92	0.989
13.10	311.1	3.26	0.1	0.33	3.23	7295.4	10955.0	0.3	150.1	882.68	0.991
13.20	311.1	3.24	0.1	0.34	3.21	7250.8	10891.1	0.3	154.5	861.92	0.991

Panel E. Firm's optimal policies for Model 1 and Model 2.											
	T_L^*	C^*	X_D	X_P	X_I	E	V^L	Lev	CRS	NBD	X_I $/X_{UI}$
Model 1	12.583	660.48	0.34	NA	3.32	4906.54	11774.25	0.58	261.72	1659.07	0.979
Model 2	12.545	403.43	0.13	0.42	3.36	6565.83	11139.71	0.41	182.03	1033.19	0.988

Table 12

Option to Innovate - Comparative Statics for the Firm with an Innovation Option

This table shows the comparative statics for the levered and unlevered firm with their optimal policies for models 1 and 2. Unlevered firm an with innovation option has optimal durability, $T = 12.77$ years, levered firm with an innovation optimizes at $T_L^* = 12.583$ years for Model 1 and $T_L^* = 12.545$ years in Model 2. Panels A and B show comparative statics for demand growth rate, α, for models 1 and 2 respectively, panels C and D show comparative static for discount rate, r, for Model 1 and Model 2 respectively, panels E and F show comparative statics for demand volatility rate, σ, for models 1 and 2 respectively, and finally, panels G and H show comparative static for tax rate, τ, for Model 1 and Model 2 respectively. The firm receives negative demand shock $\phi(T) = 1.8534 - 0.0095 * T^2$ in bankruptcy in models 1 and 2. Additionally, in Model 2 at X_P the firm receives negative demand shock $\psi(T) = 1.8534 - 0.0095 * T^2$. Credit spread, CRS, is defines as $\left(\frac{C^*}{D} - r\right)10000$ and net benefit of debt, NBD, credit spread, CRS. X_D is default boundary, $X_P = \frac{C^*(r-\alpha)}{r\Omega}$ is the boundary where consumers rationally perceive the firm's default is imminent. X_{UI} and X_I are boundaries when unlevered firm and levered firm will exercise innovation option. Lastly, the ratio X_I/X_{UI} measures the distance between these two boundaries.

	T_L^*	C^*	X_D	X_P	X_I	E	V^L	Lev	CRS	NBD
Panel A. Model 1 - Firm's option exercise policies and financing policies as demand growth rate, α, varies										
α = 0%	12.583	660.48	0.34	NA	3.32	4906.54	11774.25	0.58	261.72	1659.07
2%	12.734	1045.58	0.37	NA	2.94	7681.83	19082.92	0.60	217.09	2845.77
4%	12.856	2118.26	0.40	NA	2.61	14615.97	38889.05	0.62	172.68	6327.00
Panel B. Model 2 - Firm's option exercise policies and financing policies as demand growth rate, α, varies										
α = 0%	12.545	403.43	0.13	0.42	3.36	6565.83	11139.71	0.41	182.03	1033.19
2%	12.683	714.17	0.17	0.54	2.99	9871.48	18203.75	0.46	157.11	1984.15
4%	12.803	1611.10	0.23	0.73	2.66	17973.02	37430.56	0.52	128.01	4901.32
Panel C. Model 1 - Firm's option exercise policies and financing policies as discount rate, r varies										
r = 3%	12.823	901.39	0.28	NA	1.75	16162.18	34143.98	0.53	201.28	3744.06
5%	12.697	726.01	0.31	NA	2.56	7991.57	17818.22	0.55	238.82	2235.99
7%	12.583	660.48	0.34	NA	3.32	4906.54	11774.25	0.58	261.72	1659.07
Panel D. Model 2 - Firm's option exercise policies and financing policies as discount rate, r, varies										
r = 3%	12.826	402.75	0.10	0.43	1.88	22700.13	32102.91	0.29	128.33	1701.22
5%	12.675	392.20	0.12	0.41	2.63	10852.86	16816.15	0.35	157.69	1241.30
7%	12.545	403.43	0.13	0.42	3.36	6565.83	11139.71	0.41	182.03	1033.19

Table 12 (*Contd.*)

	T_L^*	C^*	X_D	X_P	X_I	E	V^L	Lev	CRS	NBD
Panel E. Model 1 - Firm's option exercise policies and financing policies as demand volatility rate, σ, varies										
σ = 15%	12.495	637.06	0.44	NA	2.61	3731.07	11393.44	0.67	131.41	2116.62
25%	12.583	660.48	0.34	NA	3.32	4906.54	11774.25	0.58	261.72	1659.07
35%	12.642	738.54	0.27	NA	4.21	5886.54	12453.07	0.53	424.70	1424.68
Panel F. Model 2 - Firm's option exercise policies and financing policies as demand volatility rate, σ, varies										
σ = 15%	12.445	469.93	0.19	0.49	2.64	4976.47	10840.40	0.54	101.39	1572.74
25%	12.545	403.43	0.13	0.42	3.36	6565.83	11139.71	0.41	182.03	1033.19
35%	12.630	376.20	0.10	0.40	4.28	7920.21	11769.83	0.33	277.24	744.41
Panel G. Model 1 - Firm's option exercise policies and financing policies as tax rate, τ, varies										
τ = 15%	12.776	366.29	0.19	NA	2.53	9744.86	14234.96	0.32	115.77	396.71
25%	12.678	535.04	0.27	NA	2.87	6893.96	12901.31	0.47	190.64	960.68
35%	12.583	660.48	0.34	NA	3.32	4906.54	11774.25	0.58	261.72	1659.07
Panel H. Model 1 - Firm's option exercise policies and financing policies as tax rate, τ, varies										
τ = 15%	12.789	193.05	0.06	0.20	2.53	11552.23	14052.71	0.18	72.05	213.40
25%	12.684	302.11	0.10	0.32	2.89	8832.32	12496.04	0.29	124.60	554.44
35%	12.545	403.43	0.13	0.42	3.36	6565.83	11139.71	0.41	182.03	1033.19