COMPARISON OF TAYLOR'S THEORY OF SPHERICAL BLAST TO A KNOWN NUMERICAL SOLUTION

by

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ABSTRACT

SANDIPKUMAR ROYADHIKARI. Comparison of Taylor's theory of spherical blast to a known numerical result. (Under the direction of DR. RUSSELL G. KEANINI)

Taylor's blast wave theory was the first of its kind and focused on the dynamics and thermodynamics of spherical blast waves. Taylor's model was stated as a similarity solution and determined the time-dependent blast radius, as well as the radially- and temporarily-varying pressure, density and velocity fields behind the blast wave. Due to the stiffness of the associated governing, coupled differential equations, a special geometric projection method must be used to integrate the equations. The theoretical solution is compared against a rare and recently reported numerical solution for near-ground, hemispherical blast wave propagation. While the theoretical solution for the time varying blast wave pressure jump is qualitatively quite similar to the reported numerical solution, significant quantitative differences are found. These differences are attributed to: i) poorly defined model conditions and definitions in the numerical solution, and ii) differences in the blast waves modeled, spherical versus hemispherical. It is argued that the present theoretical solution is valid since: 1) simple order of magnitude arguments indicate that predicted blast wave pressure ratios are of the correct magnitude, and 2) a numerical validation test using a simpler spherical blast wave model predicts results that are essentially identical to those obtained by the full Taylor model.

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CHAPTER1: INTRODUCTION

1.1 Introduction

Merriam Webster dictionary says a blast is “the sudden, loud, and release of energy that happens when something (such as a bomb) breaks apart in a way that sends parts flying outward”. In a military combat the heat and the shock wave generated by a blast is propagated all the direction which destroy or damage the enemy vehicle, personnel. But the advent terrorism of 20th and 21st century changed the world, especially blast in Twin tower in New York, London, Madrid or Beslan bomb blast. They try to make more injury with less effort. It has changed our vision to study the blast.

In this study a known numerical solution is compared with the Taylor’s blast wave theory. The numerical solution had studied the free blast, blast reflection, but here only free standing wave is studied. Taylor has considered the blast phenomenon is only depending on the energy released and the density of the ambient. The numerical solution has studied the primary shock wave released by the blast along with the secondary and tertiary shock waves, but for simplicity purpose here only primary shock wave is studied. Usually the initial blast is considered in two different ways, one is point blast in which the blast is considered as an infinitesimally small point and the other is isothermal blast in which the blast is considered as a high pressure sphere with certain radius. The numerical result which is taken as a reference is considered as an isothermal sphere.

In Taylor's theorem the radius of the blast wave is proportional to the total energy released by the blast and inversely proportional to the ambient density and they are determined by the similarity solution process. Now by Hugonoit shock relationship the pressure, velocity and density along the shockwave was determined. Now it is also to be remembered that the radius of the shock changes with the time, because it is propagates all the direction with the progress of time.

The numerical solution has determined the solution of a particular location and other parameters are calculated behind the shock. Here also the same location is taken as the reference and calculated the parameters behind the shock.

1.2 Literature Review

A bunch of literature had been studied while doing this research work among them the most significant are 1. Taylor's blast wave [1] and the other is 2. Numerical study of spherical blast-wave propagation and reflection by Liang, Wang and Chen [2].

G.I. Taylor work on blast wave theory is a theoretical approach. First he supposed that explosion can be idealized as the sudden release of an amount of energy concentrated at a point, and that is only dimensional parameter introduced by the explosion. Second it is supposed that the resulting disturbance will be so strong that the initial pressure and speed of the ambient air are negligible compared with the pressure and velocities produced in the disturbed flow. Then the only dimensional parameter to the ambient gas is the density. Then he did the non-dimensional calculation and found that the blast radius at any point of time after the blast is proportional to the energy released by the blast, inversely proportional to the density of the medium that is air and also proportional to the time. This is taken as the main reference to the study of the problem here. Other than Taylor, people like Vön

Neumann and Leonard Sedov had independently worked and found the same result. Then the pressure, density and velocity are calculated numerically.

In the Liang et. al[2] paper they consider the blast to be an isothermal sphere. In This paper the authors used numerical method to capture the shock and they calculated the primary, secondary and tertiary shock in two different locations calculated the pressure ration. On those particular location the pressure is increased in a initially then decreased, then the secondary and tertiary shock also increased the pressure ratio again.

In Friedlander's empirical relationship the pressure is increased rapidly when the shock passed to a particular location then gradually reduced to the atmospheric pressure then even in gone below the ambient pressure then once again increased to the atmospheric pressure.

Tai et. al.[3] worked on blast wave interaction and reflection around close ended and open ended bomb shell. The total Variation Diminishing Finite Volume method was employed to solve the three Euler's equation. The reflected shock wave patterns transit from regular reflection to Mach reflection in both bomb shelters under an unsteady situation. For regular reflection, the incident shock and reflected shock wave and the Mach stem meet is formed. The incident, the reflected shock wave and the Mach stem meet at the triple point. Shi et. al.[4] simulated the blast wave interaction with a standalone structural column. Parametric studies included the scaled distance of the blast, column stiffness, and column dimension and geometry. The formula to predict the reflected pressure, and impulse on the front and on the rear surface of the columns with different dimensions and geometry were derived. Yang et al [5] numerically studied the shock wave reflection patterns generated by a blast wave impinging on a circular cylinder. The transition from regular to Mach reflection, trajectory of the triple point, and the complex shock-on-shock interaction were discussed. Blast wave propagation and reflection has also been investigated experimentally. Takayama and Sekiquchi[6] studied the interaction of a spherical shock wave into free space in a conventional shock tube. When a spherical shock wave encounters a planar or conical wall, a transition from regular to Mach reflection takes place with the incident angle larger than the critical transition angle. Dewey and McMillin[7] used high speed photography to investigate the blast wave interaction with ideal and real surfaces. It was observed that a smooth surface induces a stronger Mach stem.

Blast wave reflection from plates had been Liang et al[2] studied the transition behavior of an unsteady cylindrical blast wave reflection from a flat plate. For the first outward-moving shock wave which is followed by expansion waves; the type of reflection transits from the regular reflection to Mach reflection. However, for the secondary shock-wave which is induced by expansion waves, the type of reflection remains the regular reflection. Colella et al[8] numerically studied the two dimensional axisymmetric reflection of a sphereical wave from a plate, which creates complex flow structures on multiple length scales.

Blast wave reflection from wedge has also been investigated, both numerically and experimentally. Olejniczak et al[9] numerically studied the steady inviscid shock interactions on the double-wedge geometries. The effects of varying second angle and Mach number on the phenomena of interaction were discussed. Five interaction types and the transition criteria between the various interactions, the transition criteria between the various interactions area identified. Ben-dor[10] simulated the reflection process of a planar shock wave over concave and convex double wedges. The pressure distributions along the two surfaces of the double wedge were investigated, and the points along the double wedges that are subjected to the highest and lowest pressures were revealed. Igra et[11] a numerically studied the reflection process of a travelling wave from wedge was studied for different dust mass loading and different dust particle diameters. It was shown that the loading and dust particle diameter affect the wave reflection patterns. It was that the dust presence affects the pressure on the wedge surface significantly. Igra et al[12]investigated blast wave reflection from wedges experimentally and numerically. In the numerical study, the two-dimensional Euler equations were solved using a Godonuv based, second order accurate scheme. A shock tube equipped with a very short driver was employed for experiments.

1.3 Spherical Blast wave Problem

The problem of interest is an isothermal blast where the initial blast is consider as a sphere where the pressure ratio 70 to the ambient pressure and the density is considered as 5 to the atmospheric density. The atmospheric considered as a 1 atm, the density and temperature and pressured as 1.225 kg/m^3 and 288.15 K respectively. The initial radius of the blast 3 meters. Now the problem is solved by Taylor's blast theory to calculate the pressure ratio at 9meters which is outside the then the result is compared with the result obtained from the numerical solution. The total energy is calculated from $h=c_p(T_0-T_{amb})$ where T_{amb} is the temperature of the environment. Then the time is determined to reach the shock at that particular location. Then the pressure ratio is calculated to the location behind the shock. The c_p is specific heat at constant pressure. The temperature inside the blast is considered as 2717K. With this temperature the specific heat is calculated through a formula provided by NASA.

1.4 Previous related works

Harold A. Brode[13] did a strong shock solution for both point blast and isothermal blast using gas motion in Lagrangian form. Von Neumann Richtmayer artificial viscosity was employed to avoid the shock discontinuity. The results include overpressure, density, particle density and position as a function of time and space. J. Lockwood Taylor[14] tried to find the problem of Taylor to get the exact solution. In the paper of the shape of blast wave: the studies of Friedlander equation, JM Dewey studied the modified Friedlander form to describe the physical properties of blast wave. Friedlander had given an empirical formulation of pressure location after the blast. The equation if quite simple but it has different modified version and it doesn't work universally according to my belief. Alex Remnnikov and Timothy Rose [15] studied the effect on a structure of a blast wave numerically. In that study they assumed there was no obstacle in between the explosion charge and building structure. They also mentioned that the effect of the blast can enhanced due to the presence of other structure in vicinity. John Von Neumannindependently studied the point blast by similarity solution. Leonard Sedov also did the similar kind of study in the same way. Bethe, Fuchs, Hirshfelder et. al did the detail study of point blast.

1.5 Purpose of Study

The purpose of this study is very significant. There are pioneers did the study the analytically or numerically, like Brode studied in 1960's numerically, whereas Taylor, Von Neumann and Sedov studied the cases in the analytically. But there is no study that is compared the results numerical with analytical at least not in my knowledge. So it is tried to study and compared the Taylor's blast wave theory with a known numerical solution.

It also opens to further study the effect of blast wave reflection and subsequent consequence of the reflection, how to react the blast, especially mechanical property and crack formation.

CHAPTER 2: SHOCK THEORY

Shock wave is a disturbance (essentially a pressure wave) that propagates through the solid, liquid or gas. It takes place when wave speed is faster than the sound wave. Like any other ordinary wave it carries energy, characterized by abruptly change of pressure, temperature and density. Shock wave can be two types first one is normal shock and the other one is oblique shock.

2.1 One dimensional Flow equation

Consider a region which represents the one dimensional flow as shown in the figure below

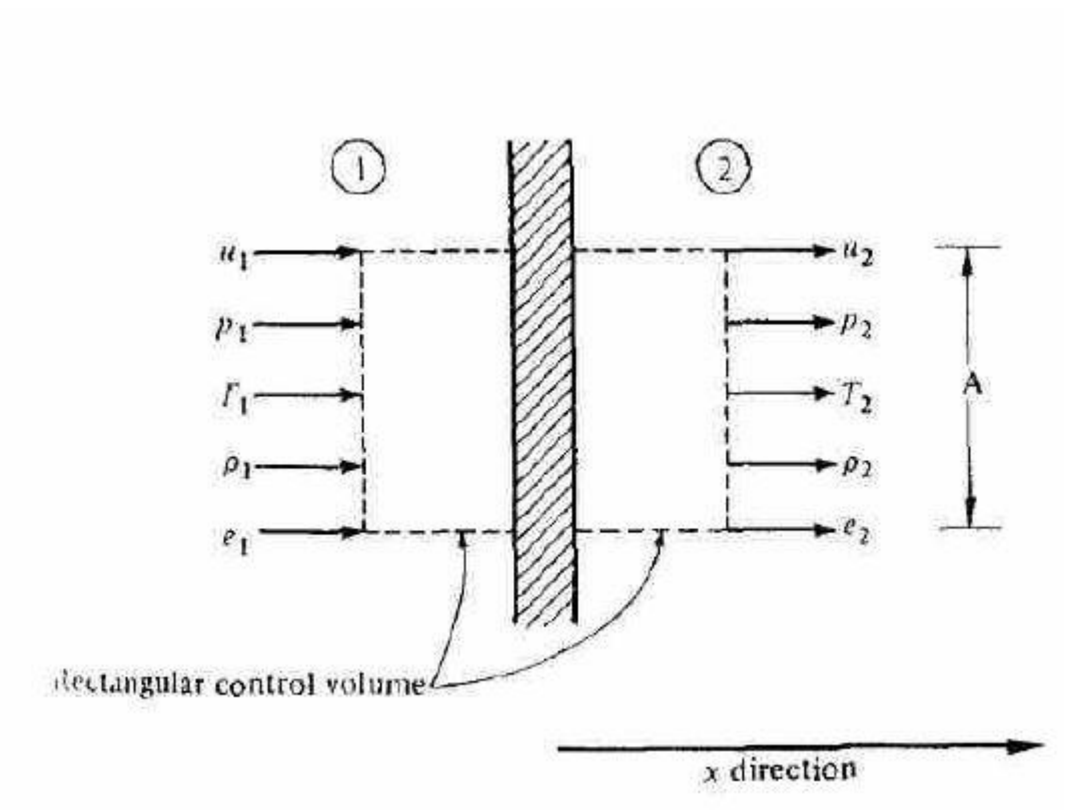

Figure 1: control volume around the shock (courtesy by ref1)

In the left region pressure, temperature, density, speed and energy are P_1, T_1, ρ_1, u_1 and e_1 respectively and in the right side pressure, temperature, density, speed and energy are p_2, T_2, ρ_2, u_2 and e_2 respectively and also assume that the flow is steady.

Now from continuity equation

$$-\oiint_{\boldsymbol{S}} \rho \boldsymbol{V}.d\boldsymbol{S} = \frac{\partial}{\partial t}\iiint_{\boldsymbol{\Omega}} \rho d\boldsymbol{\Omega}$$

Where ρ is density, **V** is speed and $\boldsymbol{\Omega}$ is volume. Since the flow is steady, hence

$$\oiint \rho \boldsymbol{V}.d\boldsymbol{S} = 0 \tag{2.1}$$

Now from the above equation

$$-\rho_1 u_1 A_1 + \rho_2 u_2 A_2 = 0$$

Since the area is same which is A.

$$\rho_1 u_1 = \rho_2 u_2 \tag{2.2}$$

Now the momentum equation is

$$\oiint_S (\rho \boldsymbol{V}.d\boldsymbol{S})\boldsymbol{V} + \iiint_\Omega \frac{\partial(\rho V)}{\partial t} d\boldsymbol{\Omega} = \iiint_\Omega \rho \boldsymbol{F} d\boldsymbol{\Omega} - \oiint_S p d\boldsymbol{S}$$

Considering steady state and no body force

$$\oiint_S (\rho \boldsymbol{V}.d\boldsymbol{S})\boldsymbol{V} = -\oiint_S p d\boldsymbol{S} \tag{2.3}$$

Since it is a one dimensional equation towards the x direction, hence

Since the flow is steady and there is no body force

$$\oiint_S (\rho V.dS)u = -\oiint_S (pds)_x \tag{2.4}$$

Now evaluating the above equation

$$\rho_1(-u_1 A)u_1 + \rho_2(u_2 A)u_2 = -(p_1 A + p_2 A)$$

or $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$ (2.5)

Momentum equation for steady state one dimensional is

$$\iiint_{\boldsymbol{\Omega}} \dot{q}\rho d\boldsymbol{\Omega} - \oiint_{\boldsymbol{S}} p\boldsymbol{V}.d\boldsymbol{S} + \iiint_\Omega \rho \boldsymbol{F}.\boldsymbol{V} d\Omega = \iiint_{\boldsymbol{\Omega}} \frac{\partial}{\partial t}\left[\rho\left(e + \frac{V^2}{2}\right)\right] d\boldsymbol{\Omega} + \oiint_{\boldsymbol{S}} \rho\left(e + \frac{V^2}{2}\right) V.d\boldsymbol{S}$$

The first represents the total heat addition and represent by $\dot{Q}$ for simplicity. The third term is zero because there is no body force and the fourth term is zero because of the steady state. Hence we have

$$\dot{Q} - \oiint_S p\boldsymbol{V}.dS = \oiint_{\boldsymbol{S}} \rho\left(e + \frac{V^2}{2}\right)\boldsymbol{V}.d\boldsymbol{S} \tag{2.6}$$

Evaluating the surface integral we have

$$\dot{Q} - (-p_1u_1A + p_2u_2A) = -\rho_1\left(e_1 + \frac{u_1^2}{2}\right)u_1A + \rho_2\left(e_2 + \frac{u_2^2}{2}\right)u_2A$$

By rearranging,

$$\frac{\dot{Q}}{A} + p_1u_1 + \rho_1\left(e_1 + \frac{u_1^2}{2}\right)u_1 = p_2u_2 + \rho_2\left(e_2 + \frac{u_2^2}{2}\right)u_2 \tag{2.7}$$

Diving (2.7) by (2.2) that is left side by $\rho_1 u_1$ and the right side by $\rho_2 u_2$

$$\frac{\dot{Q}}{\rho_1u_1A} + \frac{p_1}{\rho_1} + e_1 + \frac{u_1^2}{2} = \frac{p_2}{\rho_2} + e_2 + \frac{u_2^2}{2} \tag{2.8}$$

Now $h = e + pv$

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2} \tag{2.9}$$

2.2 Speed of sound

Sound is a pressure wave and when it reaches the eardrum then through the eardrum the wave reaches the brain, by this human being perceives the presence of sound. Let the pressure wave is passing through the gas with speed a. Now consider if someone rides on the wave then that person will see the gas is moving towards him with speed a from the front while in his back the speed will be different, though it is small , let's say the speed is $a + da$.

Hence the other properties will different as well. Let's say pressure, temperature and density corresponding to the speed a are p, T and ρ respectively and pressure , temperature and density corresponding to the speed $a + da$ are $p + dp, T + dT$ and $\rho + d\rho$ respectively as shown in the picture above.

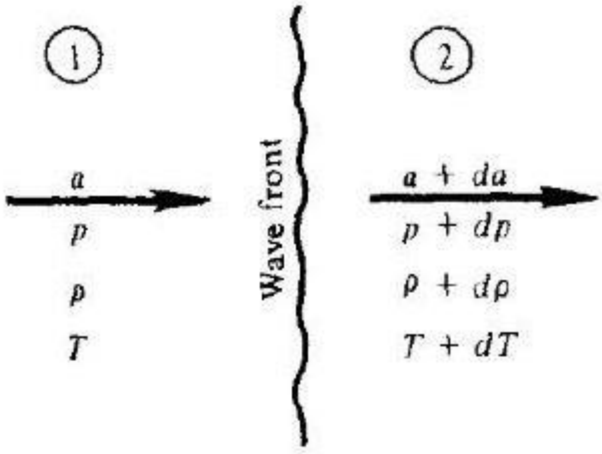

Figure 2. Schematic diagram of sound wave

Hence the other properties will different as well. Let's say pressure, temperature and density corresponding to the speed a are p, T and ρ respectively and pressure , temperature and density corresponding to the speed $a + da$ are $p + dp, T + dT$ and $\rho + d\rho$ respectively as shown in the picture above.

Now say that the area on the both sides is equal, then by continuity equation

$\rho a = (\rho + d\rho)(a + da)$

$$\rho a = \rho a + \rho da + a d\rho + d\rho da \tag{2.10}$$

Multiplication of between the differential is neglected and hence

$\rho a = \rho a + \rho da + a d\rho$

$$\text{Or } a = -\rho \frac{da}{d\rho} \tag{2.11}$$

Now from momentum equation

$$p + \rho a^2 = (p + dp) + (\rho + d\rho)(a + da)^2 \tag{2.12}$$

By ignoring the product of differential

$$dp = -2a\rho da - a^2 d\rho \tag{2.13}$$

Solve for da

$$da = \frac{dp + a^2 d\rho}{-2a\rho} \tag{2.14}$$

Substituting

$$a = -p\left(\frac{\frac{dp}{d\rho} + a^2}{-2a\rho}\right) \tag{2.15}$$

Hence for a^2

$$a^2 = \frac{dp}{d\rho} \tag{2.16}$$

Since the process is isentropic by considerations like no heat edition, no frictional work, change of properties are very small, hence the relation of sound speed equation above is the speed of sound in isentropic process and can be represented as

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s \tag{2.17}$$

Let say $\rho = 1/v$ where v is specific volume, then

$$a^2 = \left(\frac{dp}{d\rho}\right)_s = -\left(\frac{dp}{dv}\right) v^2 = -\frac{v}{(1/v)(\partial v/\partial p)_s} \tag{2.18}$$

So,

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} \tag{2.19}$$

In isentropic relationship

$$pv^{\gamma} = c$$

Where γ is the ratio of specific heat.

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} = \sqrt{\frac{\gamma p}{\rho}}$$

Now since $\frac{p}{\rho} = RT$, hence

$$a = \sqrt{\gamma RT} \tag{2.20}$$

Now Mach number (M) is the ratio of speed and sonic , so if

M> 1 supersonic

M=0 sonic

M<1 subsonic

If the kinetic and internal energy per unit mass is $V^2/2$ and e respectively then

$$\frac{V^2/2}{e} = \frac{V^2/2}{c_v T} = \frac{V^2/2}{RT/(\gamma-1)} = \frac{(\gamma/2)V^2}{a^2(\gamma-1)} = \frac{\gamma(\gamma-1)}{2} M^2$$

2.3 Some Convenient Definition

Let a flow adiabatically slow down (if M>1) or speed up (M<1) to Mach number 1, then the temperature will get change and let say the new temperature is T^*, then speed is going to be $a^* = \sqrt{\gamma R T^*}$

Then the characteristic Mach number $M^* = V/a^*$

Stagnation speed of sound $a_o = \sqrt{\gamma R T_o}$

Stagnation density $\rho_0 = p_0/RT_0$

Where R is gas constant p_o, T_0 are the stagnation pressure and temperature respectively.

2.4 Alternative Form of One Dimensional Energy Equation

Assume that there is no heat addition

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \tag{2.21}$$

Since $h = c_p T$, hence

$$c_p T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2} \tag{2.22}$$

$$\frac{\gamma R T_1}{\gamma-1} + \frac{u_1^2}{2} = \frac{\gamma R T_2}{\gamma-1} + \frac{u_2^2}{2} \tag{2.23}$$

Since $a = \sqrt{\gamma R T}$

$$\frac{a_1^2}{\gamma-1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma-1} + \frac{u_2^2}{2} \tag{2.24}$$

The above equation can also be written as

$$\frac{\gamma}{\gamma-1}\left(\frac{p_1}{\rho_1}\right) + \frac{u_1^2}{2} = \frac{\gamma}{\gamma-1}\left(\frac{p_2}{\rho_2}\right) + \frac{u_2^2}{2} \tag{2.25}$$

Or $\frac{a^2}{\gamma-1}+\frac{u^2}{2}=\frac{a^{*2}}{\gamma-1}+\frac{u^{*2}}{2}$

$$\frac{a^2}{\gamma-1}+\frac{u^2}{2}=\frac{\gamma+1}{2(\gamma-1)}a^{*2} \quad (2.26)$$

a and u are the Mach number and speed at any point.

From the definition of stagnation temperature

$$c_pT+\frac{u^2}{2}=c_pT_0 \quad (2.27)$$

Now the ratio of stagnation temperature and temperature of a point can be found by

$$\frac{T_0}{T}=1+\frac{u^2}{2c_pT}=1+\frac{u^2}{2\gamma RT/(\gamma-1)}=1+\frac{u^2}{2a^2/(\gamma-1)}=1+\frac{\gamma-1}{2}\left(\frac{u}{a}\right)=1+\frac{\gamma-1}{2}M^2 \quad (2.28)$$

Now ratio of stagnation pressure and pressure, stagnation density and density

$$\frac{p_0}{p}=(1+\frac{\gamma-1}{2}M^2)^{\gamma/\gamma-1} \quad (2.29)$$

and

$$\frac{\rho_0}{\rho}=(1+\frac{\gamma-1}{2}M^2)^{1/\gamma-1} \quad (2.30)$$

So,

$$\frac{a^2}{\gamma-1}+\frac{u^2}{2}=\frac{a_0^2}{2} \quad (2.31)$$

From (2.31) and (2.26)

$$\frac{\gamma+1}{2(\gamma-1)}a^{*2}=\frac{a_0^2}{\gamma-1} \quad (2.32)$$

Now divide (2.32) by a^*/a_0 and use (2.20)

$$(\frac{a^*}{a_0})^2=\frac{T^*}{T_0}=\frac{2}{\gamma+1} \quad (2.33)$$

By recalling the definition of p^* and ρ^* which calls for Mach=1 and from (2.29) and (2.30)

$$\frac{p^*}{p_0}=(\frac{2}{\gamma+1})^{\gamma/\gamma-1} \quad (2.34)$$

And

$$\frac{\rho^*}{\rho_0}=(\frac{2}{\gamma+1})^{1/\gamma-1} \quad (2.35)$$

Now , divide the (2.26) by u^2

$$M^2=\frac{2}{[(\gamma+1)/M^{*2}]-(\gamma-1)} \quad (2.36)$$

2.5 Normal Shock Relationship

Let's now apply the formula for normal shock. When the flow is subsonic then the flow is continued with any disturbance but when it is supersonic it produces shock as shown in the figure below. For quantitative reason all quantity ahead of shock is prescribed by subscript 1 and all quantity behind the shock is subscript by 2. There is no heat addition or extraction from the system, hence the system is adiabatic.

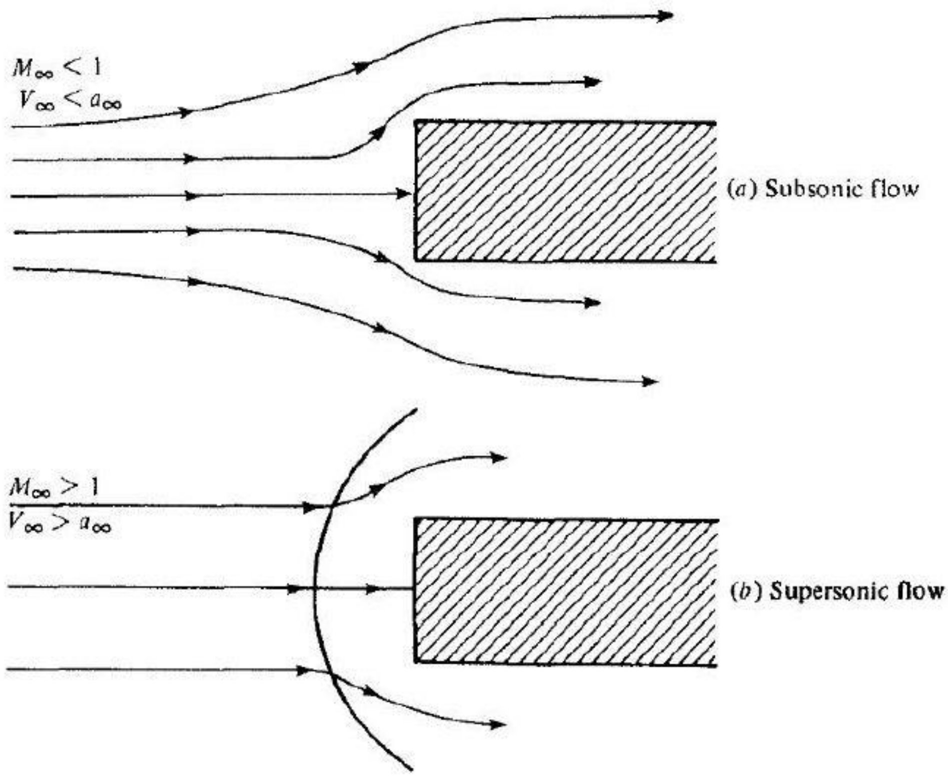

Figure 3: Flow hindered by a body. The upper one is in subsonic flow and lower one in case of supersonic flow

From continuity equation,

$$\rho_1 u_1 = \rho_2 u_2 \tag{2.37}$$

From momentum equation,

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \tag{2.38}$$

From energy equation,

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \tag{2.39}$$

Now from the equation for calorific perfect gas

$$p = \rho RT \tag{2.40}$$

$$h = c_p T \tag{2.41}$$

Now divide (2.38) by (2.37) and rearranging

$$\frac{p_1}{\rho_1 u_1} - \frac{p_2}{\rho_2 u_2} = u_2 - u_1 \tag{2.42}$$

By using $a = \sqrt{\gamma p/\rho}$ in equation (2.42)

$$\frac{a_1^2}{\gamma u_1} - \frac{a_2^2}{\gamma u_2} = u_2 - u_1 \tag{2.43}$$

By utilizing (2.43), (2.39) and (2.26)

$$a_1^2 = \frac{\gamma+1}{2} a^{*2} - \frac{\gamma-1}{2} u_1^2 \tag{2.44}$$

$$a_2^2 = \frac{\gamma+1}{2} a^{*2} - \frac{\gamma-1}{2} u_2^2 \tag{2.45}$$

Using (2.45), (2.44) and (2.43)

$$a^{*2} = u_1 u_2 \tag{2.46}$$

This is called Prandtl relation.

$$1 = \frac{u_1}{a^*} \frac{u_2}{a^*} = M_1^* M_2^*$$

$$M_1^* = \frac{1}{M_2^*} \tag{2.47}$$

This proves that the behind the normal shock the Mach number of the flow is subsonic.

Diving (2.26) by u^2

$$M^2 = \frac{2}{[(\gamma+1)/M^{*2}] - (\gamma-1)} \tag{2.48}$$

From (2.48) and (2.47)

$$M^{*2} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2} \tag{2.49}$$

From (2.49) and (2.47)

$$\frac{(\gamma+1)M_2^2}{2+(\gamma-1)M_2^2} = 1/\left[\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right] \tag{2.50}$$

From the above equation evaluate for M_2^2

$$M_2^2 = \frac{1+[(\gamma-1)/2]M_1^2}{\gamma M_1^2-(\gamma-1)/2} \tag{2.51}$$

The above equation is very powerful equation, which interprets that the downstream speed only on upstream speed. The other parameters can be found very easily

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{u_1^2}{u_1 u_2} = \frac{u_1^2}{a^{*2}} = M_1^{*2} \tag{2.52}$$

From (2.52) and (2.49)

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2} \tag{2.53}$$

From momentum equation

$$p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2$$

Combining with (2.37)

$$p_2 - p_1 = \rho_1 u_1 (u_1 - u_2) = \rho_1 u_1^2 (1 - \frac{u_2}{u_1}) \tag{2.54}$$

Dividing (2.54) by p_1 and recalling $a_1^2 = \gamma p_1/\rho_1$

$$\frac{p_2-p_1}{p_1} = \gamma M_1^2 \left(1 - \frac{u_2}{u_1}\right) \tag{2.55}$$

From (2.55) and (2.53)

$$\frac{p_2-p_1}{p_1} = \gamma M_1^2 \left(1 - \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right) \tag{2.56}$$

Simplifying (2.56)

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1) \tag{2.57}$$

The temperature ratio can be found out by equation of state $p = \rho RT$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{\rho_1}{\rho_2}\right) \tag{2.58}$$

From (2.57), (2.53) and (2.58)

$$\frac{T_2}{T_1} = \frac{h_2}{h_1} = \left[1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)\right]\left[\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right] \tag{2.59}$$

2.6 Hugoniot Equation

In the last section the properties are the functions of Mach number, but in this section the properties will be explained on thermodynamic properties.

From (2.37)

$$u_2 = u_1\left[\frac{\rho_1}{\rho_2}\right] \tag{2.60}$$

Now replace (2.60) into (2.38)

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2\left(\frac{\rho_1}{\rho_2}u_1\right)^2 \tag{2.61}$$

From (2.61)

$$u_1^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1}\right) \tag{2.62}$$

From (2.37)

$$u_1 = u_2\left(\frac{\rho_2}{\rho_1}\right)$$

hence

$$u_2^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_1}{\rho_2}\right) \tag{2.63}$$

From energy equation (2.39)

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

By definition$h = e + p/\rho$, so

$$e_1 + \frac{p_1}{\rho_1} + \frac{u_1^2}{2} = e_2 + \frac{p_2}{\rho_2} + \frac{u_2^2}{2} \tag{2.64}$$

Now, from (2.62), (2.63) and (2.64)

$$e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}\left[\frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1}\right)\right] = e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}\left[\frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_1}{\rho_2}\right)\right] \tag{2.65}$$

By simplification

$$e_2 - e_1 = \frac{(p_1+p_2)}{2}\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) \tag{2.66}$$

Or

$$e_2 - e_1 = \frac{p_1+p_2}{2}(v_1 - v_2) \tag{2.67}$$

The equation (2.67) is called Hugoniot's equation.

Now, $v = 1/\rho$ and by (2.63)

$$u_1^2 = \frac{p_2-p_1}{1/v_2-1/v_1}\left(\frac{v_1}{v_2}\right) \tag{2.68}$$

By rearranging

$$\frac{p_2-p_1}{v_2-v_1} = -(\frac{u_1}{v_1})^2 \tag{2.69}$$

Now, (2.67) replace $e = c_v T$ and $T = pv/R$

$$\frac{p_2}{p_1} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\frac{v_1}{v_2}-1}{\left(\frac{\gamma+1}{\gamma-1}\right)-\frac{v_1}{v_2}} \tag{2.70}$$

2.7 Oblique Shock

The oblique shock occurs when there is or are obstacle(s) or deviation(s) in the path, then the shock turned into itself as shown in the figure 3. At point A the stream is deflected at an angle θ upward. Consequently the flow is also deflecting to same degree of bend and the shock wave will be bending towards the main stream. All the flow the downstream of the shock will be parallel to the downstream surface. Across the shock Mach number decreases, temperature, pressure increases.

On the other hand if the surface turns away from itself then it creates expansion fans. As shown in the figure 4. So the Mach number increases and pressure, temperature decreases.

Mach waves are weak pressure wave caused by the slight pressure change in compressible flow. Mach wave can combine together and produce shock wave. Such shock waves are called Mach stem. A Mach wave propagates across the flow with an angle which is called Mach angle which is the angle formed between Mach wave front and a vector that points opposite to the vector of motion. Mach angle μ can be calculated by

$$\mu = arcsin\frac{1}{M}$$

Where M is the Mach number.

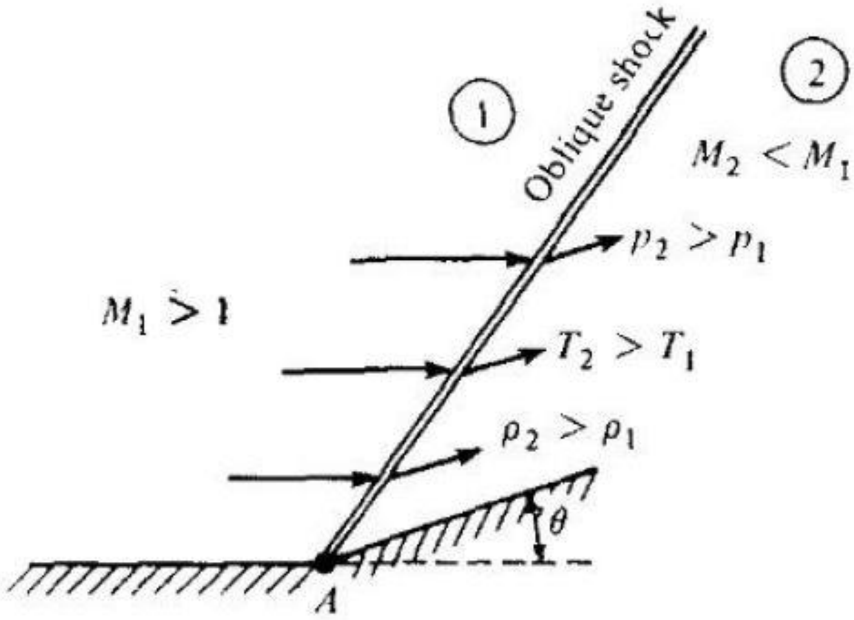

Figure 4a. Oblique shock

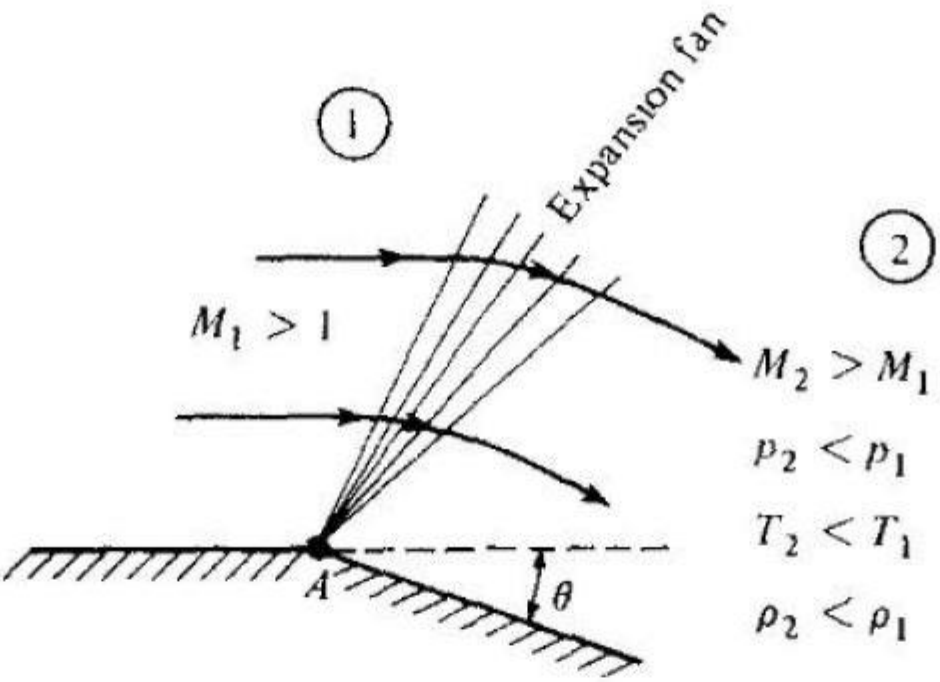

Figure 4b . Expansion fans

Figure 5. Mach Wave (courtesy Wikipedia.com)

2.8 Oblique Shock Wave Relationship

Consider a supersonic flow is hindered by a surface with an angle θ. So the shock is bending to the angle β and it is called wave angle. Now upstream to the flow the velocity is V_1 and corresponding Mach number is M_1. In the downstream direction the velocity and Mach number is V_2 and M_2 respectively. Now the normal component of upstream velocity, Mach number is u_1, M_{n1} respectively and tangential component of upstream velocity, Mach number is w_1, M_{t1} respectively. Along the downstream the normal component of velocity, Mach number is u_2, M_{n2} respectively tangential component of velocity, Mach number is w_2, M_{t2} respectively.

Let's consider two control surfaces opposite to the shock wave and calculate the different parameter. The details are as shown in figure 6. Let the area in both side is equal.

From continuity equation

$$\rho_1 u_1 = \rho_2 u_2 \tag{2.71}$$

Now from energy equation,

$$(-\rho_1 u_1) w_1 + (\rho_2 u_2) w_2 = 0 \tag{2.72}$$

Comparing the above two equation it is found that the tangential components are equal.

$$w_1 = w_2 \tag{2.73}$$

Now from Figure 6, balancing the normal components of the shock

$$(-\rho_1 u_1) u_1 + (\rho_2 u_2) u_2 = -(-p_1 + p_2)$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \tag{2.74}$$

Diving (2.75) by (2.72)

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \tag{2.76}$$

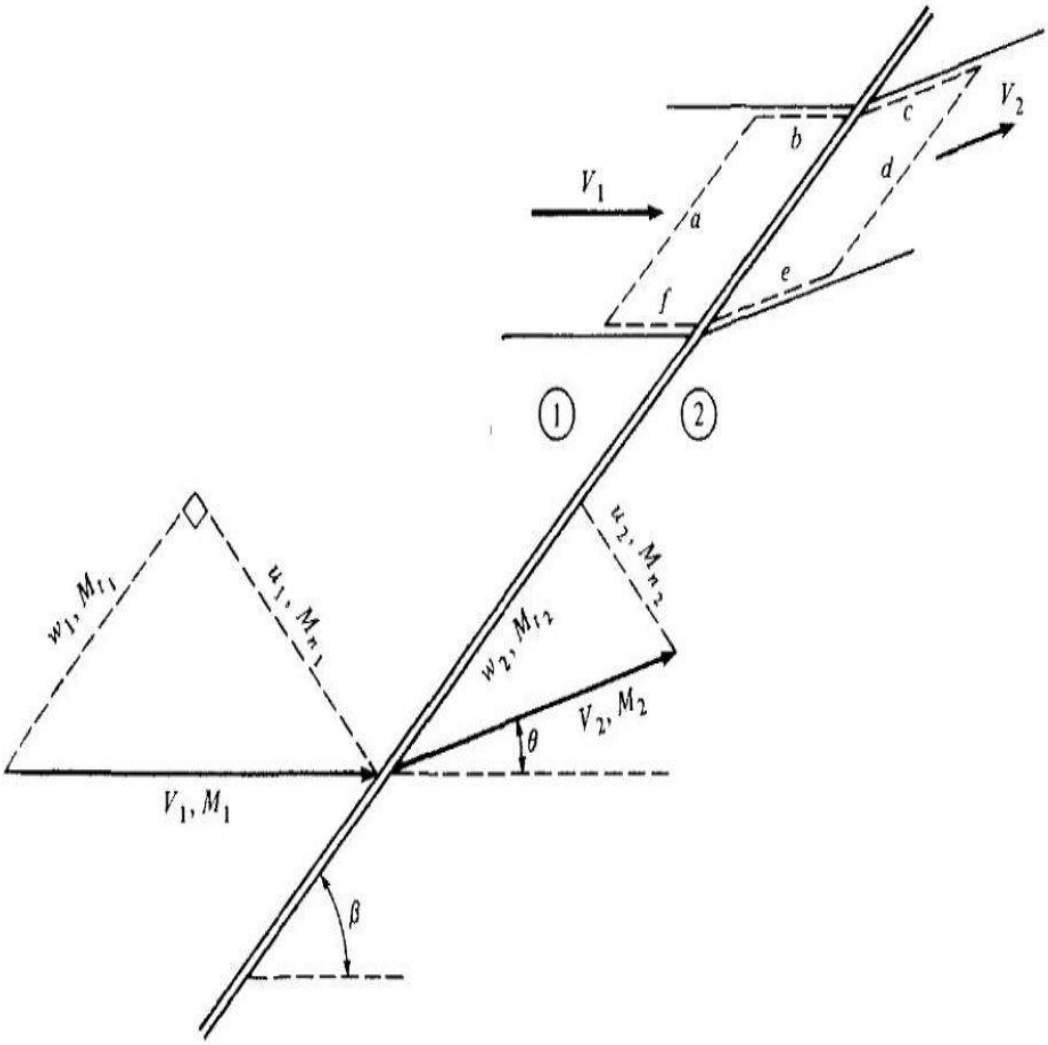

Figure 6. Oblique shock geometry

By geometry $V^2 = u^2 + w^2$ and $w_1 = w_2$

$V_1^2 - V_2^2 = (u_1^2 + w_1^2) - (u_2^2 + w_2^2) = u_1^2 - u_2^2$

Hence from (2.76)

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \tag{2.77}$$

Looking at the equations it is clear that the oblique shock wave equations are identical with normal shock wave and governed by the normal component of the flow.

$$M_{n1} = M_1 \sin\beta \tag{2.78}$$

For calorific perfect gas

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_{n1}^2}{(\gamma-1)M_{n1}^2+2} \tag{2.79}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_{n1}^2 - 1) \tag{2.80}$$

$$M_{n2}^2 = \frac{M_{n1}^2+[2/(\gamma-1)]}{[2\gamma/(\gamma-1)]M_{n1}^2-1} \tag{2.81}$$

And

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} \tag{2.82}$$

Behind the shock

$$M_2 = \frac{M_{n2}}{\sin(\beta-\theta)} \tag{2.83}$$

Now looking at the geometry

$$\tan\beta = \frac{u_1}{w_1} \tag{2.84}$$

$$\tan(\beta-\theta) = \frac{u_2}{w_2} \tag{2.85}$$

Combining (2.84), (2.85) and $w_1 = w_2$

$$\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{u_2}{u_1} \tag{2.86}$$

Combining (2.86) with equation (2.71), (2.78) and (2.79)

$$\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{2+(\gamma-1)M_1^2 sin^2\beta}{(\gamma+1)M_1^2 sin^2\beta} \tag{2.87}$$

By trigonometric manipulation

$$\tan\theta = 2\cot\theta\left[\frac{M_1^2 sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}\right] \tag{2.88}$$

The above equation is called the famous $\theta-\beta-M$ relation.

a. For any given M_1 , there is a corresponding θ_{max}. If geometry is such that $\theta > \theta_{max}$, then there will be a detached shock wave away from the distractor as shown in the figure 8.

b. If $\theta < \theta_{max}$ then there will be two values . For the larger value of , the shock is called strong shock and for the smaller one is weak shock. But the nature selects such a way that the weak shock will be exist and it is shown in the figure 9.

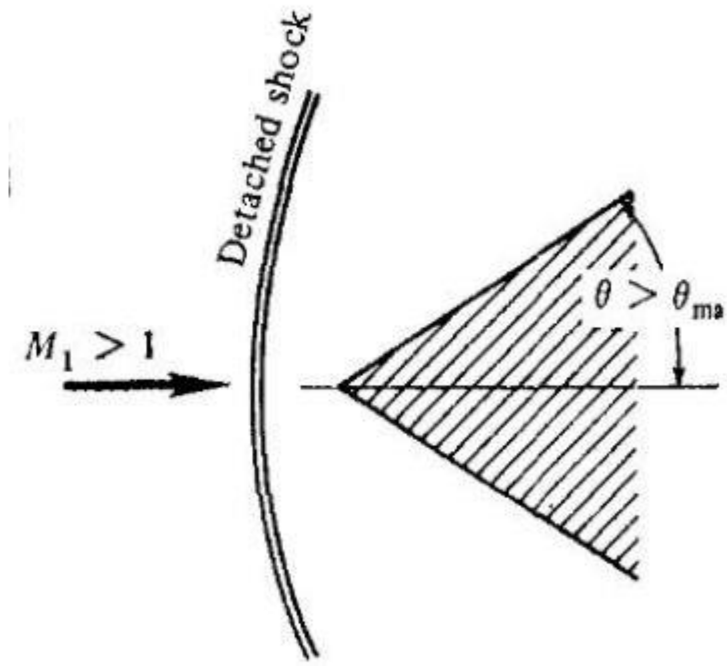

Figure 7. Detached shock for $\theta > \theta_{max}$

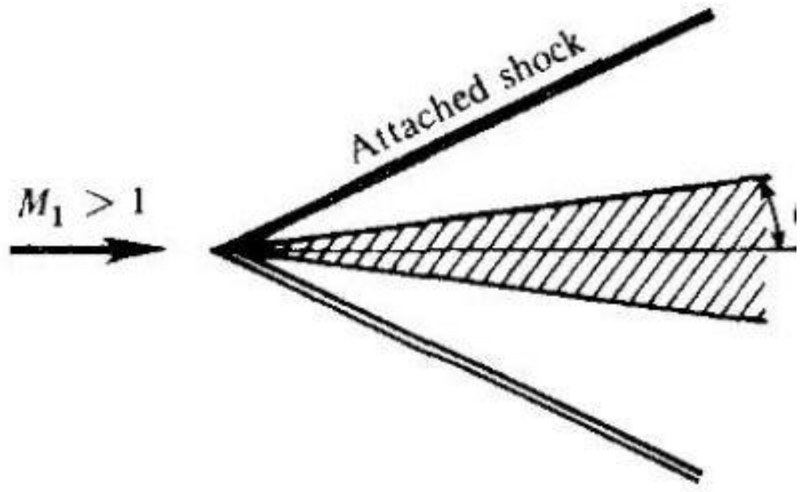

Figure 8. Oblique shock $\theta<\theta_{max}$

c. For θ=0 , there will be normal shock

2.9 Unsteady Wave Motion

So far it is considered that the shock wave is stationary. If the shock speed gas behind the moves two different opposite with same speed then the shock appears to be stationary, but if the gas behind the shock is stationary then the shock moves as demonstrated in the Figure 9.

Let us consider a shock tube, in which two different gases are enclosed in pressure and separated by a diaphragm as shown in the fig.10.

Section 4 i.e. the left handed portion of the diaphragm which is kept at higher pressure which is called the driver section and the right portion (section 1)of the diaphragm is kept at lower pressure and called the driven section.

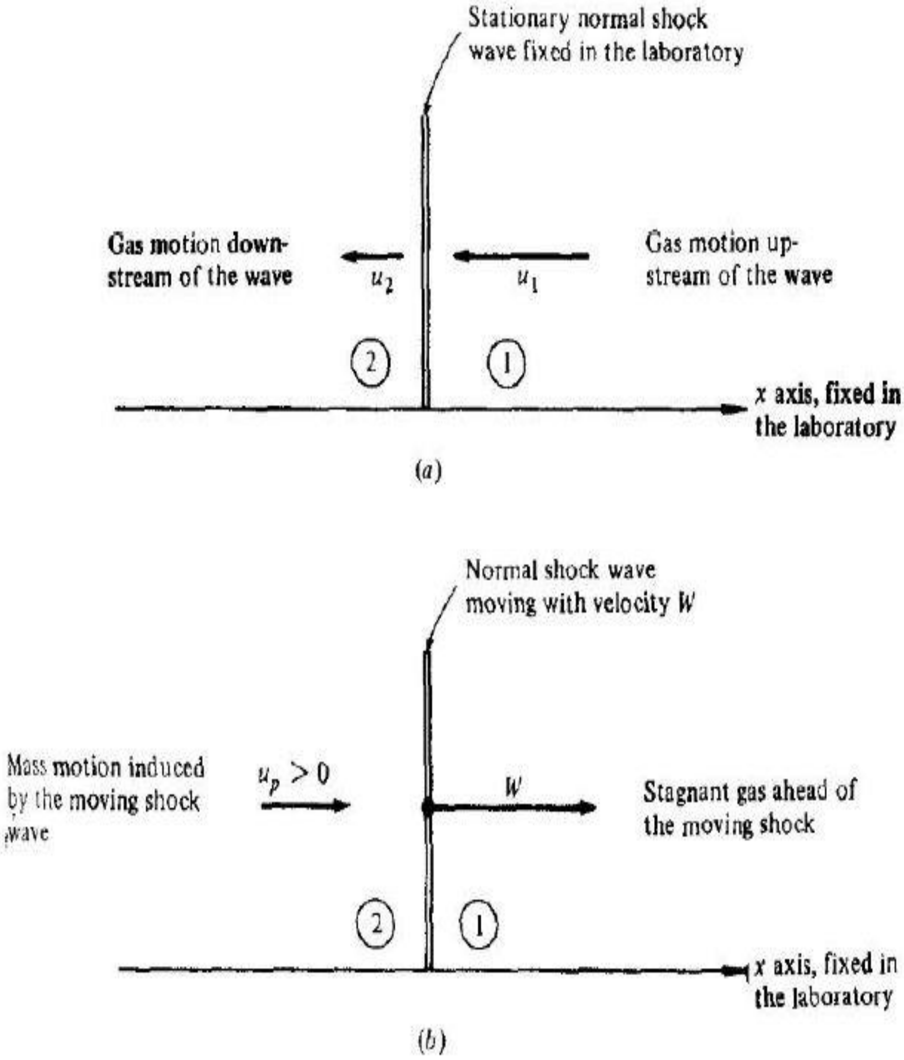

Figure 9. Schematic diagram of (a) stationary shock, (b) moving shock

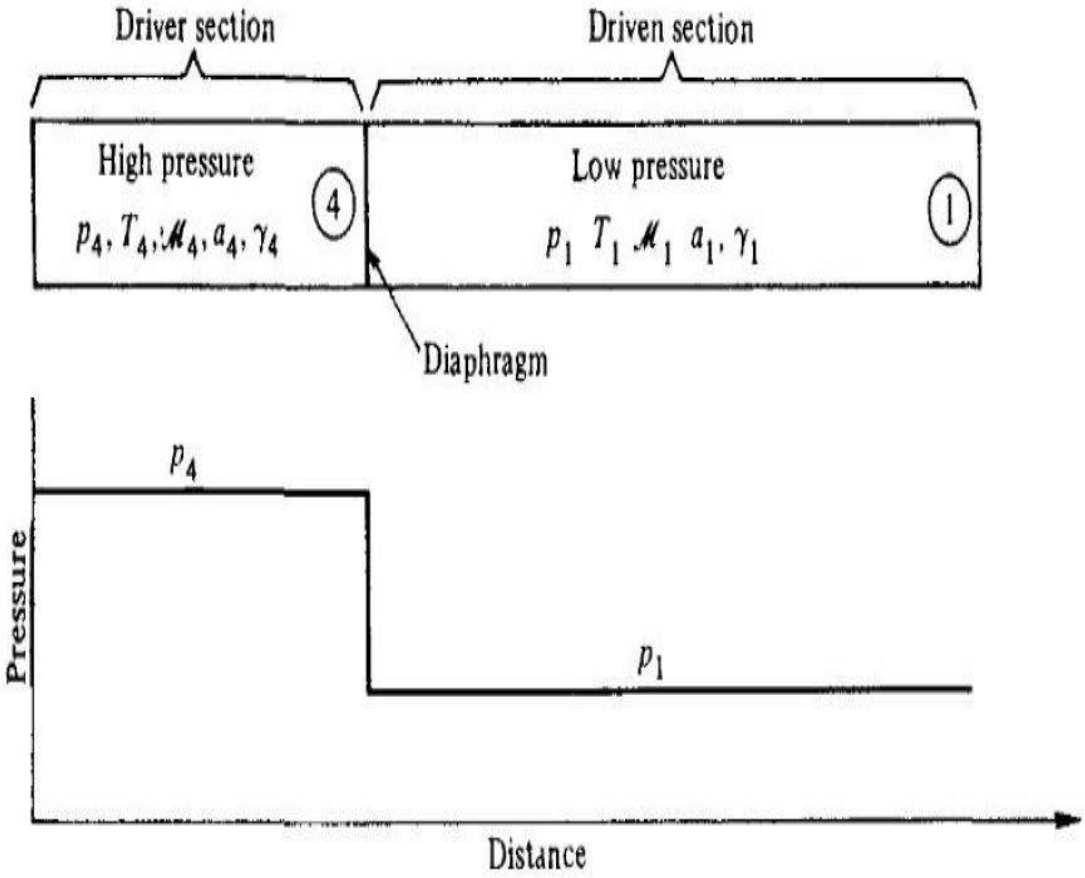

Figure 10. Shock tube at the initial condition

Now if the diaphragm is broken by any mean then the higher pressure gas will create a shock wave which will travel on the lower pressure zone with velocity w and the gas behind it will travel in the same direction with velocity u_p and expansion wave propagates towards the opposite direction. Hence the properties will be not only depending upon the location it will also depend on time ($T(x,t), \rho(x,t), u(x,t)$).
Let's say after sometime the shock wave moves to a different location as shown in the Figure 11, then $u_p = u_2 = u_3$.
Shock tube phenomena are very important to demonstrate the shock property at very high temperature and speed.

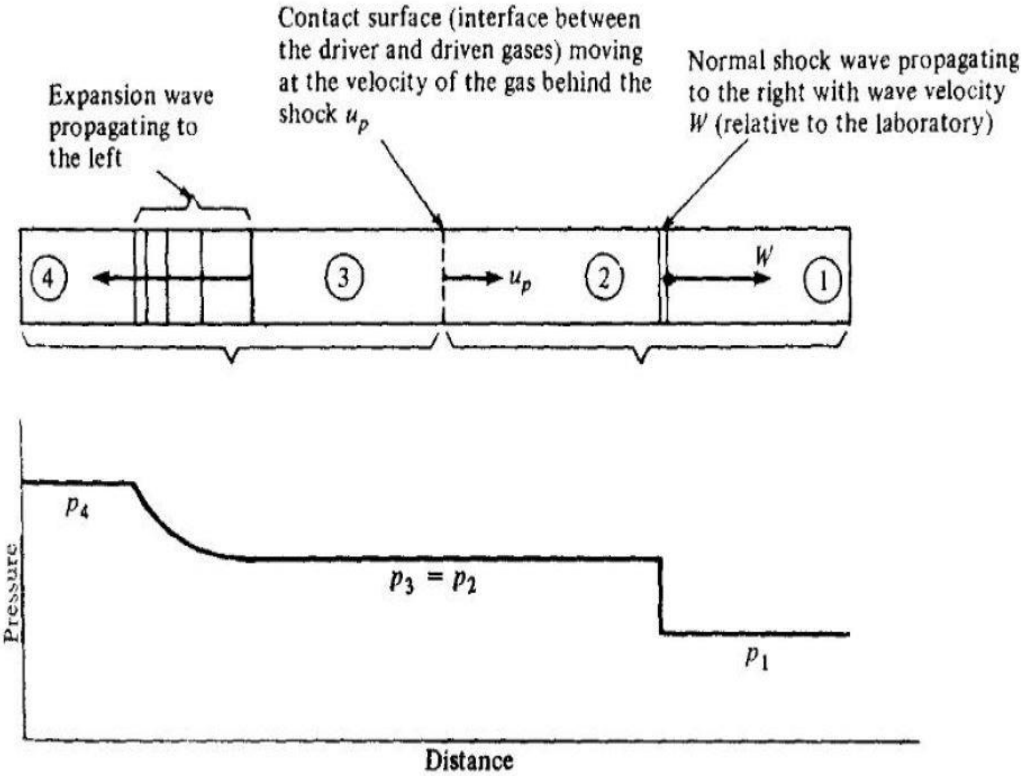

Figure 11. Shock tube wave propagation after the diaphragm is broken

2.10 Moving Normal Shock

Now let's manipulate the Normal shock wave equations for standing waves equation for continuity, momentum and energy, equation number (2.37), (2.38) and (2.39).

Let's look at the figure 10a

u_1 = Velocity of the gas ahead of the shock wave, relative to the wave

u_2 = Velocity of the gas behind the shock wave, relative to the wave

Now looking at the Figure 10b

W = Velocity of the gas ahead of the shock wave, relative to the wave

$W - u_p$ = Velocity of gas behind the shock wave, relative to the wave

Hence

$$\rho_1 W = \rho_2 (W - u_p) \tag{2.89}$$

$$p_1 + \rho_1 W^2 = p_2 + \rho_2 (W - u_p)^2 \tag{2.90}$$

$$h_1 + \frac{W^2}{2} = h_2 + \frac{(W-u_p)^2}{2} \tag{2.91}$$

Now, rearrange (2.89)

$$W - u_p = W\frac{\rho_1}{\rho_2} \tag{2.92}$$

From (2.90) and (2.89)

$$p_1 + \rho_1 W^2 = p_2 + \rho_2 W^2 (\frac{\rho_1}{\rho_2})^2 \tag{2.93}$$

By arranging

$$p_2 - p_1 = \rho_1 W^2 \left(1 - \frac{\rho_1}{\rho_2}\right)$$

$$W^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1}\right) \tag{2.94}$$

From (2.89)

$$W = (W - u_p)\frac{\rho_2}{\rho_1} \tag{2.95}$$

From (2.94) and (2.95)

$$(W - u_p)^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_1}{\rho_2}\right) \tag{2.96}$$

From equation (2.94), (2.96) and (2.91) and $h = e + p/\rho$

$$e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}\left[\frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1}\right)\right] = e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}\left[\frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_1}{\rho_2}\right)\right] \tag{2.97}$$

By simplifying (2.97)

$$e_2 - e_1 = \frac{p_1 + p_2}{2}\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$$

$$e_2 - e_1 = \frac{p_1 + p_2}{2}(v_1 - v_2) \tag{2.97a}$$

The above equation is Hugoniot equation.

If $e = c_v T$ and $v = RT/p$ and by equation (2.97a)

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}\left(\frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{1 + \frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}}\right) \tag{2.98}$$

And

$$\frac{\rho_2}{\rho_1} = \frac{1+\frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}{\frac{\gamma+1}{\gamma-1}+\left(\frac{p_2}{p_1}\right)} \tag{2.99}$$

Hence it can be concluded that unlike normal shock wave unsteady shock wave properties are the function of pressure ration.

Consider that the Mach number of the shock is M_s

$$M_s = \frac{W}{a_1}$$

Where a_1 is the sonic speed at driver section.

From (2.57)

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_s^2 - 1) \tag{2.100}$$

From (2.100)

$$M_s = \sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1} \tag{2.101}$$

$$W = a_1\sqrt{\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} - 1\right) + 1} \tag{2.102}$$

From equation (2.89)

$$u_p = W\left(1 - \frac{\rho_1}{\rho_2}\right) \tag{2.103}$$

From (2.98) , (2.102) and (2.103)

$$u_p = \frac{a_1}{\gamma}\left(\frac{p_2}{p_1} - 1\right)\sqrt{\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}} \tag{2.104}$$

Now , if the sonic speed at driven section is a_2, then

$$\frac{u_p}{a_2} = \frac{u_p}{a_1}\frac{a_1}{a_2} = \frac{u_p}{a_1}\sqrt{\frac{T_1}{T_2}} \tag{2.105}$$

From (2.99), (2.104) and (2.105)

$$\frac{u_p}{a_2} = \frac{1}{\gamma}\left(\frac{p_2}{p_1}\right)\sqrt{\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}} \frac{1+\frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)}{\frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1}\right)+\left(\frac{p_2}{p_1}\right)^2}} \tag{2.106}$$

In the unsteady shock wave unlike the steady flow the total enthalpy in both side of shock is note equal.

2.11 Reflected Shock Wave

Let a shock wave travels with a speed W and is about to incident on the endwall. Now mass motion behind the shock wave $u_1 = 0$ and behind the shock wave the mass motion is u_p towards the wall. After the shock incident on the wall, it creates a reflected shock with speed W_R(relative to the laboratory) and behind the reflected shock wave the speed $u_5 = 0$. The detail is shown in the Figure 13.

For convenience a $x - t$ diagram is drawn. As shown in the figure 14. At time $t = 0$ the shock is at the diaphragm. At $t = t_1$ the shock moves towards the right direction and the location is $x = x_1$, then at $x = x_2$ and time $t = t_2$the shock reflects with a velocity W_R , now at time $t = t_3$ the shock is at $x = x_3$.

The slope incident and reflected shock wave is $1/W$ and $1/W_R$ respectively, where $W_R < W$.

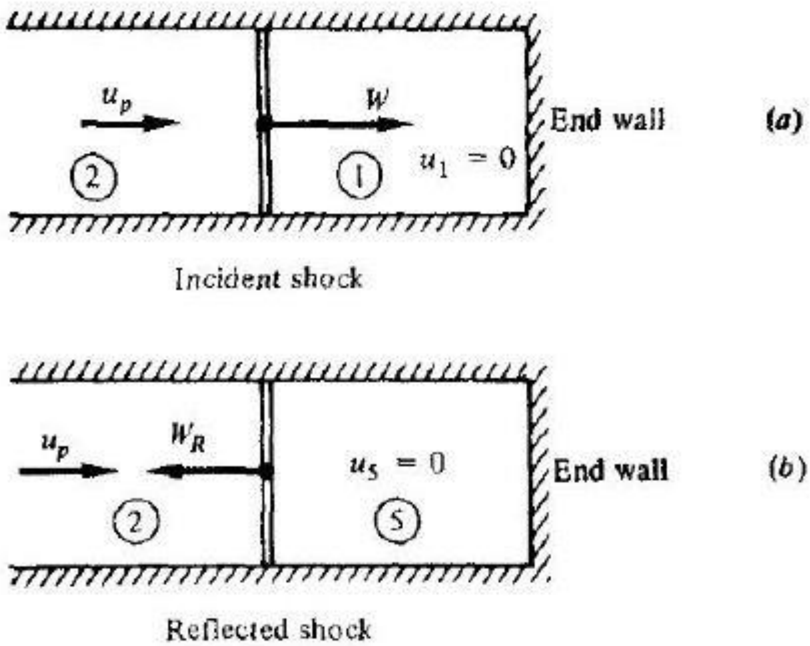

Figure 12. Incident and reflected shock wave

Now from fig 13(b)

$W_R + u_p =$ Velocity of the gas ahead of the shock relative to the wave

$W_R =$ Velocity of the gas behind the shock wave relative to the wave

Now from (2.37), (2.38) and (2.39)

$$\rho_2(W_R + u_p) = \rho_5 W_R \qquad (2.107)$$

$$p_2 + \rho_2(W_R + u_p)^2 = p_5 + \rho_5 W_R^2 \qquad (2.108)$$

$$h_2 + \frac{(W_R+u_p)^2}{2} = h_5 + \frac{W_R^2}{2} \qquad (2.109)$$

The Mach number of incident shock $M_s = W/a_1$ and Mach number of reflected shock wave$M_R = (W_R + u_p)/a_2$. From (2.89) through (2.91) and from equation (2.107) through (2.109) for a calorific perfect gas

$$\frac{M_R}{M_R^2-1} = \frac{M_S}{M_S^2-1}\sqrt{1 + \frac{2(\gamma-1)}{(\gamma+1)^2}(M_S^2 - 1)\left(\gamma + \frac{1}{M_S^2}\right)} \qquad (2.110)$$

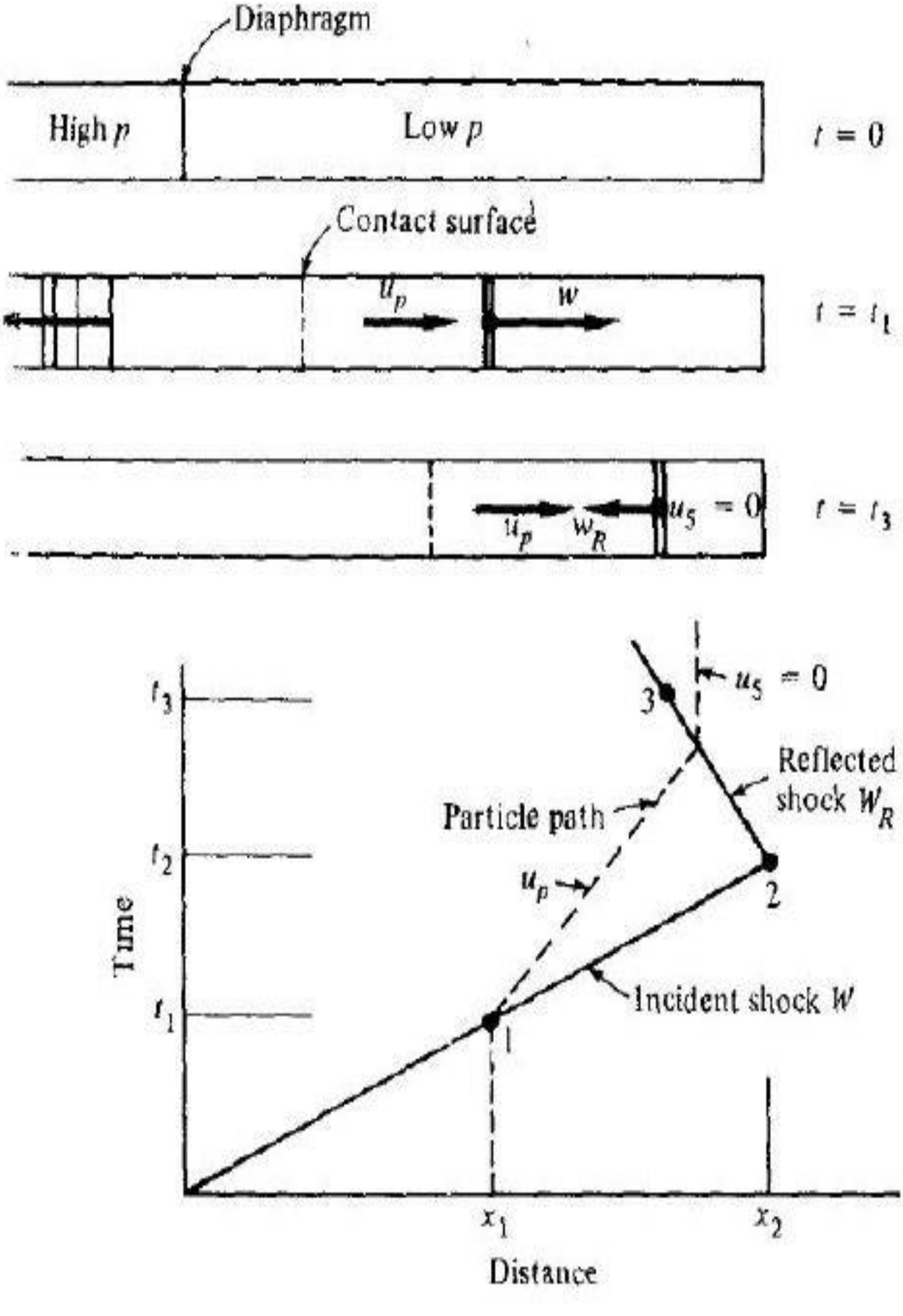

Figure 13. Wave diagram

CHAPTER 3: THEORETICAL DEVELOPMENT

3.1 Geometry

The authors had considered the blast as an isothermal sphere with radius R_0 and distance along the horizontal axis as x and vertical axis as y. Here they took the non-dimensional length.

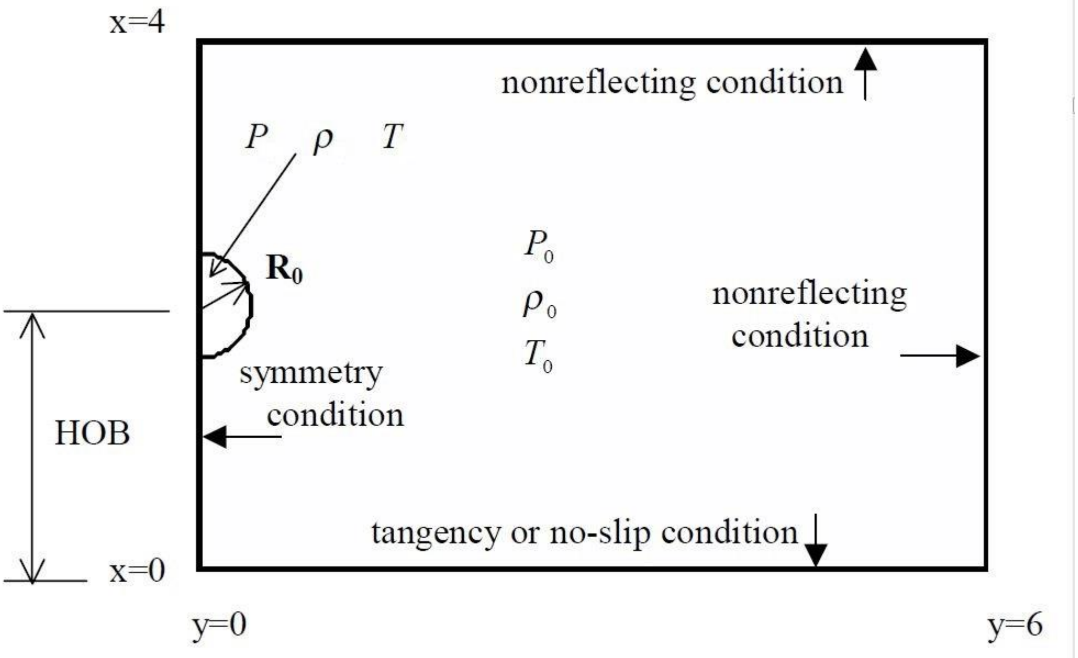

Figure 14: Computational domain and the initial and Boundary condition

HOB= Height of blast

P = Pressure inside the blast

ρ = Density inside the blast

T = Temperature of blast

P_0= Ambient pressure

ρ_0= Ambient density

T_0=Ambient temperature

The non-dimensional length along x axis from 0 to 4 while along the y- axis is taken as 0 to 6.

Now in this study x axis is 0 to 40 meter, y axis is 0 to 60 meter. The blast radius is considered as 3 meter and the focus the pressure ratio at 9 meter.

3.2 Taylor's Blast Wave Theory

G.I. Taylor considered that the blast radius to a particular time is depends upon on the total energy of the blast, density of the atmosphere and time.

The dimension of energy is ML^2/T^2, density is M/L^3 and time is T.

With the dimensional analysis time dependent radius becomes

$R(t) = S(\gamma)(E/\rho_0)^{1/5} t^{2/5}$

Where S (γ) is a constant and γ is the ratio of the specific heat.

The propagation and decay of a spherical blast in air has been studied for the case when the maximum pressure does not crosses 2 atm. The pressure far away from the center of the blast decays as in a sound wave inversely proportional to R but at the center the pressure decays faster than the R^{-1}. When the excess pressure is 0.5 atm, Taylor claimed that a logarithmic plot shows that it varies as $R^{-1.9}$, when the excess pressure is 1.5 atm, the decay is proportional to $R^{-2.8}$. The intensity of the blast at the center is very difficult because of initial shock wave raises entropy along the traverse direction by an amount which depends on the intensity of the shock wave. So the passage of the shock leaves the air in a state in which the entropy decays radially so that after the passage, when the air has returned to atmospheric pressure, the air temperature decreases with increasing distance from the site of the explosion. For this reason the density is not a single valued function of pressure in a blast wave. After passage of the blast wave, the relationship between pressure and density for any given particle of air is simply the adiabatic one corresponding with the entropy with which that particle was endowed by the shock wave during its passage past it. For this reason it is general necessary to use a form of analysis in which the initial position of each particle is retained as one of the variables. This introduces a great complexity and , in general, solutions can only be derived by using step by step method of numerical integration. But great simplicity is introduced in the spherical detonation problem, by assuming that the disturbance is similar at all time , merely increasing its linear dimensions with increasing time from initiation, gives encouragement to an attempt to apply similar principles to the blast produced by a very intense explosion in a very small volume.

Taylor considered the appropriate similarities for an expanding blast wave of constant total energy are

Pressure, $p/p_0 = y = R^{-3}f_1$ (3.1)

Density, $\rho/\rho_0 = \psi$ (3.2)

Radial velocity, $u = R^{-3/2}\phi_1$ (3.3)

Where R is the radius of the shock wave forming the outer edge of the disturbance, p_0 and ρ_0 are the pressure and density of the undisturbed atmosphere. If r is the radial co-ordinate, η=r/R and f_1, ϕ_1 and ψ are functions ofη. It is found that these assumptions are consistent with the equations of motion and continuity and with the equation of state of a perfect gas.

The equation of motion is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} = -\frac{p_0}{p}\frac{\partial y}{\partial r} \tag{3.4}$$

Substituting from (3.1), (3.2) and (3.3) in (3.4) and writing, f_1', ϕ_1' for $\frac{\partial}{\partial\eta} f_1, \frac{\partial}{\partial\eta}\phi_1$

$$-\left(\frac{3}{2}\phi_1 + \eta\phi_1'\right) R^{-\frac{5}{2}}\frac{dR}{dt} + R^{-4}\left(\phi_1\phi_1' + \frac{p_0}{\rho_0}\frac{f_1'}{\psi}\right) = 0 \tag{3.5}$$

This can be satisfied if

$$\frac{dR}{dt} = AR^{-\frac{3}{2}} \tag{3.6}$$

Where A is a constant, and

$$-A\left(\frac{3}{2}\phi_1 + \eta\phi_1'\right) + \phi_1\phi_1' + \frac{p_0}{\rho_0}\frac{f_1'}{\psi} = 0 \tag{3.7}$$

The continuity equation is

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\left(\frac{\partial u}{\partial r} + \frac{2u}{r}\right) = 0 \tag{3.8}$$

Substituting from (3.1), (3.2), (3.3) and (3.6), (3.8) becomes

$$-A\eta\psi' + \psi'\phi_1 + \psi\left(\phi_1' + \frac{2}{\eta}\phi_1\right) = 0 \tag{3.9}$$

The equation of state for a perfect gas is

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)(p\rho^{-\gamma}) = 0 \tag{3.10}$$

Where γ is the ratio of specific heats.

Substituting from (1), (2), (3) and (6), (10) becomes

$$A(3f_1 + \eta f_1') + \frac{rf_1}{\psi}\psi'(-A\eta + \phi_1) - \phi_1 f_1' = 0 \tag{3.11}$$

The equation (3.7), (3.9) and (3.11) may be reduced to a non-dimensional form by substituting

$$f = f_1 a^2/A^2 \tag{3.12}$$

$$\phi = \phi_1/A \tag{3.13}$$

Where a is the velocity of sound in air so that$a^2 = \gamma p_0/\rho_0$. The resulting equations only one parameter, namely, γ, are

$$\phi'(\eta - \phi) = \frac{1}{\eta}\frac{f'}{\psi} - \frac{3}{2}\phi \tag{3.7a}$$

$$\frac{\psi'}{\psi} = \frac{\phi' + 2\phi/\eta}{\eta - \phi} \tag{3.9}$$

$$3f + \eta f' + \frac{\gamma\psi'}{\psi} f(-\eta + \phi) - \phi f' = 0 \tag{3.11a}$$

Eliminating ψ' from (3.11a) by means of (3.7a) and (3.9a) the equation for calculating f' when f, ϕ, ψ and η are given is

$$f'\{\left(\eta - \phi\right)^2 - \frac{f}{\psi}\} = f\{-3\eta + \phi\left(3 + \frac{1}{2}\gamma\right) - \frac{2\gamma\phi^2}{\eta}\} \tag{3.14}$$

When f' has been found from (14), ϕ' can be calculated from (7a) and hence ψ' from (3.9a). Thus if for any value of η, f, ϕ and ψ are known their values can be computed step-by-step for other values of η

3.3 Shock-Wave Conditions

The conditions at the shock wave $\eta = 1$are given by Rankine-Hugoniot relations which may be reduced to the form

$$\frac{\rho_1}{\rho_0} = \frac{\gamma - 1 + (\gamma + 1)y_1}{\gamma + 1 + (\gamma - 1)y_1} \tag{3.15}$$

$$\frac{U^2}{a^2} = \frac{1}{2\gamma}\{\gamma - 1 + (\gamma + 1)y_1\} \tag{3.16}$$

$$\frac{u_1}{U} = \frac{2(y_1 - 1)}{\gamma - 1 + (\gamma + 1)y_1} \tag{3.17}$$

Where ρ_1, u_1 and y_1 represent the value of ρ, u and y immediately behind the shock wave and $U = dR/dt$ is the radial velocity of the shock wave.

The conditions cannot be satisfied consistently with the similarity assumptions represented by (3.1), (3.2) and (3.3). On the other hand, when y_1 is large so that the pressure is high compared with atmospheric pressure, (3.15), (3.16) and (3.17) assume the approximate asymptotic forms

$$\frac{\rho_1}{\rho_0} = \frac{\gamma + 1}{\gamma - 1} \tag{3.15a}$$

$$\frac{U^2}{a^2} = \frac{2\gamma}{\gamma + 1} y_1 \tag{3.16a}$$

$$\frac{u_1}{U} = \frac{2}{\gamma+1} \quad (3.17a)$$

These approximate boundary conditions are consistent with (3.1), (3.2), (3.3) and (3.6); in fact (3.15a) yields, for the conditions at $\eta = 1$

$$\psi = \frac{\gamma+1}{\gamma-1} \quad (3.15b)$$

(3.16a) yields

$$f = \frac{2\gamma}{\gamma+1} \quad (3.16b)$$

and (3.17a) yields

$$\phi = \frac{2}{\gamma+1} \quad (3.17b)$$

3.4 Energy

The total energy E of the disturbance may be regarded as consisting of two parts, the kinetic energy

$$K.E. = 4\pi \int_0^R \frac{1}{2}\rho u^2 r^2 dr$$

and the heat energy

$$H.E. = 4\pi \int_0^R \frac{pr^2}{\gamma-1} dr$$

In terms of the variables f, ϕ, ψ and η

$$E = 4\pi A^2 \{\frac{1}{2}\rho_0 \int_0^1 \psi^2\phi^2\eta^2 d\eta + (\frac{p_0}{a^2(\gamma-1)} \int_0^1 f\eta^2 d\eta\}$$

Or since $p_0 = a^2\rho_0/\gamma$, $E = B\rho_0 A^2$, where B is a function of γ only

whose value is

$$B = 2\pi \int_0^1 \psi\phi^2\eta^2 d\eta + \frac{4\pi}{\gamma(\gamma-1)} \int_0^1 f\eta^2 d\eta \quad (3.18)$$

Since the two integrals in (3.18) are both functions of γ only it seems that for a given value of γ, A^2 is simply proportional to E/ρ_0

3.5 Initial Conditions

For this calculation the

$\frac{P_1}{P_2} = 70$

Atmospheric pressure is 1 atm

P_1= 7092750 N/m^2

Temperature inside the sphere T_1=2717 K

Ambient temperature T_2= 288.15 K

$\frac{\rho_1}{\rho_2} = 5$

Density of the environment ρ_2= 1.225 Kg/m^3

Density inside the blast ρ_1= 6.125 Kg/m^3

Non-dimensional radius of the sphere R_0=3 meter

Total mass of the sphere M= $\frac{4}{3}\pi R_0^3 \rho_1 = \frac{4}{3}\pi 3^3 X\ 6.125 = 692.72\ Kg$

Ratio of specific heat at temperature 2717K is determined by the equation can be found out by the following relationship which is available in NASA website

$$\gamma = 1 + \frac{(\gamma_{perf} - 1)}{1 + \left(\gamma_{perf} - 1\right)[(\frac{\theta}{T})^2 \frac{e^{\frac{\theta}{T}}}{(e^{\frac{\theta}{T}} - 1)^2}]}$$

γ_{perf}=1.4

θ =5500° Rankine

T= 2717 K= 4890.6 Rankine

Hence we get γ =1.2929

Energy per unit mass $h = \frac{P\gamma}{\rho(1+\gamma)} = \frac{7092750 X\ 6.125}{6.125(1+1.2929)}$= 652962.711 Joule/kg

Total energy inside the sphere E = M X h

= 652962.7 X 692.72

= 452320329.164 Joule

3.5 Boundary Conditions

The boundary conditions are found by finding f, ϕ and ψ in $\eta = 1$

$$f = \frac{2\gamma}{\gamma+1} = \frac{2\ X\ 1.2929}{1.2929+1} = 1.1277$$

$$\phi = \frac{2}{\gamma+1} = 0.87$$

$$\psi = \frac{\gamma+1}{\gamma-1} = 7.828$$

Then the values of the f, ρ and ψ are determined numerically. These values are used to determine the values of the pressure, density and velocity ratio.

CHAPTER 4: DEVELOPING WORKING EQUATION

4.1 Numerical Method

When γ=1.2929 the boundary values of f, ϕ and ψ at η=1 are (3.15a), (3.16a), (3.17a), 1.1277, 0.87 and 7.828. Values of f, ϕ and ψ were calculated from η=1 to $\eta = 0.5$, using intervals in 0.02 in η. Starting each step with values of $f', \phi', \psi', f, \phi$ and ψ found in previous steps, values of f', ϕ' and ψ' at the end of the interval predicted by assuming that the previous two values form a geometrical progression with the predicted one; thus the (s+1)th term, f'_{s+1} in a series of f' was taken as $\frac{1}{2}(f'_s)^2/f'_{s-1}$. With this assumed value the mean f' is the sth interval was taken as $\frac{1}{2}(f'_{s+1} + f'_s)$ and the increment in f was taken as $(0.02)(\frac{1}{2})(f'_{s+1} + f'_s)$. The values of $f'_{s+1}, \phi'_{s+1}$and ψ'_{s+1}were then calculated from the formula (3.14), (3.7a) and (3.9a). If that is differed appreciably from the predicted values of f'_{s+1} by this new calculated value. In the early stages of calculation near η=1 two or three approximations were made, but in the later stages the estimated value was so close to the calculated one that the value of f' calculated in this first approximation was used directly in the next stage.

The results are tabulated in the table1 and are shown in the figure. These curves and the tables show three striking feature: (a) the ϕ curve rapidly settles down to a curve which is very nearly a straight line through the origin, (b) the density curve ψ rapidly approaches the axis ψ=0, (c) the pressure becomes practically constant. These facts suggest that the solution tends to a limiting as η decreases in which ϕ=cη,

ϕ'=c=constant, f, f', ψ and ψ' become small. Substituting for $\frac{1}{\gamma}\frac{f'}{\psi}$ from (3.7a), (3.14) becomes

$$\frac{f'}{f}(\eta-\phi)^2 = \gamma\phi' + \frac{3}{2}\gamma\phi - 3\eta + \left(3+\frac{1}{2}\gamma\right)\phi - \frac{2\gamma\phi^2}{\eta} \tag{4.1}$$

Table 1: Step by step calculation for γ = 1.2929

Serial No	**Location**	**f**	$\boldsymbol{\phi}$	**ψ**
1	1	1.1277	0.872	7.828
2	0.98	0.966	0.8355	5.267
3	0.96	0.8399	0.819	3.5344
4	0.94	0.7571	0.7895	2.3487
5	0.92	0.7	0.7628	1.4862
6	0.9	0.6592	0.7384	1.0128
7	0.88	0.6295	0.7159	0.7792
8	0.86	0.6074	0.695	0.5931
9	0.84	0.5908	0.675	0.487
10	0.82	0.5783	0.656	0.305
11	0.8	0.5689	0.638	0.251
12	0.78	0.5617	0.62	0.183
13	0.76	0.5562	0.6	0.141
14	0.74	0.5521	0.586	0.101
15	0.72	0.5489	0.5699	0.075
16	0.7	0.5466	0.5536	0.056
17	0.68	0.5448	0.5374	0.043
18	0.66	0.5435	0.5214	0.0325
19	0.64	0.5426	0.505	0.0245
20	0.62	0.5419	0.489	0.0166
21	0.6	0.5414	0.4735	0.0127
22	0.58	0.541	0.4577	0.0083
23	0.56	0.5408	0.4418	0.0058
24	0.54	0.5406	0.426	0.004

25	0.52	0.5406	0.41	0.0027
26	0.5	0.5404	0.3944	0.0018

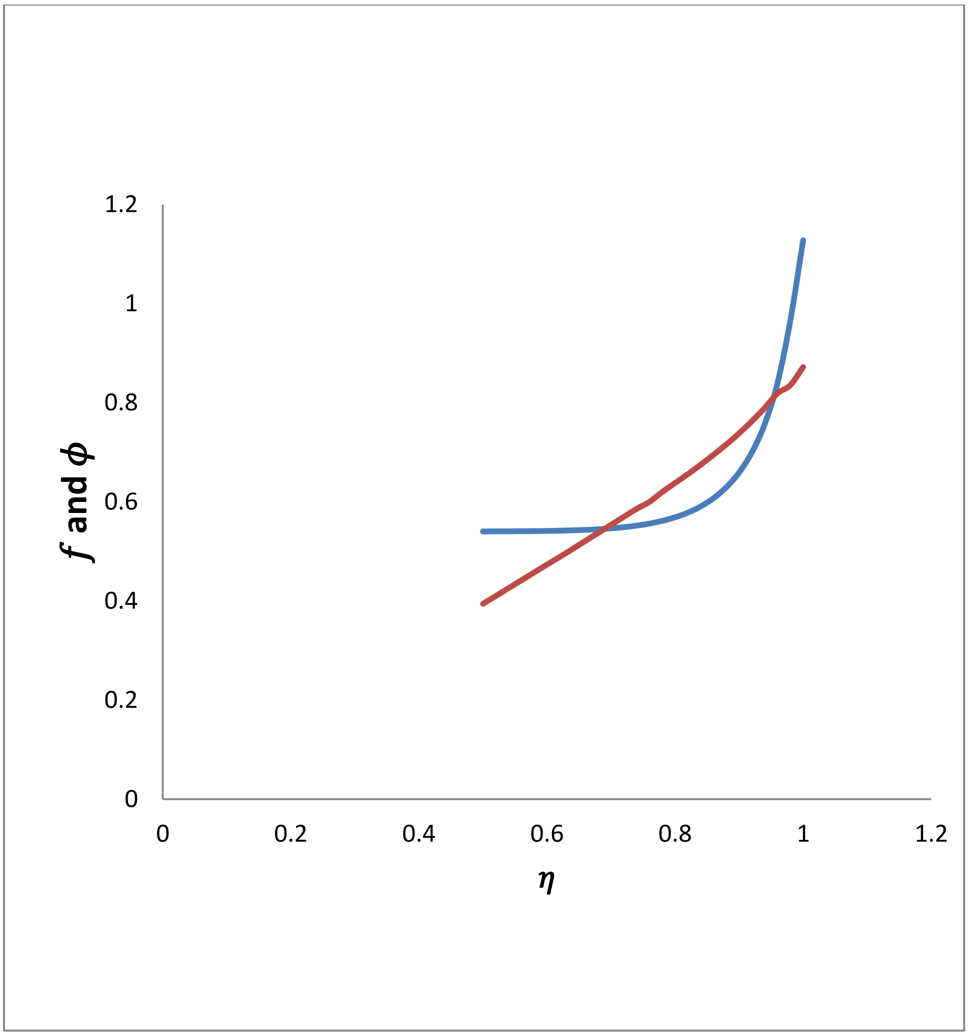

Figure15. The blue line is for different values of f and the red line is Different values of ϕ. Blue line is for f and the redline is ϕ

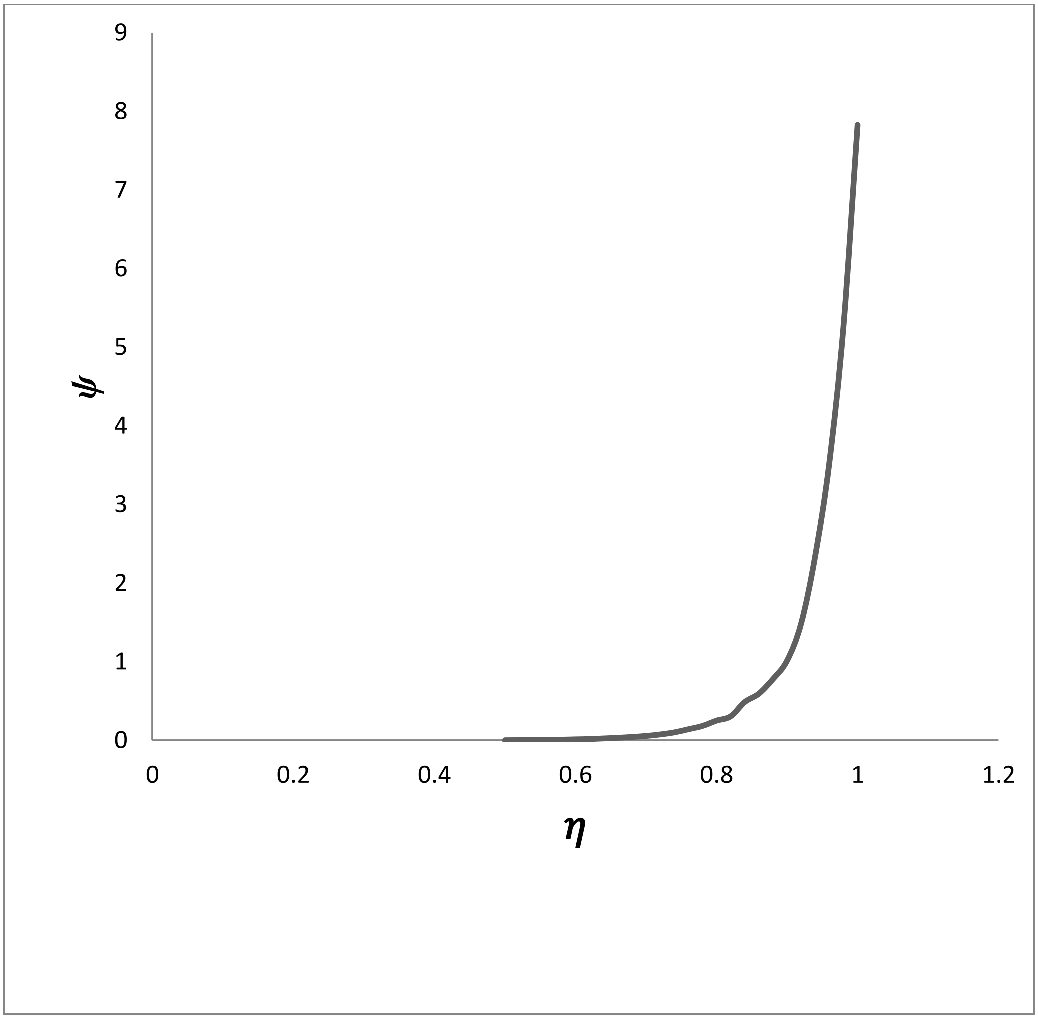

Figure 16. The blue line is for different values of ψ

Diving by (η-ϕ) (19) becomes

$$\frac{f'}{f}(\eta-\phi)=\gamma\phi'-3+\frac{2\gamma\phi}{\eta} \tag{4.2}$$

If the left-hand side which contains f'/f be neglected the approximate solution of (4.2) which ϕ vanishes at η=0 is

$$\phi=\eta/\phi \tag{4.3}$$

The line $\phi=\eta/\gamma$ is shown in figure above. It will be seen that the points are calculated by the step by step method nearly run into this line. The difference appears to be due to the accumulation of errors in calculation.

3.2 Approximate Formulae

The fact that the ϕ curve seems to leave the straight line $\phi = \eta/\gamma$ rather rapidly remaining close to it over the range η=0 to η=0.5 suggests that an approximate set of formula might be assuming

$$\phi = \frac{\eta}{\gamma} + \alpha\eta^n \quad (4.4)$$

Where n is a positive number which may be expected to be more than, say, 3 or 4. If this formula applies at η=1. Then from equation (4.4) and (3.17b)

$$\frac{1}{\gamma} + \alpha = \frac{2}{\gamma+1}$$

$$\alpha = \frac{\gamma-1}{\gamma(\gamma+1)} \quad (4.5)$$

Now differentiate the equation with respect to η

$$\phi' = \frac{1}{\gamma} + n\alpha\eta^{n-1}$$

The above value insert in equation (4.2)

$$\frac{f'}{f} = \alpha\gamma(n+2)(\gamma+1)/(\gamma-1)$$

From (3.14) and (3.15b), (3.16b), (3.17b) the true value of f'/f at η=1 is

$$\frac{2\gamma^2+7\gamma-3}{\gamma-1}$$

Equating the above two equation are

$$n = \frac{7\gamma-1}{\gamma^2-1} \quad (4.6)$$

Now substitute the (4.4) in the equation (4.2), with new value of n and α

$$\frac{f'}{f} = \frac{(n+2)\alpha\gamma^2\eta^{n-2}}{\gamma-1-\gamma\alpha\eta^{n-1}} \quad (4.7)$$

Now integrating both side of the equation (4.7)

$$\log f = \log\frac{2\gamma}{\gamma+1} - \frac{2\gamma^2+7\gamma-3}{7-\gamma}\log\left(\frac{\gamma+1}{\gamma} - \frac{\eta^{n-1}}{\gamma}\right) \quad (4.8)$$

Now insert the value of ϕ and ϕ' in the equation number (3.9a) and the new value of ψ is

$$\log\psi = \log\frac{\gamma+1}{\gamma-1} - \int_{\eta}^{1}\frac{3+(n+2)\alpha\gamma\eta^{n-1}}{(\gamma-1)\eta-\alpha\gamma\eta^{n}}\,d\eta \tag{4.9}$$

Integrating this and substituting for α from (4.5)

$$\log\psi = \log\frac{\gamma+1}{\gamma-1} + \frac{3}{\gamma-1}\log\eta - 2\,\frac{(\gamma+5)}{7-\gamma}\log\left(\frac{\gamma+1-\eta^{n-1}}{\gamma}\right) \tag{4.10}$$

When η is small this formula gives

$$\psi = D\eta^{3/(\gamma-1)} \tag{4.11}$$

Where

$$\log D = \log\frac{\gamma+1}{\gamma-1} - 2\,\frac{(\gamma+5)}{7-\gamma}\log\left(\frac{\gamma+1}{\gamma}\right) \tag{4.12}$$

When γ=1.2929, using (4.12) gives D=2.2124 so that

$$D=2.2124\,\eta^{11.986} \tag{4.13}$$

3.3 Blast Wave Expressed in Terms of the Energy of the Explosion

In the equation (3.18) it can be seen that E/ρ_0A^2 is a function of γ only. Evaluating the integral in (3.18) for γ=1.2929, and using the values for calculation, it is found

$$\int_0^1 \eta^2\phi^2\psi d\eta = 0.113$$

And

$$\int_0^1 \eta^2 f d\eta = 0.2636$$

So the total kinetic energy is as follows

$$\text{K.E.}= 2\pi(0.113)\rho_0A^2 = 0.7096\rho_0\,A^2 \tag{4.14}$$

And the Heat energy is

$$\text{H.E.}= \frac{4\pi}{1.2929X0.2929}(.2636)\rho_0A^2 = 8.743\rho_0A^2 \tag{4.15}$$

Hence the total energy

$$\text{E}= 0.7096\rho_0\,A^2 + 8.743\rho_0A^2 = 9.4436\,\rho_0A^2 \tag{4.16}$$

Pressure

The pressure p at any point is found

$$P = P_0 R^{-3} f \frac{A^2}{a^2} = R^{-3} f \frac{\rho_0 A^2}{\gamma} = 0.1059\, R^{-3} E f \qquad (4.17)$$

Now the maximum pressure is at $f = 1.1277$ at r=R . Hence the maximum pressure is

$$p_{max} = 0.1194 R^{-3} E \qquad (4.18)$$

Velocity of air and shock wave

The velocity u of the gas at any point is

$$u = R^{-\frac{3}{2}} A \phi = R^{-\frac{3}{2}} E^{\frac{1}{2}} (B\rho_0)^{-\frac{1}{2}} \phi \qquad (4.19)$$

The velocity of radial expansion of the disturbance is, from (3.6),

$$\frac{dR}{dt} = A R^{-\frac{3}{2}} \qquad (4.20)$$

So that, if t is the time since the begging of the explosion,

$$t = \frac{2}{5} R^{-\frac{5}{2}} (B\rho_0)^{\frac{1}{2}} E^{-\frac{1}{2}} \qquad (4.21)$$

Now

$$E = B\rho_0 A^2 = 9.4436\, \rho_0 A^2 \qquad (4.20a)$$

So,

$$B = 9.4436 \qquad (4.20b)$$

Insert the value B in (4.21)

$$t = \frac{2}{5} R^{\frac{5}{2}} \rho_o^{\frac{1}{2}} (9.4436)^{\frac{1}{2}} E^{-\frac{1}{2}} = 1.22922\, R^{\frac{5}{2}} \rho_o^{\frac{1}{2}} E^{-\frac{1}{2}} \qquad (4.20c)$$

It can be seen that pressure ratio is only depended on $ER^{-3}$and it is not depended on the atmospheric densityρ_0. But the time scale is depended on $\rho_0^{\frac{1}{2}}$. So now to calculate the local pressure i.e a particle will subject feel the pressure due to the blast wave passed through a given location.

If t_0 is the time since initiation taken for the wave to reach radius R_o the pressure at time *t* at R_o is given by

$$\frac{p}{p_0} = \left(\frac{R_0}{R}\right)^3 \frac{f}{[f]_{\eta=1}} \tag{4.22}$$

Where p_1 is the pressure in the shock wave as it passed over radius R_0 at the time t_0 , R is the radius of the shock wave at time *t* and $\eta = R_0/R$. $[f]_{\eta=1}$ is the maximum value of f at γ=1.2929.

Now from equation (4.21) $\eta = (t/t_0)^{2/5}$

And

$$\frac{p}{p_1} = \frac{1}{[f]_{\eta=1}} \left(\frac{t_0}{t}\right)^{6/5} [f]_{(t_0/t)^{2/5}} \tag{4.23}$$

CHAPTER 5: RESULT AND DISCUSSION

In this section the results from Analytical process is discussed and compared with the numerical result. While calculating the pressure ratio at different time step, only the primary shock wave phenomena is considered. The secondary and tertiary shocks are neglected.

To calculate the pressure ratio equation (4.40) is used. Local f is calculated by equation (4.26). The calculated value of f by equation doesn't always match with the step by step numerical value. If the graph is noticed then it can be seen that the equation holds good at after the sharp downward.

Here are the different pressure ratio calculated at different instances by the and simultaneously the results from the numerical value is shown below.

Serial No	Time	Numerical value	Analytic value
1	0.15	1	0.025647
2	0.2	5.33	0.01816
3	0.25	4.5	0.013893
4	0.3	1.4	0.011163
5	0.35	1.1	0.009278
6	0.4	1	0.007904
7	0.45	0.9	0.006862
8	0.5	0.8	0.006047
9	0.55	0.7	0.005394
10	0.6	0.7	0.004859
11	0.625	1.2	0.004627
12	0.65	1	0.004414
13	0.7	0.9	0.004038
14	0.75	0.85	0.003718
15	0.8	0.8	0.003441
16	0.85	0.7875	0.003199

Serial No	Time	Numerical value	Analytic value
17	0.9	0.775	0.002987
18	0.95	0.7625	0.002799
19	1	0.75	0.002632
20	1.05	0.75	0.002483
21	1.1	0.75	0.002348
22	1.15	0.75	0.002226
23	1.2	0.8	0.002115
24	1.25	0.8	0.002014
25	1.3	0.9	0.001921
26	1.35	0.75	0.001836
27	1.4	0.7	0.001758
28	1.45	0.7	0.001685
29	1.5	0.7	0.001618
30	1.55	0.7	0.001556
31	1.6	0.7	0.001498
32	1.65	0.7	0.001443
33	1.7	0.7	0.001393
34	1.75	0.8	0.001345
35	1.8	0.8	0.0013
36	1.85	0.8	0.001258
37	1.9	0.8	0.001219
38	1.95	0.9	0.001181
39	2	0.9	0.001146
40	2.05	0.9	0.001112
41	2.1	0.9	0.001081
42	2.15	0.9	0.001051
43	2.2	0.9	0.001022
44	2.25	0.9	0.000995
45	2.3	0.9	0.000969
46	2.35	0.9	0.000944
47	2.4	1	0.000921
48	2.5	1	0.000877
49	3	1	0.000704
50	3.5	1	0.000585
51	4	1	0.000499

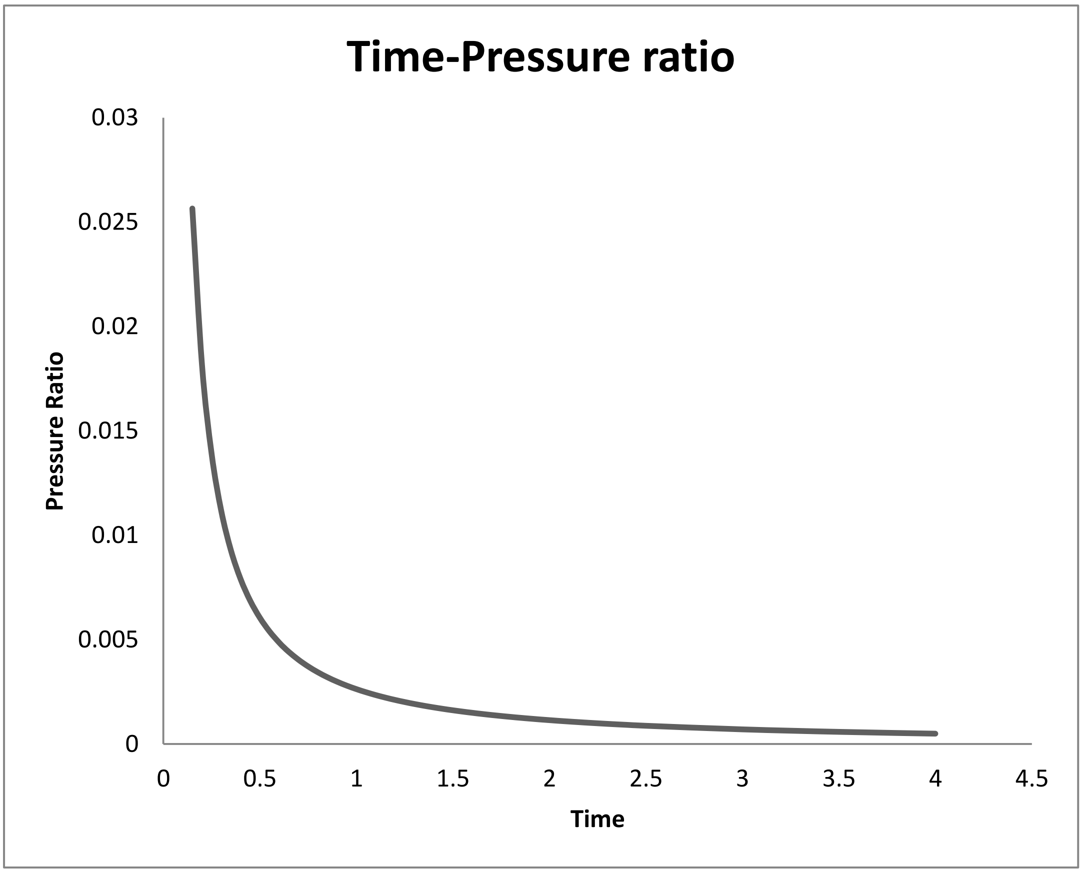

Figure17. Time versus Pressure ratio total time frame (Analytic Method)

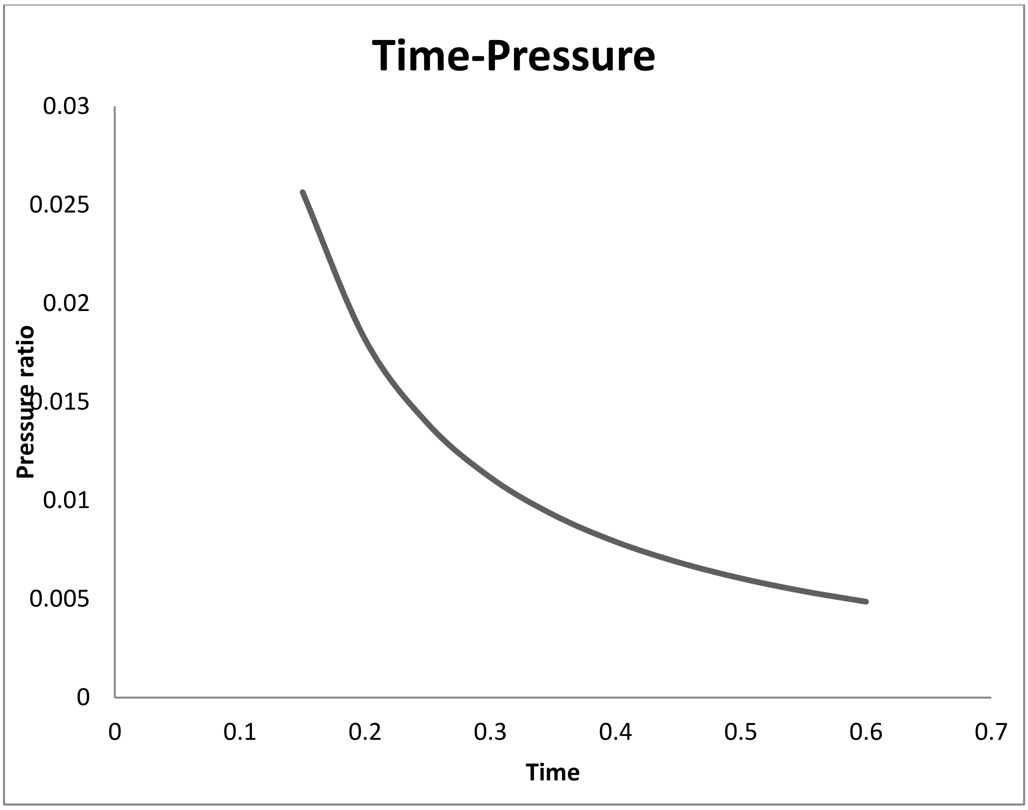

Figure18. Time versus Pressure ratio for only primary shock wave

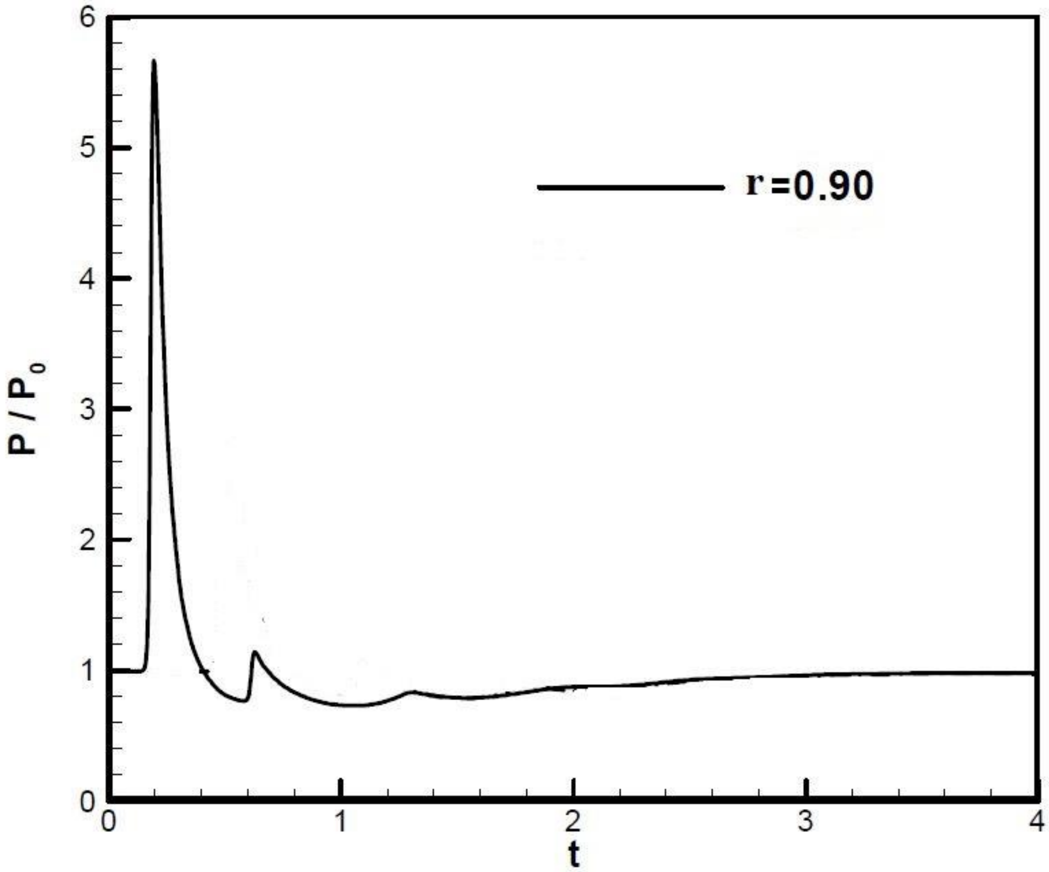

Figure19. Numerical time versus Pressure ratio

It can be seen in the numerical graph that the pressure ratio at the beginning increased rapidly and then slowed down rapidly. In the analytical results (Fig. 17)are get reduced very fast at the beginning and after that it become almost stable same things can be seen in the shock wave diminished the pressure ratio become almost constant (Fig. 19). In the figure 18, the pressure ratio is reduced very fast till 0.25 then the reduction rate gets reduced and after the portion reduction rate gets slow

CHAPTER 6: CONCLUSION & FURTHER STUDY

Looking at the numerical graph and the analytical graph has the similarity and the dissimilarity as well. Both the graph have same trend of reduction in the pressure ratio except for the secondary and tertiary phase. In the analytical section there is no secondary and tertiary shock wave (they are not calculated). When Taylor had given his famous theory, probably there was no concept of secondary and tertiary shock wave concept until in 1960's. But another very important point to look is that the results don't match at all. All the results are way different from the numerical results. The authors of the numerical experiment had claimed that they had compared their results with the experiment. So I can guess that their something in the Taylor's calculation is missing perhaps the calculation of secondary and tertiary shock wave may give some clue which has not calculated in this research paper.

In the further study the theory of secondary and tertiary shock can be brought to the picture, so that the whole result can be seen. In the main numerical paper the authors had also studied the shock reflection from a plate due to strong and weak shock. So the in the further study the same things can be studied on analytically.

One more thing is very important, while doing the numerical calculation the authors had used a numerical coding, the same numerical model can be recreated by commercial software like "Ansys Explicit Analysis".

It can be also studied the how the different kind lands element like soil, stone reacts to the explosion. Not only that the different elements which are used to make building reacts to the blast. By doing so there can possibilities to design and manufacture different kind material which can be shock resistance. This is because there are a lots of countries which suffer from regular terrorist attack, they can build houses or other constructions, so that at least some life can be saved, every single life is more precious than any jewel in the world.

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