WRITING TO UNDERSTAND, EXPLAIN, AND REFLECT: THE IMPLEMENTATION OF A WRITERS' WORKSHOP MODEL IN A FOURTH-GRADE MATHEMATICS CLASSROOM

by

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A dissertation submitted to the faculty of
The University of North Carolina at Charlotte
in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in
Curriculum and Instruction

Charlotte

2013

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ABSTRACT

CHRISTIE MARTIN. Writing to understand, explain, and reflect: the implementation of a writers' workshop model in a fourth-grade mathematics Classroom (Under the direction of DR. BRIAN KISSEL)

To understand how students use writing in mathematics to reflect their learning and problem solving and the ways in which students' writing influences teachers' lesson plans, I conducted a qualitative study that was guided by the following questions:

1. How do students use writing to reflect on their learning in mathematics?
2. How do students use writing to show how they solve mathematical problems?
3. How does the teacher adjust her lesson plans in response to writing produced by students using the writers' workshop model?

This study builds on research that suggests writing serves as a reflective tool that increases metacognition.

The study spanned six weeks and included 18 implementations of an adapted version of the Writers' Workshop in a fourth grade mathematics class. On a biweekly basis, the data were reviewed and changes made to the model. The data included students' writing, field notes, conferencing transcriptions, my journal, interviews with the students and the teachers, and classroom observations. I analyzed these data to answer the research questions above.

According to my findings, the students used writing as a tool to demonstrate their mathematical understanding and their process. Students also used writing to demonstrate

their understanding of mathematical vocabulary. Their written reflections and written explanations informed instructional practices. Their writing prompted conferencing questions, assisted in grouping decisions, and influenced decision as to whether to move to a higher level of instruction.

DEDICATION

This dissertation is dedicated to my wonderful husband Todd, our beautiful daughter Eva, and my mom. It is only with the love, support, and joy they bring into my life each day that this could be accomplished. Thank you for always believing in me and reinforcing my faith. Todd, you are my love and best friend. I am blessed to share this with you and forever grateful for our beautiful family.

ACKNOWLEDGEMENTS

I would like to acknowledge and thank the members of my dissertation committee. First and foremost, I am thankful that my chair, Dr. Kissel, guided my interest in content area writing, shared his excitement for students' writing, offered encouragement, and challenged me every step of the way. Dr. Chuang Wang, my methodologist and friend, provided endless support. I am grateful for his willingness to answer questions and talk me through every step of my research. I greatly appreciate Dr. Polly, who provided support and encouragement for my research from the pilot study through the dissertation. I have tremendous gratitude for his ability to keep me on my toes. The fourth member of my committee, Dr. Algozzine, was always a kind and supportive advocate. His sage advice and thoughtful delivery inspired me throughout this process. I am thankful for the teacher, Michelle, for inviting me into her classroom; without her support, the study could not have been completed. I must also acknowledge the Graduate Assistant Support plan that has allowed me to develop strong relationships with my committee and provided financial support. This dissertation would not be possible without this wonderful committee.

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CHAPTER 1: INTRODUCTION

Globalization and education are interwoven entities that are dependent on one another. Globalization is defined as a social and economic process that is identifiable by growing levels of financial and technological integration and interconnections in the world system (Stiglitz, 2002). Technology has enhanced our ability to communicate with the global world; this communication is often written. This increased economic interconnectedness impacts educational systems around the world (Chana, 2010), and means that people will exchange written communications with colleagues, possibly for years, without meeting face to face. The newfound power of individuals to send out their products and ideas is reshaping the flow of creativity, innovation, political mobilization, and information gathering and dissemination (Friedman, 2006). These changes in our global economic, social, and political climate require that students be capable of creating informational text. In elementary, middle, and high school grades students are being asked to compose informational texts that include communicating a level of mathematical understanding.

Christensen and Horn (2008) suggests that all countries in the world are working in an education system that is organized as a manufacturing plant, in which students are inputs, they are subjected to standardized processes, and their performance on a standardized test determines their future. Christensen and Horn feel that this problematic system can be combated with student-centered learning, which allows students to learn each subject consistent with their type of intelligences and style. Student-centered or learner-centered pedagogy relies on providing students opportunities to develop specific skills as individuals. Opportunities to write informative texts are critical for honing skills that will be essential in the globalized market.

Educational research is ongoing, and with each new study more knowledge is gained to further pedagogical practices. Research continues to support writing as a pedagogically sound practice for the classroom. The integration of writing in the mathematics classroom is important; however, its effective use is dependent on teachers' ability to provide opportunities for effective writing practices. Often, teachers have little experience using writing to convey mathematical understanding, and this makes them apprehensive about using writing in their classroom (Totten, 2005). Writing is more readily transferred to humanities and limited in its use for mathematics. To be able to support their students in the future, pre-service teachers need experience developing and using a variety of representations of mathematical ideas to model problem situations, to investigate mathematical relationships, and to justify or disprove conjectures (McCormick, 2010).

In this chapter I begin with a statement of the problem. Next, I provide a rationale for the purpose of this study, followed by the scope of the study. I then describe the theoretical framework and define the terms. Lastly, I present the limitations of the study and a summary.

Statement of the Problem

Writing across the curriculum (WAC) is a research-based strategy for supporting students' conceptual understanding (NGA/CCSSO, 2011; Pugalee, 2004). The WAC movement began in the late 1970s. In this movement, teachers integrate writing into their teaching of mathematics; however, this can be a difficult task, given that most teachers have had little experience using writing as a tool to learn and communicate their understanding of mathematics (Totten, 2005). Teachers seeking to use writing in their mathematics lessons to develop quantitative literacy may question which type of writing to employ. Many teachers struggle to link writing and mathematics and honor the integrity of both disciplines at the same time (Wilcox & Monroe, 2011). Writing in mathematics is an aspect of WAC that has rarely been researched; this lack of research limits the resources available for teachers implementing informative writing as a tool.

Writing in mathematics provides a level of reflection and analysis that allows students to focus their thinking on their own process and problem solving (Artz & Armour-Thomas, 1992; Carr & Biddlecomb, 1998; Powell, 1997; Pugalee, 2001a, 2001b). Writing, which is a traditional comprehension-enhancing strategy, has demonstrated utility in math classrooms by adding a dimension of literacy; however, writing is not used frequently in math classrooms (Baxter, Woodward, & Olson, 2005). And although frequent discussion is given to emphasizing writing across the curriculum, math is often left out of this equation (Ediger, 2006) Studies in the area of teaching and learning mathematics reveal that reflection and communication are the key processes in building understanding (Hiebert et al., 1997; MacGregor & Price, 1999; Manouchehri & Enderson, 1999; Monroe, 1996). Burns (1995) suggests that the key components of the writing process—gathering, organizing, revising, and clarifying—are skills that can be readily applied to mathematical problems. The opportunity to write in the mathematics classroom provides learners with an outlet for clarifying, refining, and consolidating their thinking.

Literacy and writing skills need to be used by teachers in their own mathematical work to model for students how to competently write and communicate mathematically (McCormick, 2010). Competent communication of mathematics includes using the symbols of the content, along with definitions and/or vocabulary, effectively (Franz & Hopper, 2007). To communicate numeric facts and patterns effectively, students should be taught to draw on concepts and skills from each of the major academic disciplines and develop quantitative literacy (Miller, 2010). Teachers have found that the integration of writing is easier in the science or social studies classroom (Varelas, Pappas, Kokkino & Ortiz, 2008). However, just as in language arts instruction, there is room for different types of writing activities in mathematics, and each type holds a specific value for the student. The recognition of the value of writing as tool for thinking is stated in the Common Core State Standards (CCSS).

The Common Core State Standards Initiative (2012) defines the CCSS as a state-led effort coordinated by the National Governors Association Center for Best Practices and the Council of Chief State School Officers (NGA/CCSSO). The standards were developed by teachers, school administrators, and other experts who collaborated to create a framework for preparing children for college and the workforce. The standards were drafted and received feedback from several sources, including teachers, postsecondary educators, and civil-rights groups. They embody the belief that consistent standards will provide appropriate benchmarks for all students regardless of location. These benchmarks have been set to ensure that high school graduates will be able to succeed in entry-level, credit-bearing academic college courses and in workforce training programs.

The CCSS signify a movement toward valuing writing across the curriculum and students' communication in various content areas. In mathematics, the CCSS include Standards for Mathematical Practice that emphasize the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning, proof, communication, representation, and connections (NGA/CCSSO, 2011). The CCSS Initiative (2012) highlights the writing standards for the English language arts (ELA), which stress the ability to write logical arguments based on substantive claims, sound reasoning, and relevant evidence. This overlaps with the aims of the standards for mathematics. The initiative states that mathematically proficient students should understand and use stated assumptions, definitions, and previously established results in constructing arguments. The initiative further defines proficient students as being able to justify their conclusions, communicate them to others, and respond to the arguments of others.

In the widely adopted CCSS for Mathematics (NGA/CCSSO, 2011), students are charged with demonstrating proficiency on grade-level content standards as well as Standards for Mathematical Practices (MPs), which focus on various mathematical behaviors related to conceptual understanding (Dacey & Polly, 2012; NGA/CCSSO, 2011). The CCSS in Mathematics underscore writing and the need for clear communication as evidence of conceptual learning. Procedural and conceptual learning represent two dichotomous types of learning; in mathematics, procedural understanding refers to memorizing with little understanding, whereas conceptual understanding represents students' ability to make connections between real-life contexts, mathematical representations, and computational work. Past work indicates that students with deep conceptual understanding can recognize and apply definitions, principles, rules, and theorems and successfully compare and contrast related concepts (Engelbrecht, Harding, & Potgieter, 2005). Students demonstrating rote procedural skills without conceptual knowledge lack an understanding of the underlying arithmetical problem and typically struggle in problem solving situations (Kaufmann, Handl, & Thony, 2003; Resnick et al., 1989; VanLehn, 1996). Historically, students in the United States have lacked conceptual understanding of essential mathematical concepts (National Center for Educational Statistics [NCES], 2008a, 2008b). Writing is a tool for demonstrating proficiency in conceptual understanding.

The NCTM (2000) called for teachers to provide students with opportunities to communicate about mathematical concepts in a clear and coherent manner. The CCSS for mathematics (CCSS-M) echo those remarks by calling for students to "construct viable arguments" and "attend to precision" in the Standards for Mathematical Practices. The CCSS-M describe mathematically proficient students engaged in the practice of constructing viable arguments as being able to use their mathematical understanding to construct plausible arguments, justify conclusions, identify flawed reasoning, and communicate these using concrete referents. The CCSS-M practice of attending to precision is evident when mathematically proficient students communicate definitions, symbols, units of measure, and their own reasoning consistently, precisely, and accurately.

The NCTM (2012) describes mathematical communication standards for Grades 3-5 as sharing, thinking, asking questions, explaining, and justifying ideas. In Grades 3-5, the standards encourage students to express and write their conjectures, questions, and solutions. The use of mathematical discourse, both in spoken and written forms, are pivotal to the construction of mathematical concepts and the development of mathematical thinking (D'Ambrosio, Johnson, & Hobbs, 1995; Koichu, Berman, & Moore, 2007).

The importance of writing in mathematics is evident in the CCSS, teachers, beginning to implement these standards, are called to provide opportunities for students to write about what they are learning in mathematics, science and social studies. It is important for research to examine the influence of mathematics journals on students' understanding as well as how to best support students' experiences writing about mathematics concepts. It is also important to examine the influence of providing opportunities for students to write across the curriculum and communicate in various content areas. Specifically, mathematics provides opportunities to share their problem solving processes as well as their mathematical thinking. In this study, elementary school students’ experiences with writing in mathematics journals will be examined to explore both the process their teacher used to implement the journals and what the journals reveal about students’ mathematical thinking.

Purpose of the Study

Previous researchers have examined writing as a learning tool in mathematics; however, the amount of writing used in mathematics instruction has rarely been studied (Ediger, 2005). Procedural learning in mathematics is prioritized in classrooms to ensure certain levels of performance on standardized tests; conceptual learning has received limited attention in the United States (NCES 2004, 2008). Students have limited opportunities to explore their mathematical thinking, conceptual learning, and to reflect on their own process.

Research indicates that writing is a tool that enhances students' ability to reflect, strategize, and communicate. It is vital that students engage in writing in mathematics to focus their own thinking and sharpen their problem solving skills (Artz & Armour-Thomas, 1992; Carr & Biddlecomb, 1998; Powell, 1997; Pugalee, 2001a, 2001b). The opportunity to write in mathematics allows for students to improve their thinking and hone their ability to convey their thinking in a clear and concise written form. Research supports the benefits of writing and, specifically, how those benefits transfer to a mathematics classroom. However, few studies have examined the process a practitioner uses to provide such opportunities for their students and or analyzed the writing produced by those students. In this study, I examined how a fourth-grade teacher used a writer's workshop model to provide writing opportunities in mathematics. Using this workshop model, students created a math journal that was used in their mathematics class several times a week. I used a Design Based Research (DBR) paradigm to gain insight into a workshop model that fosters informative writing. Using the DBR paradigm, I worked in two-week iterations with the teacher to further research the process and any changes she implemented to enhance the workshop model in her mathematics class. I investigated what happened when children engaged in consistent writing in mathematics. Furthermore, I explored how writing reflected their thinking and problem solving. Several types of data were used to answer research questions.

Scope of the Study

This study examined fourth-grade students' experiences with writing in mathematics journals. Using a workshop model, students created a math journal that was used in their mathematics class several times a week.

I was guided by three research questions:

1. How do students use writing to reflect on their learning in mathematics?
2. How do students use writing to show how they solve mathematical problems?
3. How does the teacher adjust her lesson plans in response to writing produced by students using the writers' workshop model?

The research design consisted of a three-month experiment in a fourth-grade classroom in a high-needs elementary school. Students wrote about mathematics at least three times a week in a journal. Students' work was supported by a math writing workshop model implemented by the classroom teacher. The math workshop was modeled on the writer's workshop model introduced by Calkins (1983). The model includes the following components: a whole class mini-lesson in which the teacher spends 5-10 minutes directly instructing the class, followed by 30-45 minutes of independent writing. During this independent writing time, the teacher confers with students. Finally, the students gather for 5-10 minutes to share what they wrote that day.

During the three days a week the students engaged in mathematical writing, I observed and took extensive field notes

The research design included iterations every two weeks over several months. After each iteration, journals were collected and examined by the teacher to inform the next two weeks of workshop planning. I also inductively analyzed data and served as a sounding board for the classroom teacher. Throughout the data analysis process, theories and conjectures were revised and implementation of the math journals refined for future iterations.

Theoretical Framework

I used the theory of social constructivism to frame the described study. Constructivism describes how one attains, develops, and uses cognitive processes; multiple theories, such as those of Piaget and Vygotsky, have been proposed to explain the cognitive processes involved in constructing knowledge (Airasian, 1997). Constructivists analyze thought in terms of conceptual processes located in the individual (Minick, 1989), giving priority to individual students' sensory-motor and conceptual activities (Cobb, 1994). Social constructivism is a theoretical framework that suggests that an individual constructs meaning and knowledge through their social environment and social interaction (Beck & Kosnik, 2006). Social constructivist or situated social constructivist perspective places major emphasis on the social construction of knowledge and rejects the individualistic orientation of Piagetian theory; within this perspective, knowledge is seen as being constructed through an individual's interaction with a social milieu in which he or she is situated, resulting in a change in both the individual and the milieu (Airasian, 1997). Situated social constructivist perspective stresses the inseparability of knowledge and social context.

Situated cognition theory is a theory of instruction closely aligned to the situated social constructivist framework. Situated cognition theory suggests that learning is naturally tied to authentic activity, context, and culture (Brown, Collins, & Duguid, 1989). Writing needs to be of value for the learner, arouse an intrinsic need, and connect to relevant life tasks (Vygotsky, 1980). Writing is a social transaction between the writer, a particular moment in time, the intended audience, and prior experience (Rosenblatt, 1988). This study explores the connection between writing and learning in mathematics. The purpose of this study is to gain a greater understanding of how the implementation of writing in the mathematics classroom influences student learning.

Definition of Terms

Design Based Research is a methodology that blends empirical educational research with the theory-driven design of learning environments in order to gain understanding of the how, when, and why educational innovations work in practice (Design-Based Research Collective, 2003)

Writing Across the Curriculum (WAC) is a research-based strategy for supporting students' conceptual understanding (Pugalee, 2004). The WAC movement began in the late 1970s, and as part of this movement, teachers are expected to integrate writing into their teaching of mathematics (Peterson, 2007; Pugalee, 2001). In higher education, WAC has been used in an attempt to improve students' critical thinking, analytical, and writing skills by integrating writing experiences throughout all disciplines and courses, and throughout a student's entire college course work (Dana, Hancock, & Phillips, 2011).

*Writer's workshop model***.** Calkins (1983) spent two years conducting a case study and, through her participant -observer role and analysis, the writer's workshop model was created. The structure of the writing workshop presented in the case study consisted of writing every Monday, Wednesday, and Friday. The sessions began with a mini-lesson, followed by a 15-minute workshop for writing and conferences, and a method of sharing work in process. This shaped the concept of the writing workshop and became a prevalent model for reading, writing, and even mathematics instruction.

Limitations

The study has several limitations. First, because it included only one fourth-grade class, results cannot be generalized to a larger population. Second, although I observed the writing workshop model three days a week, I did not observe the rest of the school day, which included an additional math lesson. The later math lesson could have affected the writing workshop model observed the following day. And third, researcher bias could have led me to focus on data that supported a particular hypothesis. Throughout the study, however, I continuously sought to mitigate researcher bias.

Summary

In this chapter, I provide the basis for my qualitative research study. The chapter begins with a discussion of the impact of global economic interconnectedness and education worldwide (Chana, 2010). Globalization creates a climate that requires students to be able to create informative texts that effectively communicate meaning. Christensen and Horn (2008) propose a student-centered learning environment that addresses the need for clear and informative text writing and offers multiple opportunities to engage in this practice as a vital component of learning. The essential ability to write informatively is explored in the context of mathematics. The literature supporting writing as a tool for thinking and reflection is abundant; however, despite this strong connection between writing and thinking, it is often left out of math instruction. This chapter examines the teacher's role in the implementation of writing in mathematics and the limited studies available to assist in the implementation process. The CCSS are thoroughly described and the connection between mathematics standards and ELA standards further support the purpose of this study. This chapter includes the research questions and description of the study, and establishes the theoretical framework. In the next chapter, I synthesize the current literature to examine what we know about writing in mathematics.

CHAPTER 2: REVIEW OF THE LITERATURE

This study explores the benefits of using writing as a tool for thinking in mathematics and examines the development of a writer's workshop model that provides opportunities for students to use writing in mathematics. I begin with a description of the theoretical framework and define writing experiences within this framework. In this chapter I also examine Writing Across the Curriculum (WAC), which is based on evidence that content-area writing is, in many ways, beneficial for students (McLeod & Soven, 1992). The proposed benefits of WAC are connected to the idea that writing and thinking are strongly related. I then describe the research on writing and thinking, their interconnectedness, and their relation to learning. Starting with WAC as the basis for content-area writing, I discuss different types of content-area writing and current research on these content areas. The writer's workshop model and the components of the model is also described in detail. More specifically, I focus on writing in mathematics and its purpose. The literature on the use of writing in mathematics for understanding is vast; however, there has been only limited study of methods for including writing in instruction and how to interpret students' writing to enhance instruction.

Theoretical Framework

These interactions and the students' writing will be examined through the lens of social constructivism.

Social Constructivism

Social constructivism holds that an individual constructs meaning and knowledge through their social environment and social interaction (Beck & Kosnik, 2006). Vygotsky (1978) posits that learners construct knowledge through social interactions; therefore, the learner and the social environment cannot be separated. Social constructivist or situated social constructivist perspectives emphasize the social construction of knowledge; within this perspective, an individual constructs knowledge through interactions in the social setting in which they are located, resulting in a change in both the individual and the social setting (Airasian, 1997). Situated social constructivist theorists believe in the inseparability of knowledge and social context.

Situated cognition theory contends that the act of learning is naturally tied to authentic activity, context, and culture, which is closely aligned with the situated social constructivist framework (Brown, Collins, & Duguid, 1989). Minick (1989) posits that constructivists view writing as an individual endeavor in which the individual's conceptual processes become a unit of analysis, whereas social constructivist theorists take the individual within the social interaction as their prime unit of analysis. The act of writing may appear to be an isolated activity for an individual; however, the social environment and interactions of the writer are instrumental to his or her ability to construct meaning (Beck & Kosnik, 2006). Even individuals, alone in a room, interact with past social interactions kept in their memory, which affect their thoughts and word choices as they write. Students engaged in their writing processes in school are part of a situated social context. The learning and construction of meaning inherent in their writing processes connect to the authenticity, context, and culture of the social environment (Brown, Collins, & Duguid, 1989).

Writing across the Curriculum

Over the past four decades, research on writing has focused on its value as a tool for learning, the understanding of subject-area written discourse, and the writing processes of student writers. One strategy, WAC, has influenced content-area instruction for years and is supported by a substantial amount of research (McLeod & Soven, 1992). The program dates from the mid-1970s, when the first such programs were developed in the United States (Goddard, 2003; McLeod, Miraglia, Soven, & Thaiss, 2001). WAC encourages content-area teachers to use a variety of instructional practices that incorporate writing to facilitate thinking and learning in their disciplines (Vacca, Vacca, & Mraz, 2011).

WAC may be defined as a comprehensive program that transforms the curriculum, encouraging writing to learn and learning to write in all disciplines (McLeod & Soven 1992). National Assessment of Educational Progress (NAEP) assessments emphasize the importance of writing across subject areas, with the significant inclusion of constructed response questions in national assessments of reading, civics, geography, U.S. history, foreign languages, mathematics, science, and economics (Applebee & Langer, 2006). The focus on written responses on national assessments confers a certain level of importance. WAC incorporates ideas that have influenced education policy and encouraged writing as a learning tool in every subject. The basic assumptions of WAC are that (a) writing and thinking are closely allied, (b) learning to write well involves learning particular discourse conventions, and (c) writing belongs across the spectrum of the curriculum (McLeod & Soven 1992). WAC programs promote interdisciplinary teaching of and learning about writing; included in the design are faculty development workshops that center on creating an intellectually coherent curriculum and helping students ask similar questions across the disciplines (Cargill & Kalikoff, 2007). The production of a piece of written work offers a time of reflection and further processing of material. The main tenets of WAC translate into content-area literacy, which includes reading and writing to learn. Several researchers examined the use of writing across content areas.

Content-area literacy has become more of a focus in elementary grades in recent years. Moss (2005) identifies standards-based education, emphasis on standardized-test performance, and technology as three critical factors that have increased attention to content-area literacy instruction in the early grades. The ability to use the Internet to gather, evaluate, and synthesize information is central to success in school and the future workplace (Schmar-Dobbler, 2003). Content-area writing addressees the skills and thinking processes required to meet these demands. By using content-area literacy strategies, students increase their ability to internalize course content and develop conceptual understanding of subject matter (Stephens & Brown, 2000).

Students are asked to participate in content-area writing for a variety of purposes. Content-area writing offers students an opportunity to communicate about a subject in nonfiction writing; it also serves as a motivation for students whose interests lie in content areas rather than in literacy, offers a balanced curriculum, and increases content-area learning (Duke & Bennett-Armistead, 2003: Moss, 2005; Wallace, Hand, & Prain, 2004). Content-area writing, as a basis for increased learning, relates to the knowledgetransforming model, which is based in cognitive theory (Bereiter & Scardamelia, 1987). This model suggests that skilled writers possess schemata on various text genres and are aware of the components of these genres (Hayes, 1996); writers must then access their content knowledge to produce a particular genre of writing (Bereiter & Scardamelia, 1987). The writer engages in an effortful process to access their knowledge and transform that knowledge into the format presented by a genre of writing. During this process, the writer may discover gaps in their knowledge, and at that point must construct new knowledge from outside sources (Bereiter & Scardamelia, 1987; Hillcocks, 2005). This process supports the tenets of writing to learn and WAC, as well as the increased interest in content-area writing.

Writing is one of the overarching academic skills identified as crucial for success in college and is often used as a tool for evaluation (Conley, 2007). Teachers of social studies and science are exploring content-area writing for learning purposes and are requiring that data be explained in response to assessment questions. Each discipline has distinct properties that require specific skills instruction for students to become proficient readers of challenging texts and produce higher-level writing (Englert, Okolo, & Mariage, 2009; Moje, 2008; Shanahan & Shanahan, 2008). It is important to understand what is necessary to learn in a specific discipline to become a competent reader and writer (Moje, 2006; Perin, 2007). Writing is instrumental in enhancing learning and helps students evaluate their own understanding (Bereiter & Scardamelia, 1987; Hillocks, 2005).

In particular, evidence shows that writing-to-learn has significant promise as a tool for supporting students through a process of conceptual change in elementary-school science (Peasley, 1992). In a grant-funded project, Bricker (2007) worked with three teachers as they implemented writing strategies in their science classrooms. The writing opportunities presented in these classes were connected to and inspired by books. One teacher modeled writing a journal entry that included a drawing of something from nature, with observation notes and questions to inspire further research. Another teacher used a similar format; however, she stressed the importance of gathering and recording empirical data. The third teacher focused on more detailed and accurate drawings and felt that accurately capturing information about the natural world extends learning. These three cases provided several insights and implications, and the teachers used students' entries as an assessment tool that directly influenced their instruction. Instructional practices generated by these writing activities included mini-lessons and conferences, which are major components of the writer's workshop model. High expectations were noted as an important factor in the increased depth and breadth of the students' writing. Bicker included a future plan to add a metacognitive piece, in which students would self-evaluate journal entries and set their own goals.

Peterson (2007) conducted research in an eighth-grade science class with a teacher that firmly subscribed to the tenets of WAC. The teacher began a unit on simple machines and mechanical advantage by introducing the writing assignment, which was for students to use any genre to communicate information about two or more simple machines. Students planned and gathered information through lessons and hands-on activities. During this unit, the teacher implemented conferences and mini-lessons. The teacher's belief in WAC was affirmed by the students' motivation and success in learning scientific concepts.

Teachers typically find it easier to integrate writing in a social studies classroom (Varelas et al., 2008); it is important to examine how these writing opportunities enhance comprehension and overall skill in writing. Leddy (2010) describes her experience as an elementary educator integrating writing in the social studies curriculum. In her first years of teaching, Leddy found the prompts she provided for the students were producing shallow writing that revealed limited understanding. This sparked concern and directed Leddy to collaborate with a small group of teachers to research and analyze writing, then share their observations. This small group grew to more than 100 teachers and became the Vermont Writing Collaborative. The model that emerged included using a large central idea to guide the unit, followed by learning experiences that would build deeper knowledge, reflections after each experience, specific instruction on writing strategies, and active learning examples. These components represented the foundation of the culminating writing project, which was directly connected to the large central idea. Students used the central idea to create a thesis statement for their writing. Leddy found that mediation and teacher interaction with individual students were important throughout the unit.

Johnson and Janisch (1998) identified the ways in which several elementary teachers used thematic units to combine social studies and literacy. Large, broad units were the starting point for an investigation that used many resources to gather knowledge. In this research, writing was intended to promote comprehension. Journals were used extensively for students to record information from their reading and reflect on their understanding. The authors noted that shared writing was a powerful incentive for students: Children need valid reasons to write, and sharing their writing and receiving responses served as a motivating tool. Persuasive and expository writing, in the form of accurate brochures, compositions, and research papers, were part of the writing instruction. These activities were used in conjunction with technology and supported the development skills for analyzing, synthesizing, and researching. In some of the reading activities, students read the text as writers and noted the style and craft of the author.

The value of students as evaluators is supported in research. Hansen (2001) proposed that when writers read, they are evaluating; this process is unified and instinctive. Hansen explained that as writers read material, they begin to realize that they must make decisions about the material to be included in their own writing. This appreciation furthers the students' examination of high-quality literature. Evaluation offers insight for writing ideas and exposes students to different styles, which can be emulated in their own work. Writers use this evaluative process to develop their voice, and the classroom can serve to both recognize and value the voice or to encourage unified structure over individuality. In the social studies classrooms involved in this study, students selected a topic and wrote a research paper. This process included the use of writing partners who would listen to drafts and check for clarity. These papers were published and shared with the class, so the writing was both a communicative act and a public act.

Lubig (2009) examined civic efficacy and civics instruction through a content analysis of the writers' workshop model. Lubig's content analysis included eight works by Atwell and four works by Ray who were chosen as leaders in the use of writing workshops. The analysis used units developed by a National Council for the Social Studies task force (2001). Lubig identified themes from the analysis and literature on writers' workshops as applied to civics instruction. The connected themes noted were that the workshop (a) has a clear objective, (b) emphasizes shared understanding, (c) establishes clear methods that include collaboration, (d) includes structure and modeling by teachers, (e) focuses on an authentic audience, and (f) allows time for reflection and discussion. Overall, Lubig found that the workshop model has the potential to increase civic efficacy.

The studies in content-area writing in science and social studies have implications for expository writing in mathematics. Expository writing can be defined as structured writing on a topic. Informational or expository writing includes summaries, arguments, observations, explanations, reports, comparing and contrasting, procedural writing, descriptions, personal dictionaries, and word problems (Bangert-Drowns, Hurley, & Wilkinson, 2004; Muth, 1997; Ediger, 2006; Kline & Ishii, 2008; Liedtke & Sales, 2001; Neil, 1996; Ntenza, 2006; Pugalee, 2001; Thompson & Chappell, 2007). Constructing an argument is requires critical thinking, whereas explaining something invites theoretical understanding (Klein & Kirkpatrick, 2010). Children involved in writing expository texts have a heightened awareness of how such texts are created (Littlefair, 1992). The studies of content-area writing cited above share several salient practices, including students' use of journals for information and reflection, teachers' use of broad concepts that are then broken into lessons that center on deepening knowledge and instruction on skills, partner writing and shared writing to help students remain motivated and learn from one another, individual student-teacher conferences, and publication of student writing (Bricker, 2007; Johnson & Janisch, 1998; Leddy, 2010; Peterson, 2007).

Shared writing is another way to enhance learning, and it can be implemented in the classroom in different ways. Variations include partner writing, teacher scripting, or interactive journals. Journal writing offers a personal space that is a free-flowing record of experiences, observations, thoughts, questions, and responses. In this form of writing, there need not be specific form or revision. Personal writing reflects on the experiences of the author and connects to the content (Bangert-Drowns et al., 2004; Thompson & Chappell, 2007). These practices are embedded in Calkins' (1983) Writers' Workshop Model and can be translated for the content area of mathematics to encourage conceptual learning and effective written communication.

The Writers' Workshop Model

Calkins (1983) spent two years conducting a case study and, through her participant-observer role and analysis, she created the writers' workshop model of instruction. Calkins analyzed students' growth in writing and tied this growth to the environment of the classroom and the type of instruction provided by the teacher. Mini-lessons, writing demonstrations, and conferences were critical elements of students' development. The structure of the writing workshop presented in the case study consisted of writing every Monday, Wednesday, and Friday for 80 minutes. The sessions began with a mini-lesson, followed by a fifteen-minute workshop for writing and conferences and time for sharing work in process using a structured format. This research shaped the idea of the writing workshop, and the model became a prevalent one for instruction in reading, writing, and even mathematics.

The consistency of the schedule was helpful and motivating, and enhanced student creativity. Each step of the writers' workshop provides an opportunity to learn and communicate. The workshop model contains components that flow from one step to the next; however, these components are fluid and adjust to the nonlinear nature of writing.

Mini-Lessons

One component of the writing workshop model is the mini-lesson, which consists of a brief lesson, usually at the beginning of the workshop. Mini-lessons focus on improving a particular aspect of writing, such as strategies for prewriting, revising, and editing, as well as writing skills (Au et al., 1997; Calkins, 1983). Many teachers find that beginning the workshop with a mini-lesson breaks a unit of study into several parts, and offers a more meaningful way to introduce subject matter and apply the data to real-life situations—one step at a time (Lombrado, 2006). Mini-lessons are a method for teaching skills within the writing process while allowing other skills and content to develop concurrently (Dowis & Schloss, 1992). Mini-lessons occur in small windows of time during which the teacher presents skills in a manner that is both teacher-directed and student-centered. Teachers use students' written work from a previous day as the basis for developing mini-lessons; the examination of student work is pivotal to the process (Jasmine & Weiner, 2007). Effective lessons are short, focused, gentle in tone, and responsive to students (Avery, 1992; Calkins, 2003). Through demonstration and modeling, skills are acquired and used in writing immediately and going forward, and the skills students struggle to acquire are reinforced through ample opportunities to write and student-teacher conferences.

Writing and Conferences

The workshop model continues from the mini-lesson into a period of writing and teacher conferences. Calkins' (1983) model divided the writing process into rehearsal, drafting, revising, and editing. Students gather ideas in the rehearsal stage; young writers may use illustrations at this stage (Calkins, 1983; Graves, 1983). Drawings or illustrations serve as a way to explore and organize for children with limited writing abilities. Calkins asserts, however, that teachers must monitor the effectiveness of drawing during the rehearsal period to ensure that the student does not become limited by drawing; artistic abilities, or their lack, may limit the student's choice of writing topics (Avery, 1993). If and when drawing impedes the rehearsal time, teachers can guide students as they create outlines or lists, read stories, or converse with peers to begin the writing process (Atwell, 1987; Calkins, 1983; Graves, 1983).

The next component of the writing process, as outlined by Calkins, is drafting, where the brainstorming of the rehearsal stage is focused and turned into the written word. Drafting is an ongoing endeavor, and students will return to their text more than once to reshape it (Jasmine & Weiner, 2007). Revision is a recursive process during which writers add, delete, and rearrange text so that the meaning becomes as clear as possible for readers. Calkins (1983) suggests that teachers encourage students to focus on content during their first draft and only later move on to consider spelling and grammatical errors. Teacher observations and conferences occur as the students engage in these various stages.

Atwell (1987) suggests that the teacher listen, tell back, and ask questions during a conference to allow students to discover their knowledge and meaning. Avery (1993) supports the importance of students' taking an active role in the conference, because writers have more knowledge than what may appear on the page. Calkins (1994) describes the listening role of the teacher as a magnetic force, in which students responding to a question from an intense listener will often find themselves sharing more than they had expected to. Conferencing provides an opportunity for students to reflect on and think about what they are trying to communicate.

Sharing

Sharing is another component of the workshop model. In some cases, an "Author's Chair" is used as when a student shares his or her work with the entire class (Parry & Hornsby, 1985). This component resembles a larger conference; it encourages students to strengthen their listening skills and provide feedback to their peers. The writing of the author and their peers is improved by thoughtful responses and purposeful dialogue (Atwell, 1987). Publishing is another form of sharing, and it supports the writer's development by providing reasons to revise and edit (Rhodes & Dudley-Marling, 1996). Authors' Day celebrations are also used to provide impetus to finish pieces and create a sense of authorship (Calkins, 1994). A pertinent part of sharing is the selection; Rhodes & Dudley-Marling (1996) and Graves (1983) stress that sharing should be a time for focusing on remarkable pieces; not every piece of writing has the same importance or skill level.

Writers' Workshops in the Classroom

Teachers implement writers' workshops in pre-K classrooms through college-level courses. Kissel, Hansen, Tower, and Lawrence (2011) conducted a six-year study of writing in a pre-K classroom that employed the workshop model. Data included field notes, interviews, and collected writing; however, findings related to the interactions of students who were engaging in workshop writing were obtained by coding and analyzing previously written analytic memos. The three primary interactions defined by the researchers were those that challenged identity, introduced new possibilities, or included interaction with more knowledgeable classmates. The examples provided for each type highlighted how students used their social interactions with one another and the teacher to strengthen their identity, explore, and connect with other classmates. The pre-K writing class was loud, and encouraged students to voice their opinions and reach out to others. The students were able to write about the topics of their choice and the teacher created an environment in which writing instruction included purpose, audience, and choice. Active learning occurred in each phase of the workshop, and interactions were especially meaningful for their writing. This study centered on a classroom that used a writers' workshop. Research on writers' workshops typically uses the case-study method, and the benefits are highlighted in the several pieces of data. However, Clippard and Nicaise (1998) conducted a quasi-experimental intervention study to examine, through comparison, the efficacy of a writer's workshop for students with writing deficits.

Clippard and Nicaise (1998) compared the writers' workshop model to a WAC model. The WAC model used was defined as teachers who taught writing by deciding, before beginning a unit, which theme cycles, topics, formats, projects, and activities would be used. Two groups of fourth- and fifth-grade students participated in the study. A series of pre/post tests were analyzed, and although both groups made improvements, the students using the workshop model improved significantly in number of words, number of paragraphs, size of vocabulary, extent of revisions, and overall quality of their writing. The workshop model has been used in all grade levels and has shown positive effects for academically struggling students.

James, Abbott, and Greenwood (2001) examined the effectiveness of the workshop model for a nine-year-old student named Adam. Adam was described as being from a low-income, two-parent home, and receiving special education services. At the start of the fourth grade, Adam was tested in reading and writing. The reading assessment showed that his reading ability was between first and second grade, and his writing during the writing assessment consisted of only five words. In collaboration with Adam's fourth-grade teacher, the authors designed a model that would connect state and district writing mandates and current research. The study was broken into two sessions. The first nine-week session included students that had scored at or above grade level, followed by a nine-week session with students that had scored one or more years below grade level. Adam participated in the second session. The components of the model included process writing, writers' workshop, graphic organizers, and assessment of the six traits used by good writers (ideas and content, organization, voice, word choice, sentence fluency, and conventions). Students were instructed in writing for 30 minutes a day for nine weeks. Writing time included students' choice of topic, instruction on graphic organizers, and encouragement for use of the six traits. The pre/post tests showed improvement for both groups; however, the lower-scoring group experienced greater improvement. The authors attribute the positive results to the combination of writers' workshop with graphic organizers, process writing, and the six-trait assessment. This study demonstrated the benefits of using a workshop model in conjunction with other strategies.

Helsel and Greenberg (2007) conducted a study with a student identified as a struggling writer. Helsel, who was teaching sixth grade and employed the writers' workshop model in her classroom, found that the workshop model allowed some students to flourish, while struggling writers were impeded by the freedom of the workshop model. The authors explored the Self-Regulated Strategy Development (SRSD) model in writing instruction. SRSD includes a series of stages that serve as guidelines for the incorporation of self-regulatory training in a writing program. The purpose of SRSD is to help students master high-level cognitive processes while developing effective, reflective, and self-regulated strategies. The struggling writer in the study worked one-on-one with Helsel for 45-minute sessions, during which Helsel used the SRSD model. The study recommends the SRSD model for upper- and middle school teachers working with struggling writers. Although Helsel embarked on the study as an avid user of and believer in writing workshops, the study did not explore how SRSD might be used in conjunction with a writers' workshop.

Studies of the workshop model are limited and have only minimally explored content-area writing. The workshop model offers a framework that encourages students to be reflective in their writing and receive feedback from peers and teachers. James et al. (2001) provide insight on using the workshop model in conjunction with other strategies. The strategies used in their study were research based. The strategies were selected in the planning stages of the design, however, so the teacher's perspective on possible design changes was not included.

Writing and Thinking

The workshop model provides a holistic connection between writing and thinking, and, as a framework, can be adjusted to address students' needs.

Metacognition

Flavell (1976), a leading researcher in the field of metacognition, defines metacognition in this way:

> "Metacognition" refers to one's knowledge, concerning one's own cognitive processes and products or anything related to them, e.g., the learning-relevant properties of information or data…Metacognition refers, among other things to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective. (p. 232)

Joseph Garofalo, Associate Professor and Co-Director of the Center for Technology and Teacher Education at the University of Virginia, and Frank Lester, Chancellor's Professor of Teacher Education at Indiana University, have done extensive research on metacognition, mathematical understanding, and cognitive ability. Garofalo and Lester (1985) assert that metacognition is composed of (a) knowledge and beliefs about cognitive phenomena, and (b) the regulation and control of cognitive actions. They are in agreement with Flavell's (1976) widely accepted description. They provide a clear distinction between metacognition and cognition, stating that cognition is involved in doing or acting, whereas metacognition involves choosing and planning activities, followed by monitoring those actions.

Brown and Palincsar (1982) describe metacognition as knowledge of cognition and regulation of cognition, which are closely related and complementary. Knowledge of cognition refers to the conscious access to one's own cognitive operations and reflection about those of others; regulation of cognition is described as executive control, which involves preplanning, planning, monitoring, and troubleshooting**.**

Many researchers have studied the role of writing in developing metacognitive behaviors. These behaviors are critical for students to successfully acquire problem-solving skills and retain conceptual knowledge and understanding (Bangert-Drowns et al., 2004; Muth, 1997; Ediger, 2006; Garofalo, 1985, 1987; Liedtke & Sales, 2001; Ntenza, 2006; Pugalee, 2001). Brown & Palincsar (1982) assert that poor problem-solvers lack spontaneity and flexibility in the regulation of cognition, and more specifically, the preplanning and monitoring components. Preplanning and monitoring are inherent in the writing process. Students engaged in the writing process are involved in a non-linear recursive process that requires a transaction between the author, the written work, and the potential audience; the act of writing is one of the most disciplined ways of making meaning and an effective method of monitoring one's thinking (Murray, 2004). Metacognitive behaviors can be exhibited by statements made about the problem or the problem-solving process (Artz & Armour-Thomas, 1992).

Garofalo and Lester (1985, 1987) suggest that to facilitate students' metacognition, a mathematics teacher should create an environment in which questions and assignments require reflection, analysis, and the reporting of mathematical knowledge. Engaging in writing tasks that require metacognitive reflection contributes to students' mathematical learning. Writing is recognized as a process that helps the learner to think more deeply and not only facilitates learning in content areas, but engages students in higher-level thinking and reasoning processes (Peterson, 2007; Brozo &

Simpson, 2003) There are, however, distinctions between writing to learn and learning to write that greatly affect classroom practice.

Throughout history, writing has been recognized as a process that helps learners to think more deeply about the ideas and information they encounter through reading, listening, viewing, and physically experiencing the world around them (Peterson, 2007). Writing to learn (WTL) is an essential component of literacy and learning across all disciplines, because students are often expected to demonstrate their knowledge through writing (Vacca etal., 2011). WTL and learning to write serve different purposes and produce different products. For teachers who implement writing in their content area, understanding the distinction between these strategies is essential. WTL creates a written text that is meant to be a catalyst for further learning and meaning making; it serves as an opportunity to recall, clarify, and question what the writer knows about a subject (Knipper & Duggan, 2006). WTL provides a format for students to demonstrate their personal understanding of course content (Andrews, 1997); instructional activities that support WTL are, by their nature, short and informal, with the intention of tapping into prior knowledge and exploring ideas (Vacca, Vacca, & Mraz, 2011). Learning to write is centered on the goal of publishing the work, which requires editing and revision until the text is at the desired level. Students learn to write formally from entering school and throughout their lives (Vacca et al., 2011). Published written work within a content area should include the proper discourse and formatting as set forth by the requirements of that particular field. Both writing to learn and learning to write are beneficial and essential in the course of one’s education. Writing has been used in various ways throughout content areas.

Writing in Mathematics

The literature on writing in mathematics focuses on, but is not limited to, four purposes: writing to assess (Draper, 2002; Ediger, 2006; Neil, 1996; Ntenza, 2006; Pugalee, 2001; Thompson & Chappell, 2007), writing to engage in the authentic work of mathematicians (Muth, 1997; Draper, 2002; Kline & Ishii, 2008; NCTM, 2000; Ntenza, 2006; Thompson & Chappell, 2007), writing to develop metacognition (Bangert-Drowns et al., 2004; Muth, 1997; Ediger, 2006; Garofalo & Lester, 1987; Liedtke & Sales, 2001; Ntenza, 2006; Pugalee, 2001), and writing to make meaning of the content (Bangert-Drowns et al., 2004; Muth, 1997; Ediger, 2006; Garofalo & Lester, 1987; Liedtke & Sales, 2001; Ntenza, 2006; Pugalee, 2001). Writing is an expression of the concrete thinking of the learner and a personal expression of the learner's speech (Bangert-Drowns et al., 2004). Because of this close connection between writing and thinking, researchers identify writing as the intersection where learning occurs (Muth, 1997; Kline & Ishii, 2008; Neil, 1996). Studies pertaining to teaching and learning mathematics identify reflection and communication as essential components for building understanding (Hiebert et al., 1996; MacGregor & Price, 1999; Manouchehri & Enderson, 1999; Monroe, 1996). Writing in mathematics engages students in active learning and development of mathematical concepts, vocabulary, and skills and has demonstrated its utility in math classrooms by adding a dimension of literacy (Bangert-Drowns et al., 2004; Baxter et al., 2005, Muth, 1997; Draper, 2002; Ediger, 2006; Garofalo & Lester, 1987; Kline & Ishii, 2008; Liedtke & Sales, 2001; NCTM, 2000; Neil, 1996; Ntenza, 2006; Pugalee, 2001; Thompson & Chappell, 2007). Writing about mathematical ideas is an inexpensive and nonintrusive technology that allows students and teachers to capture, examine, and respond to mathematical thinking (Powell, 1997).

The research on both writing instruction and mathematics education supports writing in mathematics to increase students' understanding of the content. The tenets of WAC are of increased interest in the field of mathematics. Mathematical communication in both speech and written form are essential to mathematical understanding and conceptualization (D'Ambrosia et al., 1995; Koichu et al., 2007). Focus on literacy strategies has increased for infusing high levels of mathematical discourse into the classroom. The NCTM created an objective in 2000 that focuses on clear and coherent math communication. The recommendations for mathematics communication state that mathematical literacy should be an integral part of instruction; mathematical language is typically confined to the mathematics classroom, which means that this is the only place where students engage in speaking, writing, listening, and reading about mathematics (Thompson & Chappell, 2007).

Building mathematical literacy into the mathematics classroom is essential for students to effectively communicate mathematical concepts, and, if not included in the mathematics classroom, students face limited opportunities to interact with mathematical literacy (Thompson & Chappell, 2007). By providing students with opportunities to work with mathematical ideas in their own language and on their own terms, writing helps them develop confidence in their understanding of mathematics and become more thoroughly engaged with mathematics (Powell, 1997).

The type of writing described by Powell (1997) and Thomson and Chappell (2007) transcends mechanical writing, which is copying or writing that does not require the writer to use his/her own words, such as fill-in-the-blank exercises (Bangert-Drowns et al., 2004; Ntenza, 2006). This type of writing is prevalent in many mathematics classrooms (Ntenza, 2006). Effective communication and interaction with mathematical literacy requires authentic writing experiences; accordingly, classrooms that emphasize writing-to-learn strategies for mathematics engage in authentic writing (Bangert-Drowns et al., 2004). Murray (2004) posits that students need to use a word at least 30 times to make it their own; simply copying words fails to internalize the word and concept. Murray argues for the increased use of language and communication in mathematics. Schuster and Anderson (2005) contend that teachers should require students to include in their written work how they came to understand a particular concept as well as the underpinnings of the concept itself. Research in teaching and learning mathematics suggests that reflection and communication are key components for increasing mathematical understanding (MacGregor & Price, 1999; Monroe, 1996). Reflection can be defined as examining one's thoughts and actions Although considered a solitary activity, reflection is enhanced by writing and talking (Heuser, 2002). Both writing and talking encourage the mental processing of experiences, solidifying vague thoughts that students can then organize and use to make connections (Zemelman, Daniels, Hyde, & Varner, 1998).

Jingzi and Normandia (2009) conducted several interviews in their study of writing in mathematics. The students interviewed stated that writing in mathematics is cognitively and linguistically challenging. They also felt that writing in mathematics is different from writing in English and the social sciences. However, despite their dislike for writing in math, due to its challenging nature, they did affirm the benefit of writing in math, and suggested that it should be included in elementary grade instruction in order for students to reach proficiency in higher grades. Rose (1989) states, “Writing down mathematical concepts, processes, and applications in order to inform, explain, or report invites students to record their understanding through written language, a process that improves fluency” (p .17). To communicate numeric facts and patterns effectively, students should be taught to draw on concepts and skills from each of the major academic disciplines and develop quantitative literacy (Miller, 2010). Numerous and varied opportunities for this integration support students as they learn to think their way into mathematics and make it their own (Zinsser, 1988). Writing in mathematics, similar to language-arts instruction, can take many forms, each of which offers a distinct benefit for student learning.

Math Journals

Mathematics autobiographies and journals are examples of personal writing (Thompson & Chappell, 2007). Some research suggests that personal writing may change or improve students’ beliefs about mathematics, which would then impact their achievement (Thompson & Chappell, 2007). Goldsby and Cozza (2002) assert that math journals provide a window into the mind of the student who is engaged in mathematical activities, providing an opportunity to see the thinking behind the process. As students engage in journal writing to explain their process, they develop a greater understanding of concepts and correctly use mathematical vocabulary (Tuttle, 2005). By using the language of mathematics in writing, students also actively participate in developing their mathematical vocabulary (Draper, 2002; NCTM, 2000; Ntenza, 2006; Thompson & Chappell, 2007). Journals become a communication channel between teacher and student and offer an environment in which comfortable individualized instruction can occur (Pugalee, 1997).

Kostos and Shin (2010) used a mixed-method action research design with second graders from a large suburb of Chicago to evaluate the effect of math journals on mathematical thinking and communication. Sixteen students were included in the study. The data included pre- and post-math assessments, students' math journals, interviews with the students, and the teacher's reflective journal. Math journal writing occurred three times a week and included 16 different prompts from *Saxon Math Two* (Larson, 2008). The teacher modeled the first three journaling sessions. Throughout the remainder of the study, 13 additional prompts were used in conjunction with mini-lessons. The results showed an increased use of mathematical vocabulary that was supported by interviews, students' journals, interviews, and teacher reflection. Post-tests demonstrated statistically significant improvement over pre-test scores, which indicate an increase in mathematical thinking. Kostos and Shin note that the limited writing capabilities of second graders did not impede their ability to show their mathematical thinking through pictures, tally marks, and words. The teacher involved in this study also found the journals to be an excellent source of assessment information and well worth the time spent collecting and reading them.

McIntosh and Draper (2001) used their years of teaching to illustrate how writing can be used in mathematics. Their research supports journal writing as valuable for both mathematical learning and assessment. They coined the term "learning log" to describe a running commentary on learning. The learning log allows students to reflect on what they are learning and to learn while they are reflecting. The researchers used prompts in the learning logs and evaluated responses for instruction. Along with Kostos and Shin (2010), McIntosh and Draper contributed research on the elementary grades. The use of prompts in math journals has since been extended beyond the elementary grades.

Jurdak and Zein (1998) studied 104 middle school students, ranging in age from 11 to 13, at the International College in Beirut. Zein was assigned to teach four mathematics classes; two were journal-writing classes (experimental group) and two classes were no-journal-writing classes (control group). The study occurred over a 12-week period. The protocol for the experimental group included a diary-like series of writing assignments or prompts given toward the end of class. The prompts called for a written response to a question, statement, or set of instructions. Students were given 7-10 minutes at the end of the class to read the prompt and respond in writing. The control group's end-of-class activity consisted of exercises from the text that minimized opportunities to write. The Mathematics Evaluation Test (MET) served as the instrument for pre- and post-tests. The dependent variables examined through a MANCOVA analysis were conceptual understanding, procedural knowledge, problem solving, mathematical communication, attitudes toward mathematics, and mathematics achievement. The results showed statistically significant improvement for the experimental group in of conceptual understanding, procedural knowledge, and mathematical communication.

Jurdak and Zein (1998) theorize that journal writing provides a self-initiated and self-controlled environment in which to process mathematical concepts, which, in turn, enhances the writer's conceptual understanding; this deeper conceptual knowledge fosters increased procedural knowledge. The authors posit that mathematical communication skills are positively influenced by the integrative nature of writing, which combines reading, comprehension, and grammar. They note the other variables'; problem solving, attitudes toward mathematics, and mathematics achievement lack of significance and, in addition, hypothesize that mathematical achievement tests are instruction-specific and fail to address broader areas that are enhanced by journal writing, such as conceptual understanding. The lack of improvement in attitudes toward mathematics contradicts the findings of studies conducted at the high school and college level; Jurdak and Zein suggest that the effect of journal writing on attitudes is controversial. In the area of problem solving, they cite Shoenfeld's (1992) four components of problem solving (resources, heuristics, control, and belief systems) and posit that journal writing increases students' conceptual and procedural knowledge, which only affects the resources component. In addition, the authors note that the literature supports the value of expository writing in mathematics to increase problem-solving skills.

Expository writing is a genre that can offer benefits for mathematics. Miller (2010) examines quantitative literacy and outlines a systematic approach to expository writing in mathematics. Miller advocates adopting the expository structure used in essay writing in language-arts classes and transferring the organization and techniques to quantitative writing. Vocabulary, analogies, and metaphors can be infused into quantitative writing to explain the relationships and directions indicated by data. The strongest descriptions of numeric patterns combine vocabulary or analogies with numeric information, because they reinforce one another and tap into different ways of explaining and visualizing patterns that will appeal to students with varied academic strengths and learning styles (Miller, 2010). Expository writing can be incorporated in journals and/or learning logs to strengthen mathematical communication and problem solving. Writing produced by students in different genres serves as an assessment tool for teachers.

Goldsby and Cozza (2002) provide in-depth analysis of four eighth-grade students' mathematical journal writing. The students were responding to a problem in which a fraction with symbols was on one side of an equal sign and a positive number on the other. The students were asked for positive numbers that could replace the symbols in two problems. Each of the students produced different strategies and explanations for solving the problem. Each explanation provided insight into instructional techniques and provided a WTL activity. Students participated in a discussion of their explanations, which made the solutions personal and meaningful. Adams (1998) offers a description of alternative assessments and emphasizes journals as a tool for assessing children's communication skills and reflections on their own capabilities, attitudes, and dispositions as well as for evaluating their ability to communicate mathematically through writing. Journals are effective because they increase metacognition (McIntosh & Draper, 2001), encourage vocabulary development (Draper, 2002; NCTM, 2000; Ntenza, 2006; Thompson & Chappell, 2007; Tuttle, 2005), and allow students to explain their process (Goldsby & Cozza, 2002). Goldsby and Cozza (2002) touched on the element of discussion or sharing, and highlighted this aspect of writing as important for meaningful metacognitive mathematical thinking.

Shared Writing

Shared writing is a strategy that offers students a chance to share their writing and receive feedback from their peers. Their classmates use this time to listen closely, give thorough feedback, and gain ideas by listening to others. Students presented with a math concept write explanations, share their ideas, and return to their writing to revise (Wilcox & Monroe, 2011. Revision can be a whole-class activity or an individual change. In this type of activity, students further their mathematical knowledge by interacting with classmates.

Pugalee (2005) cites an example from his research in which students were given a simplified rubric, which was closely aligned with the rubric used on the state-mandated writing test, to evaluate their partner’s explanation of his or her methods and conclusions for a task. In the example, one student illustrated a strong understanding of the concept of similar triangles and the use of proportions to solve for a missing distance. However, the partner noticed that the idea of corresponding parts, which was the basis for solving the problem, was not clearly explained in the writing. Although according to the rubric the response was highly rated, the sharing between partners offered insight into deepening the response by including pertinent information. This shared writing example is part of Pugalee’s model of speaking-writing mathematics. An important component of the model is the feedback loop that occurs in various classroom settings: students working in pairs or groups, the teacher’s facilitation of the lesson, or discourse involving the whole class (pp. 99-100). Interaction and sharing increase mathematical discourse or mathematical literacy, which further strengthens students’ expository writing. Mathematics journals, expository writing, and sharing have been shown to enhance mathematical learning. These components can be used in a workshop structure.

A Mathematics Writing Workshop

A mathematics writing workshop is based on the philosophy that a writer's process should be supported by discussion and collaborative writing. Introducing mathematical writing as a genre during a writing workshop resulted in worthwhile and easy-to-understand stories about mathematical thinking (Carter, 2009). Students participating in writing workshops use illustrations to explain the mathematical content of the text. Revision and practice with this strategy eventually lessen the need for illustrations and affirm that meaning resides in the text. Heuser (2000) identifies the learner-centered components of the writers' workshop as a format that could accommodate different methods and content. Heuser also identifies the mini-lesson, activity time, and student self-reflection as essential parts of a mathematics writers' workshop. These parts mirror the Writer's Workshop.

Heuser's (2000) mathematics workshop research was developed in the first- and second-grade classes he taught. The mini-lessons were constructed based on student needs and abilities. Heuser emphasizes that mini-lessons should not exceed 10 minutes. In one class, for instance, after a short lesson on estimation, students were told to choose objects of interest, spread them out, estimate the amount, and then count. The activity lasted 25-50 minutes and was an example of a directed activity period. In contrast, during undirected activity periods, students were allowed to choose any manipulatives to complete an activity and to decide whether they preferred to work independently or with a partner. The teacher's role during activity time is to observe, assess, question, and conduct individual conferences. The reflection time is described by Heuser as a time for students to process what they have learned by sharing with a partner and then writing a reflection using "Today in Math Workshop, I…" as a prompt. Both the activity and the time for reflection encourage mathematical language. Heuser highlights the development of skills, mathematical language, an awareness of varying skills, thoughtful involvement, and innovative thinking as some of the benefits attributed to mathematics workshop.

Carter (2009) conducted action research in first and second grade classrooms with similar results after finding that her students were turning in math assignments with vague explanations or blank lines. Carter noticed that her students connected with a real-life explanation of one dozen, one half dozen, and a baker's dozen from a story told by a girl in the class. This spurred Carter to focus on providing students time and tools for writing in mathematics. She implemented journals for students to write about strategy, questions, and reflections. The journal was their mathematical story, and offered Carter information that she decided to use for mini-lessons in the Writer's Workshop she was using in language arts. The mathematically themed workshops produced student writing with titles such as "The Hexagon Adventure" and "Do Math in Kindergarten and Beyond." During Author's Chair, students posed questions about the story and the mathematical concepts. Carter concluded that adding writing in reflective journals and incorporating math in the Writer's Workshop extended the students' thinking about the strategies they use to problem solve in math class.

Fernsten (2007) suggested that the workshop model furthers mathematical understanding and advances student learning by writing out the strategies used by mathematicians. Fernsten, who taught in secondary schools for many years and continues to teach graduate courses for teachers, has found that her students majoring in mathematics question the relevancy of a writing workshop in a math classroom. For the purpose of her article and to reach the students in doubt, Fernsten defined a writing workshop as a structured peer collaboration that engages participants in thoughtful and controlled discussions of written assignments. The structure was outlined to include specific prompts, rubrics, and a sharing design. The teacher provides a prompt or math problem for which students use a rubric to create a response. This activity is completed in groups of four; however, each individual has his or her own work to share. The sharing design gives specific roles to the listeners. These roles include pointing out a positive, saying back the steps or concept, questioning, and suggesting. Fernsten uses research and experience to explain the benefits of this structure.

Wilcox and Monroe (2011) provide an overall review of types of writing in mathematics, including learning logs, note-taking, shared writing, and a workshop model. Each type of writing discussed includes a sample from a student ranging from third through fifth grade. The authors describe the types of writing in mathematics and their benefits. The samples offer a real-life picture of student writing; however, the authors include only a few such samples. It is also unclear for what purpose and when a particular type of writing was used and if there were several drafts leading to the work provided.

Journals, shared writing, expository writing, and workshops are used in mathematics for many purposes and assist students in acquiring the ability to communicate effectively in mathematics. Current mathematics curricula no longer focus solely on skills, as the prevailing belief now is that the classroom should be a learning community of shared communication (Thompson & Chappell, 2007). Current education standards reflect this focus.

Current Standards in Education

The No Child Left Behind Act (NCLB) of 2001 ushered in another shift in federal policy: accountability for student results as a requirement for receipt of federal education dollars. NCLB requires that states account for overall student performance (Burke & Heritage, 2012). The act instituted high-stakes testing as a way of evaluating and ensuring that students are proficient in certain areas of study. Failure means possible retention for students and loss of funding for underperforming schools. Test scores hold high value and the pedagogy of being a strong test taker have permeated many classrooms. NCLB places phonics on an equal par with comprehension, and does not include writing as part of its standards (Calkins, Ehrenworth, & Lehman, 2012). The first decade of NCLB produced an environment in which states manipulated the law by reducing standards to ensure that they would qualify for federal money (Watt, 2011).

Reading and content-area teachers began to face a limited amount of time to teach students the standard course of study and ensure their ability to pass the yearly standardized tests. The positive learning and expressive power of writing have been reaffirmed in educational literature for decades. Time is an essential component for the classroom to reap the benefits of writing (Calkins, 1983, 1994, 2003; Graves, 1983), but the time required for writing instruction—to encourage a love of writing and learning—has been hindered by the environment created by NCLB. Now, over a decade since enactment of NCLB, there are changes on the horizon. Educators and policymakers have encouraged the development of the CCSS (Watt, 2011). Contending that the NCLB created incentives for states to manipulate the law by lowering standards, both conservative and progressive policymakers advocate the development of national standards and assessments (Watt, 2011). States have worked together to produce the CCSS, which entered the 2012 academic calendar for schools across the United States. These standards place an emphasis on formative assessments and higher-level thinking (Calkins et al., 2012).

Summary

Content-area teachers have demonstrated more interest in including literacy in their classrooms; however, many still struggle with having the knowledge base to do so (D’Arcangelo, 2002; Vacca, 2002a). Teachers of mathematics have been skeptical of the value of WAC; if literacy educators can learn how to tailor literacy instruction to serve the goals of mathematics, teachers in both communities will benefit (Siebert & Draper, 2008). An examination of writing in mathematics presents an opportunity for literacy educators to explore what is meant by text and which definition makes sense in the discipline of mathematics.

The NCTM has created an objective centered on math communication because research indicates that students typically remain at the operating or processing level with regard to mathematics. Students are able to write a step-by-step regurgitation of problem solving, but their responses lack depth of conceptual understanding. Teachers striving to increase and elevate mathematical thinking often take over the mathematical discourse of the classroom. This may appear to serve as modeling, but it actually takes the opportunity away from students to think their way through and make deeper connections. Each type of writing described above can be beneficial for expanding the theoretical understanding of mathematics.

Math journals offer a place for students to think through a problem and freely express their process. The journal is a place for reflection. Teachers implementing journals have a window into the thoughts and possible misconceptions of their students. Teachers can use the journals in a shared writing activity and give students a chance to work through their understanding together. The workshop model is a place where many genres of writing can be combined with mathematics.

The research on writing in mathematics is limited, in that it does not include a comprehensive examination of the teacher's process in developing literacy-based mathematics instruction and how the analysis of student writing enhances implementation of literacy and writing in mathematics. It is my intention to provide an in-depth description of this implementation—which will serve an additional resource for practitioners—in addition to illustrating the struggles and adjustments inherent in the application of the workshop model in mathematics. Another goal is to add to the research on student writing in mathematics by offering a thorough analysis of the learning and problem-solving strategies exhibited by the students in this case study.

CHAPTER 3: METHODOLOGY

Overview

This chapter provides a description of the research design, procedures, and limitations. The purpose of this study is to examine how students involved in a math writer's workshop use writing as a tool for thinking in mathematics and how their teacher uses their writing to inform instruction. As a result, a qualitative approach was adopted to answer the following research questions:

1. How do students use writing to reflect on their learning in mathematics?

2. How do students use writing to show how they solve mathematical problems?

3. How does the teacher adjust her lesson plans in response to writing produced by students using the writers' workshop model?

Qualitative methodology allowed me to examine complicated interactions in the classroom and the mathematical writing produced by the students. I employed a qualitative research design to gain an understanding of what happened when students engaged in a writer's workshop model in their mathematics class, how their writing reflected their learning and problem solving, and how their teacher interpreted and used their writing

Qualitative research has its origins in descriptive analysis, and is essentially an inductive process, reasoning from the specific to a general conclusion (Wiersma & Jurs, 2009). Qualitative research aims to gather data and offer a detailed and thorough analysis that enhances understanding. The analysis in qualitative research strives for depth of understanding (Merriam, 1998). The purpose of qualitative research is to learn about the social world, generate new understanding, and build understanding (Rossman & Rallis, 1988). Qualitative research, in its purest sense, follows the paradigm that research should be conducted in the natural setting and presents a holistic interpretation of the setting (Lancy, 1993). This type of research takes an interpretive perspective, focusing on meaning and processes; it regards context as interconnected with the understanding of multiple perspectives and interactions. In an interpretive paradigm, understanding the meaning or the process constitutes the knowledge to be gained from an inductive mode of inquiry (Merriam, 1998). According to Dyson and Genishi (2005), "interpretive research is reflexive; researchers' data gathering, analysis, and indeed eventual write up of others' experiences are mediated by their own lives" (p.81).

In qualitative research the researcher analyzes data for understanding. The analysis of qualitative data is inductive, grounded in particular pieces of data that are sorted and interrelated in order to comprehend the dimensions of some phenomenon enacted by intentional social actors in a time and place (Dyson & Genishi, 2005). Inductive research builds on abstractions, concepts, hypotheses, or theories rather than by testing existing theory (Merriam, 1998). I will use interpretive methods to describe the socially constructed meanings and perspectives of this naturally occurring classroom setting (Wiersma & Jurs, 2009). Creswell (2009) outlines four factors to consider when selecting a research design: the audience, background, scholarly literature, and personal approach. Creswell suggests that researchers consider the audience to whom they will report their research and the familiarity of that audience with various research designs. The personal-approach factor refers to a self-reflection of the researcher's training and experience. Scholarly literature and background information provide insight into the research design that will produce information that best addresses the research questions. After considering this outline and conducting a thorough review of the literature, I decided that a case study would be the best design for my purposes.

Case-Study Design

A case-study design was employed to understand the mathematical writing and thinking of fourth graders and the teacher's interpretation of their writing. Denscombe (1998) suggests that the aim of case-study research is to illuminate the general by looking at the particular; the complexity of human experience leads researchers to case studies in the qualitative or interpretive tradition (Erickson, 1986). Dyson and Genishi (2005) contend that cases are constructed as researchers decide how to angle their vision on places that include many stories of the human experience. They emphasize that construction of meaning and the social environment are interconnected; in case-study research, the phenomenon is explained as it is interpreted within a particular case. In this study the social environment and interactions were integral to the development of meaning. Yin (1993) identifies the ability to deal with a variety of evidence as a distinctive strength of case-study research.

Case studies are a bounded system of analysis (Stake, 1995). In education, some examples of bounded systems are districts, schools, and classrooms. Case-study analysis is an in-depth, holistic, inductive, and recursive examination of themes and patterns in the data. The purpose of case-study research is to describe, explain, and explore a particular topic (Yin, 2009). The strength of case studies is their detail, complexity, and use of multiple sources to obtain multiple perspectives (Rossman & Rallis, 2003). Stake (1995) differentiates three types of case studies: intrinsic, instrumental, and collective. The intrinsic case study contributes to the better understanding of a particular case, the instrumental case study examines a case to provide insight into an issue or draw a generalization, and the collective case study investigates a population or general condition. I used an instrumental case-study approach and connected the rich in-depth description on this single case to the general issue of literacy and mathematics.

Design-Based Research

According to Barab and Squire (2004), the commitment to examining learning in naturalistic contexts, which are designed and systematically changed by the researcher, requires the application of a design-based research (DBR) framework to derive evidence-based claims from these contexts. In this context, the research moves from observation to active implementation and involves systematically engineering the setting to improve and generate evidence-based claims about learning. This type of research provides a means for addressing the complexity that is a hallmark of educational settings (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). DBR offers several benefits: (a) research results that consider the role of social context and have better potential for influencing educational practice, tangible products, and programs that can be adopted elsewhere; and (b) research results that are validated by the consequences of their use, providing consequential evidence or validity (Messick, 1992). DBR entails both engineering particular forms of learning and studying those forms; the iterations are similar to systematic variation experiments (Cobb et al., 2003). In this case study I examined the process and changes made to the Mathematics Writers' Workshop model on a biweekly basis, worked closely with the teacher to create mini-lessons, provided instruction to a small group, and conducted conferences with students. This process was critical for gaining insight into the use of writing in mathematics.

Research Context

Description of the Setting

The study took place at Sunny Brook Elementary school (name has been changed), a public school located outside a southeastern city in the United States. The demographics for the community in which this school resides are as follows: 83% Caucasian, 12% African American, 3% Other, 1% Mixed Race, and 1% Asian. The education level of the surrounding community is 52% high school graduate, 33% high school or less, 13% bachelor's or associate's degree, and 3% graduate degree. The top three industries for employment are manufacturing, education/health, and retail/wholesale.

Sunny Brook Elementary is a Title I school that serves kindergarten through fourth grade. The estimated enrollment was 329 for the 2011-2012 school year. The student demographics at Sunny Brook are 43.47% Caucasian, 29.18% Hispanic, 25.53% African American, 1% Asian, and .91% Native American. Sunny Brook Elementary was the site for this research study for many reasons, including its participation in the Math and Science Partnership (MSP) grant, which I was affiliated with for three years.

The MSP grant lasted three years. Each year a cohort of kindergarten through fifth-grade teachers participated in a 70-hour, year-long professional development project centered on *Math Investigations*, a standards-based mathematics curriculum. The grant conducted project evaluations through teacher observations, observations of the professional development workshops, teacher videos, and statistical analysis of students' pre- and post-assessments. Several teachers in the first cohort who were identified as strong leaders became teacher leaders and facilitated professional development workshops for cohorts two and three. As part of the evaluation team, I conducted approximately 30 observations and attended most of the professional development sessions. I was able to foster a relationship with teachers during observations in both the fall and spring.

Dr. Polly, one of my committee members and a leader on the MSP grant, introduced me to Michelle (name has been changed) in 2010. Michelle, a member of cohort one, became a teacher leader and remained involved with the grant for two additional years; I was able to observe Michelle's fourth-grade math class on several occasions. My connection to the faculty at Sunny Brook, and more specifically to Michelle—though the MSP grant—allowed me to discuss my research ideas with them. Michelle participated in my pilot study in the spring of 2012; I chose her for both the pilot study and my dissertation case study for several reasons. I had observed her teaching and found her classroom to be an environment that nurtures higher-level thinking. She poses cognitively challenging questions, allows students to explain multiple strategies, and fosters collaboration between peers. Michelle also uses a writers' workshop during language-arts instruction, believes in the value of writers' workshops, and expressed enthusiasm at the prospect of translating the model into a Mathematics Writer's Workshop. We developed a relationship that extended beyond the pilot study into the summer. During the summer, Michelle and I exchanged emails, had phone conversations, and met in person to discuss plans for the dissertation study. These conversations further solidified my insider status. I documented these interactions to continually reflect on and use as analytic notes.

Pilot Study

The purpose of the pilot study was to gain a greater understanding of how the implementation of writing in the mathematics classroom influences student learning. The pilot study was guided by the following research questions:

1. How does the teacher perceive the value of the instructional approach (journal writing in math), and what are the expectations?
2. How do teachers support students' writing in a math journal?
3. How does writing in a math journal influence students' understanding of mathematics?

The pilot-study analysis was based on a social constructivist framework. The study took place in Michelle's fourth-grade class at Sunny Brook Elementary School. The three other fourth-grade teachers also used the journal prompts and submitted their students' work to me for analysis. The data included an interview and student journals; findings were a mixture of description and analysis using the theoretical framework of the study (Merriam, 1997).

During classroom instruction, teachers provided at least two writing prompts to their students each week for two months. Students had 3-10 minutes to complete prompts in their mathematics journals. Journal entries were entered into an Excel spreadsheet, read, coded, and organized according to themes. The thematic analysis began with open coding to allow categories to emerge (Ezzy, 2002). Data were futher analyzed with a constant comparison thematic analysis.

Findings

Research Question #1: How does the teacher perceive the value of the instructional approach (journal writing in math), and what are the expectations?

On March 23, 2012, I conducted the first interview to gather data for my research on math journals in four fourth-grade classes and their influence on math understanding. Michelle was interviewed three days before implementation of the journals. This interview served as a pre-research interview, which allowed Michelle to share her beliefs about writing and mathematics and her expectations for the project and what purposes the journals would serve for the students. Her perceptions of the value of the instructional approach were identified through the expression of her pre-journal thoughts, emotions, ideas, and expectations surrounding the project before its implementation.

Michelle exuded excitement and positive expectations and stated that this group of teachers is very open to trying new things. Her feelings were evident when she said, “I’m excited to try the journal and see how it works with them as their morning work. . . . We’re willing to try just about anything with this group.” She expected overall positive results from the use of journals in mathematics and said that the journals would enhance understanding of mathematical concepts, which would further solidify concepts. She expressed the idea that knowing a concept is at one level; being able to write clearly about that area is at a higher level. She expressed these ideas throughout the interview.

For instance, “We’ll get a better idea of what they’re actually understanding and where they are misstepping. . . . I think if they can explain it in writing then they really have a concrete understanding of what they are doing.” When asked about sharing journal responses, she responded, “Sharing how they solve the problems, I think . . . will help them . . . to see how their writing is, but I also think that it will—what they say may help someone else who is struggling with it, or to look at it a different way.” She expected that the sharing of the journals would foster collaboration, offer classmates insights from one another, and broaden ideas about writing in mathematics.

Research Question #2: How do teachers support students' writing in a math journal?

The implementation procedures were chosen by Michelle and included strategies for support. She explained to me that the journals would be used for two weeks for a total of nine days. Although the prompts were provided, she felt that discussion and modeling would be essential to students’ understanding of expectations. Her feelings about the structure of journal writing were apparent in this excerpt from the interview “I think the questioning or the question or the prompt would have to be something that’s very specific [so] that they would hit those different things, whether it was the patterns or the place value or the breaking apart of numbers, or . . . I think it would have to be, somewhere in the prompt, very specific in the prompt of what you would want from them.” The fourth-grade classes were grouped by ability, and her class was considered to be at a lower level for math. The amount of support and modeling were increased due to the ability grouping. She mentioned that focused prompts were a vital part of the project. Similar to the writer’s workshop format already in place, the students will also share responses from their journals. She felt that the journals would provide a window into the level of understanding attained by each student. In particular, the journal of a student who seems to understand a mathematical concept or “flies under the radar” would present evidence of their actual level. The journal can also be used for EOG (end-of-grade test) review. Michelle will use the window of information provided by the journal to adjust her lesson planning and provide more support.

Research Question #3: How does writing in a math journal influence students' understanding of mathematics?

Originally, I planned to examine the journals of the students from each of the fourth grade classrooms, but due to time constraints I examined only five journals from Michelle’s classroom. This allowed me to continue with the constant-comparison analysis plan. The themes that emerged from the journals were the role of modeling, comprehension, strategy, and reflection. It became evident that the first two journal prompts were completed with modeling and scaffolding from the teacher. The five journals analyzed had the same distinct explanations for the operations of carrying, borrowing, perimeter, and area. Example 1 illustrates the distinct responses that show the modeling effect.

Example 1

Josh – Area is the inside surface of an object. For example you would use area to decide how much carpet to buy, or grass seed to buy, our how much flooring to buy.

Matt –Area is the inside surface of an object. For example you would use are to dicide how much carpet to buy, or grass seed to buy, or how much flooring to buy to find area you would multiple the lenth times the width

Sarah – Area is the inside surface of an object. For example you would use area to decide how much carpet to buy, or grass seeds to buy, or how much flooring to buy.

The responses were lengthy and well expressed, but failed to reveal the true thought process of the individual student. However, the modeling may have provided the example that influenced future entries. The rest of the entries presented more data by the individual student. The third entry asked for an explanation of the relationship between multiplication and division and why learning these inverse operations together would be helpful. This entry varied drastically between the five students. The depth of their knowledge of these operations was apparent in their entries. Example 2 demonstrates the individual responses and depth of knowledge.

Example 2

Sarah –Even though when you multiply your numbers get bigger and when you divide they get smaller, using your multiplication facts can help you in division. I (f) you have a division problem like 24÷6 = ______ if you know 6 X 4=24 you know it has to be 4 or if you have a big one like 240÷12 = ______ you could use a multiplication fact to start:

12 x 10 = 120,

12 x10 = 120

12 x20 = 240It's soooo easy!

Josh- Multiplication and divided are oppos thing but they are almost the same. Like 10 x ____ = 50 and divided is 50÷ _____ = 5 they are like the same but a little different it is oppsit.

Nick – In a division problem to get the quotient you would multiply to get the division problem like 22 x 12 = 244, 244 ÷ 12= 12 we these operations because we can teach other people it

In nearly all of the entries, strategies were explained and examples of how these strategies could be implemented were provided. The last theme of reflection was represented in several entries. In particular, the fifth entry was a multistep problem that required a full description for solving the problem; it also included a sample of a wrong response and asked for advice for the fictional student with the wrong answer. Each one of these entries indicated a precise reflection of the processes used to solve the problem, followed by sound advice for the fictional student. The advice included reading carefully and doing one step at a time or following step by step to avoid getting mixed up. Example 3 is of the fifth entry, and the responses illustrate the use of strategy and reflection.

Example 3

1) The prompt says Marley ran 5 miles a day for 5 days on the sixth day she ran 4 miles and on the seventh day she ran 6 miles. How many total miles did she run in seven days? Marley needs to complete 30 miles a week for her training, did she complete the needed miles? If so, did she go over and by how much? If not, how many miles did she miss in her training?

   Nick – 5 x 5 = 25 + 4 + 6 = 35 She ran the miles she wanted to run, she ran extra 5 miles. I multiplied 5 times 5 and the answer was 25. I added 4 miles equals 29 miles. I added 6 miles equals 35 miles cause I added. Yes, she ran 35 but she only needed to run 30 miles. She ran 5 miles more. I can do 35 subtract 30 = 5 miles.

> He (Bert) should have added 5 extra but before that he should have 25 + 4+ 29 and 29 + 6 = 35
>
> Sarah – I timed 5 x5 = 25 miles because she ran 5 miles a day for 5 days. I added 4 +6 = 10 miles because she ran those miles. (on the last two days). I added to get my total 25 + 10 +35. 35 – 30 = 5. Marley ran 5 miles over so she complete the needed miles. I would tell him (Bert) to read the question very carefully then I would tell him to do 5 x5 =25 because she ran 5 miles for 5 days. Then add 4 +6=10 because she ran 4 miles and 6 miles on the last 2 days. Then add it together 25 +10=35 and so Marley is 5 miles over.
>
> (** Sarah also wrote notes to herself next to the problem. The notes included an arrow pointing to a section and writing multiplication and addition, on the last 2 days they had different amounts, and another arrow explaining follow then step by step you might get mixed up.)

This pilot study examined the implementation of journal writing in the mathematics classroom to gain a greater understanding of its influence on student learning. It deepened my relationship with Michelle and further confirmed Sunny Brook as the setting for the dissertation study.

Description of Participants

This case study used purposeful sampling, which according to Patton (1990) means that I selected for in-depth, information rich data. The pilot study data analysis showed that these participants and site would provide the type of data described by Patton. In the pilot study interview with Michelle, she informed me that her teaching career spanned 15.5, years, which included third through eighth grade. The semi structured interview revealed her expertise in teaching and her enthusiasm for combining literacy and mathematics. The MSP grant included formal and informal observations of Michelle's class; I examined anecdotal notes from those visits and judged her method of teaching to be conducive to implementing a workshop model. After reviewing both the observations and interview data, I chose Michelle and her fourth-grade class to participate in this case study. Michelle had 18 students in her mathematics class for the 2012-2013 school year. Fourth grade students range in age from 8 to 10.

Data Collection Methods and Process

I used multiple sources of data; this allowed me to triangulate my methods of collection and the individual data pieces in order to address the validity of my findings. Data were collected in the form of observations, field notes, unstructured interviews, and students' journals. I also used a daily journal to capture my thoughts, questions, and initial analysis and to assist in the practice of reflexivity. The collection process happened in stages. The first or planning stage began in the summer of 2012. The pilot study spurred our desire to continue examining mathematical writing in Michelle's fourth-grade class. I explained an overview of my interest in using a workshop model in mathematics, and she decided that she wanted to begin her new school year with the workshop format. We communicated regularly about this upcoming project and I assisted her in the workshop plans for the first two weeks of school. Our communication included an outline of the research process, my role as the researcher, and an approximate timeline. All of these interactions were documented and included in the analysis. These first few weeks were underway as I worked with my dissertation committee and reviewed the IRB approval for the pilot study for any necessary amendments.

In stage two, I began visiting Michelle's classroom on the three days a week that the workshop model in mathematics was used, and did this for six weeks (Table 3.1). Five days were built in every two weeks to analyze the data and make changes to the model. Observing three days a week was important, according to research on writing (Calkins, 1983; Graves, 1986; Murray, 1968). In order for writing to be beneficial, it needs to be a regular part of instruction. While conducting the case study, I interviewed students with their writing samples to gain greater understanding of their work and their experiences. In the final stage, I followed up with Michelle with a post-interview to fully explore her experience and future plans for her class.

Table 3.1: Timeline for the study

Name*	Sex	Week 1 - 10/29-10/31	Week 2 - 11/5-11/9	Week 3 - 11/12-11/16	Week 4 - 11/19-11/23	Week 5 - 11/26-11/30	Week 6 - 12/3-12/7
Andrew	Male	X	X	X	X		X
Albert	Male	X	X	X	X	X	X
Alexis	Female	X	X				
Phoebe	Female	X		X	X	X	X
Harold	Male	X	X	X	X	X	X
Ivory	Female	X	X				
Jari	Female	X	X	X	X	X	X
Joe	Male	X	X				
Kate	Female	X	X				
Linda	Female	X	X	X	X	X	X
Marsha	Female	X	X				
Mark	Male	X					
Madison	Female	X					
Oscar	Male	X	X				
Sylvia	Female	X	X	X	X	X	X
Tom	Male	X	X				
Joselyn	Female	X					

Zoe	Female	X	X	X	X	X	X
Hannah	Female	X	X	X	X	X	X

*All names have been changed.

Observations

Observations provided a context of the classroom interactions that informed the students' writing. While conducting observations, I used the iPad application Audiotorium to orally record detailed field notes on the setting, participants, lessons, and events and make notes about brief conversations and comments, which were then automatically downloaded to Dropbox. The writers' workshop model contains several components that are recursive in nature; however, each part is important. The model consists of mini-lessons, writing, conferencing, and sharing. I used an observation protocol that identified the ways in which these parts are used and constructed in the math writers' workshop (see Appendix A). I took extensive field notes of the mini-lessons and the teacher led discussions. I sat in different parts of the classroom to record the students at various tables as they shared the experience of solving problems in mathematics and engaging in the writing process. I observed the sharing component of the workshop model and noted the reactions and responses of the other students. I copied the students' mathematics notebooks to further clarify my notes for the day. I included my own thoughts and reactions in the field notes. Each day, within 24 hours of my latest observation, I accessed my field notes from Dropbox, reviewed them for accuracy, and added any relevant details. I also transcribed the recorded conversations from Audiotorium. I created analytic memos on a weekly basis, which allowed themes to emerge from the data. Every two weeks I met with Michelle to discuss students' writing;

this meeting represented another observation. I used this time to carefully note Michelle's thoughts and plans for the next two weeks. These meetings led to changes in the format of the workshop.

Collaborative conversations

This study employed a design-based research framework within the case-study design. Describing this method, Barab and Squire (2004) state:

> Design-based research is not so much *an* approach as it is a series of approaches, with the intent of producing new theories, artifacts, and practices that account for and potentially impact learning and teaching in naturalistic settings. (p. 20)

With this framework in mind, the collaborative conversations took place every two weeks. However, Michelle and I also engaged in many informal conversations throughout the study. My role as the researcher was to assist in planning and administering the following two weeks' worth of Mathematics Writers' Workshop lessons, according to Michelle's planning. As Michelle examined the journals of her students, I carefully noted her interpretations and assessments of the students' writing. I analyzed these journals myself; these interactions served to highlight how Michelle was able to use the students' writing to inform her instruction. The goal of these discussions was to identify the ways in which the journals had implications for Michelle's strategies and, if they were deemed appropriate, changes to the workshop model.

I used Audiotorium to record and take notes during these sessions, transcribed these immediately, and included a scanned copy of the journal entries had Michelle used for planning. These analytic notes and the themes of the students' journals were compared and analyzed.

Student Journals

Students used a composition notebook during the Mathematics Writers' Workshop. The workshop lessons included prior-knowledge prompts, mathematical problems related to the mini-lesson, and prompts geared toward reflection. Each of these components was typed, copied, cut to fit their notebooks, and glued into the notebook in appropriate areas for students to complete the tasks and easily refer back to their writing. The Mathematics Writer's Workshop occurred three times a week, but these journals were also used during other math lessons for students to reflect on their prior writing. Students were aware that these were their journals and that their writing would be used to inform instruction and enhance discussion, but would not be graded.

I read each of the journals and developed themes; this was an ongoing process over the six weeks. The themes from the journals were discussed during collaborative conversations.

Researcher's Journal

My researcher's journal served as a place to write my thoughts, questions, ideas, and overall feelings throughout the study. If patterns emerged from the meetings and observations, I recorded those notes in the journal. I used my researcher journal daily and shared my thoughts from this journal in peer debriefing meetings.

Table 3.2 Researcher Journal

Researcher Journal: 10/29/2012 10:50-12:20
Today's workshop centered on the idea of open arrays. The students were asked to take the problem 12 x 7 and find a way to break it into a friendlier problem (10*7) + (2*7). This problem was used as the mini-lesson to review this concept. The students explained how to break up the problem and their verbal response was turned into a written response on chart paper, this was to serve as a model for the students when they engaged in the activity and the writing. This workshop took place with half of the group for 45 minutes and the other half of the group for 45 minutes. The students met on the carpet to participate in the workshop. The carpet area was not the best area for students to work and for conferencing to take place. The students were squirming and distracted by each other. It was difficult to circulate and have conferences. The students worked on solving two questions (how many legs on 21 spiders? & How many legs on 28 horses?) Even though the students explained the concept of breaking up arrays during the mini-lesson many reverted to repeated addition. Those that multiplied struggled with explaining their steps. Students struggled with the concept of explaining their thoughts and strategies in writing. The students are considered to be of lower ability in mathematics, their writing also indicates that their literacy levels are low as well. As the students were wrapping up math and preparing for lunch Mrs. L and I talked about the dynamics of the carpet area and decided to use another table to elongate one area of desks and conduct the workshop in a desk setting the next day. The carpet may be a viable component for sharing once the students start to get the hang of the workshop.

Interviews

I talked with Michelle regularly, documented those conversations, and used the data to assist in creating a semi structured interview for the end of the study. This interview addressed the ways in which she used the journals for instruction and her future plans to use the workshop model in her class.

Semi structured Teacher Interview

1. In what ways might you use the findings from the students' writing for instruction?

2. During this process we looked at the writing and made changes to the grouping of students. We also talked about what they understood and the types of questions to include in the workshop. With all of this in mind, do you think their writing is helpful for these purposes?

3. Did you find the journals useful for you and your students?

4. What are your future plans for the workshop model?

After analyzing the students' journals, I conducted semi structured interviews with several students based on their journals and directly related to their writing. These interviews served as a form of member checking, which increased the trustworthiness of the analysis.

Semi structured Student Interview

1. How do feel about Math Writers' workshop time?

2. Can you share your writing from your journal?

3. Do you think writing in Math Writers' workshop is helpful?

4. Do you like writing in math?

5. Does writing in math help you understand a problem better?

6. Does writing in math help you use more math words?

7. What was your favorite part about writing in math?

Data Analysis

In most qualitative studies and those using a DBR framework, data analysis takes place during data collection. Data were analyzed using an inductive approach that included an in-depth and thorough description. Data analysis was a time to organize the data from the journals, collaborative meetings, observations, interviews, and my researcher's journal. The goal of organization was to identify and gain analytic insight into the dimensions and dynamics of the phenomenon being studied; this process is inductive and grounded in the collected data (Dyson & Genishi, 2005). I used a thematic analysis and began with open coding to allow categories to emerge. Data were further analyzed with a constant-comparison thematic analysis, which allowed data to be grouped and differentiated as categories were identified (Ezzy, 2002). This process required multiple readings of each piece of data in order to find patterns of reemerging themes. Open-coding led to a plethora of codes and categories, and as I became more familiar with the data, these codes were revised and collapsed.

It was important to transcribe data in a timely manner for effective analysis. I had my researcher's journal with me at all times to increase my reflexivity and capture and organize data as it occurred. It was important to engage in these activities so that themes could emerge along the way and the most pertinent themes could become evident.

Trustworthiness: Reliability and Validity of Design

An important part of observation relates to the idea of contextualization: To understand behavior, the observer must understand the context in which individuals are thinking and reacting (Wiersma & Jurs, 2009). Glesne (2011) outlines the meaning of the researcher as a participant observer; this role resides on a continuum that ranges from "primarily observation" to "primarily participation." For my research, I participated in the planning and administering of the workshop model and then gravitated toward being primarily an observer as participant during whole-group instruction. I observed the lessons several times a week and was available to attend planning sessions to develop rapport and listen to Michelle's analysis of the previous lessons and students' writing.

The researcher is at the center of qualitative research, and therefore, addressing reliability and validity is essential for credible findings. Several strategies were used for quality control of the data analysis. During this anylsis, I employed peer debriefing, which offered many benefits regarding the direction of the study, data analysis, and trustworthiness of the findings. Spall (1998) posits that peer debriefing should be conducted at crucial junctures to make the researcher aware of the influence of his or her own personal values, provide opportunities to test theories and interpretations of the data through discussion, discuss problems, and plan. Peer debriefing, which is a process of exposing oneself to a disinterested party, helped me become aware of the influence of my personal values and theoretical orientations on the collection and interpretation of the data (Ezzy, 2002). Dr. Wang, my methodologist, served as one of the peers that I frequently consulted. I shared my findings as the developed from the analysis. Dr. Wang was well aware of my personal values and helped to limit their influence over the analysis.

Another strategy for ensuring trustworthiness of data is to practice both confessional and theoretical reflexivity. Confessional reflexivity requires the researcher to turn inward in a critical manner, producing awareness of our own subjectivity and dispelling the notion of absolute truth, whereas theoretical reflexivity goes back and forth between the concrete experience and the abstract theoretical explanation of that experience (Foley, 2002). Using these methods of reflexivity during my analysis strengthened my interpretation.

Analyzing the data while the study is ongoing allowed the point of saturation to become evident. Having a point of saturation strengthened the analysis because the themes began to reappear over and over. Validity and more accurate conclusions are increased by the use of multiple sources (Yin, 2009), which are then triangulated. Triangulation is a comparison of information to determine whether or not there is corroboration; it is the search for convergence of the data on a common finding or concept (Wiersma & Jurs, 2009). I used multiple sources, including observations, interviews, journals, collaborative meeting notes, and my researcher’s journal. I provided thick description, which gives readers a greater understanding of the study. Another technique for increasing trustworthiness is member checks. I regularly shared my interpretations with Michelle to verify the accuracy of what I had heard and the meaning behind her words. Student interviews related to their math workshop writing also served as a member check.

Possible Ethical Issues

There were minimal risks to participating in the study. Participants were involved voluntarily, and the workshop model was aligned with the standard course of study. In some cases, participants may have felt coerced to participate. This concern was mitigated by Michelle's high level of involvement and collaboration. Interviews can also cause psychological stress for the participant. This concern was addresssed partly by my estabilished relationship with Michelle. I also ensured privacy and used a semistructured model for a conversational interview. I also provided Michelle with my data analysis and will give her a copy of the dissertation to assist in future instruction and team planning. Journal entries and interview data were kept in a locked storage cabinet in a locked office at UNC Charlotte. Data on computers were kept in password-protected documents. Participant names were kept confidential and, for the purpose of disseminating study findings, have been changed. Master lists were destroyed. All data will be destroyed five years after completion of the project.

Limitations

Part of demonstrating the trustworthiness of data is to realize the limitations; limitations are consistent with the partial state of knowing in social science, and elucidating limitations helps readers understand how research should read and interpreted (Glesne, 2011). In this study, limitations due to the purposive sample, possible researcher bias, and classroom setting have been addressed as fully as possible. The sample, as described earlier, included a teacher who had positive expectations for writing in mathematics; I, as the researcher, also believe in the value of writing in mathematics. Both Michelle and I may have had preconceived assumptions that influenced the interpretation of data. Reflexivity and peer debriefing were critical to addressing this potential bias.

Summary

In this case study I have sought to understand the process of creating a workshop model for mathematics and the ways in which students use mathematical writing for reflection and problem solving. The study included one fourth-grade classroom with one teacher and approximately 25 students. The study lasted six weeks, from late October to mid-December of the 2012 Fall semester. Data included student journals, observations, collaborative meeting notes, interviews, and my researcher's journal. All of these sources were used for triangulation. Data analysis was inductive in nature and included a thematic analysis along with constant comparison. The following chapter discusses the findings of the study.

CHAPTER 4: FINDINGS

The purpose of this study was to examine an alternative approach to using writing in mathematics instruction. Specifically, I examined how the Writers' Workshop components of mini-lesson, writing/conferencing, and sharing were adapted to create a mathematics workshop. I employed design-based research methodologies to examine the data on a biweekly basis and make adjustments with the teacher. The questions, data sources and findings are included in the table below:

Table 4.1 Questions, Sources and Findings

Questions	Data Sources	Findings
1. How do students use writing to reflect on their learning in mathematics?	Student journals, interviews, observations, field notes, researcher journal, and collaboration meeting notes.	Students used their written reflections to explain and reflect on their thinking. Their writing revealed their learning of mathematical concepts, strategies, and vocabulary. The students recognized the value of writing in the interviews and asserted that it was a tool for learning. This information was used by Michelle and me to differentiate and individualize instruction.

Table 4.1 (continued)

2. How do students use writing to reflect on their learning in mathematics?	Student journals, interviews, observations, field notes, researcher journal, and collaboration meeting notes.	Students used their written reflections to explain and reflect on their thinking. Their writing revealed their learning of mathematical concepts, strategies, and vocabulary. The students recognized the value of writing in the interviews and asserted that it was a tool for learning. This information was used by Michelle and me to differentiate and individualize instruction.
3. How do students use writing to show how they solve mathematical problems?	Student journals, interviews, observations, field notes, researcher journal, and collaboration meeting notes.	In the workshop model, students were encouraged to explain their responses using words, drawings, and examples. Many workshops included a section that directly asked students to explain their process. In these sections, students' writing revealed how they solved mathematical problems. The process they used to solve problems showed their level of efficiency and their understanding.
4. How does the teacher adjust her lesson plans in response to writing produced by students using the writers' workshop	Student journals, interviews, observations, field notes, researcher journal, and collaboration meeting notes.	The third finding demonstrated the ways in which Michelle and I adjusted their teaching according to the students' writing. The writing was used to inform the

model?		grouping of students and how to differentiate lessons between those groups. Student writing was the main source of information for interacting with students. Their reflections, explanations, and calculations provided insight into their thinking. Conferencing questions and topics emerged from their writing, and instruction was embedded into the conference. Notes from these conferences influenced lesson planning.

In this chapter, I discuss the three main findings and the subcategories within each finding from this study. The analysis of the findings revealed how students' writing reflected their learning and provided insight into their problem solving, and how this information was interpreted for instruction. The findings were as follows (a) Students used writing as a tool to demonstrate their mathematical understanding; (b) students' written reflections informed the teacher's instruction, and (c) students' written explanations informed instruction. The chapter concludes with a summary of findings.

Students Use Writing as a Tool to Demonstrate Their Mathematical Understandings

The workshop model implementation in this study offered students a place to develop and demonstrate their understanding of the lessons and activities. Conferencing, classroom observation, and student participation were important resources for gaining insight into student understanding. However, the writing reflected students' mathematical understanding that was not always apparent in discussion and students' answers. Through the data analysis process, I noted how difficult it was to identify students' understandings based on only their answers to problems. Also, students' writing reflected their understanding of mathematical vocabulary. The data sources related to this category had three primary subcategories (Figure 4.1).

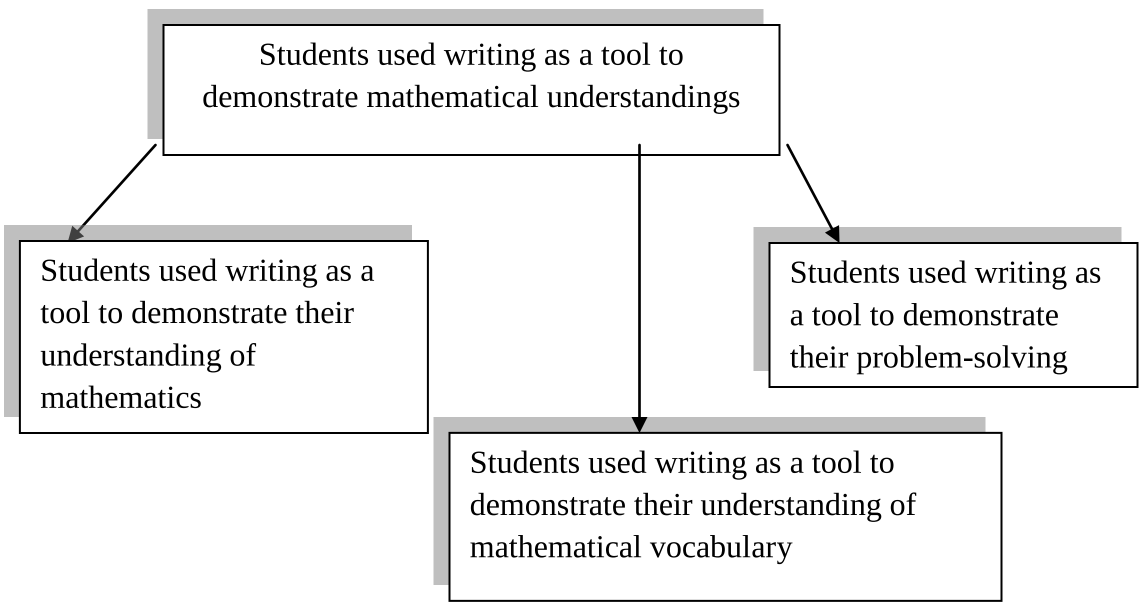


Figure 4.1 First Finding with Subcategories

Students use Writing as a Tool to Demonstrate Their Understanding of Mathematics

Students used writing to demonstrate their mathematical understandings and, in some cases, their misunderstandings. Their writing provided insight into their readiness for more challenging problems. It also highlighted connections students made between prior knowledge and the mathematical concepts presented in class.

Students' writing reflected their misunderstandings of concepts. The student writing in Figure 4.2 occurred after a mini-lesson on using the multiple towers strategy to find the answer to a division problem. This strategy encourages students to use the multiples of the divisor to add up to the dividend. For example, in the task 104 divided by 8, students would skip count by multiples of 8 until they reached 104. In this strategy, students recognize the divisor is the amount of groups they are creating out of the whole, known as the dividend. The size of each group, which is the quotient, is determined by the factor that is represented in the height of the tower; if going to the next level on the tower surpasses the dividend, the student stops and counts up or subtracts to find the remainder. This strategy can also help students solve division problems with larger numbers.

During the study one student, Albert, became more and more proficient with his multiplication skills and displayed an eagerness for challenges. Toward the end of the workshop time there was additional time and I was offering a few challenging problems. Albert said, "Make them three digits." I offered the problem 126 ÷ 25 (Figure 4.2). Albert usually solved problems successfully, but struggled with reporting his answer. In this instance, Albert looked at his tower and calculated both the quotient and the remainder, but only reported the remainder. While conferencing with Albert (Figure 4.3), he talked about his misunderstanding. We looked together at the work he had done, and when asked how to solve a problem he showed the multiple tower strategy. However, his writing showed his misunderstanding that quotients and remainders are the same thing; this is also supported by his answers, in which only one number is reported (Figure 4.2 A) despite his calculations. Albert studied his multiplication facts and often worked quickly; with only the calculations to review, I might have concluded that the recording of only the remainder represented an error made in haste. The writing indicated that Albert was unclear about the difference between quotients and remainders.

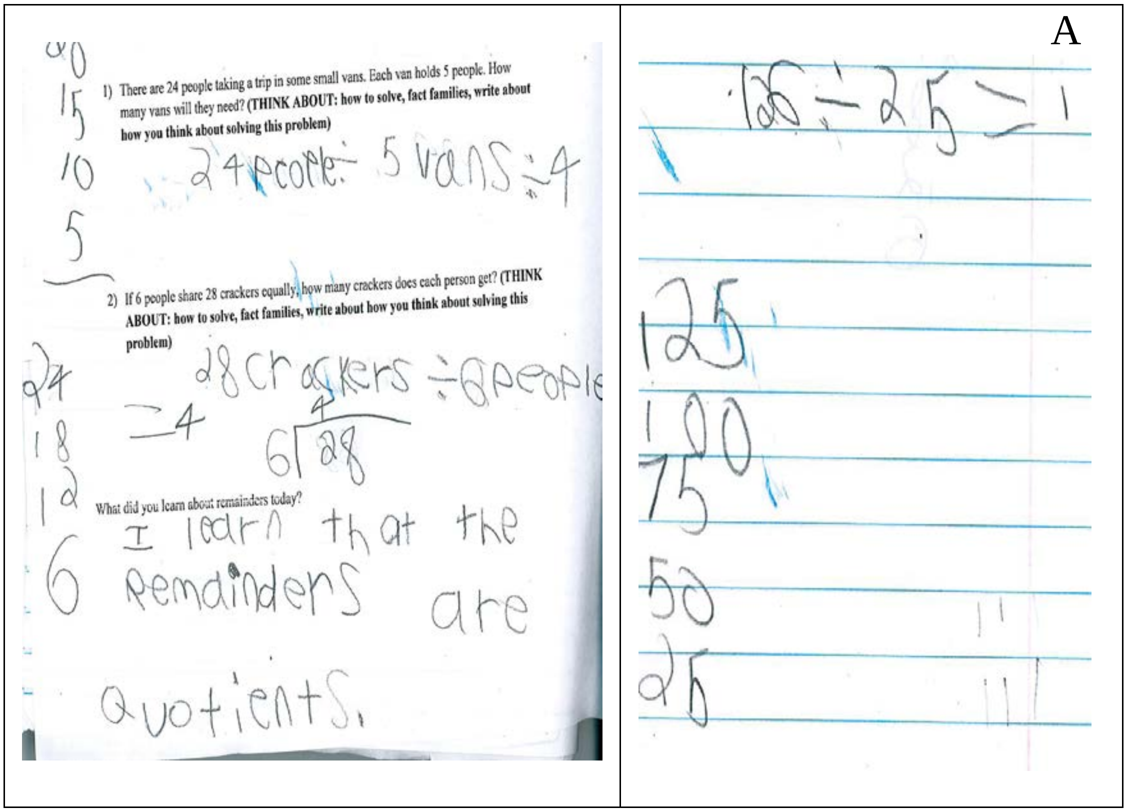


Figure 4.2. Albert's Writing Sample

- While talking with Albert he seemed full of eagerness to start work and feels confident in his multiplication skills, but he seemed stuck. I asked him what he thinks he should do to solve the division problem. He started to convey the multiple tower idea and wrote on the side in tower form 5,10,15,20; the problem was 24/5. I asked what made him stop making the tower and he said 5 * 5 = 25 so he couldn't go higher. I asked how many were left in the dividend (pointing to the 24) and he said 4. At that point I went to the next person and revisited him a few minutes later. He gravitated toward creating the towers and knowing the remainder, but struggled to decide how to report the actual answer. While we talked he seemed to understand what he needed to report.

Figure 4.3. Notes from Conference with Albert (11/9/12)

The above example of Albert’s writing (Figure 4.2) and my conference notes (Figure 4.3) revealed his misunderstanding of remainders and quotients. Albert was successful in building a multiple tower; however, when he had completed his work he was unsure about whether the answer was the quotient or the remainder. Albert’s confusion showed a lack of understanding as to why he is building the tower and how this computation affords him the answer. This information can help teachers to determine whether or not students can apply and understand various strategies. The information from the data was incorporated into the next workshop.

In the next workshop, Michelle began her instruction with three practice division problems followed by workshop time with the groups. I used the information from Albert’s writing (Figure 4.2) and conference (Figure 4.3) to create a mini-lesson for my group that outlined information for the operations of multiplication and division.

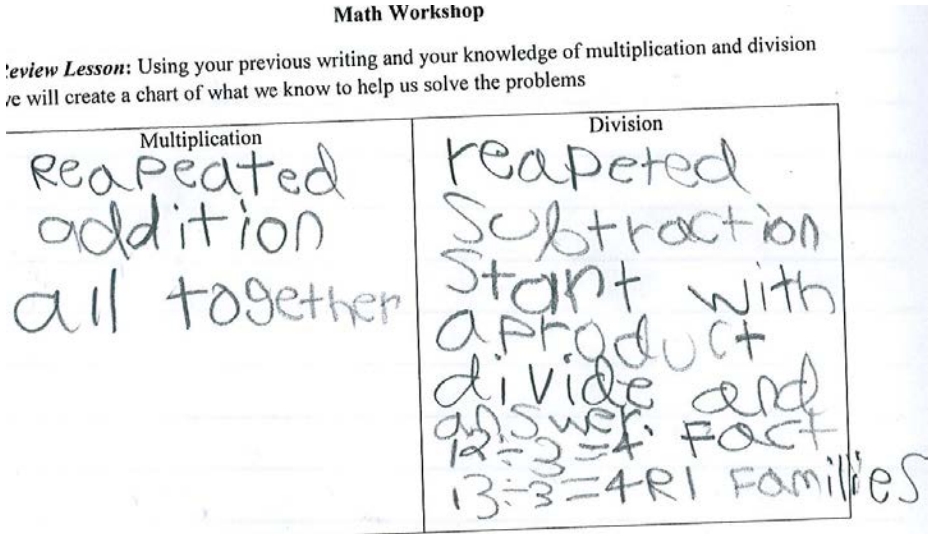
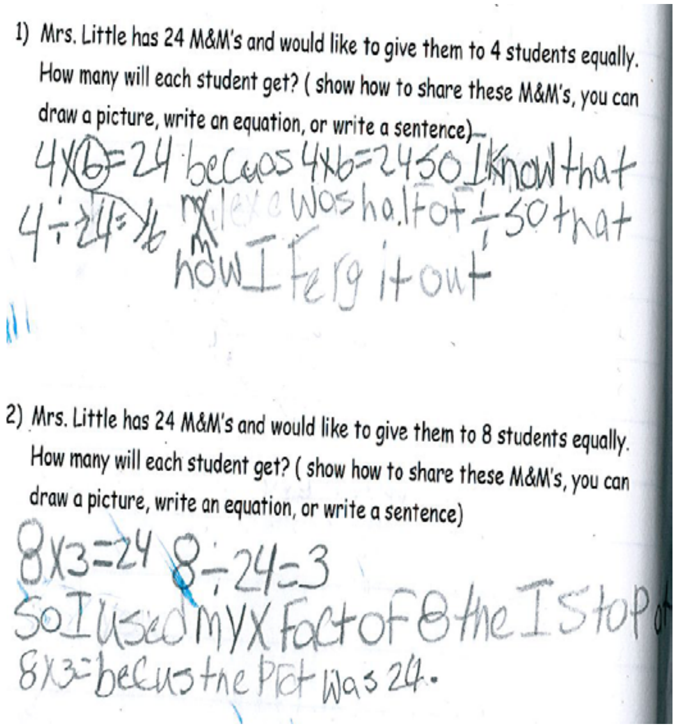


Figure 4.4. Multiplication and Division Mini-lesson 11/14/12

In the discussion and creation of the chart (Figure 4.4), we specifically explained that remainders are a component of the quotient. Albert included this information in his chart (Figure 4.4). In the next example, Linda explained her understanding of using multiplication to solve division problems. These problems included a bag of M&Ms to be used as a hands-on tactile manipulative for solving the problem. Linda’s work and description are included in Figure 4.5, while Figure 4.6 includes my conference notes made from this workshop.



1) Mrs. Little has 24 M&M's and would like to give them to 4 students equally. How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence)

4X6=24 becaus 4X6=24 so I Know that 4÷24=6 was half of ÷ so that how I ferg it out

2) Mrs. Little has 24 M&M's and would like to give them to 8 students equally. How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence)

8X3=24 8÷24=3
So I used my X Fact of 8 the I stop
8X3= becus the Pet was 24.

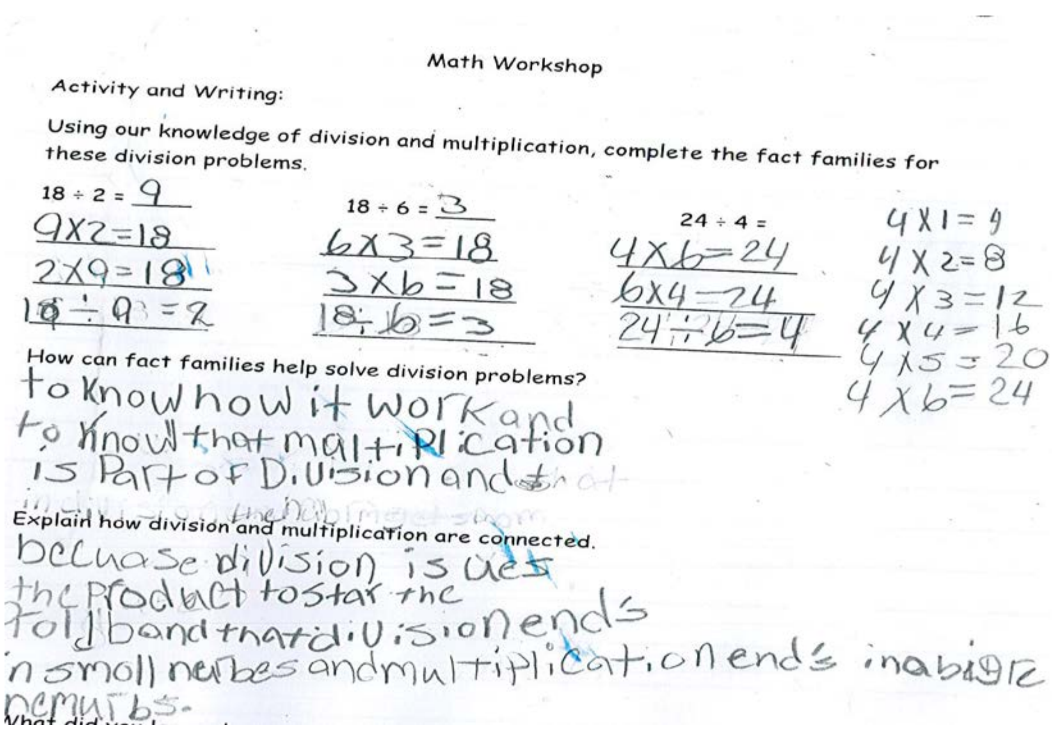
Problem 1	"Because 4 X 6 = 24 so I know that was half of division, so that how I figure it out."
Problem 2	"So I used my multiplication fact of 8 the I stop at 8 X 3 = because the problem was 24."

Figure 4.5. Linda's Writing Sample 10/30/12

M&M activity—Linda shares that she used multiplication to get the answer to the problem. The M& M bag remained intact. Linda was able to make a connection between division and multiplication to solve the problem. Her division problems are written with the numbers switched.

Figure 4.6. Conference with Linda (10/30/12)

The above example illustrates Linda's connection and understanding of the relationship between multiplication and division. The conference notes include that Linda progressed past using physical manipulatives and relied on her own mathematical knowledge. Based on her writing, Linda recognized a connection between multiplication and division; however, she still demonstrated a lack of clarity. Linda demonstrated her understanding of the connection between multiplication and division, but showed that she had applied the commutative property incorrectly by writing 8 ÷ 24 and 4 ÷ 24. The conference notes include the incorrect equation and the mini-lesson from the next day, along with the type of problems included to address this idea. Figure 4.7 shows the change in Linda's thinking about writing fact family equations.


Math Workshop

Activity and Writing:

Using our knowledge of division and multiplication, complete the fact families for these division problems.

18 ÷ 2 = 9
9X2=18
2X9=18
18 ÷ 9 = 2

18 ÷ 6 = 3
6X3=18
3Xb=18
18÷b=3

24 ÷ 4 =
4Xb=24
6X4=24
24÷b=4

4X1=4
4X2=8
4X3=12
4X4=16
4X5=20
4Xb=24

How can fact families help solve division problems?

to know how it work and to know that maltipication is Part of Division and

Explain how division and multiplication are connected.

becuase division is ues the Prodact to star the Poldband that division end's in smoll nerbes and multiplication end's in a bigre nemurbs.

"To know how it work and to know that multiplication is part of division.
Because division uses the product to start the problem and that division ends in smaller numbers and multiplication ends in bigger numbers."

Figure 4.7. Linda's Writing Sample 10/31/12

Linda moved past manipulatives and began connecting her multiplication facts for solving division problems in her writing in Figure 4.5. The conference and analysis of her work sample provided information for the following day. To expand on Linda's thinking, the workshop included a group discussion of division and multiplication. The ideas presented to the small group appeared in her written responses. In this next example, Tom’s writing reveals his understanding of multiplication and division. His writing also shows the strategies he appeared most comfortable using.

Students’ writing indicated their readiness for more challenging tasks. Figure 4.8 is a compilation of Tom’s responses to three workshops that occurred over two weeks.

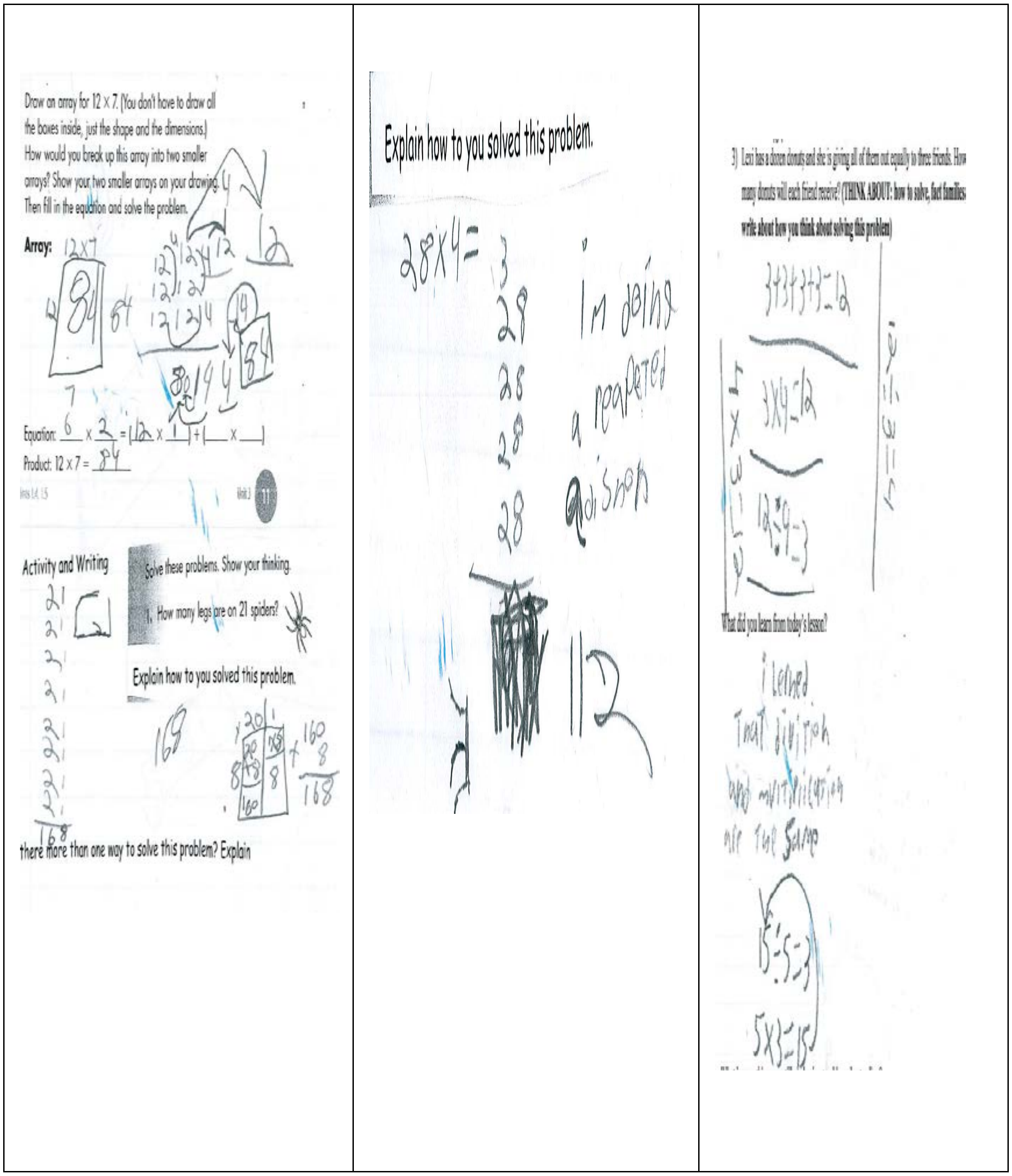


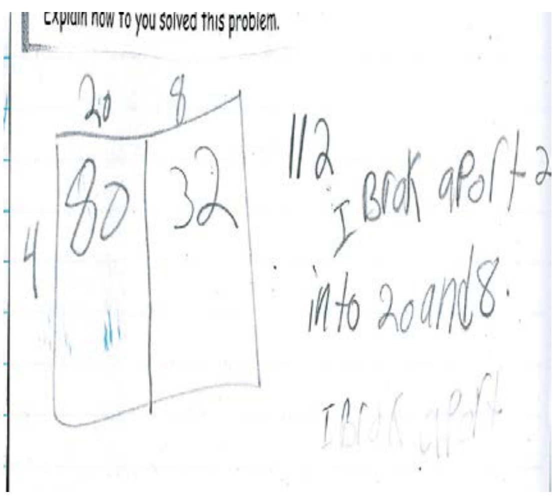
Figure 4.8. Tom’s Writing Samples 10/29/12

The lessons included in Figure 4.8 centered on multiplication and division. The multiplication workshop and lessons offered an open-array strategy or a breaking-up-arrays strategy. The array multiplication strategy takes the two factors being multiplied and breaks those numbers up into numbers that are easier to multiply. In this way one challenging multiplication problem becomes two or more easier problems. The products from the new multiplication problems are added together for the answer to the original problem.

One example to illustrate this strategy is 28 x 5. A student would draw a box split in half and multiply 20 x 5 and 8 x 5 and add the products together. The division workshop introduced “fact families” as a tool for solving division problems. A fact family is the four equations that are derived from the same three numbers included in problem. An example of a fact family is four equations using the same numbers. Tom’s math computation in the top section of Figure 4.8 shows a box with the problem 12 x 7. Students had been working on the strategy of breaking numbers up by place value, so that the left side of the box would represent 10 x 7 and the right side would represent 2 x 7. His statement—“I am doing repeated division”—and his calculations showing repeated addition indicated he was unsure how to use this strategy. In the second problem in the first section, Tom attempted to break the numbers up, but his work showed a repeated addition strategy. In the second section of Figure 4.8, he reverts back to repeated addition and describes his strategy in the explanation section as “I’m doing repeated division.” In the third section, the problem asks how to divide 12 donuts among 3 friends. Tom begins with repeated addition and continues from there to write the related fact family equations for 12 3. Tom reported that he learned that division and multiplication are the same, which indicated some misconceptions about multiplication and division. Based on his work, he appeared uncomfortable to be moving away from repeated addition.

In Tom's example, there appeared to be several misunderstandings regarding multiplication and division. Tom's writing made his confusion visible by providing information about his strategies and his definitions of multiplication and division. Tom's work illuminated his thinking and limited the amount of guesswork that may be necessary when using only numeric calculations. The inconsistencies in Tom's work indicated that he may be overwhelmed by more challenging problems at this point; however, additional practice with multiplication and division may be more appropriate.

In the next example (Figure 4.9), Hannah's computation and writing illustrate her readiness for more challenging tasks.

<table>
<tr>
<td>
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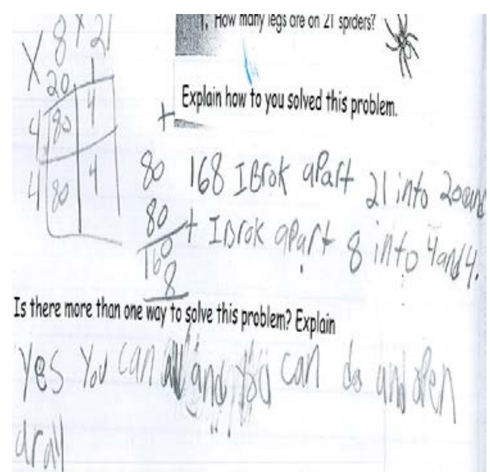
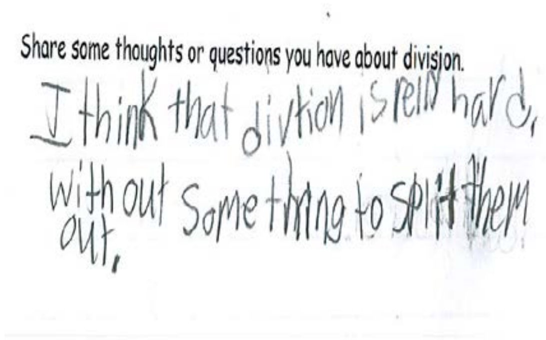
<td>
</td>
</tr>
<tr>
<td>"I break apart 28 into 20 and 8 ."</td>
<td>"168 I broke apart 21 into 20 and 1 I broke apart 8 into 4 and 4. Yes, you can add and do an open array."</td>
</tr>
</table>

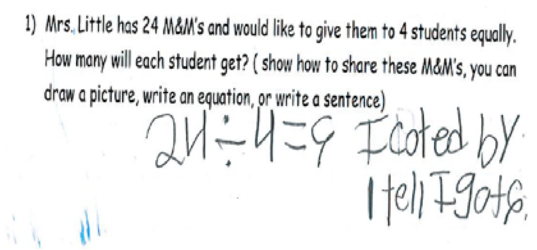
Figure 4.9. Writing Sample by Hannah 10/29/12

In Figure 4.9, Hannah showed she used the array strategy in two different ways. Her calculations are correct. She described how she solved the problem and recognized that multiplication represents repeated addition; she then noted addition was another way to solve multiplication. This work meets the instructor's expectation and objective for this lesson. Therefore, Hannah's writing seems to suggest a readiness for more challenging multiplication problems.

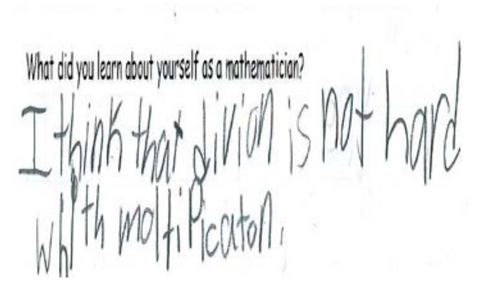
Students' writing showed their connections and understanding. Students engaged in writers' workshop in mathematics three days a week. An analysis of the students' writing showed connections from lesson to lesson. In the next example, Sylvia's writing shows changes and new connections from one workshop to the next. Figure 4.10 shows two responses, one from a workshop on division using manipulatives to create groups, and another in which students used fact family connections to solve division problems. Sylvia began a lesson on division by using a one-by-one strategy to distribute 24 M&Ms into 4 groups. In her writing she indicated that division is hard without something to split them out. In this response, she appeared to recognize the inefficiency of the one-by-one strategy. The next workshop addressed the inefficiency of this strategy with a mini-lesson focused on fact families. Students engaged in finding fact families for several multiplication equations that they had studied. Sylvia's writing on the next day seemed to indicate that she had connected the idea of fact families and division. Her responses showed that she understood how multiplication can be useful for division; this was further affirmed in her use of fact family examples for 60 ÷10.



"I think that division is really hard without something to split them out."



"I counted by one till I got 6"



"I think that division is not hard with multiplication."

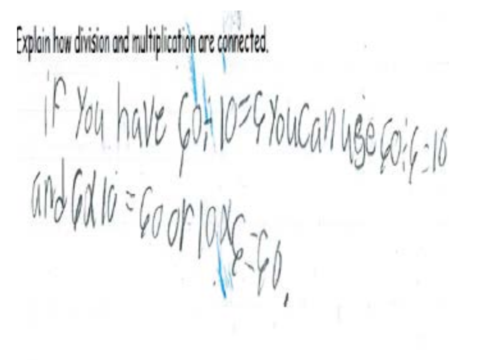


Figure 4.10. Writing Sample by Sylvia 10/31/12

The writing of Albert, Tom, Hannah, and Sylvia included in the examples above offer some insight into their understanding and misunderstanding. During the workshop on multiple towers, for instance, Albert's multiplication proficiency and eagerness to move to more difficult problems may have been enough to consider his reporting of just the remainder as a careless error. However, his writing showed a misconception about quotients and remainders that would need to be addressed to move forward. Tom's work and writing indicated a struggle with multiplication and division and a certain level of comfort with the strategy of repeated addition. Hannah's work with open array displayed proficiency with the strategy and an understanding of multiplication. In Sylvia's writing, her understanding of the connection between multiplication and division developed from one workshop to the next. She seemed to have an understanding of how this connection relates to solving division.

Student's reactions. In the interview excerpt below, Sylvia emphasized that she liked writing in math because teachers are able to read her work and gain insight into her mathematical understanding. Then, they can offer help if needed. In an interview with Hannah, she asserted that her mathematical understanding was contained in her writing, and this eliminated the chance of a teacher thinking she cheated. In her response, she referred to writing as a way for her to present her individual thinking, have ownership of her thinking, and share that thinking with her teacher. This response provides evidence that students solve problems and describe their problem-solving differently. Hannah appeared to recognize writing as a tool to show her understanding and highlight her individuality.

> Me: Do you like writing during math? Why or why not?
> Sylvia: [Pause] Um, yes, because then when the teacher checks your notebook they can see if you are doing good or if you need help.
> Me: What was your favorite part about the workshop?
> Sylvia: Um, when we would have to solve it and then write what we did.
>
> *Interview with Sylvia (12/07/12)*

> Me: Do you think writing in math is helpful?
> Hannah: Yes, because it explains the answer and it makes sure the teacher knows we haven't cheated.
>
> *Interview with Hannah (12/07/12)*

The writing examples reflected the students' mathematical understanding. The interview responses indicated that the students recognized that their writing showed their thinking and that this information will be important for a teacher to know where students needed more instruction and for teachers to be sure of the integrity of students' work. The mathematical understanding expressed in Tom and Hannah's writing suggested that instruction should include re-teaching and practice for Tom, versus extensions and more challenges for Hannah. The next subcategory reflects how students used writing to demonstrate their understanding of mathematical vocabulary.

Students Use Writing as a Tool to Demonstrate Their Understanding of Mathematical Vocabulary

Vignette 4.1

Michelle begins the class, as a whole-group discussion, with a question: "What did we do the other day?" Some students at their tables begin mumbling and whispering. From the mumbles comes the term "division." Michelle then asks the class, "How did we do it [division]?" Again, students engage in murmurs and whispers as they look at one another and around the room. Several student voices are heard in chorus saying, "We broke up the numbers." Michelle then asks the students to stand and take a count of the number of people in the room. The count includes the adults that are present. She then asks the large group to break into smaller equal groups. This activity occurs a few times. During this activity, there are several instances in which people are unable to join a group and fulfill the requirement of equal groups. When these instances occur, the terms "leftover" and "remainder" are generated from the group. Michelle asks the class, "Has that ever happened when you are sharing and you have remainders?" The class nods and seems comfortable with these terms. Everyone returns to their desk to continue with the lesson.

As each unit of mathematics was introduced, Michelle used proper terms and often displayed these terms in the classroom. The vignette above is an example of how Michelle introduced terms and incorporated those terms into a real-life activity in which students had to use division. Students were encouraged in mathematics instruction to use these vocabularies. When students used mathematical vocabularies in their writing, their understanding—or, in some cases, their misunderstanding—of the terminology became clear.

Students' writing illustrated their understanding of mathematical vocabulary. Students used mathematical vocabulary in their writing. Figure 4.11 shows two examples from two students; in each example, the student explained a mathematical idea and reflected her understanding of mathematical vocabulary.

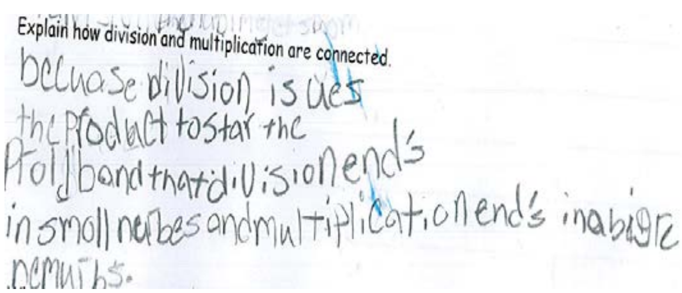
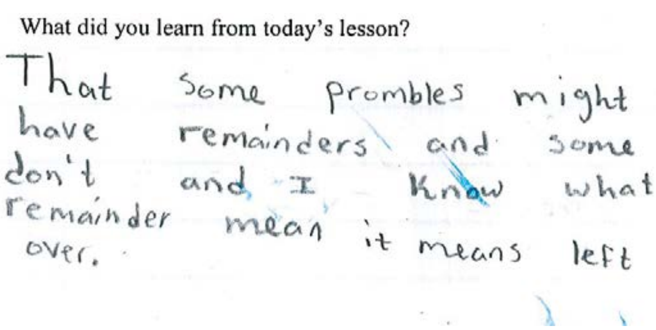
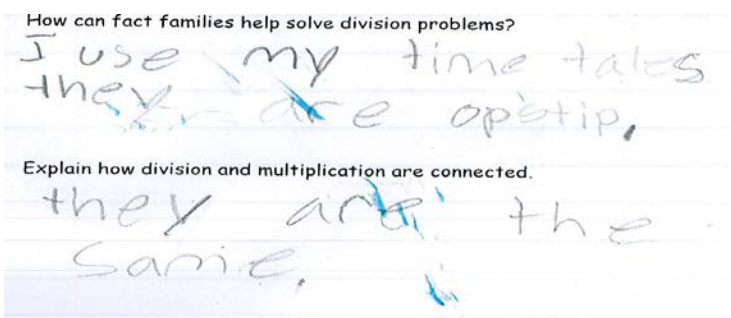
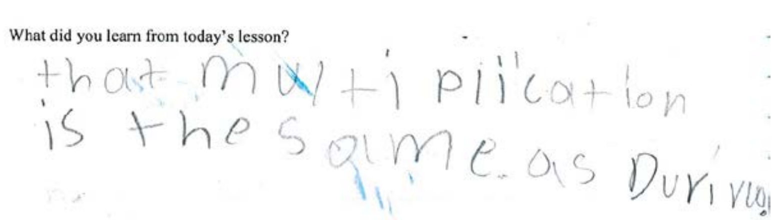
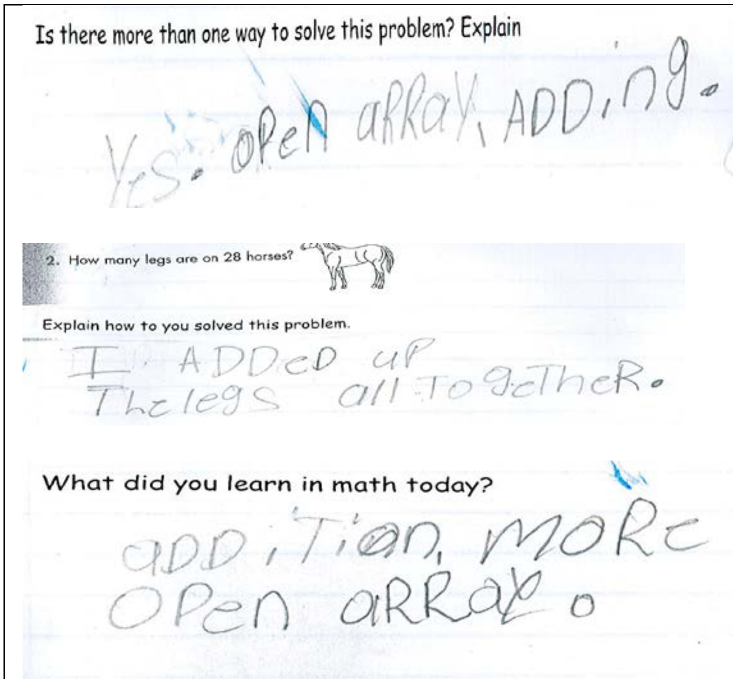
 Explain how division and multiplication are connected. becuase division is ues the product tostar the Prollband that division end's in smoll nerbes and multiplication end's inabigre nemuths.	"Because **division** uses the **product** to start the problem that division ends in small numbers and **multiplication** ends in bigger numbers." **Linda**
 What did you learn from today's lesson? That some prombles might have remainders and some don't and I know what remainder mean it means left over.	"That some problems might have **remainders** and some don't and I know what **remainder** mean it means left over." **Zoe**

Figure 4.11. Writing Samples by Linda and Zoe 10/31/12

In the first example, Linda used the mathematical terms division, product, and multiplication to explain how fact families are related. Her explanation included reasoning as to why answers in the multiplication of whole numbers are larger than the factors and why answers to whole-number division problems are smaller than the dividend. In the second example, Zoe provided a description of a remainder that included the definition and added that not all problems have remainders. Zoe's writing included terms from the activity described in Vignette 4.1. It appeared, from both girls' writing, that they have an understanding of these mathematical terms. Their understanding of these terms in writing was used as student examples for discussion in later lessons.

In the next examples (Figure 4.12), the writing, use of mathematical vocabulary, and lack thereof seem to indicate misunderstandings.

	"I use my **time tables** they are opposite, they (multiplication and division) are the same" **Joe**
	"That **multiplication** is the same as **division.**" **Oscar**
	"Yes. **Open array,** adding"



“I added up the legs all together.”

“addition, more open array”

Marsha

Figure 4.12. Writing samples by Joe, Oscar, and Marsha 10/31/12

Joe’s response above suggested that fact families can be used to solve division problems because “time tables and division are opposite”; in the next response, he indicated that multiplication and division are the same. Another student, Oscar, also wrote that multiplication and division are the same. These two students are using mathematical terms in their writing; however, it appears that they were unclear how to define these operations. Their writing indicated that they recognize a relationship between multiplication and division, but are unsure how to explain the relationship. The last writing example, from Marsha, was a short response using the term “open array.” An open array is a multiplication strategy in which students break larger numbers up, multiply, and add the products. Marsha’s written response showed limited understanding of this strategy. The problem included in example above, in which an open array could have been employed, illustrated Marsha’s decision to use addition instead of multiplication to answer how many legs are on 28 horses. This seemed to indicate a hesitation to use multiplication. Her written responses focused mainly on addition and showed little explanation about open arrays, which also suggested she was unclear about the multiplication open array strategy.

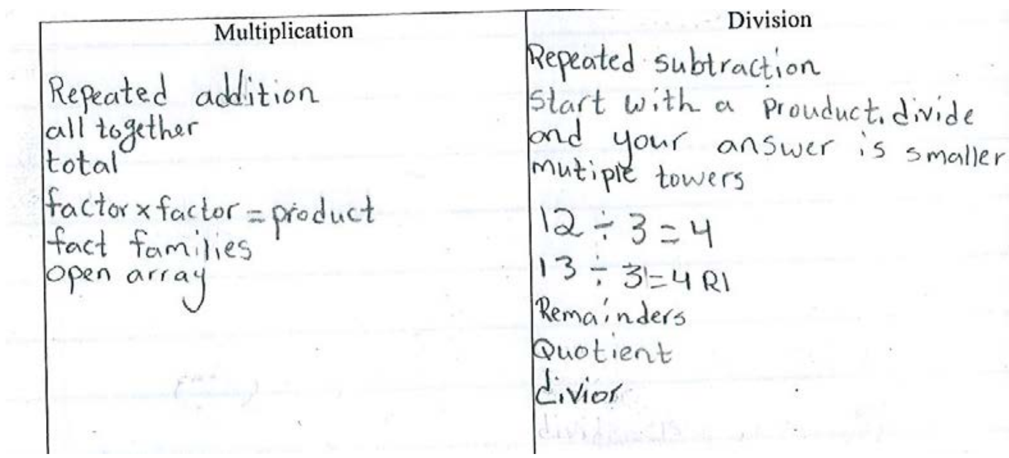
Students used mathematical vocabulary with a range of difficulty. The student writing in these examples reveals varying levels of understanding of and confidence in using mathematical terms. In Linda's response, she appeared to be noticing a pattern that she expressed using vocabulary that she felt comfortable using. In part of her response, she wrote, "Division ends in small numbers"; this wording could be replaced by the term "quotient." Although this term had already been introduced in class, it does not appear in her writing. In contrast, Zoe chose a term that she seemed to understand and explained the term clearly and succinctly. Both Linda and Zoe showed understanding of the terms; however, Linda's response indicated she was making a connection with the structure of fact families, and she described that idea—whereas Zoe defined one term that she appeared to firmly grasp, and the response does not include any other terms. Linda's response used several mathematical terms, which may be connected to why "quotient" was not included. Linda's writing indicated she was building on her conceptual mathematical knowledge by interacting with these terms. Students can accomplish the task of solving mathematical problems without knowing mathematical vocabulary; however, these terms become essential for communication and cognitive awareness. Both of the girls used writing to interact with mathematical vocabulary, and their responses indicate an understanding of these terms.

The examples from Joe and Oscar showed similar misunderstandings. Both boys wrote about division and multiplication. Joe's writing indicated he was unsure whether these terms mean the same thing or are opposite; however, Joe's minimal writing made identifying his confusion difficult. Joe also refrained from using the terms "multiplication" and "division" and replaced them with "times table" for multiplication and "they" to encompass both terms. It appears as though Joe struggled with this vocabulary and does not feel comfortable using these words in his writing. Oscar does use the terms multiplication and division; his response revealed he connected these terms in such a way that they mean the same thing. His minimal writing also made understanding his thinking more difficult. Even so, the writing by these students still provides information for instruction. Even though their responses are limited in length, their misunderstandings are evident.

Students' writing contained frequently discussed terms. Marsha's written responses used a lot of vocabulary words that had been discussed in class, even if she was unclear how to use or define the term. She used the term "open array" in her written response twice with no description of the strategy or evidence that she employed this strategy in her calculations. I also noted this desire to use new vocabulary in my field notes on mini-lessons and discussions. Students began to work with new vocabulary and started using those terms in discussion. When questioned further, they showed limited understanding of the terms. One example occurred during the first workshop, which included work on open arrays. The open array strategy is to break difficult numbers up when multiplying to create an easier equation. Place value is usually used to break the numbers up (e.g. 28 x 4, 28 = 20 + 8, 28 x 4 = (20+8) x 4, 4 x20 =80, 4 x 8 = 32, 80 + 32

= 112). As long as the numbers you break a larger number into add up to the original factor, you will arrive at the same answer (28 = 10+10+8, so 28x4 becomes (10+10+8) x 4, 10 x 4=40, 10 x 4=40, 8 x 4=32; the sum is 40 + 40 + 32 = 112). The numbers the factor is broken into become addends that must add up to the original factor in order to solve correctly. Linda used the strategy effectively, but wanted to use the term "multiples" to describe how she broke up the larger factor. She repeatedly explained that as long as they are multiples it will work. When I asked her to explain the term "multiple," she struggled. I used our conference to talk about multiples and the open-array strategy.

In the seventh workshop, the mini-lesson began with the whole group's creation of a chart about multiplication and division. The objective was to clarify the purpose of each operation, discuss the terminology used in word problems that relate to these operations, and identify the strategies for solving. The chart was filled in by the students; during this, they called out a list of terms: open array, multiple tower, factor, divisor, quotient, remainder, product, and fact family. I asked each contributor to explain the term, tell me where on the chart to place the term, and state whether the term could be applied to both multiplication and division. The group members were eager to call out the terms, but found explaining more difficult. I filled in the chart with the terms that could be explained and accurately placed on the chart, and asked the group to look back over previous workshops to gather ideas about the other terms. This review of their previous work offered further clarification, and the chart was completed (Figure 4.13).



Zoe

Figure 4.13. Multiplication and Division Chart

Student reactions. Mathematical vocabulary presented a challenge for the students, but they displayed an eagerness to interact with these new words. Capitalizing on this eagerness to use the vocabulary needed to be infused into the workshop. This idea became part of the collaborative discussions with Michelle. Many of the mathematical terms included in the lessons were limited to mathematics class. Students were interviewed, and their responses support the idea that writing had helped them to use and become familiar with new mathematical vocabulary.

> Me: Does writing in math help you use more math words?
> Linda: Yeah.
> Me: Yeah. . . . Why?
> Linda: Because it helps me use words that I didn’t know.
>
> Interview with Linda (12/7/12)

> Me: Does writing in math help you use more math words? Why or why not?
> Sylvia: Um, yes, ’cause after—if you forget the words you can look back at your writing and find them.
>
> Interview with Sylvia (12/7/12)

> Me: Okay, do you think that writing in math helps you use more math words?
> Zoe: Yes, because it helps me learn more words.
> Researcher [pointing to her work]: Do you see any math words in there?

Zoe: Yes, “remainder.”
Me: You really explained what a remainder was there.

Interview with Zoe (12/7/12)

Both Linda and Zoe asserted that writing helped them learn the words, and Sylvia notes that her writing served as a resource if she forgot the new terms she had learned. During Hannah’s interview, she didn’t recognize mathematical words or vocabulary in her writing. However, I noticed examples in which she had more advanced terms in her writing and decided to ask additional questions.

> Me: Okay, that’s great, do you think writing in math—and when I say writing, I mean your writing—so do you think your writing in math helps you use more math words or math vocabulary?
> Hannah: Sometimes, because we have to put big numbers in it and, um, break that . . . down so it’s not so long and make it where it will fit on the page.
> Me: Can you show me an example, or I might be able to show you something?
> Hannah: Like, “many”—I could have drawn a lot of these but using many is shorter.
> Me: Can you read your answer there?
> Hannah: I can’t read that word. Um . . .
> Me: What were you doing here?
> Hannah: Oh, I started with my hundreds then I went to 489 and 475 I had to go to my tens.
> Me: Did you use any math words in your writing?
> Hannah: Um….nope.
> Me: Okay, so no math words in there.
> Hannah: Not that I know of…there are hundreds and tens.
> Me: Think those are math words, hundreds and tens?
> Hannah: Sort of, because you use them for math and it’s a number.
> Me: Do you use the words hundreds and tens often outside of math?
> Hannah: No.
> Me: So they are kind of math words.

Interview with Hannah (12/7/12)

Hannah used the place value terms “hundreds” and “tens” in her writing, and it took her a moment to realize that she had used words that are primarily mathematical terms. I refrained from pointing the terms out, and she paused and noticed the terms on her own. Many students in the group used phrases like “first number” and “second number" to describe place value. Hannah appeared to have certain mathematical terms clearly defined in her mind, and considered them part of her writing and not distinctly mathematical.

Student understanding of mathematics and of mathematical vocabulary were evident in their writing. The workshop is designed for students to write about what they are doing in mathematics, and student writing offers insight into their thinking. Their writing is a tangible work sample that can be analyzed to identify their understanding of the lessons and the terms. In some of the examples, the calculations were not enough to afford a more complete picture of the student's thinking. The writing was an integral part of planning and conferencing.

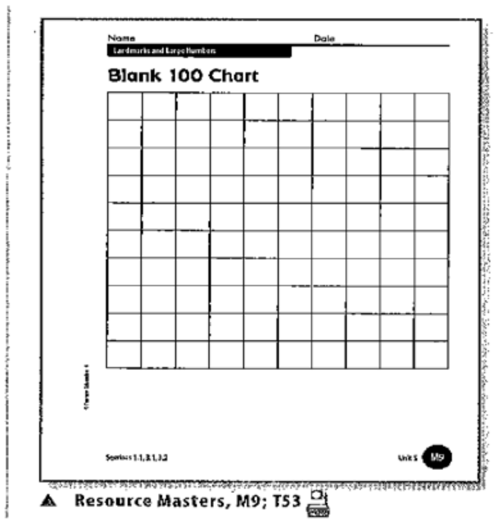
> Me: What ways might you use the findings from the students' writing for instruction?
>
> Michelle: One of the things I want to do is to sit down and really look at it and analyze where the missteps are still occurring. When I was working with some of the students after school, I did see where place value is still a huge deficit for so many of them. I also thought I could use some of their work to help develop lessons to build upon and re-teach the areas of weakness for them.
>
> Teacher Interview (12/15/12)

In the interview with Michelle, she described the writing as a resource that she refers to and analyzes to decide on instruction. This next section will explore the last subcategory of the first finding, which examined student writing and their problem-solving process.

Students use Writing as a Tool to Demonstrate Their Problem-Solving Process

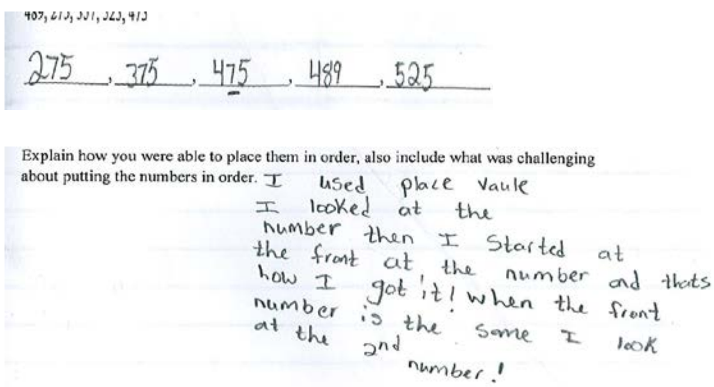
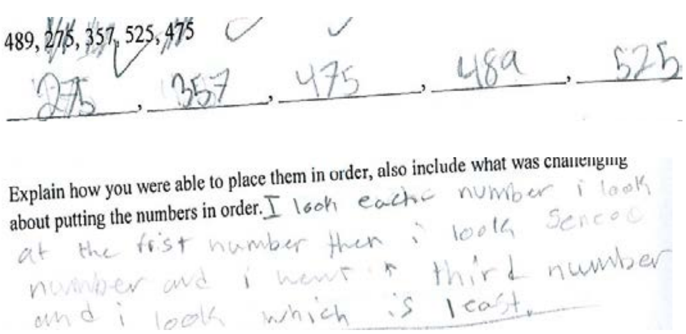
Vignette 4.2

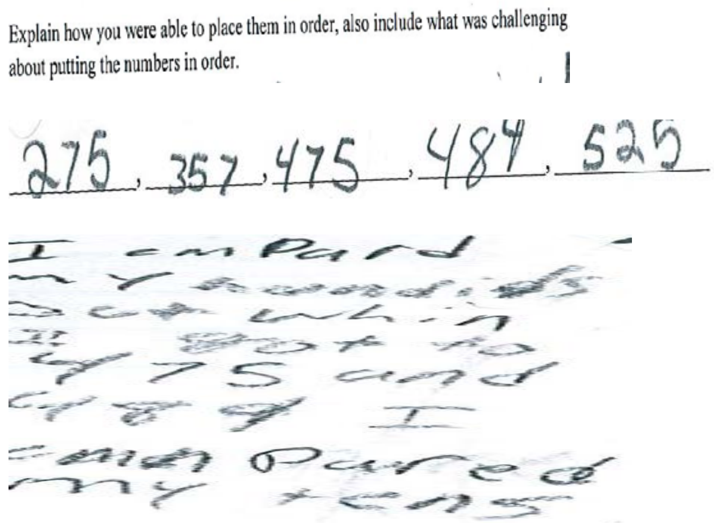
Michelle begins a mathematics class by discussing the concept of 1,000 and asking her students to think about the size of 1,000. This discussion was used before the students created their 1,000 book. The students each held in their hands 10 paper squares with the dimensions 10 x 10; the paper was graphed to show the 100 units contained within each square. The 1,000 book, when finished, would contain one square per page and a label of 0-100, 101-200, 201-300, etc. This book served as a reference for future calculations.



Michelle asked the students to talk to their neighbor about what 1,000 looks like. One student began to count the hundreds that she had in her hand. Tom responded that 1,000 would look like 10 hundreds. This response prompted others to count their papers to see if Tom's idea was correct. Other students began to test this idea by counting each of the units in their square. The problem of how to describe the number 1,000 produced several interpretations.

Student's writing demonstrates process. In the workshop format, instructors encouraged students to explain their work through words, drawings, and examples. The explanations they provided in their writing reflected their problem-solving process. In some cases, the students' writing reflected a process that was consistent among many students. In other cases, the process used to arrive at their answers was different. This information is valuable for instruction and to recognize changes in students' process for similar problems in the future. The writing showed the process used by students and how proficient they were with efficient strategies. Their writing helped guide instruction in more efficient practices. Figure 4.14 includes several examples in which the students' writing reflected a process that is consistent with each other's, and Figure 4.15 shows examples in which the process is different. In Figure 4.14, the students used the same process for putting five large numbers in order.

	"I used place value I looked at the number then started at the front at the number and that's how I got it! When the front number is the same I look at the 2nd number!" **Zoe**
	"I look each number I look at the first number then I look second number and I look at third number and I look which is least." **Phoebe**

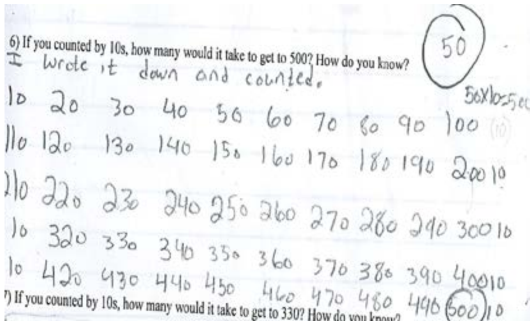
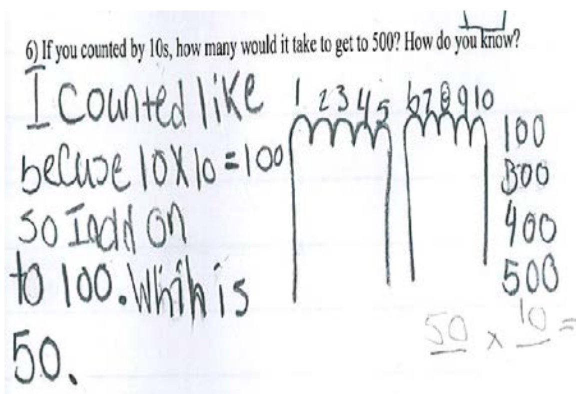


"I compared my hundreds, but when I got to 475 and 489 I compared my tens."

Hannah

Figure 4.14. Comparing and Ordering Numbers

Students' explanations vary in the level of clarity. In the last example, Hannah used the proper place value terms of hundreds and tens, whereas the other students refer to their places as front number or first number. These students successfully placed the numbers in order from least to greatest and described their process. Their description of their process is an accurate way of comparing numbers. Their writing suggests the task of comparing numbers is a skill they are comfortable with and for which they have a process in place. Their description allowed me to feel more confident about their comparing-numbers process and adjust instruction accordingly. In Figure 4.15, the students completed a problem and their descriptions revealed a different process for each student. I also included the questions I asked during the instruction to help them make the connections between the problem they were solving and what they already knew. The math problem that preceded the problem included in the examples below was, "If you counted by 100s, how many would it take to get to 500? How do you know?"

	"I wrote down and I counted." Questions asked to facilitate understanding: Do you see a connection between the first problem and this problem? Is there another way to solve this problem? **Zoe**
	"I counted because 10 X 10 = 100, so I add on to 100. Which is 50." Questions asked to facilitate understanding: What made you think to figure out how many tens were in 100 and use that to help you? What if the next problem asked how many 5s are in 500? Could this problem help you? **Linda**

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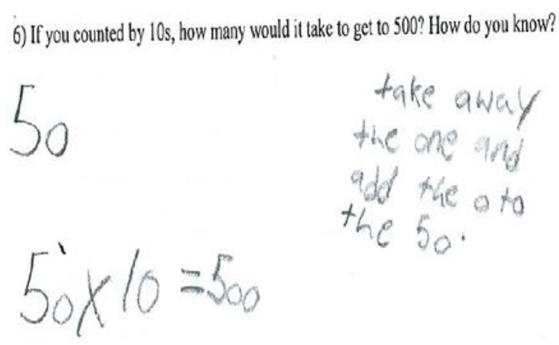
<td>
</td>
<td>“Take away the one and add the 0 to the 50.”

Questions asked to facilitate understanding:

Can you explain your process more?
How do you know that this will work?
Do you think it will always work?

Hannah</td>
</tr>
</table>

Figure 4.15. Problem Solving and Process

Students’ writing demonstrates varying degrees of efficiency in process. In Figure 4.15, each student arrived at the correct answer to the problem. However, the process they used to get to this answer was different for each. In the first example, Zoe’s process provided her with the correct answer, but it was less efficient and lacked a connection to the previous problem. The previous problem asked how many hundreds are in 500, and Zoe figured this problem out in the same way and reported that there are five one hundreds in 500. In the problem included in Figure 4.15, Zoe doesn’t connect the idea of finding out how many 10’s are in 100 as a means of solving this problem. Instead, she returns to the inefficient method of writing out all of the multiples of 10 until she arrives at 500 and then proceeds to count how many numbers she has written. In the next example, Linda shows her drawings and makes a connection to the previous problem to help her solve this problem more efficiently. Hannah appears to be using a multiplication process that is sometimes described as a trick to solve the problem; she uses this process in several examples. Hannah’s answers were correct, and she finished her work quickly. Even though Hannah’s work showed accuracy and efficiency, it was important to probe deeper into her understanding of the concept. The process exhibited by each of these students provides valuable information for the teacher. Each student's process varies in levels of efficiency, understanding, and in how much they connect one concept to prior knowledge. This information is important when deciding the questions to ask to check their understanding and for the next lesson. In Figure 4.15, I included the questions that I used in individual conferences. Michelle also noted in the interview that writing serves as a tool to keep students thinking about their process.

> Michelle: I think it was helpful to see what they understood and I think that some of them understood more than they were able to communicate and it was difficult because of their ELL status, their EC status. One of their greatest difficulties was communicating how they solved the problem. So, I think this was helpful for some of them to keep them thinking about their process.
>
> Interview (12/15/12)

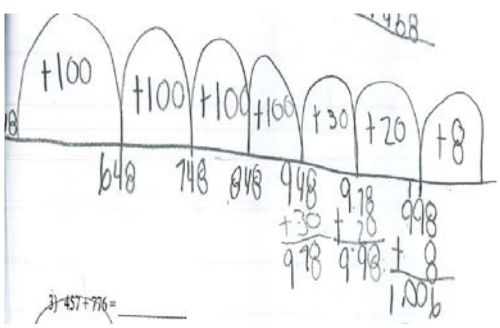
The process revealed in the writing by these students suggested a certain level of thinking and a level of efficiency that is important for instruction. Michelle stated that the writing would be used to develop lessons.

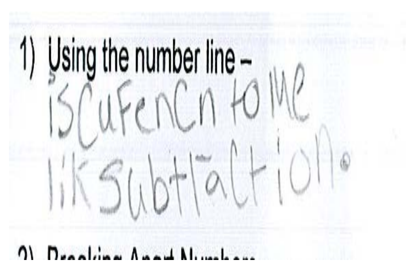
> Michelle: I also thought I could use some of their work to help develop lessons to build upon and re-teach the areas of weakness for them.
>
> Interview (12/15/12)

Student reactions. In the interview below, Linda discusses her thoughts about writing in mathematics. She notes that she likes writing because she can share her opinion about how she likes to solve problems. She seems to enjoy the independence of the workshop, which allows her to combine new ways of solving math problems with the processes that she already knows, and then express her opinion in her writing. In Figure 4.16, Linda identifies the process and expresses her opinion.

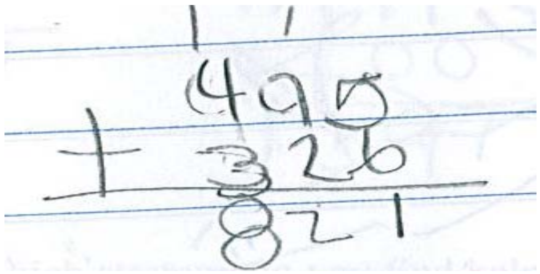
> Me: How do feel about workshop time?

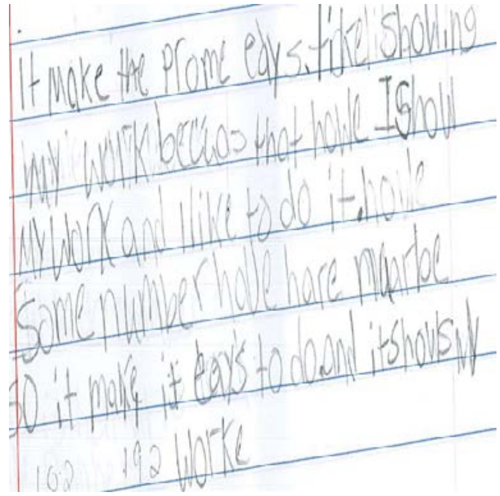
Linda: I like it.
Me: You like it—what do you like about it?
Linda: I can do my problems the way I want to.
Me: Okay.
Linda: It's fun.
Me: Do you want to share any of your writing or anything from your journal that you like?
Linda [Flipping through the pages]: I like to do a lot of problems. [Continue flipping} I want more problems.
Me: Do you think writing in math is helpful? Why or why not?
Linda: Yeah, it's helpful
Me: Why do you think it's helpful?
Linda: Because it teaches us how to write in math.
Researcher [While writing notes]: Because it teaches us how to write in math.
Linda: And explain things.
Me: Okay.
Me: Do you like writing in math?
Linda: Yes, because I like to share my opinion about things—like, if I like to do it this way or that way.





It is confusing to me, like subtraction.



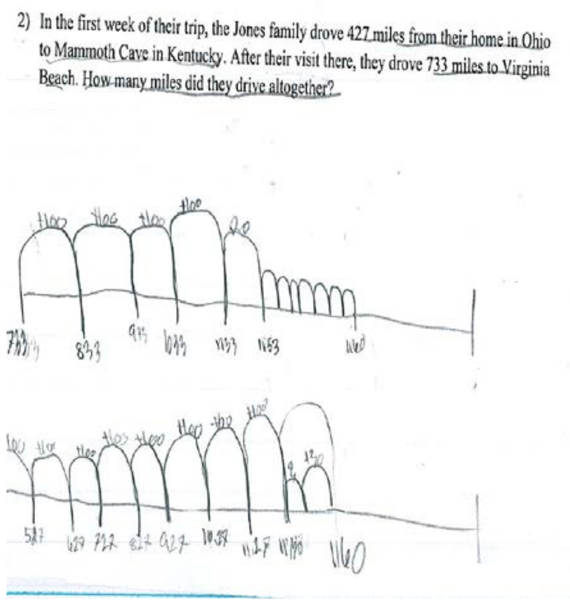


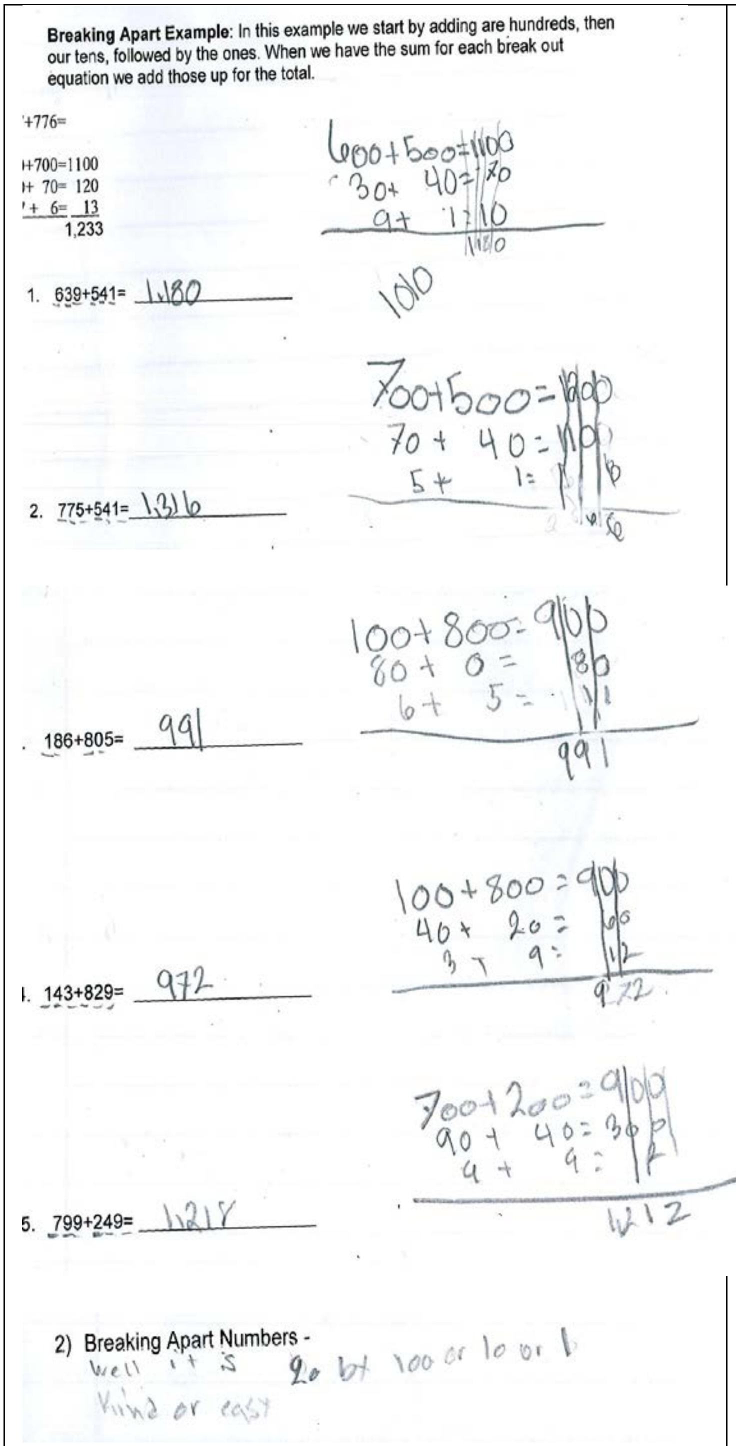
Traditional Algorithm
It make the problem easy like showing my work because that how I show my work and I like to do it. Some numbers have hard number so it makes it easy to do and it shows my work.

Linda

Figure 4.16. Process and Opinion

Linda shows that she has engaged in the process of using the number line and the process of using a traditional algorithm. She arrived at the correct answer in the problems pertaining to larger digit addition; however; her writing showed that the process of using a number line is confusing, and she compared the number-line difficulty with the difficulty level of subtraction. The number line in addition and subtraction is used as a tool to better understand place value. There is a “break-apart method” for addition that also reinforces place value. Linda’s writing and conferences with other students provided insight into the students’ struggle with the number line process. I spoke with Michelle and gave her this information. Based on this information and the progress in her group, Michelle informed me that the break-apart method would be the next step. This method for adding and recognizing place value seemed to resonate with the students and allowed them to make more progress in this area. In Figure 4.17, comparisons of these two days of instruction support this conclusion. Figure 4.17 includes analysis of the students' experiences over the two days of instruction.

	Analyses of the student's math problem-solving process: Phoebe has worked on this problem for 40 minutes and seems genuinely fatigued. The conferences with her have shown that she has difficulty keeping track. She is unable to recognize that she can take "larger jumps" along the number line, and this is leading her to complete more than 10 addition problems for one sum. **Phoebe**


Breaking Apart Example: In this example we start by adding are hundreds, then our tens, followed by the ones. When we have the sum for each break out equation we add those up for the total.

'+776=

ı+700=1100
ı+ 70= 120
'+ 6= 13
1,233

1. 639+541= 1,180

600+500=1100
30+ 40=170
9+ 1=10
1180

2. 775+541= 1,316

700+500=1200
70+ 40=110
5+ 1=6
1316

186+805= 991

100+800=900
80+ 0=80
6+ 5=11
991

143+829= 972

100+800=900
40+ 20=60
3+ 9=12
972

5. 799+249= 1218

700+200=900
90+ 40=130
4+ 9=13
1212

2) Breaking Apart Numbers -
Well it's by 100 or 10 or 1
kind or easy

"Well it's by 100 or 10 or 1 kind of easy"

Analyses of the student's math problem-solving process:

Phoebe is able to use the break-out method with greater success; she completes six problems and uses the lines to carefully mark place value. Her efficiency with this method, as compared to the number line, is much greater. When this work is completed, several of the girls, including Phoebe, grab white boards to continue practicing.

Phoebe

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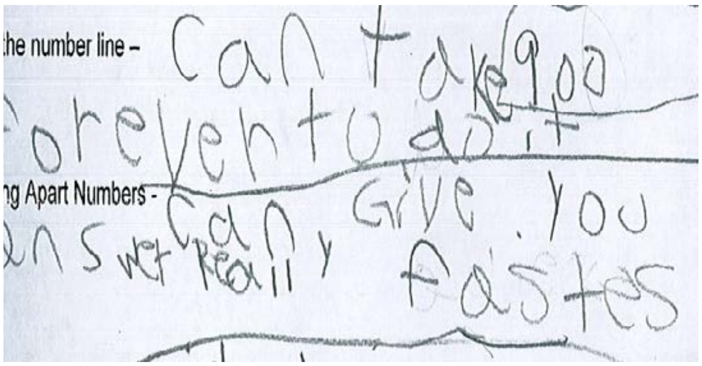
<td>

Albert
Number line – can take forever to do it.
Break Apart Numbers –can give you answer really fast.</td>
<td>Analyses of the student’s math problem-solving process:

Albert is first resistant to a new process, but begins to work quickly through the problems. This realization is expressed in his writing.

Albert</td>
</tr>
</table>

Figure 4.17. Workshops and Analysis

Students’ written explanations explicitly revealed their processes, which offered more information for examining numeric responses. Michelle mentioned the goal of efficiency to the whole group on many occasions during the classroom observations. Students’ writing provided important information that was used in the lesson-planning process to decide whether or not to introduce a new method that would address place value and addition.

Summary of Finding 1

Students used writing to demonstrate their understanding of mathematics, of mathematical vocabulary, and of mathematical processes. In some cases, examining only the numerical work failed to illuminate the students’ understanding, as was the case with Albert and quotients. In Tom’s example, his desire to use the open-array strategy is apparent in his writing, yet his calculations rely on repeated addition. His writing also revealed misunderstandings about division and multiplication. These misunderstandings impeded his readiness for the-open array strategy. Sylvia's thinking about the efficiency of dividing one-by-one versus using multiplication facts to assist in division was described in her writing. This highlighted her understanding and movement toward a more efficient method for dividing. In their interview responses, Hannah and Sylvia recognized the value of showing their understanding in their writing. The students' writing provides insight into their thinking and eliminates the likelihood of making assumptions based only on their number calculations.

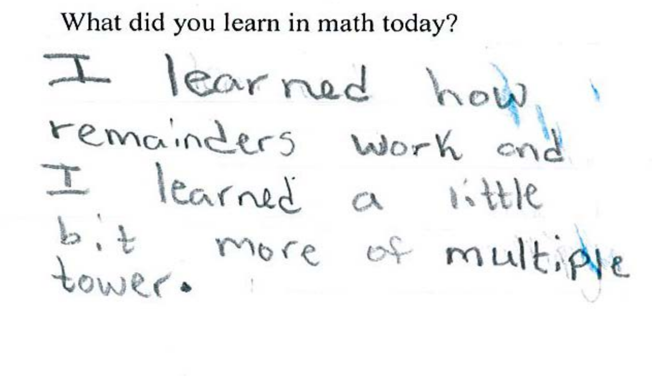
Michelle's implementation of the workshop encouraged students to write, which reflected their understanding of mathematical vocabulary. In Linda's example, she used terms related to multiplication and division to reason her way through the connection between these inverse operations. She correctly used the terms and challenged herself to use multiple terms in one response. Zoe used one term, "remainder," and showed she was secure in her understanding of its meaning in mathematics. In other examples, students revealed misconceptions about terms and showed a desire to use vocabulary that was part of instruction, even if they were unsure of the meaning. Interviews with students supported the idea that writing offered an opportunity for them to interact with mathematical vocabulary. Mathematical vocabulary is often limited to being used only in math, which makes the opportunity to interact with these terms more valuable.

The goal, expressed by Michelle on several occasions, was for students to gain efficiency in their mathematical calculations without sacrificing understanding. Students' writing illustrates their process. In the first three examples in Figure 4.14, the students explained their process for comparing numbers. The writing varied in expression; however, their process was the same. The written explanation allowed for confirmation that this particular skill did not need to be taught further; the process students used explained is the most efficient and accurate way of comparing numbers. In the next example, Figure 4.15, the students' answers are correct, but their actual process varied in efficiency. This presented an opportunity to work with students at their level in order to move them on to the next level of efficiency. Students are introduced to several ways of solving problems. Some students showed two different processes and added an opinion about their preference. This information was a factor in deciding to move on to the break-apart method mentioned in Figures 4.16 and 4.17. The next theme focuses on the reflective component of the students' writing.

Students' Written Reflections Inform Teachers' Instruction

Vignette 4.3

Today's mathematics class begins with whole-group instruction, which centers on solving division problems. Michelle uses pictures, a multiple tower strategy, and fact families to solve division problems. One division problem asks students to explain how many rows of 8 are filled by 26 students attending a movie. Several students respond in different ways. Some begin drawing chairs to represent the rows and others begin building a multiple tower. Michelle gathers information from the students and solves the problem both ways. The whole class completes another problem in this fashion before breaking into workshop groups. While in the groups, students solve another two problems and reflect on what they are learning. Zoe often looks up to the ceiling and does some fidgeting when she arrives at the reflection question. It takes a moment or two of contemplation before she begins to write. In this reflection, Zoe seems to show confidence in her understanding of remainders, but appears to be still learning the multiple tower strategy. She takes the *time of contemplation to gauge what she has learned, and her writing offers a glimpse into her confidence level with each concept.*



The teachers designed the workshop to ask questions that encouraged reflection. The teachers' purposes for asking reflective questions was to encourage students to take a moment and think about what they had learned, make connections, and ask their own questions. This finding includes three subcategories: (a) written reflections helped students identify strategies they used when figuring out mathematical problems, (b) written reflections helped teachers make decisions about moving to the next level of instruction, and (c) written reflections helped teachers engage in verbal interactions with students about their mathematical reasoning (Figure 4.18)

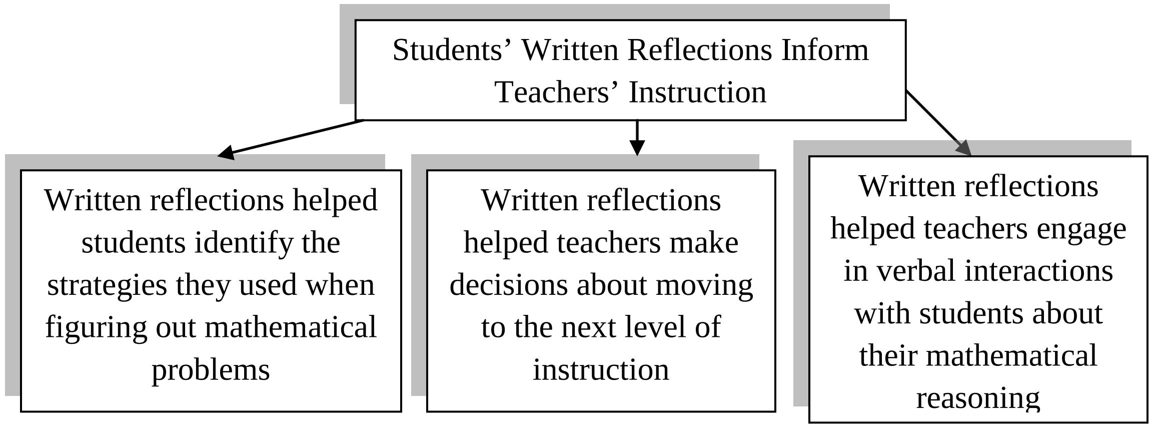


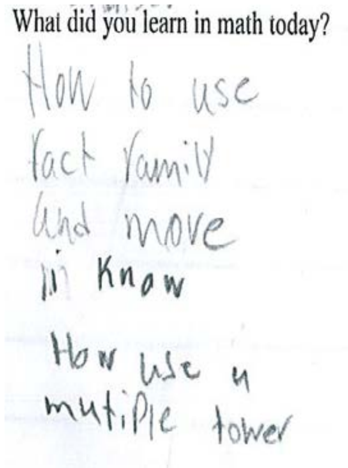
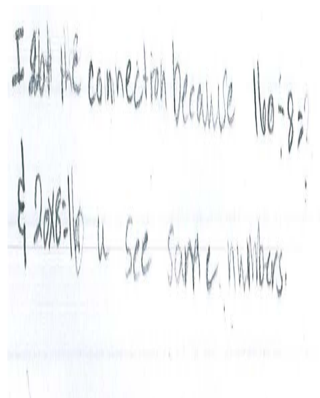
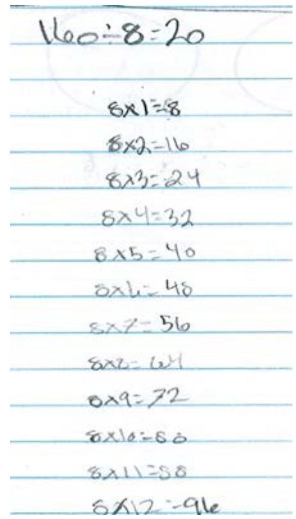
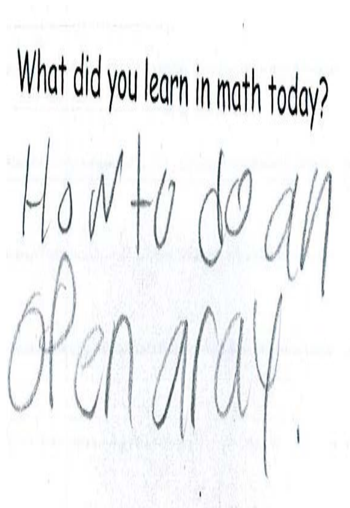
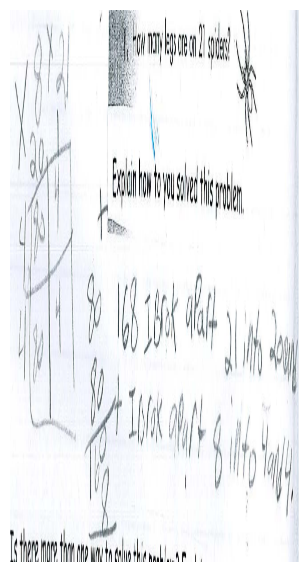
Figure 4.18 Second Finding with Subcategories

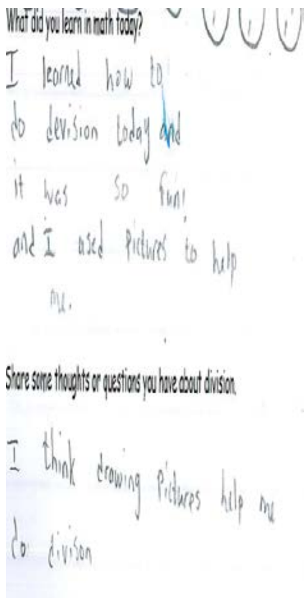
Each workshop included a reflection question and, in some cases, more than one question. These questions served as an impetus for students to think about their problem-solving strategy, process, and understanding. These reflections highlighted the strategies used, helped instructors interact with students, and became influential in deciding whether to move to the next level of instruction.

Written reflections helped students identify the strategies they used when figuring out mathematical problems.

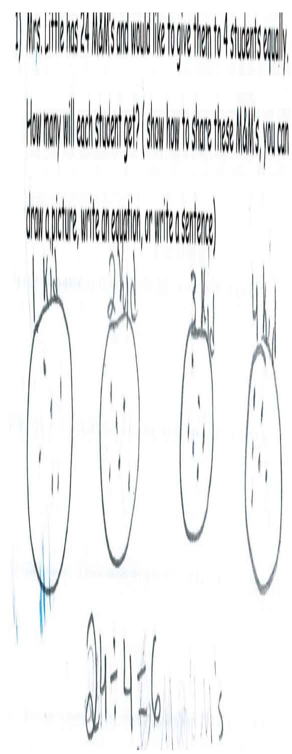
Students used reflection as a time to write what they learned and make connections. Often, these reflections included the strategy the student used to figure out the mathematical problems included in the workshop. These reflections served as a guide for the teacher to reconcile the students' written reflection and their actual use of the strategies in their calculations. There were several examples of both consistency and inconsistency between written reflections and students' strategy. Their reflections provided insight into students' understanding of the strategies presented in instruction. Student interviews supported the idea of reflective writing being helpful for increasing understanding.

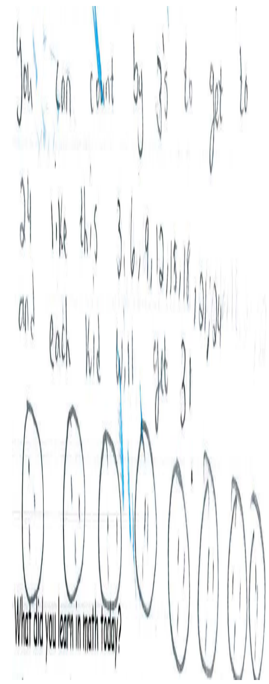
Students' written reflections were consistent with calculations. At times, the teacher started the lesson with whole-group instruction to introduce a strategy and continued in a small-group workshop. On other occasions, the workshop began almost immediately and lasted the entire 90-minute period. The workshop always began with a mini-lesson that re-instructed a strategy or added more detail to an already instructed concept. In both cases, the students moved on to solving problems, having conferences, and using writing to convey their ideas and to reflect. The strategies included in instruction were fact families, open arrays, multiple tower, and use of drawing to solve division (Figure 4.19). Fact families make a connection between multiplication and division facts. The open-array strategy breaks a large multiplication problem into two or more easier problems, followed by adding the products. The multiple tower strategy uses multiples of the divisors to reach the dividend; the height of the tower is the quotient. Finally, pictorial representations of division problems were encouraged as a way to understand that the dividend is being broken into groups. In Figure 4.19, student reflections and the use of the strategy in their calculations are consistent.

Reflection	**Calculations**	**Calculations**
 "How to use fact family and more I know how to use multiple tower."	 I got the connection because 160 8 = & 20 x 8 = 160 u see same numbers.	 **Phoebe**
 "How to do an open array."	 "I broke apart 20 into 20 and 8."	 "I broke apart 21 into 20 and 1, I broke apart 8 into 4 and 4." **Hannah**



"I learned how to do division today and it was so fun! And I used pictures to help me."
I think drawing pictures help me do division."



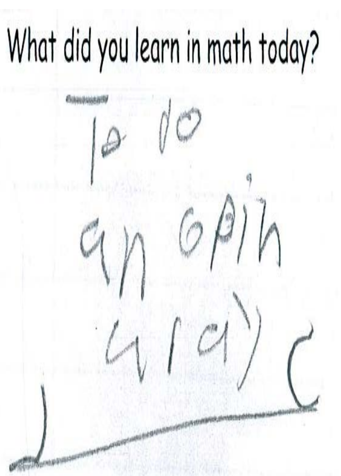
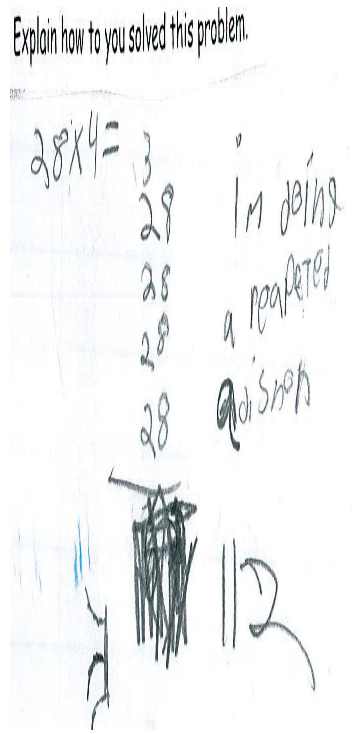


Zoe

Figure 4.19. Student Reflection and Calculations

During the observations, I noted that reflection questions seemed to require students to take a long time to express their thinking. In many examples, students identified their strategies in the reflection questions. In the first example, Phoebe affirms that she learned how to use fact families and the multiple-tower strategy. Her calculations were consistent with her affirmation; she accurately reviewed her work and clearly identified her method for problem solving. In the next example, Hannah asserted that she learned to do an open array. Her calculations revealed she was able to break the larger multiplication problem into two or four easier multiplication problems and use addition for the final answer. She recognized the strategy and the different variations in using this strategy. In the final example, Zoe connected drawing pictorial representations as a helpful method for division problems. In her second calculation, she labeled each circle (1 kid, 2 kid, 3 kid, and 4 kid) to solve the problem 24 (M&M candies) ÷ 4. She wrote that “this was helpful and made division fun.” The three examples in Figure 4.19 show a reflection that is consistent and supported by the calculations made earlier in the workshop, whereas in several other cases, students show a lack of consistency between their written reflections and their calculations.

Students’ written reflections showed lack of consistency in calculations. In Figure 4.20, the reflections and calculations are either inconsistent or lack a clear connection.

Reflection	Calculations	Calculations
 “To do an open array.”	 	**A**  **Tom**

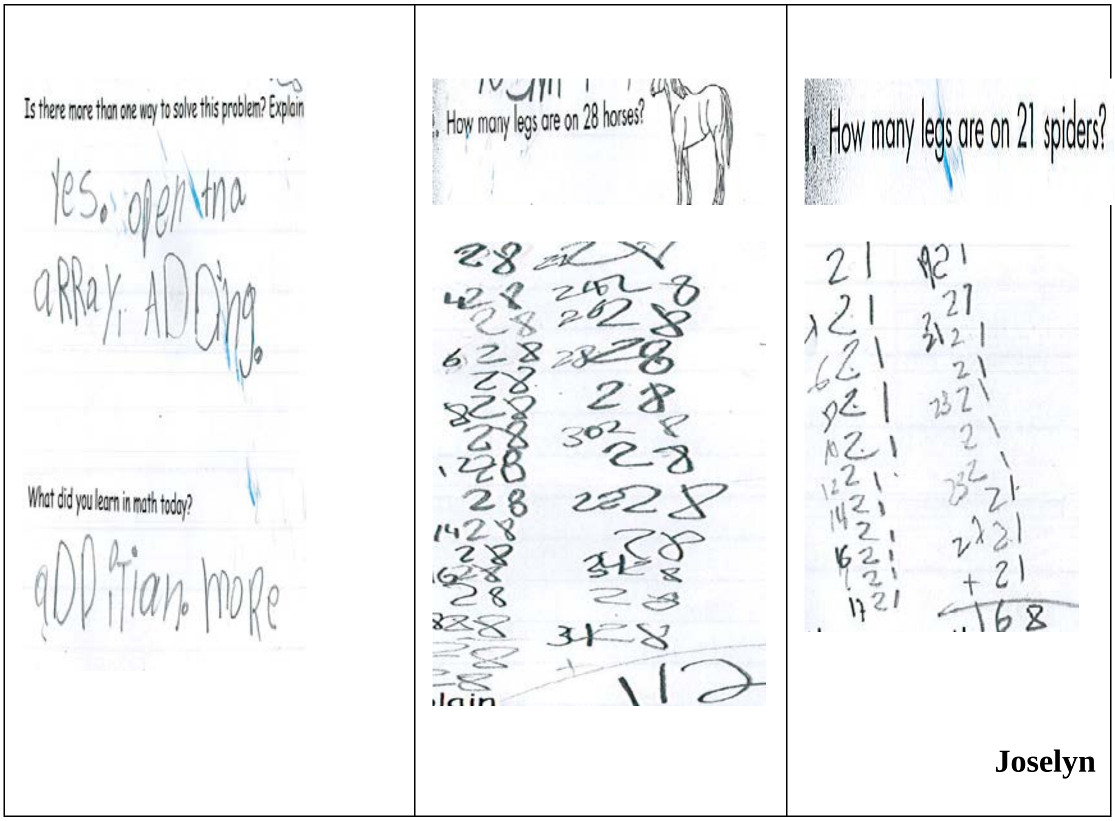


Figure 4.20. Reflection and Inconsistency

In both of these examples, Tom and Joselyn reflect on their learning and write that they learned how to do an open array. The actual calculations completed by both students showed repeated addition as their strategy rather than using an open array. Tom's work for the second problem (Figure 4.20A) showed some understanding of the open-array strategy. The open-array strategy includes drawing a box to separate the new multiplication problems. Tom attempted to draw the box; however, his dimensions remain the same as the original problem. His equations written below showed an effort to break the original equation up into easier equations. The equations he presented were 6 x 1 and 12 x 1, which did not represent the original problem. The calculations where he used arrows appeared to be instances where he broke the numbers up for easier addition problems. In Joselyn's reflection, she identified addition as what she learned in math that day; however, the lesson for several days, including this day, centered on open arrays. In her calculations, there was only addition, with no attempt at the open-array strategy. At the time of the workshop, the term “open array” was used repeatedly, and the idea that this is a strategy for multiplication seems to be part of Tom and Joselyn’s thinking.

The comparison of the reflections with the calculations provided insight for instruction. In Figure 4.19, Hannah’s assertion that she learned how do an open array was evident in her ability to use place value to break one factor apart in 28 X 4 and to break both factors in 21 X 8, one using place vale (20, 1) and one in half (4, 4). In both equations, she organized her open array, multiplied correctly, and found the correct sum. However, Joselyn and Tom asserted in their written reflection that they learned an open array, but had little to no evidence to support their assertion. The students in these examples identified strategies in their reflection, and using their reflections as a guide while reviewing their calculations provided information for future instruction. In the interview with Michelle, she stated that their workshop materials would be helpful for lesson planning.

> Michelle: I also thought I could use some of their work to help develop lessons to build upon and re-teach the areas of weakness for them.

Michelle indicated that the student writing provided information that would be helpful for developing lessons.

Student reactions. In the interviews, students indicated that they valued writing and connected writing to a greater understanding of the material.

> Me: Does writing in math help you understand a problem better?
> Linda: Yeah, because, like, it shows me what to do and what I learned and, like, how to do math problems that, like, I probably could never do.

Interview with Linda (12/7/12)

> Me: Do you think writing in math is helpful?
> Zoe: Yes.
> Me: Why?
> Zoe: Because you are learning, you are doing both at the same time, you are writing and doing math.
>
> Interview with Zoe (12/7/12)

> Me: So do you think writing in math helps you understand a problem better? Why or why not?
> Hannah: Sometimes because when you go back to the very first one, this helps me because it tells me what to do [Hannah points to the writing that is part of the problem, but not her own writing], but if it is like a number line I would have to count the number—wait, wait, like, this one I had to count 230 then 5 to get the answer because that was that plus that equals that.
> Me: I was thinking of your writing—when you write, does it help you understand problems better? Why or why not?
> Hannah: Yes, because when I write I might have the wrong answer, but then when I write it I might see a difference and change my answer.
>
> Interview with Hannah (12/7/12)

> Me: Do you think writing in math is helpful?
> Jari: Yes, because it helps your memory, memory to feel good.
> Me: Why does it make you feel good?
> Jari: Because, because whenever you have homework for math you can use the strategies that you used for math.
>
> Interview with Jari (12/7/12)

Linda stated that writing helped her to see what she learned, and this expanded her ability to solve more problems. Zoe noted that the combination of writing and solving math problems is learning, while Hannah found her writing as helpful for detecting mistakes and misconceptions that would then allow her to make corrections to her work. Jari mentioned that writing was helpful for her memory and for using strategies for problem solving in her homework. Although each student expressed different connections between reflective writing and problem solving, their connections indicated that they view writing positively. In this next excerpt, Sylvia described the workshop time as helpful because her writing served as a reference and reminder. However, when asked if writing is helpful for understanding problems better, she said no, because the reflection is just saying what you did. These answers seem contradictory; she recognized a value in having a record, but did not connect this idea with a better understanding of problem solving.

> Me: Do you think math workshop is helpful?
> Sylvia: Yes, because when—after you, you go back in your journal you can read in case you forget about.
> Me: Okay, back to that earlier question. Do you think writing in math helps you understand problems better?
> Sylvia: Um, no.
> Me: Okay. Why do you think it doesn't help?
> Sylvia: Because you are just saying what you did.
> Interview with Sylvia (12/7/12)

This response indicated that Sylvia viewed writing as an activity that is reflective of an understanding she already possesses, rather than an activity that builds understanding. Sylvia noted that her written reflection serves as a reference, but does not recognize writing as a tool for thinking. This response contrasted with Zoe's response that learning occurs in the combination of writing and solving math problems. Sylvia still recognized a value in writing and seems to reflect on her own writing at later times.

Reflections help Teachers Decide When to Move to the Next Level of Instruction

Students' written reflections showed students' confidence level. Reflection questions served as a prompt for students to think about what they had learned and to examine their connections to previous material. In the examples in Figure 4.19, the students' reflection and actual calculations were consistent; this consistency was helpful for teachers to make the decision to move these students on to the next level. The students' examples in Figure 4.20 indicated a lack of consistency between the students' reflections on strategy and actual calculations. Teachers examining this inconsistency can provide more instruction on these strategies and decide to not move forward, or to move forward while revisiting these topics for further clarification. Reflection questions gave students the opportunity to convey how they felt about a certain mathematical topic.

In Figure 4.21A, the students offered reflections that provided teachers with insight into the areas students felt they were having trouble understanding. Ivory, Tom, Elexius, and Marsha's responses below indicated they were having trouble with multiplication and division. These examples are useful for teachers to plan instruction and for students to think about their own learning. Figure 4.21B provides examples from Jari, Linda, Sylvia, and Albert to the same reflection question; however, these students' responses showed confidence in their understanding.

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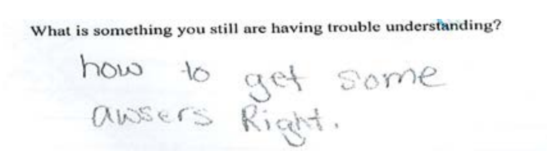
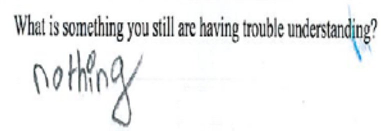
<td>A


"How to get some answers right." Ivory</td>
<td>B


"nothing" Jari</td>
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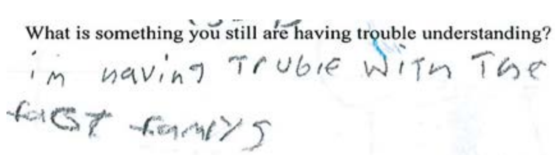
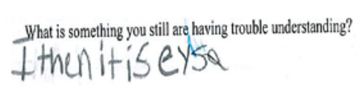
<tr>
<td>

"I'm having trouble with the fact families" Tom</td>
<td>

"Then it is easy" Linda</td>
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</table>

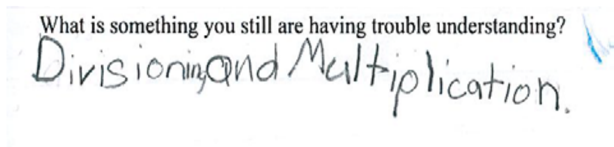
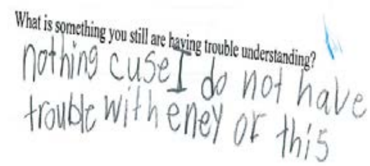
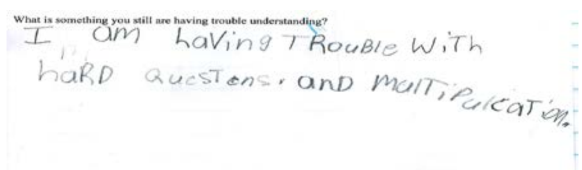
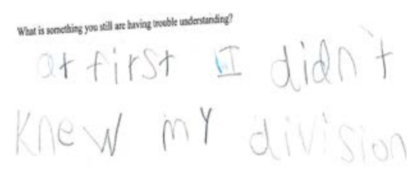
 "Division and multiplication" **Elexius**	 "Nothing cause I do not have trouble with any of this" **Sylvia**
 "I am having trouble with hard questions, and multiplication." **Marsha**	 "At first I didn't know my division." **Albert**

Figure 4.21. Expressions of Confidence or Areas of Struggle

These reflections provided information that became part of planning instruction for Michelle and me (see interviews with Michelle on p. 115). These reflections were considered for mini-lessons, conferences, and grouping. In particular, the students presented in Figure 4.21A continued to use manipulatives to solve multiplication and division problems. This group of students also received more instruction on these concepts and worked less independently.

In these examples, the process of writing allowed students to clearly state their thoughts about what they were still having trouble with; these reflections provided information for teachers to make decisions about moving forward and how to move forward. Reflections also provide material for meaningful interactions with students.

Reflections Help Instructors Interact with Students

The reflection component of the workshop provided an opportunity for students to reflect on what they learned, the strategies they used, and the questions they had about the mathematical concepts presented in the lessons. These reflections were beneficial for teachers to review students' writing and their work for consistency, to plan instruction according to the needs presented in the workshop material, and to interact with students based on the students' writing. Michelle used several pieces of data and the curriculum guides to plan the whole-group lesson and the types of questions to be included in the workshop when the class separated into groups one and two. I supported her whole-group lesson with the small-group mini-lesson and helped create writing prompts and reflections for the workshop. While working with a group of students I regularly worked with, I used their reflections to interact with them.

Reflections were helpful for conferencing. In Figure 4.20, the reflections showed students identified strategies that were not evident in their actual calculations. This created an opportunity for teachers to interact with the students and have discussions based on the writing. Tom and Joselyn asserted that they have used an open-array strategy, but their calculations did not reflect this strategy. Conferring with these students included questions about how to use an open-array strategy; these types of questions can provide insight into whether the students included this terminology because of cues in their social environment or if they equated repeated addition with the open-array strategy. The students' writing served as a gateway into finding which strategies they used. The examples in Figures 4.21A and 4.21B also provide an entry point for further conversation. In Figure 4.21A, the students' responses showed some concerns about multiplication and division; however, a conference can be constructed to have the students elaborate on these responses and offer specific details. Some students' responses—for example, "I think it is easy" and "nothing I do not have trouble with this" in Figure 4.21B—indicate some level of confidence with multiplication and division. A conference provides an opportunity to have these students elaborate on their understanding.

In some reflections, the students provided unclear responses. Conferring with these students about their writing filled in important information and avoided assumptions. In Figure 4.22, several of the students' responses necessitated further discussion in conferences.

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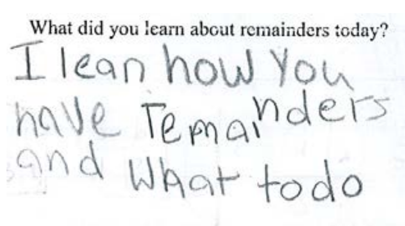
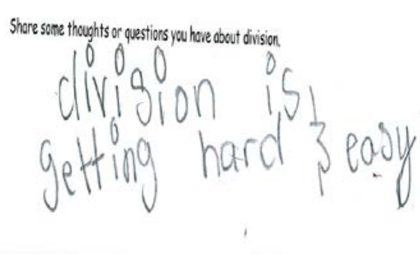
<td>


.I learn how you have remainders and what to do."
</td>
<td>
Conference Notes:
Linda is using fact families to solve the division problems in the workshop. I asked her what she meant by "I learn how you have remainders and what to do" She said, "Because crackers you can split in half or put away, but you would need another van." I asked her how she could include this interpretation in her problem; she shrugged her shoulders and seemed unsure how to express her thinking.
Linda
</td>
</tr>
<tr>
<td>


"Division is getting hard and easy."
</td>
<td>
Jari begins her problem solving with pictures of four students and uses a one-by-one strategy with the M&M's to divide 24/4. During the first 2-minute conference, I asked her about her strategy and she said, "I can use drawing to divide"; she used a similar sentence on the reflection question "What did you learn today?" In the second conference, I asked what about division is getting hard and easy. She responded, "I don't know, the drawing is easy but it's a lot."
Jari
</td>
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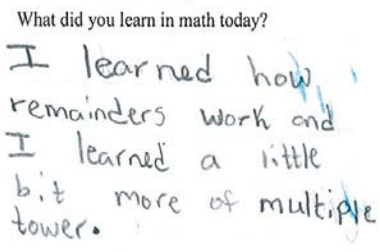
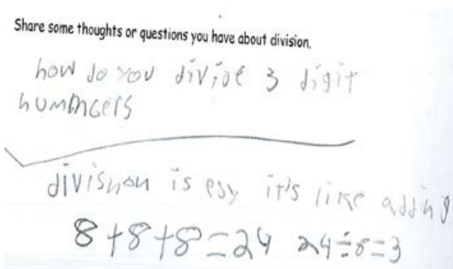
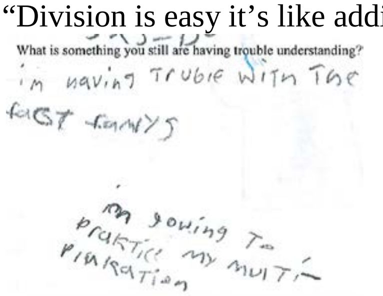
 What did you learn in math today? I learned how remainders work and I learned a little bit more of multiple tower. “I learned how remainders work and I learned little bit more of multiple tower.”	Zoe's work includes drawings to solve the division problems. During the first conference I mentioned the work she had done the day before and how she wrote out multiples of the divisor. I told her she had been using the multiple-tower strategy the day before. In the next conference I asked her about her reflection and she said, “I kind of know towers and that remainder is left over, but I'm not sure.” **Zoe**
 Share some thoughts or questions you have about division. how do you divide 3 digit numbers division is esy it's like adding 8+8+8=24 24÷8=3 “How do you divide 3 digit numbers” “Division is easy it’s like adding.”  What is something you still are having trouble understanding? i'm having trubie with the fact famlys I'm going to praktice my multiplication “I’m having trouble with the fact families. I’m going to practice my multiplication.”	Shared some family issues that happened in the morning. He seemed distracted, but willing to participate. He felt he was struggling with fact families. When I asked how he could get better, he said, “Practice.” I asked if he meant practice his multiplication facts and he said yes, and included that in his writing. **Tom**

Figure 4.22. Reflections and Conferences

In the above reflection, Linda explained that she learned about remainders and what to do. The conference question centered on exactly what is meant by “what to do,” to gain further clarification. By talking with Linda about her reflection, I discovered—by her response, “Because crackers you can split in half or put away, but you would need another van”—that she was starting to understand that remainders require interpretation. It was also evident that she was unsure how to express this interpretation in the explanation of her problem solving. The conference offered insight into her thinking; her reflection could have been understood in different ways, and the discussion clarified her response.

In the next example, Jari wrote that division was “getting hard and easy.” I addressed the ambiguity of this response in the second conference. Jari’s response—“I don't know, the drawing is easy but it’s a lot”—seemed to allude to the limitations of the drawing strategy. She used her drawings in the previous two problems to get the correct quotient, but perhaps recognized the time consuming nature of the drawing strategy. In the last reflection, Zoe appeared to be gaining a firmer understanding of division, multiple towers, and remainders. Her conference response—“I kind of know towers and that remainder is left over, but I'm not sure”—seemed to show a familiarity with the definition of “remainder” and a slight confidence building with the multiple-tower strategy. The next example of reflection is tied to the previous example from Tom in Figure 4.8. In Figure 4.8, Tom‘s work and writing indicated that he finds division and multiplication challenging. In the following reflective question, which was part of the division workshop, Tom revealed his concern. Tom’s writing illustrated his own recognition of the limitations of repeated addition as a method of solving larger problems. In the second section of Figure 4.8, Tom recognized his difficulty with fact families. During the conference I asked how he could address this area and he mentioned practicing his multiplication tables. He included this in his reflective writing. Reflections provided a place to start conversations and to have the students further reflect on their thinking as they began to verbally communicate the meaning in their writing. The conversations focused on attaining clarity and better understanding of the students' thinking.

The reflective component of the workshop served as a place for students to reflect on their strategies and as a guide for teachers to reconcile the consistency between strategies recorded in the reflection and the actual calculations. Michelle mentioned deciding to revisit the open-array strategy with Tom. She revisited the strategy with Tom after additional instruction with the strategy, and Tom appeared to be doing better. (Unfortunately, this instruction when I was not there, so it was not recorded in the field notes.) The reflections provided a place for students to more directly communicate the areas they were struggling in and where they felt more confident in their skills. This information was helpful when planning to move forward or to revisit a topic. The reflections created a place to engage students in conversation, and these conversations or conferences helped construct deeper understandings of the students' thinking.

Summary of Finding 2

The written reflection helped students identify the strategies they used, present their concerns, and affirm the areas in which they felt secure. The writing produced in their reflections helped teachers to interact with students, review reflections and actual calculations, and make informed decisions about instruction. In most workshops, the reflection question, "What did you learn today?" was used. Students often wrote about the strategy they used in their calculations. These reflections were used to compare actual calculations. Consistency between students' work and students' reflections indicated that the student was becoming more proficient in that strategy. Inconsistencies indicated that students might need more instruction.

Consistency between reflection and strategy used was informative for teachers to decide how to proceed. Another reflection question often included in workshops was, "What is something you are still having trouble understanding?" This question allowed students to directly communicate their difficulties and need for more instruction, and, in other cases, allowed students to exude a level of confidence in the material that was also helpful in deciding to move forward. Student interviews indicated that students recognized the value of reflecting and writing. Interview responses included that it helped them learn, showed them what to do, and provided a resource to use repeatedly.

The reflections also provided information for teachers to use in interactions and to gain clarification. Students' reflections were at times ambiguous, and it was important to have conversations to understand what they were expressing. The conversations sometimes revealed that the students were recognizing an idea, but were still unsure how to connect it to solving math problems. Linda was beginning to realize that remainders required interpretation and was unsure how that would be reflected in the quotient. Both Jari and Tom were recognizing the limitations of the strategies they were currently using for multiplication and division. However, the students were not equipped yet with a new strategy to remedy those limitations. In all three conversations, the students revealed their thinking, which informed instruction.

Third Finding: Students' Written Explanations Informed Instruction

During the workshop model, writing was infused into mathematics throughout mathematics instruction. Students were encouraged to write about their problem solving and their thinking, and to reflect on what they were learning. The writing in these components was informative for grouping, lesson design, and conferring. One category with three subcategories materialized from this finding (Figure 4.23).

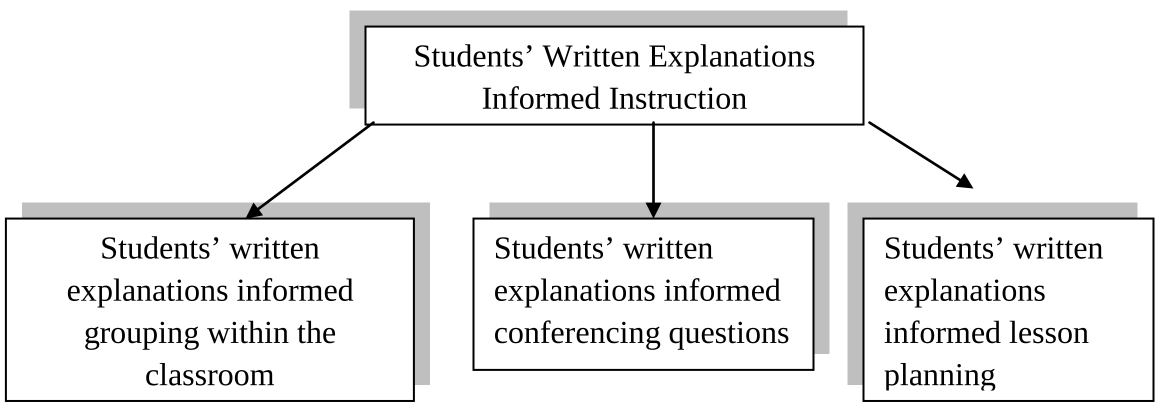


Figure 4.23. Third Finding with Subcategories

Students' Written Explanations Informed Grouping within the Classroom

Sunny Brook Elementary, as mentioned in Chapter 3, analyzed third-grade End of Grade (EOG) scores and the first two months of fourth-grade data to homogeneously group their students for mathematics instruction. There are four fourth-grade classrooms at Sunny Brook. This case study was conducted in the classroom of the lowest performing students. The grouping of the fourth-grade students occurred three weeks before this case study started. Michelle and I had met and discussed the workshop case study in the summer. However, having the group of students with lower mathematics scores and the pressure for students to show improvement on the state EOG test added pressure for Michelle.

Students' writing showed differences in their mathematical thinking

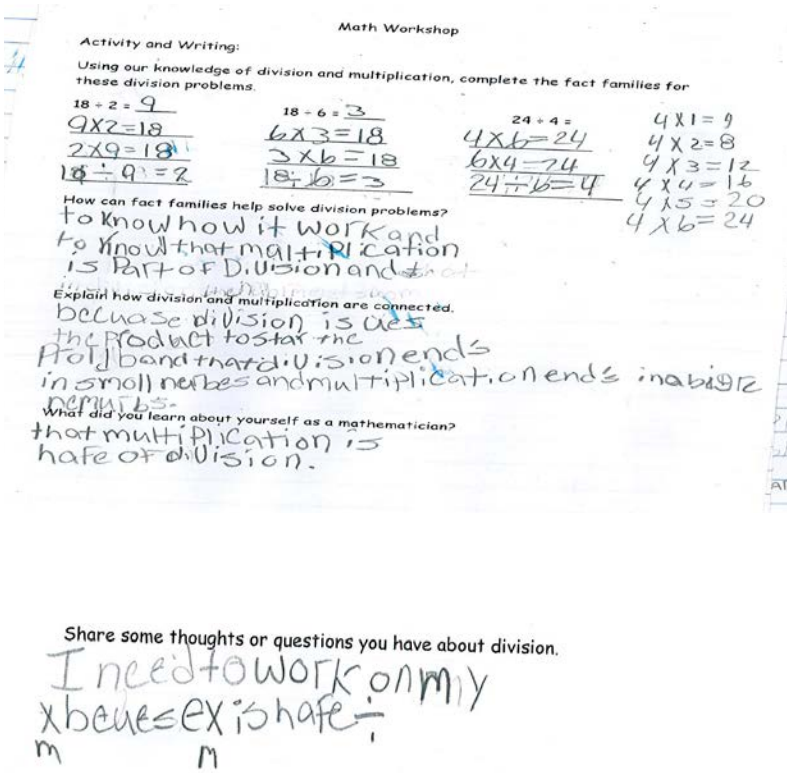
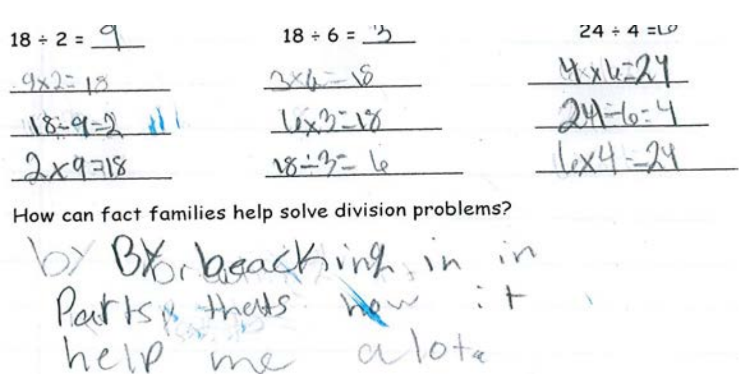
Michelle mentioned concerns about the amount of time devoted to the workshop, and feared that this particular group of students would need more instruction. She expressed this toward the end of the first two weeks in an e-mail:

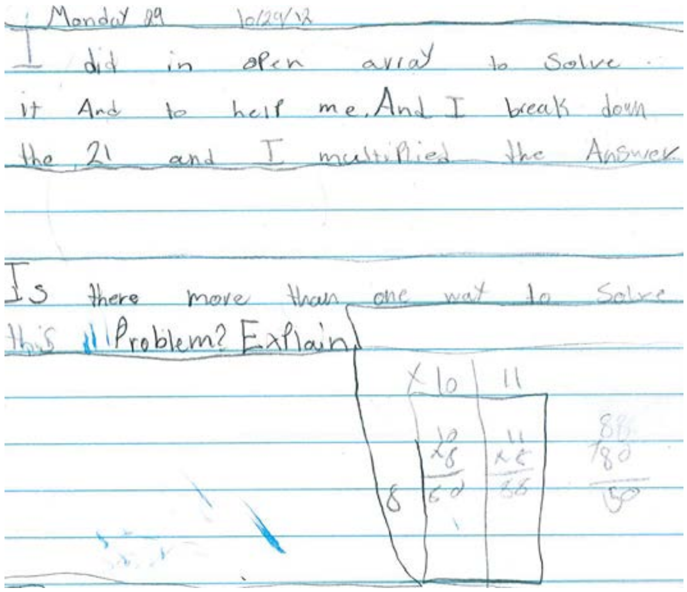
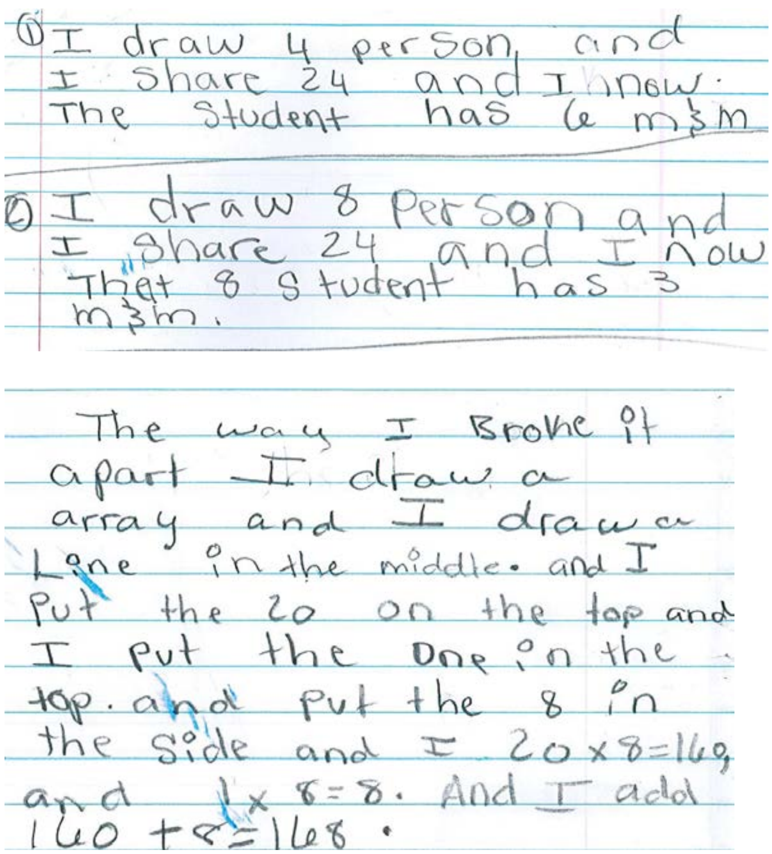
> The problem is that we do need to be moving on, and there doesn't seem to be enough instructional time. What if we did 2 quick writes on general topics a week. For example, what did you learn today? How did you solve this problem? I like the idea of writers' workshop, I just don't think that it is a good practice with this particular group. I also think that some topics lend themselves better to writing than others. Just some thoughts as we get into this further. (11/7/12)

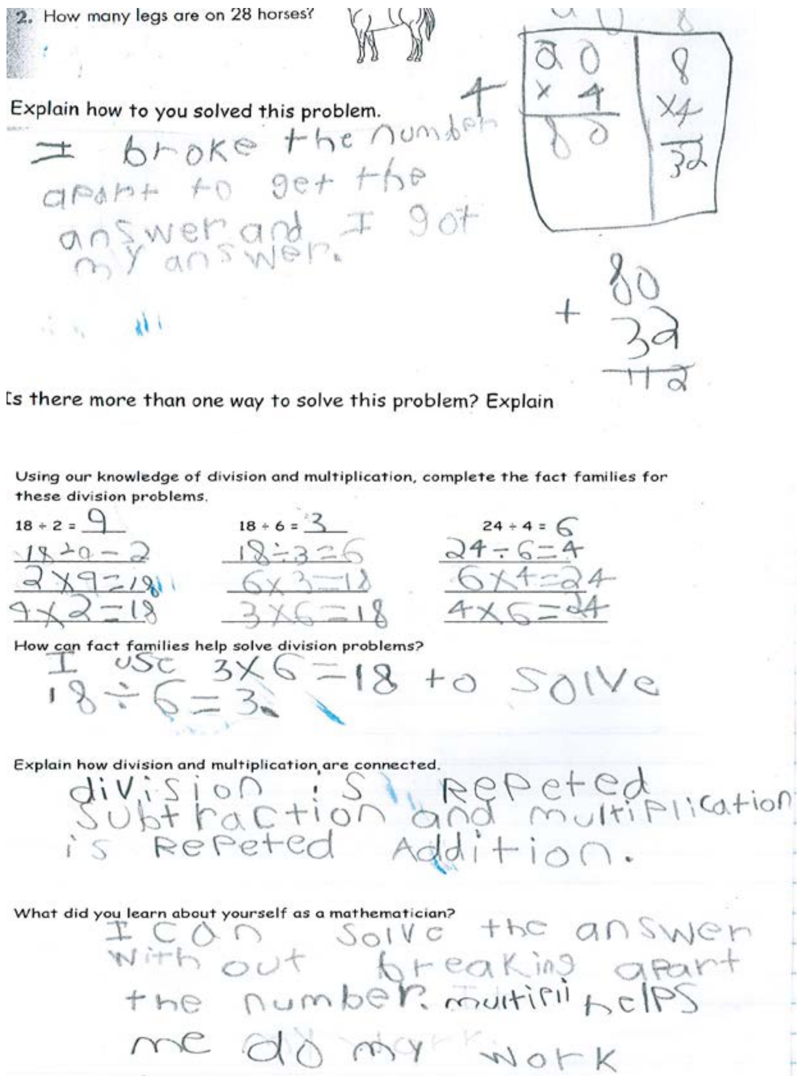
According to the case-study method, the end of the first two weeks was a time for analysis and possible restructuring of the workshop. I feared that reducing the model to two quick writes would eliminate opportunities to learn from student writing and individualize instruction. However, Michelle takes full responsibility for her students' learning, and needed to know whether her students were benefiting from this experience.

I began to analyze the six workshops completed over the first two weeks of the case study. Students' writing and the mathematical calculations included in these workshops highlighted differences between the students. Of the 18 students in the class, seven appeared to have stronger mathematical computation skills, and their writing seemed to show a greater understanding of the mathematical concepts. Two Spanish-speaking students seemed more engaged in workshop due to the materials in the workshop having been translated into Spanish. The analysis of data from the other nine students was aligned with the fears Michelle had expressed. I presented the findings of the groupings (Figures 4.24 and 4.25) and presented a new model for Michelle to consider for the next two weeks. Included in Figures 4.24 and 4.25 is the analysis of the students' progress over the two weeks of instruction. Michelle accepted this model based on the analysis of the first two weeks included in the figures and her own findings with her small-group and whole-class instruction.

The proposed model for the writers' workshop format was continued with the seven students that seemed ahead in certain aspects and the two Spanish-speaking students. The other nine students worked directly with Michelle in a small group. Michelle's group focused on math instruction and practice without the writing component. In most workshops, the class would begin as a whole group and Michelle would provide instruction before breaking into groups. I would then work with the nine students in group one; workshops began with a mini-lesson that reiterated the lesson just presented by Michelle. The workshop writing data was only collected from the nine students in the workshop group going forward. Figure 4.24 includes seven of the students in the group and the analysis of their work from the beginning two weeks. Andres and Hernando were the other two students who were part of the group that continued with the workshop; both only speak and read Spanish.

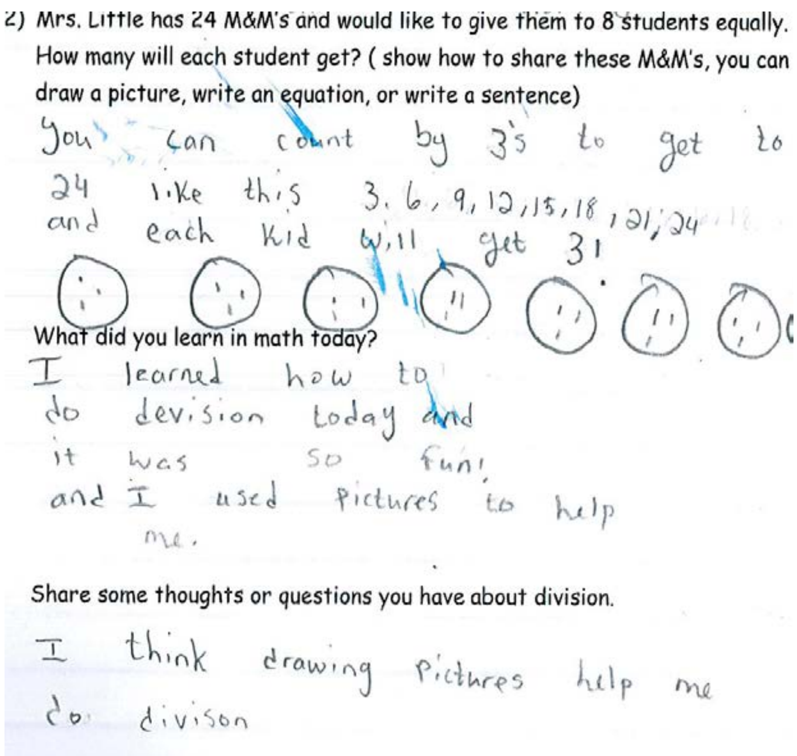
 Math Workshop Activity and Writing: Using our knowledge of division and multiplication, complete the fact families for these division problems. 18 ÷ 2 = 9 9x2=18 2x9=18 18÷9=2 18 ÷ 6 = 3 6x3=18 3x6=18 18÷6=3 24 ÷ 4 = 4x6=24 6x4=24 24÷6=4 4x1=4 4x2=8 4x3=12 4x4=16 4x5=20 4x6=24 How can fact families help solve division problems? to know how it work and to know that multiplication is Part of Division and Explain how division and multiplication are connected. becuase division is uses the product to star the fold band that division ends in smoll nerbes and multiplication ends in a bigre nembrs. What did you learn about yourself as a mathematician? that multiplication is hafe of division. Share some thoughts or questions you have about division. I need to work on my x beues ex is hafe ÷	Analysis of first two weeks of case study data: Linda – She appears ready to use her multiplication facts to solve division problem. She is able to construct fact families with ample time for writing and she offers explanations of her work during conferences. She asserts below that she needs to study her facts; I believe she would be willing and able to continue in workshop.
 18 ÷ 2 = 9 9x2=18 18÷9=2 2x9=18 18 ÷ 6 = 3 3x6=18 6x3=18 18÷3=6 24 ÷ 4 = 6 4x6=24 24÷6=4 6x4=24 How can fact families help solve division problems? by breaking in in Parts thats how it help me a lot.	Analysis of first two weeks of case study data: Phoebe - She seems to be doing well with the fact families and her writing indicates she is reflecting on what is helpful for her to solve problems. She also

 Monday 29 10/29/18 I did in open array to Solve it And to help me. And I break down the 21 and I multiplied the Answer Is there more than one way to Solve this Problem? Explain	expressed her work with an open array clearly and was able to show another strategy.
 ① I draw 4 person, and I share 24 and I know. The Student has 6 m$m ② I draw 8 Person and I share 24 and I now That 8 Student has 3 m$m. The way I Broke it apart I draw a array and I draw a Line in the middle. and I Put the 20 on the top and I put the One in the top. and put the 8 in the Side and I 20 x 8 = 160, and 1 x 8 = 8. And I add 160 + 8 = 168.	Analysis of first two weeks of case study data: Jari – She seems to articulate the way she solves division and open arrays well in the examples below. Her writing has offered a place to begin the conversation when we are conferencing. She also seemed eager in today’s work (11/8), to do more, and move forward. She attempted the 3rd problem on her own and was able to talk it through clearly.

<table>
<tr><td></td><td></td></tr>
<tr><td>
2. How many legs are on 28 horses?
20 × 4 = 80 | 8 × 4 = 32
Explain how to you solved this problem.
I broke the number apart to get the answer and I got my answer.
80 + 32 = 112
Is there more than one way to solve this problem? Explain
Using our knowledge of division and multiplication, complete the fact families for these division problems.
18 ÷ 2 = 9 | 18 ÷ 6 = 3 | 24 ÷ 4 = 6
18÷9=2 | 18÷3=6 | 24÷6=4
2×9=18 | 6×3=18 | 6×4=24
9×2=18 | 3×6=18 | 4×6=24
How can fact families help solve division problems?
I use 3×6=18 to solve 18÷6=3.
Explain how division and multiplication are connected.
division is repeted subtraction and multiplication is repeted Addition.
What did you learn about yourself as a mathematician?
I can solve the answer with out breaking apart the number. multipli helps me do my work</td><td>Analysis of first two weeks of case study data:

Albert – He seems to have a strong handle on his multiplication facts and is making connections to how these facts can help him solve problems. In the first workshop he refrained from writing too much about his thought process; each workshop thereafter, he has increased his description through both words and pictures.</td></tr>
<tr><td></td><td>Analysis of first two weeks of case study data:

Zoe – She seems to be expressive in her writing and this has helped with conferencing. She stumbled across a multiple strategy during the</td></tr>
</table>


2) Mrs. Little has 24 M&M's and would like to give them to 8 students equally. How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence)

You can count by 3's to get to 24 like this 3, 6, 9, 12, 15, 18, 21, 24 and each kid will get 3!

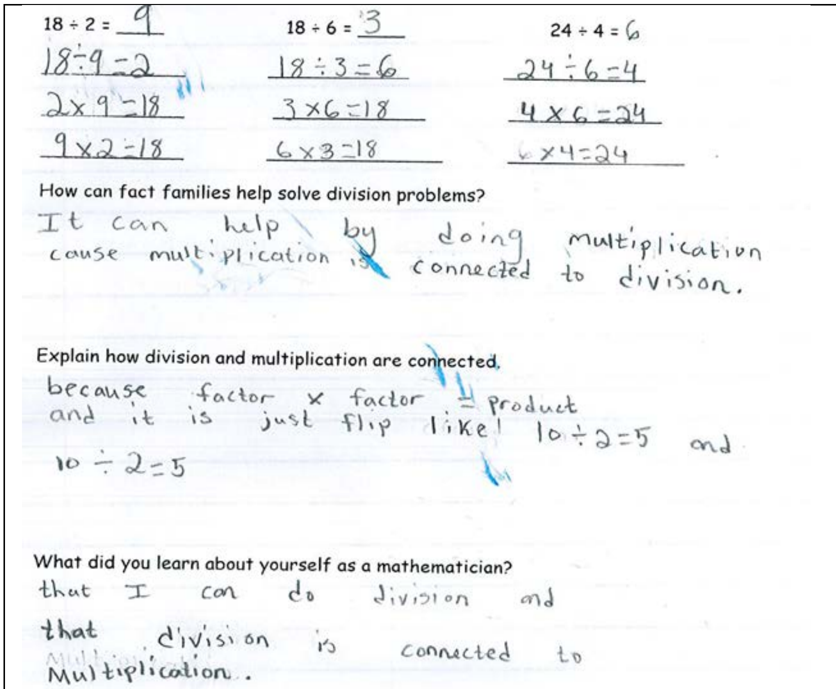
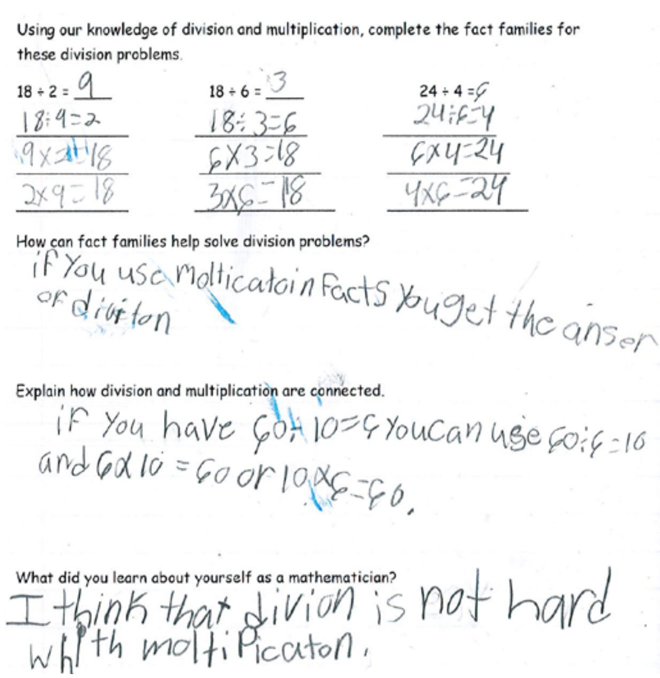
What did you learn in math today?

I learned how to do devision today and it was so fun! and I used pictures to help me.

Share some thoughts or questions you have about division.

I think drawing pictures help me do divison

Wednesday workshop (11/7), and this was such a great conversation that lent itself to today's multiple tower work. I think she can benefit from one on one because she appears to be thinking about her strategies and writing those thoughts down, and is excited to share. From her writing, I have been adding instruction through questioning and repeating back her ideas. Today (11/8), during whole group, she didn't revisit her strategy. She saw 32/10 and shouted a few times in class, "4*8=32." I think that if she had been tackling that problem in a small group, she could have been directed to her previous writing and had her strategy reinforced in the one-on-one conference.

 18 ÷ 2 = 9 · 18 ÷ 6 = 3 · 24 ÷ 4 = 6 18÷9=2 · 18÷3=6 · 24÷6=4 2x9=18 · 3x6=18 · 4x6=24 9x2=18 · 6x3=18 · 6x4=24 How can fact families help solve division problems? It can help by doing multiplication cause multiplication is connected to division. Explain how division and multiplication are connected. because factor x factor = product and it is just flip like! 10÷2=5 and 10÷2=5 What did you learn about yourself as a mathematician? that I can do division and that division is connected to Multiplication.	
 Using our knowledge of division and multiplication, complete the fact families for these division problems. 18 ÷ 2 = 9 · 18 ÷ 6 = 3 · 24 ÷ 4 = 6 18÷9=2 · 18÷3=6 · 24÷6=4 9x2=18 · 6X3=18 · 6x4=24 2x9=18 · 3X6=18 · 4X6=24 How can fact families help solve division problems? if You usa moltiCatoin Facts You get the anser of divi ton Explain how division and multiplication are connected. iF You have 60÷10=6 You can use 60÷6=10 and 6x10=60 or 10x6=60. What did you learn about yourself as a mathematician? I think that divion is not hard whith moltiPicaton.	Analysis of first two weeks of case study data: Sylvia – Shows strong understanding of fact families and is making connections between multiplication and division. She provides an example of 60/10=6 and 60/6=10 to show that she can transfer from examples in the activity to outside problems. Her writing is clear and offers a great starting point for conferencing.

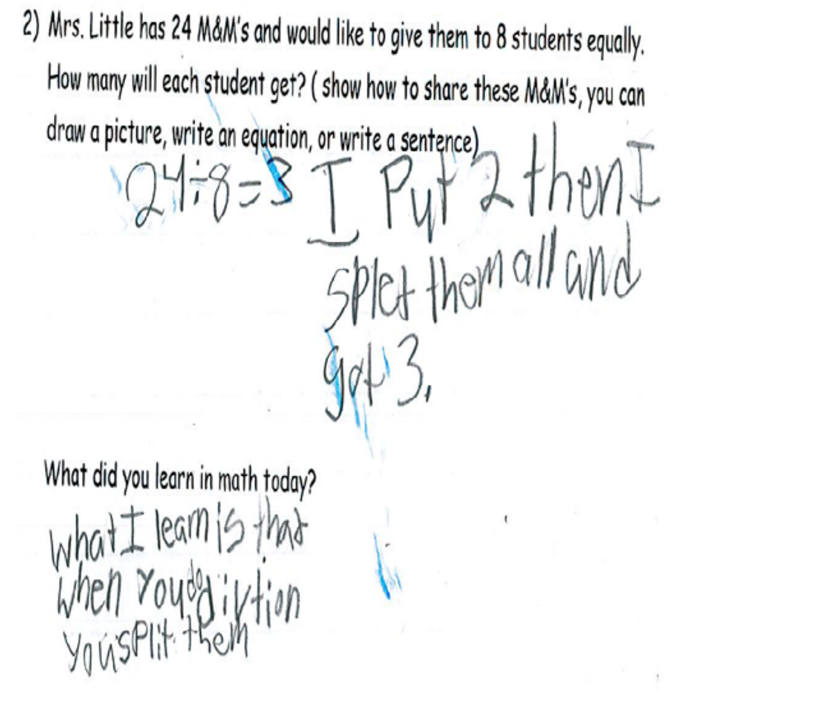
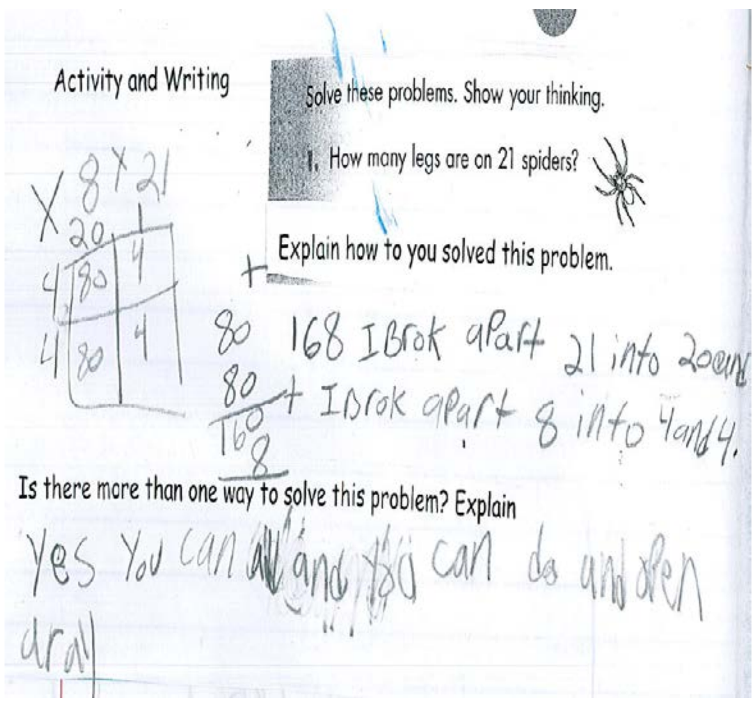
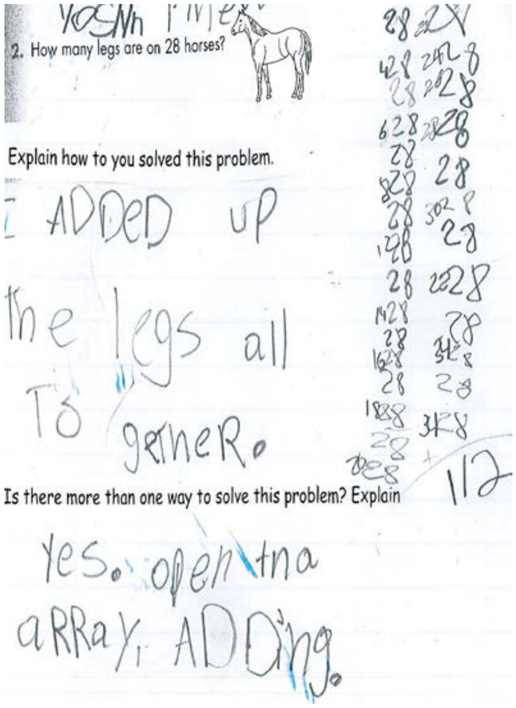
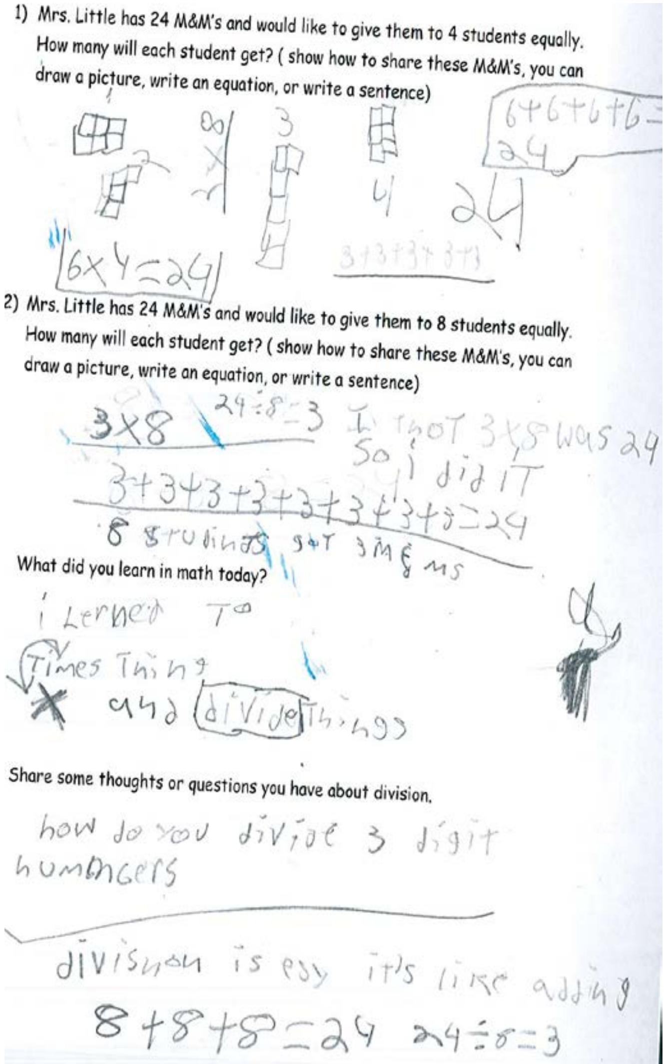
 2) Mrs. Little has 24 M&M's and would like to give them to 8 students equally. How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence) 24÷8=3 I Put 2 then I splet them all and get 3. What did you learn in math today? what I learn is that when you do divition you split them	
 Activity and Writing Solve these problems. Show your thinking. 1. How many legs are on 21 spiders? Explain how to you solved this problem. 8 X 21 168 I Brok apart 21 into 20 and 1 I Brok apart 8 into 4 and 4. Is there more than one way to solve this problem? Explain Yes You can add and You can do and open draw	Analysis of first two weeks of case study data: Hannah – Her computation skills seem to be strong and her writing suggests she is able to think through what she did to attain her answer and express those steps.

Figure 4.24. Analysis of Group One

Group Two	Analysis of first two weeks of case study data:
 	Joselyn – She seems to know that an open array is another way to solve the problem, but is reluctant to complete the problem with multiplication.



1) Mrs. Little has 24 M&M's and would like to give them to 4 students equally. How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence)

6+6+6+6= 24

6x4=24

2) Mrs. Little has 24 M&M's and would like to give them to 8 students equally. How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence)

3x8 24÷8=3 I thot 3x8 was 24 so I did it

3+3+3+3+3+3+3+3=24

8 studints get 3 M&Ms

What did you learn in math today?

i Lerned To Times Things and divide Things

Share some thoughts or questions you have about division.

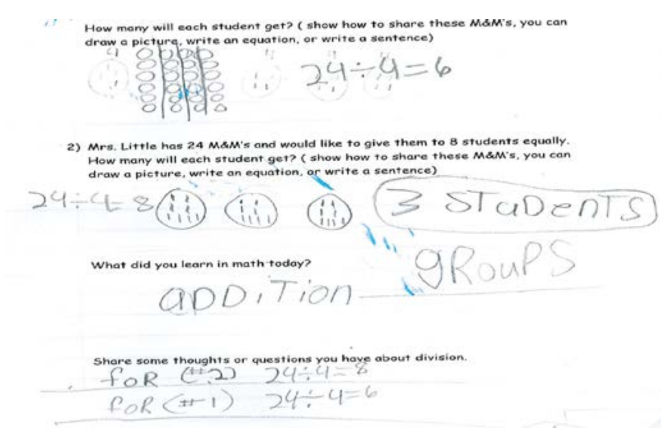
how do you divide 3 digit numbers

divison is esy it's like adding

8+8+8=24 24÷8=3

Analysis of first two weeks of case study data:

Tom – His thoughts appear scattered and his understanding of concepts seems shaky. He also held tightly to repeated addition to solve larger problems. His computations skills show accuracy, but it is hard for him to express his own thinking.



How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence)

24÷4=6

2) Mrs. Little has 24 M&M's and would like to give them to 8 students equally. How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence)

24÷4=8 3 STuDenTS

What did you learn in math today?

aDDiTion gRouPS

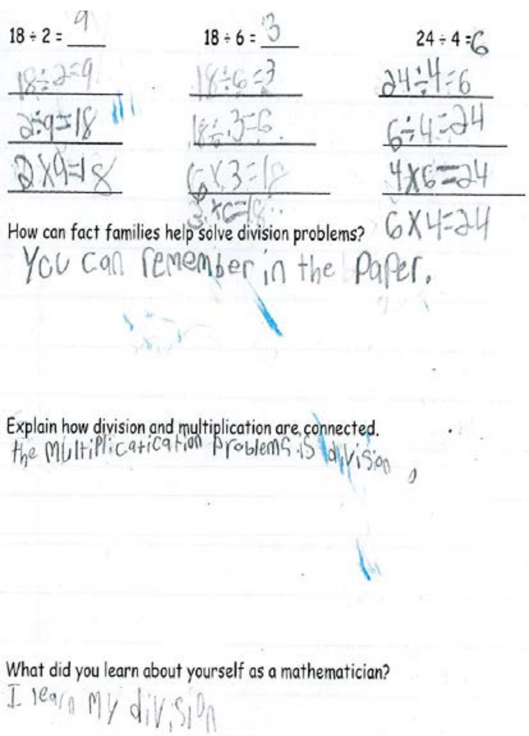
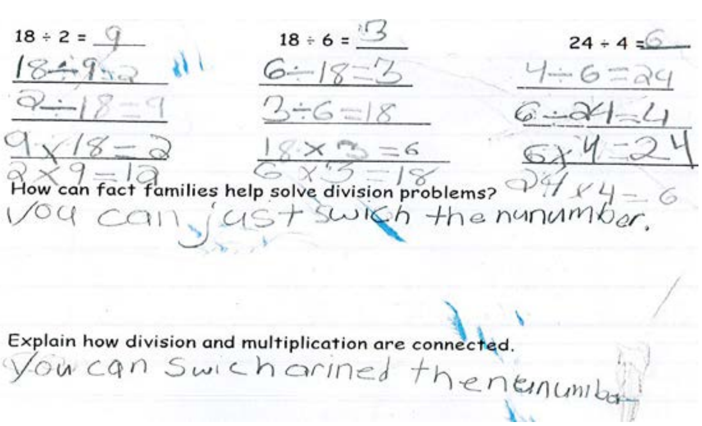
Share some thoughts or questions you have about division.

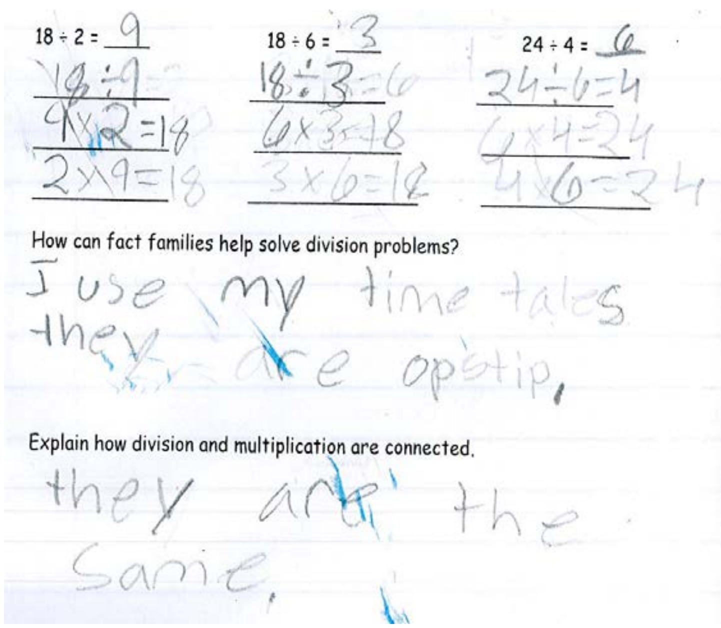
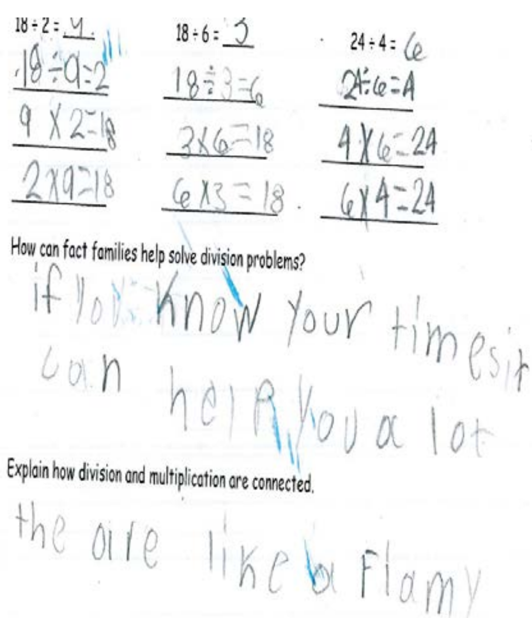
foR (#2) 24÷4=8

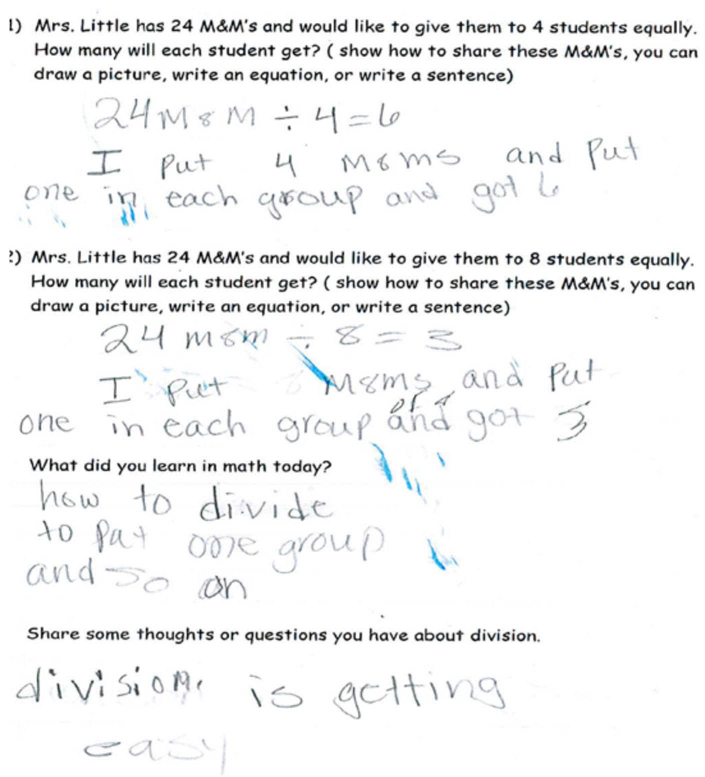
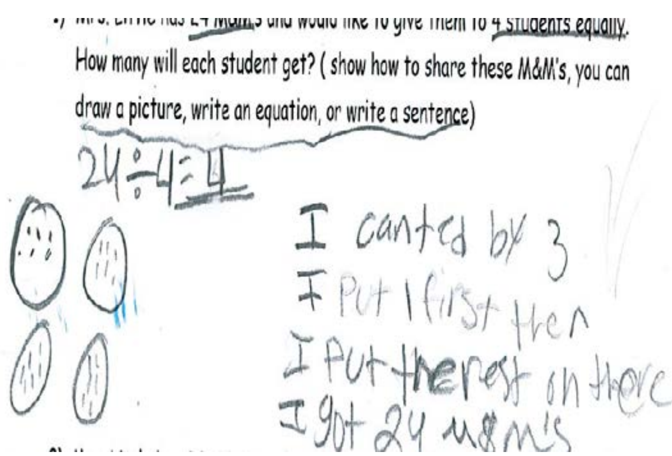
foR (#1) 24÷4=6

Analysis of first two weeks of case study data:

Marsha –She uses pictures to help her complete both problems, and in some cases the answers and the pictures are different. She appears to struggle with her computation and has difficulty communicating her understanding in

	conferences and in her writing.
 18 ÷ 2 = 9 18 ÷ 6 = 3 24 ÷ 4 = 6 How can fact families help solve division problems? You can remember in the Paper. Explain how division and multiplication are connected. the Multiplication Problems is division What did you learn about yourself as a mathematician? I learn My division	Analysis of first two weeks of case study data: Mark – He seems to be lacking computation skills, and his understanding of multiplication and division is unclear. He seems distracted in whole group and workshop. The one-on-one conversation are a challenge, but he does try to explain his reasoning.
 18 ÷ 2 = 9 18 ÷ 6 = 3 24 ÷ 4 = 6 How can fact families help solve division problems? you can just swich the nunumber. Explain how division and multiplication are connected. you can swich arined the nenumber	Analysis of first two weeks of case study data: Alexis – She seems to have a limited understanding of multiplication and division. Her fact families have several crucial errors and it appears that she struggles with computation. Her level of engagement may be

	directly correlated with her math foundational knowledge.
	Analysis of first two weeks of case study data: Joe – He expresses an idea that multiplication and division are opposite operations, then in the next sentence states that they are the same. In the other workshop he participated in, he produced minimal writing. It appears he is struggling with multiplication and division.
	Analysis of first two weeks of case study data: Oscar –He seems to grasp the fact families; he was extremely eager to fill in the blank when I gave some challenge problems at the end of lesson after sharing (32/ ? = 8) he could see the missing family member. He appeared to struggle with his multiplication facts, and his responses indicate that he may also be struggling in literacy.

 1) Mrs. Little has 24 M&M's and would like to give them to 4 students equally. How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence) 24 M&M ÷ 4 = 6 I put 4 M&ms and put one in each group and got 6 2) Mrs. Little has 24 M&M's and would like to give them to 8 students equally. How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence) 24 m&m ÷ 8 = 3 I put 8 M&ms and put one in each group and got 3 What did you learn in math today? how to divide to put one group and so on Share some thoughts or questions you have about division. division is getting easy	Analysis of first two weeks of case study data: Ivory – She seems to be enjoying writing in math and gives details to explain her thinking. Her computations seem to be directly connected to the use of manipulatives to solve problems, and she seems comfortable when she has something tangible.
 How many will each student get? (show how to share these M&M's, you can draw a picture, write an equation, or write a sentence) 24÷4=4 I canted by 3 I Put 1 first then I put the rest in there I got 24 M&M's	Analysis of first two weeks of case study data: Kate – She writes her strategy for the problems and uses pictures to help her divide. From the conferences, I believe she is still unsure of these concepts and will be using pictures a little longer.

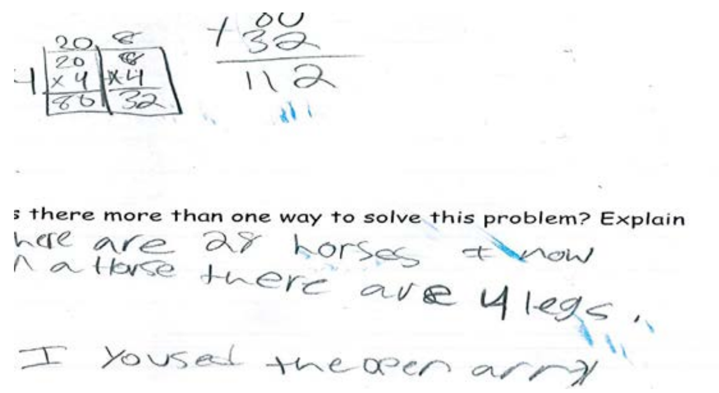


Figure 4.25 Group Two Analysis

This analysis above occurred after two weeks of the workshop. The writing and conferencing that are part of the workshop provided in the examples above offered information that revealed differences in the class. Michelle confirmed these initial findings and allowed the students in Group One to continue with a workshop model that encouraged writing. She felt better about working directly with Group Two and providing more instruction and using manipulatives and completing activities as a whole group. Although I found the writing from Group Two helpful for mini-lessons and conferences, this compromise was important for Michelle, who is ultimately responsible for student performance. In our interview, Michelle revealed more details about her fears and the basis for her concerns for the students in Group Two.

Me: Do you feel that the group that did not participate in the workshop would benefit from incorporating writing in math at another time?
Michelle: I actually feel that they may be more frustrated by it.
Me: Uh huh.
Michelle: Because they are so far below grade level, I can see them, I can see a couple of them really being frustrated.
Me: Okay.
Michelle: And that's not to say that you couldn't throw out a question that—as an exit ticket or something, and have them respond.
Me: Uh huh.

Michelle: If that makes sense, I think because they are so far below they don't have some of the confidence that some of the other group has, and I also think that because some of their skills are so low, I think they would just confuse themselves by trying to write their thoughts.
Me: Uh huh.
Michelle: And use words that they don't know how to necessarily use.

Interview (12/15/12)

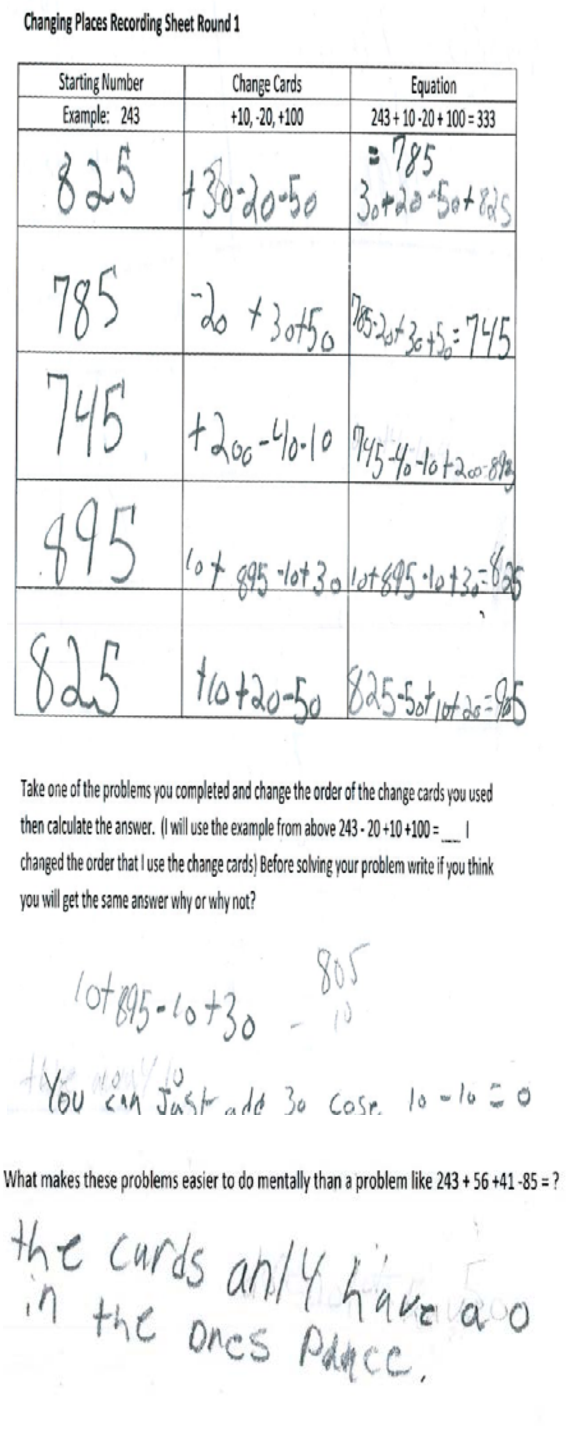
Michelle seemed to believe that Group Two would be frustrated by writing. She indicated that the students in Group Two would be further confused by trying to write their thoughts, and might use words that they do not know how to use. Her response reflects an assumption that struggling students would not benefit from writing.

The students' writing provided a way to monitor how they were progressing. The students in Group One, who continued with the workshop model, were able to complete activities independently. The sense of independence that the workshop model provided appeared to be valued by this group. In an open-ended interview question, Sylvia identified this sense of autonomy as beneficial for her productivity.

> Me: All right, how do you feel about workshop time?
> Sylvia: That it is good, because when we work, like, alone we can get more done.
>
> *Interview with Sylvia (12/07/12)*

Sylvia's positive feelings about the workshop were also supported by the students' actions. The workshop time required a table to be moved to provide space for the group of nine students to gather. Students in this group would walk into the classroom for mathematics and almost immediately move the table and want to sit in the workshop seats. Students showed enthusiasm while in the workshop. Sylvia's response offers some explanation for the enthusiasm. She feels that having more freedom to progress at her own pace is more productive than completing activities as a whole class.

Students’ written explanations helped to create partners. Their writing continued to provide information about their progress, and helped to create groupings or pairs within the group based on their writing. I reviewed their writing throughout the project. The writing and problem solving of Hayley and Sylvia suggested that they would be good partners for the Changing Places game. Hayley’s work and writing showed connections, efficiency, and some previous instruction on mathematical shortcuts. Sylvia’s work and writing indicated connections and efficiency in some workshops, and in other workshops she struggled. The pairing of these students for a mathematics game allowed them to share their thinking and learn from one another. The Changing Places game is meant to advance mental math, make connections to place values that contain zeros, and to grasp moving 10s and 100s on their 1,000 chart. Figure 4.26 shows Hannah’s first page and Sylvia’s second page of work during the game. It should be noted that their answers are the same for each round of the game. The writing provided by Hannah and Sylvia indicate that their partnership highlighted their individual concepts, which allowed each person to gain more of the ideas the activity was meant to teach. Hannah showed that the change cards can be reordered before beginning the problem to reduce the amount of addition and subtraction. She also recognized that the zero in the 1s place made her calculations easier. Sylvia noted that “this game is easy to play because you are moving along your 1,000s chart to find the correct answer.” This grouping created an environment in which different perceptions added to their learning.



Changing Places Recording Sheet Round 1

Starting Number	Change Cards	Equation
Example: 243	+10, -20, +100	243 + 10 -20 + 100 = 333
825	+30-20-50	30+20-50+825 = 785
785	-20 +30+50	785-20+30+50=745
745	+200-40-10	745-40-10+200=895
895	10+ 895 -10+30	10+895-10+30=825
825	+10+20-50	825-50+10+20=[illegible]

Take one of the problems you completed and change the order of the change cards you used then calculate the answer. (I will use the example from above 243 - 20 +10 +100 = ___ I changed the order that I use the change cards) Before solving your problem write if you think you will get the same answer why or why not?

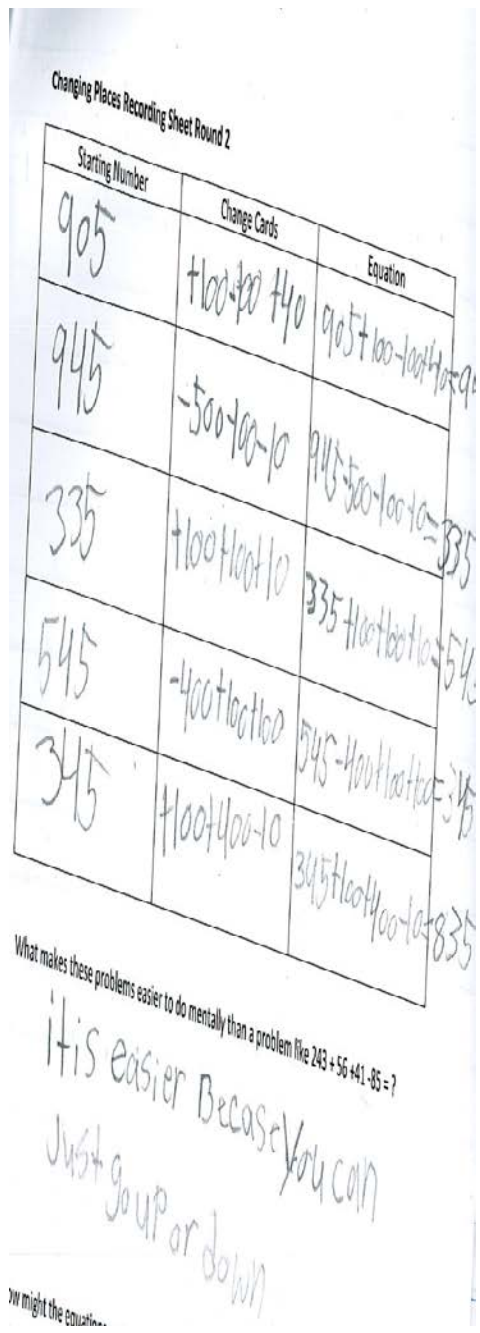
10+895-10+30 805

You can Just add 30 cose 10 - 10 = 0

What makes these problems easier to do mentally than a problem like 243 + 56 +41 -85 = ?

the cards anly have a 0 in the ones Pance.

Hannah



Changing Places Recording Sheet Round 2

Starting Number	Change Cards	Equation
905	+100-100 +40	905+100-100+40=9[illegible]
945	-500+00-10	945-500-100-10=335
335	+100+100+10	335+100+100+10=54[illegible]
545	-400+100+100	545-400+100+100=345
345	+100+400-10	345+100+400-10=835

What makes these problems easier to do mentally than a problem like 243 + 56 +41 -85 = ?

it is easier Becase you can Just go up or down

[illegible] might the equation...

Sylvia

Figure 4.26 Changing Places Game

Students' Written Explanation Informed Conferencing Questions

Calkins' (1983) case study offers a detailed examination of the workshop model. One component of the workshop model is conferencing. This time is devoted to listening, questioning, and instructing for the teacher. It typically occurs with four or five students during a workshop. Each student typically confers with the teacher for about 5-10 minutes, and on many occasions not all students are able to engage in a conference with the teacher. In the math workshop, conferencing took on a different format. Instead of a full 5-10 minute conference at one time, students would conference with a teacher for two minutes at a time, and usually more than once during the workshop.

Students' written explanations were a starting point for conferencing. The change in conferencing was based on students' needs. Some students struggled to get started on the problems and needed to talk out their thinking just to begin, while others wanted to begin, and then have a teacher listen as they reviewed their strategy. Some students benefited from having a conference that allowed them to talk through their thinking and put into words their ideas, strategies, and future questions. In the example in Figure 4.27, Ivory wrote only an equation and wasn't sure what to do next. Through the conversation, she moved from her original ideas of drawing pictures or using an open array into using multiplication facts to solve the problem. The shift from using manipulatives and pictures to using multiplication facts increased her efficiency in solving math problems.

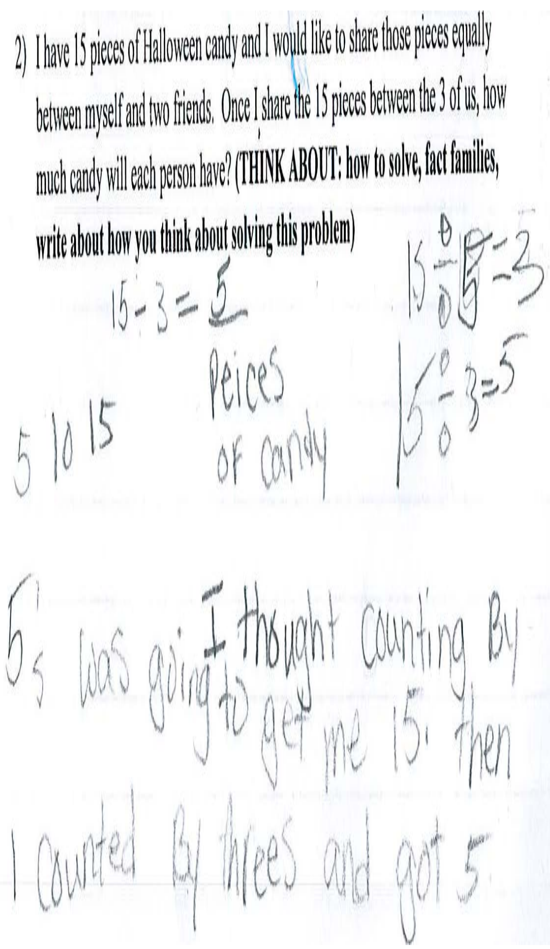
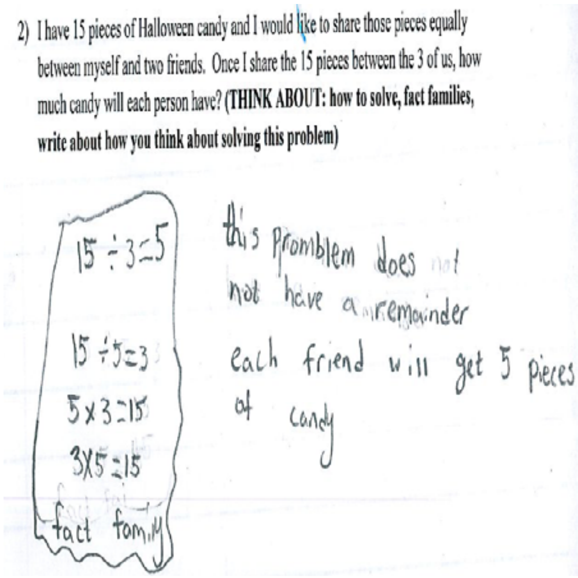
	**Me:** That's exactly the equation—that's awesome. What do you think you want to do now? **Ivory**: I think I want to draw a picture. **Me:** I think I have some of your work where you drew great pictures. **Ivory:** I know what to do now—an open array. **Me:** Is an open an array for multiplication or division? **Ivory:** It's for multiplication. **Me:** And what do you have here? **Ivory:** Division. **Me:** This is kind of interesting what are you doing here, counting by what? **Ivory:** Fives. **Me:** Yeah, and there's three groups, right? So three groups of 5. **Iyehsa:** 3,6,9,12,15. **Me:** Right. So how many are in each group? **Ivory:** 5. **Me:** And you didn't even need a picture. So what made you start with 5,10,15? **Ivory:** Because at first I thought 5 was the answer, because 5 and 3—because I thought 5 was going to help me get to 15.

Figure 4.27. Conferencing with Ivory

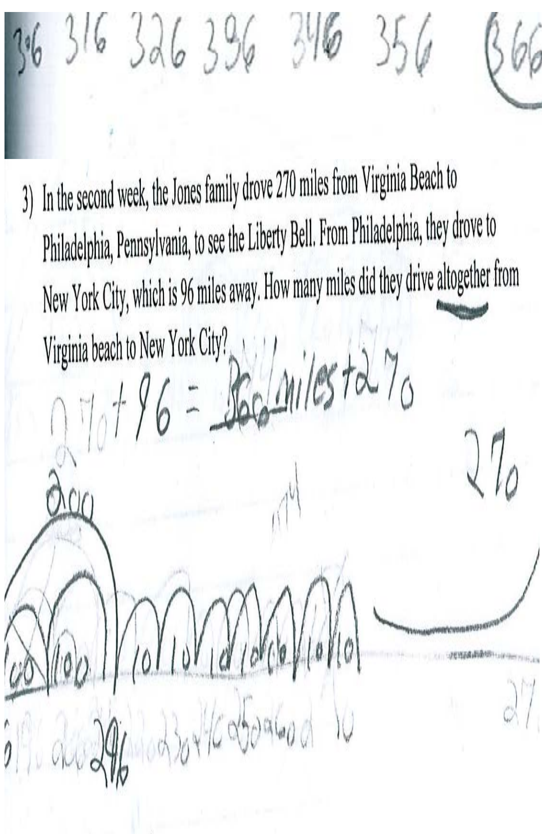
Zoe was not sure of the equation for this problem. While conferencing (Figure 4.28), she was able to make connections with additional questions and instruction. This conference lasted two minutes, and Zoe went on to the next problem without assistance.



Me: What do you think the equation is? 15 pieces of Halloween candy, and you want to share it with yourself and your two friends, so that's the three of you—so what is that equation, you think?
Zoe: 5.
Me: Interesting—you said five. What do you think five is as part of this problem
Zoe: It's actually 3.
Me: There's 3 people, right, 15 pieces of candy, and 5 is in there. So something is going on in your mind; you're seeing a relationship. Try to think it through. because I think you are on to something. [Pause] Think about your equation—you had your total up here right, then you wanted to divide it, which you did, then you got your answer.
Zoe: 15 3 = 5.
Me: So five was sticking in your head, probably because you know the fact family. Think about the fact family now—what goes with this?
Zoe : [Writes down another division equation with the same numbers}
Me: Awesome. What are the multiplication sentences that go with this?
Zoe: 5 x 3 = 15 and 3 X 5 =15.
Me: Yes, it does.

Figure 4.28 Conferencing with Zoe

This short conference helped Zoe move forward in her understanding of fact families. Zoe included the fact family in her writing, which she referred to the next day during the workshop. In the next conference (Figure 4.29), Hannah's work was difficult to understand at first glance, but it did provide enough information to begin the conference. The conference began with Hannah suggesting that she should change the order of her addition problem. In the conversation, the misconception she had about how to use the number line with this type of problem was revealed. It allowed me to talk to her about her ideas and point out her previous work to help her revise her thinking and work out the problem successfully. Hannah voiced a misconception that was challenging for other students as well.

	**Hannah**: So I need to switch the numbers around **Me:** [Hannah originally had 270 + 96 = 174, which appears as though she is solving the difference] Your problem says you start with 270 and add 96 to that number and your sum is 174. **Hannah:** [Moves to change the problem] **Me:** Hold on—let’s think this through. Okay, let’s leave your equation and I will look at your work. You start at 96, jump to 196, and then jump to— **Hannah:** And then you can’t add another 100 because you will be past 270 and land on 296. **Me:** But remember: Where you land on the number line for this problem is unknown [pointing to a previous problem]. See? You didn’t know where you were going to land. **Hannah**: Oh, yeah. **Me:** You’re hopping to see where you land; that’s the big question mark. **Hannah:** So you don’t add to get to that number. **Me:** Right—you are starting with one number and adding the other and being kind of surprised where you land on the number line. **Hannah**: Would it be 320? **Me:** Um, 320. Um.

	Hannah: 330. **Me:** I think you are just guessing now. I would like to see your number line so I know—okay, take a moment. [On the return conference, Hannah reached 366.]

Figure 4.29 Conference with Hannah

The conferences were grounded in the students' responses and their previous written reflections and work. Their writing provided the information for specific questions and also highlighted the need for broader questions. The conferences allowed students to have instruction meet them at the point of difficulty. Sylvia mentioned in her interview that the workshop "is good because when we work, like, alone we can get more done." Conferencing also allowed students to skip further instruction in areas where they were not struggling. This may have attributed to Sylvia's reference to higher productivity.

Students' Written Explanations Informed Instruction

Students' writing informed instruction and can be used in instruction. The analysis of all 18 students' writing after the first two weeks informed instruction for the following weeks. Group One moved on with instruction that included the writing component and refrained from using manipulatives. Group Two received more direct instruction and used manipulatives, and the writing component was eliminated due to time constraints.

Students' writing informed immediate instruction and the planning of future instruction. The examples presented earlier of students' writing that reflected their understanding of mathematics, mathematical vocabulary, and mathematical processes were used in instruction. Students' writing revealed a desire to use mathematical vocabulary without a clear understanding of the meaning of the terms. This changed instruction to include more questioning when students used words in their writing, conferencing, and class discussions. If the terms were unclear, it presented an opportunity to re-teach. Figure 4.30 shows an example from earlier followed by the next problem completed by Zoe. Zoe's process is inefficient, but she is arriving at the correct answer. In order to build on what she had established as a method for finding the answer, I asked her on the second problem to place a box around the amount of 10s she wrote to get to 100. She started to make the connection that there are 10 tens in 100 and that that information could help her solve the problem more efficiently. Zoe's work and writing allowed me to build on her understanding gradually.

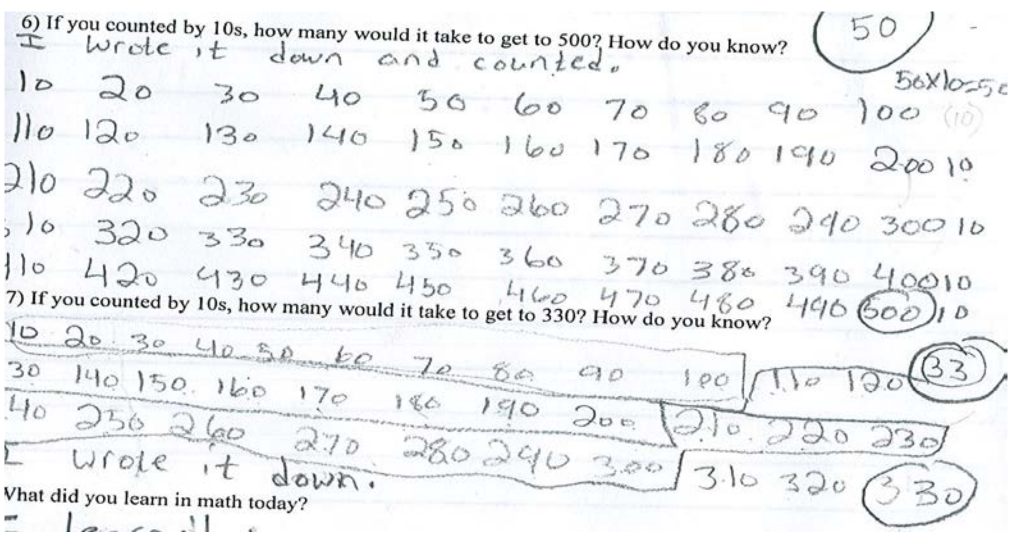


Figure 4.30 Zoe's Calculations

Students benefitted from their writing being used in instruction. In Week 3 of the case study, the concept of using multiplication or fact families to solve division problems was part of the main whole-group lesson. In the fourth week of the case study, the use of a multiple tower was introduced to solve division problems with larger numbers. In Figure 4.31, Zoe’s writing in Week 3 alluded to the multiple-tower strategy that was to be introduced the following week. I shared this with Michelle, who mentioned this in her whole-group instruction on the multiple-tower strategy. When we broke into groups, I used Zoe’s work exclusively for the mini-lesson and had her explain her work from Week 3 to the group. I reiterated her explanation and confirmed the idea that the multiples of the divisor would build up to the dividend; the height of the tower is the quotient. The group seemed more engaged when the strategy was explained by a fellow classmate.

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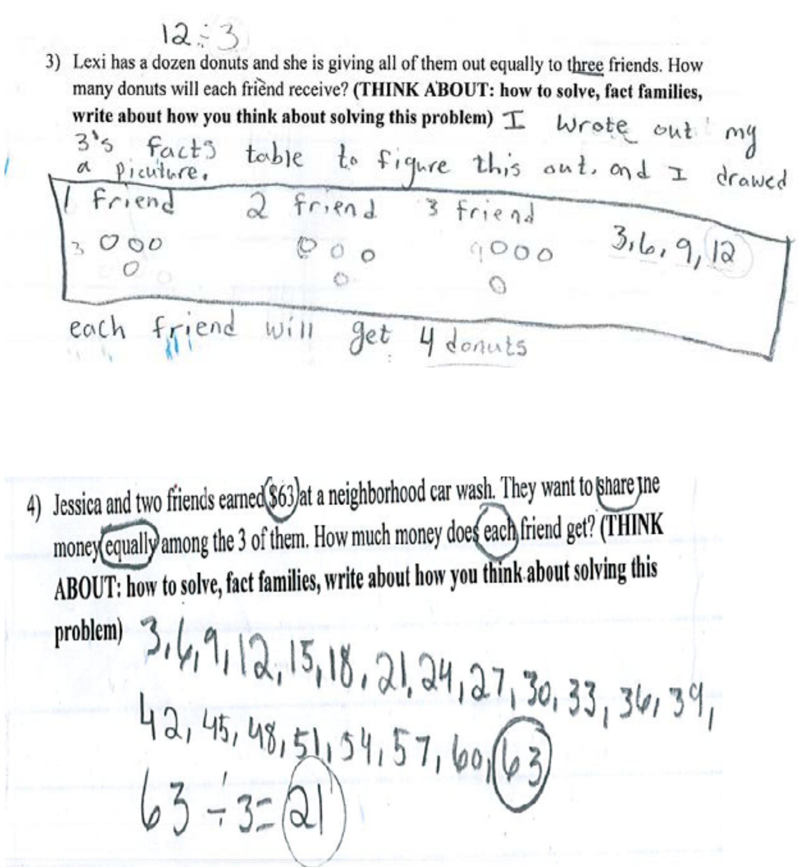
<td>
</td>
<td>Field Notes:

Zoe, Week 3:She begins to write out the multiples of the divisor until she gets to the dividend and then counts how many multiples she has written. This will be great to introduce with multiple towers.</td>
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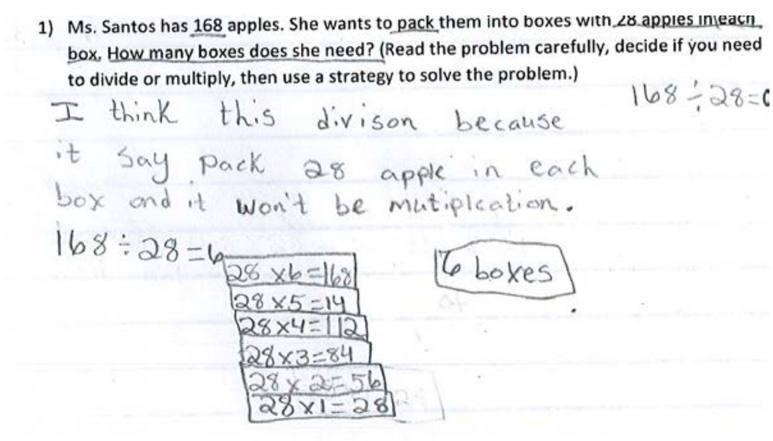

Field Notes:

Zoe: Does a great job deciding that this is a division problem and understands how to build the multiple tower to find the answer.

Figure 4.31. Writing Used for Instruction

Students had difficulty making sense of fact families involving multiplication and division. During a mini-lesson on fact families, I used Linda's explanation to re-teach the concept. Figure 4.32 contains an example from Jari, who was having difficulty with this concept; however, this can only be seen by the faint eraser marks. She made corrections based on discussions that used Linda's explanation. In both experiences, the writing of the students helped inform instruction and make instruction more engaging for them.

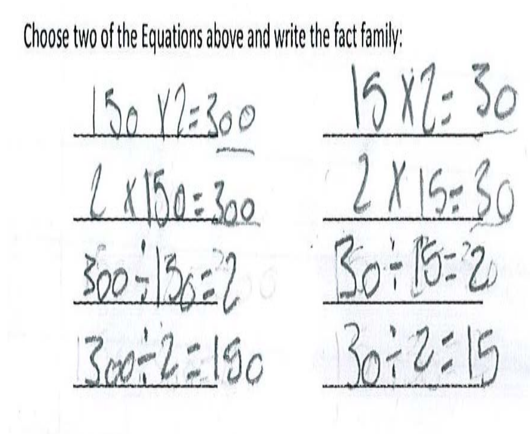	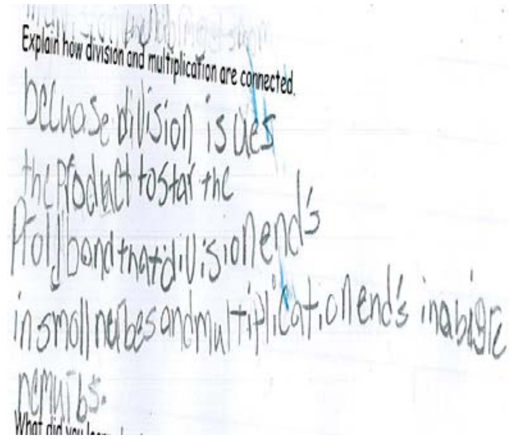
The division sentences for both fact families are where Jari erased and made corrections. **Jari**	*“Because division uses the product to start the problem and that division ends in small numbers and multiplication ends in bigger numbers.”* **Linda**

Figure 4.32 Writing used in Instruction

Summary of Finding 3

Students’ writing informed the grouping of students, conferencing topics, and the planning of future instruction. The writing served as the primary data used in the decision to split the class into two groups. The writing illustrated a difference in students’ mathematical understanding; Michelle agreed with the analysis from the writing and continued with this grouping. The workshop continued, and student writing showed differences between students' thinking and processes that suggested that certain pairings would be beneficial.

The workshop model included conferencing with students, these conferences presented opportunities for teachers to interact with students and provide instruction. Conference questions were derived directly from students' writing. The writing allowed the interactions to center specifically on the individual student's work. Instruction was given in the specific area of concern, and students were able to have more individualized instruction.

Students' writing was analyzed daily and had direct influences on lesson planning. Feedback from the writing was offered to Michelle, and she often incorporated it into whole-group instruction. It was also used in mini-lessons in the smaller groups. Students' written explanations and reflections provided information pertaining to their understanding of mathematics, vocabulary, strategy, and processes. These were discussed at length in the first and second findings. These findings were analyzed for instructional purposes and influenced lesson planning.

Answers to Research Questions

Research Question #1: How do students use writing to reflect on their learning in mathematics?

The first and second findings highlight the ways students used writing to reflect their learning in mathematics. In one component of the first finding, students' writing showed their understanding of mathematics. Albert's writing indicated a misunderstanding between quotients and remainders. Albert appeared to have learned that quotients are the same as remainders, and therefore reported only the remainder as his answer in division. This was noted by the instructors and used for planning and is described further in Research Question #3. Tom's writing revealed the difficulties he was having distinguishing between multiplication and division. He seemed to be more comfortable with repeated addition. Although he seemed more comfortable with using repeated addition to solve problems, he recognized in his writing that this was limiting in solving more challenging problems. Sylvia's writing showed that she had learned that multiplication and division are connected and that this connection could be used to solve problems more efficiently. Their writing provided insight into their learning and what they understood.

Another aspect of the first finding illustrated students' learning of mathematical vocabulary. The examples in this section displayed students' interaction with the words they learned. In some cases, students' writing indicated that they had a desire to use the terms they had heard, but their usage revealed a certain level of confusion. In other cases, students used the words correctly and with varying degrees of difficulty. Their writing reflected what they learned about these terms and demonstrated any misunderstandings. In the interviews, students supported the idea that writing helped them use mathematical vocabulary and reflected what they had learned. They also noted that writing helped them learn the vocabulary.

The second finding, which centered on the analysis of the reflective writing questions included in the workshop, highlighted the students' learning of mathematical strategies. This section included several examples of students' writing about the strategies they had learned and used in the workshop. Michelle and I verified the learning reflected in their writing with their calculations. For some students, there was consistency with what they asserted they had learned and their calculations, while other students' writing and calculations revealed inconsistencies and their misconceptions. Their reflections showed their learning, and when interviewed, students articulated that writing helped them learn and showed them how they were thinking.

Students used their written reflections to explain and reflect on their thinking. Their writing revealed their learning of mathematical concepts, strategies, and vocabulary. The students recognized the value of writing in the interviews and asserted that it was a tool for learning. This information was used by Michelle and me to differentiate and individualize instruction.

Research Question #2: How do students use writing to show how they solve mathematical problems?

This research question was addressed in both the first and second findings. In the workshop model, students were encouraged to explain their responses using words, drawings, and examples. Many workshops included a section that directly asked students to explain their process. In these sections, students' writing revealed how they solved mathematical problems. The process they used to solve problems showed their level of efficiency and their understanding. In the comparing-numbers example presented earlier, students' writing indicated that they had a secure and efficient process in place for solving this problem. In other examples, students' writing revealed that they had varying problem-solving processes in place that afforded them the correct answer with varying degrees of efficiency.

The workshop model included questions that required students to explain more than one problem-solving process for a particular task. This section showed students' thinking about problem solving, and students presented their opinions regarding which process was most helpful for solving problems. The students' reflections included in the second finding provided writing from the students that detailed the strategies they had used, or believed they had used, in previous problems. These sections allowed students to convey their problem-solving methods, strategies, and opinions. This information helped to guide instruction and limited the assumptions teachers had to make about the students' problem-solving methods.

Research Question #3: How do teachers adjust their lesson plans in response to writing produced by students in the writers' workshop model?

The third finding demonstrated the ways in which Michelle and I adjusted their teaching according to the students' writing. The writing was used to inform the grouping of students and how to differentiate lessons between those groups. The fourth-grade students who participated in this case study were the lowest-performing students in the grade level, based on previous EOG scores. Michelle was concerned with the amount of time spent on writing. After two weeks of the workshop, student writing was analyzed and revealed differences in the mathematical abilities within the class. From this analysis, the case study continued with one group of nine, whose writing seemed to show a greater understanding of the lesson material. The analysis of data from the first two weeks of the study was the predominant factor in this grouping. As the workshop continued with the nine students, student writing was used to pair students who appeared to complement one another. Writing from the workshops served as a tool for lesson plans throughout the case study.

Student writing was the main source of information for interacting with students. Their reflections, explanations, and calculations provided insight into their thinking. Conferencing questions and topics emerged from their writing, and instruction was embedded into the conference. Notes from these conferences influenced lesson planning. Student writing was used in lessons; both Linda's explanation of the relationship between multiplication and division and Zoe's work with multiples was used to engage and provide instruction for classmates.

The student writing described in findings one and two informed planning. Students presented their number-line process along with their thoughts about the process. This writing impacted the decision to move on to another strategy that would convey the concept of place value in a way that might resonate more with the students. *Math Investigations* is a curriculum that presents concepts in more than one unit. The analysis of students' writing produced data that will be useful when revisiting mathematical strategies, vocabulary, and processes in the spring.

Summary

Overall, the students 'writing offered insight into their mathematical learning and problem solving. This information served to enhance instruction. In several examples, students conveyed their ideas in a limited amount of writing. However, even one sentence such as "Quotients and remainders are the same" or "Multiplication and division are the same" illustrated a misunderstanding that may have been difficult to recognize without their clear words. The writing did not need to be extensive to be valuable.

Student interviews provided further validation for writing in math being helpful, both for learning and to serve as a resource for future problems. The interview responses below illustrate a certain appreciation for the opportunity to express their opinions and describe their process.

> Me: What was your favorite part about writing in math?
> Linda: How I can express what I like.
> *Interview with Linda (12/07/12)*

> Me: What was your favorite part about the workshop?
> Sylvia: Um, when we would have to solve it and then write what we did.
> *Interview with Sylvia (12/07/12)*

CHAPTER 5: DISCUSSION OF FINDINGS

In this study, I sought to explore how the use of an adapted writers' workshop model in a fourth-grade mathematics class offered students opportunities to write about their mathematical thinking. I also wanted to examine how the students' writing connected to planning of future instruction. Previous research on content-area writing in social studies and science examined the use of strategies such as journal writing, reflection, teachers breaking large units into smaller components, offering instruction in skills, teacher-student conferences, and students sharing their writing (Bricker, 2007; Johnson & Janisch, 1998; Leddy, 2010; Peterson, 2007). Many of these strategies are nested in Calkins' (1983) Writers' Workshop model, which was developed from an extensive case study. The use of this model in writing instruction has been part of several research studies (Clippard & Nicaise, 1998; Helsel & Greenberg, 2007; James et al., 2001; Kissel et al., 2011). Many of these studies made adaptations to the workshop model to address the students' needs.

Research on writing in the content area of mathematics includes studies in several grade levels using math journals (Goldsby & Cozza, 2002; Jurdak & Zein, 1998; Kostos & Shin, 2010; Pugalee, 1997), shared writing (Pugalee, 2005; Wilcox & Monroe, 2010) and a workshop model (Carter, 2009; Heuser, 2000). However, few studies have examined the process a practitioner undertakes to provide opportunities for mathematical writing and the analysis of the writing produced by those students. In an attempt to better understand these processes and to address the gap in the literature, my case study examined the following research questions:

1. How do students use writing to reflect on their learning in mathematics?
2. How do students use writing to show how they solve mathematical problems?
3. How do teachers adjust their lesson plans in response to writing produced by students using the writers' workshop model?

Data were collected over a period of nine weeks in a fourth grade classroom. Data included observations, recorded field notes, conferences and interviews with both the teacher and students, and the students' writing. Analysis of the data resulted in three findings: (a) students use writing as a tool to demonstrate their understanding, (b) students' written reflections inform the teacher's instruction, and (c) student's written explanations inform instruction.

Discussion of Findings

The fourth-grade students in this case study used writing in mathematics on a limited basis prior to the study. The workshop model, which infused and encouraged writing throughout the mathematics lesson, was a new activity for this class. Students in this study had three disadvantages entering the study: They had low scores on their mathematics assessments, as indicated on their End-of Grade (EOG) test; they were new to writing extensively in mathematics; and they struggled in developing literacy skills. Garofalo and Lester (1985, 1987) suggest that students use reflective writing and reporting mathematical knowledge to facilitate the development of metacognition. Several researchers assert that metacognitive behaviors are critical for students to develop problem-solving skills (Bangert-Drowns et al., 2004; Muth, 1997; Ediger, 2006; Garofalo, 1985, 1987; Liedtke & Sales, 2001; Ntenza, 2006; Pugalee, 2001). The metacognitive skills discussed in research connect to the skills embedded in the writing process (Brozo & Simpson, 2003; Murray, 2004; Peterson, 2007). The research suggests that writing is a tool that can build understanding and is not limited to particular types of students.

During this case study, the benefits of mathematical writing suggested in the research—such as that writing helps students convey meaning and interact with mathematical vocabulary, enhances reflection, and aids in instruction—were supported in the data. I also encountered a certain level of concern from the teacher about the students' mathematical and literacy skills encumbering them and ultimately leading to frustration. These concerns led to a design change for the case study. The nine students included supported the findings in the interview responses and exhibited an appreciation for their writing. The positive feedback from the nine students that continued indicates the experience may not have been frustrating for the other nine students and may have offered the same benefits.

First finding: Students use writing as a tool to demonstrate their understanding

In the classroom studied, students engaged in mathematical writing that reflected both their understandings and misunderstandings. Students' writing reflected their understanding of mathematical vocabulary as they used new terms to reason through their thinking. The students' writing highlighted changes in their thinking and illustrated a movement toward more efficient and sophisticated calculations. In some cases, students recognized a limitation in their method for solving a problem and expressed this idea in their writing. The growth in their mathematical thinking was represented in their writing. Jurdak and Zein (1998) propose that the journal writing opportunities afforded to the middle school students in their study provided an environment in which to process mathematical concepts and increase their conceptual understanding. This was supported by statistically significant differences between test scores of students involved in mathematical writing versus students who completed additional problems with no written explanation.

This study, similar to Kostos and Shin's (2010) research, reflected this beginning state as students' writing was, in some instances, only a sentence or two and included drawings. Although the students' responses were limited in detail and included grammatical and spelling errors, the information contained in their writing provided insight into their thinking and perhaps laid the foundation for future proficiency in higher grades. Jingzi and Normandia (2009) conducted several interviews about writing in mathematics with older students. These students noted that writing in mathematics was more difficult, but they also recognized the value and suggested it be included in elementary instruction to increase proficiency in later years. In the Jingzi and Normandia study the students expressed the idea that writing in mathematics is challenging and should be introduced in the elementary grades in order to reach proficiency later. This idea indicates that the elementary grades represent the beginning stages of mathematical writing.

The students in this case study produced writing which illustrated these ideas. Students' writing revealed a thoughtful consideration of what they had learned, the process they underwent, and the strategies employed. In one particular interview, a student mentioned she was learning because she was writing and doing mathematics at the same time. Brown & Palincsar (1982) noted that pre-planning and monitoring are key metacognitive behaviors, inherent in the writing process, that facilitate learning. This study produced evidence that supports the connection between writing and metacognition.

Thompson and Chappell (2007) suggest that building mathematical literacy into the mathematics classroom is essential for effective communication of mathematical concepts, and consider the exclusion of mathematical literacy a practice that ultimately limits opportunity to communicate mathematically. The workshop time, in this study, encouraged written mathematical communication and the use of specific mathematical terms by including terms in mini-lesson discussion, providing lists of terms in instructions, and using conferencing as a time to encourage writing. Murray (2004) asserts that students need to use a word at least 30 times to make it their own. Mathematical vocabulary is often limited to the mathematics classroom, which makes writing in mathematics important for creating an opportunity to interact with these terms at least 30 times. The students' responses included varying amounts of mathematical vocabulary. In some cases, students' writing indicated a desire to use a word or phrase that had been used in instruction repeatedly, even if they were unsure of its meaning. In other cases, specific mathematical terms were used as a way to reason through their thinking or to show a clear understanding of the terms. The workshop model used in this study attempted to create an authentic writing environment that emphasized writing to learn (Bangert-Drowns et al., 2004) and provide opportunities for students to use their own language and terms to build confidence (Powell, 1997). The results from this study were aligned with ideas presented in previous research. Powell (1997) alludes to the idea of writing as a tool for building confidence; this was also evident in my study, in which the connection between writing and confidence arose during my interview with the teacher, Michelle; however, this interview occurred after student interviews and therefore could not be confirmed with students which limited the ability to triangulate this idea.

> I think . . . these students are never given the chance to shine and feel like leaders in the classroom because of the literacy, so they have never been able to shine, but I think this is something that gave those particular students something to feel special about. . . . So I think that because of their struggles with math, this was a huge confidence booster for them. I think it also helped them develop some of their skills in communicating their thoughts in a very nonthreatening way. They were making—they had this feeling of, you know, "We're the better ones in the room," or . . . "We're kind of like the leaders," so I think they felt more comfortable sharing.

In this excerpt, Michelle seems to recognize the authentic, nonthreatening environment created by the workshop and its effect on students' communication skills and confidence. This quote is important and will be discussed later in relations to implications for future research.

The students' writing revealed their problem solving process; the writing showed their metacognitive reflective thinking as they solved problems. Their writing illustrated the efficiency of their process and whether they connected with and built on previous skills. Pugalee (1997) describes mathematics journals as a channel between teacher and student that offers a place for individualized instruction to occur. The results from this study support this view. In the example where the students compared five numbers and placed them in order from least to greatest, all the students used an accurate and efficient strategy to solve the problem. This allowed Michelle and I to feel comfortable moving beyond this skill; however, the next examples, which involved division, revealed that students used different methods. Individualized instruction was used to assist students in building on their method to attain a higher level of efficiency.

The writing time during mathematics allowed the students to demonstrate their understanding of mathematics and mathematical vocabulary. Their writing offered insight into their process and level of efficiency. The students' writing provided information that assisted Michelle and I in making instructional decisions. These findings support previous research studies (Bangert-Drowns et al., 2004, Thompson & Chappell, 2007).

Second finding: Students' written reflections inform the teacher's instruction

In this study, the students' reflections were used to examine their work for consistencies between their reflected understanding and their actual calculations. This act of reconciling their written reflection of their understanding highlighted important information for instruction. It also showed social influences as students constructed meaning their writing reflected the words of the teachers and their peers. These reflections provided information that assisted in decisions on moving to the next level and offered opportunities for teachers to engage in verbal interactions concerning the students' mathematical reasoning. Heusser (2000) identified that mini-lessons, activity time, and students' self reflections are essential parts of a mathematics writers' workshop. This study mirrored the writers' workshop model with the components listed above. Research on mathematics teaching and learning indicated that reflection and communication are key components to increasing mathematical understanding (MacGregor & Price, 1999; Monroe, 1996). In previous research, student reflection increased their understanding as they first reflect on what they have learned and shared this information through writing (MacGregor & Price, 1999).

In several examples, students used one strategy in their calculations and then, in a reflection question, asserted that they had used another. It appears that the students realized that a strategy included in instruction should be used in solving the problems, but had limited understanding of the new strategy. In their reflections, they wrote about using the strategy they felt was socially expected from the teacher and their peers, rather than reflecting on their own calculations. The reflections that conflicted with actual strategy revealed the social context that had influenced the students' writing.

The consistency and lack thereof between calculations and reflections were used in deciding on instructional practices. Instructional practices such as using manipulatives, re-teaching, and less independent time were included for students whose writing showed misconceptions. Posing more challenging problems, having students only work with pencil and paper, and more independent time were some of the practices used for those showing clear understanding. The reflections also provided a starting point for conversations with students about their thinking.

The reflection questions often caused students to report their thinking and the strategies they had learned. As previously mentioned, the workshop increased the amount of writing in mathematics, which was new for the students. Their written responses were limited at times; however, there was an overall increase in the frequency of writing. One example of a sentence with limited detail was "I learned how you have remainders and what to do"; this type of reflection presented an opportunity to use verbal communication to learn more about the student's thinking. The reflective aspect of the workshop provided these types of opportunities three times a week. Pugalee's (2005) model of speaking-writing mathematics highlights the value of feedback that occurs through the interaction among students in pairs or groups, teacher, and whole-class discourse. These interactions increased mathematical discourse and literacy.

The written reflections of the students revealed their strategies and the social influences of their environment and informed instruction. The time during which students thought about the reflection questions and constructed a response enhanced their metacognitive skills. Their writing provided opportunities for teachers to engage in verbal conferences and design instruction.

Third finding: Students' written explanations inform instruction

McIntosh and Draper (2001) support journal writing as a worthwhile endeavor for students to display their mathematical learning and for teachers to elicit anecdotal data for assessment. In their study, the students' written explanations of their work were important for decision-making regarding grouping, conferring, and lesson planning. The study used the format of design-based-research methodologies to analyze the data biweekly and used the results to make changes to the design of the workshop.

After the first two weeks, Michelle was concerned about the amount of instructional time the students received and thought the writing component might have consumed too much time. In later interviews, Michelle revealed her concern that students would become frustrated with writing due to their low ability with both mathematics and literacy. Accordingly, I conducted an analysis of the students' writing over the preceding two weeks. The writing indicated that one group of students grasped the strategies, completed the problems, and had ample time to write. The writing from the other group of students showed difficulty with the strategies and, often, minimal writing. Michelle, as the classroom teacher, was ultimately responsible and felt more comfortable allowing only the first group to continue with the workshop.

Helsel and Greenberg's (2007) study used a writers' workshop model in a sixth-grade language-arts class. The authors found that some students thrived, while struggling students were hindered by the freedom of the model. The study explored Self-Regulated Strategy Development (SRSD), which contains a series of more structured stages to increase reflective, self-regulated strategies. In both the Helsel and Greenberg study and my study, concerns about struggling students and their writing progress led to a change in the format. In both cases, the format change was a complete separation from the workshop model for some students rather than an adjustment or combination. My study continued with half of the class using the writers' workshop model, and the written explanations assisted with conferencing and lesson development. In my interviews with them, the students who had remained in the workshop group reported that they valued their writing experiences. The students that remained in the workshop group recognized the value of their writing in the interview responses. The other group missed the opportunity to participate in the writers' workshop in mathematics for the remaining four weeks of the study and this may have limited their ability to share their thinking through writing and have their writing impact instruction.

The independent writing time began with problem solving connected to the mini-lesson. Students wrote their calculations. This writing provided conferencing questions that assisted students in working through their thinking, getting started with a problem, and making connections to previous work in order to solve more efficiently. This type of verbal interaction differed from the interactions generated by the reflective writing opportunities. These conferences allowed students to talk through their specific needs while solving problems, rather than going through problems as a whole group and waiting for a specific question to be answered. One student in the workshop group stated that she was able get more done in the workshop format. Students' explanations of their calculations were used to develop lessons and provided information for the focus of future mini-lessons.

The movement from one strategy to a more efficient strategy occurred in stages based on students' written explanations. While immersed in division, for instance, students wrote out every multiple of the divisor to get to the dividend. They then proceeded to count how many numbers they had written to get the quotient. This method arrived at the correct answer; however, it was inefficient. Their work was used as a starting point in a mini-lesson to discuss how they could start farther along in the multiple list of the divisor based on their knowledge of multiplication. This mini-lesson allowed students to work more efficiently on their long division problems. Their list of multiples to get to the quotient became shorter; there were still stages to progress through in order to increase efficiency. The analysis of their progression was used in subsequent instruction.

The students' written explanations informed the design, grouping, and pairing of students for mathematics instruction. The students explained their thinking in their journals, and, while conferencing, used their written words to help them think through their verbal communications. This analysis of their writing and the conferencing informed instruction and design of the workshop.

Implications for Practice and Policy

For several decades, research in writing has focused on writing as a valuable tool for learning across content areas. Evidence of this research appears in education initiatives, educational standards, and curricula. Common Core State Standards (CCSS) were developed from state-led committees that formulated college and career-ready standards that are clear and consistent, include rigorous content, and require the application of knowledge through high-order skills (Watt, 2011). The CCSS and the National Council of Teachers of Mathematics' (NCTM) standards emphasize content-area writing. The CCSS Initiative includes standards for mathematical practice that emphasizes the NCTM's (2000) process standards of problem solving, reasoning, proof, communication, representation, and connections.

The NCTM (2000) called for teachers to provide students with opportunities to communicate about mathematical concepts in a clear and coherent manner. The document states that mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. The document further defines proficient students as being able to justify their conclusions, communicate them to others, and respond to the arguments of others. The CCSS Standards for Mathematical Practices (CCSS-M) include "construct viable arguments" and "attend to precision" as practices for proficiency. The CCSS-M describe mathematically proficient students engaged in the practice of constructing viable arguments as being able to use their mathematical understanding to construct plausible arguments, justify conclusions, identify flawed reasoning, and communicate these abilities using concrete referents. The CCSS-M practice of attending to precision is evident when mathematically proficient students communicate definitions, symbols, units of measure, and their own reasoning consistently, precisely, and accurately. NCTM (2012) has defined mathematical communication standards for Grades 3-5 as sharing thinking, asking questions, explaining, and justifying ideas. In Grades 3-5, the standards encourage students to express and write their conjectures, questions, and solutions.

In the widely adopted CCSS-M (NGA/CCSSO, 2011), proficiency on grade-level content standards and Standards for Mathematical Practices is aligned with mathematical behaviors related to conceptual understanding (Dacey & Polly, 2012; NGA/CCSSO, 2011). In mathematics, procedural understanding refers to memorizing with little understanding, whereas conceptual understanding means students' ability to make connections between real-life contexts, mathematical representations, and computational work. Teachers should work to develop students' procedural understanding and conceptual understanding simultaneously (National Research Council, 2001). Students demonstrating rote procedural skills without conceptual knowledge lack the understanding of the underlying arithmetical problem and typically struggle in problem-solving situations (Kaufmann et al., 2003; Resnick, 1982; VanLehn, 1990). The CCSS standards for writing are focused on argument and informative/explanatory and narrative writing; the standards clearly indicate that teaching writing belongs to all teachers, including math, social studies, and science (Calkins et al., 2012). The CCSS include a movement in the direction of formative assessments that encourage feedback and development (Long, 2011).

The focus on writing in mathematics is evident in the CCSS; however, achieving the level of precision in mathematical communication described in these standards is a process. This study included students with low-test scores for mathematics and literacy and minimal experience with writing in mathematics. The data collected in this case study illustrated students' reasoning, misconceptions, understanding, language difficulties, and mathematical vocabulary. The responses at times were difficult to read and required a certain level of decoding. Discussion and conferencing increased response length and enhanced students' ability to talk through their learning. Many students used examples and pictures rather than full sentences. Their writing, even with its flaws, offered insight into the students' thinking. The description of mathematical writing found in the CCSS and often used as examples in research studies is of a finished and refined piece of writing. This exploratory study generated data pertaining to the beginning phases of introducing writing in a mathematics classroom. It examined the process and showed students' writing in the introductory phase.

This study also highlighted the process and development that occurred as students began to interact with writing in mathematics on a regular basis. Jingzi and Normandia (2009) conducted interviews in which older students recognized the need to engage in writing in mathematics at an elementary level to reach proficiency in older grades. As seen in the data presented in this study, there is a foundational period during which students are just beginning to interact with writing in mathematics. The continuation of the practice of writing in mathematics is critical for reaching the level of proficiency described by the CCSS standards.

This study can be used for the purposes of professional development. Small pilot studies of teachers may be assembled to examine the data and process described in this study. The ideas generated from teacher leaders in the field would then serve to further this model and be implemented in more classrooms.

In this study, one particular group of students did not participate in the workshop. Michelle's fears about the students' ability to reach mathematical benchmarks on their standardized assessments, along with the possibility of a high frustration level, prevented them from engaging in writing. Michelle's concerns also stemmed from administrative pressure for students to reach goals measured by standardized tests. However, Michelle's earlier statement (p. 166) suggests that this group missed the confidence-building experience of the workshop group. More importantly, the non-workshop group missed the opportunity to write, and this may impede their movement toward the proficient writing described in the CCSS. The pressure of reaching benchmarks weighed heavily on Michelle, and played a role in changing the format to include only half of the class. In order for the goals of the CCSS to be achieved classroom, teachers need to feel comfortable creating all-inclusive writing environments for their students. School administrators can play an important role in making the CCSS goals a priority and ensuring that teachers can incorporate opportunities for students to write. Teachers and administrators should keep in mind that mathematical writing in elementary grades may be a foundational stage for students with limited writing experience. Expectations and writing instructions should be adjusted accordingly. The students in this study engaged in mathematical writing that they recognized to be helpful for their understanding and that allowed their teacher to gain knowledge about students' thinking.

Implications for Future Study

The research suggests many benefits to writing in mathematics, such as growth in skills (Heusser, 2000), increased use of mathematical vocabulary (Tuttle, 2005), and insight for instruction (Goldsby & Cozza, 2002). Several research studies (Jingzi & Normandia, 2009: Pugalee, 1997) include examples from middle school students. The examples show writing that is representative of the standards described in the CCSS. Other content-area research studies (Bricker, 2007; Johnson & Janisch, 1998; Leddy, 2010; Peterson, 2007) highlight how the incorporation of writing facilitates learning and understanding. These studies included changes made to the format used in content-area writing instruction; however, the researchers did not explain the reasons for those changes. My findings, as well as those from previous work, offer several areas to consider for future research.

The processes behind instruction practices and the description of the analysis prior to instructional decisions are needed to provide practitioners with information for implementation. Previous studies of content-area writing have included changes to format without explaining why. The conferencing described in Calkins' (1983) workshop model was altered in this study, based on the students' need to talk through their thinking more frequently. However, changes made by previous researchers and my alteration of the conferencing format for this study present opportunities for further research. It would be helpful to closely examine conferencing in mathematics and identify the different types of conferencing questions that are most useful. Research centered on how data are used to design writing opportunities in content areas would provide insight for teachers embarking on this process.

The exploratory nature of this study focused on the beginning stages of mathematical writing. The students' responses included pictures and, at times, a minimal number of words. Research to examine the next instructional steps that should be taken to move students at this stage of mathematical writing on to more in-depth responses would be helpful as teachers begin working with CCSS. This type of information might be best obtained through a longitudinal study of a classroom that engages in content-area writing throughout the academic year.

Although CCSS has been a national effort my educators across the nation, standardized testing remains a large factor in instructional practices. This study reveals the pressures attached to grade levels that include standardized testing. Examining a writers' workshop model in kindergarten through second grade might provide an environment in which teachers feel more inclined to use this format with all students for a longer duration. These grade levels can offer a place for a lengthier study—even more valuable, as research on writing in mathematics for these grades is limited.

Another area for future research is the connection between writing and confidence, which was touched on in this study. Michelle noted in an interview that she thought the students participating felt a sense of leadership, had a chance to shine, and felt comfortable sharing. Future studies should examine the connection between mathematical writing and self-efficacy.

Summary

In this study, I sought to understand how students' writing reflected their mathematical understanding and problem solving, and how teachers interpreted and used this information. An adapted writers' workshop was used to provide writing opportunities. The data show that students used writing as a tool to demonstrate their understanding of mathematical processes, vocabulary, and strategies. Their writing presented opportunities for instructors to conference with students multiple times in one lesson. Data analysis was instrumental in instructional decisions. Students interviewed in this study conveyed a sense of appreciation for writing and asserted that it was valuable in many ways. This study supports research on the benefits of writing in mathematics; it also demonstrates that the beginning stages of writing in mathematics may not reflect the writing described in the CCSS. These stages are necessary to reach the level of proficiency in the standards. For this tool to be used regularly and include all students, teachers must view writing in mathematics as beneficial. The pressures of standardized and benchmark testing and the writing produced by students in the beginning can deter teachers from writing. However, the benefits of writing and the movement to more formative assessments should offer more support for teachers who engage their students in writing.

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APPENDIX A: OBSERVATION PROTOCOL

Unit of Study ____

Day and Time ____

# of students ____

Special Circumstances

____

____

Description of Workshop Components

Mini-Lesson

____

____

Writing-Conferencing

____

____

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Sharing

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Additional Details:

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