EMBEDDING NUMERACY INSTRUCTION WITHIN STANDARDS-BASED ALGEBRA LESSONS FOR SECONDARY STUDENTS WITH EXTENSIVE SUPPORT NEEDS

by

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ABSTRACT

AMY MARIE CLAUSEN. Embedding numeracy instruction within standards-based algebra lessons for secondary students with extensive support needs. (Under the direction of DR. FRED SPOONER)

Federal legislation requires every student receive access to and make progress in the general curriculum. Teachers have reported difficulties in meeting this requirement for students with extensive support needs (ESN) who do not yet demonstrate foundational academic skills, such as in the areas of literacy and numeracy. Once students enter high school, the gap between the mathematics skills a student currently demonstrates and those required to engage in grade-level, standards-based mathematics, widens considerably. Researchers have evaluated different interventions to teach both foundational and grade-level skills to students with ESN in the area of mathematics. This study builds on that research by evaluating the effects of an intervention package comprising modified schema-based instruction (MSBI) and embedded simultaneous prompting (SP) to teach secondary students with ESN who do not demonstrate numeracy skills to solve simple linear equations (e.g., $3 + x = 9$) and to identify numerals 1–9 concurrently. The experimental design was a single-case multiple probe across numeral sets replicated across participants. Two high school males with ESN participated, along with instructors, a paraeducator and a special education teacher who implemented the intervention. The intervention was not effective for teaching numeral identification or solving simple linear equations, nor did the students generalize numeral identification to real-world settings. However, the instructors did find the study procedures to be socially acceptable and hypothesized that the students would eventually reach mastery criterion, given additional time in the intervention. The findings from

this study can be used to guide future research designed to support the needs of secondary students with ESN who do not yet demonstrate foundational numeracy skills.

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LIST OF ABBREVIATIONS

AA-AAS	alternate assessments based on alternate achievement standards
BST	behavioral skills training
CCSS	Common Core State Standards
CCSSM	Common Core State Standards for Mathematics
COVID-19	Coronavirus disease 2019
CTD	constant time delay
DV	dependent variable
EBP	evidence-based practice
EE	expressions and equations
ESEA	Elementary and Secondary Education Act
ESN	extensive support needs
ESSA	Every Student Succeeds Act
ID	intellectual disability
IDEA	Individuals with Disabilities Education Act
IEP	individualized education program
IOA	interobserver agreement
IRB	Institutional Review Board
MAFS	Mathematics Florida Standards
MD	measurement and data
MSBI	modified schema-based instruction
NBT	numbers in base ten
NCTM	National Council of Teachers of Mathematics
OA	operations and algebraic thinking

PARC	Pennsylvania Association for Retarded Citizens
RP	ratios and proportions
SBI	schema-based instruction
SIS-C™	*Supports Intensity Scale—Children's Version*™
SP	simultaneous prompting
USDOE	United States Department of Education

CHAPTER 1: INTRODUCTION

The first public education classroom for students with extensive support needs (ESN) opened in 1896, 261 years after the first public school opened in the United States (Boston Latin School, n.d.; Scheerenberger, 1983). The goal for these students was to prepare them for life in institutions. This philosophy is a marked difference from today's requirement of standards-based instruction and rigorous academics for students with ESN. In this chapter, I present a brief history of public education for students with ESN, an introduction to mathematics instruction for students with ESN, and a discussion of meaningful access to the general education curriculum.

History of Education for Students with ESN

Students with ESN refer to the approximately 1% of students who have significant cognitive disabilities and pervasive support needs across domains (Kurth et al., 2019; Taub et al., 2017). These students often have disability labels of autism, intellectual disability (ID), multiple disabilities, or deaf-blindness and are typically assessed on their state's alternate assessment based on alternate achievement standards (AA-AAS; Kurth et al., 2019; Taub et al., 2017). Historical terms for individuals with ESN have included *idiocy*, *trainable mentally retarded*, *severely and profoundly handicapped*, and *students with moderate and severe disabilities* (Scheerenberger, 1983; Schalock et al., 2007; Spooner, 2022). These terms, however, emphasized an individual's disability, suggesting an inherent deficit (Spooner & Brown, 2017). Instead, the term *extensive support needs* represents a shift from the traditional medical model of disability, in which disability is viewed as something needing to be fixed or cured, to a social-ecological understanding of disability in which the focus is on the level of supports an individual requires to be successful in their environment (Hogan, 2019; Schalock et al., 2007). Support needs refer to "a psychological construct referring to the pattern and intensity of supports necessary for a person to participate in activities linked with normative human functioning" (Thompson et al., 2009, p. 135). The term *extensive support needs* describes an individual who requires pervasive and ongoing supports across multiple domains of their life, including at home, in the community, in school, within health and safety activities, during social activities, and in advocating for themselves (Thompson et al., 2016).

Public education for students with ESN in the United States can be traced to 1896, when the first special education class opened in Providence, RI. Additional programs were opened in city centers across the northeast, where children with disabilities were placed in ungraded special education classrooms. However, it is important to note that many of these programs were reserved for students with less intensive support needs, and those with ESN were often excluded (Scheerenberger, 1983). By 1930, 16 states had laws requiring special education services in public schools, and by the 1950s, most schools across the country accepted students labeled "trainable mentally retarded" (Scheerenberger, 1983).

In 1971, the court ruled in *Pennsylvania Association for Retarded Children (PARC)* v. *the Commonwealth of Pennsylvania* that every child, including those with ESN, had the right to a free public education in Pennsylvania. In 1972, the court upheld *PARC* v. *Pennsylvania* in *Mills* v. *Board of Education of the District of Columbia*, ensuring free, public education in D.C. for all students. It was not until 1975 with the passing of PL 94-142, known today as the Individuals with Disabilities Education Act (IDEA), that the provision of special education services became federally mandated, and all students with ESN were accepted in schools.

Over the past 50 years, special education for students with ESN has progressed through multiple curricular foci. In the 1970s, the focus was on a developmental curriculum, in which education was based on a student's mental age rather than chronological age (Browder et al., 2020). The 1980s were characterized by a focus on functional skills, with the "criterion of ultimate functioning" being the goal of special education (Browder et al., 2020; Brown et al., 1976). In the 1990s, students with ESN were included in the general education classroom for the purpose of social inclusion but not inclusion in the general curriculum (Browder et al., 2020).

It was not until the late 1990s/early 2000s that standards-based academic instruction became the expectation for students with ESN. In 1997, with the reauthorization of IDEA, all students were expected to participate in statewide assessments. Students with ESN were expected to participate in the AA-AAS; however, alternate achievement standards were often far removed from the standards for students without disabilities. States were expected to align alternate achievement standards to grade-level content standards with the 2001 reauthorization of the Elementary and Secondary Education Act (ESEA), penned as the No Child Left Behind Act.

The 2010s can be characterized by an emphasis on access to the general curriculum (Browder et al., 2020). The goal for students with ESN now is to engage in the same curriculum as their peers without disabilities, with accommodations and modifications as needed. ESEA was again reauthorized in 2015, with the newly named Every Student Succeeds Act (ESSA) raising expectations for all students, including those with disabilities. Furthermore, in 2017, the Supreme Court of the United States ruled in *Endrew F. v. Douglas County School District* that mere access (i.e., exposure to academic material) was insufficient; instead, the expectation must be progress in the general curriculum.

Regardless of federal legislation, many educational professionals believe all students deserve access to the general curriculum (Browder et al., 2007; Courtade et al., 2012; Knight et al., 2010). When students with ESN are given the opportunity to participate in the general curriculum, they also have increased school-based opportunities (Spooner et al., 2006). Their academic skills improve, and they perform better on AA-AAS (Roach & Elliott, 2006; Spooner et al., 2006). Additionally, students have increased opportunities to engage with their peers in inclusive settings when participating in the same curriculum (Ruppar & Olson, 2017; Spooner et al., 2006).

Beyond school, participation in the general curriculum prepares students for adult life (Browder et al., 2007; Common Core State Standards [CCSS], 2022; Courtade et al., 2012). The general curriculum prepares students for careers and independence and provides them with the means to communicate about the world around them (Courtade et al., 2012). Students who participate in the general curriculum also have improved self-determination and social relationships (Browder et al., 2007; Knight et al., 2010; Spooner et al., 2006).

Mathematics for Students with ESN

The increased emphasis on the general curriculum in the 2000s corresponded with an increased focus on academic instruction for students with ESN (Spooner & Browder, 2015). Much of this research focused on reading, particularly sight word instruction (Browder et al., 2006). Research on mathematics instruction, however, has increased in the past 15 years. In 2008, Browder et al. located 19 quality studies addressing mathematics instruction for students with ESN, addressing only two content areas (number and operations, measurement; National Council of Teachers of Mathematics [NCTM], 2000). In 2019, Spooner et al. located 22 additional quality studies addressing mathematics instruction for students with ESN, addressing each mathematical content area (number and operations, algebra, geometry, measurement, and data analysis and probability; NCTM, 2000). However, research in standards-based mathematics instruction for students with ESN is still limited.

Standards-Based Mathematics

When defining high-quality, standards-based instruction, it is necessary to first define academic standards. Academic standards refer to a collection of goals that describe what students should learn in each grade (National Governors Association for Best Practices, 2010). In the United States, these goals are set by each state's department of education and exist for each content area (e.g., English language arts, mathematics, science; ESSA, 2015). Academic standards promote equality, ensuring that all students have access to the same high expectations, regardless of socioeconomic status, parental education, or geographical location (Ravitch, 1995). The goal of standards for mathematics instruction is to support students in becoming problem solvers, reasoning mathematically, valuing mathematics, gaining confidence, and communicating mathematically (Maccini & Gagnon, 2002). In the United States, standards for mathematics instruction are typically drawn from either NCTM (2000) or the Common Core State Standards for Mathematics (CCSSM; National Governors Association for Best Practices, 2010). Both sets of standards address content (i.e., the skills of mathematics) and process (i.e., how to apply those skills).

The NCTM (2000) standards address five content areas: (a) number and operations; (b) algebra; (c) geometry; (d) measurement; and (e) data analysis and probability. The content standards for NCTM span all grade levels, whereas the content standards for CCSSM are unique to grade bands. For example, in Grades K–5, domains include (a) counting and cardinality, (b) operations and algebraic thinking, (c) number and operations in base ten, (d) number and operations—fractions, (e) measurement and data, and (f) geometry. In Grades 6–8, domains include (a) ratios and proportional relationships, (b) the number system, (c) expressions and equations, (d) functions, (e) statistics and probability, and (f) geometry. Finally, in Grades 9–12, domains include (a) number and quantity, (b) algebra, (c) functions, (d) modeling, (e) geometry, and (f) statistics and probability.

Both the NCTM process standards (2000) and the CCSS Standards for Mathematical Practice (National Governors Association Center for Best Practices, 2010) address necessary proficiencies for mathematical thinkers and are applied to students in every grade. Within NCTM, process standards include (a) problem solving, (b) reasoning and proof, (c) communication, (d) connections, and (e) representation. Like the content standards, the process standards listed in CCSS, known as the Standards for Mathematical Practice, are similar to NCTM but with some expansion: (a) make sense and persevere in solving problems, (b) reason abstractly and quantitatively, (c) construct viable arguments and critique the reasoning of others, (d) model with mathematics, (e) use appropriate tools strategically, (f) attend to precision, (g) look for and make use of structure, and (h) look for and express regularity in repeated reasoning.

These standards for mathematics were developed to ensure that students graduating high school would have the skills and processes necessary to succeed in college, career, and life (National Governors Association Center for Best Practices, 2010). Standards-based instruction is a process in which teachers begin by reviewing the standards for each grade and content area and then develop lessons aligned with those learning goals. NCTM (2000) defines five components necessary for high-quality standards-based mathematics instruction. First, high-quality standards-based instruction in mathematics must promote equity by holding each student to high expectations. Second, standards-based mathematics instruction requires a clear, coherent, and well-sequenced curriculum. Third, the instruction should promote active learning in which students build upon their prior knowledge. Fourth, it should incorporate assessment that provides important data to students and teachers. Finally, high-quality standards-based instruction should promote technology that enhances student learning (NCTM, 2000).

Standards-Based Mathematics for Students with ESN

Access to high-quality, standards-based mathematics instruction is crucial for students with ESN. It is often argued that students with ESN require instruction in mathematics skills traditionally termed “functional” (e.g., money, time, cooking; Ayres et al., 2011). The skills students learn in school mathematics, however, are indeed functional, and can be applied throughout daily life (Ahlgrim-Delzell et al., 2009). For example, students must recognize numbers and apply operations when dialing a phone, participating in sports activities, or creating a budget. Algebraic skills are necessary to calculate costs at the produce stand or analyze relationships between variables. When students navigate their community, planning their daily route, geometry is essential. Data analysis is required for goal setting and planning activities based on the weather. Finally, students will use measurement when cooking, gardening, or cleaning (Ahlgrim-Delzell et al., 2009). The process skills students learn in school mathematics are also important for developing critical thinking and learning problem-solving skills (Cox & Jimenez, 2020; Goya et al., 2019). Proficiency in mathematics is necessary for students to gain employment and live independently in their adult lives (Creech-Galloway et al., 2013).

Despite the importance of these skills, students with ESN have historically received reduced access to mathematics instruction (Cox & Jimenez, 2020). This gap in access can be attributed to many factors, including teachers’ perceptions of their students’ abilities; a traditional emphasis on “functional” mathematics skills, like time and money; and limited access to teachers with content expertise in mathematics (Browder et al., 2008; Goya et al., 2019). To determine the prevalence of experimental studies investigating mathematics for students with

ESN, I analyzed past literature reviews investigating mathematics instruction for students with ESN, autism, or intellectual and developmental disabilities more broadly.

I located 11 reviews comprising 181 experimental studies published between 1972 and 2020 (i.e., Ahlgrim-Delzell et al., 2009; Bowman et al., 2019; Browder et al., 2008; Clausen et al., 2021; Goya et al., 2019; Hart Barnett & Cleary, 2015; Hudson et al., 2018; Root, Jimenez, et al., 2020; Mastropieri et al., 1991; Schnepel & Aunio, 2021; Spooner et al., 2019). I conducted a brief review and found 28 additional studies published between 2017 and 2021 not included in any of these previous reviews. I included 152 studies in my analysis, using the definition of students with ESN provided by Kurth et al. (i.e., "those students who need ongoing pervasive supports across academic and daily living domains; who may be classified with disabilities including autism, intellectual disability, and multiple disabilities; and who are eligible to take their state's alternate assessment" 2019, p. 3). Of those studies, only 16 explicitly addressed standards-based mathematics (Table 1). Skills that could be aligned to the standards are certainly addressed in other studies; however, for true standards-based instruction to occur, instructors and researchers must intentionally plan their lessons by first selecting a standard and then creating the instructional plan (Clayton et al., 2006).

Table 1

Studies Addressing Standards-Based Mathematics

Study	Grade(s)	Source	Standard
Bowman et al. (2020)	1st, 4th, and 5th	Dynamic Learning Maps	1.OA.1a: Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, or acting out situations. 4.OA.3: Solve one-step real-world problems using addition or subtraction within 100. 5.NBT.5: Multiply whole numbers up to 5 x 5.
Browder, Jimenez, and Trela (2012)	Middle school	State Standards	Use and evaluate algebraic expressions. Solve simple (one- and two-step) equations or inequalities. Represent problem situations with geometric models. Identify, predict, and describe dilations in the coordinate plane. Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, and paper and pencil. Collect, organize, analyze, and display data to solve problems.
Browder, Trela, et al. (2012)	Middle and high school	NCTM	Not listed [a]. Targeted skills in algebra, geometry, measurement, and data analysis/probability.
Collins et al. (2011)	Middle school	State Standards	Apply the order of operations using addition and multiplication.
Creech-Galloway et al. (2013)	9th and 11th	State Standards	Use the Pythagorean theorem to solve a problem accurately
Heinrich et al. (2016)	10th	State Alternate Achievement Standards	Identify geometric figures. Solve linear equations in one variable.
Jimenez et al. (2008)	High school	NCTM	Not listed [a]. Taught students to solve a simple linear equation.
Jimenez and Staples (2015)	3rd, 4th, and 5th	CCSSM	Graph points on the coordinate plane to solve real-world and math problems. Use the four operations with whole numbers to solve problems. Draw and identify lines and angles, and classify shapes by properties of their lines and angles. Generate and analyze patterns.

Study	Grade(s)	Source	Standard
Karl et al. (2013)	Secondary	Kentucky State Alternate Achievement Standards	Students solve real world problems involving percent increase or percent decrease.
Orihuela et al. (2019)	3rd, 4th, and 5th	Kentucky State Alternate Achievement Standards	Identification of geometric shapes.
Polychronis et al. (2004)	2nd and 5th	Utah State Standards	Read, tell, and write time. Represents mathematical ideas with objects, pictures, and symbols.
Root & Browder (2019)	6th and 7th	CCSSM	6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number. 7.EE.B.4: Use variables to represent quantities in real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
Root, Browder, et al. (2017)	2nd, 3rd, and 4th	CCSSM	2.NBT.B.7: Add and subtract within 1000 using concrete models or drawings. 2.OA.A.1: Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of comparing. 3.OA.D.8: Solve two-step word problems using the four operations. 4.OA.3: Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations.
Root et al. (2019)	4th and 5th	Florida State Standards	MAFS.4.MD.2: Represent and interpret data. MAFS.5.MD.2: Use place value understanding and properties of operations to perform multi-digit arithmetic. MAFS.5.NBT.2: Perform operations with multi-digit whole numbers.
Root, Cox, et al. (2018)	High school	Florida State Alternate Achievement Standards	Not listed [a]. Taught students to solve percent of change word problems.

Study	Grade(s)	Source	Standard
Root, Cox, et al. (2020)	6th and 7th	CCSSM	6.RP.A.3.C: Solve problems involving finding the whole, given a part and the percent. 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems.

Note. Grade levels are listed in the second column. When exact grade levels are not provided, grade bands (e.g., middle, high, secondary) are listed. The source of the standard used in the study is listed in the third column. When the state is provided in the article, it is included in this table. The exact wording of the standard is listed in the fourth column. When included in the article, the standard code number is provided. NCTM = National Council of Teachers of Mathematics; CCSSM = Common Core State Standards for Mathematics; OA = Operations and Algebraic Thinking; NBT = Numbers in Base Ten; EE = Expressions and Equations; MAFS = Mathematics Florida Standards; MD = Measurement and Data; RP = Ratios and Proportions.
[a]Three studies did not provide explicit standards used when developing the intervention. However, these studies did include the source in the narrative and therefore have been included in this analysis.

Although the increase in studies investigating standards-based mathematics instruction for students with ESN is promising, much more work is needed. Students with disabilities historically have had fewer opportunities to engage in meaningful mathematics instruction than their same-age peers without disabilities (Lambert & Tan, 2017). In only six studies did research teams investigate standards-based mathematics instruction for high school students with ESN, which is particularly concerning when only 41% of students eligible for AA-AAS in the 2016–2017 school year achieved mastery in high school mathematics (Wu & Thurlow, 2019).

Additional work is needed to ensure students with ESN have meaningful access to and make progress in standards-based mathematics.

Meaningful Access to the Mathematics Curriculum

In 2017, the Supreme Court of the United States ruled that the progress of students with disabilities must be more than the *de minimus* standard set in previous rulings (*Hendrick Hudson Dist. Bd. of Ed. v. Rowley*, 1982; *Endrew F. v. Douglas County School District RE-1*, 2017). Although teachers were already required to set high expectations for their students through “access to the general education curriculum in the regular classroom, to the maximum extent possible” (20 U.S.C. § 1400.[c][5][A]), the decision made by the Supreme Court in the *Endrew F.* case clearly stated that access is not enough—meaningful progress must be made. Schools must now ensure progress rather than simply enabling or permitting progress as was the standard before this ruling (Turnbull et al., 2018).

Teachers often find it challenging to create meaningful access to the general curriculum for students with ESN due to difficulties bridging the gap between foundational skills (e.g., literacy, numeracy) and standards-based instruction (Browder et al., 2007). In a qualitative study, Timberlake (2014) found that teachers felt their students with ESN needed to acquire prerequisite foundational skills before accessing the general curriculum. Agran et al. (2002) surveyed teachers of students with ESN and found that most did not feel the curriculum was appropriate for their students. The NCTM, however, developed mathematics standards for all students, regardless of whether they could demonstrate foundational skills, such as computational fluency (1989). The doubt teachers express regarding the appropriateness of standards-based instruction for their students with ESN is a significant barrier to general curriculum access (Ryndak et al., 2014).

Foundational Numeracy Instruction

All students require a strong foundation, or understanding, of numeracy to achieve success in more complex mathematics skills (e.g., algebra, geometry, statistics; National Mathematics Advisory Panel, 2008; Clements & Sarama, 2009). Numeracy is the foundational basis for mathematics and includes such skills as numeral identification, rote counting, one-to-one correspondence, number conservation, composing and decomposing numbers, place value, early measurement concepts, and patterning (Browder, Jimenez, Spooner, et al., 2012). Most students with ESN require explicit instruction in numeracy, unlike their peers without disabilities, who typically enter school with a beginning understanding of number (Browder, Jimenez, Spooner, et al., 2012). Numeracy skills are critical for students with ESN to access standards-based mathematics meaningfully (Saunders et al., 2019).

Finding a Balance

Although it is certainly age-appropriate to address numeracy skills in elementary school, particularly in the primary grades, stand-alone numeracy instruction becomes problematic as students age. Teachers must provide instruction based on grade-level standards, yet many high school students with ESN require continued instruction on foundational numeracy skills (Browder, Jimenez, Spooner, et al., 2012). One method to address the continued need for instruction in foundational skills while still addressing age-appropriate content is through the use of embedded instruction (Browder & Spooner, 2014; Hunt et al., 2012). Although the premise of embedding numeracy instruction within secondary standards-based mathematics lessons has yet to be investigated, researchers have successfully used embedded instruction to teach vocabulary, comprehension, functional skills, and other foundational skills (e.g., Brosh et al., 2018; Ruppar et al., 2017).

For example, Ruppar et al. (2017) used a multiple baseline across conditions design to evaluate the effectiveness of comprehension instruction embedded within shared story reading for a 16-year-old girl with multiple disabilities who was included in a general education 9th grade class. The class was reading *The Odyssey* by Homer, a complex text. The student's teachers wanted to promote general curriculum access while still addressing her goals of answering "wh" comprehension questions. The researchers embedded a systematic response prompting strategy, constant time delay (CTD), to target the student's comprehension goals within the context of the general curriculum lesson. Using an adapted text and embedded instruction, the target student met her foundational comprehension goals while still engaging with age-appropriate materials and content.

Most like my proposed study, Brosh et al. (2018) embedded foundational literacy skills within a mathematics lesson. Brosh et al. evaluated the use of modified schema-based instruction (MSBI) and embedded CTD to teach elementary students with ESN. MSBI is a cognitive-based strategy incorporating explicit and systematic instruction, graphic organizers, and chants to help students with ESN solve mathematical world problems (Spooner et al., 2017). Using embedded CTD, the researchers also targeted a foundational literacy skill, identifying nouns and verbs. The students made progress in both the grade-aligned mathematics skill and the foundational literacy skill.

Statement of Purpose and Research Questions

In contrast to Brosh et al. (2018), who investigated embedded literacy instruction within mathematical word-problem solving using CTD, I investigated embedded numeracy instruction within standards-based mathematics lessons for high school students with ESN, using simultaneous prompting (SP). Specifically, the purpose of my study is to investigate the feasibility and effectiveness of embedding foundational skill instruction (i.e., numeral identification) within a standards-based, grade-aligned mathematics lesson (i.e., solving linear equations) for secondary students with ESN. My research questions are:

1. To what extent does an intervention package comprising MSBI and embedded SP improve foundational numeral identification skills for secondary students with ESN?
2. To what extent does an intervention package comprising MSBI and embedded SP improve the independent algebraic problem-solving skills of secondary students with ESN?
3. To what extent can secondary students with ESN generalize numeral identification skills to real-world settings and situations?
4. To what extent is addressing numeracy skills within grade-level mathematics content considered an acceptable, effective, and efficient practice by classroom teachers of secondary students with ESN?

Significance of Study

This study contributes to the research on mathematics instruction for students with ESN in two ways. This is the first study to investigate the use of MSBI with students who do not yet have foundational numeracy skills. The results of this study suggest that numeracy skills may in fact be a true prerequisite when using MSBI to solve mathematical word problems. Neither student in this study acquired the skill of mathematical word problem solving, though the generalizability of this study is limited. Additionally, I investigated an intervention in which foundational and standards-based skills were addressed in tandem. Although the students in this study did not meet mastery criterion, the instructors did rate the intervention as acceptable and effective, and hypothesized the students would have made progress in the numeracy skills given additional time. Finally, the students' instructors, rather than outside researchers, implemented the intervention within the students' classroom. Relatively few studies have been published in which natural change agents implemented MSBI. The instructors in this study had high levels of fidelity given scripted lesson plans and premade instructional materials.

Delimitations

This study has several delimitations. First, I used a multiple probe across number sets single-case design, replicated across participants to investigate the intervention. By nature, single-case designs have a small *n*, reducing the potential for generality. While I originally planned to apply this intervention to a group of three to five participants, due to difficulties with recruitment related to teacher shortages and the impact of the COVID-19 pandemic, I recruited only two participants. Second, I conducted this study with students with ESN who are educated in self-contained settings. The research suggests that inclusive placements are the most beneficial for students (Gee et al., 2020). The majority of students with ESN, however, are still educated in self-contained placements, including the sample of students who were available to me for participation in this study.

Third, mathematical word problem solving requires students to simultaneously engage in multiple cognitive processes. Students must read and understand the text of the problem, create a representation of the problem, identify a process for solving the problem, solve the problem, and determine if the solution is reasonable (Jitendra et al., 2013). To reduce the cognitive demands required in mathematical word problem solving, sometimes called cognitive load, I presented only one problem type (i.e., combine schema). Combine schemas refer to those word problems in which the student *combines* two groups of small items (e.g., balls and dolls) to make a big group (e.g., toys; Powell & Fuchs, 2018). Student participants were not required to discriminate amongst problem types, which is the goal of schema-based instruction (SBI). This study instead addressed only a portion of the schema-based, or MSBI intervention. Additionally, I restricted combine problems to those with the second *small group* unknown (e.g., $3 + x = 9$).

Finally, this study required multiple instructors to serve as interventionists, as most schools have only one or two students who meet my eligibility criteria. To reduce the impact of this possible confounding variable, I provided scripted lessons and behavioral skills training (BST; Miltenberger, 2016; Parsons & Reid, 1995) to each instructor.

Definition of Terms

The following terms will be important to understand within the context of this study. Definitions of these terms are provided.

Academic Content Standards: A set of guidelines or goals that describe what teachers are supposed to teach and what students are expected to learn within a grade level (e.g., 4th grade) or content area (e.g., Algebra I; Ravitch, 1995)

Algebraic Problem Solving: Solving mathematical equations in which a term is unknown (e.g., $x + 2 = 5$; NCTM, 2000).

Behavioral Skills Training (*BST*): A research-supported training package that includes four parts: (a) instruction: describing the skill; (b) modeling: demonstrating the skill; (c) rehearsal: practicing the skill; and (d) feedback: reviewing correct and incorrect implementation of the skill (DiGennaro Reed et al., 2018; Miltenberger, 2016; Parsons & Reid, 1995).

Contextualized Instruction: Teaching an academic skill (e.g., algebra) in context of a real-world story or problem (Bottge et al., 2001; Cognition and Technology Group at Vanderbilt, 1990).

Embedded Instruction: A method of instruction in which instructional trials are distributed across activities (Jameson et al., 2020).

Explicit Instruction: An approach to instruction that promotes student engagement by chunking complex skills, incorporating modeling and think-alouds, systematically fading support, providing multiple opportunities to respond, and creating opportunities for practice (Hughes et al., 2017). A common approach within explicit instruction used with students with ESN is *model-lead-test*, in which an instructor first provides a model of the task, then leads the student through completing the task with decreasing levels of support, and finally tests the student by providing them an opportunity to complete the task independently (Gibson & Schuster, 1992).

Extensive Support Needs (ESN): Individuals who have pervasive support needs across academic and daily living domains, who are likely eligible to take their state's AA-AAS, and who may receive special education services under the IDEA classifications of autism, ID, multiple disabilities, or deaf-blindness (Kurth et al., 2019).

General Curriculum Access: Participation in the same curriculum delivered to same-age peers without disabilities, ideally in the same environment (Ryndak et al., 2008–2009).

Linear Equation: An algebraic equation presented as $y = mx + b$, where b is a constant.

Modified Schema-Based Instruction (*MSBI*): An instructional strategy designed to support students with ESN as they solve a mathematical word problem. Modifications to schema-based instruction include task analyses to replace mnemonic heuristics, schematic diagrams presented as graphic organizers, systematic instruction added to explicit instruction, and metacognitive instruction enhanced by chants and hand motions (Spooner et al., 2017).

Numeracy: The development of number concepts, which includes numeral identification, counting, one-to-one correspondence, composing and decomposing numbers, and place value (Browder et al., 2020).

Schema: In mathematics, a framework or diagram that is used to organize thinking when solving a mathematical word problem (Powell, 2011).

Simultaneous Prompting (*SP*): A systematic instructional strategy in which the teacher probes the student first by presenting the instructional stimuli with no prompts or reinforcement, followed by teaching trials in which the teacher delivers the task direction and immediately prompts the student to respond correctly (Collins, 2022; Gibson & Schuster, 1992).

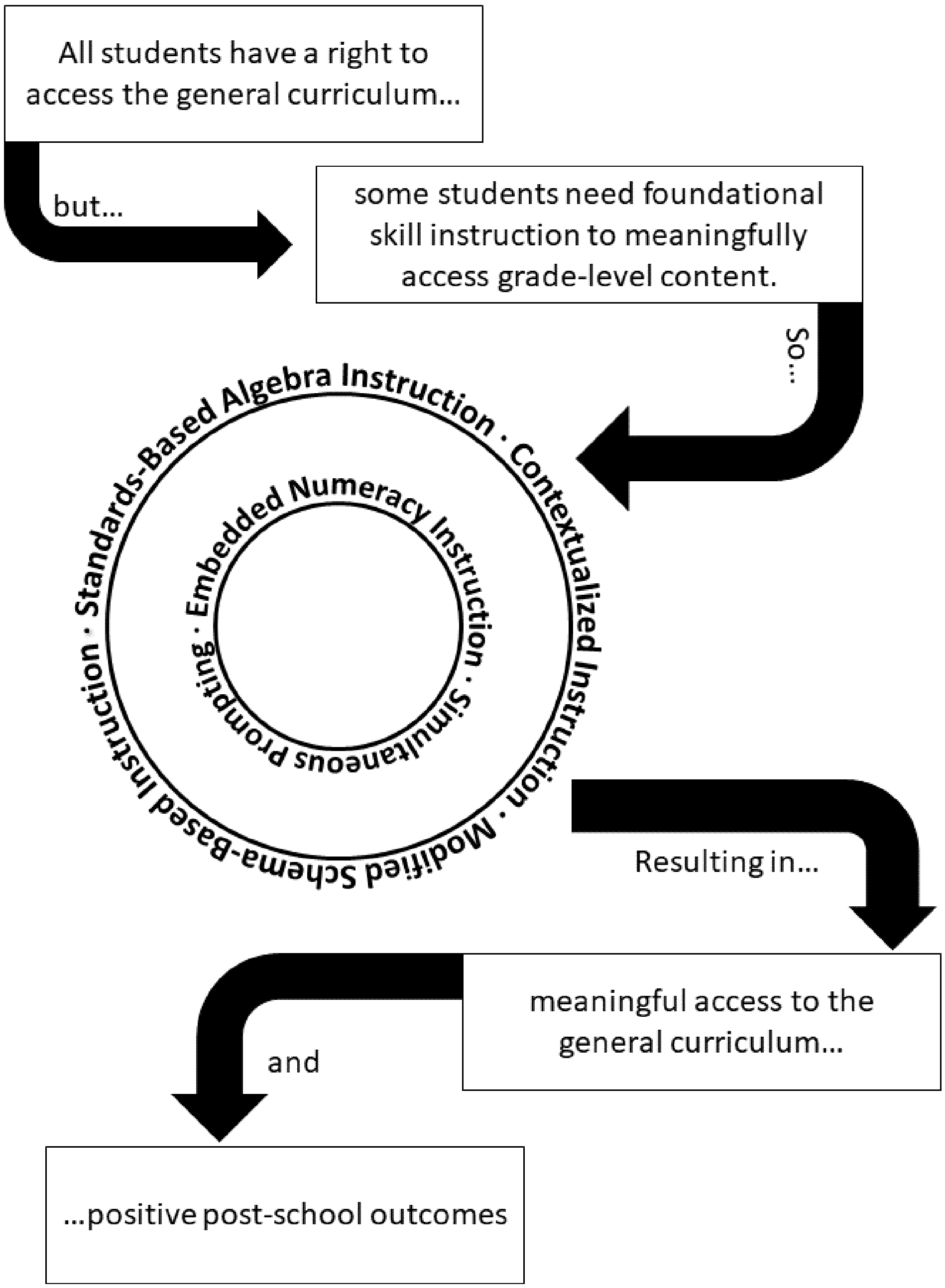
Systematic Instruction: A set of evidence-based instructional strategies based on the principles of behavior analysis, in which observable and measurable responses are taught using prompting systems, including graduated guidance, most-to-least prompting, system-of-least prompts, time delay, or SP (Collins, 2022; Snell, 1978).

CHAPTER 2: LITERATURE REVIEW

Federal law requires that all students have access to and progress in the general curriculum (ESSA, 2015; IDEA, 2004). Students with ESN have often not had the opportunity to acquire foundational skills necessary to meaningfully access the general curriculum. This chapter will explore the research base for the proposed intervention package: embedding foundational numeracy instruction within standards-based algebra lessons for secondary students with ESN. First, I will present a history and current perspective of general curriculum access. Next, I will review the importance of standards-based algebra instruction, contextualizing instruction when teaching core content, and the use of MSBI to teach grade-level, standards-based algebra content. Finally, I will propose a method to embed foundational skills such as numeracy within these standards-based algebra lessons using SP. Figure 1 presents the theory of change model for this proposed intervention.

Figure 1

Theory of Change Model



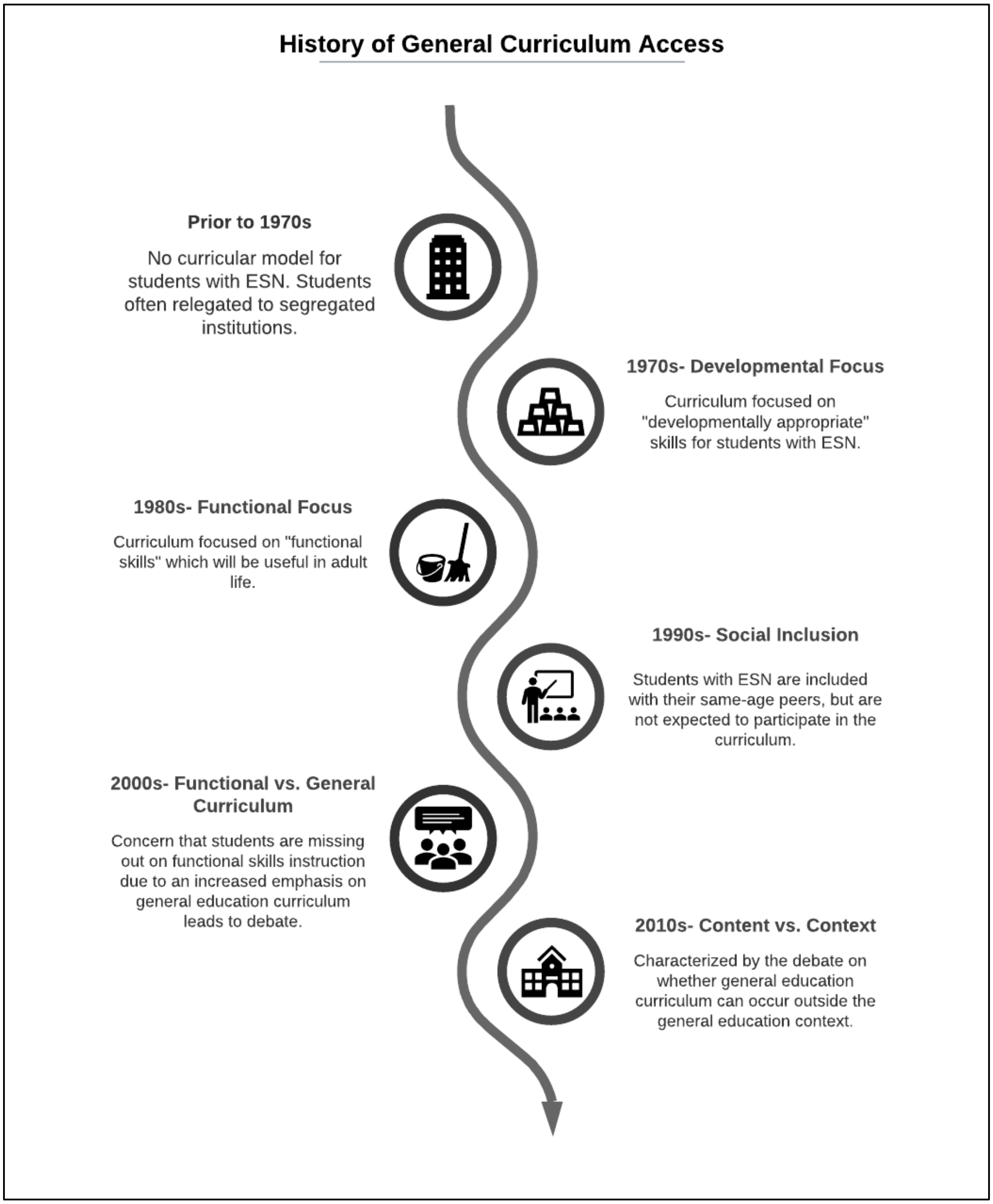
General Curriculum Access

When discussing *general curriculum access*, the referred population is typically students with ESN. Students with ESN represent 1% or less of all students (Kleinert et al., 2015). These students are typically identified with the exceptionalities of autism, ID, multiple disabilities, and deaf-blindness (Kleinert et al., 2015). However, it is important to note that not every child with these labels has ESN. Students with ESN require extensive support across multiple domains, including learning, communication, independent living, self-care, and employment (Meyer et al., 1991; Thompson et al., 2018). In the school setting specifically, students with ESN may require extensive support due to challenges with communication, short-term memory, generalization, and executive functioning (Copeland & Cosbey, 2008–2009; Hughes et al., 1994).

One reason the discussion around general curriculum access is often focused on students with ESN is that these students have limited access to the settings in which the general curriculum is typically taught (i.e., general education classrooms). In general, students with disabilities are served in less restrictive settings today, with 64.8% of all students with disabilities being taught in the regular class 80% or more of the day during the 2020–2021 school year (i.e., full inclusion; United States Department of Education [USDOE], 2022). That percentage is significantly lower for students who may have ESN. Only 39.8% of students with autism, 26.5% of students with deaf-blindness, 16.6% of students with ID, and 14.3% of students with multiple disabilities were fully included in the general education classroom (USDOE, 2022). Kleinert et al. (2015) looked at placements specifically for students who were eligible for their state’s AA-AAS (i.e., had ESN). Fewer than 3% were fully included in the general education setting. Regardless of placement, federal law now requires students to have access to the general curriculum; however, that has not always been the case (Figure 2).

Figure 2

Historical Timeline of General Curriculum Access



Note. Adapted from Browder et al., 2020, p. 9

Historical Perspective of Curriculum Access

The provision of public special education for students with ESN became law in the United States in 1975, with the passage of PL 94-142. Prior to the 1970s, there was no concept of curriculum for students with disabilities. Many individuals with ESN were expected to reside in segregated institutions when they left school, if they attended school at all (Blatt & Kaplan, 1974; Rivera, 1972; Wolfensberger, 1975). There was no expectation that students with ESN could learn, and thus they were not given the opportunity to do so (Goldberg & Cruickshank, 1958; Haring & Pious, 1976; Sontag & Haring, 1996).

When students with ESN began to attend schools more reliably in the 1970s, educators followed a developmental approach to curriculum (Browder et al., 2020; Dymond & Orelove, 2001; Jackson et al., 2008–2009). Students were taught based on their developmental age, and as a result, most students spent their entire educational career learning early childhood skills. This approach to curriculum, unfortunately, lacked both meaningfulness and functionality for students with ESN.

In 1976, Brown et al. introduced the "criterion of ultimate functioning" as an alternative to the developmental approach. The approach aimed to promote functioning in as many environments with peers without disabilities as possible (Brown, Branston-McClean, et al., 1979). Educators emphasized the skills students would need after they left school rather than those primarily relevant to school environments (i.e., academics; Brown, Branston, et al., 1979). Problematically, this approach resulted in a parallel functional curriculum. Students with ESN receiving this curriculum were relegated to separate settings dedicated to this special functional curriculum (Trela & Jimenez, 2013).

Curriculum in the 1980s transitioned from a focus on functional curriculum to one that was based on a student's chronological age (Browder et al., 2020; Donnellan, 1984). This change in focus can be attributed to the concept of the least dangerous assumption, conceptualized by Donnellan in 1984. According to this concept, educators should make decisions for their students based on the assumption that, if they are wrong, it will have the least dangerous effect on the student. Donnellan asserts that the educational placements and priorities that will be "least dangerous" for students with ESN are those in which students receive the same education in the same placement as their peers without disabilities.

In the 1980s, curriculum changed for all students with the release of the report, *A Nation at Risk* (National Commission on Excellence in Education, 1983). In this report, the authors called for comprehensive reform in schools. Students needed to be held to more rigorous standards to continue competing on a world stage. This call to action led to the introduction of state standards for education. Although these standards were not yet applied to students with ESN, the stage was set for future educational reform (Browder et al., 2006). For example, in 1992, Kentucky was the first state to assess its students with ESN based on an alternate set of standards, known as alternate achievement standards (Towles-Reeves et al., 2009).

Students with ESN in the 1990s had increased access to general education settings but still had limited access to the general curriculum (Browder et al., 2006). The focus of this period was on 'social inclusion' wherein students with ESN were physically in the same classroom as their same-age peers but were not expected to make progress in the curriculum. This focus shifted with the reauthorization of IDEA in 1997, in which students with ESN were expected to receive the general education curriculum in their least restrictive environment (Wehmeyer et al., 2020).

The increased emphasis on access to the general curriculum in the 2000s aligned with a debate about the role of functional skills in school settings (Ayres et al., 2011; Ford et al., 2001). There was a concern that students with ESN were missing instruction on “meaningful” functional skills due to the emphasis on the general curriculum. This debate can be best characterized by the exchange of position papers by Ayres et al. and Courtade et al. in 2011 and 2012, with Ayres et al. arguing for a functional approach and Courtade et al. arguing for academic instruction. Many authors, such as Browder and Cooper-Duffy (2003), argued for a balanced approach, suggesting educators teach the general curriculum content in a functional skills context. This balanced approach has shifted in priority today, with researchers now recommending a focus on general curriculum content and contexts, with functional and foundational skills embedded within the lesson (Browder et al., 2020)

Defining General Curriculum Access Today

To define general curriculum access in the context of the 2010s and 2020s, it is necessary first to define *general curriculum*. Curriculum can be defined as the content to be taught, including materials and texts used in the classroom (Browder et al., 2004; Restorff et al., 2012). Kurz et al. (2010) defined curriculum in three parts: the intended curriculum referring to the state standards, the enacted curriculum referring to what is actually taught, and the assessed curriculum referring to the content measured by assessments. The general curriculum refers to the curriculum that the state or local educational agency has adopted and includes the goals, objectives, instructional materials and methods, and assessment that an educator is expected to deliver to their students (Hitchcock et al., 2002; Wehmeyer et al., 2001).

General curriculum access has been interpreted as access to the general education classroom (i.e., context), access to the general education content, and access to both content and context (Ryndak et al., 2008–2009). This content and context must be aligned to a student's chronological age rather than their instructional level (Knight et al., 2010). Hitchcock et al. (2002) defined access as not simply being presented with access to information or activities but access to a plan for learning. Similarly, Wehmeyer (2006) stressed the importance of moving beyond *access to* toward *progress in* the general curriculum. As evidenced by the Supreme Court's ruling in *Endrew F. v. Douglas County School District* (2017), it is not enough for students to merely be presented with the curriculum. Educators must ensure that students make progress in the curriculum.

Most recent definitions of general curriculum access emphasize the importance of both content (what they are taught) and context (where they are taught). This emphasis on context is due to data showing that students taught in restrictive settings have less access to rigorous standards-based instruction and are more passively engaged in instruction. In contrast, students taught in general education classrooms have improved access to the general curriculum and highly qualified teachers (Kurth et al., 2019). For example, in an analysis of student progress, Gee et al. (2020) found that students with ESN educated in inclusive placements made significant gains in communication, literacy, and mathematics compared to their peers educated in self-contained settings. Mansouri et al. (2022) conducted a literature review of studies in which outcomes related to educational placement for students with ESN were compared and found there were more positive outcomes for students who received education in inclusive settings. This research supports assertions by experts in the field, (e.g., Ryndak et al., 2008–2009; Saunders et al., 2019) who argue that the triangulation of context, content, and learning is necessary for full access to the general curriculum.

Many authors have proposed definitions of access because, although federal legislation requires general curriculum access, there is no explicit definition for access. For example, IDEA (2004) requires that individualized education program (IEP) goals "enable the child to be involved in and make progress in the general education curriculum" §1414(d)(1)(A)(II)(aa). Similarly, ESSA (2015) states that any alternate achievement standards designed for ESN must promote involvement and progress in the general education curriculum. The Supreme Court recently confirmed that the preferred environment for students to access the general curriculum is the general education classroom (*Endrew F*., 2017). The clearest definition for access was provided by Yudin et al. (2015) in a "Dear Colleague" letter to Congress, in which they defined access as "the curriculum that is based on a State's academic content standards for the grade in which the child is enrolled" (p. 3).

The lack of an explicit definition of access, either in law or by prominent researchers in the field, can lead to confusion for teachers. This confusion is evident in qualitative investigations of teachers' definitions of access. For example, Dymond et al. (2007) interviewed 25 high school teachers to determine their definition of general curriculum access. The authors noted that most participants had difficulty defining access. Of those who could define access, there was a marked difference between general and special education teachers. Like Saunders et al. (2019), the general education teachers defined *access* as general education curriculum in the general education classroom. In contrast, special education teachers defined access as participating in a curriculum that is meaningful and relevant to the student. As most students with ESN are educated by special education teachers in special education classrooms, this is particularly concerning.

Timberlake (2014) interviewed teachers of students with ESN specifically and found that similarly to Dymond et al. (2007), the teachers prioritized functionality and relevance in defining access to the curriculum. To address gaps in students' foundational knowledge, teachers often aligned instruction to standards from lower grade levels rather than the grade in which the student was enrolled. In contrast, Olson et al. (2016) conducted a case study with teachers at an inclusive middle school and found that the teachers believed context was important for access and instruction of students with ESN should occur in the general education classroom. Like Wehmeyer (2006), these teachers moved beyond simple access to focus instead on progress in the curriculum. It seems clear that access to inclusive settings impacts not only students' access to the general curriculum but also teachers' perceptions of access to the general curriculum.

Importance of Access

Regardless of the complexities of defining access, research has shown that access to the general curriculum is crucial for student success in school and after graduation. For all students, the general curriculum is designed to promote learning and prepare students for college and career (Courtade et al., 2012; Yudin et al., 2015). Access to the general curriculum is an important predictor of post-school success (Gilley et al., 2021; Shogren et al., 2018; Test et al., 2009). Success post-school, which includes access to postsecondary education and competitive employment, typically requires a high school diploma (Kearns et al., 2011; Mazzotti et al., 2021). According to a USDOE report, in the 2018–2019 school year, only 71.4% of students with autism, 68.1% of students with deaf-blindness, 47.3% of students with ID, and 44.8% of students with multiple disabilities graduated with a regular high school diploma (USDOE, 2022). In most states, students who participate in curriculum aligned to alternate achievement standards, rather than the general education curriculum are not eligible to earn a high school diploma, severely limiting their post-school opportunities.

Regardless of whether a student meets the requirements to earn a high school diploma, students with ESN still have a right to access the general curriculum (Courtade et al., 2012). Academic knowledge improves students' competence for adult living, increases opportunities for jobs and leisure activities, and promotes self-determination (Courtade et al., 2012; Knight et al., 2010). As Browder et al. (2020) claim, education is necessary for equality, and every student has the right to participate in a full educational program.

Summary

Students with ESN have various support needs across academic, social, vocational, and independent living domains (Meyer et al., 1991; Thompson et al., 2018). These students represent less than 1% of the population and are typically educated in segregated settings (Kleinert et al., 2015). Historically, students with ESN have not been given the opportunity to engage in high-quality, standards-based instruction due in part to perceived deficits in communication and cognition (Cox & Jimenez, 2020; Rockwell et al., 2011).

Prior to the 1970s, students with ESN were given few opportunities to participate in any formal curriculum or learning environment (Goldberg & Cruickshank, 1958). After the passing of PL 94-142 in 1975, students with ESN had the federal right to be educated in school settings and have since experienced multiple curricula changes. The 1970s were characterized by a developmental focus, in which students were educated based on their developmental age (Dymond & Orelove, 2001). With the introduction of the criterion of ultimate functioning in 1976 by Brown and colleagues, the 1980s were characterized by a functional approach. Donnellan conceptualized the least dangerous assumption in 1984, which led to a focus on age-appropriate activities in age-appropriate settings in the 1980s. In the 1990s, students were typically included with their same-age peers but not expected to make progress in the same curriculum (Browder et al., 2006). IDEA was reauthorized in 1997, requiring students to receive access to the general education curriculum in their least restrictive environment. This legislation led to debates on the importance of general education curriculum *vs*. functional skills in the 2000s (Ayres et al., 2011; Courtade et al., 2012).

Legislation, such as IDEA (2004) and ESSA (2015), and judicial rulings, such as *Endrew F.* (2017), have further promoted access to the general education curriculum for all students, including those with ESN. Researchers today have defined access as access to both the instructional content and context (Ryndak et al., 2008–2009; Saunders et al., 2019). In contrast, qualitative investigations with general and special education teachers demonstrate that their perceptions of access differ from that of researchers and legislators, emphasizing the relevance and meaning of instructional content delivered in separate settings (Dymond et al., 2007; Timberlake, 2014). Regardless of these differences in defining access and prioritizing curricular areas, access to the general curriculum is important. Access is not only a right but is crucial for post-school success (Courtade et al., 2012; Gilley et al., 2021; Test et al., 2009). For students with ESN to meaningfully access the general curriculum, however, they require high-quality instruction in standards-based content areas and remediation in foundational skills they have yet to master (Browder et al., 2020).

Standards-Based Algebra Instruction

One component of the general curriculum that is particularly important is algebra. It is important because algebra is needed to succeed in future mathematics coursework, postsecondary education, and the workplace (Fuchs et al., 2014; NCTM, 2000). Algebra comprises analyzing relationships between variables and quantities and finding patterns, including functions and comparing rates (Ahlgrim-Delzell et al., 2009; Fuchs et al., 2014; NCTM, 2000; Rodriguez, 2016). High school algebra typically requires students to demonstrate mastery in symbols and expressions, linear equations, quadratic equations, functions, polynomials, and probability (Geary et al., 2008). Both previous and concurrent mathematics instruction must stress conceptual understanding, fluency in numbers and operations, and mathematical problem-solving skills to prepare students for success in algebra (Geary et al., 2008).

Algebra Instruction for Students with ESN

Historically, students with ESN have had limited access to complex areas of mathematics, like algebra instruction (Cox & Jimenez, 2020). This lack of access is often attributed to the perceived deficits of these students. Indeed, students with ESN may require exceptional support in communication, language, executive functioning, numeracy, and mathematical problem-solving, making accessing complex mathematics concepts difficult (Hart Barnett & Cleary, 2015; King et al., 2017). Additionally, many students with ESN are educated in self-contained classrooms, where they have reduced access to mathematics content experts (Cox & Jimenez, 2020).

Regardless of these perceived deficits and challenges, standards-based algebra instruction is extremely important for students with ESN. The skills students learn in school mathematics apply to all domains of daily life (Ahlgrim-Delzell et al., 2009). For example, algebraic skills are necessary when calculating costs or analyzing relationships between variables. The process skills students learn in algebra are also important for developing critical thinking and problem-solving skills (Cox & Jimenez, 2020; Goya et al., 2019). Proficiency in algebra is necessary for students to gain employment and live independently in their adult life (Creech-Galloway et al., 2013).

Limited research has been conducted on algebra instruction for students with ESN, particularly at the high school level. In the first study of its kind, Jimenez et al. (2008) taught three high school students with ESN to solve simple linear equations using a multiple probe across participants design. The multicomponent intervention comprised graphic organizers, task analytic instruction, and systematic prompting. All three students communicated using oral speech and could count to nine and identify numbers one through nine before starting the study. All three participants reached mastery; two had near-immediate success, with graphs representing a stable and increasing trend. The third participant demonstrated some variability at the beginning but showed success once given a visual of the task analysis in addition to increased wait time.

Most of the research investigating algebra instruction for students with ESN has been an extension of Jimenez et al. (2008). For example, Baker et al. (2015) taught three middle school students with ESN to solve a linear equation, using the same intervention package as Jimenez et al. The researchers demonstrated a functional relation, with all students making progress in the skill of solving simple linear equations. Chapman et al. (2019) extended Jimenez et al.'s research by presenting linear equations in the context of a real-world scenario and using realistic manipulatives. The three high school students with ESN acquired the skill and reached mastery.

Contextualized Instruction

Research has suggested that students with ESN can learn algebra given graphic organizers, task analytic instruction, and systematic prompting (Baker et al., 2015; Chapman et al., 2019; Jimenez et al., 2008). These students, however, often have trouble generalizing skills from the classroom environment to the real world (Root, Knight, & Mims, 2017). Access to the general curriculum, and algebra specifically, is crucial for student success post-school, but only when taught in a meaningful context.

Contextualized instruction, sometimes called anchored or authentic instruction, emphasizes academic instruction within a real-life activity or natural routine (Bottge et al., 2001; Cognition and Technology Group at Vanderbilt, 1990; Root, Cox, et al., 2018). Teaching skills like algebra in context of a meaningful, real-world activity helps all students understand the purpose of the skill (Ahlgrim-Delzell et al., 2009; Bowman et al., 2019). Contextualized instruction helps support the transition from concrete to abstract reasoning, deepening students' conceptual understanding of the skill (Stephan et al., 2020; Yang, 2006).

Educators can support generalization through contextualized instruction (Bowman et al., 2019; Hunt et al., 2012; Saunders et al., 2013, 2019). When teaching skills in contexts that replicate students' real-world experiences, educators promote generalization by programming common stimuli and using sufficient exemplars (Root, Cox, et al., 2018; Root, Knight, & Mims, 2017; Stokes & Baer, 1977). Students who can apply their knowledge of algebra to real-world settings are better prepared for post-school employment, education, and living.

Contextualized instruction entails selecting a grade-level standard, identifying real-life applications for the skill, and teaching explicitly with plans for generalization (Root, Cox, et al., 2018). Educators should attempt to make the skill culturally and personally relevant, using familiar people, settings, and objects when presenting the problem (Bartell et al., 2017; Gallivan, 2020; Trela & Jimenez, 2013). Realistic stimuli, such as objects, photographs, or videos, can also improve contextualized instructional scenarios (Hunt et al., 2012; Root, Knight, & Mims, 2017). For example, a teacher may present a problem in which the student needs to go to the local grocery store to purchase ingredients for their favorite meal. The problem can be enhanced by adding images of the actual ingredients along with their corresponding prices.

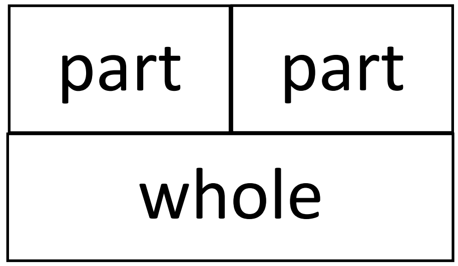
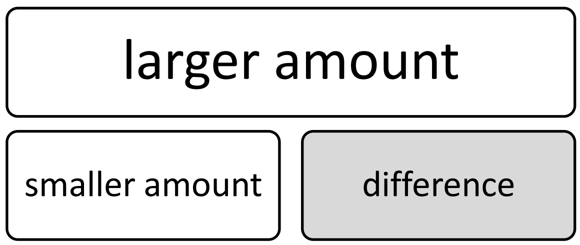
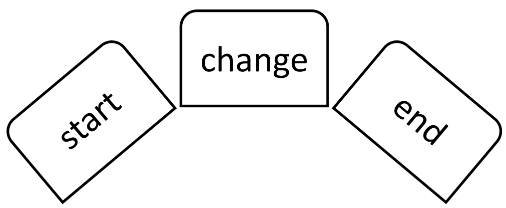
Story problems have been frequently cited in the literature as a way to provide contextualized instruction in mathematics (Bowman et al., 2019). Story problems provide students the opportunity to apply mathematics skills in context (Browder, Jimenez, & Trela, 2012). For example, Chapman et al. (2019) used contextualized instruction by presenting the algebra skill in the context of a real-world problem based on a vocational task the students were completing (e.g., stocking a shelf). The students acquired the skill of solving a mathematical word problem and generalized to a real-world setting (i.e., their jobsite). One mathematics instruction strategy that incorporates story problems is MSBI (Browder et al., 2018).

Modified Schema-Based Instruction

MSBI is grounded in SBI, a cognitive-based mathematical problem-solving strategy (Jitendra et al., 2015). Students learn to categorize mathematical word problems into schemas based on the structure of the problem. Then, they can apply set procedures to the word problems in a specific category, or schema, to arrive at a conceptually and procedurally sound answer (Jitendra et al., 2013). These categories include additive (including both addition and subtraction operations) and multiplicative (including both multiplication and division operations) schemas. When teaching students to solve mathematical word problems, additive schemas are typically introduced first (Root, Saunders, et al., 2022). The additive schema include combine, compare, and change problem types (see Table 2; Bouck et al., 2021; Powell & Fuchs, 2018). In each of these schemas, problems may be presented so that the sum is unknown (e.g., $1 + 2 = x$) or either addend is unknown (e.g., $1 + x = 3$ or $x + 2 = 3$).

Table 2

Additive Schemas

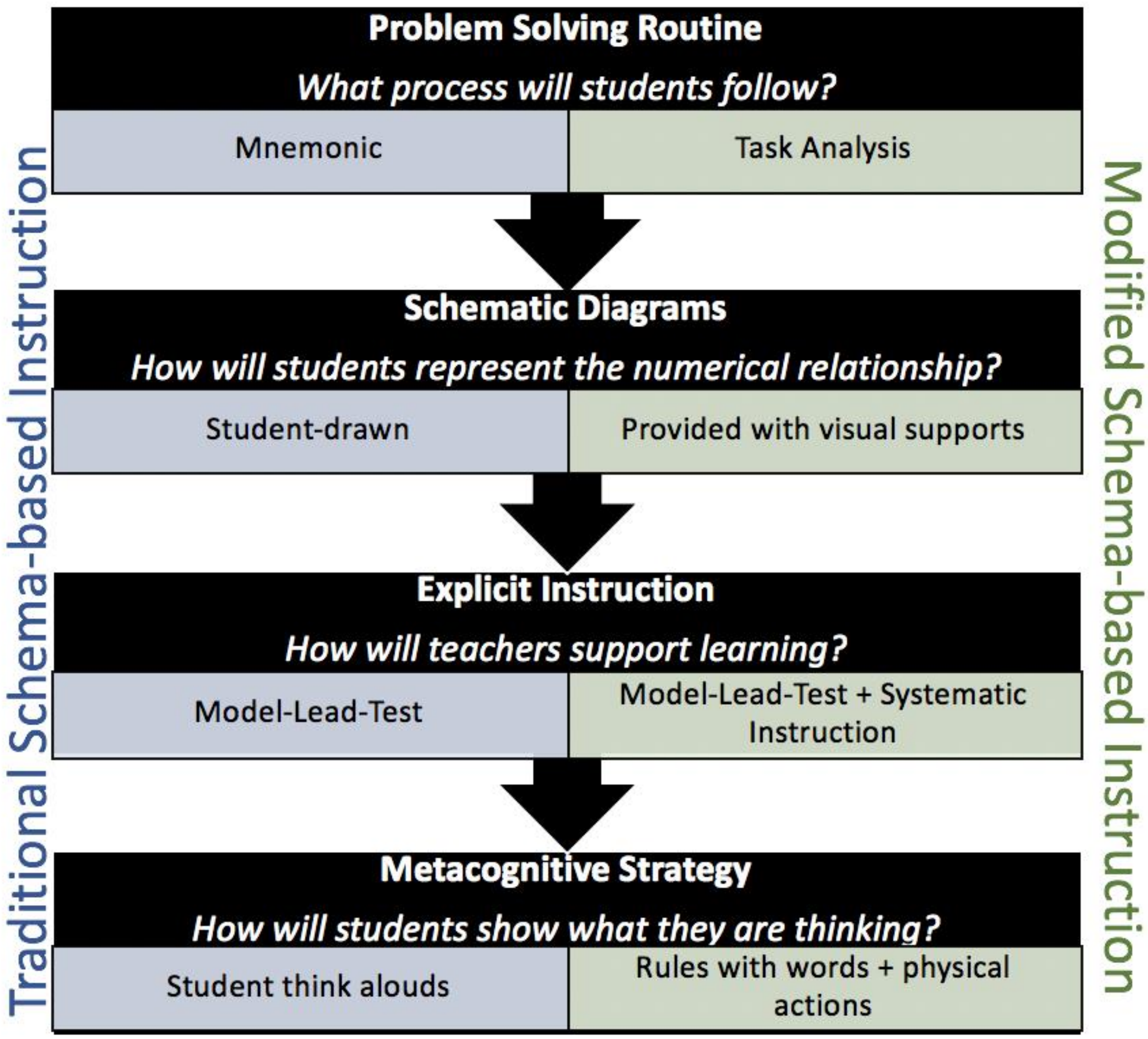
Schema	Potential Visual Representation	Example
Combine		Miley went to the library. She checked out 4 fiction books. She also checked out 3 biographies. How many books did Miley check out?
Compare		Matthew and Cole went to their job at the hotel. Matthew cleaned 8 rooms. Cole cleaned 6 rooms. How many more rooms did Matthew clean?
Change		Zoe is volunteering at the animal shelter. She has 10 dog treats. She gives 4 treats away. How many dog treats does she have left?

Note. Adapted from Powell & Fuchs, 2018. Reprinted with permission from Bouck et al., 2021. See permission letter in Appendix A.

SBI includes four critical elements: (a) heuristics, (b) schematic diagrams, (c) explicit instruction, and (d) metacognitive strategy instruction (Jitendra et al., 2013). Students with ESN, however, may require additional supports in reading, language, numeracy, and working memory to access these elements (Spooner et al., 2017). These support needs led Spooner et al. to modify SBI by replacing or supplementing the four components of SBI (Figure 3). In MSBI, mnemonic heuristics are replaced with task analyses, schematic diagrams are presented as graphic organizers, systematic instruction is added to explicit instruction, and metacognitive strategy instruction is enhanced by chants and hand motions (Spooner et al., 2017).

Figure 3

Modifications to Schema-Based Instruction



Note. Reprinted with permission from Bouck et al. (2021). See permission letter in Appendix A.

The conceptual framework presented by Spooner et al. (2017) was influenced by Saunders' 2014 dissertation and the Solutions Project (Browder et al., 2013–2017). Saunders was the first to evaluate the use of MSBI with students with ESN. She provided three elementary students with autism and moderate ID (who were considered to have ESN) with word problems requiring the use of combine or change schemas, a 12-step task analysis, graphic organizers, and manipulatives. Additionally, Saunders incorporated technology through video models and virtual manipulatives. Saunders required that students be able to identify numerals, count, and create sets up to 10 before participating in the study. Results demonstrated a functional relation between the MSBI and the students' mathematical word problem-solving.

Browder et al. (2013–2017) published a guide for practitioners based on their findings from the Solutions Project, in which they sought to teach mathematical word problem solving to students with ESN. Within the project, they taught students to solve combine, compare, and change schema types and to discriminate between types. All problems required the student to solve for the final sum (i.e., $a + b = x$). They developed a 12-step task analysis to be used across schema types:

1. Read the problem
2. Circle the "whats" (i.e., the addend labels in the problem)
3. Write the label (i.e., the sum label; what they were solving to find)
4. Same? Different? More/Fewer? (i.e., were the "whats" same or different labels)
5. Choose the graphic organizer (e.g., combine, compare, change)
6. Say the rule (e.g., combine= "*Small group, Small group. Big group!"*)
7. Circle the numbers
8. Fill in the number sentence

9. Write + or –
10. Make sets
11. Solve and write answer (Browder et al., 2013–2017, p. 53).

Additionally, they provided graphic organizers incorporating color coding (i.e., green and red for addends, blue for sum) and space for students to manipulate objects representing the quantities in the word problem. Finally, Browder et al. (2013–2017) recommended that practitioners use a consistent structure when writing word problems. For example, when writing a combine word problem, begin with an anchor sentence introducing the problem (e.g., *Jason goes to the pet store.*), then introduce the first "what" (e.g., *He buys 2 cat toys*), then the second what (e.g., *He buys 1 dog food bowl*), and then the question with the label to solve (e.g., *How many pet supplies does Jason buy?)*. Additionally, they recommend incorporating familiar names and the student's interests within the word problem to increase student engagement.

Since Saunders' dissertation in 2014, additional research teams have investigated MSBI for teaching word problem solving for students with different support needs. Root et al. (2021) conducted a review of MSBI studies in which participants had a diagnosis of autism and determined it was an evidence-based practice (EBP) for this population. Not all students with autism, however, also have ESN. Clausen et al. (2021) reviewed 11 MSBI studies in which participants were determined to have ESN. Within those studies, thirty-nine students participated, ranging in age from 7 to 21 yrs. Thirteen students were in elementary school, 22 in middle, and 3 in high school. All the studies were conducted in either a self-contained special education classroom or a separate setting (e.g., a hallway). The researcher served as the interventionist in nine studies. Natural change agents (e.g., teachers, peers) served as the interventionists in only two studies though, since this review, Root, Cox, and McConomy (2022) published an additional study in which teachers served as change agents. Only four of the studies included in this review were explicitly aligned to mathematics standards, specifically those related to arithmetic and algebra (e.g., number and operations in base ten; operations and algebraic thinking; expressions and equations; ratios and proportional relationships; algebra-reasoning with equations and inequalities). Additionally, students in 10 of the 11 studies were required to demonstrate foundational numeracy skills prior to their inclusion in the intervention. Although Clausen et al. did not locate a sufficient dispersion of studies (e.g., across research teams, geographic regions) to classify MSBI as an EBP for students with ESN, they did report a very large effect size, suggesting MSBI may be a promising practice for students with ESN.

Little research has been conducted on the application of MSBI in algebraic contexts. Root, Henning, and Boccumini (2018) taught three elementary students with autism and mild ID to solve algebraic word problems using MSBI. The researchers presented word problems with either a missing “part” (e.g., $15 + x = 25$) or missing “whole” (e.g., $15 + 10 = x$), requiring the combine problem schema. Although the students did not have ESN, the intervention was effective, a functional relation was present, and a large effect size was demonstrated. Root and Browder (2019) taught three middle school students with ESN to solve algebraic word problems using MSBI. The tasks aligned to sixth and seventh grade standards and used combine schemas with either missing part or missing whole. The students could identify numerals 1 to 10, make sets of quantities up to 10, and count with one-to-one correspondence before inclusion in the study. A functional relation was demonstrated between MSBI and the students’ ability to solve algebraic word problems.

At this time, no research teams have investigated MSBI to teach standards-based algebra skills to high school students with ESN, nor have any research teams investigated whether MSBI is effective for students who do not yet exhibit foundational numeracy skills. MSBI is a promising strategy for teaching mathematics skills like algebra to students with ESN. More research is needed with students with ESN in high school settings, particularly those who do not yet exhibit foundational numeracy skills.

Summary

Standards-based algebra instruction is necessary for all students to succeed post-school (Fuchs et al., 2014; NCTM, 2000). Historically, students with ESN have not been provided the opportunity to participate in algebra instruction (Cox & Jimenez, 2020). This is problematic as algebra is an important life skill that can be applied across all domains (Ahlgrim-Delzell et al., 2009). Jimenez et al. conducted the first investigation of algebra instruction for students with ESN in 2008. Their method of using graphic organizers, task analytic instruction, and systematic prompting has been replicated and extended since then (e.g., Baker et al., 2015; Chapman et al., 2019). Contextualized instruction is necessary to provide meaning to algebra instruction and support students' generalization of skills (Root, Cox, et al., 2018). MSBI incorporates contextualized instruction and has been demonstrated to be effective with students with ESN (Clausen et al., 2021; Spooner et al., 2017). Thus far, MSBI has only been applied in algebraic contexts in two studies, one of which was implemented with elementary students with autism and mild ID (Root, Henning, & Boccumini, 2018) and the other with middle school students with ESN (Root & Browder, 2019), all of whom demonstrated prerequisite foundational numeracy skills. More research is needed to investigate MSBI to teach standards-based algebraic problem-solving skills to high school students with ESN who do not yet exhibit foundational numeracy skills.

Foundational Numeracy Instruction

Numeracy is a foundational mathematics skill necessary for students to access more complex mathematics (Browder et al., 2020). Numeracy skills include, but are not limited to, identification of numbers, counting, one-to-one correspondence, subitizing, estimating, making comparisons between different quantities, and place value (Clements & Sarama, 2009; Geary et al., 2008; Jordan et al., 2009; Whitacre et al., 2020). Numeral identification is a key numeracy skill in which students either point (i.e., receptive identification) or name (i.e., expressive identification) the numeral (Browder et al., 2020). Without a way to communicate numbers, access to mathematics becomes significantly limited (Clements & Sarama, 2009).

An understanding of numeracy contributes to improved opportunities for employment, independent living, postsecondary education, and leisure activities (Browder et al., 2020). Numeracy is necessary for organizing finances, interpreting data, and making decisions as an informed citizen (Goos et al., 2012). Numeral identification is necessary for dialing a telephone, telling time, using paper money, or calling an elevator (Akmanoglu & Batu, 2004). Additionally, success with early mathematics leads to greater confidence, which can impact later learning (Judge & Watson, 2011).

All future mathematical learning, for example, algebra, requires a strong understanding of number (NCTM, 2000). Claessens and Engel (2013) analyzed longitudinal data of young children and found that early numeracy skills were the most important predictors for later achievement in reading and mathematics. Understanding foundational concepts like numeracy are necessary for a deeper understanding of complex mathematics skills and problem-solving (Claessens & Engel, 2013).

Typical Development of Numeracy Skills

Most children develop early numeracy skills with little need for direct instruction (Gersten & Chard, 1999). Clements and Sarama (2009) offered a developmental progression for mathematics skills of children aged 1–7 years. In their progression, children at age 1 begin to demonstrate counting skills by chanting in a “sing-song” voice without actually using number words. By age 2, children with typical language development begin vocally counting to five, though not necessarily in the correct order. By age 3, children begin counting to 10, with some errors. They also demonstrate one-to-one correspondence, in which each object is assigned a number (e.g., button one, button two, button three). Children three years of age begin to answer “how many?” questions but often answer incorrectly.

At age 4, children begin to count up to 10 accurately. They may be able to identify and write numerals 1–10. These children can answer the “how many” question by restating the last number counted (e.g., “one, two, three… three buttons!”). They also can produce quantities up to five. By age 5, children begin to demonstrate an understanding of number up to about 30, counting and producing quantities to 30. These children can count objects in a random array, keeping track of which objects have been counted. At age 5, they also can count backward from 10, identify errors in their counting, and make corrections.

Around six years of age, children can count from any number (e.g., “four, five, six…”) to 100. They can learn to skip count by 10s, 100s, 2s, and 5s (e.g., “five, ten, fifteen, twenty…”). They also demonstrate an initial understanding of base-ten place value concepts and can “count on” (e.g., starting from a set of 10 and counting on “eleven, twelve, thirteen…). Finally, at age 7, children can conserve number, meaning that they recognize that a set quantity remains the same even when the distribution changes to take up a larger space (e.g., eight objects close together are the same as eight objects spread far apart). They can count forward, or backward starting at any number and recognize similarities across decades (e.g., twenty-*two*, twenty-*three*, thirty-*two*, thirty-*three;* Clements & Sarama, 2009).

Numeracy for Students with ESN

In contrast to students with typical language and cognitive development, students with ESN often enter school without numeracy skills (Browder, Jimenez, Spooner, et al., 2012). According to a survey conducted by Kearns et al. (2011), approximately 12% to 17% of students with ESN demonstrated no understanding of numeracy. Even as students progress through school, they may fail to acquire numeracy skills, not necessarily because of an innate deficit but because they have had limited exposure or high-quality instruction in this area (Browder, Jimenez, Spooner, et al., 2012; Gee et al., 2020; Jimenez & Staples, 2015). For students who exhibit challenging behaviors, for example, instruction in their early years may have focused on social and behavioral skills, so they missed out on the opportunity to engage in numeracy activities (Cox & Jimenez, 2020). Traditionally, mathematics instruction for students with ESN has focused on skills labeled as functional (e.g., time, money, purchasing) rather than a focus on the conceptual, foundational understanding of number emphasized in standards-based instruction for students in early elementary (Browder et al., 2008).

When students with ESN do not have the opportunity to acquire numeracy skills, it can affect their quality of life in adulthood (Tzanakaki et al., 2014). For example, identifying numerals is a prerequisite skill for using the telephone, telling time, or boarding the correct bus (Akmanoglu & Batu, 2004). Counting and comparing quantities is necessary for using money or following recipes (Greer & Erickson, 2019). Additionally, numeracy is necessary for students to access the general curriculum and standards-based mathematics instruction (Jimenez & Staples, 2015). Even if students learn to follow a task analysis to solve a mathematics problem procedurally, conceptual learning is hindered without the foundational understanding of number (Jimenez & Kemmery, 2013).

Fortunately, research has suggested that students with ESN can learn numeracy skills when provided with explicit and systematic instruction (Saunders et al., 2019). Numeracy instruction should be presented in the context of real-world activities or mathematical problems so that students learn when and how to apply numeracy skills (Browder, Jimenez, Spooner, et al., 2012; Saunders et al., 2019). When teaching counting skills to students with ESN, Greer and Erickson (2019) suggest beginning with numeral identification (i.e., naming numerals). Identifying numerals is not a prerequisite to counting/cardinality for children with typically developing language skills, but for students who communicate through augmentative and alternative communication (AAC), understanding number names is a necessity (Greer & Erickson, 2019).

Teaching Numeral Identification

Browder et al. (2020) suggested using systematic instruction to teach numeral identification. Systematic instruction is a set of EBPs for teaching academic skills, specifically mathematics, to students with ESN (Snell, 1978; Spooner et al., 2012, 2019). Systematic instruction is based on the principles of behavior analysis and dates back to the 1970s in the first research on instructional strategies for individuals with ESN (Snell, 1978). Within systematic instruction are stimulus and response prompting strategies. Stimulus prompting strategies pair the prompt with the instructional stimuli (Cooper et al., 2020). For example, when teaching the color “red,” the instructor might present the written word “red” in red color. Response prompts refer to the presentation of some cue (e.g., verbal, model, physical) after the presentation of the instructional stimulus, resulting in the student selecting the correct response (Cooper et al., 2020). Over time, these prompts are faded systematically so stimulus control is transferred from the prompt to the instructional stimuli with minimal errors (i.e., errorless learning; Cooper et al., 2020). Response prompting systems include graduated guidance, most-to-least prompting, system of least prompts, CTD, and SP (Browder et al., 2014).

Graduated guidance, most-to-least prompting, and system of least prompts are the oldest forms of response prompting investigated in special education literature (Azrin et al., 1976; Brown et al., 1972; Collins, 2022; Foxx & Azrin, 1973; Horner & Keilitz, 1975). In each of these systems, the intrusiveness of the prompt level is either decreased (i.e., graduated guidance, most-to-least prompting) or increased (i.e., system of least prompts) depending on the students' needs. With graduated guidance, the instructor provides full physical support, fading to partial physical, then finally removing all physical support as the student becomes more independent (e.g., Jimenez & Alamer, 2018). In most-to-least prompting, the teacher systematically fades prompt levels based on a hierarchy of at least three levels (e.g., model, verbal, independent; Collins, 2022; Wolery et al., 1992). Most-to-least prompting is often used to teach new and difficult skills in which the learner may experience frustration or even danger without intensive support, such as for teaching swimming (e.g., Yilmaz et al., 2010).

Like most-to-least, the system of least prompts procedure also comprises a prompting hierarchy, but students are first given the opportunity to respond independently before the instructor provides more intrusive support (Collins, 2022). The prompting hierarchy is dependent on the skill and the student, but a commonly used hierarchy is independent—verbal—model—physical. Shepley et al. (2019) conducted a review of 123 studies investigating system of least prompts. They found the procedure to be an EBP for teaching individuals with ID to complete community-based, self-care, and vocational chained skills. CTD is another EBP that is an efficient way to teach many skills including academic skills (Browder et al., 2009; Courtade et al., 2014). In CTD, the instructor begins by presenting the instructional stimulus and immediately provides the answer (i.e., 0 s delay). The instructor then inserts a delay, allowing the student to respond independently before providing the response prompt (Snell & Gast, 1981; Touchette, 1971).

SP is the most recently conceptualized form of response prompting (Gibson & Schuster, 1992; Schuster et al., 1992). SP comprises probe and teaching trials. In probe trials, the instructor assesses the student's current level of performance, without providing prompting, reinforcement, or error correction. In teaching trials, the instructor delivers the target stimulus and immediately prompts the student to respond correctly. The instructor reinforces correct responses and error correction, if necessary, during teaching trials. (Collins, 2022).

Selecting a Response Prompting System

CTD and SP are effective response prompting procedures to teach academic skills like numeral identification (Collins, 2022; Spooner et al., 2019). Schuster et al. (1992) introduced SP as an alternative, potentially more efficient, procedure to CTD. They hypothesized it would be more efficient because (a) teaching trials all have the same procedures, so teachers are not required to change their behavior (in CTD, teachers change from no delay [e.g., 0 s] procedures to delay [e.g., 3 s] procedures); (b) differential reinforcement is not required as the correct answer is always prompted; and (c) students do not need to be taught a "waiting" behavior (in CTD, they are taught to wait for prompting if they do not know the answer [Snell & Gast, 1981]). Schuster et al. taught four students with ESN to identify pictures of commonly used grocery words using both CTD and SP. They found that both procedures were effective, and SP was more efficient, with students reaching mastery in fewer trials with fewer errors. These differences, however, were minimal.

Since 1992, additional research teams have evaluated the relative effectiveness and efficiency of SP and CTD with students with ESN through comparative studies. For example, Riesen et al. (2003) taught vocabulary words to junior high students, Akmanoglu et al. (2015) taught elementary students their personal information, Swain et al. (2015) taught restaurant words to elementary students, and Ackerlund Brandt et al. (2016) taught a variety of academic skills to children in early childhood programs. In each of these comparison studies, it was found that SP and CTD were both effective procedures. Although results were mixed regarding which procedure was more efficient or resulted in fewer errors, differences were minimal.

As both procedures seem to be effective and efficient for teaching academic skills, the instructor should consider logistical factors when choosing between SP and CTD (Waugh et al., 2009). If an instructor is unfamiliar with systematic prompting strategies, SP may be easier to implement, as procedures remain consistent throughout the instructional procedure, reducing possibilities for instructor error (Tekin-Iftar et al., 2019). Additionally, SP requires less direct instructional time than CTD, as the instructor does not spend time correcting errors or providing reinforcement for correct responses (Schuster et al., 1992).

Simultaneous Prompting

Tekin-Iftar et al. (2019) conducted a review of experimental studies evaluating the use of SP from 1990 to 2017. They located 41 studies, 27 of which met quality indicator standards for inclusion in their descriptive analysis. Of the articles reviewed, investigators applied SP to mathematics skills in seven studies. Additionally, I located seven studies published between 2016 and 2022 not included in Tekin-Iftar et al. in which investigators applied SP to mathematics skills in four studies. A list of the studies in which mathematics was addressed is presented in Table 3.

Table 3

Simultaneous Prompting in Mathematics

Study	Mathematics Skill	Participant Characteristics	Interventionist
Ackerlund-Brandt et al. (2016)	Expressive addition facts (e.g., 1 + 2 =)	6-year-old child with autism	Therapist at university-based clinic
Akmanoglu & Batu (2004)	Receptive identification of numerals 1–9	17-year-old with autism and ID 12-year-old with autism and ID 6-year-old with autism	Researcher
Aydın & Cavkaytar (2019)	Number-object matching, identification of geometric shapes, creating patterns	7-year-old with autism and mild ID	Parent
Birkan (2005)	Receptive identification of numerals 1–9	6-year-old with mild ID	Researcher
Bowman et al. (2020)	Solve one-step word problems with sums up to five (change and group schemas)	7-year-old with developmental delay 11-year-old with ID 10-year-old with autism	General ed teachers
Creech-Galloway et al. (2013)	Follow a task analysis to solve a problem with the Pythagorean Theorem	Four students ranging from 15 to 17 years old with moderate ID	Teacher-Researcher
Drevon & Reynolds (2018)	Expressive multiplication facts	9-year-old with no identified disability	Researcher
Gursel et al. (2006)	Expressively identify mathematical symbols (e.g., +, −, =)	12-year-old with mild ID	Researcher

Study	Mathematics Skill	Participant Characteristics	Interventionist
Heinrich et al. (2016)	Solve a simple linear equation; Expressively identify geometric shapes (e.g., cube, sphere, cone)	17-year-old with visual impairment and moderate ID	Teacher-Researcher
Jimenez & Saunders (2019)	Subitizing quantities up to 10	10-year-old with moderate ID 12-year-old with moderate ID 8-year-old with multiple disabilities	Teacher
Karl et al. (2013)	Calculate price of groceries, given a percentage decrease	15-year-old with moderate ID 16-year-old with moderate ID 18-year-old with moderate ID	Teacher-Researcher
Öztürk & Yıkmış (2020)	Number-object matching	7-year-old with mild ID 7-year-old with ID 6-year-old with mild ID	Computer software
Ramirez et al. (2014)	Calculate elapsed time	Two 12-year-olds with autism 14-year-old with autism	Researcher
Sönmez & Alptekin (2020)	Expressive multiplication facts	11-year-old with no identified disability	Teacher

Note. ID = intellectual disability

Only two research teams have evaluated the use of SP to teach numeral identification. Birkan (2005) taught a six-year-old with mild ID to receptively identify numerals. The researchers used a multiple probe design across behaviors (i.e., three sets of three numerals) to evaluate the intervention. The student acquired the skill across all three sets of numerals and maintained it up to 25 days after the intervention. Similarly, Akmanoglu and Batu (2004) taught three students with ESN to identify numerals 1 through 9 by pointing using SP. The students ranged in age from 6 to 17 years. All students acquired the skill and generalized it to the natural environment, in this case, identifying numbers on a calendar. Apart from one participant in Akmanoglu and Batu, all students included in these studies were in elementary grades. When teaching students in elementary grades, it is appropriate to teach numeracy skills in isolation, as the mathematics standards in these grades explicitly address numeracy. For students in secondary grades, however, numeracy instruction should be embedded within grade-aligned, standards-based instruction (Saunders et al., 2019).

Embedded Instruction

Embedded instruction refers to the purposeful distribution of instructional trials across activities and naturally occurring routines (Jameson et al., 2020; Ruppar et al., 2017). Embedding foundational skills, such as numeracy, within grade-level content, such as algebra, is an effective and efficient way to target individualized learning goals without reducing access to the general curriculum (Bowman et al., 2020; Collins et al., 2011). Educators are often overwhelmed by the number of skills they are expected to address in a limited time (Rock et al., 2016). Embedded instruction allows educators to address multiple skills within a single lesson or activity.

Skills such as numeracy are often considered prerequisite skills—students must master these skills before moving on to more advanced mathematics. Students with ESN, however, may require additional years of instruction to acquire these skills and should not be prevented from accessing grade-level mathematics contingent on acquiring foundational skills (Courtade et al., 2012). Instead, educators should embed numeracy instruction within standards-based lessons (Bowman et al., 2020; Hunt et al., 2012). Additionally, embedding numeracy instruction in general curriculum content promotes generalization as students have increased opportunities to practice these skills in the natural context (Peterson, 2016).

Embedding Foundational Skills in General Curriculum

Traditionally, researchers have embedded instructional trials across non-academic times, such as during breaks or transitions (Ruppar et al., 2017). For example, McDonnell et al. (2002) taught four high school students with ESN to read and define words necessary to participate in the general curriculum. They embedded the instruction using CTD during opening and closing activities, activity transitions, and breaks. The students acquired the skills, improving access to the general curriculum.

Browder, Jimenez, Spooner, et al. (2012) taught seven elementary students with ESN numeracy skills using embedded CTD in the context of an inclusive mathematics classroom. Like McDonnell et al. (2002), the instructors embedded trials during natural breaks in the lesson but also embedded the trials during the lesson itself, prompting students to identify numerals on worksheets. All students acquired the skills in the general education environment.

Brosh et al. (2018) embedded foundational skills in context of a general curriculum lesson by inserting non-targeted information in the instructional feedback. They taught three elementary students with ESN to identify and define nouns and verbs in context of a mathematical word problem. Identifying nouns and verbs is a foundational skill necessary to read and understand mathematical word problems. They found that simply providing information about the literacy skills through instructional feedback was not sufficient, but the students did acquire the targeted skills once the researchers embedded CTD within the mathematics lesson. Additionally, the students all learned to solve the mathematical word problems, given MSBI.

Few researchers have investigated the use of SP within embedded instruction. For example, Bowman et al. (2020) taught three elementary students with ESN to solve mathematical word problems using SP embedded during naturally occurring breaks in the inclusive mathematics classroom. At preselected times during the class (e.g., during transitions, when the other students were completing independent work), the general education teacher would present each step of the task analysis to solve the word problem, and then the student would complete the step. The students acquired the word problem solving skill and maintained it three to five weeks after the intervention ended. Karl et al. (2013) embedded SP during a small-group cooking activity to teach reading, mathematics, and science skills to four secondary students with ESN. Before each cooking activity, the teacher conducted the SP probes, and then embedded the SP teaching trials during the cooking activity. The students learned, generalized, and maintained the skills.

Most like my study, Rivera et al. (2017) successfully taught three elementary students with ESN vocabulary words using SP embedded within a shared story intervention. Throughout the week, the researchers would conduct probes to determine the students' acquisition of the vocabulary words. Then, at predetermined intervals during the shared story reading activity, they would present an image and the vocabulary word and direct the student to touch the picture as they repeated the word (i.e., SP teach trial). The students acquired and maintained the vocabulary words. No researchers to date, however, have investigated SP procedures to teach numeracy skills to high school students with ESN embedded within standards-based algebra lessons.

Summary

Numeracy skills are foundational mathematics skills that include numeral identification, counting, one-to-one correspondence, subitizing, estimating, making comparisons, and place value (Clements & Sarama, 2009; Geary et al., 2008; Jordan et al., 2009; Whitacre et al., 2020). Communicating numerals is crucial to engaging with mathematics tasks (Clements & Sarama, 2009). For students who use augmentative and alternative communication systems, identifying numerals is necessary to communicate numbers. Most children acquire numeracy skills with minimal direct instruction (Gersten & Chard, 1999). When teaching foundational skills, like numeracy, to older students with ESN, it is important to embed instruction in the context of grade-aligned, standards-based lessons (Saunders et al., 2019). Embedded instruction refers to the purposeful distribution of instructional trials across activities and naturally occurring routines (Jameson et al., 2020; Ruppar et al., 2017). SP is a systematic instructional strategy that can be used with embedded instruction to teach academic skills to students with ESN (Bowman et al., 2020; Collins, 2022; Karl et al., 2013; Rivera et al., 2017).

Summary of the Review of the Literature

In this chapter, I presented a brief history of general curriculum access, including current definitions of the concept. Next, I reviewed standards-based algebra instruction and proposed using contextualized MSBI to teach this content to students with ESN. Finally, I discussed the importance of foundational numeracy skills and how to embed instruction using SP within standards-based lessons for high school students with ESN.

In summary, students with ESN have a right to high-quality standards-based instruction (Courtade et al., 2012). Access to the general curriculum is an important predictor of post-school success (Gilley et al., 2021; Shogren et al., 2018). Additionally, access to the general curriculum is required by law for all students (ESSA, 2015; IDEA, 2004). Students with ESN, however, are not always presented with opportunities to engage in all areas of the general curriculum (Cox & Jimenez, 2020). Algebra, for example, is one such content area.

Algebra is an important curricular area necessary for success in future mathematics coursework, postsecondary education, and the workplace (Fuchs et al., 2014; NCTM, 2000). Algebraic content should be taught in context of real-world activities to ensure meaningful access to the general curriculum (Ahlgrim-Delzell et al., 2009; Bowman et al., 2019). MSBI is a strategy to teach algebra content that incorporates contextualized instruction, systematic instruction, and explicit instruction (Spooner et al., 2017). One limitation of previous studies investigating MSBI is that all participants demonstrated prerequisite numeracy skills (Clausen et al., 2021).

For older students with ESN, foundational numeracy skills instruction should be embedded within standards-based algebra instruction (Bowman et al., 2020; Hunt et al., 2012). SP is one instructional strategy to teach numeracy skills, such as numeral identification (Akmanoglu & Batu, 2004). No research to date has been conducted investigating embedding numeracy instruction in standards-based algebra instruction for high school students with ESN.

Numeracy is necessary for mathematical understanding, yet approximately 12–17% of students with ESN do not demonstrate numeracy skills (Kearns et al., 2011; NCTM, 2000). It is not appropriate to limit a student’s access to the general curriculum contingent on their mastery of numeracy skills (Hunt et al., 2012). Therefore, there is a need to combine foundational and standards-based instruction to promote meaningful access to the general curriculum, which will result in positive post-school outcomes for students with ESN.

CHAPTER 3: METHOD

The purpose of this investigation was to analyze the effects of an intervention package comprising MSBI and embedded SP for secondary students with ESN using an experimental, single-case multiple probe across number sets design (Horner & Baer, 1978; Ledford & Gast, 2018). Specifically, the outcomes addressed were numeral identification (i.e., receptive identification of numerals 1–9) and algebraic problem solving, given a word problem requiring a simple linear equation (i.e., $a + x = c$).

The research questions were:

1. To what extent does an intervention package comprising MSBI and embedded SP improve foundational numeral identification skills for secondary students with ESN?
2. To what extent does an intervention package comprising MSBI and embedded SP improve the independent algebraic problem-solving skills of students with ESN?
3. To what extent can secondary students with ESN generalize numeral identification skills to real-world settings and situations?
4. To what extent is addressing numeracy skills within grade-level mathematics content considered an acceptable and effective practice by classroom teachers of students with ESN?

Participants

I recruited two high school students with ESN and their teachers or paraeducators to participate in this study using convenience sampling. I selected the student participants based on the following inclusion criteria: (a) received special education services under the IDEA exceptionality categories of autism, ID, or multiple disabilities; (b) participated in AA-AAS; (c) enrolled in a high school algebra course or equivalent (i.e., Math I); (d) reported inability to identify numerals 1–10; (e) was able to physically manipulate materials (e.g., grasp, point, slide); and (f) attended school in-person, face-to-face. Inclusion criteria for the instructor participants were as follows: (a) teacher of record OR paraeducator assigned to work in the classroom with the student participant; (b) willing to participate in BST outside of school hours, which may take up to two hours; (c) willing to implement intervention daily, using a scripted lesson plan, barring student absence or personnel shortages; and (d) willing to record sessions and upload to a cloud-based service daily. Prior knowledge of or training in SP, embedded instruction, or MSBI was not required as an inclusion or exclusion criterion.

I provided consent forms to legal guardians of the potential participants prior to the beginning of the study. See Appendix B for the parental consent form. Following receipt of the signed consent form, the participating instructors asked the student to provide assent. See Appendix C for the student assent form. As the student participants communicated primarily through non-vocal means, I asked that the instructors consider the presence of challenging behaviors (e.g., aggression, elopement, refusal to transition) as declining assent. Additionally, I asked the participating instructors to provide informed consent. See Appendix D for the instructor consent form.

Screening Procedures

After the student and their guardians provided assent and consent, I asked the instructors to conduct the *Early Numeracy Assessment* (Jimenez et al., 2013). The *Early Numeracy Assessment* was developed by Jimenez et al. as a part of the *Early Numeracy Curriculum*. The *Early Numeracy Curriculum* was developed as a part of the Project MASTERY grant (Browder & Spooner, 2008–2011). The assessment can be used to assess students' skills in counting, one-to-one correspondence, numeral identification, and creating and adding sets. As this study focuses on numeral identification, I only required that the instructors complete the relevant portions of the assessment using a data sheet provided in Appendix E (i.e., the sections on numeral identification). The eligibility criteria for continued participation in this study was a score of 20% or less on the *Early Numeracy Assessment*.

Student Participant Information Gathering Procedures

In addition to traditional demographic information, such as student gender, race, and disability, I asked each student's instructor and their parent or guardian to complete the *Supports Intensity Scale—Children's Version*™ (*SIS-C*™; Thompson et al., 2016). The *SIS-C*™ is a norm-referenced assessment tool that can be used to describe children based on their support needs rather than their ability level or IQ. The *SIS-C*™ is normed against other children with ID, providing a relative understanding of the level of supports needed. Cronbach's alpha ranges from 0.927 to 0.948 for the subscales of the assessment, indicating strong internal consistency. Test-retest reliability scores range from .855 to .936 across subscales, indicating excellent test-retest reliability; Pearson r correlations for all three measures on all seven subscales ranged from .762 to .947, suggesting high consistency across repetitions. In addition, there is demonstrated validity across content validity, criterion validity, and construct validity (Thompson et al., 2016).

Information requested on the *SIS-C*™ includes (a) family's primary language spoken at home; (b) student age; (c) gender; (d) race and ethnicity; (e) diagnoses/exceptionality; (f) IQ and adaptive behavior scores; (g) medical needs; (h) behavioral needs; and (i) support needs across home life activities, community and neighborhood activities, school participation activities, school learning activities, health and safety activities, social activities, and advocacy activities. For each support need area, the respondents are asked to rate the type of support required (e.g., monitoring, verbal prompting, physical assistance), the frequency in which support is required (e.g., infrequently, very frequently, always), and the daily support time required (e.g., 30 min; 2 to 4 hrs; 4 hrs or more). The mean rating (on a scale of 0 to 4) is translated to a standard score (scale 0 to 16) and a corresponding percentile rank based on the student's age.

In addition to the *SIS—C*, I asked the instructors to describe each student's current IEP goals in the areas of mathematics and the degree to which they participate in grade-aligned mathematics instruction. This information was useful in determining the students' history of access to the general curriculum. Finally, I asked the instructors to complete an informal preference assessment in which they provided information about the students' interests (e.g., favorite foods, movies) and names of preferred people in each student's life (e.g., classmates, family members). I used this information to design personally relevant mathematical word problems for each student. See Appendix F for the preference assessment.

Dyad 1

Dyad 1 included student participant Shaquille and instructor participant Rachel. Pseudonyms have been assigned to each participant to ensure confidentiality. Shaquille was 15 years old and in the 9th grade. According to his school records, he was identified as male and Black and received special education services under the eligibility category of autism. During the 2021–2022 school year, Shaquille was working toward two mathematics goals on his IEP, both addressing functional mathematics skills (i.e., money). The first goal was "When purchasing two items of his choice at the store or mock store at school, Shaquille will be able to exchange the items for the correct dollar up amount independently in four out of five trials for seven consecutive trials." The second goal was "When presented with real or fake money, Shaquille will identify the one-, five-, and ten-dollar bill in a field of three by pointing to the correct bill in four out of five trials for seven consecutive trials." His classroom teacher reported that his instruction was based on his IEP goals. On his most recent high-stakes assessment (the NCEXTEND 1), he received a score of “not proficient” on the mathematics portion. Shaquille scored 7.1% accuracy on the selected portions of the *Early Numeracy Assessment* administered at the beginning of this study.

Shaquille lived at home with his mother, father, older sister, and twin sister. I interviewed his instructor, Rachel, and his mother using the *SIS—C* to determine the intensity of Shaquille’s support needs across both home and school settings (see Figure 4). Shaquille’s support needs were most intense in school learning activities (e.g., learning academic skills, using educational technologies, accessing grade-level content). In this area, he had a mean rating of 3.78 (on a scale of 0 to 4), standard score 13 (on a scale of 0 to 16), and percentile rank of 84.1, suggesting his support needs in this area are more intense than 84.1% of other individuals with intellectual and developmental disabilities, ages 15–16. Next in terms of intensity of supports were community and neighborhood and home life activities. For support needs in community and neighborhood activities (e.g., moving around the community, shopping, complying with laws), Shaquille had a mean rating of 2.63, a standard score of 10, and a percentile rank of 50. For support needs in home life activities (e.g., completing household chores, dressing, keeping self-occupied), he had a mean rating of 2.07, a standard score of 10, and a percentile rank of 50. Following these areas in terms of intensity of support required were the areas of health and safety and school participation. For support needs in health and safety activities (e.g., maintaining physical health, responding in emergency situations, protecting self from abuse), Shaquille’s mean rating was 2.50, standard score was 9, and a percentile rank was 36.9. For support needs in school participation activities (e.g., inclusion in general education classroom, keeping track of schedule, participating in assessments), his mean rating was 2.37, standard score was 9, and percentile rank was 36.9. Finally, the areas in which the respondents described Shaquille as needing the least intensive supports were social and advocacy activities. For support needs in social activities (e.g., maintaining positive relationships, communicating with others in social situations, coping with changes), Shaquille's mean rating was 1.59, standard score was 7, and percentile rank was 15.9. For support needs in advocacy activities (e.g., making choices, setting goals, using problem-solving strategies), his mean rating was 1.70, standard score was 6, and percentile rank was 9.1. Shaquille does not have any exceptional medical or behavioral needs. Shaquille's overall support needs index score, which averaged his ratings across areas, was a mean rating of 2.38, standard score of 91 (range 51–125), and percentile rank of 27.4, suggesting his overall support needs are more intense than 27.4% of other individuals with intellectual and developmental disabilities, ages 15–16.

Figure 4

SIS—C Support Needs Profile: Shaquille

Home Life	**Commu-nity & Neighbo-rhood**	**School Particip-ation**	**School Learning**	**Health & Safety**	**Social**	**Advoca-cy**	***SIS-C* Support Needs Index**
16	16	16	16	16	16	16	≥124
15	15	15	15	15	15	15	120–123
14	14	14	14	14	14	14	116–119
13	13	13	13	13	13	13	112–115
12	12	12	12	12	12	12	108–111
11	11	11	11	11	11	11	104–107
10	10	10	10	10	10	10	100–103
9	9	9	9	9	9	9	96–99
8	8	8	8	8	8	8	92–95
7	7	7	7	7	7	7	88–91
6	6	6	6	6	6	6	84–87
5	5	5	5	5	5	5	80–83
4	4	4	4	4	4	4	76–79
3	3	3	3	3	3	3	72–75
2	2	2	2	2	2	2	68–71
0–1	0–1	0–1	0–1	0–1	0–1	0–1	≤67

Note. Adapted from Thompson et al. (2015). Shaded cells represent the standard score for Shaquille in the area identified. The mean standard score for individuals in the 15–16 age cohort was 10/100.

Shaquille's instructor, Rachel was a special education paraeducator in Shaquille's class. She identified as a White female. This year was her first year as a paraeducator. She had been working with Shaquille since October (i.e., approximately three months at the start of the study). Rachel earned a bachelor's degree in history and was working towards certification as a Registered Behavior Technician® at the time of this study.

Dyad 2

Dyad 2 included student participant Jackson and instructor participant Cindy. Jackson was 15 years old and in the 9th grade. He was identified as male and African American/White according to school records and received special education services under the eligibility category of Intellectual Disability- Severe. He also had a diagnosis of autism. During the 2021–2022 school year, Jackson was working toward a foundational mathematics goal on his IEP: "In 36 weeks, given a numeral 1–10 and objects or picture of objects, Jackson will correctly match the numeral to its corresponding quantity with 80% accuracy in 4 out of 5 recording days". He had been working on IEP goals related to numeral identification or recognition of quantities for the past seven years. Cindy reported that she tried to align her instruction to the North Carolina Extended Standards, but most of their instructional time is spent on IEP goals. Additionally, they used resources from the Unique Learning platform, a commonly used alternate curriculum for students with ESN, for instruction on addition, subtraction, money, and time. Jackson scored a 14.2% on the selected portions of the *Early Numeracy Assessment* administered at the beginning of this study.

Jackson resided in a group home setting, along with four other teenage males with ESN, at which he received 24/7 monitoring. I interviewed both his teacher, Cindy, and the group home manager using the *SIS—C* to determine the intensity of Jackson's support needs across both settings (see Figure 5). Jackson's support needs were most intense in the areas of health and safety and advocacy activities. For support needs in health and safety activities (e.g., maintaining physical health, responding in emergency situations, protecting self from abuse), Jackson had a mean rating was 3.67 (on a scale of 0 to 4), standard score 13 (on a scale of 0 to 16), and percentile rank 84.1, suggesting his support needs in this area are more intense than 84.1% of other individuals with intellectual and developmental disabilities ages 15–16. For support needs in advocacy activities (e.g., making choices, setting goals, using problem-solving strategies), his mean rating was 3.48, standard score was 13, and percentile rank was 84.1. The areas in which his teacher and caregiver rated him as needing slightly less intense supports were home life and school learning activities. For support needs in home life activities (e.g., completing household chores, dressing, keeping self-occupied), Jackson had a mean rating of 2.52, a standard score of 12, and a percentile rank of 74.8. For support needs in school learning activities (e.g., learning academic skills, using educational technologies, accessing grade-level content), his mean rating was 3.70, standard score was 12, and percentile rank was 74.8. Finally, the areas in which both his teacher and caregiver rated Jackson as needing the least intense supports were community and neighborhood, school participation, and social activities. For support needs in community and neighborhood activities (e.g., moving around the community, shopping, complying with laws), his mean rating was 2.54, standard score was 10, and percentile rank was 50.0. For support needs in school participation activities (e.g., inclusion in general education classroom, keeping track of schedule, participating in assessments), his mean rating was 2.63, standard score was 10, and percentile rank was 50.0. For support needs in social activities (e.g., maintaining positive relationships, communicating with others in social situations, coping with changes), his mean rating was 2.70, his standard score was 10, and percentile rank was 50.0. Jackson has no exceptional medical needs but is considered to have exceptional behavioral needs. Finally, Jackson's overall support needs index score, which averages his ratings across areas, was a mean rating of 3.03, standard score 107 (range 51–125), and percentile rank 68.0, suggesting his overall support needs are more intense than 68.0% of other individuals with intellectual and developmental disabilities, ages 15–16.

Figure 5

SIS-C Support Needs Profile: Jackson

Home Life	Commu-nity & Neighbo-rhood	School Particip-ation	School Learning	Health & Safety	Social	Advoca-cy	*SIS-C* Support Needs Index
16	16	16	16	16	16	16	≥124
15	15	15	15	15	15	15	120–123
14	14	14	14	14	14	14	116–119
13	13	13	13	13	13	13	112–115
12	12	12	12	12	12	12	108–111
11	11	11	11	11	11	11	104–107
10	10	10	10	10	10	10	100–103
9	9	9	9	9	9	9	96–99
8	8	8	8	8	8	8	92–95
7	7	7	7	7	7	7	88–91
6	6	6	6	6	6	6	84–87
5	5	5	5	5	5	5	80–83
4	4	4	4	4	4	4	76–79
3	3	3	3	3	3	3	72–75
2	2	2	2	2	2	2	68–71
0–1	0–1	0–1	0–1	0–1	0–1	0–1	≤67

Note. Adapted from Thompson et al. (2015). Shaded cells represent the standard score for Jackson in the area identified. The mean standard score for individuals in the 15–16 age cohort was 10/100.

Jackson's instructor, Cindy was a special education teacher. She identified as a White female. This was her first year as a teacher, but she had worked with Jackson in the previous year as a paraeducator. She served as a paraeducator for ten years prior to becoming a teacher. Cindy held a bachelor's degree in education and had licensure in the area of Special Education: General Curriculum. She was not licensed to teach students with ESN at the time of the study, but was working toward the required licensure of Special Education: Adapted Curriculum.

Setting

This study took place in two self-contained special education classrooms in two local, rural public-school districts. Shaquille attended a K–21 public separate school for students with ESN. Twenty-nine students were enrolled in the program, with four certified special educators and 20 paraeducators. The majority of students were male (*n* = 26) and in high school (*n* = 18). Twenty-one students received free or reduced lunch. Shaquille's class included six other high school students with ESN, one certified special educator, and two paraeducators, including Rachel, the instructor participant in this study. Rachel conducted the sessions in a small room used for 1-1 instruction at a table with two chairs. The sessions were conducted at a time convenient to the school, typically after recess in the afternoon. Generalization sessions took place in the school's life-skills kitchen, with a microwave hung at Shaquille's eye level.

Jackson attended a 9–12 public high school in a rural school district. During the 2020–2021 school year, 323 students were enrolled, 57.7% of whom were considered economically disadvantaged (North Carolina Department of Public Instruction, 2021). The school had an 89.5% graduation rate. Fewer than 5% of students met grade-level proficiency or career and college readiness standards in Math I. Jackson's class included three other high school students with ESN, one certified special educator (Cindy, the participating instructor), and one paraeducator. Cindy conducted the sessions at a table in the center of the classroom while other students engaged in instructional or leisure activities. The sessions were conducted at a time convenient to the teacher, typically during the afternoon. Toward the end of the school year, Cindy began conducting two sessions daily, once in the morning and once in the afternoon to allow Jackson maximum access to the intervention.

As the experimenter, I conducted BST at a location of each of the instructor's choosing. Rachel preferred in-person training. I provided BST after school hours in the same classroom where the sessions were conducted. Cindy preferred virtual training, so I provided BST sessions via Zoom, either before or after school.

Investigator

I, Amy Clausen, served as the primary investigator for this study. At this time of this study, I was a doctoral candidate certified in special education for students with ESN in Grades K–12. I taught students with ESN in Grades 7–8 for two years and served as a program coordinator and instructional coach for three years before entering the doctoral program. Additionally, I had a master's degree in behavior analysis. As the primary investigator, I recruited participants, conducted BST, coordinated data collection across all phases, and served as primary observer when collecting procedural fidelity data. Additionally, I developed the lesson plans and instructional materials required for the study. A doctoral student in special education served as the secondary experimenter, calculating interobserver agreement (IOA) across dependent variables and collecting procedural fidelity data.

Materials and Equipment

Standards-Based Lessons

The instructors followed scripted lesson plans throughout the study (Appendix G). I developed the lesson plans and asked experts (both in mathematics and special education) to review them for content validity, and made changes based on their feedback. Additionally, I provided all necessary manipulatives, including number lines, word problem-solving mats, color-coded game pawns, and the talking task analysis. The lesson plans were aligned to the North Carolina Math I algebra standard: *Solve linear equations and inequalities in one variable*

(NC.M1:A-REI.3). The lesson plans included procedures for the numeracy probe, the generalization probe, and the MSBI lessons, including model, lead, and test procedures. The same lesson plan set was used across multiple sets of word problems. The repetition reduced the burden on the instructor to learn new lesson plans and increased consistency and predictability for the student participants.

Talking Photo Album

To promote student independence, as the participants in this study were not yet readers, I placed the steps of the task analysis for solving the simple linear equation in a Talking Photo Album (see https://www.attainmentcompany.com/talking-photo-album; see Appendix H). The album had room for 20 separate inserts with audio recordings 10 s in duration. A textual version of the task analysis is presented in Table 4. I selected this product due to the relative cost effectiveness ($69.00).

Table 4

Task Analysis for Solving Algebraic Word Problems

Step	Expected Student Response
1. Read the problem.	Student activates the GoTalkButton to read aloud the problem.
2. Circle the terms.	Student circles the labels of the relevant terms in the problem (e.g., shirt, pants, clothes).
3. Same, Different, More/Fewer?	Student selects the icon that best describes the terms (e.g., *shirts* and *pants* are different).
4. Choose the problem mat.	Student selects the appropriate graphic organizer which aligns with the problem type.
5. Use my rule.	Student chants and/or uses the hand motions for the appropriate problem type (e.g., “small group, small group, COMBINE into big group!”).
6. Put the terms on the mat.	Student moves the Velcro labels from step 2 to the appropriate line on the graphic organizer.
7. Circle the numbers.	Student circles the numbers found in the word problem.
8. Put the numbers on the mat.	Student places numeral cards representing the numbers from step 7 on the problem mat. Student also uses the variable x to represent the unknown term.
9. Plus or minus?	Student determines if the problem requires addition or subtraction and places the appropriate operation icon on the graphic organizer.
10. Put what you know on the number line.	Student places green chips to represent the first known addend in the problem. Student places a blue barrier over the final term of the problem.
11. Count to find what’s missing.	Student adds red chips to the number line to count up from the last green chip to the blue barrier.
12. Solve to find x.	Student moves red chips (representing the unknown addend) from the first number line to a second, so they may count to determine the quantity of x.
13. Write the answer.	Student places a numeral card corresponding to the number of red chips on the equation x =_.

Numeracy Probe Cards

I developed a set of numeracy probe cards with numerals 1 through 9 printed on index cards, sized 3 in by 5 in. To promote generalization, I printed the numerals in various fonts and colors (see Appendix I).

Number Line

I provided each student with two number lines with movable manipulatives to support 1-1 correspondence and counting (see Appendix J). The student participants in this study were not required to demonstrate numeracy skills such as 1-1 correspondence or counting to be eligible for participation, so the number lines were created to support their conceptual understanding of number when solving the linear equations. Additionally, I provided color-coded game pieces with Velcro dots able to be affixed to the number line to promote 1-1 correspondence.

Word Problems

I created three sets of word problems for each student participant based on the results of their preference assessment to ensure instruction is relevant to their personal experiences. In SBI, students are taught to differentiate between problem types (e.g., combine, compare, change) when selecting the correct procedures to solve the problem. To reduce cognitive load, I presented students with only one schema throughout the intervention (i.e., combine). The three sets corresponded to the groups of numerals presented at each stage of the intervention. The first set of problems included numerals 3, 4, and 9 (i.e., $3 + x = 9$; $3 + x = 4$; $4 + x = 9$), the second set included numerals 1, 5, and 8, and the third set included numerals 2, 6, and 7. Unfortunately, there was no mathematical possibility to include a unique set of numerals in each word problem, but this organization ensured that in each set of word problems, the numerals were presented an equal number of times.

As per the recommendations of Browder et al. (Browder et al., 2013–2017), I used the same format in each word problem. For example:

Willow bought 3 shirts at the mall. (Known initial “small group”)

She also bought some pants. (Missing medial small group)

She bought 9 clothes all together. (Known “big group”)

How many pants did she buy? (Problem statement)

The student participants were not required to demonstrate reading or writing skills prior to the study. Therefore, I supported their comprehension and independence by including movable icons for the relevant terms of the problem (e.g., *shirt*, *pants*, and *clothes* in the problem above). The students were able to transfer these icons to the graphic organizer word problem mats. Sample word problems are presented in Appendix K.

GoTalkButton

I provided a GoTalkButton (see https://www.attainmentcompany.com/gotalk-button) to each student participant. The GoTalkButton allows the user to record a 20 s message, which can be replayed by pushing the button. The instructors recorded themselves reading the word problem each day before beginning instruction. The button allowed the students to complete step 1 of the task analysis (read the problem) by activating the audio button, increasing student independence. Similar to the Talking Photo Album, I chose this assistive technology support for its relative cost effectiveness ($19.00).

Graphic Organizer

In MSBI, schematic diagrams are provided to the student through graphic organizers. I created graphic organizers using the schematic designs presented in Powell and Fuchs (2018; see Appendix L). Although each word problem presented required the combine schema, the students had access to three additive schema graphic organizers (i.e., combine, compare, change) to encourage differentiation in future instruction. As student participants were not required to demonstrate reading or writing skills, I also provided icons and number cards affixed with Velcro along with the graphic organizers.

Video Camera

The instructor videotaped each session so we could measure procedural fidelity and calculate IOA. The video camera was angled so that the participating student, instructor, and materials were visible, but no other students in the classroom were visible. The instructors uploaded the videos to a shared drive (i.e., Dropbox) daily, so timely coaching could occur if fidelity dropped below 80%.

Real World Numeracy Stimuli

Each instructor selected stimuli that could be used to assess the student's generalization of numeral identification to their natural environment. Shaquille identified numerals using a microwave, as he was also working towards a goal of making his food. Jackson identified numerals using a calculator and an analog clock that he used during functional mathematics instruction.

Dependent Variables and Measurement

There are four dependent variables of interest in this study. The primary dependent variable is the percent correct of numeral identification for numerals 1–9. The secondary dependent variable is the percent correct of steps on a task analysis to solve a simple linear equation. The third dependent variable is the percent correct of numeral identification for numerals 1–9 using natural stimuli (i.e., the generalization measure). The final dependent variable is a measure of social validity using a five-point Likert-type scale completed by the participating instructors.

Numeral Identification

The primary dependent variable was the percent correct of receptive identification for numerals 1–9. During the full probe sessions, the instructor assessed the student's identification of all nine numerals by presenting them randomly in an array of five (see data sheet in Appendix M). The student identified each numeral by pointing to or handing the card to their instructor. During the daily probes, the instructor assessed only three numerals (3, 4, 9 in Numeral Set A; 1, 5, 8 in Numeral Set B; and 2, 6, 7 in Numeral Set C) but presented each in an array of five. An array of five was selected as it reduced the probability that the student would randomly select the correct card from 33% (in a field of three) to 25%.

Solving Algebraic Word Problems

The secondary dependent variable was the percentage of steps correct on a task analysis when given a simple linear equation. A simple linear equation is a number sentence in which a constant (e.g., 3) is added to a variable to equal a known sum (e.g., $3 + x = 9$). Instructors used explicit instruction (i.e., model, lead, test) to teach students to solve a simple linear equation. Data were collected during the test sessions, described further in the procedures section. See the data sheet in Appendix N.

Generalization to Real-World Stimuli

The third dependent variable was the percent correct of numeral identification for numerals 1–9 using natural stimuli (i.e., the generalization measure). The instructor measured generalization on the same days they conducted the test sessions for the simple linear equation. During each generalization session, the instructor probed all nine numerals. For Shaquille, the instructor asked him to identify numerals on a microwave keypad as he was working towards a transition goal of preparing his food. For Jackson, the instructor asked him to identify numerals on both a calculator and an analog clock.

Social Validity- Instructor

The final dependent variable was the instructors' rating on a social validity measure. I administered a social validity questionnaire via Google Forms before BST and on the last day of the study to determine if their perceptions of the intervention and its outcomes changed. The questionnaire included six questions—two of which addressed social significance of the goals, two which addressed social acceptability of the intervention, and two which addressed social importance of the outcomes (Wolf, 1978). The instructors responded to each question on a Likert-type scale of 1 (strongly disagree) to 5 (strongly agree). See the questionnaire in Appendix O.

Experimental Design

I used a multiple probe design (conditions) across number sets (Horner & Baer, 1978; Ledford & Gast, 2018) to evaluate the effectiveness of the embedded SP on the students' numeracy skills (DV #1). Originally, I planned to use a multiple probe across participants design; however, due to difficulties recruiting participants during this era of teacher shortages and burnout, I switched the design to multiple probe across number sets prior to baseline. I selected a multiple probe design over a multiple baseline design as a probe design requires fewer assessments, resulting in reduced potential for frustration when repeatedly assessing a student who has not yet received instruction. I set mastery criterion for moving tiers at 100% accuracy over three consecutive sessions for the DV #1. During intervention, it appeared the embedded SP was ineffective. I added a criterion to introduce a modification after data remained at or below baseline levels for at least five consecutive sessions.

I used an A-B design (Birnbrauer et al., 1974; Ledford & Gast, 2018) to evaluate the effectiveness of the MSBI on the student's completion of the linear equation task analysis (DV #2). I graphed the generalization measure within the multiple probe graph using a unique data marker (i.e., an open triangle; DV #3). I present the results of the social validity questionnaire in a table format (DV #4).

Procedures

Pre-Baseline Training

Prior to baseline, I used BST (Miltenberger, 2016; Parsons & Reid, 1995) to prepare instructors to implement the study procedures. BST is an evidence-based training package comprising instruction, modeling, rehearsal, and feedback (DiGennaro Reed et al., 2018). This training took approximately 1 hr. Based on each instructor's preferences, I conducted the BST with Rachel in person and with Cindy via a video conferencing platform (i.e., Zoom).

At the start of the training session, I asked the instructors to complete the pre-intervention social validity questionnaire (see Appendix O). I used BST to prepare the instructors to implement the baseline components of the study. I briefly described how to conduct the numeracy probe using SP and the test session for the linear equation. The BST protocol for baseline is included in Appendix P. Then, I modeled the baseline components of the study, both for the numeracy probe and the linear equation assessment. I also created a video model of each component so the instructors could refer to it as needed. Then, I rehearsed the procedures with the instructors, acting as the student. I used the procedural fidelity worksheets (Appendix Q) to guide this rehearsal. The rehearsal stage continued until the instructor reached 100% fidelity. I provided supportive and corrective feedback throughout rehearsal to support the instructors' learning.

Baseline

During baseline, the instructors conducted full numeracy probes, assessing numerals 1–9 in random order, presented in an array of five. The instructor conducted three sessions during each full probe phase using SP probe procedures. Using the numeracy probe cards (see Appendix I), the instructor presented the student five cards in a random array. The instructor then asked the student to identify a numeral between 1 and 9, using their preferred communicative mode. Shaquille selected the card by pointing with one finger and repeating the number word aloud. Jackson selected the card by picking up the card and placing it in a basket. When the student answered accurately, the instructor marked a + on the data sheet but did not provide any reinforcement or acknowledgment of the correct response. The data sheet is provided in Appendix M. When the student was incorrect or did not make a response in the predetermined response interval (i.e., 3 s for Shaquille, 5 s for Jackson), the instructor marked a – on the data sheet. The instructor did not provide any error correction or acknowledgment of the incorrect response during the full probe. Upon completing the probe, the instructor reinforced the student's attention to the task by praising and providing a known social reinforcer, such as a high-five.

The instructors also measured participating students' percentage correct of steps completed on a 13-step task analysis for a simple linear equation. The instructors presented the materials (e.g., talking photo album, graphic organizer word problem mats, manipulatives, number lines, word problems) and delivered the instructional cue ("Solve a simple linear equation on your own"). If the student completed the step correctly, the instructor marked a + on the data sheet (see Appendix N). If the student completed the step incorrectly or did not make a response in the predetermined response interval, the instructor marked a – on the data sheet and then completed the step for the student. A multiple opportunity method was used, meaning that the students had an opportunity to perform each step of the task analysis independently, regardless of their performance on the preceding steps (Cooper et al., 2020). No reinforcement or error correction was provided during baseline. After completing the 13-step task analysis, the instructor reinforced the student’s attention to task by praising and providing a known social reinforcer, such as a high-five.

Pre-Intervention Training

Prior to intervention, I used BST to prepare instructors to implement the intervention procedures. The training took approximately 1 hr and took place in person for Rachel and via Zoom for Cindy, based on their personal preferences. See Appendix R for the BST intervention protocol.

First, I described the intervention process, reviewing the procedures for MSBI, including explicit instruction, the system of least prompts, and embedded SP. I provided a rationale for the study and a brief review of the research for each instructional procedure. Next, I demonstrated the scripted lesson plan for model, with the instructor acting as the student participant. I also created a video model of the lesson plan procedures for the instructors to reference throughout the study. The instructor then rehearsed the model phase with me acting as a student. I used the procedural fidelity worksheets (see Appendix Q) to guide the rehearsal. We then repeated these procedures for the lead phase of the lesson plan. The rehearsal continued until the teacher reached 100% fidelity. I provided supportive and corrective feedback throughout rehearsal to support the instructors’ learning. If at any point during the intervention the instructor’s fidelity dropped below 80%, we met for a follow-up coaching session in which I provided a model and we rehearsed until the instructor reached 100% fidelity.

Intervention

I provided the participating instructors with the scripted lesson plan used across various combine schema algebraic word problems (see Appendix G). These lessons were intended to be delivered daily, taking approximately 15–20 min. Each session, the instructor conducted the daily SP numeracy probe, assessing three numerals (3, 4, 9 in Numeral Set A; 1, 5, 8 in Numeral

Set B; and 2, 6, 7 in Numeral Set C). During this probe, the instructor did not reinforce correct responses, nor correct errors. Immediately after conducting the numeracy probe, the instructor presented the algebra lesson—following either the model, lead, or test lesson plan, depending on the schedule of instruction.

MSBI combines explicit instruction (i.e., model-lead-test) and systematic instruction (i.e., system of least prompts). During the model phase, the instructor provided verbal instructions (e.g., "Terms are the pieces of the algebra problem. Help me circle our terms.") and modeled or pointed to the correct response. The student was expected to mimic the response (e.g., moving the icon to the correct spot on the problem mat). Embedded within the model lesson were 20 opportunities for the instructor to present the targeted numerals to the student (i.e., the SP teach trials). For example, after the student read the problem by activating the GoTalkButton (step 1), the instructor prompted the student to identify the known addend in the word problem (e.g., numeral 3) and then the known sum in the word problem (i.e., numeral 9) using SP teach procedures.

During the lead sessions, the instructor prompted the student to solve a simple linear equation using a modified system of least prompts. Following the instructional cue or the completion of the previous step, the student was given a predetermined response interval (e.g., 3 s) to complete the step of the task analysis. If the student responded correctly, the instructor praised the student. If the student did not respond or responded incorrectly, the instructor provided a nonspecific verbal prompt (e.g., "What's next?") while gesturally prompting the student to activate the talk-aloud button on the photo album. If the student completed the step correctly, the instructor provided a moderate level of praise and waited for the student to complete the next step. If the student did not respond or responded incorrectly, the instructor provided a specific verbal prompt (i.e., telling the student what to do). If the student completed the step correctly, the instructor provided a low level of praise and waited for the student to complete the next step. If the student still did not respond or responded incorrectly, the instructor provided a model prompt showing the student what to do. The model prompt was expected to serve as a controlling prompt (the least intrusive prompt expected to elicit the behavior; Collins, 2022). For Jackson, a physical prompt was occasionally necessary to elicit the correct response. Similar to the model session, there were 20 embedded opportunities to present instruction on numerals relevant to the problem using embedded SP during the lead sessions.

Finally, during the test sessions, the instructor followed the same procedures as in baseline. The instructor presented the materials and provided the instructional cue ("Solve the linear equation by yourself"). The test was presented as a multiple-opportunity probe. If the student completed a step correctly, the instructor marked a + on the data sheet but did not reinforce the response. If the student answered incorrectly or did not respond in a predetermined response interval (e.g., 3 s), the instructor completed the step for the student. There were no embedded opportunities for numeral identification during the test sessions.

Ideally, the instructor would present each phase of the instruction in each session. However, like many students in self-contained settings, the students in this study were typically only expected to work for a short period (e.g., 15 min) before engaging in preferred tasks. Therefore, I required the instructor to only present one phase of the model-lead-test procedure in each session. Originally, I planned to ask the instructor to present the model lesson on Mondays, the lead lesson on Tuesdays through Thursdays, and the test session on Fridays. During BST, however, both Rachel and Cindy expressed the belief that their students would be more successful with multiple presentations of the model lesson prior to transitioning to the lead lesson. To accommodate their requests, I adjusted the schedule of instruction so that they presented the model phase until both the instructor and I, as the researcher, agreed that the student was beginning to *anticipate* the next step, defined as reaching toward the materials prior to the instructor's prompt. Then, the instructor transitioned to the lead phase and conducted a test session every six sessions.

Modifications to Intervention Procedures. Originally, the instructors prompted their students to identify the numeral in context of the materials. For example, during the embedded SP procedures, the instructor would point to the numeral on the number line or in the word problem, prompting the student to identify the numeral. Neither Shaquille nor Jackson appeared to transfer this identification to the numeracy probe conducted at the start of each session. I hypothesized that the students' accuracy might improve if the stimuli presented during the embedded SP trials were more salient. Thus, I modified the procedures so the instructor would present the same numeral card used during the numeracy probe during the instructional trials. Rather than having the student point to the numeral in context of the word problem text, the instructor would present the numeral card to the student and prompt him to identify it.

Unfortunately, the intervention continued to be ineffective. I did not have adequate time to introduce another modification to Jackson's instruction; but I was able to introduce a second modification to Shaquille's instruction. I selected SP as the prompting procedure as the literature suggests it can be simpler for instructors to implement (Schuster et al., 1992). Instructors are not required to increase wait intervals in SP nor must they differentially reinforce responses as in CTD (Schuster et al., 1992). However, Shaquille tended to select cards in the same position during each trial. I hypothesized that this behavior was reinforced by the removal of the trial, regardless of his selection. I changed the response prompting procedure from SP to CTD. See revised lesson plans in Appendix S. During the embedded trials, the instructor now presented five cards in a random array and provided the instructional cue, “Touch (*numeral*).”. The instructor waited a predetermined response interval (e.g., 3 s). If the student answered correctly, the instructor praised the response and marked a + on the data sheet. If the student answered incorrectly or did not respond in the predetermined interval, the instructor gesturally prompted the student to select the correct numeral card and marked a – on the revised data sheet (see Appendix T). This data collection replaced the daily numeracy probe procedures. Although the number of embedded instructional trials remained the same in this modification (i.e., 20 trials), the procedures for data collection changed significantly. Rather than presenting each of the three numerals assessed in the tier once, the instructor presented two of the three numerals 10 times each. For example, on Monday, she would present a word problem requiring the linear equation (i.e., $3 + x = 9$). Within the lesson, there would be 10 opportunities to conduct at CTD trial with numerals 3 and 9. Then on Tuesday, should would present numerals 4 and 9 (i.e., $4 + x = 9$), and on Wednesday, numerals 3 and 4 (i.e., $3 + x = 4$). Therefore, the denominator for calculating percentage correct changed from three (during SP probes) to 20 (for embedded CTD trials).

Generalization of Number Identification Skills

Throughout the study procedures, I promoted generalization by training sufficient exemplars and programming common stimuli (Stokes & Baer, 1977). I accomplished this by creating five sets of numeral cards. I used a commonly available sans serif font (i.e., Arial) to create the first set of numeral cards. I used a font typically seen on digital clocks for the second set of numeral cards. I took images of the numerals on a calculator typically used in high school settings (i.e., TI-83) for the third set. In the fourth set, I took a screenshot of the numerals used on the phone app of an Android cellphone. For the final set of numeral cards, I took an image of the buttons on an elevator. The numerals on the materials used in the instruction also used the sans serif font. As stated in the previous section on modifications to the intervention, however, I hypothesized that the use of multiple exemplars resulted in the delayed acquisition of the number identification skill, and so I began only using the first set of numeral cards.

The instructors assessed the students' generalization of numeral identification by using materials available in the students' natural environment. For Shaquille, the natural material was the number keypad of a microwave. For Jackson, the natural materials included an analog clock and a calculator. The instructor conducted the generalization probe once during baseline and multiple times throughout the intervention period.

Reliability and Procedural Fidelity

Interobserver Agreement

A doctoral student supported this research by calculating IOA on 33% of the numeracy probes and linear equation test sessions. She used the trial-by-trial method to measure IOA ($\frac{\text{Number of trials agreement}}{\text{Total number of trials}} \times 100$; Cooper et al., 2020). I trained her using the recommendations presented in Ledford and Gast (2018). First, I provided definitions of the behavior and then explained the procedures, verbally and in writing. Then we practiced coding a video together and discussed any discrepancies. Next, we each independently coded three videos and discussed discrepancies. This process continued until we reached 100% agreement. The mean IOA across both dyads was 100% during numeracy probes, 100% during test sessions, and 97.8% during generalization probes (range 89–100%).

Rachel and Shaquille. Regarding numeral identification (DV #1), the doctoral student watched and calculated IOA in 40% (2 out of 5 sessions) of the initial full numeracy probes. IOA indicated 100% agreement. She calculated IOA in 38% (5 out of 13 sessions) of the daily numeracy probes. IOA indicated 100% agreement. She calculated IOA for numeral identification probes within the revised lead sessions, in which CTD replaced SP in 33.3% of sessions (8 out of 24 sessions). IOA indicated 100% agreement. She calculated IOA for 37.5% of generalization sessions (3 out of 8). IOA indicated 100% agreement. Regarding completion of the task analysis for solving a simple linear equation (DV #2), the doctoral student watched and calculated IOA in 40% of test sessions during baseline (2 out of 5 sessions). IOA indicated 100% agreement. She calculated IOA in 50% of test sessions during intervention (2 out of 4 sessions; i.e., those sessions occurring after the model and lead sessions had been presented). IOA indicated 100% agreement.

Cindy and Jackson. Regarding numeral identification (DV #1), the doctoral student watched and calculated IOA in 43% (3 out of 7 sessions) of the full numeracy probes. IOA indicated 100% agreement. For the daily numeracy probes, she calculated IOA in 35% (7 out of 20 sessions). IOA indicated 100% agreement. She calculated IOA for 50% of generalization sessions (2 out of 4). IOA indicated 94.5% agreement (range 89–100). For the session where IOA was not 100, I also watched to confirm, and then once the doctoral student and I were in agreement, I changed the data to reflect the revised accuracy. Regarding completing the task analysis for solving a simple linear equation (DV #2), the doctoral student watched and calculated IOA in 33% of test sessions (1 out of 3) during baseline. IOA indicated 100% agreement. She calculated IOA in 33% of test sessions during intervention (1 out of 3; i.e., those sessions occurring after the model and lead sessions had been presented). IOA indicated 100% agreement.

Procedural Fidelity

To control threats to procedural fidelity, I provided the instructors with scripted lessons. To verify the degree to which the scripted lessons were delivered as designed, both I and a secondary observer assessed procedural fidelity across 50% or more of sessions across each phase of the intervention for each dyad. We collected procedural fidelity data using permanent product (video) recording and the procedural fidelity checklist (see Appendix Q). Procedural fidelity was calculated by dividing the total number of steps implemented correctly by the total number of steps delivered and multiplied by 100. If an instructor's procedural fidelity dropped below 80%, I provided coaching on the missed steps. The mean procedural fidelity for both instructors during numeracy probes was 100%. The mean procedural fidelity for both instructors during model lessons was 90.1% (range 80–100). The mean procedural fidelity for both instructors during lead lessons was 85.9% (range 50–97). The mean procedural fidelity for both instructors during test sessions was 98.7% (range 86–100). Finally, the mean procedural fidelity for both instructors during generalization was 96.5% (range 89–100).

Rachel and Shaquille. During the full numeracy probes delivered by Rachel, we watched the videos and assessed procedural fidelity in 80% of sessions (4 out of 5 sessions). Rachel conducted these sessions with 100% fidelity. We assessed procedural fidelity for 100% of sessions (5 out of 5) for the test lessons delivered during baseline. Rachel conducted these sessions with 100% fidelity. We assessed procedural fidelity during 92% of the daily numeracy probe sessions (12 out of 13). Rachel conducted these sessions with 100% fidelity. During the initial set of model lessons, in which no modifications were delivered, we assessed procedural fidelity in 86% of sessions (6 out of 7). Rachel conducted these sessions with a mean of 87.2% fidelity (range 80–97). During the next set of model lessons, in which the first modification was introduced (i.e., presenting numeral cards within the SP procedure), we assessed procedural fidelity in 100% of the sessions (3 out of 3 sessions). Rachel conducted these sessions with a mean of 90.3% fidelity (range 86–94). During the lead lessons, in which modification 1 continued, we assessed procedural fidelity in 100% of sessions (4 out of 4). Rachel conducted these sessions with a mean of 84.0% fidelity (range 74–94). During the lead revised sessions, when CTD replaced SP procedures, we assessed procedural fidelity in 50% of sessions (12 out of 24 sessions). Rachel conducted these sessions with a mean of 91.7% fidelity (range 85–97). We assessed procedural fidelity for 50% of sessions (2 out of 4 sessions) for the test lessons delivered during intervention. Rachel conducted these sessions with 100% fidelity. Finally, we assessed procedural fidelity in 50% of generalization sessions (4 out of 8 sessions). Rachel conducted these sessions with 97.5% fidelity (range 90–100).

Cindy and Jackson. During the full numeracy probes delivered by Cindy, we watched the videos and assessed procedural fidelity in 55% of sessions (5 out of 9 sessions). Cindy conducted these sessions with 100% fidelity. We assessed procedural fidelity for 67% of sessions (2 out of 3 sessions) for the test lessons delivered during baseline. Cindy conducted these sessions with 93.0% fidelity (range 86–100). We assessed procedural fidelity during 65% of the daily numeracy probe sessions (13 out of 20 sessions). Cindy conducted these sessions with 100% fidelity. During the initial set of model lessons, in which no modifications were delivered, we assessed procedural fidelity in 80% of sessions (4 out of 5 sessions). Cindy conducted these sessions with a mean of 94.8% fidelity (range 86–100). During the next set of model lessons, in which the first modification was introduced (i.e., presenting numeral cards within the SP procedure), we assessed procedural fidelity in 100% of sessions (3 out of 3 sessions). Cindy conducted these sessions with a mean of 89.7% fidelity (range 86–94). During the lead lessons, in which modification 1 continued, we assessed procedural fidelity in 50% of sessions (6 out of 12 sessions). Cindy conducted these sessions with a mean of 75.6% fidelity (range 50–93). The reason for this decrease in fidelity was that she did not continue the modification of presenting the numeral card and mistakenly reverted to the initial procedures in which she prompted Jackson to point to the numeral in context of the word problem for the first three sessions. Due to scheduling difficulties, I was not able to meet with Cindy for a follow-up coaching session until the third session in which her fidelity was below 80%, but following coaching, her fidelity rose to 93%. The coaching session consisted of a brief description of the error and then rehearsal with the Cindy until she delivered the intervention with 100% fidelity. We assessed procedural fidelity for 67% of sessions (2 out of 3 sessions) for the test lessons delivered during intervention. Cindy conducted these sessions with 100% fidelity. Finally, we assessed procedural fidelity in 50% of generalization sessions (2 out of 4 sessions). Cindy conducted these sessions with 94.4% fidelity (range 89–100).

Social Validity

I assessed social validity through subjective evaluations completed by the participating instructors at the start and end of the study (i.e., DV #4). Evaluations addressed the relevance of the skills addressed, the comfortability of the intervention, and the importance and personal relevance of the skills addressed for each student. See Appendix O. I compared the pre- and post-evaluations to determine if the intervention influenced the instructors' perceptions of algebra instruction and embedded foundational skill instruction for secondary students with ESN.

I also assessed social validity from the perspective of the target students and their caregivers. To assess social validity of the target students' parents, I requested that they complete a social validity questionnaire at the end of the intervention (see Appendix U). To assess social validity from the students' perspective, I asked the instructor to record their perceptions of each student's affect, either positive or negative, at the completion of each session on the numeracy probe data sheet (see Appendices M & S).

CHAPTER 4: RESULTS

Results for Question 1: To what extent does an intervention package comprising MSBI and embedded SP improve foundational numeral identification skills for secondary students with ESN?

Shaquille

In the first set of full numeracy probes (i.e., baseline), Shaquille responded at a low level across numeral sets, ranging from 0 to 33% accuracy (see Figure 6). He continued to respond at a low level, ranging from 0 to 33% accuracy through intervention for Numeral Set A (numerals 3, 4, 9). After five sessions, we introduced the first modification, switching from identifying numerals in context of the materials to the instructor presenting the numeral cards used during the probes. Responding continued at or below 33% accuracy for seven sessions.

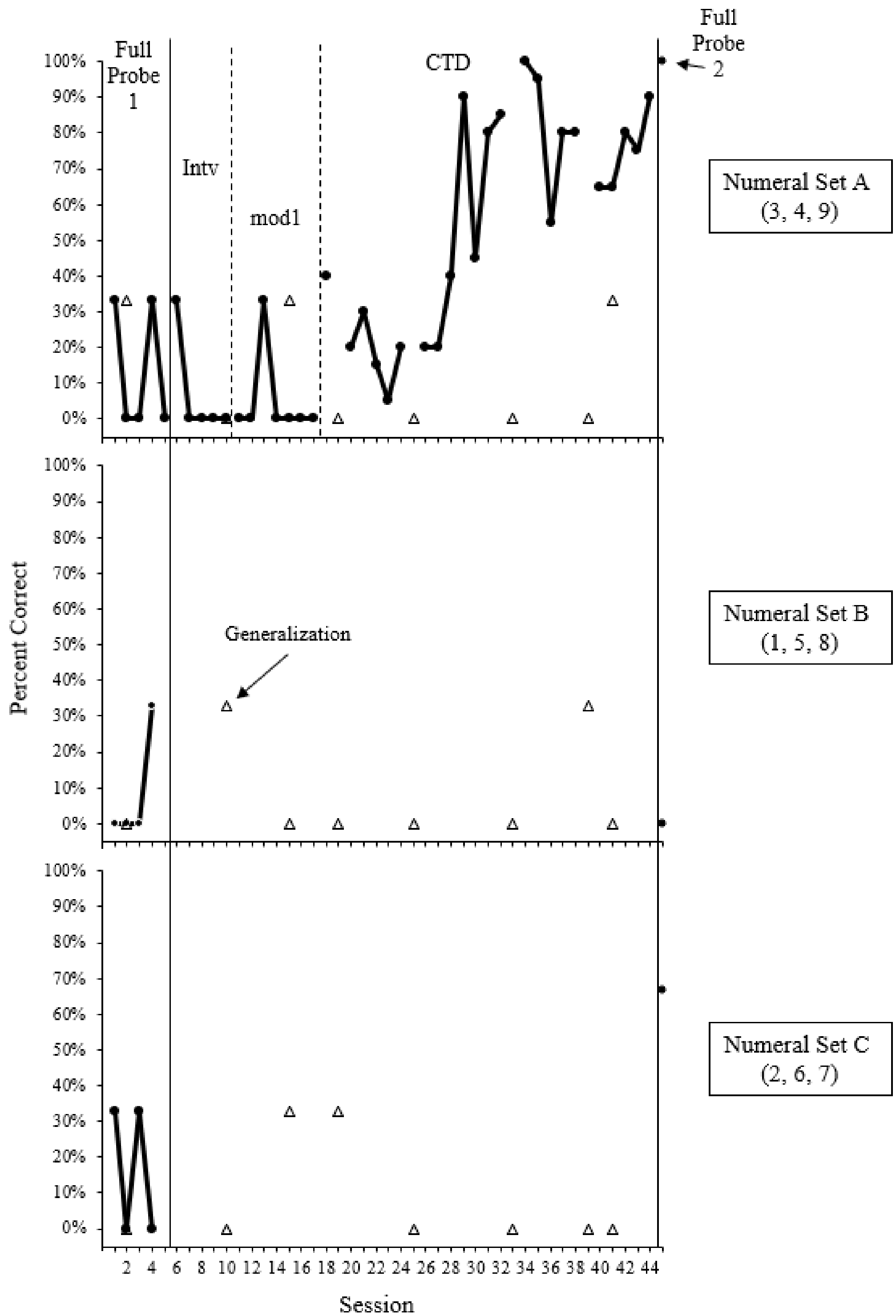
At this point, we shifted to the second modification, in which the instructor used embedded CTD rather than SP. Notedly, this meant that the denominator switched from three to 20. During SP, the instructor probed the three numerals in Set A once daily; thus, the denominator was three. During CTD, the instructor presented two of the three numerals 10 times each throughout the lesson; thus, the denominator was 20. Across three sessions, each numeral was presented at least 20 times. For example, in session 20, Rachel presented numerals 3 and 9 (i.e., $3 + x = 9$), in session 21, she presented numerals 4 and 9 (i.e., $4 + x = 9$), and in session 22, she presented numerals 3 and 4 (i.e., $3 + x = 4$).

Once CTD was introduced, the percentage of Shaquille's accurate responses remained low, ranging from 5 to 40%, with a mean of 23.3%. However, on the tenth day of CTD instruction, his accuracy improved to 90%. Throughout the remainder of the study, his accuracy was variable but demonstrated an increasing trend overall. Between the tenth and final intervention sessions, Shaquille's accuracy remained above baseline, ranging from 55 to 100%, with a mean of 75.4%.

Although he had not reached mastery by the end of the school year, I asked Rachel to conduct a full numeracy probe on the final day of the study, session 45. This probe served as a maintenance check for Numeral Set A (numerals 3, 4, 9) and as a continued baseline probe for Numeral Sets B (numerals 1, 5, 8) and C (numerals 2, 6, 7). She only presented each numeral once during the session, so the denominator returned to three. For Numeral Set A, Shaquille responded with 100% accuracy (i.e., accurately identified numerals 3, 4, and 9). For Numeral Set B, Shaquille responded with 0% accuracy (i.e., did not accurately identify numerals 1, 5, or 8), and he responded with 66% accuracy in Numeral Set C (i.e., identified numerals 6 and 7). Although the effects of this intervention would have been strengthened had Numeral Set C remained at or below baseline levels (i.e., 33% or below), it was clear that Numeral Set A was under stimulus control. Not only did Shaquille accurately identify these numerals when asked, but he did not identify the numerals when given other discriminative stimuli. When presented numerals 6 and 7, which had not yet been taught, Shaquille accurately pointed to the correct stimulus; however, he also pointed to these numerals when given other stimuli. For example, when given the discriminative stimulus "Touch 2", Shaquille touched 6, and when given the discriminative stimulus, "Touch 8", he touched 7. In contrast, Shaquille identified numerals 3, 4, and 9 when, and only when, the respective discriminative stimuli were presented. In total, Shaquille received 35, 15 min intervention sessions (both embedded SP and CTD) delivered over 103 calendar days. Using Warren et al.'s (2007) definition of cumulative intervention intensity, Shaquille was exposed to 700 total teaching episodes (i.e., each SP teach trial or CTD trial).

Figure 6

Shaquille's Accurate Identification of Numerals 1–9



Note. Intv = intervention; mod1= first modification; CTD = constant time delay modification

Jackson

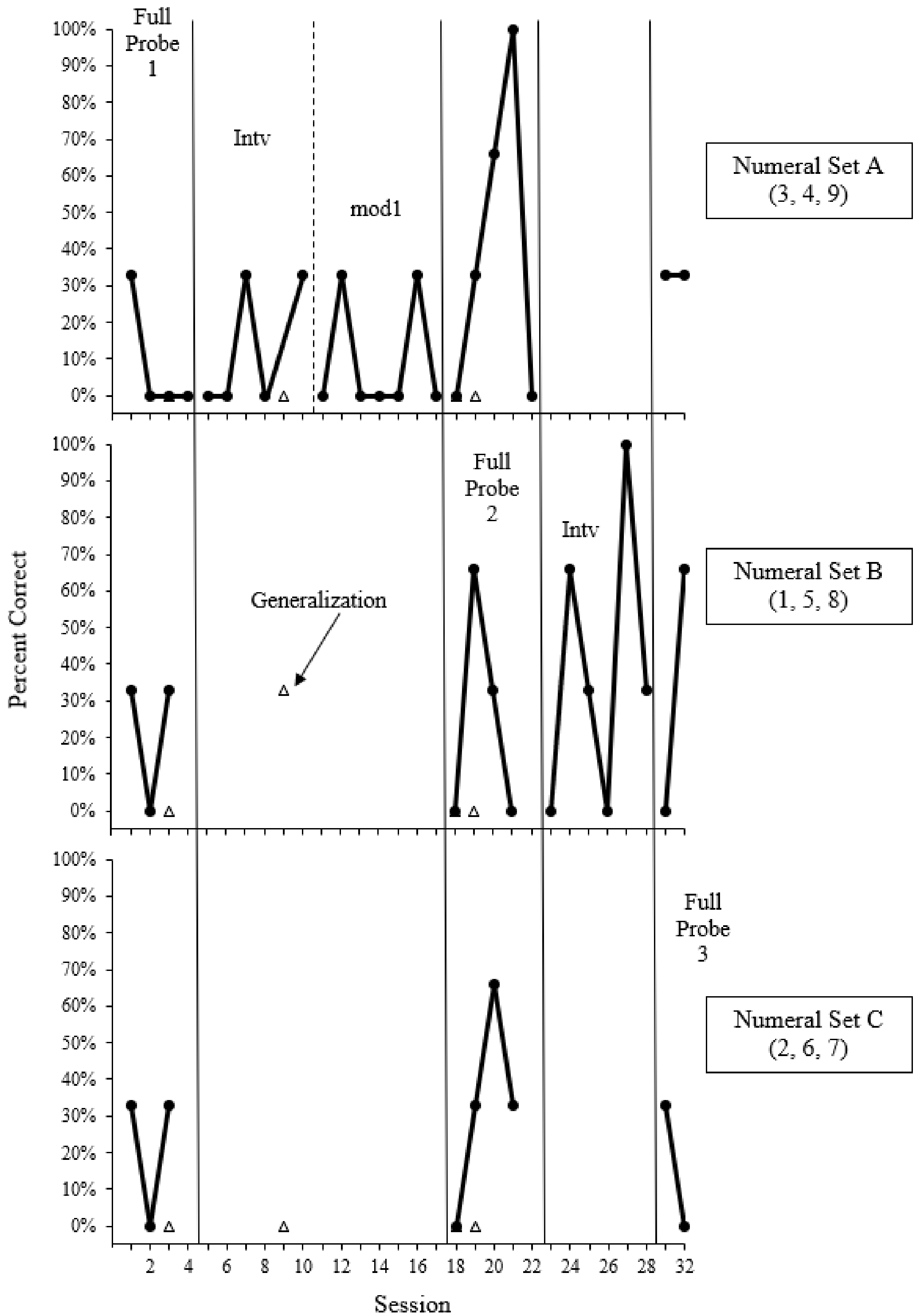
In the first set of full numeracy probes (i.e., baseline), Jackson responded at a low level, with a decreasing trend (see Figure 7). He continued to respond at a low, variable level through intervention for Numeral Set A (numerals 3, 4, 9). After five sessions of responding at or below 33% accuracy (i.e., the highest level reached during baseline), we introduced the first modification, switching from identifying numerals in context of the materials to the instructor presenting the numeral cards used during the probes. Responding continued at or below 33% accuracy for seven sessions. At this point, I concluded that the intervention was insufficient to achieve the goal of numeral identification. I wanted to confirm this through replication across numeral sets, and so I made the decision to introduce the intervention to Numeral Set B at this point.

Similar to the design used in other studies where SP was investigated (e.g., Gibson & Schuster, 1992; Schuster et al., 1992), we presented a second full probe across tiers. For Numeral Set A, this second full probe served as a maintenance check, whereas for Numeral Sets B and C, it served as additional baseline probes. During the second set of full numeral probes, Jackson accurately identified Numeral Set A with 100% accuracy in one session, which was unexpected as he had not responded with more than 33% accuracy during the intervention sessions in which instruction occurred. However, accuracy returned to 0% in the following session, suggesting that the increasing trend observed during the second full numeracy probe was aberrant.

The second full numeracy probe served as additional baseline probes for Numeral Sets B and C. In both tiers, Jackson's percentage of correct responses varied from 0% to 66% accuracy. The trend was variable. Mean responding in the second numeracy probe was higher than mean responding in the first numeracy probe even though no instruction had been provided. We introduced the intervention using Numeral Set B (1, 5, 8) on the 23rd session. Cindy conducted six intervention sessions before the school year ended. He continued to respond variably, achieving 100% accuracy in one session but dropping back to 33% in the next session. I asked Cindy to complete two final numeracy probes on the last day of school. These probes served as maintenance checks for Numeral Sets A and B and as additional baseline probes for Numeral Set C. In Numeral Set A, Jackson continued to respond with 33% accuracy, similar to levels observed both in baseline and intervention. In Numeral Set B, Jackson responded first with 0% and then with 33% accuracy, similar to both baseline and intervention levels. Finally, in Numeral Set C, Jackson responded with 33% and then 0% accuracy, similar to previous baseline probes. In total, Jackson received 17, 15 min sessions, delivered over 45 calendar days (for both Numeral Sets A and B). Using Warren et al.'s (2007) definition of cumulative intervention intensity, Jackson was exposed to 340 total teaching episodes (i.e., each SP teach trial).

Figure 7

Jackson's Accurate Identification of Numerals 1–9



Note. Intv = intervention; mod1= first modification

Summary

No functional relation between the intervention and numeral identification was observed. This is primarily due to the limited potential demonstrations of effect. For Jackson, there were only two potential opportunities to demonstrate an effect; however, no effect was observed. Therefore, there was no functional relation. For Shaquille, there was only one potential opportunity to demonstrate an effect. An effect was not observed given the embedded SP procedures, but an effect was observed given the CTD modification. There was still no functional relation, though, because there was no intraparticipant replication. To further explicate this, I completed a visual analysis using the worksheet provided by Ledford and Gast (2018; see Figure 8).

Figure 8

Visual Analysis Worksheet

	Shaquille	Jackson
Level		
Consistent level established prior to condition change?	YES (+)	YES (+)
Consistent level change between conditions, in the expected direction?	YES (+)	NO (−)
Trend		
Are unexpected trends present?	NO (+)	YES (−)
Consistent change in trend across conditions?	NO (−)	NO (−)
Variability		
Does unexpected variability exist?	NO (+)	YES (−)
Does within-condition variability impede determinations?	NO (+)	YES (−)
Consistency		
Are data within and changes between conditions consistent?	NO (−)	NO (−)
If no, was that expected?	NO (−)	NO (−)
Does inconsistency impede confidence in functional relation?	YES (−)	YES (−)
Overlap		
Are data highly overlapping?	YES (−)	YES (−)
If yes, does overlap improve over time?	YES (+)	NO (−)
Is overlap consistent across comparisons?	N/A (−)	YES (+)
Was overlap expected a priori?	YES (+)	YES (+)
Does presence of overlap impede confidence in a functional relation?	NO (+)	YES (−)
Immediacy		
Are changes between tiers immediate, in the intended direction?	N/A (−)	NO (−)
If no, are delays in change consistent across tiers?	N/A (−)	YES (+)
Does lack of immediacy impede confidence in a functional relation?	YES (+)	YES (+)
Functional Relation		
Is a functional relation present?	**NO**	**NO**

Note. Adapted from Ledford and Gast (2018).

Results for Question 2: To what extent does an intervention package comprising MSBI and embedded SP improve the independent algebraic problem-solving skills of students with ESN?

Unfortunately, the MSBI and embedded SP intervention package did not appear to have a meaningful impact on the participating students' independent mathematical word problem-solving skills. Looking exclusively at the data presented in Figures 9 and 10, the students responded with 0% independence during baseline and continued to respond with 0% independence following introduction of the intervention. Anecdotally, both students began to respond with more independence during the lead sessions. For example, at the start of the study, Shaquille required intensive modeling to move each green pawn to the number line to complete step 12 of the task analysis. Rachel would point to each pawn and then to each spot on the number line. By the end of the study, Rachel simply needed to gesture to the pawns, and Shaquille would then move the pawns independently. Similarly, Jackson required intensive modeling and, occasionally, physical prompting to move the Velcro icons in step 8 of the task analysis. By the end of the study, Cindy simply gestured to the icon on the word problem and then to the problem mat.

Figure 9

Shaquille's Independent Algebraic Problem Solving

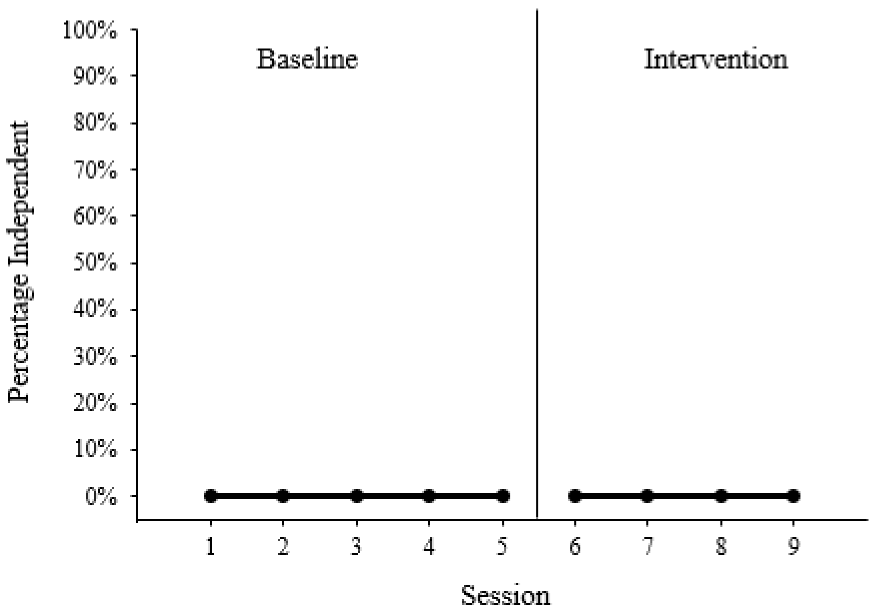
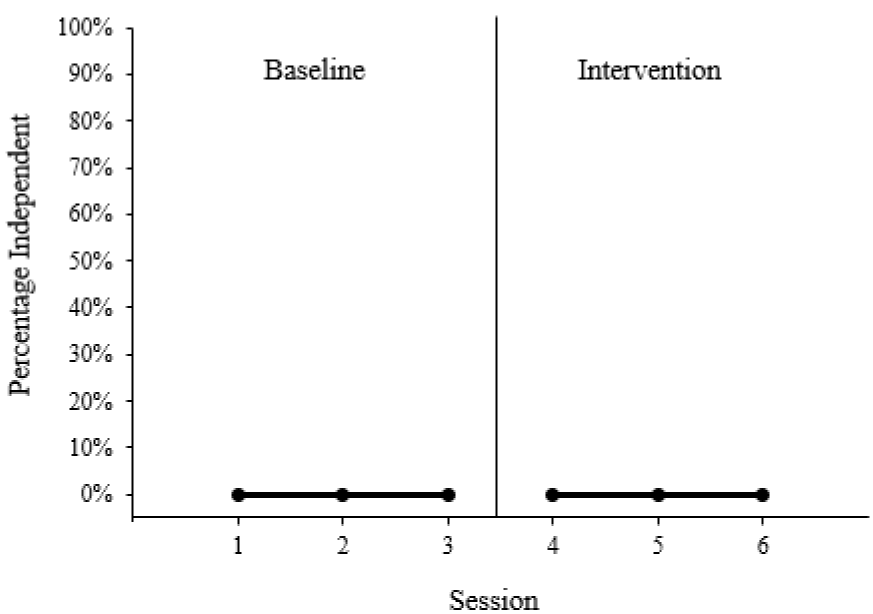


Figure 10

Jackson's Independent Algebraic Problem Solving



I graphed only the independent responses during the test sessions conducted at the beginning of the study (i.e., baseline) and following the presentation of the model and lead sessions (i.e., intervention). During these test sessions, if the student did not respond independently within 3 s, the instructor completed the step for the student. Therefore, there was no opportunity for the student to respond to the nonspecific verbal or verbal prompts. To determine whether there was an increase in independent responding, I created additional graphs (Figures 11 and 12) with the students' responses using a weighted point system, in which responses to less intrusive prompts are awarded more points (Ault et al., 2013). Thus, a model prompt was worth 0 points, a verbal was worth 1, a nonspecific verbal was worth 2, and an independent response was worth 3. As there were 13 steps in the task analysis, there was a total possible 39 points to be earned during each lead session. Neither Shaquille nor Jackson made progress in the degree of independence. Shaquille's prompted correct score was low and relatively stable, ranging from 0 to 5 (out of 39; $M = 1.65$). Similarly, Jackson's prompted correct score remained low and relatively stable throughout the intervention, ranging from 0 to 3 (out of 39; $M = 0.45$). In total, Shaquille received 35, 15 min intervention sessions, delivered over 106 calendar days (i.e., nine model sessions; 26 lead sessions). Jackson received 19, 15 min intervention sessions, delivered over 46 calendar days (i.e., eight model sessions; 11 lead sessions).

Figure 11

Shaquille's Prompted Algebraic Problem Solving

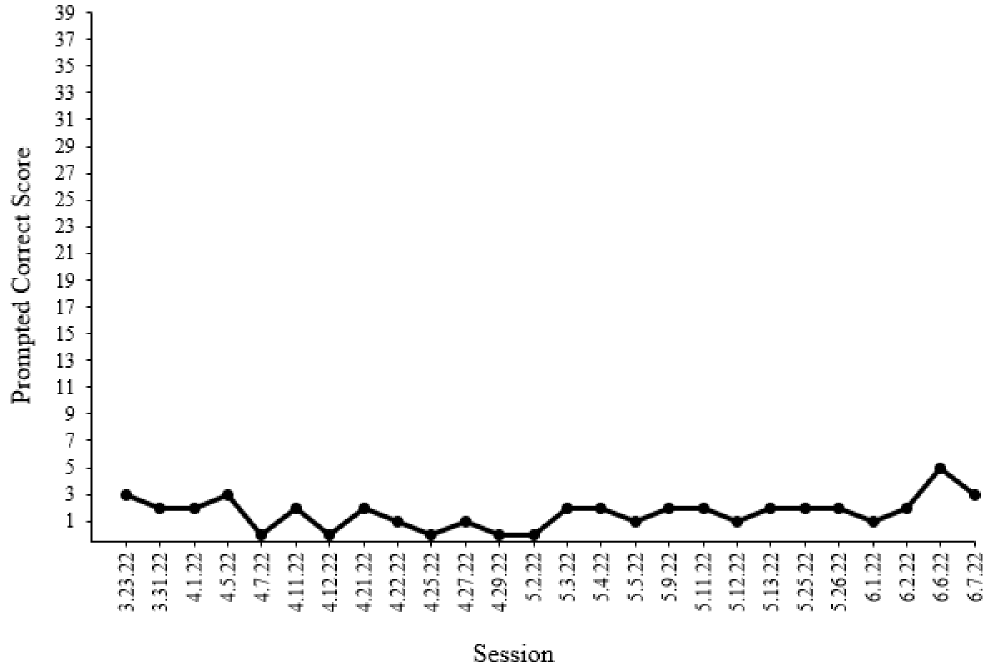
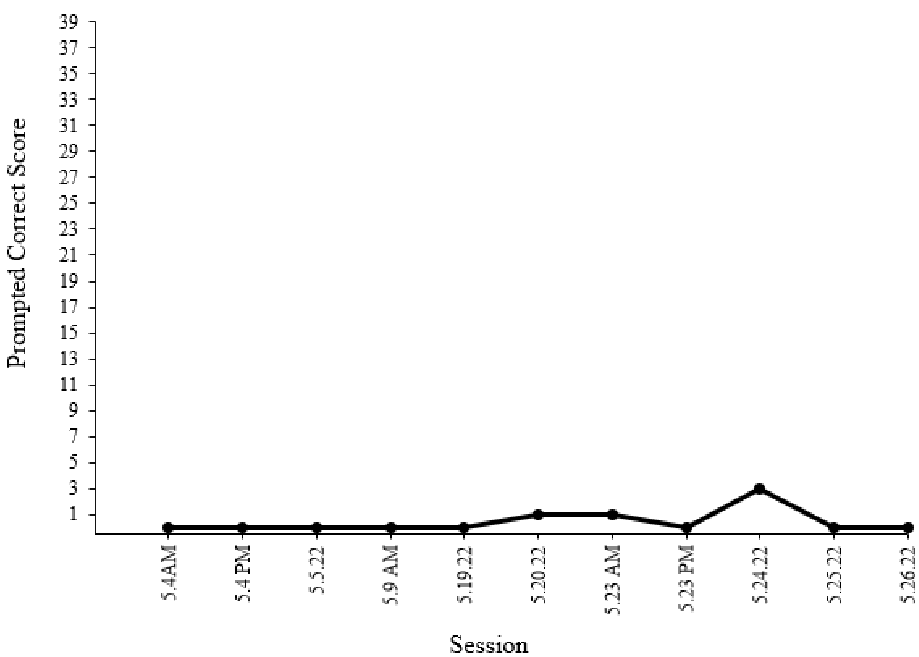


Figure 12

Jackson's Prompted Algebraic Problem Solving



Results for Question 3: To what extent can secondary students with ESN generalize numeral identification skills to real world-settings and situations?

Neither student demonstrated generalization, though it is important to note that neither student reached mastery in the acquisition phase. Rachel directed Shaquille to identify numerals on the microwave keypad during generalization probes (see Figure 6). Shaquille's accurate identification remained low in each probe across each set of numerals. For Numeral Set A, Shaquille responded with 33% accuracy during the baseline probe. During the probes conducted throughout the intervention, his accuracy varied from 0 to 33%, with no apparent trend. For both Numeral Sets B and C, Shaquille responded with 0% accuracy during the initial baseline probe and continued to respond at either 0 or 33% accuracy throughout the study.

Cindy presented Jackson with either an analog clock or a calculator to assess his generalization (see Figure 7). Jackson's accurate identification remained low in each probe across each set of numerals. For Numeral Set A, Jackson responded with 0% accuracy during the baseline probe and continued to respond with 0% accuracy in the probes conducted throughout the intervention. For Numeral Set B, Jackson responded at either 0 or 33% accuracy throughout the study. Finally, for Numeral Set C, he responded with 0% accuracy during each probe. Importantly, for both students, all nine numerals (including distractors such as the numerals 11 and 12 on the clock or the symbols × or = on the calculator) were presented during generalization probes, as opposed to numeracy probes when only five cards were presented.

Results for Question 4: To what extent is addressing numeracy skills within grade-level mathematics content considered an acceptable and effective practice by classroom teachers of students with ESN?

I asked Rachel and Cindy to complete a researcher-developed social validity inventory at the beginning of the study, before the first BST session, and at the end of the study, following the collection of the final data point. Overall, Rachel and Cindy had positive responses to the questions asked. I present the results of these pre- and post-surveys in Table 5. A truncated version of each question is included in the table. A full copy of the social validity questionnaires can be found in Appendix O.

Table 5

Results from Instructor Social Validity Questionnaire

	Pre		Post	
Question	Rachel	Cindy	Rachel	Cindy
1. It is important to teach grade-level academics.	3	5	2	3
2. It is important to teach foundational academic skills.	5	5	5	5
3. Embedding numeracy in standards-based instruction is effective.	4	5	4	5
4. Embedding numeracy in standards-based instruction is efficient.	4	5	4	5
5. The student can identify 1–9.	1	1	1	2
6. The student can solve linear equations.	1	1	1	2

Note. 5 = strongly agree; 1 = strongly disagree

The study ended before the students reached mastery. Thus, I asked two additional questions in the post-study survey: (a) “Had the study continued, I believe the target student would have learned to identify the numbers 1 through 9” and (b) “Had the study continued, I believe the target student would have learned to solve simple linear equations.” To the first question (numeral identification), both Cindy and Rachel responded 5 (strongly agree). To the second question (linear equations), however, Cindy responded 5 (strongly agree) but Rachel responded 1 (strongly disagree). I also asked if the instructor planned to use the procedures from the study in future instruction, either with the same student or with other students. Cindy responded 5 (strongly agree), and Rachel responded 4 (agree).

Within the survey, I provided an option for the instructors to provide comments or additional feedback. Rachel chose not to provide additional feedback, but Cindy shared the following:

> During these lessons, [Jackson] has become more engaged with making choices that involve numeral identification than he had been… I was surprised by how much he seemed to enjoy completing the study. As soon as the materials were brought to the table, he would come over, sit down, and wait for me to get everything set up. These materials were perfect, and I look forward to using their ideas with all of my students in the fall.

Although not explicitly related to Research Question #4, I was also interested in the opinions of the students and their caregivers on the acceptability of this study. Throughout the study, I asked the instructors to record their perceptions of the students' affect, either positive or negative. In every session, Rachel recorded that Shaquille appeared to have a positive affect. Anecdotally, he is observed to be laughing and smiling in most of the videos. Cindy recorded that Jackson appeared to have a positive affect in 30 of the 33 sessions (91%). Anecdotally, Cindy reported that Jackson appeared to enjoy participating in the lessons. Typically, he required verbal and gestural prompting to transition to the table for academic tasks. By the end of the school year, he would independently transition to the table as soon as Cindy began to set out the materials.

Finally, I asked each student's parent or guardian to respond to a series of questions related to the study. The full survey can be found in Appendix U. A truncated version of the questions is presented in Table 6.

Table 6

Parent/Guardian Social Validity

Question	Shaquille's mother	Jackson's guardian
1. It is important for my child/ward to learn grade-level academics.	5	5
2. It is important for my child/ward to learn foundational academic skills.	5	5
3. Teaching numbers in context of algebra lessons is effective for my child/ward.	5	4
4. It is better to teach numbers in context, rather than in isolation.	I don't know	5
5. My child/ward can identify numbers 1–9.	3	I don't know
6. Learning math is important for post-school success.	5	4

Note. 5 = strongly agree; 1 = strongly disagree

CHAPTER 5: DISCUSSION

The purpose of this investigation was to evaluate the effects of a treatment package comprising MSBI and embedded SP on number identification and algebraic word problem-solving skills of high school students with ESN using a multiple probe across number sets replicated across participants design. I measured number identification skills through SP probe trials to answer the first research question. To answer the second research question, I measured algebraic word problem-solving skills using a task analysis. To answer the third research question, I measured the generalization of number identification using SP probes and novel stimuli. Finally, to answer the fourth research question, I evaluated social validity through checklists completed by the teachers, parents, and students to determine their perception of the acceptability of the intervention and the importance of numeracy and algebraic problem-solving skills. In this chapter, I will explore the outcomes of the treatment package and hypothesize some reasons for these findings. Further, I will explore the contributions this study adds to the research on students with ESN, the limitations of this study, as well as recommendations for future research and implications for practice.

Summary of Results

Research Question 1: *To what extent does an intervention package comprising MSBI and embedded SP improve foundational numeral identification skills for secondary students with ESN?*

Neither student reached mastery on the first dependent variable, numeral identification. Due to recruitment challenges resulting in only two participants, I changed the experimental design from a multiple probe across participants to a multiple probe across numeral sets, replicated across participants. Unfortunately, I could still not demonstrate effect across the three numeral sets before the school year ended. Shaquille only received intervention for Numeral Set A (3, 4, 9). During baseline, he responded at low levels with some variability. This pattern of responding continued through the intervention of embedded SP. After 12 days of no progress, I switched from embedded SP to embedded CTD. At this point, Shaquille began responding with an increasing trend, with mild variability, reaching 100% in one session. He did not reach mastery criterion (100% over three consecutive sessions) before the end of the school year.

Due to difficulties with recruitment and consent, I could not begin the study with Jackson until the end of March 2022. The percentage of accurate responses in baseline ranged from 0 to 33% across numeral sets. During intervention sessions, his percentage of accurate responses did not increase. We removed the intervention in the second full probe, and Jackson's percentage of accurate responses unexpectedly increased to 100% before dropping back to 0% for Numeral Set A and reached 66% for Numeral Sets B and C. During intervention for Numeral Set B, the percentage of Jackson's accurate responses was highly variable, ranging from 0 to 100% accuracy, ending at 33%. The school year ended before we could intervene in Numeral Set C.

A functional relation between the intervention of embedded SP (and, later, embedded CTD) and the dependent variable of numeral identification was not present. I can hypothesize that if the study continued, Shaquille would eventually reach mastery across the three numeral sets, given embedded CTD. It is more difficult to hypothesize for Jackson as there were insufficient data to establish patterns.

Research Question 2: To what extent does an intervention package comprising MSBI and embedded SP improve the independent algebraic problem-solving skills of students with ESN?

MSBI was not effective in teaching these two students to solve simple linear equations in the context of a word problem. The students and context in this study differed from previous studies in which researchers investigated MSBI. Additional modifications may be needed for this strategy to be accessible to students like Shaquille and Jackson. This study was likely the first to apply MSBI to high school algebra standards. Additionally, as Clausen et al. (2021) noted, in all previous investigations of MSBI, the participating students demonstrated numeracy prerequisites, which was not a requirement in this study. Access to mathematics can be significantly limited when students cannot communicate numbers, as was the case in this study (Clements & Sarama, 2009). Regardless, students with ESN are expected to engage in grade-level mathematics; thus, additional research needs to be conducted to determine how to include students without numeracy skills meaningfully in high school algebra. For example, perhaps it is necessary to increase the dosage of the intervention, or the incorporation of technology may alleviate some of the barriers related to limited numeracy skills.

Research Question 3: To what extent can secondary students with ESN generalize numeral identification skills to real world-settings and situations?

Neither student demonstrated generalization of the numerals to real-world stimuli. The lack of generalization is unsurprising because neither student reached acquisition. Shaquille did, with relative consistency, accurately identify the numerals 3, 4, and 9 (i.e., Numeral Set A). However, he still could not generalize to the real-world stimuli of a microwave and a calculator. Anecdotally, I noticed that even once Shaquille acquired the numeral identification skill using the numeral cards on the cookie sheet, he did not generalize to the numerals within the word problem. For example, he did not accurately locate “3” on the number cards sheet used in step 8: Put the numbers on the mat.

Although generalization has been a primary goal of instruction for students with ESN for decades (e.g., Baer et al., 1968; Stokes & Baer, 1977), many studies still depict poor outcomes for generalization measures for this population (McDonnell et al., 2020). McDonnell et al. hypothesize that one of the reasons for this is the continued instruction of students with ESN in segregated settings. They argue that students in general education settings have access to a variety of instructional activities, formats, and materials that are often not present in self-contained settings. Indeed, to reduce cognitive load and encourage procedural understanding, each lesson in this study used the same instructional materials and script. To promote concept development, however, it is important to incorporate multiple stimuli from the same class, which I did not do in this study, perhaps explaining the lack of generalization (Cooper et al., 2020).

Research Question 4: To what extent is addressing numeracy skills within grade-level mathematics content considered an acceptable and effective practice by classroom teachers of students with ESN?

To determine if the instructors felt that the intervention was socially valid, I administered a researcher-developed questionnaire at the beginning and the conclusion of the study. Wolf (1978) conceptualized social validity in three parts: social significance of the goals of the investigation, social acceptability of the intervention, and social importance of the outcomes. The first two questions I developed addressed the first component, social significance. This study had two goals, to teach students to solve a simple linear equation (i.e., a grade-level academic skill) and to identify numerals (i.e., a foundational academic skill). At the start of the study, Rachel rated the first goal (grade-level academics) as “3” or “neutral” concerning significance. At the end of the study, she rated it as “2” or “disagree.” This is likely because Shaquille did not make progress on this goal. It also is possible that Rachel was unfamiliar with grade-level content at the start of the study. As a paraeducator, it is her responsibility to implement instruction designed by the classroom teacher. According to the teacher, Shaquille was not receiving any instruction aligned with grade-level standards. The lesson I developed in this study was likely Rachel's first exposure to grade-level content as an instructor. Similarly, Cindy's perception of the significance of this goal decreased over the course of the study, from 5 (strongly agree) at the start to 3 (neutral) by the end of the study. In contrast, Rachel and Cindy rated the goal of foundational academics as 5 (strongly agree) at the beginning and conclusion of the study. This finding is unsurprising, as many educators value skills they view as more functional (including foundational academic skills) over skills they consider primarily of use in the school environment (e.g., algebra; Ayres et al., 2011; Timberlake, 2014).

The second set of questions addressed social acceptability. I asked the instructors if they felt embedding numeracy instruction in standards-based algebra instruction was effective and efficient. Wolery et al. (1992) define *effectiveness* as whether the intervention results in positive effects and *efficiency* as whether the intervention results in positive effects more easily or quickly than other interventions. Rachel responded "agree" to these questions at the start and conclusion of the study, and Cindy responded "strongly agree" both times. It was somewhat surprising that they considered the intervention to be effective and efficient, as the students did not reach their goals. However, Rachel and Cindy responded positively to a question asking if they would use these procedures in the future or with other students. These findings encourage me to continue investigating this intervention, making modifications as necessary to ensure mastery in the future.

Finally, I asked the instructors if the students met their goals of identifying numerals and solving simple linear equations (i.e., if the effects were socially important). At the start, Rachel and Cindy responded, "strongly disagree." Rachel's response remained "strongly disagree" at the end of the study, though Cindy changed her response to "disagree." Again, this was unsurprising as the students did not reach mastery. In the final social validity survey, I did ask the instructors to predict whether the students would meet their goals if the study had continued. Rachel and Cindy strongly agreed in reference to numeral identification. Cindy strongly agreed in reference to the linear equations, but Rachel strongly disagreed.

Themes Derived from Research

Upon completing this research, I have made a set of hypotheses to explain the results of this study, as well as predictions to improve results in future research. In this section, I will describe the difficulties with conducting school-based research in 2022 and the impact it had on the recruitment of participants and the length of intervention. Then I will theorize why CTD was more effective for Shaquille than SP and why the students were unable to generalize to novel stimuli. Next, I suggest that the inclusion of the embedded instruction may have negatively impacted the students' acquisition of the chained task, solving simple linear equations. Finally, I theorize that the students' limited access to the general curriculum significantly impacted their success in this study.

School-Based Research in 2022

Since March 2020, the nature of schooling in the United States has changed due to the COVID-19 pandemic. Students and teachers have navigated virtual, in-person, and hybrid learning shifts that negatively impacted students, particularly students with disabilities (Lane et al., 2021; Turner & Klein, 2021). Additionally, many schools closed their doors to outside personnel (i.e., university researchers; families) to ensure the health and safety of their students and faculty. Somewhat related to the COVID-19 pandemic, the last few years also have been characterized by a significant shortage of educators across the country (U.S. Department of Education, March 25, 2022). In certain cases, teachers are covering multiple classrooms and juggling increased responsibilities as they attempt to serve their students with reduced resources. As a result of both the changes in school delivery methods and the shortage of qualified educators, student achievement has suffered, particularly in mathematics (Kuhfeld et al., 2020). Participating in educational research projects, in which the practices being evaluated may or may not be successful, has not been a priority for teachers or administrators at this time (Lane et al., 2021). For example, although I sent recruitment letters to administrators from 37 schools and districts in the state, only three districts expressed interest in participating in this study. Many administrators who responded declined, stated an unwillingness to place additional responsibilities on their educators at this time.

Potentially the biggest barrier to students reaching mastery in this study was the limited amount of time they had to participate in the intervention. Both instructors felt their students would have met mastery criterion if they had more time to implement the intervention. Ideally, I would have begun the study earlier in the school year to allow for this time. This was not possible, however, due to several factors outside my control.

In mid-September, I began recruiting districts. I proposed my dissertation and submitted it to the Institutional Review Board (IRB) in mid-October (see Figure 13). During this time, three districts expressed interest: Districts A, B, and C. District B had two teachers who wanted to participate. Between the time I met with the teachers to further describe the study and when I received approval from the IRB, one of those teachers had resigned from teaching. I received IRB approval on December 15, 2022. As with many other job openings, the university had difficulty filling two open positions in the research office, resulting in delayed approval. By the time approval was given, students were preparing for winter break.

I received consent from the interested instructors in late December/early January. Once consent was obtained, I asked the instructors to send home parental consent requests. District A received parental consent almost immediately. District B sent home requests for parental consent multiple times but did not receive a signed letter, resulting in their eventual decision to drop from the study on February 7, 2022. District C required that the parental consent letter first be approved by the board, which I submitted on January 5, 2022. After numerous follow-up requests, the board approved the letter on February 25, 2022, and the student's guardians returned the letter in mid-March.

Scheduling BST was another barrier, particularly with District A, where a paraeducator served as the instructor. Special education teachers, particularly those in self-contained settings, often do not have a planning period in which meetings can be scheduled, so I had to ask the participating instructors to meet with me before or after school. In general, teachers arrive early and stay late, but it is not reasonable to ask paraeducators, who are paid hourly, to work beyond their paid hours. Fortunately, paraeducators were paid to stay after students were dismissed for 30 min each day in District A. However, that time was prescheduled for instructional planning and other school-wide meetings. Thus, it took several weeks to schedule the first BST with Rachel (i.e., District A paraeducator) and the follow-up training, which was needed because we exceeded the 30 min time allotted.

Rachel began implementing the study with Shaquille (District A) on February 14, 2022. Between that day and the end of the school year, spring break occurred (five instructional days missed) and Rachel was pulled to support students with more intensive behavioral support needs in other classes (10 instructional days missed). Additionally, Shaquille's parents pulled him out for a family vacation (6 instructional days missed). Shaquille's last day of school was June 6,

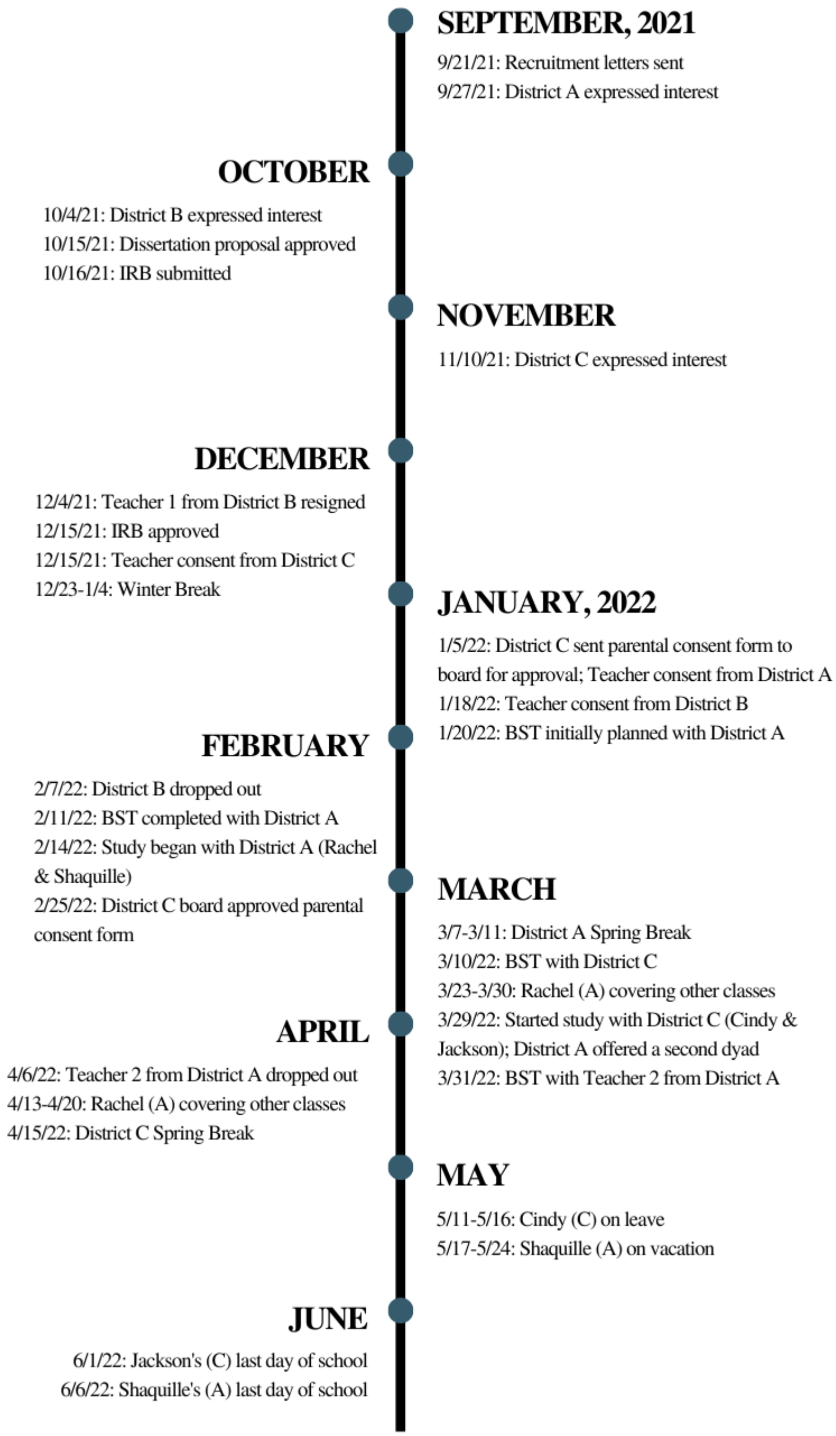
2022. In total, Shaquille participated in 35 intervention sessions (i.e., 9 model; 26 lead sessions). Despite these difficulties, District A expressed interest in implementing the study procedures with a second teacher-student dyad. However, after receiving BST, the interested teacher chose not to continue with the study due to job-related pressures.

Cindy began implementing the study with Jackson (District C) on March 29, 2022. Spring break occurred between that day and the end of the school year (5 instructional days missed), and Cindy took leave for bereavement (3 instructional days). Jackson's last day of school was scheduled to be June 8, 2022, but his guardians chose to end his school year on June 1, 2022. In total, Jackson participated in 19 intervention sessions (i.e., 8 model; 11 lead sessions).

Although I do believe these students would have made more progress, given additional time, it is important to note that the dosage they did receive was higher than those reported in previous MSBI studies. I reviewed the 12 studies included in the Clausen et al. (2021) MSBI review and found that the number of model lessons ranged from one to three, and the number of lead lessons required to reach mastery ranged from three to 22. Regarding numeral identification. the dosage received also was higher than previous studies in which SP was investigated (i.e., Akmanoglu & Batu, 2004; Birkan, 2005). The students in this study each received a dose of 20 trials per session for 12 sessions (i.e., 240 cumulative teaching opportunities), compared to 6 trials per session for 5 to 16 sessions in Akmanoglu & Batu (i.e., 30 to 96 cumulative teaching opportunities), and 9 trials per session for 5 to 15 sessions in Birkan (i.e., 45 to 135 cumulative teaching opportunities).

Figure 13

Study Timeline



Differential Reinforcement- Simultaneous Prompting vs. Time Delay

Although embedded SP was ineffective, it appears that embedding a different response prompting procedure (i.e., CTD) did have promise. Once again, I could not confirm these findings due to time constraints, but Shaquille made progress with embedded CTD and reached 100% accuracy in one session. I switched to CTD after noticing in the videos that Shaquille appeared to select cards in the same position, regardless of the stimulus. After switching to CTD, this positional bias appeared to fade. I hypothesize that the removal of the instructional trial negatively reinforced Shaquille's behavior of selecting a card at random. Conversely, in CTD, his discrimination was positively reinforced with social reinforcement (e.g., "nice work!"), while the repetition of the trial positively punished the behavior of random selection. Additionally, Shaquille seemed to enjoy the social reinforcement. After making a correct response once CTD was implemented, he would lean toward Rachel and smile as she praised him, which aligns with Wolery et al.'s (1992) hypothesis that students may enjoy the positive interactions they engage in with their instructors during CTD procedures.

I originally selected SP as the response prompting procedure based on Schuster et al.'s (1992) assertion that SP was easier to implement than CTD. When Schuster et al. developed procedures for SP, they theorized it would be easier as the instructors did not have to change their behavior, as they do when switching from 0 s to delay sessions, nor did they need to provide reinforcement differentially. In SP, no reinforcement or error correction is provided in the probe sessions, and the correct response is immediately prompted in instructional sessions. Conversely, in CTD, the instructor must reinforce correct responses and correct errors during the delay session. Additionally, data collection procedures were more intensive. The instructor was required to collect data during the linear equation lesson, as opposed to in isolation during SP probes. Regardless of these differences in implementation ease, the students did not make progress when provided SP, and thus I decided to switch to CTD.

CTD is an EBP for teaching academic skills to students with ESN (Browder et al., 2009; Courtade et al., 2014). Furthermore, studies comparing CTD and SP have found minimal or mixed differences in student performance (e.g., Ackerlund Brandt et al., 2016; Akmanoglu et al., 2015; Schuster et al., 1992; Seward et al., 2014; Swain et al., 2015; Riesen et al., 2003). Indeed, it may be that previous researchers have found SP ineffective but their research was not published due to publication bias or the “file drawer” problem in which researchers do not submit studies in which the intervention was ineffective (Tincani & Travers, 2019). In conclusion, it is impossible to generalize the results because of the relatively small *n* (only two students) and the fact that I was unable to demonstrate experimental control. Teachers, however, may interpret these results to suggest that when one response prompting procedure, such as SP, appears ineffective, they should try a different method, such as CTD.

Salience of Stimuli and Generalization

Stokes and Baer (1977) described nine techniques or instructional approaches used by researchers to plan for and assess generalization. This list included ineffective strategies, such as “train and hope,” in which researchers do not explicitly plan for generalization but simply probe to determine if the participant can generalize the skill at the conclusion of the study. From this list, Cooper et al. (2020) suggest five techniques to promote generalization: “(a) teach the full range of relevant stimulus conditions and response conditions; (b) make the instructional setting similar to the generalization setting; (c) maximize the target behavior’s contact with reinforcement in the generalization setting; (d) mediate generalization; and (e) train to generalize” (p. 724).

In this study, I planned to incorporate multiple stimulus examples by presenting multiple sets of numeral cards (i.e., teach the full range) and programming common stimuli by using real-world, personally relevant word problems (i.e., make the setting similar; Cooper et al., 2020). During the probes, I provided multiple sets of numeral cards representing stimuli that the students may encounter during their typical days (e.g., numerals on a calculator; elevator buttons; dial buttons on a smartphone; see Appendix I). I noticed in the video recordings, however, that when each instructor prompted their student to point to a numeral in the context of the algebra lesson materials (e.g., in the word problem; on the number line), the student often pointed to other stimuli, such as a word or image. Even though this should have been errorless, as the instructor provided an immediate prompt, each student still erred. One potential modification could have been to change from a model to a physical prompt. However, Rachel reported that Shaquille found physical prompts aversive, and Jackson engaged in hand mouthing and biting, such that physical prompting was unsafe for Cindy to engage in due to the presence of saliva on his hands. Therefore, I modified the procedure so that the instructor presented a numeral card (from a single set) to see if perhaps increasing the salience of the stimuli would improve the students' numeral identification skills. This modification did not result in improved identification, but I did not reintroduce the multiple numeral card sets after transitioning to the second modification, CTD (Shaquille only). It is possible that had I reintroduced those multiple exemplars, the students' generalization may have improved.

Interruption of Chained Task

It is plausible that the inclusion of the instructor-directed embedded instruction trials interfered with the students' independent completion of the task analysis. Traditionally, embedded instruction occurs during non-instructional times (e.g., during transitions; when the teacher is taking attendance; Jameson et al., 2020; Ruppar et al., 2017). In this study, the numeracy instruction was embedded within the chained task of solving a simple linear equation, essentially breaking the chain. Typically, within a chained task, the completion of the previous step serves as the discriminative stimulus for the following step (Cooper et al., 2020). In this situation, the completion of the previous step served as the discriminative stimulus in some situations (e.g., between steps 12 and 13), but the completion of the embedded instruction trial was meant to serve as the discriminative stimulus in other situations (e.g., between steps 3 and 4). This issue could be remediated either by including the opportunities for embedded instruction within the student's task analysis or by reducing the latency between completion of a step and the instructor's presentation of the embedded trial such that an instructor's expectant pause becomes the discriminative stimulus for the student's initiation of the next step on the task analysis. Interestingly, in other studies in which instruction was embedded in instructional tasks, there was not an adverse effect on the chained task being targeted (e.g., Brosh et al., 2018; Ruppar et al., 2017).

Regarding latency, another finding in this study was the relatively poor fluency in the instructors' delivery of the scripted lessons, particularly in the case of Cindy's delivery. Both instructors consistently met fidelity goals, but I only assessed their delivery of the scripts and their incorporation of the response interval prior to providing a more intensive prompt within the lead lessons. I did not measure their fluency, which I define as the speed of their delivery of the script following the student's completion of the previous step. Anecdotally, I noticed in the video recordings that the instructors often lost their place when reading the script or required additional time preparing materials during the lesson. Indeed, in an effort to support the students' communication, reading, and mathematics support needs, I provided significantly more materials than had been used in previous MSBI studies (e.g., Talking Photo Album, GoTalk Button, number lines, number cards), Anecdotally, the instructors reported it was difficult to manage all the materials at the start of the study, though they shared it became easier as they became more familiar with the procedures. Providing more intensive training during the BST procedures, particularly increasing the opportunities for rehearsal, may have improved the instructors' fluency, reducing the latency between steps. (Wolery et al., 1992).

In retrospect, it may have been more prudent to simply evaluate MSBI with this population of students without incorporating embedded instruction. I did not do this, however, because I knew my target students would not be able to reach 100% mastery without numeracy skills, as steps 7, 8, 10, and 13 all require the skill of numeral identification (see task analysis in Appendix H). Even without independently completing those steps, it still would have been possible for the students to achieve 69% accuracy, and the students in this study consistently responded with 0% independence. I hypothesized that embedding numeracy instruction would have promoted conceptual understanding, whereas just the MSBI without the numeracy instruction would have only promoted procedural understanding, but additional research comparing the two approaches is necessary.

Access to the General Curriculum

Schools have been required to provide access to the general curriculum for students with ESN in their least restrictive environment since the reauthorization of IDEA in 1997 (Wehmeyer et al., 2020). Regardless, neither student in this study was reported to be working towards standards-based instruction or had IEP goals aligned to grade-level standards. Unfortunately, this is still the norm for many students with ESN, particularly in the area of mathematics instruction (Cox & Jimenez, 2020). The first study in which standards-based algebra instruction was investigated was not published until 11 years after the 1997 reauthorization of IDEA (Jimenez et al., 2008). It is possible that these students' teachers believed, similar to those interviewed by Timberlake (2014), that their students need to demonstrate prerequisite skills before being exposed to the general curriculum. Indeed, numeracy skills are considered a prerequisite, foundational skill necessary for accessing higher-level mathematics content (Clements & Sarama, 2009; Saunders et al., 2019). However, even the NCTM, which designed the first set of mathematical standards in 1989, stated that students should have access to standards-based instruction even if they do not yet demonstrate foundational skills that should have been acquired in previous grades.

Additionally, both students were educated in highly segregated settings. Shaquille attended a special school with other students with ESN. Although Jackson did attend a traditional high school, he was educated in a self-contained classroom with no access to inclusive settings. Researchers have suggested that for true general curriculum access to occur, students must receive instruction in the general education classroom (Gee et al., 2020; Ryndak et al., 2008–2009; Saunders et al., 2019). For example, Cosier et al. (2013) found that time in the general education classroom correlated to increased mathematics achievement for students with high-incidence disabilities, such as learning disabilities. Special education teachers are, by certification, experts in specially designed instruction, but not necessarily in specific content areas, such as high school algebra (Kurth et al., 2019).

Although previous research teams have demonstrated that MSBI can be used to teach standards-based skills to students with ESN (e.g., Root & Browder, 2019; Root, Browder, et al., 2017; Root et al., 2019; Root, Cox, et al., 2018, 2020), they did not report if those students were already receiving standards-based instruction. Had Shaquille and Jackson been receiving instruction aligned to grade-level standards throughout their school careers, the intervention may have been more effective. Increased accountability measures, both to ensure teachers are prepared to provide access to the general curriculum and that they do indeed provide said access, should be considered.

Contributions to the Field

I designed this study for three reasons. First, I believe all students deserve access to the general curriculum, but I recognize that, particularly in high school, a lack of foundational skills can be a barrier to meaningful access. I hoped to develop a protocol in which students would concurrently learn foundational (i.e., numeral identification) and standards-based academic skills (i.e., solving a simple linear equation). I was largely unsuccessful in this attempt. On the other hand, there is some evidence to support the use of embedded CTD to teach foundational academic skills. What is lacking, however, is evidence to suggest that the students can acquire the standards-based academic skill in which the foundational skill is embedded. Minimal research has been conducted investigating embedding instruction in context of an academic activity (e.g., Brosh et al., 2018; Browder, Jimenez, Spooner, et al., 2012; Rivera et al., 2017). Although the students in these studies made progress in the embedded skill as well as in the instruction in which the skill was embedded, the students were all in elementary school, whereas the students in this study were in high school. It may be more appropriate to embed foundational skills in elementary classrooms wherein the foundational and grade-level skills are closer in complexity.

My second reason for conducting this research was to expand the literature base on the use of MSBI to solve word problems. It is important to show not only for whom an intervention is successful but also for whom it is not (Tincani & Travers, 2019). Previous research has suggested that MSBI is appropriate for teaching students with autism or moderate ID who exhibit prerequisite numeracy skills to solve mathematical word problems (Clausen et al., 2021). One student in this study, Jackson, was identified as having a severe ID (Shaquille's ID severity was not reported). Both students had high levels of support needs reported by their instructors in the area of school learning activities, with Shaquille requiring more intense supports than 84.1% of his peers with ID and Jackson requiring more intense supports than 74.8% of his peers with ID. Neither student demonstrated numeracy skills, scoring less than 20% on the selected sections of the *Early Numeracy Assessment* administered at the beginning of this study (Jimenez et al., 2013).

Tincani and Travers (2018) provide a set of guidelines necessary for "failed" studies to demarcate boundary conditions for interventions which had been successful in previous demonstrations. These guidelines address procedural fidelity and intervention intensity. In this study, procedural fidelity ranged from 50 to 100% across conditions and interventionists, but averaged 94.5%. Only five of the 91 sessions in which we calculated fidelity were below 80% fidelity. Regarding intervention intensity, Shaquille received nine model and 26 lead lessons, and Jackson received eight model and 11 lead lessons, compared to the one to three model lessons and three to 22 lead lessons presented in previous MSBI studies. The dose, or the number of word problems presented in each session, was one in this study, and ranged from one to three in previous MSBI studies. These findings suggest that MSBI may not be effective for students with severe ID without numeracy skills, though more research is needed.

Finally, I designed this research so that the natural change agents in the environment were responsible for implementing the intervention. The prevalence of research studies conducted by outside personnel (e.g., university-based researchers) rather than by those who are most familiar with the school system and the students they teach (e.g., teachers, paraeducators) is often cited as a contribution to the research-to-practice gap (Greenwood & Abbott, 2001). In this study, a paraeducator intervened with Shaquille, and a special education teacher intervened with Jackson. Both instructors had high levels of fidelity and rated that study as acceptable, stating they would use the procedures in the future. These data contribute to the research base of teacher-implemented interventions, particularly when provided with pre-made instructional materials and scripted lessons.

Limitations

The primary limitation of this research is the lack of experimental control. Due to issues with recruitment and consent, this study took place over a shorter period, with fewer participants than I had originally intended. This, combined with the ineffectiveness of the embedded SP procedure, reduced the potential for intra- and inter-participant replications. It may have been more prudent to continue with my original plan of conducting a multiple probe across participants single-case design. I did not do this because I did not want to delay the start of the study until I had at least three participants. Recent advice from experts in single-case design suggests that nonconcurrent multiple baseline designs can be appropriate, particularly in situations where the participants have no contact with each other (e.g., they attend different schools; Ledford & Zimmerman, 2022; Slocum et al., 2022). Had I continued to recruit a third participant and used a nonconcurrent design, it may have been possible to demonstrate replication across the three participants.

Somewhat related is the lack of stability in Jackson's data. I chose to move from the first to second numeral set after Jackson's responding remained low in an effort to replicate the noneffect. However, when I withdrew the intervention during the second set of full probes, Jackson's responding unexpectedly increased. I should have continued probing until the data stabilized before reintroducing the intervention. Few studies have been published showing noneffects, due to publication bias (Tincani & Travers, 2018). As such, I did not have a model which I could refer to when making decisions regarding a failed intervention in a multiple probe across stimuli design. Although replication and experimental control are considered necessary for quality single case design (Horner et al., 2005), Tincani and Travers argue that other factors, including stability of data before changing phases, are just as important, particularly when an intervention is ineffective.

Another limitation of this study was the presence of the video camera. One participant, Shaquille, appeared distracted by the video camera. It is possible this could be attributed to the principles of observer reactivity or the Hawthorne effect, in which the presence of an observer affects the observed (Baum et al., 1975; Ledford & Gast, 2018). I hypothesize however, that it was the presence of a highly preferred item that resulted in the distraction, rather than the presence of an observer. Shaquille's mother shared that his favorite activity is taking photos on his phone. In the recordings, it is possible to see Shaquille looking at the camera and smiling. Rachel used her smartphone to take the recordings, and even though she attempted to place it in an inconspicuous location outside Shaquille's direct sightline, he often turned to face it. Perhaps a more traditional camcorder would have reduced this distraction, as Shaquille is most familiar with the camera feature on a smartphone. Regardless, I was unable to determine if the presence of the camera had a negative impact on Shaquille's attention to and acquisition of the skills taught in this study.

Finally, issues with staff availability was a limitation, particularly with Shaquille. His instructor, Rachel, was often reassigned to work with other students and thus was unable to conduct the study procedures on those days. Jackson's instructor, Cindy, also could not conduct the study on certain days when her time and attention were required to support other students in the classroom who required intensive behavioral support. Because of these issues, there were often gaps in instruction, with the students losing between one and six consecutive days of intervention at a time. Training additional staff members to conduct the study procedures would have been a possible solution. As the researcher, I offered to serve as the interventionist when needed to help alleviate staff pressures. Unfortunately, I was not permitted to do so due to school regulations limiting outside visitors to reduce the spread of COVID-19.

Recommendations for Future Research

There are multiple opportunities for continued research on this topic, both by myself and by other researchers. I plan to continue this study next year, with slight modifications. Both Rachel and Cindy expressed a desire to continue working with Shaquille and Jackson on this study in the fall. If we can do so, I will adjust the dependent variables so that the study will be novel rather than a continuation. I will change the first dependent variable from numeral identification to quantity identification (i.e., when asked to identify "4", the student will point to the card depicting four dots). I will change the second dependent variable from solving a simple linear equation given a combine word problem schema to a simple linear equation given a change word problem schema. I will also begin by embedding CTD rather than SP, unless there is data to suggest from the student's teaching history that SP may be more effective.

I also would like to conduct a study in which I modify the MSBI procedures. MSBI incorporates model-lead-test (i.e., explicit instruction) and system of least prompts (i.e., systematic instruction). These two instructional frameworks are somewhat at odds, however, with model-lead-test representing a gradual release of instructional control to students and with system of least prompts representing a gradual increase in support from the teacher. Rather than combining these two instructional procedures, which increases complexity for the instructor and the student, I propose that most-to-least prompting may be more effective, particularly for students with more extensive support needs, though it is important to always take into consideration each student's needs and history with prompting when selecting a prompting system.

Researchers and practitioners typically prefer the system of least prompts over most-to-least prompting when teaching academic skills to students with ESN, as students have earlier and increased opportunities to complete the task independently (Collins, 2022). Historically, however, Billingsley and Romer (1983) suggested that decreasing assistance over time (i.e., a most-to-least prompting procedure) may be more effective and efficient for students with ESN. Few studies, however, have been conducted comparing system of least prompts and most-to-least prompting, and those that have been conducted compare the procedures in context of teaching leisure skills (e.g., Billingsley & Romer, 1983; Demchak, 1989; Libby et al., 2008). Wolery et al. (1992) suggest that the two procedures are relatively equivalent in terms of effectiveness and efficiency. Modifications in the form of probes can be made to ensure students are not receiving more prompting than necessary. Most-to-least prompting is recommended when students are learning a new and difficult task, as they were in this study, and when they need more frequent contact with reinforcement, as these two students appeared to require (Wolery et al., 1992). Model-lead-test is essentially a most-to-least procedure, with instructor support fading from most intensive in model to least intensive in test. Rather than embedding an SLP procedure within the model-lead-test, it may be more effective and efficient to replace model-lead-test with most-to-least prompting procedures.

If future studies incorporating the modifications suggested in this section are successful, there are many opportunities to expand this research. For example, future researchers may investigate these procedures in different contexts, such as in small groups or inclusive settings. Different foundational skills, including other numeracy skills, but also, for example, literacy skills can be embedded. The academic skill in which the foundational instruction is embedded can also be adjusted, perhaps addressing other mathematical content areas like geometry or statistics. Finally, the relevance of this instruction can be improved by conducting generalization probes in real-world settings, such as within the community, where students can apply both numeracy and algebraic skills.

Implications for Practice

In this study, I investigated the use of embedded SP to teach numeral identification but found CTD to be more effective. This finding does not mean that CTD is more effective than SP for every individual or skill. Instead, practitioners should collect data to evaluate the effectiveness of different response prompting systems. Through data-based decision-making, practitioners can and should act as researchers in their classrooms, making modifications necessary to ensure student success (Collins, 2022).

The students in this study did not acquire the grade-level standards-based skill of solving a simple linear equation. Practitioners should not interpret this as evidence to suggest that standards-based instruction is inappropriate for students with ESN. I provided several hypotheses as to why this intervention was unsuccessful. One hypothesis I will reiterate here is the lack of exposure these students had to the general curriculum before participating in this study. Had these students received high-quality instruction aligned to the standards throughout their school career, they may have experienced more success with this intervention.

Finally, the instructors shared that they appreciated the scripted lessons and instructional materials I created for them. As a paraeducator, Rachel should be receiving instructional materials and lessons from the teacher of record, but as the classroom teacher, Cindy is responsible for creating her own lessons. Since the beginning of the study, Cindy expressed interest in learning more about the procedures and was appreciative of the support she received from me. She did not feel adequately prepared to teach students with ESN and specifically to provide access to the general curriculum. Many teachers similarly report feeling unprepared (Ruppar et al., 2016). Educator preparation providers should ensure their graduates are prepared to teach all students, including those with ESN, and districts should provide more support, including instructional materials, to their teachers of students with ESN.

Summary

In summary, it is not possible to determine if an intervention package comprising MSBI and embedded SP is or is not effective to teach numeral identification and algebraic problem solving to secondary students with ESN based on the results of this study. However, the results suggest that CTD may be more effective than SP for some students, and MSBI might require additional modifications when implemented with students who do not yet demonstrate foundational numeracy skills. I discussed various hypotheses for why this intervention was not effective, including the length of the study, salience of the stimuli, effects of embedded instruction on interrupting a chained task, and the students’ history of access to the general curriculum. The procedures developed in this study can be used in future investigations to improve meaningful access to the general curriculum for secondary students with ESN.

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Appendix A

Reprint Permission Letter

University of North Carolina at Charlotte
Cato College of Education
9201 University City Boulevard
Charlotte, NC 29223-0001
Suite 301, Office 307

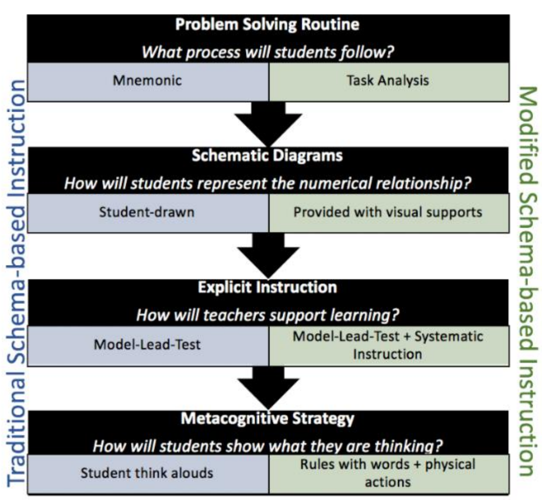
May 4, 2022

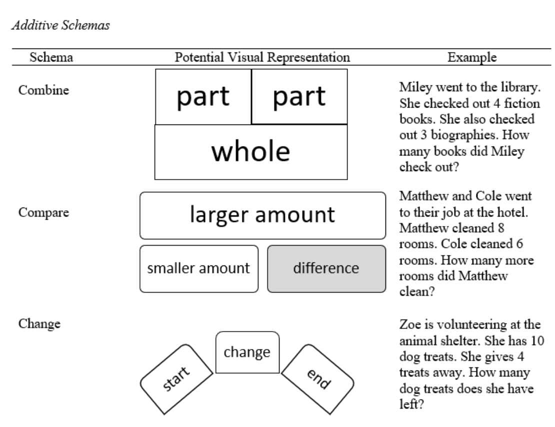
Emily Bouck, Executive Director
Division on Autism and Developmental Disabilities
Michigan State University
College of Education
343A Erickson Hall
East Lansing, MI 48824-1034

Dear Dr. Bouck:
I am completing a doctoral dissertation at the University of North Carolina at Charlotte entitled "*Embedding Numeracy Instruction within Standards-Based Algebra Lessons for Secondary Students with Extensive Support Needs*." I would like your permission to reprint in my dissertation an excerpt from the following:

> Bouck, E. C., Root, J. R., & Jimenez, B. A. (2021). *Mathematics education and students with autism, intellectual disability, and other developmental disabilities*. Division on Autism and Developmental Disabilities.

The excerpts to be reproduced are the following figures included in the chapter I co-authored entitled *Teaching Problem Solving Using Modified Schema-Based Instruction*:





Note. Adapted from Powell & Fuchs, 2018. Reprinted with permission from Bouck et al., 2021.

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If these arrangements meet with your approval, please sign this letter where indicated below and return it to me in the enclosed return envelope. Thank you very much.

Sincerely,

Amy Clausen

PERMISSION GRANTED FOR THE USE REQUESTED ABOVE:

DocuSigned by:

Emily Bouck

82341141E9A2497...

Emily Bouck

Date: 06/20/2022 | 4:59 AM PDT

Appendix B

Parental Recruitment and Consent

Parent or Legal Guardian Consent Email

[Date]

Dear Sir or Madam,

Your child/legal ward is invited to participate in a voluntary research study as part of a research project I am conducting at the University of North Carolina at Charlotte. The purpose of this study is to investigate the effects of an intervention in which your child will learn both number identification and algebra. Your child has been selected to participate in this study because they are enrolled in a high school mathematics course, but do not yet know their numbers.

In this study, your child/legal ward will be asked to participate in daily instruction lessons that will be delivered by his/her teacher during regularly scheduled instructional times. The lesson will embed number identification within age-appropriate algebra lessons.

Your child's instructional sessions will be audio and video recorded so that I can collect and analyze the data and ensure the quality of the intervention. There is nothing your child/legal ward will need to do differently as a result of being videotaped or audiotaped. All information will be kept confidential. Please see the [included/attached] Parental/Legal Guardian Consent Form for additional information related to the study.

Due to restrictions presented by COVID-19, the consent and student assent process for this study will be fully electronic. Should you have any questions regarding the study and/or your child's/legal ward's participation, I can be reached electronically through phone (985.290.6249), email (aclause1@uncc.edu), or video conference.

Thank you for your time.

Sincerely,

Amy M. Clausen, M.Ed.
Doctoral Candidate
Department of Special Education and Child Development
University of North Carolina at Charlotte
9201 University City Blvd
Charlotte, NC 28223

UNC CHARLOTTE
Cato College of Education
Department of Special Education and Child Development
9201 University City Blvd, Charlotte, NC 28223-0001
t/ 704.687.8828 f/ 704.687.1625 www.uncc.edu

Parent or Legal Guardian Consent for Child/Minor Participation in Research

Title of the Project: Embedding Numeracy Instruction within Standards-Based Algebra Lessons for Secondary Students with Extensive Support Needs
Principal Investigator: Amy Clausen, M.Ed., University of North Carolina at Charlotte
Co-investigator: Dr. Fred Spooner, Ph.D., University of North Carolina at Charlotte

Your child/legal ward is invited to participate in a research study. Your child's/legal ward's participation in this research study is voluntary. The information provided is to help you decide whether or not to allow your child/legal ward to participate. If you have any questions, please ask.

Important Information You Need to Know

- The purpose of this study is to find out if it is feasible and effective to teach both number identification and algebra (how to solve a liner equation) to a secondary student with extensive support needs.
- Your child/legal ward is being asked to participate in this study if he or she is a student in ninth through twelfth grades at [SCHOOL NAME]. Their teacher has identified them as a student who does not yet have number identification skills.
- The instruction will take place in the student's normal classroom, during their mathematics block, and will be delivered by their teacher. Their teacher will receive training in how to implement intervention by the researcher, Amy Clausen.
- Instructional sessions will be video and audio recorded so that data collection can occur. Your child's confidentiality will be protected and their names will not be used in any viewings of the videos.
- We do not believe that your child/legal ward will experience any risk from participating in this study. The instructional methods used in this intervention will be similar to those used in the child's educational career. If at any point, your child demonstrates a desire to stop participation in the study (verbally refuses, leaves the environment, engages in challenging behavior), the intervention will be stopped. We will not resume the intervention until the research team has discussed the intervention and any potential modifications with you, the parent/guardian of the child.

- Your child/legal ward will still take part in normal classroom learning and activities, even if you decide to not let them participate in this study.
- Due to restrictions presented by COVID-19, the consent process for this study will be fully electronic. Should you have any questions, a member from the research team can be reached electronically through phone, email, or video conference.
- Please read this form and ask any questions you may have before you decide whether to allow your child/legal ward to participate in this research study.

Why are we doing this study?
The purpose of this study is to investigate the feasibility and effectiveness of embedding foundational skill instruction (i.e., numeral identification) within a standards-based, grade-aligned mathematics lesson (i.e., solving linear equations) for secondary students with extensive support needs.

Why is your child/legal ward being asked to be in this research study?
You are being asked to allow your child/legal ward to participate in this study because he or she is in a student at [NAME OF SCHOOL], is enrolled in high school mathematics, and does not yet demonstrate number identification skills.

What will children do in this study?
Your child/legal ward will be asked to participate in daily mathematics lessons that will be delivered by his/her teacher during regularly scheduled instructional times. The lesson will entail solving a real-world linear equation, while also learning number identification skills. Student achievement will be recorded on data collection sheets. Prior to the start of the intervention, we will also ask your child's teacher to conduct a preference assessment, so that the algebra problems can be tailored to the student's personal environment. Additionally, we will ask you to provide information regarding your child's support needs using an assessment titled the *Supports Intensity Scale- Child's Version*. At the completion of the study, we will ask you to answer a short questionnaire to share your thoughts on the intervention and your child's skill levels.

Your child's instructional sessions will be audio and video recorded so that I can collect and analyze the data and ensure the quality of the intervention. There is nothing your child/legal ward will need to do differently as a result of being videotaped or audiotaped. All information will be kept confidential. I may use segments from the videotapes and audiotapes to demonstrate the effects of the intervention to other research team members or staff at the school. All identifying information will be removed. No one other than myself or members of the research team will be able to identify your child/legal ward. The videotapes and audiotapes may be used for teacher training and educational purposes, if you provide permission.

What benefits might children experience?
Although there is no guaranteed benefit, participation in this study may improve your student's mathematics skills, in both number identification and real-world algebra.

What risks might children experience?

There are minimal risks that your child/legal ward may experience because this study will occur as part of routine classroom teaching. Potential, but rare, risks of this study include emotional distress. If at any point your child demonstrates emotional distress (verbally refuses, leaves the environment, engages in challenging behavior), the intervention will be stopped. We will not resume the intervention until the research team has discussed the intervention and any potential modifications with you, the parent/guardian of the child.

How will information be protected?

We will not use your child's/legal ward's name. Instead, we will use a pseudonym (fake name). Paper materials will be stored in a locked filing cabinet and electronic materials will be stored in a University Dropbox folder that the researcher team can access. Only the research team will have routine access to the study information. Other people with approval from the Investigator may need to see the information we collect, including people who work for UNC Charlotte and other agencies as required by law or allowed by federal regulations.

How will information be used after the study is over?

After this study is complete, study data may be shared with other researchers for use in other studies without asking for consent again or as may be needed as part of publishing our results. The data we share will NOT include information that could identify your child.

Will my child/legal ward receive an incentive for taking part in this study?

Your child/legal ward will not receive any payment for being in this study.

What other choices are there if I don't want my child/legal ward to take part in this study?

If you decide not to let your child/legal ward take part in this study, he/she will still take part in the routine classroom activities as he/she would on a normal day. The classroom teacher will still teach all students the daily lessons.

What are my child's/legal ward's rights if they take part in this study?

Participating in this study is voluntary. Even if you decide to allow your child/legal ward to be part of the study now, you may change your mind and stop his/her participation at any time. You and your child/legal ward will not lose any benefits to which you are entitled.

Who can answer my questions about this study and participant rights?

For questions about this research, you may contact Amy Clausen at 985.290.6249 or aclause1@uncc.edu or Dr. Fred Spooner (responsible faculty) at fred.spooner@uncc.edu.

If you have questions about research participant's rights, or wish to obtain information, ask questions, or discuss any concerns about this study with someone other than the researcher(s), please contact the Office of Research Compliance at 704.687.1871 or uncc-irb@uncc.edu.

Parent or Legally Authorized Representative Consent

By signing this document, you are agreeing to your child's/legal ward's participation in this study. Make sure you understand what the study is about before you sign. You will receive a copy of this document for your records. If you have any questions about the study after you sign this document, you can contact the study team using the information provided above.

I understand what the study is about, and my questions so far have been answered. I agree for my child/legal ward to take part in this study.

I consent to my child's/legal ward's participation in "Embedding Numeracy Instruction within Standards-Based Algebra Lessons for Secondary Students with Extensive Support Needs": ____ Yes _____No

I consent to the use of video and audio recordings in the study: ____ Yes _____No (Please see a separate videotape and audiotape consent form)

____
Participant Name (PRINT)

____
Parent/Legally Authorized Representative Name and Relationship to Participant (PRINT)

____
Signature Date

____
Name and Signature of person obtaining consent Date

Multi Use Video/Audio Release Form (Student)

I hereby consent and agree to my child/legal ward being photographed, audio recorded, and/or videotaped during instructional delivery and coaching sessions by the University of North Carolina at Charlotte (herein "UNC Charlotte") or anyone authorized by UNC Charlotte, including but not limited to Principal Investigators and researchers (herein "Agents"), during my participating in the research study Embedding Numeracy Instruction within Standards-Based Algebra Lessons for Secondary Students with Extensive Support Needs" (herein "Research"). I give permission to UNC Charlotte and its Agents to use or reproduce any such videos or recordings for the following purposes (initial):

_______ Scholarship and the dissemination of research findings; and/or

_______ Classroom and professional training and education.

I agree that the use herein may be without compensation. I hereby waive any right to inspect or approve the finished recordings and expressly release UNC Charlotte and its Agents, from any and all claims which I may have for invasion of privacy, right of publicity, defamation, copyright infringement, or any other causes of action arising out of the use, adaptation, reproduction, distribution, broadcast, or exhibition of such photographs or video recordings.

I understand that my name <u>will not</u> be associated with the any videos or recordings and that all recordings will be maintained in compliance with University Policies on Records Management, Retention, and Disposition. I further understand that I have the right to revoke this permission, which must be in writing. However, any such revocation shall not affect disclosures or publications previously made by UNC Charlotte and its Agents prior to the receipt of such written revocation.

I HAVE READ THIS AGREEMENT, I UNDERSTAND IT AND I AGREE TO BE BOUND BY IT.

______________________________ ______________________________
(Signature) (Date)

____
(Printed Name)

Appendix C

Student Assent

Student Assent for
Embedding Numeracy Instruction within Standards-Based Algebra Lessons for Secondary Students with Extensive Support Needs

Month Day, 2022

To be delivered by the teacher of record:

“Do you want to do a project with me? We will work on learning your numbers and algebra. We’ll be using real-life stories. We will work together during math time. I’ll also make a video of us so we can show how great you are doing! You don’t have to do this project. It’s your choice and no one will made at you if you don’t want to work on this project. Do you want to do this project with me?”

An adult has read this to me. My choice is:

YES	NO
	

______________________________ ______________________________ ______

Student Name Student Signature Date
(stamp and/or student response recorded, if unable to sign)

______________________________ ______________________________ ______

Researcher’s Signature Date

This form was approved for use by the UNCC internal Review Board on ______, expires _____.

Appendix D

Instructor Consent

UNC CHARLOTTE
Cato College of Education
Department of Special Education and Child Development
9201 University City Blvd, Charlotte, NC 28223-0001
t/ 704.687.8828 f/ 704.687.1625 www.uncc.edu

Teacher Consent to Participate in a Research Study

Title of the Project: Embedding Numeracy Instruction within Standards-Based Algebra Lessons for Secondary Students with Extensive Support Needs
Principal Investigator: Amy Clausen, M.Ed., University of North Carolina at Charlotte
Co-investigator: Dr. Fred Spooner, Ph.D., University of North Carolina at Charlotte

You are invited to participate in a research study. Participation in this research study is voluntary. The information provided is to help you decide whether or not to participate. If you have any questions, please ask.

Important Information You Need to Know

- The purpose of this study is to investigate the feasibility and effectiveness of embedding foundational skill instruction (i.e., numeral identification) within a standards-based, grade-aligned mathematics lesson (i.e., solving linear equations) for secondary students with ESN.
- You have been recommended for participation in this study by your principal and the assistant supervisor of special education.
- You will be asked to participate in a virtual training, implement a scripted mathematics lesson each day, and collect data on the student's numeral identification skills and ability to solve a simple linear equation. You will also be asked to video record each session and upload to a cloud-based service daily.
- The consent process for this study will be fully electronic. Should you have any questions, a member from the research team can be reached electronically through phone, email, or video conference.
- Please read this form and ask any questions you may have before you decide whether to participate in this research study.

Why are we doing this study?

The purpose of this study is to investigate the feasibility and effectiveness of embedding foundational skill instruction (i.e., numeral identification) within a standards-based, grade-aligned mathematics lesson (i.e., solving linear equations) for secondary students with extensive support needs.

Why am I being asked to be in this research study?
You are being asked to be in this study because you are a teacher of a high school student with extensive support needs who does not yet demonstrate numeral identification skills.

What will students do in this study? What is my role?
This study will involve teacher implementation of a scripted lesson plan during the mathematics block. Student's numeral identification skills and ability to solve a simple linear equation will be measured.

If you agree to participate, your role will include:

1. Participate in two, 1.5 hour training sessions to learn how to implement the intervention.
2. Deliver instruction using a scripted lesson plan daily during your mathematics block.
3. Video record and upload the instructional session daily.
4. Collect data daily on the student's numeral identification skills and ability to solve a simple linear equation.

The training and coaching sessions will be video, and audio recorded so that I can collect and analyze the data and ensure the quality of the training and of the intervention. There is nothing you will need to do differently as a result of being videotaped or audiotaped. All information will be kept confidential. I may use segments from the videotapes and audiotapes to demonstrate the effects of the intervention to other research team members. All identifying information will be removed. No one other than myself or members of the research team will be able to identify you. The videotapes and audiotapes may be used for teacher training and educational purposes, if you provide permission.

What benefits might students experience?
Although there is no guaranteed benefit, your students may increase their mathematics skills, both in foundational numeracy skills and standards-based algebra skills. Additionally, findings from this study may benefit other students with extensive support needs as we learn more about how to support students with significant discrepancies in their instructional and grade level. You may gain knowledge of strategies to help improve students' academic achievement through the use of modified schema-based instruction and simultaneous prompting.

What risks might I experience?
There are minimal risks to participate in this study. Potential, but rare, risks of this study include loss of confidentiality. To minimize the potential risk, the researcher will keep inclusion criteria confidential, providing participants with a consent process that effectively communicates what the study entails and enables participants to make the decision that is best for them, and protect data through storage methods only accessible with Niner Credentials (University Dropbox). All hard copy data will be stored in a separate locked cabinet from consent forms and any identifying information will be redacted from video- and audio-recording transcripts.

How will information be protected?

We will not use your name. Instead, we will use a pseudonym and class ID code. Video and audio recordings will be shared with the research team and used for training other teachers in the future if you provide permission. Paper materials will be stored in a locked filing cabinet and electronic materials will be stored in a university password-protected Dropbox folder that the researcher team can access. Only the research team will have routine access to the study information. Other people with approval from the Investigator may need to see the information we collect, including people who work for UNC Charlotte and other agencies as required by law or allowed by federal regulations.

How will information be used after the study is over?

We may use the video and audio recordings after the study is over to demonstrate the effectiveness of the intervention, when working with other teachers, preparing pre-service personnel, and in presentations at professional education conferences. The data may be shared through publication of our results. The data shared for publication will NOT include information that could identify you and your students.

Will I receive an incentive for taking part in this study?

You will not receive any direct incentives for taking part in this study. However, the training provided to you in this study may be beneficial when working with other students in your classroom.

What other choices are there if I don't want to take part in this study?

If you decline participation or choose to stop, you and your students will not be penalized, and you will not lose any benefits to which you are otherwise entitled.

What are my rights if I take part in this study?

Participating in this study is voluntary. Even if you decide to be part of the study now, you may change your mind and stop your participation at any time. You and your students will not lose any benefits to which you are entitled.

Who can answer my questions about this study and participant rights?

For questions about this research, you may contact Amy Clausen at 985.290.6249 or aclause1@uncc.edu or Dr. Fred Spooner (responsible faculty) at fred.spooner@uncc.edu.

If you have questions about research participant's rights, or wish to obtain information, ask questions, or discuss any concerns about this study with someone other than the researcher(s), please contact the Office of Research Compliance at 704.687.1871 or uncc-irb@uncc.edu.

Teacher Consent

By signing this document, you are agreeing to participate in this study. Make sure you understand what the study is about before you sign. You will receive a copy of this document for your records. If you have any questions about the study after you sign this document, you can contact the study team using the information provided above.

I understand what the study is about, and my questions so far have been answered.

I consent to my participation in "Embedding Numeracy Instruction within Standards-Based Algebra Lessons for Secondary Students with Extensive Support Needs": ____ Yes _____No

I consent to the use of video and audio recordings in the study: ____ Yes _____No (Please see a separate videotape and audiotape consent form)

____

Participant Name (PRINT)

____

Signature Date

____

Name and Signature of person obtaining consent Date

Multi Use Video/Audio Release Form (Adult)

I hereby consent and agree to be photographed, audio recorded, and/or videotaped during instructional delivery and coaching sessions by the University of North Carolina at Charlotte (herein "UNC Charlotte") or anyone authorized by UNC Charlotte, including but not limited to Principal Investigators and researchers (herein "Agents"), during my participating in the research study Embedding Numeracy Instruction within Standards-Based Algebra Lessons for Secondary Students with Extensive Support Needs" (herein "Research"). I give permission to UNC Charlotte and its Agents to use or reproduce any such videos or recordings for the following purposes (initial):

_______ Scholarship and the dissemination of research findings; and/or

_______ Classroom and professional training and education.

I agree that the use herein may be without compensation. I hereby waive any right to inspect or approve the finished recordings and expressly release UNC Charlotte and its Agents, from any and all claims which I may have for invasion of privacy, right of publicity, defamation, copyright infringement, or any other causes of action arising out of the use, adaptation, reproduction, distribution, broadcast, or exhibition of such photographs or video recordings.

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_________________________________ _________________________________
(Signature) (Date)

____
(Printed Name)

Appendix E

Preassessment Data Sheet

Early Numeracy Pre-Assessment

Student Code: __________________
Response Mode: ________________
Date Administered: _______________

Page	Numeral	+/−	Notes
Page 61	Point to number 3		
	Point to number 1		
	Point to number 4		
	Point to number 2		
	Point to number 5		
Page 117	Point to number 10		
	Point to number 6		
	Point to number 4		
	Point to number 5		
	Point to number 7		
	Point to number 9		
	Point to number 8		
	Point to number 3		
	Point to number 2		
The next section requires that the student communicates expressively, using vocal communication or an AAC device. If the student does not communicate vocally or with an AAC device, skip this section.			
Page 173	What number? (2)		
	What number? (3)		
	What number? (1)		
	What number? (5)		
	What number? (4)		
Page 231	What number? (7)		
	What number? (10)		
	What number? (8)		
	What number? (6)		
	What number? (9)		
	What number? (4)		
	What number? (5)		
	What number? (3)		
	What number? (2)		

Key:
+: correct −: incorrect

Note. Adapted from Jimenez et al. (2013)

Appendix F

Preference Assessment

Student Name: Click or tap here to enter text.

Person Filling Out this Form: Click or tap here to enter text.

Directions: Answer the questions below by providing single word answers or short phrases. This will help us to design personally relevant word problems for the student to engage with.

What does the student do for fun? (e.g., Play sports, watch videos, listen to music). Click or tap here to enter text.

Where does the student go in the community? (Bus stop, Target, Walmart, grocery store, church). Click or tap here to enter text.

Where does the student go in school? (Cafeteria, Art room, Ms. Smith's class). Click or tap here to enter text.

Who are important people in the student's life? (Family, classmates, teachers). Click or tap here to enter text.

What are the student's favorite foods? Click or tap here to enter text.

What are the student's favorite characters/people? (Spongebob, Taylor Swift, Shaq). Click or tap here to enter text.

What are the student's goals for employment? (Filing, Custodial service, Post office, Cashier). Click or tap here to enter text.

Are there any words/phrases/places that we should avoid using? Click or tap here to enter text.

Appendix G

Lesson Plan

Solving Simple Linear Equations (Combine Schema)

Standard: NC.M1:A-REI.3: Solve linear equations and inequalities in one variable.

Numeracy Probe

Each day, you will assess the student's numeral identification skills before beginning the lesson. You will not provide any reinforcement or error correction during this probe. During full probe sessions (completed on "test" days), you will assess all nine numerals. During intervention Phase A, you will assess only numerals 9, 4, and 3. During intervention Phase B, you will assess numerals 8, 5, and 1. During Intervention Phase C, you will assess numerals 7, 6, and 2.

Instructional Cue	Student Response	Teacher Response
Deliver attentional cue: **"Let's warm up our numbers! Show me you are ready."**	Student attends to materials.	**"Let's get started!"**
Place five numeral cards in random array in front of the student. Probe first numeral (in random order). **"Touch (#)"**	Student touches correct numeral.	Mark + on the data sheet. Do not provide reinforcement.
	Student does not touch correct numeral.	Mark – on the data sheet. Do not provide error correction.
Repeat process for each numeral (in random order). Be sure to shuffle the placement of the correct card each time.		
Reinforce for participation: **"Thank you for working with me today!"** If BIP requires, provide tangible reinforcer and/or break.		

Numeracy Generalization Probe

Throughout the study, you will assess the student's numeral identification skills. You will not provide any reinforcement or error correction during this probe. During generalization, you will assess all nine numerals. On the data sheet, be sure to record the type of material used in the generalization probe (e.g., calculator, calendar, phone, microwave, clock, etc.)

<table>
<tr><th>Instructional Cue</th><th>Student Response</th><th>Teacher Response</th></tr>
<tr><td>Deliver attentional cue: "Today we're going to find numbers on a (calendar, clock, calculator, etc.)"</td><td>Student attends to materials.</td><td>"Let's get started!"</td></tr>
<tr><td rowspan="2">Point to a numeral between 1–9 on the generalization stimulus. Probe the numerals in random order. "Touch (#)"</td><td>Student touches correct numeral.</td><td>Mark + on the data sheet. Do not provide reinforcement.</td></tr>
<tr><td>Student does not touch correct numeral.</td><td>Mark – on the data sheet. Do not provide error correction.</td></tr>
<tr><td colspan="3">Repeat process for each numeral (in random order).</td></tr>
<tr><td>Reinforce for participation: "Thank you for working with me today!" If BIP requires, provide tangible reinforcer and/or break.</td><td></td><td></td></tr>
</table>

Model

Next you will model how to solve a simple linear equation. You will do this on the first day of each intervention phase. The student does not need to respond to each step of the task analysis, but you should ensure they are attending to the instruction. There will be 20 opportunities for embedded simultaneous prompting where you will prompt the student to receptively identify numerals.

In Phase A, you will use one of three word problems each time you present this lesson, either $4 + x = 9$, $3 + x = 9$, *or* $3 + x = 4$. *In Phase B, you will use* $5 + x = 8$, $1 + x = 8$, *or* $1 + x = 5$. *In Phase C, you will use* $6 + x = 7$, $2 + x = 7$, *or* $2 + x = 6$.

Instructional Cue	Student Response	Teacher Response
Deliver attentional cue: **"We're going to use a linear equation to solve a real-world math problem. Show me you are ready!"**	Student attends to materials.	**"Let's get started!"**
Present materials to student. **"Here are our materials. We have our task analysis,** (present talking photo album) **our word problem,** (present word problem) **our problem mats,** (present graphic organizers) **and our number line and manipulatives** (present manipulatives and number line).	Student attends to materials.	n/a
Step 1: **"Let's listen to step 1. Press the button."** (Point to audio button on page 1 of the talking photo album).	Student presses audio button. ►*Read the problem*	Provide physical prompting if necessary.
Point to problem. **"Here is the problem. Press this button to read the problem."**	Student presses the green and gold button.	Provide physical prompting if necessary.

Instructional Cue	Student Response	Teacher Response
Embedded Instruction: **“*Read first line.* Touch (*first number*)”** Simultaneously point to number.	Student touches numeral in word problem.	Provide descriptive feedback: **“That’s (#)!”**
Embedded Instruction: **“*Read third line.* Touch (*last number*)”** Simultaneously point to the number.	Student touches numeral in word problem.	Provide descriptive feedback: **“That’s (#)!” “Let’s turn the page and listen to our next step.”**
Step 2: **“Press the button.”** (Point to audio button on page 2 of talking photo album).	Student presses audio button. ►*Circle the terms*	Provide physical prompting if necessary.
“Terms are the pieces of the algebra problem. Help me circle/**mark our terms. In this problem, our terms are (_____),** (prompt student to circle first term), **(_____)** (prompt student to circle second term), **and (____)”** (prompt student to circle last term).	Student circles terms.	If student cannot hold marker, have them point to the terms and circle for them.
Embedded Instruction: **“We have (#) of our first term. Touch (*first number*).”** Simultaneously point to number.	Student touches numeral in word problem.	Provide descriptive feedback: **“That’s (#)!”**
Embedded Instruction: **“We don’t know how many we have of our second term. But we do know we have (#) of our last term. Touch (*last number*).”** Simultaneously point to last number.	Student touches numeral in word problem.	Provide descriptive feedback: **“That’s (#)! Let’s listen to our next step.”**
Step 3: **“Press the button.”** Point to audio button on page 3 of talking photo album.	Student presses audio button. ►*Same, different, more or fewer?*	Provide physical prompting if necessary.

Instructional Cue	Student Response	Teacher Response
"(*First term*) and (*second term*) are different. Point to 'different' on our task analysis." Point to different.	Student points to 'different' on task analysis.	Provide physical prompting if necessary. **"Let's turn the page and listen to our next step."**
Step 4: **"Press the button."** (Point to audio button on page 4 of talking photo album).	Student presses audio button. ►*Choose the problem mat*	Provide physical prompting if necessary.
"We need the problem mat for 'different'. Combine problems are about two small groups of different things we *combine* into one big group. The Combine problem mat is the one for different. Find Combine." Point to the Combine mat.	Student selects the Combine mat.	Provide physical prompting if necessary.
Embedded Instruction: **"Great! We're combining (# *first term*) and some (*second term*) to make (# *last term*). Touch (*first number*)."** Simultaneously point to number.	Student touches numeral in word problem.	Provide descriptive feedback: **"That's (#)!"**
Embedded Instruction: **"Touch (*last number*)."** Simultaneously point to number.	Student touches numeral in word problem.	Provide descriptive feedback: **"That's (#)! Let's listen to our next step."**
Step 5: **"Press the button."** (Point to audio button on page 5 of talking photo album).	Student presses audio button. ►*Use my rule*	Provide physical prompting if necessary.

Instructional Cue	Student Response	Teacher Response
"This problem is about two small groups of different things we combine into a big group. That means it is a combine problem. The rule for combine is small group, small group, COMBINE into big group!" Show hand motions as you say the rule.	No expected student response.	**"Now, say it with me!"**
Prompt student to say with you. **"The rule for combine is small group, small group, COMBINE into big group!"** Show hand motions as you say the rule.	Student says rule (if able) and shows hand motions.	Provide physical prompting for hand motions if necessary. **"Great! Let's turn the page and listen to our next step."**
Step 6: **"Press the button."** (Point to audio button on page 6 of talking photo album).	Student presses audio button. ►*Put the terms on the mat.*	Provide physical prompting if necessary.
"Help me label the problem mat with the terms from our problem. (*Read first sentence*). That's a small group." Use left hand motion for 'small group'. "**Put (_____) on the mat."**	Student moves Velcro label to the mat.	Provide physical prompting if necessary.
Embedded Instruction: **"Touch (*first number*)."** Simultaneously point to number.	Student touches numeral in word problem.	Provide descriptive feedback: **"That's (#)!"**
"(*Read second sentence*). That's another small group." Use right hand motion for 'small group'. **"Put (____) on the mat."**	Student moves Velcro label to the mat.	Provide physical prompting if necessary.

Instructional Cue	Student Response	Teacher Response
“(*First term*) and (*second term*) combine to make our big group, (# *last term*).” Use hand motion for ‘big group.’ **“Put (____) on the mat.”**	Student moves Velcro label to the mat.	Provide physical prompting if necessary.
Embedded Instruction: **“Touch *(last number*).”** Simultaneously point to number.	Student touches numeral in word problem.	Provide descriptive feedback: **“That’s (#)! Let’s listen to our next step.”**
Step 7: **“Press the button.”** (Point to audio button on page 7 of talking photo album).	Student presses audio button. ►*Circle the numbers*	Provide physical prompting if necessary.
Embedded Instruction: **“(*Read first sentence*). Touch (*first number*).”** Simultaneously point to number.	Student touches numeral in word problem.	Provide descriptive feedback: **“That’s (#)!”**
“(#) is a number. Help me circle/mark **(#).”** Prompt student to circle or mark the number.	Student circles number.	If student cannot hold a marker, have the student point to the number and circle for the student. **“Great! Let’s see if there are more numbers in our problem.”**
Embedded Instruction: **“(*Read second sentence*). I don’t see any numbers in this sentence. Let’s keep reading. (*Read third sentence*). Touch (*last number*).”**	Student touches numeral in word problem.	Provide descriptive feedback: **“That’s (#)!”**
“(#) is a number. Help me circle/mark **(#).”** Prompt student to circle or mark the number.	Student circles number.	Provide physical prompting if necessary. **“You circled the numbers! Let’s turn the page and listen to our next step.”**
Step 8: **“Press the button.”** (Point to audio button on page 8 of talking photo album).	Student presses audio button. ►*Put the numbers on the mat.*	Provide physical prompting if necessary.

Instructional Cue	Student Response	Teacher Response
Embedded Instruction: **"Let's look at our mat. Our first term is (______). How many (______)? (# _____). Touch (*first number*)."** Simultaneously point to the first number.	Student touches numeral on number cards.	Provide descriptive feedback: **"That's (#)!"**
"Help me move (#) to our equation." Prompt student to move Velcro number.	Student moves numeral to mat.	**"Great work!"**
"Our second term is (______). How many (_____)? Some (_____). That means we don't know how many (_____). We can use a variable, x, when we don't know how many. Help me move the x to the equation." Prompt student to move Velcro x to mat.	Student moves variable to mat.	n/a
Embedded Instruction: **"Our third term is (____). How many (_____)? (#). Touch (*last number*)."** Simultaneously point to the last number.	Student touches numeral on number cards.	Provide descriptive feedback: **"That's (#)!"**
"Help me move (#) to our equation." Prompt student to move Velcro number to mat.	Student moves numeral to mat.	**"Let's listen to our next step."**
Step 9: **"Press the button."** (Point to audio button on page 9 of talking photo album).	Student presses audio button. ►*Plus or minus?*	Provide physical prompting if necessary.

Instructional Cue	Student Response	Teacher Response
"What is our rule? '*Small group, small group, COMBINE into big group*.'" (Use hand motions). **"Small group *plus* small group combines into big group. That means we're adding. We need to use the plus sign. Help me move the plus sign to the equation."** Prompt student to move Velcro "+" to the mat.	Student moves operation to equation.	**"Let's turn the page and listen to our next step."**
Embedded Instruction: **"Great! We're going to add (__) plus x which equals (__)! Touch (*first number*)."** Simultaneously point to the first number.	Student touches numeral on problem mat.	Provide descriptive feedback: **"That's (#)!"**
Embedded Instruction: **"Touch (*last number*)."** Simultaneously point to last number.	Student touches numeral on problem mat.	Provide descriptive feedback: **"That's (#)! Let's see what's next!"**
Step 10: **"Press the button."** (Point to audio button on page 10 of talking photo album).	Student presses audio button. ►*Put what you know on the number line.*	Provide physical prompting if necessary.
Embedded Instruction: **"(*Read first sentence*). Touch (*first number*)."** Simultaneously point to first number.	Student touches numeral on number line.	Provide descriptive feedback. **"That's (#)!"**
"Let's move (#) green chips to the number line. Move the chips as I count. *1, 2, 3, 4*"	Student moves the chips to the mat.	Provide physical prompting if necessary. Praise appropriate 1-1 correspondence.
"(*Read second sentence*). We don't know how many (____). Let's keep looking."	No expected student response	n/a

Instructional Cue	Student Response	Teacher Response
Embedded Instruction: **"(*Read third sentence*). Touch (*last number*)."** Simultaneously point to the last number.	Student touches numeral on number line.	Provide descriptive feedback. **"That's (#)!"**
"Put the blue marker on (#) so we know where to stop." Point to (#) on the number line.	Student moves the marker to the mat.	Provide physical prompting if necessary. **"Let's listen to our next step."**
Step 11: **"Press the button."** Point to audio button on page 11 of the talking photo album.	Student presses audio button. ► *Count to find what's missing.*	Provide physical prompting if necessary.
Embedded Instruction: **"We have (# _____). Touch (*first number*)."** Simultaneously point to first number.	Student touches numeral on number line.	Provide descriptive feedback. **"That's (#)!"**
Embedded Instruction: **"And (# _____) all together. Touch (*last number*)."** Simultaneously point to the last number.	Student touches numeral on number line.	Provide descriptive feedback. **"That's (#)!"**
"Let's add chips to the number line to find how many ______. Move the red chips as I count. *5…6…7…8…9*."	Student adds red chips to the number line.	Provide physical prompting if necessary. Praise appropriate 1-1 correspondence. **"Great! Let's turn the page and listen to our next step."**
Step 12: **"Press the button."** Point to the audio button on page 12 of the talking photo album.	Student presses audio button, ►Solve to find x	Provide physical prompting if necessary.
"Remember, x is a variable. We used x because we didn't know how many ____. We can find x by counting the red chips. Move the red chips to the second number line as I count. *1…2…3…4…5*."	Student moves red chips to the second number line.	Provide physical prompting if necessary. Praise appropriate 1-1 correspondence.

Instructional Cue	Student Response	Teacher Response
Step 13: **"Press the button."** (Point to audio button on page 13 of the talking photo album).	Student presses audio button. ►*Write the answer*	Provide physical prompting if necessary.
"Help me move (*answer*) to our equation." Prompt student to move Velcro "#" to equation.	Student moves numeral to equation.	**"x = (#)! Great work!"**
Embedded Instruction: **(*Read equation:* # __ *plus* # __ *equals* # ___)! Touch (*first number*)."** Simultaneously point to first number.	Student touches numeral on problem mat.	Provide descriptive feedback. **"That's (#)!"**
Embedded Instruction: **"Touch (*last number*)."** Simultaneously point to the last number.	Student touches numeral on problem mat.	Provide descriptive feedback. **"That's (#)!" Great work solving a real-world problem using a linear equation!"**

Lead

During the remaining intervention sessions, you will lead the student through the process to solve a simple linear equation. If the student does not answer, or answers incorrectly, you will first provide a ***nonspecific verbal prompt*** *(read the step on the task analysis), then a* ***specific verbal prompt*** *(tell the student what to do), and then a* ***model prompt*** *(show the student what to do). Additionally, there will be 20 opportunities for embedded simultaneous prompting where you will prompt the student to receptively identify numerals.*

In Phase A, you will use one of three word problems each time you present this lesson, either $4 + x = 9$*,* $3 + x = 9$*, or* $3 + x = 4$*. In Phase B, you will use* $5 + x = 8$*,* $1 + x = 8$*, or* $1 + x = 5$*. In Phase C, you will use* $6 + x = 7$*,* $2 + x = 7$*, or* $2 + x = 6$*. For the purpose of this script, we will use the following word problem:* Jared has 3 dogs. He also has some (x) cats. Jared has 4 pets all together. How many cats does he have?

Instructional Cue	Student Response	Teacher Response
Deliver attentional cue: **"Now it's your turn to solve a real-world math problem using a linear equation. Show me you are ready!"**	Student attends to materials.	**"Let's get started!"**
Step 1: "**Here are your materials."** Present materials (task analysis, word problem, problem mats, number line, manipulatives) to the student. Open the talking photo album to the first page.	Student reads the problem or activates the read-aloud function	Provide high-quality praise. Move to embedded instruction.
	No response	Nonspecific verbal prompt: "**What's our first step? Press the button."** Prompt the student to activate the talk aloud button on the photo album: *Read the problem.*
	Student reads the word problem or activates the read-aloud function.	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: "**Touch the green button to read it aloud**."
	Student reads the word problem or activates the read-aloud function.	Provide low-quality praise. Move to embedded instruction.
	No response	Model prompt: **Touch this button to read the problem.** Point to the GoTalk button. Move to embedded instruction.

Instructional Cue	Student Response	Teacher Response
Embedded Instruction: **“Jared has 3 dogs. Touch 3.”** Simultaneously point to 3.	Student touches numeral in word problem.	Provide descriptive feedback. **“That’s 3! 3 dogs.”**
Embedded Instruction: “**He has 4 pets in all. Touch 4.”** Simultaneously point to 4.	Student touches numeral in word problem.	Provide descriptive feedback: **“That’s 4!”**
Step 2: (Turn photo album page if student is unable. If student is unable to hold a marker, have them point to the terms, and you will circle them).	Student circles the terms.	Provide high-quality praise. Move to step 3.
	No response	Nonspecific verbal prompt: **“What’s next? Turn the page and press the button!”** Prompt student to activate the talk aloud button on the photo album: *Circle the terms.*
	Student circles the terms.	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: **“Terms are the pieces of our algebra problem. In this problem, I see three terms: dogs, cats, and pets**.
	Student circles the terms.	Provide low-quality praise. Move to embedded instruction.
	No response	Model prompt: Guide the student to circle *dogs*, *cats* and *pets*. Move to embedded instruction.
Embedded Instruction: **“Jared has 3 of our first term, dogs. Touch 3.”** Simultaneously point to 3.	Student touches numeral in word problem.	Provide descriptive feedback: **“3 dogs!”**
Embedded Instruction: **“We don’t know how many Jared has of our second term, cats. But we do know Jared has 4 of our last term, pets. Touch 4.”** Simultaneously point to 4.	Student touches numeral in word problem.	Provide descriptive feedback: **“4 pets!”** Move to step 3.

Instructional Cue	Student Response	Teacher Response
Step 3: (Turn photo album page if student is unable).	Student points to correct schema (different) on task analysis.	Provide high-quality praise. Move to step 4.
	No response.	Nonspecific verbal prompt: **"What's next? Press the button."** Prompt student to activate the talk aloud button on the photo album: *Same, different, more or fewer?*
	Student points to correct schema (different) on task analysis.	Provide moderate praise. Move to step 4.
	No response.	Specific verbal prompt: "**Dogs and cats are different. Touch *different*.**"
	Student points to correct schema (different) on task analysis.	Provide low-quality praise. Move to step 4.
	No response.	Model prompt: Gesture to "different" on the task analysis. Move to step 4.
Step 4: (Turn photo album page if student is unable).	Student selects correct problem mat (combine).	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **"What's next? Turn the page and press the button."** Prompt student to activate the talk aloud button on the photo album: *Choose the problem mat*.
	Student selects correct problem mat (combine).	Provide moderate praise. Move to embedded instruction.

Instructional Cue	Student Response	Teacher Response
(Step 4 continued)	No response.	Specific verbal prompt: **"We need the problem mat for 'different'. Combine problems are about two small groups of different things we combine into one big group. The Combine problem mat is the one for different."**
	Student selects correct problem mat (combine).	Provide low-quality praise. Move to embedded instruction.
	No response.	Model prompt: Point to correct problem mat (combine). Move to embedded instruction.
Embedded Instruction: **"Good work! We are combining 3 dogs and some cats to make 4 pets. Touch 3."** Simultaneously point to 3.	Student touches numeral in word problem.	Provide descriptive feedback: **"3 dogs!"**
Embedded Instruction: **"Touch 4."** Simultaneously point to 4.	Student touches numeral in word problem.	Provide descriptive feedback: **"4 pets!"** Mover to step 5.
Step 5: (Turn photo album page if student is unable).	Student says the rule and/or uses hand motions.	Provide high-quality praise. Move to step 6.
	No response.	Nonspecific verbal prompt: **"What's next? Press the button."** Prompt student to activate the talk aloud button on the photo album: *Use my rule*.
	Student says the rule and/or uses hand motions.	Provide moderate praise. Move to step 6.

Instructional Cue	Student Response	Teacher Response
	No response.	Specific verbal prompt: **"This problem is about two small groups of different things we combine into a big group. That means it is a combine problem. The rule for combine is '*small group, small group, COMBINE into big group!*'"**
	Student says the rule and/or uses hand motions.	Provide low-quality praise. Move to step 6.
	No response.	Model prompt: Guide the student to make the hand motions for the rule.
Step 6: (Turn photo album page if student is unable).	Student puts the terms on the mat.	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **"What's next? Turn the page and press the button."** Prompt the student to activate the talk aloud button on the photo album: *Put the terms on the mat.*
	Student puts the terms on the mat.	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: **"Label the problem mat with the terms from the problem. Jared has dogs. That's a small group." "He has cats. That's another small group." "Dogs and cats combine to make the big group, pets."**
	Student puts the terms on the mat.	Provide low-quality praise. Move to embedded instruction.

Instructional Cue	Student Response	Teacher Response
	No response	Model prompt: Guide the student to move the “dogs” icon, “cats” icon, and “pets” icon to the mat. Move to embedded instruction.
Embedded Instruction: **“Our first small group is 3 dogs. Touch 3.”** Simultaneously point to 3.	Student touches numeral in word problem.	Provide descriptive feedback: **“3 dogs is our small group!”**
Embedded Instruction: **“The big group is 4 pets. Touch 4.”** Simultaneously point to 4.	Student touches numeral in word problem.	Provide descriptive feedback: **“4 pets!”** Move to step 7.
Step 7: (Turn photo album page if student is unable. If student is unable to hold a marker, have them point to the numbers and circle for them).	Student circles the numbers.	Provide high-quality praise. Move to embedded instruction.
	No response	Nonspecific verbal prompt: **“What’s next? Press the button.”** Prompt student to activate the talk aloud button on the photo album: *Circle the numbers*.
	Student circles the numbers.	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: **“Jared has 3 dogs. 3 is a number. Circle 3.”** Pause. **“He has 4 pets. 4 is a number. Circle 4”**
	Student circles the numbers.	Provide low-quality praise. Move to embedded instruction.
	No response	Model prompt: Guide the student to circle “3” and “4”. Move to embedded instruction.
Embedded Instruction: **“Jared has 3 dogs. Touch 3.”** Simultaneously point to 3.	Student touches numeral in word problem.	Provide descriptive feedback: **“That’s 3!”**
Embedded Instruction: **“He has 4 pets. Touch 4.”** Simultaneously point to 4.	Student touches numeral in word problem.	Provide descriptive feedback: **“That’s 4!”** Move to step 8

Instructional Cue	Student Response	Teacher Response
Step 8: (Turn photo album page if student is unable).	Student puts numbers on the mat.	Provide high-quality praise. Move to embedded instruction.
	No response	Nonspecific verbal prompt: **"What's next? Turn the page and press the button."** Prompt student to activate the talk aloud button on the photo album: *Put the numbers on the mat*.
	Student puts numbers on the mat.	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: **"Let's look at our mat. Our first term is dogs. How many dogs? 3 dogs. Move 3 to the mat."** Pause. If student does not move numeral, provide model prompt. If student does move numeral, continue with specific verbal prompt. **"Our second term is cats. How many cats? Some cats. That means we don't know how many cats. We can use a variable, x, when we don't know how many. Move the x to the mat."** Pause. If student does not move numeral, provide model prompt. If student does move numeral, continue with specific verbal prompt. **"Our third term is 'pets'. How many pets? 4 pets. Move 4 to the mat."**

Instructional Cue	Student Response	Teacher Response
(Step 8 continued)	Student puts numbers on the mat.	Provide low-quality praise. Move to embedded instruction.
	No response	Model prompt: Guide the student to put the numbers on the mat. Move to embedded instruction.
Embedded Instruction: **"He has 3 dogs. Touch 3."** Simultaneously point to 3.	Student touches numeral on the mat.	Provide descriptive feedback: **"That's 3!"**
Embedded Instruction: **"He has 4 pets. Touch 4."** Simultaneously point to 4.	Student touches numeral on the mat.	Provide descriptive feedback: **"That's 4!"** Move to step 9.
Step 9: (Turn photo album page if student is unable).	Student chooses correct operation (+)	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **"What's next? Press the button."** Prompt student to activate the talk aloud button on the photo album: *Plus or minus?*
	Student chooses correct operation (+)	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: **"What is our rule? *'Small group, small group, COMBINE into big group'.*"** (Use hand motions). **"Small group plus small group combines into big group. That means we're adding. We need to use the plus sign."**
	Student chooses correct operation (+).	Provide low-quality praise. Move to embedded instruction.

Instructional Cue	Student Response	Teacher Response
	No response	Model prompt: Guide the student to move the Velcro plus sign (+) to the mat. Move to embedded instruction.
Embedded Instruction: **“Awesome! We’re going to add 3 plus x which equals 4! Touch 3.”** Simultaneously point to 3.	Student touches numeral on problem mat.	Provide descriptive feedback: **“That’s 3!”**
Embedded Instruction: **“Touch 4.”** Simultaneously point to 4.	Student touches numeral on problem mat.	Provide descriptive feedback: **“That’s 4!”** Move to step 10.
Step 10: (Turn photo album page if student is unable).	Student places the green chips and blue marker on the number line.	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **“What’s next? Turn the page and press the button.”** Prompt student to activate the talk aloud button on the photo album: *Put what you know on the number line*.
	Student places the green chips and blue marker on the number line.	Provide moderate praise. Move to embedded instruction.
	No response.	Specific verbal prompt: **“Put what you know on the number line. Jared has 3 dogs. Put 3 green chips on the number line.”** Pause. If student does not put chips on number line, provide model prompt. If student does move chips, continue with specific verbal prompt. **“Jared has 4 pets in all. Put the blue marker on 4 so we know where to stop.”** If student does not put marker on number line, move to model prompt.

Instructional Cue	Student Response	Teacher Response
	Student places the green chips and blue marker on the number line.	Provide low-quality praise. Move to embedded instruction.
	No response.	Model prompt: Guide the student to move manipulatives to the number line as you count aloud.
Embedded Instruction: **"He has 3 dogs. Touch 3."** Simultaneously point to 3.	Student touches numeral on number line.	Provide descriptive feedback: **"That's 3!"**
Embedded Instruction: **"He has 4 pets all together. Touch 4."** Simultaneously point to 4.	Student touches numeral on number line.	Provide descriptive feedback: **"That's 4!"** Move to step 11.
Step 11: (Turn photo album page if student is unable).	Student counts to find what's missing.	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **"What's next? Press the button."** Prompt student to activate the talk aloud button on the photo album: *Count to find what's missing.*
	Student counts to find what's missing.	Provide moderate praise. Move to embedded instruction.
	No response.	Specific verbal prompt: **"Add red chips to the number line to find how many cats."**
	Student counts to find what's missing.	Provide low-quality praise. Move to embedded instruction.
	No response.	Model prompt: Guide the student to add 1 red chip to the number line. Count aloud as you add: **"4!"** Move to embedded instruction.

Instructional Cue	Student Response	Teacher Response
Embedded Instruction: **“We have 3 dogs. Touch 3.”** Simultaneously point to 3.	Student touches numeral on number line.	Provide descriptive feedback. **“That’s 3!”**
Embedded Instruction: **“And 4 pets all together. Touch 4.”** Simultaneously point to 4.	Student touches numeral on number line.	Provide descriptive feedback. **“That’s 4!”** Move to step 12.
Step 12: (Turn photo album page if student is unable).	Student moves red chips to second number line.	Provide high-quality praise. Move to step 13.
	No response.	Nonspecific verbal prompt: **“What’s next? Turn the page and press the button.”** Prompt student to activate the talk aloud button on the photo album: *Solve to find x.*
	Student moves red chips to second number line.	Provide moderate praise. Move to step 13.
	No response.	Specific verbal prompt: **“Remember, x is a variable. We used x because we didn’t know how many cats. We can find x by counting the red chips. Move the red chips to the second number line to find x.”**
	Student moves red chips to second number line.	Provide low-quality praise. Move to step 13.
	No response.	Model prompt: Guide the student to move the red chip to the second number line, counting as they move: **“1”** Move to step 13.
Step 13: (Turn photo album page if student is unable).	Student moves Velcro “1” to $x =$.	Provide high-quality praise. Move to embedded instruction.

Instructional Cue	Student Response	Teacher Response
	No response.	Nonspecific verbal prompt: **“What’s next? Press the button.”** Prompt student to activate the talk aloud button on the photo album: *Write the answer*.
	Student moves Velcro “1” to $x =$.	Provide moderate praise. Move to embedded instruction.
	No response.	Specific verbal prompt: **“How many cats? Let’s count. 1. 1 cat. Move 1 to $\boldsymbol{x} =$.”**
	Student moves Velcro “1” to $x =$.	Provide low-quality praise. Move to embedded instruction.
	No response.	Model prompt: Guide the student to move the Velcro “1” to $x =$. Move to embedded instruction.
Embedded Instruction: **“3 dogs plus 1 cat equals 4 pets! Touch 3.”** Simultaneously point to 3.	Student touches numeral on problem mat.	Provide descriptive feedback: **“That’s 3!”**
Embedded Instruction: **“Touch 4.”** Simultaneously point to 4.	Student touches numeral on problem mat.	Provide descriptive feedback: **“That’s 4! Awesome job solving a real-world problem using a linear equation!”** End session

Test

On generalization days, you will test the student on their ability to solve a simple linear equation. You will not provide any prompts or reinforcement. There will be no embedded simultaneous prompting. You will use a different word problem each time you present this lesson.

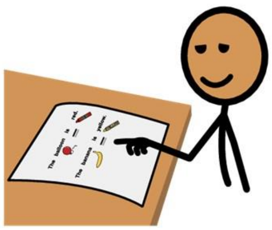
Instructional Cue	Student Response	Teacher Response
Present all instructional materials and deliver attentional cue: **“Now you are going to solve a real-world problem using a linear equation all by yourself. Show me you are ready!”**	Student attends to materials.	**“Let’s get started!”**
Step 1: (Open photo album to first page if student is unable).	Student reads the problem or activates the read-aloud function.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Read the problem aloud, or activate the read-aloud function.
Step 2: (Turn photo album page if student is unable).	Student circles the terms.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Circle the terms for the student (do not use physical prompting).
Step 3: (Turn photo album page if student is unable).	Student points to “different” on the task analysis.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Point to “different”.
Step 4: (Turn photo album page if student is unable).	Student selects combine problem mat.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Select the combine problem mat.
Step 5: (Turn photo album page if student is unable).	Student says the rule and/or uses hand motions.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Say **“small group, small group, Combine into big group!”** with hand motions.

Instructional Cue	Student Response	Teacher Response
Step 6: (Turn photo album page if student is unable).	Student puts the terms on the mat.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Put the terms on the mat for the student.
Step 7: (Turn photo album page if student is unable).	Student circles the numbers.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Circle the numbers for the student.
Step 8: (Turn photo album page if student is unable).	Student puts the numbers on the mat.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Put the numbers on the mat for the student.
Step 9: (Turn photo album page if student is unable).	Student selects (+).	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Move the (+) to the equation.
Step 10: (Turn photo album page if student is unable).	Students puts green chips and the blue marker on the number line.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Move the manipulatives to the number line.
Step 11: (Turn photo album page if student is unable).	Student counts up to find what's missing.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Move the red chips to the number line.
Step 12: (Turn photo album page if student is unable).	Student moves the red chips to the second number line.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Move the red chips to the number line.
Step 13: (Turn photo album page if student is unable).	Student moves the number to solve the question.	Mark a + on the data sheet.
	No response or incorrect.	Mark a – on the data sheet. Move the Velcro number to the $x =$. Provide praise for completion. End session.

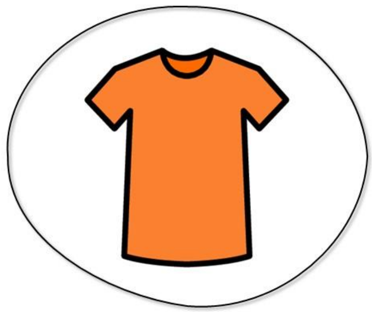
Appendix H

Task Analysis

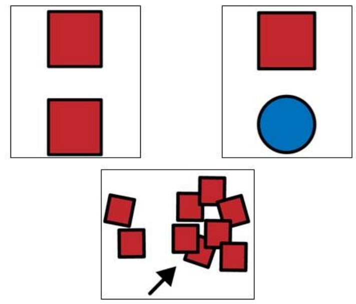
Read the problem.



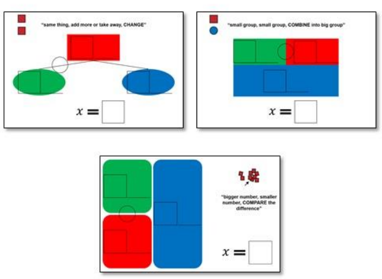
Circle the terms.



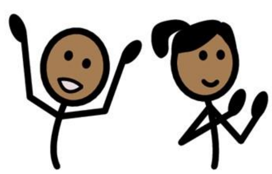
Same, Different, More/Fewer?



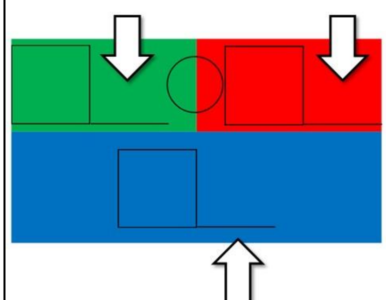
Choose the problem mat.



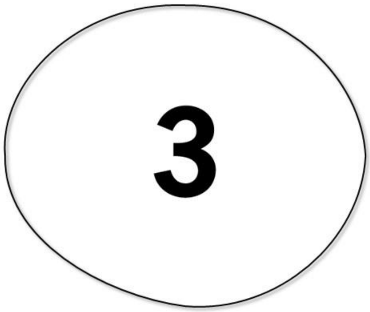
Use my rule.



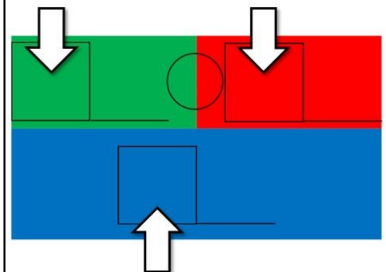
Put the terms on the mat.



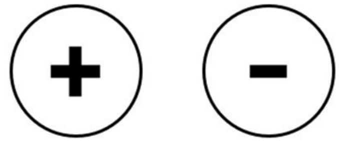
Circle the numbers.



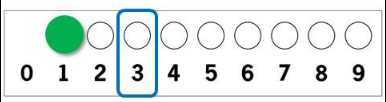
Put the numbers on the mat.



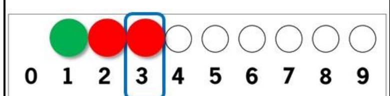
+ or −?



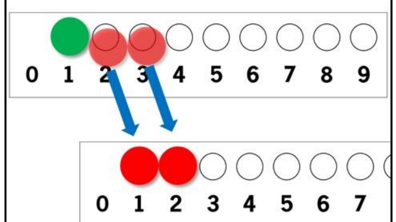
Put what you know on the number line.



Count to find what's missing.

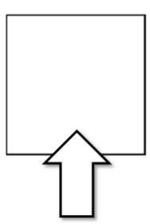


Solve to find x.



Write the answer.

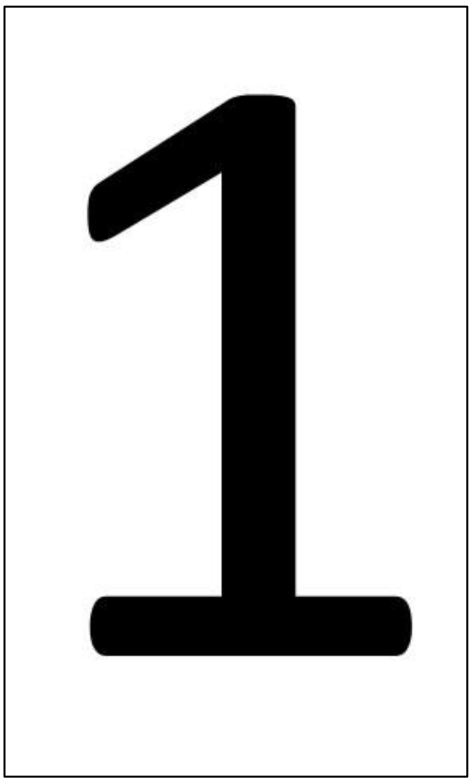
$x =$

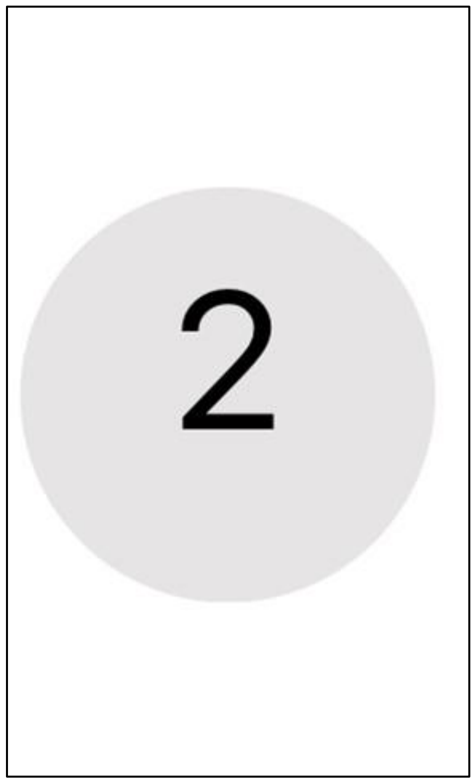


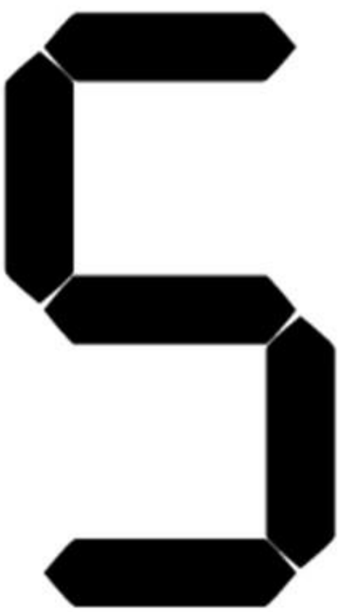
Appendix I

Numeracy Probe Cards

Each numeral (1–9) is included in each set. The sets include numerals from the number line in this intervention, numerals from an iPhone call number pad, numerals from a TI-84 calculator, numerals from an elevator, and numerals from a digital alarm clock. Additional numerals relevant to each participant's environment will be added once the preference assessment is complete.







6

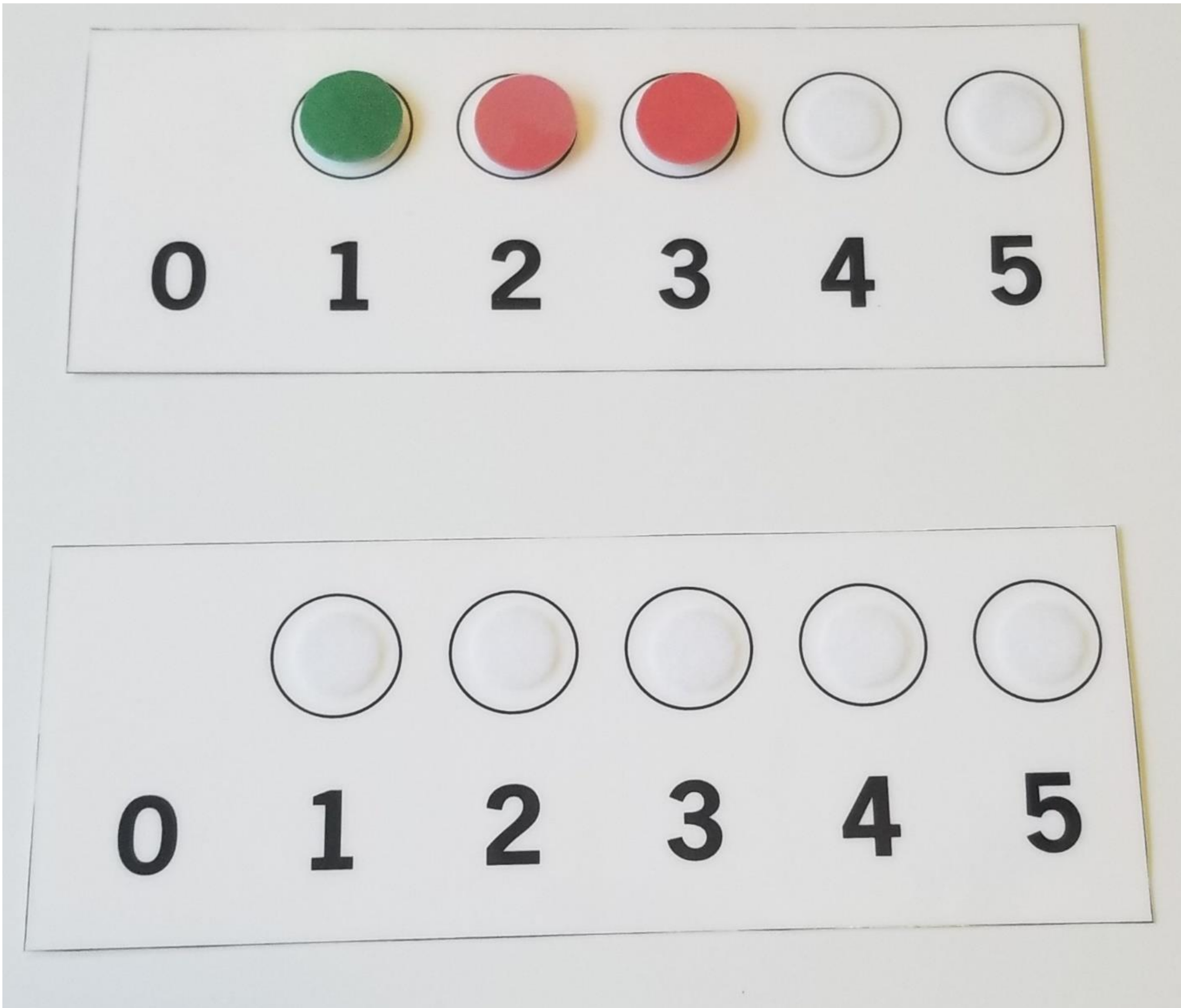

7


8


9

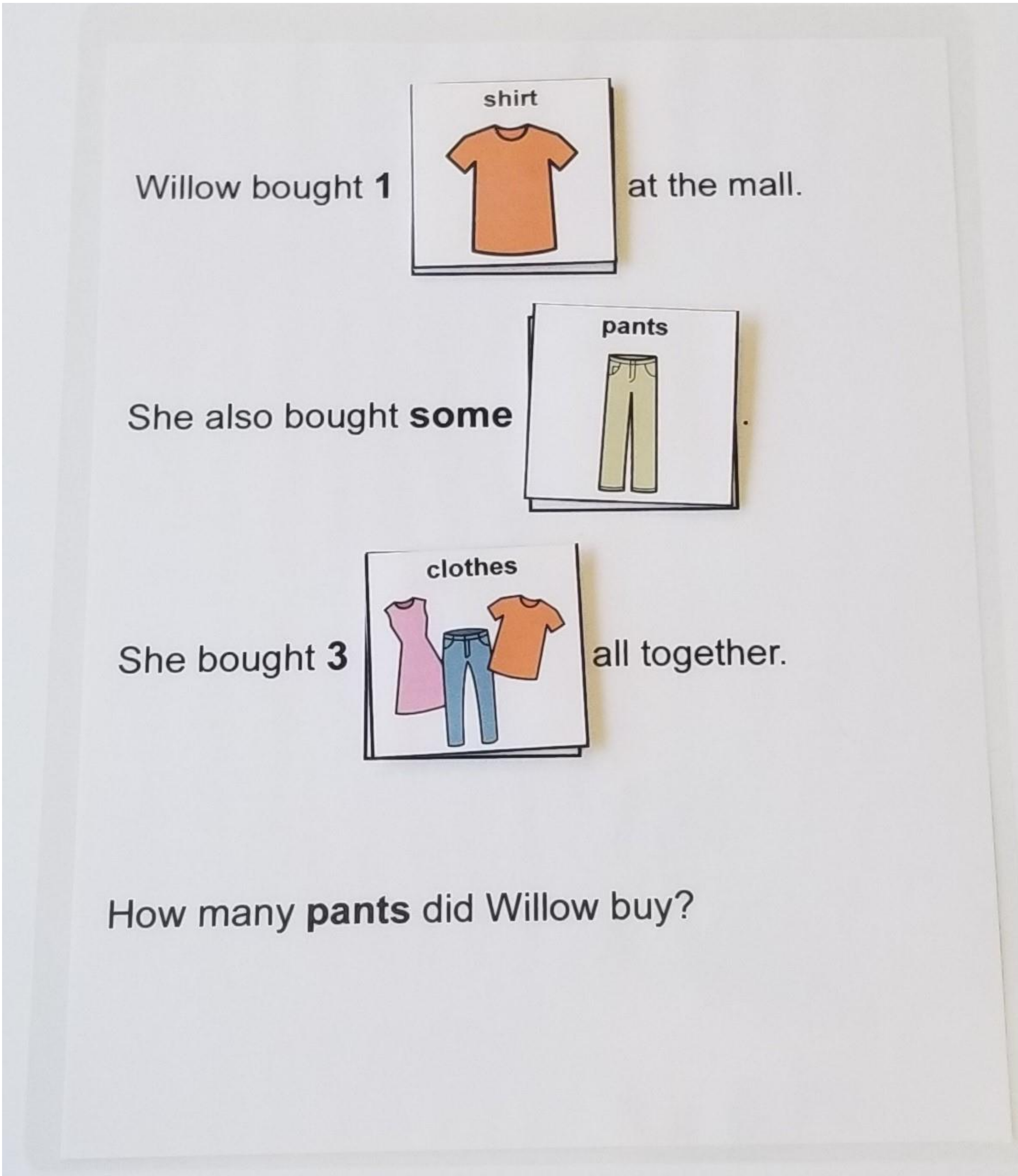
Appendix J

Number Line



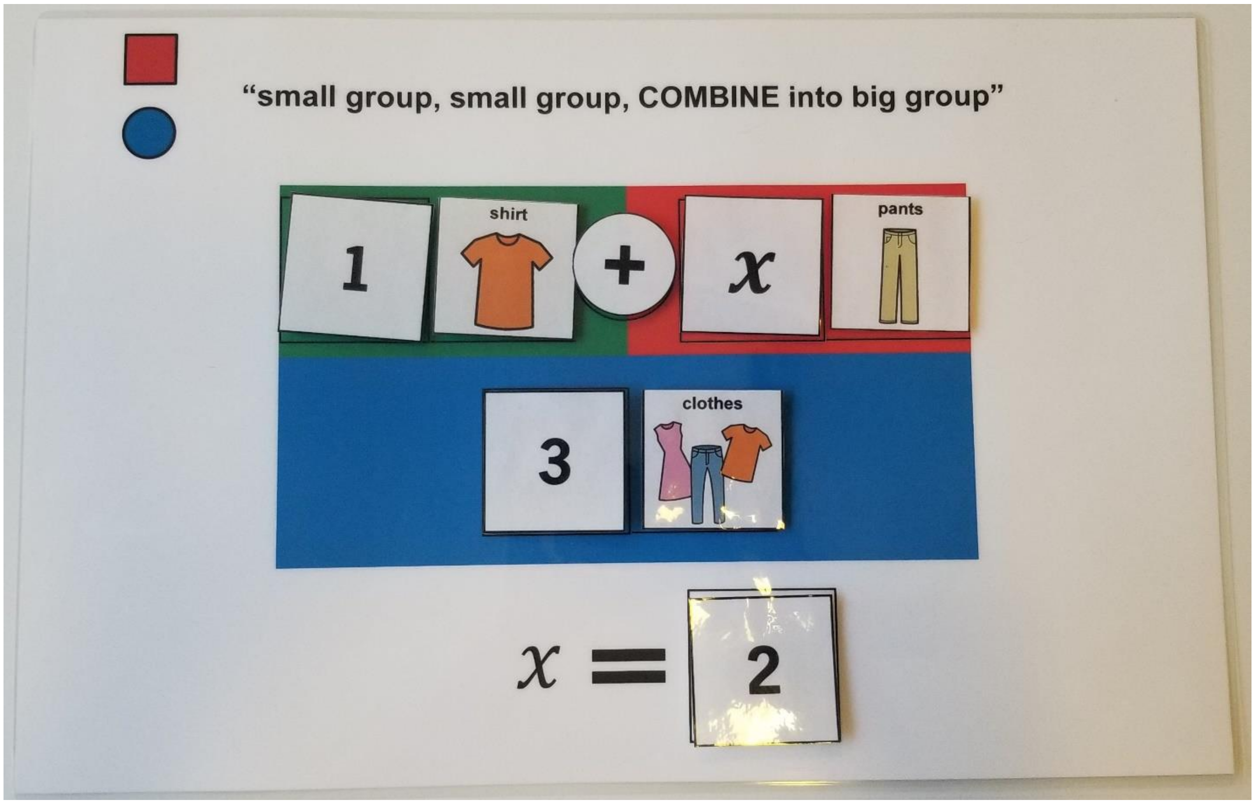
Appendix K

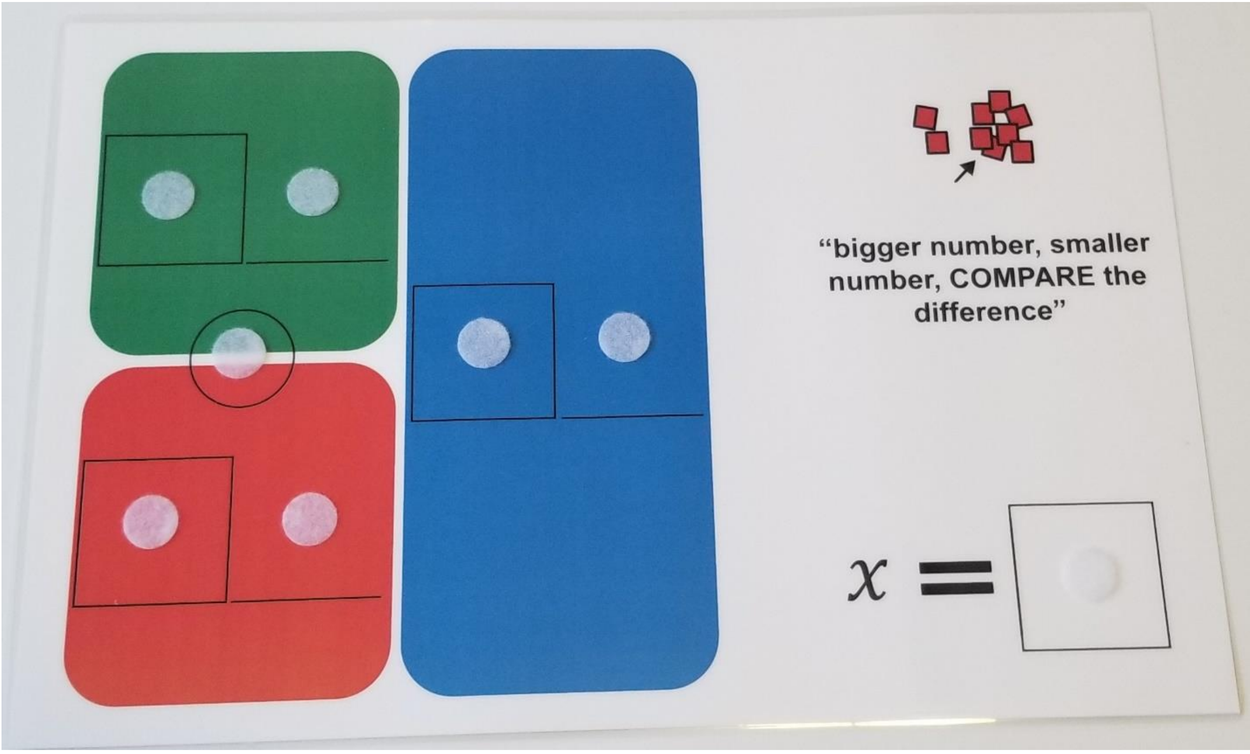
Sample Word Problem

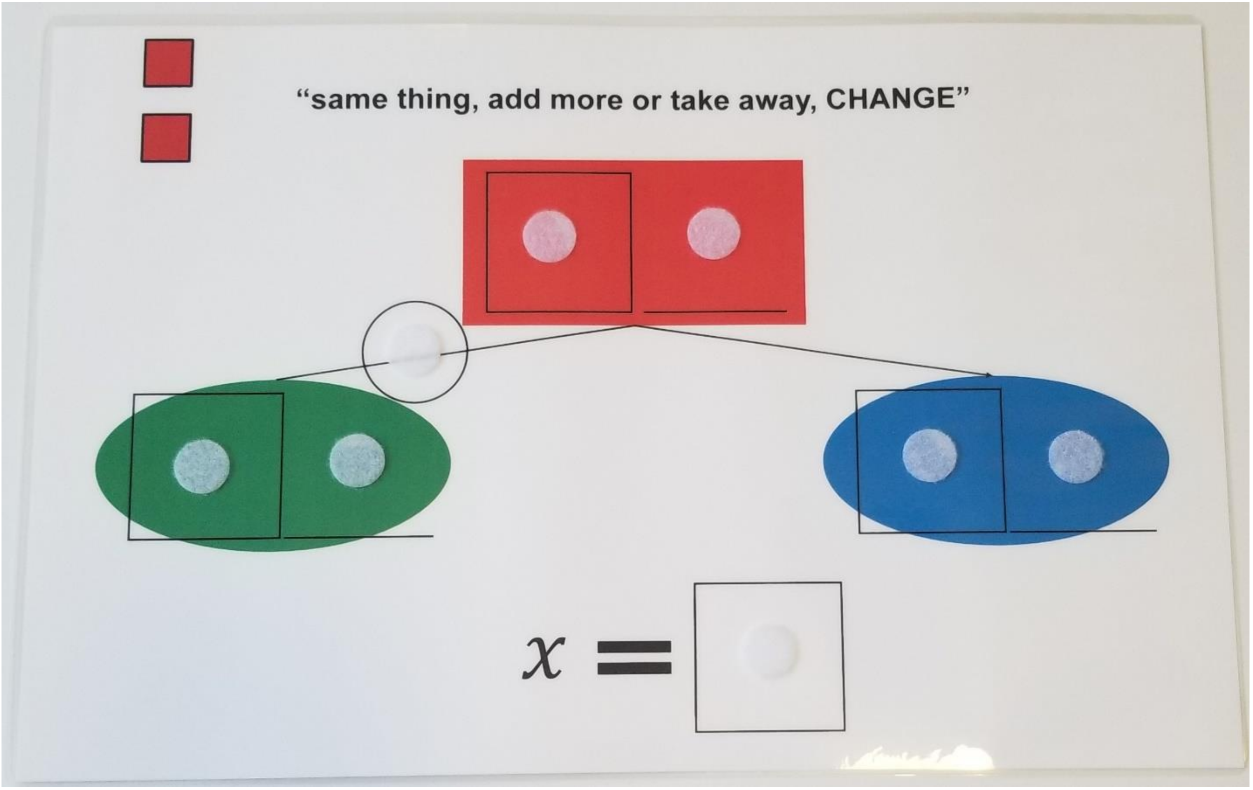


Appendix L

Graphic Organizers






"same thing, add more or take away, CHANGE"
$x =$

Appendix M

Numeral Identification Data Sheet

Student Code: ___________________

Response Mode: _________________

Date									
Card Set									
1									
2									
3									
4									
5									
6									
7									
8									
9									
+									
–									
Student Affect (P or N)									

Key:
+: correct
−: incorrect

P: Positive Affect (Smiling, Engaged, Happy)
N: Negative Affect (Upset, Trying to escape, Frowning)

Appendix N

Linear Equation Data Sheet

When given a word problem requiring a simple linear equation, defined as an equation in which the variable is always the second addend and the absence of coefficients, the student will use a visual task analysis to solve for x.

Student Code: ______________________________ Response Mode: ____________________________	Date:	Date:	Date:	Date:
Step 1: Read the problem (ER): Student activates read-aloud function				
Step 2: Find the terms (ER): Student circles terms (3 total)				
Step 3: Same, different, more/fewer (ER): Student points to the icon representing different				
Step 4: Choose problem mat (ER): Student selects "combine" problem mat				
Step 5: Use my rule (ER): Student says the rule and demonstrates hand motions				
Step 6: Put terms on mat (ER): Student moves the Velcro labels to the mat				
Step 7: Circle the numbers (ER): Student circles the numbers (2 total)				
Step 8: Put numbers on the mat (ER): Student moves the Velcro numbers and variable to the mat				
Step 9: + or − (ER): Student moves the Velcro + to the mat				
Step 10: Put what you know on the number line (ER): Student puts green chips and blue marker on number line				
Step 11: Count to find what's missing (ER): Student counts up to solve, adding red chips				
Step 12: Solve to find x (ER): Student moves red chips to second number line				
Step 12: Write answer (ER): Student moves Velcro number to $x =$				
# correct				

Key:

ER (expected response): How the student is expected to respond
+: Correct response
−: Incorrect response/No response

Appendix O

Instructor Social Validity Questionnaire

Pre-Study Social Validity Questionnaire

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
It is important to teach grade-level academics like algebra to students with ESN.	1	2	3	4	5
It is important to teach foundational academic skills like numeracy to high school students with ESN.	1	2	3	4	5
Embedding numeracy instruction in standards-based lessons is an *effective* instructional strategy for high school students with ESN.	1	2	3	4	5
Embedding numeracy instruction in standards-based lessons is an *efficient* instructional strategy for high school students with ESN.	1	2	3	4	5
The target students in this study can reliably identify the numbers 1–9.	1	2	3	4	5
The target student made progress in algebra skills.	1	2	3	4	5

Describe how the student currently interacts with numerals in their environment:

Post-Study Social Validity Questionnaire

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree
It is important to teach grade-level academics like algebra to students with ESN.	1	2	3	4	5
It is important to teach foundational academic skills like numeracy to high school students with ESN.	1	2	3	4	5
Embedding numeracy instruction in standards-based lessons is an *effective* instructional strategy for high school students with ESN.	1	2	3	4	5
Embedding numeracy instruction in standards-based lessons is an *efficient* instructional strategy for high school students with ESN.	1	2	3	4	5
The target students in this study can reliably identify the numbers 1–9.	1	2	3	4	5
Had the study continued, I believe the target student would have learned to identify the numbers 1 through 9.	1	2	3	4	5
The target student can solve simple linear equations (i.e., 1 + x = 3), given supports like a task analysis, graphic organizer, and manipulatives.	1	2	3	4	5
Had the study continued, I believe the student would have learned to solve simple linear equations, given supports like a task analysis, graphic organizer, and manipulatives	1	2	3	4	5
I will use the strategies and procedures I learned in this study with future students when teaching numeral identification and/or linear equations	1	2	3	4	5

Is there anything else you would like to share?

Appendix P

Baseline Behavioral Skills Training Protocol

Describe, Part 1

1. **Numeracy Probe:** Researcher meets with participating teachers and provides brief description of baseline procedures. (10 min).
    a. "Baseline essentially refers to where the students are performing, without any intervention. It is important for us to collect baseline data to see what the student's need instruction on. It is also important for us as researchers to collect baseline data to demonstrate whether or not the intervention works. We will be using a multiple probe design for this study. That means each student will begin intervention at a different time. Student 1, for example, may begin intervention in the first week of October. Student 2, maybe not till the third week of October, Student 3, middle of November, and so on. We do need you to continue taking baseline during this time—not every day, but at least once a week.
    Everybody will collect the first data point on the same day. After that, I will let you know which day you will collect. Per research standards, we need at least 5 days of data, 3 of which are consecutive. I know it might seem strange to take such a long time before starting instruction, but it is important to help show that the intervention is working.
    b. You will conduct two separate probes during baseline, a numeracy probe, and a linear equation probe. For the numeracy probe, you will see if the student can receptively identify numerals. You will have different sets of cards (1–9). We want to present multiple fonts/colors/backgrounds so they student doesn't just learn to identify one version of the numeral. Each time you probe, use a different set of cards. Be sure to list which set of cards you are using on the data sheet (see Appendix M).
    You will select three cards at random and place them in front of the student. You will then ask the student to identify a numeral. Don't go in order (1, 2, 3…), be sure to mix them up. You will tell the student "touch _." If the student gets it correct, mark a + on the data sheet. If the student gets it incorrect, or if they don't respond within 3 s, mark a – on the data sheet. Don't praise the student or correct them. Just move on to the next numeral. Between trials, you will remove all the cards, and choose three more at random. Make sure that you also change the position of the correct numeral (e.g., don't always place it in the middle).

Model, Part 1

2. **Numeracy Probe**: Researcher models numeracy probe with the teacher acting as the student (10 min).
    a. Model numeracy probe following scripted lesson plan; see Appendix G.
    b. Present video model (0:00–2:12) for numeracy probe
    c. Present video model (2:13–3:27) for generalization probe

Rehearsal, Part 1

3. **Numeracy Probe**: Teacher rehearses numeracy probe with the researcher acting as the student. Researcher provides corrective and supportive feedback. Rehearsal continues until teacher reaches 100% fidelity in one session (see Appendix Q). (5 min).

Describe, Part 2

4. **Linear Equation Probe**: Researcher meets with participating teachers and provides brief description of baseline procedures. (10 min).
    a. Also during baseline, you will collect data on the student's ability to solve a simple linear equation. It is likely that the student will not know how to do this, because we have not taught it yet! It is still important to collect this data, however. This phase of the study may seem a little strange to you as well, as I haven't taught you how to teach it yet. We will go over that before we move in to baseline.
    b. At the start of this probe, you will present the materials to the student. These include a talking photo album with the task analysis, the word problem, with movable icons, three problem mats with movable number cards, and two number lines, with green chips, red chips, and a blue marker. Then you will tell them to solve the problem. It is likely that the student will not follow the steps of the task analysis. However, you will give them the opportunity to complete each step, by waiting 3 s each time. If the student does not complete the step within 3 s, or if they answer incorrectly, you will complete the step for the student. You will not prompt them to complete the step, you will simply do it for them. Then you will wait 3 more seconds for the next step.
    c. The steps of the task analysis are as follows:
        1) Read the problem. If the student does not do this step, you will complete it for them, by activating this read-aloud button. (Important—before each day's session, you will want to record your voice, reading the word problem aloud!)
        2) Circle the terms. If the student does not do this step, you will complete it for them, by circling the terms in the word problem using a dry erase marker. The terms will be easy to identify, because they will be velcroed icons on the word problem.
        3) Same, Different, More/Fewer. In this study, every word problem will have terms that are different. If the student does not do this step, you will complete it by pointing to the icon "different" on the task analysis.
        4) Choose the problem mat. In this study, every word problem will require the Combine problem mat. If the student does not do this step, you will place the Combine mat in front of them and remove the other two mats.
        5) Use my rule. In this study, you will use the rule, "small group, small group, COMBINE into big group" with these hand motions (*demonstrate*). If the student does not do this step, you will chant the rule with the hand motions.

6) Put terms on the mat. If the student does not do this step, you will complete it for them by moving the Velcro icons from the word problem to the mat. The first term (the known small group) will go on the green line. The second term (the unknown small group) will go on the red line. The third term (the big group) will go on the blue line.
7) Circle the numbers. If the student does not do this step, you will complete it for them, by circling the numbers in the word problem using a dry erase marker.
8) Put the numbers on the mat. If the student does not do this step, you will complete it for them by moving the Velcro number cards to the mat. You will place the first number in the green box. You will place the variable x in the red box, and the last number in the blue box.
9) Plus or minus? In this study, every word problem will require addition. If the student does not do this step, you will complete it by moving the "plus" icon from the response board to the mat.
10) Put what you know on the number line. If the student does not do this step, you will complete it for them, first by placing the appropriate number of green chips on the number line. Then, you will place the blue marker box around the "big group" number on the number line.
11) Count to find what's missing. If the student does not do this step, you will complete it for them by placing the red chips on the number line to solve the problem.
12) Solve to find x. If the student does not do this step, you will complete it for them by moving the red chips from the first number line to the second number line.
13) Write the answer. If the student does not do this step, you will complete it for them by moving the correct numeral card from the response board to the equation on the mat.

Upon completion of the probe, you may provide reinforcement for completion (e.g., "Thank you for working with me!").

Model, Part 2

5. **Linear Equation Probe**: Researcher models linear equation probe with the teacher acting as the student (10 min).
    a. Model linear equation probe following scripted lesson plan; see Appendix G.
    b. Present video model (21:04–25:34) for Test

Rehearsal, Part 2

6. **Linear Equation Probe**: Teacher rehearses linear equation probe with the researcher acting as the student. Researcher provides corrective and supportive feedback. Rehearsal continues until teacher reaches 100% fidelity in one session (see Appendix Q). (5 min).

Appendix Q

Procedural Fidelity Data Sheets

Numeracy Probe

Component	Expectation	Fidelity
Attentional Cue	Deliver attentional cue	
Trial 1	Present 5 cards in random array. Probe. Collect data.	
Trial 2	Present 5 cards in random array. Probe. Collect data.	
Trial 3	Present 5 cards in random array. Probe. Collect data.	
Trial 4	Present 5 cards in random array. Probe. Collect data.	
Trial 5	Present 5 cards in random array. Probe. Collect data.	
Trial 6	Present 5 cards in random array. Probe. Collect data.	
Trial 7	Present 5 cards in random array. Probe. Collect data.	
Trial 8	Present 5 cards in random array. Probe. Collect data.	
Trial 9	Present 5 cards in random array. Probe. Collect data.	
	Total +	
	Total Opportunities	
	% Fidelity	

Model

Component	Expectation	Fidelity
Attentional Cue	Deliver attentional cue	
Present materials	Present materials	
Step 1	Model step 1	
Embedded Trial 1	Simultaneously provide controlling prompt.	
Embedded Trial 2	Simultaneously provide controlling prompt.	
Step 2	Model step 2	
Step 3	Model step 3	
Step 4	Model step 4	
Step 5	Model step 5	
Step 6	Model step 6	
Step 7	Model step 7	
Embedded Trial 3	Simultaneously provide controlling prompt.	
Embedded Trial 4	Simultaneously provide controlling prompt.	
Step 8	Model step 8	
Embedded Trial 5	Simultaneously provide controlling prompt.	
Embedded Trial 6	Simultaneously provide controlling prompt.	
Step 9	Model step 9	
Step 10	Model step 10	
Embedded Trial 7	Simultaneously provide controlling prompt.	
Embedded Trial 8	Simultaneously provide controlling prompt.	
Step 11	Model step 11	
Embedded Trial 9	Simultaneously provide controlling prompt.	
Embedded Trial 10	Simultaneously provide controlling prompt.	
Step 12	Model step 12	
Embedded Trial 11	Simultaneously provide controlling prompt.	
Step 13	Model step 13	
Embedded Trial 12	Simultaneously provide controlling prompt.	
Embedded Trial 13	Simultaneously provide controlling prompt	
Embedded Trial 14	Simultaneously provide controlling prompt.	
Embedded Trial 15	Simultaneously provide controlling prompt.	
	Total +	
	Total Opportunities	
	% Fidelity	

Lead

Component	Expectation	Fidelity
Attentional Cue	Deliver attentional cue	
Present Materials	Present materials	
Step 1: Read the problem	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 1	Simultaneously provide controlling prompt	
Embedded Trial 2	Simultaneously provide controlling prompt	
Step 2: Circle the terms	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 3	Simultaneously provide controlling prompt	
Embedded Trial 4	Simultaneously provide controlling prompt	
Step 3: Same, Different, More/Fewer	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Step 4: Choose problem mat	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 5	Simultaneously provide controlling prompt	
Embedded Trial 6	Simultaneously provide controlling prompt	
Step 5: Use my rule	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Step 6: Put terms on mat	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 7	Simultaneously provide controlling prompt	
Embedded Trial 8	Simultaneously provide controlling prompt	
Step 7: Circle numbers	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 9	Simultaneously provide controlling prompt	
Embedded Trial 10	Simultaneously provide controlling prompt	

Step 8: Put numbers on mat	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 11	Simultaneously provide controlling prompt	
Embedded Trial 12	Simultaneously provide controlling prompt	
Step 9: Plus or minus	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 13	Simultaneously provide controlling prompt	
Embedded Trial 14	Simultaneously provide controlling prompt	
Step 10: Put what you know on the number line	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 15	Simultaneously provide controlling prompt	
Embedded Trial 16	Simultaneously provide controlling prompt	
Step 11: Count to find	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 17	Simultaneously provide controlling prompt	
Embedded Trial 18	Simultaneously provide controlling prompt	
Step 12: Solve	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Step 13: Write the answer	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 19	Simultaneously provide controlling prompt	
Embedded Trial 20	Simultaneously provide controlling prompt	
	Total +	
	Total Opportunities	
	% Fidelity	

Test

Component	Expectation	Fidelity
Attentional Cue	Deliver attentional cue	
Step 1	Complete step if student does not. Collect data.	
Step 2	Complete step if student does not. Collect data.	
Step 3	Complete step if student does not. Collect data.	
Step 4	Complete step if student does not. Collect data.	
Step 5	Complete step if student does not. Collect data.	
Step 6	Complete step if student does not. Collect data.	
Step 7	Complete step if student does not. Collect data.	
Step 8	Complete step if student does not. Collect data.	
Step 9	Complete step if student does not. Collect data.	
Step 10	Complete step if student does not. Collect data.	
Step 11	Complete step if student does not. Collect data.	
Step 12	Complete step if student does not. Collect data.	
Step 13	Complete step if student does not. Collect data.	
	Total +	
	Total Opportunities	
	% Fidelity	

Lead - Revised

Component	Expectation	Fidelity
Attentional Cue	Deliver attentional cue	
Present Materials	Present materials	
Step 1: Read the problem	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 1	Reinforce if correct. Prompt if incorrect.	
Embedded Trial 2	Reinforce if correct. Prompt if incorrect.	
Step 2: Circle the terms	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 3	Reinforce if correct. Prompt if incorrect.	
Embedded Trial 4	Reinforce if correct. Prompt if incorrect.	
Step 3: Same, Different, More/Fewer	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Step 4: Choose problem mat	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 5	Reinforce if correct. Prompt if incorrect.	
Embedded Trial 6	Reinforce if correct. Prompt if incorrect.	
Step 5: Use my rule	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Step 6: Put terms on mat	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 7	Reinforce if correct. Prompt if incorrect.	
Embedded Trial 8	Reinforce if correct. Prompt if incorrect.	
Step 7: Circle numbers	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 9	Reinforce if correct. Prompt if incorrect.	
Embedded Trial 10	Reinforce if correct. Prompt if incorrect.	

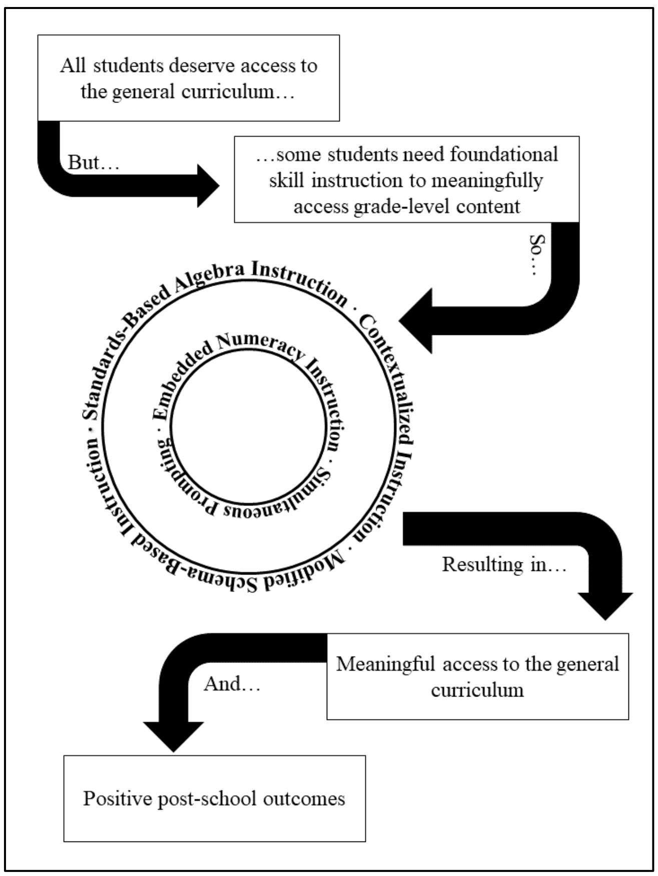
Step 8: Put numbers on mat	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 11	Reinforce if correct. Prompt if incorrect.	
Embedded Trial 12	Reinforce if correct. Prompt if incorrect.	
Step 9: Plus or minus	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 13	Reinforce if correct. Prompt if incorrect.	
Embedded Trial 14	Reinforce if correct. Prompt if incorrect.	
Step 10: Put what you know on the number line	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 15	Reinforce if correct. Prompt if incorrect.	
Embedded Trial 16	Reinforce if correct. Prompt if incorrect.	
Step 11: Count to find	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 17	Reinforce if correct. Prompt if incorrect.	
Embedded Trial 18	Reinforce if correct. Prompt if incorrect.	
Step 12: Solve	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Step 13: Write the answer	Wait 3 seconds	
	Nonspecific verbal prompt	
	Specific verbal prompt	
	Model prompt	
Embedded Trial 19	Reinforce if correct. Prompt if incorrect.	
Embedded Trial 20	Reinforce if correct. Prompt if incorrect.	
	Total +	
	Total Opportunities	
	% Fidelity	

Appendix R

Intervention BST Protocol

Describe

1. Researcher meets with participating teachers and provides rationale for study and brief explanation of research base. (5 min)
    a. **Introduction of Idea:** "As a classroom teacher, I often struggled to bridge the gap between my students' instructional levels and the grade-aligned standards, particularly in mathematics. Like you, I had students in my class who had not yet mastered basic numeracy skills, such as numeral identification. Yet, I knew it was important to also address the standards. This challenge led me to my research idea." *Present theory of change model:*



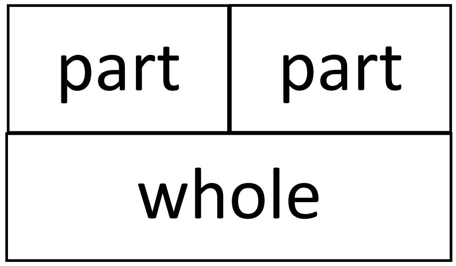
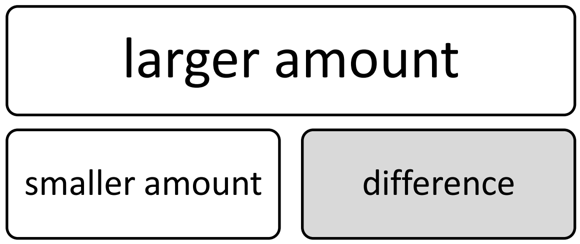
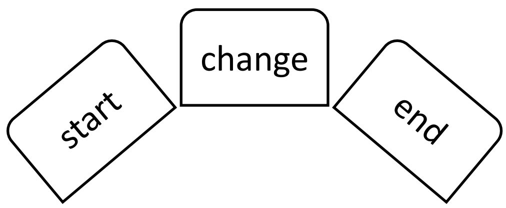
    b. **Research Base for General Curriculum Access:** "Research has shown that access to the general curriculum is crucial for student success, both in school, and after graduation. For all students, the general curriculum is designed to promote learning and to prepare students for college and career (Courtade et al., 2012; Yudin et al., 2015). Access to the general curriculum is an important predictor of post-school success (Gilley et al., 2021; Shogren et al., 2018; Test et al., 2009). Success post-school, which includes access to postsecondary education and competitive employment, typically requires a high school diploma (Kearns et al., 2011; Mazzotti et al., 2021). According to a US Department of Education report in 2020, however, only 70% of students with autism, 53.3% of students with deaf-blindness, 42.3% of students with intellectual disability, and 45.8% of students with multiple disabilities graduated with a regular high school diploma. Students with ESN will never have the opportunity to earn a high school diploma, severely limiting their post-school opportunities, if they do not first have access to the general curriculum.

Regardless if a student meets the requirements to earn a high school diploma, students with ESN still have a right to access the general curriculum (Courtade et al., 2012). Academic knowledge improves students' competence for adult living, increases opportunities for jobs and leisure activities, and promotes self-determination (Courtade et al., 2012; Knight et al., 2010). Education is necessary for equality, and every student deserves the opportunity to participate in a full educational program.

2. Researcher describes rationale and procedures for modified schema-based instruction (15 min)
    a. "Standards-based algebra instruction is extremely important for students with ESN. The skills students learn in school mathematics are applicable to all domains of daily life (Ahlgrim-Delzell et al., 2009). For example, algebraic skills are necessary when calculating costs at the produce stand or when analyzing relationships between variables. The process skills students learn in algebra are also important for developing critical thinking and learning problem-solving skills (Cox & Jimenez, 2020; Goya et al., 2019). Proficiency in algebra is necessary for students to gain employment and live independently in their adult life (Creech-Galloway et al., 2013).

       Research has suggested students with ESN can learn algebra, given graphic organizers, task analytic instruction, and systematic prompting (Baker et al., 2015; Chapman et al., 2019; Jimenez et al., 2008). These students, however, often have trouble generalizing skills from the classroom environment to the real-world (Root, Knight, & Mims, 2017). We can make algebra more meaningful to students by contextualizing it in story problems that are relevant to the student's life (Browder, Jimenez, & Trela, 2012).

       One instructional strategy that incorporates story problems is modified schema-based instruction (MSBI; Browder et al., 2018; Spooner et al., 2017). MSBI is a way for students to learn how to solve different types of word problems. For example, there are 3 types of additive word problems, or schemas." *Present schema types table*

Schema	Potential Visual Representation	Example
Combine		Miley went to the library. She checked out 4 fiction books. She also checked out 3 biographies. How many books did Miley check out?
Compare		Matthew and Cole went to their job at the hotel. Matthew cleaned 8 rooms. Cole cleaned 6 rooms. How many more rooms did Matthew clean?
Change		Zoe is volunteering at the animal shelter. She has 10 dog treats. She gives 4 treats away. How many dog treats does she have left?

Note. Adapted from Powell & Fuchs, 2018. Reprinted with permission from Bouck et al., 2021.

In this study, you are going to teach your student to solve combine problems, with the second, "part" missing. For example, instead of "Miley went to the library. She checked out 4 fiction books. She also checked out 3 biographies. How many books did Miley check out?", our problem would look like "Miley checked out 4 fiction books. She also checked out some biographies. She checked out 7 books in all. How many biographies did she check out?"

You will support student's understanding of the problem by providing a task analysis (*show*) and graphic organizers (*show*), by using chants with hand motions (e.g., "small group, small group, COMBINE into big group!"), and by using explicit instruction (model-lead-test), and systematic instruction (system of least prompts). In this study, you'll model how to use MSBI on Mondays, lead a student to use MSBI using system of least prompts on Tuesdays, Wednesdays, and Thursdays, and then test the student and collect data on Fridays.

The steps to solve a combine word problem are: (*present task analysis)*

1) Read the problem. You will have a recordable button so students can "read" the problem by activating the button.
2) Circle the terms (i.e., the labels of the items we're adding).
3) Same, Different, More/Fewer: Determine if the terms are same, different, or if we're looking at a word problem that uses the words "more" or "fewer".

4) Choose the problem mat (*present graphic organizers*). The change mat is used for "same", the combine mat is used for "different", and the compare graph is used for "more/fewer".
5) Use my rule. The rule for change is "same thing, add more or take away, change". The rule for combine is "small group, small group, combine into big group". The rule for compare is "bigger number, small number, compare the difference".
6) Put terms on the mat. Now that you have chosen the mat and you know your rule, you can label it. You'll put the first small group on the green line, the second small group on the red line, and the big group on the blue line. As you can see (*present word problem*), the labels on the word problem are velcroed so students can label the mat without having to write with a pencil.
7) Circle the numbers.
8) Put the numbers on the mat. Students will then put the numbers on the mat. You'll have number cards that can be used to eliminate the need for writing with a pencil. During this step, you'll also explain that when we don't know an amount, we'll use a variable (x).
9) Plus or minus? You'll explain that we're adding the small groups to combine into a big group and the student will place the (+) on the mat.
10) Put what you know on the number line. (*Present number line and manipulatives*). The student will use green chips (corresponding with the mat) to put the amount we know (the first part) on the number line. They will also use a blue marker to mark the whole, so they know where to stop counting in the next step.
11) Count to find what's missing. The student will place red chips on the number line up to the amount marked with the blue marker. This represents the unknown "part".
12) Solve to find x. Students will move the red chips to a second number line so they can more easily find the correct answer.
13) Write the answer. Students will use the number cards to solve the equation $x =$_."

During the "Lead" component of MSBI, you will use the system of least prompts, a systematic instructional strategy which uses a prompting hierarchy to support student understanding (Collins, 2012). First, the student will have an opportunity to complete the step independently. You will not provide any direction or prompting. If they do not complete the step within 3 seconds of finishing the last step, or if they make an error, you will use a 3-step prompting hierarchy. First, you will provide a nonspecific verbal prompt. For example, you will ask the student "What's our first step?" and prompt the student to activate the talk aloud button on the photo album. If they still do not respond, or if they make an error, you will then provide a specific verbal prompt. You provide instruction and tell the what to do. For example, for step 2, circle the terms, the specific verbal prompt is "Terms are the pieces of our algebra problem. In this problem, I see three terms: dogs, cats, and pets.". If they still do not respond, or if they make an error, you will provide a model prompt, in which you show the student what to do, by pointing to the next step, or by physically guiding the student to complete the step. We will also be using a concept known as differential reinforcement. This means you will differentiate the amount of reinforcement you provide based on the student's independence. If they complete the step independently, provide high-quality praise. If they need a nonspecific verbal prompt, provide moderate praise. If they need a specific verbal prompt, provide low-quality praise, and if they need a model prompt, simply move on to the next step of the problem.

3. Researcher describes rationale and procedures for embedded simultaneous prompting. (5 minutes)
    a. "As you are probably thinking, the steps above require an understanding of numeracy that your students cannot yet demonstrate. Regardless, they are still required to participate in this standards-based algebra instruction, per federal law. That's why you will also be teaching the student to identify numerals within the algebra lesson. We will use a process called embedded simultaneous prompting. Embedded instruction refers to the purposeful distribution of instructional trials across activities and naturally occurring routines (Jameson et al., 2020). It provides a way for you to address multiple skills within a single lesson or activity. Simultaneous prompting is a systematic instructional strategy in which there are probe and teaching trials (Collins, 2012). In the probe trials, you assess the student's performance, without prompting, providing reinforcement, or error correction. This is what you did during the baseline phase of our study. You'll continue to probe each day at the start of your algebra lesson. In the teaching trials, you deliver the task direction and then immediately prompt the student to respond correctly. For example, "Touch 3", and then I'll immediately prompt you to touch 3, either by pointing to it, or by using hand-over-hand prompting. Throughout both the Model and the Lead components of MSBI, you will have 15 opportunities to embed simultaneous prompting—5 trials for each numeral relevant to the problem. For example, after step 1 of the task analysis (read the problem), you will embed two simultaneous prompting trials. So, after listening to the word problem (*Willow bought 1 shirt at the mall. She also bought some pants. She bought 3 clothes all together. How many pants did she buy*), you will teach "1" by saying "Willow bought 1 shirt. Touch 1." and immediately prompt the student to touch "1" in the word problem, by providing a gesture or physical prompt. Each of these trials are already planned for you on your lesson plan.

Model, Part 1

4. **Numeracy Probe:** Researcher models numeracy probe with the teacher acting as the student. (5 min)
    a. At the start of each lesson (Monday through Thursday), you will conduct a numeracy probe. This is just like the procedures in baseline. The only difference now, is you will only probe numerals 1–3. Once the student masters those numerals, we'll add more. Let's run through one numeracy probe. I'll be the

teacher, and you will be the student. (Model numeracy probe following scripted lesson plan; see Appendix G).

Rehearsal, Part 1

5. **Numeracy Probe:** Teacher rehearses numeracy probe with the researcher acting as the student. Researcher provides corrective and supportive feedback. Rehearsal continues until teacher reaches 100% fidelity in one session (see Appendix Q). (5 min).

Model, Part 2

6. **Model**: Researcher models "model" component of lesson plan with the teacher acting as the student (15 min).
    a. On the first day of the week (typically Monday), you will conduct the "model" portion of the lesson plan. The student does not need to respond to each step of the task analysis, but you should ensure they are attending to the instruction. There will be 15 opportunities for embedded simultaneous prompting where you will prompt the student to receptively identify numerals. You will use a different word problem each time you present this lesson. The word problem I will model is: *Willow bought 3 clothes at the store. She bought 1 shirt and some pants. How many pants did she buy?* (Model "model" component following scripted lesson plan; see Appendix G).

Rehearsal, Part 2

7. **Model:** Teacher rehearses "model" component with the researcher acting as the student. Researcher provides corrective and supportive feedback. Rehearsal continues until teacher reaches 100% fidelity in one session (see Appendix Q). (15 min).

Model, Part 3

8. **Lead**: Researcher models "lead" component of lesson plan with the teacher acting as the student (15 min).
    a. During the middle of the week (typically Tuesday, Wednesday, and Thursday), you will conduct the "lead" portion of the lesson plan. If the student does not complete a step, or completes the step incorrectly, you will use the system of least prompts. Remember, for this intervention, our prompt hierarchy will be nonspecific verbal prompt ("What's next?), specific verbal prompt (tell the student what to do), and model (show the student what to do). Just like the model portion, you will have 15 opportunities for embedded simultaneous prompting where you will prompt the student to receptively identify numerals relevant to the problem. (Model "lead' component following scripted lesson plan; see Appendix G).

Rehearsal, Part 3

9. **Lead**: Teacher rehearses "lead" component with the researcher acting as the student. Researcher provides corrective and supportive feedback. Rehearsal continues until teacher reaches 100% fidelity in one session (see Appendix Q). (15 min).

Model, Part 4

10. **Test**: Researcher models "test" component of lesson plan with the teacher acting as the student (5 min).
    a. On the last day of the week (typically Fridays), you will conduct the "test" portion of the lesson plan. This is just like the procedures in baseline. You will not provide any prompts or reinforcement. If the student does not complete a step in the allotted wait time (e.g., 3 s), you will complete the step for the student. You will also collect data during this portion. There will not be any embedded simultaneous prompting in this lesson. (Model "test" component following scripted lesson plan; see Appendix G).

Rehearsal, Part 4

11. **Test**: Teacher rehearses "test" component with the researcher acting as the student. Researcher provides corrective and supportive feedback. Rehearsal continues until teacher reaches 100% fidelity in one session (see Appendix Q). (5 min).

Ongoing Feedback

Throughout study, procedural fidelity data will be collected. When procedural fidelity drops below 90%, additional feedback will be provided to the teacher.

Appendix S

Revised Lesson Plans

Lesson Plan

Solving Simple Linear Equations (Combine Schema)

Standard: NC.M1:A-REI.3: Solve linear equations and inequalities in one variable.

Lead

*During the remaining intervention sessions, you will lead the student through the process to solve a simple linear equation. If the student does not answer, or answers incorrectly, you will first provide a **nonspecific verbal prompt** (read the step on the task analysis), then a **specific verbal prompt** (tell the student what to do), and then a **model prompt** (show the student what to do). Additionally, there will be 20 opportunities for embedded constant time delay where you will ask the student to receptively identify numerals.*

In Phase A, you will use one of three word problems each time you present this lesson, either $4 + x = 9$, $3 + x = 9$, *or* $3 + x = 4$. *In Phase B, you will use* $5 + x = 8$, $1 + x = 8$, *or* $1 + x = 5$. *In Phase C, you will use* $6 + x = 7$, $2 + x = 7$, *or* $2 + x = 6$. *For the purpose of this script, we will use the following word problem:* Jared has 3 dogs. He also has some (x) cats. Jared has 4 pets all together. How many cats does he have?

Instructional Cue	Student Response	Teacher Response
Deliver attentional cue: **"Now it's your turn to solve a real-world math problem using a linear equation. Show me you are ready!"**	Student attends to materials.	**"Let's get started!"**
Step 1: "**Here are your materials."** Present materials (task analysis, word problem, problem mats, number line, manipulatives) to the student. Open the talking photo album to the first page.	Student reads the problem or activates the read-aloud function	Provide high-quality praise. Move to embedded instruction.
	No response	Nonspecific verbal prompt: "**What's our first step? Press the button."** Prompt the student to activate the talk aloud button on the photo album: *Read the problem.*
	Student reads the word problem or activates the read-aloud function.	Provide moderate praise. Move to embedded instruction.

Instructional Cue	Student Response	Teacher Response
	No response	Specific verbal prompt: "**Touch the green button to read it aloud**."
	Student reads the word problem or activates the read-aloud function.	Provide low-quality praise. Move to embedded instruction.
	No response	Model prompt: **Touch this button to read the problem.** Point to the GoTalk button. Move to embedded instruction.
Embedded Instruction: Present five number cards. **"Jared has 3 dogs. Touch 3."**	Student touches correct numeral.	Provide descriptive feedback. **"That's 3! 3 dogs."** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 3. Touch 3."** Mark a – on the data sheet.
Embedded Instruction: Mix up cards. "**He has 4 pets in all. Touch 4."**	Student touches correct numeral.	Provide descriptive feedback: **"That's 4! Great work!"** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 4. Touch 4."** Mark a – on the data sheet.
Step 2: (Turn photo album page if student is unable. If student is unable to hold a marker, have them point to the terms, and you will circle them).	Student circles the terms.	Provide high-quality praise. Move to step 3.
	No response	Nonspecific verbal prompt: **"What's next? Turn the page and press the button!"** Prompt student to activate the talk aloud button on the photo album: *Circle the terms.*
	Student circles the terms.	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: **"Terms are the pieces of our algebra problem. In this problem, I see three terms: dogs, cats, and pets**.

Instructional Cue	Student Response	Teacher Response
	Student circles the terms.	Provide low-quality praise. Move to embedded instruction.
	No response	Model prompt: Guide the student to circle *dogs*, *cats* and *pets*. Move to embedded instruction.
Embedded Instruction: Present five number cards. **“Jared has 3 of our first term, dogs. Touch 3.”**	Student touches correct numeral.	Provide descriptive feedback: **“3 dogs! Awesome!”** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **“This is 3. Touch 3.”** Mark a – on the data sheet.
Embedded Instruction: Mix up cards. **“Jared has 4 of our last term, pets. Touch 4.”**	Student touches correct numeral.	Provide descriptive feedback: **“That’s 4! Great work!”** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **“This is 4. Touch 4.”** Mark a – on the data sheet.
Step 3: (Turn photo album page if student is unable).	Student points to correct schema (different) on task analysis.	Provide high-quality praise. Move to step 4.
	No response.	Nonspecific verbal prompt: **“What’s next? Press the button.”** Prompt student to activate the talk aloud button on the photo album: *Same, different, more or fewer?*
	Student points to correct schema (different) on task analysis.	Provide moderate praise. Move to step 4.
	No response.	Specific verbal prompt: “**Dogs and cats are different. Touch *different*.**”

Instructional Cue	Student Response	Teacher Response
	Student points to correct schema (different) on task analysis.	Provide low-quality praise. Move to step 4.
	No response.	Model prompt: Gesture to "different" on the task analysis. Move to step 4.
Step 4: (Turn photo album page if student is unable).	Student selects correct problem mat (combine).	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **"What's next? Turn the page and press the button."** Prompt student to activate the talk aloud button on the photo album: *Choose the problem mat*.
	Student selects correct problem mat (combine).	Provide moderate praise. Move to embedded instruction.
	No response.	Specific verbal prompt: **"We need the problem mat for 'different'. Combine problems are about two small groups of different things we combine into one big group. The Combine problem mat is the one for different."**
	Student selects correct problem mat (combine).	Provide low-quality praise. Move to embedded instruction.
	No response.	Model prompt: Point to correct problem mat (combine). Move to embedded instruction.
Embedded Instruction: **"Good work! We are combining 3 dogs and some cats. Touch 3."**	Student touches correct numeral.	Provide descriptive feedback. **"That's 3! Amazing!"** Mark a + on the data sheet.

Instructional Cue	Student Response	Teacher Response
	Student makes incorrect response, or no response.	Provide error correction. **“This is 3. Touch 3.”** Mark a – on the data sheet.
Embedded Instruction: Mix up cards. **“We combine to make 4 pets. Touch 4.”**	Student touches correct numeral.	Provide descriptive feedback: **“That’s 4! Nice!”** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **“This is 4. Touch 4.”** Mark a – on the data sheet.
Step 5: (Turn photo album page if student is unable).	Student says the rule and/or uses hand motions.	Provide high-quality praise. Move to step 6.
	No response.	Nonspecific verbal prompt: **“What’s next? Press the button.”** Prompt student to activate the talk aloud button on the photo album: *Use my rule*.
	Student says the rule and/or uses hand motions.	Provide moderate praise. Move to step 6.
	No response.	Specific verbal prompt: **“This problem is about two small groups of different things we combine into a big group. That means it is a combine problem. The rule for combine is ‘*small group, small group, COMBINE into big group!*’”**
	Student says the rule and/or uses hand motions.	Provide low-quality praise. Move to step 6.
	No response.	Model prompt: Guide the student to make the hand motions for the rule.

Instructional Cue	Student Response	Teacher Response
Step 6: (Turn photo album page if student is unable).	Student puts the terms on the mat.	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **“What’s next? Turn the page and press the button.”** Prompt the student to activate the talk aloud button on the photo album: *Put the terms on the mat.*
Step 6, continued	Student puts the terms on the mat.	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: **“Label the problem mat with the terms from the problem. Jared has dogs. That’s a small group.” “He has cats. That’s another small group.” “Dogs and cats combine to make the big group, pets.”**
	Student puts the terms on the mat.	Provide low-quality praise. Move to embedded instruction.
	No response	Model prompt: Guide the student to move the “dogs” icon, “cats” icon, and “pets” icon to the mat. Move to embedded instruction.
Embedded Instruction: Present five number cards. **“Our first small group is 3 dogs. Touch 3.”**	Student touches correct numeral.	Provide descriptive feedback. **“That’s 3! Good!”** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **“This is 3. Touch 3.”** Mark a – on the data sheet.
	Student touches correct numeral.	Provide descriptive feedback: **“That’s 4! Nice**

Instructional Cue	Student Response	Teacher Response
Embedded Instruction: Mix up cards. **"The big group is 4 pets. Touch 4."**		**job!"** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 4. Touch 4."** Mark a – on the data sheet.
Step 7: (If student is unable to hold a marker, have them point to the numbers and circle for them).	Student circles the numbers.	Provide high-quality praise. Move to embedded instruction.
Step 7 Continued	No response	Nonspecific verbal prompt: **"What's next? Press the button."** Prompt student to activate the talk aloud button on the photo album: *Circle the numbers*.
	Student circles the numbers.	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: **"Jared has 3 dogs. 3 is a number. Circle 3."** Pause. **"He has 4 pets. 4 is a number. Circle 4"**
	Student circles the numbers.	Provide low-quality praise. Move to embedded instruction.
	No response	Model prompt: Guide the student to circle "3" and "4". Move to embedded instruction.
Embedded Instruction: Present five number cards. **"Jared has 3 dogs. Touch 3."**	Student touches correct numeral.	Provide descriptive feedback. **"That's 3! Awesome!"** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 3. Touch 3."** Mark a – on the data sheet.
Embedded Instruction: Mix up cards. **"He has 4 pets. Touch 4."**	Student touches correct numeral.	Provide descriptive feedback: **"That's 4! Great work!"** Mark a + on the data sheet.

Instructional Cue	Student Response	Teacher Response
	Student makes incorrect response, or no response.	Provide error correction. **“This is 4. Touch 4.”** Mark a – on the data sheet.
Step 8: (Turn photo album page if student is unable).	Student puts numbers on the mat.	Provide high-quality praise. Move to embedded instruction.
	No response	Nonspecific verbal prompt: **“What’s next? Turn the page and press the button.”** Prompt student to activate the talk aloud button on the photo album: *Put the numbers on the mat*.
	Student puts numbers on the mat.	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: **“Let’s look at our mat. Our first term is dogs. How many dogs? 3 dogs. Move 3 to the mat.”** Pause. If student does not move numeral, provide model prompt. If student does move numeral, continue with specific verbal prompt. **“Our second term is cats. How many cats? Some cats. That means we don’t know how many cats. We can use a variable, x, when we don’t know how many. Move the x to the mat.”** Pause. If student does not move numeral, provide model prompt. If student does move numeral, continue with

Instructional Cue	Student Response	Teacher Response
		specific verbal prompt. **"Our third term is 'pets'. How many pets? 4 pets. Move 4 to the mat."**
	Student puts numbers on the mat.	Provide low-quality praise. Move to embedded instruction.
	No response	Model prompt: Guide the student to put the numbers on the mat. Move to embedded instruction.
Embedded Instruction: Present number cards. **"He has 3 dogs. Touch 3."**	Student touches correct numeral.	Provide descriptive feedback. **"That's 3! Nice!"** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 3. Touch 3."** Mark a – on the data sheet.
Embedded Instruction: Mix up cards. "**He has 4 pets in all. Touch 4."**	Student touches correct numeral.	Provide descriptive feedback: **"That's 4! Great work!"** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 4. Touch 4."** Mark a – on the data sheet.
Step 9: (Turn photo album page if student is unable).	Student chooses correct operation (+)	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **"What's next? Press the button."** Prompt student to activate the talk aloud button on the photo album: *Plus or minus?*
	Student chooses correct operation (+)	Provide moderate praise. Move to embedded instruction.
	No response	Specific verbal prompt: **"What is our rule? *'Small group, small group, COMBINE into big group'.*"** (Use hand motions). **"Small group**

Instructional Cue	Student Response	Teacher Response
		plus small group combines into big group. That means we're adding. We need to use the plus sign."
	Student chooses correct operation (+).	Provide low-quality praise. Move to embedded instruction.
	No response	Model prompt: Guide the student to move the Velcro plus sign (+) to the mat. Move to embedded instruction.
Embedded Instruction: **"Awesome! We're going to add 3 dogs and some cats. Touch 3."**	Student touches correct numeral.	Provide descriptive feedback. **"That's 3! Great!"** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 3. Touch 3."** Mark a – on the data sheet.
Embedded Instruction: Mix up cards. **"And we''ll get 4 pets."**	Student touches correct numeral.	Provide descriptive feedback: **"That's 4! God job!"** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 4. Touch 4."** Mark a – on the data sheet.
Step 10: (Turn photo album page if student is unable).	Student places the green chips and blue marker on the number line.	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **"What's next? Turn the page and press the button."** Prompt student to activate the talk aloud button on the photo album: *Put what you know on the number line*.
	Student places the green chips and blue marker on the number line.	Provide moderate praise. Move to embedded instruction.

Instructional Cue	Student Response	Teacher Response
	No response.	Specific verbal prompt: **“Put what you know on the number line. Jared has 3 dogs. Put 3 green chips on the number line.”** Pause. If student does not put chips on number line, provide model prompt. If student does move chips, continue with specific verbal prompt. **“Jared has 4 pets in all. Put the blue marker on 4 so we know where to stop.”** If student does not put marker on number line, move to model prompt.
	Student places the green chips and blue marker on the number line.	Provide low-quality praise. Move to embedded instruction.
	No response.	Model prompt: Guide the student to move manipulatives to the number line as you count aloud.
Embedded Instruction: Present five number cards. **“He has 3 dogs. Touch 3.”**	Student touches correct numeral.	Provide descriptive feedback. **“That’s 3! Amazing!”** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **“This is 3. Touch 3.”** Mark a – on the data sheet.
Embedded Instruction: Mix up cards. **“He has 4 pets all together. Touch 4.”**	Student touches correct numeral.	Provide descriptive feedback: **“That’s 4! Awesome!”** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **“This is 4. Touch 4.”** Mark a – on the data sheet.

Instructional Cue	Student Response	Teacher Response
Step 11: (Turn photo album page if student is unable).	Student counts to find what's missing.	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **"What's next? Press the button."** Prompt student to activate the talk aloud button on the photo album: *Count to find what's missing.*
	Student counts to find what's missing.	Provide moderate praise. Move to embedded instruction.
	No response.	Specific verbal prompt: **"Add red chips to the number line to find how many cats."**
	Student counts to find what's missing.	Provide low-quality praise. Move to embedded instruction.
	No response.	Model prompt: Guide the student to add 1 red chip to the number line. Count aloud as you add: **"4!"** Move to embedded instruction.
Embedded Instruction: Present five number cards. **"We have 3 dogs. Touch 3."**	Student touches correct numeral.	Provide descriptive feedback. **"That's 3! You did it!"** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 3. Touch 3."** Mark a – on the data sheet.
Embedded Instruction: Mix up cards. **"And 4 pets all together. Touch 4."**	Student touches correct numeral.	Provide descriptive feedback: **"That's 4! Great work!"** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 4. Touch 4."** Mark a – on the data sheet.

Instructional Cue	Student Response	Teacher Response
Step 12: (Turn photo album page if student is unable).	Student moves red chips to second number line.	Provide high-quality praise. Move to step 13.
	No response.	Nonspecific verbal prompt: **"What's next? Turn the page and press the button."** Prompt student to activate the talk aloud button on the photo album: *Solve to find x.*
	Student moves red chips to second number line.	Provide moderate praise. Move to step 13.
	No response.	Specific verbal prompt: **"Remember, x is a variable. We used x because we didn't know how many cats. We can find x by counting the red chips. Move the red chips to the second number line to find x."**
	Student moves red chips to second number line.	Provide low-quality praise. Move to step 13.
	No response.	Model prompt: Guide the student to move the red chip to the second number line, counting as they move: **"1"** Move to step 13.
Step 13: (Turn photo album page if student is unable).	Student moves Velcro "1" to $x =$.	Provide high-quality praise. Move to embedded instruction.
	No response.	Nonspecific verbal prompt: **"What's next? Press the button."** Prompt student to activate the talk aloud button on the photo album: *Write the answer*.
	Student moves Velcro "1" to $x =$.	Provide moderate praise. Move to embedded instruction.
	No response.	Specific verbal prompt: **"How many cats? Let's count. 1. 1 cat. Move 1 to $x =$."**

Instructional Cue	Student Response	Teacher Response
Step 13, continued	Student moves Velcro "1" to $x =$.	Provide low-quality praise. Move to embedded instruction.
	No response.	Model prompt: Guide the student to move the Velcro "1" to $x =$. Move to embedded instruction.
Embedded Instruction: Present five number cards. **"3 dogs plus 1 cat equals 4 pets! Touch 3."**	Student touches correct numeral.	Provide descriptive feedback. **"That's 3! 3 dogs."** Mark a + on the data sheet.
	Student makes incorrect response, or no response.	Provide error correction. **"This is 3. Touch 3."** Mark a – on the data sheet.
Embedded Instruction: Mix up cards. **"Touch 4."**	Student touches correct numeral.	Provide descriptive feedback: **"That's 4! 4 pets! Awesome job solving a real-world problem using a linear equation!"** Mark a + on the data sheet. End session.
	Student makes incorrect response, or no response.	Provide error correction: **"This is 4. Touch 4. Awesome job solving a real-world problem using a linear equation!"** Mark a – on the data sheet. End session.

Appendix T

Revised Numeral Identification Data Sheet

Date										
Session										
Problem										
1										
2										
3										
4										
5										
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7										
8										
9										
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11										
12										
13										
14										
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16										
17										
18										
19										
20										
+										
−										
Student Affect										

Key:
+: correct
−: incorrect
😊: positive student affect
🙁: negative student affect

Session Key:
FP: Full Probe (numerals 1–9)
IA: Intervention A (numerals 3, 4, 9)
IB: Intervention B (numerals 1, 5, 8)
IC: Intervention C (numerals 2, 6, 7)
Gen: Generalization (numerals 1–9, on microwave)

Appendix U

Caregiver Social Validity Questionnaire

	Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree	I Don't Know
It is important for my child to learn grade-level academics like algebra.	1	2	3	4	5	X
It is important for my child to learn foundational academic skills, like identifying numbers.	1	2	3	4	5	X
Teaching number identification within the context of an algebra lesson is an effective way to teach my child.	1	2	3	4	5	X
It is better to teach number identification within the context of an algebra lesson, rather than teaching the skill in isolation.	1	2	3	4	5	X
My child can reliably identify the numbers 1 through 9.	1	2	3	4	5	X
Learning math is important for my child to be successful after graduating high school.	1	2	3	4	5	X

Please describe some ways in which your child currently interacts with numbers in their environment