STABILITY AND PERFORMANCE OF NETWORKED CONTROL SYSTEMS

by

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ABSTRACT

LIBIN BAI. Stability and performance of networked control systems. (Under the direction of DR. SHENG-GUO WANG)

Network control systems (NCSs), as one of the most active research areas, are arousing comprehensive concerns along with the rapid development of network. This dissertation mainly discusses the stability and performance of NCSs into the following two parts.

In the first part, a new approach is proposed to reduce the data transmitted in networked control systems (NCSs) via model reduction method. Up to our best knowledge, we are the first to propose this new approach in the scientific and engineering society. The "unimportant" information of system states vector is truncated by balanced truncation method (BTM) before sending to the networked controller via network based on the balance property of the remote controlled plant controllability and observability. Then, the exponential stability condition of the truncated NCSs is derived via linear matrix inequality (LMI) forms. This method of data truncation can usually reduce the time delay and further improve the performance of the NCSs. In addition, all the above results are extended to the switched NCSs.

The second part presents a new robust sliding mode control (SMC) method for general uncertain time-varying delay stochastic systems with structural uncertainties and the Brownian noise (Wiener process). The key features of the proposed method are to apply singular value decomposition (SVD) to all structural uncertainties, to introduce adjustable parameters for control design along with the SMC method, and new Lyapunov-type functional. Then, a less-conservative condition for robust stability and a

new robust controller for the general uncertain stochastic systems are derived via linear matrix inequality (LMI) forms. The system states are able to reach the SMC switching surface as guaranteed in probability 1 by the proposed control rule. Furthermore, the novel Lyapunov-type functional for the uncertain stochastic systems is used to design a new robust control for the general case where the derivative of time-varying delay can be any bounded value (e.g., greater than one). It is theoretically proved that the conservatism of the proposed method is less than the previous methods.

All theoretical proofs are presented in the dissertation. The simulations validate the correctness of the theoretical results and have better performance than the existing results.

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CHAPTER 1: INTRODUCTION

Time delay systems appear in practical control systems and applications, e.g., remote control systems (Hokayem and Spong, 2006), unmanned aircrafts (Zeitlin and McLaughlin, 2006), industrial control systems (Brooks, 1986, Kaloust et al., 2004, Umeno and Hori, 1991) and networked control systems (Chow and Tipsuwan, 2001, Kuperman and Zhong, 2011). For decades, it has aroused a lot of research attention in the robust control of time delay systems (John and Jacques, 2007, Emilia and Uri, 2003). It is because of the fact that dynamic systems usually have uncertainties in the face of changing environment and disturbances, especially their time delays are also uncertain in many cases, of that we may only know the upper bounds. The uncertainties make those systems difficult to be described by accurate mathematic models, and are often the source of instability.

Network control systems (NCSs), as one of the most active research areas, is arousing comprehensive concerns along with the rapid development of network. A lot of applications of NCSs can be found in different areas, e.g., telesurgery, factory automation, autonomous robots, etc. There are several advantages to use network as the shared media to transfer signal of remote control systems, e.g., flexibility, low installation cost, easy diagnosis and easy maintenance (Walsh and Yeo, 2001). However, bandwidth is always limited on network. What's worse, the data might collide on the shared medium. Therefore, the network could be congested because of the large data arrival rate. A large data arrival rate could results in degradation of the overall performance of NCSs, because it is difficult for a controller to track the plant state in real-time due to the amount of data and bandwidth limitation. Moreover, for most of the networks, there is no quality and time guarantee in exchanging of information between the plant and the controller over the shared transmission channel. The network-induced delay, either being constant or uncertain, might destabilize the whole system and meanwhile degrade the performance. For example, TCP/IP protocol involves packet resending if packet loss happens before it reaches the destination. The resending would with no doubt increase the delay in transmission. Even worse, during busy period, on the networks, like Ethernet with CSMA/CD (carrier sense multiple access with collision detection) strategy, the data exchanging quality largely depends on the number of users concurrently, because the shared medium can be used for sending data by only one user at one time. There are a lot of research about the NCSs. e.g., Xu et al. (2012) discussed the stochastic optimal control of linear NCS with uncertain system dynamics, network random delays, and packet losses using an adaptive estimator and Q-learning to solve the stochastic optimal and suboptimal regulation control of NCS. Ulusoy et al. (2011) presented the time-triggered wireless NCSs over cooperative wireless network with a model-based predictive controller. Shi and Yu (2011) investigated the two-mode-dependent robust synthesis of NCSs with norm-bounded uncertainties and random delays in both forward controller-to-actuator and feedback sensor-to-controller links modeled as Markov chains. Then, the stochastic stability, $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms are discussed. The authors also noticed that the research of real-time performance and its application of NCS is an open problem currently (Yang, 2006, Gupta and Chow, 2010, Wu et al., 2002). Recently, Wang and Bai (2012) discussed general time delay systems with

Brownian motion and structural uncertainties by the SVD and LMI.

Stochastic process models are used in many applications, especially on networks, in which both variables and processes randomly and dynamically change with the time. Thus, robust control on stochastic process has become an active research area, e.g., Wang and Stengel (2000) described a robust stochastic control for hypersonic aircraft, and Xu et al. (2006) investigated robust stochastic stabilization and robust H_∞ control in stochastic neutral time-delay systems with uncertainties. Wu et al. (2010) studied delay-dependent robust control for a class of uncertain stochastic systems with time-delays for two cases.

Switched systems, as a class of the hybrid dynamical system consists of a family of subsystems governed by the switching signal. It has raised a lot of attention in the past decade (Lin and Antsaklis, 2005, Zhao et al., 2009, Hou et al., 2012). The switch feature is an intrinsic part of many industrial systems. For example, (i) real-time adjustment of system structure parameters, (ii) change of system structure by easily damaged parts, (iii) approximation of nonlinear system by linear system. Lin and Antsaklis (2005) described the switched system on NCSs in discretized model, and discussed the time delay and packet loss following certain probability distributions via stochastic model. Fan et al. (2013) investigated exponential stabilization of a dual-rate linear control system via switched systems and the input delay approach.

There are a lot of research done in model reduction, such as Balanced Truncation Method (BTM), which is one of the most common model order reduction schemes (Heydari and Pedram, 2006, Wang and Wang, 2008), Asymtotic Waveform Evaluation for Timing Analysis based on Padé approximation (Wang et al., 2011c), Krylov-based model order reduction (Michiels et al., 2011), etc. It is noticed that the BTM is a useful method in model reduction (Reis and Stykel, 2010). The major advantage of BTM is its guaranteed reduction error bound over the whole frequency domain. It is generally accepted that network induced delay could degrade the performance and even causes instability of NCSs. Meanwhile, the reduced data size usually helps to reduce the network induced delay during the packet transmission.

To deal with uncertainties, sliding mode control (SMC) has been frequently used in uncertain systems. The system structure switches based on the system state vector in order to force the system trajectory toward and/or to stay in a predefined subspace. Because of its excellent performance in the presence of external disturbances and parametric deviations, SMC has been widely adopted in many areas, especially, in robust control of uncertain systems. For example, Roh and Oh (1999) proposed a SMC method for the robust stabilization of uncertain linear input-delay systems with nonlinear parametric perturbations. A robust control law was derived to reduce the effect of both delay and uncertainty. Xia and Jia (2003) introduced a delay-independent robust SMC for time-delay systems with mismatched structural uncertainties. Furthermore, Niu et al. (2005) proposed an LMI condition to ensure the reachability to the sliding mode surface and the robust stochastic stability for general uncertain stochastic systems with time-varying delay based on the constructed integral sliding mode surface. The simulation of their results is very good. Yu and Chu (1999) also developed an LMI approach to guaranteed cost control of linear time-delay systems. De Souza and Li (1999) discussed delay-dependent robust $H\infty$ control of uncertain linear state delayed systems. Gouaisbaut et al. (2002) discussed the sliding mode control of uncertain systems with state-delays and additive perturbations, and designed a sliding surface to maximize the set of admissible delays and build LMIs for optimization. Choi (1999) proposed a sliding surface design for systems with mismatched unstructured uncertainties, and showed SMC as an effective robust control method for both uncertain systems and reduced order systems. Choi (2003) further extended the SMC strategy to multivariable uncertain systems with mismatched uncertainties. Late, Wu and Ho (2010) discussed the SMC of nonlinear singular stochastic systems with Markovian switching without time delay .

The partial major contributions of the chapter 3 in this dissertation include that: (i) Up to our best knowledge, we are the first to introduce the model reduction into NCS to reduce data transmission; (ii) The exponential stability condition is given based on the NCS with reduced data via the BTM; (iii) The performance optimization method is presented to setup the BTM parameter; and (iv) We further extended our theory and method to switched NCSs. The main results of this chapter have been published in (Bai and Wang, 2014a, Bai and Wang, 2014b).

The main contributions of the chapter 4 include: (i) to propose less-conservative methods for general uncertain stochastic systems with time-varying delay and structural uncertainties by introducing the SVD on the structural matrices of uncertainties for uncertain time-delay systems as a first time in the literature; (ii) to derive the robustly stochastic stability conditions by developing new Lyapunov-type functional for uncertain stochastic time-varying delay systems with general cases of time-delay derivative $\dot{\tau}(t) \leq h < +\infty$, where $\tau(t)$ is the time-varying delay, and the bound h may be greater than one; and (iii) to show our new results are less conservative than the existing results by theoretical analysis, proof and examples. The main results of this chapter have been published in (Wang and Bai, 2012, Wang and Bai, 2014).

Besides, the author of this dissertation has several other publications in image process (Bian et al., 2011, Bian et al., 2012), traffic modeling (Wang et al., 2011b, Wang et al., 2011a), and smart grid (Rahman et al., 2012).

The rest of this dissertation is organized as follows. Chapter 2 is the survey of the networked control systems, including a brief description of the NCS structure and current research. Chapter 3 mainly presents the performance optimization of remote networked control systems via model reduction method. Chapter 4 discusses the integral robust sliding mode control of the more general time-varying delay stochastic systems with structural uncertainties. Chapter 5 presents concludes the dissertation.

Notations*:* Euclidean norm of a vector is denoted by $\|\cdot\|$ in the following section. The minimum and maximum eigenvalue of matrix X are denoted by $\lambda_{min}(X)$ and $\lambda_{max}(X)$ respectively.

CHAPTER 2: NETWORKED CONTROL SYSTEMS

This chapter is a survey mainly about the history and methodologies in the research of NCSs. This chapter is organized as follows. As a more general form of NCSs, time-delay systems are firstly presented in section A about the mathematical forms and the basic concept of stability. Section B provides the research history of NCSs. Section C discusses two kinds of structure of NCSs. Since the time-delay is one of the major considerations of NCSs, Section D presents the delay analysis in the NCSs. The following section E is about the current compensation of the network-induced delay. The control methodologies in NCSs are discussed in section F. The rest two sections are the concept of stability and some open problems respectively.

2.1 Summary of Time-Delay Systems

NCS is a special form of time-delay systems, most of the methodologies and concepts are similar. The theory of time-delay systems is very important in the modeling and analyzing of NCSs. Therefore, before going through the NCS, this chapter first starts with the time-delay systems. The mathematic forms, some important definitions (e.g., stability and controllability) and some important theorems will be briefly summarized.

Time-delay system (also called systems with aftereffect or dead-time, hereditary systems, equations with deviating argument or differential-difference equations) appears in many kinds of control systems. Time-delay effect is usually an intrinsic part of some systems (e.g. many industrial systems and the recent X-37B unpiloted space plane) and cannot be ignored in the analyzing. Time-delay, in most of the cases, can degrade the performance of a system and is the source of instability. Moreover, time-delay systems usually are infinite dimensional and are generally much harder to analyze than ordinary differential equations. Therefore, a careful attention needs to be devoted in modeling and analyzing.

2.1.1 History of Time-Delay systems

TABLE 1 shows the important events in the development of the theory of time-delay systems (Richard, 2003).

TABLE 1: The important events of the time-delay system

Period	Event
18th century	The delay equations were firstly considered in the literature (e.g. the works of Bernoulli, Euler or Condorcet)(Emilia and Uri, 2003).
1920's to the end of 1940's	Systematic study of delay equations by V. Volterra , A. Myshkis and R. Bellman(Emilia and Uri, 2003).
1940's	Stability of time-delay systems became a formal subject of study with the contribution of Pontryagin and Bellman(Pontryagin, 1954, Gu et al., 2003, Pontryagin, 1966, Bellman and Dreyfus, 1959, Bellman and Zadeh, 1970).
The end of 1950's	Lyapunov's second method for the stability of delay systems was developed by N. Krasovskii and by B. Razumikhin (Krasovskii, 1963, Razumikhin, 1960, Emilia and Uri, 2003).
1959	Smith controller was invented (Smith, 1959).

1960's	The monographs of Pinney and Bellman and Cooke with a particular interest in the complex-doman approach and related frequency-domain techniques and methods (Michiels and Niculescu, 2007).
1970's	The theory arrived to some degree of maturity (Michiels and Niculescu, 2007).
After 1990's	A lot of research was on analysis and synthesis of uncertain systems with time-delay. Based on Lyapunov stability theory, plenty of outcomes have been obtained. e.g. finite-dimensional sufficient conditions.(Emilia and Uri, 2003)

Time-delay systems have received a great deal of attention the areas such as mathematics and control engineering after 1950s. The invention of Lyapunov function of stability and its improvement developed by N. Krasovskii and by B. Razumikhin result in many theories such as the sufficient conditions of stability (Emilia and Uri, 2003, Michiels and Niculescu, 2007, John and Jacques, 2007).

Recently, a lot of researchers are interested in robust control, which treats delays in dynamic systems as uncertainty (John and Jacques, 2007).

2.1.2 Mathematical Forms of Time-Delay Systems

2.1.2.1 General Model of Retarded Systems

The retarded time-delay systems can be described by functional differential equations (FDEs),

$$\dot{x}(x) = f(x_t, t, u_t), \qquad t \geq 0$$

$$y(t) = g(x_t, t, u_t),$$

$$x_t(\theta) = x(t+\theta), \quad -h \leq \theta \leq 0,$$

$$u_t(\theta) = u(t+\theta), \quad -h \leq \theta \leq 0,$$

$$x(t_0+\theta) = \varphi(\theta), \quad -h \leq \theta \leq 0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state variable at time t, $f: \mathbb{C}^n \times \mathbb{R} \times \mathbb{C}^n \rightarrow \mathbb{R}^n$, $\varphi(\theta)$ is the initial condition function mapping $[-h, 0]$ to $\mathbb{R}^n$, for which can be simplified to $C = C([t_0 - h, t_0], \mathbb{R}^n)$.

2.1.2.2 General Model of Neutral Systems

$$\dot{x}(x) = f(x_t, t, \dot{x}_t, u_t), \qquad t \geq 0 \tag{2}$$

The difference between retarded system and neutral system is the introduction of $\dot{x}_t$, which means the current state not only depends on the past state but also the derivative of past state.

The formula (2) could also be rewritten in Hale's form

$$F\dot{x}_t = \frac{dFx_t}{dt} = f(x_t, t, u_t) \tag{3}$$

2.1.2.3 Model of LTI Systems

The linear time-invariant time-delay system (LTI) can be represented as (Richard, 2003):

$$\dot{x}(t) + \sum_{l=1}^{q} D_l \dot{x}(t - w_l)$$

$$= \sum_{i=1}^{k} \left(A_i x(t - h_i) + B_i(t - h_i) \right) + \sum_{j=1}^{r} \int_{t-\tau_j}^{t} \left(G_j(\theta) x(\theta) + H_j(\theta) u(\theta) \right) d\theta$$

$$y(t) = \sum_{i=0}^{k} C_i x(t - h_i) + \sum_{j=1}^{r} \int_{t-\tau_j}^{t} N_j(\theta) x(\theta) d\theta \tag{4}$$

where the matrices D_l make the neutral part; A_i and B_i represents discrete-delay phenomena; the sum of integral corresponds to distributed delay effects; $y(t)$ is the output.

2.1.2.4 Some Typical Time-delay Systems

A system with a single delay on state variable can be represented as follows:

$$\dot{x}(t) = Ax(t) + A_1 x(t - h)$$
$$x(t_0 + \theta) = \varphi(\theta), \quad -h \leq \theta \leq 0 \tag{5}$$

where $A, A_1 \in R^{n \times n}$; h is a nonnegative delay; the initial states from the segment $[-h, 0]$ is needed to be defined as $\varphi(\theta)$ in order to get a unique solution.

As an extension, a system with multiple time-delays can be represented in the following form:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{k} A_\mathrm{i} \mathrm{x}(\mathrm{t} - \mathrm{h_i})$$
$$x(t_0 + \theta) = \varphi(\theta), \quad -\max_{i=1..k}(h_i) \leq \theta \leq 0 \tag{6}$$

As a special case, a LTI system with the following form is called time-delay systems with commensurate delays.

$$\dot{x}(t) = A_0 x(t) + \sum_{i=1}^{q} A_i x(t - ih), \quad h \geq 0 \tag{7}$$

where all the delays is the multiple of h.

Otherwise, the LTI system is called time-delay systems with incommensurate delays. Basically, the stability analyzing of incommensurate delays system is much harder than commensurate case. What’s more, many stability problem of incommensurate cases are proved to be NP hard (Gu et al., 2003, Kharitonov, 1999).

A more general form is distributed delay system as stated in (Kharitonov, 1999):

$$\dot{x}(t) = \int_{-h}^{0} [dF(\theta)]x(t+\theta)$$

$$\int_{-h}^{0} \|dF(\theta)\| < \infty$$

$$x(t_0+\theta) = \varphi(\theta), \qquad -h \le \theta \le 0 \tag{8}$$

The initial state $\varphi(\theta)$ need to be specified in the region $[-h, 0]$. With the appropriate choosing of $F(\theta)$, system (8) could be converted into system (5) and (6).

2.1.3 Stability of Time-Delay Systems

This part is about the conception of stability and criteria.

2.1.3.1 Stability Criteria

Definition 1. The system (2) is said to be Lyapunov stable if for every positive ε there exit a positive δ such that if $\|\varphi(\cdot)\| < \delta$, then $\|x(t, t_0, \varphi(\cdot))\| < \delta$ for all $t \geq= t_0$.

Remark 1. If the value δ from the above definition may be chosen such that in addition $x(t, t_0, \varphi(\cdot)) \to 0$ when $t \to \infty$, then the system (2) is said to be asymptotic stable.

Definition 2. The system (2) is said to be exponential stable if there exist positive constant μ such that every solution of the system satisfy the following inequality

$$\|x(t, t_0, \varphi(\cdot))\| \le \mu \|\varphi(\cdot)\| e^{-\alpha(t-t_0)} \text{ for all } t \ge t_0. \tag{9}$$

The above two definitions and remark can be found in (Kharitonov, 1999).

There are two kinds of uncertainty that might affect the stability in time-delay systems. The first one is in system structure description, e.g., ΔA and ΔA_1 in the simple linear time-delay system (10). The second one is the uncertainty in delay constant, e.g., Δh

and Δh_1 in system (10).

$$\dot{x}(t) = (A + \Delta A)x(t) + (A_1 + \Delta A_1)x\big(t - (h + \Delta h)\big) + (B + \Delta B)u\big(t - (h_1 + \Delta h_1)\big) \quad (10)$$

Definition 3. Given a time-delay system with uncertainty description $\mathcal{D}$ (time-delay) and $\mathcal{S}$(system matrices), the system is said to be robustly stable if it is stable for all delay elements from $\mathcal{D}$ and for all system matrices from $\mathcal{S}$ (Kharitonov, 1999).

A system is called uncertainty free system if both uncertainty description sets $\mathcal{D}$ and $\mathcal{S}$ contain only one element. As stated in (Özbay, 1996), the systems with time-delay uncertainty are much more complex to analyze than those with system uncertainty.

2.1.3.2 Lyapunov-Krasovskii Theorem

Theorem 1. The time-delay system (2) with $\mathrm{u_t} = 0$ is asymptotical stable if there exists a continuous functional $v(\cdot)$ such that (Su et al., 2007, Krasovskii, 1963)

i. $v_1(\|\varphi(0)\|) \leq v(t_0, \varphi) \leq v_2(\|\varphi(\theta)\|)$

ii. $\frac{d}{dt}v(x_t) \leq -\beta\|x(t)\|^2$, for a positive constant β.

Generally, the construction and analysis of functional $v(\cdot)$ for a system is a very difficult task. It is often used for deriving sufficient conditions.

2.1.3.3 Lyapunov-Razumikhin Theorem

Theorem 2. The system (2) with $u_t = 0$ is said to be stable if there exit a continuous function $v(t, x_t)$ and an increasing function $p(t) > 0$ such that (Razumikhin, 1960, Kharitonov, 1999)

i. $v(\varsigma, x_\varsigma) < p\big(v(t, x_t)\big),\ \ \varsigma \in [t - h, t], t > t_0$

ii. $\dot{v}(t, x_t) \leq -\gamma(\|x\|) + 2\varepsilon$

where ε is nonnegative constant, and satisfy $2\varepsilon < lim_{r\to 0}\,\gamma(r) \coloneqq \ell$.

Compare to Theorem 1, this theorem can obtain an upper bound of the time derivative independent of past states and therefore reduces the computing complexity, however, it also brings some conservative (Kharitonov, 1999).

2.1.3.4 Frequency Domain Robust Stability Analysis

As the classical method, frequency domain stability analysis uses the method like Nyquist test and root-locus. "With the aid of the small gain theorem, frequency domain tests have become increasingly more prevalent in stability analysis, and have played especially a central role in the theory of robust control." (Gu et al., 2003)

The stability of an LTI retarded delay systems can be analyzed by position of poles in s plane, which is zeros or roots of characteristic function.

Consider a LTI retarded system (6), its characteristic function is in the form of

$$p(s; e^{-h_1 s}, \dots, e^{-h_k s}) = det\left(sI - A - \sum_{i=1}^{k} A_k e^{-h_i s}\right) \quad (11)$$

Specifically, the sufficient and necessary condition of stable is $p(s; e^{-h_1 s}, \dots, e^{-h_k s})$ has no zero with nonnegative real parts.

In addition, for a neural type time-delay system, the stability conditions require that: for all the characteristic roots λ_i, $Re(\lambda_i) \leq -\alpha$ for some $\alpha > 0$.

2.1.3.5 Stability Independent of Delay

A time-delay system is said to be stability independent of delay (i.o.d) if for all the possible nonnegative delay the system is stable. Otherwise, if time-delay systems are stable for only a subset of nonnegative delay, this kind of system is called dependent of delay (d.d). Typically, if the system (6) is stable with delays $h_i, i = 1, .., k$, this system can remain be stable in a certain neighborhood area of those delays. That is within this area the zeros of $p(s; e^{-h_1 s}, \ldots, e^{-h_k s})$ stay in the left half plane.

For time-delay system with commensurate delays, it has the following theorem about stability.

Theorem 3. The system (7) is i.o.d stable if and only if

i. A is stable.

ii. $A + \sum_{i=1}^{k} A_i$ is stable, and

$$\rho\big(M_k(j\omega)\big) < 1, \quad \forall \omega > 0 \tag{12}$$

where

$$M_k(s) \coloneqq \begin{pmatrix} (sI-A)^{-1}A_1 & \ldots & (sI-A)^{-1}A_{k-1} & (sI-A)^{-1}A_k \\ I & \ldots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & I & 0 \end{pmatrix}$$

and ρ(·) is spectral radius. See (Gu et al., 2003)

2.1.4 Controllability

It has to be mentioned the definitions of controllability of time-delay system are different from those with time-delay free.

i. In delay free system, the state controllability means the ability to move from one state to any other states in a finite interval, where controllability in delay system means the ability to move from a vector $\{x(\theta_0), \theta_0 \in [t_0,\ t_0 + max(h_i)]\}$ to other vectors $\{x(\theta_1), \theta_1 \in [t_1, t_1 +\ max(h_i)]\}$.

ii. The introduction of delay also introduced the minimum reaching time. e.g., the system $\dot{x} = x(t) + u(t-1)$ cannot be controlled within 1sec (Richard, 2003).

2.2 The Earlier History of NCSs

The typical early controller is centralized controller. The components (e.g. sensor, observer, controller, and actuator) are connected directly by wire. In this period, researchers did not need to worry about time delay and signal loses. With the development of system, the controllers become much complex. The large-scale system is divided into several sub-systems with interconnections between each other as depicted in FIGURE 1. This is decentralized control scheme, which was the only solution for some applications in that time. The controllers are connected to their corresponding subsystems, however, the controllers cannot exchange signal between each other in this structure.

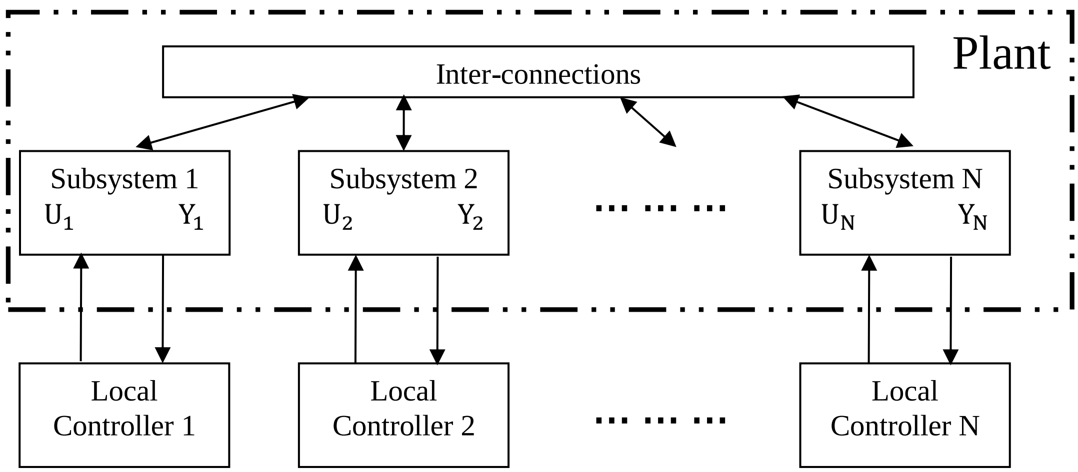

FIGURE 1: Decentralized control scheme (Yang, 2006)

This structure was very useful at that time. However, it has fatal defects. It has been proved that for some systems, there is no decentralized controller which could make the systems stable. To solve this problem, quasi-decentralized control scheme as depicted in FIGURE 2 was proposed (Yang et al., 2000). In this control scheme, local controllers have very limited ability to exchange data with other local controllers. Because the remote communication is expensive before 1990s, the exchanging data between local controllers need to be minimized.

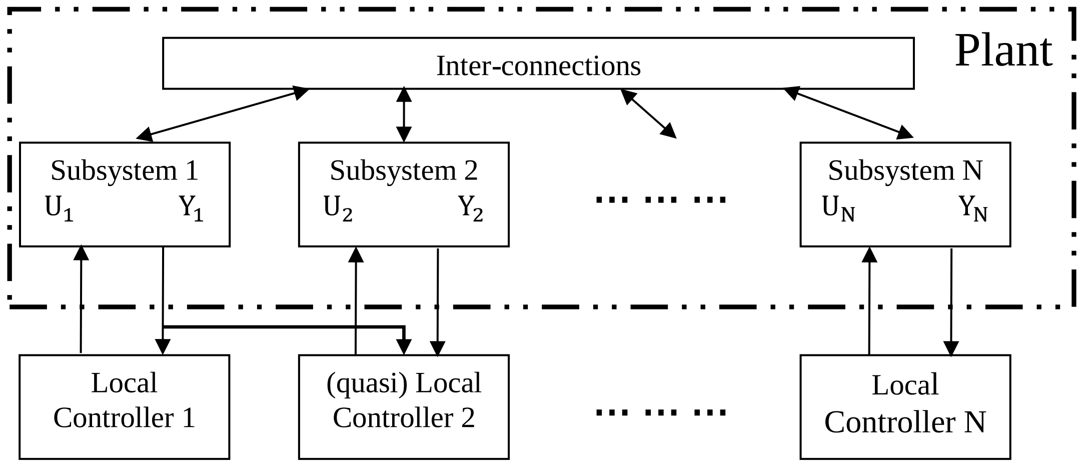

FIGURE 2: Quasi-decentralized control scheme (Yang, 2006)

In decades, the fast development of network with far better performance and less cost greatly make the application of distributed systems more practical. Because of its wide applications, NCS becomes a very promising research field. For more detail about the history of NCS, please refer to (Yang, 2006) .

Recently, although there are a lot of research on NCSs, the research of NCSs could be categorized into two parts (Gupta and Chow, 2010):

iii. Control of network: e.g., congestion control, routing control and etc.

iv. Control over network: It includes control strategy, control system design and stability condition.

2.3 Structure of NCS

Despite the slight difference in applications, generally, there are two kinds of structure of NCS. One is direct structure, the other is hierarchal structure (Tipsuwan and Chow, 2003). Those two kinds of structure will be presented in this part.

2.3.1 Direct Structure

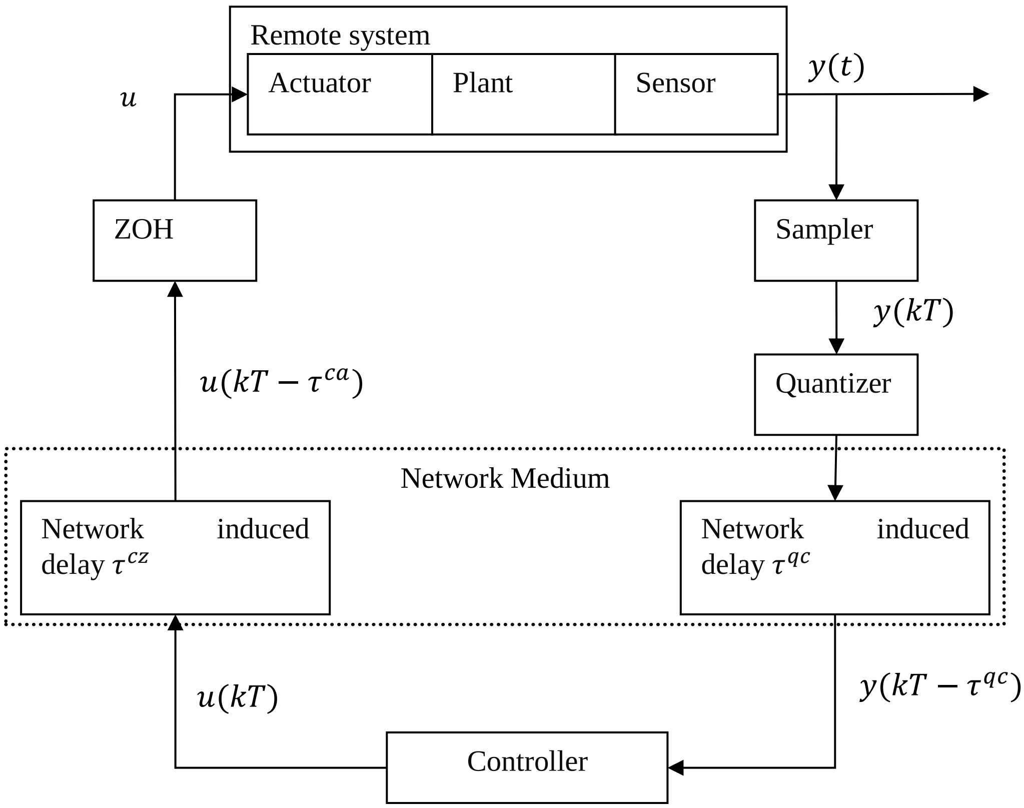

FIGURE 3: Direct structure NCS

This configuration, as depicted in FIGURE 3, mainly includes two parts: *remote system* and *controller*. Those two parts communicate by exiting network. The state or output of the plant (e.g. temperature, speed, and position et al.) is converted into electric signal by sensors. Since continuous analogy signal usually cannot be transferred on network, those signals are then discretized by sampler. The *quantizer* is used to convert signal into digital code which could be transferred on network. When *controller* get the feedback signal $y(kT)$, as depicted in FIGURE 3, control signal $u(kT)$ is then generated and sent to *ZOH* (zero-order hold), which can continuous output one value until a new input comes. Usually, the time-delay on *sampler* and *quantizer* could be neglected, since it's usually too small compare to the network induced delay.

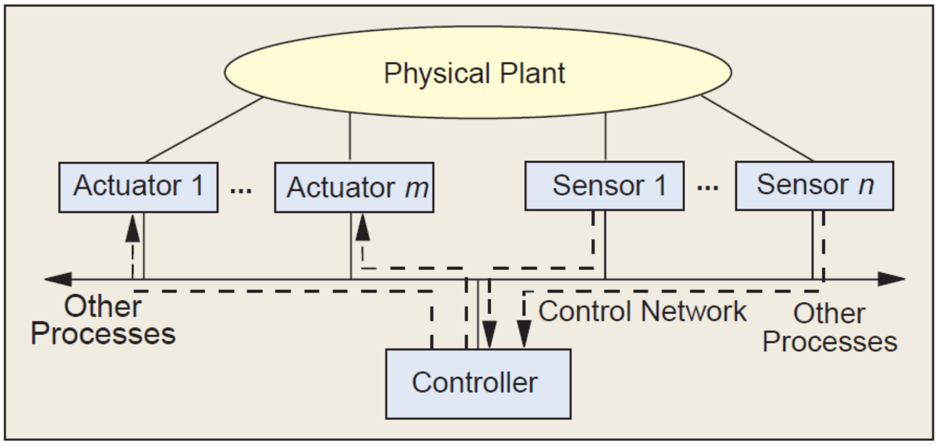

FIGURE 4: A NCS system with multiple sensors and actuators (Zhang et al., 2001)

Particularly, for some plants with multiple sensors and actuators, as depicted in FIGURE 4, this structure also belongs to direct structure. A lot of systems belong to this case in application. For example, a car might have to sample its engine temperature and exhaust to its controller by using several sensors.

A more general framework of direct structure is presented in (Yang, 2006).

2.3.2 Hierarchical Structure

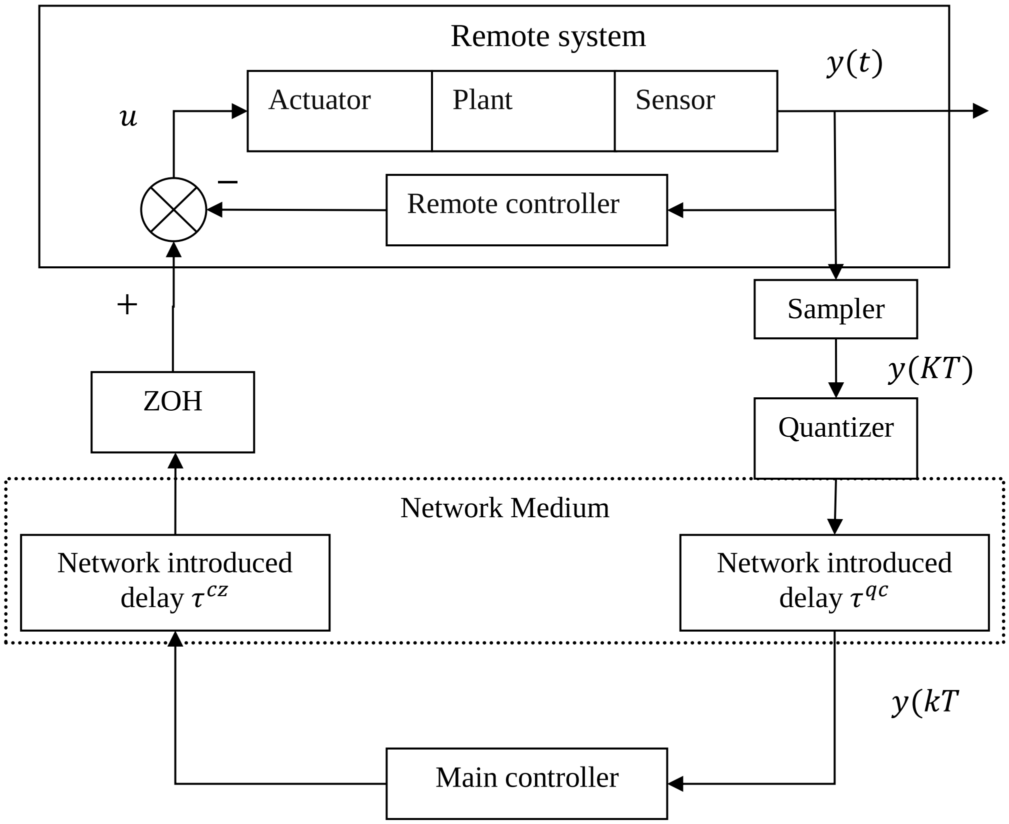

FIGURE 5: Hierarchical structure NCS

Unlike direct structure NCS without any controller in the plant side, hierarchical structure as depicted in FIGURE 5 consists of two controllers: *remote controller* in remote side with plant and the *main controller*. The *remote controller* is used here to perform local closed-loop control. For *remote controller*, the time-delay in exchanging data with plant could be neglected. Compare to the *remote controller*, the delay in exchanging data between the *main controller* and the *plant* usually needs to be taken into consideration. The outputs of two controllers are combined together as reference signal of plant. Hierarchical structure NCS can be implemented to handle multiple networked control loops for several remote systems. There are several applications of this structure, e.g., mobile robot

(Tipsuwan and Chow, 2002) or teleoperation (Xi and Tarn, 1998, Hokayem and Spong, 2006).

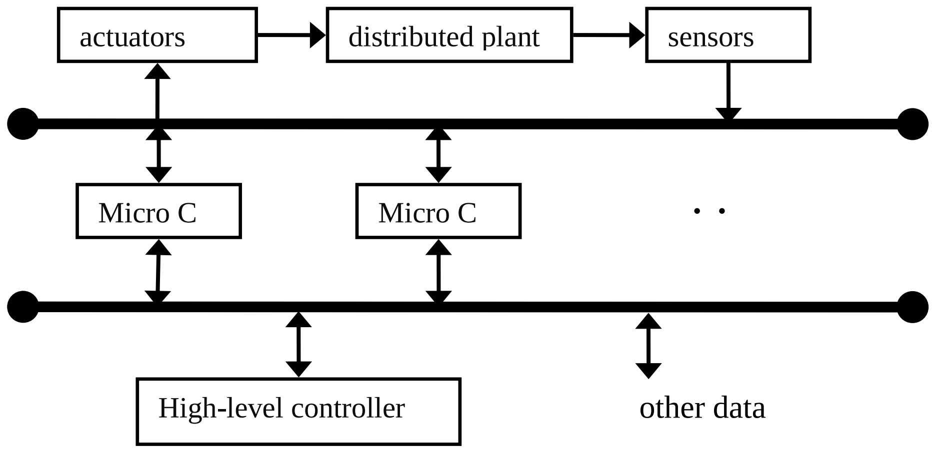

FIGURE 6: NCS with two-level communications (Yang, 2006)

Another hierarchical structure introduced by Yang (Yang, 2006, Yang et al., 2004) is depicted in FIGURE 6. Instead of controlling directly to the plant in FIGURE 5, high-level controller controls microcontrollers in L2C. The two networks L1C (level 1 communication) and L2C (level 2 communication) are independent from each other. This kind of structure can be used in many applications, e.g. industrial process plant with several embedded microcontrollers supervised by computers as stated in (Yang, 2006).

2.4 Analysis of Delay

Generally, there are four kinds of delays introduced by network (Wikipedia, 2009).

i. Computing delay: time spent to process data, e.g., encryption/decryption process.

ii. Waiting/Queuing delay: time spent in service / routing queues.

iii. Transmission delay: time spent in placing a frame/packet on link.

iv. Propagation delay: time spent for a frame/packet travelling through the physical link.

In most of the applications, the computation delay can be neglected, because it is usually very minor compare to other kinds of delay in NCSs or it can be lumped into delay from controller to actuator in analysis. The three kinds of delay (waiting/Queuing delay, transmission delay, and propagation delay) can be lumped into network induced delay. e.g., τ^{qc} and τ^{cz} in FIGURE 3 and FIGURE 5, where τ^{qc} means the time-delay from *quantizer* to *controller* and τ^{cz} means time-delay from the *controller* to the *ZOH*. It needs to be mentioned that the delays on the *sampler*, *quantizer* and *ZOH* are ignored in the following part, therefore τ^{sc} (delay from the *sensor* to the *controller*) is equal to τ^{qc}, and τ^{ca} (time-delay from controller to actuator) is equal to τ^{cz}.

The network induced time-delay has different characteristics depends on the type of network used for exchanging data.

i. Cyclic-service network. Data on cyclic-service network exchange in a cyclic order, therefore the time-delays τ^{sc} and τ^{ca} are deterministic or even constant (ideal case) in this kind of network. However, in practice there are small uncertainties of delay for various reasons. The protocols in this category include token passing (e.g. IEEE 802.4, SAE token bus, PROFIBUS, Fieldbus), token ring (e.g., IEEE 802.5, SAE token ring), and polling (e.g., MIL-STD-1553B, FIP, etc.). For more information please refer to (Chow and Tipsuwan, 2001, Tipsuwan and Chow, 2003, Yang, 2006).

ii. Random access network. Time-delay on this kind of network involves much more uncertainties, because of the presence of queuing and collision. Moreover, the networks such as TCP/IP often change the routing path while exchanging package. It definitely increases the uncertainty on delay. Most of the protocols belong to this case.

It's a generally accepted fact that network induced delay could degrade the performance and even causes instability of NCSs. The time-delay might change the poles of a system and it usually has infinite order. Network-induced delay could be modeled in many methods, e.g., independent random delay, delay with known probability distribution governed by Markov chain (Gupta and Chow, 2010).

2.5 Compensation for Network-Induced Delay

The network induced delay could to some extend be compensated in pre-estimated approach. An approach has been proposed in (Zhang et al., 2001). Generally, the delay from quantizer to controller τ^{qc} could be detected by controller. For example, in TCP/IP protocol, a timestamp is set for each packet when it begins to be sent, if the clocks of all components are synchronized, the delay τ^{qc} is very easy to be calculated by letting the reach time to controller minus its timestamp. Therefore, the pre-estimator could use τ^{qc} and pasting states to predict current state of plant without delay. However, the time-delay from controller to actuator is hard to be predicted by controller, as stated in (Zhang et al., 2001).

An example of how to design a pre-estimator to compensate time-delay is shown below.

A system without network induced delay defined in (Zhang et al., 2001) is modeled as:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) \tag{13}$$

If NCS has a full-state feedback, which is $y(kh) = x(kh)$, the discrete controller is defined as:

$$u(kh) = -Kx(kh), k = 0,1,2, \dots. \tag{14}$$

The plant state estimate at the time $y(kh)$ received is defined as:

$$\bar{x}(kh + \tau_k^{sc}) = x(kh + \tau_k^{sc}) = e^{A\tau_k^{sc}} x(kh) + \int_{kh}^{kh+\tau_k^{sc}} e^{A\left(kh+\tau_k^{sc}-s\right)} Bu(s)ds \tag{15}$$

Then the control law is computed by

$$u(kh + \tau_k^{sc}) = -K\bar{x}(kh + \tau_k^{sc}). \tag{16}$$

Using this control law, the closed-loop system becomes

$$x\big((k+1)h + \tau_k^{sc}\big) = \widetilde{\Phi}(\delta_k) x(kh + \delta_k) \tag{17}$$

where

$$\delta_k = h + \tau_{k+1}^{sc} - \tau_k^{sc},$$
$$\widetilde{\Phi}(\delta_k) = \Phi(\delta_k) - \Gamma(\delta_k)K,$$
$$\Phi(\delta_k) = e^{A\delta_k},$$
$$\Gamma(\delta_k) = \int_0^{\delta_k} e^{As} B ds.$$

It can be inferred from (17), since the τ_k^{sc} and τ_{k+1}^{sc} are known by controller, the time-delay effect from sensor to controller is fully compensated.

For the system with output feedback, the pre-estimator is slightly different. Assume the plant state is $y(kh) = Cx(kh)$, the estimator can be described as:

$$\bar{x}\big((k+1)h\big) = \hat{x}\big((k+1)h\big) + L_c\Big(y\big((k+1)h\big) - C\hat{x}\big((k+1)h\big)\Big)$$

$$\hat{x}\big((k+1)h\big) = \Phi\bar{x}(kh) + \Gamma u(kh) \tag{18}$$

where L_c is the current estimator gain.

Using the two formulas defined above, the closed-loop system with the estimator is

$$z\big((k+1)h + \tau_{k+1}\big) = \overline{\Phi}(\delta_k) z(kh + \tau_k) \tag{19}$$

where

$$\bar{\Phi}(\delta_k) = \begin{bmatrix} \Phi(\delta_k) & -\Gamma(\delta_k)K \\ 0 & \Phi(\delta_k) - L_c H \Phi(\delta_k) \end{bmatrix}.$$

2.6 Control Methodologies

2.6.1 Stochastic Lyapunov Function

This method was originally used on multi-loop control systems with feedback loop closed through a time-shared digital computer. Because of the similarity, stochastic delay-differential equation could be used to model NCS with a randomly distribute traffic on the network (Wu et al., 2002, Isle and Kozin, 1972, Isle, 1975).

2.6.2 Augmented Deterministic Discrete-Time Model Methodology

This method was firstly proposed by halevi and Ray (1988). The system is represented by an augmented state vector consisted of two parts: (i) past values of the plant input and output, and (ii) current states of both the plant and controller. A finite-dimensional and time-varying discrete model is formed. For more information please refer to (Wu et al., 2002, Tipsuwan and Chow, 2003, Liou and Ray, 1991a, Liou and Ray, 1991b)

2.6.3 Queuing Method

Queuing method (or buffering method) is a popular method. This method could convert a NCS into a time-invariant system. It has several advantage such as (i) no need to redesign the existing predictive controller; (ii) no need of clock synchronization; and (iii) slightly influence of bad network condition (Gupta and Chow, 2010).

Lucka and Ray (1990) at first proposed queuing theory into NCS, this method uses queue and the received plant state sequence to predict current state by controller.

Chan and Ozguner (1994) proposed a new kind of estimator design based on queuing theory. The configuration of is depicted in FIGURE 7. The assumptions of this system include:

i. The queue is FIFO (first in first out) /FCFS (first come first serve)

ii. The sensor will send data by communication link as long as the queue is not empty.

iii. The communication link is shared.

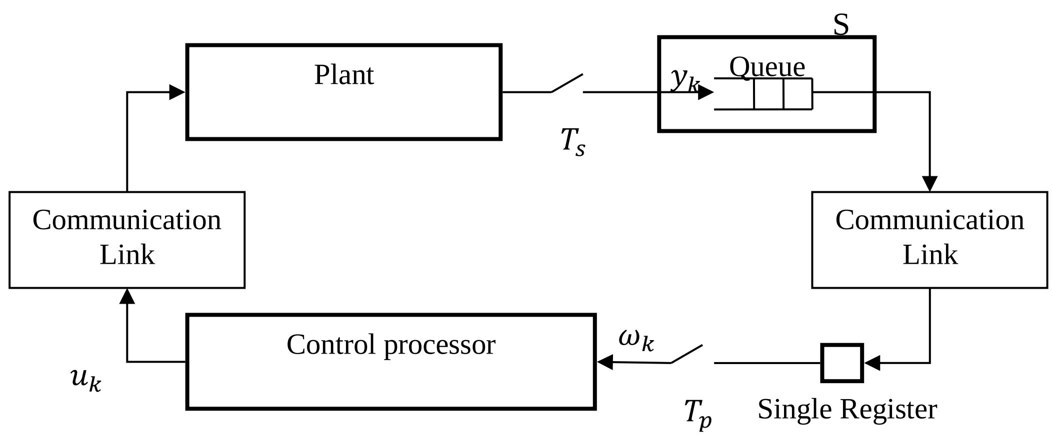

FIGURE 7: Block diagram of the controlled system involving queue and communication link (Chan and Ozguner, 1994)

Define n as the queue length, T_s as the sensor sample period and T_p as the processor sampling period. Let $T_p = T_s = T$ in this NCS. In the data sending process, sensor will send the current length of queue n attached with sampled data. Therefore the queue length is known by control processor. The assertion defined is: "At the kth processor sampling instant, if it is know that the queue length n is equal to j, then the corresponding data is either y_{k-j} or y_{k-j+1}, $1 \leq j \leq m$ (where m is the queue buffer size)".

Consider a plant as:

$$x_{k+1} = Ax_k + Bu_k \qquad k = 0,1,2,\ldots \tag{20}$$

Assume the full state is transmitted. The received state of control processor is x_{k-j}

or x_{k-j+1} (j is equal to queue size n) the prediction of current state of plant is:

$$\hat{x}_k = P_0\left(A^{j-1}\omega_k + W_j\right) + P_1\left(A^j\omega_k + W_{j+1}\right) \quad (21)$$

where

$$W_1 = 0$$

$$W_i = [B \quad AB \quad \ldots \quad A^{i-2}B]\begin{bmatrix} u_{k-1} \\ u_{k-2} \\ \vdots \\ u_{k-i+1} \end{bmatrix}, \quad (22)$$

P_0, P_1 are weighting matrices.

The calculation of optimal P_0, P_1 and the feedback without full state information feedback is also discussed in (Chan and Ozguner, 1994). What's more, the observer is introduced if this feedback system is not full state feedback.

2.6.4 Optimal Stochastic Control Methodology

John Nilsson proposed an approach that treat NCSs with random network-induced delay as LQG (Linear-Quadratic-Gaussian) problem, which is one of the most fundamental optimal control problem (Nilsson, 1998, Nilsson et al., 1996, Athans, 1971).

This approach is based on following assumptions:

i. $\tau_k^{sc} + \tau_k^{ca} < h$, which means the summation of delay from sensor to actuator and delay from actuator and sensor is less that sampling period h.

ii. The set $\{\tau_0^{sc}, \ldots, \tau_k^{sc}, \tau_0^{ca}, \ldots, \tau_{k-1}^{ca}\}$ is known by controller. This is actually very easy to be obtained by applying timestamp.

Consider a controlled process be

$$\frac{dx}{dt} = Ax(t) + Bu(t) + v(t) \quad (23)$$

where $v(t)$ is white noise with zero mean.

Discretization of system (23) with the consideration of time-delay gives

$$x_{k+1} = \Phi x_k + \Gamma_0(\tau_k^{sc}, \tau_k^{ca})u_k + \Gamma_1(\tau_k^{sc}, \tau_k^{ca})u_{k-1} + v_k$$
$$y_k = Cx_k + w_k \tag{24}$$

where

$$\Phi = e^{Ah}$$

$$\Gamma_0(\tau_k^{sc}, \tau_k^{ca}) = \int_0^{h-\tau_k^{sc}-\tau_k^{ca}} e^{As} ds\, B,$$

$$\Gamma_1(\tau_k^{sc}, \tau_k^{ca}) = \int_{h-\tau_k^{sc}-\tau_k^{ca}}^{0} e^{As} ds\, B,$$

and v_k and w_k are unrelated white noise.

In order to derive the optimal controller, John Nilsson defined an object function

$$J_N = E[x_N^T Q_N x_N] + E\left[\sum_{k=0}^{N-1} \begin{bmatrix} x_k \\ u_k \end{bmatrix} Q \begin{bmatrix} x_k \\ u_k \end{bmatrix}\right], \tag{25}$$

where Q is symmetric and positive semi-definite.

Q has the following form:

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix}, \tag{26}$$

where Q_{22} is positive definite,.

The optimal controller should minimize J_N. Based on J_N and assume $y_k = x_k$, the optimal control law of system (23) is given by:

$$u_k = -L_k(\tau_k^{sc}, r_k)\begin{bmatrix} x_k \\ u_{k-1} \end{bmatrix}, \tag{27}$$

where, for $r_k = i, i = 1, \dots, s$, we have

$$L_k(\tau_k^{sc}, i) = \left(Q_{22} + \tilde{S}_i^{22}(k+1)\right)^{-1} [Q_{12}^T + \tilde{S}_i^{21}(k+1) \quad \tilde{S}_i^{23}(k+1)],$$

$$\tilde{S}_i(k+1) = E_{\tau_k^{ca}}\left(G^T \sum_{j=1}^{s} q_{ij} S_j(k+1) G \,|\, \tau_k^{sc}, r_k = i\right),$$

$$G = \begin{bmatrix} \Phi & \Gamma_0(\tau_k^{sc}, \tau_k^{ca}) & \Gamma_1(\tau_k^{sc}, \tau_k^{ca}) \\ 0 & I & 0 \end{bmatrix},$$

$$S_k = E_{\tau_k^{ca}}\left(F_1^T Q F_1 + F_2^T \widetilde{S_\iota}(k+1) F_2 | r_k = i\right),$$

$$F_1 = \begin{bmatrix} I \quad 0 \\ -L_k(\tau_k^{sc}, r_k) \end{bmatrix},$$

$$F_2 = \begin{bmatrix} I \quad 0 \\ -L_k(\tau_k^{sc}, r_k) \\ 0 \quad I \end{bmatrix},$$

$$S_N = \begin{bmatrix} Q_N & 0 \\ 0 & 0 \end{bmatrix}.$$

In equation (27) the r_k is the Markov state, $S_i(k)$ is the symmetric matrix represented the cost to go at time k if $r_k = i$, $\left(\tilde{S}_i^{ab}\right)(k)$ is block (a, b) of symmetric matrix $\widetilde{S_\iota}(k)$, and Q_{ab} is the block (a, b) of symmetric constant matrix Q.

This approach has shown better performance than deterministic predictor-based method in example of (Nilsson et al., 1996) . However, as stated by Tipsuwan and Chow (2003), this approach require large amount of memory in controller to store a large amount of past information. Moreover, this approach might be not effective for fast response systems.

2.6.5 Gain Scheduler Middleware

Since the controller and strategy are usually needed to be re-designed for another specific application or environment, this would definitely increase cost and time. Tipsuwan and Chow (2004b) introduced Gain Scheduler Middleware (GSM) to use exiting delay-free controllers for network control. As depicted in FIGURE 8, the GSM includes: network traffic estimator, feedback preprocessor and gain scheduler. Feedback preprocessor is used to preprocess the feedback data from remote system before forwarding the signal to the controller. Gain scheduler is used to modify the controller output with respect to the current network conditions. Network traffic estimator is used to estimate the current network traffic condition. In the experiment of (Tipsuwan and Chow, 2004b, Tipsuwan and Chow, 2004a), it is used to measure the distribution of network time-delay and find the best parameter for feedback preprocessor and gain scheduler.

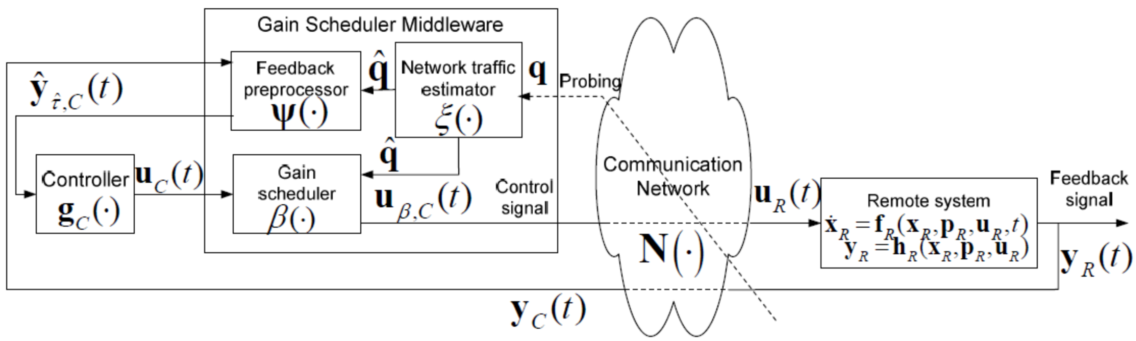

FIGURE 8: Block diagram of GSM module designed by Tipsuwan and Chow for network delay compensation (Tipsuwan and Chow, 2004b)

2.6.6 Other Important Control Methodologies

Robust control methodology applies robust control theory into the design of network controller. The main advantage of this approach is there is no need for priori information of network delays probability distributions as stated in (Tipsuwan and Chow, 2003, Goktas, 2000).

Fuzzy logic modulation methodology proposed by Almutairi applied fuzz logic into the compensation of network induced delay in controlling a DC motor (Zadeh, 1965, Almutairi et al., 2001, Almutairi and Chow, 2001).

2.7 Stability Condition

Like time-delay systems, the stability condition of NCS is usually inferred from Lyapunov functional.

Zhang and Branicky el al. (2001) discussed the stability of NCSs with n sensors as depicted in FIGURE 4. The analysis is divided in two cases: total network induced delay($\tau = \tau^{sc} + \tau^{ca}$) less than sample interval and total network induce delay longer than sample interval. The conclusions and stability region τ are given in each case mentioned above. The case of sample interval shorter than network induced delay is much more complicated than other one.

Gao and Chen et al. (2008) discussed a direct structure NCS with bounded delays. This article firstly defined a new Lyapunov-krasovskii functional which make full use on $\tau^{sc}(t)$ and $\tau^{ca}(t)$. Based on a network model with logarithmic quantizer and bounded disturbance, the new stability conditions are given in LMI form.

2.8 Some Open Problems

Since the research on NCS is still in infancy, there are a lot of opening problems. However, most of the research on NCS so far is on stability condition. The performance and the design of controller has been overlooked as stated in (Antsaklis and Baillieul, 2007).

It's mentioned in (Wu et al., 2002, Gupta and Chow, 2010, Yang, 2006) that the further research might includes:

i. Real-time performance is very important part of NCS. Because of the bandwidth, dropout, end bit error of NCS, there are a lot of problems need to solve with different protocol.

ii. Techniques for evaluating the structure of the NCS.

iii. Inter-operability and integration techniques.

iv. Dynamic neuron-fuzzy modeling for NCSs.

v. Design theory and optimal scheduling method.

vi. Security and reliability of NCSs, especially real-time secured control. The network, especially wireless network, is very easy to be intercepted. The security need to be great concerned in the application e.g., nuclear reactor power plant, space projects. The survey of those security issues in industrial automation is given in (Dacfey Dzung et al., 2005).

vii. Middleware technical of NCS. It is the base technology of intelligent space as depicted in FIGURE 9. The compatible OSs of NCS middleware include AlphaOS (Yang et al., 2005), Autosar (Heinecke and Bielefeld, 2006).

viii. Multi-agent traffic control: With the help of technologies e.g., GPS, electronics atlas (Google map). E.g. safe vehicle operation in multithreaded environment stated in (Murray, 2009).

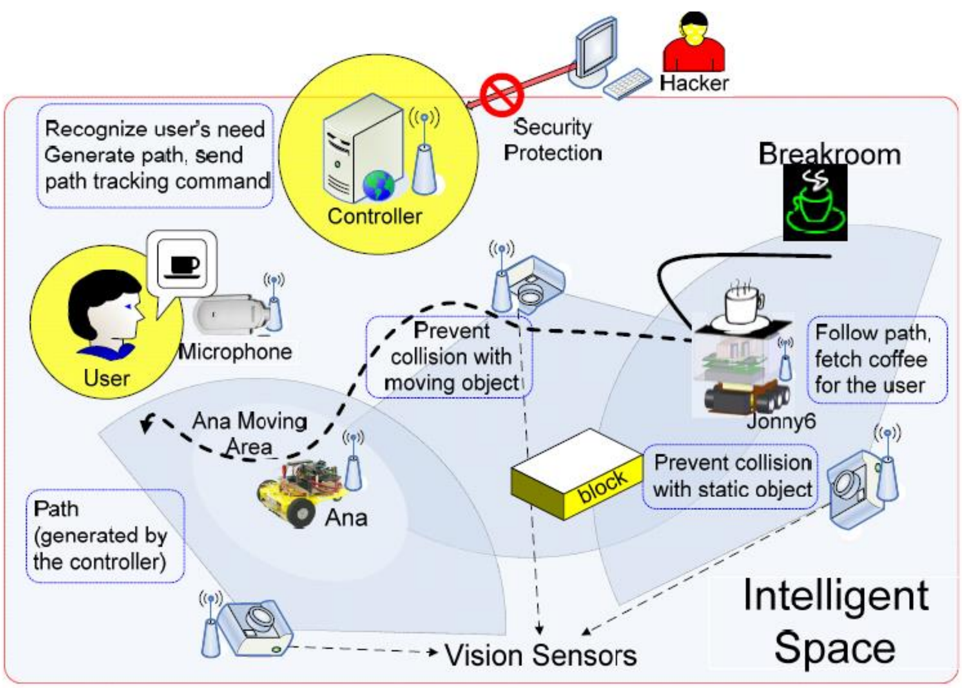

FIGURE 9: Conceptual diagram of iSpace – another example/application NCS (Gupta et al., 2008)

CHAPTER 3: PERFORMANCE OPTIMIZATION OF REMOTE NETWORKED CONTROL SYSTEMS VIA MODEL REDUCTION METHOD

This chapter is organized as follows. Section A provides the problem formulation, including a brief description of the switched NCS structure with our MRDC. Section B mainly presents the our theorems on the exponential stability condition of non-switched NCS with MDRC in B.1, and the exponential stability condition of switched NCS with MDRC in B.2. Section C discusses the optimization problem and methods. Section D presents examples to demonstrate the results.

3.1 Problem Formulation

This dissertation considers a typical switched NCSs structure with our *Model Reduction Data Compressor* (MRDC) depicted as FIGURE 10. A similar non-switched NCS application without *MRDC* was applied in (Gao et al., 2008). Our *MRDC* contains three parts: (1) *Balancing Transform*, (2) *Truncator*, and (3) *Balancing Inverse Transform*. In this closed-loop NCS, *sampler* is assumed to be clock-driven, while other components like *quantizer*, *balancing transform, truncator, balancing inverse transform, controller, network* and *zero-order hold (ZOH)* are all event-driven.

This dissertation considers the remote system:

$$\dot{x}(t) = A_{\beta(t)}x(t) + B_{\beta(t)}u(t)$$

$$y(t) = C_{\beta(t)}x(t) + D_{\beta(t)}u(t) \tag{28}$$

where $x(t) \in \mathbb{R}^{n\times 1}$ is the system state vector at the time t; matrices $A_\beta \in \mathbb{R}^{n\times n}$, $B_\beta \in \mathbb{R}^{n\times l}$,$C_\beta \in \mathbb{R}^{m\times n}$,$D_\beta \in \mathbb{R}^{m\times l}$ are the system matrices; $u(t) \in \mathbb{R}^{l}$ is the control input; $y(t)$ is the output of this plant; $\beta(t) = i, i \in \mathcal{N}$ is switching signal with $\mathcal{N} = \{1,2,\dots,N\}$. For the simplicity of denotation, β is equivalent to $\beta(t)$ without explanation, and $\beta_{\mathrm{d}1}$ is the delayed switching signal over the *downlink*.

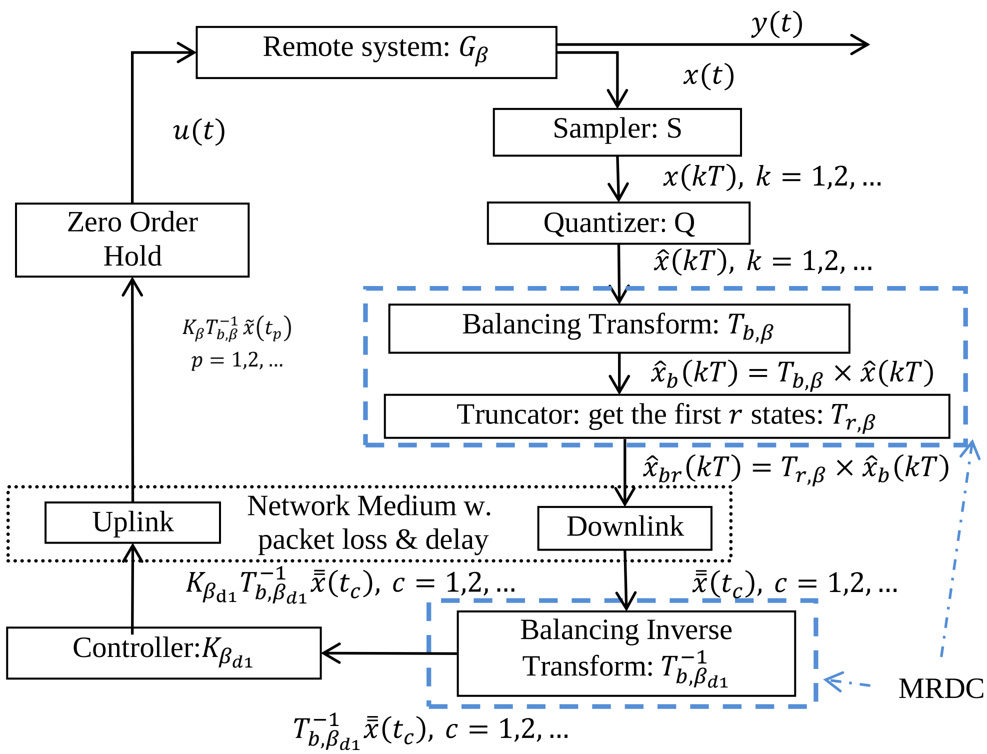

FIGURE 10: Structure NCS with Model Reduction

Without loss of generality, the following two assumptions are held for subsystem β of (28):

i. (A_β, B_β) is controllable, and

ii. B_β has full column rank.

The switching signal throughout the closed-loop NCS satisfies the switching sequence $\{(i_0, t_0), (i_1, t_1), \dots, (i_k, t_k), \dots \mid i_k \in \mathcal{N}, k = 0,1,\dots\}$ with $t_0 = 0$, which means the i-th subsystem is activated when $t \in [t_k, t_{k+1})$.

The sampling period of *sampler* is denoted by T, which is a positive real constant. We assume the k-th sample of the feedback states vector $x(t)$ is $x(kT), k = 0, 1, \dots, \infty$. It is assumed that the measured k-th sample $x(kT)$ is immediately sent to quantizer for the quantization process without delay.

To digitize the value of the sampled state vector, the quantization process is achieved via the *quantizer*. The quantization function of the sampled state vector is characterized by

$$q(\cdot) = [q_1(\cdot) \quad q_2(\cdot) \quad \cdots \quad q_n(\cdot)]^T, \tag{29}$$

where symmetric function $q_j(v)$, $j = 1, \dots, n$ is the quantizer function for j-th state satisfying $q_j(-v) = -q_j(v)$.

Then, for each sampling instant kT, we have the quantization output

$$\hat{x}(kT) = \left[q_1\big(x_1(kT)\big) \quad q_2\big(x_2(kT)\big) \quad \dots \quad q_n\big(x_n(kT)\big)\right]^T. \tag{30}$$

Now denote the quantization levels of $q_j(v)$ by a set $\mathcal{S}_j$

$$\mathcal{S}_j = \{0, \pm u_i^j \mid i = 0, \pm 1, \pm 2, \dots, \pm L_j\}, j = 1, \cdots, n \tag{31}$$

where $\pm L_j$ are the upper & lower bounds of quantized levels.

In this dissertation, we choose the *logarithmic* quantizer similar to (Elia and Mitter, 2001, Fu and Xie, 2005, Gao et al., 2008). Our *logarithmic* quantization levels of $q_j(v)$ has the form

$$\mathcal{S}_j = \{0, \quad \pm u_0^j, \quad \pm u_i^j \mid u_i^j = \rho_j u_0^j, i = \pm 1, \pm 2, \dots \pm L_j\},$$

$$0 < \rho_j < 1, \quad u_0^j > 0 \tag{32}$$

The associated quantizer $q_j(v)$ of the j-th state is described as follows:

$$q_j(v) = \begin{cases} u_i^j & \frac{1}{1+\sigma_j} u_i^j < v \le \frac{1}{1-\sigma_j} u_i^j, \quad v > 0 \\ 0 & v = 0 \\ -q_j(-v) & v < 0 \end{cases} \tag{33}$$

where $j = 1, 2, \dots, n$, and

$$\sigma_j = \frac{1-\rho_j}{1+\rho_j}, \ \frac{1}{1+\sigma_j} = \frac{1+\rho_j}{2}, \ \frac{1}{1-\sigma_j} = \frac{1+\rho_j}{2\rho_j}.$$

Then, every quantized sample $\hat{x}(kT) = q\big(x(kT)\big)$ can be represented by $\sum_{j=1}^{n} ceil\big(\log_2(2L_j + 3)\big)$ bits binary code, where $ceil(\cdot)$ returns the next round up integer value, e.g., $ceil(2.1)$ returns integer 3.

Before introducing *MRDC*, we firstly briefly introduce the balanced-truncation method (BTM) (Wang and Wang, 2008, Zhou and Doyle, 1998, Gugercin and Antoulas, 2004). The brief algorithm of BTM is as follows.

Consider an n-th order system $G = \{A, B, C, D\}$.

i. Compute controllability Gramian P_c by solving $AP_c + P_cA^T + BB^T = 0_n$;

ii. Compute observability Gramian Q_c by solving $A^TQ_c + Q_cA + C^TC = 0_n$;

iii. Decompose P_c into $R_c' \times R_c$;

iv. Apply singular value decomposition on $R_cQ_cR_c' = U_c\Sigma^2U_c^T$, where $\Sigma = \text{diag}\{\sigma_i I_{s_i}\}_{i=1,\dots k}$ is a diagonal matrix with $\sigma_i > \sigma_j, \forall i < j$, and $\sum_{i=1}^{k} s_i = n$;

v. The balancing transform matrix $T_b = \Sigma^{1/2}U_c^TR_c^{-1}$, and $T_b^{-1} = R_cU_c\Sigma^{-1/2}$;

vi. The balanced realization G_b of the system G is denoted by $G_b = \{A_b, B_b, C_b, D\} = \{T_bAT_b^{-1}, T_bB, CT_b^{-1}, D\}$.

vii. Then, the truncated r-th order reduced system is $G_{br} = \{A_{br}, B_{br}, C_{br}, D\} = \big\{\{A_b\}_{(1:r)\times(1:r)}, \{B_b\}_{(1:r)}, \{C_b\}_{(1:r)}, D\big\}$, where A_{br} consists of the first r rows and first r columns of A_b, B_{br} consists of the first r rows of B_b, C_{br} consists the first r colmuns of C_b.$r = \sum_{i=1}^{v} s_i$;

viii. It has been proved that the error E_r between the G and the reduced system G_{br} satisfies

$$E_r = \|T(s) - T_{br}(s)\|_\infty \leq 2\sum_{i=v+1}^{k} \sigma_i \quad (34)$$

where $T(s)$ and $T_{br}(s)$ are the transfer functions of G and G_{br} respectively.

Remark 2: From the algorithm of BTM, denote the state vector of the original system $G = \{A, B, C, D\}$ by $x(t)$. It can be easily derived that the state vector of the r-th order truncated system G_{br} is $x_{br}(t) = T_r T_b x(t)$, where $T_r = diag\{I_{r\times r}, 0_{(n-r)\times(n-r)}\}$.

Remark 3: The transformation $T_b x(t)$ generally is to transform the state vector of the original system based on the "importance". The $T_r T_b x(t)$ is to remove the "unimportant" information from the vector state in the view of the system structure. The smaller r of T_r results larger E_r in (34). As for $r = n$, $E_r = 0$.

Remark 4: In FIGURE 10, the structure of the original remote plant is not altered by the *MRDC*. The ideas behind *MRDC* are (i) to calculate the balanced transform matrices $T_{b,\beta}, \beta \in \mathcal{N}$ for each switching sub-system; (ii) to transform the quantized sample state vector $\hat{x}(kT),\ k = 1,2,\ldots$ by balancing transform matrices $T_{r,\beta}, \beta \in \mathcal{N}$ based on current sub-system; (iii) to truncate the "unimportant" information from the $T_{b,\beta}\hat{x}(kT), \beta \in \mathcal{N},\ k = 1,2,\ldots$; (iv) to transmit the truncated data; (v) to recover the truncated state vector before sending to controller by appending $n - r$ zeros to the truncated state vector and pre-multiplying $T_{b,\beta_{d1}}^{-1}$.

In the switched systems, we can apply the above BTM algorithm to each sub-system β to calculate the balancing transform matrices $T_{b,\beta}, \beta \in \mathcal{N}$. The truncator matrix of sub-system $\beta \in \mathcal{N}$ is denoted by $T_{r,\beta} = diag\left\{I_{r_\beta \times r_\beta}, 0_{(n_\beta - r_\beta)\times(n_\beta - r_\beta)}\right\} \in \mathbb{R}^{n\times n}$, where $r_\beta \in \{0,1,2,\dots,n\}$ is truncated order of the sub-system β.

It needs to be noticed that the switching signal has to be transmitted with every packet over the *downlink & uplink*. As for the controlled system with only one sub-system, the switching signal does not need to be attached to the packet.

The truncated data $\hat{x}_{br}(kT)$ is then packed into a packet with time stamp, and β value, and transmitted via the networks. In the special case that switched systems have only one sub-system, then, the β value can be ignored. The packet in network medium might be delayed or lost due to connection or congestion issue. Without loss of generality, it is assumed that (i) the networks are best-effort delivery, e.g., UDP protocol; (ii) if a packet with a larger time stamp than the expected time stamp arrives, we use it and treat the expected packet as the lost packet; and (iii) if a packet with smaller time stamp than the received one arrives, we just disregard it because it is a too late (i.e., old) packet.

Assume there is no delay on the *balancing inverse transform* and the *controller* computation, or they are so small and can be omitted. Once the data is arrived, both the *balancing inverse transform* and the *controller* will be triggered. Denote $\bar{\bar{x}}(t_c)$ to be the *c*-th successfully arrived states vector, where $t_c, c = 1,2,\dots,$ is the updating instance of both the *balancing inverse transform* and the *controller*. We assume $\bar{\bar{x}}(t_c)$ is automatically appended $n - r$ zeros before the *balancing inverse transform*. An example of packet transmission from the *truncator* to the *balancing inverse transform* over the *downlink* is depicted as FIGURE 11. In this example, there is a packet loss between $c = 2$ and $c = 3$, it can be caused by either $\hat{x}_{br}(3T)$ arrive earlier than $\hat{x}_{br}(2T)$ or the packet contains

$\hat{x}_{br}(2T)$ dropouts during the transmission.

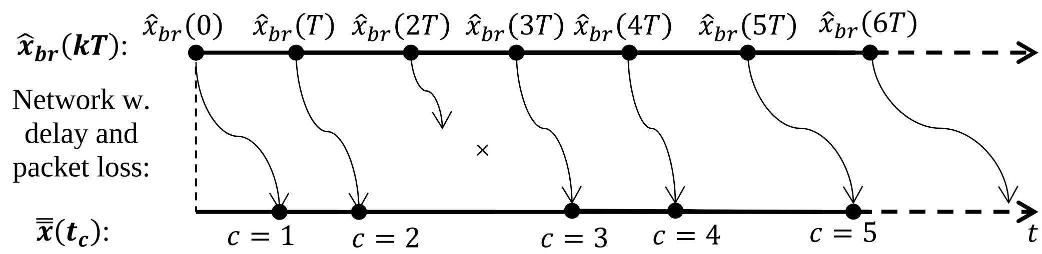

FIGURE 11: Data transmission from *truncator* to the *balancing inverse transform* over the *downlink*

Denote the *balancing inverse transform* and the *controller* by $T_{b,\beta_{d1}}^{-1} \in \mathbb{R}^{n\times n}$ and $K_{\beta_{d1}} \in \mathbb{R}^{l\times n}$with $\beta_{d1} \in \mathcal{N}$ respectively. Then, the output of the controller at the updating instant t_c, $c = 1,2, \ldots$ is calculated by $K_{\beta_{d1}} T_{b,\beta_{d1}}^{-1} \bar{\bar{x}}(t_c) \in \mathbb{R}^l$. Similarly, this control input data is then packed into a packet with a timestamp and sent to *ZOH* over the *uplink*. It needs to be noticed that there is no treatment on control signal, i.e., the data transmitted on *uplink*. In application, the control signal size *l* is usually much less than the system size *n*, especially for large-scale systems.

Assume the *uplink* (receiver side) will drop the control input data which is not same to its current running subsystem. Then, from the input-output point of view, the model from the *truncator* to the *ZOH* in FIGURE 10 is equivalent to the model depicted in FIGURE 12. The rest of this dissertation will be based on this equivalent model without explanation. Furthermore, all the components with switching signal can be regarded to work in the same switching signal under this assumption.

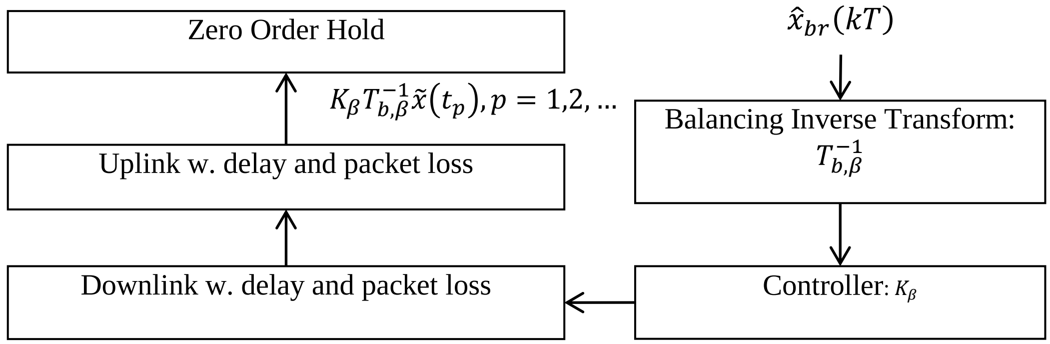

FIGURE 12: Equivalent model from the *truncator* to the *ZOH* from the input-output data point of view

The p-th updating instant of the *ZOH* in FIGURE 10 is denoted by $t_p, p = 1, 2, \ldots$. The p-th successful arrived control signal over the *downlink & uplink* is described by $K_{\beta(t_p)}T_{b,\beta(t_p)}^{-1}\tilde{x}(t_p)$, where $\tilde{x}(t_p)$ mathematically represents the p-th successful transmitted states vector over the *downlink & uplink* (however, $\tilde{x}(t_p)$ is not really transmitted over the *uplink*, only the control signal is transmitted). It needs to be noticed here that we consider this denotation from *truncator* to *ZOH* over the *downlink* & *uplink*. Because the *ZOH* is event-driven, the *ZOH* will hold the value of $K_{\beta(t_p)}T_{b,\beta(t_p)}^{-1}\tilde{x}(t_p)$ until a new value is updated. In the real application, the time-delay of switched systems is much smaller compare to the average dwell time of each sub-system. Meanwhile, we have assumed earlier that the receiver side of the *uplink* will drop the control signal which is not same to its current running subsystem. Therefore, the control input $u(t)$ that the value *ZOH* holds satisfies

$$u(t) = K_\beta T_{b,\beta}^{-1}\tilde{x}(t_p), t_p \le t < t_{p+1} \tag{35}$$

An example of the data transmission from the *truncator* to the *ZOH* on network medium is depicted as FIGURE 13. This example is related to the example in FIGURE 11. From these two figures, it is noticed that the packet $k = 2$ drops on the *downlink*, and packet $k = 4$ drops on the *uplink*.

Since we assume that the time-delays on all the components except networks are ignored, the delays on the closed-loop NCS are from the networked induced delays.

We denote the network induced delay of the p-th successfully arrived packet from *truncator* to *ZOH* by d_p. It satisfies

$$0 \le d_{m,\beta} \le d_p \le d_{M,\beta} \tag{36}$$

where $d_{m,\beta}$ is the minimum delay, $d_{M,\beta}$ denotes the maximum delay of the p-th packet from the *quantifier* to the *ZOH*. The *truncator order* $T_{r,\beta}$ of each sub-system can be different. It might results in different packet size between each sub-system. In most of the existing networks, the average packet transfer time is increased by the packet size. It could result in different network time delay range among sub-systems due to their different packet sizes.

Therefore, the control input $u(t)$ satisfies

$$u(t) = K_\beta T_{b,\beta}^{-1} T_{r,\beta} T_{b,\beta} \cdot q\left(x\left(t_p - d_p\right)\right), t_p \le t < t_{p+1} \tag{37}$$

where t_p is the p-th updating instant of ZOH, $T_{b,\beta}$ is the balancing transform matrix of the subsystem β, $T_{r,\beta}$ is the truncator order of the sub-system β.

For simplicity, we denote

$$T_{c,\beta} = T_{b,\beta}^{-1} T_{r,\beta} T_{b,\beta}. \tag{38}$$

Then, the closed-loop system can be represented by

$$\dot{x}(t) = A_\beta x(t) + B_\beta K_\beta T_{c,\beta} \cdot q\left(x\left(t_p - d_p\right)\right) \tag{39}$$

with $t_p \le t < t_{p+1}$.

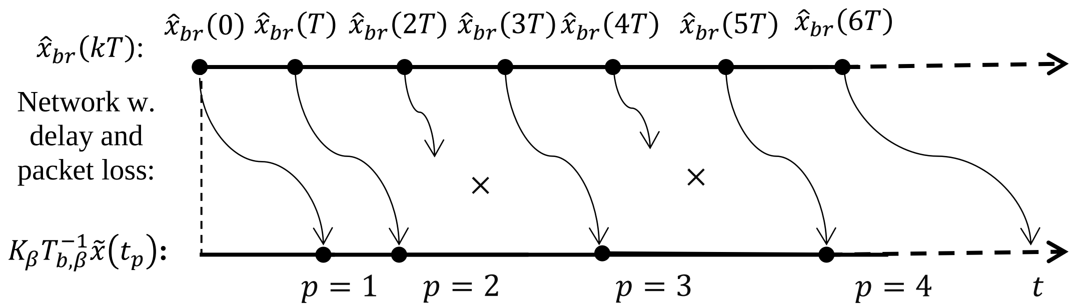

FIGURE 13: Data transmission from the *truncator* to *ZOH* over the *downlink & uplink*

Remark 5: In the *MRDC*, if the *truncator* $T_{r,\beta} = n$, i.e., no data is truncated, then $T_{c,\beta} = T_{b,\beta}^{-1} T_{r,\beta} T_{b,\beta} = I$.

Denote the accumulated packet loss at the updating instant t_{p+1} since t_p by ϑ_{p+1} satisifying $\vartheta_{p+1} \leq \bar{\vartheta}$, where $\bar{\vartheta}$ is the maximum accumulated packet loss over the *downlink & uplink* from *truncator* to *ZOH*.

Considering the packet loss between the p-th and the $(p+1)$-th packets at *ZOH*, we can easily derive that the updating interval satisfies

$$t_{p+1} - t_p = (\vartheta_{p+1} + 1)T + d_{p+1} - d_p \tag{40}$$

where T is the sampling interval, d_p and d_{p+1} are network induced delays of the p-th and the $(p+1)$-th successfully arrived packet from the *truncator* to the *ZOH* respectively.

It can be inferred from (39) that

$$\dot{x}(t) = A_\beta x(t) + B_\beta K_\beta T_{c,\beta} \cdot q\left(x\left(t - d_{m,\beta} - d_\beta(t)\right)\right) \tag{41}$$

where $d_\beta(t) = t - t_p + d_p - d_{m,\beta}$, and $0 \leq d_\beta(t) \leq \bar{d}_\beta = (1 + \bar{\vartheta})T + d_{M,\beta} - d_{m,\beta}$ according to (39) and (40), T is the sampling interval.

According to *logarithmic* quantizer (30)-(33) and (Elia and Mitter, 2001, Gao et al., 2008, Fu and Xie, 2005), (41) can be represented by

$$\dot{x}(t) = A_\beta x(t) + B_\beta K_\beta T_{c,\beta} \cdot (I + \Lambda(t))x\left(t - d_{m,\beta} - d_\beta(t)\right) \tag{42}$$

where

$$\Lambda(t) = diag\{\Lambda_1(t), \Lambda_2(t), \dots, \Lambda_n(t)\}, \Lambda_j(t) \in [-\sigma_j, \sigma_j]. \quad (43)$$

Remark 6: The NCS described in (Gao et al., 2008) is our special case with our number of subsystems $N = 1$, and $T_{c,\beta} = I$.

The following definitions and Lemmas will be used in the proof of main results.

Definition 4: (Sun et al., 2006) The equilibrium $x^* = 0$ of the switched systems (41) is said to be exponentially stable under $\beta(t)$ if the trajectory of the solution $x(t)$ satisfies

$$\|x(t)\| \le \kappa \|\varphi(t_0)\|_s e^{-\lambda(t-t_0)}, \qquad \forall t \ge t_0 \quad (44)$$

for $\kappa \ge 0$, $\lambda > 0$, and $\|\varphi(t)\|_s = \sup_{-\max_{i\in\mathcal{N}}(d_{m,i}+\bar{d}_i)\le\theta\le 0}\{x(t+\theta),\ \dot{x}(t+\theta)\}$.

Definition 5: (Liberzon, 2003, Wu et al., 2011) For any $T_2 > T_1 \ge 0$, let $N_\beta(T_1, T_2)$ denote the number of switching of $\beta(t)$ over (T_1, T_2). If $N_\beta(T_1, T_2) = N_0 + (T_2 - T_1)/T_a$ hold for $T_a > 0$ and $N_0 \ge 0$, T_a is called the average dwell time.

As it is commonly used in literatures, we choose $N_0 = 0$ in the Definition 5.

Lemma 1: (Schur, 1917) Given a symmetric matrix $L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$, where its sub-matrices have respective appropriate dimensions, and $L_{12} = L_{21}^T$, then the following two properties are equivalent:

$$L < 0 \quad (45)$$

$$L_{22} < 0, L_{11} - L_{12}L_{22}^{-1}L_{21} < 0 \quad (46)$$

Lemma 2: (Gao et al., 2008, Sanchez and Perez, 1999) For appropriate dimension matrices $L_3 > 0$ and $L_4 > 0$, $(L_4 - L_3)L_4^{-1}(L_4 - L_3) \ge 0$ leads to inequality $-L_3L_4^{-1}L_3 \le L_4 - 2L_3$.

3.2 Stability Analysis

This section presents the exponential stability condition of the NCSs.

3.2.1 Non-Switched System

We firstly discuss our non-switched NCS with the *MRDC*.

Non-switched NCS is a special case of switched NCSs in (42) with $\beta(t) \equiv 1$ or $\mathcal{N} = \{1\}$. From (42) we can get the non-switched NCS with the *MRDC* as follows

$$\dot{x}(t) = Ax(t) + BKT_c \cdot \big(I + \Lambda(t)\big) \cdot x\big(t - d_m - d(t)\big) \tag{47}$$

with $0 \leq d(t) \leq \bar{d} = (1 + \bar{\vartheta})T + d_M - d_m$ and $x(t) = \varphi(t),\ t \in \left[-d_m - \bar{d}, 0\right]$, β is omitted because it is a constant value in the non-switched NCS.

This subsection presents two cases for the non-switched NCS. In the first case, the controller is fixed. In applications, the controllers of some systems can be high cost to be adjusted or replaced. For this kind of system, our *MRDC* behaves like a "booster", i.e., other components including controller do not need to do any adjustment at all. The adjustment parameter is the r (*truncator order*) of *MRDC*. The second case is that the controller K is adjustable in the design process, i.e., both r and K can be adjusted/searched for the performance optimization.

The first theorem is the exponential stability condition for the NCS with the *MRDC* and fixed controller K. This theorem has been accepted (Bai and Wang, 2014a, Bai and Wang, 2014b) but without the proof.

Theorem 4: For a given constant $\alpha > 0$, the closed loop-system (47) with controller K is exponentially stable, if there exist positive definite matrices X,P_1, P_2, Q_1,Q_2, Q_3, G, and any appropriate dimensional matrices T, U, V, W, and satisfying

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & \Psi_1 \\ * & M_{22} & 0 & 0 \\ * & * & M_{33} & \Psi_2 \\ * & * & * & -G \end{bmatrix} < 0. \tag{48}$$

where

$$M_{11} = \Xi_1^T + \Xi_1 + \begin{bmatrix} XA + A^TX + P_1 + P_2 + \alpha X & 0 & XBKT_c & 0 \\ * & -e^{-\alpha d_m}P_1 & 0 & 0 \\ * & * & \Lambda^2 G & 0 \\ * & * & * & -e^{-\alpha(d_m+\bar{d})}P_2 \end{bmatrix},$$

$$M_{12} = [T \quad U \quad V \quad W],$$

$$M_{13} = [A \quad 0 \quad BKT_c \quad 0]^T[Q_1 \quad Q_2 \quad Q_3],$$

$$M_{22} = diag\left\{-d_m^{-1}R_1, -\bar{d}^{\,-1}R_2, -\bar{d}^{-1}R_2, -\left(d_m + \bar{d}\right)^{-1}R_3\right\},$$

$$M_{33} = diag\left\{-d_m^{-1}Q_1, -\bar{d}^{-1}Q_2, -\left(d_m + \bar{d}\right)^{-1}Q_3\right\},$$

$$\Xi_1 = [T + W \quad U - T \quad V - U \quad -V - W]^T,$$

$$\Psi_1 = [T_c^TK^TB^TX \quad O \quad O \quad O]^T, \Psi_2 = [Q_1 \quad Q_2 \quad Q_3]^TBKT_c,$$

$$R_1 = e^{-\alpha d_m}Q_1, R_2 = e^{-\alpha(d_m+\bar{d})}Q_2,\ R_3 = e^{-\alpha(d_m+\bar{d})}Q_3.$$

Proof:

Denote a Lyapunov-Krasovskii functional as

$$\mathrm{V}(t) = V_1(t) + V_2(t) + V_3(t) \tag{49}$$

where

$$V_1(t) = x^T(t)Xx(t)$$

$$V_2(t) = \int_{t-d_m}^{t} x^T(s)e^{\alpha(s-t)}P_1x(s)ds + \int_{t-d_m-\bar{d}}^{t} x^T(s)e^{\alpha(s-t)}P_2x(s)ds$$

$$V_3(t) = \int_{-d_m}^{0}\int_{t+p}^{t} \dot{x}^T(s)e^{\alpha(s-t)}Q_1\dot{x}(s)ds\,dp$$

$$+ \int_{-d_m-\bar{d}}^{-d_m}\int_{t+p}^{t} \dot{x}^T(s)e^{\alpha(s-t)}Q_2\dot{x}(\alpha)ds\,dp + \int_{-d_m-\bar{d}}^{0}\int_{t+p}^{t} \dot{x}^T(s)e^{\alpha(s-t)}Q_3\dot{x}(s)ds\,dp$$

The derivative of $V(t)$:

$$\dot{V}_1(t) = 2x^T(t)X\dot{x}(t)$$

$$= 2x^T(t)X\left[Ax(t) + BKT_c \cdot \left(I + \Lambda(t)\right) \cdot x\left(t - d_m - d(t)\right)\right] \quad (50)$$

$$\dot{V}_2(t) = -\alpha V_2(t) + x^T(t)P_1x(t) - x^T(t - d_m)e^{-\alpha d_m}P_1x(t - d_m)$$

$$+x^T(t)P_2x(t) - x^T\left(t - d_m - \bar{d}\right)e^{-\alpha(d_m+\bar{d})}P_2x\left(t - d_m - \bar{d}\right) \quad (51)$$

$$\dot{V}_3(t) \le -\alpha V_3(t) + \dot{x}^T(t)\left[d_mQ_1 + \bar{d}Q_2 + \left(d_m + \bar{d}\right)Q_3\right]\dot{x}(t)$$

$$-\int_{t-d_m}^{t}[\dot{x}^T(s)e^{-\alpha d_m}Q_1\dot{x}(s)]ds - \int_{t-d_m-d(t)}^{t-d_m}\left[\dot{x}^T(s)e^{-\alpha(d_m+\bar{d})}Q_2\dot{x}(s)\right]ds$$

$$-\int_{t-d_m-\bar{d}}^{t-d_m-d(t)}\left[\dot{x}^T(s)e^{-\alpha(d_m+\bar{d})}Q_2\dot{x}(s)\right]ds - \int_{t-d_m-\bar{d}}^{t}\left[\dot{x}^T(s)e^{-\alpha(d_m+\bar{d})}Q_3\dot{x}(s)\right]ds \quad (52)$$

For appropriately dimensioned matrices T, U, V, W, $\gamma_i = 0$, $i = 1,2,3,4$ via Newton-Leibniz formula.

$$\gamma_1 \triangleq \xi^T(t)T\left(x(t) - x(t - d_m) - \int_{t-d_m}^{t}\dot{x}(\alpha)d\alpha\right)$$

$$\gamma_2 \triangleq \xi^T(t)U\left(x(t - d_m) - x\left(t - d_m - d(t)\right) - \int_{t-d_m-d(t)}^{t-d_m}\dot{x}(\alpha)d\alpha\right)$$

$$\gamma_3 \triangleq \xi^T(t)V\left(x\left(t - d_m - d(t)\right) - x\left(t - d_m - \bar{d}\right) - \int_{t-d_m-\bar{d}}^{t-d_m-d(t)}\dot{x}(\alpha)d\alpha\right)$$

$$\gamma_4 \triangleq \xi^T(t)W\left(x(t) - x\left(t - d_m - \bar{d}\right) - \int_{t-d_m-\bar{d}}^{t}\dot{x}(\alpha)d\alpha\right)$$

$$\xi^T(t) = \left[x^T(t) \quad x^T(t - d_m) \quad x^T\left(t - d_m - d(t)\right) \quad x^T\left(t - d_m - \bar{d}\right)\right] \quad (53)$$

Then we have

$$\Theta_i \ge 0, i = 1,2,3,4 \quad (54)$$

where

$\Theta_1 = \int_{t-d_m}^{t} H_1^T(R_1)^{-1}H_1\, ds,$ where $H_1 = T^T\xi(t) + R_1\dot{x}(s),$

$\Theta_2 = \int_{t-d_m-d(t)}^{t-d_m} H_2^T(R_2)^{-1}H_2\, ds,$ where $H_2 = U^T\xi(t) + R_2\dot{x}(s),$

$$\Theta_3 = \int_{t-d_m-\bar{d}}^{t-d_m-d(t)} H_3^T (R_2)^{-1} H_3 \, ds, \text{ where } H_3 = V^T \xi(t) + R_2 \dot{x}(s),$$

$$\Theta_4 = \int_{t-d_m-\bar{d}}^{t} H_4^T (R_3)^{-1} H_4 \, ds, \text{ where } H_4 = W^T \xi(t) + R_3 \dot{x}(s).$$

Our first goal is to prove

$$\dot{V}(t) + \alpha V(t) \leq 0 \tag{55}$$

It is straightforward that

$$\dot{V}(t) + \alpha V(t) \leq \dot{V}(t) + \alpha V(t) + \sum_i \Theta_i + \sum_i \gamma_i \tag{56}$$

By Shur's complement, $\dot{V}(t) + \alpha V(t) + \sum_i \Theta_i + \sum_i \gamma_i < 0$ is equivalent to

$$M_1 = \begin{bmatrix} M_{11}^{(1)} & M_{12} & M_{13}^{(1)} \\ * & M_{22} & 0 \\ * & * & M_{33} \end{bmatrix} < 0 \tag{57}$$

where

$$M_{11}^{(1)} = \Xi_1^T + \Xi_1$$

$$+ \begin{bmatrix} XA + A^T X + P_1 + P_2 + \alpha X & 0 & XBKT_c \cdot \big(I + \Lambda(t)\big) & 0 \\ * & -e^{-\alpha d_m} P_1 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & -e^{-\alpha(d_m + \bar{d})} P_2 \end{bmatrix}$$

$$M_{13}^{(1)} = \begin{bmatrix} A & 0 & BKT_c \cdot \big(I + \Lambda(t)\big) & 0 \end{bmatrix}^T [Q_1 \quad Q_2 \quad Q_3],$$

M_{12}, M_{33} and M_{33} are from (48).

For any matrix $G > 0$, the following inequality is held.

$$\Sigma_1 \Sigma_2 + \Sigma_2^T \Sigma_1^T \leq \Sigma_1 G^{-1} \Sigma_1^T + \Sigma_2^T G \Sigma_2 \tag{58}$$

where $\Sigma_1 = [\Psi_1^T \quad O \quad \Psi_2^T]^T$, $\Sigma_2 = [\Psi_3 \quad 0 \quad 0 \quad 0]$, $\Psi_3 = [0\ 0\ \Lambda(t)\ 0\ ]$.

It can be inferred from (57) and (58) that $M < 0$ leads to $M_1 < 0$ and (55) by Schur complement.

Our next goal is to prove the exponential stability. The inequality (55) can be derived from $M < 0$. By integrate both side of (55), it can be easily derived that

$$V(t) \le e^{-\alpha(t-t_0)}V(t_0). \tag{59}$$

Then, it gives

$$b\|x(t)\|^2 \le V(t) \le e^{-\alpha(t-t_0)}V(t_0) \le c\|\varphi(t_0)\|_s e^{-\alpha(t-t_0)} \tag{60}$$

where

$$b = \lambda_{min}(X) \tag{61}$$

$$\begin{aligned} c = \lambda_{max}(X) + d_m\lambda_{max}(P_1) + (d_m + \bar{d})\lambda_{max}(P_2) + \frac{d_m^2}{2}\lambda_{max}(Q_1) \\ + \left(\frac{\bar{d}^2}{2} + \bar{d}d_m\right)\lambda_{max}(Q_2) + \frac{(d_m + \bar{d})^2}{2}\lambda_{max}(Q_3) \end{aligned} \tag{62}$$

It leads to the following inequality

$$\|x(t)\| \le \sqrt{\frac{c}{b}}\, e^{-\frac{\alpha}{2}(t-t_0)}\|\varphi(t_0)\|_s, \quad t > 0 \tag{63}$$

where initial time t_0 is usually defined to be 0.

Therefore, the closed loop system (47) with controller K is exponentially stable. ■

The second theorem can be used to find a controller K under the exponential stability condition.

Theorem 5: For a given constant $\alpha > 0$, the closed loop system (47) with $T_r = I$ is exponentially stable, if there exist positive definite matrices $\tilde{X}$,$\tilde{P}_1$, $\tilde{P}_2$, $\tilde{Q}_1$, $\tilde{Q}_2$, $\tilde{Q}_3$, $\tilde{G}$, and any appropriate dimensional matrices $\tilde{T}$, $\tilde{U}$, $\tilde{V}$, $\tilde{W}$, $\tilde{K}$, and satisfying

$$M_2 = \begin{bmatrix} M_{11}^{(2)} & M_{12}^{(2)} & M_{13}^{(2)} & \Psi_1^{(2)} \\ * & M_{22}^{(2)} & O & O \\ * & * & M_{33}^{(2)} & \Psi_2^{(2)} \\ * & * & * & -\tilde{G} \end{bmatrix} < 0, \tag{64}$$

where controller K is solved to be $K = \tilde{K}\tilde{X}^{-1}$, and

$$M_{11}^{(2)} = \begin{bmatrix} A\tilde{X} + \tilde{X}A^T + \tilde{P}_1 + \tilde{P}_2 + \alpha\tilde{X} & 0 & B\tilde{K} & 0 \\ * & -e^{-\alpha d_m}\tilde{P}_1 & 0 & 0 \\ * & * & \Lambda^2\tilde{G} & 0 \\ * & * & * & -e^{-\alpha(d_m+\bar{d})}\tilde{P}_2 \end{bmatrix} + \tilde{\Xi}_1^T + \tilde{\Xi}_1,$$

$$\tilde{\Xi}_1 = [\tilde{T} + \tilde{W} \quad \tilde{U} - \tilde{T} \quad \tilde{V} - \tilde{U} \quad -\tilde{V} - \tilde{W}]^T,$$

$$M_{22}^{(2)} = \ diag\{d_m^{-1}e^{-\alpha d_m}(\tilde{Q}_1 - 2\tilde{X}), \bar{d}^{\,-1}e^{-\alpha(d_m+\bar{d})}(\tilde{Q}_2 - 2\tilde{X}),$$

$$\bar{d}^{\,-1}e^{-\alpha(d_m+\bar{d})}(\tilde{Q}_2 - 2\tilde{X}),\ (d_m + \bar{d})^{-1}e^{-\alpha(d_m+\bar{d})}(\tilde{Q}_3 - 2\tilde{X})\},$$

$$M_{33}^{(2)} = diag\left\{-d_m^{-1}\tilde{Q}_1, -\bar{d}^{-1}\tilde{Q}_2, -(d_m + \bar{d})^{-1}\tilde{Q}_3\right\},$$

$$M_{12}^{(2)} = [\tilde{T} \quad \tilde{U} \quad \tilde{V} \quad \tilde{W}], M_{13}^{(2)} = [A\tilde{X}^T \quad 0 \quad B\tilde{K} \quad 0]^T[I \quad I \quad I],$$

$$\Psi_1^{(2)} = [\tilde{K}^T B^T \quad 0 \quad 0 \quad 0]^T, \Psi_2^{(2)} = [I \quad I \quad I]^T B\tilde{K}.$$

Similarly to Theorem 4, denote

$$\tilde{b} = \lambda_{min}(X) \tag{65}$$

$$\tilde{c} = \lambda_{max}(X) + d_m\lambda_{max}(P_1) + (d_m + \bar{d})\lambda_{max}(P_2) + \frac{d_m^2}{2}\lambda_{max}(Q_1) + \left(\frac{\bar{d}^2}{2} + \bar{d}d_m\right)\lambda_{max}(Q_2) + \frac{(d_m + \bar{d})^2}{2}\lambda_{max}(Q_3) \tag{66}$$

where $X = \tilde{X}^{-1}$, $P_j = X\tilde{P}_jX,\ j = 1, 2,\ Q_j = \tilde{Q}_j^{-1}, i = 1, 2, 3.$

Then, the following inequality holds

$$\|x(t)\| \le \sqrt{\frac{\tilde{c}}{\tilde{b}}}e^{-\frac{\alpha}{2}(t-t_0)}\|\varphi(t_0)\|_s, \quad t > 0. \tag{67}$$

Proof:

For the full-order system $T_r = I$, it has $T_c = T_b^{-1}T_rT_b = I$.

Pre and post multiply $diag\{J_1, J_2, J_3, J_4\}$ to (48), where $J_1 = J_2 = diag\{X^{-1}, X^{-1}, X^{-1}, X^{-1}\},\ J_3 = diag\{Q_1^{-1}, Q_2^{-1}, Q_3^{-1}\}, J_4 = X^{-1}$.

Denote $\tilde{X} = X^{-1}$; $\tilde{P}_j = X^{-1}P_jX^{-1}, j = 1, 2$; $\tilde{Q}_j = Q_j^{-1}, i = 1,2,3$; $\tilde{G} = X^{-1}GX^{-1}$;

$\tilde{K} = K\tilde{X}$; $[\tilde{T} \quad \tilde{U} \quad \tilde{V} \quad \tilde{W}] = J_1[T \quad U \quad V \quad W]J_2$.

Then, inequality (48) is equivalent to

$$M_3 = \begin{bmatrix} M_{11}^{(2)} & M_{12}^{(2)} & M_{13}^{(2)} & \Psi_1^{(2)} \\ * & M_{22}^{(3)} & O & O \\ * & * & M_{33}^{(2)} & \Psi_2^{(2)} \\ * & * & * & -\tilde{G} \end{bmatrix} < 0 \tag{68}$$

where

$M_{22}^{(3)} =$

$diag\left\{-d_m^{-1}e^{-\alpha d_m}\tilde{X}\tilde{Q}_1^T\tilde{X},\ -\bar{d}^{\,-1}e^{-\alpha(d_m+\bar{d})}\tilde{X}\tilde{Q}_2^{-1}\tilde{X},\ -\bar{d}^{\,-1}e^{-\alpha(d_m+\bar{d})}\tilde{X}\tilde{Q}_2^{-1}\tilde{X}, -\left(d_m + \bar{d}\right)^{-1}e^{-\alpha(d_m+\bar{d})}\tilde{X}\tilde{Q}_3^{-1}\tilde{X}\right\}$.

By Lemma 2, $-\tilde{X}\tilde{Q}_j^T\tilde{X} \le \left(\tilde{Q}_j - 2\tilde{X}\right)$, $j = 1, 2, 3$, then $M_{22}^{(3)} \le M_{22}^{(2)}$, and $M_3 \le M_2$.■

After the K is designed by Theorem 5, the Theorem 4 can be applied to design the *truncator order* r.

3.2.2 Switched Systems

We now consider the switched NCS. Similarly to the non-switched NCS, we firstly give the switched system with fixed controller K_i in Theorem 6.

In the following Theorems 6 and 7 we firstly denote the minimum delay and maximum delay of the switched NCSs by

$$d_m = \min_{i\in\mathcal{N}}\left(d_{m,i}\right) \tag{69}$$

$$\bar{d} = \max_{i\in\mathcal{N}}\left(d_{m,i} + \bar{d}_i\right) - d_m \tag{70}$$

Then, the delay of each subsystem is between d_m and $\bar{d}$.

Theorem 6: For the given constants $\alpha_i \ge 0$, $i \in \mathcal{N}$, the switched NCS (42) with controller

K_i is exponentially stable, if (i) there exist positive definite matrices X_i,$P_{1,i}$, $P_{2,\mathrm{i}}$, $Q_{1,i}$, $Q_{2,i}$, $Q_{3,i}$, G_i, and any appropriate dimensional matrices T_i, U_i, V_i, W_i, and satisfying

$$M_{4,i}=\begin{bmatrix} M_{11,i}^{(4)} & M_{12,i}^{(4)} & M_{13,i}^{(4)} & \Psi_{1,i}^{(4)} \\ * & M_{22,i}^{(4)} & 0 & 0 \\ * & * & M_{33,i}^{(4)} & \Psi_{2,i}^{(4)} \\ * & * & * & -G_i \end{bmatrix}<0, i\in\mathcal{N}, \quad (71)$$

and (ii) average dwell time of switching signal satisfying

$$T_a>\frac{\ln\mu}{\alpha}, \mu\geq 1, \alpha=min(\alpha_i), \text{ and} \quad (72)$$

$$X_i\leq\mu X_l, P_{1,i}\leq\mu P_{1,l}, P_{2,i}\leq\mu P_{2,l}, Q_{1,i}\leq\mu Q_{1,l}, Q_{2,i}\leq\mu Q_{2,l}, Q_{3,i}\leq\mu Q_{3,l}, \quad (73)$$
$$\forall i,l\in\mathcal{N}.$$

where

$$M_{11,i}^{(4)}=\Xi_{1,i}^T+\Xi_{1,i}$$
$$+\begin{bmatrix} X_iA_i+A_i^TX_i+P_{1,i}+P_{2,i}+\alpha_iX_i & 0 & X_iB_iK_iT_{c,i} & 0 \\ * & -e^{-\alpha_id_m}P_{1,i} & 0 & 0 \\ * & * & \Lambda_i^2G_i & 0 \\ * & * & * & -e^{-\alpha_i(d_m+\bar{d})}P_{2,i} \end{bmatrix},$$

$$M_{12,i}^{(4)}=[T_i \quad U_i \quad V_i \quad W_i], M_{13,i}^{(4)}=[A_i \quad 0 \quad B_iK_iT_{c,i} \quad 0]^T[Q_{1,i} \quad Q_{2,i} \quad Q_{3,i}],$$

$$M_{33}^{(4)}=diag\left\{-d_m^{-1}Q_{1,i}, -\bar{d}^{-1}Q_{2,i}, -\left(d_m+\bar{d}\right)^{-1}Q_{3,i}\right\},$$

$$\Xi_{1,i}=[T_i+W_i \quad U_i-T_i \quad V_i-U_i \quad -V_i-W_i]^T,$$

$$M_{22,i}^{(4)}=diag\left\{-d_m^{-1}e^{-\alpha_id_m}Q_{1,i}, -\bar{d}^{-1}e^{-\alpha_i(d_m+\bar{d})}Q_{2,i}, -\bar{d}_i^{-1}e^{-\alpha_i(d_m+\bar{d})}Q_{2,i},\right.$$
$$\left.-\left(d_m+\bar{d}\right)^{-1}e^{-\alpha_i(d_m+\bar{d})}Q_{3,i}\right\}, \Psi_{1,i}^{(4)}=\left[T_{c,i}^TK_i^TB_i^TX_i \quad O \quad O \quad O\right]^T,$$

$$\Psi_{2,i}^{(4)}=[Q_{1,i} \quad Q_{2,i} \quad Q_{3,i}]^TB_iK_iT_{c,i}.$$

Proof:

Define a piecewise Lyapunov-Krasovskii functional candidate as

$$V_{\beta(t)}(t) = V_{1,\beta(t)}(t) + V_{2,\beta(t)}(t) + V_{3,\beta(t)}(t) \tag{74}$$

where

$$V_{1,\beta(t)}(t) = x^T(t)X_\beta x(t)$$

$$V_{2,\beta(t)}(t) = \int_{t-d_m}^{t} x^T(s)e^{\alpha(s-t)}P_{1,\beta}x(s)ds + \int_{t-d_m-\bar{d}}^{t} x^T(s)e^{\alpha(s-t)}P_{2,\beta}x(s)ds$$

$$V_{3,\beta(t)}(t) = \int_{-d_m}^{0}\int_{t+p}^{t} \dot{x}^T(s)e^{\alpha(s-t)}Q_{1,\beta}\dot{x}(s)ds\,dp$$

$$+\int_{-d_m-\bar{d}}^{-d_m}\int_{t+p}^{t} \dot{x}^T(s)e^{\alpha(s-t)}Q_{2,\beta}\dot{x}(\alpha)ds\,dp + \int_{-d_m-\bar{d}}^{0}\int_{t+p}^{t} \dot{x}^T(s)e^{\alpha(s-t)}Q_{3,\beta}\dot{x}(s)ds\,dp$$

Also we define another piecewise Lyapunov-Krasovskii functional candidate as

$$\tilde{V}_{\beta(t)}(t) = V_{1,\beta(t)}(t) + \tilde{V}_{2,\beta(t)}(t) + \tilde{V}_{3,\beta(t)}(t) \tag{75}$$

where

$$\tilde{V}_{2,\beta(t)}(t) = \int_{t-d_m}^{t} x^T(s)e^{\alpha_\beta.(s-t)}P_{1,\beta}x(s)ds + \int_{t-d_m-\bar{d}}^{t} x^T(s)e^{\alpha_\beta\cdot(s-t)}P_{2,\beta}x(s)ds$$

$$\tilde{V}_{3,\beta(t)}(t) = \int_{-d_m}^{0}\int_{t+p}^{t} \dot{x}^T(s)e^{\alpha_\beta\cdot(s-t)}Q_{1,\beta}\dot{x}(s)ds\,dp$$

$$+\int_{-d_m-\bar{d}}^{-d_m}\int_{t+p}^{t} \dot{x}^T(s)e^{\alpha_\beta\cdot(s-t)}Q_{2,\beta}\dot{x}(\alpha)ds\,dp + \int_{-d_m-\bar{d}}^{0}\int_{t+p}^{t} \dot{x}^T(s)e^{\alpha_\beta\cdot(s-t)}Q_{3,\beta}\dot{x}(s)ds\,dp$$

During the time range $t \in [t_k, t_{k+1})$, it is obvious that $V_{\beta(t)}(t) \geq \tilde{V}_{\beta(t)}(t) \geq 0$. Furthermore, similar to (49) - (52), we can easily derive $\dot{V}_{\beta(t)}(t) + \alpha V_{\beta(t)}(t) \leq \frac{d\tilde{V}_{\beta(t)}(t)}{dt} + \alpha_\beta \tilde{V}_{\beta(t)}(t)$. The satisfaction of LMIs (71) can guarantee $\frac{d\tilde{V}_{\beta(t)}(t)}{dt} + \alpha_{\beta(t)}\tilde{V}_{\beta(t)}(t) < 0$, which results in

$$\dot{V}_{\beta(t)}(t) + \alpha V_{\beta(t)}(t) < 0,\ t \in [t_k, t_{k+1}) \tag{76}$$

According to (59), (71), (72) and (76), during the time range $t \in [t_k, t_{k+1})$

$$V_{\beta(t)}(t) \leq e^{-\alpha(t-t_k)}V_{\beta(t_k)}(t_k) \tag{77}$$

For any switching instance t_i, we have

$$V_{\beta(t_i)}(t_i) \le \mu V_{\beta(t_i^-)}(t_i^-), i = 1,2, \dots \tag{78}$$

and

$$V_{\beta(t)}(t) \le e^{-\alpha(t-t_k)}\mu V_{\beta(t_k^-)}(t_k^-) \le e^{-\alpha(t-t_{k-1})}\mu^2 V_{\beta(t_{k-1}^-)}(t_{k-1}^-)$$

$$\le \cdots \le e^{-\alpha(t-t_0)}\mu^k V_{\beta(t_0)}(t_0) \le e^{-\left(\alpha-\frac{ln(\mu)}{T_\alpha}\right)(t-t_0)} V_{\beta(t_0)}(t_0) \tag{79}$$

with $k = N_\beta(t_0, t) \le (t - t_0)/T_a$.

Denote

$$b_s = \min_{\forall i \in \mathcal{N}} \lambda_{min}(X_i) \tag{80}$$

$$c_s = \max_{\forall i \in \mathcal{N}} \lambda_{max}(X_i) + d_m \max_{\forall i \in \mathcal{N}} \lambda_{max}\left(P_{1,i}\right) + \left(d_m + \bar{d}\right) \max_{\forall i \in \mathcal{N}} \lambda_{max}\left(P_{2,i}\right)$$

$$+ \frac{d_m^2}{2} \max_{\forall i \in \mathcal{N}} \lambda_{max}\left(Q_{1,i}\right) + \left(\frac{\bar{d}^2}{2} + \bar{d} d_m\right) \max_{\forall i \in \mathcal{N}} \lambda_{max}\left(Q_{2,i}\right) \tag{81}$$

$$+ \frac{1}{2}\left(d_m + \bar{d}\right)^2 \max_{\forall i \in \mathcal{N}} \lambda_{max}\left(Q_{3,i}\right)$$

We can derive:

$$V_{\beta(t)}(t) \ge b_s \|x(t)\|^2 \tag{82}$$

$$V_{\beta(t_0)}(t_0) \le c_s \|\varphi(t_0)\|_s^2 \tag{83}$$

From (79)-(83) we have

$$\|x(t)\|^2 \le \frac{1}{b_s} V_{\beta(t)}(t) \le \frac{1}{b_s} e^{-\left(\alpha-\frac{ln\,\mu}{T_a}\right)(t-t_0)} V_{\beta(t_0)}(t_0)$$

$$\le \frac{c_s}{b_s} e^{-\left(\alpha-\frac{ln\,\mu}{T_a}\right)(t-t_0)} \|\varphi(t_0)\|_s^2 \tag{84}$$

■

Remark 7: From (84), it is obvious that the state trajectory decay satisfying

$$\|x(t)\| \le \sqrt{\frac{c_s}{b_s}} e^{-\frac{1}{2}\left(\alpha-\frac{\ln\mu}{T_a}\right)(t-t_0)} \|\varphi(t_0)\|_s \tag{85}$$

Remark 8: For switched systems satisfying Theorem 6, the Lyapunov-Krasovskii functional hold the following inequality.

$$\dot{V}_i(t) + \alpha V_i(t) < 0, i \in \mathcal{N}. \tag{86}$$

The following theorem is to solve the controller K_i for the switched NCSs (42) in the case that the controller is adjustable.

Theorem 7: For the given constants $\alpha_i > 0$, $i \in \mathcal{N}$, the switched NCS (42) with $T_r = I$ is exponentially stable, if (i) there exist positive definite matrices $\tilde{X}_i$,$\tilde{P}_{1,i}$, $\tilde{P}_{2,\mathrm{i}}$, $\tilde{Q}_{1,i}$, $\tilde{Q}_{2,i}$, $\tilde{Q}_{3,i}$, $\tilde{G}_i$, and any appropriate dimensional matrices $\tilde{T}_i$, $\tilde{U}_i$, $\tilde{V}_i$, $\tilde{W}_i$,$\tilde{K}_i$ and satisfying

$$M_{5,i} = \begin{bmatrix} M_{11,i}^{(5)} & M_{12,i}^{(5)} & M_{13,i}^{(5)} & \Psi_{1,i}^{(5)} \\ * & M_{22,i}^{(5)} & 0 & 0 \\ * & * & M_{33,i}^{(5)} & \Psi_{2,i}^{(5)} \\ * & * & * & -G_i \end{bmatrix} < 0, i \in \mathcal{N}, \tag{87}$$

and (ii) average dwell time of switching signal satisfying

$$T_a > \frac{\ln \mu}{\alpha},\ \mu \geq 1, \alpha = \min(\alpha_i), \text{ and} \tag{88}$$

$$\tilde{X}_i \leq \mu \tilde{X}_l,\ \tilde{P}_{1,i} \leq \mu \tilde{P}_{1,l}, \tilde{P}_{2,i} \leq \mu \tilde{P}_{2,l}, \tilde{Q}_{1,i} \leq \mu \tilde{Q}_{1,l}, \tilde{Q}_{2,i} \leq \mu \tilde{Q}_{2,l}, \tilde{Q}_{3,i} \leq \mu \tilde{Q}_{3,l},$$

$$\forall i, l \in \mathcal{N}. \tag{89}$$

where

$$M_{11}^{(5)} = \begin{bmatrix} A_i\tilde{X}_i + \tilde{X}_i A_i^T \ + \tilde{P}_{1,i} + \tilde{P}_{2,i} + \alpha_i \tilde{X}_i & 0 & B_i\tilde{K}_i & 0 \\ * & -e^{-\alpha_i d_m}\tilde{P}_{1,i} & 0 & 0 \\ * & * & \Lambda^2 \tilde{G}_i & 0 \\ * & * & * & -e^{-\alpha_i(d_m+\bar{d})}\tilde{P}_{2,i} \end{bmatrix} +$$

$$\tilde{\Xi}_{1,i}^T + \tilde{\Xi}_{1,i}, \tilde{\Xi}_1 = [\tilde{T}_i + \tilde{W}_i \quad \tilde{U}_i - \tilde{T}_i \quad \tilde{V}_i - \tilde{U}_i \quad -\tilde{V}_i - \tilde{W}_i]^T,$$

$$M_{22}^{(5)} =$$

$$diag\left\{d_m^{-1} e^{-\alpha_i d_m}\left(\tilde{Q}_{1,i} - 2\tilde{X}_i\right), \bar{d}^{-1} e^{-\alpha_i(d_m+\bar{d})}\left(\tilde{Q}_{2,i} - 2\tilde{X}_i\right),\ \bar{d}^{-1} e^{-\alpha_i(d_m+\bar{d})}\left(\tilde{Q}_{2,i} - 2\tilde{X}_i\right),\right.$$

$$\left.\left(d_m + \bar{d}\right)^{-1} e^{-\alpha_i(d_m+\bar{d})}\left(\tilde{Q}_{3,i} - 2\tilde{X}_i\right)\right\},$$

$$M_{33}^{(5)} = diag\left\{-d_m^{-1}\tilde{Q}_{1,i}, -\bar{d}^{-1}\tilde{Q}_{2,i}, -\left(d_m + \bar{d}\right)^{-1}\tilde{Q}_{3,i}\right\},$$

$$M_{12}^{(5)} = [\tilde{T}_i \quad \widetilde{U}_i \quad \tilde{V}_i \quad \widetilde{W}_i],\ M_{13}^{(5)} = [A_i\tilde{X}_i^T \quad 0 \quad B_i\widetilde{K}_i \quad 0]^T[I \quad I \quad I],$$

$$M_{12}^{(5)} = M_{12}, \Psi_1^{(5)} = [\widetilde{K}_i^T B_i^T \quad 0 \quad 0 \quad 0]^T, \Psi_2^{(5)} = [I \quad I \quad I]^T B_i\widetilde{K}_i.$$

Then, the controllers of switched NCS is designed to be $K_i = \widetilde{K}_i\tilde{X}_i^{-1}$.

Proof: The proof is similar with the proofs of theorems 5 and 6. ■

3.3 Performance Optimization Analysis

It is obviously that truncating more order will result in less data transmission. In the network, a shorter data needs less time to transmit. Meanwhile, in the shared network, this saved network bandwidth can be utilized for more tasks. For some packet based networks, especially TCP/IP, the longer packets will usually result in larger packet loss rate (Korhonen and Wang, 2005).

Our objective is to maximize α of the switched NCSs with the right *truncator order* vector for each subsystem, or maximize α of non-switched NCS with the right *truncator order.*

In our general system depicted in FIGURE 10, the system state vector $x(t)$ is converted by matrix $T_{b,\beta}$, then the unimportant system states are removed by the *truncator*. Without loss of generality, we assume that the network delay is a function of packet size. The packet size of each system is a function of r_i, $i \in \mathcal{N}$, which is the remaining order of the states. Then the network delay is the function of reduced order r_i.

Notation $d_m(r)$ is the minimum delay, and $\bar{d}(r)$ is the upper bound of network delay. Our optimization problem becomes how to find the optimized *truncator order* sequence r_i, $i \in \mathcal{N}$ to maximize the decay rate α for stability.

We use α to evaluate the performance of the NCS, because larger α usually means larger decay rate of $V(t)$ and $x(t)$ in view of (44).

We consider two kinds of cases in this dissertation. In the first case, the controller $K_i, i \in \mathcal{N}$ is fixed. It might be because of the high cost of adjusting/changing. In this case, our *MRDC* is "booster" to this kind of system. It can boost the performance without changing any devices. In the second case, the controller $K_i, i \in \mathcal{N}$ can be altered. The following algorithms are all offline search algorithm.

The algorithm to find the optimized r_i for the fixed controllers $K_i, i \in \mathcal{N}$ is as follows:

PROC find_the_optimized_r_i
Denote the order of all the sub-systems by vector $[r_1, r_2, \dots, r_N]$
Permute all the possible order sequence and save to set ORDERS
max_α = -1
FOR EACH sequence $[r_1, r_2, \dots, r_N]$ in ORDERS
Evaluate d_m, $\bar{d}$ for switched NCSs in vector $[r_1, r_2, \dots, r_N]$ by Eqs. (69) and (70)
Initialize a vector $[\alpha_1, \alpha_2, \dots, \alpha_N]$ for switched NCSs
*FOR EACH $i \in \mathcal{N}$ /*For each system i */*
α_i = Calculate_the_largest_$\alpha(i, r_i, d_m, \bar{d})$ / time efficiency can be greatly improved by caching the previous results */*
END FOR
Store the pair $\{[\alpha_1, \alpha_2, \dots, \alpha_N], [r_1, r_2, \dots, r_N]\}$ into set S
END FOR
*α_tmp = -1 /*temp variable*/*
FOR EACH pair $\{[\alpha_1, \alpha_2, \dots, \alpha_N], [r_1, r_2, \dots, r_N]\}$ in set S
*α_min =min($[\alpha_1, \alpha_2, \dots, \alpha_N]$) /*Calculate the minimum α */*
IF $\alpha\_tmp < \alpha\_min$
$\alpha\_tmp = \alpha\_min$
Cache the order sequence to $[r_1, r_2, \dots, r_N]_{cached}$
END IF
END FOR
IF $\alpha\_tmp \neq$ -1
*RETURN α_tmp and $[r_1, r_2, \dots, r_N]_{cached}$ /*return the best orders sequence*/*
ELSE
*RETURN -1 /*instable switched NCSs*/*
END IF
END PROC

The function *Calculate_the_largest_*α is defined as

```
/*system_index ∈ 𝒩; order: order to be tested; d_m: minimal delay; d̄: maximum delay range; K: controller*/
FUNCTION Calculate_the_largest_α (system_index, order, d_m, d̄, K)
    /*Initialize*/
    lower_bound = 0    /*lower bound to search α */
    upper_bound = 10e5    /*upper_bound to search α */
    DO
        mid = (lower_bound + upper_bound)/2        /*Bisection Search*/
        IF "α=mid" satisfies the Theorem 6 for system[system_index]
            lower_bound = mid
        ELSE
            upper_bound = mid
        END IF
    WHILE upper_bound - lower_bound > 1e-4 /*stop criteria*/
If "α=lower_bound" satisfies the Theorem 6 for system[system_index]
RETURN lower_bound
ELSE
RETURN -1        /*instable system return -1*/
END IF
END PROC
```

For the second case with adjustable controller in the design process, the algorithm to find the optimized r_i sequence is as follows:

```
PROC find_the_optimized_r_i_with_adjustable_Controller
Denote the order of all the sub-systems by vector [r_1, r_2, ..., r_N]
    Permute all the possible order sequence and save to set ORDERS
max_α = -1
    FOR EACH sequence [r_1, r_2, ..., r_N] in ORDERS
        Evaluate d_m, d̄ for switched NCSs in vector [r_1, r_2, ..., r_N] by Eqs. (69) and (70)
        Initialize a vector [α_1, α_2, ..., α_N] for switched NCSs
        FOR EACH i ∈ 𝒩    /*For each system i */
            K_i = find_K_i(i, d_m, d̄)
            α_i = Calculate_the_largest_α(i, r_i, d_m, d̄, K_i)    /* time efficiency can be greatly improved by caching the
        previous results */
        END FOR
        Store triplet {[α_1, α_2, ..., α_N], [r_1, r_2, ..., r_N], [K_1, K_2, ..., K_N]} into set S
    END FOR
    α_tmp = -1        /*temp variable*/
    FOR EACH triplet {[α_1, α_2, ..., α_N], [r_1, r_2, ..., r_N], [K_1, K_2, ..., K_N]} in set S
```

```
        α_min =min([α_1, α_2, ... , α_N])  /*Calculate the minimum α */
        IF    α_tmp < α_min
            α_tmp =  α_min
            Cache the pair to {[r_1, r_2, ... , r_N], [K_1, K_2, ... , K_N]}_cached
        END IF
    END FOR
    IF α_tmp ≠  -1
        RETURN α_tmp and  {[r_1, r_2, ... , r_N], [K_1, K_2, ... , K_N]}_cached  /*return the best orders sequence & controllers */
    ELSE
        RETURN -1   /*instable switched NCSs*/
    END IF
END PROC
```

The function $find\_K_i$ is defined as

```
PROC find_K_i(i, d_m, d̄)
/*Initialize*/
lower_bound = 0    /* lower bound of  α */
upper_bound = 10e5  /* upper_bound of α  */
DO
mid = (lower_bound + upper_bound) / 2  /*Bisection Search*/
IF  "α=mid" satisfies the Theorem 7 for system[i] /*Theorem 2 for non-switched NCS*/
            Get K_i for system i by Theorem 7
            lower_bound = mid
        ELSE
            upper_bound = mid
        END IF
WHILE upper_bound - lower_bound <1e-4  /*stop criteria*/
RETURN   K_i
END PROC
```

3.4 Examples

To illustrate the theorems of this dissertation, we choose two examples. The first example is non-switched NCS. It can be regarded as switched NCS with only one subsystem. The second example is switched NCSs with two subsystems.

3.4.1 Example 1

We firstly consider non-switching case. In applications, the concerned systems are usually of very high order, e.g., there are hundreds of orders in aircraft. The benefit of truncated data can be very significant. Due to the page limit, we choose a part from an aircraft to be controlled. The remote control plant transfer function considered in this example is a 6th order representation of the C5A aircraft wing referred from a NASA report (Harvey and Pope, 1978).

$$A = \begin{bmatrix} -0.1192 & 0.5806 & 4.758 & -1.464 & 2.060 & 1.640 \\ -0.4412 & -0.04414 & -0.1014 & 1.343 & -0.4941 & -0.5637 \\ -5.366 & 0.5039 & -0.9381 & -2.174 & 4.632 & 3.238 \\ 0.7003 & -0.8856 & 0.1491 & -1.232 & 4.452 & 5.533 \\ -0.9315 & -0.3954 & -0.1598 & -0.4563 & -6.579 & -2.592 \\ 0.02980 & -0.2697 & 0.02673 & -0.4245 & -0.4385 & -7.364 \end{bmatrix}$$

$B^T =$

$$\begin{bmatrix} -2.577\cdot 10^8 & 1.865\cdot 10^8 & -2.491\cdot 10^7 & -1.875\cdot 10^7 & -1.139\cdot 10^7 & -3.218\cdot 10^6 \\ 2.985\cdot 10^8 & 2.345\cdot 10^8 & -8.587\cdot 10^7 & -2.817\cdot 10^7 & -1.851\cdot 10^7 & -2.683\cdot 10^6 \end{bmatrix},$$

$$C_l = I, D_l = 0_{6\times 2},\ u = \begin{bmatrix} Aileron\ command \\ Elevator\ command \end{bmatrix},$$

$$x^T(\theta) = [0.5 \quad 1 \quad -0.3 \quad 0.4 \quad 0.3 \quad -1],\ \theta \in \left[-d_m - \bar{d}, 0\right]$$

The data link and physic layer of the *downlink & uplink* we assume are based on long range RF media. The upper level protocol is best-effort packet delivery. We assume the max total packet loss on both the *downlink & uplink* is $\bar{\vartheta} = 2$. As it is described earlier, either the packet loss on the *downlink & uplink* will be considered to be total packet loss. For the similarity of comparison, we do not consider the relationship between packet size and drop rate, although the reduced packet size can usually improve the drop rate. On the *downlink*, we assume (1) the head of the packet, including packet counter, time stamp, and switching signal, takes 36 bits; (2) every state takes 16 bits; (3) the bit rate is $\nu = 1.6 bit/ms$; (4) the propagation delay (include the time like queue waiting time) of a successful packet is between 5ms and 8ms; (5) sampling interval is $T = 2ms$. Then, the packet size on the *uplink* of the above full-order system is $\xi = (36 + 16 \times 6)\ bits$.

Because the packet size on the *uplink* is fixed, we assume the packet transmit time on the *uplink* is between 23ms and 30ms. The total transmission time over the *downlink & uplink* is calculated as follows.

Assume the transmission delay of the p-th successful transmitted packet d_p satisfies

$$d_m = 5ms + 23ms + \frac{\xi}{v} \leq d_p \leq d_M = 8ms + 30ms + \frac{\xi}{v} \tag{90}$$

By considering the drop loss and (41), the *ZOH* updating interval $d_m + d(t)$ satisfies

$$d_m \leq d_m + d(t) \leq d_m + \bar{d} \tag{91}$$

where $\bar{d} = 3T + d_M - d_m$.

3.4.1.1 Fixed Controller

We firstly consider a fixed controller

$$K = 10^{-8} \times \begin{bmatrix} -3.44 & 0.32 & 2.96 & -2.84 & 1.25 & -0.66 \\ -5.77 & 0.62 & 1.88 & -3.17 & -0.34 & -1.65 \end{bmatrix} \tag{92}$$

For the better comparison, the time-delay variation and packet drop rate are pre-generated and are the same in each simulation. Moreover, the parameters of quantizer $q(\cdot)$ are chosen to be $\rho_i = 0.98$.

Firstly, calculate the T_b and T_b^{-1} by BTM method introduced in Section A. The next step is to apply *PROC find_the_optimized_*r_i in section C. The result shows the maximum $\alpha = 2.793$ can be achieved at $r = 4$, the data transmitted saved by *24.24%* on the *downlink*. The $x(t)$ settling time is reduced from 3.19s to 3.06s. The system vector trajectories $x(t)$ are depicted in Figures 14 & 15.

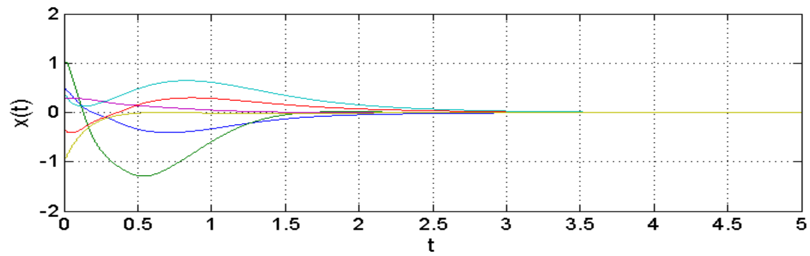

FIGURE 14: The $x(t)$ trajectory of the original 6th order system

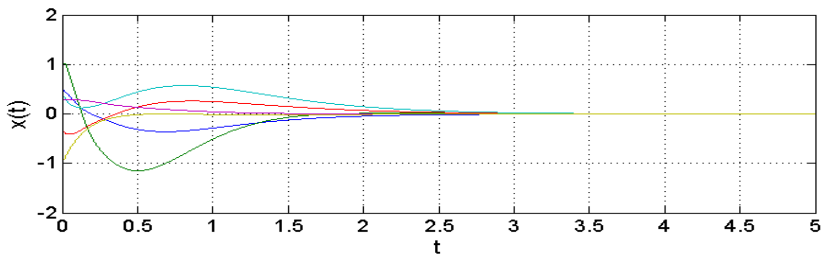

FIGURE 15: The $x(t)$ trajectory of with *MRDC*

In this example, we can find that the introduction of *MRDC* can significantly save data transmitted without changing any other parameters in other parts.

3.4.1.2 Adjustable Controller

Here we consider the case with adjustable controller. From the offline calculation, the maximum $\alpha = 2.998$ can be achieved at $r = 3$ with data reduced by *36.4%* on the *downlink*. The settling time of $x(t)$ is further reduced to 2.23s. The K is solved to be

$$K = 10^{-8} \times \begin{bmatrix} -0.63 & -0.37 & 1.64 & -1.18 & 0.95 & -0.14 \\ -2.78 & -0.04 & 0.62 & -1.44 & 0.09 & -0.77 \end{bmatrix} \quad (93)$$

The system vector trajectories $x(t)$ are depicted in FIGURE 16.

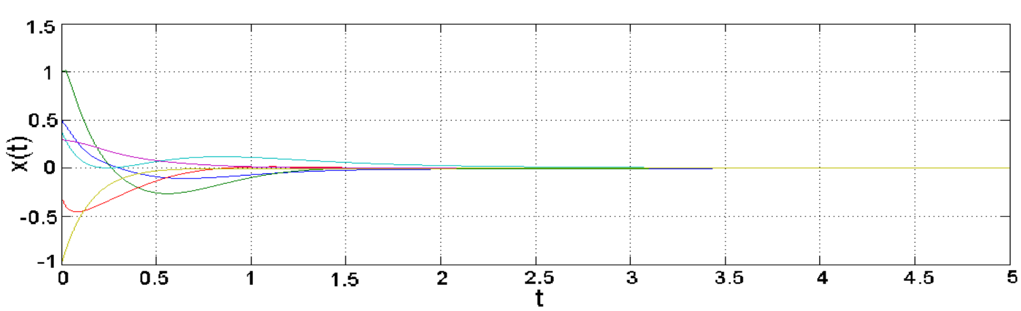

FIGURE 16: The $x(t)$ trajectory of the adjustable controller with *MRDC* at $r = 3$

3.4.2 Example 2

We consider the switched NCS example with two sub-systems from (Safonov and Chiang, 1989).

In the first sub-system,

$$A_1 = \begin{bmatrix} -6 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -10 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -13 & -3 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -14 & -9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

$$B_1^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 10^{-3} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 10^{-3} \end{bmatrix}, C_1 = I, D_1 = 0_{10\times 2}$$

The second sub-system $\{A_2, B_2, C_2, D_2\}$ is same as the first sub-system $\{A_1, B_1, C_1, D_1\}$ except the element $A_2(8,9) = -1$.

The initial states of switched NCS is assumed to be

$$x^T(\theta) = [-1 \quad 0.6 \quad -0.6 \quad 0.4 \quad -0.3 \quad 0.5 \quad 1 \quad -0.3 \quad -0.5 \quad 0],$$

$$\theta \in [-d_m - \bar{d}, 0]$$

The *downlink & uplink* are via a controller area network (CAN) with 1,000 meters long in the interference. The upper level protocol is best-effort packet delivery. The maximum total packet loss on both *downlink & uplink* is $\bar{\vartheta} = 2$. On the *downlink*, we assume (1) the packet head size is same as in the example 1; (2) the bit rate to be $\nu = 2bit/ms$;(3) the propagation delay (include queue waiting time) of a successful packet is between 5ms and 7ms; (4) sampling interval $T = 1ms$. The packet size on the *downlink* of the above full-order system is $\xi = (36 + 16 \times 10)\ bits$. On the *uplink* we assume the time needed is between 10ms and 14ms. The parameters of quantizer $q(\cdot)$ are $\rho_i = 0.98$. The simulation time is from 0s to 30s.

3.4.2.1 Fixed Controllers

We first consider the fixed controller. Controllers of the two sub-systems K_1 and K_2 are assumed to be

$$K_1 = \begin{bmatrix} 0.132 & 0.267 & -5.23\cdot 10^{-7} & 4.57\cdot 10^{-7} & 1.76 \\ -3.86\cdot 10^{-8} & -1.26\cdot 10^{-7} & 0.830 & -0.692 & -8.10\cdot 10^{-7} \end{bmatrix}$$

$$\begin{bmatrix} 1.58 & -3.92 & -1.03\cdot 10^{-6} & -2.06\cdot 10^{-6} & -1.34\cdot 10^{-3} \\ -7.27\cdot 10^{-7} & 1.86\cdot 10^{-6} & 1.92 & 3.87 & -1.09\cdot 10^{-3} \end{bmatrix} \quad (94)$$

$$K_2 = \begin{bmatrix} 0.130 & 0.271 & -5.22\cdot 10^{-7} & 4.23\cdot 10^{-7} & 1.76 \\ -1.86\cdot 10^{-8} & -1.23\cdot 10^{-7} & 0.77 & -0.606 & -8.17\cdot 10^{-7} \end{bmatrix}$$

$$\begin{bmatrix} 1.58 & -3.93 & -1.23\cdot 10^{-6} & -2.91\cdot 10^{-7} & -1.37\cdot 10^{-3} \\ -7.15\cdot 10^{-7} & 1.84\cdot 10^{-6} & 2.07 & 0.498 & -1.07\cdot 10^{-3} \end{bmatrix} \quad (95)$$

Calculate the $T_{b,i}$ and $T_{b,i}^{-1}$, $i = 1,2$. Then, apply the optimization algorithm for the fixed controller in the Section C. The maximum $\alpha = \min(\alpha_1, \alpha_2) = 3.994$ can be achieved at truncator order sequence ($r_1 = 1$, $r_2 = 1$) of this switched NCSs.

In the simulation, we choose the switch signal as

$$\beta(t) = \begin{cases} 1 & 0s \le t < 0.5s, \quad 1s \le t < 9s, \quad 17s \le t < 25s \\ 2 & 0.5s \le t < 1s, \quad 9s \le t < 17s, \quad 25s \le t \le 30 \end{cases} \quad (96)$$

By comparing with the original systems, the simulation result shows that the settling time is reduced to 0.845s from 0.940s while the transmitted data are reduced by 73.5% as depicted in Figures 17& 18.

It needs to be noticed that the simulation time is from 0s to 30s, however, the x axle of the following figures are from 0s to 2.5s because the $x(t)$ trajectory converge before 1.5s.

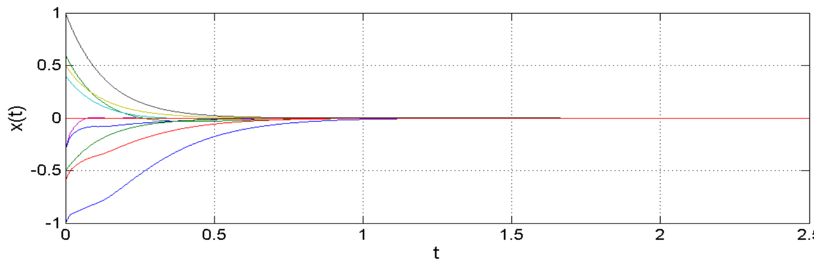

FIGURE 17: The $x(t)$ trajectory of the fixed controller without the *MRDC*

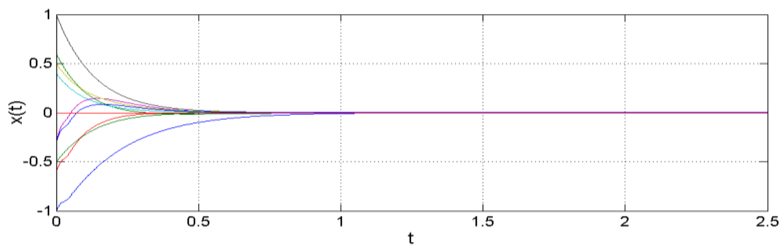

FIGURE 18: The $x(t)$ trajectory of the fixed controller w. the *MRDC* at $r_1 = 1$ and $r_2 = 1$

3.4.2.2 Adjustable Controllers

We then illustrate the switched NCS with adjustable controllers in the design process.

The largest $\alpha = \min(\alpha_1, \alpha_2) = 5.176$ can be achieved at the truncator order combination ($r_1 = 8$, $r_2 = 9$) of this switched NCS.

$$K_1 = \left[\begin{matrix} -0.226 & 0.245 & -1.94 \cdot 10^{-2} & -4.10 \cdot 10^{-2} & 0.97 \\ 9.66 \cdot 10^{-5} & -8.58 \cdot 10^{-3} & 0.452 & -0.562 & -0.150 \end{matrix}\right.$$

$$\left.\begin{matrix} 1.38 & -2.68 & -1.33 & -0.525 & -1.01 \cdot 10^{3} \\ -0.218 & 0.523 & 1.210 & 3.370 & -1.24 \cdot 10^{3} \end{matrix}\right] \quad (97)$$

$$K_2 = \left[\begin{matrix} -0.407 & 0.318 & 6.53 \cdot 10^{-4} & -1.55 \cdot 10^{-3} & 1.15 \\ -2.33 \cdot 10^{-3} & -2.73 \cdot 10^{-4} & 1.08 \cdot 10^{-3} & -1.33 \cdot 10^{-3} & 1.57 \cdot 10^{-3} \end{matrix}\right.$$

$$\left.\begin{matrix} 1.52 & -3.16 & 2.78 \cdot 10^{-3} & 7.84 \cdot 10^{-4} & -9.55 \cdot 10^{2} \\ 1.98 \cdot 10^{-3} & -4.47 \cdot 10^{-3} & 6.13 \cdot 10^{-3} & 2.87 \cdot 10^{-3} & -4.85 \end{matrix}\right] \quad (98)$$

The convergence time to 0.02 is sharply reduced to 0.635s from 9.940s by FIGURE 19.

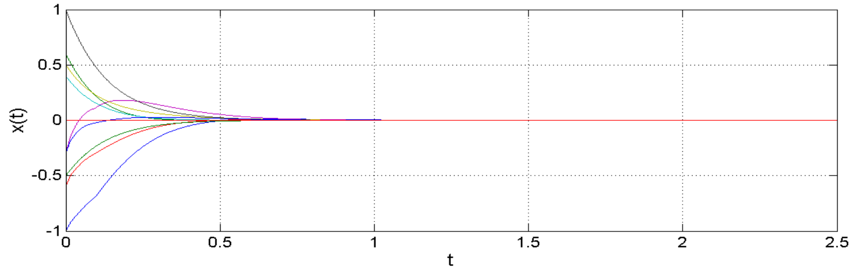

FIGURE 19: The $x(t)$ trajectory of the flexible controller of switched NCS with the *MRDC*

CHAPTER 4: ROBUST SLIDING MODE CONTROL OF GENERAL TIME-VARYING DELAY STOCHASTIC SYSTEMS WITH STRUCTURAL UNCERTAINTIES

This chapter is about the robust sliding mode control of general time-varying delay stochastic systems with structural uncertainties. This chapter is organized as follows. Section A provides the problem formulation, including a brief description of the considered general uncertain stochastic system and the robustly stochastic stability. The Section B presents the SVD method for system structural uncertainties in B.1, the sliding mode controller design in B.2, the theorems on the robust stability condition in B.3. The reachability analysis of the switching surface in Section C. Comparisons between the proposed methods and the existing methods are provided in Section D. Section E presents examples for the results.

4.1 Problem Formulation

In this section, we present the problem formulation of general uncertain stochastic systems that we are treating. It is a kind of general stochastic systems with time-varying delay and structural uncertainties established on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ as described in (Niu et al., 2005) with the Itô form as

$$dx(t) = [(A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t - \tau(t)) + B(u(t) + f(x(t), t))]dt$$
$$+D[(C + \Delta C(t))x(t) + (C_d + \Delta C_d(t))x(t - \tau(t))]dw(t)$$
$$x(t) = \varphi(t), \qquad t \in [-d, 0], \tag{99}$$

where $x(t) \in \mathbb{R}^{n\times 1}$ is the system state vector; $u(t) \in \mathbb{R}^{m\times 1}$ is the control input; $w(t)$ is a one-dimensional Brownian motion with $\mathcal{E}\{dw(t)\} = 0$ and $\mathcal{E}\{[dw(t)]^2\} = dt$; matrices $A \in \mathbb{R}^{n\times n}$, $A_d \in \mathbb{R}^{n\times n}$, $C \in \mathbb{R}^{m\times n}$, $C_d \in \mathbb{R}^{m\times n}$, and $D \in \mathbb{R}^{n\times m}$ are known constant matrices; matrix B is a full column rank matrix; matrices $\Delta A(t) \in \mathbb{R}^{n\times n}$, $\Delta A_d(t) \in \mathbb{R}^{n\times n}$, $\Delta C(t) \in \mathbb{R}^{m\times n}$, and $\Delta C_d(t) \in \mathbb{R}^{m\times n}$ are unknown time-varying uncertain matrices; $f(x(t), t) \in \mathbb{R}^n$ represents unknown state-dependent uncertain non-linear function vector; and $\tau(t)$ is the time-varying delay.

Without loss of generality, the following four assumptions are held for system (1):

1) $\|f(x(t), t)\| \le \gamma\|x(t)\|$, with constant $\gamma > 0$;

2) $0 < \tau(t) \le \bar{d} < \infty$, $\dot{\tau}(t) \le h < +\infty$;

3) $(A, B)$is controllable (100)

The above-mentioned structural uncertainties of system (99) are represented in practical and flexible forms as follows:

$$\Delta A(t) = \sum_{i=1}^{l_A} a_i(t)A_i, \qquad |a_i(t)| \le 1 \tag{101}$$

$$\Delta A_d(t) = \sum_{j=1}^{l_{A_d}} a_{d,j}(t)A_{dj}, \quad |a_{d,j}(t)| \le 1 \tag{102}$$

$$\Delta C(t) = \sum_{h=1}^{l_C} c_h(t)C_h, \qquad |c_h(t)| \le 1 \tag{103}$$

$$\Delta C_d(t) = \sum_{s=1}^{l_{C_d}} c_{d,s}(t)C_{ds}, \quad |c_{d,s}(t)| \le 1 \tag{104}$$

where A_i, A_{dj}, C_h, and C_{ds} are constant matrices representing the uncertainty structures; $a_i(t)$, $a_{d,j}(t)$, $c_h(t)$ and $c_{d,s}(t)$ are time-varying uncertain parameters. Without loss of generality, the absolute values of the time-varying uncertain parameters are bounded by 1. The goal is to design a controller with SMC to robustly stabilize the uncertain time-delay stochastic system (99). The stability of stochastic systems is defined below, where Definition 6 can be found in (Xu and Chen, 2002), but we modify it by $\|x(t)\|$ to replace

$|x(t)|$. The operator $\|\cdot\|$ denotes $\|\cdot\|_2$ in the rest of the dissertation.

Definition 6: The nominal system of the uncertain system (99) with $u(t) = 0$ is said to be mean-square stable if for any $\varepsilon > 0$ there is a $\delta(\varepsilon) > 0$ such that $\mathcal{E}[\|x(t)\|^2] < \varepsilon$ for all $t > 0$when $\sup_{-d \le s \le 0} \mathcal{E}[\|\varphi(s)\|] < \delta(\varepsilon)$. If, in addition, $\lim_{t\to\infty} \mathcal{E}[\|x(t)\|^2] = 0$ for all initial conditions, then the nominal time-delay stochastic system is said to be mean-square asymptotically stable.

Definition 7: The uncertain stochastic system in (99) is said to be robustly stochastically stable if the system is mean-square asymptotically stable for all admissible uncertainties.

The following two Lemmas will be used in the proof of main results.

Lemma 3. (Utkin, 1993) Let P_1 and P_2 be matrices with approximate dimensions, and P_3 be a symmetric matrix satisfying $\varepsilon I - P_3 > 0$ and $\varepsilon > 0$. Then the following inequality exists:

$$P_1^T P_3 P_2 + P_2^T P_3 P_1 + P_2^T P_3 P_2 \le P_1^T P_3 (\varepsilon I - P_3)^{-1} P_3 P_1 + \varepsilon P_2^T P_2 \qquad (105)$$

4.2 Stability Analysis for Uncertain Stochastic Systems

This section presents new less-conservative control method for system (99). At first, we apply the SVD to structural uncertainties in section B.1, and introduce a design of sliding mode control in B.2. Then, several new less-conservative conditions are derived to find robust controller for system (99) in section B.3. We prove the switching surface $s(t) = 0$ reachable with probability 1 in B.4.

4.2.1 Efficient Structural Uncertainties Decomposition

We apply the SVD method to the structural uncertainties in (101)-(104) as in (Wang et al., 1993, Wang et al., 2001, Wang, 2003). Then, the structural uncertainty $\Delta A(t)$ in (101) can be decomposed and represented as follows:

$$\Delta A(t) = J_A R_A(t) U_A \tag{106}$$

where $J_A = [J_{A,1} \quad J_{A,2} \quad \cdots \quad J_{A,l_A}]$, $U_A = \begin{bmatrix} U_{A,1}^T & U_{A,2}^T & \cdots & U_{A,l_A}^T \end{bmatrix}^T$, $A_i = J_{A,i} U_{A,i}$, $i = 1, \cdots, l_A$, from the SVD method, $R_A(t) = diag\{ a_1(t)I, a_2(t)I, \dots, a_{l_A}(t)I \}$, and the identity matrix I has its appropriate dimension.

Similarly to (106), other uncertainty structural matrices $\Delta A_d(t)$, $\Delta C(t)$ and $\Delta C_d(t)$ can also be decomposed and represented as:

$$\Delta A_d(t) = J_{A_d} R_{A_d}(t) U_{A_d},\ A_{dj} = J_{A_d,j} U_{A_d,j}, j = 1, \dots, l_{A_d} \tag{107}$$

$$\Delta C(t) = J_C R_C(t) U_C,\ \ C_h = J_{C,h} U_{C,h}, h = 1, \dots, l_C \tag{108}$$

$$\Delta C_d(t) = J_{C_d} R_{C_d}(t) U_{C_d}, C_{ds} = J_{C_d,s} U_{C_d,s}, s = 1, \dots, l_{C_d} \tag{109}$$

where

$$R_{A_d} = diag\left\{a_{d,1}(t)I, a_{d,2}(t)I, \dots, a_{d,l_{A_d}}(t)I\right\},$$

$$R_C = diag\left\{c_1(t)I, c_2(t)I, \dots, c_{l_C}(t)I\right\},$$

$$R_{C_d} = diag\left\{c_{d,1}(t)I, c_{d,2}(t)I, \dots, c_{d,l_{C_d}}(t)I\right\}.$$

Remark 9.The uncertainty treatment in (106)–(108) is more flexible and less conservative than the normal treatment as pointed out by Wang et al. (1998) in the literature, and that leads to less conservative result.

It should be noticed that the description of structural uncertainties in (101)-(104) or (106)–(108) (e.g., $\Delta A(t) = \sum_{i=1}^{l_A} a_i(t) A_i$) may seem to have relation with the $EF(t)H$ form as commonly used in H_∞ control (de Souza and Li, 1999, Zhou et al., 1995, Xie et al., 1992). However, as shown in (Wang and Bai, 2012, Wang et al., 1998), they are not equivalent because the common EFH form binds different structural uncertainties, e.g., ΔA, ΔA_d, ΔB and/or so on, that leads to conservatism and large size matrices. Furthermore, their matrices E and H are not from the SVD. Thus, our treatment makes difference and leads to less conservative results.

4.2.2 Sliding Mode Controller Design

As the first step of the SMC design, its switching surface function is defined as the following form, similar to (Niu et al., 2005, Chang and Wang, 1999),

$$s(t) = Gx(t) - \int_0^t G(A + BK)x(s)ds \quad (110)$$

where $s(t) = [s_1(t) \quad s_2(t) \quad \cdots \quad s_m(t)]^T \in \mathbb{R}^{m\times 1}$; $x(t)$ is the system state vector; A and B are the matrices in (99); G is chosen to make GB as non-singular as specified in (111) below; and K is the control feedback matrix to be determined. The value of $s(t)$ depends not only on the current states but also on the historical states from time 0 to the current time *t*.

The following stochastic integral equation is derived by substituting $dx(t)$ in (1a) into (110)

$$\begin{aligned} s(t) = Gx(0) + \int_0^t [\,(G\Delta A(s) - GBK)x(s) + G\big(A_d + \Delta A_d(s)\big)x\big(s - \tau(s)\big) \\ +GB\big(u(s) + f(x(s), s)\big)]ds + \int_0^t GD[\,\big(C + \Delta C(s)\big)x(s) \\ + \big(C_d + \Delta C_d(s)\big)x\big(s - \tau(s)\big)]dw(s) \end{aligned} \quad (111)$$

The Browne motion part of (111) can be eliminated by choosing $GD = B^T XD = 0$. Then we have

$$\dot{s}(t) = (G\Delta A(t) - GBK)x(t) + G\big(A_d + \Delta A_d(t)\big)x\big(t - \tau(t)\big) + GB\big(u(t) + f(x(t), t)\big) \quad (112)$$

Similar to (Niu et al., 2005), matrix G is chosen to be the following form to guarantee the non-singularity of GB as

$$G = B^T X, \quad X > 0 \tag{113}$$

Once the system state vector reaches to the predefined switching surface $s(t) = 0$, the system will enter the sliding mode with $\dot{s}(t) = 0$. Hence, by calculating $u(t)$from $\dot{s}(t) = 0$, the equivalent control law $u_{eq}(t)$ is

$$u_{eq}(t) = -(GB)^{-1}G[\Delta A(t)x(t)$$
$$+\big(A_d + \Delta A_d(t)\big)x\big(t - \tau(t)\big)] + Kx(t) - f(x(t), t) \tag{114}$$

By substituting $u(t)$ in system (99) by $u_{eq}(t)$ in (114), the system dynamic equation is

$$dx(t) = \big[\Phi_1(t)x(t) + \Phi_2(t)x\big(t - \tau(t)\big)\big]dt$$
$$+D\big[\big(C + \Delta C(t)\big)x(t) + \big(C_d + \Delta C_d(t)\big)x\big(t - \tau(t)\big)\big]dw$$
$$= \Phi(t)dt + G(t)dw \tag{115}$$

where

$$\Phi_1(t) = A + BK + \Delta A(t) - B(GB)^{-1}G\Delta A(t)$$

$$\Phi_2(t) = A_d + \Delta A_d(t) - B(GB)^{-1}G\big(A_d + \Delta A_d(t)\big)$$

$$\Phi(t) = \Phi_1(t)x(t) + \Phi_2(t)x(t - \tau(t))$$

$$G_1(t) = \big(C + \Delta C(t)\big)x(t), G_2(t) = (C_d + \Delta C_d(t))x(t - \tau(t))$$

$$G(t) = D[G_1(t) + G_2(t)]$$

In view of (114), we select the control in (116) for the sliding mode controller as follows

$$u(t) = u_{eqc}(t) + u_s(t) \tag{116}$$

$$u_{eqc}(t) = -(GB)^{-1}GA_d x\big(t - \tau(t)\big) + Kx(t) \tag{117}$$

$$u_s(t) = -\rho(t)sgn\big(s(t)\big) \tag{118}$$

where $u_{eqc}(t)$ is derived from the equivalent control rule in (114) for the nominal system of (1) without all uncertainties, $\rho(t)$ is the switching gain, $u_s(t)$ is the switching control part, that is able to overcome the system uncertainties in sliding mode control, and $sgn(\cdot)$ is the sign function.

Due to $u_{eqc}(t) \neq u_{eq}(t)$ for the uncertain system (un-nominal system), the states cannot retain on the switching surface. However, $u_s(t)$ can force the state vector toward $s(t) = 0$. Therefore, the total control input (116) can hold the states within a certain range around the switching surface.

Remark 10. Here, without loss of generality, we assume that $x\big(t - \tau(t)\big)$ is known by the controller, either for the case of known time delay $\tau(t)$ as a time trigger signal, or for the case of unknown time delay but as an event trigger signal. The latter means that the value of $\tau(t)$ may not need to be exactly known, while the value $x\big(t - \tau(t)\big)$ is known or received.

Remark 11. In (111), the condition of $GB = B^T XB$ nonsingular needs matrix B to be a full-column rank matrix. It is because X is a non-singular matrix. The condition $GD = B^T XD = 0$ implies that D is not a full rank matrix. In practical applications, we may release this constraint by choosing the trace of GD to be a much small value.

The SMC design needs to achieve two goals: (i) the system states trajectory should globally reach the switching surface $s(t) = 0$, *i.e.*, $s(t)\dot{s}(t) < 0$, and(ii) the system dynamics (19) on the switching surface is robustly stochastically stable. We address these two goals in the following sections.

4.2.3 Robust Sliding Mode Control via LMI Approach

Based on the above SVD on uncertainty structures in B.1, we derive the following stability theorems. First, we treat a case with $\dot{\tau}(t) \leq h < 1$ in Theorem 8, and then we treat a general case with $\dot{\tau}(t) \le h < +\infty$ in Theorems 9-11

Theorem 8. The system (99)-(100) with control (114) and $\dot{\tau}(t) \le h < 1$ is robustly stochastically stable, if there exist positive definite matrices X and Q, and positive adjustable scalars ε_{1i}, ε_{2j}, ε_3, ε_{4i}, ε_{5j} ($i = 1, \dots, l_A$ and $j = 1, \dots, l_{A_d}$) satisfying

$$M_1 = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & O \\ * & M_{22} & O & O & O \\ * & * & M_{33} & O & M_{35} \\ * & * & * & M_{44} & O \\ * & * & * & * & M_{55} \end{bmatrix} < 0, \text{ and } GD = B^T XD = 0 \qquad (119)$$

where matrix M_1 is a symmetric matrix with symbol$*$ as its symmetric part for simplicity,

$$M_{11} = \begin{bmatrix} \Pi_1 & XA_d \\ * & \Pi_2 \end{bmatrix} + diag\left\{\sum_{i=1}^{l_A} \varepsilon_{4i} U_{A,i}^T U_{A,i}, \ \sum_{j=1}^{l_{A_d}} \varepsilon_{5j} U_{A_d,j}^T U_{A_d,j}\right\},$$

$$M_{12} = [\bar{C}^T D^T X \quad \bar{C}^T D^T XD[J_C \quad J_{C_d}]],$$

$$M_{13} = \begin{bmatrix} O & O & \sqrt{2}XB \\ O & A_d^T X & O \end{bmatrix}, M_{14} = \begin{bmatrix} XJ_A & XJ_{A_d} \\ O & O \end{bmatrix}, M_{35} = \begin{bmatrix} XJ_A & 0 & 0 \\ 0 & XJ_{A_d} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$M_{22} = diag\{-X, -\Pi_3\}, M_{33} = diag\{-X, -X, -B^T XB\},$$

$$M_{44} = diag\left\{-\varepsilon_{11}I, \ \cdots, -\varepsilon_{1l_A}I, -\varepsilon_{21}I, \cdots, -\varepsilon_{2l_{A_d}}I\right\},$$

$$M_{55} = diag\left\{-\varepsilon_{41}I, \cdots, -\varepsilon_{4l_A}I, -\varepsilon_{51}I, \cdots, -\varepsilon_{5l_{A_d}}I\right\},$$

$$\Pi_1 = X(A + BK) + (A + BK)^T X + \sum_{i=1}^{l_A}\left(\varepsilon_{1i} U_{A,i}^T U_{A,i}\right) + Q + \varepsilon_3 U_C^T U_C,$$

$$\Pi_2 = \sum_{j=1}^{l_{A_d}}\left(\varepsilon_{2j} U_{A_d,j}^T U_{A_d,j}\right) - (1 - h)Q + \varepsilon_3 U_{C_d}^T U_{C_d},$$

$$\Pi_3 = \varepsilon_3 I - [J_C \quad J_{C_d}]^T D^T XD[J_C \quad J_{C_d}], \quad \bar{C} = [C \quad C_d]$$

Proof: Choose a Lyapunov function candidate for system (1) as

$$V(x(t), t) = V_1(x(t), t) + V_2(x(t), t) \qquad (120)$$

where

$$V_1(x(t),t) = x^T(t)Xx(t)$$

$$V_2(x(t),t) = \int_{t-\tau(t)}^{t} x^T(s)Qx(s)ds$$

According to Itô's formula, the following stochastic differential equation can be derived:

$$dV(x(t),t) = dV_1(x(t),t) + dV_2(x(t),t) \tag{121}$$

where

$$dV_1(x(t),t) = 2x^T(t)Xdx(t) + \frac{1}{2}\frac{\partial x^T(t)}{\partial w}X\frac{\partial x(t)}{\partial w}dt$$

$$= 2x^T(t)X\big[\Phi_1(t)x(t) + \Phi_2(t)x\big(t-\tau(t)\big)\big]dt$$

$$+ \xi^T(t)[D\bar{C} + D\Delta\bar{C}]^T X[D\bar{C} + D\Delta\bar{C}]\xi(t)dt$$

$$+ 2x^T(t)XD\big[\big(C+\Delta C(t)\big)x(t) + \big(C_d + \Delta C_d(t)\big)x\big(t-\tau(t)\big)\big]dw(t)$$

$$dV_2(x(t),t) = x^T(t)Qx(t) - \big(1-\dot{\tau}(t)\big)x^T\big(t-\tau(t)\big)Qx\big(t-\tau(t)\big)$$

$\xi(t) = \big[x^T(t) \quad x^T\big(t-\tau(t)\big)\big]^T$, $\Delta\bar{C} = [\Delta C \quad \Delta C_d(t)] = \big[J_C R_C(t) U_C J_{C_d} R_{C_d}(t) U_{C_d}\big]$,

and $J_C, R_C(t),\ U_C,\ J_{C_d},\ R_{C_d}(t), U_{C_d}$ are in (106) –(109).

Denote:

$$\mathcal{L}V(x(t),t) = 2x^T(t)X\Phi_1(t)x(t) + 2x^T(t)X\Phi_2(t)x\big(t-\tau(t)\big) + x^T(t)Qx(t)$$

$$+\xi^T(t)[D\bar{C} + D\Delta\bar{C}]^T X[D\bar{C} + D\Delta\bar{C}]\xi(t) - \big(1-\dot{\tau}(t)\big)x^T\big(t-\tau(t)\big)Qx\big(t-\tau(t)\big) \tag{122}$$

For any $\varepsilon_{1i} > 0, i = 1, \dots, l_A$ and $\varepsilon_{2j} > 0, j = 1, \dots, l_{A_d}$, the following matrix inequalities (123)– (126) hold in view of (101)– (109) and (113), similar to (Niu et al., 2005):

$$-2x^T(t)XB(GB)^{-1}G\Delta Ax(t) \le x^T(t)XB(B^TXB)^{-1}B^TXx(t) + x^T(t)\Delta A^T(t)X\Delta Ax(t) \tag{123}$$

$$-2x^T(t)XB(GB)^{-1}G\big(A_d+\Delta A_d(t)\big)x\big(t-\tau(t)\big) \le x^T(t)XB(B^TXB)^{-1}B^TXx(t)$$
$$+x^T\big(t-\tau(t)\big)\cdot\big(A_d+\Delta A_d(t)\big)^TX\big(A_d+\Delta A_d(t)\big)x\big(t-\tau(t)\big) \tag{124}$$

$$2x^T(t)X\Delta A(t)x(t) \le \sum_{i=1}^{l_A}\Big(\varepsilon_{1i}^{-1}x^T(t)XJ_{A,i}J_{A,i}^TXx(t)\Big)+\sum_{i=1}^{l_A}\Big(\varepsilon_{1i}x^T(t)U_{A,i}^TU_{A,i}x(t)\Big) \tag{125}$$

$$2x^T(t)X\Delta A_d(t)x\big(t-\tau(t)\big)$$
$$\le \sum_{j=1}^{l_{A_d}}\Big(\varepsilon_{2j}^{-1}x^T(t)XJ_{A_d,j}J_{A_d,j}^TXx(t)\Big)$$
$$+\sum_{j=1}^{l_{A_d}}\Big(\varepsilon_{2j}x^T\big(t-\tau(t)\big)U_{A_d,i}^TU_{A_d,i}x\big(t-\tau(t)\big)\Big) \tag{126}$$

In (119), let $\Pi_3 > 0$. By selecting a suitable $\varepsilon_3 > 0$, we have the following inequality from Lemma 3.

$$[D\bar{C}+D\Delta\bar{C}]^TX[D\bar{C}+D\Delta\bar{C}] \;\le\; \bar{C}^TD^TXD\bar{C}$$
$$+\bar{C}^TD^TXD[J_C \quad J_{C_d}]\Pi_3{}^{-1}[J_C \quad J_{C_d}]^TD^TXD\bar{C}+\varepsilon_3\begin{bmatrix}U_C & 0\\ 0 & U_{C_d}\end{bmatrix}^T\begin{bmatrix}U_C & 0\\ 0 & U_{C_d}\end{bmatrix} \tag{127}$$

Substituting (123)–(127) into $\mathcal{L}V(x(t),t)$, and applying Schur complement, we have

$$\mathcal{L}V(x(t),t) \le \xi^T(t)\Sigma\xi(t) \tag{128}$$

where

$$\Sigma = \begin{bmatrix}\Pi_4 & XA_d\\ * & \Pi_5\end{bmatrix}+\varepsilon_3\begin{bmatrix}U_C^TU_C & 0\\ 0 & U_{C_d}^TU_{C_d}\end{bmatrix}+\bar{C}^TD^TXD\bar{C}$$
$$+\bar{C}^TD^TXD\,[J_C \quad J_{C_d}]\Pi_3{}^{-1}[J_C \quad J_{C_d}]^TD^TXD\bar{C}$$

$$\Pi_4 = X(A+BK)+(A+BK)^TX+2XB(B^TXB)^{-1}B^TX+\Delta A^TX\Delta A$$
$$+\sum_{i=1}^{l_A}\big(\varepsilon_{1i}^{-1}XJ_{A,i}J_{A,i}^TX\big)+\sum_{i=1}^{l_A}\big(\varepsilon_{1i}U_{A,i}^TU_{A,i}\big)+\sum_{j=1}^{l_{A_d}}\big(\varepsilon_{2j}^{-1}XJ_{A_d,j}J_{A_d,j}^TX\big)+Q$$

$$\Pi_5 = \sum_{j=1}^{l_{A_d}} \left(\varepsilon_{2j} U_{A_d,j}^T U_{A_d,j}\right) - (1-h)Q + \left(A_d + \Delta A_d(t)\right)^T X \left(A_d + \Delta A_d(t)\right).$$

Since $-X < 0$, by applying Schur complement, inequality $\Sigma < 0$ is equivalent to the following LMI:

$$M_2 = \begin{bmatrix} M_{11}^{(2)} & M_{12}^{(2)} & M_{13}^{(2)} & M_{14}^{(2)} \\ * & M_{22}^{(2)} & O & O \\ * & * & M_{33}^{(2)} & O \\ * & * & * & M_{44}^{(2)} \end{bmatrix} < 0 \tag{129}$$

where

$$M_{11}^{(2)} = \begin{bmatrix} \Pi_1 & XA_d \\ * & \Pi_2 \end{bmatrix},$$

$$M_{12}^{(2)} = M_{12} = [\bar{C}^T D^T X \quad \bar{C}^T D^T X D[J_C \quad J_{C_d}]],$$

$$M_{13}^{(2)} = \begin{bmatrix} \Delta A^T X & 0 & \sqrt{2} XB \\ 0 & \left(A_d^T + \Delta A_d^T(t)\right) X & 0 \end{bmatrix},$$

$$M_{14}^{(2)} = M_{14} = \begin{bmatrix} XJ_A & XJ_{A_d} \\ O & O \end{bmatrix},$$

$$M_{22}^{(2)} = M_{22} = diag\{-X, -\Pi_3\},$$

$$M_{33}^{(2)} = M_{33} = diag\{-X, -X, -B^T XB\},$$

$$M_{44}^{(2)} = M_{44} = diag\left\{-\varepsilon_{11} I, \quad \cdots, -\varepsilon_{1l_A} I, -\varepsilon_{21} I, \cdots, -\varepsilon_{2l_{A_d}} I\right\}.$$

The uncertain part of matrix M_2 satisfies

$$\sum_{i=1}^{l_A} a_i(t) \left(\Xi_{A,i}\Theta_{A,i} + \left(\Xi_{A,i}\Theta_{A,i}\right)^T\right) + \sum_{j=1}^{l_{A_d}} a_{d,j}(t) \left(\Xi_{A_d,j}\Theta_{A_d,j} + \left(\Xi_{A_d,j}\Theta_{A_d,j}\right)^T\right)$$

$$\leq \sum_{i=1}^{l_A} \left(\varepsilon_{4i}\Xi_{A,i}\Xi_{A,i}^T + \varepsilon_{4i}^{-1}\Theta_{A,i}^T\Theta_{A,i}\right) + \sum_{j=1}^{l_{A_d}} \left(\varepsilon_{5\mathrm{j}}\, \Xi_{A_d,j}\Xi_{A_d,j}^T + \varepsilon_{5\mathrm{i}}^{-1}\Theta_{A_d,j}^T\Theta_{A_d,j}\right) \tag{130}$$

where

$$\Xi_{\mathrm{A,i}} = \left[diag\{\mathrm{U}_{\mathrm{A,i}}, 0\} \quad O \quad O \quad O\right]^T, \Theta_{A,i} = \left[O \quad O \quad diag\{J_{A,i}^T X, 0,0\} \quad O\right]$$

$$\Xi_{A_d,j} = \left[diag\{0, U_{A_d,j}\} \quad O \quad O \quad O\right]^{\mathrm{T}}, \Theta_{A_d,j} = \left[O \quad O \quad diag\{0, J_{A_d,j}^T X, 0\} \quad O\right].$$

Therefore, we have inequality (131) from (129) –(130):

$$M_2 \leq M_3 = \begin{bmatrix} M_{11}^{(3)} & M_{12}^{(3)} & M_{13}^{(3)} & M_{14}^{(3)} \\ * & M_{22}^{(3)} & O & O \\ * & * & M_{33}^{(3)} & O \\ * & * & * & M_{44}^{(3)} \end{bmatrix} < 0 \tag{131}$$

where

$$M_{11}^{(3)} = M_{11}^{(2)} + diag\left\{\sum_{i=1}^{l_A} \varepsilon_{4i} U_{A,i}^T U_{A,i}, \sum_{j=1}^{l_{A_d}} \varepsilon_{5j} U_{A_d,j}^T U_{A_d,j}\right\},$$

$$M_{33}^{(3)} = M_{33} + diag\left\{\sum_{i=1}^{l_A} \varepsilon_{4i}^{-1} X J_{A,i} J_{A,i}^T X, \sum_{j=1}^{l_{A_d}} \varepsilon_{5j}^{-1} X J_{A_d,j} J_{A_d,j}^T X, 0\right\},$$

$$M_{12}^{(3)} = M_{12}, M_{13}^{(3)} = M_{13}^{(2)}, M_{14}^{(3)} = M_{14}, M_{44}^{(3)} = M_{44}.$$

Then, LMI $M_3 < 0$ is equivalent to $M_1 < 0$ by Schur complement in Lemma 1. Therefore, when (119) holds, the control (114) guarantees the selected Lyapunov function (120) to have its derivative (121) negative along the state trajectory for all admissive uncertainties and time varying delay, i.e., system (99) is robustly stable. It completes the proof. ■

Now, we consider the general case with $\dot{\tau}(t) \leq h < +\infty$, where $h > 1$ is allowed. When $h > 1$, it makes the term $-(1 - \dot{\tau}(t))x^T(t - \tau(t))Qx(t - \tau(t))$ in (121) as a positive definite or semi-positive definite term in dV_2, which is related to the time delay from the Lyapunov functional. Thus, we develop a new Lyapunov-type functional for this general case. First, we derive Theorem 9 below.

Theorem 9. (Wang, 2013) Consider system (99)-(100) with $\dot{\tau}(t) \leq h < +\infty$ and $\tau(t) \leq$

$\bar{d}$. If we choose a Lyapunov-type functional

$$V_3(x(t),t) = \int_{-\bar{d}}^{0}\int_{t+\beta}^{t} dx^T(\alpha)Zdx(\alpha) \tag{132}$$

then the following inequality holds

$$\begin{aligned} dV_3(x(t),t) \le \bar{d}dx^T(t)Zdx(t) + 2\xi^T(t)W\left(x(t) - x\big(t-\tau(t)\big)\right)dt \\ +\bar{d}\xi^T(t)WZ^{-1}W^T(t)\xi(t)dt \end{aligned} \tag{133}$$

where matrix $Z > 0$,matrix $W = [W_1^T \quad W_2^T]^T$ is any appropriate size matrix, and

$$\bar{d}dx^T(t)Zdx(t) = \bar{d}\xi^T(t)[D\bar{C} + D\Delta\bar{C}]^T Z[D\bar{C} + D\Delta\bar{C}]\,\xi(t)dt,$$

$$\xi(t) = \left[x^T(t) \quad x^T\big(t-\tau(t)\big)\right]^T,\ \bar{C} = [C \quad C_d],\ \Delta\bar{C} = [\Delta C \quad \Delta C_d(t)]$$

Proof: For general system in (99)-(100), we take a new Lyapunov-type functional $V_3(x(t),t)$ in (132). For any appropriate size matrix $W = [W_1^T \quad W_2^T]^T$, it is true to have $\eta = 0$ as

$$\eta = 2dt\xi^T(t)W\left[x(t) - x\big(t-\tau(t)\big) - \int_{t-\tau(t)}^{t} dx(\alpha)\right] = 0 \tag{134}$$

Then, let

$$H = dtW^T\xi(t) + Zdx(\alpha) \tag{135}$$

By applying Itô's integral rule and (134)– (135), we have

$$\begin{aligned} dV_3(x(t),t) &\le \bar{d}dx^T(t)Zdx(t) + \eta - \int_{t-\tau(t)}^{t} H^T Z^{-1} H \\ &\quad +2dt\xi^T(t)W\int_{t-\tau(t)}^{t} dx(\alpha) + \bar{d}\xi^T(t)WZ^{-1}W^T(t)\xi(t)dt \\ &\le \bar{d}dx^T(t)Zdx(t) + 2\xi^T(t)W\left(x(t) - x\big(t-\tau(t)\big)\right)dt \\ &\quad + \bar{d}\xi^T(t)WZ^{-1}W^T(t)\xi(t)dt \end{aligned} \tag{136}$$

where

$$\bar{d}dx^T(t)Zdx(t) = \bar{d}\xi^T(t)[D\bar{C} + D\Delta\bar{C}]^T Z[D\bar{C} + D\Delta\bar{C}]\,\xi(t)dt$$

if we drop higher order infinitesimal items. It completes the proof. ■

Now we present a new result for the general case of time-varying delay with any finite change rate, i.e., $\dot{\tau}(t) \leq h < +\infty$.

Theorem 10. The system (99)-(100) with control (114) and $\dot{\tau}(t) \leq h < +\infty$ is robustly stochastically stable, if there exist matrices $X > 0$, $Q > 0$, and $Z > 0$, any appropriate size matrix $W = [W_1^T \quad W_2^T]^T$, and positive adjustable scalars ε_{1i}, ε_{2j}, ε_3, ε_{4i}, ε_{5j}, ε_6 ($i = 1, \dots, l_A$ and $j = 1, \dots, l_{A_d}$) satisfying

$$M_4 = \begin{bmatrix} M_{11}^{(4)} & M_{12}^{(4)} & M_{13}^{(4)} \\ * & M_{22}^{(4)} & O \\ * & * & M_{33}^{(4)} \end{bmatrix} < 0, \text{ and } GD = B^T XD = 0 \tag{137}$$

where

$$M_{11}^{(4)} = M_{11} + \begin{bmatrix} W_1^T + W_1 & -W_1 + W_2^T \\ * & -W_2^T - W_2 \end{bmatrix} + diag\{\bar{d}\varepsilon_6 U_C^T U_C, \ \bar{d}\varepsilon_6 U_{C_d}^T U_{C_d}\},$$

$$M_{12}^{(4)} = [M_{12} \quad M_{13} \quad M_{14} \quad O], M_{13}^{(4)} = \sqrt{\bar{d}}[\bar{C}^T D^T Z \quad W \quad \bar{C}^T D^T Z D [J_C \quad J_{C_d}]],$$

$$M_{22}^{(4)} = \begin{bmatrix} M_{22} & O & O & O \\ * & M_{33} & O & M_{35} \\ * & * & M_{44} & O \\ * & * & * & M_{55} \end{bmatrix}, M_{33}^{(4)} = diag\{-Z, \ -Z, \ -\Pi_6\},$$

$$\Pi_6 = \varepsilon_6 I - [J_C \quad J_{C_d}]^T D^T Z D [J_C \quad J_{C_d}].$$

M_{11}, M_{12}, M_{13}, M_{14}, M_{22}, M_{33}, M_{35}, M_{44}, and M_{55} are the same as in Theorem 8.

Proof: Choose a Lyapunov-type functional candidate for system (99)-(100) as

$$V_g(x(t), t) = \mathrm{V}_1(x(t), t) + \mathrm{V}_2(x(t), t) + V_3(x(t), t) \tag{138}$$

where $V_1(x(t), t)$ and $\mathrm{V}_2(x(t), t)$ are in (120), and $V_3(x(t), t)$ in (132).

Thus, from (136) in Theorem 9 and (138), the following inequality holds:

$$dV_g(x(t),t) \leq \mathcal{L}V_g(x(t),t)dt$$
$$+2x^T(t)XD[(C+\Delta C(t))x(t)+(C_d+\Delta C_d(t))x(t-\tau(t))]dw(t) \tag{139}$$

where

$$\mathcal{L}V_g(x(t),t) = \mathcal{L}V(x(t),t) + 2\xi^T(t)W\left(x(t)-x(t-\tau(t))\right)$$
$$+\bar{d}\xi^T(t)WZ^{-1}W^T(t)\xi(t) + \bar{d}\xi^T(t)[D\bar{C}+D\Delta\bar{C}]^T Z[D\bar{C}+D\Delta\bar{C}]\,\xi(t)$$

Let $\Pi_6 > 0$ in (134). By selecting a suitable adjustable scalar $\varepsilon_6 > 0$, we have a similar result to (127) as

$$[D\bar{C}+D\Delta\bar{C}]^T Z[D\bar{C}+D\Delta\bar{C}] \leq \bar{C}^T D^T ZD\bar{C}$$
$$+\bar{C}^T D^T ZD\,[J_C \quad J_{C_d}]\Pi_6{}^{-1}[J_C \quad J_{C_d}]^T D^T ZD\bar{C} + \varepsilon_6\begin{bmatrix} U_C & 0 \\ 0 & U_{C_d}\end{bmatrix}^T\begin{bmatrix} U_C & 0 \\ 0 & U_{C_d}\end{bmatrix} \tag{140}$$

Substitute (140) into (139) and apply Schur complement to it. Then we have

$$\mathcal{L}V_g(x(t),t) = \xi^T(t)\Sigma_2\xi(t) \tag{141}$$

and want to prove $\Sigma_2 < 0$, where

$$\Sigma_2 = \Sigma + \begin{bmatrix} W_1+W_1^T & -W_1+W_2^T \\ * & -W_2-W_2^T\end{bmatrix} + \bar{d}\bar{C}^T D^T ZD\bar{C} + \bar{d}\varepsilon_6\begin{bmatrix} U_C^T U_C & 0 \\ 0 & U_{C_d}^T U_{C_d}\end{bmatrix}$$
$$+\bar{d}WZ^{-1}W^T(t) + \bar{d}\bar{C}^T D^T ZD\,[J_C \quad J_{C_d}]\Pi_6{}^{-1}[J_C \quad J_{C_d}]^T D^T ZD\bar{C},$$

Σ is in (128).

Since $-\Pi_6 < 0$, and $-Z < 0$, the inequality $\Sigma_2 < 0$ is equivalent to $M_4 < 0$ by Schur complement. It completes the proof. ▪

In view of Theorem 9 that can deal with general time-varying delay, the Lyapunov-type functional can also be chosen as $V_p(x(t),t) = \mathrm{V}_1(x(t),t) + V_3(x(t),t)$, i.e., without above $V_2(x(t),t)$.Then we can derive the following theorem.

Theorem 11. The system (99)-(100) with control (114) and $\dot{\tau}(t) \leq h < +\infty$ is robustly stochastically stable, if there exist matrices $X > 0$, $Z > 0$, any appropriate size matrix $W = [W_1^T \quad W_2^T]^T$, and positive adjustable scalars ε_{1i}, ε_{2j}, ε_3, ε_{4i}, ε_{5j}, ε_6 ($i = 1, \dots, l_A$ and $j = 1, \dots, l_{A_d}$) satisfying

$$M_5 = \begin{bmatrix} M_{11}^{(5)} & M_{12}^{(5)} & M_{13}^{(5)} \\ * & M_{22}^{(5)} & O \\ * & * & M_{33}^{(5)} \end{bmatrix} < 0, \text{ and } GD = B^T XD = 0 \qquad (142)$$

where

$$M_{11}^{(5)} = \begin{bmatrix} \Pi_7 & XA_d \\ * & \Pi_8 \end{bmatrix} + \begin{bmatrix} W_1^T + W_1 & -W_1 + W_2^T \\ * & -W_2^T - W_2 \end{bmatrix} + diag\{\bar{d}\varepsilon_6 U_C^T U_C, \ \bar{d}\varepsilon_6 U_{C_d}^T U_{C_d}\}$$

$$+ diag\left\{\sum_{i=1}^{l_A} \varepsilon_{4i} U_{A,i}^T U_{A,i}, \ \sum_{j=1}^{l_{A_d}} \varepsilon_{5j} U_{A_d,j}^T U_{A_d,j}\right\},$$

$$\Pi_7 = X(A + BK) + (A + BK)^T X + \sum_{i=1}^{l_A} \left(\varepsilon_{1i} U_{A,i}^T U_{A,i}\right) + \varepsilon_3 U_C^T U_C,$$

$$\Pi_8 = \sum_{j=1}^{l_{A_d}} \left(\varepsilon_{2j} U_{A_d,j}^T U_{A_d,j}\right) + \varepsilon_3 U_{C_d}^T U_{C_d},$$

$$M_{12}^{(5)} = M_{12}^{(4)}, M_{13}^{(5)} = M_{13}^{(4)}, M_{22}^{(5)} = M_{22}^{(4)}, M_{33}^{(5)} = M_{33}^{(4)}$$

$M_{12}^{(4)}$, $M_{13}^{(4)}$, $M_{22}^{(4)}$ and $M_{33}^{(4)}$ are in Theorem 10.

Proof: Because the Lyapunov-type functional is selected as $V_1(x(t), t) + V_3(x(t), t)$, it leads to setting $Q = 0$ in Theorem 10. Thus, the remaining proof is straightforward. Matrices Π_7 and Π_8 are equivalent to Π_1 and Π_2 with $Q = 0$ in $M_{11}^{(4)}$, i.e., in M_{11} (24), respectively. And, the all remaining block-matrices in M_5 are the same as those corresponding ones in M_4 of Theorem 10. ■

4.3 Controller and Reachability Analysis

The above sub-section discusses the robustly stochastic stability of the system (99) on the switching surface $s(t) = 0$. Here we design the controller to globally drive the system states trajectory to the switching surface.

Theorem 12. The reachability in probability 1 to the switching surface $s(t) = 0$ for the states of system (99)-(100) can be guaranteed by controller (116) with $\rho(t)$ as:

$$\rho(t) = \lambda + \|(GB)^{-1}GJ_A\| \cdot \|U_A x(t)\| + \left\|(GB)^{-1}GJ_{A_d}\right\| \cdot \left\|U_{A_d} x\big(t-\tau(t)\big)\right\| + \gamma\|x(t)\|, \;\; \lambda > 0 \tag{143}$$

Proof: Define a Lyapunov functional candidate $V_s(t) = \frac{1}{2}s^T(t)(GB)^{-1}s(t)$. The $V_s(t)$ is nonnegative from its definition. It is straightforward to derive:

$$\begin{aligned}\dot{V}_s(t) &= s^T(t)(GB)^{-1}G[\Delta A(t)x(t) + \Delta A_d(t)x(t-\tau)] \\ &\quad + s^T(t)\Big(-\rho(t)sgn\big(s(t)\big) + f\big(x(t),t\big)\Big) \\ &\le \|s(t)\| \cdot [\|(GB)^{-1}GJ_A\| \cdot \|U_A x(t)\| + \left\|(GB)^{-1}GJ_{A_d}\right\| \\ &\quad \cdot \left\|U_{A_d} x\big(t-\tau(t)\big)\right\|] - \rho(t)\|s(t)\| + \gamma\|s(t)\| \cdot \|x(t)\|\end{aligned} \tag{144}$$

because of $\|s(t)\|_1 \ge \|s(t)\|$. By substituting $\rho(t)$ in (143) into (144), it leads to $\dot{V}_s \le -\lambda\|s(t)\| < 0$, if $s(t) \ne 0$, because $\lambda > 0$. Thus, the control in (116)-(118) with $\rho(t)$ in (143) guarantees the system states toward the switching surface $s(t) = 0$. It completes the proof. ■

Theorem 12 states that the state vector of system (99)-(100) will globally be driven to the pre-determined switching surface $s(t) = 0$ by the control in (116)-(118) and (144).

In this dissertation, we choose two kinds of λ. One is positive constant $\lambda = const > 0$, e.g., 0.5. In this choice, the obvious high frequency chatter in $u(t)$ with a main amplitude λ may be observed in the face of time varying uncertainties. The chattering depends on the system uncertainties and their time varying behavior. To reduce the chatter, we choose another form of λ as

$$\lambda = c_1\|s(t)\|^{1/c_2}, \tag{145}$$

where $s(t)$ is the switching surface function, and c_1 and c_2 are positive constants. This choice makes $\lambda > 0$ as the state vector is not on the switching surface $s(t) = 0$. As the state vector is close to $s(t) = 0$, λ is also reduced in (145), and not as constant, that leads to small chatter amplitudes. Therefore, the chatter of $u(t)$ can also be reduced.

4.4 Comparison

In the above sections, we have described the structural uncertainties in a practical and flexible form, and introduced a group of adjustable parameters in Theorem 8 and Theorems 9-10 to easily find the available matrix X in Theorems by the LMI for the controller design. To show the advantage of the method, we compare our method with the previous methods by a theoretical analysis as follows.

In the existing methods, the uncertainties are usually described in a binding format as usually noticed in H∞ control and robust control (Niu et al., 2005, Yu and Chu, 1999, de Souza and Li, 1999, Xie et al., 1992), e.g.,

$$[\Delta A \;\; \Delta A_d] = E_1 F_1(t)[H_a H_{ad}], \tag{146}$$

$$[\Delta C \;\; \Delta C_d] = E_2 F_2(t)[H_c H_{cd}], \tag{147}$$

$$F_i^T(t) F_i(t) \leq I, i = 1,2. \tag{148}$$

Because this treatment lets the uncertainties ΔA and ΔA_d share a same E_1, and ΔC and ΔC_d share a same E_2, therefore it has less flexibility than our proposed method. When the uncertainties have a practical format as (101)-(104), similar to (Wang et al., 2001), the previous methods need to treat structural matrices $E_1, E_2, H_a, H_{ad}, H_c$ and H_{cd} in larger sizes in their format than our SVD decomposed matrices $J_{A,i}$, $U_{A,i}$, $J_{A_d,j}$, $U_{A_d,j}$, $J_{C,h}$, $U_{C,h}$, $J_{C_d,s}$, $U_{C_d,s}$, and even J_A, U_A, J_{A_d}, U_{A_d}, J_C, U_C, J_{C_d}, U_{C_d} in (106)-(109). For example, we can have the followings

$$E_1 = [J_A \quad J_{Ad}], \quad E_2 = [J_c \quad J_{Cd}], \tag{149}$$

$$H_a = \begin{bmatrix} U_A \\ 0 \end{bmatrix}, H_{ad} = \begin{bmatrix} 0 \\ U_{A_d} \end{bmatrix}, H_c = \begin{bmatrix} U_C \\ 0 \end{bmatrix}, H_{cd} = \begin{bmatrix} 0 \\ U_{C_d} \end{bmatrix}, \tag{150}$$

where 0 is appropriate dimensional zero matrix, and the above (149)-(150) clearly show that the proposed method reduces the matrices sizes for the treatment.

To avoid the confliction of variable names, the variable names in the previous methods are marked with "~" on the head.

Proposition 1: The method in (Niu et al., 2005) with (149)-(150) may be observed as a special case of the proposed method in view of new Theorem 8 as

$$\tilde{\varepsilon}_1 = \varepsilon_{1i}, \tilde{\varepsilon}_2 = \varepsilon_{2j}, \ \tilde{\varepsilon}_3 = \varepsilon_3, \quad \tilde{\varepsilon}_4 = \varepsilon_{4i} = \varepsilon_{5j}, i = 1, \dots, l_A, \quad j = 1, \dots, l_{Ad} \tag{151}$$

Proof: The proof is straightforward in (119) by substituting scalars $\varepsilon_{1i}, \varepsilon_{2j}, \varepsilon_3, \varepsilon_{4i}$, ε_{5j} ($i = 1, \dots, l_A$ and $j = 1, \dots, l_{A_d}$) with $\tilde{\varepsilon}_1$, $\tilde{\varepsilon}_2$, $\tilde{\varepsilon}_3$, and $\tilde{\varepsilon}_4$, in (145). It is observed by comparing (119) with the equation (13) in (Niu et al., 2005). ■

Remark 12. Proposition 1 clearly shows that the previous method has restrictions on the adjustable parameters, i.e., it forces to use the same one adjustable parameter $\tilde{\varepsilon}_1$ for all various uncertainty structural matrices in ΔA, and one same $\tilde{\varepsilon}_2$ for all various uncertainty structural matrices in ΔA_d, and even one $\tilde{\varepsilon}_4$ for all different uncertainty structural matrices in both ΔA and ΔA_d. However, our method treats them by various adjustable parameters (i) $\varepsilon_{1i}, i = 1, \dots, l_A$, instead of one $\varepsilon_1 (or\ \tilde{\varepsilon}_1)$, (ii) $\varepsilon_{2j}, j = 1, \dots, l_{Ad}$, instead of one ε_2 $(or\ \tilde{\varepsilon}_2)$, and (iii) $\varepsilon_{4i}, i = 1, \dots, l_A$, and $\varepsilon_{5j}, j = 1, \dots, l_{Ad}$, instead of one ε_4 (or $\tilde{\varepsilon}_4$). Even for a very special situation, e.g., $l_A = 1$ and $l_{Ad} = 1$, the proposed method is still more flexible in view of ε_4 and ε_5 for ΔA and ΔA_d, respectively, to replace one ε_4 for both ΔA and ΔA_d.

Remark 13. The new proposed method further deletes one additional LMI condition (14) of

(Niu et al., 2005) to search solution matrix X in Theorem 8. It makes the method further effective. That additional condition (14) is to check matrices X, E_1X and $\varepsilon_4 I$.

Corollary 1. The proposed method Theorem 8 also improves the previous methods in (Yu and Chu, 1999, de Souza and Li, 1999) by reducing the conservatism and the matrix sizes of structural uncertainties, when the problems can be treated by these methods and models.

Proof: The proof is similar to the proof in Proposition 1. In view of (149)-(150), it is observed that the matrix size of structural uncertainties will be reduced. ■

Remark 14. For Theorem 10 in (Yu and Chu, 1999), our Theorem 8 can release the combination of ΔA and ΔA_d (ΔA_1 in (Yu and Chu, 1999), and further provide flexibility by various adjustable parameters $\varepsilon_{1i}, i = 1, \dots, l_A$, and $\varepsilon_{2j}, j = 1, \dots, l_{Ad}$, instead of one ε (or $\tilde{\varepsilon}$).

Remark 15. For Theorem 3.2 in (de Souza and Li, 1999), our Theorem 8 may provide flexibility by various adjustable parameters $\varepsilon_{1i}, i = 1, \dots, l_A$, instead of one $\varepsilon_1 (or\ \alpha_1 = \tau\varepsilon_1)$, and $\varepsilon_{2j}, j = 1, \dots, l_{Ad}$, instead of one ε_2 $(or\ \alpha_2 = \tau\varepsilon_2)$ in (de Souza and Li, 1999).

After we have derived Theorem 9, we notice that Chen et al. (2008) Chen et al. (2008) also considered general case of $\dot{\tau}(t) > 1$. However, the approach to solve this problem is different. The following Remark 16 states the difference as a brief highlight. In Section 4.5 Examples, a performance comparison on their example is presented.

Remark 16. It is noticed that the Lyapunov functional $V_1(x(t), t)$ and $V_2(x(t), t)$in this dissertation and (Chen et al., 2008), as well as many other papers (which usually consider the case of $\dot{\tau}(t) \leq 1$) in the literature, are respectively same as common. However, in order to deal with the general case of $\dot{\tau}(t) > 1$, our Theorem 9 (with Theorems 10–11) introduces a new Lyapunov-type functional for stochastic time-delay systems as

$$V_3(x(t),t) = \int_{-\bar{d}}^{0}\int_{t+\beta}^{t} dx^T(\alpha)Zdx(\alpha) \tag{152}$$

that considers whole $dx(t)$ during the period with Brownian motion part. In (Chen et al., 2008), their corresponding $V_3(x(t),t)$ is as

$$V_{3\varphi}(x(t),t) = \bar{d}\int_{-\bar{d}}^{0}\int_{t+\beta}^{t} \Phi^T(\alpha)Z\Phi(\alpha)d\alpha d\beta \tag{153}$$

where $\Phi(t)$ is in (115), that considers only the part without Brownian motion.

It is better to consider the whole $dx^T(t)Zdx(t)$ than only the partial $\Phi^T(t)Z\Phi(t)$ in (115) because the considered system is a stochastic system which is also driven and affected by the Brownian motion part, and the goal of selecting Lyapunov-type functional is to check the stability and/or the speed toward the stable state. Thus, to consider $dx^T(\alpha)Zdx(\alpha)$ is physically meaningful for dynamic systems and stochastic systems, where Z is a positive-definite matrix, $x(t)$ is the system state vector, and $dx(t)$ represents the state-vector change rate. So, the development of new Lyapunov-type functional (152) is practical and suitable, especially for the analysis and synthesis of general stochastic systems with time-varying delay and uncertainties.

4.5 Examples

In this section, we present examples to show the proposed method and to compare it with other methods.

First, consider a system (99)-(100) with its parameters as follows:

$$A = \begin{bmatrix} -0.8 & 2.0 & 0.11 \\ -0.2 & -0.3 & -0.6 \\ 0.2 & 0.3 & -0.6 \end{bmatrix}, A_d = \begin{bmatrix} -0.2 & 0.3 & 0.2 \\ 0.1 & 0 & 0.4 \\ 0.2 & 0.3 & -0.3 \end{bmatrix}, B = \begin{bmatrix} 0.7 & 2 \\ 1 & 1.5 \\ 6 & 3.6 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.1 & 0.2 & -0.1 \\ -0.2 & 0.5 & -0.04 \end{bmatrix}, C_d = \begin{bmatrix} 0 & 0.2 & 0.01 \\ 0.3 & 0.2 & 0.1 \end{bmatrix}, D = \begin{bmatrix} -0.2 & 0.1 \\ 0.4 & 0.2 \\ 0.1 & -0.1 \end{bmatrix},$$

$$\Delta A(t) = 0.3\sin(t)\begin{bmatrix} 0 & -0.01 & 0 \\ -0.005 & 0 & 0 \\ -0.05 & 0 & 0 \end{bmatrix} + 0.2\cos(t) \times \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$+0.3\sin(\mathrm{t})\cos(t)\begin{bmatrix} 0 & -0.2 & 0.02 \\ 0 & 0 & -0.05 \\ 0 & 0 & -0.05 \end{bmatrix},$$

$$\Delta A_d(t) = 0.4\cos(t)\begin{bmatrix} 0 & 0.255 & 0 \\ 0 & -0.005 & 0.01 \\ 0 & 0 & 0 \end{bmatrix} + 0.5\sin(t) \times \begin{bmatrix} 0 & 0 & -0.05 \\ 0 & 0 & -0.2 \\ 0 & 0.005 & 0.01 \end{bmatrix}$$

$$+0.5\sin(\mathrm{t})\cos(t)\begin{bmatrix} 0 & -0.05 & -0.001 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Delta C(t) = 0.3\sin(t)\begin{bmatrix} 0.002 & 0.004 & 0.002 \\ 0.002 & 0.004 & 0.002 \end{bmatrix} + 0.4\cos(t) \times \begin{bmatrix} -0.02 & -0.02 & 0.06 \\ 0.02 & -0.02 & 0.06 \end{bmatrix}$$

$$+0.3\sin(\mathrm{t})\cos(t)\begin{bmatrix} 0 & 0.002 & 0.06 \\ 0 & 0.002 & -0.06 \end{bmatrix},$$

$$\Delta C_d(t) = 0.3\sin(t)\cos(t)\begin{bmatrix} 0.0001 & 0 & 0 \\ 0 & 0 & 0.001 \end{bmatrix} + 0.3\cos(t)$$

$$\times \begin{bmatrix} 0.0001 & 0.0001 & 0 \\ 0 & 0 & 0.0001 \end{bmatrix} + 0.4\sin(\mathrm{t})\begin{bmatrix} 0 & 0.0001 & 0.0001 \\ 0 & 0 & 0.0001 \end{bmatrix}$$

$$f(x(t), x) = [0.3x_1 \quad 0.5x_2],\ \bar{d} = 1,$$

The matrix $A + BK$ needs to be Hurwitz stable, so we choose K as

$$K = \begin{bmatrix} -1.5 & 5 & -4 \\ -3 & -3.5 & 3 \end{bmatrix}.$$

Matrices $J_A, J_{Ad}, J_C, J_{C_d}$, U_A, U_{A_d}, U_C and U_{C_d} are calculated by the SVD method (106)-(109).

In Example 1.A, we apply Theorem 8 to a case with a time-varying delay $\tau(t) = 0.5\sin(t) + 0.5$, i.e., $\dot{\tau}(t) \le h = 0.5 < 1$.

4.5.1 Example 1.A

Consider the above described uncertain time-varying delay stochastic system and check the stability by Theorem 8. To solve controller in (116), we convert the linear equation $B^T XD = 0$ in (119) into an LMI form. It is equivalent to $tr[(B^T XD)^T B^T XD] = 0$. Then, it is to find the minimum β of the matrix inequality $(B^T XD)^T B^T XD \leq \beta I$, which can be solved by "mincx" solver in the Matlab LMI toolbox (Gahinet et al., 2011).

In order to compare it with the previous methods, we select the matrices according to (149)-(150) in Section D. The "mincx" solver in Matlab cannot find X and Q for a solution by the previous methods (Niu et al., 2005, de Souza and Li, 1999). However, the new method in Theorem 8 is able to find the result as:

$$X = \begin{bmatrix} 1.70 & -2.34 & 0.260 \\ -2.34 & 4.19 & -0.476 \\ 0.260 & -0.476 & 0.283 \end{bmatrix} \times 10^{-4}, Q = \begin{bmatrix} 0.200 & -0.139 & 0.181 \\ -0.139 & 5.92 & -0.210 \\ 0.181 & -0.210 & 0.484 \end{bmatrix} \times 10^{-3},$$

and all adjustable scalars ε_{1i}, ε_{2j}, ε_3, ε_{4i}, ε_{5j} ($i = 1, \dots, l_A$ and $j = 1, \dots, l_{A_d}$) are positive.

Also, $\beta = 2.92 \cdot 10^{-10}$, so the constraint $B^T XD = 0$approximately holds.

Take the control of (116)-(118) and (143) with the calculated X and $G = B^T X$ from Theorems 8 and 12, and $\gamma = 0.6$.

In simulation, we take the initial value $x(t) = [0.6 \quad 0.2 \quad -1]^T$, $t \in [-1, 0]$, and the simulation time as $[0, 3]$ sec. The simulation results are shown in the following figures. Three different $w(t)$ are randomly generated by random seeds 1, 2, and 3. Two kinds of λ are chosen as above described in section C. Firstly, let $\lambda = 0.5$. The trajectories of the state vector $x(t)$ are shown in FIGURE 20.

Figures 21 and 22 show $s(t)$and $u(t)$, respectively. The high frequency chatter of $u(t)$with amplitude 0.5 can be observed after $t = 3$ in FIGURE 20. To reduce the chatter, we choose $\lambda = 3\|s(t)\|^{1/4}$, i.e., $c_1 = 3, c_2 = 4$ in (145). Its simulation results are shown in Figures 23-25. They show that the time-varying delay stochastic system with structural uncertainties is robustly stable. The chatter of $u(t)$ is significantly reduced, however, it takes a little bit more time to reach to the switching surface $s(t) = 0$ than a constant $\lambda = 0.5$.

Further simulation in the above example without $dw(t)$ and $f(x(t), t)$ has been done for the methods in (Yu and Chu, 1999, de Souza and Li, 1999) without ΔB such that the simplified example fits all these methods in view of differences. The simulation results show that (i) No LMI solution in Theorem 3.2 of (de Souza and Li, 1999); and (ii) Our method has some improvement in the performance compared with the result from (Yu and Chu, 1999) (e.g., the convergence speed is increased, the settling time is reduced by 4%, and the overshot is reduced by 11%), that is from our flexible solution space and different control rule (Yu and Chu, 1999, de Souza and Li, 1999).

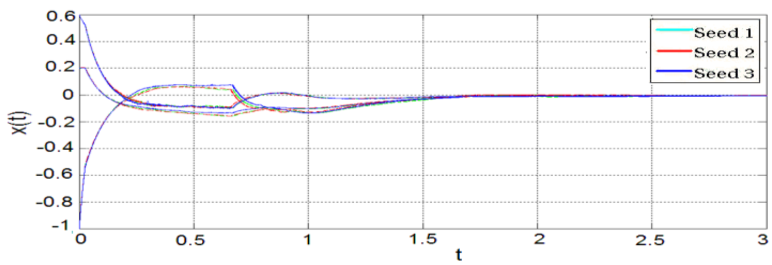

FIGURE 20: $x(t)$ with $\lambda = 0.5$ and $w(t)$in random seeds 1, 2, 3 and without chattering reduction

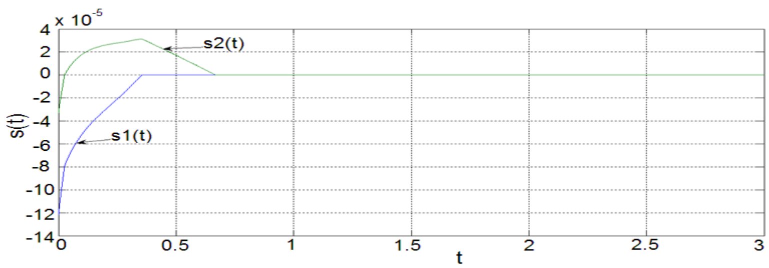

FIGURE 21: $s(t)$ with $\lambda = 0.5$ and $w(t)$ in random seed 3 and without chattering reduction

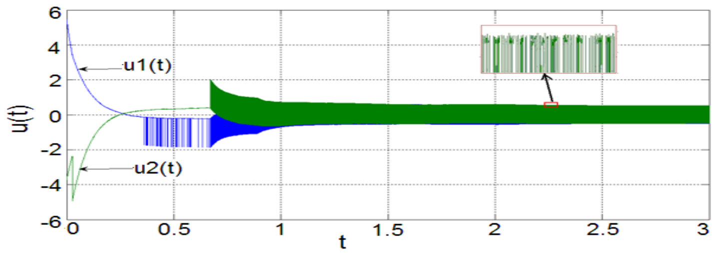

FIGURE 22: $u(t)$ with $\lambda = 0.5$ and $w(t)$ in random seed 3 and without chattering reduction

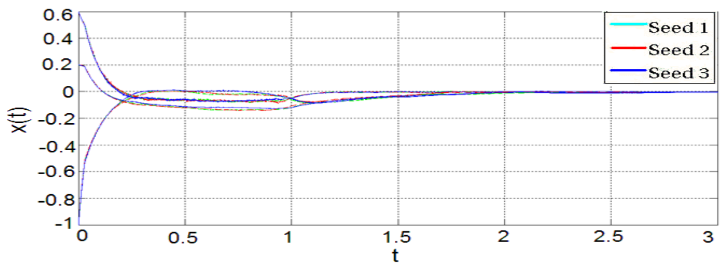

FIGURE 23: Trajectory $x(t)$ with various $w(t)$ in random seeds 1, 2, 3 and chattering reduction

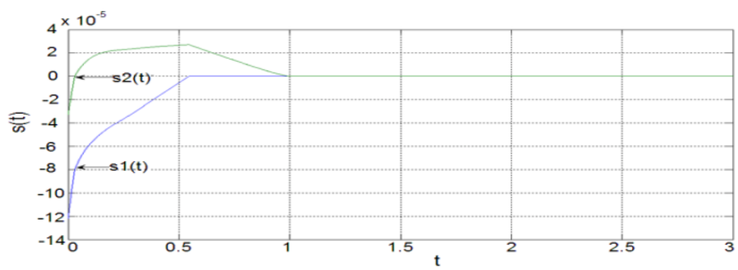

FIGURE 24: $s(t)$with a flexible λ and $w(t)$in random seed 3 and chattering reduction

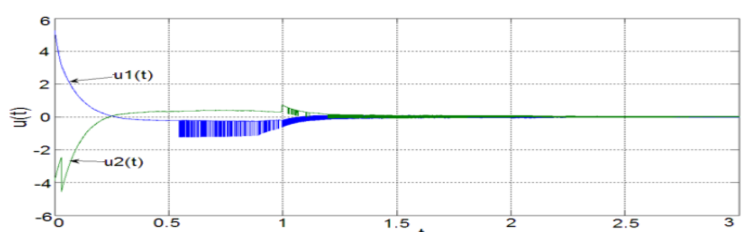

FIGURE 25: $u(t)$ with a flexible λ and $w(t)$ in random seed 3 and chattering reduction

4.5.2 Example 1.B

Consider the same system in Example 1.A, and apply Theorem 10 to the problem. We can get $(B^T XD)^T B^T XD \leq \beta I$ with $\beta = 2.6575 \cdot 10^{-8}$, so the constraint $B^T XD = 0$

approximately holds.

The solutions of X, Q, Z, and W are:

$$X = \begin{bmatrix} 2.5 & -3.7 & 0.39 \\ -3.7 & 6.9 & -0.71 \\ 0.39 & -0.71 & 0.283 \end{bmatrix} \times 10^{-3}, \quad Q = \begin{bmatrix} 1.9 & -3.4 & 0.90 \\ -3.4 & 8.4 & -2.5 \\ 0.90 & -2.5 & 0.87 \end{bmatrix} \times 10^{-3}.$$

$$Z = \begin{bmatrix} 0.43 & 1.71 & -0.43 \\ 1.71 & 6.9 & -1.71 \\ -0.43 & -1.71 & 0.43 \end{bmatrix} \times 10^{8}, W_1 = \begin{bmatrix} -0.14 & -0.57 & 0.14 \\ -0.57 & -2.3 & 0.57 \\ 0.14 & 0.57 & -0.14 \end{bmatrix} \times 10^{8},$$

$$W_2 = \begin{bmatrix} 0.14 & 0.57 & -0.14 \\ 0.57 & 2.3 & -0.57 \\ -0.14 & -0.57 & 0.14 \end{bmatrix} \times 10^{8},$$

and all adjustable scalars ε_{1i}, ε_{2j}, ε_3, ε_{4i}, ε_{5j}, ε_6 $(i = 1, \dots, l_A$ and $j = 1, \dots, l_{A_d})$ are positive. The Trajectory $x(t)$ with $w(t)$ in random seed 3 is shown in FIGURE 26.

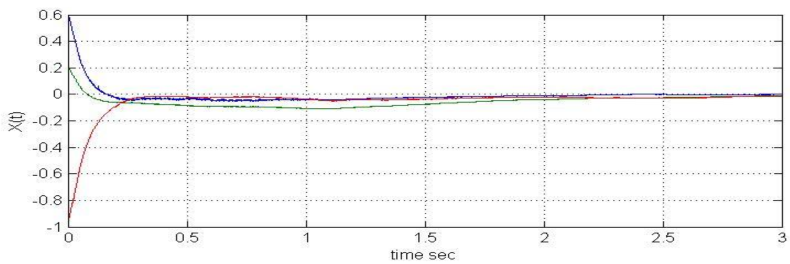

FIGURE 26: Trajectory $x(t)$ with $w(t)$in random seed 3 and chattering reduction

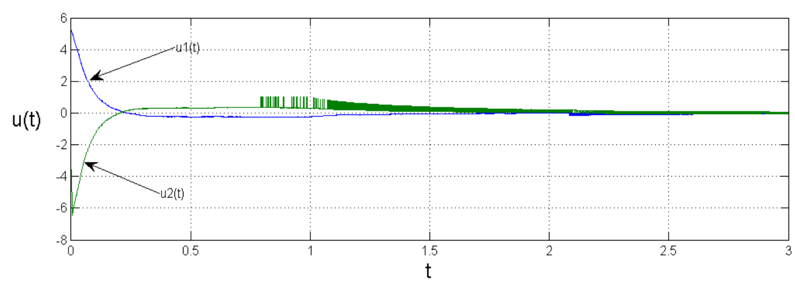

FIGURE 27: $u(t)$ with a flexible λ and $w(t)$ in random seed 3 and chattering reduction

Remark 17. It is noticed that the overshoot of trajectory $x(t)$ by Theorem 10 in this case is reduced by comparing with Theorem 8.

4.5.3 Example 2

Consider a system with the same parameters as in Example 1, except $\tau(t) =$

$0.5\sin(4t) + 0.5$, which leads $\dot{\tau}(t) \le h$ and $h = 2 > 1$.Now, we apply Theorem 10 to this example. In this case, we have $(B^T XD)^T B^T XD \le \beta I$ with $\beta = 1.1858 \cdot 10^{-8}$, so the constraint $B^T XD = 0$ approximately holds.

The solutions of X, Q, Z, and $W$are:

$$X = \begin{bmatrix} 1.7 & -2.4 & 0.26 \\ -2.4 & 4.6 & -0.46 \\ 0.26 & -0.47 & 0.18 \end{bmatrix} \times 10^{-3}, \quad Q = \begin{bmatrix} 0.31 & -0.51 & 0.14 \\ -0.51 & 1.2 & -0.38 \\ 0.14 & -0.38 & 0.13 \end{bmatrix} \times 10^{-3}.$$

$$Z = \begin{bmatrix} 0.43 & 1.71 & -0.43 \\ 1.71 & 6.9 & -1.71 \\ -0.43 & -1.71 & 0.43 \end{bmatrix} \times 10^{8}, W_1 = \begin{bmatrix} -0.14 & -0.57 & 0.14 \\ -0.57 & -2.3 & 0.57 \\ 0.14 & 0.57 & -0.14 \end{bmatrix} \times 10^{8},$$

$$W_2 = \begin{bmatrix} 0.14 & 0.57 & -0.14 \\ 0.57 & 2.3 & -0.57 \\ -0.14 & -0.57 & 0.14 \end{bmatrix} \times 10^{8},$$

and all adjustable scalars ε_{1i}, ε_{2j}, ε_3, ε_{4i}, ε_{5j}, ε_6 $(i = 1, \dots, l_A$ and $j = 1, \dots, l_{A_d})$ are positive.

The simulation parameters, including random seed 3 and chatter reduction treatment, are chosen as same as in Example 1, except $\tau(t)$ as mentioned above. Figures 28 and 29 show the simulation results by Theorem 10.

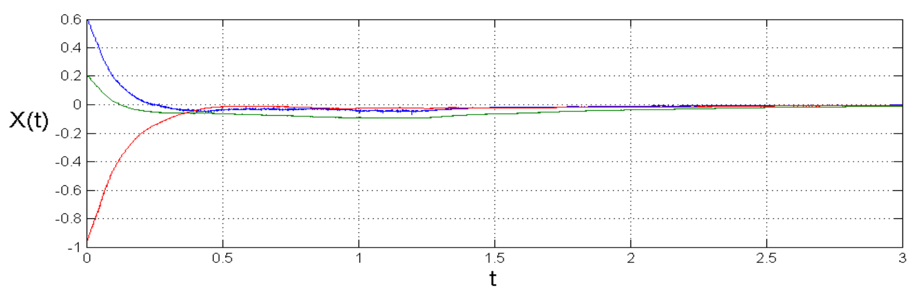

FIGURE 28: Trajectory $x(t)$ with $w(t)$ in random seed 3 and chattering reduction

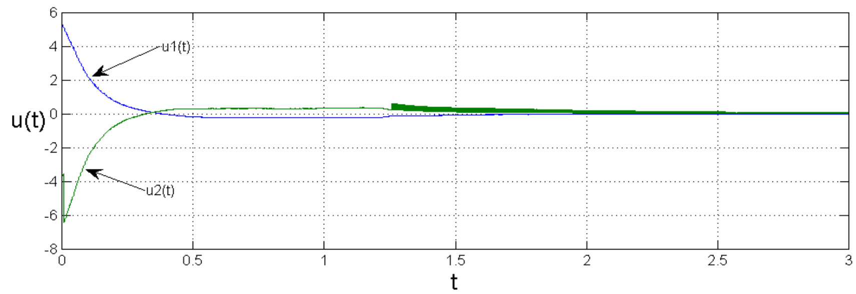

FIGURE 29: $u(t)$ with a flexible λ and $w(t)$ in random seed 3 and chattering reduction

The above simulation results in Figures 28 and 29 demonstrate that the proposed theoretical results of Theorems 9 and 10 work very well on stability of the uncertain stochastic systems with general time varying delay.

4.5.4 Example 3 (Chen et al., 2008)

Consider a system from (99)-(100) without control $u(t)$ and nonlinear uncertain item $f(x(t),t)$, i.e., as same as in example 1 of (Chen et al., 2008):

$$dx(t) = \left[\left(A + \Delta A(t)\right)x(t) + \left(A_d + \Delta A_d(t)\right)x\left(t - \tau(t)\right)\right]$$

$$+\left[\left(C + \Delta C(t)\right)x(t) + \left(C_d + \Delta C_d(t)\right)x\left(t - \tau(t)\right)\right]dw(t), \quad x(t) = \varphi(t), \quad t \in \left[-\bar{d}, 0\right]$$

$$\mathrm{A} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ -0.2 & -1.2 \end{bmatrix},$$

$$[\Delta A \quad \Delta A_1 \quad \Delta C \quad \Delta C_d] = LF(t)[0.2I \quad 0.2I \quad 0.2I \quad 0.2I], L = \mathrm{I}, \mathrm{F}^{\mathrm{T}}(\mathrm{t})\mathrm{F}(\mathrm{t}) \leq \mathrm{I}.$$

In this example, the goal is to find the admissible upper bound $\bar{d}$ of time-varying delay for two cases:

Case I: time-varying delay $\tau(t)$ is differentiable with an upper bound h of the delay derivative, i.e., $\dot{\tau}(t) \leq h < \infty$, and

Case II: time-varying delay $\tau(t)$ is continuous and uniformly bounded by $\bar{d}$ as in (100).

We apply our Theorem 9 to the above problem, i.e., to use our $V_3(x(t),t)$ in (152) instead of their $V_{3\varphi}(x(t),t)$ in (153) as stated in Remark 16. It may also be seen as applying a simplified Theorem 10 to the system (99)-(100) with $u(t) = 0$, $f(x(t),t) = 0$, $B = 0$, and simple uncertainties for this example (no SVD is needed). The comparison of the maximal admissible bound $\bar{d}$ of time-varying delay is listed in TABLE 2.

TABLE 2**:** The maximal admissible bound $\bar{d}$ of time-varying delay

$\bar{d}$	h =0	h =0.5	h =0.9	$h \geq 1$	Any h
$\bar{\text{d}}$ via Theorem 9, Case I	9.7249	6.9185	4.9631	4.6548	—
$\bar{\text{d}}$ via Theorem 9, Case II	—	—	—	—	4.6548
$\bar{d}$, Case I (Chen et al., 2008)	1.7075	1.1398	0.7678	0.6769	—
$\bar{d}$,Case II (Chen et al., 2008)	—	—	—	—	0.6769
$\bar{d}$,Case I (Gao et al., 2003, Yue and Han, 2005)	No conclusion	No conclusion	No conclusion		

Remark 18. It is noticed that our new results from Theorem 9 (and Theorem 10) are better than those in (Chen et al., 2008), and those only to treat the case with $\dot{\tau}(t) \leq h \leq 1$ in (Gao et al., 2003, Yue and Han, 2005) where no conclusion can be made as cited in (Chen et al., 2008). Our new method also leads to better and larger admissible bounds $\bar{d}$ for other examples in (Chen et al., 2008) than the existing methods in the literature.

CHAPTER 5: CONCLUSION

In this dissertation, the brief survey of NCSs is presented in chapter 2.

In chapter 3, a BTM based data compressor is proposed. The sufficient condition of the exponential stability has been derived for both the switched NCSs and non-switched NCS with the *MRDC* in fixed controller and/or flexible controller cases. Furthermore, we show that the introduction of the *MRDC* with the reduced order search is no worse than the original NCSs. Furthermore, an optimization of exponential decay α is discussed. The example shows that the introduction of *MRDC* can not only save the bandwidth, but also boost the performance of NCS.

The chapter 4 discusses a flexible robust SMC method for general uncertain stochastic systems with time-varying delay and structural uncertainties is developed to reduce conservatism. By the SVD decomposition on the structural uncertainties, a less-conservative condition in LMI for the robust control design has been derived. The robust control in the proposed method can globally drive the state trajectories toward the preselected switching surface. Furthermore, a new Lyapunov-type functional for general stochastic systems with general time-varying delay is proposed, and based on that new robust stability results are derived via Itô's integral rule. It is proved that the new methods have more flexibility and less conservatism.

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