INTRODUCING NON-LINEARITIES AND INTERACTION TERMS IN A CONDITIONAL ASSET PRICING MODEL

by

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ABSTRACT

KRISTOFFER RASK. Introducing non-linearities and interaction terms in a conditional asset pricing model. (Under the direction of DR. CRAIG A. DEPKEN II)

Throughout history of the conditional asset pricing literature the goal has been to find the best possible model to explain what determines a firm's expected stock return. In Dickson (2015) the variables that prove to be best at explaining a firm's stock return is book-to-market, market capitalization, gross profitability, investment, short-term reversal, and momentum. The aim of this study is to further examine improvements in Dickson (2015) by changing the functional form and adding interaction terms between the variables. The chosen methodology is a version of the popular Fama-Macbeth regressions which are well documented in the literature to determine the added risk premium associated with firm characteristics. By allowing for the possibility of non-linear characteristics and interaction terms, this study shows that market capitalization follows a significant non-linear relationship with the average stock return and by adding the squared regressor to the model, the explanatory power and risk premium for market capitalization improves. The study further shows that including the interaction between investment and market capitalization improves the explanatory power of the investment variable and the market capitalization variable.

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CHAPTER 1: INTRODUCTION

This study is an extension of the first chapter in Dickson (2015) with the sole purpose of exploring new insights into how the risk premiums in a conditional asset-pricing model may be better explained. This will be done by either including non-linear forms of the firm characteristic regressors or by including interaction terms with the other variables in the model. This study examines the additional explanatory power these variables contribute with by looking at the regression statistics. The choice of variables is based on those included in Dickson (2015) who, after careful consideration, determined those variables that best explain the excess return of a firm's stock. The included variables are book-to-market, market capitalization, investment, and gross profitability between the years 1963 and 2013. The historical stock price data comes from all firms in the exchanges: NYSE, NASDAQ and AMEX and the accounting data used for calculating the firm characteristic regressors is gathered from Compustat. Based on the findings in Dickson (2015) this study uses the same variables, but changes the functional form and includes interaction terms to determine if it is possible to improve the explanatory power of Dickson's (2015) model. If successful, by changing the functional form or including interaction terms of the regressors, future trading strategies can use these findings and better explain the predicted return. The majority of the new regressors does not improve the explanatory power, but when including the squared market capitalization and the interaction term between market capitalization and investment, the model is better explained.

It is important to understand that the results generated from this study can not be used as an implementable trading strategy. Fama-Macbeth regressions are not set up in a way that work as a trading strategy. The benefit with the regressions is that it is possible to determine the individual firm characteristic's risk premium which findings can be added to the selection process when constructing the trading portfolio.

The comparison will be possible because the same methodological procedure will be used, specifically, Fama-Macbeth regressions are performed following Dickson (2015). The results from this study will be possible to include in both current conditional asset pricing models and add new insights to the asset pricing literature.

In the current literature there is only a limited number of studies in this area. The discussion exists but there is no paper that explicitly takes a model and includes new variables that are either new functional forms or interaction terms. The aim of this study is to use the well documented Fama-Macbeth regression approach and include different functional forms and determine if non-linear marginal effects exist in the chosen firm characteristic variable. The results show how the slopes and t-statistics for the regressors change when adding squared terms and interaction terms to the regression. This study will conclude if this is possible by answering the following question: Is it possible to improve the explanatory power of the risk premium in a conditional asset pricing model by allowing for interaction terms between the regressors and non-linear functional forms?

CHAPTER 2: BACKGROUND

Dickson (2015) investigated issues from previous asset pricing literature and focused on generating a systematic portfolio choice solution with better return predictability. In his first chapter, Dickson (2015) investigates in a multivariate cross-sectional regression which firm characteristics variables that are best at predicting a firm's stock return. With the generated model he builds tradable portfolios to compare with benchmarks to see by how much the portfolios outperform the market. In his second chapter Dickson (2015) continues the portfolio analysis using individual stock data. Results show that his approach of using naive diversification outperforms the use of traditional active diversification. The third chapter finalizes the analysis by investigating how well conditional asset pricing models can construct tradable stock portfolios. To improve the performance of tradable stock portfolios Dickson (2015) discusses the importance of looking at improvements in return predictability by finding a functional form of firm characteristics that is better at explaining the predicted return. By starting off with the popular Fama-Macbeth regression, Dickson (2015) uses predictive regressions that are linear models, with no squared terms or interaction terms.

The Fama-Macbeth regression is a cross-sectional regression estimating risk premiums for asset pricing models. The method was introduced by Fama and MacBeth (1973) using the Capital Asset Pricing Model (CAPM) with a panel data of multiple firm characteristics for a number of firms over a specific time period. The Fama-Macbeth regression combines one regression estimating the betas (risk premiums) for every firm characteristic over time. The second step is to regress stock returns on these estimated betas for a fixed period of time and then iterate this procedure for every time period in the sample. The betas from this regression are the estimated risk premiums for each firm characteristic.

Slope coefficients in the regressions can be interpreted as returns when used in characteristic-based portfolios and hence, by generating a higher slope coefficient for a regressor, the regressor's marginal risk premium awarded to the firm's stock price increaes. In other words, if it is possible to interpret the slope coefficients in the characteristic-based portfolios as returns, the t-statistic, which is a statistical measure testing the significance of an included regressor coefficient, can be used as proxy for the Sharpe ratio, a commonly used measure of return considering the risk of an investment. The higher the t-statistic, the higher the Sharpe ratio. As a portfolio manager you want to achieve as high return as possible with the lowest possible risk, therefore the Sharpe ratio is a good measure when comparing different investments with different risk profiles.

2.1 Literature Review

In their pioneering asset pricing paper Fama and MacBeth (1973) laid the foundation for the standard model for asset pricing. In their initial stochastic model for stock return Fama and MacBeth (1973) tested for non-linearities in all firm characteristic variables by including squared regressors of the variables in the model. The included squared regressors worked as a test for linearity between expected return on a firm's stock return and the firm characteristic. Fama and MacBeth (1973) argue if the price formation on tradable firms is shifting based on investors attempts to hold efficient portfolios, the linearity condition must hold. Fama and MacBeth (1973) found that in the two-parameter model for their dataset (1935-1968), they were not able to reject the hypothesis that the relationship between the expected return and the risk factor is linear even though they find evidence that there existed stochastic nonlinearities from period to period.

Fama and French (1992) saw some connection between the size variable and the book-to-market variable, i.e. testing for a statistically significant interaction term between these two regressors. They found that small firms tend to have lower stock price (higher book-to-market ratio) due to higher uncertainty about their future profits. Large firms on the other hand tend to have higher stock prices, based on the opposite argument - more positive future outlook and thus a lower book-to-market ratio. In their data they find a negative correlation between the logarithmic value of market capitalization and the logarithmic value of book-to-market for individual stocks of -0.26, hence, individual slope coefficients will differ depending on whether one includes the interaction variable in the regression. Fama and French (1992) argue that the interaction term between size and book-to-market seems to absorb the roles of leverage. In their preliminary tests, they capture a pattern in the data where the natural logarithm of the leverage variable is a good functional form which captures the relationship between leverage and average returns. Using the logarithmic transformation of the regressor instead of using levels smooth's the observation distribution, which reduces the impact of extreme values.

In a more recent paper, Fama and French (2008) argue that the standard Fama-Macbeth regression faces potential problems when the sample data are sorted into portfolios. They argue that the assigned risk premium in the regression for each firm characteristic will not represent the true marginal effect they have on average stock return. The second issue when using sorted portfolios is that the estimated functional form of the firm characteristics will not be very precisely estimated compared with an estimation using a large data sample. Fama and French (2008) analyzed how accurate this estimation is in the full data sample by looking at simple diagnostics of the regression residuals. They find that by using a large data sample the functional form can be measured more precisely.

Fama and French (2008) separate stocks in three groups based on firm size before running the Fama-Macbeth regressions (microcap stocks, small stocks, and large stocks). They do this separation to more precisely examine how firm characteristics behave in different sized firms. The smallest stocks are most likely to have extreme firm characteristics and extreme stock returns. When running the regression using the full data sample, the smallest stocks contribute more to a firm characteristic's risk premium than the average firm in the full data sample, so the slope coefficients (risk premiums) are not as accurately estimated. Fama and French (2008) compare the size-sorted output with the output using the full data sample. Minor differences exist, but they are still able to draw the same conclusions from each regression about which firm characteristics are good at explaining a firm's stock return. Fama and French (2008) finish by saying, using an alternative functional form of the firm characteristics in the full data sample regression can potentially capture the minor difference in the output.

The Fama and French (2008) results indicate that the average residuals from the above-mentioned regressions do identify a minor functional form problem for the following variables: net stock issues, momentum, accruals, profitability, and asset growth. The problem lies in the extreme values. In other words by using the linear functional form, the extreme values are not fully captured in the model. Fama and French (2008) finish their paper by saying that there is no point in trying to find a better explanation of average returns by changing the functional form. This is because many of the anomalous returns are caused by the extremes, and changing the functional form will not produce a better explanation of the excess return.

Fama and French (2015) discuss the potential relation between size and investment. When sorting portfolios based on size and investment there is a size effect present when looking at how much investment affects the average return. Small stocks have higher average return and small firms with low investment have much higher average return than small firms with high investment. But looking at the portfolio with the highest amount of investment (top qunitle) the size effect is not present. This relationship can be tested by including an interaction term between the market cap and investment.

Cordis and Kirby (2015) use an alternative approach of capturing the relationship between a firm characteristic and a firm's average stock return. Their approach is entirely built on a one step cross-sectional regression, compared to the more commonly used two-step approach found in this study, where the object is to use cross-sectional estimates in a time-series specification.

Cordis and Kirby (2015) conclude that an advantage of using their alternative method is that the method is valid independent of the number of firm characteristics under consideration. This means that the model can easily be extended and include any number of firm characteristics, because the method isolates the relationship between each firm characteristic and the firm's average stock return by controlling for the cross-sectional correlation between the regressors. They also introduce a brief discussion about how including nonlinear functional forms of the firm characteristics in the model can generate new insights in the relationship between average returns and the firm characteristic variables. Cordis and Kirby (2015) mention that including the squared functional form of the firm characteristic regressor is a simple test for a quadratic relationship between a firm characteristic and the firm's average stock return.

CHAPTER 3: DATA AND FIRM CHARACTERISTIC VARIABLES

The regressions use a stock price data from the years 1963 and 2013. The historical stock price data have been collected from the Center for Research in Security Prices (CRSP) and the accounting data have been collected from Compustat. The data have previously been used in Dickson (2015) and the same data are used in this study. The historical stock price data comes from the exchanges: NYSE, NASDAQ, and AMEX (share codes 10 and 11). The paper will use all available data and include all firms in the regressions. The microcap stock definition will be the same as in Fama and French (2008), i.e., stocks with a market equity below the 20th percentile of the NASDAQ market capitalization distribution. Microcap stocks are roughly 50% of all stocks traded at NYSE, NASDAQ, and AMEX and represent 3% of total market capitalization.

The accounting data from Compustat will be used to calculate the firm characteristic regressors. In order to make sure the model is only trading on public data, the tests will only use data that are available at time t. In addition to this, the data will be lagged a minimum 6-month, max 18-month to make sure all the information used is publicly available. The firm characteristic betas are a weighted average across the time periods and the firm characteristics are the actual observation at a specific time:

The book-to-market regressor is calculated as the book value of the firm's equity over the market capitalization of the firm. The book value of equity is calculated as the shareholder's equity, plus deferred taxes, and less preferred stock. The market capitalization of the firm, price per share times the number of shares outstanding, is taken from the month of June each year.

The market capitalization regressor is calculated by the same method as the denominator in the book-to-market regressor, specifically it is the share price times the number of shares outstanding for the firm in June each year.

The investment regressor is the growth in investments from one year to the next. The regressor is calculated as the change in total assets by dividing the change in total assets in year I_{t-1} by the total assets in year I_{t-2}.

The Gross Profitability regressor is the ratio of the firm's gross profits to its total assets. The gross profits are calculated as total revenue less cost of goods sold (COGS).

The Short-term reversal regressor is the 1 month lagged return for time period t.

The Momentum regressor is the previous year's 11 month return. The 12th month return is excluded in order to not capture any Short-term reversal effect in the regressor.

3.1 Spurious Regression and Data Mining

In the asset pricing litterature it is important to understand how the explanatory variables covary with the expected return of the model. Situations when a variable has high autocorrelation the resulting statistics will overestimate the quality of the variable in a time series regression. This characteristic called spurious regression will yield higher, more significant test statistics as pointed out by Yule (1926) and Granger (1974). The regression model will show more significant relations than what actually exists. Ferson, Sarkissian, and Simin (2006) restate the previous research that stock return data has strong persistence and that, for instance, the dividend yield spread of the SandP 500 index has an autocorrelation of 0.97 which may cause the time series regression to be spurious. They also argue that even though stock returns are not considered as highly autocorrelated, the expected return consists of unobserved expected return and unobserved noise, so the expected returns could be persistent time series and the risk of spurious regression exists. Looking at this the other way around, i.e., if the expected returns are not persistent, the estimated results will not be biased and spurious regression does not exist even though the regressors could be highly autocorrelated (because the autocorrelation of the errors are the same as the autocorrelation of the expected return). They continue to restate that this phenomenon causes the t-test to generate statistically significant values in some cases when the regressors actually are independent. In their tests they conclude that the problem arises from the biased estimate of the standard error.

In this study the explanatory variables are all fairly stationary and due to the use of Fama-Macbeth regression methodology (sequential cross-sectional regression, not time series regression) the assumption will be made that the regressions are not spurious.

A second issue when working with conditional asset pricing models is data mining, the process of finding variables that have a systematic and consistent relationship with the dependent variable. The variables that in history have shown to be significant may not be of value in the future and this is a crucial problem to solve when trying to find a model that tries to predict excess stock returns in future periods. In this study, the included variables have been closely examined so that they are chosen based on economic theory and not solely based on the fact that the variables in historic periods seem to explain the excess return.

CHAPTER 4: METHODOLOGY

4.1 Fama-Macbeth Replication

In the asset pricing literature the most common method for testing firm characteristic variables, that could potentially explain anomalous returns, is by modeling a regression based on Fama and MacBeth (1973). Fama and MacBeth (1973) tested their theory on the Capital Asset Pricing Model (CAPM) and by combining two regressions were able to explain the premium rewarded to one of the included risk factor's exposure to the market. The breakthrough combined two regressions, one cross sectional regression capturing the firm characteristics effects on an individual stock return and a time-series regression that makes predictions about the risk premium for each factor.

Step 1. For each period of time estimate a cross sectional regression relating the stock return of a company to the chosen firm characteristics for the same time period. By using a standard OLS regression the slope coefficient, beta, for the individual risk factor, $x_{i,t}$ can be captured.

$$r_{i,t} = \alpha + \beta x_{i,t} + \epsilon_{i,t}; \qquad i = 1, 2, ..., N \quad ; \ t = 1, 2, ..., T. \tag{1}$$

This procedure will generate a vector for each intercept and slope coefficient that allow one to compute the average of the slope coefficients over time. The elements that are statistically significant, i.e., have predictive power in explaining stock returns, are kept as explanatory variables for the second step of the model.

Further discussion from Cordis and Kirby (2015) confirms that the firm characteristics are usually lagged in order to make sure that the changes in the variables reflect public information at the beginning of the holding period. Cordis and Kirby (2015) also add that even though the linearity assumption of the firm characteristics is violated, the relation between the firm characteristic and the firm's stock return only needs to be monotonic (strictly increasing or strictly decreasing) to prove that the firm characteristic's contribution to the model is statistically significant when using a large cross-section of firms.

The errors in a standard OLS regression are correlated across the individual firms. This creates a problem because the t-statistics assume that the error terms are uncorrelated. There exist two options to tackle this issue, either use Fama-Macbeth standard errors or use a model that uses the cross sectional correlation. Petersen (2005) discusses the properties of the Fama-Macbeth standard errors and adds that the errors are biased downward and increase along with the magnitude of the firm effect. Because the Fama-Macbeth regression tries to explain the time effect, not the firm effect, the Fama-Macbeth standard errors are unbiased and more efficient than the OLS standard errors.

Step 2. The following time series regression uses the time estimated intercepts and the time estimated betas generated from the OLS regressions, by using the previously discussed Fama-Macbeth standard errors. Equation (2) captures the time series average:

$$E(r_i) = E(\alpha) + \sum_{j=1}^{K} E(\beta_j)E(x_j). \tag{2}$$

Some firms in the cross-sectional average $\hat{\beta}$ vector have a higher explanatory variable than the average and some firms have a lower explanatory variable than the average. Taking the estimated value of this vector, $E(\hat{\beta})$, the sum is zero and can be thought of as a self-financing portfolio, ”zero-cost hedge portfolio”, because you will buy the firms with the higher expected return and finance the purchases by shorting the firms with lower expected return.

By using the testable form of the CAPM model, in a multi-factor equilibrium setting,

$\tilde{R}_i$ is defined as:

$$\tilde{R}_{i,t} = R_f + \sum_{j=1}^{K} \beta_{i,j}[\tilde{R}_{j,t} - R_f] + \tilde{\epsilon}_{i,t}. \tag{3}$$

For each firm i and average j, we can continue discussing the relationship between the average firm characteristic β_j in time-series regressions and the average firm characteristic β_j in cross-sectional regressions. But before I discuss this it is crucial to state several assumptions when using time-series regressions in an asset-pricing model. Dickson (2015) lists the assumptions in the following order: 1) The multi-factor versions of the market model must hold every period; 2) the equilibrium relationship must hold every period; and 3) the $\hat{\beta}'s$ are stable over time. In equilibrium we have $\hat{\beta}'s$ in both the cross-sectional setting and in the time-series setting that are stable over time. In the multi-factor asset pricing model, equation (3), $\hat{\beta}_j$ is defined as a zero-cost hedge portfolio. In equilibrium, taking the average firm characteristic beta, $\hat{\beta}_j$, relationship between the models into account with the assumption mentioned above that $\hat{\beta}_j's$ are stable over time, it is possible to assume that the firm characteristic beta, $\hat{\beta}_j$ in the time-series regression is equivalent to the firm characteristics in the cross sectional regressions. Because of the earlier explained characteristics of $E(\hat{\beta})$ in equation (2), $\hat{\beta}_j$, is defined as the zero-cost hedge portfolio and with $\tilde{R}_{j,t}$ in equation (3) that in asset pricing litterature is defined as the zero-cost hedge portfolio it is possible to construct portfolios on the basis of factor loadings, when the factors are constructed as zero-cost hedge portfolios of the same firm characteristics which is the case in these regressions.

4.2 Model Specification

The aproach I will use when trying to better explain the risk premium generated in Dickson (2015) is to first use his existing regression model and experiment with the functional form of the regressors and, second, measure how including interaction terms changes the explanatory power.

The method to generate the output consists of a cross-sectional regression and a time-series regression described in section 4.1 above.

I will explain the cross sectional regression models that are used in Dickson (2015) and introduce the changes made in order to determine if it is possible to increase the explanatory power.

The cross sectional regression model is specified as:

$$\begin{aligned} r_{i,t+1} = \alpha + \beta_1 ln(BE/ME)_{i,t} + \beta_2 ln(ME)_{i,t} + \beta_3 GPdat_{i,t} \\ +\beta_4 INV_{i,t} + \beta_5 R1to0_{i,t} + \beta_6 R12to2_{i,t} + \epsilon_{i,t}. \end{aligned} \tag{4}$$

Where the α and β's are parameters to be estimated and $\epsilon_{i,t}$ is a zero mean error term. The dependent variable is the value of stock i in period t and the included explanatory variables are book-to-market (ln(BE/ME)), market capitalization (ln(ME)), gross profitability (GPdat), investment (INV), short-term reversal (R1to0) and, momentum (R12to2).

I will modify the model by including the new interaction terms and changing the functional form of the regressors in the cross sectional model. A rolling average of the monthly data sample will be used:

$$\begin{aligned} \hat{r}_{i,t+1} = \hat{\alpha}_s + \hat{\beta}_{1,s} ln(BE/ME)_{i,t} + \hat{\beta}_{2,s} ln(ME)_{i,t} + \hat{\beta}_{3,s} GPdat_{i,t} + \\ \hat{\beta}_{4,s} INV_{i,t} + \hat{\beta}_{5,s} R1to0_{i,t} + \hat{\beta}_{6,s} R12to2_{i,t} + \epsilon_{i,t}. \end{aligned} \tag{5}$$

In equation (5), all the variables are included. i.e., all the variables Dickson (2015) finds have the best explanatory power when looking at individual excess stock return. The included regressors can briefly be divided between the slow moving level variables (ME, BE/ME, GPdat, and INV) and the short-lived regressors (short-term reversal and momentum) whose persistence is more short-lived and their values are based on historic stock returns causing higher variation and higher turnover in a portfolio.

The slow moving variables will be squared and interaction terms created between every variable, except between log(BE/ME) and log(ME) because market equity is simply the denominator of the log(BE/ME) variable. The output will generate a total of nine new regressions, one regression for each new variable, summarized in the Appendix.

4.3 Functional Form

When trying to find a functional form of an independent variable that better fits the data, previous theory is a good starting point. When no previous theory exists, looking at the data and experiment is the best way to determine if a functional form, other than the linear functions, is better at explaining the data. I will experiment with the included regressors and by looking at the generated statistics determine if there could exist an alternative functional form that increases the variable's explanatory power.

Previously mentioned is that Fama and MacBeth (1973) found nonlinearity characteristics in the data from period to period, but not throughout the whole data set. I am working with a different data set, which means that these nonlinearities could be present. By including squared terms the nonlinearity condition can be tested. This approach allows me to find new insights into how excess returns vary with firm characteristics.

4.4 Interaction Terms

When arguing for interaction terms that have not yet been discussed in the literature, it is harder to argue for each interaction term (opposed to the functional form). It is not possible to plot the data to determine if there exist another interaction term that may improve the model. The approach I will use is to experiment by including interaction terms for all the regressors and then subsequently drop one after one and look at the resulting statistics and by that approach determine which interaction terms increase the explanatory power and those variables that do not.

4.5 Incremental Contribution

Once the regressions have been set up with the new variables, the results will be specified in a table where I will be able to determine the effect each new regressor has on the excess return and its explanatory power. In order to determine each individual variable's incremental contribution to the overall result I will include several statistical variables for each specific model. For instance, the R^2 provides information how the explanatory variables explain the overall variance of the model. A higher R^2 is indicated by a closer estimation of the independent values to the actual data and indicates a larger fraction of the model's overall return volatility.

I will be using the same R^2 decomposition as in Greene (2002):

$$
\begin{aligned}
R^2 &= (\rho(y, \hat{y}))^2 \\
R^2 &= (\rho(y, \hat{y}))^2 = \hat{\beta}
\end{aligned} \tag{6}
$$

$$
R^2 = (\rho(y, \hat{y}))^2 = \frac{cov(\hat{y}, y)^2}{var(\hat{y})var(y)} \tag{7}
$$

$$
\hat{\beta} = \frac{cov(\hat{y}, y)}{var(y)} \tag{8}
$$

The panel data set contains a different number of observations for each month of data and in order to adjust R^2 for this, I have to multiply the covariance for each month with that month's degrees of freedom, equation (9). The same procedure is done for the variance of y, equation (10). This procedure will generate the appropriate sum of squares.

$$
SS_m = (N - 1) \times cov(\hat{y}, y) \tag{9}
$$

$$
SS_t = (N - 1) \times var(y) \tag{10}
$$

The sum of squares is added up and divided by the total number of sum of squares. This will give me the approprate R^2.

Even though the R^2 is an important variable to look at the more interesting thing to look at is how much each variable contributes to the excess return. All the variables have already been proven in Dickson (2015) to have predictive power, so that part of the analysis will be ignored. The focus will instead be on whether the new interaction variables and the functional forms improve the existing variables explanatory power and risk premium.

Because I will use various functional forms, I will conduct several tests similar to Clarke (2014a), i.e., I will start with the full model, including all variables and the interaction terms and then drop each interaction term. The output from each test will show how much incremental contribution these variables have.

I define an increase in a firm characteristic's incremental contribution to the model by a better estimated risk premium for the firm characteristic, which is caused by a lower estimated standard error and a higher t-statistic, see equation (11). As an example, if the true relationship between a firm's market capitalization and a firm's average stock return is non-linear, including the squared regressor of market capitalization in the regression will improve the explanatory power in how much of a firm's risk premium is caused by the market capitalization of the firm. With a better estimated regressor, the standard error will be lower, hence, the t-statistic for the market capitalization regressor will be higher. The opposite, or no effect, will occur if the included regressor does not improve the explanatory power. In order to easily compare the added explanatory power in each new regression model, the t-statistic is presented in parenthesis under each intercept and regressor, see output tables in the Appendix. With the previously mentioned possibility to interpret the slope coefficient as the marginal return in a characteristic-based portfolio, the corresponding t-statistic can be used as proxy for the Sharpe ratio for that slope coefficient. The reason to look at returns when considering risk is because a higher return is not always good. If the higher return is achieved by undertaking much higher risk, the higher return may no longer look very attractive. Therefore the Sharpe ratio, i.e., the risk-adjusted return, is a good measure to use when comparing risk premiums, see equation (12).

$$
\begin{aligned}
t &= \frac{\hat{\beta}}{\sigma(\hat{\beta})/\sqrt{N}} \\
\sigma &= \frac{s}{\sqrt{N}}.
\end{aligned} \tag{11}
$$

From the t-statistic it is possible to calculate the risk adjusted return, that is, the Sharpe ratio:

$$
\begin{aligned}
SR &= \frac{\hat{\beta}}{\sigma(\hat{\beta})} \\
SR &= \frac{t}{\sqrt{N}}.
\end{aligned} \tag{12}
$$

CHAPTER 5: RESULTS

The first results reported replicate Dickson (2015) to make sure I use the same procedure which will allow me to compare the original results with my new regressions. It is important to restate that the results generated from this study, a version of the Fama-Macbeth procedure, can not be used as an implementable trading strategy. For instance, the generated expected return, the dependent variable, is calculated at $t + 1$, hence, can not be used in an actual trading strategy. Discussed by Fama (1976) the slopes of Fama-Macbeth regressions can be proxies for the returns on characteristic-based stock portfolios. Using these proxies, the following conclusion can be made as well; the t-statistic for the slopes (proxy for return) can be used as a proxy for the Sharpe ratio (risk adjusted return) of characteristic based stock portfolios. Higher t-statistics means higher Sharpe ratio for the portfolio.

Once again, the aim of this study was not to generate a trading strategy, but to explore the possibility that the chosen stock characteristic variables have non-linear relationship with stock returns. Furthermore, these characteristics have properties that better explain each variable's added risk premium. By comparing my results, coefficients and t-statistics when including the new variables to Dickson's original model, I am able to draw conclusions about additional properties in these variables that previously have not been documented. Specifically, if the coefficient of the original regressor increases when including the quadratic regressor, the regressor's marginal effect on the expected return is non-linear. If also the t-statistic increases in absolute values, the explanatory power of the risk premium becomes more significant. The results will be used as a proxy for deciding if the variables add significant value to an implementable trading strategy. For example, lets say one of the new variables is statistically significant and improves the t-statistics of the original model, this will indicate that trading strategies including the linear variable should consider also including the new non-linear version of the variable. The second benefit with the chosen methodology is to allow for comparison with other studies that also use Fama-Macbeth regressions.

The following results refer to the tables in the Appendix, where I have located all the regression outputs with corresponding t-statistics and R^2. Highlighting a few things in the data, in Panel A of Table (2), the mean return over the full sample is 1.273% per month, where the highest mean regressor value stems from market capitalization of 4.750 and the lowest mean value from book-to-market, -0.523. It is only in the 75th percentile and lower that the mean value for book-to-market is positive. The remaining variables range between 0.124-0.332 as their mean values.

In Panel B of Table (2), the highest standard deviation among the variables is for investment, in the 99th percentile of 2.924, much higher than the other variables in the 99th percentile, ranging from 0.058-0.961. In the full sample, market capitalization has the highest standard deviation of 2.051 compared to the other variables ranging from 0.262-0.854 with a full sample return standard deviation of 16.110.

Looking at the correlation between the variables, Panel C of Table (2), the lowest correlation is between book-to-market and market capitalization of -0.321, not surprisingly because market capitalization is the denominator in the book-to-market ratio. There is no strong positive correlation between the variables; the highest positive correlation of 0.057 is between momentum and market capitalization. The positive correlation is not surprising because momentum is calculated as the firm's previous year's 11 month stock return and market capitalization is the stock price times the number of shares outstanding, hence, if the stock price appreciates over a long period of time, the value of both regressors increase.

The initial model, Table (3), replicates Dickson (2015) and generates the same slope coefficients and t-statistics. The interpretation of statistical significance in this study is at

5% with a two-sided critical t-value of 1.96. All the variables with an absolute t-value over 1.96 are considered statistical significant. This study comes to the same conclusion as in Dickson (2015) that the market capitalization variable is not statistically significant, but will still be kept in the model due to it is fundamental importance in previous asset pricing literature. The model generates positive and significant risk premium for book-to-market, profitability, and momentum. The t-statistic for book-to-market and profitability is roughly the same (6.52 and 6.05, respectively) which means that their Sharpe ratios are roughly the same and have the same power in the model. The risk premiums for investment and short-term reversal are negative and significant.

5.1 Functional Form

The first new regressor added to the model is squared market capitalization, see Table (4). The risk premium for the market capitalization variable decreases from -0.066 to -0.244 and the t-statistic increases from the previous -1.723 to -2.175. The market capitalization risk premium is no longer insignificant, also the squared market capitalization risk premium has a significant t-statistic of 2.2. These results says that the market capitalization variable has non-linear marginal effects on the expected return and is better at explaining the variability in the return related to the size of the firm. Once the linear market capitalization regressor is removed from the regression, the t-statistic for the squared term decreases to (-1.753) with a coefficient close to zero (-0.006). This reinforces the non-linear marginal effect that market capitalization has on expected return. A potential reason for this could be that small cap stocks have a higher variation in returns than large cap stocks. The non-linear movement with a negative linear coefficient indicate that to a certain breakpoint market capitalization, the return decreases with the size of the firm, but the non-linear effect shows that the negative effect of the size of the firm changes and at a breakpoint, larger firms yields higher return. Another significant effect is the change in the intercept; from 1.387 to 1.726, both significant t-statistics (4.649 and 4.462, respectively). The intercept, the average return not explained by the regressors, increases. If the aim of the model is to increase the average return (intercept), also called the abnormal return, including the squared market capitalization regressor clearly improves the model.

Removing the squared market capitalization variable and instead including the squared book-to-market variable, Table (5), the outcome is just a slight effect in the book-to-market regressor. The risk premium decreases from 0.360 to 0.349 with a change in t-statistics from 6.52 to 5.71. The squared book-to-market variable is only significant when excluding the linear book-to-market variable and with no change in the intercept. The added explanatory power of having the squared term is zero. This shows that there is no non-linear marginal effect in the book-to-market variable.

When including the squared gross profitability regressor, Table (6), the gross profitability regressor becomes less significant, but still significant (t-statistics go from 6.047 to 2.121) and with a slightly lower risk premium (0.608). Also, the squared term is not significant, showing that adding the squared term does not add any value to the regression, i.e. the marginal effect in gross profitability is not non-linear. The result does not show any major effect in the intercept (the significant intercept goes from 1.387 to 1.360). The same kind of result is generated when including the squared investment regressor, Table (7); no t-statistic equal to significant effect in the risk premium or in the intercept. The squared regressor is not significant (0.947) which indicates investment does not have a non-linear marginal effect on excess return.

5.2 Interaction Terms

Looking at how interaction terms affect the explanatory power with the interaction between market capitalization and gross profitability, Table (8). The market capitalization regressor becomes significant (t = -2.159) with a risk premium of (-0.091), but the gross profitability regressor becomes insignificant (t = 1.817), when it was previously significant. The interaction term itself is not significant and does not add any explanatory value to the regression. The only situation where the interaction term is significant is when the linear gross profitability regressor is excluded (t = 5.905).

The second interaction term is between market capitalization and investment, Table (9). The previous insignificant market capitalization regressor becomes significant (t = -2.118) with a slightly lower risk premium (from -0.066 to -0.083). The risk premium for investment decreases as well (from -0.559 to -1.269), the interaction term itself has a positive and significant risk premium (t = 4.383) and when included, the significant intercept increases (from 1.387 to 1.453). The interaction term has explanatory power and a positive risk premium. This relationship was discussed by Fama and French (2015) and explains that the risk premium of investment decreases with the size of the firm. In other words, the marginal effect of increased investment decreases with the size of the firm. The reason why the market capitalization regressor becomes signifiant could be that more investment increases the potential for future growth and the discounted value of this growth increases the value of the firm, hence the risk premium for the market capitalization regressor is better represented and the regressor becomes significant.

If looking at the interaction term between book-to-market and gross profitability, Table (10), the new variable does not add any significant value itself (t = -1.694). The only significant effect is in the intercept, which increases to 1.414. In order to get the interaction term significant, either book-to-market or gross profitability needs to be excluded from the regression. Similar results are found when including the interaction between gross profitability and investment, Table (11), a slightly higher intercept, but the interaction term itself is not significant (1.292).

The final interaction term, book-to-market and investment, Table (12), has the same intercept as without the interaction term. Looking at the effect in investment, the t-statistic decreases in absolute values (from -7.597 to -6.690) and for book-to-market, there is no substantial effect in the coefficient or the t-statistic. The interaction term itself is not statistically significant (t = -1.509).

The final output, Table (13) includes all variables, i.e., all the original variables and all of the new variables. The only significant new variables are the squared market capitalization and the interaction between market capitalization and investment (t = 1.971 and 3.649, respectively). These results are in line with the previous results when the variables were included separately, see Table (3) and Table (9).

CHAPTER 6: CONCLUSIONS

The purpose of this study was to take the existing conditional asset pricing model in Dickson (2015) and explore if it was possible to better explain the added risk premium for each firm characteristic variable. This was possible because of allowing the model to include non-linear functional forms of the variables and/or by including interaction terms between them. Previous literature discusses this potential relation between average return and a firm characteristic's added risk premium, but the idea has not been exclusively tested on an actual model. Previous studies are using linear variables and no interaction terms. This is why this study adds new insights to the asset pricing literature. Using Fama-Macbeth regressions, the same methodology as in Dickson (2015), including the new variables, and looking at the regression statistics it has been possible to evaluate and conclude that risk premiums can be better explained when allowing for interaction terms and non-linear functional forms.

The importance of this study is also in the easiness of replication. With the commonly used Fama-Macbeth regressions as methodology, this study allow others using Fama-Macbeth regressions to replicate the test into their conditional asset pricing model, even if the included firm characteristics is not the same as in this study.

The only two new variables that improved the explanatory power of Dickson's model are the squared market capitalization regressor and the interaction between market capitalization and investment. The squared market capitalization regressor adds a non-linear marginal effect to the risk premium in the linear market capitalization regressor and makes the regressor significant (previously insignificant). These findings are important because if only including the linear market capitalization regressor the interpretation is that the size of the firm does not add a significant risk premium to the average return of the firm. The interaction between market capitalization and investment makes the risk premium for market capitalization significant and the interaction term itself adds a significant risk premium.

Most of previous literature includes market capitalization in the model even if their results show an insignificant risk premium with the argument that market capitalization is an intuitively important variable and should be kept in the model. By including the interaction term or the squared variable, the argumentation for including market capitalization can be made based on intuition and because the variable is statistically significant. Apart from the above two regressors, no other regressor improved the original model in any way (risk premium or t-statistic). The findings in this study can be directly implemented in future conditional asset pricing studies and be a good reference in the discussion on how firm characteristic risk premiums vary with stock returns. A future avenue in this field is to use the findings from this study in an actual simulated trading strategy. This is crucial to try because it is important to see how the regressors behave in a realistic setting, considering for instance transaction costs and portfolio turnover.

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APPENDIX A: TABLES

Table 1: Data description

All available firms between 1963 to 2013 for the exchanges; NYSE, NASDAQ, and AMEX have been used. Microcap stocks are roughly 50% of all stocks traded at NYSE, NASDAQ, and AMEX and represent 3% of total market capitalization. The historical stock price data have been collected from the Center for Research in Security Prices (CRSP) and the accounting data have been collected from Compustat. Annual accounting data for time period t is matched with the stock price data for June t+1 to July t+2.

Log(BE/ME)	The book-to-market regressor is calculated as the book value of the firm's equity over the market capitalization of the firm. The book value of equity is calculated as the shareholder's equity, plus deferred taxes, and less preferred stock. The market capitalization of the firm, price per share times the number of shares outstanding, is taken from the month of June each year.
Log(Me)	The market capitalization regressor is calculated by the same method as the denominator in the book-to-market regressor, specifically it is the share price times the number of shares outstanding for the firm in June each year.
GPdat	The gross profitability regressor is the ratio of the firm's gross profits to its total assets. The gross profits are calculated as total revenue less cost of goods sold (COGS).
Inv	The investment regressor is the growth in investments from one year to the next. The regressor is calculated as the change in total assets by dividing the change in total assets in year I_{t-1} by the total assets in year I_{t-2}.
R1to0	The short-term reversal regressor is the 1 month lagged return for time period t.
R12to2	The momentum regressor is the previous year's 11 month return. The 12th month return is excluded in order to not capture any Short-term reversal effect in the regressor.

Table 2: Summary statistics

Panel A: Regressor values sorted in percentiles

	Mean	1st	10th	25th	50th	75th	90th	99th
Log(BE/ME)	-0.523	-2.927	-1.649	-1.023	-0.442	0.065	0.483	1.193
Log(Me)	4.750	0.665	2.133	3.227	4.630	6.194	7.509	9.673
Gpdat	0.332	-0.204	0.040	0.130	0.303	0.485	0.685	1.076
Inv	0.223	-0.359	-0.092	0.004	0.091	0.227	0.537	2.883
R12to2	0.124	-0.749	-0.421	-0.186	0.060	0.330	0.690	1.927
Return	1.273	-35.840	-14.174	-6.186	0.000	7.206	16.667	51.613

Panel B: Standard deviations for full sample and percentiles

	Full sample	1st	10th	25th	50th	75th	90th	99th
Log(BE/ME)	0.854	0.811	0.657	0.539	0.422	0.334	0.274	0.187
Log(Me)	2.051	2.013	1.816	1.598	1.302	1.008	0.786	0.349
Gpdat	0.262	0.254	0.241	0.222	0.196	0.173	0.142	0.058
Inv	0.678	0.678	0.698	0.749	0.884	1.161	1.603	2.924
R12to2	0.523	0.517	0.491	0.480	0.497	0.559	0.659	0.961
Return	16.110	15.453	14.253	14.092	15.231	18.146	23.637	47.117

Panel C: Correlation matrix

	Log(BE/ME)	Log(Me)	Gpdat	Inv	R12to2	Return
Log(BE/ME)	1.000	-0.321	-0.163	-0.181	-0.133	0.034
Log(Me)	-0.321	1.000	-0.106	0.032	0.057	-0.018
Gpdat	-0.163	-0.106	1.000	-0.056	0.042	0.012
Inv	-0.181	0.032	-0.056	1.000	-0.068	-0.025
R12to2	-0.133	0.057	0.042	-0.068	1.000	0.006
Return	0.034	-0.018	0.012	-0.025	0.006	1.000

Table 3: Statistical significance of the average estimated marginal effects

All firms Dickson (2015)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Intercept	1.387	1.513	1.096	1.634	1.302	1.440	1.508
	(4.649)	(5.12)	(5.76)	(5.496)	(4.334)	(4.734)	(4.739)
Log(BE/ME)	0.360	.	0.446	0.302	0.426	0.351	0.280
	(6.52)	(.)	(8.06)	(5.49)	(7.254)	(6.106)	(4.602)
Log(Me)	-0.067	-0.106	.	-0.077	-0.066	-0.083	-0.055
	(-1.723)	(-2.848)	(.)	(-2.007)	(-1.698)	(-2.068)	(-1.359)
Gpdat	0.676	0.414	0.788	.	0.748	0.655	0.677
	(6.047)	(3.62)	(6.808)	(.)	(6.777)	(5.847)	(5.879)
Inv	-0.559	-0.746	-0.517	-0.580	.	-0.519	-0.622
	(-7.597)	(-8.276)	(-6.772)	(-7.946)	(.)	(-6.946)	(-7.857)
R1to0	-5.720	-5.523	-5.293	-5.616	-5.633	.	-5.541
	(-14.181)	(-13.356)	(-12.452)	(-13.883)	(-13.855)	(.)	(-12.809)
R12to2	0.750	0.676	0.841	0.761	0.807	0.757	.
	(3.985)	(3.557)	(4.279)	(4.037)	(4.238)	(3.925)	(.)
R2	0.046	0.041	0.034	0.043	0.044	0.039	0.037

Table 4: Statistical significance when introducing squared market capitalization regressor

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	1.726	1.943	1.243	1.999	1.723	1.851	1.780	1.387
	(4.462)	(5.11)	(5.2)	(5.276)	(4.456)	(4.671)	(4.48)	(4.649)
Log(BE/ME)	0.351	.	0.374	0.293	0.413	0.341	0.271	0.360
	(6.358)	(.)	(6.853)	(5.357)	(7.052)	(5.937)	(4.469)	(6.52)
Log(Me)	-0.244	-0.326	.	-0.268	-0.280	-0.296	-0.193	-0.066
	(-2.175)	(-2.95)	(.)	(-2.413)	(-2.496)	(-2.574)	(-1.726)	(-1.723)
Gpdat	0.665	0.408	0.698	.	0.734	0.643	0.668	0.675
	(5.891)	(3.558)	(6.312)	(.)	(6.563)	(5.681)	(5.756)	(6.047)
Inv	-0.541	-0.720	-0.563	-0.560	.	-0.498	-0.608	-0.559
	(-7.488)	(-8.122)	(-7.637)	(-7.811)	(.)	(-6.819)	(-7.794)	(-7.597)
R1to0	-5.689	-5.489	-5.690	-5.585	-5.602	.	-5.512	-5.719
	(-14.207)	(-13.371)	(-14.024)	(-13.903)	(-13.887)	(.)	(-12.821)	(-14.181)
R12to2	0.753	0.684	0.755	0.765	0.812	0.762	.	0.749
	(3.987)	(3.582)	(4.012)	(4.042)	(4.25)	(3.939)	(.)	(3.985)
Sqrd(Me)	0.019	0.023	-0.006	0.020	0.022	0.022	0.014	.
	(2.2)	(2.707)	(-1.753)	(2.371)	(2.619)	(2.593)	(1.686)	(.)
R2	0.049	0.044	0.044	0.046	0.047	0.042	0.040	0.045

Table 5: Statistical significance when introducing squared book-to-market regressor

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	1.389	1.533	1.078	1.631	1.303	1.438	1.511	1.387
	(4.715)	(5.175)	(5.716)	(5.561)	(4.39)	(4.796)	(4.819)	(4.649)
Log(BE/ME)	0.349	.	0.454	0.274	0.414	0.344	0.273	0.360
	(5.711)	(.)	(6.532)	(4.441)	(6.44)	(5.53)	(4.016)	(6.52)
Log(Me)	-0.068	-0.098	.	-0.079	-0.067	-0.084	-0.056	-0.066
	(-1.785)	(-2.569)	(.)	(-2.086)	(-1.754)	(-2.123)	(-1.427)	(-1.723)
Gpdat	0.653	0.513	0.761	.	0.727	0.633	0.653	0.675
	(5.877)	(4.576)	(6.621)	(.)	(6.619)	(5.679)	(5.703)	(6.047)
Inv	-0.561	-0.661	-0.519	-0.582	.	-0.520	-0.624	-0.559
	(-7.622)	(-7.927)	(-6.819)	(-7.976)	(.)	(-6.962)	(-7.887)	(-7.597)
R1to0	-5.744	-5.654	-5.331	-5.644	-5.658	.	-5.574	-5.719
	(-14.297)	(-13.987)	(-12.627)	(-14.007)	(-13.97)	(.)	(-12.956)	(-14.181)
R12to2	0.742	0.721	0.830	0.753	0.800	0.751	.	0.749
	(3.975)	(3.816)	(4.259)	(4.026)	(4.235)	(3.93)	(.)	(3.985)
Sqrd(BE/ME)	0.010	-0.105	0.043	0.000	0.010	0.016	0.013	.
	(0.464)	(-4.488)	(1.603)	(0.008)	(0.445)	(0.712)	(0.566)	(.)
R2	0.047	0.044	0.035	0.044	0.045	0.039	0.038	0.045

Table 6: Statistical significance when introducing squared gross profitability regressor

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	1.360	1.495	1.067	1.470	1.274	1.411	1.476	1.387
	(4.66)	(5.157)	(5.764)	(4.947)	(4.337)	(4.736)	(4.781)	(4.649)
Log(BE/ME)	0.367	.	0.448	0.362	0.433	0.358	0.284	0.360
	(6.733)	(.)	(8.114)	(6.575)	(7.472)	(6.328)	(4.725)	(6.52)
Log(Me)	-0.062	-0.103	.	-0.064	-0.060	-0.078	-0.050	-0.066
	(-1.628)	(-2.825)	(.)	(-1.669)	(-1.594)	(-1.978)	(-1.281)	(-1.723)
Gpdat	0.608	0.437	0.835	.	0.675	0.577	0.677	0.675
	(2.121)	(1.481)	(2.595)	(.)	(2.358)	(2.006)	(2.296)	(6.047)
Inv	-0.565	-0.752	-0.527	-0.552	.	-0.524	-0.630	-0.559
	(-7.735)	(-8.426)	(-7.045)	(-7.566)	(.)	(-7.079)	(-7.998)	(-7.597)
R1to0	-5.777	-5.589	-5.366	-5.697	-5.692	.	-5.602	-5.719
	(-14.382)	(-13.591)	(-12.716)	(-14.108)	(-14.06)	(.)	(-13.017)	(-14.181)
R12to2	0.745	0.666	0.835	0.753	0.803	0.753	.	0.749
	(3.978)	(3.518)	(4.278)	(4.005)	(4.234)	(3.922)	(.)	(3.985)
Sqrd(Gpdat)	0.137	-0.004	0.018	0.728	0.146	0.150	0.045	.
	(0.543)	(-0.017)	(0.065)	(7.114)	(0.575)	(0.593)	(0.172)	(.)
R2	0.047	0.042	0.036	0.045	0.045	0.040	0.038	0.045

Table 7: Statistical significance when introducing squared investment regressor

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	1.388	1.514	1.122	1.635	1.353	1.440	1.508	1.387
	(4.683)	(5.155)	(5.877)	(5.523)	(4.55)	(4.767)	(4.776)	(4.649)
Log(BE/ME)	0.350	.	0.431	0.292	0.393	0.340	0.270	0.360
	(6.381)	(.)	(7.914)	(5.346)	(6.945)	(5.979)	(4.481)	(6.52)
Log(Me)	-0.062	-0.099	.	-0.073	-0.070	-0.079	-0.050	-0.066
	(-1.644)	(-2.722)	(.)	(-1.94)	(-1.827)	(-1.997)	(-1.274)	(-1.723)
Gpdat	0.679	0.426	0.791	.	0.694	0.658	0.680	0.675
	(6.076)	(3.729)	(6.791)	(.)	(6.224)	(5.872)	(5.885)	(6.047)
Inv	-0.845	-1.090	-0.941	-0.838	.	-0.793	-0.911	-0.559
	(-6.12)	(-7.251)	(-5.909)	(-6.068)	(.)	(-5.673)	(-6.352)	(-7.597)
R1to0	-5.748	-5.556	-5.338	-5.642	-5.687	.	-5.574	-5.719
	(-14.301)	(-13.507)	(-12.61)	(-13.997)	(-14.037)	(.)	(-12.964)	(-14.181)
R12to2	0.740	0.667	0.831	0.753	0.780	0.748	.	0.749
	(3.942)	(3.525)	(4.233)	(3.998)	(4.138)	(3.896)	(.)	(3.985)
Sqrd(Inv)	0.149	0.165	0.422	0.131	-0.338	0.138	0.113	.
	(0.947)	(1.04)	(1.966)	(0.85)	(-3.95)	(0.885)	(0.714)	(.)
R2	0.047	0.042	0.035	0.044	0.045	0.039	0.037	0.045

Table 8: Statistical significance when introducing interaction term between market capitalization and gross profitability

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	1.497	1.498	1.116	1.617	1.427	1.562	1.587	1.387
	(4.886)	(4.822)	(5.823)	(5.46)	(4.635)	(5.007)	(4.852)	(4.649)
Log(BE/ME)	0.364	.	0.388	0.368	0.430	0.356	0.282	0.360
	(6.576)	(.)	(7.097)	(6.66)	(7.297)	(6.17)	(4.613)	(6.52)
Log(Me)	-0.091	-0.103	.	-0.112	-0.093	-0.110	-0.072	-0.066
	(-2.159)	(-2.455)	(.)	(-2.927)	(-2.208)	(-2.526)	(-1.647)	(-1.723)
Gpdat	0.368	0.478	1.021	.	0.395	0.310	0.471	0.675
	(1.817)	(2.373)	(3.177)	(.)	(1.953)	(1.504)	(2.285)	(6.047)
Inv	-0.553	-0.742	-0.552	-0.559	.	-0.512	-0.616	-0.559
	(-7.539)	(-8.264)	(-7.427)	(-7.604)	(.)	(-6.88)	(-7.801)	(-7.597)
R1to0	-5.736	-5.543	-5.581	-5.720	-5.651	.	-5.557	-5.719
	(-14.248)	(-13.421)	(-13.51)	(-14.176)	(-13.928)	(.)	(-12.872)	(-14.181)
R12to2	0.748	0.674	0.783	0.756	0.804	0.756	.	0.749
	(3.977)	(3.548)	(4.096)	(4.02)	(4.228)	(3.925)	(.)	(3.985)
Log(Me)*Gpdat	0.071	-0.011	-0.087	0.138	0.081	0.080	0.049	.
	(1.762)	(-0.288)	(-1.244)	(5.905)	(1.983)	(1.97)	(1.203)	(.)
R2	0.047	0.042	0.041	0.045	0.045	0.039	0.037	0.045

Table 9: Statistical significance when introducing interaction term between market capitalization and investment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	1.453	1.562	1.115	1.700	1.314	1.502	1.583	1.387
	(4.82)	(5.219)	(5.805)	(5.657)	(4.382)	(4.889)	(4.918)	(4.649)
Log(BE/ME)	0.365	.	0.429	0.307	0.371	0.355	0.286	0.360
	(6.646)	(.)	(8.208)	(5.631)	(6.774)	(6.218)	(4.732)	(6.52)
Log(Me)	-0.083	-0.117	.	-0.094	-0.053	-0.098	-0.073	-0.066
	(-2.118)	(-3.124)	(.)	(-2.414)	(-1.369)	(-2.426)	(-1.792)	(-1.723)
Gpdat	0.669	0.405	0.763	.	0.687	0.648	0.669	0.675
	(5.986)	(3.551)	(6.738)	(.)	(6.154)	(5.788)	(5.816)	(6.047)
Inv	-1.269	-1.319	-0.498	-1.317	.	-1.207	-1.398	-0.559
	(-7.594)	(-7.855)	(-1.888)	(-7.881)	(.)	(-7.158)	(-8.193)	(-7.597)
R1to0	-5.729	-5.534	-5.388	-5.626	-5.715	.	-5.553	-5.719
	(-14.215)	(-13.393)	(-12.795)	(-13.918)	(-14.162)	(.)	(-12.85)	(-14.181)
R12to2	0.746	0.671	0.816	0.757	0.764	0.753	.	0.749
	(3.968)	(3.539)	(4.199)	(4.02)	(4.062)	(3.911)	(.)	(3.985)
Log(Me)*Inv	0.165	0.127	-0.054	0.172	-0.097	0.160	0.181	.
	(4.383)	(3.245)	(-0.803)	(4.543)	(-5.374)	(4.222)	(4.682)	(.)
R2	0.046	0.042	0.037	0.044	0.045	0.039	0.037	0.045

Table 10: Statistical significance when introducing interaction between book-to-market and gross profitability

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	1.414	1.317	1.111	1.633	1.331	1.460	1.550	1.387
	(4.802)	(4.333)	(5.964)	(5.496)	(4.495)	(4.88)	(4.948)	(4.649)
Log(BE/ME)	0.424	.	0.488	0.482	0.490	0.406	0.362	0.360
	(6.279)	(.)	(7.219)	(7.465)	(6.903)	(5.753)	(4.96)	(6.52)
Log(Me)	-0.068	-0.080	.	-0.078	-0.067	-0.084	-0.057	-0.066
	(-1.778)	(-2.137)	(.)	(-2.028)	(-1.756)	(-2.117)	(-1.435)	(-1.723)
Gpdat	0.625	0.960	0.745	.	0.690	0.620	0.586	0.675
	(4.581)	(7.288)	(5.069)	(.)	(5.11)	(4.516)	(4.174)	(6.047)
Inv	-0.560	-0.662	-0.516	-0.569	.	-0.519	-0.622	-0.559
	(-7.639)	(-8.379)	(-6.796)	(-7.81)	(.)	(-6.985)	(-7.912)	(-7.597)
R1to0	-5.735	-5.621	-5.324	-5.658	-5.650	.	-5.558	-5.719
	(-14.26)	(-13.7)	(-12.579)	(-14.017)	(-13.939)	(.)	(-12.889)	(-14.181)
R12to2	0.747	0.713	0.835	0.755	0.804	0.756	.	0.749
	(3.986)	(3.788)	(4.27)	(4.014)	(4.24)	(3.933)	(.)	(3.985)
Log(BE/ME)*Gpdat	-0.143	0.527	-0.092	-0.434	-0.145	-0.122	-0.192	.
	(-1.694)	(6.119)	(-0.981)	(-5.804)	(-1.708)	(-1.417)	(-2.202)	(.)
R2	0.046	0.043	0.035	0.044	0.044	0.039	0.037	0.045

Table 11: Statistical significance when introducing interaction term between gross profitability and investment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	1.402	1.520	1.113	1.623	1.291	1.453	1.520	1.387
	(4.691)	(5.125)	(5.825)	(5.474)	(4.292)	(4.772)	(4.772)	(4.649)
Log(BE/ME)	0.362	.	0.448	0.320	0.388	0.353	0.282	0.360
	(6.571)	(.)	(8.111)	(5.831)	(6.862)	(6.148)	(4.64)	(6.52)
Log(Me)	-0.066	-0.105	.	-0.074	-0.065	-0.083	-0.054	-0.066
	(-1.726)	(-2.852)	(.)	(-1.952)	(-1.694)	(-2.073)	(-1.361)	(-1.723)
Gpdat	0.630	0.391	0.732	.	0.864	0.612	0.640	0.675
	(5.66)	(3.463)	(6.367)	(.)	(8.025)	(5.507)	(5.624)	(6.047)
Inv	-0.642	-0.777	-0.635	-0.802	.	-0.593	-0.694	-0.559
	(-6.128)	(-6.979)	(-5.857)	(-7.545)	(.)	(-5.603)	(-6.455)	(-7.597)
R1to0	-5.725	-5.528	-5.298	-5.650	-5.671	.	-5.546	-5.719
	(-14.197)	(-13.373)	(-12.467)	(-13.985)	(-13.991)	(.)	(-12.825)	(-14.181)
R12to2	0.750	0.675	0.841	0.759	0.779	0.757	.	0.749
	(3.99)	(3.555)	(4.283)	(4.037)	(4.112)	(3.932)	(.)	(3.985)
Gpdat*Inv	0.271	0.092	0.363	0.745	-0.958	0.248	0.238	.
	(1.292)	(0.432)	(1.7)	(3.085)	(-5.936)	(1.174)	(1.12)	(.)
R2	0.046	0.041	0.034	0.044	0.045	0.039	0.037	0.045

Table 12: Statistical significance when introducing interaction term between book-to-market and investment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	1.387	1.490	1.098	1.631	1.318	1.438	1.513	1.387
	(4.666)	(5.023)	(5.785)	(5.511)	(4.396)	(4.749)	(4.773)	(4.649)
Log(BE/ME)	0.373	.	0.464	0.314	0.365	0.362	0.303	0.360
	(6.568)	(.)	(8.059)	(5.571)	(6.522)	(6.113)	(4.875)	(6.52)
Log(Me)	-0.066	-0.103	.	-0.076	-0.067	-0.082	-0.054	-0.066
	(-1.71)	(-2.793)	(.)	(-1.996)	(-1.747)	(-2.057)	(-1.345)	(-1.723)
Gpdat	0.672	0.431	0.782	.	0.707	0.651	0.674	0.675
	(6.022)	(3.83)	(6.778)	(.)	(6.363)	(5.822)	(5.859)	(6.047)
Inv	-0.603	-0.568	-0.574	-0.620	.	-0.555	-0.706	-0.559
	(-6.69)	(-6.365)	(-6.157)	(-6.893)	(.)	(-6.103)	(-7.364)	(-7.597)
R1to0	-5.725	-5.548	-5.298	-5.622	-5.659	.	-5.547	-5.719
	(-14.208)	(-13.434)	(-12.48)	(-13.911)	(-13.96)	(.)	(-12.84)	(-14.181)
R12to2	0.744	0.681	0.836	0.756	0.795	0.752	.	0.749
	(3.959)	(3.591)	(4.256)	(4.015)	(4.188)	(3.906)	(.)	(3.985)
Log(BE/ME)*Inv	-0.099	0.155	-0.142	-0.100	0.165	-0.087	-0.145	.
	(-1.509)	(1.9)	(-2.05)	(-1.508)	(2.911)	(-1.323)	(-2.202)	(.)
R2	0.046	0.042	0.034	0.044	0.045	0.039	0.037	0.045

Table 13: Summary of statistical significance including the total number of new regressors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Intercept	1.791	1.985	1.229	1.926	1.700	1.928	1.797	1.488	1.781	1.863	1.805	1.733	1.750	1.825	1.777	1.793
	(4.664)	(5.211)	(5.462)	(5.114)	(4.45)	(4.902)	(4.614)	(5.108)	(4.543)	(4.802)	(4.687)	(4.644)	(4.569)	(4.757)	(4.643)	(4.658)
Log(BE/ME)	0.356	.	0.413	0.328	0.407	0.333	0.309	0.381	0.416	0.334	0.376	0.359	0.363	0.310	0.360	0.355
	(5.057)	(.)	(5.915)	(4.776)	(5.734)	(4.619)	(4.078)	(5.383)	(5.874)	(4.763)	(5.32)	(5.162)	(5.173)	(5.098)	(5.196)	(5.137)
Log(Me)	-0.248	-0.303	.	-0.272	-0.252	-0.305	-0.185	-0.088	-0.246	-0.263	-0.262	-0.234	-0.240	-0.250	-0.245	-0.248
	(-2.175)	(-2.697)	(.)	(-2.458)	(-2.213)	(-2.597)	(-1.619)	(-2.146)	(-2.15)	(-2.306)	(-2.294)	(-2.118)	(-2.102)	(-2.203)	(-2.152)	(-2.169)
Gpdat	0.463	0.105	0.859	.	0.548	0.362	0.684	0.502	0.500	0.342	0.450	0.614	0.462	0.291	0.506	0.463
	(1.309)	(0.308)	(2.459)	(.)	(1.559)	(1.016)	(1.886)	(1.443)	(1.418)	(1.626)	(1.272)	(2.073)	(1.307)	(0.862)	(1.449)	(1.311)
Inv	-1.431	-1.722	-1.434	-1.463	.	-1.353	-1.527	-1.492	-1.422	-1.424	-1.114	-1.437	-0.845	-1.495	-1.406	-1.456
	(-5.92)	(-6.979)	(-5.965)	(-6.08)	(.)	(-5.55)	(-6.244)	(-6.148)	(-5.849)	(-5.929)	(-5.918)	(-5.914)	(-4.852)	(-6.246)	(-6.858)	(-6.414)
R1to0	-5.846	-5.774	-5.860	-5.801	-5.825	.	-5.691	-5.879	-5.819	-5.793	-5.821	-5.835	-5.834	-5.834	-5.841	-5.840
	(-14.848)	(-14.569)	(-14.672)	(-14.685)	(-14.767)	(.)	(-13.556)	(-14.804)	(-14.721)	(-14.648)	(-14.746)	(-14.81)	(-14.809)	(-14.782)	(-14.837)	(-14.817)
R12to2	0.726	0.719	0.728	0.734	0.750	0.741	.	0.721	0.730	0.732	0.735	0.729	0.728	0.727	0.726	0.727
	(3.921)	(3.859)	(3.941)	(3.954)	(4.042)	(3.92)	(.)	(3.907)	(3.902)	(3.935)	(3.964)	(3.937)	(3.928)	(3.91)	(3.914)	(3.922)
Sqrd(Me)	0.017	0.019	-0.005	0.017	0.019	0.020	0.012	.	0.017	0.017	0.018	0.017	0.017	0.016	0.016	0.017
	(1.971)	(2.252)	(-1.666)	(2.078)	(2.258)	(2.358)	(1.411)	(.)	(1.993)	(2.033)	(2.11)	(1.981)	(2.03)	(1.941)	(1.944)	(1.959)
Sqrd(BE/ME)	-0.018	0.055	0.053	0.056	0.056	0.050	0.049	0.054	0.056	0.056	0.056	0.056	0.056	0.056	0.057	0.056
	(-0.784)	(-2.718)	(0.075)	(-0.91)	(-0.218)	(-0.455)	(-0.781)	(-0.526)	(.)	(-0.789)	(-0.699)	(-0.776)	(-0.656)	(-0.322)	(-0.805)	(-0.879)
Sqrd(Gpdat)	-0.003	0.228	-0.053	0.209	0.016	0.031	-0.132	0.009	-0.026	.	0.027	0.017	-0.005	0.092	-0.03	-0.004
	(-0.012)	(0.89)	(-0.204)	(1.4)	(0.064)	(0.121)	(-0.495)	(0.035)	(-0.103)	(.)	(0.105)	(0.066)	(-0.019)	(0.368)	(-0.12)	(-0.016)
Sqrd(Inv)	0.142	0.212	0.204	0.117	-0.092	0.127	0.1	0.162	0.156	0.128	.	0.145	0.127	0.138	0.147	0.121
	(0.822)	(1.224)	(1.152)	(0.687)	(-0.69)	(0.741)	(0.579)	(0.938)	(0.904)	(0.744)	(.)	(0.836)	(0.76)	(0.798)	(0.869)	(0.77)
Log(Me)*GPdat	0.04	0.089	-0.043	0.092	0.038	0.057	0.003	0.03	0.041	0.06	0.043	.	0.045	0.055	0.038	0.041
	(0.977)	(2.154)	(-0.995)	(2.453)	(0.913)	(1.357)	(0.092)	(0.742)	(0.981)	(1.441)	(1.034)	(.)	(1.087)	(1.422)	(0.93)	(0.99)
Log(ME)*Inv	0.142	0.153	0.13	0.141	-0.043	0.137	0.15	0.151	0.143	0.144	0.134	0.139	.	0.139	0.146	0.143
	(3.649)	(3.906)	(3.29)	(3.627)	(-1.462)	(3.514)	(3.781)	(3.804)	(3.647)	(3.683)	(3.527)	(3.566)	(.)	(3.592)	(3.819)	(3.892)
Log(BE/ME)*Gpdat	-0.112	0.313	-0.157	-0.038	-0.144	-0.066	-0.201	-0.122	-0.134	-0.064	-0.112	-0.124	-0.105	.	-0.131	-0.11
	(-1.12)	(3.459)	(-1.58)	(-0.413)	(-1.447)	(-0.648)	(-1.929)	(-1.216)	(-1.389)	(-0.673)	(-1.12)	(-1.347)	(-1.056)	(.)	(-1.367)	(-1.109)
Gpdat*Inv	0.13	0.647	0.084	0.281	-0.516	0.151	0.003	0.116	0.106	0.105	0.043	0.169	0.075	0.352	.	0.13
	(0.534)	(2.711)	(0.35)	(1.175)	(-2.567)	(0.616)	(0.013)	(0.475)	(0.437)	(0.444)	(0.183)	(0.701)	(0.315)	(1.56)	(.)	(0.565)
Log(BE/ME)*Inv	0.012	0.107	0.03	0.009	-0.03	0.027	-0.037	0.019	0.015	0	-0.007	0.011	-0.054	0.018	0.023	.
	(0.156)	(1.36)	(0.381)	(0.126)	(-0.416)	(0.344)	(-0.468)	(0.24)	(0.205)	(-0.012)	(-0.105)	(0.15)	(-0.745)	(0.243)	(0.317)	(.)
R2	0.057	0.055	0.053	0.056	0.056	0.05	0.049	0.054	0.056	0.056	0.056	0.056	0.056	0.056	0.057	0.056