BULLWHIP EFFECT IN PRICING IN VARYING SUPPLY CHAIN STRUCTURES AND CONTRACTS USING A GAME THEORETICAL FRAMEWORK

by

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ABSTRACT

ZIAUL HAQ ADNAN. Bullwhip effect in pricing in varying supply chain structures and contracts using a game theoretical frameworks (Under the direction of Dr. E. C. OZELKAN)

Bullwhip effect in Pricing (BP) refers to the amplified variability of prices in a supply chain. When the amplification takes place from the upstream (i.e. supplier's side) towards the downstream (i.e. retail side) of a supply chain, this is referred as the Reverse Bullwhip effect in Pricing (RBP). On the other hand, if an absorption in price variability takes place from the upstream towards the downstream of a supply chain, we refer this phenomenon as the Forward Bullwhip effect in Pricing (FBP).

In this research, we analyze the occurrence of BP in the case of different game structures and supply chain contracts. We consider three game scenarios (e.g. simultaneous, wholesale-leading, and retail-leading) and two supply chain contracts (e.g. buyback and revenue-sharing). We analyze the occurrence of BP for some common demand functions (e.g. log-concave, linear, isoelastic, negative exponential, logarithmic, logit etc.). We consider some common pricing practices such as a fixed-dollar and fixed-percentage markup pricing and the optimal pricing game.

We discuss the conditions for the occurrence of BP based on the concavity coefficient and the cost-pass-through. We analyze the price variation analytically and then illustrate the results through numerical simulations. We extend the cost-pass-through analysis for a N-stage supply chain and conjecture the BP ratios for a N-stage supply chain. We compute cost-pass-through under both a buyback and a revenue-sharing contract. We compared the BP ratios between a revenue-sharing contract and a no-contract cases. We

include both the deterministic and stochastic demand functions with an additive and a multiplicative uncertainty.

The results indicate that the occurrence of BP depends on the concavity coefficient of the demand functions. For example: RBP occurs for an isoelastic demand, FBP occurs for a linear demand, No BP occurs for a negative exponential demand etc. This study also shows that, FBP and RBP occur in varying magnitude for different types of games and supply chain contracts. The comparison between the stochastic model and the risk-less model shows that the additive or multiplicative uncertainty changes the price fluctuation. The comparison between contract and no-contract cases shows that the contract minimizes FBP or RBP in some cases.

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CHAPTER 1: INTRODUCTION

1.1 Introduction

Pricing decision is critical as it is responsible for significant share (e.g. up to 90%) of the final product cost (Davenport & Kalagnanam, 2001). Pricing is directly related to sales, revenues, and profits. In order to improve the customer service or to attract more customers, many companies apply dynamic pricing strategy. Many a times, companies fluctuate price to adjust with the supply or to cope up with the competition in the market. Thus, it benefits both the seller and the buyer (Dugar, Jain, Rajawat, & Bhattacharya, 2015). However, fluctuation of prices can lead to market speculation and increased uncertainty. It creates information distortion in order quantity and inventory (also known as the 'Bullwhip Effect') which adversely affects the supply chain in terms of excess inventories, backorders, inefficient use of resources etc (Lee, Padmanabhan, and Whang, 2004). Therefore, it is necessary to study the fluctuation of price in the supply chain.

Price variation may occur due to internal or external factors such as managerial decisions, cost changes, scarcity of resources, supplier quantity discounts, promotional sales, or future market speculations. In this study, we consider external cost changes and then analyze the impact of the cost change on the supply chain optimal pricing.

Interestingly, price variation does not remain constant always across the various stages of supply chain. It may propagate in an increased or decreased fashion towards downstream (i.e. customer side) supply chain depending on the demand function, supply chain structure etc. We name the amplified or absorbed variability of prices as the 'Bullwhip effect in Pricing (BP)'. If variability of price is increased towards the downstream supply chain, then researchers named it as 'Reverse Bullwhip effect in Pricing (RBP)' (Özelkan & Çakanyıldırım, 2009). On the other hand, if variability of price is absorbed towards the downstream supply chain, we name it as 'Forward Bullwhip effect in Pricing (FBP)'. The 'reverse' and 'forward' directions refer to the direction of the classical Bullwhip effect in order quantity and inventory decision. In classical bullwhip effect, the variability of order information towards upstream is higher. Hence, if the variability of price towards downstream is higher, then the direction is referred as 'reverse'; on the other hand, if the variability of price towards downstream is less, then the direction is referred as 'forward'.

Using real market data, figure 1.1 and 1.2 shows the empirical evidences of an amplified and reduced variation in price respectively. Figure 1.1 shows amplified variability in the case of U.S. beef market and potato prices in Chicago, IL. This is an example of RBP. Figure 1.2 shows decreased variability in oil retail prices. This is an example of FBP. Empirical research in U.S. coffee market shows, a 10% increase in the cost resulting a 3% increase in the retail price (Leibtag, Nakamura, Nakamura, & Zerom, 2007). German coffee market also shows reduced variability in retail price (Bonnet, Dubois, Villas Boas, & Klapper, 2013). We can say, FBP occurs in the case of coffee market.

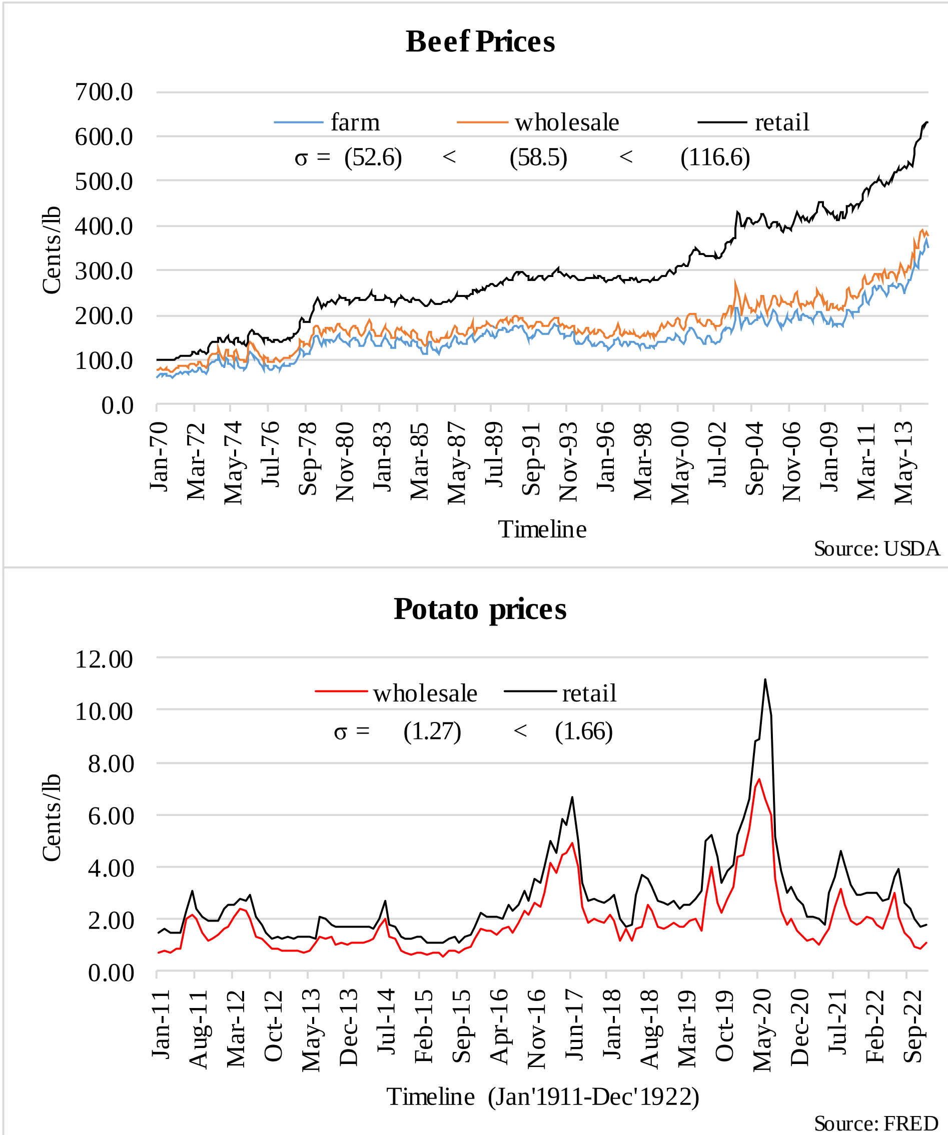

Figure 1.1: Amplified variability[1] in beef and potato prices towards downstream supply chain

[1] In this figure, we compare the standard deviations (σ) of the real beef price data from USDA and potato price data from FRED. Similar conclusion can be drawn by comparing the price index data (e.g. CPI, PPI etc.) from Bureau of Labor Statistic database (Ozelkan and Lim, 2008).

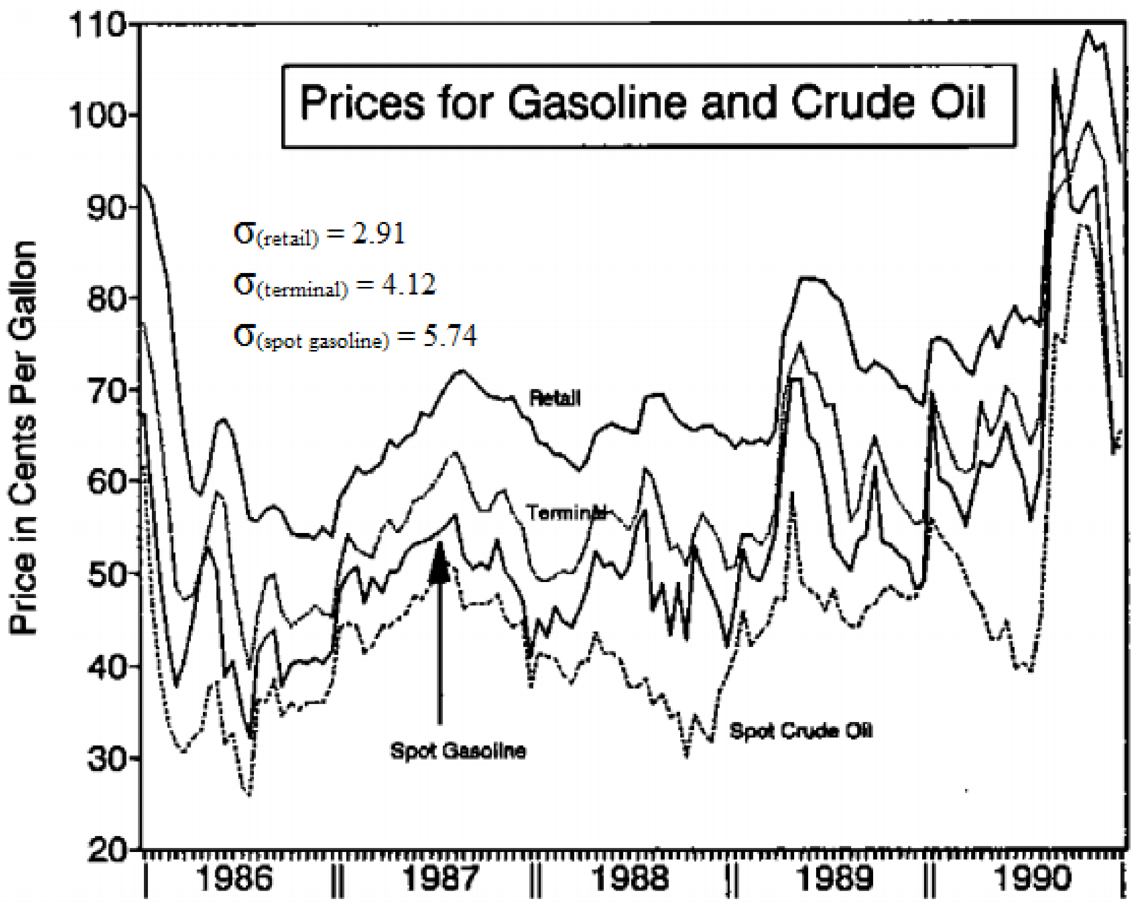

Figure 1.2: Decreased fluctuation of retail oil price. [Image adapted from Borenstein and Cameron (1992) and further edited]

Existing research is centered on the occurrence of RBP. Therefore, one research question may be asked, "can bullwhip effect in pricing propagate towards forward direction?". This same question can be rephrased as, "Does Forward Bullwhip effect in Pricing (FBP) occur?". In order to figure out the required conditions for the occurrence of RBP, existing literature considered game theoretic model of a multi-stage linear supply chain, where a leader-follower type 'Stackelberg' game was considered in which a supplier or an upstream supply chain player act as the leader. However, previous research did not consider the reverse direction of game where powerful retailers (or downstream supply chain players) may act as leaders too. A simultaneous game structure was not considered either. The occurrence of BP in the case of advanced supply chain contracts[2] was also unanswered. Based on previous studies, research questions that we are trying to answer can be summarized as follows:

1. Can bullwhip effect in pricing propagate in the forward direction? Alternately, does FBP occur?
2. Does bullwhip effect in pricing exist if retailers or downstream supply chain players act as leaders in 'Stackelberg' game?
3. How does BP occur in the case of simultaneous supply chain pricing games?
4. What is the effect of Buyback contracts on BP?
5. What is the effect of Revenue-sharing contracts on BP?

The objective of this research is to analyze the price variability across the supply chain stages considering various game structures and supply chain contracts. In next chapter, we review the literatures. Then in Chapter 3, we analyze the conditions for the occurrence of BP and conclude the occurrence of BP for some common demand functions and pricing practices. In Chapter 4, we analyze the occurrence of BP for optimal pricing in three game-settings. After that, to analyze the occurrence of BP in the case of supply chain contracts[3], we consider buyback (Chapter 5) and revenue-sharing contracts (Chapter 6 and 7) in our model. In the case of buyback contract[4], the demand is stochastic and the problem

[2] Supply chain contracts enables earning more profit. G. P. Cachon (2003) reviewed various contracts' performance in coordinating the supply chain.
[3] Some popular supply chain contracts are revenue-sharing, buyback/return/markdown, cost-plus, sales rebate, quantity discount, price-discount/bill-back, quantity flexibility etc. (G. P. Cachon, 2003)
[4] In the case buyback contract, there is no deterministic demand case. Because, for deterministic demand, there is no need of return-policy/buyback.

is modeled as a newsvendor case (Chapter 5). In the case of revenue-sharing contract we consider both deterministic and stochastic demand. In the deterministic demand case (Chapter 6), the problem is modeled as markup-pricing games (similar to Chapter 4). In the stochastic demand case (Chapter 7), the problem is modeled as a newsvendor case (similar to Chapter 5). In stochastic demand cases (Chapter 5 and 7), we consider both additive and multiplicative type demand uncertainties. We conduct analytical analysis and illustrate the results with numerical simulations in each of the chapters (4,5,6, and 7). Then finally, in Chapter 8, we summarize the major research, discuss the limitations and suggest future directions.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

In this chapter, we review several streams of literatures related to the research such as the effect of price variation on bullwhip effect, bullwhip effect in pricing, price variation, pricing database, game theory applications in supply chains, newsvendor model, buyback contracts, and revenue-sharing contracts.

2.1 Effect of Price Variation on Bullwhip effect

The term 'Bullwhip effect' was originally introduced by H. L. Lee, Padmanabhan, and Whang (1997). Since then, it has been a buzzword in the supply chain analysis. There are numerous analytical (L. Chen & Lee, 2009; Ma, Wang, Che, Huang, & Xu, 2013) and empirical analysis to quantify and reduce the bullwhip effect in various supply chain structure. For a recent comprehensive review about bullwhip effect, the reader may check the review paper by X. Wang and Disney (2015). F. Chen, Drezner, Ryan, and Simchi-Levi (2000) quantified the Bullwhip effect in supply chain considering simple supply chain. They also illustrated the existence of bullwhip effect even considering centralized demand. H. L. Lee, Padmanabhan, and Whang (2004) identified four sources of bullwhip effect (e.g. demand signal processing, rationing game, order batching, and price variations). Later, other researchers found many other sources of bullwhip effect (Bhattacharya & Bandyopadhyay, 2011). Among various causes, Paik and Bagchi (2007) considered price variations as one of three most significant causes of bullwhip effect in order quantity and inventory. Therefore, reducing price variation may reduce bullwhip effect. Mujaj, Leukel, and Kirn (2007) also suggested that pricing strategy (e.g. reverse pricing) could reduce the bullwhip effect in order quantity. They used agent-based simulation in their analysis to support their claim.

2.2 Bullwhip effect in pricing

Researchers identified amplified fluctuation in prices towards downstream supply chain and referred it as 'Reverse Bullwhip effect in Pricing (RBP)' (Özelkan & Çakanyıldırım, 2009; Özelkan & Lim, 2008). Özelkan and Çakanyıldırım (2009) considered leader-follower game framework in the supply chain and related the cost-pass-through to capture the ratio of price-variances. They derived the conditions on price-sensitive demand function for which price-variation may be amplified. Özelkan and Lim (2008) extended the previous analysis[5] considering stochastic demand function and added some stronger and weaker conditions on the demand function. Both of these papers focused on the reverse bullwhip effect in pricing but did not consider the plausibility of forward direction of bullwhip effect in pricing. Literature related bullwhip effect in pricing is very limited. To our best knowledge, no other paper discusses bullwhip effect in pricing, however, there are numerous papers that discussed the concept from dynamic pricing and cost-pass-through perspectives which are reviewed in the next section.

[5] The paper of Özelkan and Çakanyıldırım (2009) was originally published online on 2007 that was cited by Özelkan and Lim (2008)

2.3 Price Variation

Among the literatures of dynamic pricing, there are analytical models, as well as empirical models.

In the analytical analysis of price change, cost-pass-through is a great economic tool (Weyl, 2008). Cost-pass-through is the marginal rate of price-changes in cost. The cost-pass-through reflects the retailer's optimal pricing response to manufacturer's price change. Tyagi (1999) shows the conditions on customer demand to conclude about the cost-pass-through. Based on the cost-pass-through, Weyl (2008) extracted conclusions about profits and markup in simultaneous and wholesale leading game. However, he did not consider the retail leading game. He also differentiated between cost amplifying and absorbing, increasing and decreasing cost-pass-through. Weyl (2008) considered canonical simple supply chain structure with two stages (retailer and manufacturer). Unlike that, Gaudin (2016) calculated pass-through in vertical contracts considering bargaining power. While Fabinger and Weyl (2012) discussed the cost-pass-through; Cowan (2004) discussed demand curvature; Spengler (1950) talked about profit margin in double marginalization; Bresnahan and Reiss (1985) compared the margins between retailer and wholesaler; Adachi and Ebina (2014) connected the work of Weyl-Fabinger and Cowan with the work of Spengler and Bresnahan-Reiss. Adachi and Ebina (2014) related the cost-pass-through with profit margins in double marginalization.

Villas-Boas (2007) empirically analyzed price-variations in yogurt market. They use the data from IRI set and considered vertical relations, various supply chain structures, linear and non-linear pricing. E. Nakamura and Zerom (2010) analyzed the incomplete cost-pass-through empirically in coffee industry. Bonnet et al. (2013) did empirical analysis of cost-pass-through in German coffee market. Some researchers used large-scale dataset to analyze price dynamics at grocery level (A. O. Nakamura, Nakamura, & Nakamura, 2011).

2.4 Pricing Database

In order to study empirical examples of price variation, we look for dataset of retail prices, wholesale prices, commodity prices etc. ERS division of USDA compared the farm price, wholesale price and retail price by commodity types (e.g. beef, orange, broccoli etc.). Federal Reserve Economic Data (FRED) by Bank of St. Louis provides economic data in various categories including commodity prices at various frequency level (e.g. weekly, monthly, annual etc.). The US Bureau of Labor Statistics (BLS) provides price indexes (e.g. Consumer Price Index (CPI) and Producer Price Index (PPI)) for various categories of products.

A good database for academicians is IRI dataset[6]. It contains store data (e.g. sales, pricing, promotion etc.) at UPC level for 11 years in 47 markets (e.g. 11,300 grocery stores; 7,500 drug stores). Advertising data is also available for some early years. Bronnenberg, Kruger, and Mela (2008) discussed about this dataset in details.

Kilt Center for Marketing from The University of Chicago Booth School of Business maintains and promotes both public and subscription-based databases for academic researchers[7]. For academic purpose, public databases (e.g. Dominick's, ERIM, Bayesm etc.) are good resources. The Dominick's Finer Foods database[8] is popular for

[6] IRI academic dataset: https://www.iriworldwide.com/en-US/solutions/Academic-Data-Set ;
Processing and handling charge: $1000; Data: 350+ gigabyte; Media: USB drive;
Key measures and application of IRI dataset: http://www.whartonwrds.com/datasets/iri/

[7] Marketing Databases: https://research.chicagobooth.edu/kilts/marketing-databases

[8] Dominick's dataset: https://research.chicagobooth.edu/kilts/marketing-databases/dominicks/general-files

academic research. This database contain data from a single retail chain (E. Nakamura, 2008).

Various research reports (e.g. eMarketer, Statista, ThomsonONE etc.) use Nielson (formerly known as AC Nielsen) data. It is a rich (in terms of size, scope, breadth, longitudinal timeframe etc.) commercial dataset that provides scanner panel data of retail prices at UPC (Universal Product Code) level. The academic version of this dataset is referred as 'Nielsen Datasets at the Kilts Center for Marketing'[9], which is a partnership between 'The University of Chicago Booth School of Business' and 'The Nielsen Company'. The Kilts Center has been licensed by Nielsen to provide approved academics (around the world) with access to several Nielsen datasets. This dataset contains consumer panel data (consisting of 40 to 60 thousand US households) since 2004 and retail scanner data (e.g. prices, point of sales information etc. of 90 retail chains) since 2006. Nielson mostly contains data from the large retail chains (except Wal-Mart), but not from the independent supermarkets, which is a major share of U.S. markets (E. Nakamura, 2008). Moreover, household buys less amount of a particular UPC and often shifts among UPCs (of the same types of product); therefore, the data represents very small cross-section of identical items. (E. Nakamura, 2008). Broda and Weinstein (2010) discussed about Nielsen datasets in details.

Unlike retail price data, wholesale prices are not readily available. Wholesale trade deals are more complex and confidential. Wholesale/manufacturer prices of some grocery chains (from 50+ markets) are available from PromoData and commodity prices can be

[9] Nielsen Datasets at the Kilt Center for Marketing: https://research.chicagobooth.edu/nielsen/

available from New York Board of Trade or New York Physicals market data (Leibtag et al., 2007; E. Nakamura & Zerom, 2010).

2.5 Games Theory Applications in Supply Chains

Game theoretical framework is commonly used in supply chain analysis. The game rule can be applied among players within the same echelon of supply chain (e.g. retailer vs retailer, supplier vs supplier etc.) (see examples in Dowrick (1986); Gal-Or (1985); Y. Li (2014) etc.) or different echelon of supply chain (e.g. wholesaler vs retailer) (see examples in Cai, Zhang, and Zhang (2009); E. Lee and Staelin (1997); Moorthy and Fader (1989) etc.). The former type is called a horizontal game and the latter is called a vertical game. A combination of the horizontal and vertical game is also seen in the supply chain literature (Yu & Huang, 2010). The game players can decide on their strategies simultaneously or one player can decide after the other player had committed on its strategy (i.e. sequential move). Simultaneous game is often referred as Nash game and sequential leader-follower type game is referred as Stackelberg game. Stackelberg game can be wholesale leading or retail leading depending on who is committing first on its strategy. The leadership role can be endogenous or exogenous (i.e. defined by the market type). The game could be quantity setting or price setting or a combination of these two (e.g. wholesaler decides on wholesale price and retailer decides on order quantity) (Ingene & Parry, 1998; Yang & Zhou, 2006). The cost information can be unknown or a common knowledge (Albæk, 1992).

Gerard P. Cachon and Netessine (2004) provided a comprehensive review of game theory application in supply chain management. Kogan and Tapiero (2008) discussed the application of supply chain games from an operation management and risk valuation perspective. He, Prasad, Sethi, and Gutierrez (2007) reviewed the applications of Stackelberg differential game in supply and marketing channel.

Followings are some of the examples of game application in supply chain analysis from the literatures. Ingene and Parry (1998) applied game theory to decide on optimal wholesale price policy considering competing retailers. Yang and Zhou (2006) considered wholesaler as a Stackelberg leader and then among the competing retailers' they considered three types of competing behaviors (e.g. Cournot, Collusion and Stackelberg). Cai et al. (2009) analyzed a dual channel competition from three game-theoretical perspectives- supplier-stackelberg, retailer-stackelberg and nash game. They compared between two situations where the supplier enters in a direct channel or the supplier operates through a retail channel. Tsao et al. (2014) applied a Retailer-Stackelberg game in the supply chain of category products where manufacturers offer trade allowances. Amin-Naseri and Khojasteh (2015) showed the application of the Stackelberg game between two supply chain and also between two players of the same supply chain. They considered both the manufacturer-leading and retail-leading game. Lantz (2009) applied the game theory to solve the double marginalization problem of transfer pricing and recommended a two-part tariff. Leng and Parlar (2010) applied a cooperative and a non-cooperative game in an assembly supply chain. X. Y. Zhang and Huang (2010) applied Nash bargaining model between one platform-product manufacturer and multiple cooperative suppliers. They developed an iterative algorithm to find the subgame perfect equilibrium. Yu and Huang (2010) applied dual simultaneous non-cooperative game framework in vendor-managed inventory. They developed the model as a dual Nash game model (two sub-games – retailer-retailer, and manufacturer-retailers). They applied Genetic Algorithm to find out the Nash equilibrium. SeyedEsfahani et al. (2011) applied Nash and Stackelberg (wholesale and retail lead) games in a vertically cooperative pricing and advertising decision. Nie (2012) showed the application of Stackelberg game with leadership in-turn under open loop and close loop information system. Widodo, Pujawan, Santosa, Takahashi, and Morikawa (2013) applied adjusted-Stackelberg game in their analysis of dual channel supply chain. Y. Li (2014) applied a simultaneous and a sequential game in vertically differentiated market (i.e. products with higher and lower quality). Konur and Geunes (2016) applied Stackelberg game between the supplier and retail chain considering horizontal centralization and joint procurement.

A relevant question may occur in the readers' mind if there is any advantage or disadvantage in leadership of the Stackelberg game. Researchers commented on this issue. Dowrick (1986) argued that in the case of horizontal pricing game, if the reaction function is downward sloping, both firms prefer to be leader in order to get more profit. On the other hand, in the case of upward sloping reaction function, both firms prefer to be follower. In such case, if the firms are allowed to choose their leadership role, they cannot agree. Similarly, if the leadership is assigned exogenously, the Stackelberg leader gets greater (or less) profits than the follower if the reaction functions of the players are downward (or upward) sloping respectively (Gal-Or, 1985). Cyrenne (1997) considered horizontal game (between manufacturer-manufacture and retailer-retailer) with vertical relations (manufacturer-retailer) and showed that the price leadership is not always advantageous in the case of vertical relationship. In the case of vertical pricing game, if the decision of the wholesaler and retailer are strategic substitutes (i.e. if one raises margin, then other finds it optimal to reduce), then the leader gets advantage. On the other hand, if one finds it optimal to increase its margin more when the other had increased the margin (i.e. strategic complements); then the follower gets advantage (E. Lee & Staelin, 1997; Moorthy & Fader, 1989). Albæk (1992) analyzed the emergence of endogenous leadership in the case of unknown cost information and argued that the assumption of unknown cost may create incentives for the leadership role; however, there will be situation when the supply chain players cannot agree on the leadership role. Konur and Geunes (2016) also commented on the advantage or disadvantage of leadership in the supply chain of one wholesaler and coordinated retail chain.

2.6 Newsvendor Model

In our research, we consider a price-setting newsvendor model to model the contracts with stochastic demand (Chapter 5 and 7). Newsvendor model is primarily used for inventory management of perishable products. This model can also be applied to other seasonal products having short-lifecycle such as fashion goods (Petruzzi & Dada, 1999; Stalk Jr & Hout, 1990). The original idea came from the concept of a 'Newsboy'[10] case, where a seller buys certain amount of newspaper at the beginning of the day and he sells those newspapers within that day, otherwise the newspaper become obsolete. Therefore, the seller needs to forecast the demand of the day accurately. If he runs out of order (i.e. understocking), then he loses potential sales that may impact his goodwill (e.g. losing customer). The loss of goodwill can be considered as a penalty cost. On the other hand, in

[10] Historically, Edgeworth (1888) was the first to discuss the newsvendor problem in a bank industry to satisfy the demand of cash flows. He suggested using the normal distribution to satisfy an 'enough' potion of the demand. Later Morse and Kimball (1951) introduced the term 'Newsboy'. Among researchers, this problem was also known as 'Christmas Tree Problem' and 'Newsperson Problem'. Currently, the term 'Newsvendor' (suggested by Matthew Sobel) is commonly used (Porteus 2008).

the case of overstocking, he may incur a complete loss or may receive a salvage for the leftovers. There are many versions of the newsvendor model. The basic version compares the cost of overstocking and the cost of understocking. Thus, it calculates the optimal service level that generates maximum payoff for the company. Newsvendor model usually consider a single product for a single period.

Many researches have been done in the field of newsvendor model. Choi (2012), Qin, Wang, Vakharia, Chen, and Seref (2011), and Khouja (1999) provided extensive reviews of newsvendor model. In the case of newsvendor model, the demand can be either price-independent or price-sensitive (Jammernegg & Kischka, 2013). It is to be noted, a suboptimal decision may generate if the price-sensitivity of demand is not considered (Ye & Sun, 2016). In the price-setting newsvendor model, the demand is price sensitive. In such model, the newsvendor decides on optimal order quantity & price. Examples of earlier works in the price setting newsvendor are Whitin (1955), Zabel (1970), Thowsen (1975), Mills (1959), Karlin and Carr (1962), Young (1978) etc. Petruzzi and Dada (1999) reviewed price-setting newsvendor, and considered both additive and multiplicative uncertainty types. In Petruzzi and Dada (1999)'s model, joint decision of stocking quantity and selling price were considered. The demand or supply can be uncertain in the price-setting newsvendor model. M. Xu, Chen, and Xu (2010) analyzed the effects of uncertain demand, and M. Xu and Lu (2013) analyzed the effects of uncertain supply in a price-setting newsvendor model. Hsieh, Chang, and Wu (2014) also considered the demand uncertainty in their price-setting newsvendor model along with competing manufacturers and a retailer. Yao, Chen, and Yan (2006) considered an additive uncertainty in the demand. Jammernegg and Kischka (2013), and X. Xu, Cai, and Chen (2011) considered a multiplicative uncertainty in the demand. Abad (2014) and Kocabiyikoglu and Popescu (2011) considered both types of additive and multiplicative uncertainty. It is to be mentioned, the additive uncertainty has constant variance. In the case of multiplicative uncertainty, the variance is price-dependent but the coefficient of variation is constant (Abad, 2014; Petruzzi and Dada,1999). Additive type model is easier to analyze and explore (Abad, 2014).

X. Xu et al. (2011) provided a solution framework for the price setting newsvendor problem considering a general demand setting. Jammernegg and Kischka (2013) assumed quasi-concavity of the objective function to narrow the range of enumeration. They calculated the optimal stocking factor and provided conditions for the existence of solution for both price-independent and price-sensitive demands. Many researchers derived necessary and sufficient conditions for unimodality of the objective function in the price-setting newsvendor model (Kocabiyikoglu & Popescu, 2011; Lu & Simchi-Levi, 2013).

In newsvendor modeling, the service level approach is preferable than the shortage cost approach, because the shortage cost is difficult to forecast and it is product-specific (Abad, 2014). Both of Lu and Simchi-Levi (2013) and Kocabiyikoglu and Popescu (2011) did not use shortage cost in their model. Kocabiyikoglu and Popescu (2011) introduced lost-sale elasticity in their model. Abad (2014) focused on service level approach to determine optimal policy for price-setting newsvendor problem. Jammernegg and Kischka (2013) considered the service level and probability of negative profit as constraints in solving for the optimal price and order quantity.

Typically, newsvendor model considers single product for single season. However, there are models that consider multiple complementary and substitute products (Kachani

& Shmatov, 2011). Hsieh, Chang, and Wu (2014) also considered differentiated products from multiple manufacturers. In the case of single product, price sensitive demand is only sensitive to its own price but in the case of availability of complementary and substitute products, the cross price-sensitivity should also be considered. Kachani and Shmatov (2011) considered sensitivities to own price, to competitor's price, and to other products' price.

Ye and Sun (2016) incorporated strategic behavior of consumers in price-sensitive newsvendor model. The strategic and forward thinking consumer tend to delay their order until the products are available at salvage or discounted price. Ye and Sun (2016) analyzed the effect of additive and multiplicative type price-sensitivity of demand, and determined optimal selling price and stock quantity that maximize the profit. The results indicated that, the strategic behavior of consumer impacts newsvendor's profit positively (Ye & Sun, 2016). Like the strategic consumers, strategic retailers can also postpone their ordering or pricing decisions. Strategic retailers may set the price immediately after experiencing the demand uncertainty. Granot and Yin (2008) analyzed the effect of price postponement and order postponement in decentralized newsvendor model. The demand was price-sensitive and the uncertainty was of multiplicative type.

In order to boost up sales or profits, supply chain experts often promote various contracts that may eventually increase the overall supply chain profit. Various popular contracts include but not limited to buyback, revenue-sharing, cost-plus, sales rebate, quantity discount, franchise-contracts etc. We are considering buyback and revenue share contract in our analysis. Under any contract, a supply chain is said to be coordinated if individual's best action improves the overall profit of the supply chain. Contracts inspire participation among decentralized firms, such that they behave like a centralized coherent system (Giannoccaro & Pontrandolfo, 2004). Moreover, the supply chain players would be interested to participate in any contract if their individual profit increases under contract situation compared to no-contract situation. Gérard P. Cachon (2003) reviewed various contracts' performance considering newsvendor model (both fixed-price and price-setting types). He also discussed the scenarios when simpler contracts (i.e. sub-optimal actions) with less administrative cost is preferred over a perfect coordination. Gérard P. Cachon (2003) also analyzed the joint consideration of price and quantity decision in newsvendor model and concluded that coordination with contract is difficult in such cases, because of conflicting incentives. In next sections, we discuss two popular contracts- buyback and revenue sharing.

2.7 Buyback contract

Buyback contract is suitable for products with limited life expectancy (Höhn, 2010). This contract is very popular in markets like books, pharmaceuticals, apparels, computers, newspapers etc. (Padmanabhan & Png, 1995). 30-35% of the new hardcover books are returned to the publisher (Cachon & Terwiesch, 2012; Chopra & Meindl, 2015). Other markets and companies that practice the buyback contract includes but not limited to toys company such as DoodleTop (Leccese, 1993), computer companies such as HP and IBM (Anonymous, 2001), Intel (Roos, 2003; Spiegel, 2002), apparel industry (Choi, 2013; Xiao & Jin, 2011) etc.

In the case of buyback practice, the geographical location plays an important because of the associated shipping cost. Hence, local suppliers may offer this contract as an added service in a competitive supplier market (e.g. Choi (2013).). For distant suppliers, a modified version of the buyback can be implemented where the retailer need not to return the good physically, but salvages at the retailer's location and the wholesaler credits an amount back for the leftovers (Cachon, 2003). In Apparel industry, the 'buyback credit' is known as 'markdown money' that is offered as a subsidy for the clearance items. For examples, manufacturers like Tommy Hilfiger, Liz Claiborne, Ralph Lauren, Jones Apparel Group etc. offer markdown money to retailers like Federated (also known as Macy's), Dillard's, Saks, Kohl's, J.C. Penney etc. (Kratz, 2005; Rozhon, Petutschnig, Wrzaczek, & Jonak, 2005; Wang & Webster, 2007).

Researchers applied the buyback contract in various supply chain structures such as a single supply chain (Wang & Webster, 2007); a supply chain with two production modes (Donohue, 2000); a supply chain with effort dependent demand (Cachon, 2003; Taylor, 2002); a supply chain with loss-averse retailer (Wang & Webster, 2007); a supply chain of mass-customization etc.

Two of the main objectives of applying supply chain contracts is to coordinate[11] the supply chain and to increase the profitability of the supply chain. In the case of a fixed price model, the buyback contract coordinates the supply chain (Pasternack, 1985), hence eliminates the double marginalization problem. In the case of a price-setting newsvendor model, the buyback contract cannot coordinate the system[12] (Kandel, 1996). However, the

[11] A supply chain is referred as coordinated if each members' optimal action optimizes the overall supply chain. That means, each members' profit function should be an affine transformation of the system's profit function (Cachon, 2003).

[12] A modified version of buyback (e.g. price-discount contract) may coordinate the price-setting newsvendor model where the wholesaler dictates the retail price (e.g. retail price maintenance) (Kandel (1996); Cachon (2003)). Moreover, it is to be mentioned, according to Marvel and Peck (1995) and Bernstein and Federgruen (2005), buyback can coordinate the supply chain if the supplier earns zero profit (Höhn, 2010). Giri, Bardhan, and Maiti (2016) claimed that their composite contract (a combination of the buyback contract, a sales rebate,

contract still incurs greater profits for the retailer and wholesaler. Emmons and Gilbert (1998) analyzed the application of buyback contract in a single supply chain with multiplicative demand uncertainty and showed that the buyback contract can increase the wholesaler's profit. Padmanabhan (2004) applied buyback contract in a market of one manufacturer and multiple competing retailers with demand uncertainty and showed that buyback (also referred as a return policy) improves manufacturer's profitability. Hsieh and Lu (2010) also applied return policy in the context of manufacturer-Stackelberg game and competing risk-averse retailers. Wu (2013) showed that buyback is profitable in both cases of single supply chain or a competing supply chain. He assumed a vertical integration and a Stackelberg game. In both cases, the buyback turned out to be profitable. There are examples of modified versions of buyback contracts as well (Cachon, 2003; Giri et al., 2016). Cachon (2003) discussed the price discount contract as a modified version of the buyback contract. Giri et al. (2016) combined the buyback contract with a sales rebate and a penalty contracts.

In this research, we are considering a single supply chain with stochastic demand (e.g. newsvendor model) where the wholesaler offers the buyback contract. Since, the buyback contract is widely practiced in the supply chain market; we are interested to analyze the price variation in this case.

2.8 Revenue-sharing contract

Revenue-sharing contract is very popular in video rental industry. Gérard P Cachon and Lariviere (2005) discussed the application, strengths, and limitations of the revenue-

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and a penalty contracts) coordinates the decentralized three-layer supply chain with stochastic demand and random yield.

sharing contract considering a newsvendor model. They showed that revenue-sharing contract is equivalent to buy-back (or price-discount) contract in the case of fixed-price (or price-setting) newsvendor model. Pfeiffer (2016) compared the revenue-sharing contract with conventional wholesale-price contract and cost-plus contract and concluded that in the case of greater cost-uncertainty, revenue-sharing contract outperforms the wholesale-price contract. Many researchers showed the application of revenue-sharing contract in coordinating the supply chain (Gérard P Cachon & Lariviere, 2005; Giannoccaro & Pontrandolfo, 2004; Hu, Meng, Xu, & Son, 2016; Kebing, Chengxiu, & Yan, 2007; S. Li, Zhu, & Huang, 2009; W.-G. Zhang, Fu, Li, & Xu, 2012)

In the case of revenue-sharing contracts, retailers share their private information (e.g. sales) with the wholesaler; therefore, there is risk of potential cheating (e.g. underreporting sales). However, supplier's audit limits the cheating of the retailer. Thus, a revenue-sharing contract requires administrative investments. Therefore, this contract is popular in video rental and book industry, where tracking of retail sales is cheap administratively. Heese and Kemahlıoğlu-Ziya (2016) analyzed revenue-sharing contract with asymmetric information and dishonest retailer.

Revenue-sharing contract is more applicable to the type of industries where sales are less dependent on retailer's effort (e.g. local promotion, advertisement, solicitation etc.). In such industries, sales are mostly influenced by the national brand effect. Thus, availability of goods in the retail shops is important to satisfy the customer demand. Revenue-sharing contract inspires retailers to order more; hence, market availability of the product increases. Under revenue share contract, wholesaler sells the products at a cheaper rate and get a share from the sales revenue. The share percentage is mutually agreed upon, and often influenced by the bargaining power of the supply chain players. However, in the case of fixed retail price model, the optimized share percentage that maximizes the overall profit, can also be calculated and agreed upon (Giannoccaro & Pontrandolfo, 2009; S. Li et al., 2009; Pfeiffer, 2016). Revenue-sharing contract reduces prices and inspires the retailer to order more. Thus, market availability and sales are increased under the revenue-sharing contract. In the literature of revenue-sharing contract, two-echelon supply chain is commonly considered; however, the analysis can be extended for three-stage (Giannoccaro & Pontrandolfo, 2004; Hu et al., 2016) or n-stage supply chain (Feng, Moon, & Ryu, 2014) as well.

Researchers have introduced several variations of revenue-sharing contracts recently. Feng et al. (2014) analyzed Revenue-sharing contract considering the reliability of the firms (RCR) and concluded that in some cases, their modified approach gives more profit than the classical revenue-sharing contract. In that approach, the arbitrary profit sharing allocation is adjusted based on the comparative reliability of the firms, hence it inspires the firms to improve their reliability. Vafa Arani, Rabbani, and Rafiei (2016) merged the option contract with revenue-sharing contract and claimed that the profit of the supply chain is increased and the double marginalization effect is reduced. They considered various leadership role (e.g. wholesale-leading, retail-leading etc.) in the game analysis for different types of market. Hu et al. (2016) applied revenue-sharing contract, compared the coordination of the supply chain between two scenarios- loss averse vs loss neutral retailer, and concluded that loss-neutral retailer gains greater profits and a greater utility compared to loss-averse scenario. S. Li et al. (2009) considered revenue-sharing contract along with consignment contract (which is popular in online markets). In their Nash bargaining model, the retailer decides on the share percentage and the manufacturer decides on retail price and order quantity.

Even if the revenue-sharing contract coordinates (i.e. maximizes the total profit) the supply chain, but the supply chain players may not be agreed on the parameters of the contracts (e.g. profit allocation etc.). Considering such case, Giannoccaro and Pontrandolfo (2009) applied agent based simulation to figure out the scenarios (i.e. parameters of revenue-sharing contract) that inspire the firms to participate under revenue-sharing contract. Chauhan and Proth (2005) suggested supply chain partnership by applying revenue-sharing contract where the profit allocation is based on the associated risk of the firms.

2.9 Conclusions and Contribution of this research

In this chapter, we reviewed the literatures on bullwhip effects, price variation, game theory applications in supply chain, and various supply chain contracts. Existing researches of bullwhip effect in pricing (Özelkan & Çakanyıldırım, 2009; Özelkan & Lim, 2008) considered a Stackelberg wholesale leading game, a wholesale-price contract, and a linear supply chain. In our best knowledge, no researcher considered retail-leading or simultaneous game, buyback and revenue-sharing contract in the analysis of bullwhip effect in pricing. This research aims at contributing in these issues. Moreover, existing research of cost-pass-through is mostly limited in wholesale leading 2-stage supply chain; this research also aims at extending the analysis for a n-stage supply chain along with considering the retail leading and simultaneous type game relations.

Primarily, we follow Özelkan and Çakanyıldırım (2009)'s methodology of relating cost-pass-through to conjecture the price variation ratio. We extend the analysis by considering different types (retail leading and simultaneous) of games, buyback-newsvendor model, and revenue-sharing contract. In order to consider simultaneous and retail leading game, we model markup pricing game (J.-C. Wang, Lau, & Lau, 2013).

For the cost-pass-through calculations, we are following the methodology of Tyagi (1999) and Weyl (2008). We extend their analysis in the case of n-stage supply chain and relate that with the bullwhip effect in pricing.

In the case of buyback contract, we consider a price-setting newsvendor model. We adapt Petruzzi-Dada's (1999) model where the retailer decides on both order quantity and price for a given wholesale price and a buyback price. After deciding on optimal actions, we analyze the optimal price variation for the changing wholesale price.

In the case of revenue-sharing contract with deterministic demand, we follow a supply chain structure similar to Gaudin (2016), but the game rules are different. Gaudin (2016) only considered wholesale leading game in a 2-stage supply chain. We analyze retail leading and simultaneous games as well. After that, we benchmark the results with no-contract situation.

In the case of a revenue-sharing contract with stochastic demand, we model the supply chain as a price-setting newsvendor model, and analyze the price variation for different values of the revenue-share percentage.

Hence, we contribute the literature in several directions by analyzing bullwhip effect in pricing considering three game structures, two contracts, various demand functions, and two types of demand uncertainty.

CHAPTER 3: CONDITIONS FOR THE OCCURRENCE OF BP

3.1 Introduction:

In this chapter, we identify the conditions for the occurrence of Bullwhip effect in Pricing (BP) and relate it with the concavity coefficient and the cost-pass-through. After that, we discuss the occurrence of BP for some common demand functions. We also show numerical illustrations of BP in the case of two markup pricing strategies.

3.2 Conditions for the occurrence of BP:

We relate the conditions with both cost-pass-through of prices and concavity coefficient of the demand functions. The discussion is as follows-

3.2.1 Cost-pass-through and The Occurrence of BP:

In order to quantify the Bullwhip effect in Price (BP), we check the ratios of standard deviations of prices between two stages ($\frac{\sigma_n}{\sigma_{n+1}}$), referred as BP ratios. Özelkan and Çakanyıldırım (2009) related the ratios of the standard deviations with the cost-pass-through (i.e. rate of change of prices with respect to cost).

The relation between the cost-pass-through and the BP ratio can be explained using a simple example case. Let assume, $p = ac + b$ and $w = Ac + B$, where p denotes the retail price, w is the wholesale price, c is the cost, and $\{a, b, A, B\}$ are constants. Hence, $\frac{dp}{dc} = a, \frac{dw}{dc} = A, Var(p) = a^2 \times Var(c), Var(w) = A^2 \times Var(c)$. Therefore, $\frac{\sigma_p}{\sigma_c} = a$ and

$\frac{\sigma_w}{\sigma_c} = A$. Then, algebraically, we can show, $\frac{\sigma_p}{\sigma_w} = \frac{a}{A}$. Thus, we can conjecture the BP ratio from the cost-pass-through. For a formal and detail proof of the relation, please check the proposition 8 of Özelkan and Çakanyıldırım (2009). In their analysis, they assumed p and w as random variables and related as $p = g(w)$. They concluded, if $\frac{dg(x)}{dx}$ is greater or equal to a constant (for all $x > 0$), then $\frac{\sigma_p}{\sigma_w}$ is also greater or equal to that constant (Özelkan and Çakanyıldırım, 2009).

Accordingly, if the cost-pass-through is greater than one, then the BP ratio is also greater than one, hence 'Reverse Bullwhip effect in Pricing' (RBP) occurs (Özelkan and Lim 2008; Özelkan & Çakanyıldırım, 2009). Similarly, if the cost-pass-through or BP ratio is less than one, then we conclude that FBP occurs. If the BP ratio equals to one, we conclude that no BP occurs.

3.2.1 Concavity Coefficient and The Occurrence of BP:

Tyagi (1999) defined the concavity coefficient as $\Phi = \frac{qq''}{(q')^2}$, where q' and q'' are the first order and second order derivative of the demand function, q in price p respectively[13]. Cowan (2004) referred this term as the 'relative curvature'. The second order condition on the profit function (i.e. profit function to be concave in price) ensures that the concavity coefficient, Φ is less than two.[14] However, based on the structure of the demand function, the concavity coefficient Φ can be greater/less/equal to one. Tyagi (1999)

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[13] Tyagi (1999)'s original notation was $\varphi = \frac{q_p q''_p}{\left(q'_p\right)^2}$

[14] Let, $p =$ retail price, $w =$ wholesale price, and $\pi =$ retail profit. The demand q is a decreasing function in price, therefore, $q' < 0$. The retail profit, $\pi = (p - w)q$. The first order condition follows: $\frac{d\pi}{dp} = 0 \Rightarrow (p - w) = \frac{-q}{q'}$. Then, the second order condition follows: $\frac{d^2\pi}{dp^2} < 0 \Rightarrow \frac{qq''}{(q')^2} < 2$.

related the cost-pass-through as $\frac{dp}{dw} = (2 - \Phi)^{-1}$. Hence, if Φ is between 1 and 2, $\frac{dp}{dw}$ is greater than one, which results in RBP (Özelkan & Çakanyıldırım, 2009). Here, in addition, we recognize that if $\Phi < 1$ is less than one, then, the cost-pass-through, $\frac{dp}{dw}$ and the BP ratio, $\frac{\sigma_p}{\sigma_w}$ are also less than one; thus, FBP occurs. Similarly, if $\Phi = 1$, then, the cost-pass-through, $\frac{dp}{dw}$ and the BP ratio, $\frac{\sigma_p}{\sigma_w}$ are equals to one which results no BP.

Proposition 1: For a linear supply chain with one retailer and one wholesaler in a wholesale leading game framework,

a. If $\Phi = \frac{qq''}{(q')^2} < 1$, then, $\frac{dp}{dw} < 1$ and $\frac{\sigma_p}{\sigma_w} < 1$; thus, FBP occurs.
b. If $\Phi = \frac{qq''}{(q')^2} = 1$, then, $\frac{dp}{dw} = 1$ and $\frac{\sigma_p}{\sigma_w} = 1$; thus, no BP occur.

Here, q' and q'' are the first and second order derivatives of the demand function, q in the retail price, p.

3.3 Occurrence of BP for some common demand functions:

Concavity coefficients, cost-pass-throughs and occurrence of BP for some commonly used demand functions are shown in Table 3.1. It is to be mentioned, some of the results are adapted from Özelkan and Çakanyıldırım (2009) and Adachi and Ebina (2014)[15]. Ozelkan and Cakayindirim (2009) discussed that for isoelastic demand, RBP always occur; for logarithmic demand, RBP occurs if $ue^{-b-1} < p < ue^{-1}$; for linear and logit demands, RBP do not occur. However, they didn't focus on the occurrence of FBP or

[15] Adachi and Ebina (2014) discussed the amplifying and absorbing cost-pass-throughs at retail and wholesale stages.

no BP which are included in the following table along with some additional demand functions.

Table 3.1: BP in some common demand functions

Demand Functions	**Concavity Coefficients,** $\boldsymbol{\Phi = \frac{qq''}{(q')^2}}$	**Cost-pass-through,** $\boldsymbol{\frac{dp}{dw} = (2-\Phi)^{-1}}$	**Occurrence of BP**
Log-concave, $(a-bp)^{1/v}$ $0 < a, b$	$1 - v < 1$	$\frac{1}{1+v} < 1$	FBP
Linear, $a - bp$	0	$1/2$	FBP
[16]Logit, $a\frac{e^{u-p}}{1+e^{u-p}}$ $0 < a;\ p < u$	$1 - \exp(u-p) < 1$	$\frac{1}{1+\exp(u-p)} < 1$	FBP
Type I extreme value distribution[17], $1 - e^{-e^{a-p}}$ $; a > p > 0$	$-e^{-a}\left(-1 + e^{e^{a-p}}\right)(e^a - e^p)$	$\frac{1}{2 + e^{-a}(-1 + e^{e^{a-p}})(e^a - e^p)} < 1$	FBP
Isoelastic, ap^{-b} $0 < a; 1 < b$	$1 < \frac{b+1}{b} < 2$	$1 < \frac{b}{b-1}$	RBP
Logarithmic, $a\left(-\ln\frac{p}{u}\right)^b$ $0 < a, b;\ p < u$	$1 - \frac{1+\ln\frac{p}{u}}{b}$	$\left(1 + \frac{1+\ln\frac{p}{u}}{b}\right)^{-1}$	RBP, FBP, No BP
Negative Exponential, $a\exp\left(\frac{-p}{b}\right)$	1	1	No BP

Proposition 2: Occurrence of BP for some common demand functions are as follows-

a. For a log-concave, linear, logit, and Type I extreme value distribution type demand functions, FBP occurs.

b. For an isoelastic demand function, RBP occurs

[16] Alternate representation: $a\frac{1}{1+\exp(p-u)}$ (logistic demand). See example in Adachi and Ebina (2014).

[17] See example in Cowan (2012) and Adachi and Ebina (2014); $1 - e^{-e^{a-p}} \in (0,1); a > p > 0$. Type I extreme value distribution is also known as Gumbel distribution.

c. For a logarithmic demand function, RBP occurs if $ue^{-b-1} < p^* < ue^{-1}$; FBP occurs if $ue^{-1} < p^*$; and no BP occurs if $p^* = ue^{-1}$

d. For a negative exponential demand function, no BP occurs.

3.4 Occurrence of BP in the common markup-pricing practices

Let's consider a common pricing strategy 'markup pricing'. There are two types of markup pricing: dollar-markup and percentage-markup (J.-C. Wang, Lau, & Lau, 2013). Dollar-markup is common for high cost products such as jewelry (Lewison & DeLozier, 1989), while percentage-markup is common in retailing (Clower, Graves, & Sexton, 1988). It is to be mentioned that a fixed markup (dollar or percentage) strategy is sub-optimal (Lee & Staelin, 1997). While fixed dollar and percentage markup pricing is discussed in the following sub-sections, optimal markup pricing strategies in a game theoretical framework will be investigated in the next chapter.

3.4.1 BP in Fixed Dollar-Markup Pricing

Let's assume, both the retailer and the wholesaler add a fixed markup (\$$u$) with their per-unit cost. Thus, the per unit wholesale price would be $w = c + u$ and the per unit retail price would be $p = w + u = c + 2u$. Therefore, the cost-pass-through is 1 (i.e. $\frac{dp}{dw} = \frac{dw}{dc} = \frac{dp}{dc} = 1$). Furthermore, it is relatively easy to verify that the standard deviation of prices and cost are same, $\frac{\sigma_p}{\sigma_w} = \frac{\sigma_w}{\sigma_c} = \frac{\sigma_p}{\sigma_c} = 1$ [see Figure 3.1 for a numerical illustration]. Hence, the price variability is constant. Therefore, we conclude no BP occur in the case of fixed dollar-markup pricing.

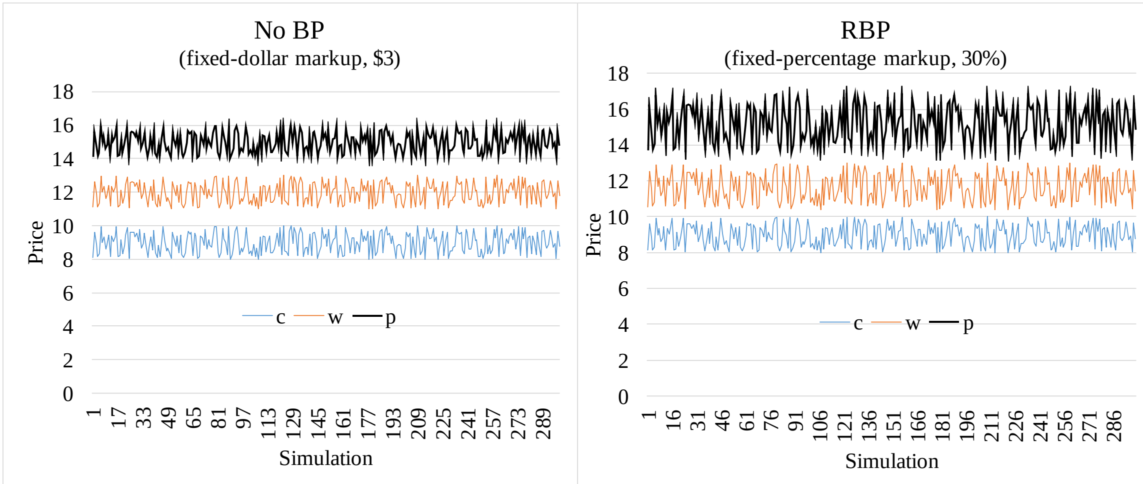

Figure 3.1: Constant or amplified variability of retail prices in the case of fixed-dollar (left) and fixed-percentage (right) markup pricing. [*p* = retail price, *w* = wholesale price, *c* = cost ($8~$10), uniform distribution, 300 simulation run].

3.4.2 BP in Fixed Percentage-Markup Pricing

Let's assume, both the retailer and wholesaler add a fixed percentage-markup (*100u%*) with their cost. Thus, the per unit wholesale price would be $w = c(1+u)$ and the per unit retail price would be $p = w(1+u) = c(1+u)^2$. Therefore, the cost-pass-through is greater than one.

$$\frac{dp}{dw} = \frac{dw}{dc} = 1+u > 1 \; ; \; \frac{dp}{dc} = (1+u)^2 > 1$$

Which indicates,

$$\frac{\sigma_p}{\sigma_w} = \frac{\sigma_w}{\sigma_c} > 1 \; ; \; \sigma_p > \sigma_w > \sigma_c$$

Figure 3.1 presents simulation results which show that the standard deviation of the retail price is more than that of the wholesale price and the standard deviation of the wholesale price is more than that of the cost [Figure 3.1]. Hence, the price variation is amplifying towards downstream supply chain. We conclude, RBP occurs in the case of fixed percentage markup pricing.

3.5 Conclusion:

In this chapter, we discussed the conditions for the occurrence of BP. If the concavity coefficient is less than one, then the cost-pass-through is also less than one that eventually creates FBP. If the concavity coefficient equals to one, then the cost-pass-through also equals one that results no BP. We discussed occurrence of BP in some common demand functions- such as isoelastic demand gives RBP, log-concave (or linear as a special case) and logit demand gives FBP, negative exponential demand gives no BP, logarithmic demand gives RBP, FBP, or no BP based on the range of the optimal price. We also discussed the occurrence of BP is a sub-optimal markup-pricing model. In the case of fixed dollar-markup pricing, no BP occur; in the case of fixed percentage-markup pricing, RBP occurs. It is to be mentioned, in this chapter, the concavity coefficient and the cost-pass-through rates are calculated assuming a single supply chain with deterministic demand following a wholesale leading Stackelberg game model. Other game structures (e.g. simultaneous and retail leading) are considered in the next chapter.

CHAPTER 4: BP IN DIFFERENT GAME STRUCTURES

4.1 Introduction

In this chapter, we consider a simple linear supply chain with centralized demand (Chen, Drezner, Ryan, & Simchi-Levi, 2000). We consider the game theory model to identify the optimal markup pricing. If the associated manufacturing/procurement cost changes due to external reasons (e.g. tax increment, change of exchange rate, scarcity of resources etc.), then the optimal prices will also change accordingly. Thus, both the retail and wholesale prices will fluctuate because of the cost changes. We analyze the fluctuation of prices and conclude whether RBP or FBP occur in different game structures.

We are interested in a price-setting game, where supply chain firms (e.g. wholesaler, retailer etc.) decide on their prices to maximize their profit. We consider three types of games- simultaneous, wholesale leading, and retail leading game. The leadership role (i.e. Who is committing first?) is exogenously determined by the market. In our analysis, we consider three common[18] types of demand functions- isoelastic ($q = ap^{-l}$), negative exponential ($q = a\exp(-p/b)$), and a log-concave type[19] ($q = (a - bp)^{1/v}$).

[18] Linear, isoelastic, and negative exponential demand functions are very commonly used among researchers because these demand forms are tractable and give constant pass-throughs (Bulow & Pfleiderer, 1983; Fabinger & Weyl, 2012). Empirical examples can be found in the literature for linear demand in the automobile market (Bresnahan & Reiss, 1985), and for isoelastic demand in beer market (Ornstein, 1980; Phelps, 1988; Weimer & Vining, 2015).

[19] Log-concave type demand ($q = (a - bp)^{1/v}$) takes the form of linear (if $v = 1$), concave (if $v > 1$) and convex (if $v < 1$) demand (See example in SeyedEsfahani, Biazaran, & Gharakhani (2011)).

In the next section, we discuss the game theoretic model. Then we conduct analytical analysis for 2-stage (section 4.3) and N-stage (section 4.4) supply chain. After that, we show some numerical examples for illustration purposes (section 4.5). After discussing the results and illustrations, we derive conclusions.

4.2 Mark-Up Pricing Game Description

We are considering a price-setting game where the wholesaler and the retailer decide on their per-unit markups 'u_w' *and* 'u_r', respectively. Thus, per-unit wholesale price 'w' is the sum of the manufacturing cost 'c' and the wholesale markup 'u_w'. Similarly, per-unit retail price 'p' is the sum of wholesale price 'w' and the retail markup 'u_r'. Demand 'q' is a decreasing function in retail price 'p' (i.e. $\frac{dq}{dp} < 0$). As, $p = w + u_r$ and $w = c + u_w$, we can write the demand function 'q' as $q(p)$ or $q(w, u_r)$ or $q(c, u_w, u_r)$ interchangeably. Manufacturing cost 'c' is known to both parties (i.e. wholesaler and retailer). Both the retailer and wholesaler intend to maximize their own profit 'Π_w' and 'Π_r', respectively by charging higher markups. On the other hand, higher markup results to higher price that adversely affects the demand quantity and eventually affects the earned profit. Moreover, each of their decision affects both of their profits. Therefore, both the wholesaler and the retailer need to consider the reaction function of their decision.

We consider three types of game scenarios (e.g. simultaneous, wholesale leading and retail leading) between the wholesaler and the retailer. In a simultaneous game, we solve for the Nash equilibrium where both wholesaler and retailer decide on their optimal markup considering other player's markup as unknown. In a sequential game, we solve for the Stackelberg equilibrium, considering one player (i.e. wholesaler or retailer) as the leader and another as the follower in decision-making. In the case of a wholesale leading game, the wholesaler declares its markup first then the retailer decides on its markup. In the case of a retail leading game, the retailer announces its markup first, and then the wholesaler sets its markup. Detail game descriptions are available in Appendix 1.

Analytical results of the cost-pass-throughs and BP ratios for 2-stage and N-stage are discussed in the following sections.

4.3 Two-stage supply chain

We consider a two-stage supply chain (i.e. One retailer and one wholesaler) and solve for specific demand functions (e.g. Log-concave, Isoelastic and Negative exponential) considering three different game scenarios. Table 4.1 shows the cost pass-throughs[20] (e.g. $\frac{dw}{dc}$ and $\frac{dp}{dc}$) and Table 4.2 shows the BP ratio.

For the log-concave type demand function[21] [e.g. $q=(a-bp)^{1/v}$], the cost-pass-throughs at wholesale and retail prices are less than one, and their interrelation can be expressed as $\frac{dp}{dc}<\frac{dw}{dc}<1$. In the case of the wholesale-leading and the retail-leading games, for linear demand, the cost-pass-through at retail price is 0.25; for convex demand, it is between 0.25 and 1; and for concave demand, it is less than 0.25. That means, for \$1 change in cost, the retail price will be changed by \$0.25 for linear demand (or less than \$0.25 for concave demand). In the case of the simultaneous game, the cost-pass-through at retail

[20] Cost-pass-throughs reflect the changes in prices for a unit change in cost. We refer $\frac{dw}{dc}$ as the cost-pass-through at wholesale price and $\frac{dp}{dc}$ as the cost-pass-through at retail price.

[21] For log-concave demand function, the concavity coefficient ($\varphi=\frac{qq''}{(q')^2}$) is less than one. For linear demand, φ is zero.

price is 1/3 for linear demand. For convex demand, it is between 1/3 and 1; for concave demand, it is less than 1/3. Other values of cost-pass-throughs are interpreted in similar fashion. (Table 4.1)

Table 4.1: Cost-pass-through (2-stage)

Demand function		Simultaneous game		Wholesale leading game		Retail leading game		Relation	RBP or FBP?
		$\frac{dw}{dc}$	$\frac{dp}{dc}$	$\frac{dw}{dc}$	$\frac{dp}{dc}$	$\frac{dw}{dc}$	$\frac{dp}{dc}$		
Log-concave, $q=(a-bp)^{1/v}$	Linear $(v=1)$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{dp}{dc}<\frac{dw}{dc}<1$	(<1) FBP
	Convex $(v<1)$	$>\frac{2}{3}$	$>\frac{1}{3}$	$>\frac{1}{2}$	$>\frac{1}{4}$	$>\frac{3}{4}$	$>\frac{1}{4}$		
	Concave $(v>1)$	$<\frac{2}{3}$	$<\frac{1}{3}$	$<\frac{1}{2}$	$<\frac{1}{4}$	$<\frac{3}{4}$	$<\frac{1}{4}$		
Iso-elastic, $q=ap^{-l}$, $(l>2)$		$\frac{l-1}{l-2}$	$\frac{l}{l-2}$	$\frac{l}{l-1}$	$\left(\frac{l}{l-1}\right)^2$	$\frac{l^2-l+1}{(l-1)^2}$	$\left(\frac{l}{l-1}\right)^2$	$1<\frac{dw}{dc}<\frac{dp}{dc}$	(>1) RBP
Negative Exponential, $q=a\exp\left(\frac{-p}{b}\right)$		1						$1=\frac{dw}{dc}=\frac{dp}{dc}$	(=1) No RBP /FBP

Table 4.2: BP ratio between the retail price and the wholesale price

Demand function	Simultaneous game	Wholesale leading game	Retail leading game	RBP or FBP?
	$\frac{\sigma_P}{\sigma_W}$	$\frac{\sigma_P}{\sigma_W}$	$\frac{\sigma_P}{\sigma_W}$	
Linear, $q=a-bp$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	(<1) FBP
Iso-elastic, $q=ap^{-l}$, $(l>2)$	$\frac{l}{l-1}$	$\frac{l}{l-1}$	$\frac{l^2}{l^2-(l-1)}$	(>1) RBP
Negative Exponential, $q=a\exp\left(\frac{-p}{b}\right)$	1			(=1) No RBP/FBP

For isoelastic demand function[22], the cost-pass-throughs at wholesale and retail prices are greater than one, and the interrelation can be expressed as $1 < \frac{dw}{dc} < \frac{dp}{dc}$. In the case of the wholesale leading and the retail leading game, for isoelastic demand, the cost-pass-through at retail price is $\left(\frac{l}{l-1}\right)^2$ that is greater than one but the value varies based on the elasticity, l. That means, if $l = 3$, then for \$1 change in cost, the retail price will be changed by \$2.25 $\left(=\left(\frac{3}{2}\right)^2\right)$. In the case of the simultaneous game, the cost-pass-through at retail price is $\frac{l}{l-2}$. If $l = 3$, then for \$1 increase/decrease in cost, the retail price will be increased/decreased by \$3. Other values of cost-pass-throughs are interpreted in similar fashion. (Table 4.1)

For negative exponential demand function (e.g. $q = a\exp(-p/b)$), the cost-pass-throughs at wholesale price and retail price are equal to one in all game scenarios[23]. For \$1 change in cost, the wholesale and retail prices will be changed by \$1.[24]

From the quantitative values of $\frac{dw}{dc}$ and $\frac{dp}{dc}$ (Table 3.1), we can conjecture the values of $\frac{\sigma_W}{\sigma_C}$ and $\frac{\sigma_P}{\sigma_C}$. Then, algebraically, we can calculate the value of $\frac{\sigma_P}{\sigma_W}$ (Table

[22] For isoelastic demand function, the concavity coefficient is greater than one.
[23] For negative exponential demand, the concavity coefficient equals to one. Moreover, for this demand function, the optimal markup for both parties (the wholesaler and retailer) is constant (i.e. \$$b$) (Fabinger & Weyl, 2012). Thus, for this demand function, optimal markup pricing is equivalent to the fixed dollar (\$$b$) markup pricing (similar to the example provided in Section 4.1). Hence, no RBP or FBP occur.
[24]Our results conform Tyagi (1999)'s conclusion. Tyagi (1999) considered wholesale leading game, derived conditions on demand function, and concluded that for linear and concave demand functions, the cost-pass-through is less than one but for a subset of convex demand (e.g. isoelastic demand), the cost-pass-through is greater than one.

4.2). The BP ratios (e.g. $\frac{\sigma_W}{\sigma_C}$, $\frac{\sigma_P}{\sigma_C}$, $\frac{\sigma_P}{\sigma_W}$ etc.) are less, greater or equal to one for linear, isoelastic, or negative exponential demand functions, respectively.

For linear demand, the retail price fluctuates less than the wholesale price (Table 4.2). In the case of the simultaneous and wholesale leading game, the BP ratio between retail and wholesale price is ½. We interpret this result as the retail price fluctuates less (i.e. 50%) compared to the wholesale price. In the case of the retail leading game, the BP ratio between the retail and wholesale price is 1/3; that means, the retail price's fluctuation is one third of the fluctuation of the wholesale price.

For isoelastic demand, the retail price fluctuates more than the wholesale price. In the case of the simultaneous and wholesale leading game, the BP ratio between the retail and wholesale price is $\frac{l}{l-1}$, where l is the elasticity of the demand function. In the case of the retail leading game, the BP ratio is $\frac{l^2}{l^2-(l-1)}$, which is also greater than one.

For negative exponential demand, the retail price fluctuates at the same rate with respect to the wholesale price (i.e. $\frac{\sigma_P}{\sigma_W}=1$).

4.4 N-stage supply chain

In this section, we extend the results of section 4.3 for N-stage supply chain (Table 4.3 and 4.4). *N* is the total numbers of stages in the supply chain and *n* refers to any stage in the supply chain. *n=1* refers to the bottom stage and *n=N* refers to the top stage.

For $q=(a-bp)^{1/v}$ type demand function (or linear demand as a special case), the cost pass through at any stage (i.e. $\frac{dp_n}{dc}$) is less than one and decreasing towards downward. In the case of the wholesale leading and the retail leading game, the cost-pass-through at retail price (i.e. $\frac{dp_1}{dc}$) is $\frac{1}{(v+1)^N}$ (or $\frac{1}{2^N}$ for linear demand). In the case of simultaneous game, the cost-pass-through at retail price (i.e. $\frac{dp_1}{dc}$) is $\frac{1}{1+Nv}$ (or $\frac{1}{1+N}$ for linear demand).

For isoelastic demand function, in the case of wholesale-leading and retail-leading game, the cost-pass-through at retail price is $\left(\frac{l}{l-1}\right)^N$. In the case of simultaneous game, it is $\frac{l}{l-N}$. Let assume, elasticity, $l=5$ and the total number of stages in the supply chain, $N=4$. Then, the cost-pass-through at retail price would be $\left(\frac{5}{4}\right)^4 = 2.44$, in the case of the wholesale-leading and retail-leading game. In the case of the simultaneous game, it would be $\frac{5}{5-4}=5$. That means, $1 increase in cost will result $2.44 increase in the retail price in the case of wholesale-leading and retail-leading game. In the case of simultaneous game, the retail price will be increased by $5 for $1 increase in cost.

Table 4.3. Cost pass-through (N-stage)
(Total stage *N*, any stage *n*, top stage *n=N*, bottom stage *n=1*)
[Detail version of this table is available in Appendix 2a]

Demand function	$\frac{dp_n}{dc}$			Relation	RBP or FBP?
	Simultaneous game	Wholesale leading	Retail leading		
$q=(a-bp)^{1/v}$	$\frac{1+(n-1)v}{1+Nv}$	$\frac{1}{(v+1)^{N-n+1}}$	$1-\sum_{i=n...N}\frac{v}{(v+1)^i}$	$\frac{dp_1}{dc}<...<\frac{dp_N}{dc}<1$	FBP
Linear, $q=a-bp$	$\frac{n}{1+N}$	$\frac{1}{2^{N-n+1}}$	$1-\sum_{i=n...N}\frac{1}{2^i}$		
Isoelastic, $q=ap^{-l}$ $(l>2)$	$\frac{l-(n-1)}{l-N}$; $l>N$	$\left(\frac{l}{l-1}\right)^{N-n+1}$; $l>1$	$1+\sum_{i=n...N}\frac{1}{l-1}\left(\frac{l}{l-1}\right)^{i-1}$; $l>1$	$1<\frac{dp_N}{dc}<...<\frac{dp_1}{dc}$	RBP
Negative exponential, $q=a\exp\left(\frac{-p}{b}\right)$	1			$\frac{dp_N}{dc}=...=\frac{dp_1}{dc}=1$	No BP

Table 4.4: BP ratio between two consecutive stages (N-stage)
(Total stage *N*, any stage *n*, top stage *n=N*, bottom stage *n=1*)
[Detail version of this table is available in Appendix 2b, 2c]

Demand function	$\frac{\sigma_n}{\sigma_{n+1}}$			Relation	RBP or FBP?
	Simultaneous game	Wholesale leading	Retail leading		
Linear, $q = a - bp$	$\frac{n}{n+1}$	$\frac{1}{2}$	$\frac{1+2^N - 2^{N-n+1}}{1+2^N - 2^{N-n}}$	$\frac{\sigma_n}{\sigma_{n+1}} < 1$; Increasing in *n*	FBP
Isoelastic, $q = ap^{-l}$ $(l > 2)$	$\frac{l-n+1}{l-n}$	$\frac{l}{l-1}$	$\frac{l^N - (l-1)^{N-n+1}\left(l^{n-1} - (l-1)^{n-1}\right)}{l^N - (l-1)^{N-n}\left(l^n - (l-1)^n\right)}$	$\frac{\sigma_n}{\sigma_{n+1}} > 1$; Increasing in *n*	RBP
Negative exponential, $q = ae^{\frac{-p}{b}}$	1			$\frac{\sigma_n}{\sigma_{n+1}} = 1$	No BP

Based on the value of the cost-pass-through ($\frac{dp_n}{dc}$), the BP ratio between two consecutive stages ($\frac{\sigma_n}{\sigma_{n+1}}$) is calculated (Table 4.4). For both linear and isoelastic demand functions, the ratio is less than one. In the case of wholesale-leading game, the BP ratios are constant. For the linear demand function, it is ½ and for the isoelastic demand function, it is $\frac{l}{l-1}$. In the case of simultaneous and retail leading game, the ratio is decreasing in *n*. In the case of simultaneous game, the BP ratio does not depend on the number of total stages. That means, in the case of simultaneous game, irrespective of the total numbers (e.g. 2, 3..or N), the BP ratio between the retail and the wholesale price (i.e. between the bottom two stages, $\frac{\sigma_1}{\sigma_2}$ or $\frac{\sigma_P}{\sigma_W}$) will be same. Figure 4.1 illustrates the BP ratios for a 4-stage supply chain.

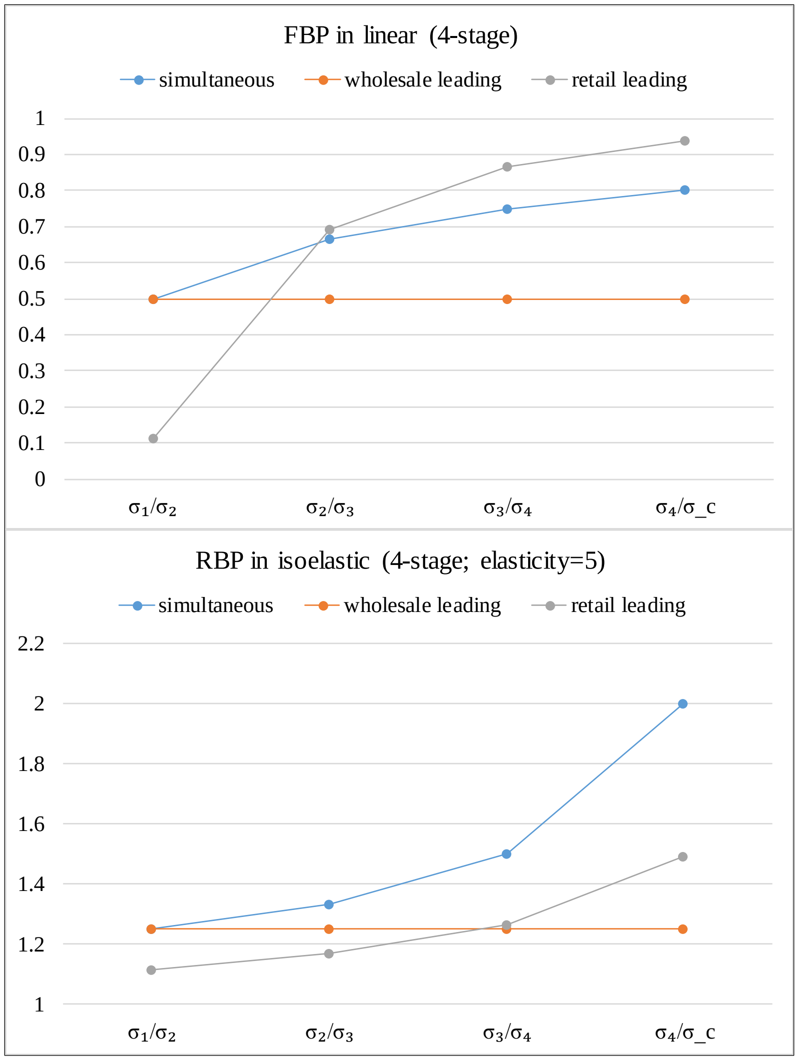

Figure 4.1: BP ratios (4-stage) [1 is the bottom stage; 4 is the top supplier stage]

4.5 Simulation Results

In this section, we run simulations to illustrate the analytical results of previous sections. We consider a two-stage supply chain (retailer and wholesaler). We randomly fluctuate the cost, calculate the optimal wholesale and retail price for each random cost. The parameters (e.g. distribution function, demand function parameters, upper or lower limit of cost, number of stages etc.) for the simulation are chosen randomly (but within the limit of the constraints) for illustration purpose. Similar results can be obtained for other parameters as well. In this simulation, the cost is uniformly distributed between \$8~\$10. The demand functions are $q = 20 - p$ (linear), $q = ap^{-2.5}$ (isoelastic) and $q = a\exp(-p/8)$ (negative exponential). We run the simulation for 300 times. Then, finally compare the standard deviation of the costs, the wholesale prices and the retail prices.

Here, we consider nine scenarios (three demand functions and three game structures). The results of this simulation are summarized in Table 4.5 and illustrated in Figure 4.2 and 4.3.

Table 4.5: Results of simulation (Markup pricing game)

		Simultaneous			Wholesale leading			Retail leading		
	σ_C	σ_W		σ_P	σ_W		σ_P	σ_W		σ_P
Linear, $q = 20 - p$	0.605	0.403	>	0.202	0.302	>	0.151	0.454	>	0.151
Isoelastic, $q = ap^{-2.5}$		1.814	<	3.024	1.008	<	1.680	1.277	<	1.680
Negative Exponential, $q = a\exp(-p/8)$		0.605								

For linear demand, the standard deviation of the retail price is less than the standard deviation of the wholesale price and the cost. Hence, price variation absorbed. In the case of simultaneous, wholesale-leading, and retail-leading game, the ratios of standard deviation of retail price to wholesale price are $\frac{0.202}{0.403} = 0.501$, $\frac{0.151}{0.302} = 0.5$, and $\frac{0.151}{0.454} = 0.33$ respectively. These ratios match very closely with the BP ratio mentioned in Table 4.2 as expected.

For isoelastic demand, the standard deviation of retail price is greater than the standard deviation of the wholesale price and the cost. Hence, price variation is amplified. In the case of simultaneous, wholesale-leading, and retail-leading game, the ratios of standard deviation of retail price to wholesale price are $\frac{3.024}{1.814} = 1.667$, $\frac{1.68}{1.008} = 1.667$, and

$\frac{1.68}{1.277} = 1.316$ respectively. Analytical results of BP ratios from Table 4.2 are $\frac{l}{l-1} = \frac{2.5}{2.5-1} = 1.667$ and $\frac{l^2}{l^2-(l-1)} = \frac{(2.5)^2}{(2.5)^2-(2.5-1)} = 1.316$. The results of Table 4.5 match the results of Table 4.2 as expected.

From figure 4.2, it is clearly visible that, for linear (or isoelastic) demand, the price variability is decreased (or amplified) towards downstream supply chain. For negative exponential demand, the variability of the cost, wholesale price, and retail price remain constant (Figure 4.3).

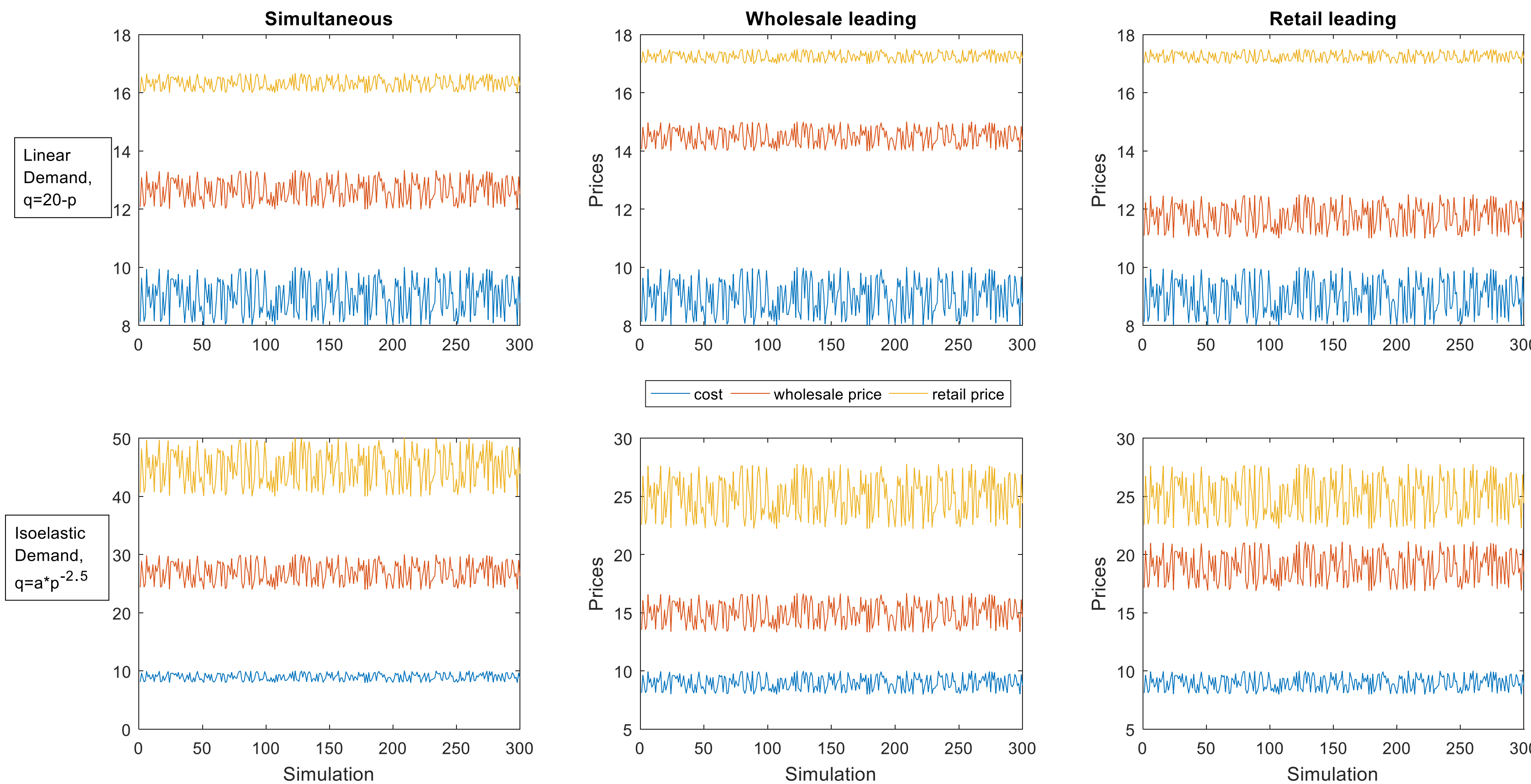

Figure 4.2: Price variation (Markup pricing game)

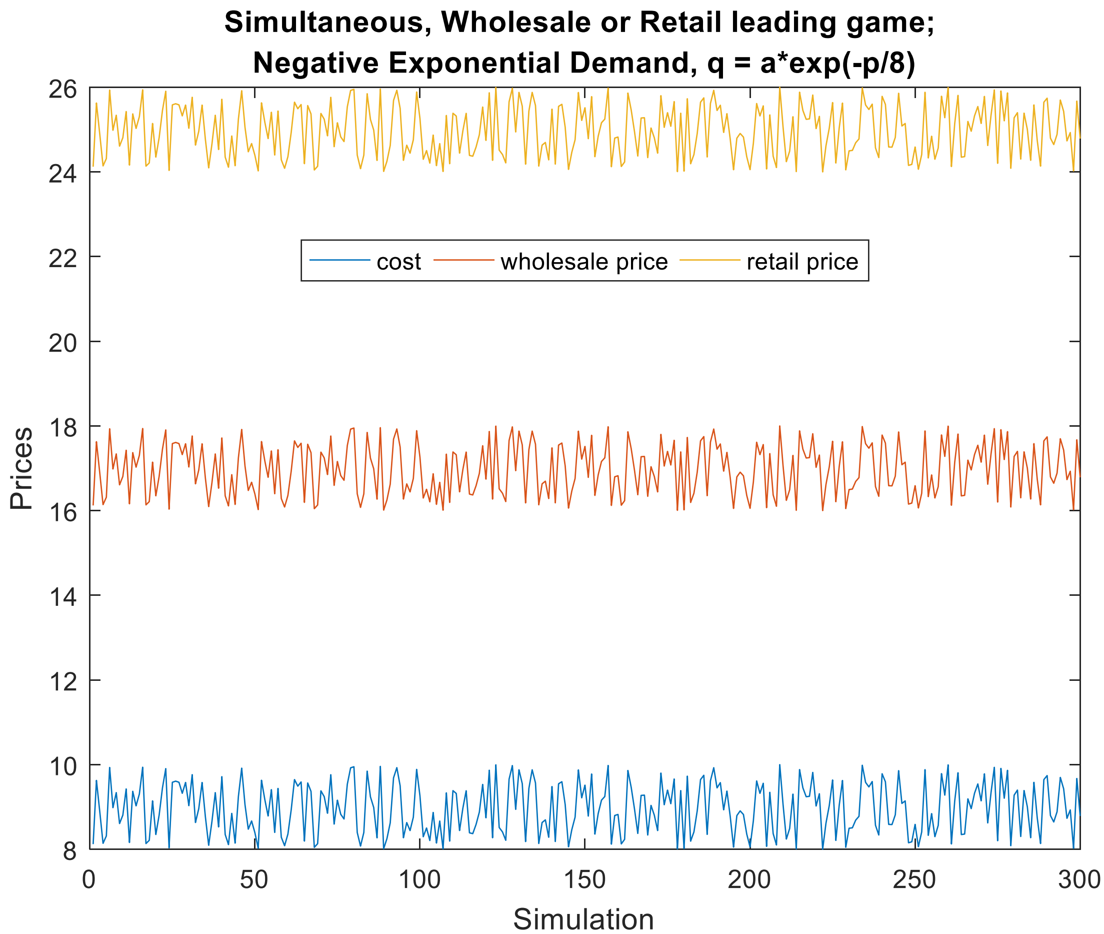

Figure 4.3: Price variation (Markup pricing game; Negative Exponential Demand)

4.6 Price variation, markups, game structure:

If we compare the price variation among various game structures, from figure 4.2 and 4.3, and table 4.5 it is seen that for linear (or isoelastic) demand, the retail price variability is same in the case of wholesale leading and retail leading game but more (or less) in the case of simultaneous game. The reason behind is that for linear (or isoelastic) demand, the optimal retail price is less (or more) in the case of simultaneous game compared to the case of wholesale leading and retail leading game. Moreover, it is also visible, that for linear (or isoelastic) demand, the more the markups the less (or more) the variability of prices. In other words, the closer the price to the cost, it captures more of the variability of the cost. For linear (or isoelastic) demand, the far the price from the cost, the variability is absorbed (amplified) more. This phenomenon contributes to the different values of cost-pass-through and BP ratios for different game structures.

4.7 Summary and Conclusion:

In this research, we analyzed the price variation analytically and then simulated the results. We considered markup-pricing model, three game rules (e.g. simultaneous, wholesale leading, and retail leading) and three types of demand functions (e.g. log-concave type (linear as a special case), isoelastic, and negative exponential). We extend the cost-pass-through analysis to N-stage supply chain and conjecture the BP ratios for N-stage supply chain. We compared the BP ratios among various game scenarios. The results can be summarized as follows-

- The cost-pass-throughs are less than one for $q = (a - bp)^{1/v}$ type demand function. For Isoelastic demand function, the cost-pass-throughs are greater than one. For negative exponential demand function, the cost-pass-throughs equal one. Cost-pass-through at retail price (i.e. $\frac{dp}{dc}$ or $\frac{dp_1}{dc}$) is same in the case of wholesale-leading and retail-leading game. It is $\frac{1}{2^N}$ and $\left(\frac{l}{l-1}\right)^N$ for linear and isoelastic demand respectively. The cost-pass-throughs are also absorbing or amplifying towards downstream supply chain for $q = (a - bp)^{1/v}$ type demand or isoelastic demand function respectively.

- BP ratio at retail and wholesale price between two consecutive stages $(\frac{\sigma_P}{\sigma_W} or \frac{\sigma_n}{\sigma_{n+1}})$ is constant in the case of wholesale-leading game for linear and isoelastic demand function. In the case of simultaneous and retail leading game, it is decreasing in *n*.
- The standard deviation of the retail (i.e. most bottom stage) price remains same for the wholesale-leading and retail-leading games but differs for the simultaneous game. The standard deviation of prices is absorbed or amplified towards downstream supply chain for linear or isoelastic demand respectively.

The analytical and simulation results help us to understand the nature of price variation for various supply chain structures.

CHAPTER 5: BP UNDER A BUYBACK CONTRACT

5.1 Introduction:

In this chapter, we model the retailer's problem in the case of a buyback contract where a newsvendor model dictates the inventory replenishment decisions. In a newsvendor model, a retailer commits the order quantity before the start of the selling season based on the demand forecast which is stochastic. Hence, if the realized demand is less than the order quantity, then the retailer salvages the leftover at a lower price. On the other hand, if the realized demand is more than the order, then the retailer incurs a cost due to shortage or loss of goodwill. Therefore, the retailer makes a tradeoff between the overage and underage cost; and thus, decides on the order quantity. This is what is called the traditional 'Newsvendor Problem'[25] (Edgeworth, 1888; Morse & Kimball, 1951; Porteus, 1990, 2008). Typically, in such a problem, the retail price is considered as exogenous. A variation of the newsvendor model is the price-setting newsvendor model where the retailer decides both the order quantity and the retail price (Mills, 1959, 1962; Whitin, 1955). In such case, the stochastic demand is price-sensitive. Another variation of the price-setting newsvendor model is the consideration of supply chain contracts (Cachon, 2003).

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[25] Other names for the 'Newsvendor Problem' are as follows - Newsboy (Morse & Kimball, 1951), Newsperson, Christmas Tree problem etc. (Porteus 2008)

Buyback contract is quite popular in industries[26] such as the book industry and textiles with brand-fashion items (Höhn, 2010). In a buyback contract, the wholesaler buys the leftover goods back at a price greater than the salvage price. This contract is also called a return policy (Cachon, 2003). Following such contract, the wholesaler incites the retailer to order more because the return practice offsets some of the retailer's risk associated with the leftover. The increased order size increases the expected profit of both the wholesaler and retailer. Brand reputation also motivates to apply this contract, where companies don't want their product to be placed in the salvage shelf of the store (Padmanabhan & Png, 1995). Stock rebalancing can be another motivation for applying a buyback contract (Höhn, 2010).

Let's consider a retailer who order q number of goods, pays the wholesaler $\$w$ per unit and sells it at the price of $\$p$ per unit. The demand can be expressed as $D = y + \epsilon$ (additive) or $D = y\epsilon$ (multiplicative) where y is the deterministic part and ϵ is the uncertain part of the demand D. We are considering both additive (Mills, 1959) and multiplicative (Emmons & Gilbert, 1998) uncertainties here. We also assume, ϵ is distributed on the interval $[A, B]$, μ is the expected value of ϵ and σ^2 is the variance of ϵ.

The wholesaler buys the leftover goods back at a unit-price β. It is necessary to assume that the buyback price β is less than the wholesale price w, otherwise the retailer would order infinite number of goods and return it back to the wholesaler while earning a positive amount of profit for each unsold item. However, another variation of the buyback contract can be such as – the retailer need not to return the goods physically, but salvages

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[26] The application of buyback contract in various industries (e.g. books, apparels etc.) are discussed elaborately in Chapter 2.7

it at his own location at a price v, then the wholesaler credits an amount β_1 per unit back for the leftover/salvaged items. Thus, the retailer earns the amount $\beta = v + \beta_1$ for each unsold product. In the case of physical return of the goods, the wholesaler pays $\beta = v + \beta_1$ to the retailer and salvages the leftover at the wholesaler's location. In both cases, the retailer's payoff for each unsold product remains the same as β. Our model captures both types of buyback contracts.

We adapt the price-setting newsvendor model of Petruzzi and Dada (1999) and modify the model to include the buyback policy. We determine the optimal actions; then, we compare the retail price variation with respect to the wholesale price variation by analyzing the cost-pass-through[27]. Following the analytical modeling, we also conduct numerical analysis for illustration purpose.

5.2 Model:

In the case of price-setting newsvendor model, the retailer's profit can be expressed as following,

$$\pi_r = \begin{matrix} pD - wq + \beta(q - D) & ; D \leq q \\ pq - wq - S(D - q) & ; D > q \end{matrix} \qquad (1)$$

Since, $D = y + \epsilon$ (additive case) or $D = y\epsilon$ (multiplicative case), by assuming[28] $z = q - y$ (additive case) or $z = q/y$ (multiplicative case), the retailer's profit $\pi_r(q,p)$ can be expressed as $\pi_r(z,p)$. The corresponding optimal policy is the order quantity, $q^* = y(p^*) + z^*$ (additive case) or $q^* = z^* y(p^*)$ (multiplicative case). Here z is called the stocking factor and can be expressed as $z = \mu + \sigma *$ (safety factor)

[27] Cost-pass-through refers to the change in price for marginal change in cost. If the retail price is p and the wholesale price is w, then the retail cost-pass-through is $\frac{dp}{dw}$.
[28] Such assumption provides mathematical convenience. We adapt this solution method from Petruzzi and Dada (1999).

The retailer's objective is to maximize its expected profit, $E[\pi_r(z,p)]$ where $\pi_r(z,p)$ is the retailer's profit and the optimal action is to determine z^* and p^*. This is a joint optimization problem in p and z. Therefore, we take partial derivatives of the expected profit in p and z, and check if the second order conditions are fulfilled. If $E[\pi_r]$ is concave in z for a given p (i.e. $\frac{\partial^2}{\partial z^2}(E[\pi_r(z|p)]) < 0$) and concave in p for a given z (i.e. $\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)]) < 0$), then we can solve the joint optimization problem following either the stocking decision approach or the pricing decision approach as follows-

Stocking decision approach: By replacing $p^*(z)$, the expected profit equation would be transformed into a single variable problem in z (Zabel 1970). Following Zabel's (1970) method, Petruzzi and Dada (1999) derived conditions for the existence of unique optimal actions in the case of newsvendor model. They showed that $E\left[\pi_r\left(z,p^*(z)\right)\right]$ reaches its maximum at the unique value of $z \neq B$ that satisfies, $\frac{d}{dz}E\left[\pi_r\left(z,p^*(z)\right)\right] = 0$. The conditions are fulfilled by exponential, uniform (Zabel, 1970), normal (Nevins, 1966), log-normal (Young 1978) distributions etc. Petruzzi and Dada's (1999) conditions are slightly more general. Their theorem is analogous in our buyback setting. We refer this method as the stocking decision approach.

Pricing decision approach: Another method of solving the joint optimization problem is to replace $z^*(p)$ into the expected profit equation; then the expected profit equation would be transformed into a single variable problem in p (Whitin 1955, Porteus 1990). We refer this method as the pricing decision approach. Emmons and Gilbert (1998) followed pricing decision method in buyback-newsvendor setting assuming multiplicative uncertainty, uniform distribution and linear demand form with no shortage cost.

Both stocking decision approach and pricing decision approach give the same optimal results. Stocking decision approach is mathematically convenient and pricing decision approach has managerial application. For price variation comparison, the pricing decision approach is convenient sometimes.

Table 5.1: Description of Parameters

Notation	**Description**
π_r	Retailer's profit
p	Retail price
w	Wholesale price
D	Demand For additive case, $D = y + \epsilon$ For multiplicative case, $D = y\epsilon$
y	Deterministic part of the demand
ϵ	Random part of the demand
q	Order quantity
β	Buyback price
S	Shortage cost
z	Stocking factor For additive case, $z = q - y$ For multiplicative case, $z = q/y$
b	Elasticity of the demand function
μ	Expected value of the random variable ϵ
$\Theta(z)$	$\int_z^B (u - z) f(u) du$ For additive case, E[shortage] = $\Theta(z)$ For multiplicative case, E[shortage] = y$\Theta(z)$
$\Lambda(z)$	$\int_A^z (z - u) f(u) du$ For additive case, E[leftover] = $\Lambda(z)$ For multiplicative case, E[leftover] = y$\Lambda(z)$
$\mu - \Theta(z) = z - \Lambda$	Expected sales
$\frac{\Lambda}{(\mu - \Theta)}$	$\frac{E[leftover]}{E[sales]}$
$F(z)$	Cumulative Distribution Function
$f(z)$	Probability Density Function
$r(z) = \frac{f(z)}{1 - F(z)}$	Hazard rate
$\frac{d\Lambda}{dz} = F(z)$; $\frac{d\Theta}{dz^*} = -[1 - F(z)]$	Derivatives of Λ and Θ in z

We conduct the analytical and numerical analysis considering two types of (additive and multiplicative) demand uncertainty. We assume a linear and isoelastic demand form with additive and multiplicative uncertainty following a uniform distribution. The detail problem formulations and solutions are discussed in Appendix 1A (Additive case) and Appendix 2A (Multiplicative case). The parameters are introduced in Table 5.1 and lemmas and propositions are mentioned in the following subsections.

5.2.1 Additive Demand Uncertainty Case:

Lemma 1a: Following the pricing decision approach for the single period buyback-newsvendor model with additive demand uncertainty, the optimal stocking factor z^* is determined as, $z^*(p) = F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]$ and the optimal p^* is the p that satisfies $\frac{d}{dp}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 0$. Hence:

1. For linear demand, $p^*(w) = \left\{p \middle| -p + \frac{a+bw}{2b} + \frac{\mu - \Theta(z^*(p))}{2b} = 0\right\}$
2. For isoelastic demand, $p^*(w) = \left\{p \middle| -p + \frac{b}{(b-1)}w + \frac{\mu - \Theta\left(z^*(p)\right)}{(b-1)ap^{-b-1}} = 0\right\}$

Proof: Appendix 1-B-i.

Lemma 1b: Following the stocking decision approach for the single period buyback-newsvendor model considering linear[29] demand with additive uncertainty, the optimal price p^* is determined as[30] $p^*(z) = \frac{a+bw+\mu}{2b} - \frac{\Theta(z)}{2b}$ and the optimal z^* is the unique z in the region $[A, B]$ that satisfies $\frac{dE\left[\Pi_r\left(z, p(z)\right)\right]}{dz} = 0$. Hence,

[29] Following the stocking decision approach, it is difficult to obtain a close-form solution of $p^*(z)$ in the case of an isoelastic demand with an additive uncertainty. However, we can solve the problem following the pricing decision approach which is mentioned in Lemma 1a.
[30] The optimal price in the case of additive certainty is less than the risk-less price. This result was shown by Mills (1959) and Petruzzi and Dada (1999).

$$z^*(w) = \left\{ z \middle| -(w-\beta) + \left(\frac{a+bw}{2b} + \frac{\mu - \Theta(z)}{2b} + S - \beta \right) [1 - F(z)] = 0 \right\}$$

Proof: Appendix 1-B-ii.

Proposition 1: In the case of a buyback-newsvendor model with additive demand uncertainty, the retail cost-pass-through is as follows-

1. For a linear demand $(y = a - bp)$, $\frac{dp^*}{dw} = \frac{1}{2}\left(1 - \frac{1+F}{2b(p^*+S-\beta)r-(1-F)}\right)$
2. For an isoelastic demand $(y = ap^{-b})$, $\frac{dp^*}{dw} = \frac{\frac{1}{r(p+S-\beta)} - bap^{-1-b}}{\frac{1-F}{r(p+S-\beta)} + abp^{-2-b}\left((b-1)p-(b+1)w\right)}$

Here, $F(.)$ is the cumulative distribution function, $f(.)$ is the probability density function, $r(.) = \frac{f(.)}{1-F(.)}$ is the hazard rate.

Proof: Appendix 1-D.

Corollary 1a: For $2b(p^* + S - \beta)\frac{r}{(1-F)} > 1$, FBP occurs in the case of buyback-newsvendor model under linear demand with additive uncertainty.

Proof: For $2b(p^* + S - \beta)\frac{r}{(1-F)} > 1$, $\frac{dp^*}{dw} < \frac{1}{2} < 1$; hence, FBP occur

Corollary 1b: Occurrence of FBP or RBP in the case of buyback-newsvendor model under isoelastic demand with additive uncertainty, depends on the parametric values.

5.2.2 Multiplicative Demand Uncertainty Case:

Lemma 2a: Following the pricing decision approach for the single period buyback-newsvendor model with multiplicative demand uncertainty, the optimal stocking factor z^* is determined as, $z^*(p) = F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]$ and the optimal p^* is the p that satisfies $\frac{d}{dp}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 0$. Hence:

1. For a linear demand, $p^*(w) = \left\{ p \middle| -p + \frac{a+bw}{2b} + \frac{1}{2} * X\left(z^*(p)\right) = 0 \right\}$

2. For an isoelastic demand, $p^*(w) = \left\{p \middle| -p + \frac{b}{b-1}w + \frac{b}{b-1} * X\big(z^*(p)\big) = 0\right\}$

Here, $X = \frac{(w-\beta)\Lambda(z^*(p)) + S\Theta(z^*(p))}{\big(\mu - \Theta(z^*(p))\big)}$

Proof: Appendix 2-B-i

Lemma 2b: Following the stocking decision approach for the single period buyback-newsvendor model with multiplicative uncertainty, the optimal p^* is determined as:

1. For linear demand, $p^*(z) = \frac{a+bw}{2b} + \frac{1}{2}X(z)$
2. For isoelastic demand, $p^*(z) = \frac{b}{b-1}w + \frac{b}{b-1}X(z)$

And the optimal z^* is the z is that satisfies $\frac{dE\left[\Pi_r\big(z,p^*(z)\big)\right]}{dz} = 0.$

$$z^*(w) = \left\{z \middle| y\left(-(w-\beta) + (p^*(z) + S - \beta)\big(1 - F(z)\big)\right) = 0\right\}$$

Here, $y = ap^{*(-b)}$ for isoelastic demand and $y = a - bp^*$ for linear demand.

Proof: Appendix 2-B-ii

Lemma 3: Let's define, $X = \frac{(w-\beta)*E[leftover] + S*E[shortage]}{E[sales]}$ and $W = \frac{1-F}{f(p+S-\beta)} * \frac{\partial X}{\partial z}$

Then it follows-

1. If $S = 0$, then $\frac{\partial X}{\partial z} > 0$
2. If $S > 0$, then
    a. $\frac{\partial X}{\partial z} > 0$ if $(w-\beta)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) > S\frac{\mu}{(\mu-\Theta)}$
    b. $\frac{\partial X}{\partial z} < 0$ if $(w-\beta)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) < S\frac{\mu}{(\mu-\Theta)}$
3. W follows the sign of $\frac{\partial X}{\partial z}$.

Here, $\frac{F}{(1-F)} > \frac{\Lambda}{(\mu-\Theta)}$ is given

Proof: Appendix 2-C

Remark: In further discussion, we will be using these two variables X and W.

Proposition 2: The expected profit, $E\left[\Pi_r\left(p, z^*(p)\right)\right]$ is concave in p or $E\left[\Pi_r\left(z, p^*(z)\right)\right]$ is concave in z for the given conditions-

1. For $y = a - bp, \frac{1}{2}W < 1$
2. For $y = ap^{-b}, b > 1,\ \frac{b}{b-1}W < 1$

where, W is defined in Lemma 3.

Proof: Appendix 2-D-i (pricing decision approach) and Appendix 2-D-ii (stocking decision approach).

Proposition 3: In the case of buyback-newsvendor model with multiplicative uncertainty, the retail cost-pass-through is as follows-

1. For linear demand (i.e. $D = (a - bp)\epsilon$), $\frac{dp^*}{dw} = \frac{1}{2}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{1}{2}-\frac{1}{1-F}\right)W}{1-\frac{1}{2}W}\right)$
2. For isoelastic demand (i.e. $D = (ap^{-b})\epsilon$), $\frac{dp^*}{dw} = \frac{b}{b-1}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{b}{b-1}-\frac{1}{1-F}\right)W}{1-\frac{b}{b-1}W}\right)$

Where, W is defined in Lemma 3 and Proposition 2, $F(.)$ is the cumulative distribution function

Proof: Appendix 2-E-i (pricing decision approach) and Appendix 2-E-ii (stocking decision approach).

Corollary 2: Comparisons of the retail cost-pass-throughs between the case of buyback newsvendor model with multiplicative demand uncertainty and the risk-less model are as follows in table 5.2.

Table 5.2: Comparison of the cost-pass-through between the optimal price and the risk-less price.

	Retail Cost-pass-through	Condition
Linear Demand	$\frac{dp^*}{dw} < 1$	$0 < \frac{W}{2} < 1$
	$\frac{dp^*}{dw} > \frac{1}{2}$	$\frac{W}{2} < \frac{\Lambda}{(\mu - \Theta)} * \frac{(1-F)}{(1+F)} < 1$
	$\frac{dp^*}{dw} < \frac{1}{2}$	$\frac{\Lambda}{(\mu - \Theta)} * \frac{(1-F)}{(1+F)} < \frac{W}{2} < 1$
	$\frac{dp^*}{dw} = \frac{1}{2}$	$\frac{W}{2} = \frac{\Lambda}{(\mu - \Theta)} * \frac{(1-F)}{(1+F)} < 1$
Isoelastic Demand	$\frac{dp^*}{dw} > \frac{b}{b-1}$	$\frac{b}{(b-1)} W < \frac{\Lambda}{(\mu - \Theta)} \frac{(1-F)}{\left(F - \frac{1}{b}\right)}$
	$\frac{dp^*}{dw} < \frac{b}{b-1}$	$\frac{\Lambda}{(\mu - \Theta)} \frac{(1-F)}{\left(F - \frac{1}{b}\right)} < \frac{b}{(b-1)} W < 1$
	$\frac{dp^*}{dw} = \frac{b}{b-1}$	$\frac{b}{(b-1)} W = \frac{\Lambda}{(\mu - \Theta)} \frac{(1-F)}{\left(F - \frac{1}{b}\right)} < 1$

Proof: Appendix 2-E-iii.

5.2.3. Discussion on the Propositions and the Corollary:

We are interested to analyze the change of p^* in w which is mentioned in Proposition 1 and 3 in term of cost-pass-through for additive and multiplicative case respectively. From Chapter 3, we know that the cost-pass-through is related with the BP ratio. If $\frac{dp^*}{dw} < 1$, then retail price fluctuates less than the wholesale price (i.e. FBP occur) and if $\frac{dp^*}{dw} > 1$, then the retail price fluctuates more than the wholesale price (i.e. RBP occur).

For a linear demand, the cost-pass-through in the case of risk-less-model (i.e. no newsvendor) is ½.[31] Lemma 1 tells that the optimal price is less than the risk-less-price; hence, the cost-pass-through is less than ½ which is conformed by Proposition 1. Since, in the case of buyback-newsvendor model for linear demand with additive uncertainty, $\frac{dp^*}{dw} < \frac{1}{2} < 1$, hence, FBP occur in this setting.

In the case of a multiplicative demand uncertainty, $\frac{dp^*}{dw}$ can be less or greater than $\frac{1}{2}$. However, the value of $\frac{dp^*}{dw}$ cannot exceed 1 for linear demand if $0 < W$. [Corollary 2]. Hence, FBP occurs in the case of a linear demand with multiplicative uncertainty.

For isoelastic demand, in the case of a risk-less model, the cost-pass-through is clearly greater than one (i.e. $\frac{dp^0}{dw} = \frac{b}{b-1} > 1$). Considering the risk associated terms, $\frac{dp^*}{dw}$ can be less or greater than $\frac{b}{b-1}$. In order to conclude for any valid condition that would make $\frac{dp^*}{dw}$ less than one for isoelastic demand, the argument $-1 < \frac{\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{b}{b-1}-\frac{1}{1-F}\right)W}{1-\frac{b}{b-1}W} < -\frac{1}{b}$ is needed to be verified where $0 < \frac{\Lambda}{(\mu-\Theta)} < 1$, $\frac{b}{b-1}W < 1$, $b > 2$, and $0 < F < 1$ are given. [Appendix 2-E-iii]. Otherwise, $\frac{dp^*}{dw}$ is greater than one for isoelastic demand. Hence, RBP occurs.

[31] In the case of risk-less model (for linear demand), $p^0 = \frac{a+bw+\mu}{2b} \Rightarrow \frac{dp^0}{dw} = \frac{1}{2} < 1$

5.3 Numerical analysis:

The price fluctuation, cost-pass-through rates and the occurrence of FBP can be illustrated through numerical analysis. The parameters are chosen randomly for illustration purpose (Table 5.3).

Table 5.3: Parameters used in the numerical simulation

Deterministic part, y	Uncertainty type	Distribution, ϵ	Shortage cost, S	Buyback price, β	Price Range
Linear ($y = 100 - p$)	Additive	Uniform$[-5,5]$, Uniform$[-10,10]$	10	15, 70	N/A
Isoelastic ($y = 10^6 p^{-3}$)	Additive	Uniform$[-5,5]$	10	15	Smaller and larger price
Linear ($y = 50 - p$)	Multiplicative	Uniform[1,5]	2	1	N/A
Isoelastic ($y = ap^{-3}$)	Multiplicative	Uniform[1,5]	2	1	N/A

5.3.1 Additive Uncertainty Case [Details are in Appendix 1-E]:

Let's assume, the deterministic part of the demand follows a linear form, $y = 100 - p$, the additive uncertainty is uniformly distributed on the interval $[-5,5]$ or $[-10,10]$, buyback price, $\beta = 15\ or\ 70$, shortage cost, $S = 10$. We consider two uniform distributions and two buyback prices for comparison purpose.

Optimal retail prices and optimal base prices for varying wholesale prices are illustrated in Figure 5.1 for $\beta = 15\ \&\ 70$ and the corresponding cost-pass-through is illustrated in Figure 5.2. Optimal prices are calculated for two uniform distributions (e.g. $[-5,5]$ and $[-10,10]$). The base price corresponds to the optimal price in the case of a risk-less model. Figure 5.1 and 5.2 shows that the optimal retail price is less than the base price (Mills 1959, Petruzzi & Dada 1999) and the cost-pass-through of the optimal price is less than ½. In Figure 5.3, for randomized values of stocking factor, we plot the corresponding wholesale prices and the optimal retail prices and base prices. It shows that the retail price fluctuates less than the wholesale price, hence FBP occurs in this setting.

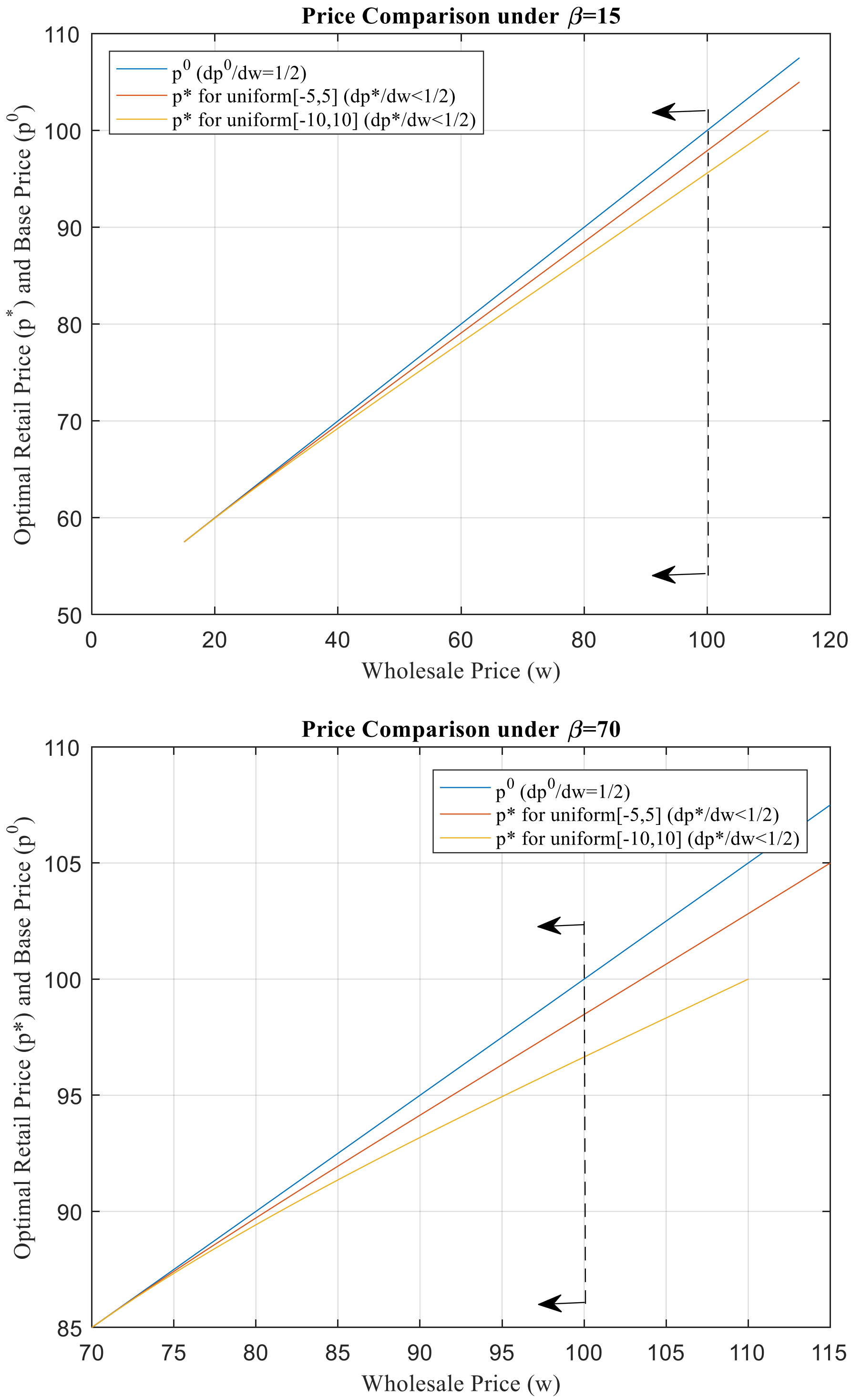

Figure 5.1: Price comparison in Buyback Newsvendor Model (linear demand, additive uncertainty)

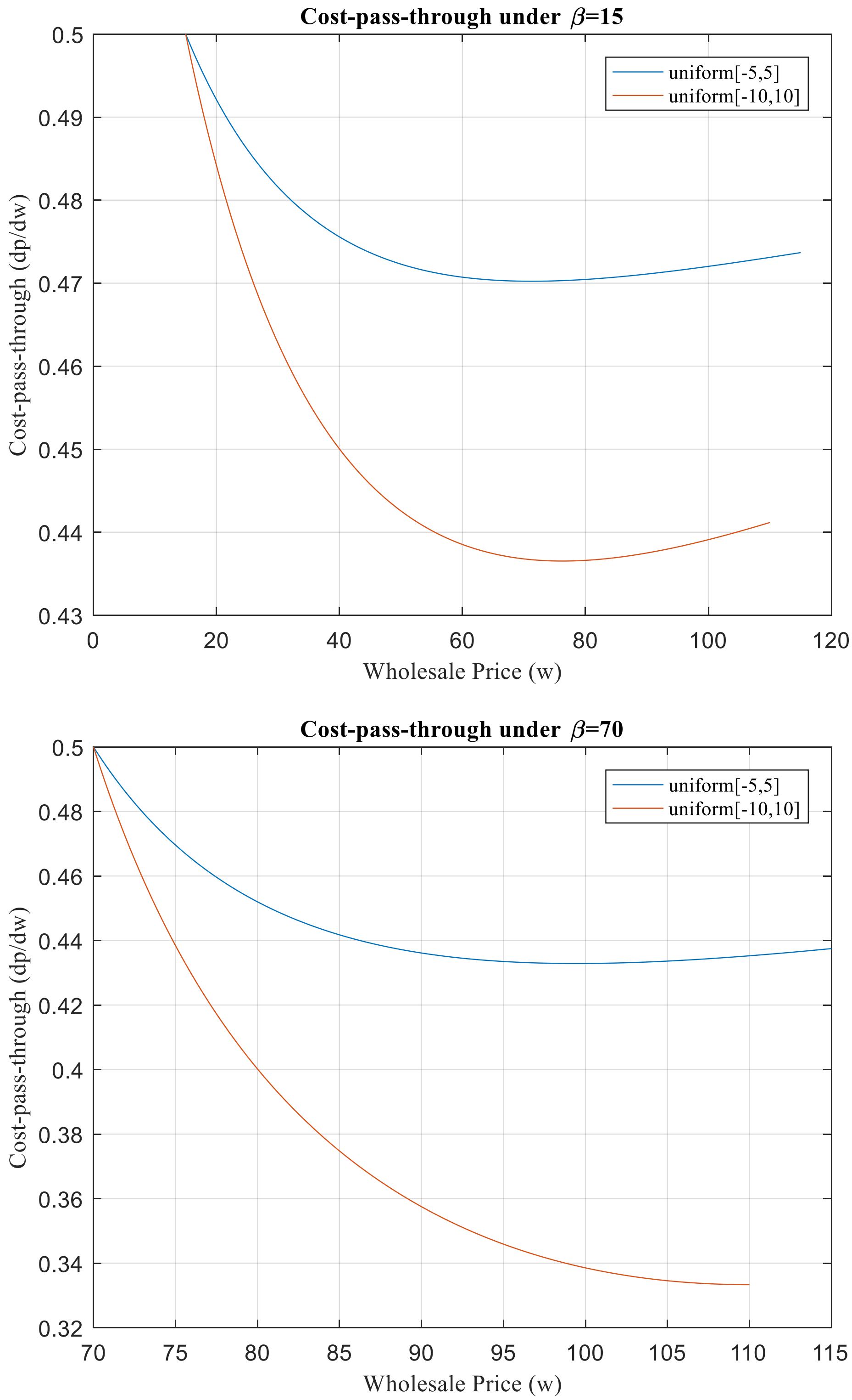

Figure 5.2: Cost-pass-through in Buyback Newsvendor Model (linear demand, additive uncertainty)

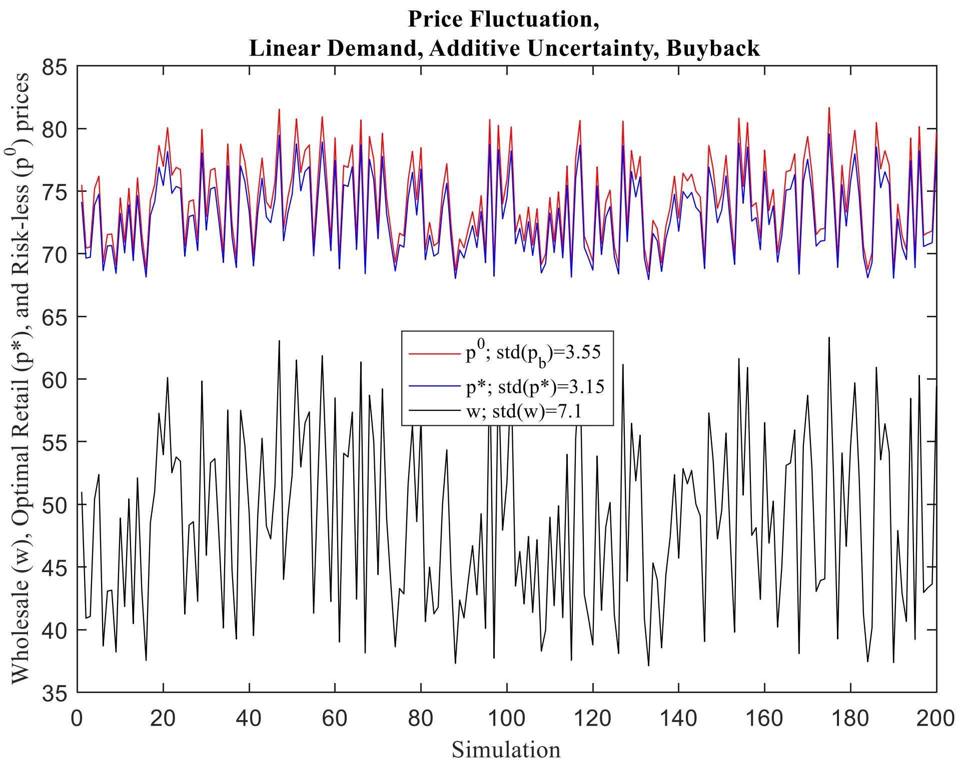

Figure 5.3: Occurrence of FBP (Linear demand, additive uncertainty)

We also consider an isoelastic form (e.g. $y = \frac{10^6}{p^3}$) for the deterministic part of the demand. Figure 5.4 shows the price comparison that reflects the optimal price is less than the risk-less price and Figure 5.5 shows the corresponding cost-pass-through. From figure 5.5, we see that the cost-pass-through changing from greater to less than one. Hence, based on the value of the wholesale price, both RBP and FBP can occur in the case of isoelastic demand with additive uncertainty. Figure 5.6 also shows similar conclusion in terms of standard deviations. Figure 5.6 shows, occurrence of RBP and FBP for two different range of the wholesale price. In the case of the selected parameters, when the wholesale price is close to \$25, then RBP occurs; when the wholesale price is close to \$45, then FBP occurs.

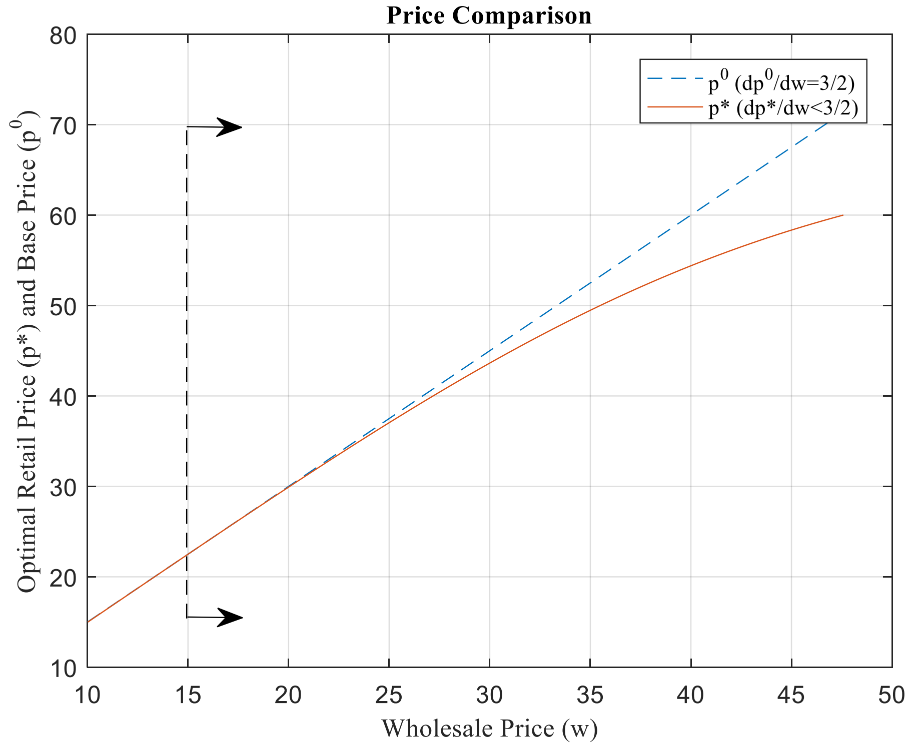

Figure 5.4: Price comparison in Buyback Newsvendor Model (Isoelastic demand, additive uncertainty)

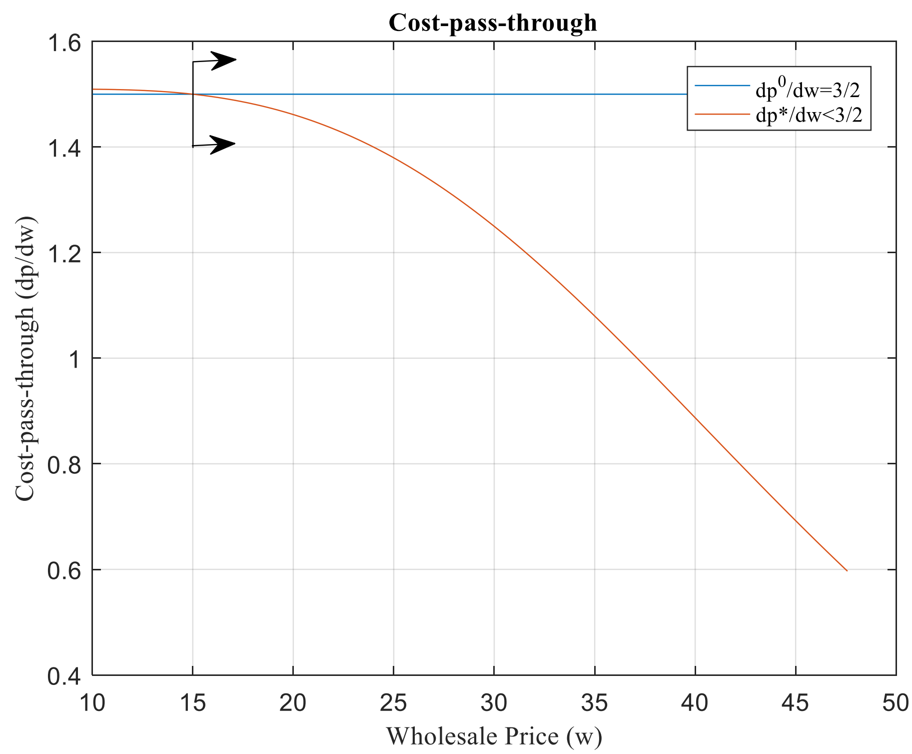

Figure 5.5: Cost-pass-through in Buyback Newsvendor Model (Isoelastic demand, additive uncertainty)

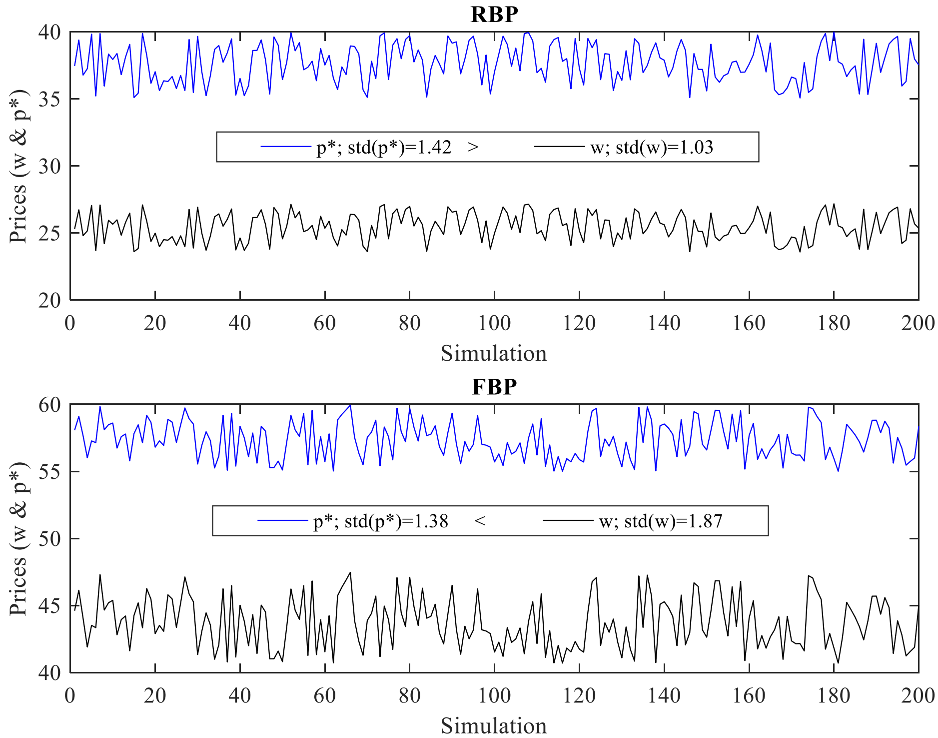

Figure 5.6: Occurrence of FBP (Isoelastic demand, additive uncertainty)

5.3.2 Multiplicative Uncertainty Case [Details are in Appendix 2-F]:

Let's assume, the multiplicative uncertainty is uniformly distributed on the interval [1,5][32], shortage price, $S = 2$, buyback price, $\beta = 1$. The minimum value of the wholesale price is the buyback price and the maximum wholesale price[33] is that price for which the corresponding demand is zero. We consider two forms (linear ($y = 100 - p$) and isoelastic ($y = 1000p^{-3}$)) for the deterministic part of the demand. The optimal results are discussed in the following subsections.

[32] The multiplicative case (with constant elasticity) require $A > 0$ in order to avoid the occurrence of negative demand (Petruzzi and Dada 1999). It is to be mentioned, Emmons and Gilbert (1998) assumed uniform distribution on the interval [0,2] with mean=1 for simplification; that worked there, because they assumed a linear form of demand.

[33] For isoelastic demand, the maximum wholesale price is $+\infty$ and the corresponding demand $\rightarrow 0$

5.3.2.1 Linear Demand:

Considering a linear demand with multiplicative uncertainty (e.g. $D = (100 - p)\epsilon$), figure 5.7 illustrates the optimal retail prices and base prices for varying wholesale prices. Figure 5.8 illustrates the corresponding cost-pass-through. Figure 5.9 shows the price fluctuations. Figure 5.7 shows that the optimal price is greater than the risk-less price (because of the multiplicative uncertainty) (Karlin & Carr, 1962, Petruzzi & Dada, 1999). The corresponding varying cost-pass-through remain less than one (Figure 5.8) for a linear demand. Hence, FBP occurs, which is shown in Figure 5.9. From Figure 5.9, we can also see how variance is absorbed in retail price compared to the wholesale price.

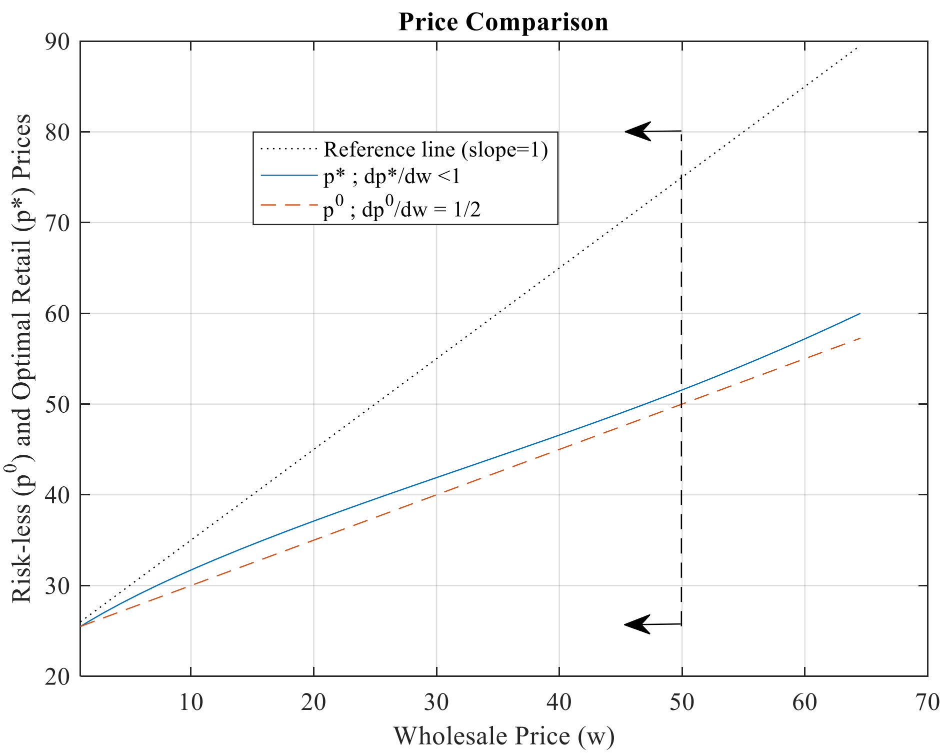

Figure 5.7: Price comparison in buyback-newsvendor model (linear demand with multiplicative uncertainty)

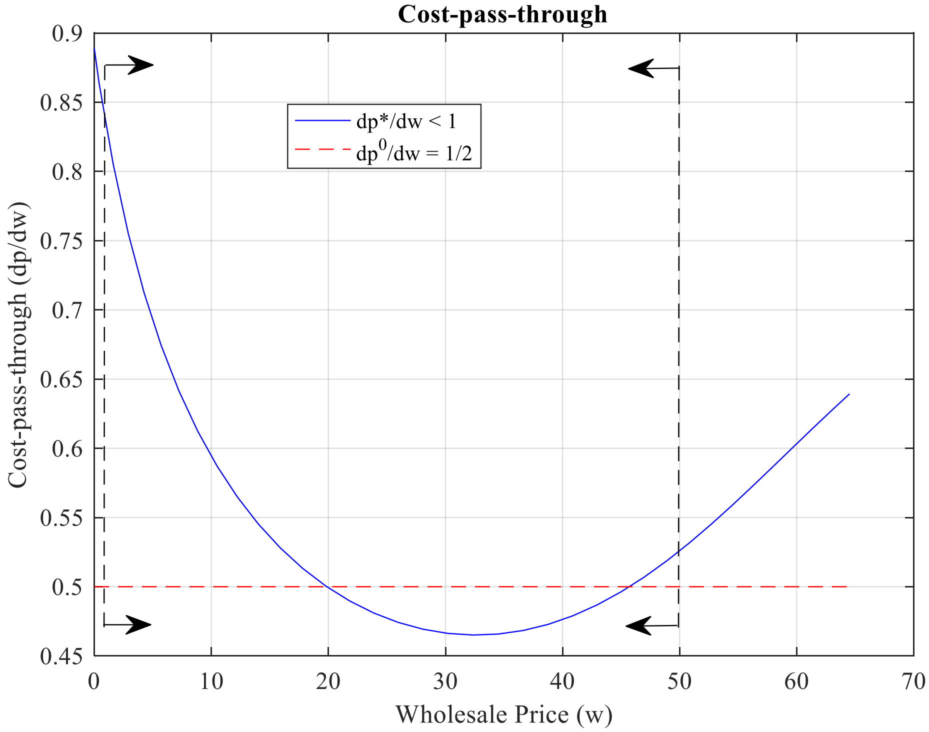

Figure 5.8: Cost-pass-through in buyback-newsvendor model (Linear demand with multiplicative uncertainty)

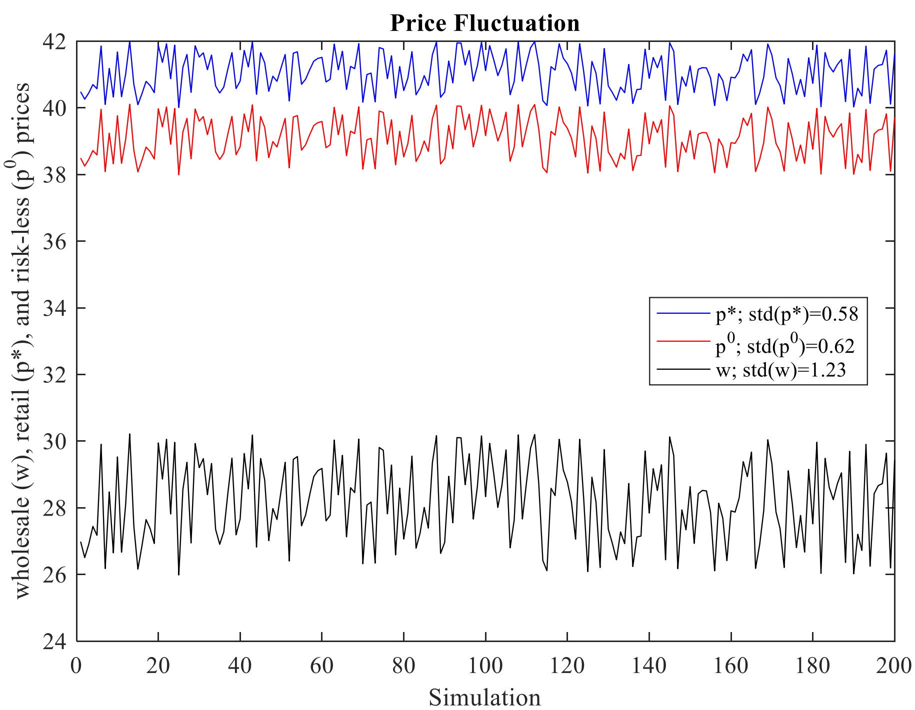

Figure 5.9: Occurrence of FBP under a buyback newsvendor model (Linear demand with multiplicative uncertainty)

5.3.2.2 Isoelastic demand:

Considering an isoelastic demand with multiplicative uncertainty (e.g. $D = (1000p^{-3})\epsilon$), the optimal prices are illustrated in Figure 5.10. Figure 5.11 shows the corresponding cost-pass-through and Figure 5.12 illustrates the price fluctuation. Figure 5.10 shows that the optimal price is greater than the risk-less price; and the corresponding cost-pass-through (figure 5.11) is greater than 3/2 which is the cost-pass-through of the risk-less price. Therefore, the cost-pass-through remains greater than 1. Hence, RBP occurs for isoelastic demand with multiplicative uncertainty with greater BP ratio. Figure 5.12 shows an example for the amplified variability in price.

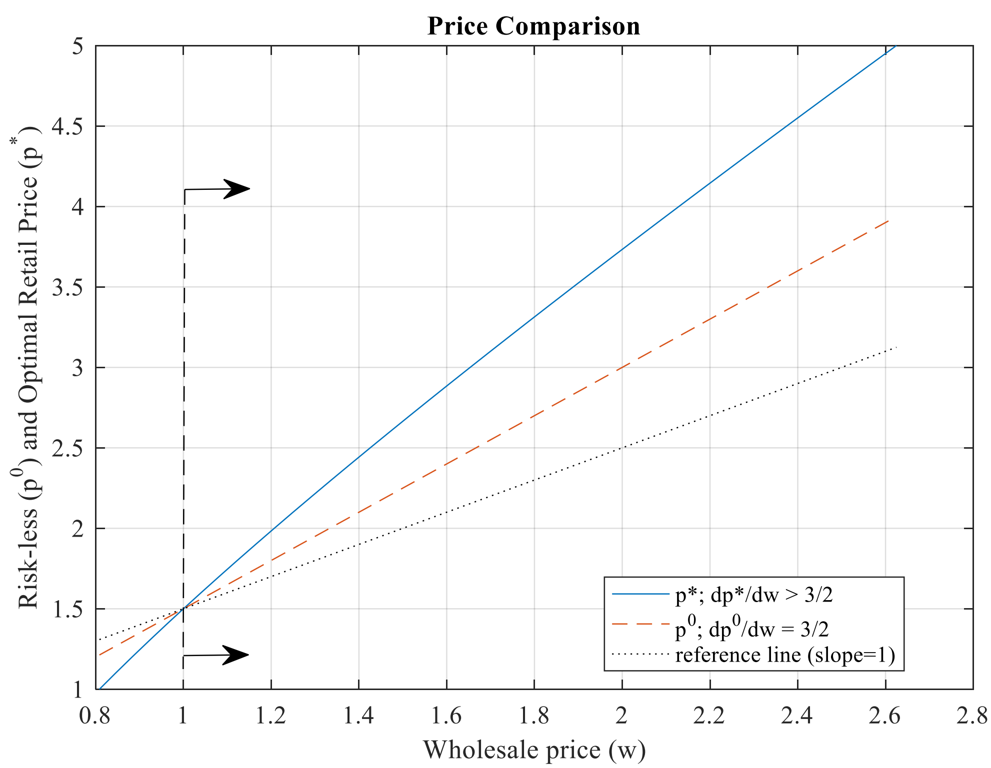

Figure 5.10: Price Comparison under a buyback contract (Isoelastic demand with multiplicative uncertainty)

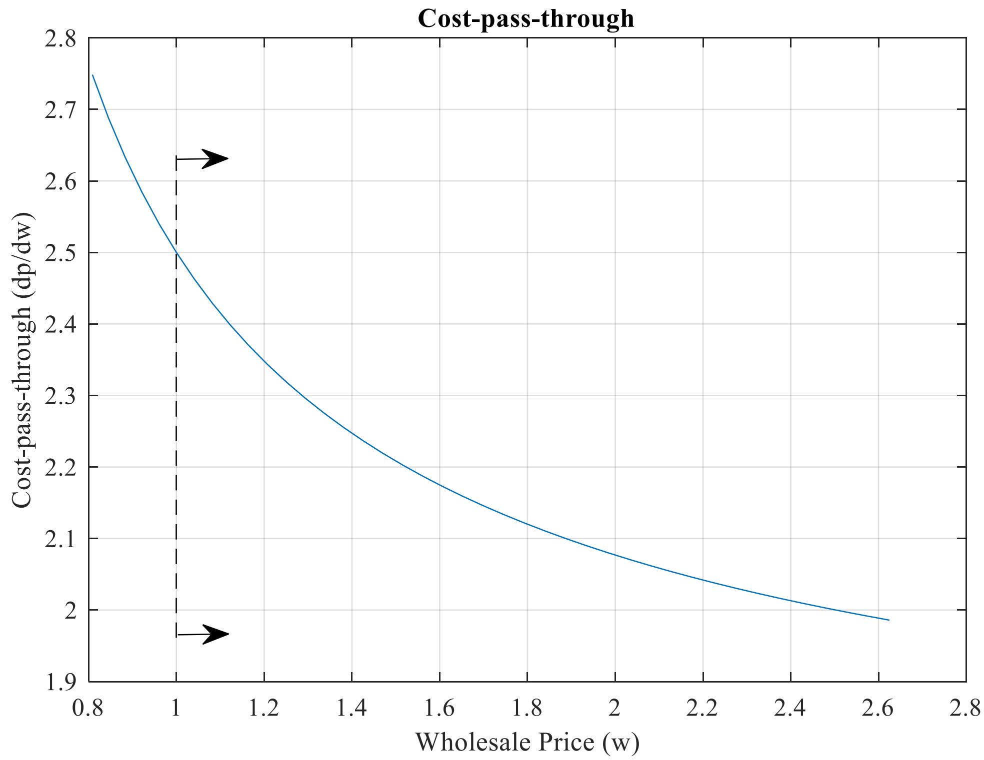

Figure 5.11: Cost-pass-through in a buyback newsvendor model (Isoelastic demand with multiplicative uncertainty)

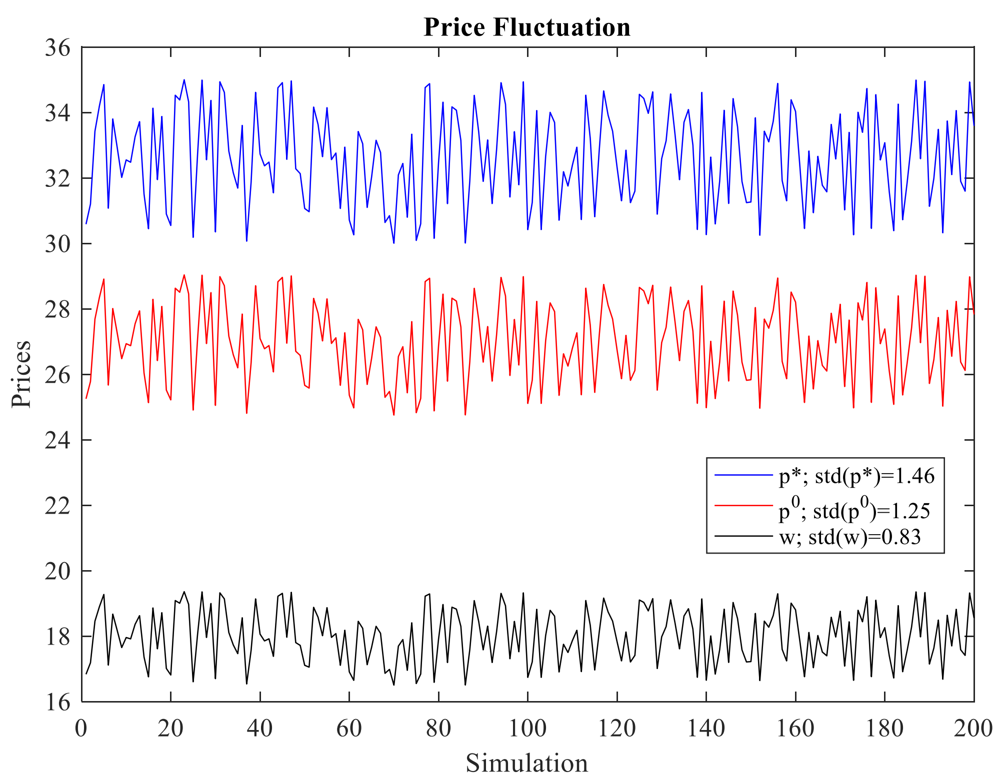

Figure 5.12: RBP under buyback contract for an isoelastic demand with multiplicative uncertainty

5.4 Conclusion:

Comparing the results of this chapter with the results of chapter 4, we see that consideration of buyback contract doesn’t change the occurrence of BP. However, the BP ratio changes because of the demand uncertainty. In the case of additive demand uncertainty, as the price decreases than the risk-less price, the cost-pass-through is also reduces. In the case of multiplicative demand uncertainty, the optimal price is higher than the risk-less price, but the cost-pass-through changes from less to greater than the risk-less cost-pass-through.

CHAPTER 6: BP IN REVENUE SHARE CONTRACT (DETERMINISTIC DEMAND)

6.1 Introduction

In this chapter, we are considering a simple supply chain of one wholesaler and one retailer operating under a revenue-sharing contract. Both the wholesaler and the retailer are independent and decide on their optimal prices that maximize their own profits. The profit allocation is exogenous. The retailer shares a portion (i.e. $0 < k < 1$) of its revenue with the wholesaler. The share portion 'k' is mutually agreed upon and constant in this game. The wholesaler decides on the per unit wholesale price, w and retailer decides on the per unit retail markup, u_r. Both intend to maximize their own profit Π_w and Π_r respectively. Per unit retail price 'p' is the sum of the wholesale price 'w' and the retail markup 'u_r'. Demand 'q' is endogenous and is a decreasing function in retail price 'p' (i.e. $\frac{dq}{dp} < 0$). As, $p = w + u_r$, we can rewrite the endogenous demand function 'q' as $q(p)$ or $q(w, u_r)$ interchangeably. Manufacturing cost 'c' is known to both parties (i.e. wholesaler and retailer). [34]

$$\max_{u_r} \Pi_R = ((1-k)\times p - w)\times q(p) = \{(1-k)\times(w+u_r) - w\}\times q(w,u_r)$$

$$\text{s.t. } ((1-k)\times(w+u_r) - w) > 0 \text{ and } q > 0$$

$$\max_{w} \Pi_W = (kp + w - c)\times q(p) = \{k\times(w+u_r) + w - c\}\times q(w,u_r)$$

$$\text{s.t. } (w-c) > 0 \text{ and } q > 0$$

[34] The game (w, u_r) in this chapter is equivalent to the game (u_w, u_r) in the chapter 4; because, the manufacturing cost, c is known to both parties. Therefore, the retailer can calculate the wholesale markup, u_w from the declared wholesale price, w.

Similar to Chapter 4, we consider three types of game scenarios (e.g. simultaneous, wholesale-leading and retail-leading) between the wholesaler and the retailer. Three types of demand functions (e.g. linear, isoelastic, and negative exponential) are considered.

6.1 Analytical results

We identify the conditions for which the optimality holds and the non-negativity constraints (of profits and demand) are satisfied (Table 6.1). We solve for the optimal prices and then calculate the cost-pass-throughs (Table 6.2) and BP ratios (Table 6.3). From the cost-pass-throughs and the BP ratios, we conclude if RBP or FBP occurs. Given the conditions on the value of 'k' from Table 6.1, the cost pass-throughs and BP ratios are less than one for linear demand and greater than one for isoelastic demand function. Moreover, for linear demand, the cost-pass-through at the retail price is less than the cost-pass-through at the wholesale price; and for isoelastic demand, the cost-pass-through at retail price is greater than the cost-pass-through at wholesale price for isoelastic demand. (Table 6.2). Thus, we conclude, FBP occurs for linear demand and RBP occurs for isoelastic demand. For negative exponential demand, the cost-pass-through at the retail price is one but the cost-pass-through at the wholesale price is less than one (Table 6.2) and the BP ratio between the retail and the wholesale price is greater than one (Table 6.3). Hence, for a negative exponential demand function (in the case of revenue-sharing contract), no BP occurs between the cost and the retail price, but FBP occurs between the cost and the wholesale price and RBP occurs between the wholesale price and the retail price.

Then, we compare the results (e.g. cost-pass-throughs and BP ratios) between the contract versus no-contract cases (Table 6.2 and 6.3). The results indicate that a revenue-sharing contract doesn't affect the cost-pass-through of the retail price (i.e. $\frac{dp}{dc}$) in case of a simultaneous price-setting game; but it does affect the cost-pass-through in the case of a sequential game. For a linear demand, the cost-pass-through of the retail price increases in contract situation in the case of a wholesale-leading game. Similarly, for a linear demand, the cost-pass-through of the retail price decreases in contract situation in the case of a retail-leading game. For an isoelastic demand, the cost-pass-through of the retail price decreases with a contract structure in the case of a wholesale-leading game; similarly, the cost-pass-through of the retail price increases with a contract structure in the case of a retail-leading game. For a negative exponential demand, the cost-pass-through of the retail price remains constant (=1) irrespective of the game changing structures and contracts. The change of cost-pass-throughs of the retail price can be explained by the value of optimal retail prices in various situations. For a linear demand under the wholesale leading game, the optimal retail price in a contract structure is less than the optimal retail price with no-contract. As we know, for linear demand, the cost-pass-through of retail price is absorbing. Hence, for lower retail prices (i.e. closer to the cost), the corresponding cost-pass-through is expected to be greater. That's why, in case of wholesale leading game and linear demand, the cost-pass-through of retail price in contract is greater than that of no-contract case. Other changes in the cost-pass-through of the retail prices can be explained in a similar fashion.

The cost-pass-through at wholesale price (i.e. $\frac{dw}{dc}$) is reduced for the contract case (Table 6.2). In case of a simultaneous game, $\frac{dw}{dc}$ is reduced by $1-k$ times for linear, isoelastic and negative exponential demand functions. In case of a wholesale-leading and retail-leading game, $\frac{dw}{dc}$ is also reduced. Hence, we can conclude that the wholesale price fluctuates less in revenue-sharing contracts compared to the no-contract case.

BP ratios between the retail-price and the wholesale-price are increased by $\frac{1}{1-k}$ times in revenue-sharing contract compared to the no-contract case for simultaneous and wholesale leading games (Table 6.3). In the case of retail leading game, comparing between revenue-sharing and no-contract cases, the BP ratio remains same for a linear demand but increased for isoelastic and negative exponential demand functions.

Table 6.1: Conditions for optimality[35] and non-negativity constraints[36] (Revenue-sharing contract)

Demand function	Simultaneous game	Wholesale leading game	Retail leading game
Linear, $q = a - bp$; $c < \frac{a}{b}$; $\{a,b,c\} \neq 0$	$0 < k < \frac{a-bc}{a+2bc} < 1$	$0 < k < \left(1 - \sqrt{\frac{bc}{a}}\right) < 1$	$0 < k < \frac{\sqrt{3a^2 + b^2c^2} - (a+bc)}{2a} < 1$
Isoelastic, $q = ap^{-l}$	$0 < k < \frac{1}{l-1} < 1$; $l > 2$	$0 < k < \frac{1}{l+1} < 1$; $l > 1$	$0 < k < \frac{1}{l-1} < 1$; $l > 1+k$
Negative Exponential $q = ae^{\frac{-p}{b}}$; $\{a,b,c\} \neq 0$	$0 < k < \frac{b}{b+c} < 1$	$0 < k < \frac{2b + c - \sqrt{c(4b+c)}}{2b} < 1$	$0 < k < \frac{\sqrt{5b^2 + 2bc + c^2} - b - c}{2b} < 1$

[35] We check the second order condition if the profit equation is concave in price.

[36] $((1-k)\times(w+u_r)-w) > 0$, $(w-c) > 0$, and $q > 0$

Table 6.2: Cost pass-through in the case of revenue-sharing contract
[Results for no-contract situation are adapted from Table 4.1]

Demand function	Simultaneous game		Wholesale leading game		Retail leading game		Contract ?	Relation (given the condition in Table 5.1)	RBP or FBP ?
	$\frac{dw}{dc}$	$\frac{dp}{dc}$	$\frac{dw}{dc}$	$\frac{dp}{dc}$	$\frac{dw}{dc}$	$\frac{dp}{dc}$			
Linear, $q = a - bp$	$\frac{2}{3}(1-k)$	$\frac{1}{3}$	$\frac{1-k}{2-k}$	$\frac{1}{2(2-k)}$	$\frac{3}{2(2+k)}$	$\frac{1}{2(2+k)}$	Revenue sharing	$\frac{dp}{dc} < \frac{dw}{dc} < 1$	FBP
	$<$	$=$	$<$	$>$	$<$	$<$			
	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	No-contract		
Isoelastic, $q = ap^{-l}$	$(1-k)\left(\frac{l-1}{l-2}\right)$	$\frac{l}{l-2}$	$(1-k)\frac{l}{k+l-1}$	$\left(\frac{l}{l-1}\right)\left(\frac{l}{k+l-1}\right)$	$\frac{(l^2-l+1)-k(l^2-1)}{(l-1)(l-k-1)}$	$\frac{l^2}{(l-1)(l-k-1)}$	Revenue sharing	$1 < \frac{dw}{dc} < \frac{dp}{dc}$	RBP
	$<$	$=$	$<$	$<$	$<$	$>$			
	$\frac{l-1}{l-2}$	$\frac{l}{l-2}$	$\frac{l}{l-1}$	$\left(\frac{l}{l-1}\right)^2$	$\frac{l^2-l+1}{(l-1)^2}$	$\left(\frac{l}{l-1}\right)^2$	No-contract		
Negative Exponential, $q = a\exp(-p/b)$	$1-k$	1	$1-k$	1	$1-k$	1	Revenue sharing	$\frac{dw}{dc} < 1; \frac{dp}{dc} = 1;$	
	$<$	$=$	$<$	$=$	$<$	$=$			
	1	1	1	1	1	1	No-contract	$\frac{dp}{dc} = \frac{dw}{dc} = 1$	No RBP /FBP -

Table 6.3: BP ratio between the retail and wholesale price in the case of revenue-sharing contract
[Results for no-contract situation are adapted from Table 4.2]

Demand function	$\frac{\sigma_P}{\sigma_W}$			Contract?	RBP or FBP? [within the limit of 'k']
	Simultaneous game	Wholesale leading game	Retail leading game		
Linear, $q = a - bp$	$\frac{1}{2(1-k)}$	$\frac{1}{2(1-k)}$	$\frac{1}{3}$	Revenue-sharing	(<1) FBP
	$\vee$	$\vee$	$=$		
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$	No contract	
Isoelastic, $q = ap^{-l}$	$\left(\frac{l}{l-1}\right)\frac{1}{(1-k)}$	$\left(\frac{l}{l-1}\right)\frac{1}{(1-k)}$	$\frac{l^2}{l^2-(l-1)-k(l^2-1)}$	Revenue-sharing	(>1) RBP
	$\vee$	$\vee$	$\vee$		
	$\frac{l}{l-1}$	$\frac{l}{l-1}$	$\frac{l^2}{l^2-(l-1)}$	No contract	
Negative Exponential, $q = ae^{\frac{-p}{b}}$	$\frac{1}{1-k}$			Revenue-sharing	(>1) RBP
	$\vee$				
	1			No contract	(=1) No RBP/FBP

6.2 Simulation results

Numerical simulations were run to illustrate the analytical results from the previous section. Simulation parameters were selected to satisfy the conditions shown in Table 6.3. Cost is assumed to be uniformly distributed between \$8 and \$10. It is to be mentioned, similar analysis can also be done assuming other types of distribution functions (i.e. Normal, Weibull etc.). Simulations were run for 300 times. Thus, we have 300 randomized cost data. Then we calculate the optimal wholesale and retail prices for the corresponding costs. Then, we compare the standard deviation of the cost and the prices. We consider linear ($q = 20 - p$), isoelastic ($q = ap^{-2.5}$), and negative exponential $\left(q = a\exp\left(-\frac{p}{8}\right)\right)$ demand functions. Table 6.4 shows the allowable limit of revenue share 'k' based on the parameter of this simulation. The limits are calculated based on Table 6.1. In this simulation, we consider 10% revenue-sharing (i.e. $k = 0.1$) that satisfies the condition (Table 6.4) for all nine possible situations (i.e. 3 demand functions in 3 game settings). Figure 6.1 shows the fluctuation of prices for linear, isoelastic, and negative exponential demand in different game settings, considering revenue share contracts.

Table 6.5 shows that the standard deviation of prices is gradually decreasing for a linear demand and increasing for a isoelastic demand. In other words, the fluctuation is damping for linear demand and amplifying for isoelastic demand functions. If we recall from section 6.3, the cost-pass-through for linear demand is less than one and for isoelastic demand, it is greater than one. Hence, FBP (i.e. decreased fluctuation) occurs for linear demand and RBP (i.e. amplified fluctuation) occurs for isoelastic demand when revenue-sharing contracts are considered. These conclusions of contract cases are similar to that of no-contract cases. However, in the case of a negative exponential demand, there is a remarkable difference in the fluctuation behavior. For a negative exponential demand function, we see the standard deviation is decreased when the wholesale price is compared to the cost, and then increased when the retail price compared to the wholesale price (Table 6.5). The standard deviation of the retail price is same as of cost. Thus, we can say that the wholesale price fluctuates less than the cost, and the retail price fluctuates more than the wholesale price but at the same rate as the cost. Figure 6.1 also shows, in case of a negative exponential demand, the fluctuation is reduced and then increased from cost to the wholesale price to the retail price. Moreover, the ratios of the standard deviations from simulation (Table 6.5) also match with the analytical values of the cost-pass-throughs (Table 6.2) as expected. (e.g. for linear demand with wholesale leading game, $\frac{\sigma_P}{\sigma_C} = \frac{0.159}{0.605} = 0.263$; $\frac{dp}{dc} = \frac{1}{4-2k} = \frac{1}{4-2(0.1)} = 0.263$).

Let's compare the results of simulation between revenue-sharing contract versus no-contract cases (Table 6.5). We use the same random seed for both simulations. Therefore, the randomized cost data is same; hence, the standard deviation of the cost is same ($\sigma_C = 0.605$). The standard deviation of the wholesale price is reduced under a contract situation compared to the no-contract situation. The standard deviation of the retail price remains same under both contract and no-contract situations in case of simultaneous games. In the case of wholesale-leading game, the standard deviation of the retail price is increased under a contract situation for linear demand and the standard deviation of the retail price is decreased under a contract situation for isoelastic demand. In case of the retail-leading game, it is vice versa. For a negative exponential demand, the standard deviation of the retail price remains the same as of the cost.

Table 6.4. Based on selected parameter, allowed value of 'k'

Demand function	Simultaneous game	Wholesale leading game	Retail leading game
Linear, $q = 20 - p$; $c < \frac{a}{b}$	$0 < k < 0.25$	$0 < k < 0.293$	$0 < k < 0.15$
Isoelastic, $q = ap^{-2.5}$	$0 < k < 0.67$	$0 < k < 0.285$	$0 < k < 0.67$
Negative Exponential, $q = a\exp(-p/8)$	$0 < k < 0.44$	$0 < k < 0.344$	$0 < k < 0.38$

Table 6.5. Results of simulation (10% revenue-sharing. i.e. $k = 0.1$)

	Simultaneous					Wholesale leading					Retail leading					Contacts/ No-contracts
	σ_C		σ_W		σ_P	σ_C		σ_W		σ_P	σ_C		σ_W		σ_P	
Linear, $q = 20 - p$	0.605	>	0.363	>	0.202	0.605	>	0.286	>	0.159	0.605	>	0.432	>	0.144	10% revenue share
		>	0.403	>	0.202		>	0.302	>	0.151		>	0.454	>	0.151	No contract
Isoelastic, $q = ap^{-2.5}$		<	1.633	<	3.024		<	0.851	<	1.575		<	1.217	<	1.8	10% revenue share
		<	1.814	<	3.024		<	1.008	<	1.680		<	1.277	<	1.680	No contract
Negative Exponential, $q = a\exp(-p/8)$		>	0.544	<	0.605		>	0.544	<	0.605		>	0.544	<	0.605	10% revenue share
		=	0.605	=	0.605		=	0.605	=	0.605		=	0.605	=	0.605	No contract

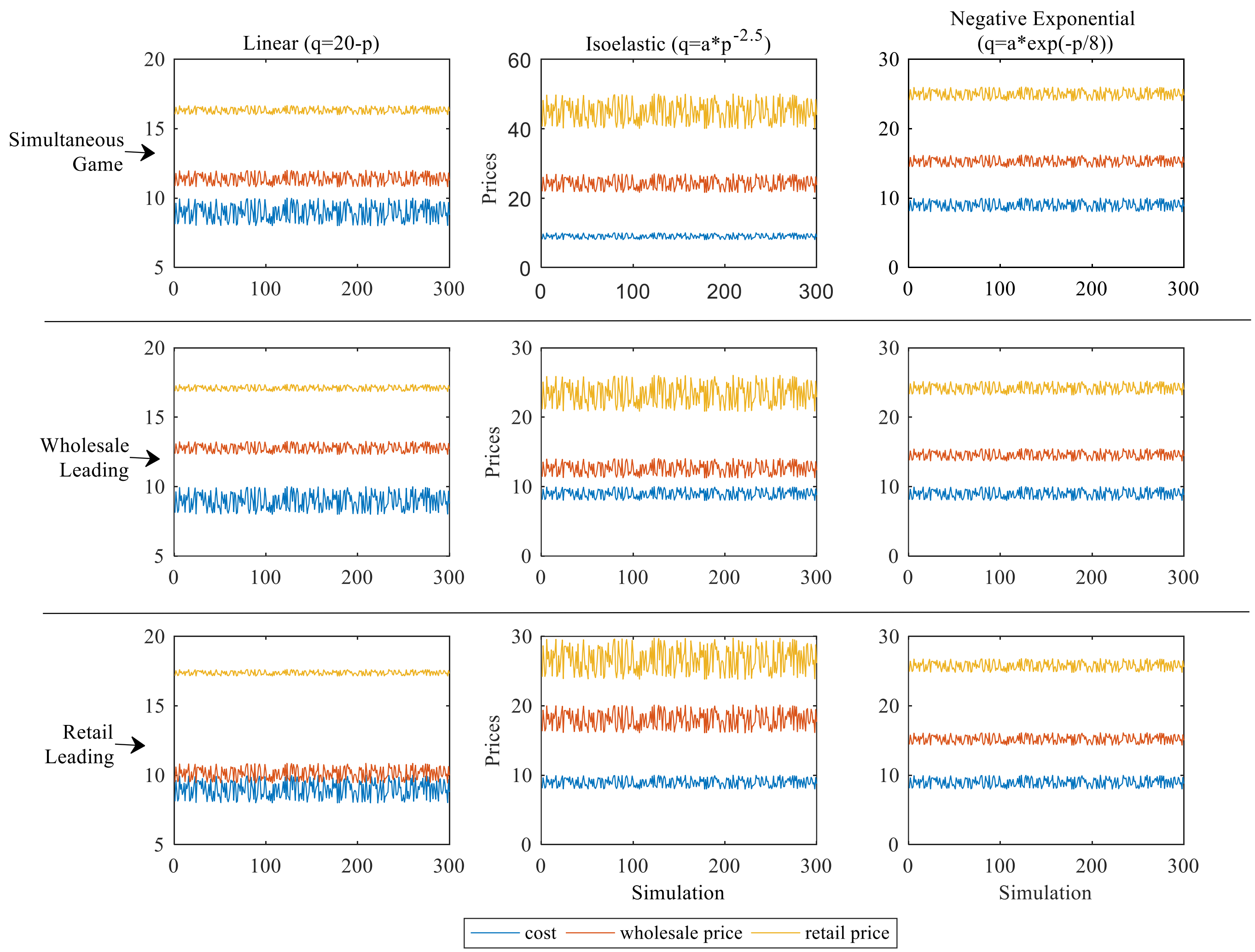

Figure 6.1: Price variation (Revenue-sharing Contract, $k = 0.1$)

6.3 Conclusion

In this chapter, we calculate the cost-pass-through and BP ratio considering a revenue-sharing contract and benchmark the results with that of the no-contract case. RBP occurs for the isoelastic demand function and FBP occurs for the linear demand function. For a negative exponential demand, FBP occurs at the wholesale stage and RBP occurs at the retail stage. The revenue share percentage also affects the quantitative value of the BP ratio. The fluctuation of the wholesale price is smoothed in case of a revenue-sharing contract. The standard deviation of the retail price remains same in the case of a simultaneous game. In the case of a wholesale-leading game, the standard deviation of the retail price decreases for an isoelastic demand and increases for a linear demand. In the case of a retail-leading game, it is vice versa.

CHAPTER 7: BP IN REVENUE-SHARE CONTRACT (STOCHASTIC DEMAND)

7.1 Introduction:

In this chapter, we consider the retailer problem as a price-setting newsvendor model[37] under a revenue sharing contract. The problem formulation for this newsvendor model is similar to what has been discussed in Chapter 5. The difference is the consideration of a revenue sharing contract. In such contract, the retailer keeps ϕ portion[38] of the revenue and shares $1-\phi$ portion with the wholesaler. It is to be mentioned, $0 < \phi < 1$. We apply a revenue-sharing contract in the Petruzzi-Dada's (1999) price-setting newsvendor model along with additive and multiplicative uncertainty. We deduce the optimal prices, analyze the retail cost-pass-through, and compare the price variation between the retail and the wholesale prices.

7.2 Model:

The problem formulation and the solution procedure[39] is similar to that of Chapter 5. However, the results are significantly different because of the adaptation of the revenue-sharing contract. The key difference under revenue-sharing practice is that the retailer

[37] Price-setting newsvendor model is discussed in Chapter 5.
[38] In chapter 6, the retailer keeps $1-k$ portion of the revenue. Hence, the results of this section (i.e. Chapter 7) can be benchmarked with the results of Chapter 6, according to the following relation: $\phi = 1-k$
[39] Stocking decision approach and pricing decision approach

shares sales revenue, and salvages at a reduced price v if leftover occurs. The retailer's profit can be expressed as following,

$$\pi_r = \begin{matrix} \phi pD - wq + v(q-D) & ; D \le q \\ \phi pq - wq - S(D-q) & ; D > q \end{matrix} \tag{1}$$

Table 7.1: Description of Parameters

Notation	**Description**
π_r	Retailer's profit
ϕ	Retailer's share from the sales revenue
p	Retail price
w	Wholesale price
D	Demand For additive case, $D = y + \epsilon$ For multiplicative case, $D = y\epsilon$
y	Deterministic part of the demand
ϵ	Random part of the demand
q	Order quantity
v	Salvage price
S	Shortage cost
z	Stocking factor For additive case, $z = q - y$ For multiplicative case, $z = q/y$
b	Elasticity of the demand function
μ	Expected value of the random variable ϵ
$\Theta(z)$	$\int_z^B (u-z)f(u)du$ For additive case, E[shortage] = $\Theta(z)$ For multiplicative case, E[shortage] = y$\Theta(z)$
$\Lambda(z)$	$\int_A^z (z-u)f(u)du$ For additive case, E[leftover] = $\Lambda(z)$ For multiplicative case, E[leftover] = y$\Lambda(z)$
$\mu - \Theta(\mathrm{z})$	Expected sales
$\frac{\Lambda}{(\mu - \Theta)}$	$\frac{E[leftover]}{E[sales]}$
$F(z)$	Cumulative Distribution Function
$f(z)$	Probability Density Function
$r(z) = \frac{f(z)}{1-F(z)}$	Hazard rate
$\frac{d\Lambda}{dz} = F(z)\,;\ \frac{d\Theta}{dz} = -[1-F(z)]$	Derivatives of Λ and Θ in z

The details of the problem formulation is mentioned in Appendix 1-A (additive demand uncertainty case) and 2-A (multiplicative demand uncertainty case), and the model parameters are described in Table 7.1. The lemmas and propositions are mentioned in the following sub-sections. In our analysis, we are using two forms of demand function such as a linear ($y = a - bp$) and an isoelastic ($y = ap^{-b}$) demand function.

7.2.1 Additive Demand Uncertainty Case:

Lemma 1a: Following the pricing decision approach for the single period buyback-newsvendor model with additive demand uncertainty, the optimal stocking factor z^* is determined as, $z^*(p) = F^{-1}\left[\frac{\phi p+S-w}{\phi p+S-v}\right]$; and the optimal p^* is the p that satisfies $\frac{d}{dp}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 0$. Hence,

1. For a linear demand, $p^*(w) = \left\{p \middle| -p + \frac{\phi a+bw}{2\phi b} + \frac{\mu - \Theta(z^*(p))}{2b} = 0\right\}$;
2. For an isoelastic demand, $p^*(w) = \left\{p \middle| -p + \frac{b}{\phi(b-1)} w + \frac{\mu-\Theta\left(z^*(p)\right)}{(b-1)ap^{-b-1}} = 0\right\}$.

Proof: Appendix 1-B-i

Lemma 1b: Following the stocking decision approach for the single period revenue-sharing-newsvendor model under a linear[40] demand with an additive uncertainty, the optimal price p^* is determined as $p^*(z) = \frac{\phi a+bw}{2\phi b} + \frac{\mu-\Theta(z)}{2b}$ and the optimal z^* is the unique z in the region $[A, B]$ that satisfies $\frac{dE\left[\Pi_r\left(z,p(z)\right)\right]}{dz} = 0$. Hence,

$$z^*(w) = \left\{z \middle| -(w - v) + \left(\frac{\phi a+bw}{2b} + \frac{\phi\left(\mu-\Theta(z)\right)}{2b} + S - v\right)[1 - F(z)] = 0\right\}$$

[40] For an isoelastic demand with additive uncertainty, the stocking decision approach is difficult due to lack of a closed-form solution of $p^*(z)$.

Proof: Appendix 1-B-ii

Proposition 1: In the case of a revenue-sharing-newsvendor model with an additive demand uncertainty, the retail cost-pass-throughs are as follows-

1. For a linear demand (i.e. $D = a - bp + \epsilon$), $\frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 - \frac{\phi(1+F)}{2br(\phi p+S-v)-\phi(1-F)}\right)$

2. For an isoelastic demand (i.e. $D = ap^{-b} + \epsilon$),

   $\frac{dp^*}{dw} = \frac{(1-F)^2 p^{2+b}\phi - abpf(w-v)}{(1-F)^3 p^{2+b}\phi^2 - abf(w-v)\big(w(b+1)-p\phi(b-1)\big)}$

where, $f(.)$ is the probability density function, $F(.)$ is the cumulative distribution function, $r(.) = \frac{f(.)}{1-F(.)}$ is the hazard rate, ϕ is the retailer's share

Proof: Appendix 1-D.

Corollary 1a: For $2b(\phi p^* + S - v)\frac{r}{\phi(1-F)} > 1$, FBP occurs in the case of revenue-sharing newsvendor model under linear demand with additive uncertainty.

Corollary 1b: The occurrence of RBP or FBP depends on the parametric values in the case of revenue-sharing newsvendor model under an isoelastic demand with additive uncertainty.

7.2.2 Multiplicative Demand Uncertainty Case:

Lemma 2a**:** Following the pricing decision approach for the single period revenue-sharing newsvendor model with a multiplicative demand uncertainty, the optimal stocking factor z^* is determined as, $z^*(p) = F^{-1}\left[\frac{\phi p+S-w}{\phi p+S-v}\right]$ and the optimal p^* is the p that satisfies $\frac{d}{dp}\left(E\left[\pi_r\big(p, z^*(p)\big)\right]\right) = 0$. Hence:

1. For a linear demand, $p^* = \left\{p \middle| -p + \frac{\phi a+bw}{2\phi b} + \frac{1}{2\phi} * X\big(w, z^*(p)\big) = 0\right\}$

2. For an isoelastic demand, $p^* = \left\{p \middle| -p + \frac{b}{(b-1)\phi} w + \frac{b}{(b-1)\phi} * X\big(w, z^*(p)\big) = 0\right\}$

Where, $X = \frac{(w-v)\Lambda(z^*(p)) + S\Theta(z^*(p))}{\left(\mu - \Theta(z^*(p))\right)}$

Proof: Appendix 2-B-i.

Lemma 2b: Following the stocking decision approach for the single period revenue-sharing newsvendor model with a multiplicative demand uncertainty, the optimal p^* is determined as:

1. For a linear demand, $p^*(z) = \frac{\phi a + bw}{2\phi b} + \frac{1}{2\phi} * X(z)$
2. For an isoelastic demand, $p^*(z) = \frac{b}{(b-1)\phi} w + \frac{b}{(b-1)\phi} * X(z)$

And the optimal z^* is the z is that satisfies $\frac{d}{dz}\left(E\left[\Pi_r\big(z, p^*(z)\big)\right]\right) = 0.$

$$z^*(w) = \left\{z \middle| y\left(-(w-v) + (\phi p^*(z) + S - v)\big(1 - F(z)\big)\right) = 0\right\}$$

Here, $y = ap^{*(-b)}$ for an isoelastic demand and $y = a - bp^*$ for a linear demand.

Proof: Appendix 2-B-ii

Lemma 3: Let's define, $X = \frac{(w-v)*E[leftover] + S*E[shortage]}{E[sales]}$ and $W = \frac{\phi(1-F)}{f(\phi p + S - \beta)} * \frac{\partial X}{\partial z}$

Then it follows-

1. If $S = 0$, then $W > 0$
2. If $S > 0$, then
    a. $W > 0$ if $(w-v)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) > S\frac{\mu}{(\mu-\Theta)}$
    b. $W < 0$ if $(w-v)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) < S\frac{\mu}{(\mu-\Theta)}$
3. W follows the sign of $\frac{\partial X}{\partial z}$.

Here, $\frac{F}{(1-F)} > \frac{\Lambda}{(\mu-\Theta)}$ is given.

Proof: Appendix 2C

Remark: In further discussion, we will be using these two parameters X and W frequently.

Proposition 2: The expected profit, $E\left[\Pi_r\left(p, z^*(p)\right)\right]$ is concave in p; or $E\left[\Pi_r\left(z, p^*(z)\right)\right]$ is concave in z for the given conditions-

1. For $y = a - bp, \frac{1}{2\phi} W < 1$
2. For $y = ap^{-b}, b > 1,\ \frac{b}{(b-1)\phi} W < 1$

where, W is defined in Lemma 3.

Proof: Appendix 2-D-i and 2-D-ii

Proposition 3: In the case of revenue-sharing newsvendor model with a multiplicative uncertainty in demand, the retail cost-pass-throughs are as follows-

1. For linear demand (i.e. $D = (a - bp)\epsilon$), $\frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{(1-F)} - \frac{1}{2}\right) * \frac{W}{\phi}}{1 - \frac{1}{2} * \frac{W}{\phi}}\right)$
2. For isoelastic demand (i.e. $D = (ap^{-b})\epsilon$), $\frac{dp^*}{dw} = \frac{b}{(b-1)\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{1-F} - \frac{b}{(b-1)}\right)\frac{W}{\phi}}{1 - \frac{b}{(b-1)} * \frac{W}{\phi}}\right)$

where, W is defined in Lemma 3 and Proposition 2, and $F(.)$ is the cumulative distribution function.

Proof: Appendix 2-E-i (pricing decision approach) and Appendix 2-E-ii (stocking decision approach).

Corollary 2: Comparisons of the retail cost-pass-throughs between the case of a revenue-sharing newsvendor model with multiplicative demand uncertainty and a revenue-sharing risk-less model are as follows in Table 7.2.

Table 7.2: Comparison of the cost-pass-through between the optimal price and the risk-less price.

	Retail Cost-pass-through	Condition
Linear Demand	$\frac{dp^*}{dw} < 1$	$0 < \frac{W}{2} < 1$
	$\frac{dp^*}{dw} > \frac{1}{2\phi}$	$\frac{W}{2\phi} < \frac{\Lambda}{(\mu - \Theta)} * \frac{(1-F)}{(1+F)} < 1$
	$\frac{dp^*}{dw} < \frac{1}{2\phi}$	$0 < \frac{\Lambda}{(\mu - \Theta)} * \frac{(1-F)}{(1+F)} < \frac{W}{2\phi} < 1$
	$\frac{dp^*}{dw} = \frac{1}{2\phi}$	$0 < \frac{W}{2\phi} = \frac{\Lambda}{(\mu - \Theta)} * \frac{(1-F)}{(1+F)} < 1$
Isoelastic Demand	$\frac{dp^*}{dw} > \frac{b}{(b-1)\phi}$	$\frac{\Lambda}{(\mu - \Theta)} > \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi}$
	$\frac{dp^*}{dw} < \frac{b}{(b-1)\phi}$	$0 < \frac{\Lambda}{(\mu - \Theta)} < \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi}$
	$\frac{dp^*}{dw} = \frac{b}{(b-1)\phi}$	$0 < \frac{\Lambda}{(\mu - \Theta)} = \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi} < 1$

Proof: Appendix 2-E-iii

7.2.3 Discussion on the propositions and corollary:

We analyze the change of p^* in w by exploring the cost-pass-through rates which are mentioned in Proposition 1 and 3, and Corollary 1 and 2.

The cost-pass-through of the risk-less-price for a linear demand under a revenue-share contract is $\frac{1}{2\phi}$.[41] In the case of an additive demand uncertainty, the optimal price is

[41] $\frac{dp^0}{dw} = \frac{d}{dw}\left(\frac{\phi a + bw}{2\phi b}\right) = \frac{1}{2\phi}$; It also conforms the results of chapter 6 in the case of wholesale leading game.

less than the risk-less-price [see Lemma 1]; hence, the cost-pass-through of the optimal price is less than $\frac{1}{2\phi}$ [see Proposition 1]. Therefore, the cost-pass-through doesn't exceed 1. Therefore, FBP occurs for the linear demand with an additive uncertainty. In the case of a multiplicative demand uncertainty, the cost-pass-through of the optimal price varies but remains less than one [Corollary 2]. Therefore, FBP occur in this setting as well.

For an isoelastic demand, the cost-pass-through of the risk-less price in the case of a revenue-sharing contract is $\frac{b}{(b-1)\phi} > 1$. The cost-pass-through of the optimal price varies from less than $\frac{b}{(b-1)\phi}$ to greater than $\frac{b}{(b-1)\phi}$. When, the cost-pass-through is equal or greater than $\frac{b}{(b-1)\phi}$, it is clearly greater than one. Hence, RBP occurs. If $\frac{dp^*}{dw}$ is less than $\frac{b}{(b-1)\phi}$, there are two possible scenarios such as $\frac{dp^*}{dw}$ remains greater than one or it reduces to less than one. In order to conclude for any valid condition that would make $\frac{dp^*}{dw}$ less than one, the argument $0 < \frac{1+\frac{\Lambda}{(\mu-\Theta)}-\left(\frac{1}{1-F}\right)\frac{W}{\phi}}{1-\left(\frac{b}{b-1}\right)\frac{W}{\phi}} < \frac{(b-1)\phi}{b}$ is needed to be verified where $0 < \frac{\Lambda}{(\mu-\Theta)} < 1$, $\left(\frac{b}{b-1}\right)\frac{W}{\phi} < 1$, $b > 1$, and $0 < F < 1$ are given. Otherwise, $\frac{dp^*}{dw}$ is greater than one for isoelastic demand in all possible scenarios. Hence, RBP occurs for isoelastic demand under a revenue sharing contract.

7.3 Numerical analysis:

In this section, we present the results of the numerical analysis for the revenue sharing contract case [Details are provided in Appendix 1-E and 2-F]. In the case of additive uncertainty case, we follow the stocking decision approach for a linear demand and the pricing decision approach for an isoelastic demand. For the multiplicative demand uncertainty case, we follow the pricing decision approach for both a linear and an isoelastic demand. We consider three different values of the revenue-share percentage (e.g. $\phi = \{1, 0.85, 0.7\}$)[42]. The parameters are chosen randomly for illustration purpose (Table 7.3). The results are illustrated in the following sub-sections.

Table 7.3: Parameters used in the numerical simulation

Deterministic part	Uncertainty	Distribution, ϵ	Shortage cost, S	Salvage price, v	Retailer's share, ϕ	Price Range
Linear ($y = 100 - p$)	Additive	Uniform$[-10,10]$	10	15	0.7, 0.85, 1	N/A
Isoelastic ($y = 10^6 p^{-3}$)	Additive	Uniform$[-5,5]$	10	15	0.7, 0.85, 1	Smaller and larger price
Linear ($y = 100 - p$)	Multiplicative	Uniform$[1,3]$	20	5	0.8, 0.9, 1	N/A
Isoelastic ($y = ap^{-3}$)	Multiplicative	Uniform$[1,3]$	20	5	0.8, 0.9, 1	N/A

7.3.1 Additive Demand Uncertainty Case:

Let's assume a linear demand ($D = 100 - p + \epsilon$) and an isoelastic demand $\left(D = \frac{10^6}{p^3} + \epsilon\right)$, a shortage cost of $S = 10$, and a salvage price of $v = 15$, and a uniform distribution on the interval $[-10,10]$ for the linear case and $[-5,5]$ for the isoelastic case. The optimal results are as follows-

7.3.1.1 Linear demand:

Figure 7.1 shows the price comparison and Figure 7.2 shows the corresponding cost past through for three different revenue-sharing percentages. It also shows that the cost-pass-through of the optimal price is less than that of the risk-less price. The cost-pass-

[42] $\phi = 1$ corresponds to no-revenue share contract; $\phi = 0.85$ refers to a revenue-sharing contract where retailer keeps 85% of the sales revenue.

through remains less than 1 for various values of the revenue-share percentages. Figure 7.3 illustrates the price fluctuations that shows the variability is absorbed in retail price compared to the wholesale price. Hence, FBP occurs for the linear demand with an additive demand uncertainty.

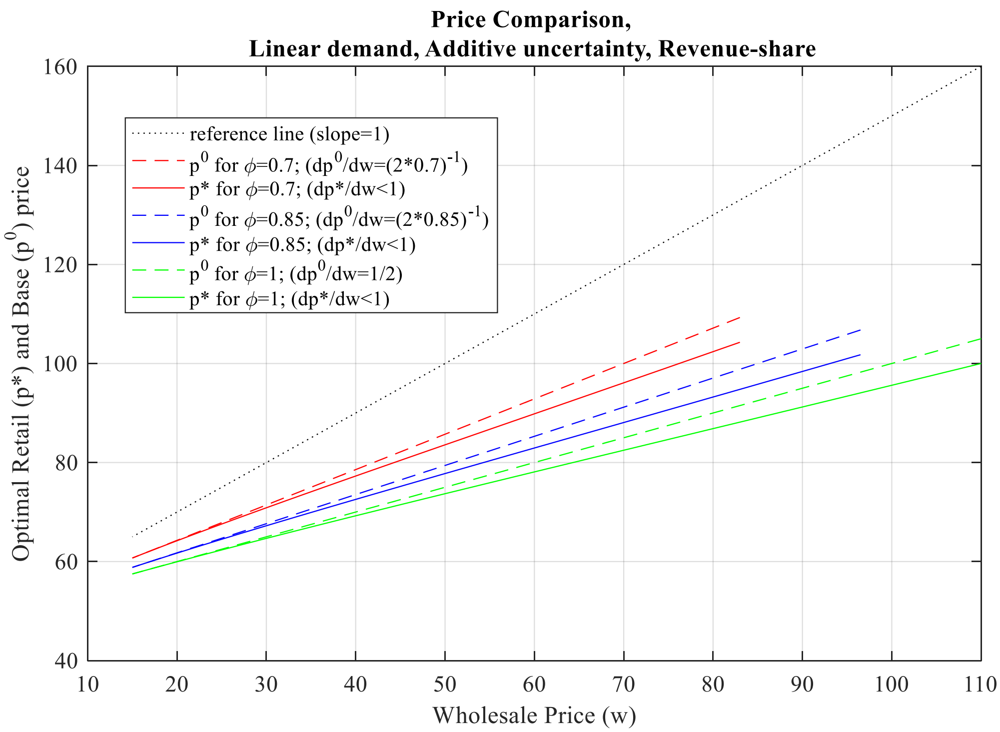

Figure 7.1: Price Comparison (Linear Demand; additive uncertainty)

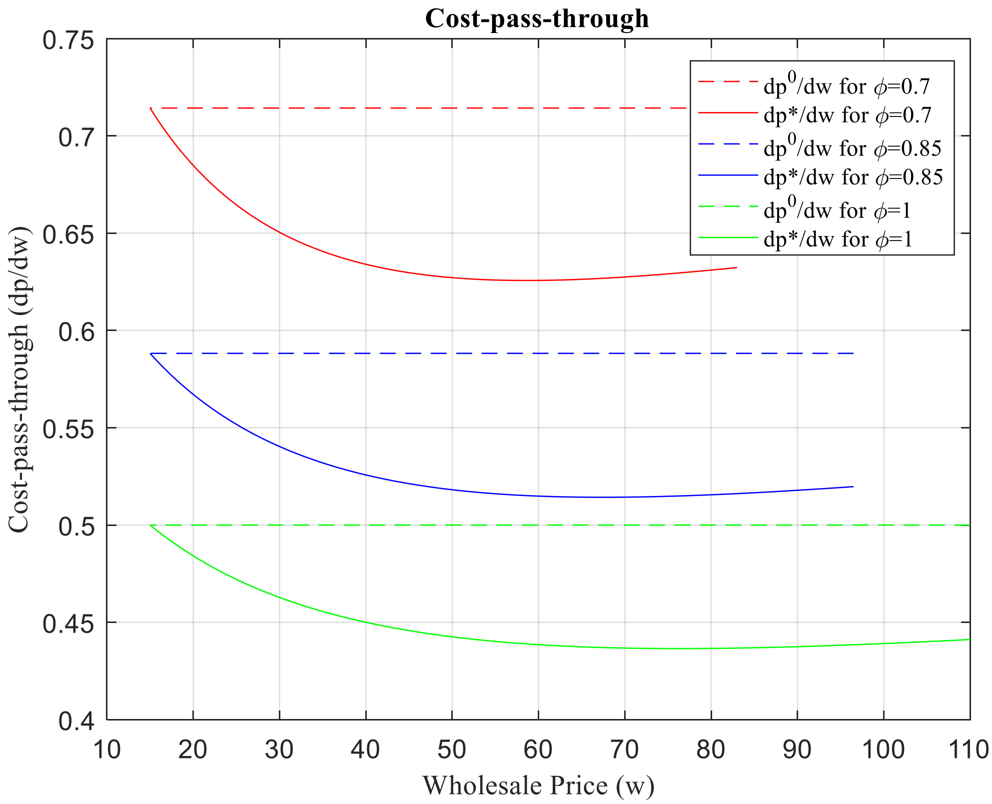

Figure 7.2: Cost-pass-through (Linear Demand, Additive Uncertainty)

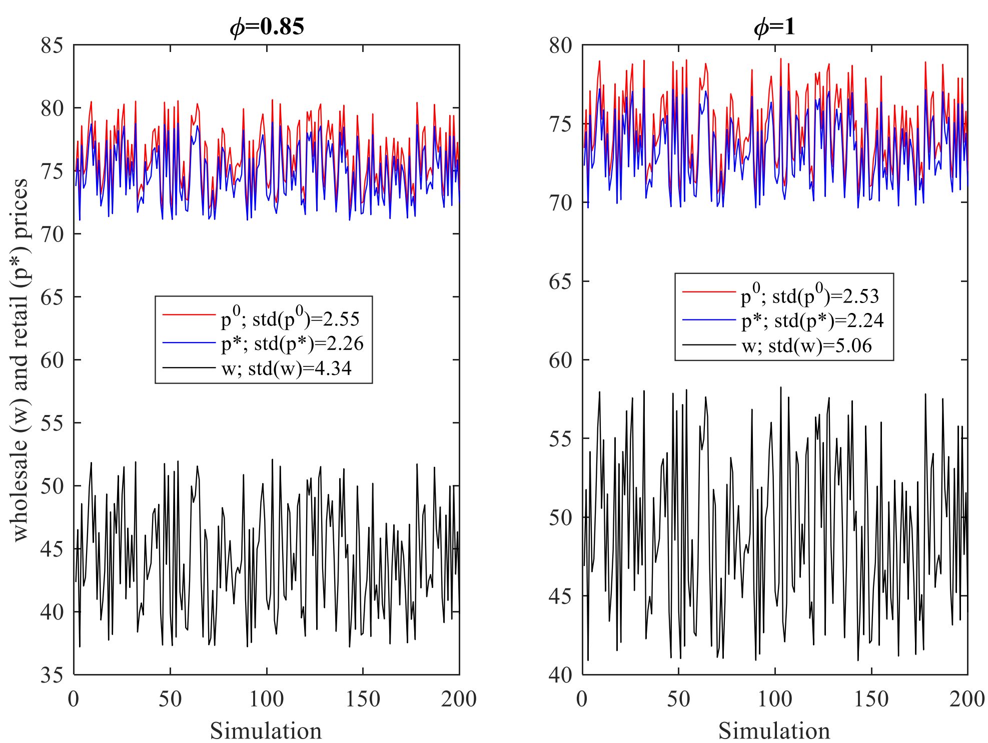

Figure 7.3: Occurrence of FBP (Linear demand; additive uncertainty)

7.3.1.2 Isoelastic demand:

Figure 7.4 shows the price comparison that reflects optimal retail price is less than risk-less price as expected. The dotted line shows the risk-less price. The slope of the risk-less is price is a constant but the slope of the optimal retail prices is concave. It is evident from the figure, that the slope is gradually decreasing in price. Figure 7.5 shows more clearly, that the cost-pass-through is decreasing and cross the horizontal line with slope=1 at certain point. This phenomenon is crucial. Because it reflects, that the cost-pass-through is greater than for some price but less than one for some other range of prices. Hence, isoelastic demand with additive uncertainty shows both RBP and FBP (Ozelkan and Lim 2008). For different revenue-share percentages, the cost-pass-through changes.

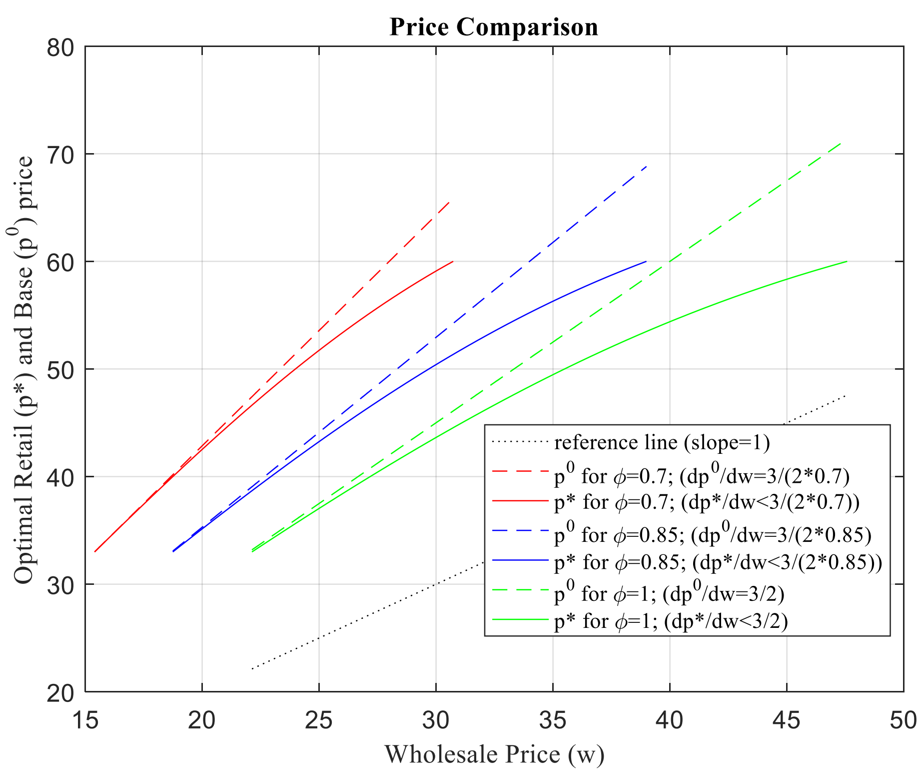

Figure 7.4: Price Comparison (Isoelastic Demand; additive uncertainty)

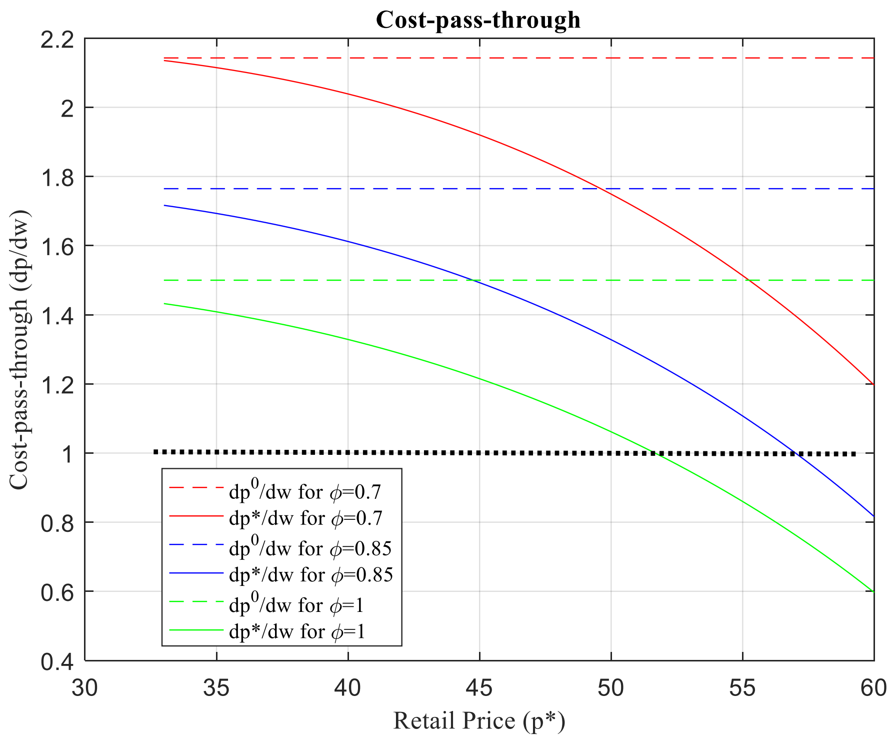

Figure 7.5: Cost-pass-through (Isoelastic Demand; additive uncertainty)

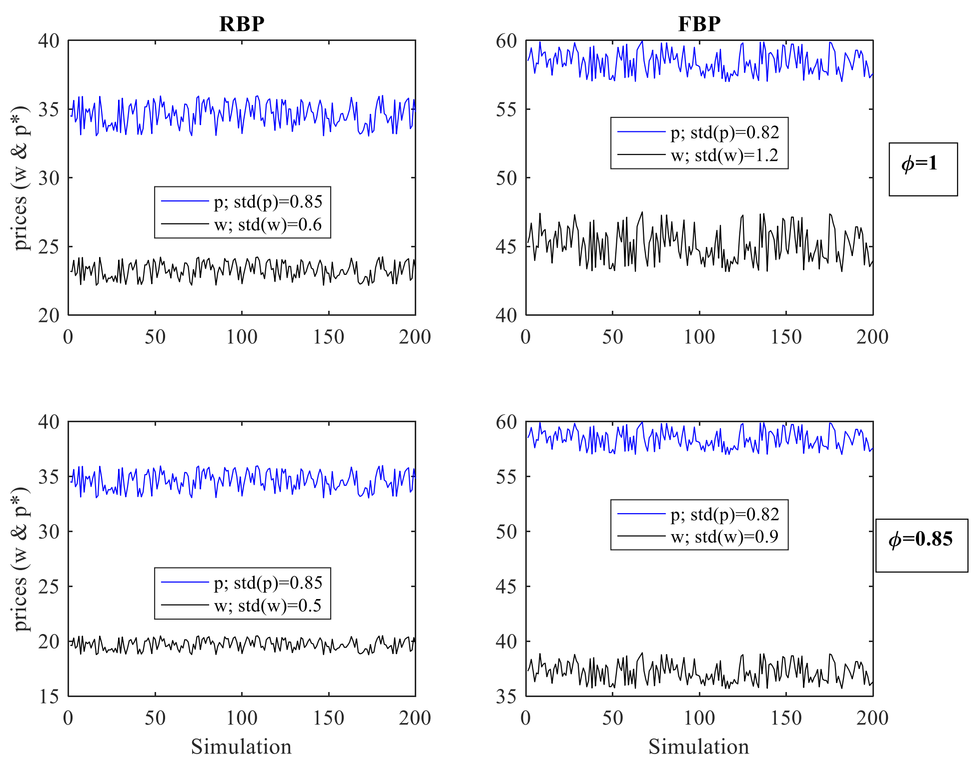

Figure 7.6: RBP and FBP (Isoelastic Demand; additive uncertainty)

It is seen from figure 7.5 that RBP and FBP both exist for different ranges of the optimal price; similarly, figure 7.6 illustrates the phenomenon in terms of standard deviations. Figure 7.6 compares two cases of larger and smaller retail prices for both with ($\phi = 0.85$) or without ($\phi = 1$) a revenue-sharing contract.

7.3.2 Multiplicative Demand Uncertainty Case:

We assume a shortage cost of $S = 20$, salvage price of $v = 5$, and a uniform distribution on the interval $[1,3]$. We consider a linear ($D = (100 - p)\epsilon$) and an isoelastic ($y = (ap^{-3})\epsilon$) demand.

7.3.2.1 Linear Demand:

Figure 7.7 and 7.8 shows the price comparison and the corresponding cost-pass-through for a linear demand ($y = 100 - p$) with multiplicative uncertainty (uniform[1,3]). As Corollary 2 dictates, the cost pass through varies from less than half to greater than half. However, the cost-pass-through remains less than one for $\phi = 0.8, 0.9$ and 1. Hence, FBP occurs in these settings as well which is also illustrated in Figure 7.9.

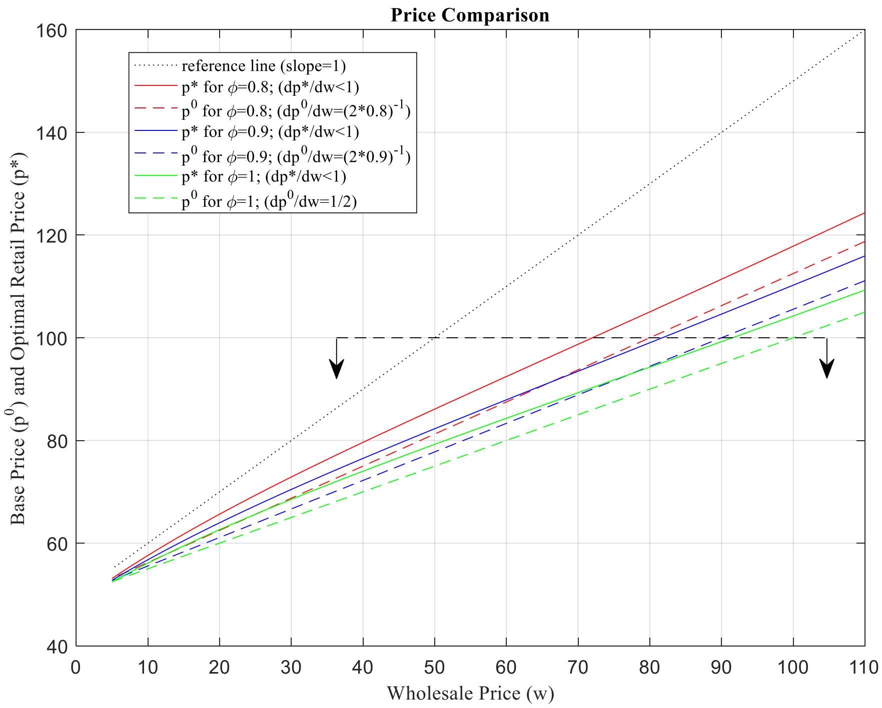

Figure 7.7: Price Comparison (Linear demand; Multiplicative uncertainty)

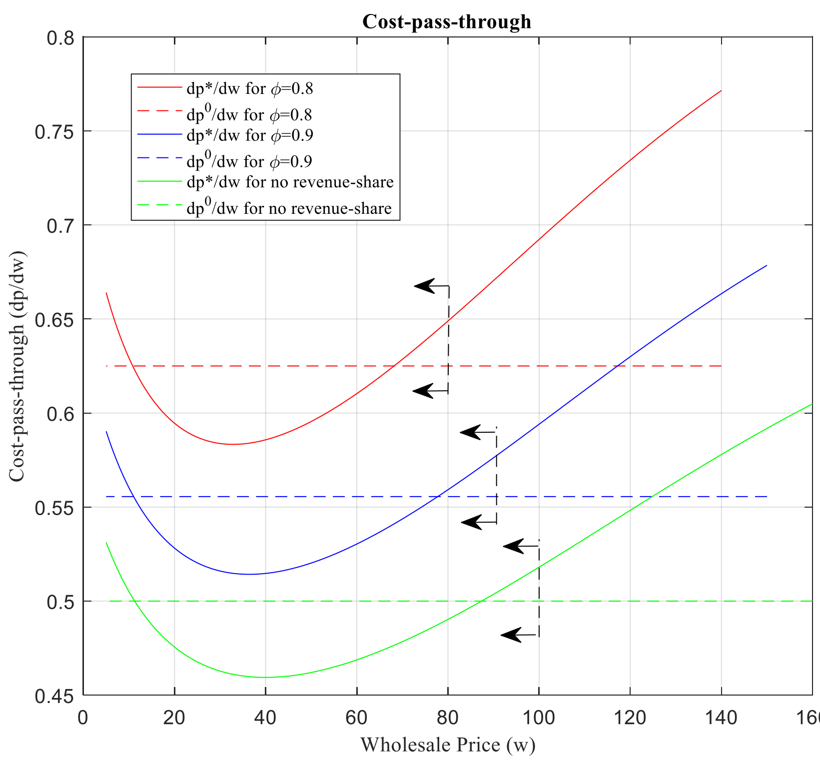

Figure 7.8: Cost-pass-through (Linear Demand; Multiplicative uncertainty; Revenue-share)

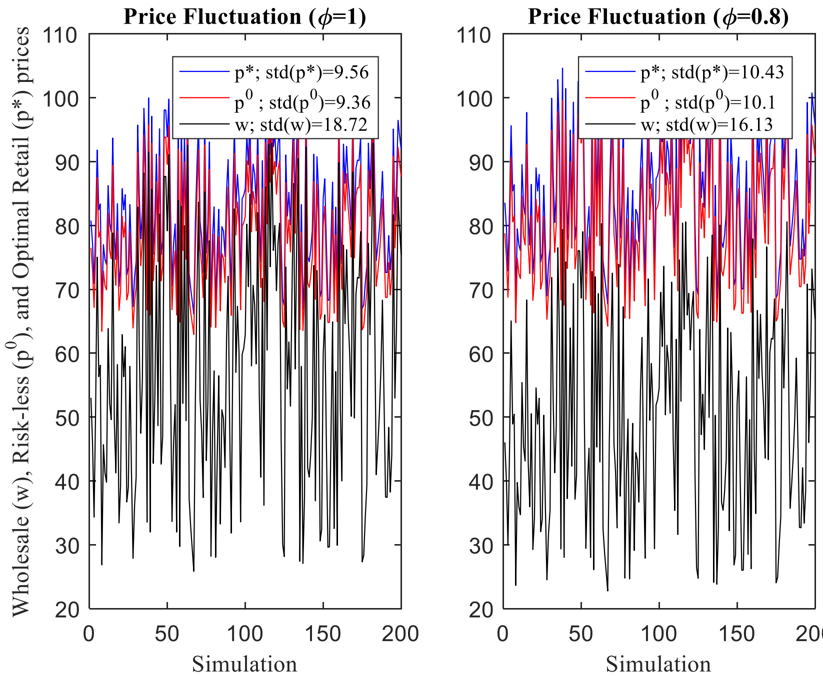

Figure 7.9: Occurrence of FBP (Linear Demand; Multiplicative uncertainty; Revenue-sharing)

7.3.2.2 Isoelastic Demand:

Here, we consider an isoelastic demand ($y = 1000p^{-3}$) with multiplicative uncertainty. For $b = 3$ (as we assumed in this numerical analysis), the cost-pass-through of the risk-less price is $\frac{3}{2\phi}$ which is greater than one. Figure 7.10 shows, the cost-pass-through of the optimal price is more than that of the risk-less-price; hence, remains more than one (Figure 7.11). Therefore, RBP occurs for isoelastic demand (Figure 7.12). Figure 7.10, 7.11, and 7.12 also shows that the cost-pass-through is increasing for decreasing values of ϕ (i.e. retailer's share).

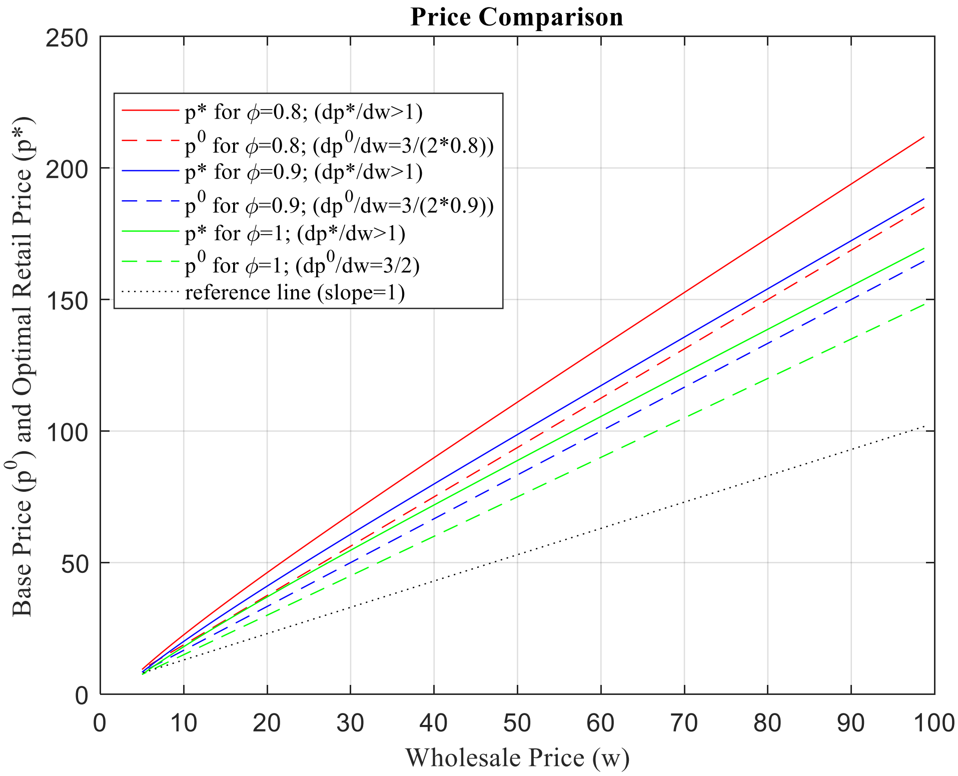

Figure 7.10: Price Comparison (Isoelastic demand; Multiplicative Uncertainty)

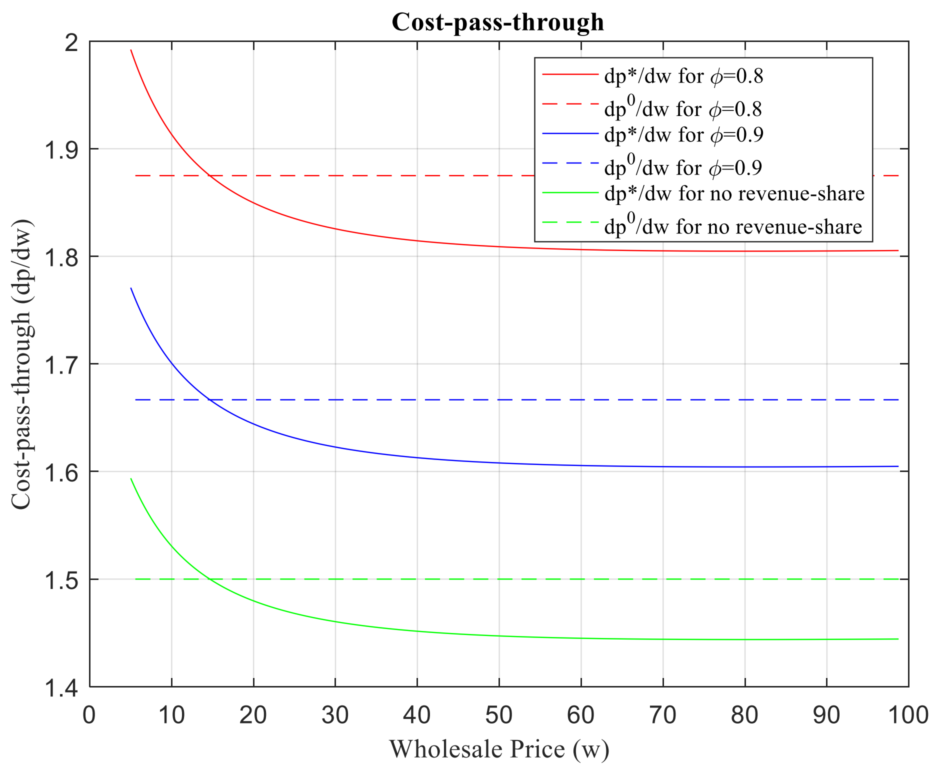

Figure 7.11: Cost-pass-through (Isoelastic demand, Multiplicative Uncertainty)

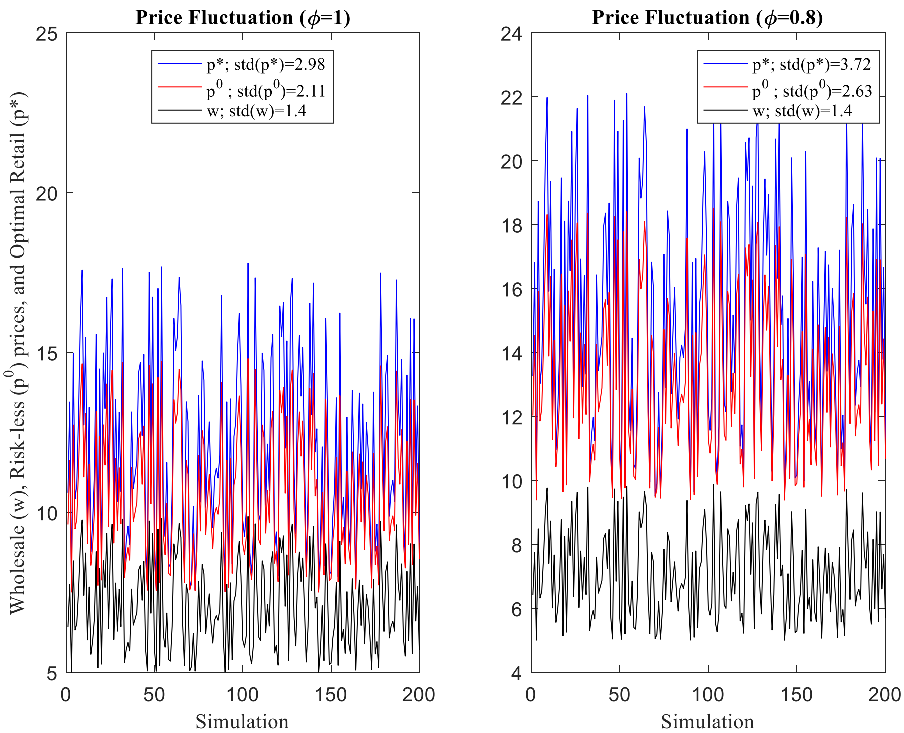

Figure 7.12: Occurrence of RBP (Isoelastic Demand; Multiplicative Uncertainty)

From simulations, we observe that the cost-pass-through is inversely proportional to the value of ϕ (Figure 7.2, 7.5, 7.8, and 7.11). Increased cost-pass-through implies that price is changing more for marginal change in cost. Therefore, the retail price fluctuation is increased for a revenue share contract (Figure 7.3, 7.6, 7.9, and 7.12).

7.5 Conclusion:

We analyze the cost-pass-through in the case of a revenue-sharing contract with a stochastic demand, and conclude on the occurrence of Bullwhip effect in Price (BP). Compared to the deterministic case, the stochasticity increases or decreases the optimal price in the case of a multiplicative or an additive demand uncertainty cases respectively.

Hence, the cost-pass-through also changes. However, the value of cost-pass-through doesn't exceed one for a linear demand or doesn't reduce to less than one for isoelastic demand. Therefore, the occurrence of BP doesn't change with the consideration of stochasticity. However, the value of the cost-pass-through and the BP ratio changes for different values of revenue-share percentage. Compared to a no-revenue share case, retail price fluctuation increases for the case of revenue share contract.

CHAPTER 8: SUMMARY, CONCLUSIONS, LIMITATIONS, AND FUTURE RESEARCH

In this section, we summarize the research outcomes, identify limitations, and suggest some future directions.

8.1 Summary and Major Conclusions

In this research, we analyzed both the amplified and the absorbed fluctuations in a product's price in a linear supply chain, and named it as the Bullwhip effect in Pricing (BP). We showed some empirical evidences of BP from beef, potato, oil, and coffee markets.

We analyzed the conditions for the occurrence of BP (Chapter 3), and found that the demand functions with a 'concavity coefficient' less than one results in a 'cost-pass-through' that is less than one, which eventually creates Forward Bullwhip Effect in Pricing (i.e. absorbed variability in retail prices). If the 'concavity coefficient' is equal to one, then no BP occurs (i.e. price fluctuation neither amplified nor absorbed).

We discussed the occurrence of BP for some common demand functions. The analysis showed that FBP occurs for log-concave (e.g. a linear demand as a special case), logit, and logistic demand functions, and RBP occurs for isoelastic demand functions. A logarithmic demand function can result in RBP, FBP or no BP depending on the range of the optimal price. Negative exponential demand creates no BP.

We also discussed the occurrence of BP in some common pricing practices such as the fixed-dollar markup, and the fixed-percentage markup pricing. A fixed dollar-markup pricing gives no BP; on the other hand, a fixed-percentage markup pricing strategy creates RBP. It is to be mentioned, in the case of the fixed markup pricing, the demand functions do not play any role in the pricing.

In the case of optimal pricing, the pricing decision is directly related to the demand functions and the supply chain structures. Primarily, the condition for the occurrence of BP was identified based on the concavity coefficient. We assumed a linear two-firm model where a retailer and a wholesaler interact following the Stackelberg wholesale-leading game structure. Later, we also considered a Nash (simultaneous) game and a Stackelberg retail-leading game structures. The pricing results for different game structures gives different rates of the cost-pass-through that eventually change the BP ratios. However, the direction of BP remains the same. FBP occurs for a log-concave, and a linear demand; RBP occurs for an isoelastic demand; no BP occurs for a negative exponential demand. For different game structures, BP ratios change. For example, under a linear demand, in the case of a simultaneous and a wholesale leading game, the retail price fluctuates 50% less than the wholesale price; while in the case of a retail leading game, the retail price fluctuation is 33.33% of the wholesale price fluctuation. In contrast, under an isoelastic demand with a price-elasticity of 3, the retail price fluctuates 1.5 times more than the wholesale price in the case of simultaneous and wholesale-leading games, while in the case of a retail-leading game, the retail price fluctuates 1.28 times more than the wholesale price.

We also extended the cost-pass-through and BP-ratio analysis for an N-stage linear supply chain. We see that the cost-pass-throughs are decreasing towards downward for a linear demand, increasing towards downward for an isoelastic demand, and remaining constant at 1 for a Negative exponential demand. The BP ratio between two consecutive stages is a constant for a wholesale leading game; but for a simultaneous and a retail leading game, the BP ratio changes with the number of the stages.

To analyze the effect of supply chain contracts on BP, we considered two popular contracts: the Buyback and the Revenue-sharing contracts. To apply the buyback contract, we considered a stochastic model (It is to be mentioned, there is no need of applying a buyback/return policy for a deterministic model). After that, we analyzed the revenue sharing contract in both deterministic and stochastic demand cases.

We considered a newsvendor model under a buyback contract with two types of demand uncertainties, additive and multiplicative types. Compared to a risk-less model (i.e. no uncertainty in demand), an additive uncertainty decreases the optimal price and a multiplicative uncertainty increases the optimal price. Hence, the cost-pass-throughs and BP ratios changes. We benchmarked the cost-pass-throughs of the buyback model with that of a risk-less model. We identified conditions for various ranges of the cost-pass-through and hence decided on the occurrence of BP. We presented numerical simulations considering two buyback prices and two uniform distributions. While, RBP and FBP still occur under a buyback contract for an isoelastic and a linear demand, respectively, the cost-pass-throughs and BP ratios vary based on the selected parameters.

In the case of a revenue-sharing contract with a deterministic demand, the results are similar to the optimal markup pricing games discussed earlier. However, the cost-pass-through rates and BP ratios are inversely proportional to the retailer’s share from the sales revenue. A remarkable difference occurs in the case of a negative exponential demand, where FBP occurs at the wholesale stage and RBP occurs at the retail stage. Compared to a no-contract cases, the variability of the retail and wholesale prices are also changed under a revenue-sharing contract that is explained by the corresponding markups under the contract situation.

In the case of a revenue-sharing contract with a stochastic demand, we followed a newsvendor model. We proposed the condition to identify the range of the cost-pass-throughs under revenue-sharing contracts with additive and multiplicative demand uncertainties. We analyzed the effect of different revenue-share percentages on the cost-pass-throughs under revenue-sharing contracts with a stochastic demand. We benchmarked the price variations of the optimal retail and base prices; we also benchmarked the price-variation between the revenue-sharing contract case and the no-contract case.

Hence, we analyzed the conditions for the occurrence of BP in different game structures and supply chain contracts cases. We illustrated examples in each case through numerical simulations. In the following sub-sections, we discuss some of the limitations of our research and suggest the future directions.

8.2 Limitations of This Research and Future Directions:

In our model, we considered a single objective of profit maximization. Other objectives beside the profit maximization (e.g. economic, environmental and social performances etc.) can also be considered. Multi-objective decision making may be applicable here (e.g. Boukherroub, Ruiz, Guinet, & Fondrevelle, 2015; Chankong & Haimes, 2008).

In our game model, we assumed the participants as risk-neutral. Consideration of human behaviors (e.g. risk aversion or seeking) may make the model more realistic. (Anupindi & Bassok, 1999; Wu, Roundy, Storer, & Martin-Vega, 1997).

We considered two popular contracts: Buyback and Revenue-Sharing contracts. Effects of other contracts (e.g. quantity discount, price discount, sales rebate etc.) on BP may be analyzed in future.

In the numerical analysis, we selected various values of parameters randomly. A better 'design of experiments' can be implemented while selecting the parameters for the simulation.

In chapters 4 and 6, we explained some of the amplification and absorption in variations by analyzing the corresponding per-unit markups. More attention can be given to explore the formal relationship between the bullwhip effect in pricing and the supply chain profit. In tying the relationship between profit and the occurrence of BP, our results can be merged with the result of with Adachi and Ebina (2014b). Adachi and Ebina (2014b) related the cost-pass-through with the retail markup and wholesale markup. However, they only considered a wholesale leading game. To match with our results, simultaneous and retail leading games, and the supply chain contracts cases need to be considered as well.

As a future research direction, we suggest to do the analysis for competing retailers. In the case of pricing competition among retailers, Moorthy (2005)'s methodology can be followed to calculate the cost-pass-through. Moorthy (2005) considered two retailers with catalog goods and brand items; then calculated cost-pass-throughs assuming Bertrand competition (i.e. pricing decision). Adachi and Ebina (2014a) calculated cost-pass-throughs for competing retailers under Cournot competition (i.e. quantity decision). Both models can be helpful in analyzing BP for competing retailers.

We also suggest to analyze the interaction between BP and BW effect in order quantity. In tying the relationship between BP and BW, researchers may follow the argument of Lee, Padmanabhan, and Whang (2004) where Lee argued the price variation as a cause for BW.

We conclude that this dissertation helps us to understand the occurrence and the directions of the Bullwhip effect in Pricing (BP). Since, empirical data from various markets shows different types of price fluctuations, the analytical models of this dissertation can help to explore the price fluctuations in various market structures. The amplified or absorbed price fluctuation reflects retailer responses to wholesaler's trade deals. We also explained how the cost-change passes through various stages of the supply chain in the case of pricing decisions. The models (e.g. different games, contracts, and demand functions) of this dissertation set the foundation for future analytical and empirical analysis of price fluctuations for various products and markets. Finally, we believe that further research along with analytical models and numerical simulations can shed more lights in the analysis of the Bullwhip effect in Pricing.

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APPENDIX

Appendix for Chapter 4: BP in Varying Game Structures

1. Game description:

Simultaneous game:

Both wholesaler and the retailer decide on their markup prices simultaneously. Retailer's objective: $\max_{u_r} \Pi_R = u_r * q(w, u_r)$ subject to, $u_r, q(w, u_r) > 0$.
Wholesaler's objective: $\max_{u_w} \Pi_W = u_w * q(c, u_w, u_r)$ subject to, $u_w, q(c, u_w, u_r) > 0$.
Decisions:

$$u_r|_{\frac{\partial \Pi_R}{\partial u_r}=0} = \frac{-q(w,u_r)}{\frac{\partial q(w,u_r)}{\partial u_r}} = \frac{-q(c,u_w,u_r)}{\frac{\partial q(c,u_w,u_r)}{\partial u_r}}. \quad 3(1)$$

$$u_w|_{\frac{\partial \Pi_W}{\partial u_w}=0} = \frac{-q(c,u_w,u_r)}{\frac{\partial q(c,u_w,u_r)}{\partial u_w}}. \quad 3(2)$$

Solving Equation 3(1) and 3(2), we get $\frac{u_r^*}{u_w^*} = \frac{\frac{\partial q(c,u_w^*,u_r^*)}{\partial u_w^*}}{\frac{\partial q(c,u_w^*,u_r^*)}{\partial u_r^*}} = 1$. As, $w = c + u_w^*(c)$ and $p = c + u_w^*(c) + u_r^*(c)$, therefore we can calculate $\frac{dw}{dc}$ and $\frac{dp}{dc}$.

Wholesale leading game:

The wholesaler declares its wholesale markup first, then the retailer decides on its retail markup price considering the wholesale price (i.e. sum of cost and wholesale markup) as given. As the retail price governs the demand function, the wholesaler considers the reaction function of the retailer for its own decision (i.e. wholesale markup price). The wholesaler anticipates the retailer's reaction as- $\max_{\overline{u_r}} \overline{\Pi_R} = \overline{u_r} q(w, \overline{u_r})$ subject to, $\overline{u_r}, q(w, \overline{u_r}) > 0$. Hence,

$$\overline{u_r}|_{\frac{\partial \overline{\Pi_R}}{\partial u_r}=0} = \frac{-q(w,\overline{u_r})}{\frac{\partial q(w,\overline{u_r})}{\partial \overline{u_r}}} = \frac{-q(c,u_w,\overline{u_r})}{\frac{\partial q(c,u_w,\overline{u_r})}{\partial u_r}}. \quad 3(3)$$

Wholesaler's objective: $\max_{u_w} \Pi_W = u_w q(c, u_w, \overline{u_r}(u_w))$ s.t. $u_w, q(c, u_w, \overline{u_r}(u_w)) > 0$

$$u_w^*|_{\frac{\partial \Pi_W}{\partial u_w}=0} = \frac{-q(c,u_w,\overline{u_r}(w))}{\frac{\partial q(c,u_w,\overline{u_r}(w))}{\partial u_w}}. \quad 3(4)$$

In Equation 3(3), by setting $u_w = u_w^*$, we calculate the retailer's optimal decision, $u_r^* = \frac{-q(c,u_w^*,u_r^*)}{\frac{\partial q(c,u_w^*,u_r^*)}{\partial u_r^*}}$. As, $w = c + u_w^*(c)$and $p = w(c) + u_w^*(c)$, therefore we can calculate $\frac{dw}{dc}$ and $\frac{dp}{dc}$. Using the chain rule, we can also calculate $\frac{dp}{dw}$, because p can be expressed as a function of w in case of wholesale leading game.

Retail leading game:

Retailer declares it retail markup first, then the wholesaler decides on its wholesale markup price considering the cost and the retail markup as given. As wholesale price affects the retailer's profit, therefore the retailer considers the reaction function of the wholesaler for its own decision (i.e. retail markup price). In the case of a retail leading game, the decision order and functional relation can be written as,

$$c \rightarrow u_r^*(c) \rightarrow u_W^*(c, u_r^*) \rightarrow w(c, u_W^*) \rightarrow p(w, u_r^*) \equiv p\left(w\left(c, u_W^*\big(c, u_r^*(c)\big)\right), u_r^*(c)\right)$$

Retailer anticipates the wholesaler's reaction as, $\max_{u_W} \Pi_W = u_W * q(c, u_W, u_r)$ subject to, $u_W, q(c, u_W, u_r) > 0$

$$u_w\Big|_{\frac{\partial \Pi_W}{\partial u_w}=0} = \frac{-q(c, u_w, u_r)}{\frac{\partial q(c, u_w, u_r)}{\partial u_w}} \qquad 3(5)$$

The solution of equation 3(5) is $\overline{u_W}(c, u_r)$. Hence, the retailer faces the demand as $q(c, \overline{u_W}(c, u_r), u_r)$ that can be written as $q(c, u_r)$.

Retailer's decision problem: $\max_{u_r} \Pi_R = u_r q(c, u_r)$ subject to, $u_r \geq 0$, $q(c, u_r) \geq 0$

$$u_r\Big|_{\frac{\partial \Pi_R}{\partial u_r}=0} = \frac{-q(c, u_r)}{\frac{\partial q(c, u_r)}{\partial u_r}} \qquad 3(6)$$

The solution of equation 3(6) is $u_r^*(c)$. In Equation 3(5), by setting $u_r = u_r^*$, we calculate the wholesaler's optimal decision as,

$$u_w^* = \frac{-q(c, u_w^*, u_r^*)}{\frac{\partial q(c, u_w^*, u_r^*)}{\partial u_w^*}} \qquad 3(7)$$

The solution of equation 3(7) is $u_W^*\big(c, u_r^*(c)\big)$.

Since, $w(c) = c + u_W^*\big(c, u_r^*(c)\big)$ and $p(c) = c + u_W^*\big(c, u_r^*(c)\big) + u_r^*(c)$ and, therefore we can calculate $\frac{dw}{dc}$ and $\frac{dp}{dc}$. It is to be noted, unlike a wholesale leading game, we cannot apply chain rule to obtain $\frac{dp}{dw}$ in the case of a retail leading Stackelberg game, because p cannot be expressed as complete function of w. However, partial derivative can be obtained from the equation $p = w + u_r$ but that does not give much insight about the pass-through between the wholesale and retail price.

2. Cost pass-throughs and BP ratios for N-stage markup price

a. Cost pass-through (Total stage N, Any stage n, top stage n=N, bottom stage n=1)

Demand function	Game structure	$\frac{dp_N}{dc}$	$\frac{dp_n}{dc}$	$\frac{dp_1}{dc}$	Relation
$q=(a-bp)^{1/v}$	Simultaneous	$\frac{1+(N-1)v}{1+Nv}$	$\frac{1+(n-1)v}{1+Nv}$	$\frac{1}{1+Nv}$	$1>\frac{dp_N}{dc}>...>\frac{dp_1}{dc}$
	Wholesale leading	$\frac{1}{v+1}$	$\frac{1}{(v+1)^{N-n+1}}$	$\frac{1}{(v+1)^N}$	
	Retail leading	$1-\frac{v}{(v+1)^N}$	$1-\sum_{i=n...N}\frac{v}{(v+1)^i}$	$\frac{1}{(v+1)^N}$	
Linear, $q=a-bp$	Simultaneous	$\frac{N}{1+N}$	$\frac{n}{1+N}$	$\frac{1}{1+N}$	
	Wholesale leading	$\frac{1}{2}$	$\frac{1}{2^{N-n+1}}$	$\frac{1}{2^N}$	
	Retail leading	$1-\frac{1}{2^N}$	$1-\sum_{i=n...N}\frac{1}{2^i}$	$\frac{1}{2^N}$	
Isoelastic, $q=ap^{-l}$	Simultaneous	$\frac{l-(N-1)}{l-N}$	$\frac{l-(n-1)}{l-N}$; $l>N$	$\frac{l}{l-N}$	$1<\frac{dp_N}{dc}<...<\frac{dp_1}{dc}$
	Wholesale leading	$\frac{l}{l-1}$	$\left(\frac{l}{l-1}\right)^{N-n+1}$; $l>1$	$\left(\frac{l}{l-1}\right)^N$	
	Retail leading	$1+\frac{1}{l-1}\left(\frac{l}{l-1}\right)^{N-1}$	$1+\sum_{i=n...N}\frac{1}{l-1}\left(\frac{l}{l-1}\right)^{i-1}$; $l>1$	$\left(\frac{l}{l-1}\right)^N$	
Negative Exponential, $q=ae^{\frac{-p}{b}}$	Simultaneous, wholesale or retail leading	1			$1=\frac{dp_N}{dc}=...=\frac{dp_1}{dc}$

b. FBP for linear demand.

	BP ratio (< 1) between two consecutive stages	Simultaneous,	Wholesale leading	Retail leading
n	$\frac{\sigma_n}{\sigma_{n+1}}$	$\frac{n}{n+1}$	$\frac{1}{2}$	$\frac{1+2^N-2^{N-n+1}}{1+2^N-2^{N-n}}$
1	$\frac{\sigma_1}{\sigma_2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{1+2^{N-1}}$
2	$\frac{\sigma_2}{\sigma_3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2^N-\left(2^{N-1}-1\right)}{2^N-\left(2^{N-2}-1\right)}$
---	---	---	---	---
---	---	---	---	---
$N-2$	$\frac{\sigma_{N-2}}{\sigma_{N-1}}$	$\frac{N-2}{N-1}$	$\frac{1}{2}$	$\frac{2^N-7}{2^N-3}$
$N-1$	$\frac{\sigma_{N-1}}{\sigma_N}$	$\frac{N-1}{N}$	$\frac{1}{2}$	$\frac{2^N-3}{2^N-1}$
N	$\frac{\sigma_N}{\sigma_{N+1}}$ or $\frac{\sigma_N}{\sigma_c}$	$\frac{N}{N+1}$	$\frac{1}{2}$	$\frac{2^N-1}{2^N}$

c. RBP for isoelasitc demand.

	BP ratio (> 1) between two consecutive stages	Simultaneous,	Wholesale leading	Retail leading
n	$\frac{\sigma_n}{\sigma_{n+1}}$	$\frac{l-n+1}{l-n}$	$\frac{l}{l-1}$	$\frac{l^N-(l-1)^{N-n+1}\left(l^{n-1}-(l-1)^{n-1}\right)}{l^N-(l-1)^{N-n}\left(l^n-(l-1)^n\right)}$
1	$\frac{\sigma_1}{\sigma_2}$	$\frac{l}{l-1}$	$\frac{l}{l-1}$	$\frac{l^N}{l^N-(l-1)^{N-1}}$
2	$\frac{\sigma_2}{\sigma_3}$	$\frac{l-1}{l-2}$	$\frac{l}{l-1}$	$\frac{l^N-(l-1)^{N-1}}{l^N-(l-1)^{N-2}\left(l^2-(l-1)^2\right)}$
---	---	---	---	---
---	---	---	---	---
$N-2$	$\frac{\sigma_{N-2}}{\sigma_{N-1}}$	$\frac{l-N+3}{l-N+2}$	$\frac{l}{l-1}$	$\frac{l^N-(l-1)^3\left(l^{N-3}-(l-1)^{N-3}\right)}{l^N-(l-1)^2\left(l^{N-2}-(l-1)^{N-2}\right)}$
$N-1$	$\frac{\sigma_{N-1}}{\sigma_N}$	$\frac{l-N+2}{l-N+1}$	$\frac{l}{l-1}$	$\frac{l^N-(l-1)^2\left(l^{N-2}-(l-1)^{N-2}\right)}{l^N-(l-1)\left(l^{N-1}-(l-1)^{N-1}\right)}$
N	$\frac{\sigma_N}{\sigma_{N+1}}$ or $\frac{\sigma_N}{\sigma_c}$	$\frac{l-N+1}{l-N}$	$\frac{l}{l-1}$	$\frac{l^N-(l-1)\left(l^{N-1}-(l-1)^{N-1}\right)}{(l-1)^N}$

d. Markups

Table: Markups for different game structures and demand functions

Demand function	Simultaneous game			Wholesale leading game			Retail leading game		
	u_w^*		u_r^*	u_w^*		u_r^*	u_w^*		u_r^*
Log-concave, $q=(a-bp)^{1/v}$	$\frac{v}{1+2v}\left(\frac{a}{b}-c\right)$	=	$\frac{v}{1+2v}\left(\frac{a}{b}-c\right)$	$\frac{v}{(1+v)}\left(\frac{a}{b}-c\right)$	>	$\frac{v}{(1+v)^2}\left(\frac{a}{b}-c\right)$	$\frac{v}{(1+v)^2}\left(\frac{a}{b}-c\right)$	<	$\frac{v}{(1+v)}\left(\frac{a}{b}-c\right)$
Linear ($q=a-bp$)	$\frac{1}{3}\left(\frac{a}{b}-c\right)$	=	$\frac{1}{3}\left(\frac{a}{b}-c\right)$	$\frac{1}{2}\left(\frac{a}{b}-c\right)$	>	$\frac{1}{4}\left(\frac{a}{b}-c\right)$	$\frac{1}{4}\left(\frac{a}{b}-c\right)$	<	$\frac{1}{2}\left(\frac{a}{b}-c\right)$
Isoelastic, $q=ap^{-l};(l>2)$	$\frac{c}{l-2}$	=	$\frac{c}{l-2}$	$\frac{c}{l-1}$	<	$c\frac{l}{(l-1)^2}$	$c\frac{l}{(l-1)^2}$	>	$\frac{c}{l-1}$
Negative exponential, $q=ae^{\frac{-p}{b}}$	b								

e. Order quantity

Demand function	Order quantity		
	Simultaneous game	Wholesale leading game	Retail leading game
Log-concave, $q=(a-bp)^{1/v}$	$\left(\frac{a-bc}{1+2v}\right)^{\frac{1}{v}}$	$\left(\frac{a-bc}{(1+v)^2}\right)^{\frac{1}{v}}$	
Linear, ($q=a-bp$)	$\frac{a-bc}{3}$	$\frac{a-bc}{4}$	
Isoelastic, $q=ap^{-l}$; $(l>2)$	$a\left(c\frac{l}{l-2}\right)^{-l}$	$ac^{-l}\left(\frac{l}{l-1}\right)^{-2l}$	
Negative exponential, $q=ae^{\frac{-p}{b}}$	$ae^{\frac{-c-2b}{b}}$		

f. profits

Table: Profit comparison across various game structure

Demand function	Profit, $\Pi = u \times q$	Wholesale leading		Simultaneous		Retail leading
Linear /concave /convex, $q=(a-bp)^{\frac{1}{v}}$	Π_W	$\frac{v}{b}\left(\frac{a-bc}{(1+v)^2}\right)^{\frac{1}{v}+1}(1+v)$	$>$	$\frac{v}{b}\left(\frac{a-bc}{1+2v}\right)^{\frac{1}{v}+1}$	$>$	$\frac{v}{b}\left(\frac{a-bc}{(1+v)^2}\right)^{\frac{1}{v}+1}$
		$\vee$		$\Vert$		$\wedge$
	Π_R	$\frac{v}{b}\left(\frac{a-bc}{(1+v)^2}\right)^{\frac{1}{v}+1}$	$<$	$\frac{v}{b}\left(\frac{a-bc}{1+2v}\right)^{\frac{1}{v}+1}$	$<$	$\frac{v}{b}\left(\frac{a-bc}{(1+v)^2}\right)^{\frac{1}{v}+1}(1+v)$
Linear $q=a-bp$	Π_W	$\frac{(a-bc)^2}{8b}$	$>$	$\frac{(a-bc)^2}{9b}$	$>$	$\frac{(a-bc)^2}{16b}$
		$\vee$		$\Vert$		$\wedge$
	Π_R	$\frac{(a-bc)^2}{16b}$	$<$	$\frac{(a-bc)^2}{9b}$	$<$	$\frac{(a-bc)^2}{8b}$
[43]Iso-elastic, $q=ap^{-l}$	Π_W	$\frac{ac^{1-l}}{l^{2l}(l-1)^{1-2l}}$	$>$	$\frac{ac^{1-l}}{l^{l}(l-2)^{1-l}}$	$<$	$\left(\frac{ac^{1-l}}{l^{2l}(l-1)^{1-2l}}\right)\frac{l}{(l-1)}$
		$\wedge$		$\Vert$		$\vee$
	Π_R	$\left(\frac{ac^{1-l}}{l^{2l}(l-1)^{1-2l}}\right)\frac{l}{(l-1)}$	$>$	$\frac{ac^{1-l}}{l^{l}(l-2)^{1-l}}$	$<$	$\frac{ac^{1-l}}{l^{2l}(l-1)^{1-2l}}$
Negative exponential, $q=ae^{\frac{-p}{b}}$	Π_W	$abe^{\frac{-c-2b}{b}}$				
		$\Vert$				
	Π_R	$abe^{\frac{-c-2b}{b}}$				

43 $\frac{(\Pi_W)_{simultaneous}}{(\Pi_W)_{wh\_leading}} = \frac{\frac{l^{-l}}{(l-2)^{1-l}}}{\frac{l^{-2l}}{(l-1)^{1-2l}}} = \frac{l^{-l}(l-1)^{1-2l}}{l^{-2l}(l-2)^{1-l}} = \frac{(l-1)^{1-2l}l^{l}}{(l-2)^{1-l}} = \frac{\left(\frac{l}{l-1}\right)^{l}}{\left(\frac{l-1}{l-2}\right)^{l-1}} < 1$; Numerically verified.

Appendix for Chapter 5: BP in the case of Buyback Newsvendor Model

1. Additive Demand Uncertainty Case:

A. Problem formulation:

The retailer's profit can be expressed as following,

$$\pi_r = \begin{matrix} pD - wq + \beta(q - D) & ; D \le q \\ pq - wq - S(D - q) & ; D > q \end{matrix} \quad (1)$$

Assuming additive uncertainty, the demand can be expressed as $D = y + \epsilon$. Let's assume[44] $z = q - y$, where z is called the stocking factor and can be expressed as $z = \mu + \sigma *$ (safety factor). Then the retailer's profit can be expressed as Equation 2 and the corresponding optimal policy is the order quantity, $q^* = y(p^*) + z^*$.

$$\pi_r = \begin{matrix} p(y+\epsilon) - w(y+z) + \beta(z - \epsilon) & ; \epsilon \le z & \rightarrow leftover \\ p(y+z) - w(y+z) - S(\epsilon - z) & ; \epsilon > z & \rightarrow shortage \end{matrix} \quad (2)$$

From Equation 2, the expected retail profit,

$$\begin{aligned} E[\pi_r] = \int_A^z [p(y+u) + \beta(z-u)]f(u)du + \int_z^B [p(y+z) - S(u-z)]f(u)du \\ - w(y+z) \\ = (p-w)(y+\mu) - [(w-\beta)\Lambda(z) + (p+S-w)\Theta(z)] \\ = \Psi(p) - L(z,p) \end{aligned} \quad (3)$$

Hence, the expected profit is the sum of the riskless profit $\Psi(p) = (p-w)(y+\mu)$ minus the loss due to uncertainty, $L(z,p) = [(w-\beta)\Lambda(z) + (p+S-w)\Theta(z)]$ (i.e. we obtain the expected profit by subtracting the loss function from the riskless profit). Here, $\Lambda(z) = \int_A^z (z-u)f(u)du =$ expected leftover and $\Theta(z) = \int_z^B (u-z)f(u)du =$ expected shortage. The loss function is the sum of the overstocking and understocking cost (i.e. $Loss\ function\ =\ overage\ cost\ *\ E(leftover)\ +\ underage\ cost * E(shortage)$).

The retailer's objective is to maximize

$$E[\pi_r(z,p)] = (p-w)(y+\mu) - [(w-\beta)\Lambda(z) + (p+S-w)\Theta(z)] \quad (4)$$

Taking partial derivatives in z and p -

$$\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = -(w-\beta) + (p+S-\beta)[1-F(z)] \quad (5)$$

$$\frac{\partial^2}{\partial z^2}(E[\pi_r(z|p)]) = -(p+S-\beta)f(z) < 0 \quad (6)$$

$$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = \frac{d}{dp}[\Psi(p)] - \frac{\partial}{\partial p}[L(z,p)] = y'\left[p - w + \frac{y}{y'}\right] + \mu - \Theta(z) \quad (7)$$

For a linear demand, $y = a - bp$, (8)

$$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = -2b\left[p - \frac{a+bw}{2b}\right] + \mu - \Theta(z)$$

[44] Such assumption provides mathematical convenience. We adapt this solution method from Petruzzi and Dada (1999).

$$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)]) = -2b < 0 \qquad (9)$$

Equation 6 tells us that $E[\pi_r]$ is concave in z for a given p. Equation 9 tells us that $E[\pi_r]$ is concave in p for a given z.

B. Proof of Lemma 1

i. Pricing decision approach

Setting $\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = 0$, we obtain,

$F[z^*(p)] = \frac{p+S-w}{p+S-\beta} \Rightarrow z^*(p) = F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]$

Linear demand:

Replacing the $z^*(p)$ into $\partial E/\partial p$:

$\frac{d}{dp}(E[\pi_r(p, z^*(p)]) = 2b\left[-p + \frac{a+bw}{2b} + \frac{\mu - \Theta(z^*(p))}{2b}\right]$

Hence, the optimal p^* is the p that satisfies $\frac{d}{dp}(E[\pi_r(p, z^*(p)]) = 0$.

Since $2b > 0$,

$p^*(w) = \left\{p \middle| -p + \frac{a+bw}{2b} + \frac{\mu - \Theta(z^*(p))}{2b} = 0\right\}$

The derivation of the optimal p^* requires to prove that the expected profit, $E\left[\pi_r\left(p, z^*(p)\right)\right]$ is concave and unimodal in p which is mentioned in Appendix 1-C-i.

Isoelastic demand:

For an isoelastic demand, $y = ap^{-b}$,

$\frac{d}{dp}[\Psi(p)] = -bap^{-b-1}\left[p - w + \frac{ap^{-b}}{-bap^{-b-1}}\right] + \mu = -bap^{-b-1}\left[p\left(\frac{b-1}{b}\right) - w\right] + \mu = -(b-1)ap^{-b-1}\left[p - \frac{b}{b-1}w\right] + \mu$

$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = -(b-1)ap^{-b-1}\left[p - \frac{b}{b-1}w\right] + \mu - \Theta(z)$

Replacing the $z^*(p)$ into $\partial E/\partial p$:

$\frac{d}{dp}(E[\pi_r(p, z^*(p)]) = \left(-p + \frac{b}{(b-1)}w\right)(b-1)ap^{-b-1} + \mu - \Theta\left(z^*(p)\right)$

Hence, the optimal p^* is the p that satisfies $\frac{d}{dp}(E[\pi_r(p, z^*(p)]) = 0$.

$p^*(w) = \left\{p \middle| \left(-p + \frac{b}{(b-1)}w\right)(b-1)ap^{-b-1} + \mu - \Theta\left(z^*(p)\right) = 0\right\}$

The derivation of the optimal p^* requires to prove that the expected profit, $E\left[\pi_r\left(p, z^*(p)\right)\right]$ is concave and unimodal in p which is mentioned in Appendix 1-C-ii.

ii. Stocking decision approach:

Solving $\frac{\partial}{\partial p}(E[\pi_r]) = -2b\left[p - \frac{a+bw}{2b}\right] + \mu - \Theta(z) = 0$, we can obtain $p^* = \frac{a+bw+\mu}{2b} - \frac{\Theta(z)}{2b}$. Then replacing p^* into the equation $\frac{\partial}{\partial z}(E[\pi_r]) = 0$ would give the single variable equation in z^*:

$\frac{d}{dz}(E[\pi_r(z,p^*(z))]) = -(w-\beta) + \left(\frac{a+bw}{2b} - \frac{\mu-\Theta(z)}{2b} + S - \beta\right)[1-F(z)] = 0$
The derivation of optimal z^* requires to prove that the expected profit, $E\left[\pi_r\left(z,p^*(z)\right)\right]$ is concave and unimodal in z which is mentioned in Appendix 1-C-i. □

C. Condition for concavity

i. Linear demand:

Stocking decision approach:

We need to show that the expected profit, $E\left[\Pi_r\left(z,p(z)\right)\right]$ is concave and unimodal in z. We adapt the proof from Petruzzi and Dada (1999) and edit it to reflect our setting. In buyback contract, the retailer's problem remains similar to the newsvendor model of Petruzzi-Dada (1999). From retailer's perspective, the difference is that the single period holding cost is replaced by a non-negative buyback price less than the wholesale price ($0 < \beta < w$). Interested readers may check the proof of Theorem 1 of Petruzzi and Dada (1999)[45] and replace their holding cost parameter '$-h$' by the buyback price parameter 'β' to obtain the proof required in our setting. For readers' convenience, we showed the detail proof here as follows-
Replacing $p^*(z)$ into the expected profit equation,
$E[\pi_r(z,p^*(z))] = (p^*-w)(a-bp^*+\mu) - [(w-\beta)\Lambda(z) + (p^*+S-w)\Theta(z)]$
Taking derivative in z,

$$\frac{d}{dz}E\left[\pi_r\left(z,p^*(z)\right)\right] = \frac{dp^*}{dz}(a-bp^*+\mu) + (p^*-w)\left(-b\frac{dp^*}{dz}\right) - \Big[(w-\beta)F(z) + \frac{dp^*}{dz}\Theta(z) - (p^*+S-w)\big(1-F(z)\big)\Big]$$

$$= \frac{\left(1-F(z)\right)}{2b}(a-bp^*+\mu) + (p^*-w)\left(-\frac{\left(1-F(z)\right)}{2}\right) - \Big[(w-\beta)F(z) + \frac{\left(1-F(z)\right)}{2b}\Theta(z) - (p^*+S-w)\big(1-F(z)\big)\Big]$$

$$= \big(1-\mathrm{F}(z)\big)\left(\frac{a}{2b} - \frac{p^*}{2} + \frac{\mu}{2b} - \frac{p^*}{2} + \frac{w}{2} - \frac{\Theta(z)}{2b} + p^* + S - w\right) - (w-\beta)\mathrm{F}(z)$$

$$= \big(1-\mathrm{F}(z)\big)\left(\frac{a}{2b} + \frac{\mu}{2b} + \frac{w}{2} - \frac{\Theta(z)}{2b} + S - w\right) - (w-\beta) + (w-\beta) - (w-\beta)\mathrm{F}(z)$$

---

[45] Theorem 1 of Petruzzi-Dada (1999) stated that-

"…z^* is determined according to the following:

a) If $F(.)$ is an arbitrary distribution function, then an exhaustive search over all values of z in the region $[A,B]$ will determine z^*.
b) If $F(.)$ is a distribution function satisfying the condition $2r(z)^2 + \frac{d}{dz}r(z) > 0$ for $A \le z \le B$, where $r(.) \equiv \frac{f(.)}{1-F(.)}$ is the Hazard rate, then z^* is the largest z in the region $[A,B]$ that satisfies $\frac{dE\left[\Pi_r\left(z,p(z)\right)\right]}{dz} = 0$."

If the condition for (b) is met and $a - b(w-2S) + A > 0$, then z^* is the unique z in the region $[A,B]$ that satisfies $\frac{dE\left[\Pi_r\left(z,p(z)\right)\right]}{dz} = 0$."

$$= \left(1 - \mathrm{F}(z)\right)\left(\frac{a+bw+\mu}{2b} - \frac{\Theta(z)}{2b} + S - w\right) - (w-\beta) + (w-\beta)\left(1-\mathrm{F}(z)\right)$$
$$= -(w-\beta) + \left(1 - \mathrm{F}(z)\right)\left(\frac{a+bw+\mu}{2b} - \frac{\Theta(z)}{2b} + S - \beta\right)$$

Alternate method: Instead of substituting p^* into the expected profit equation, substituting p^* into $\frac{\partial}{\partial z}E[\pi_r]$, also gives the same result.

$$\frac{d}{dz}\left(E[\pi_r(z, p^*(z))]\right)$$
$$= -(w-\beta) + (p^* + S - \beta)[1 - F(z)]$$
$$= -(w-\beta) + (\frac{a+bw+\mu}{2b} - \frac{\Theta(z)}{2b} + S - \beta)[1 - F(z)]$$

$$\frac{d^2}{dz^2}\left(E[\pi_r(z, p^*(z))]\right)$$
$$= -\left(\frac{a+bw+\mu}{2b} + S - \beta - \frac{\Theta(z)}{2b}\right)f(z) + \frac{[1-F(z)]^2}{2b}$$
$$= -\frac{f(z)}{2b}\left\{2b\left(\frac{a+bw+\mu}{2b} + S - \beta\right) - \Theta(z) + \frac{1-F(z)}{\left(\frac{f(z)}{1-F(z)}\right)}\right\}$$

$$\frac{d^3}{dz^3}\left(E[\pi_r(z, p^*(z))]\right)$$
$$= \frac{df(z)}{dz}\left[\left(\frac{1}{f(z)}\right)\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right] - \frac{f(z)}{2b}\left\{[1-F(z)] + \frac{d}{dz}\left(\frac{1-F(z)}{\left(\frac{f(z)}{1-F(z)}\right)}\right)\right\}$$
$$= \frac{df(z)}{dz}\left[\left(\frac{1}{f(z)}\right)\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right] - \frac{f(z)}{2b}\left\{[1-F(z)] + \frac{f(z)}{\left(\frac{f(z)}{1-F(z)}\right)} + \frac{1-F(z)}{\left(\frac{f(z)}{1-F(z)}\right)^2} * \frac{d}{dz}\left(\frac{f(z)}{1-F(z)}\right)\right\}$$
$$= \frac{df(z)}{dz}\left[\left(\frac{1}{f(z)}\right)\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right] - \frac{f(z)[1-\mathrm{F}(z)]}{2b\left(\frac{f(z)}{1-F(z)}\right)^2}\left\{2\left(\frac{f(z)}{1-F(z)}\right)^2 + \frac{d}{dz}\left(\frac{f(z)}{1-F(z)}\right)\right\}$$

$$\frac{d^3}{dz^3}\left(E[\pi_r(z, p^*(z))]\right)\Big|_{\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}=0} = -\frac{f(z)[1-\mathrm{F}(z)]}{2b\left(\frac{f(z)}{1-F(z)}\right)^2}\left\{2\left(\frac{f(z)}{1-F(z)}\right)^2 + \frac{d}{dz}\left(\frac{f(z)}{1-F(z)}\right)\right\}$$

As argued by Petruzzi and Dada (1999), if $\left\{2\left(\frac{f(z)}{1-F(z)}\right)^2 + \frac{d}{dz}\left(\frac{f(z)}{1-F(z)}\right)\right\} > 0$, then $\frac{d}{dz}\left(E[\pi_r(z, p^*(z))]\right)$ is monotone or unimodal and thus having at most two roots. Moreover, for $z = B$, $\frac{d}{dz}\left(E[\pi_r(z, p^*(z))]\right) = -(w-\beta) < 0$. Therefore, if $\frac{d}{dz}\left(E[\pi_r(z, p^*(z))]\right)$ has only one root, it indicates a change in sign from positive to negative. It corresponds to a local maximum of $E[\pi_r(z, p^*(z))]$

If $\frac{d}{dz}\left(E[\pi_r(z, p^*(z))]\right)$ has two roots, the larger (smaller) of the two corresponds to a local maximum (minimum) of $E[\pi_r(z, p^*(z))]$. In either case, $E[\pi_r(z, p^*(z))]$ has only one local maximum, identified either as the unique value (or as the larger of two values) of z that satisfies $\frac{d}{dz}\left(E[\pi_r(z, p^*(z))]\right) = 0$. Since $E[\pi_r(z, p^*(z))]$ is unimodal if $\frac{d}{dz}\left(E[\pi_r(z, p^*(z))]\right)$ has only one root (assuming $\left\{2\left(\frac{f(z)}{1-F(z)}\right)^2 + \frac{d}{dz}\left(\frac{f(z)}{1-F(z)}\right)\right\} > 0$), a sufficient condition for unimodality of $E[\pi_r(z, p^*(z))]$ is $\frac{d}{dz}\left(E[\pi_r(z, p^*(z))]\right)\Big|_{z=A} > 0$ or equivalently,

$$2b * \frac{d}{dz}(E[\pi_r(z, p^*(z))])\Big|_{z=A} > 0,$$
$$\Rightarrow -2b(w-\beta) + \left(2b\left(\frac{a+bw+\mu}{2b} + S - \beta\right) - \Theta(A)\right)[1 - F(A)] > 0$$
$$\Rightarrow -2b(w-\beta) + \left(a + bw + \mu + 2b(S-\beta) - (\mu - A)\right) > 0$$
$$\Rightarrow a - b(w - 2S) + A > 0$$

Petruzzi & Dada (1999) summarized the conditions for concavity and unimodality. Similar conditions were proposed by Ernest (1970), Young (1978), Bulow and Proschan (1975). It is to be mentioned, PF2 distributions and log-normal distributions (that have non-decreasing hazard rate, $r(.) = \frac{f(.)}{1-F(.)}$) satisfy the above-mentioned conditions.

Hence, if $2\left(\frac{f(z)}{1-F(z)}\right)^2 + \frac{d}{dz}\left(\frac{f(z)}{1-F(z)}\right) > 0$, then $E[\pi_r(z, p^*(z))]$ is concave and unimodal in z.

Pricing decision approach:

Differentiating $F[z^*(p)] = \frac{p+S-w}{p+S-\beta}$ with respect to p,

$$f * \frac{dz^*}{dp} = \frac{1}{p+S-\beta} - \frac{p+S-w}{(p+S-\beta)^2} = \frac{1}{p+S-\beta} - \frac{F}{(p+S-\beta)} = \frac{1-F}{(p+S-\beta)}$$

Therefore, $\frac{dz^*}{dp} = \frac{1-F}{f(p+S-\beta)} = \frac{1}{r(p+S-\beta)}$

Here, $r = \frac{f}{1-F} =$ hazard rate

We need to show that $E\left[\Pi_r\left(p, z^*(p)\right)\right]$ is concave in p.

Replacing the $z^*(p)$ into $\partial E/\partial p$: $\frac{d}{dp}(E[\pi_r(p, z^*(p)]) = 2b\left[-p + \frac{a+bw}{2b} + \frac{\mu - \Theta(z^*(p))}{2b}\right]$

We need to find zeros of $\frac{d}{dp}(E[\pi_r])$:

$$\frac{d^2}{dp^2}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 2b\left[-1 + \frac{\left(1-F(z^*)\right)\frac{dz^*}{dp}}{2b}\right] = -2b + \frac{(1-F)}{r(p+S-\beta)}$$

$$\frac{d^3}{dp^3}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = \frac{d}{dp}\left[\frac{(1-F)}{r(p+S-\beta)}\right] = \frac{-f*\frac{dz^*}{dp}}{r(p+S-\beta)} - \frac{(1-F)}{r^2(p+S-\beta)^2}\frac{d}{dp}[r(p+S-\beta)] = \frac{-f}{r^2(p+S-\beta)^2} - \frac{(1-F)}{r^2(p+S-\beta)^2}\left[\frac{dr}{dz^*} * \frac{dz^*}{dp}(p+S-\beta) + r\right] = -\frac{f}{r^2(p+S-\beta)^2} - \frac{(1-F)\left[r + \frac{1}{r}*\frac{dr}{dz^*}\right]}{r^2(p+S-\beta)^2} = -\frac{f + (1-F)\left[r + \frac{1}{r}*\frac{dr}{dz^*}\right]}{r^2(p+S-\beta)^2} = -\frac{(1-F)\left[\frac{f}{1-F} + r + \frac{1}{r}*\frac{dr}{dz^*}\right]}{r^2(p+S-\beta)^2} = -\frac{(1-F)\left[2r + \frac{1}{r}\frac{dr}{dz^*}\right]}{r^2(p+S-\beta)^2} = -\frac{(1-F)\frac{1}{r}\left[2r^2 + \frac{dr}{dz^*}\right]}{r^2(p+S-\beta)^2} = -\frac{(1-F)\left[2r^2 + \frac{dr}{dz^*}\right]}{r^3(p+S-\beta)^2}$$

Since $r = \frac{f}{1-F} > 0$, therefore, for $2r^2 + \frac{dr}{dz^*} > 0$, $\frac{d^3}{dp^3}(E[\pi_r]) < 0$ that follows that $\frac{d}{dp}(E[\pi_r])$ is either monotone or unimodal.

It is to be mentioned, the condition $2r^2 + \frac{dr}{dz^*} > 0$ is the same condition from the stocking decision approach as expected.

ii. Isoelastic Demand:

We adapt the proof from Arcelus et al. 2005

$\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = -(w-\beta) + (p+S-\beta)[1-F(z)]$

$\frac{\partial^2}{\partial z^2}(E[\pi_r(z|p)]) = -(p+S-\beta)f(z) < 0$

Setting $\frac{\partial E[\pi_r]}{\partial z} = 0$, we obtain $1 - F = \frac{w-\beta}{p+S-\beta}$. Therefore, $\frac{\partial^2 E[\pi_r]}{\partial z^2} = -\frac{w-\beta}{1-F} f$

$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = \frac{d}{dp}[\Psi(p)] - \frac{\partial}{\partial p}[L(z,p)] = y'(p-w) + y + \mu - \Theta(z)$

setting $\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = 0$, we obtain, $\Rightarrow (p-w) = \frac{\Theta(z)-y-\mu}{y'} = -\frac{1}{y'}(y+\mu-\Theta)$

$\frac{\partial^2 E[\pi_r]}{\partial p^2} = y''(p-w) + 2y' = -\frac{y''}{y'}(y+\mu-\Theta) + 2y' = -\frac{yy''}{y'}\left(1+\frac{\mu-\Theta}{y}\right) + 2y' = y'\left(2 - \frac{yy''}{(y')^2}\left(1+\frac{\mu-\Theta}{y}\right)\right)$

Since, $y' < 0$, therefore, $\frac{\partial^2 E[\pi_r]}{\partial p^2} < 0$ if $\frac{yy''}{(y')^2}\left(1+\frac{\mu-\Theta}{y}\right) < 2$

$\frac{\partial^2 E[\pi_r]}{\partial p \partial z} = \frac{\partial}{\partial p}\left(\frac{\partial E[\pi_r]}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial E[\pi_r]}{\partial p}\right) = 1 - F > 0$

$\frac{\partial^2 E[\pi_r]}{\partial z^2} \times \frac{\partial^2 E[\pi_r]}{\partial p^2} - \left(\frac{\partial^2 E[\pi_r]}{\partial p \partial z}\right)^2 = -\frac{w-\beta}{1-F} f y'\left(2 - \frac{yy''}{(y')^2}\left(1+\frac{\mu-\Theta}{y}\right)\right) - (1-F)^2$

$\frac{\partial^2 E[\pi_r]}{\partial z^2} \times \frac{\partial^2 E[\pi_r]}{\partial p^2} - \left(\frac{\partial^2 E[\pi_r]}{\partial p \partial z}\right)^2 > 0$ if

$\Rightarrow -\frac{w-\beta}{1-F} f y'\left(2 - \frac{yy''}{(y')^2}\left(1+\frac{\mu-\Theta}{y}\right)\right) > (1-F)^2 \Rightarrow f > \frac{(1-F)^3}{-(w-\beta)y'\left(2-\frac{yy''}{(y')^2}\left(1+\frac{\mu-\Theta}{y}\right)\right)}$

It is to be mentioned, the denominator is positive, because $y' < 0$ and $\frac{yy''}{(y')^2}\left(1+\frac{\mu-\Theta}{y}\right) < 2$. Moreover, since, $f \le 1$, we can write the condition as

$1 \ge f > \frac{(1-F)^3}{-(w-\beta)y'\left(2-\frac{yy''}{(y')^2}\left(1+\frac{\mu-\Theta}{y}\right)\right)}$

For an isoelastic demand, $y = ap^{-b}$, $y' = -bap^{-b-1} = \frac{-by}{p} < 0$,

$y'' = \frac{-by'}{p} - \frac{-by}{p^2} = -\frac{y'}{p}(b+1) = \frac{by}{p^2}(b+1) > 0$, $\frac{yy''}{(y')^2} = \frac{y\frac{by}{p^2}(b+1)}{\left(\frac{-by}{p}\right)^2} = \frac{b+1}{b} > 1$

For $b > 1$, $1 < \frac{b+1}{b} < 2$, therefore, $1 < \frac{yy''}{(y')^2} < 2$

Hence, for $y = ap^{-b}$, the condition is,

$\frac{b+1}{b}\left(1+\frac{\mu-\Theta}{ap^{-b}}\right) < 2$ and $1 \ge f > \frac{(1-F)^3}{(w-\beta)\frac{by}{p}\left(2-\frac{b+1}{b}\left(1+\frac{\mu-\Theta}{ap^{-b}}\right)\right)} > 0$

D. Cost-pass-through

i. Linear demand:

i. Stocking decision approach

z^* has to satisfy $\frac{d}{dz}\left(E[\pi_r(z,p^*(z))]\right)=0,$

$\Rightarrow -(w-\beta)+\left(\frac{a+bw+\mu}{2b}-\frac{\Theta(z^*)}{2b}+S-\beta\right)[1-F(z^*)]=0$

Differentiating it with respect to w,

$-1+\left(\frac{1}{2}+\frac{1-F}{2b}*\frac{dz^*}{dw}\right)(1-F)-\left(\frac{a+bw+\mu}{2b}-\frac{\Theta(z^*)}{2b}+S-\beta\right)f\frac{dz^*}{dw}=0$

$\Rightarrow -1+\frac{(1-F)}{2}+\frac{(1-F)^2}{2b}*\frac{dz^*}{dw}-\left(\frac{a+bw+\mu}{2b}-\frac{\Theta(z^*)}{2b}+S-\beta\right)f\frac{dz^*}{dw}=0$

$\Rightarrow -1+\frac{(1-F)}{2}=\left[\left(\frac{a+bw+\mu}{2b}-\frac{\Theta(z^*)}{2b}+S-\beta\right)f-\frac{(1-F)^2}{2b}\right]*\frac{dz^*}{dw}$

$\Rightarrow \frac{-1-F}{2}=(1-F)\left[\left(\frac{a+bw+\mu}{2b}-\frac{\Theta(z^*)}{2b}+S-\beta\right)r-\frac{(1-F)}{2b}\right]*\frac{dz^*}{dw}$

$\Rightarrow \frac{dz^*}{dw}=-\frac{1+F}{2(1-F)\left[\left(\frac{a+bw+\mu}{2b}-\frac{\Theta(z^*)}{2b}+S-\beta\right)r-\frac{(1-F)}{2b}\right]}$

From $\frac{d}{dp}\left(E[\pi_r]\right)=0,$

$p^*(w,z)=\frac{a+bw+\mu}{2b}-\frac{\Theta(z^*)}{2b}$

Differentiating with respect to w, $\frac{dp^*}{dw}=\frac{1}{2}+\frac{1-\mathrm{F}}{2b}*\frac{dz^*}{dw}$

$\frac{dp^*}{dw}=\frac{1}{2}-\frac{1-\mathrm{F}}{2b}*\frac{1+F}{2(1-F)\left[\left(\frac{a+bw+\mu}{2b}-\frac{\Theta(z^*)}{2b}+S-\beta\right)r-\frac{(1-F)}{2b}\right]}=\frac{1}{2}-\frac{1}{2}*\frac{1+F}{[2b(p^*+S-\beta)r-(1-F)]}$

$\Rightarrow \frac{dp^*}{dw}=\frac{1}{2}\left(1-\frac{1+F}{2b(p^*+S-\beta)r-(1-F)}\right)$

ii. Pricing Decision approach:

From, the standard newsvendor result, $F[z^*(p,w)]=\frac{p+S-w}{p+S-\beta}$

Taking derivative in w,

$f\left(z^*(p,w)\right)\frac{d}{dw}\left(z^*(p,w)\right)=\frac{\left(\frac{dp}{dw}-1\right)}{(p+S-\beta)}-\frac{p+S-w}{(p+S-\beta)^2}*\frac{dp}{dw}=\frac{\left(\frac{dp}{dw}-1\right)}{(p+S-\beta)}-\frac{F[z^*(p,w)]*\frac{dp}{dw}}{(p+S-\beta)}=$
$\frac{(1-F[z^*(p,w)])}{(p+S-\beta)}*\frac{dp}{dw}-\frac{1}{(p+S-\beta)}$

$\Rightarrow \frac{dz^*}{dw}=\frac{1-F}{f(p+S-\beta)}*\frac{dp}{dw}-\frac{1}{f(p+S-\beta)}=\frac{1-F}{f(p+S-\beta)}\left[\frac{dp}{dw}-\frac{1}{1-F}\right]$

For a linear demand, $y=a-bp$, p^* has to satisfy this equation,

$-p^*+\frac{a+bw}{2b}+\frac{\mu-\Theta(z^*)}{2b}=0$

Taking derivative in w, $-\frac{dp^*}{dw}+\frac{1}{2}+\frac{1}{2b}(1-\mathrm{F})\frac{\mathrm{d}z^*}{dw}=0$

$\Rightarrow -\frac{dp^*}{dw}+\frac{1}{2}+\frac{1}{2b}*\frac{(1-F)}{r(p+S-\beta)}\left[\frac{dp}{dw}-\frac{1}{1-F}\right]=0$

$\Rightarrow \frac{1}{2}-\frac{1}{2b}*\frac{1}{r(p+S-\beta)}=\frac{dp^*}{dw}\left(1-\frac{1}{2b}*\frac{(1-F)}{r(p+S-\beta)}\right)$

$$\Rightarrow \frac{dp^*}{dw} = \frac{1}{2}\frac{\left(1-\frac{1}{b}*\frac{1}{r(p+S-\beta)}\right)}{\left(1-\frac{1}{2b}*\frac{(1-F)}{r(p+S-\beta)}\right)} = \frac{1}{2}\frac{\left(1-\frac{1}{2b}*\frac{(1-F)}{r(p+S-\beta)}\pm\frac{1}{2b}*\frac{(1-F)}{r(p+S-\beta)}-\frac{1}{b}*\frac{1}{r(p+S-\beta)}\right)}{\left(1-\frac{1}{2b}*\frac{(1-F)}{r(p+S-\beta)}\right)}$$

$$= \frac{1}{2}\left(1-\frac{\left(\frac{1+F}{2br(p+S-\beta)}\right)}{\left(1-\frac{1-F}{2br(p+S-\beta)}\right)}\right) = \frac{1}{2}\left(1-\frac{1+F}{2br(p+S-\beta)-(1-F)}\right)$$

ii. Isoelastic demand:

We will be following pricing decision approach here. Therefore, from the previous appendix, we adapt the result of $\frac{dz^*}{dw} = \frac{1}{r(p+S-\beta)}\left(\frac{dp}{dw}-\frac{1}{1-F}\right)$ where, $= \frac{f}{1-F}$. It is to be mentioned the result of $\frac{dz^*}{dw}$ doesn't depend on the form of demand function.

For an isoelastic demand, $y = ap^{-b}$, p^* satisfy this equation,

$$\left(-p+\frac{b}{(b-1)}w\right)(b-1)ap^{-b-1}+\mu-\Theta\left(z^*(p)\right)=0$$

Taking derivative in w,

$$\left(-\frac{dp}{dw}+\frac{b}{(b-1)}\right)(b-1)ap^{-b-1}+\left(-p+\frac{b}{(b-1)}w\right)(-b-1)(b-1)ap^{-b-2}\frac{dp}{dw}+(1-F)\frac{dz^*}{dw}=0$$

Substituting $\frac{dz^*}{dw}$,

$$\Rightarrow \left(-\frac{dp}{dw}+\frac{b}{(b-1)}\right)(b-1)ap^{-b-1}+\left(-p+\frac{b}{(b-1)}w\right)(-b-1)(b-1)ap^{-b-2}\frac{dp}{dw}+\frac{1-F}{r(p+S-\beta)}\left(\frac{dp}{dw}-\frac{1}{1-F}\right)=0$$

$$\Rightarrow -\frac{dp}{dw}(b-1)ap^{-b-1}+bap^{-b-1}+\left(-p+\frac{b}{(b-1)}w\right)(-b-1)(b-1)ap^{-b-2}\frac{dp}{dw}+\frac{dp}{dw}*\frac{1-F}{r(p+S-\beta)}-\frac{1}{r(p+S-\beta)}=0$$

$$\Rightarrow \left(-p+\frac{b}{(b-1)}w\right)(-b-1)(b-1)ap^{-b-2}\frac{dp}{dw}+\frac{dp}{dw}*\frac{1-F}{r(p+S-\beta)}-\frac{dp}{dw}(b-1)ap^{-b-1}=\frac{1}{r(p+S-\beta)}-bap^{-b-1}$$

$$\Rightarrow \frac{dp}{dw} = \frac{\frac{1}{r(p+S-\beta)}-bap^{-1-b}}{\frac{1-F}{r(p+S-\beta)}-(b-1)ap^{-1-b}-a(b+1)(b-1)p^{-2-b}\left(-p+\frac{bw}{b-1}\right)} = \frac{\frac{1}{r(p+S-\beta)}-bap^{-1-b}}{\frac{1-F}{r(p+S-\beta)}+ab(b-1)p^{-1-b}-ab(b+1)wp^{-2-b}} = \frac{\frac{1}{r(p+S-\beta)}-bap^{-1-b}}{\frac{1-F}{r(p+S-\beta)}+a(b+1)(b-1)p^{-2-b}\left(\frac{b}{b+1}p-\frac{b}{b-1}w\right)}$$

$$\Rightarrow \frac{dp}{dw} = \frac{\frac{1}{r(p+S-\beta)}-bap^{-1-b}}{\frac{1-F}{r(p+S-\beta)}+abp^{-2-b}\left((b-1)p-(b+1)w\right)}$$

E. Numerical Analysis
i. Linear demand:

Case 1: Uniform[-5,5]
$a = 100, b = 1, S = 10$, uniform$[-5{,}5]$
$\mu = 0; f(u) = 0.1; F(u) = \frac{u+5}{10}; F[z^*] = \frac{z^*+5}{10}; 1 + F = \frac{z^*+15}{10}; 1 - F = \frac{5-z^*}{10}; r = \frac{f}{1-F} = \frac{1}{5-z^*};\ \Theta(z^*(p)) = \frac{1}{10}\int_{z^*}^{5}(u - z^*)du = \frac{1}{20}(z^* - 5)^2;$

Following stocking decision approach-
Condition: $a - b(w - 2S) + A > 0 \Rightarrow 100 - (w - 20) - 5 > 0 \Rightarrow 115 > w$

Let's consider a buyback price of $\beta = 15$
z^* satisfy:
$$-(w-\beta) + \left(\frac{a+bw}{2b} + \frac{\mu - \Theta(z)}{2b} + S - \beta\right)[1 - F(z)] = 0$$
$$\Rightarrow -(w-15) + \left(\frac{100+w}{2} + \frac{-\frac{1}{20}(z^*-5)^2}{2} + 10 - 15\right)\left[1 - \frac{z^*+5}{10}\right] = 0$$
$$\Rightarrow -(w-15) + \left(45 + \frac{w}{2} - \frac{1}{40}(z^*-5)^2\right)\left(\frac{5-z^*}{10}\right) = 0$$
The solution of this equation: $z^* = 5 - \frac{-5400-60w}{615^{1/3}X^{1/3}} + \frac{25^{1/3}X^{1/3}}{3^{2/3}}$
Where, $X = (-675 + 45w + \sqrt{15}\sqrt{-698625 - 28350w - 135w^2 - w^3})$
This expression is tedious and let's define $w = g(z^*)$, then the solution can be expressed as follows-
$$-(w-15) + \left(45 + \frac{w}{2} - \frac{1}{40}(z^*-5)^2\right)\left(\frac{5-z^*}{10}\right) = 0$$
$$\Rightarrow -w + 15 + \frac{w}{2}\left(\frac{5-z^*}{10}\right) + \left(45 - \frac{1}{40}(z^*-5)^2\right)\left(\frac{5-z^*}{10}\right) = 0$$
$$\Rightarrow 15 + \left(45 - \frac{1}{40}(z^*-5)^2\right)\left(\frac{5-z^*}{10}\right) = w\left(1 - \frac{5-z^*}{20}\right)$$
$$\Rightarrow \boldsymbol{w} = \left(\mathbf{15} + \left(\mathbf{45} - \frac{(\boldsymbol{z}^*-\mathbf{5})^2}{\mathbf{40}}\right)\left(\frac{\mathbf{5}-\boldsymbol{z}^*}{\mathbf{10}}\right)\right)\left(\frac{\mathbf{20}}{\mathbf{15}+\boldsymbol{z}^*}\right)$$
$$w|_{z^*=-5} = \left(15 + \left(45 - \frac{(-5-5)^2}{40}\right)\left(\frac{5+5}{10}\right)\right)\left(\frac{20}{15-5}\right) = 115$$
$$w|_{z^*=5} = \left(15 + \left(45 - \frac{(5-5)^2}{40}\right)\left(\frac{5-5}{10}\right)\right)\left(\frac{20}{15+5}\right) = 15$$
We know the range of z^* is $[-5{,}5]$. Using that, we obtain the range of w as $[15{,}115]$. Then we calculate the corresponding retail price.
p^* can be obtained as, $p^*(z) = \frac{a+bw}{2b} + \frac{\mu-\Theta(z)}{2b} = \frac{100+w}{2} - \frac{(z^*-5)^2}{40}$
The corresponding cost-pass-through,

$$\frac{dp^*}{dw} = \frac{1}{2}\left(1 - \frac{1+F}{2b(p^*+S-\beta)r-(1-F)}\right) = \frac{1}{2}\left(1 - \frac{\frac{z^*+15}{10}}{2(p^*-5)\frac{1}{5-z^*}-\frac{5-z^*}{10}}\right)$$

$$\left.\frac{dp^*}{dw}\right|_{z^*=-5} = \frac{1}{2}\left(1 - \frac{\frac{-5+15}{10}}{2(p^*-5)\frac{1}{5+5}-\frac{5+5}{10}}\right) = \frac{1}{2}\left(1 - \frac{1}{\frac{1}{5}(p^*-5)-1}\right) < \frac{1}{2}$$

$$\left.\frac{dp^*}{dw}\right|_{z^*=4<5} = \frac{1}{2}\left(1 - \frac{\frac{4+15}{10}}{2(p^*-5)\frac{1}{5-4}-\frac{5-4}{10}}\right) = \frac{1}{2}\left(1 - \frac{\frac{19}{10}}{2(p^*-5)-\frac{1}{10}}\right) < \frac{1}{2}$$

Let's consider a buyback price of $\beta = 70$

z^* satisfy: $-(w-\beta) + \left(\frac{a+bw}{2b} + \frac{\mu-\Theta(z)}{2b} + S - \beta\right)[1-F(z)] = 0$

$$\Rightarrow -(w-70) + \left(\frac{100+w}{2} + \frac{-\frac{1}{20}(z^*-5)^2}{2} + 10 - 70\right)\left(1 - \frac{z^*+5}{10}\right) = 0$$

$$\Rightarrow \boldsymbol{w} = \left(\mathbf{70} + \left(\mathbf{-10} - \frac{(\boldsymbol{z}^*-\mathbf{5})^2}{\mathbf{40}}\right)\left(\frac{\mathbf{5}-\boldsymbol{z}^*}{\mathbf{10}}\right)\right)\left(\frac{\mathbf{20}}{\mathbf{15}+\boldsymbol{z}^*}\right)$$

$$w(z^* = -5) = \left(70 + \left(-10 - \frac{(-5-5)^2}{40}\right)\left(\frac{5+5}{10}\right)\right)\left(\frac{20}{15-5}\right) = 115$$

$$w(z^* = 5) = \left(70 + \left(-10 - \frac{(5-5)^2}{40}\right)\left(\frac{5-5}{10}\right)\right)\left(\frac{20}{15+5}\right) = 70$$

We see that lower limit of w is determined by the buyback price and upper limit of w is determined by the shortage cost.

p^* can be obtained as, $p^*(z) = \frac{a+bw}{2b} + \frac{\mu-\Theta(z)}{2b} = \frac{100+w}{2} - \frac{(z^*-5)^2}{40}$

The corresponding cost-pass-through,

$$\frac{dp^*}{dw} = \frac{1}{2}\left(1 - \frac{1+F}{2b(p^*+S-\beta)r-(1-F)}\right) = \frac{1}{2}\left(1 - \frac{\frac{z^*+15}{10}}{2(p^*-60)\frac{1}{5-z^*}-\frac{5-z^*}{10}}\right)$$

$$\left.\frac{dp^*}{dw}\right|_{z^*=-5} = \frac{1}{2}\left(1 - \frac{\frac{-5+15}{10}}{2(p^*-60)\frac{1}{5+5}-\frac{5+5}{10}}\right) = \frac{1}{2}\left(1 - \frac{1}{\frac{1}{5}(p^*-60)-1}\right) < \frac{1}{2}$$

$$\left.\frac{dp^*}{dw}\right|_{z^*=4<5} = \frac{1}{2}\left(1 - \frac{\frac{4+15}{10}}{2(p^*-60)\frac{1}{5-4}-\frac{5-4}{10}}\right) = \frac{1}{2}\left(1 - \frac{\frac{19}{10}}{2(p^*-60)-\frac{1}{10}}\right) < \frac{1}{2}$$

Case 2: Uniform[-10,10]

$a = 100, b = 1, S = 10$, uniform$[-10,10]$
$\mu = 0;\ f(u) = \frac{1}{20};\ F(u) = \frac{u+10}{20}\ ;\ F[z^*] = \frac{z^*+10}{20};\ 1 + F = \frac{z^*+30}{20};\ 1 - F = \frac{10-z^*}{20}\ ;\ r = \frac{f}{1-F} = \frac{1}{10-z^*}$
$\Theta(z^*(p)) = \frac{1}{20}\int_{z^*}^{10}(u - z^*)du = \frac{1}{40}(z^* - 10)^2;$
Following stocking decision approach-
Condition (Petruzzi and Dada, 1999): $a - b(w - 2S) + A > 0 \Rightarrow 100 - (w - 20) - 10 > 0 \Rightarrow 110 > w$

Let's consider a buyback price of $\beta = 15$

z^* satisfy: $-(w - \beta) + \left(\frac{a+bw}{2b} + \frac{\mu-\Theta(z)}{2b} + S - \beta\right)[1 - F(z)] = 0$
$\Rightarrow -(w - 15) + \left(\frac{100+w}{2} + \frac{-\frac{1}{40}(z^*-10)^2}{2} + 10 - 15\right)\left(1 - \frac{z^*+10}{20}\right) = 0$
$\Rightarrow w = \left(15 + \left(45 - \frac{1}{80}(z^* - 10)^2\right)\left(\frac{10-z^*}{20}\right)\right)\left(\frac{40}{30+z^*}\right)$
$\Rightarrow w(z^* = -10) = \left(15 + \left(45 - \frac{1}{80}(-10 - 10)^2\right)\left(\frac{10+10}{20}\right)\right)\left(\frac{40}{30-10}\right) = 110$
$\Rightarrow w(z^* = 10) = 15$
p^* can be obtained as, $p^*(z) = \frac{a+bw}{2b} + \frac{\mu-\Theta(z)}{2b} = \frac{100+w}{2} - \frac{(z^*-10)^2}{80}$
The corresponding cost-pass-through,
$\frac{dp^*}{dw} = \frac{1}{2}\left(1 - \frac{1+F}{2b(p^*+S-\beta)r-(1-F)}\right) = \frac{1}{2}\left(1 - \frac{\frac{z^*+30}{20}}{2(p^*-5)\frac{1}{10-z^*} - \frac{10-z^*}{20}}\right)$
$\left.\frac{dp^*}{dw}\right|_{z^*=-10} = \frac{1}{2}\left(1 - \frac{\frac{-10+30}{20}}{2(p^*-5)\frac{1}{10+10} - \frac{10+10}{20}}\right) = \frac{1}{2}\left(1 - \frac{1}{\frac{1}{10}(p^*-5)-1}\right) < \frac{1}{2}$
$\left.\frac{dp^*}{dw}\right|_{z^*=9<10} = \frac{1}{2}\left(1 - \frac{\frac{9+30}{20}}{2(p^*-5)\frac{1}{10-9} - \frac{10-9}{20}}\right) = \frac{1}{2}\left(1 - \frac{\frac{39}{20}}{2(p^*-5)-1}\right) < \frac{1}{2}$
Let's consider a buyback price of $\beta = 70$ where the retail, wholesale, and buyback prices are closer.

z^* satisfy: $-(w - \beta) + \left(\frac{a+bw}{2b} + \frac{\mu-\Theta(z)}{2b} + S - \beta\right)[1 - F(z)] = 0$

$\Rightarrow -(w - 70) + \left(\frac{100+w}{2} + \frac{-\frac{1}{40}(z^*-10)^2}{2} + 10 - 70\right)\left(1 - \frac{z^*+10}{20}\right) = 0$

$\Rightarrow w = \left(70 + \left(-10 - \frac{1}{80}(z^* - 10)^2\right)\left(\frac{10-z^*}{20}\right)\right)\left(\frac{40}{30+z^*}\right)$

$$w(z^* = -10) = \left(70 + \left(-10 - \frac{1}{80}(-10-10)^2\right)\left(\frac{10+10}{20}\right)\right)\left(\frac{40}{30-10}\right) = 110$$

$$w(z^* = 10) = 70$$

p^* can be obtained as, $p^*(z) = \frac{a+bw}{2b} + \frac{\mu-\Theta(z)}{2b} = \frac{100+w}{2} - \frac{(z^*-10)^2}{80}$

The corresponding cost-pass-through,

$$\frac{dp^*}{dw} = \frac{1}{2}\left(1 - \frac{1+F}{2b(p^*+S-\beta)r-(1-F)}\right) = \frac{1}{2}\left(1 - \frac{\frac{z^*+30}{20}}{2(p^*-60)\frac{1}{10-z^*} - \frac{10-z^*}{20}}\right)$$

$$\left.\frac{dp^*}{dw}\right|_{z^*=-10} = \frac{1}{2}\left(1 - \frac{\frac{-10+30}{20}}{2(p^*-60)\frac{1}{10+10} - \frac{10+10}{20}}\right) = \frac{1}{2}\left(1 - \frac{1}{\frac{1}{10}(p^*-60)-1}\right) < \frac{1}{2}$$

$$\left.\frac{dp^*}{dw}\right|_{z^*=9<10} = \frac{1}{2}\left(1 - \frac{\frac{9+30}{20}}{2(p^*-60)\frac{1}{10-9} - \frac{10-9}{20}}\right) = \frac{1}{2}\left(1 - \frac{\frac{39}{20}}{2(p^*-60) - \frac{1}{20}}\right) < \frac{1}{2}$$

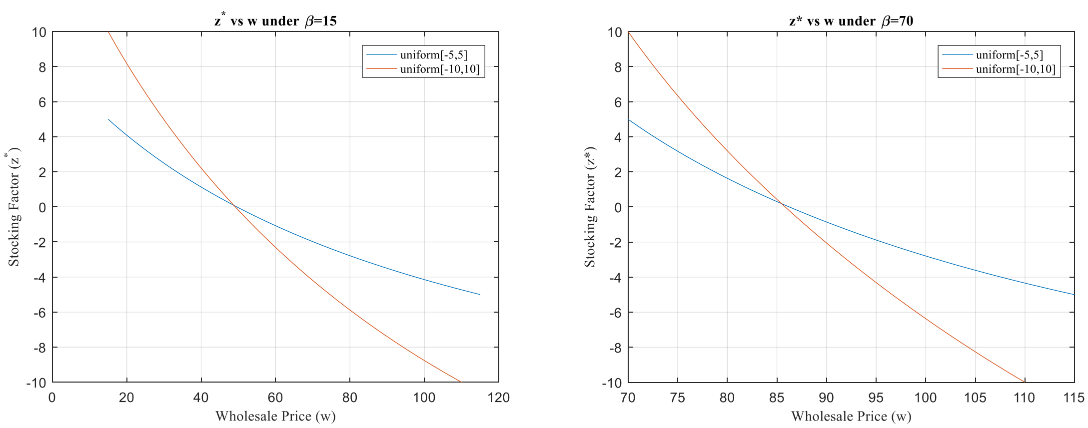

Figure: Optimal stocking factor
(linear demand, additive uncertainty, buyback price \$15 (left) and \$70 (right))

ii. Isoelastic demand

Let assume an isoelastic demand function $y = ap^{-3}$, a per unit shortage cost of $S = 10$, a per-unit buyback price of $\beta = 15$, and the uncertainty is uniformly distributed on the interval $[-5,5]$. Therefore, it follows,

$\mu = 0;\ f(u) = 0.1;\ F(u) = \frac{u+5}{10}$;

$F[z^*] = \frac{z^*+5}{10} = \frac{p+10-w}{p+10-15} \Rightarrow z^* = \frac{5(25+p-2w)}{-5+p}$

$\Theta(z^*(p)) = \frac{1}{10}\int_{z^*}^{5}(u - z^*)du = \frac{1}{20}(z^*-5)^2 = \frac{1}{20}\left(\frac{5(25+p-2w)}{-5+p} - 5\right)^2 = \frac{5(-15+w)^2}{(-5+p)^2}$;

Following pricing decision approach,

p^*satisfy, $\left(-p+\frac{b}{(b-1)}w\right)(b-1)ap^{-b-1}+\mu-\Theta\big(z^*(p)\big)=0$

$\Rightarrow\left(-p+\frac{3}{2}w\right)2ap^{-4}-\frac{5(-15+w)^2}{(-5+p)^2}=0$

The solution in p is very tedious, therefore, obtaining the solution as an inverse function, $w=g(p^*)\Rightarrow p^*=g^{-1}(w)$

$$w=-\frac{1}{10}(-5+p)^2\left(-\frac{150}{(-5+p)^2}-\frac{3a}{p^4}\pm\sqrt{\frac{a(9a(-5+p)^2+20(45-2p)p^4)}{(-5+p)^2p^8}}\right)$$

Accepting the solution that satisfy $w\le p$

$w=-\frac{1}{10}(-5+p)^2\left(-\frac{150}{(-5+p)^2}-\frac{3a}{p^4}+\sqrt{\frac{a(9a(-5+p)^2+20(45-2p)p^4)}{(-5+p)^2p^8}}\right)$

Assuming $a=1000000$,

$w=-\frac{1}{10}(-5+p)^2\left(-\frac{150}{(-5+p)^2}-\frac{3000000}{p^4}+1000\sqrt{\frac{9000000(-5+p)^2+20(45-2p)p^4}{(-5+p)^2p^8}}\right)$

The minimum value of the wholesale price is the buyback price $\beta=15$ and the maximum value is $+\infty$. It is to be mentioned, as $w\to\infty$, the demand $y\to 0$.

For $p\in[15{,}60]$, the illustration plot is shown in Figure 5.4.

The corresponding cost-pass-through is $\frac{dp}{dw}=\frac{\frac{1}{r(p+S-\beta)}-bap^{-1-b}}{\frac{1-F}{r(p+S-\beta)}+abp^{-2-b}\big((b-1)p-(b+1)w\big)}$

Substituting, $=\frac{p+S-w}{p+S-\beta}; 1-F=\frac{w-\beta}{p+S-\beta}; r=\frac{f}{1-F}=\frac{p+S-\beta}{10(w-\beta)}$, we obtain,

$\Rightarrow\frac{dp}{dw}=\frac{-abp^{-1-b}+\frac{10(w-\beta)}{(p+S-\beta)^2}}{abp^{-2-b}((-1+b)p-(1+b)w)+\frac{10(w-\beta)^2}{(p+S-\beta)^3}}$

Substituting $S=10,\beta=15,\Rightarrow\frac{dp}{dw}=\frac{-abp^{-1-b}+\frac{10(w-15)}{(p-5)^2}}{\frac{10(w-15)^2}{(p-5)^3}+abp^{-2-b}((-1+b)p-(1+b)w)}$

For, $a=1000000$ and $p\in[15{,}60]$, the cost-pass-through is illustrated in Figure 5.5.

From Appendix 1c-ii, the condition for optimality is

$\frac{b+1}{b}\left(1+\frac{\mu-\Theta}{ap^{-b}}\right)<2$ and $1\ge f>\frac{(1-F)^3}{(w-\beta)\frac{by}{p}\left(2-\frac{b+1}{b}\left(1+\frac{\mu-\Theta}{ap^{-b}}\right)\right)}>0$

For our selected parameter, the condition is

$\frac{4}{3}\left(1+\frac{p^3\left(\frac{1}{10}-\frac{5(-15+w)^2}{(-5+p)^2}\right)}{10^6}\right)<2$ and

$1\ge\frac{1}{10}>\frac{\left(\frac{w-15}{p-5}\right)^3}{(w-15)\frac{b10^6p^{-3}}{p}\left(2-\frac{4}{3}\left(1+\frac{p^3\left(\frac{1}{10}-\frac{5(-15+w)^2}{(-5+p)^2}\right)}{10^6}\right)\right)}>0$

Or equivalently,

$$\Rightarrow 1 > \frac{p^4(-15+w)^2}{10^5 * b(-5+p)^3\left(2-\frac{4}{3}\left(1+\frac{p^3\left(\frac{1}{10}-\frac{5(-15+w)^2}{(-5+p)^2}\right)}{10^6}\right)\right)} > 0$$

Let define, $C1 = \frac{4}{3}\left(1 + \frac{p^3\left(\frac{1}{10}-\frac{5(-15+w)^2}{(-5+p)^2}\right)}{10^6}\right)$ and $C2 = \frac{p^4(-15+w)^2}{10^5 * b(-5+p)^3\left(2-\frac{4}{3}\left(1+\frac{p^3\left(\frac{1}{10}-\frac{5(-15+w)^2}{(-5+p)^2}\right)}{10^6}\right)\right)}$

Hence, the equivalent conditions are $C1 < 2$ and $1 > C2 > 0$. For $p \in [15,60]$, it can be plotted as follows that shows that the conditions are fulfilled for the selected parameters.

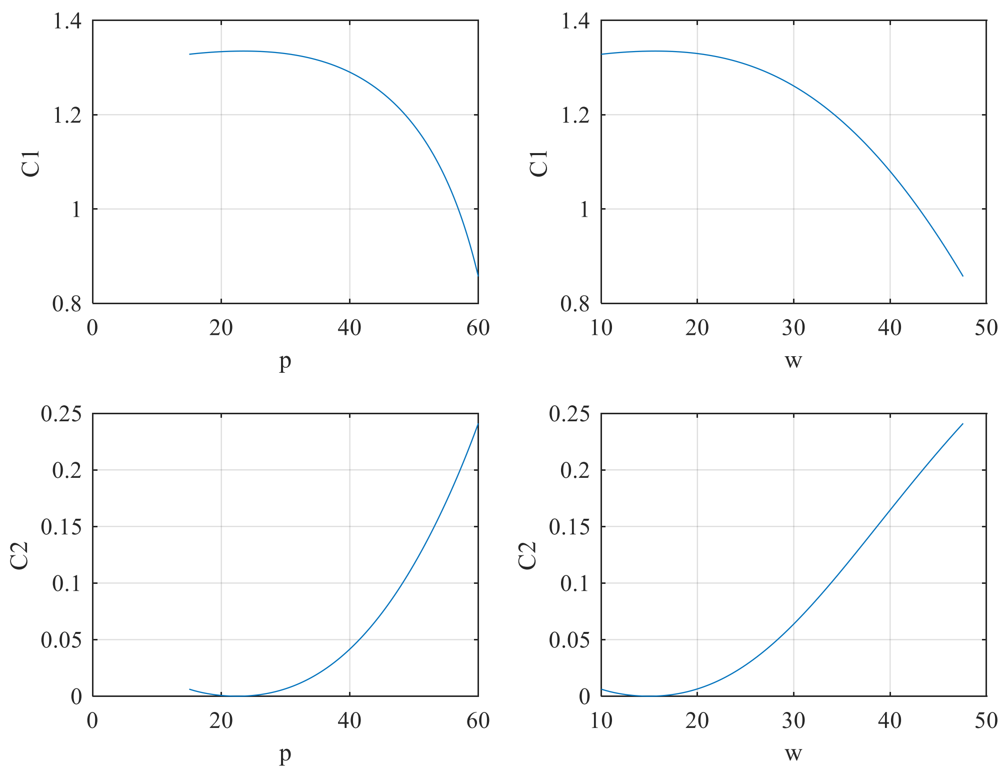

Figure: Conditions for the optimality for the selected parameters

2. Multiplicative Demand Uncertainty Case:

A. Problem Formulation:

From equation 1, the retailer's profit is,

$$\pi_r = \begin{matrix} pD - wq + \beta(q - D) & ; D \leq q \\ pq - wq - S(D - q) & ; D > q \end{matrix}$$

Assuming multiplicative uncertainty, the demand can be expressed as $D = y\epsilon$. Let's assume $z = q/y$, where z is called the stocking factor and can be expressed as $z = \mu + \sigma * (safety\,factor)$. Then the retailer's profit can be expressed as Equation 2 and the corresponding optimal policy is the order quantity, $q^* = y(p^*)z^*$.

$$\pi_r = \begin{matrix} py\epsilon - wyz + \beta y(z-\epsilon) & ;\epsilon \le z & \to leftover \\ pyz - wyz - Sy(\epsilon - z) & ;\epsilon > z & \to shortage \end{matrix} \tag{2}$$

From Equation 2a, the expected retail profit,

$$\begin{aligned} E[\pi_r] = \int_A^z [pyu + (\beta)y(z-u)]f(u)du + \int_z^B [pyz - Sy(u-z)]f(u)du \\ - wyz = (p-w)y\mu - [(w-\beta)\mathrm{y}\Lambda(z) + (p+S-w)\mathrm{y}\Theta(z)] \\ = \Psi(p) - L(z,p) \end{aligned} \tag{3a}$$

Hence, the expected profit is the sum of the riskless profit $\Psi(p) = (p-w)y\mu$ minus the loss due to uncertainty, $L(z,p) = [(w-\beta)\mathrm{y}\Lambda(z) + (p+S-w)\mathrm{y}\Theta(z)]$ (i.e. subtracting the loss function from the riskless profit). Here, $\mathrm{y}\Lambda(z) = y\int_A^z (z-u)f(u)du =$ expected leftover and $\mathrm{y}\Theta(z) = y\int_z^B (u-z)f(u)du =$ expected shortage. The loss function is the sum of the overstocking and understocking cost (i.e. $Loss\ function\ =\ overage\ cost\ * E(leftover)\ +\ underage\ cost * E(shortage)$).

The retailer's objective is to maximize,

$$E[\pi_r(z,p)] = (p-w)y\mu - [(w-\beta)\mathrm{y}\Lambda(z) + (p+S-w)\mathrm{y}\Theta(z)] \tag{4a}$$

This is a joint optimization problem in p and z. Therefore, we take partial derivatives of the expected profit in p and z, and check if the second order conditions are fulfilled.

$$\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = y[-(w-\beta) + (p+S-\beta)[1-F(z)]] \tag{5a}$$

$$\frac{\partial^2}{\partial z^2}(E[\pi_r(z|p)]) = -y(p+S-\beta)f(z) < 0 \tag{6a}$$

$$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = \frac{d}{dp}[\Psi(p)] - \frac{\partial}{\partial p}[L(z,p)] = y'\mu\left[p - w + \frac{y}{y'}\right] - \mathrm{y}\Theta(z) - y'[(w-\beta)\Lambda(z) + (p+S-w)\Theta(z)] \tag{7a}$$

For, $y = a - bp$,

$$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = 2b\big(\mu - \Theta(z)\big)\left[-p + p^0 + \frac{1}{2} * \frac{(w-\beta)\Lambda(z) + S\Theta(z)}{(\mu - \Theta(z))}\right] \tag{8a}$$

where $p^0 = \frac{a+bw}{2b}$

$$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)]) = -2b\big(\mu - \Theta(z)\big) < 0 \tag{9a}$$

For, $y = ap^{-b}$,

$$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = (b-1)ap^{-b-1}\{\mu - \Theta(z)\}\left[-p + p^0 + \frac{b}{b-1}\left\{\frac{(w-\beta)\Lambda(z) + S\Theta(z)}{(\mu - \Theta(z))}\right\}\right], \tag{10a}$$

where $p^0 = \frac{b}{b-1}w$

$$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)]) = -\frac{b+1}{p} * \frac{\partial}{\partial p}(E[\pi_r(p|z)]) - (b-1)ap^{-b-1}\{\mu - \Theta(z)\} \tag{11a}$$

$$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)])\bigg|_{\frac{\partial}{\partial p}(E[\pi_r(p|z)])=0} = -(b-1)ap^{-b-1}\{\mu - \Theta(z)\} < 0 \tag{12a}$$

Equation 6a tells us that $E[\pi_r]$ is concave in z for a given p. In equation 8a and 10, p^0 is the price that maximizes the riskless profit. We can obtain the riskless optimal price as p^0 by setting the $\frac{d}{dp}[\Psi(p)] = 0$. In equation 9a and 12a, the non-negativities hold because $\mu - \Theta(z) \geq \mu - \Theta(A) = A > 0$. Therefore, $E[\pi_r]$ is concave in p for a given z.

B. Proof of Lemma 2

i. Lemma 2a:

Setting $\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = 0$, we obtain,

$$\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = y[-(w-\beta) + (p+S-\beta)[1-F(z)]] = 0 \Rightarrow z^*(p) = F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]$$

This is the standard newsvendor result of stocking factor, $z^*(p)$ when p is fixed (Porteus 1990). Then substituting $z^*(p)$ into the expected profit (Eq. 4a), it will convert the joint optimization problem into a single variable decision problem[46]. Alternatively, we can also substitute $z^*(p)$ into the partial derivative of the expected profit equation with respect to p (Eq. 7a),

$$\frac{d}{dp}(E[\pi_r(z^*(p),p)])$$
$$= y'\mu\left[p - w + \frac{y}{y'}\right] - y\Theta(z^*) - y'[(w-\beta)\Lambda(z^*) + (p+S-w)\Theta(z^*)]$$

For $y = a - bp$ (from Eq. 8a),

$$\frac{d}{dp}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 2b\left(\mu - \Theta(z^*)\right)\left[-p + \frac{a+bw}{2b} + \frac{1}{2} * \frac{(w-\beta)\Lambda(z^*) + S\Theta(z^*)}{\left(\mu - \Theta(z^*)\right)}\right]$$

For $y = ap^{-b}$ (from Eq.10a),

$$\frac{d}{dp}(E[\pi_r(p, z^*(p))]) = (b-1)ap^{-b-1}\left(\mu - \Theta(z^*)\right)\left[-p + \frac{b}{b-1}w + \frac{b}{b-1}\left\{\frac{(w-\beta)\Lambda(z^*) + S\Theta(z^*)}{(\mu - \Theta(z^*))}\right\}\right]$$

If $E[\pi_r(p, z^*(p)]$ is concave in p [Proposition 2], then p^* is the p that satisfies $\frac{d}{dp}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 0$. Since $\mu - \Theta(z^*) > 0$,

For $y = a - bp$, $p^*(w) = \left\{p \middle| -p + \frac{a+bw}{2b} + \frac{1}{2} * \frac{(w-\beta)\Lambda(z^*(p)) + S\Theta(z^*(p))}{\left(\mu - \Theta(z^*(p))\right)} = 0\right\}$

For $y = ap^{-b}$, $p^*(w) = \left\{p \middle| -p + \frac{b}{b-1}w + \frac{b}{b-1} * \frac{(w-\beta)\Lambda(z^*(p)) + S\Theta(z^*(p))}{\left(\mu - \Theta(z^*(p))\right)} = 0\right\}$

□

[46] Single variable decision problem in p:

$$E[\pi_r(z^*(p),p)] = y\left[(p-w)\mu - (w-\beta)\int_A^{F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]}\left(F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right] - u\right)f(u)du - (p+S-w)\int_{F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]}^{B}\left(u - F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]\right)f(u)du\right]$$

ii. Lemma 2b:

Solving $\frac{\partial}{\partial p}(E[\pi_r]) = 0$ (Eq. 8a and 10a), we can obtain, $p^*(z) = \frac{a+bw}{2b} + \frac{1}{2}X(z)$ [for linear demand] or $p^*(z) = \frac{b}{b-1}w + \frac{b}{b-1}X(z)$ [for isoelastic demand] where, $(z) = \frac{(w-\beta)\Lambda(z)+S\Theta(z)}{(\mu-\Theta(z))}$.

Then replacing $p^*(z)$ into the equation $\frac{\partial}{\partial z}(E[\pi_r]) = 0$ (Eq. 5a) would give the single variable equation in z^*:

$$\frac{d}{dz}(E[\pi_r(z,p^*(z))]) = y(p^*(z))\Big(-(w-\beta) + (p^*(z)+S-\beta)\big(1-F(z)\big)\Big) = 0$$

The derivation of the optimal z^* requires to prove that the expected profit, $E\big[\pi_r\big(z,p^*(z)\big)\big]$ is concave in z which is analyzed in Proposition 2. □

C. Proof of Lemma 3: Defining X and W

Let's define,

$$X = \left\{\frac{(w-\beta)E[leftover]+S*E[shortage]}{E[sales]}\right\} = \left\{\frac{(w-\beta)y\Lambda(z)+Sy\Theta(z)}{y\big(\mu-\Theta(z)\big)}\right\} = \left\{\frac{(w-\beta)\Lambda+S\Theta}{\mu-\Theta}\right\}$$

$$\frac{\partial X}{\partial z} = \frac{(w-\beta)F-S(1-F)}{(\mu-\Theta)} - \frac{(w-\beta)\Lambda+S\Theta}{(\mu-\Theta)^2}(1-F) =$$
$$\frac{(w-\beta)F(\mu-\Theta)-S(1-F)(\mu-\Theta)-(w-\beta)\Lambda(1-F)-S\Theta(1-F)}{(\mu-\Theta)^2} = \frac{(w-\beta)\big(F(\mu-\Theta)-\Lambda(1-F)\big)-S(1-F)\mu}{(\mu-\Theta)^2} =$$
$$\frac{(w-\beta)(\mu-\Theta)(1-F)\Big(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\Big)-S(1-F)\mu}{(\mu-\Theta)^2} = \frac{(1-F)(\mu-\Theta)}{(\mu-\Theta)^2}\left[(w-\beta)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right)-S\frac{\mu}{(\mu-\Theta)}\right] =$$
$$\frac{(1-F)}{(\mu-\Theta)}\left[(w-\beta)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right)-S\frac{\mu}{(\mu-\Theta)}\right]$$

Here, $\frac{CDF}{1-CDF} = \frac{F}{(1-F)} > 1 > \frac{\Lambda}{(\mu-\Theta)} = \frac{E[leftover]}{E[sales]}$.

Therefore, for zero shortage cost (i.e. $S = 0$), $\frac{\partial X}{\partial z}$ is positive. For non-negative shortage cost, $\frac{\partial X}{\partial z}$ is positive if $(w-\beta)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right) > S\frac{\mu}{(\mu-\Theta)}$; otherwise, $\frac{\partial X}{\partial z}$ is negative if $(w-\beta)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right) < S\frac{\mu}{(\mu-\Theta)}$.

Now let's define, $W = \frac{1-F}{f(p+S-\beta)} * \frac{\partial X}{\partial z}$

Since $(1-F), f, (p+S-\beta)$ are non-negative terms, therefore, the sign of W follows the sign of $\frac{\partial X}{\partial z}$. It is to be mentioned, following the pricing decision approach, $\frac{\partial X}{\partial p} = \frac{\partial X}{\partial z} * \frac{\partial z}{\partial p}$ takes the value of W.

In further derivation, we will be using these two variables X and W frequently.

D. Condition for concavity

i. Pricing decision approach:

Proposition:

$E\left[\Pi_r\left(p, z^*(p)\right)\right]$ is concave in p for the given conditions-

1. For $y = a - bp, \frac{1}{2}W < 1$
2. For $y = ap^{-b}, b > 1, \ \frac{b}{b-1}W < 1$

where, $W = \frac{1-F}{f(p+S-\beta)} * \frac{\partial X}{\partial z}$. Hence, the optimal p^* is the p that satisfies $\frac{dE\left[\Pi_r\left(p,z^*(p)\right)\right]}{dp} = 0$.

Proof:

We obtain $z^*(p)$ from, $F\left(z^*(p)\right) = \frac{p+S-w}{p+S-\beta}$

Taking derivative in p,

$f * \frac{dz^*}{dp} = \frac{1}{p+S-\beta} - \frac{p+S-w}{(p+S-\beta)^2} = \frac{1}{p+S-\beta} - \frac{F}{(p+S-\beta)} = \frac{1-F}{(p+S-\beta)}$

$\Rightarrow \frac{dz^*}{dp} = \frac{1-F}{f(p+S-\beta)}$

Replacing the $z^*(p)$ into $\partial E/\partial p$:

Linear Demand

For $y = a - bp$,

$\frac{d}{dp}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 2b\left(\mu - \Theta(z^*)\right)\left[-p + \frac{a+bw}{2b} + \frac{1}{2} * X\left(z^*(p)\right)\right]$

$\frac{d^2}{dp^2}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 2b(1-F)\frac{dz^*}{dp}\left[-p + \frac{a+bw}{2b} + \frac{1}{2} * X\right] + 2b(\mu - \Theta)\left[-1 + \frac{1}{2} * \frac{dX}{dz^*} * \frac{dz^*}{dp}\right]$

$\frac{d^2}{dp^2}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = \frac{(1-F)}{(\mu-\Theta)} * \frac{dz^*}{dp} * \frac{d(E[\pi_r])}{dp} + 2b(\mu - \Theta)\left[-1 + \frac{1}{2} * W\right]$

Here, $W = \frac{dX}{dz^*} * \frac{dz^*}{dp} = \frac{1-F}{f(p+S-\beta)} * \frac{(1-F)}{(\mu-\Theta)}\left[(w-\beta)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) - S\frac{\mu}{(\mu-\Theta)}\right]$

$\frac{d^2\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp^2}\bigg|_{\frac{d\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp}=0} = 2b(\mu - \Theta)\left[-1 + \frac{1}{2} * W\right]$

Since $2b(\mu - \Theta) > 0$, therefore for $\frac{W}{2} < 1$, $\frac{d^2\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp^2}\bigg|_{\frac{d\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp}=0} < 0$

Hence, $E\left[\Pi_r\left(p, z^*(p)\right)\right]$ is concave in p and the optimal p^* is the p that satisfies $\frac{dE\left[\Pi_r\left(p,z^*(p)\right)\right]}{dp} = 0$.

Isoelastic Demand

For $y = ap^{-b}$,

$\frac{d}{dp}(E[\pi_r(p, z^*(p))]) = (b-1)ap^{-b-1}\{\mu - \Theta(z^*(p))\}\left[-p + \frac{b}{b-1}w + \frac{b}{b-1}X\left(z^*(p)\right)\right]$

Let's define, $R(p) = -p + \frac{b}{b-1}w + \frac{b}{b-1}X$

$\frac{d}{dp}(E[\pi_r(p, z^*(p))]) = (b-1)ap^{-b-1}\{\mu - \Theta\}R(p)$

$\frac{d^2}{dp^2}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = (b-1)\left[-(b+1)ap^{-b-1-1}(\mu - \Theta)R + ap^{-b-1}(1 - F)\frac{dz^*}{dp}R + ap^{-b-1}(\mu - \Theta)\frac{dR}{dp}\right]$

$= (b-1)ap^{-b-1}\left[-(b+1)p^{-1}(\mu - \Theta)R + (1 - F)\frac{dz^*}{dp}R + (\mu - \Theta)\frac{dR}{dp}\right]$

$\frac{dR(p)}{dp} = -1 + \frac{b}{b-1} * \frac{dX}{dz^*} * \frac{dz^*}{dp} = -1 + \frac{b}{b-1} * W$

Here, $W = \frac{dX}{dz^*} * \frac{dz^*}{dp} = \frac{1-F}{f(p+S-\beta)} * \frac{(1-F)}{(\mu-\Theta)}\left[(w-\beta)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) - S\frac{\mu}{(\mu-\Theta)}\right]$

Substituting $\frac{dR(p)}{dp}$,

$\frac{d^2}{dp^2}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = (b-1)ap^{-b-1}\left[-(b+1)p^{-1}(\mu - \Theta)R + (1 - F)\frac{dz^*}{dp}R + (\mu - \Theta)\left(-1 + \frac{b}{b-1}W\right)\right]$

$= -(b+1)p^{-1}\frac{d}{dp}(E[\pi_r]) + \frac{(1-F)}{(\mu-\Theta)} * \frac{dz^*}{dp} * \frac{d}{dp}(E[\pi_r]) + (b-1)ap^{-b-1}(\mu - \Theta)\left(-1 + \frac{b}{b-1}W\right)$

$\frac{d^2\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp^2}\Bigg|_{\frac{d\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp}=0} = (b-1)ap^{-b-1}(\mu - \Theta)\left(-1 + \frac{b}{b-1}W\right)$

Since $(b-1)ap^{-b-1}(\mu - \Theta) > 0$, therefore for $\frac{b}{b-1}W < 1$,

$\frac{d^2\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp^2}\Bigg|_{\frac{d\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp}=0} < 0$

Hence, $E\left[\Pi_r\left(p, z^*(p)\right)\right]$ is concave in p for the given condition and the optimal p^* is the p that satisfies $\frac{dE\left[\Pi_r\left(p,z^*(p)\right)\right]}{dp} = 0$.

ii. Stocking Decision Approach:

Proposition:

$E\left[\Pi_r\left(z, p^*(z)\right)\right]$ is concave in z for the given conditions-

1. For $y = a - bp$, $\frac{1}{2}W < 1$

2. For $y = ap^{-b}$, $b > 1$, $\frac{b}{b-1}W < 1$

where, $W = \frac{1-F}{f(p+S-\beta)} * \frac{\partial X}{\partial z}$. Hence, the optimal z^* is the z that satisfies $\frac{dE[\Pi_r(z,p^*(z))]}{dz} = 0$.

Proof:

Linear Demand Form:

Replacing the $p^*(z)$ into $\partial E/\partial z$:

$$\frac{d}{dz}E\left[\pi_r\left(z,p^*(z)\right)\right] = (a - bp^*)\left(-(w-\beta) + (p^* + S - \beta)\left(1 - F(z)\right)\right)$$

$$\frac{d^2}{dz^2}\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right) = -b\frac{dp^*}{dz} * \frac{\frac{d}{dz}E[\pi_r]}{a-bp^*} + (a - bp^*)\left(\left(\frac{dp^*}{dz}\right)(1-F) - (p^* + S - \beta)f\right)$$

$$\left.\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right|_{\frac{d\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz}=0} = (a - bp^*)\left(\left(\frac{dp^*}{dz}\right)(1-F) - (p^* + S - \beta)f\right) =$$
$$-(a - bp^*)(p^* + S - \beta)f\left(1 - \frac{(1-F)}{(p^*+S-\beta)f}\left(\frac{dp^*}{dz}\right)\right)$$

From, $p^*(z) = \frac{a}{2b} + \frac{w+X}{2}$,

$$\frac{dp^*}{dz} = \frac{1}{2}\frac{dX(z)}{dz} = \frac{1}{2} * \frac{(1-F)}{(\mu-\Theta)}\left[(w-\beta)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) - S\frac{\mu}{(\mu-\Theta)}\right]$$

Hence, substituting $\frac{dp^*}{dz}$,

$$\left.\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right|_{\frac{d\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz}=0} = -(a - bp^*)(p^* + S - \beta)f\left(1 - \frac{1}{2} * \frac{(1-F)}{(p^*+S-\beta)f} *\right.$$
$$\left.\left(\frac{dX(z)}{dz}\right)\right) = -(a - bp^*)(p^* + S - \beta)f\left(1 - \frac{1}{2} * W\right)$$

Since, $(a - bp^*)(p^* + S - \beta)f > 0$, therefore, for $\frac{1}{2} * W < 1$,

$$\left.\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right|_{\frac{d\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz}=0} < 0$$

Therefore, $E\left[\pi_r\left(z,p^*(z)\right)\right]$ is concave in z and the optimal z^* is the z that satisfy $\frac{d}{dz}E\left[\pi_r\left(z,p^*(z)\right)\right] = 0$.

Isoelastic Demand Form:

Replacing the $p^*(z)$ into $\partial E/\partial z$:

$$\frac{d}{dz}E\left[\pi_r\left(z,p^*(z)\right)\right]=\left(ap^{*(-b)}\right)\left(-(w-\beta)+(p^*+S-\beta)\left(1-F(z)\right)\right)$$

$$\frac{d^2}{dz^2}\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)=-bap^{*(-b-1)}\frac{dp^*}{dz}*\frac{\frac{d}{dz}E[\pi_r]}{ap^{*(-b)}}+\left(ap^{*(-b)}\right)\left(\left(\frac{dp^*}{dz}\right)(1-F)-\right.$$
$$\left.(p^*+S-\beta)f\right)$$

$$\left.\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right|_{\frac{d\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz}=0}=\left(ap^{*(-b)}\right)\left(\left(\frac{dp^*}{dz}\right)(1-F)-(p^*+S-\beta)f\right)=$$
$$-\left(ap^{*(-b)}\right)(p^*+S-\beta)f\left(1-\frac{(1-F)}{(p^*+S-\beta)f}\left(\frac{dp^*}{dz}\right)\right)$$

From, $p^*(z)=\frac{b}{b-1}(w+X)$,

$$\frac{dp^*}{dz}=\frac{b}{b-1}\frac{dX(z)}{dz}=\frac{b}{b-1}*\frac{(1-F)}{(\mu-\Theta)}\left[(w-\beta)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right)-S\frac{\mu}{(\mu-\Theta)}\right]$$

Hence, substituting $\frac{dp^*}{dz}$,

$$\left.\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right|_{\frac{d\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz}=0}=-\left(ap^{*(-b)}\right)(p^*+S-\beta)f\left(1-\frac{b}{b-1}*\frac{(1-F)}{(p^*+S-\beta)f}*\right.$$
$$\left.\left(\frac{dX(z)}{dz}\right)\right)=-\left(ap^{*(-b)}\right)(p^*+S-\beta)f\left(1-\frac{b}{b-1}*W\right)$$

Since, $\left(ap^{*(-b)}\right)(p^*+S-\beta)f>0$, therefore, for $\frac{b}{b-1}*W<1$,

$$\left.\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right|_{\frac{d\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz}=0}<0$$

Therefore, $E\left[\pi_r\left(z,p^*(z)\right)\right]$ is concave in z and the optimal z^* is the z that satisfy $\frac{d}{dz}E\left[\pi_r\left(z,p^*(z)\right)\right]=0$.

E. Cost-pass-through

Proposition: In the case of newsvendor model with multiplicative uncertainty, the retail cost-pass-through is as follows-

1. For linear demand (i.e. $D = (a - bp)\epsilon$), $\frac{dp^*}{dw} = \frac{1}{2}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} + \left(\frac{1}{2} - \frac{1}{1-F}\right)W}{1 - \frac{1}{2}W}\right)$
2. For isoelastic demand (i.e. $D = (ap^{-b})\epsilon$), $\frac{dp^*}{dw} = \frac{b}{b-1}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} + \left(\frac{b}{b-1} - \frac{1}{1-F}\right)W}{1 - \frac{b}{b-1}W}\right)$

where, $W = \frac{(1-F)}{f(p+S-\beta)} * \frac{\partial X}{\partial z}$

Proof:

i. Pricing Decision Approach:

$F[z^*(p,w)] = \frac{p+S-w}{p+S-\beta}$

Taking derivative in w,

$f\left(z^*(p,w)\right)\frac{d}{dw}\left(z^*(p,w)\right) = \frac{\left(\frac{dp}{dw}-1\right)}{(p+S-\beta)} - \frac{p+S-w}{(p+S-\beta)^2} * \frac{dp}{dw} = \frac{\left(\frac{dp}{dw}-1\right)}{(p+S-\beta)} - \frac{F[z^*]*\frac{dp}{dw}}{(p+S-\beta)} = \frac{(1-F[z^*])}{(p+S-\beta)} * \frac{dp}{dw} - \frac{1}{(p+S-\beta)}$

$\Rightarrow \frac{dz^*}{dw} = \frac{1-F}{f(p+S-\beta)} * \frac{dp}{dw} - \frac{1}{f(p+S-\beta)} = \frac{1-F}{f(p+S-\beta)}\left[\frac{dp}{dw} - \frac{1}{1-F}\right]$

We defined,

$X\left(w, z^*(p,w)\right) = \frac{(w-v)\Lambda(z^*(p)) + S\Theta(z^*(p))}{\mu - \Theta(z^*(p))}$

$\frac{dX}{dw} = \frac{\partial X}{\partial w} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw} = \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{1-F}{f(p+S-\beta)}\left[\frac{dp}{dw} - \frac{1}{1-F}\right]$

$\Rightarrow \frac{dX}{dw} = \frac{\Lambda}{(\mu-\Theta)} + W\left[\frac{dp}{dw} - \frac{1}{1-F}\right]$

Linear Demand:

For $y = a - bp$, p^* has to satisfy this equation (from Lemma 2a),

$-p^* + \frac{a+bw}{2b} + \frac{1}{2} * X\left(w, z^*(p^*, w)\right) = 0$

Taking derivative in w,

$-\frac{dp^*}{dw} + \frac{1}{2} + \frac{1}{2} * \frac{dX}{dw} = 0$ (B-a-1)

Then substituting "$\frac{dX}{dw}$" back into the equation B-a-1,

$-\frac{dp^*}{dw} + \frac{1}{2} + \frac{1}{2}\left[\frac{\Lambda}{(\mu-\Theta)} - \frac{W}{1-F} + W * \frac{dp^*}{dw}\right] = 0$

$\Rightarrow \frac{1}{2}\left(1 + \frac{\Lambda}{(\mu-\Theta)} - \frac{W}{(1-F)}\right) = \left(1 - \frac{W}{2}\right)\frac{dp^*}{dw}$

$\Rightarrow \frac{dp^*}{dw} = \frac{\frac{1}{2}\left(1 + \frac{\Lambda}{(\mu-\Theta)} - \frac{W}{(1-F)}\right)}{\left(1 - \frac{W}{2}\right)} = \frac{1}{2}\frac{\left(1 - \frac{W}{2} + \frac{\Lambda}{(\mu-\Theta)} - \frac{W}{(1-F)} + \frac{W}{2}\right)}{\left(1 - \frac{W}{2}\right)} = \frac{1}{2}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} + \left(\frac{1}{2} - \frac{1}{1-F}\right)W}{\left(1 - \frac{1}{2}W\right)}\right)$

Isoelastic Demand:

For $y = ap^{-b}$, p^* has to satisfy this equation,
$-p^* + \frac{b}{b-1}w + \frac{b}{b-1} * X\big(w, z^*(p^*, w)\big) = 0$
Taking derivative in w,
$-\frac{dp^*}{dw} + \frac{b}{b-1} + \frac{b}{b-1} * \frac{dX}{dw} = 0$
Substituting "$\frac{dX}{dw}$",
$-\frac{dp^*}{dw} + \frac{b}{b-1} + \frac{b}{b-1} * \left[\frac{\Lambda}{(\mu-\Theta)} - \frac{W}{1-F} + W * \frac{dp^*}{dw}\right] = 0$
$\Rightarrow \frac{b}{b-1} * \left[1 + \frac{\Lambda}{(\mu-\Theta)} - \frac{W}{1-F}\right] = \left[1 - \frac{b}{b-1} * W\right] \frac{dp^*}{dw}$
$\Rightarrow \frac{dp^*}{dw} = \frac{b}{b-1}\left(\frac{1+\frac{\Lambda}{(\mu-\Theta)}-\frac{W}{1-F}}{1-\frac{b}{b-1}*W}\right) = \frac{b}{b-1}\left(\frac{1-\frac{bW}{b-1}+\frac{\Lambda}{(\mu-\Theta)}+\frac{bW}{b-1}-\frac{W}{1-F}}{1-\frac{b}{b-1}W}\right) = \frac{b}{b-1}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{b}{b-1}-\frac{1}{1-F}\right)W}{1-\frac{b}{b-1}W}\right)$

ii. Stocking Decision Approach:

$\frac{dX(w,z^*)}{dw} = \frac{\partial X}{\partial w} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw} = \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw} = \frac{\Lambda}{(\mu-\Theta)} + \frac{(1-F)}{(\mu-\Theta)}\left[(w-\beta)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) - S\frac{\mu}{(\mu-\Theta)}\right] * \frac{dz^*}{dw}$

Linear Demand:

Optimal price, $p^*(z^*) = \frac{a+bw}{2b} + \frac{1}{2}\left\{\frac{(w-\beta)\Lambda(z^*)+S\Theta(z^*)}{\mu-\Theta(z^*)}\right\} \Rightarrow p^*(z) = \frac{a}{2b} + \frac{w+X}{2}$

Taking derivatives in w, $\frac{dp^*}{dw} = \frac{1}{2}\left(1 + \frac{dX}{dw}\right) = \frac{1}{2}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw}\right)$

z^* has to satisfy- $\frac{d}{dz}E\left[\pi_r\big(z^*, p^*(z^*)\big)\right] = 0$

$\Rightarrow (a - bp^*)[-(w-\beta) + (p^* + S - \beta)(1-F)] = 0$

Since, $y = a - bp^* > 0$,

$\Rightarrow [-(w-\beta) + (p^* + S - \beta)(1-F)] = 0$

Therefore, z^* has to satisfy, $\left[-(w-\beta) + \left(\frac{1}{2}(w+X) + S - \beta\right)(1-F)\right] = 0$

Differentiating this equation w.r.t. w,

$-1 + \frac{1}{2}\left(1 + \frac{dX}{dw}\right)(1-F) - \left(\frac{1}{2}(w+X) + S - \beta\right) f\frac{dz^*}{dw} = 0$

$$\Rightarrow -1+\frac{1}{2}\left(1+\frac{\Lambda}{(\mu-\Theta)}+\frac{\partial X}{\partial z^*}*\frac{dz^*}{dw}\right)(1-F)-(p^*+S-\beta)f\frac{dz^*}{dw}=0$$

$$\Rightarrow -1+\frac{1}{2}\left(1+\frac{\Lambda}{(\mu-\Theta)}\right)(1-F)=\left((p^*+S-\beta)f-\frac{1}{2}\left(\frac{\partial X}{\partial z^*}\right)(1-F)\right)\frac{dz^*}{dw}$$

$$\Rightarrow -\frac{1}{(1-F)}+\frac{1}{2}\left(1+\frac{\Lambda}{(\mu-\Theta)}\right)=\left((p^*+S-\beta)\frac{f}{1-F}-\frac{1}{2}\left(\frac{\partial X}{\partial z^*}\right)\right)\frac{dz^*}{dw}$$

$$\Rightarrow \frac{dz^*}{dw}=\frac{\frac{1}{2}\left(1+\frac{\Lambda}{(\mu-\Theta)}\right)-\frac{1}{(1-F)}}{(p^*+S-\beta)\frac{f}{1-F}-\frac{1}{2}\left(\frac{\partial X}{\partial z^*}\right)}=-\frac{\frac{1}{2}\left(1+\frac{\Lambda}{(\mu-\Theta)}\right)-\frac{1}{(1-F)}}{\frac{1}{2}\left(\frac{\partial X}{\partial z^*}\right)-(p^*+S-\beta)\frac{f}{1-F}}=-\frac{\frac{1}{2}\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{1}{2}-\frac{1}{(1-F)}\right)}{\frac{1}{2}\left(\frac{\partial X}{\partial z^*}\right)-(p^*+S-\beta)\frac{f}{1-F}}=\frac{1-F}{(p^*+S-\beta)f}*\frac{-\frac{1}{2}\frac{\Lambda}{(\mu-\Theta)}-\left(\frac{1}{2}-\frac{1}{(1-F)}\right)}{\left[\frac{1}{2}*\frac{1-F}{(p^*+S-\beta)f}\left(\frac{\partial X}{\partial z^*}\right)-1\right]}=\frac{1-F}{(p^*+S-\beta)f}*\frac{\frac{1}{2}\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{1}{2}-\frac{1}{(1-F)}\right)}{\left[1-\frac{1}{2}*W\right]}$$

Hence,

$$\frac{\partial X}{\partial z^*}*\frac{dz^*}{dw}=\frac{\partial X}{\partial z^*}*\frac{1-F}{(p^*+S-\beta)f}*\frac{\frac{1}{2}\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{1}{2}-\frac{1}{(1-F)}\right)}{\left[1-\frac{1}{2}*W\right]}=W*\frac{\frac{1}{2}\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{1}{2}-\frac{1}{(1-F)}\right)}{\left[1-\frac{1}{2}*W\right]}$$

$$\frac{dp^*}{dw}=\frac{1}{2}\left(1+\frac{\Lambda}{(\mu-\Theta)}+\frac{\partial X}{\partial z^*}*\frac{dz^*}{dw}\right)=\frac{1}{2}\left(1+\frac{\Lambda}{(\mu-\Theta)}+W*\frac{\frac{1}{2}\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{1}{2}-\frac{1}{(1-F)}\right)}{\left[1-\frac{1}{2}*W\right]}\right)=\frac{1}{2}\left(1+\frac{\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{1}{2}-\frac{1}{(1-F)}\right)W}{1-\frac{1}{2}*W}\right)$$

Isoelastic Demand Form:

Optimal price, $p^*(z^*)=\frac{bw}{(b-1)}+\frac{b}{b-1}\left\{\frac{(w-\beta)\Lambda(z^*)+S\Theta(z^*)}{\mu-\Theta(z^*)}\right\}\Rightarrow p^*(z)=\frac{b}{b-1}(w+X)$

Taking derivatives in w, $\frac{dp^*}{dw}=\frac{b}{b-1}\left(1+\frac{dX}{dw}\right)=\frac{b}{b-1}\left(1+\frac{\Lambda}{(\mu-\Theta)}+\frac{\partial X}{\partial z^*}*\frac{dz^*}{dw}\right)$

z^* has to satisfy, $\frac{d}{dz}E\left[\pi_r\left(z^*,p^*(z^*)\right)\right]=0$

$$\Rightarrow ap^{*(-b)}[-(w-\beta)+(p^*+S-\beta)(1-F)]=0$$

Since, $y=ap^{*(-b)}>0$,

$$\Rightarrow [-(w-\beta)+(p^*+S-\beta)(1-F)]=0$$

Therefore, z^* has to satisfy-

$$\Rightarrow \left[-(w-\beta)+\left(\frac{b}{b-1}(w+X)+S-\beta\right)(1-F)\right]=0$$

Differentiating this equation w.r.t. w,

$$\Rightarrow -1 + (1-F)\frac{b}{b-1}\left(1 + \frac{dX}{dw}\right) - \left(\frac{b}{b-1}(w+X) + S - \beta\right) f \frac{dz^*}{dw} = 0$$

$$\Rightarrow -1 + (1-F)\frac{b}{b-1}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw}\right) - \left(\frac{b}{b-1}(w+X) + S - \beta\right) f \frac{dz^*}{dw} = 0$$

$$\Rightarrow -1 + (1-F)\frac{b}{b-1}\left(1 + \frac{\Lambda}{(\mu-\Theta)}\right) = \left[\left(\frac{b}{b-1}(w+X) + S - \beta\right) f - (1-F)\frac{b}{b-1}\frac{\partial X}{\partial z^*}\right]\frac{dz^*}{dw}$$

$$\Rightarrow -1 + \frac{b}{b-1}(1-F)\left(1 + \frac{\Lambda}{(\mu-\Theta)}\right) = \left[(p^* + S - \beta) f - \frac{b}{b-1}(1-F)\frac{\partial X}{\partial z^*}\right]\frac{dz^*}{dw}$$

$$\Rightarrow -\frac{1}{(1-F)} + \frac{b}{b-1}\left(1 + \frac{\Lambda}{(\mu-\Theta)}\right) = \left[(p^* + S - \beta)\frac{f}{(1-F)} - \frac{b}{b-1}\frac{\partial X}{\partial z^*}\right]\frac{dz^*}{dw}$$

$$\Rightarrow \frac{dz^*}{dw} = \frac{-\frac{1}{(1-F)} + \frac{b}{b-1}\left(1+\frac{\Lambda}{(\mu-\Theta)}\right)}{(p^*+S-\beta)\frac{f}{(1-F)} - \frac{b}{b-1}\frac{\partial X}{\partial z^*}} = -\frac{\frac{b}{b-1}\left(1+\frac{\Lambda}{(\mu-\Theta)}\right) - \frac{1}{(1-F)}}{\frac{b}{b-1}\left(\frac{\partial X}{\partial z^*}\right) - (p^*+S-\beta)\frac{f}{(1-F)}} = -\frac{\frac{b}{b-1}\left(\frac{\Lambda}{(\mu-\Theta)}\right) + \left(\frac{b}{b-1} - \frac{1}{(1-F)}\right)}{\frac{b}{b-1}\left(\frac{\partial X}{\partial z^*}\right) - (p^*+S-\beta)\frac{f}{(1-F)}} =$$

$$\frac{1-F}{(p^*+S-\beta)f}\frac{\frac{b}{b-1}\left(\frac{\Lambda}{(\mu-\Theta)}\right) + \left(\frac{b}{b-1} - \frac{1}{(1-F)}\right)}{1 - \frac{b}{b-1}\frac{1-F}{(p^*+S-\beta)f}\left(\frac{\partial X}{\partial z^*}\right)}$$

$$\Rightarrow \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw} = \frac{1-F}{(p^*+S-\beta)f}\frac{\partial X}{\partial z^*} * \frac{\frac{b}{b-1}\left(\frac{\Lambda}{(\mu-\Theta)}\right) + \left(\frac{b}{b-1} - \frac{1}{(1-F)}\right)}{1 - \frac{b}{b-1}\frac{1-F}{(p^*+S-\beta)f}\left(\frac{\partial X}{\partial z^*}\right)} = W * \frac{\frac{b}{b-1}\left(\frac{\Lambda}{(\mu-\Theta)}\right) + \left(\frac{b}{b-1} - \frac{1}{(1-F)}\right)}{1 - \frac{b}{b-1}W}$$

Hence,

$$\frac{dp^*}{dw} = \frac{b}{b-1}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw}\right) = \frac{b}{b-1}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + W * \frac{\frac{b}{b-1}\left(\frac{\Lambda}{(\mu-\Theta)}\right) + \left(\frac{b}{b-1} - \frac{1}{(1-F)}\right)}{1 - \frac{b}{b-1}W}\right) =$$

$$\frac{b}{b-1}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} + \left(\frac{b}{b-1} - \frac{1}{(1-F)}\right)W}{1 - \frac{b}{b-1}W}\right)$$

iii. Proof of Corollary 2

Linear demand:

$$\frac{dp^*}{dw} = \frac{1}{2}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} + \left(\frac{1}{2} - \frac{1}{1-F}\right)W}{\left(1 - \frac{1}{2}W\right)}\right) = \frac{1}{2}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \frac{(1+F)}{(1-F)} * \frac{W}{2}}{1 - \frac{W}{2}}\right)$$

In order to decide if $\frac{dp^*}{dw}$ is less or greater than $\frac{1}{2}$, which is the cost-pass-through in risk less situation for linear demand, we need to check if $\frac{\frac{\Lambda}{(\mu-\Theta)} - \frac{(1+F)}{(1-F)} * \frac{W}{2}}{1 - \frac{W}{2}}$ is positive or negative. It is to be mentioned, $0 < \frac{\Lambda}{(\mu-\Theta)} = \frac{E(leftover)}{E(sales)} < 1$ and $\frac{W}{2} < 1$ [Appendix 2D]. W can be positive or negative.

The denominator, $1-\frac{W}{2}$ has a positive value for both positive and negative W, because $\frac{W}{2}<1$. Therefore, the numerator $\frac{\Lambda}{(\mu-\Theta)}-\frac{(1+F)}{(1-F)}*\frac{W}{2}$ determine the sign of the term.

Condition for $\frac{dp^*}{dw}>\frac{1}{2}$:

$\frac{\Lambda}{(\mu-\Theta)}-\frac{(1+F)}{(1-F)}*\frac{W}{2}>0 \Rightarrow \frac{W}{2}<\frac{\Lambda}{(\mu-\Theta)}*\frac{(1-F)}{(1+F)} \Rightarrow \frac{W}{2}<\frac{\Lambda}{(\mu-\Theta)}*\frac{(1-F)}{(1+F)}<1$

This condition is satisfied by all negative values and some positive values of W.

Now we are interested to check if $\frac{dp^*}{dw}$ can exceed 1!

For positive W, the numerator $\left(\frac{\Lambda}{(\mu-\Theta)}-\frac{(1+F)}{(1-F)}*\frac{W}{2}\right)$ is less than the denominator $\left(1-\frac{W}{2}\right)$ because $\frac{\Lambda}{(\mu-\Theta)}<1$ and $\frac{(1+F)}{(1-F)}*\frac{W}{2}>\frac{W}{2}$. Therefore, the value of $\frac{dp^*}{dw}$ cannot exceed 1 for linear demand in the case of $0<W$.

Now let's see if $\frac{dp^*}{dw}$ can exceed 1 for negative W. Let's define, $Z=-W$

$\Rightarrow \frac{dp^*}{dw}=\frac{1}{2}\left(1+\frac{\frac{\Lambda}{(\mu-\Theta)}+\frac{(1+F)}{(1-F)}*\frac{Z}{2}}{1+\frac{Z}{2}}\right)=\frac{1}{2}\left(1+\frac{\frac{\Lambda}{(\mu-\Theta)}+\frac{2F}{(1-F)}*\frac{Z}{2}+\frac{Z}{2}}{1+\frac{Z}{2}}\right)$

$\frac{dp^*}{dw}$ can exceed 1 if

$\frac{\Lambda}{(\mu-\Theta)}+\frac{(1+F)}{(1-F)}*\frac{Z}{2}>1+\frac{Z}{2} \Rightarrow \left(\frac{(1+F)}{(1-F)}-1\right)\frac{Z}{2}>1-\frac{\Lambda}{(\mu-\Theta)} \Rightarrow \frac{F}{(1-F)}Z>1-\frac{\Lambda}{(\mu-\Theta)} \Rightarrow$

$-\frac{F}{(1-F)}W>1-\frac{\Lambda}{(\mu-\Theta)}$

$\Rightarrow W<-\left(1-\frac{\Lambda}{\mu-\Theta}\right)\frac{1-F}{F}$

This is the required condition for which $\frac{dp^*}{dw}$ can exceed 1 in the case of linear demand. However, it is unusual to obtain a cost-pass-through greater than one in the case of linear demand. Therefore, we recommend further verification of the plausibility of the condition $\left(W<-\left(1-\frac{\Lambda}{\mu-\Theta}\right)\frac{1-F}{F}\right)$.

Condition for $\frac{dp^*}{dw}<\frac{1}{2}$: $\frac{\Lambda}{(\mu-\Theta)}-\frac{(1+F)}{(1-F)}*\frac{W}{2}<0 \Rightarrow \frac{W}{2}>\frac{\Lambda}{(\mu-\Theta)}*\frac{(1-F)}{(1+F)}$

$\Rightarrow \frac{\Lambda}{(\mu-\Theta)}*\frac{(1-F)}{(1+F)}<\frac{W}{2}<1$

This condition is satisfied by some positive values of W

Hence for some positive values of W, $\frac{dp^*}{dw}$ increases from less than half to greater than half.

Condition for $\frac{dp^*}{dw}=\frac{1}{2}$: $\frac{\Lambda}{(\mu-\Theta)}-\frac{(1+F)}{(1-F)}*\frac{W}{2}=0 \Rightarrow \frac{W}{2}=\frac{\Lambda}{(\mu-\Theta)}*\frac{(1-F)}{(1+F)}<1$

Isoelastic demand:

$$\frac{dp^*}{dw}=\frac{b}{b-1}\left(1+\frac{\frac{\Lambda}{(\mu-\Theta)}+\left(\frac{b}{b-1}-\frac{1}{1-F}\right)W}{1-\frac{b}{b-1}W}\right)=\frac{b}{b-1}\left(1+\frac{\frac{\Lambda}{(\mu-\Theta)}-\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W}{1-\frac{b}{b-1}W}\right)$$

In order to decide if $\frac{dp^*}{dw}$ is less or greater than $\frac{b}{b-1}$, which is the cost-pass-through in the risk less situation for isoelastic demand, we need to check if $\frac{\frac{\Lambda}{(\mu-\Theta)}-\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W}{1-\frac{b}{b-1}W}$ is positive or negative. It is to be mentioned, $0<\frac{\Lambda}{(\mu-\Theta)}=\frac{E(leftover)}{E(sales)}<1$ and $\frac{b}{b-1}W<1$ [Appendix 2D]. W can be positive or negative [Appendix 2C]. The denominator, $1-\frac{b}{b-1}W$ has a positive value for both positive and negative W, because $\frac{b}{b-1}W<1$. Therefore, the numerator $\frac{\Lambda}{(\mu-\Theta)}-\frac{\left(F-\frac{1}{b}\right)}{(1-F)}\frac{b}{(b-1)}W$ determine the sign of the term.

Let's assume, b is sufficiently large so that $F-\frac{1}{b}>1-F>0$

Condition for $\frac{dp^*}{dw}>\frac{b}{b-1}$: $\frac{\Lambda}{(\mu-\Theta)}-\frac{\left(F-\frac{1}{b}\right)}{(1-F)}\frac{b}{(b-1)}W>0 \Rightarrow \frac{\Lambda}{(\mu-\Theta)}\frac{(1-F)}{\left(F-\frac{1}{b}\right)}>\frac{b}{(b-1)}W$

Condition for $\frac{dp^*}{dw}<\frac{b}{b-1}$: $\frac{\Lambda}{(\mu-\Theta)}-\frac{\left(F-\frac{1}{b}\right)}{(1-F)}\frac{b}{(b-1)}W<0 \Rightarrow \frac{\Lambda}{(\mu-\Theta)}\frac{(1-F)}{\left(F-\frac{1}{b}\right)}<\frac{b}{(b-1)}W<1$

Condition for $\frac{dp^*}{dw}=\frac{b}{b-1}$: $\frac{\Lambda}{(\mu-\Theta)}-\frac{\left(F-\frac{1}{b}\right)}{(1-F)}\frac{b}{(b-1)}W=0 \Rightarrow \frac{b}{(b-1)}W=\frac{\Lambda}{(\mu-\Theta)}\frac{(1-F)}{\left(F-\frac{1}{b}\right)}$

We are interested to check if check if $\frac{dp^*}{dw}$ can reduce below 1!

Required condition: $\frac{dp^*}{dw}=\frac{b}{b-1}\left(1+\frac{\frac{\Lambda}{(\mu-\Theta)}-\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W}{1-\frac{b}{b-1}W}\right)<1 \Rightarrow \frac{\frac{\Lambda}{(\mu-\Theta)}-\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W}{1-\frac{b}{b-1}W}<-\frac{1}{b} \Rightarrow$ $-\frac{\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W-\frac{\Lambda}{(\mu-\Theta)}}{1-\frac{b}{b-1}W}<-\frac{1}{b} \Rightarrow \frac{\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W-\frac{\Lambda}{(\mu-\Theta)}}{1-\frac{b}{b-1}W}>\frac{1}{b} \Rightarrow \frac{b\left(F-\frac{1}{b}\right)b}{(b-1)(1-F)}W-\frac{b\Lambda}{(\mu-\Theta)}>1-\frac{b}{b-1}W \Rightarrow$ $\frac{\left(F-\frac{1}{b}\right)b}{(1-F)}\left(\frac{b}{(b-1)}W-\frac{(1-F)}{\left(F-\frac{1}{b}\right)}\frac{\Lambda}{(\mu-\Theta)}\right)>1-\frac{b}{b-1}W$

Let's check the validity of $\frac{\frac{\Lambda}{(\mu-\Theta)}-\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W}{1-\frac{b}{b-1}W}<-\frac{1}{b}$,

where $0<\frac{\Lambda}{(\mu-\Theta)}<1$, $\frac{b}{b-1}W<1$, $b>2$, $0<F<1$, and $-1<\frac{\frac{\Lambda}{(\mu-\Theta)}-\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W}{1-\frac{b}{b-1}W}$ are given.

$$\frac{\frac{\Lambda}{(\mu-\Theta)}-\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W}{1-\frac{b}{b-1}W}=-\frac{-\frac{\Lambda}{(\mu-\Theta)}+\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W}{b\left(\frac{1}{b}-\frac{1}{b-1}W\right)}=-\frac{1}{b}*\frac{\left(\frac{1}{b}-\frac{1}{b-1}W\right)-\frac{1}{b}-\frac{\Lambda}{(\mu-\Theta)}+\frac{b\left(F-\frac{1}{b}\right)+(1-F)}{(b-1)(1-F)}W}{\left(\frac{1}{b}-\frac{1}{b-1}W\right)}=-\frac{1}{b}*$$
$$\left\{1+\frac{-\frac{1}{b}-\frac{\Lambda}{(\mu-\Theta)}+\frac{b\left(F-\frac{1}{b}\right)+(1-F)}{(b-1)(1-F)}W}{\left(\frac{1}{b}-\frac{1}{b-1}W\right)}\right\}=-\frac{1}{b}*\left\{1+\frac{-1-\frac{\Lambda}{(\mu-\Theta)}b+b\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W+\frac{b}{(b-1)}W}{\left(1-\frac{b}{b-1}W\right)}\right\}=-\frac{1}{b}*\left\{1+\right.$$
$$\left.\frac{-1+\left(\frac{b}{(b-1)}W-\frac{\Lambda}{(\mu-\Theta)}*\frac{(1-F)}{\left(F-\frac{1}{b}\right)}\right)\frac{b\left(F-\frac{1}{b}\right)}{(1-F)}+\frac{b}{(b-1)}W}{\left(1-\frac{b}{b-1}W\right)}\right\}=-\frac{1}{b}*\left\{1-\frac{1-\frac{b}{(b-1)}W-\left(\frac{b}{(b-1)}W-\frac{\Lambda}{(\mu-\Theta)}*\frac{(1-F)}{\left(F-\frac{1}{b}\right)}\right)\frac{b\left(F-\frac{1}{b}\right)}{(1-F)}}{\left(1-\frac{b}{b-1}W\right)}\right\}$$

$$\frac{\frac{\Lambda}{(\mu-\Theta)}-\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W}{1-\frac{b}{b-1}W}=-\frac{-\frac{\Lambda}{(\mu-\Theta)}+\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W}{b\left(\frac{1}{b}-\frac{1}{b-1}W\right)}=-\frac{1}{b}*\frac{\left(\frac{1}{b}-\frac{1}{b-1}W\right)-\frac{1}{b}-\frac{\Lambda}{(\mu-\Theta)}+\frac{b\left(F-\frac{1}{b}\right)+(1-F)}{(b-1)(1-F)}W}{\left(\frac{1}{b}-\frac{1}{b-1}W\right)}=-\frac{1}{b}*$$
$$\left\{1+\frac{-\frac{1}{b}-\frac{\Lambda}{(\mu-\Theta)}+\frac{b\left(F-\frac{1}{b}\right)+(1-F)}{(b-1)(1-F)}W}{\left(\frac{1}{b}-\frac{1}{b-1}W\right)}\right\}=-\frac{1}{b}*\left\{1+\frac{-1-\frac{\Lambda}{(\mu-\Theta)}b+b\frac{b\left(F-\frac{1}{b}\right)}{(b-1)(1-F)}W+\frac{b}{(b-1)}W}{\left(1-\frac{b}{b-1}W\right)}\right\}$$

It is very difficult to analyze analytically; numerical analysis may be helpful.

F. Numerical Analysis

We are following the pricing decision approach here. Let's assume: shortage price, $S = 2$, buyback price, $\beta = 1$, and a uniform distribution on the interval [1,5],[47]

$\mu = 3;\ f(u) = \frac{1}{4};\ F(u) = \frac{u-1}{4}\ ;$

$F\left(z^*(p)\right) = \left[\frac{p+S-w}{p+S-\beta}\right] \Rightarrow \frac{z^*-1}{4} = \frac{p+S-w}{p+S-\beta} \Rightarrow z^*(p) = 1 + 4\,\frac{p+S-w}{p+S-\beta}$

$\Lambda(z^*(p)) = \int_1^{z^*(p)} (z^*(p) - u) f(u) du = \frac{(z^*(p)-1)^2}{8} = 2\left(\frac{p+S-w}{p+S-\beta}\right)^2 = 2\left(\frac{p+2-w}{p+1}\right)^2$

$\Theta(z^*(p)) = \int_{z^*(p)}^{5} (u - z) f(u) du = \frac{(z^*(p)-5)^2}{8} = 2\left(\frac{-w+\beta}{p+S-\beta}\right)^2 = 2\left(\frac{-w+1}{p+1}\right)^2$

$\frac{\Lambda}{(\mu-\Theta)} = \frac{2\left(\frac{p+2-w}{p+1}\right)^2}{3-2\left(\frac{-w+1}{p+1}\right)^2} = \frac{2(2+p-w)^2}{1+6p+3p^2+4w-2w^2}$

$F(z) = \frac{p+2-w}{p+1}\ ;\ (1-F) = 1 - \frac{p+2-w}{p+1} = \frac{-1+w}{1+p}$

[47] The multiplicative case (with constant elasticity) require $A > 0$ in order to avoid the occurrence of negative demand (Petruzzi and Dada 1999). However, Emmons and Gilbert (1998) assumes uniform distribution on the interval [0,2] with mean=1 for simplification. That worked there, because their model didn't have shortage cost parameter (i.e. S=0).

; $\frac{(1+F)}{(1-F)} = \frac{1+\frac{p+2-w}{p+1}}{1-\frac{p+2-w}{p+1}} = \frac{3+2p-w}{-1+w}$; $\frac{F}{(1-F)} = \frac{\frac{p+2-w}{p+1}}{1-\frac{p+2-w}{p+1}} = \frac{2+p-w}{-1+w}$

i. Linear Demand:
In the case of linear demand ($y = a - bp$), let's assume $= 50, b = 1$.
P^* satisfy the following equation-

$$-p + \frac{a+bw}{2b} + \frac{1}{2} * \frac{(w-\beta)\Lambda(z^*(p)) + S\Theta(z^*(p))}{(\mu - \Theta(z^*(p)))} = 0$$

$$-p + \frac{50+w}{2} + \frac{1}{2} * \frac{(w-1)*2*(p+2-w)^2 + 2*2*(-w+1)^2}{3(p+1)^2 - 2(-w+1)^2} = 0$$

$$-p + 25 + \frac{w}{2} + \frac{(w-1)(p+2-w)^2 + 2(-w+1)^2}{3(p+1)^2 - 2(-w+1)^2} = 0$$

The solution in p is tedious[48], therefore, defining $w = g(p^*)$, the solution can be written as-

$$w = \frac{1}{204}\left(209 + 10p + 5p^2 \pm (1+p)\sqrt{62449 - 2398p + 25p^2}\right)$$

We accept one of the roots because the other root gives greater wholesale price than the retail price which is not acceptable.

$$w = \frac{1}{204}\left(209 + 10p + 5p^2 - (1+p)\sqrt{62449 - 2398p + 25p^2}\right)$$

From the risk-less part of the price ($p^0 = \frac{a+bw}{2b} = \frac{50+w}{2}$), the highest value of w and p^0 is 50 for which the corresponding deterministic demand is zero ($y = a - bp^0 = 0$). The minimum value of p^0 is 25 for $w = 0$. Hence, we conduct the numerical analysis within this limit. However, we avoid the boundary value in the simulation considering the effect of the random part.
Solving numerically, for $w = \{1.61 \sim 44.99\}, p = \{26 \sim 49\}, p^0 = \{25.8 \sim 47.496\}, \Delta w = 43.38, \Delta p = 23, \Delta p^0 = 21.696$
The results are illustrated in the main section.

The corresponding cost-pass-through: $\frac{dp^*}{dw} = \frac{1}{2}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \frac{(1+F)}{(1-F)} * \frac{W}{2}}{1 - \frac{W}{2}}\right)$

$$W = \frac{(1-F)^2}{(\mu-\theta)f(p+S-\beta)}\left[(w-\beta)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\theta)}\right) - S\frac{\mu}{(\mu-\theta)}\right]$$

$$W = \frac{\left(\frac{-1+w}{1+p}\right)^2}{\left(3-2\left(\frac{-w+1}{p+1}\right)^2\right)\frac{1}{4}(p+1)}\left[(w-1)\left(\frac{2+p-w}{-1+w} - \frac{2(2+p-w)^2}{1+6p+3p^2+4w-2w^2}\right) - 2\frac{3}{\left(3-2\left(\frac{-w+1}{p+1}\right)^2\right)}\right]$$

$$= \frac{4(-1+w)^2(4+5p+3p^2-9w-5pw+2w^2)}{(-1-6p-3p^2-4w+2w^2)^2}$$

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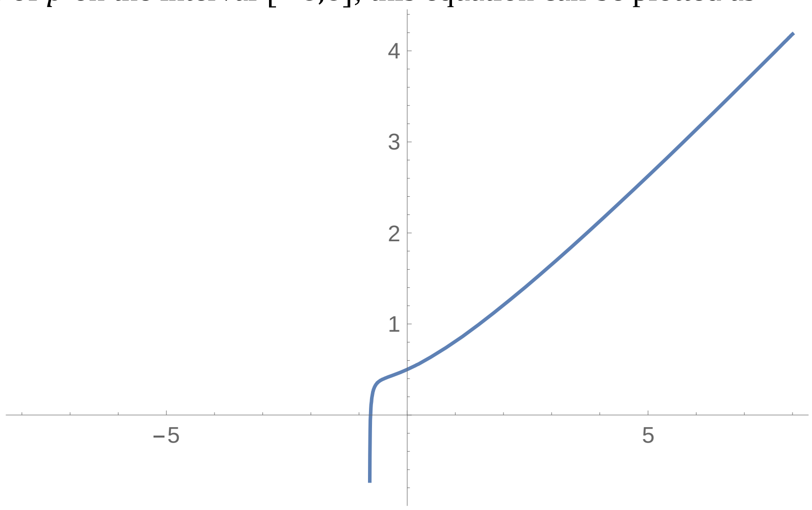
$$p = \frac{1}{18}(136 + 5w) + \frac{(-23716 - 1540w - 25w^2)}{18X} - \frac{1}{18}X$$

Where, $X = \left(-3602692 - 454884w + 38022w^2 - 125w^3 + 54\sqrt{34}\sqrt{-3627478 + 6849644w - 2803618w^2 - 431909w^3 + 13486w^4 - 125w^5}\right)^{1/3}$

$$\frac{dp^*}{dw} = \frac{1}{2}\left(1 + \frac{\frac{2(2+p-w)^2}{1+6p+3p^2+4w-2w^2} - \frac{3+2p-w}{-1+w} * \frac{2(-1+w)^2(4+5p+3p^2-9w-5pw+2w^2)}{(-1-6p-3p^2-4w+2w^2)^2}}{1 - \frac{2(-1+w)^2(4+5p+3p^2-9w-5pw+2w^2)}{(-1-6p-3p^2-4w+2w^2)^2}}\right)$$
$$= \frac{(1+p)(25+54p+15p^2-20w-24pw+10w^2)}{2(-7+9p+27p^2+9p^3+42w+36pw-36w^2-18pw^2+10w^3)}$$

ii. Isoelastic demand:
In the case of isoelastic demand ($y = ap^{-b}$), assuming $b = 3$, p^* is the p that satisfies–
$$-p + \frac{b}{b-1}w + \frac{b}{b-1} * \frac{(w-\beta)\Lambda(z^*(p)) + S\Theta(z^*(p))}{(\mu - \Theta(z^*(p)))} = 0$$
$$\Rightarrow -p + \frac{3}{2}w + 3 * \frac{(w-1)(p+2-w)^2 + 2(-w+1)^2}{3(p+1)^2 - 2(-w+1)^2} = 0$$
The solution of this equation in p is tedious[49]. Let's define, $w = g(p*)$; then, the solution of this equation can be written as,
$$w_1 = \frac{27+46p+15p^2-\sqrt{3}\sqrt{(1+p)^2(147+198p+11p^2)}}{4(3+4p)},\ w_2 = \frac{27+46p+15p^2+\sqrt{3}\sqrt{(1+p)^2(147+198p+11p^2)}}{4(3+4p)}$$
Accepting one of the roots (that satisfies $p > w$),
$$w = \frac{27+46p+15p^2-\sqrt{3}\sqrt{(1+p)^2(147+198p+11p^2)}}{4(3+4p)}$$
For the range of p on the interval $[-8,8]$, this equation can be plotted as-



Solving numerically, for $w = \{1 \sim 2.62\}$, we obtain $p = \{1.5 \sim 5\}, p^0 = \{1.5 \sim 3.94\}$. We are interested to analyze the change is prices which are $\Delta w = 69.52, \Delta p = 115, \Delta p^0 = 104.28$. The figure (in main section) shows the cost-pass-through for both the riskless situation and the buyback-newsvendor situation.

The corresponding cost-pass-through: $\frac{dp^*}{dw} = \frac{b}{b-1}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} + \left(\frac{b}{b-1} - \frac{1}{1-F}\right)W}{1 - \frac{b}{b-1}W}\right)$

---

[49] $p^* = \frac{1}{12}\left(-11 + 9w + \frac{47-114w-33w^2}{X^{1/3}} - X^{1/3}\right)$, where
$X = \left(-145 + 909w - 1683w^2 - 81w^3 + 12\sqrt{3}\sqrt{-(-1+w)^2(-289+1781w-2927w^2+367w^3+68w^4)}\right)$

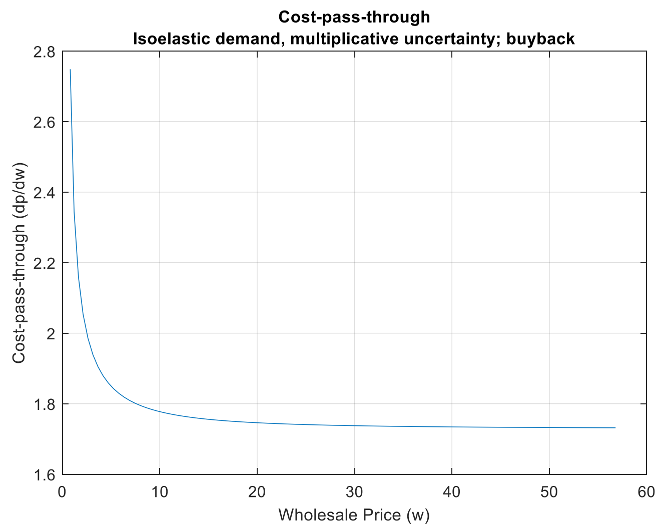
$$W = \frac{(1-F)^2}{(\mu-\Theta)f(p+S-\beta)}\left[(w-\beta)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) - S\frac{\mu}{(\mu-\Theta)}\right]$$

$$W = \frac{\left(\frac{-1+w}{1+p}\right)^2}{\left(3-2\left(\frac{-w+1}{p+1}\right)^2\right)\frac{1}{4}(p+1)}\left[(w-1)\left(\frac{2+p-w}{-1+w} - \frac{2(2+p-w)^2}{1+6p+3p^2+4w-2w^2}\right) - 2\frac{3}{\left(3-2\left(\frac{-w+1}{p+1}\right)^2\right)}\right]$$

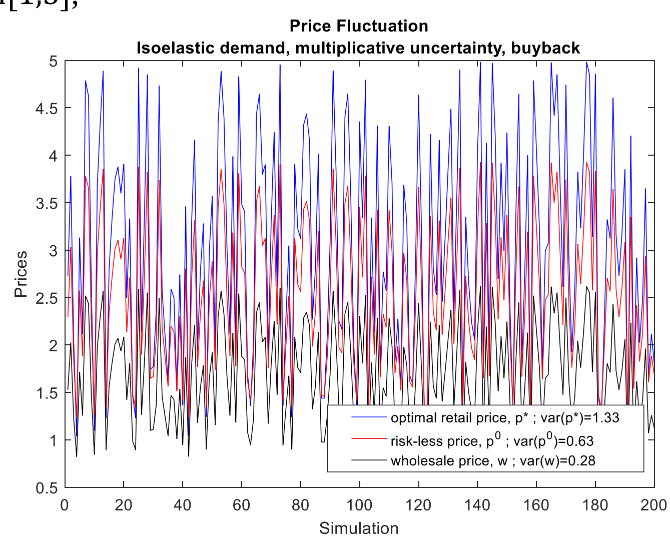
$$= \frac{4(-1+w)^2(4+5p+3p^2-9w-5pw+2w^2)}{(-1-6p-3p^2-4w+2w^2)^2}$$

$$\frac{dp^*}{dw} = \frac{3}{2}\left(1 + \frac{\frac{2(2+p-w)^2}{1+6p+3p^2+4w-2w^2}+\left(\frac{3}{2}-\frac{1+p}{-1+w}\right)\frac{4(-1+w)^2(4+5p+3p^2-9w-5pw+2w^2)}{(-1-6p-3p^2-4w+2w^2)^2}}{1-\frac{3}{2}*\frac{4(-1+w)^2(4+5p+3p^2-9w-5pw+2w^2)}{(-1-6p-3p^2-4w+2w^2)^2}}\right)$$

$$= \frac{3(1+p)^2(15p^2-6p(-9+4w)+5(5-4w+2w^2))}{2(-23+36p^3+9p^4+110w-132w^2+62w^3-8w^4+p^2(24+60w-30w^2)+6p(-3+23w-19w^2+5w^3))}$$



For $p^* = \text{uniform}[1,5]$,



which is too clumsy to visualize properly, therefore, for better illustration, let's assume $p^* = \text{uniform}[30,35]$. The corresponding figure is mentioned in the main section.

G. Detail calculation of Equations

$$\pi_r = \begin{matrix} pD - wq + (\beta)(q-D) & ; D \leq q \\ pq - wq - S(D-q) & ; D > q \end{matrix} \quad (1)$$

$$\pi_r = \begin{matrix} py\epsilon - wyz + (\beta)y(z-\epsilon) & ; \epsilon \leq z & \rightarrow leftover \\ pyz - wyz - Sy(\epsilon - z) & ; \epsilon > z & \rightarrow shortage \end{matrix} \quad (2)$$

$E[\pi_r] = \int_A^z [pyu + (\beta)y(z-u)]f(u)du + \int_z^B [pyz - Sy(u-z)]f(u)du - wyz$ (3)

$= y\left[\int_A^z [pu + (\beta)(z-u)]f(u)du + \int_z^B [pz - S(u-z)]f(u)du - wz\right]$

$= y\left[p\int_A^z uf(u)du + (\beta)\int_A^z (z-u)f(u)du + p\int_z^B zf(u)du - S\int_z^B (u-z)f(u)du - w\mu - w(z-\mu)\right]$

$= y\left[p\int_A^z uf(u)du + p\int_z^B uf(u)du - p\int_z^B uf(u)du + p\int_z^B zf(u)du + (\beta)\int_A^z (z-u)f(u)du - S\int_z^B (u-z)f(u)du - w\mu - w\left(\int_A^z (z-u)f(u)du + \int_z^B (z-u)f(u)du\right)\right]$

$= y\left[p\mu - p\int_z^B (u-z)f(u)du + (\beta)\int_A^z (z-u)f(u)du - S\int_z^B (u-z)f(u)du - w\mu - w\int_A^z (z-u)f(u)du + w\int_z^B (u-z)f(u)du\right]$

$= y\left[p\mu - w\mu - w\int_A^z (z-u)f(u)du + (\beta)\int_A^z (z-u)f(u)du - p\int_z^B (u-z)f(u)du - S\int_z^B (u-z)f(u)du + w\int_z^B (u-z)f(u)du\right]$

$= (p-w)y\mu - (w-\beta)y\int_A^z (z-u)f(u)du - (p+S-w)y\int_z^B (u-z)f(u)du$

$= (p-w)y\mu - [(w-\beta)\mathrm{y}\Lambda(z) + (p+S-w)\mathrm{y}\Theta(z)]$

$= \Psi(p) - L(z,p)$

$E[\pi_r(z,p)] = (p-w)y\mu - [(w-\beta)\mathrm{y}\Lambda(z) + (p+S-w)\mathrm{y}\Theta(z)]$ (4)

$\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = -y\big[(w-\beta)\mathrm{F}(z) - (p+S-w)\big(1-\mathrm{F}(z)\big)\big] = -y[(w-\beta+p+S-w)F(z) - (p+S-w)] = y[p+s-w-(p+S-\beta)F(z)] = y[-(w-\beta)+(p+S-\beta)[1-F(z)]]$ (5)

$\frac{\partial^2}{\partial z^2}(E[\pi_r(z|p)]) = -y(p+S-\beta)f(z) < 0$ (6)

$\frac{d}{dp}[\Psi(p)] = y'\mu\left[p - w + \frac{y}{y'}\right]$ (7)

$\frac{\partial}{\partial p}[L(z,p)] = \frac{\partial}{\partial p}\big[y[(w-\beta)\Lambda(z) + (p+S-w)\Theta(z)]\big] = [(w-\beta)\Lambda(z)y' + \mathrm{y}\Theta(z) + (p+S-w)\Theta(z)y'] = \mathrm{y}\Theta(z) + y'[(w-\beta)\Lambda(z) + (p+S-w)\Theta(z)]$

$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = \frac{d}{dp}[\Psi(p)] - \frac{\partial}{\partial p}[L(z,p)] = y'\mu\left[p - w + \frac{y}{y'}\right] - \mathrm{y}\Theta(z) - y'[(w-\beta)\Lambda(z) + (p+S-w)\Theta(z)]$

For, $y = a - bp$, (8)

$\frac{d}{dp}[\Psi(p)] = -b\mu\left[p - w + \frac{a-bp}{-b}\right] = \mu[-2bp + bw + a] = -2b\mu\left[p - \frac{a+bw}{2b}\right] = -2b\mu[p - p^0]$

$\frac{\partial}{\partial p}(E[\pi_r(z,p)]) = -2b\mu[p - p^0] - (a - bp)\Theta(z) + b[(w - \beta)\Lambda(z) + (p + S - w)\Theta(z)]$

$= b\left[-2\mu p + 2\mu p^0 - \frac{a}{b}\Theta(z) + p\Theta(z) + (w - \beta)\Lambda(z) + p\Theta(z) + S\Theta(z) - w\Theta(z)\right]$

$= b\left[-2\mu p + 2\mu p^0 - (\frac{a}{b} + w)\Theta(z) + 2p\Theta(z) + (w - \beta)\Lambda(z) + S\Theta(z)\right]$

$= b[-2\mu p + 2\mu p^0 - 2p^0\Theta(z) + 2p\Theta(z) + (w - \beta)\Lambda(z) + S\Theta(z)]$

$= b\left[2(p^0 - p)\big(\mu - \Theta(z)\big) + (w - \beta)\Lambda(z) + S\Theta(z)\right]$

$= 2b\big(\mu - \Theta(z)\big)\left[-p + p^0 + \frac{1}{2} * \frac{(w-\beta)\Lambda(z)+S\Theta(z)}{(\mu-\Theta(z))}\right]$

where $p^0 = \frac{a+bw}{2b}$

$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)]) = -2b\big(\mu - \Theta(z)\big) < 0$ (9)

For, $y = ap^{-b}$, (10)

$\frac{d}{dp}[\Psi(p)] = -bap^{-b-1}\mu\left[p - w + \frac{ap^{-b}}{-bap^{-b-1}}\right] = -bap^{-b-1}\mu\left[p - w + \frac{p}{-b}\right] = ap^{-b-1}\mu[-bp + bw + p] = \mu ap^{-b-1}[bw - (b - 1)p] = -(b - 1)\mu ap^{-b-1}\left[p - \frac{bw}{b-1}\right] = -(b - 1)\mu ap^{-b-1}[p - p^0]$

Here, $p^0 = \frac{bw}{b-1}$ is the price that maximizes the riskless profit.[50] We can obtain the riskless optimal price as $\frac{bw}{b-1}$ by setting the $\frac{d}{dp}[\Psi(p)] = 0$ where $y = ap^{-b}$.

$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = -(b - 1)\mu ap^{-b-1}[p - p^0] - ap^{-b} * \Theta(z) + bap^{-b-1}[(w - \beta)\Lambda(z) + (p + S - w)\Theta(z)]$

$= (b - 1)ap^{-b-1}\left[-\mu[p - p^0] - \frac{p}{b-1} * \Theta(z) + \frac{b}{b-1}[(w - \beta)\Lambda(z) + (p + S - w)\Theta(z)]\right]$

$= (b - 1)ap^{-b-1}\left[-\mu p + \mu p^0 - \frac{p}{b-1} * \Theta(z) + \frac{b}{b-1}(w - \beta)\Lambda(z) + \frac{b}{b-1}p\Theta(z) + \frac{b}{b-1}S\Theta(z) - \frac{b}{b-1}w\Theta(z)\right]$

$= (b - 1)ap^{-b-1}\left[-\mu p + \mu p^0 - \frac{p}{b-1} * \Theta(z) + \frac{b}{b-1}p\Theta(z) + \frac{b}{b-1}(w - \beta)\Lambda(z) + \frac{b}{b-1}S\Theta(z) - p^0\Theta(z)\right]$

[50] Petruzzi and Dada argued that "$\Psi(p)$ reached its maximum at $p^0 = \frac{bw}{b-1}$, because $-(b - 1)\mu ap^{-b-1} < 0$ for $p < \infty$, thus $\Psi(p)$ is increasing for $0 < p < \frac{bw}{b-1}$ and decreasing for $\frac{bw}{b-1} < p < \infty$" (Petruzzi and Dada, 1999).

$= (b-1)ap^{-b-1}\left[\mu p^0 - p^0\Theta(z) - \mu p + p\Theta(z) + \frac{b}{b-1}\{(w-\beta)\Lambda(z) + S\Theta(z)\}\right]$

$= (b-1)ap^{-b-1}\left[\big(\mu-\Theta(z)\big)(p^0-p) + \frac{b}{b-1}\{(w-\beta)\Lambda(z) + S\Theta(z)\}\right]$

$= (b-1)ap^{-b-1}\{\mu-\Theta(z)\}\left[-p+p^0+\frac{b}{b-1}\left\{\frac{(w-\beta)\Lambda(z)+S\Theta(z)}{\big(\mu-\Theta(z)\big)}\right\}\right]$

where $p^0 = \frac{b}{b-1}w$

$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)]) = -\frac{b+1}{p} * \frac{\partial}{\partial p}(E[\pi_r(p|z)]) - (b-1)ap^{-b-1}\{\mu-\Theta(z)\}$ (11)

$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)])\Big|_{\frac{\partial}{\partial p}(E[\pi_r(p|z)])=0} = -(b-1)ap^{-b-1}\{\mu-\Theta(z)\} < 0$ (12)

$\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = y[-(w-\beta) + (p+S-\beta)[1-F(z)]] = 0$ (13)

$\Rightarrow z^*(p) = F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]$

$E[\pi_r(z^*(p),p)] = y\left[(p-w)\mu - (w-\beta)\int_A^{F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]}\left(F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right] - u\right)f(u)du - (p+S-w)\int_{F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]}^{B}\left(u - F^{-1}\left[\frac{p+S-w}{p+S-\beta}\right]\right)f(u)du\right]$ (14)

$\frac{\partial}{\partial p}(E[\pi_r(z^*(p),p)]) = y'\mu\left[p-w+\frac{y}{y'}\right] - \mathrm{y}\Theta(z^*) - y'[(w-\beta)\Lambda(z^*) + (p+S-w)\Theta(z^*)]$ (15)

For $y = a - bp$, (16)

$\frac{\partial}{\partial p}\big(E\big[\pi_r\big(p,z^*(p)\big)\big]\big) = 2b\big(\mu-\Theta(z^*)\big)\left[-p+p^0+\frac{1}{2}*\frac{(w-\beta)\Lambda(z^*(p))+S\Theta(z^*(p))}{\big(\mu-\Theta(z^*)\big)}\right]$

where, $p^0 = \frac{a+bw}{2b}$

Since $\mu - \Theta(z^*) > 0$, (17)

$p^*(w) = \left\{p\,\middle|\, -p + \frac{a+bw}{2b} + \frac{1}{2}*\frac{(w-\beta)\Lambda(z^*(p))+S\Theta(z^*(p))}{\big(\mu-\Theta(z^*(p))\big)} = 0\right\}$

Similarly, for $y = ap^{-b}$, (18)

$p^*(w) = \left\{p\,\middle|\, -p + \frac{b}{b-1}w + \frac{b}{b-1}*\frac{(w-\beta)\Lambda(z^*(p))+S\Theta(z^*(p))}{\big(\mu-\Theta(z^*(p))\big)} = 0\right\}$

Appendix for Chapter 7: BP in the case of Revenue Sharing Contract (Stochastic Demand)

1. Additive Demand Uncertainty Case

A. Problem Formulation:

Let's assume $z = q - y$, where z is called the stocking factor and can be expressed as $z = \mu + \sigma *$ (safety factor). Then the retailer's profit can be expressed as Equation 2 and the corresponding optimal policy is the order quantity, $q^* = y(p^*) + z^*$.

$$\pi_r = \begin{matrix} \phi p(y+\epsilon) - w(y+z) + v(z-\epsilon) & ;\epsilon \leq z & \rightarrow leftover \\ \phi p(y+z) - w(y+z) - S(\epsilon - z) & ;\epsilon > z & \rightarrow shortage \end{matrix} \quad (2)$$

From Equation 2, the expected retail profit,

$$\begin{aligned} E[\pi_r] &= \int_A^z [\phi p(y+u) + v(z-u)]f(u)du \\ &\quad + \int_z^B [\phi p(y+z) - S(u-z)]f(u)du - w(y+z) \quad (3) \\ &= (\phi p - w)(y+\mu) - [(w-v)\Lambda(z) + (\phi p + S - w)\Theta(z)] \\ &= \Psi(p) - L(z,p) \end{aligned}$$

Hence, the expected profit is the sum of the riskless profit $\Psi(p) = (\phi p - w)(y+\mu)$ minus the loss due to uncertainty, $L(z,p) = [(w-v)\Lambda(z) + (\phi p + S - w)\Theta(z)]$ (i.e. we obtain the expected profit by subtracting the loss function from the riskless profit). Here, $\Lambda(z) = \int_A^z (z-u)f(u)du =$ expected leftover and $\Theta(z) = \int_z^B (u-z)f(u)du =$ expected shortage. The loss function is the sum of the overstocking and understocking cost (i.e. $Loss\ function = overage\ cost * E(leftover) + underage\ cost * E(shortage)$). The retailer's objective is to maximize,

$$E[\pi_r(z,p)] = (\phi p - w)(y+\mu) - [(w-v)\Lambda(z) + (\phi p + S - w)\Theta(z)] \quad (4)$$

This is a joint optimization problem in p and z. Therefore, we take partial derivatives of the expected profit in p and z, and also check if the second order conditions are fulfilled.

$$\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = -(w-v) + (\phi p + S - v)[1 - F(z)] \quad (5)$$

$$\frac{\partial^2}{\partial z^2}(E[\pi_r(z|p)]) = -(\phi p + S - v)f(z) < 0 \quad (6)$$

$$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = \frac{d}{dp}[\Psi(p)] - \frac{\partial}{\partial p}[L(z,p)] = \phi\left[y'\left(p - \frac{w}{\phi} + \frac{y}{y'}\right) + \mu - \Theta(z)\right] \quad (7)$$

For, $y = a - bp$,

$$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = -2\phi b\left[p - \frac{\phi a + bw}{2\phi b} - \frac{\mu - \Theta(z)}{2b}\right] \quad (8)$$

$$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)]) = -2\phi b < 0 \tag{9}$$

Equation 6 tells us that $E[\pi_r]$ is concave in z for a given p. Equation 9 tells us that $E[\pi_r]$ is concave in p for a given z.

B. Proof of Lemma 1:

i. Pricing decision approach

Setting $\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = 0$, we obtain, $F[z^*(p)] = \frac{\phi p+S-w}{\phi p+S-v} \Rightarrow z^*(p) = F^{-1}\left[\frac{\phi p+S-w}{\phi p+S-v}\right]$

Replacing the $z^*(p)$ into $\partial E/\partial p$: $\frac{dE[\pi_r(p,z^*(p))]}{dp} = y'(\phi p - w) + \phi\big(y + \mu - \Theta(z^*(p))\big)$

Linear demand:

For a linear demand, $y = a - bp$, $\frac{dE[\pi_r(p,z^*(p)]}{dp} = 2\phi b\left[-p + \frac{\phi a+bw}{2\phi b} + \frac{\mu-\Theta\left(z^*(p)\right)}{2b}\right]$. Hence, the optimal p^* is the p that satisfies $\frac{dE[\pi_r(p,z^*(p)]}{dp} = 0$.

Since $2\phi b > 0$, $p^*(w) = \left\{p\middle|-p + \frac{\phi a+bw}{2\phi b} + \frac{\mu-\Theta(z^*(p))}{2b} = 0\right\}$.

Isoelastic demand:

For an isoelastic demand, $y = ap^{-b}$,

$\frac{dE[\pi_r(p,z^*(p))]}{dp} = -\frac{bap^{-b}}{p}(\phi p - w) + \phi\Big(ap^{-b} + \mu - \Theta(z)\Big) = (-\Theta + \mu)\phi + ap^{-1-b}\big(p\phi + b(w - p\phi)\big) = (-\Theta + \mu)\phi + \phi(b-1)ap^{-1-b}\left(-p + \frac{b}{\phi(b-1)}w\right) = \phi\left[\left(-p + \frac{b}{\phi(b-1)}w\right)(b-1)ap^{-1-b} + (\mu - \Theta)\right]$.

The optimal p^* is the p that satisfies $\frac{d}{dp}(E[\pi_r(p,z^*(p)]) = 0$. Since, $\phi > 0$, hence, $p^*(w) = \left\{p\middle|\left(-p + \frac{b}{\phi(b-1)}w\right)(b-1)ap^{-b-1} + \mu - \Theta\big(z^*(p)\big) = 0\right\}$.

Or, equivalently, $p^*(w) = \left\{p\middle|-p + \frac{b}{\phi(b-1)}w + \frac{\mu-\Theta\left(z^*(p)\right)}{(b-1)ap^{-b-1}} = 0\right\}$.

The derivation of the optimal p^* requires to prove that the expected profit, $E\big[\pi_r\big(p,z^*(p)\big)\big]$ is concave and unimodal in p which is mentioned in Appendix 1-C-2 and 1-C-3.

ii. Stocking decision approach

Solving $\frac{\partial}{\partial p}(E[\pi_r]) = -2\phi b\left[p - \frac{\phi a+bw}{2\phi b} - \frac{\mu-\Theta(z)}{2b}\right] = 0$, we can obtain $p^* = \frac{\phi a+bw}{2\phi b} + \frac{\mu-\Theta(z)}{2b}$. Then replacing p^* into the equation, $\frac{\partial}{\partial z}(E[\pi_r]) = 0$ would give the single variable equation in z^*,

$$\frac{d}{dz}(E[\pi_r(z,p^*(z))]) = -(w-v) + \left(\frac{\phi a+bw}{2b} + \frac{\phi\big(\mu-\Theta(z)\big)}{2b} + S - v\right)[1-F(z)] = 0$$

The derivation of optimal z^* requires to prove that the expected profit, $E\big[\Pi_r\big(z,p(z)\big)\big]$ is concave and unimodal in z which is mentioned in Appendix 1-C-1.

C. Condition for concavity

1. Stocking decision approach:

We need to show that the expected profit, $E\left[\Pi_r\left(z, p(z)\right)\right]$ is concave and unimodal in z. We adapt the proof (in case of newsvendor model) from Petruzzi and Dada (1999) and modify it to reflect our revenue-sharing contract setting. Interested readers may check the proof of Theorem 1 of Petruzzi and Dada (1999)[51] and replace their holding cost parameter '$-h$' by the salvage price parameter 'v' and retail revenue parameter 'p' by 'ϕp' to obtain the proof required in our setting. For readers' convenience, we showed the detail proof here as follows-

From lemma 1, $p^*(z) = \frac{\phi a+bw}{2\phi b} + \frac{\mu-\Theta(z)}{2b} \Rightarrow \frac{dp^*}{dz} = \frac{(1-F(z))}{2b}$

Replacing $p^*(z)$ into the expected profit equation,

$E[\pi_r(z, p^*(z))] = (\phi p^* - w)(y+\mu) - [(w-v)\Lambda(z) + (\phi p^* + S - w)\Theta(z)]$

Taking derivative in z,

$$\frac{d}{dz}E\left[\pi_r\left(z, p^*(z)\right)\right] = \phi\frac{dp^*}{dz}(a - bp^* + \mu) + (\phi p^* - w)\left(-b\frac{dp^*}{dz}\right) - \left[(w-v)F(z) + \phi\frac{dp^*}{dz}\Theta(z) - (\phi p^* + S - w)\left(1 - F(z)\right)\right]$$

$$= \frac{\left(1-\mathrm{F}(z)\right)}{2b}\phi(a - bp^* + \mu) + (\phi p^* - w)\left(-\frac{\left(1-\mathrm{F}(z)\right)}{2}\right) - \left[(w-v)\mathrm{F}(z) + \phi\frac{\left(1-\mathrm{F}(z)\right)}{2b}\Theta(z) - (\phi p^* + S - w)\left(1-\mathrm{F}(z)\right)\right]$$

$$= \phi\left(1-\mathrm{F}(z)\right)\left(\frac{a}{2b} - \frac{p^*}{2} + \frac{\mu}{2b} - \frac{p^*}{2} + \frac{w}{2\phi} - \frac{\Theta(z)}{2b} + p^* + \frac{S}{\phi} - \frac{w}{\phi}\right) - (w-v)\mathrm{F}(z)$$

$$= \phi\left(1-\mathrm{F}(z)\right)\left(\frac{a}{2b} + \frac{w}{2\phi} + \frac{\mu}{2b} - \frac{\Theta(z)}{2b} + \frac{S}{\phi} - \frac{w}{\phi}\right) - (w-v) + (w-v) - (w-v)\mathrm{F}(z)$$

$$= \phi\left(1-\mathrm{F}(z)\right)\left(\frac{\phi a+bw}{2b\phi} + \frac{\mu-\Theta(z)}{2b} + \frac{S}{\phi} - \frac{w}{\phi}\right) - (w-v) + (w-v)\left(1-\mathrm{F}(z)\right)$$

$$= \phi\left(1-\mathrm{F}(z)\right)\left(p^* + \frac{S}{\phi} - \frac{w}{\phi}\right) - (w-v) + (w-v)\left(1-\mathrm{F}(z)\right)$$

$$= \left(1-\mathrm{F}(z)\right)(\phi p^* + S - w) - (w-v) + (w-v)\left(1-\mathrm{F}(z)\right)$$

$$= -(w-v) + \left(1-F(z)\right)(\phi p^* + S - v)$$

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[51] Theorem 1 of Petruzzi-Dada (1999) stated that-

"…z^* is determined according to the following:

c) If $F(.)$ is an arbitrary distribution function, then an exhaustive search over all values of z in the region $[A, B]$ will determine z^*.
d) If $F(.)$ is a distribution function satisfying the condition $2r(z)^2 + \frac{d}{dz}r(z) > 0$ for $A \le z \le B$, where $r(.) \equiv \frac{f(.)}{1-F(.)}$ is the Hazard rate, then z^* is the largest z in the region $[A, B]$ that satisfies $\frac{dE\left[\Pi_r\left(z,p(z)\right)\right]}{dz} = 0$."

If the condition for (b) is met and $a - b(w - 2S) + A > 0$, then z^* is the unique z in the region $[A, B]$ that satisfies $\frac{dE\left[\Pi_r\left(z,p(z)\right)\right]}{dz} = 0$."

Alternate method: Instead of substituting p^* into the expected profit equation, substituting p^* into $\frac{\partial}{\partial z}E[\pi_r]$, also gives the same result.

$\frac{d}{dz}(E[\pi_r(z,p^*(z))]) = -(w-v) + (\phi p^*(z) + S - v)[1-F(z)]$

$\frac{d^2}{dz^2}(E[\pi_r(z,p^*(z))]) = \phi\frac{dp^*(z)}{dz}[1-F(z)] - (\phi p^*(z)+S-v)f(z) = \phi\frac{[1-F(z)]^2}{2b} - (\phi p^*(z)+S-v)f(z) = -\frac{f(z)}{2b}\left\{2b(\phi p^*(z)+S-v) - \phi\frac{1-F(z)}{r}\right\}$

Here, $r = \left(\frac{f(z)}{1-F(z)}\right)$ =hazard rate

$$\begin{aligned}
&\frac{d^3}{dz^3}(E[\pi_r(z,p^*(z))]) \\
&= \frac{df(z)}{dz}\left[\left(\frac{1}{f(z)}\right)\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right] - \frac{f(z)}{2b}\left\{2b*\phi*\frac{[1-F(z)]}{2b} - \phi\frac{d}{dz}\left(\frac{1-F(z)}{r}\right)\right\} \\
&= \frac{df(z)}{dz}\left[\left(\frac{1}{f(z)}\right)\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right] - \frac{f(z)}{2b}\left\{\phi[1-F(z)] - \phi\left(-\frac{f(z)}{r} - \frac{1-F(z)}{r^2}\frac{dr}{dz}\right)\right\} \\
&= \frac{df(z)}{dz}\left[\left(\frac{1}{f(z)}\right)\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right] - \phi\frac{f(z)}{2b}\left\{1-F(z) + \frac{f(z)}{r} + \frac{1-F(z)}{r^2}\frac{dr}{dz}\right\} \\
&= \frac{df(z)}{dz}\left[\left(\frac{1}{f(z)}\right)\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right] - \phi\frac{f(z)[1-F(z)]}{2br^2}\left\{r^2 + \frac{f(z)}{1-F(z)}r + \frac{1}{r^2}\frac{dr}{dz}\right\} \\
&= \frac{df(z)}{dz}\left[\left(\frac{1}{f(z)}\right)\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\right] - \phi\frac{f(z)[1-F(z)]}{2br^2}\left\{2r^2 + \frac{dr}{dz}\right\}
\end{aligned}$$

$$\frac{d^3}{dz^3}(E[\pi_r(z,p^*(z))])\Big|_{\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}=0} = -\phi\frac{f(z)[1-\mathrm{F}(z)]}{2b\left(\frac{f(z)}{1-F(z)}\right)^2}\left\{2\left(\frac{f(z)}{1-F(z)}\right)^2 + \frac{d}{dz}\left(\frac{f(z)}{1-F(z)}\right)\right\}$$

Following Petruzzi and Dada (1999)'s argument analogously in a revenue-sharing setting, if $\left\{2\left(\frac{f(z)}{1-F(z)}\right)^2 + \frac{d}{dz}\left(\frac{f(z)}{1-F(z)}\right)\right\} > 0$, then $\frac{d}{dz}(E[\pi_r(z,p^*(z))])$ is monotone or unimodal and thus having at most two roots. Moreover, for $z = B$, $\frac{d}{dz}(E[\pi_r(z,p^*(z))]) = -(w-v) < 0$. Therefore, if $\frac{d}{dz}(E[\pi_r(z,p^*(z))])$ has only one root, it indicates a change in sign from positive to negative. It corresponds to a local maximum of $E[\pi_r(z,p^*(z))]$

If $\frac{d}{dz}(E[\pi_r(z,p^*(z))])$ has two roots, the larger (smaller) of the two corresponds to a local maximum (minimum) of $E[\pi_r(z,p^*(z))]$. In either case, $E[\pi_r(z,p^*(z))]$ has only one local maximum, identified either as the unique value (or as the larger of two values) of z that satisfies $\frac{d}{dz}(E[\pi_r(z,p^*(z))]) = 0$. Since $E[\pi_r(z,p^*(z))]$ is unimodal if $\frac{d}{dz}(E[\pi_r(z,p^*(z))])$ has only one root (assuming $\left\{2\left(\frac{f(z)}{1-F(z)}\right)^2 + \frac{d}{dz}\left(\frac{f(z)}{1-F(z)}\right)\right\} > 0$), a sufficient condition for unimodality of $E[\pi_r(z,p^*(z))]$ is $\frac{d}{dz}(E[\pi_r(z,p^*(z))])\Big|_{z=A} > 0$ or equivalently, $2b*\frac{d}{dz}(E[\pi_r(z,p^*(z))])\Big|_{z=A} > 0$,

$\Rightarrow -2b(w-v) + 2b\left(\frac{\phi a+bw}{2b} + \frac{\phi(\mu-\Theta(A))}{2b} + S - v\right)[1-F(A)] > 0$

$\Rightarrow -2b(w-v) + \left(\phi a + bw + \phi(\mu - \mu + A) + 2b(S-v)\right) > 0$

$\Rightarrow \phi a - b(w-2S) + \phi A > 0$

Petruzzi & Dada (1999) summarized the conditions for concavity and unimodality. Similar conditions were proposed by Ernest (1970), Young (1978), Bulow and Proschan (1975). It is to be mentioned, PF2 distributions and log-normal distributions (that have non-decreasing hazard rate, $r(.) = \frac{f(.)}{1-F(.)}$) satisfy the above mentioned conditions.

Hence, if $2\left(\frac{f(z)}{1-F(z)}\right)^2 + \frac{d}{dz}\left(\frac{f(z)}{1-F(z)}\right) > 0$, then $E[\pi_r(z, p^*(z))]$ is concave and unimodal in z.

2. Pricing decision approach:

We need to show that $E\left[\Pi_r\left(p, z^*(p)\right)\right]$ is concave in p.

$F[z^*(p)] = \frac{\phi p+S-w}{\phi p+S-v} \Rightarrow z^*(p) = F^{-1}\left[\frac{\phi p+S-w}{\phi p+S-v}\right]$

Differentiating $F[z^*(p)] = \frac{\phi p+S-w}{\phi p+S-v}$ with respect to p,

$f * \frac{dz^*}{dp} = \frac{\phi}{\phi p+S-v} - \phi\frac{\phi p+S-w}{(\phi p+S-v)^2} = \phi\left(\frac{1}{\phi p+S-v} - \frac{F}{\phi p+S-v}\right) = \phi\frac{1-F}{(\phi p+S-v)}$

Therefore, $\frac{dz^*(p)}{dp} = \phi\frac{1-F}{f(\phi p+S-v)} = \frac{\phi}{r(\phi p+S-v)}$

Here, $r = \frac{f}{1-F}$ = hazard rate

Replacing the $z^*(p)$ into $\partial E/\partial p$: $\frac{d}{dp}(E[\pi_r(p, z^*(p)]) = 2\phi b\left[-p + \frac{\phi a+bw}{2\phi b} + \frac{\mu - \Theta\left(z^*(p)\right)}{2b}\right]$

We need to find zeros of $\frac{d}{dp}(E[\pi_r])$:

$\frac{d^2}{dp^2}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 2\phi b\left[-1 + \frac{\left(1-F(z^*)\right)\frac{dz^*}{dp}}{2b}\right] = -2b + \frac{\phi(1-F)}{r(\phi p+S-v)}$

$\frac{d^3}{dp^3}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = \frac{d}{dp}\left[\frac{\phi(1-F)}{r(\phi p+S-v)}\right] = \phi\left\{\frac{-f*\frac{dz^*}{dp}}{r(\phi p+S-v)} - \frac{(1-F)}{r^2(\phi p+S-v)^2}\frac{d}{dp}[r(\phi p + S - v)]\right\} = \phi\left\{\frac{-\phi f}{r^2(\phi p+S-v)^2} - \frac{(1-F)}{r^2(\phi p+S-v)^2}\left[\frac{dr}{dz^*} * \frac{dz^*}{dp} * (\phi p + S - v) + \phi r\right]\right\} =$

$\phi\left\{-\frac{\phi f}{r^2(\phi p+S-v)^2} - \frac{(1-F)\left[\phi r+\frac{\phi}{r}*\frac{dr}{dz^*}\right]}{r^2(\phi p+S-v)^2}\right\} = -\phi^2\left\{\frac{f+(1-F)\left[r+\frac{1}{r}*\frac{dr}{dz^*}\right]}{r^2(p+S-v)^2}\right\} = -\frac{\phi^2(1-F)\left[\frac{f}{1-F}+r+\frac{1}{r}*\frac{dr}{dz^*}\right]}{r^2(p+S-v)^2} = -\frac{\phi^2(1-F)\left[2r+\frac{1}{r}*\frac{dr}{dz^*}\right]}{r^2(p+S-v)^2} = -\frac{\phi^2(1-F)\frac{1}{r}\left[2r^2+\frac{dr}{dz^*}\right]}{r^2(p+S-v)^2} = -\frac{\phi^2(1-F)\left[2r^2+\frac{dr}{dz^*}\right]}{r^3(p+S-v)^2}$

Since $r = \frac{f}{1-F} > 0$, therefore, for $2r^2 + \frac{dr}{dz^*} > 0$, $\frac{d^3}{dp^3}(E[\pi_r]) < 0$ that follows that $\frac{d}{dp}(E[\pi_r])$ is either monotone or unimodal.

It is to be mentioned, the condition $2r^2 + \frac{dr}{dz^*} > 0$ is the same condition from the stocking decision approach as expected.

3. Another method (Arcelus et al. 2005)

We modify the proof from Arcelus et al. 2005 according to the revenue-sharing contract.

$\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = -(w - v) + (\phi p + S - v)[1 - F(z)]$

$\frac{\partial^2}{\partial z^2}(E[\pi_r(z|p)]) = -(\phi p + S - v)f(z) < 0$

Setting $\frac{\partial E[\pi_r]}{\partial z} = 0$, we obtain $1 - F = \frac{w-v}{\phi p + S - v}$. Therefore, $\frac{\partial^2 E[\pi_r]}{\partial z^2} = -\frac{w-v}{1-F} f$

$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = \frac{d}{dp}[\Psi(p)] - \frac{\partial}{\partial p}[L(z,p)] = y'(\phi p - w) + \phi\big(y + \mu - \Theta(z)\big)$

Setting $\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = 0$, we obtain, $\Rightarrow (\phi p - w) = -\frac{\phi}{y'}(y + \mu - \Theta)$

$\frac{\partial^2 E[\pi_r]}{\partial p^2} = y''(\phi p - w) + 2\phi y' = -\frac{y''\phi}{y'}(y + \mu - \Theta) + 2\phi y' = \phi\left[-\frac{yy''}{y'}\left(1 + \frac{\mu - \Theta}{y}\right) + 2y'\right] = \phi y'\left(2 - \frac{yy''}{(y')^2}\left(1 + \frac{\mu-\Theta}{y}\right)\right)$

Since, $\phi > 0$ and $y' < 0$, therefore, $\frac{\partial^2 E[\pi_r]}{\partial p^2} < 0$ if $\frac{yy''}{(y')^2}\left(1 + \frac{\mu-\Theta}{y}\right) < 2$

$\frac{\partial^2 E[\pi_r]}{\partial p \partial z} = \frac{\partial}{\partial p}\left(\frac{\partial E[\pi_r]}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial E[\pi_r]}{\partial p}\right) = \phi(1 - F) > 0$

$\frac{\partial^2 E[\pi_r]}{\partial z^2} \times \frac{\partial^2 E[\pi_r]}{\partial p^2} - \left(\frac{\partial^2 E[\pi_r]}{\partial p \partial z}\right)^2 = -\frac{w-v}{1-F} f\phi y'\left(2 - \frac{yy''}{(y')^2}\left(1 + \frac{\mu-\Theta}{y}\right)\right) - \phi^2(1 - F)^2$

$\frac{\partial^2 E[\pi_r]}{\partial z^2} \times \frac{\partial^2 E[\pi_r]}{\partial p^2} - \left(\frac{\partial^2 E[\pi_r]}{\partial p \partial z}\right)^2 > 0$ if

$\Rightarrow -\frac{w-v}{1-F} f\phi y'\left(2 - \frac{yy''}{(y')^2}\left(1 + \frac{\mu-\Theta}{y}\right)\right) > \phi^2(1 - F)^2 \Rightarrow f > \frac{\phi(1-F)^3}{-(w-v)y'\left(2 - \frac{yy''}{(y')^2}\left(1 + \frac{\mu-\Theta}{y}\right)\right)}$

It is to be mentioned, the denominator is positive, because $y' < 0$ and $\frac{yy''}{(y')^2}\left(1 + \frac{\mu-\Theta}{y}\right) < 2$. Moreover, since, $f \leq 1$, we can write the condition as $1 \geq f > \frac{\phi(1-F)^3}{-(w-v)y'\left(2 - \frac{yy''}{(y')^2}\left(1 + \frac{\mu-\Theta}{y}\right)\right)}$

For a linear demand, $y = a - bp$, $y' = -b < 0$, $y'' = 0$, $\frac{yy''}{(y')^2} = 0$

Hence, for $y = a - bp$, the condition is, $1 \geq f > \frac{\phi(1-F)^3}{2b(w-v)}$

For an isoelastic demand, $y = ap^{-b}$, $y' = -bap^{-b-1} = \frac{-by}{p} < 0$, $y'' = \frac{-by'}{p} - \frac{-by}{p^2} = -\frac{y'}{p}(b + 1) = \frac{by}{p^2}(b + 1) > 0$, $\frac{yy''}{(y')^2} = \frac{y\frac{by}{p^2}(b+1)}{\left(\frac{-by}{p}\right)^2} = \frac{b+1}{b} > 1$

For $b > 1$, $1 < \frac{b+1}{b} < 2$, therefore, $1 < \frac{yy''}{(y')^2} < 2$

Hence, for $y = ap^{-b}$, the condition is,

$\frac{b+1}{b}\left(1 + \frac{\mu-\Theta}{ap^{-b}}\right) < 2$ and $1 \geq f > \frac{\phi(1-F)^3}{(w-v)\frac{by}{p}\left(2 - \frac{b+1}{b}\left(1 + \frac{\mu-\Theta}{ap^{-b}}\right)\right)} > 0$

D. Cost-pass-through
1. Linear demand

i. Stocking decision approach

z^* has to satisfy $\frac{d}{dz}(E[\pi_r(z, p^*(z))]) = 0$,
$\Rightarrow -(w-v) + (\phi p^*(z) + S - v)[1 - F(z)] = 0$
$\Rightarrow -(w-v) + \left(\frac{\phi a + bw}{2b} + \frac{\phi(\mu - \Theta(z))}{2b} + S - v\right)[1 - F(z^*)] = 0$
Differentiating it with respect to w,
$-1 + \left(\frac{1}{2} + \phi\frac{1-F}{2b} * \frac{dz^*}{dw}\right)(1-F) - \left(\frac{\phi a + bw}{2b} + \frac{\phi(\mu - \Theta(z))}{2b} + S - v\right) f\frac{dz^*}{dw} = 0$
$\Rightarrow -1 + \frac{(1-F)}{2} + \phi\frac{(1-F)^2}{2b} * \frac{dz^*}{dw} - \left(\frac{\phi a + bw}{2b} + \frac{\phi(\mu - \Theta(z))}{2b} + S - v\right) f\frac{dz^*}{dw} = 0$
$\Rightarrow -1 + \frac{(1-F)}{2} = \left[\left(\frac{\phi a + bw}{2b} + \frac{\phi(\mu - \Theta(z))}{2b} + S - v\right) f - \phi\frac{(1-F)^2}{2b}\right] * \frac{dz^*}{dw}$
$\Rightarrow \frac{-1-F}{2} = (1-F)\left[\left(\frac{\phi a + bw}{2b} + \frac{\phi(\mu - \Theta(z))}{2b} + S - v\right) r - \phi\frac{(1-F)}{2b}\right] * \frac{dz^*}{dw}$
$\Rightarrow \frac{dz^*}{dw} = -\frac{1+F}{2(1-F)\left[\left(\frac{\phi a + bw}{2b} + \frac{\phi(\mu - \Theta(z))}{2b} + S - v\right) r - \phi\frac{(1-F)}{2b}\right]}$
From $\frac{d}{dp}(E[\pi_r)]) = 0$, $p^*(w, z) = \frac{\phi a + bw}{2\phi b} + \frac{\mu - \Theta(z)}{2b}$
Differentiating with respect to w,
$\frac{dp^*}{dw} = \frac{1}{2\phi} + \frac{1-\mathrm{F}}{2b} * \frac{dz^*}{dw}$
$\Rightarrow \frac{dp^*}{dw} = \frac{1}{2\phi} - \frac{1-F}{2b} * \frac{1+F}{2(1-F)\left[\left(\frac{\phi a + bw}{2b} + \frac{\phi(\mu - \Theta(z))}{2b} + S - v\right) r - \phi\frac{(1-F)}{2b}\right]} = \frac{1}{2\phi} - \frac{1}{2b} *$
$\frac{1+F}{2\left[(\phi p^* + S - v) r - \phi\frac{(1-F)}{2b}\right]} = \frac{1}{2\phi} - \frac{1}{2} * \frac{1+F}{[2b(\phi p^* + S - v) r - \phi(1-F)]} = \frac{1}{2\phi} - \frac{1}{2\phi} *$
$\frac{\phi(1+F)}{[2b(\phi p^* + S - v) r - \phi(1-F)]} = \frac{1}{2\phi}\left(1 - \frac{\phi(1+F)}{2br(\phi p + S - v) - \phi(1-F)}\right)$

ii. Pricing Decision approach:
$F[z^*(p, w)] = \frac{\phi p + S - w}{\phi p + S - v}$
Differentiating with respect to w,
$f\left(z^*(p, w)\right)\frac{d}{dw}\left(z^*(p, w)\right) = \frac{\left(\phi\frac{dp}{dw} - 1\right)}{(\phi p + S - v)} - \frac{\phi p + S - w}{(\phi p + S - v)^2} * \phi\frac{dp}{dw} = \frac{\left(\phi\frac{dp}{dw} - 1\right)}{(\phi p + S - v)} - \frac{F[z^*(p,w)] * \phi\frac{dp}{dw}}{(\phi p + S - v)} =$
$\frac{(1 - F[z^*(p, w)])}{(\phi p + S - v)} * \phi\frac{dp}{dw} - \frac{1}{(\phi p + S - v)}$
$\Rightarrow \frac{dz^*}{dw} = \frac{1-F}{f(\phi p + S - v)} * \phi\frac{dp}{dw} - \frac{1}{f(\phi p + S - v)} = \frac{1-F}{f(\phi p + S - v)}\left[\phi\frac{dp}{dw} - \frac{1}{1-F}\right] = \frac{1}{r(\phi p + S - v)}\left[\phi\frac{dp}{dw} - \frac{1}{1-F}\right]$
For $y = a - bp$, p^* has to satisfy this equation, $-p^* + \frac{\phi a + bw}{2\phi b} + \frac{\mu - \Theta\left(z^*(p^*)\right)}{2b} = 0$
Taking derivative in w, $-\frac{dp^*}{dw} + \frac{1}{2\phi} + \frac{1}{2b}(1-F)\frac{dz^*}{dw} = 0$
Substituting $\frac{dz^*}{dw}$,

$$\Rightarrow -\frac{dp^*}{dw} + \frac{1}{2\phi} + \frac{1}{2b} * \frac{(1-F)}{r(\phi p+S-v)}\left[\phi\frac{dp}{dw} - \frac{1}{1-F}\right] = 0$$
$$\Rightarrow \frac{1}{2\phi} - \frac{1}{2b} * \frac{1}{r(\phi p+S-v)} = \frac{dp^*}{dw}\left(1 - \frac{\phi}{2b} * \frac{(1-F)}{r(\phi p+S-v)}\right)$$
$$\Rightarrow \frac{1}{2\phi}\left(1 - \frac{\phi}{b} * \frac{1}{r(\phi p+S-v)}\right) = \frac{dp^*}{dw}\left(1 - \frac{\phi}{2b} * \frac{(1-F)}{r(\phi p+S-v)}\right)$$
$$\Rightarrow \frac{dp^*}{dw} = \frac{1}{2\phi} * \frac{\left(1-\frac{\phi}{b}*\frac{1}{r(\phi p+S-v)}\right)}{\left(1-\frac{\phi}{2b}*\frac{(1-F)}{r(\phi p+S-v)}\right)} = \frac{1}{2\phi} * \frac{\left(1-\frac{\phi}{2b}*\frac{(1-F)}{r(\phi p+S-v)}-1+\frac{\phi}{2b}*\frac{(1-F)}{r(\phi p+S-v)}+1-\frac{\phi}{b}*\frac{1}{r(\phi p+S-v)}\right)}{\left(1-\frac{\phi}{2b}*\frac{(1-F)}{r(\phi p+S-v)}\right)} = \frac{1}{2\phi} *$$
$$\left(1 + \frac{\left(\frac{\phi}{2b}*\frac{(1-F)}{r(\phi p+S-v)}-\frac{\phi}{b}*\frac{1}{r(\phi p+S-v)}\right)}{\left(1-\frac{\phi}{2b}*\frac{(1-F)}{r(\phi p+S-v)}\right)}\right) = \frac{1}{2\phi} * \left(1 + \frac{(\phi(1-F)-2\phi)}{2br(\phi p+S-v)-\phi(1-F)}\right) = \frac{1}{2\phi}\left(1 - \frac{\phi(1+F)}{2br(\phi p+S-v)-\phi(1-F)}\right)$$

Substituting $(\phi p + S - v) = \frac{w-v}{1-F}$,
$$\Rightarrow \frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 - \frac{\phi(1+F)}{2b\frac{f}{1-F}*\frac{w-v}{1-F}-\phi(1-F)}\right) = \frac{1}{2\phi}\left(1 - \frac{(1-F)^2(1+F)\phi}{2bf(w-v)-(1-F)^3\phi}\right)$$

2. Isoelastic demand:

From, $\frac{dE[\pi_r]}{dz} = -(w-v) + (\phi p + S - v)[1 - F(z)] = 0$, taking derivative in w,
$$-1 + \phi\frac{dp}{dw}[1-F] - (\phi p + S - v)f\frac{dz^*}{dw} = 0 \Rightarrow \phi\frac{dp}{dw} - \frac{1}{1-F} = (\phi p + S - v)r\frac{dz^*}{dw}$$
$$\Rightarrow \frac{dz^*}{dw} = \frac{1}{(\phi p+S-v)r}\left(\phi\frac{dp}{dw} - \frac{1}{1-F}\right) = \frac{1-F}{(w-v)r}\left(\phi\frac{dp}{dw} - \frac{1}{1-F}\right) = \frac{(1-F)^2}{(w-v)f}\left(\phi\frac{dp}{dw} - \frac{1}{1-F}\right)$$
For an isoelastic demand, $y = ap^{-b}$, p^* satisfy this equation,
$$\left(-p + \frac{b}{\phi(b-1)}w\right)(b-1)ap^{-b-1} + \mu - \Theta\left(z^*(p)\right) = 0$$
Taking derivative in w,
$$\left(-\frac{dp}{dw} + \frac{b}{\phi(b-1)}\right)(b-1)ap^{-b-1} + \left(-p + \frac{b}{\phi(b-1)}w\right)(-b-1)(b-1)ap^{-b-2}\frac{dp}{dw} + (1-F)\frac{dz^*}{dw} = 0$$
Substituting $\frac{dz^*}{dw}$,
$$\Rightarrow \left(-\frac{dp}{dw} + \frac{b}{\phi(b-1)}\right)(b-1)ap^{-b-1} + \left(-p + \frac{b}{\phi(b-1)}w\right)(-b-1)(b-1)ap^{-b-2}\frac{dp}{dw} + \frac{(1-F)^3}{(w-v)f}\left(\phi\frac{dp}{dw} - \frac{1}{1-F}\right) = 0$$
$$\Rightarrow \frac{dp}{dw} = -\frac{p\left(abf(v-w)+(-1+F)^2p^{1+b}\phi\right)}{(-1+F)^3p^{2+b}\phi^2-abf(v-w)(w+bw+p\phi-bp\phi)} =$$
$$\frac{(1-F)^2p^{2+b}\phi-abpf(w-v)}{(1-F)^3p^{2+b}\phi^2-abf(w-v)\left(w(b+1)-p\phi(b-1)\right)} = \frac{(1-F)p^{2+b}\phi-abpr(w-v)}{(1-F)^2p^{2+b}\phi^2-abr(w-v)\left(w(b+1)-p\phi(b-1)\right)}$$

E: Numerical Analysis

1. Linear demand:

$a = 100, b = 1, S = 10, v = 15$, uniform$[-10{,}10]$

$\mu = 0; f(u) = \frac{1}{20}; F(u) = \frac{u+10}{20}; F[z^*] = \frac{z^*+10}{20};$

$\Theta(z^*) = \frac{1}{20}\int_{z^*}^{10}(u - z^*)du = \frac{1}{40}(z^* - 10)^2;$

Following stocking decision approach-

Condition: $\phi a - b(w - 2S) + \phi A > 0 \Rightarrow 0.9 * 100 - (w - 20) - 0.9 * 10 > 0 \Rightarrow 101 > w$

z^* satisfy:

$-(w - v) + (\phi p^*(z^*) + S - v)[1 - F(z^*)] = 0$

$\Rightarrow -(w - v) + \left(\frac{\phi a + bw}{2b} + \phi\frac{\mu - \Theta(z^*)}{2b} + S - v\right)[1 - F(z^*)] = 0$

$\Rightarrow -w + 15 + \left(50\phi + \frac{w}{2} - \phi\frac{(z^*-10)^2}{80} - 5\right)\left(\frac{10-z^*}{20}\right) = 0$

$\Rightarrow w = \left(15 + \left(50\phi - \frac{\phi}{80}(z^* - 10)^2 - 5\right)\left(\frac{10-z^*}{20}\right)\right)\left(\frac{40}{30+z^*}\right)$

The solution of this equation in $z^*$is tedious, therefore, we express the solution in w defining $w = g(z^*)$.

p^* can be obtained as, $p^*(z) = \frac{\phi a + bw}{2\phi b} + \frac{\mu - \Theta(z)}{2b} = 50 + \frac{w}{2\phi} - \frac{(z^*-10)^2}{80}$

The corresponding cost-pass-through,

$\frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 - \frac{\phi(1+F)}{2br(\phi p + S - v) - \phi(1-F)}\right) = \frac{1}{2\phi}\left(1 - \frac{\phi\frac{z^*+30}{20}}{2(\phi p - 5)\frac{1}{10-z^*} - \phi\frac{10-z^*}{20}}\right)$

For $\phi = 0.85$,

$$w = \left(15 + \left(40 - \frac{0.85(z^*-10)^2}{80}\right)\left(\frac{10-z^*}{20}\right)\right)\left(\frac{40}{30+z^*}\right)$$

$$w|_{z^*=-10} = \left(15 + \left(40 - \frac{0.85(-10-10)^2}{80}\right)\left(\frac{10+10}{20}\right)\right)\left(\frac{40}{30-10}\right) = 101$$

$$w|_{z^*=10} = \left(15 + \left(40 - \frac{0.85(10-10)^2}{80}\right)\left(\frac{10-10}{20}\right)\right)\left(\frac{40}{30+10}\right) = 15$$

$$p^*(z) = 50 + \frac{w}{2*0.85} - \frac{(z^*-10)^2}{80}$$

$$\frac{dp^*}{dw} = \frac{1}{2*0.85}\left(1 - \frac{0.85*\frac{z^*+30}{20}}{2(0.85*p - 5)\frac{1}{10-z^*} - 0.85*\frac{10-z^*}{20}}\right)$$

2. Isoelastic demand:

Let assume an isoelastic demand function $y = ap^{-3}$, a per unit shortage cost of $S = 10$, a per-unit salvage price of $v = 15$, and the uncertainty is uniformly distributed on the interval $[-5{,}5]$. Therefore, it follows,

$\mu = 0; f(u) = 0.1; F(u) = \frac{u+5}{10};$

$F[z^*] = \frac{z^*+5}{10} = \frac{\phi p + 10 - w}{\phi p + 10 - 15} \Rightarrow z^* = \frac{5(25 + p - 2w)}{-5 + p} = \frac{5(25 - 2w + p\phi)}{-5 + p\phi}\ ; 1 - F = \frac{w-15}{\phi p - 5}$

$\Theta(z^*(p)) = \frac{1}{10}\int_{z^*}^{5}(u - z^*)du = \frac{1}{20}(z^* - 5)^2 = \frac{1}{20}\left(\frac{5(25-2w+p\phi)}{-5+p\phi} - 5\right)^2 = \frac{5(-15+w)^2}{(-5+p\phi)^2}$;

Following the pricing decision approach,

p^*satisfy, $\left(-p + \frac{b}{\phi(b-1)}w\right)(b-1)ap^{-b-1} + \mu - \Theta\big(z^*(p)\big) = 0$

$\Rightarrow \left(-p + \frac{3}{2\phi}w\right)2ap^{-4} - \frac{5(-15+w)^2}{(-5+p\phi)^2} = 0 \Rightarrow \frac{a(3w-2p\phi)}{p^4\phi} - \frac{5(-15+w)^2}{(-5+p\phi)^2} = 0$

The solution in p is very tedious, therefore, we obtain the solution as an inverse function, $w = g(p^*) \Rightarrow p^* = g^{-1}(w)$. The solution in w, gives two roots, we accept the one that satisfy $w \leq p$.

For $\phi = 1$,

$$w = -\frac{1}{10}(-5+p)^2\left(-\frac{150}{(-5+p)^2} - \frac{3a}{p^4} + \sqrt{\frac{a(9a(-5+p)^2+20(45-2p)p^4)}{(-5+p)^2p^8}}\right)$$

For $\phi = 0.85$,

$$w = -\frac{1}{10}\left(5 - \frac{17p}{20}\right)^2\left(-\frac{150}{\left(5-\frac{17p}{20}\right)^2} - \frac{60a}{17p^4} + \frac{20}{17}\sqrt{\frac{a\big(9a(100-17p)^2-680p^4(-450+17p)\big)}{(100-17p)^2p^8}}\right)$$

For $\phi = 0.7$,

$$w = -\frac{1}{10}\left(5 - \frac{7p}{10}\right)^2\left(-\frac{15000}{(50-7p)^2} - \frac{30a}{7p^4} + \frac{10}{7}\sqrt{\frac{a(9a(50-7p)^2+280(225-7p)p^4)}{(50-7p)^2p^8}}\right)$$

The minimum value of the wholesale price is the salvage price $v = 15$ and the maximum value is $+\infty$. It is to be mentioned, as $w \to \infty$, the demand $y \to 0$. We select a range of retail prices such that the corresponding wholesale price remain greater than the salvage price 15. Assuming $a = 1000000$, for $p \in [33,60]$, the price-comparison plot is shown in Figure 7.4.

The corresponding cost-pass-through is $\Rightarrow \frac{dp}{dw} = \frac{(1-F)^2p^{2+b}\phi - abpf(w-v)}{(1-F)^3p^{2+b}\phi^2 - abf(w-v)(w(b+1)-p\phi(b-1))} =$

$$\frac{\left(\frac{w-15}{\phi p-5}\right)^2p^5\phi - \frac{3a}{10}p(w-15)}{\left(\frac{w-15}{\phi p-5}\right)^3p^5\phi^2 - \frac{3a}{10}(w-15)(4w-2\phi p)} = -\frac{\big(p(-5+p\phi)(-10p^4(-15+w)\phi+3a(-5+p\phi)^2)\big)}{\big(2(5p^5(-15+w)^2\phi^2-3a(2w-p\phi)(-5+p\phi)^3)\big)}$$

For, $\phi = \{0.7, 0.85, 1\}$, $a = 1000000$ and $p \in [33,60]$, the cost-pass-through and price fluctuation plots are illustrated in Figure 7.5 and 7.6 respectively.

It is enough to check the condition for optimality for $\phi = 1$ only, because both conditions have 'less than' constraints. For $\phi = 1$, the conditions are similar to what is presented in the numerical section of Chapter 5.

F. Detail calculation of equations

$$\pi_r = \begin{matrix} \phi pD - wq + v(q-D) & ; D \leq q \\ \phi pq - wq - S(D-q) & ; D > q \end{matrix} \qquad (1)$$

$$\pi_r = \begin{matrix} \phi p(y+\epsilon) - w(y+z) + v(z-\epsilon) & ; \epsilon \leq z & \rightarrow leftover \\ \phi p(y+z) - w(y+z) - S(\epsilon - z) & ; \epsilon > z & \rightarrow shortage \end{matrix} \qquad (2)$$

$E[\pi_r] = \int_A^z [\phi p(y+u) + v(z-u)] f(u)du + \int_z^B [\phi p(y+z) - S(u-z)] f(u)du - w(y+z)$ (3)

$= \int_A^z \phi p(y+u) f(u)du + \int_A^z v(z-u) f(u)du + \int_z^B \phi p(y+u-u+z) f(u)du - \int_z^B S(u-z) f(u)du - w(y+\mu-\mu+z)$

$= \int_A^z \phi p(y+u) f(u)du + \int_A^z v(z-u) f(u)du + \int_z^B \phi p(y+u) f(u)du - \int_z^B \phi p(u-z) f(u)du - \int_z^B S(u-z) f(u)du - w(y+\mu) - w(z-\mu)$

$= \phi p(y+\mu) - w(y+\mu) + \int_A^z v(z-u) f(u)du - \int_z^B (\phi p + S)(u-z) f(u)du - w(z-\mu)$

$= (\phi p - w)(y+\mu) + \int_A^z v(z-u) f(u)du - \int_A^z w(z-u) f(u)du - \int_z^B w(z-u) f(u)du - \int_z^B (\phi p + S)(u-z) f(u)du$

$= (\phi p - w)(y+\mu) - \int_A^z (w-v)(z-u) f(u)du + \int_z^B w(u-z) f(u)du - \int_z^B (\phi p + S)(u-z) f(u)du$

$= (\phi p - w)(y+\mu) - (w-v)\int_A^z (z-u) f(u)du - (\phi p + S - w)\int_z^B (u-z) f(u)du$

$= (\phi p - w)(y+\mu) - [(w-v)\Lambda(z) + (\phi p + S - w)\Theta(z)]$

$= \Psi(p) - L(z,p)$

$$E[\pi_r(z,p)] = (\phi p - w)(y+\mu) - [(w-v)\mathit{\Lambda}(z) + (\phi p + S - w)\mathit{\Theta}(z)] \qquad (4)$$

$\frac{\partial}{\partial z}(E[\pi_r])$ (5)

$= -\left[(w-v)F(z) - (\phi p + S - w)\left(1 - F(z)\right)\right]$

$= -[(w - v + \phi p + S - w)F(z) - (\phi p + S - w)]$

$= -[(\phi p + S - v)F(z) - (\phi p + S - v - w + v)]$

$= -[(\phi p + S - v)F(z) - (\phi p + S - v) + (w - v)\ ]$

$= -(w - v) + (\phi p + S - v)[1 - F(z)]$

$\frac{\partial^2}{\partial z^2}(E[\pi_r(z|p)]) = -(\phi p + S - v)f(z) < 0$, because $\phi p > v$ (6)

$\frac{d}{dp}[\Psi(p)] = \phi(y + \mu) + (\phi p - w)y' = \phi\left[y'\left(p - \frac{w}{\phi} + \frac{y}{y'}\right) + \mu\right]$ (7)

$\frac{\partial}{\partial p}[L(z,p)] = \frac{\partial}{\partial p}[(w - v)\Lambda(z) + (\phi p + S - w)\Theta(z)] = \phi\Theta(z)$

$\frac{\partial}{\partial p}(E[\pi_r(\phi p|z)]) = \frac{d}{dp}[\Psi(\phi p)] - \frac{\partial}{\partial p}[L(z,\phi p)] = \phi\left[y'\left(p - \frac{w}{\phi} + \frac{y}{y'}\right) + \mu - \Theta(z)\right]$

For $y = a - bp$, (8)

$\frac{d}{dp}[\Psi(p)] = \phi\left[-b\left(p - \frac{w}{\phi} + \frac{a-bp}{-b}\right) + \mu\right] = -2\phi b\left[p - \frac{\phi a+bw}{2\phi b}\right] + \phi\mu$

$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = \phi\left[-2b\left[p - \frac{\phi a+bw}{2\phi b}\right] + \mu - \Theta(z)\right] = -2\phi b\left[p - \frac{\phi a+bw}{2\phi b} - \frac{\mu-\Theta(z)}{2b}\right]$

$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)]) = -2\phi b < 0$ (9)

2. Multiplicative Demand Uncertainty Case

A. Problem formulation

Let's assume $z = q/y$, where z is called the stocking factor and can be expressed as $z = \mu + \sigma * (safety\ factor)$. Then the retailer's profit can be expressed as Equation 2 and the corresponding optimal policy is the order quantity, $q^* = y(p^*)z^*$.

$$\pi_r = \begin{matrix} \phi py\epsilon - wyz + vy(z-\epsilon) & ;\epsilon \le z & \rightarrow leftover \\ \phi pyz - wyz - Sy(\epsilon - z) & ;\epsilon > z & \rightarrow shortage \end{matrix} \quad (2a)$$

From Equation 2, the expected retail profit,

$$E[\pi_r] = \int_A^z [\phi pyu + vy(z-u)]f(u)du + \int_z^B [\phi pyz - Sy(u-z)]f(u)du - wyz = (\phi p - w)y\mu - [(w-v)\mathrm{y}\Lambda(z) + (\phi p + S - w)\mathrm{y}\Theta(z)] = \Psi(p) - L(z,p) \quad (3a)$$

Hence, the expected profit is the sum of the riskless profit $\Psi(p) = (\phi p - w)y\mu$ minus the loss due to uncertainty, $L(z,p) = [(w-v)\mathrm{y}\Lambda(z) + (\phi p + S - w)\mathrm{y}\Theta(z)]$ (i.e. subtracting the loss function from the riskless profit). Here, $\mathrm{y}\Lambda(z) = y\int_A^z (z-u)f(u)du =$ expected leftover and $\mathrm{y}\Theta(z) = y\int_z^B (u-z)f(u)du =$ expected shortage. The loss function is the sum of the overstocking and understocking cost (i.e. $Loss\ function\ =\ overage\ cost\ * E(leftover)\ +\ underage\ cost * E(shortage)$).

The retailer's objective is to maximize,

$$E[\pi_r(z,p)] = (\phi p - w)y\mu - [(w-v)y\Lambda(z) + (\phi p + S - w)y\Theta(z)] \quad (4a)$$

This is a joint optimization problem in p and z. Therefore, we take partial derivatives of the expected profit in p and z, and check if the second order conditions are fulfilled.

$$\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = y[-(w-v) + (\phi p + S - v)[1 - F(z)]] \quad (5a)$$

$$\frac{\partial^2}{\partial z^2}(E[\pi_r(z|p)]) = -y(p + S - v)f(z) < 0 \quad (6a)$$

$$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = \frac{d}{dp}[\Psi(p)] - \frac{\partial}{\partial p}[L(z,p)] = \phi y'\mu\left[p - \frac{w}{\phi} + \frac{y}{y'}\right] - \phi y\Theta(z) - y'[(w-v)\Lambda(z) + (\phi p + S - w)\Theta(z)] \quad (7a)$$

For, $y = a - bp$,

$$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = 2\phi b(\mu - \Theta(z))\left[-p + p^0 + \frac{1}{2\phi} * \frac{[(w-v)\Lambda(z) + S\Theta(z)]}{(\mu - \Theta(z))}\right] \quad (8a)$$

where $p^0 = \frac{\phi a + bw}{2\phi b}$

$$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)]) = -2\phi b\big(\mu - \Theta(z)\big) < 0 \tag{9a}$$

For, $y = ap^{-b}$,

$$\frac{\partial}{\partial p}(E[\pi_r(p|z)]) = \phi(b-1)ap^{-b-1}\{\mu - \Theta(z)\}\left[-p + p^0 + \frac{b}{(b-1)\phi}\left\{\frac{(w-v)\Lambda(z) + S\Theta(z)}{\mu - \Theta(z)}\right\}\right] \text{, where } p^0 = \frac{bw}{(b-1)\phi} \tag{10a}$$

$$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)]) = -\frac{b+1}{p} * \frac{\partial}{\partial p}(E[\pi_r]) - \phi(b-1)ap^{-b-1}\{\mu - \Theta(z)\} \tag{11a}$$

$$\frac{\partial^2}{\partial p^2}(E[\pi_r(p|z)])\Big|_{\frac{\partial}{\partial p}(E[\pi_r(p|z)])=0} = -\phi(b-1)ap^{-b-1}\{\mu - \Theta(z)\} < 0 \tag{12a}$$

Equation 6a tells us that $E[\pi_r]$ is concave in z for a given p. In equation 8a and 10a, p^0 is the price that maximizes the riskless profit. We can obtain the riskless optimal price as p^0 by setting the $\frac{d}{dp}[\Psi(p)] = 0$. In equation 9a and 12a, the non-negativities hold because $\mu - \Theta(z) \geq \mu - \Theta(A) = A > 0$. Therefore, $E[\pi_r]$ is concave in p for a given z.

B. Proof of Lemma 2

i. Lemma 2a:

Setting $\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = 0$, we obtain the standard newsvendor result of stocking factor, $z^*(p)$ when p is fixed (Porteus 1990).

$$\frac{\partial}{\partial z}(E[\pi_r(z|p)]) = y\big[-(w-v) + (\phi p + S - v)[1 - F(z)]\big] = 0$$

$$\Rightarrow z^*(p) = F^{-1}\left[\frac{\phi p + S - w}{(\phi p + S - v)}\right]$$

Then substituting $z^*(p)$ into the expected profit (Eq. 4a) will convert the joint optimization problem into a single variable decision problem[52]. Alternatively, we can also substitute $z^*(p)$ into the partial derivative of the expected profit equation with respect to p (Eq. 7a),

$$\frac{d}{dp}(E[\pi_r(z^*(p), p)]) = y'\mu\left[\phi p - w + \phi\frac{y}{y'}\right] - y\phi\Theta(z^*) - y'[(w-v)\Lambda(z^*) + (\phi p + S - w)\Theta(z^*)] \tag{13a}$$

[52] Single variable decision problem in p:

$$E[\pi_r(z^*(p), p)] = y\left((\phi p - w)\mu - (w-v)\int_A^{F^{-1}\left[\frac{\phi p+S-w}{(\phi p+S-v)}\right]}\left(F^{-1}\left[\frac{\phi p+S-w}{(\phi p+S-v)}\right] - u\right)f(u)du - (\phi p + S - w)\int_{F^{-1}\left[\frac{\phi p+S-w}{(\phi p+S-v)}\right]}^{B}\left(u - F^{-1}\left[\frac{\phi p+S-w}{(\phi p+S-v)}\right]\right)f(u)du\right)$$

For $y = a - bp$ (from Eq. 8a),

$$\frac{d}{dp}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 2\phi b(\mu - \Theta(z^*))\left[-p + \frac{\phi a + bw}{2\phi b} + \frac{1}{2\phi} * \frac{(w-v)\Lambda(z^*) + S\Theta(z^*)}{\mu - \Theta(z^*)}\right] \tag{14a}$$

For $y = ap^{-b}$ (from Eq.10)

$$\frac{d}{dp}(E[\pi_r(p, z^*(p))]) = \phi(b-1)ap^{-b-1}\{\mu - \Theta(z^*)\}\left[-p + \frac{bw}{(b-1)\phi} + \frac{b}{(b-1)\phi}\left\{\frac{(w-v)\Lambda(z) + S\Theta(z)}{\mu - \Theta(z)}\right\}\right]$$

If $E[\pi_r(p, z^*(p)]$ is concave in p [Proposition 2], then p^* is the p that satisfies $\frac{d}{dp}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 0$. Since $\mu - \Theta(z^*) > 0$,

For $y = a - bp,\ p^*(w) = \left\{p \middle| -p + \frac{\phi a + bw}{2\phi b} + \frac{1}{2\phi} * \frac{(w-v)\Lambda(z^*(p)) + S\Theta(z^*(p))}{\mu - \Theta(z^*(p))} = 0\right\}$

For $y = ap^{-b},\ p^*(w) = \left\{p \middle| -p + \frac{b}{(b-1)\phi}w + \frac{b}{(b-1)\phi} * \frac{(w-v)\Lambda(z^*(p)) + S\Theta(z^*(p))}{\left(\mu - \Theta(z^*(p))\right)} = 0\right\}$

□

ii. Lemma 2b:

Solving $\frac{\partial}{\partial p}(E[\pi_r]) = 0$ (Eq. 8a and 10a), we can obtain, $p^*(z) = \frac{\phi a + bw}{2\phi b} + \frac{1}{2\phi}X(z)$ [for linear demand] or $p^*(z) = \frac{bw}{(b-1)\phi} + \frac{b}{(b-1)\phi}X(z)$ [for isoelastic demand] where, $(z) = \frac{(w-v)\Lambda(z) + S\Theta(z)}{(\mu - \Theta(z))}$.

Then replacing $p^*(z)$ into the equation $\frac{\partial}{\partial z}(E[\pi_r]) = 0$ (Eq. 5a) would give the single variable equation in z^*:

$$\frac{d}{dz}(E[\pi_r(z, p^*(z))]) = y(p^*(z))\left(-(w-v) + (\phi p^*(z) + S - v)\left(1 - F(z)\right)\right) = 0$$

The derivation of the optimal z^* requires to prove that the expected profit, $E\left[\pi_r\left(z, p^*(z)\right)\right]$ is concave in z which is analyzed in Proposition 2. □

C. Proof of Lemma 3: Defining X and W

Let's define, $X = \left\{\frac{(w-v)E[leftover] + S*E[shortage]}{E[sales]}\right\} = \left\{\frac{(w-v)y\Lambda(z) + Sy\Theta(z)}{y\left(\mu - \Theta(z)\right)}\right\} = \left\{\frac{(w-v)\Lambda + S\Theta}{\mu - \Theta}\right\}$

$$\frac{\partial X}{\partial z^*} = \frac{(w-v)F(z^*) - S\left(1 - F(z^*)\right)}{\left(\mu - \Theta(z^*)\right)} - \frac{[(w-v)\Lambda(z^*) + S\Theta(z^*)] * \left(1 - F(z^*)\right)}{\left(\mu - \Theta(z^*)\right)^2} =$$
$$\frac{(w-v)F(\mu - \Theta) - S(1-F)(\mu - \Theta) - (w-v)\Lambda(1-F) - S\Theta(1-F)}{(\mu - \Theta)^2} = \frac{(w-v)\left(F(\mu - \Theta) - \Lambda(1-F)\right) - S(1-F)\mu}{(\mu - \Theta)^2} =$$

$\frac{(w-v)(\mu-\Theta)(1-F)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right)-S(1-F)\mu}{(\mu-\Theta)^2} = \frac{(1-F)(\mu-\Theta)}{(\mu-\Theta)^2}\left[(w-v)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right)-S\frac{\mu}{(\mu-\Theta)}\right] = \frac{(1-F)}{(\mu-\Theta)}\left[(w-v)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right)-S\frac{\mu}{(\mu-\Theta)}\right]$

Here, $\frac{CDF}{1-CDF} = \frac{F}{(1-F)} > 1 > \frac{\Lambda}{(\mu-\Theta)} = \frac{E[leftover]}{E[sales]}$.
Therefore, for zero shortage cost (i.e. $S = 0$), $\frac{\partial X}{\partial z}$ is positive. For positive shortage cost, $\frac{\partial X}{\partial z}$ is positive if $(w-v)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right) > S\frac{\mu}{(\mu-\Theta)}$; otherwise, $\frac{\partial X}{\partial z}$ is negative if $(w-v)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right) < S\frac{\mu}{(\mu-\Theta)}$.

Now let's define, $W = \frac{\phi(1-F)}{f(\phi p+S-v)}\frac{\partial X}{\partial z^*}$
Since $\phi, (1-F), f, (p+S-\beta)$ are non-negative terms, therefore, the sign of W follows the sign of $\frac{\partial X}{\partial z}$. It is to be mentioned, following the pricing decision approach, $\frac{\partial X}{\partial p} = \frac{\partial X}{\partial z} * \frac{\partial z}{\partial p}$ takes the value of W.

In further calculations, we will be using these two variables X and W frequently.

D. Condition for concavity

i. Pricing Decision Approach
Proposition:
$E\left[\Pi_r\left(p, z^*(p)\right)\right]$ is concave in p for the given conditions-

1. For $y = a - bp$, $\frac{1}{2\phi}W < 1$
2. For $y = ap^{-b}$, $b > 1$, $\frac{b}{(b-1)\phi}W < 1$

where, $W = \frac{\phi(1-F)}{f(\phi p+S-\beta)} * \frac{\partial X}{\partial z}$.

Hence, the optimal p^* is the p that satisfies $\frac{dE\left[\Pi_r\left(p,z^*(p)\right)\right]}{dp} = 0$.

Proof:
We can obtain $z^*(p)$ from $F[z^*(p)] = \frac{\phi p+S-w}{(\phi p+S-v)}$
Differentiating $F[z^*(p)] = \frac{\phi p+S-w}{(\phi p+S-v)}$ with respect to p,
$f * \frac{dz^*}{dp} = \frac{\phi}{\phi p+S-v} - \phi\frac{\phi p+S-w}{(\phi p+S-v)^2} = \frac{\phi}{\phi p+S-v} - \frac{\phi F}{\phi p+S-v} = \phi\frac{1-F}{(\phi p+S-v)}$
Therefore, $\frac{dz^*}{dp} = \phi\frac{1-F}{f(\phi p+S-v)} > 0$. Hence z^* is increasing in p

Replacing the $z^*(p)$ into $\partial E/\partial p$:

Linear Demand

For $y = a - bp$,

$$\frac{d}{dp}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 2\phi b\left(\mu - \Theta\left(z^*(p)\right)\right)\left[-p + \frac{\phi a + bw}{2\phi b} + \frac{1}{2\phi} * X\left(z^*(p)\right)\right]$$

$$\frac{d^2}{dp^2}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = 2\phi b(1-F)\frac{dz^*}{dp}\left[-p + \frac{\phi a + bw}{2\phi b} + \frac{1}{2\phi} * X\right] + 2\phi b(\mu - \Theta)\frac{d}{dp}\left[-p + \frac{\phi a + bw}{2b\phi} + \frac{1}{2\phi} * X\right]$$

$$\frac{d^2}{dp^2}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = \frac{(1-F)}{(\mu-\Theta)} * \frac{dz^*}{dp} * \frac{d(E[\pi_r])}{dp} + 2\phi b(\mu - \Theta)\left[-1 + \frac{1}{2\phi}\frac{dX}{dz^*} * \frac{dz^*}{dp}\right] = \frac{(1-F)}{(\mu-\Theta)} * \frac{dz^*}{dp} * \frac{d(E[\pi_r])}{dp} + 2\phi b(\mu - \Theta)\left[-1 + \frac{1}{2\phi}W\right]$$

Here, $W = \frac{dX}{dz^*} * \frac{dz^*}{dp} = \frac{\phi(1-F)}{f(\phi p + S - \beta)} * \frac{(1-F)}{(\mu-\Theta)}\left[(w-\beta)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) - S\frac{\mu}{(\mu-\Theta)}\right]$

$$\left.\frac{d^2\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right)}{dp^2}\right|_{\frac{d\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right)}{dp}=0} = 2\phi b(\mu - \Theta)\left[-1 + \frac{1}{2\phi}W\right]$$

Since $2b(\mu - \Theta) > 0$, therefore for $\frac{1}{2\phi}W < 1$, $\left.\frac{d^2\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right)}{dp^2}\right|_{\frac{d\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right)}{dp}=0} < 0$

Hence, $E\left[\Pi_r\left(p, z^*(p)\right)\right]$ is concave in p for the given condition and the optimal p^* is the p that satisfies $\frac{dE\left[\Pi_r\left(p, z^*(p)\right)\right]}{dp} = 0$.

Isoelastic Demand

For $y = ap^{-b}$,

$$\frac{d}{dp}\left(E[\pi_r(p, z^*(p))]\right) = \phi(b-1)ap^{-b-1}\{\mu - \Theta(z^*)\}\left[-p + \frac{bw}{(b-1)\phi} + \frac{b}{(b-1)\phi}X\left(z^*(p)\right)\right]$$

Let's define, $R(p) = -p + \frac{bw}{(b-1)\phi} + \frac{b}{(b-1)\phi}X$

$$\frac{d}{dp}\left(E[\pi_r(p, z^*(p))]\right) = \phi(b-1)ap^{-b-1}\{\mu - \Theta\}R(p)$$

$$\frac{d^2}{dp^2}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = \phi(b-1)\left[-(b+1)ap^{-b-1-1}(\mu-\Theta)R + ap^{-b-1}(1-F)\frac{dz^*}{dp}R + ap^{-b-1}(\mu-\Theta)\frac{dR}{dp}\right]$$

$$= \phi(b-1)ap^{-b-1}\left[-(b+1)p^{-1}(\mu-\Theta)R + (1-F)\frac{dz^*}{dp}R + (\mu-\Theta)\frac{dR}{dp}\right]$$

$$\frac{dR(p)}{dp} = -1 + \frac{b}{(b-1)\phi}\frac{dX}{dz^*} * \frac{dz^*}{dp} = -1 + \frac{b}{(b-1)\phi}W$$

Substituting $\frac{dR(p)}{dp}$,

$\frac{d^2}{dp^2}\left(E\left[\pi_r\left(p, z^*(p)\right)\right]\right) = \phi(b-1)ap^{-b-1}\left[-(b+1)p^{-1}(\mu-\Theta)R + (1-F)\frac{dz^*}{dp}R + (\mu-\Theta)\left(-1+\frac{b}{(b-1)\phi}W\right)\right]$

$= -(b+1)p^{-1}\frac{d}{dp}(E[\pi_r]) + (1-F) * \frac{dz^*}{dp} * \frac{d}{dp}(E[\pi_r]) + \phi(b-1)ap^{-b-1}(\mu-\Theta)\left(-1+\frac{b}{(b-1)\phi}W\right)$

$\frac{d^2\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp^2}\Bigg|_{\frac{d\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp}=0} = \phi(b-1)ap^{-b-1}(\mu-\Theta)\left(-1+\frac{b}{(b-1)\phi}W\right)$

Since $\phi(b-1)ap^{-b-1}(\mu-\Theta) > 0,$ therefore for $\frac{b}{(b-1)\phi}W < 1,$ $\frac{d^2\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp^2}\Bigg|_{\frac{d\left(E\left[\pi_r\left(p,z^*(p)\right)\right]\right)}{dp}=0} < 0$

Hence, $E\left[\Pi_r\left(p, z^*(p)\right)\right]$ is concave in p for the given condition and the optimal p^* is the p that satisfies $\frac{dE\left[\Pi_r\left(p,z^*(p)\right)\right]}{dp} = 0.$

ii. Stocking Decision Approach:

Proposition:

$E\left[\Pi_r\left(z, p^*(z)\right)\right]$ is concave in z for the given conditions-

1. For $y = a - bp, \frac{1}{2\phi}W < 1$
2. For $y = ap^{-b}, b > 1, \frac{b}{(b-1)\phi}W < 1$

where, $W = \frac{\phi(1-F)}{f(\phi p+S-\beta)} * \frac{\partial X}{\partial z}.$

Hence, the optimal z^* is the z that satisfies $\frac{dE\left[\Pi_r\left(z,p^*(z)\right)\right]}{dz} = 0.$

Proof:

Linear Demand Form:

Replacing the $p^*(z)$ into $\partial E/\partial z$:

$\frac{d}{dz}E\left[\pi_r\left(z, p^*(z)\right)\right] = (a - bp^*)\left(-(w-\beta) + (\phi p^* + S - \beta)\left(1 - F(z)\right)\right)$

$\frac{d^2}{dz^2}\left(E\left[\pi_r\left(z, p^*(z)\right)\right]\right) = -b\frac{dp^*}{dz} * \frac{\frac{d}{dz}E[\pi_r]}{a-bp^*} + (a - bp^*)\left(\phi\left(\frac{dp^*}{dz}\right)(1-F) - (\phi p^* + S - \beta)f\right)$

$$\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\Bigg|_{\frac{d\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz}=0} = (a-bp^*)\left(\phi\left(\frac{dp^*}{dz}\right)(1-F)-(\phi p^*+S-\beta)f\right) = -(a-bp^*)(\phi p^*+S-\beta)f\left(1-\frac{\phi(1-F)}{(\phi p^*+S-\beta)f}\left(\frac{dp^*}{dz}\right)\right)$$

From, $p^*(z)=\frac{\phi a+bw}{2\phi b}+\frac{X}{2\phi}$,

$$\frac{dp^*}{dz}=\frac{1}{2\phi}\frac{dX(z)}{dz}=\frac{1}{2\phi}*\frac{(1-F)}{(\mu-\Theta)}\left[(w-\beta)\left(\frac{F}{(1-F)}-\frac{\Lambda}{(\mu-\Theta)}\right)-S\frac{\mu}{(\mu-\Theta)}\right]$$

Hence, substituting $\frac{dp^*}{dz}$,

$$\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\Bigg|_{\frac{d\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz}=0} = -(a-bp^*)(\phi p^*+S-\beta)f\left(1-\frac{1}{2\phi}*\frac{\phi(1-F)}{(\phi p^*+S-\beta)f}*\left(\frac{dX(z)}{dz}\right)\right) = -(a-bp^*)(\phi p^*+S-\beta)f\left(1-\frac{1}{2\phi}*W\right)$$

Since, $(a-bp^*)(\phi p^*+S-\beta)f>0$, therefore, for $\frac{1}{2\phi}*W<1$,

$$\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\Bigg|_{\frac{d\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz}=0} < 0$$

Therefore, $E\left[\pi_r\left(z,p^*(z)\right)\right]$ is concave in z and the optimal z^* is the z that satisfy $\frac{d}{dz}E\left[\pi_r\left(z,p^*(z)\right)\right]=0$.

Isoelastic Demand Form:
Replacing the $p^*(z)$ into $\partial E/\partial z$:

$$\frac{d}{dz}E\left[\pi_r\left(z,p^*(z)\right)\right]=\left(ap^{*(-b)}\right)\left(-(w-\beta)+(\phi p^*+S-\beta)\left(1-F(z)\right)\right)$$

$$\frac{d^2}{dz^2}\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)=-bap^{*(-b-1)}\frac{dp^*}{dz}*\frac{\frac{d}{dz}E[\pi_r]}{ap^{*(-b)}}+\left(ap^{*(-b)}\right)\left(\phi\left(\frac{dp^*}{dz}\right)(1-F)-(\phi p^*+S-\beta)f\right)$$

$$\frac{d^2\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz^2}\Bigg|_{\frac{d\left(E\left[\pi_r\left(z,p^*(z)\right)\right]\right)}{dz}=0} = \left(ap^{*(-b)}\right)\left(\phi\left(\frac{dp^*}{dz}\right)(1-F)-(\phi p^*+S-\beta)f\right) = -\left(ap^{*(-b)}\right)(\phi p^*+S-\beta)f\left(1-\frac{\phi(1-F)}{(\phi p^*+S-\beta)f}\left(\frac{dp^*}{dz}\right)\right)$$

From, $p^*(z) = \frac{b}{(b-1)\phi}(w + X)$,

$$\frac{dp^*}{dz} = \frac{b}{(b-1)\phi}\frac{dX(z)}{dz} = \frac{b}{(b-1)\phi} * \frac{(1-F)}{(\mu-\Theta)}\left[(w-\beta)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) - S\frac{\mu}{(\mu-\Theta)}\right]$$

Hence, substituting $\frac{dp^*}{dz}$,

$$\frac{d^2(E[\pi_r(z,p^*(z))])}{dz^2}\Bigg|_{\frac{d(E[\pi_r(z,p^*(z))])}{dz}=0} = -\left(ap^{*(-b)}\right)(\phi p^* + S - \beta)f\left(1 - \frac{b}{(b-1)\phi} * \frac{\phi(1-F)}{(\phi p^*+S-\beta)f} * \frac{dX(z)}{dz}\right) = -\left(ap^{*(-b)}\right)(\phi p^* + S - \beta)f\left(1 - \frac{b}{(b-1)\phi} * W\right)$$

Since, $\left(ap^{*(-b)}\right)(\phi p^* + S - \beta)f > 0$, therefore, for $\frac{b}{(b-1)\phi} * W < 1$,

$$\frac{d^2(E[\pi_r(z,p^*(z))])}{dz^2}\Bigg|_{\frac{d(E[\pi_r(z,p^*(z))])}{dz}=0} < 0$$

Therefore, $E[\pi_r(z,p^*(z))]$ is concave in z and the optimal z^* is the z that satisfy $\frac{d}{dz}E[\pi_r(z,p^*(z))] = 0$.

E. Cost-pass-through:

Proposition 3: In the case of revenue-sharing newsvendor model with multiplicative uncertainty in demand, the retail cost-pass-through is as follows-

1. For linear demand (i.e. $D = (a - bp)\epsilon$), $\frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{(1-F)} - \frac{1}{2\phi}\right)W}{1 - \frac{1}{2\phi}W}\right)$
2. For isoelastic demand (i.e. $D = (ap^{-b})\epsilon$), $\frac{dp^*}{dw} = \frac{b}{(b-1)\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{1-F} - \frac{b}{(b-1)\phi}\right)W}{1 - \frac{b}{(b-1)\phi}W}\right)$

where, $W = \frac{\phi(1-F)}{f(\phi p+S-v)} * \frac{\partial X}{\partial z}$

Proof:

i. Pricing Decision Approach:

$F[z^*(p,w)] = \frac{\phi p+S-w}{\phi p+S-v}$

Taking derivative in w,

$$f(z^*(p,w))\frac{d}{dw}(z^*(p,w)) = \frac{\left(\phi\frac{dp}{dw}-1\right)}{(\phi p+S-v)} - \frac{\phi p+S-w}{(\phi p+S-v)^2} * \phi\frac{dp}{dw} = \frac{\phi\frac{dp}{dw}}{(\phi p+S-v)} - \frac{1}{(\phi p+S-v)} - \frac{F*\phi\frac{dp}{dw}}{(\phi p+S-v)} = \frac{\phi\frac{dp}{dw}(1-F)}{(\phi p+S-v)} - \frac{1}{(\phi p+S-v)}$$

$$\Rightarrow \frac{dz^*(w,p)}{dw} = \frac{\phi(1-F)}{(\phi p+S-v)f} * \frac{dp}{dw} - \frac{1}{(\phi p+S-v)f} = \frac{\phi(1-F)}{(\phi p+S-v)f}\left(\frac{dp}{dw} - \frac{1}{\phi(1-F)}\right)$$

We defined, $X\big(w, z^*(p,w)\big) = \frac{(w-v)\Lambda(z^*(p))+S\Theta(z^*(p))}{\mu-\Theta(z^*(p))}$

$$\frac{dX}{dw} = \frac{\partial X}{\partial w} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw} = \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{\phi(1-F)}{f(\phi p+S-v)}\left[\frac{dp}{dw} - \frac{1}{\phi(1-F)}\right]$$

$$\Rightarrow \frac{dX}{dw} = \frac{\Lambda}{(\mu-\Theta)} + W\left[\frac{dp}{dw} - \frac{1}{\phi(1-F)}\right]$$

Linear Demand:

For $y = a - bp$, p^* has to satisfy the following equation (from Lemma 2a),

$$-p^* + \frac{\phi a+bw}{2\phi b} + \frac{1}{2\phi} * X\big(w, z^*(p^*, w)\big) = 0$$

Taking derivative in w, $-\frac{dp^*}{dw} + \frac{1}{2\phi} + \frac{1}{2\phi} * \frac{dX}{dw} = 0$

By substituting $\frac{dX}{dw}$, $\quad -\frac{dp^*}{dw} + \frac{1}{2\phi} + \frac{1}{2\phi} * \left(\frac{\Lambda}{(\mu-\Theta)} - \frac{W}{\phi(1-F)} + W * \frac{dp^*}{dw}\right) = 0$

$$\Rightarrow \frac{1}{2\phi}\left(1 + \frac{\Lambda}{(\mu-\Theta)} - \frac{W}{\phi(1-F)}\right) = \left(1 - \frac{W}{2\phi}\right)\frac{dp^*}{dw}$$

$$\Rightarrow \frac{dp^*}{dw} = \frac{\frac{1}{2\phi}\left(1+\frac{\Lambda}{(\mu-\Theta)}-\frac{W}{\phi(1-F)}\right)}{\left(1-\frac{W}{2\phi}\right)} = \frac{1}{2\phi}\frac{\left(1-\frac{W}{2\phi}+\frac{W}{2\phi}-\frac{W}{\phi(1-F)}+\frac{\Lambda}{(\mu-\Theta)}\right)}{\left(1-\frac{W}{2\phi}\right)} = \frac{1}{2\phi}\left(1 + \frac{\left(\frac{1}{2\phi}-\frac{1}{\phi(1-F)}\right)W+\frac{\Lambda}{(\mu-\Theta)}}{\left(1-\frac{W}{2\phi}\right)}\right)$$

$$\Rightarrow \frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)}-\left(\frac{1}{1-F}-\frac{1}{2}\right)\frac{W}{\phi}}{1-\frac{1}{2\phi}W}\right)$$

Isoelastic Demand:

For $y = ap^{-b}$, p^* has to satisfy this equation,

$$-p^* + \frac{b}{(b-1)\phi}w + \frac{b}{(b-1)\phi} * X\big(w, z^*(p^*, w)\big) = 0$$

Taking derivative in w, $-\frac{dp^*}{dw} + \frac{b}{(b-1)\phi} + \frac{b}{(b-1)\phi} * \frac{dX}{dw} = 0$

Substituting “$\frac{dX}{dw}$”,

$$-\frac{dp^*}{dw} + \frac{b}{(b-1)\phi} + \frac{b}{(b-1)\phi} * \left(\frac{\Lambda}{(\mu-\Theta)} - \frac{W}{\phi(1-F)} + W * \frac{dp^*}{dw}\right) = 0$$

$$\Rightarrow \frac{b}{(b-1)\phi} * \left[1 + \frac{\Lambda}{(\mu-\Theta)} - \frac{W}{\phi(1-F)}\right] = \left[1 - \frac{b}{(b-1)\phi} * W\right]\frac{dp^*}{dw}$$

$$\Rightarrow \frac{dp^*}{dw} = \frac{b}{(b-1)\phi}\left(\frac{1+\frac{\Lambda}{(\mu-\Theta)}-\frac{W}{\phi(1-F)}}{1-\frac{b}{(b-1)\phi}*W}\right) = \frac{b}{(b-1)\phi}\left(\frac{1-\frac{bW}{(b-1)\phi}+\frac{\Lambda}{(\mu-\Theta)}+\frac{bW}{(b-1)\phi}-\frac{W}{\phi(1-F)}}{1-\frac{b}{(b-1)\phi}W}\right) = \frac{b}{(b-1)\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)}-\left(\frac{1}{1-F}-\frac{b}{(b-1)}\right)\frac{W}{\phi}}{1-\frac{b}{(b-1)\phi}W}\right)$$

ii. Stocking Decision Approach:

We define, $X\big(w, z^*(w)\big) = \frac{(w-v)\Lambda(z^*)+S\Theta(z^*)}{\mu-\Theta(z^*)}$

$\Rightarrow \frac{dX}{dw} = \frac{\partial X}{\partial w} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw} = \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw}$

Linear Demand:

z^* has to satisfy, $\frac{d}{dz} E\big[\pi_r\big(z^*, p^*(z^*)\big)\big] = 0$

$\Rightarrow (a - bp^*)[-(w-\beta) + (\phi p^*(z^*) + S - \beta)(1-F)] = 0$

Since, $y = a - bp^* > 0$,

$\Rightarrow [-(w-\beta) + (\phi p^*(w, z^*) + S - \beta)(1-F)] = 0$

Differentiating this equation w.r.t. w,

$$-1 + \left(\phi \frac{dp^*\big(w, z^*(w)\big)}{dw}\right)(1-F) - (\phi p^*(z^*) + S - \beta) f \frac{dz^*}{dw} = 0$$

$$\Rightarrow -1 + \left(\phi \frac{dp^*\big(w, z^*(w)\big)}{dw}\right)(1-F) = (\phi p^*(z^*) + S - \beta) f \frac{dz^*}{dw}$$

$$\Rightarrow \frac{dz^*}{dw} = -\frac{1}{(\phi p^*(z^*)+S-\beta)f} + \frac{\phi(1-F)}{(\phi p^*(z^*)+S-\beta)f} * \frac{dp^*\big(w, z^*(w)\big)}{dw}$$

Optimal price, $p^*(w, z^*(w)) = \frac{\phi a + bw}{2\phi b} + \frac{1}{2\phi} * X(w, z^*)$

Taking derivatives in w,

$$\frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 + \frac{d}{dw} X(w, z^*)\right) = \frac{1}{2\phi}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw}\right)$$

Substituting $\frac{dz^*}{dw}$,

$$\frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \left(-\frac{1}{(\phi p^*+S-\beta)f} + \frac{\phi(1-F)}{(\phi p^*+S-\beta)f} * \frac{dp^*}{dw}\right)\right)$$

$$\Rightarrow \frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \left(-\frac{\frac{\partial X}{\partial z^*}}{(\phi p^*+S-\beta)f} + \frac{\phi(1-F)\frac{\partial X}{\partial z^*}}{(\phi p^*+S-\beta)f} * \frac{dp^*}{dw}\right)\right) = \frac{1}{2\phi}\left(1 + \frac{\Lambda}{(\mu-\Theta)} - \frac{\frac{\partial X}{\partial z^*}}{(\phi p^*+S-\beta)f} + W * \frac{dp^*}{dw}\right)$$

$$\Rightarrow \frac{dp^*}{dw} - \frac{W}{2\phi}\left(\frac{dp^*}{dw}\right) = \frac{1}{2\phi}\left(1 + \frac{\Lambda}{(\mu-\Theta)} - \frac{\frac{\partial X}{\partial z^*}}{(\phi p^*+S-\beta)f}\right)$$

$$\Rightarrow \frac{dp^*}{dw} = \frac{\frac{1}{2\phi}\left(1+\frac{\Lambda}{(\mu-\Theta)}-\frac{\frac{\partial X}{\partial z^*}}{(\phi p^*+S-\beta)f}\right)}{1-\frac{W}{2\phi}} = \frac{\frac{1}{2\phi}\left(1+\frac{\Lambda}{(\mu-\Theta)}-\frac{1}{\phi(1-F)}W\right)}{1-\frac{W}{2\phi}} = \frac{\frac{1}{2\phi}\left(1-\frac{W}{2\phi}+\frac{W}{2\phi}+\frac{\Lambda}{(\mu-\Theta)}-\frac{1}{\phi(1-F)}W\right)}{1-\frac{W}{2\phi}} =$$
$$\frac{1}{2\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)}-\left(\frac{1}{(1-F)}-\frac{1}{2}\right)\frac{W}{\phi}}{1-\frac{W}{2\phi}}\right)$$

Isoelastic Demand Form:

Optimal price, $p^*(z^*) = \frac{b}{(b-1)\phi}\left(w + X(w,z^*)\right)$

Taking derivatives in *w*,

$$\frac{dp^*}{dw} = \frac{b}{(b-1)\phi}\left(1 + \frac{dX}{dw}\right) = \frac{b}{(b-1)\phi}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw}\right)$$

z^* has to satisfy: $\frac{d}{dz}E\left[\pi_r\left(z^*, p^*(z^*)\right)\right] = 0$

$$\Rightarrow ap^{*(-b)}[-(w-\beta) + (\phi p^* + S - \beta)(1-F)] = 0$$

Since, $y = ap^{*(-b)} > 0$,

$$\Rightarrow [-(w-\beta) + (\phi p^* + S - \beta)(1-F)] = 0$$

Differentiating this equation w.r.t. *w*,

$$-1 + \left(\phi\frac{dp^*}{dw}\right)(1-F) - (\phi p^* + S - \beta)f\frac{dz^*}{dw} = 0$$

$$\Rightarrow -1 + \frac{b}{(b-1)}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw}\right)(1-F) - (\phi p^* + S - \beta)f\frac{dz^*}{dw} = 0$$

$$\Rightarrow -\frac{1}{1-F} + \frac{b}{(b-1)}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw}\right) - \frac{(\phi p^*+S-\beta)f}{1-F}\frac{dz^*}{dw} = 0$$

$$\Rightarrow -\frac{1}{1-F} + \frac{b}{(b-1)}\left(1 + \frac{\Lambda}{(\mu-\Theta)}\right) = \frac{(\phi p^*+S-\beta)f}{1-F}\frac{dz^*}{dw} - \frac{b}{(b-1)}\frac{\partial X}{\partial z^*} * \frac{dz^*}{dw}$$

$$\Rightarrow \frac{dz^*}{dw} = \frac{-\frac{1}{1-F}+\frac{b}{(b-1)}\left(1+\frac{\Lambda}{(\mu-\Theta)}\right)}{\frac{(\phi p^*+S-\beta)f}{1-F}-\frac{b}{(b-1)}\frac{\partial X}{\partial z^*}} = \frac{1}{\frac{(\phi p^*+S-\beta)f}{1-F}} * \frac{-\frac{1}{1-F}+\frac{b}{(b-1)}\left(1+\frac{\Lambda}{(\mu-\Theta)}\right)}{1-\frac{b}{(b-1)}*\frac{1-F}{(\phi p^*+S-\beta)f}*\frac{\partial X}{\partial z^*}} = \frac{1-F}{(\phi p^*+S-\beta)f} *$$
$$\frac{\frac{b}{(b-1)}*\frac{\Lambda}{(\mu-\Theta)}-\frac{1}{1-F}+\frac{b}{(b-1)}}{1-\frac{b}{(b-1)\phi}*W}$$

$$\Rightarrow \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw} = \frac{\partial X}{\partial z^*} * \frac{1-F}{(\phi p^* + S - \beta)f} * \frac{\frac{b}{(b-1)} * \frac{\Lambda}{(\mu-\Theta)} - \frac{1}{1-F} + \frac{b}{(b-1)}}{1 - \frac{b}{(b-1)\phi} * W} = \frac{W}{\phi} * \frac{\frac{b}{(b-1)} * \frac{\Lambda}{(\mu-\Theta)} - \frac{1}{1-F} + \frac{b}{(b-1)}}{1 - \frac{b}{(b-1)\phi} * W}$$

Hence,

$$\frac{dp^*}{dw} = \frac{b}{(b-1)\phi}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \frac{\partial X}{\partial z^*} * \frac{dz^*}{dw}\right) = \frac{b}{(b-1)\phi}\left(1 + \frac{\Lambda}{(\mu-\Theta)} + \frac{W}{\phi} * \frac{\frac{b}{(b-1)} * \frac{\Lambda}{(\mu-\Theta)} - \frac{1}{1-F} + \frac{b}{(b-1)}}{1 - \frac{b}{(b-1)\phi} * W}\right) = \frac{b}{(b-1)\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi}}{1 - \frac{b}{(b-1)\phi} * W}\right)$$

iii. Proof of Corollary 2

Linear Demand:

$$\frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{(1-F)} - \frac{1}{2}\right)\frac{W}{\phi}}{1 - \frac{W}{2\phi}}\right) = \frac{1}{2\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi}}{1 - \frac{W}{2\phi}}\right)$$

In order to decide if $\frac{dp^*}{dw}$ is less or greater than $\frac{1}{2\phi}$, which is the cost-pass-through in risk-less situation for linear demand, we need to check if $\frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi}}{1 - \frac{W}{2\phi}}$ is positive or negative. It is to be mentioned, $0 < \frac{\Lambda}{(\mu-\Theta)} = \frac{E(leftover)}{E(sales)} < 1$ and $\frac{1}{2\phi}W < 1$ [Appendix D]. W can be positive or negative [Appendix C]. The denominator, $1 - \frac{1}{2\phi}W$ has a positive value for both positive and negative W, because $\frac{1}{2\phi}W < 1$. Therefore, the numerator $\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi}$ determine the sign of the term.

Condition for $\frac{dp^*}{dw} > \frac{1}{2\phi}$:

$$\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi} > 0 \Rightarrow \frac{\Lambda}{(\mu-\Theta)} > \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi} \Rightarrow \frac{W}{2\phi} < \frac{\frac{\Lambda}{(\mu-\Theta)}}{\left(\frac{1+F}{1-F}\right)}$$

$$\Rightarrow \frac{W}{2\phi} < \frac{\frac{\Lambda}{(\mu-\Theta)}}{\left(\frac{1+F}{1-F}\right)} < \frac{\Lambda}{(\mu-\Theta)} < 1$$

This condition is satisfied by some positive values and all negative values of W

Condition for $\frac{dp^*}{dw} < \frac{1}{2\phi}$:

$$\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi} < 0 \Rightarrow \frac{\Lambda}{(\mu-\Theta)} < \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi} \Rightarrow \frac{\frac{\Lambda}{(\mu-\Theta)}}{\left(\frac{1+F}{1-F}\right)} < \frac{W}{2\phi}$$

$$\Rightarrow 0 < \frac{\frac{\Lambda}{(\mu-\Theta)}}{\left(\frac{1+F}{1-F}\right)} < \frac{W}{2\phi} < 1$$

This condition is satisfied by some positive values of W

Hence, for some positive values of W, $\frac{dp^*}{dw}$ increases from less than $\frac{1}{2\phi}$ to greater than $\frac{1}{2\phi}$.

Condition for $\frac{dp^*}{dw} = \frac{1}{2\phi}$ **:**

$$\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi} = 0 \Rightarrow \frac{\Lambda}{(\mu-\Theta)} = \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi}$$

$$\Rightarrow 0 < \frac{W}{2\phi} = \frac{\frac{\Lambda}{(\mu-\Theta)}}{\left(\frac{1+F}{1-F}\right)} < 1$$

Hence, for positive values of W, $\frac{dp^*}{dw}$ changes from less than $\frac{1}{2\phi}$ to equals $\frac{1}{2\phi}$ to greater than $\frac{1}{2\phi}$. We are interested to know, if $\frac{dp^*}{dw}$ can exceed 1. Assuming positive values of W, from $\frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi}}{1-\frac{W}{2\phi}}\right)$, the numerator of the right term inside the parenthesis, $\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1+F}{1-F}\right)\frac{W}{2\phi}$ is greater than the denominator $1 - \frac{W}{2\phi}$ because, $\frac{\Lambda}{(\mu-\Theta)} < 1$ and $\left(\frac{1+F}{1-F}\right) > 1$. Therefore, $\frac{dp^*}{dw}$ cannot exceed 1 for linear demand.

Isoelastic Demand:

$$\frac{dp^*}{dw} = \frac{b}{(b-1)\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi}}{1 - \frac{b}{(b-1)} * \frac{W}{\phi}}\right)$$

In order to decide if $\frac{dp^*}{dw}$ is less or greater than $\frac{b}{(b-1)\phi}$, which is the cost-pass-through in risk less situation for isoelastic demand in the case of revenue-sharing contract, we need to check if $\frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi}}{1 - \frac{b}{(b-1)} * \frac{W}{\phi}}$ is positive or negative. It is to be mentioned, $0 < \frac{\Lambda}{(\mu-\Theta)} = \frac{E(leftover)}{E(sales)} < 1$ and $\frac{b}{(b-1)\phi}W < 1$ [Appendix D]. W can be positive or negative [Appendix C]. The denominator, $1 - \frac{b}{(b-1)\phi}W$ has a positive value for both positive and negative W, because $\frac{b}{(b-1)\phi}W < 1$. Therefore, the numerator $\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi}$ determine the sign of the term.

Condition for $\frac{dp^*}{dw} > \frac{b}{(b-1)\phi}$:

$$\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi} > 0 \Rightarrow \frac{\Lambda}{(\mu-\Theta)} > \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi}$$

Equivalently,

$$\Rightarrow \frac{\Lambda}{(\mu-\Theta)} > \frac{bF-1}{b(1-F)} * \frac{b}{b-1} * \frac{W}{\phi}$$

Condition for $\frac{dp^*}{dw} = \frac{b}{(b-1)\phi}$:

$\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi} = 0 \Rightarrow \frac{\Lambda}{(\mu-\Theta)} = \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi}$

Condition for $\frac{dp^*}{dw} < \frac{b}{(b-1)\phi}$:

$\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi} < 0 \Rightarrow \frac{\Lambda}{(\mu-\Theta)} < \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi}$

We are interested to know, if $\frac{dp^*}{dw}$ can reduce to less than one. Required condition for that is $0 < \frac{dp}{dw} < 1$. That follows- $0 < \frac{b}{(b-1)\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{1-F} - \frac{b}{b-1}\right)\frac{W}{\phi}}{1 - \frac{b}{(b-1)} * \frac{W}{\phi}}\right) < 1$

$\Rightarrow 0 < \frac{1 + \frac{\Lambda}{(\mu-\Theta)} - \frac{1}{1-F}\frac{W}{\phi}}{1 - \frac{b}{(b-1)} * \frac{W}{\phi}} < \frac{(b-1)\phi}{b}$

where $0 < \frac{\Lambda}{(\mu-\Theta)} < 1$, $\left(\frac{b}{b-1}\right)\frac{W}{\phi} < 1$, $b > 1$, and $0 < F < 1$ are given.

F. Numerical Analysis

We are following the stocking decision approach here. Let's assume: shortage price, $S = 20$, buyback price, $v = 5$, and a uniform distribution on the interval [1,3],[53]

$\mu = 2; f(u) = \frac{1}{2}; F(u) = \frac{u-1}{2}; r = \frac{f}{1-F} = \frac{1}{3-z}$

$F(z^*) = \frac{z^*-1}{2}; 1 + F = \frac{1+z^*}{2}; 1 - F = \frac{3-z^*}{2}$; $\frac{F}{(1-F)} = \frac{z^*-1}{3-z^*}$; $\frac{(1+F)}{(1-F)} = \frac{1+z^*}{3-z^*}$

$\Lambda(z^*) = \int_1^{z^*}(z^* - u)f(u)du = \frac{(z^*-1)^2}{4}$; $\Theta(z^*) = \int_{z^*}^{3}(u - z)f(u)du = \frac{(z^*-3)^2}{4}$

$\frac{\Lambda}{(\mu-\Theta)} = \frac{\frac{(z^*-1)^2}{4}}{2 - \frac{(z^*-3)^2}{4}} = \frac{(z^*-1)^2}{8-(z^*-3)^2}$; $\frac{\mu}{(\mu-\Theta)} = \frac{2}{2 - \frac{(z^*-3)^2}{4}} = \frac{8}{8-(z^*-3)^2}$

$(w - v)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) = (w-5)\left(\frac{z^*-1}{3-z^*} - \frac{(z^*-1)^2}{8-(z^*-3)^2}\right) = (w-5)\frac{2(z^2-1)}{(z-3)(z^2-6z+1)}$; this expression is greater than zero for $1 \le z < 3$

$S\frac{\mu}{(\mu-\Theta)} = S * \frac{8}{8-(z^*-3)^2} = S * \frac{8}{-(z^2-6z+1)}$; this expression is greater than zero for $1 \le z < 3$

For $S = 0$, $(w-5)\frac{2(z^2-1)}{(z-3)(z^2-6z+1)} > S * \frac{8}{-(z^2-6z+1)}$

For a large value of S (e.g. 20), the LHS can be either greater or less than the RHS depending on the value of w.

[53] The multiplicative case (with constant elasticity) require $A > 0$ to avoid the occurrence of negative demand (Petruzzi and Dada 1999). However, Emmons and Gilbert (1998) assumed a linear demand with uniform distribution on the interval [0,2] with mean=1 for simplification.

$$X(z) = \frac{(w-v)\Lambda + S\Theta}{\mu - \Theta} = \frac{(w-5)\frac{(z^*-1)^2}{4} + 20 * \frac{(z^*-3)^2}{4}}{2 - \frac{(z^*-3)^2}{4}} = \frac{20(-3+z)^2 + (-5+w)(-1+z)^2}{8-(z^*-3)^2}$$

$$\frac{\partial X}{\partial z} = \frac{(1-F)}{(\mu-\Theta)}\left[(w-v)\left(\frac{F}{(1-F)} - \frac{\Lambda}{(\mu-\Theta)}\right) - S\frac{\mu}{(\mu-\Theta)}\right]$$

$$\Rightarrow \frac{\partial X}{\partial z} = \frac{\frac{3-z^*}{2}}{2-\frac{(z^*-3)^2}{4}}\left[(w-5)\frac{2(z^2-1)}{(z-3)(z^2-6z+1)} - \frac{20*8}{-(z^2-6z+1)}\right] = \frac{4(w(-1+z^2)-5(47-16z+z^2))}{(1-6z+z^2)^2}$$

$$W = \frac{\phi(1-F)}{f(\phi p+S-\beta)} * \frac{\partial X}{\partial z} = \phi * \frac{3-z}{(\phi p+15)} * \frac{4w(-1+z^2)-20(47-16z+z^2)}{(1-6z+z^2)^2}$$

Linear Demand:
Following the stocking decision approach-
For a linear demand ($D = (100-p)\epsilon$),
p^* can be obtained as, $p^*(z) = \frac{\phi a+bw}{2\phi b} + \frac{1}{2\phi} * X(z)$

$$\Rightarrow p^* = 50 + \frac{1}{2\phi}(w+X) = 50 + \frac{1}{2\phi}\left(w + \frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z^*-3)^2}\right)$$

The corresponding cost-pass-through,

$$\frac{dp^*}{dw} = \frac{1}{2\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)} - \left(\frac{1}{(1-F)} - \frac{1}{2}\right) * \frac{W}{\phi}}{1 - \frac{1}{2} * \frac{W}{\phi}}\right)$$

$$= \frac{1}{2\phi}\left(1 + \frac{\frac{(z-1)^2}{8-(z-3)^2} - \left(\frac{2}{3-z} - \frac{1}{2}\right) * \frac{3-z}{(\phi p+15)} * \frac{4w(-1+z^2)-20(47-16z+z^2)}{(1-6z+z^2)^2}}{1 - \frac{1}{2} * \frac{3-z}{(\phi p+15)} * \frac{4w(-1+z^2)-20(47-16z+z^2)}{(1-6z+z^2)^2}}\right)$$

z^* satisfy: $-(w-v) + (\phi p^*(z) + S - v)(1-F) = 0$

$$\Rightarrow -(w-5) + \left(50\phi + \frac{1}{2}\left(w + \frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z-3)^2}\right) + 15\right)\frac{3-z}{2} = 0$$

Even after assuming numeric values of ϕ, the solution in z is very tedious that may lead to erroneous solution as well (due to exceeding the numerical ability of advanced software). Therefore, we define the solution as $w = g(z^*) \Rightarrow z^* = g^{-1}(w)$.
For $\phi = 1$,

z^* satisfy: $-(w-5) + \left(50 + \frac{1}{2}\left(w + \frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z-3)^2}\right) + 15\right)\frac{3-z}{2} = 0$

$w = \frac{5(23+417z-207z^2+23z^3)}{-4+12z}$; $w|_{z=3} = 5$; $w|_{z=1} = 160$

Optimal price, $p^* = 50 + \frac{1}{2}\left(w + \frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z^*-3)^2}\right)$

The corresponding cost-pass-through,

$$\frac{dp^*}{dw} = \frac{1}{2}\left(1 + \frac{\frac{(-1+z)^2}{(-11+z)^2} - \frac{2(1+z)\left(w(-1+z^2)-5(47-16z+z^2)\right)}{(15+p)\left(1-6z+z^2\right)^2}}{1 + \frac{2(-3+z)\left(w(-1+z^2)-5(47-16z+z^2)\right)}{(15+p)\left(1-6z+z^2\right)^2}}\right)$$

Substituting w,

$$\frac{dp^*}{dw} = \frac{2(p(-61+561z-1268z^2+420z^3-55z^4+3z^5)-10(10074-5984z-1585z^2+1493z^3-205z^4+7z^5))}{(-11+z)^2(2p(-1+9z-19z^2+3z^3)+5(-501+384z+38z^2-120z^3+23z^4))}$$

$$\left.\frac{dp^*}{dw}\right|_{z=1} = \frac{95+p}{110+2p} < 1 \text{ ; } \left.\frac{dp^*}{dw}\right|_{z=3} = \frac{17}{32} < 1$$

For $\phi = 0.9$,

z^* satisfy: $-(w-5)+\left(50*0.9+\frac{1}{2}\left(w+\frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z-3)^2}\right)+15\right)\frac{3-z}{2}=0$

$w=\frac{5(29+379z-189z^2+21z^3)}{-4+12z}$; $w|_{z=3}=5$; $w|_{z=1}=150$

Optimal price, $p^*=50+\frac{10}{2*9}\left(w+\frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z^*-3)^2}\right)$

$$\frac{dp^*}{dw}=\frac{5}{9}\left(1+\frac{\frac{(-1+z)^2}{(-11+z)^2}-\frac{20(1+z)\left(w(-1+z^2)-5(47-16z+z^2)\right)}{3(50+3p)(1-6z+z^2)^2}}{1+\frac{20(-3+z)\left(w(-1+z^2)-5(47-16z+z^2)\right)}{3(50+3p)(1-6z+z^2)^2}}\right)$$

Substituting w,

$$\frac{dp^*}{dw}=\frac{10(9p(-61+561z-1268z^2+420z^3-55z^4+3z^5)-100(9711-5797z-1247z^2+1307z^3-180z^4+6z^5))}{9(-11+z)^2(9p(-1+9z-19z^2+3z^3)+25(-483+372z+22z^2-108z^3+21z^4))}$$

$\left.\frac{dp^*}{dw}\right|_{z=1}=\frac{5(950+9p)}{9(550+9p)}<1$ [Numerically, it can be shown that for $p>0$, $\frac{dp^*}{dw}<1$]

$\left.\frac{dp^*}{dw}\right|_{z=3}=\frac{85}{144}<1$

For $\phi=0.8$,

z^* satisfy: $-(w-5)+\left(50*0.8+\frac{1}{2}\left(w+\frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z-3)^2}\right)+15\right)\frac{3-z}{2}=0$

$w=\frac{5(35+341z-171z^2+19z^3)}{-4+12z}$; $w|_{z=3}=5$; $w|_{z=1}=140$

Optimal price, $p^*=50+\frac{10}{2*8}\left(w+\frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z^*-3)^2}\right)$

Cost-pass-through,

$$\frac{dp^*}{dw}=\frac{5}{8}\left(1+\frac{\frac{(-1+z)^2}{(11-z)^2}-\frac{\left(-\frac{1}{2}+\frac{2}{3-z}\right)(3-z)\left(4w(-1+z^2)-20(47-16z+z^2)\right)}{\left(15+\frac{4p}{5}\right)(1-6z+z^2)^2}}{1-\frac{(3-z)\left(4w(-1+z^2)-20(47-16z+z^2)\right)}{2\left(15+\frac{4p}{5}\right)(1-6z+z^2)^2}}\right)$$

Substituting $w=g(z^*)$,

$$\frac{dp^*}{dw}=\frac{5(2p(-61+561z-1268z^2+420z^3-55z^4+3z^5)-25(9348-5610z-909z^2+1121z^3-155z^4+5z^5))}{(-11+z)^2(8p(-1+9z-19z^2+3z^3)+25(-465+360z+6z^2-96z^3+19z^4))}$$

$\left.\frac{dp^*}{dw}\right|_{z=1}=\frac{5(475+4p)}{8(275+4p)}$; $\left.\frac{dp^*}{dw}\right|_{z=3}=\frac{85}{128}<1$

Hence, for $z^*\in[1,3]$, we can obtain the values of w , p^*, and $\frac{dp^*}{dw}$; that are plotted in Figure 7.7 and 7.8. The minimum value of w is the salvage price of $v=5$, and the maximum feasible value of w is $\{100,90,80\}$ for $\phi=\{1,0.9,0.8\}$ respectively. We calculate the maximum feasible value of w from these two equations: $y=100-p^0$ and $p^0=\frac{100\phi+w}{2\phi b}$. Here, the maximum p^0 is 100 for which the corresponding deterministic demand is zero ($y=100-p^0=0$).

To illustrate the fluctuation in terms of the standard deviation, we consider some randomized values of z^*. For randomized drawing, we can use the original range $[1,3]$ or any range in between such as $[1.5,2.5]$. The spread of the boundary values will change the corresponding standard deviations but the BP ratio will remain same. Figure 7.9 illustrates the price fluctuation and the corresponding standard deviations of the wholesale price and the retail price.

Isoelastic demand:
Following the stocking decision approach-
For an isoelastic linear demand ($D = (ap^{-3})\epsilon$),
p^* can be obtained as,

$$p^*(z) = \frac{b}{(b-1)\phi}\left(w + X(z)\right) = \frac{3}{2\phi}\left(w + \frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z^*-3)^2}\right)$$

The corresponding cost-pass-through,

$$\frac{dp^*}{dw} = \frac{b}{(b-1)\phi}\left(1 + \frac{\frac{\Lambda}{(\mu-\Theta)}-\left(\frac{1}{(1-F)}-\frac{b}{(b-1)}\right)*\frac{W}{\phi}}{1-\frac{b}{(b-1)}*\frac{W}{\phi}}\right)$$

$$= \frac{3}{2\phi}\left(1 + \frac{\frac{(z-1)^2}{8-(z-3)^2}-\left(\frac{2}{3-z}-\frac{3}{2}\right)*\frac{3-z}{(\phi p+15)}*\frac{4w(-1+z^2)-20(47-16z+z^2)}{(1-6z+z^2)^2}}{1-\frac{3}{2}*\frac{3-z}{(\phi p+15)}*\frac{4w(-1+z^2)-20(47-16z+z^2)}{(1-6z+z^2)^2}}\right)$$

$$= \frac{3}{2\phi}\left(1 + \frac{\frac{(-1+z)^2}{(-11+z)^2}-\frac{2(-5+3z)(w(-1+z^2)-5(47-16z+z^2))}{(1-6z+z^2)^2(15+p\phi)}}{1+\frac{6(-3+z)(w(-1+z^2)-5(47-16z+z^2))}{(1-6z+z^2)^2(15+p\phi)}}\right)$$

z^* satisfy,

$$-(w-v) + (\phi p^*(z) + S - v)(1-F) = 0$$

$$\Rightarrow -(w-5) + \left(\frac{3}{2}\left(w + \frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z-3)^2}\right) + 15\right)\frac{3-z}{2} = 0$$

The solution in z is very tedious[54] that may lead to erroneous solution as well (due to exceeding the numerical ability of advanced software). Therefore, we define the solution as $w = g(z^*) \Rightarrow z^* = g^{-1}(w)$.

$$\Rightarrow \frac{4w(-1-3z+2z^2)+5(-293+165z-35z^2+3z^3)}{4(1-6z+z^2)} = 0$$

For $1 - 6z + z^2 \neq 0 \Rightarrow z \neq 3 \pm 2\sqrt{2}$,

$$\Rightarrow w = -\frac{5(-293+165z-35z^2+3z^3)}{4(-1-3z+2z^2)} \;;\; w|_{z=3} = 5\;;\; w|_{z=2} = \frac{395}{4}\;;\; w|_{z=1} = -100$$

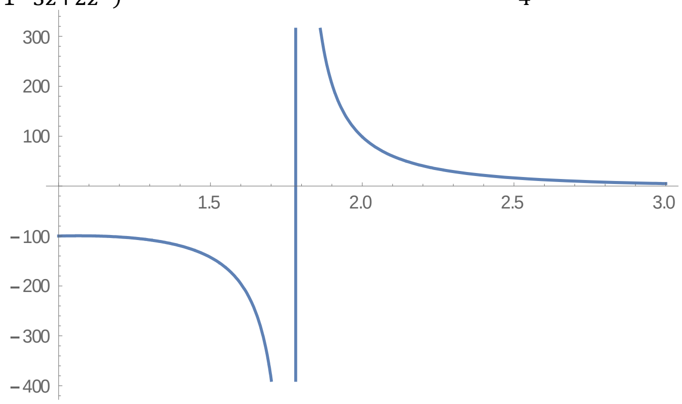

Figure: w vs z^*

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$$z^* = \frac{1}{45}(175-8w) - \frac{6500+2260w-64w^2}{90x^{1/3}} + \frac{2}{45}x^{1/3}$$

Where, $x = \left(8000 - 16950w + 3390w^2 - 64w^3 + 45\sqrt{3}\sqrt{716875 + 692125w + 291525w^2 - 3905w^3 - 68w^4}\right)$

We don't accept the values of z^* for which the corresponding wholesale price w is negative. The minimum value of w is the salvage price of $v = 5$ for $z^* = 3$. The maximum value of w tends to $+\infty$ for $z^* \to \frac{1}{4}\left(3+\sqrt{17}\right) = 1.781$. Therefore, we consider $1.781 < z^* \leq 3$.

For $\phi = 1$,
Optimal price, $p^* = \frac{3}{2}\left(w + \frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z^*-3)^2}\right)$
The corresponding cost-pass-through,

$$\frac{dp^*}{dw} = \frac{3}{2}\left(1 + \frac{\frac{(-1+z)^2}{(-11+z)^2} - \frac{2(-5+3z)\left(w(-1+z^2)-5(47-16z+z^2)\right)}{(15+p)\left(1-6z+z^2\right)^2}}{1+\frac{6(-3+z)\left(w(-1+z^2)-5(47-16z+z^2)\right)}{(15+p)\left(1-6z+z^2\right)^2}}\right)$$

For $\phi = 0.9$,
Optimal price, $p^* = \frac{3*10}{2*9}\left(w + \frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z^*-3)^2}\right)$

$$\frac{dp^*}{dw} = \frac{5}{3}\left(1 + \frac{\frac{(-1+z)^2}{(-11+z)^2} - \frac{2(-5+3z)\left(w(-1+z^2)-5(47-16z+z^2)\right)}{\left(15+\frac{9p}{10}\right)\left(1-6z+z^2\right)^2}}{1+\frac{6(-3+z)\left(w(-1+z^2)-5(47-16z+z^2)\right)}{\left(15+\frac{9p}{10}\right)\left(1-6z+z^2\right)^2}}\right)$$

For $\phi = 0.8$,
Optimal price, $p^* = \frac{3*10}{2*8}\left(w + \frac{20(-3+z)^2+(-5+w)(-1+z)^2}{8-(z^*-3)^2}\right)$
Cost-pass-through,

$$\frac{dp^*}{dw} = \frac{15}{8}\left(1 + \frac{\frac{(-1+z)^2}{(-11+z)^2} - \frac{2(-5+3z)\left(w(-1+z^2)-5(47-16z+z^2)\right)}{\left(15+\frac{4p}{5}\right)\left(1-6z+z^2\right)^2}}{1+\frac{6(-3+z)\left(w(-1+z^2)-5(47-16z+z^2)\right)}{\left(15+\frac{4p}{5}\right)\left(1-6z+z^2\right)^2}}\right)$$

Hence, for $z^* \in [2,3]$, we can obtain the values of w , p^*, and $\frac{dp^*}{dw}$; that are plotted in Figure 7.10 and 7.11.

To illustrate the fluctuation in terms of the standard deviation, we consider some randomized values of z^*. For randomized drawing, we can use the previous range $[2,3]$ or a smaller range in between such as $[2.7,3]$ which gives better illustration of the price fluctuation. The spread of the boundary values change the standard deviations but the corresponding BP ratios give similar conclusions. Figure 7.12 illustrates the price fluctuation and the corresponding standard deviations of the wholesale price and the retail price.